University of New South Wales

School of Economics

Honours Thesis

Pay To Win:

Micro-Transactions in Video-Games

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Bachelor of Economics (Economics) (Honours)

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Bachelor of Science (Material Science)

22nd November, 2019

Declaration

I declare that this thesis is my own work and that, to the best of my knowledge, it contains no material which has been written by another person or persons, except where acknowledgement has been made. This thesis has not been submitted for the award of any degree or diploma at the University of New South Wales Sydney, or at any other institute of higher education.

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22nd November, 2019

Acknowledgements

First and foremost, I thank my two supervisors Keiichi and JC. To Keiichi, from the first day up until the last day you where there helping, guiding, challenging and bettering me. I would have been not able to complete this work without you. If not for you challenging me in Micro 2 and then again in Managerial, I dare say I may not have found my passion for Economics, Thank you. No one could ask for a better supervisor/mentor. To JC, your guidance and insights this year has been invaluable. Your patience whenever I got confused or got off track have steadied me throughout the year and have contributed much to the work I have produced, thank you.

To my family, thank you for your support throughout this entire year. To Mum and Dad, your love and support are unmatched by anyone. You continually inspire me to be a better person. To Courtney, thank you for your last-minute checks and for continuously pushing me to be better. To Daniel, thank you for always being there when I needed someone to talk to and dealing with my frustrations.

To my Friends, thank you for putting up with me talk non-stop about micro-transactions. To Peter, your advice and conjectures on this topic have enormously enriched my understanding. To Harrison, thank you for reading and re-reading my work. To Brendan, your support throughout this year has helped get me through it.

Finally, to the Honours cohort, Thank you for making this year that little bit easier and $\forall \mathcal{B}$ memes.

Contents

Abstract

I consider the emerging pricing mechanism of micro-transactions (also known as an add-on) in the video-game industry. I construct a one-shot contest between two players where prior to the contest, a firm offers an add-on which increases players' probability of winning the contest. The firm also chooses the players probability of winning function. I demonstrate that the firm can extract almost all the players' surplus when unconstrained in its choice of players' probability of winning. I then show that even when the firm is constrained over its choice of players' probability of winning the firm can still secure the same profit as unconstrained so long as players' cost of effort is sufficiently low. I compare the add-on pricing mechanism to an old pricing mechanism of only an entry fee, finding that when the video-game is structured such that the players' return to effort is high and players' have a non-negative and non-zero cost of effort, the firm maximises its profits by offering an add-on.

Section 1
Introduction

There has been a rise in the use of micro-transactions as a pricing mechanism in video-games over the past ten years. Micro-transactions are in-game purchases that increase a player's ability and probability of winning a match against an opponent. For example, in a first-person shooter video-game, the add-on may be buying a gun that inflicts more damage on your opponents. Micro-transactions (also known as an add-on) have been incredibly lucrative for video-game companies with in-game purchases for console and PC games worth AUD750 million in 2018 (Bungard, 2019). An add-on affect each players' payoffs via changing their probability of winning. This modifies the way a player makes decisions regarding the amount of effort to exert when playing the video-game.

An add-on creates a interesting dimension to the usual competition that we see in most contest models. In most contest models there are a certain number of players who choose how much effort to exert. A player's choice of effort affect their probability of winning the contest. In the video-game scenario the contest designer is selling an add-on to the players that they can choose whether to buy or not. The contest designer only profits through sales of the add-on. The video-game designer also chooses how the add-on and effort affect a player's probability of winning. This is my main contribution to the literature. I explore how a contest designer can use the pricing mechanism of an add-on to extract surplus from players. Of interest are what aspects and features of the add-on are desirable to the firm and how this allows the firm to price the add-on to maximise their profit.

Intuitively, firms may have only started creating add-ons because new technology has allowed them to. However, the model I construct predicts that add-ons have increased as a pricing mechanism in video-games with a high return to effort because the add-on alters the equilibrium path such that neither player exerts effort.

In Section 2, I review the related literature. In Section 3, I construct the model to represent the nature of a video-game in which the firm only sells one add-on. The aim is to build intuition behind how the firm will design and price such an add-on. In Section 4, I analyse the game where the firm has almost total control over each player's probability of winning. I show that the firm can secure the same profit regardless of the cost of effort that players have. However, the way in which the firm achieves this profit is different for various costs of effort. In Section 5, I constrain the firm's power over the contest outcomes, such that the independent effect of the add-on is never greater than the independent effect of effort, when facing players with symmetric costs of effort. Sensible intuition might be that limiting the firm's ability to design the add-on will restrict the flexibility that the firm has in setting the price for the add-on. However, when the firm is facing players with a low cost of effort, the firm's profits remain unchanged. The firm's profits only decrease when facing players with a high cost of effort. I then compare the add-on pricing mechanism to the pricing mechanism of only an entry fee. I show that if the video-game is structured such that players' return to effort is high and players have a non-negative and non-zero cost of effort, then the add-on pricing mechanism maximises the firm's profits. This occurs because the add-on alters the equilibrium path such that neither player exerts effort. By doing so the firm can capture more of the players' surplus for a win. This may be why firms such as *Blizzard* (Riekki, 2016) and *MindArk* (Olsson and Sidenblom, 2010) have increased their use of add-ons.

Section 2
Literature Review

The question of how a firm can externally profit from a contest of two players is not well studied. The firm externally profits because it does not profit from the players' choice of effort. The only revenue the firm gains is through add-on sales. However, there is a wealth of literature on contests and the affect of head-starts on future contests (Clark and Nilssen, 2018), (Clark et al., 2019) and the optimal design of a contest for a contest designer (Franke et al., 2018). Understanding the affect of head-starts and biases on the contest is important as it forms the basis of how a firm can price an add-on as it affects players' probability of winning the contest. This forms the demand for the add-on.

Clark and Nilssen (2018) consider two consecutive contests where one player has a head-start in the first contest. The player who wins the first contest gains a prize that is determined by the contest designer and an advantage in the second contest that follows. The winner of the second contest wins a prize set by the contest designer. Clark and Nilssen (2018) find if the head-start can be overcome by the disadvantaged player, then the contest designer prefers to run two contests, dividing the prize between the two contests. In some sense this head-start idea is like the analyses I focus on throughout the paper. However, a key difference is that I analyse a contest designer whose objective is to maximise profit through selling the head-start, not maximise total effort exerted by players in the contest. In some sense this contest designer is more external to the contest. Another difference is that I analyse a one-shot contest. However, extending the model to a dynamic multiple contest set up is an area for future research.

Franke et al. (2018) analyse the revenue-enhancing potential of favouring a specific contestant in a complete-information all-pay action and lottery contest with several heterogeneous contestants. They considered two instruments of favouritism: head-starts that add to the bids of a specific contestant; and multiplicative biases that give disproportionate weights to the bids. They found head-starts are more effective in the all pay auction than biases but combining both instruments yield the first best revenue. Biases are more effective than head-starts in the lottery contest because they level the playing field which encourages the entry of non-active weak players. Combining both instruments cannot increase revenue further. This is in some sense like the problem a video-game firm faces. The firm is designing the contest giving both players the potential to access a head-start for some price. However again the main difference to with this paper is the contest designer only mains to maximise total exert exerted. I only focus on a firm who only maximises profits through the sale of an add-on and does not generate profit from players exerting effort.

Clark et al. (2019) analysed a two consecutive contest model in which the player who wins the first contest gains an advantage over their opponent in the second contest. The advantage affects his cost of exerting effort and his effort productivity in the second contest. My focus will be on how much a player is willing to pay for an advantage that only affects their probability of winning, and this is a key difference to this paper. The players both have equal access to a certain advantage that only affects the probability of winning the contest that follows. While in Clark et al. (2019) only one player gets access to an uncertain advantage, by winning the first contest.

Section 3
The Model

There is one firm and two players $i = 1, 2$. The firm chooses price $p \in \mathbb{R}_+$ for an add-on and the winning probability function $\mu(\mathbf{e}|\mathbf{a})$ from $\mathcal{M}$ that will be explained momentarily. The players choose whether to play the video-game to have a chance to win the gross payoff of 1, or not and receive the payoff zero. If both decide to play the game, then they decide whether to buy an add-on and whether to exert effort.

After observing the price p and the winning probability function set by the firm, each player i decides whether to purchase an add-on $a_i = 1$ at the cost of p or not $a_i = 0$. After observing the add-on choice by the other player, each player i chooses whether to exert effort $e_i = 1$ and incurs cost $c \in \mathbb{R}$ (commonly known) or not to exert effort $e_i = 0$.[1] Then a player becomes either the winner and receives the payoff of 1, or the loser and receives the payoff 0. The winner is chosen with probability $\mu(\mathbf{e}|\mathbf{a})$. Since there is no tie, I only specify the probability that player 1 becomes the winner by $\mu(\mathbf{e}|\mathbf{a})$, where $\mathbf{e} = (e_1, e_2)$ and $\mathbf{a} = (a_1, a_2)$. For notational simplicity, I use $\mathbf{a}_{a_1 a_2}$ to denote $\mathbf{a} = (a_1, a_2)$; and $\mathbf{e}_{e_1 e_2}$ to denote $\mathbf{e} = (e_1, e_2)$. For example, $\mathbf{e} = (1, 0)$ is denoted as $\mathbf{e}_{10}$.

I impose a few restrictions on $\mathcal{M}$, i.e. the set of the winning probability functions from which the firm can choose. More specifically I assume that $\mu(\mathbf{e}|\mathbf{a}) \in \mathcal{M}$ if and only if μ satisfies the following three properties:

1. Monotonicity: $\mu(\mathbf{e}|\mathbf{a})$ is (weakly) increasing in e_1 and a_1.

2. Anonymity: $\mu(\mathbf{e}_{e_1 e_2}|\mathbf{a}_{a_1 a_2}) = 1 - \mu(\mathbf{e}_{e_2 e_1}|\mathbf{a}_{a_2 a_1})$.

3. No-sure-win: $\mu(\mathbf{e}|\mathbf{a}) \geq \varepsilon$ for some $\varepsilon > 0$, $\forall$ $\mathbf{e}, \mathbf{a}$.

[1]When $c < 0$, a player enjoys exerting effort.

4. Technology constraint: The independent effect of effort is greater than the independent effect of the add-on

$$\begin{aligned} &\mu(\mathbf{e}_{10}|\mathbf{a}_{00}) - \mu(\mathbf{e}_{00}|\mathbf{a}_{00}) \geq \mu(\mathbf{e}_{00}|\mathbf{a}_{10}) - \mu(\mathbf{e}_{00}|\mathbf{a}_{00}) \\ \Longleftrightarrow &\mu(\mathbf{e}_{00}|\mathbf{a}_{01}) \geq \mu(\mathbf{e}_{01}|\mathbf{a}_{00}) \end{aligned} \tag{3.1}$$

Assumption four comes from a technology constraint on the firm. As the video-game already existed and had a strong following the firm has then introduced the add-on but the structure of the game is such that the level boost the add-on gives players, cannot be greater than the level boost effort gives players. This level boost of effort is fixed. This constrains the firm's control over $\mu \in \mathcal{M}$. The firm can only sell an add-on if the add-on's independent effect on $\mu \in \mathcal{M}$ is no more than the independent effect of effort on $\mu \in \mathcal{M}$. As level boost effort is fixed, I fix $\mu(\mathbf{e}_{01}|\mathbf{a}_{00})$. I define it as such.

Definition 1. $\overline{\mu} \in \mathcal{M}$ *such that*

$$\frac{1}{2} \geq \mu(\mathbf{e}_{01}|\mathbf{a}_{00}) = \overline{\mu} \geq \varepsilon$$

Note that the lower $\overline{\mu}$ the higher players (gross) return from effort is. A player's return to effort is given as:

$$\mu(\mathbf{e}_{10}|\mathbf{a}_{00}) - \mu(\mathbf{e}_{00}|\mathbf{a}_{00}) \Longleftrightarrow \frac{1}{2} - \overline{\mu}$$

Assumptions 1–3 make intuitive sense. Neither players individual effort nor add-on should hurt their probability of winning. No-sure-wins allows for weaker players to still have a chance of winning. This makes sense as when playing a video-game there is some degree of randomness which mean a player will never win with certainty.

I will first analyse the benchmark case where the firm must only satisfy assumptions 1–3 on $\mu \in \mathcal{M}$. Then I constrain the firm to also choose $\mu \in \mathcal{M}$ such that the technology constraint is also satisfied.

The firm's profit is equal to the revenue from the add-on sales:

$$\pi\left(p, \mu, \mathbf{e}, \mathbf{a}\right) = p \times \left(a_1 + a_2\right).$$

I am assuming zero marginal costs for the add-on and no fixed costs of production. It is not a strong assumption to assume zero marginal cost add-on as this is a digital goods and often digital goods have zero marginal cost (Goldfarb and Tucker, 2019). Assuming zero fixed costs of production will not affect the analyses of the design of the add-on and it as a pricing mechanism, as they will be sunk costs. Also, because a video-game already exists fixed costs to produce the video-game are sunk.

The payoffs of player 1 and 2 are respectively,

$$\begin{aligned} u_1\left(p, \mu, \mathbf{e}, \mathbf{a}\right) &= \mu\left(\mathbf{e}|\mathbf{a}\right) - p \times a_1 - c \times e_1; \\ u_2\left(p, \mu, \mathbf{e}, \mathbf{a}\right) &= \left(1 - \mu\left(\mathbf{e}|\mathbf{a}\right)\right) - p \times a_2 - c \times e_2. \end{aligned}$$

The game proceeds as follows:

1. The firm announces p and chooses $\mu\left(\mathbf{e}|\mathbf{a}\right)$.
2. Players 1 and 2 simultaneously decide whether to play the video-game or not.
3. Players 1 and 2 simultaneously make add-on choices a_1 and a_2, respectively.
4. The add-on choices become public information.
5. Player 1 and 2 simultaneously make effort choices e_1 and e_2, respectively.
6. Outcomes are determined by $\mu(\mathbf{e}|\mathbf{a})$ and payoffs are realised.

By identifying the sub-game perfect equilibrium of this game, I characterise the profit maximizing add-on price and design.

There are three sub-games in which players and the firm take the following actions.

1. Firm choose add-on price p and $\mu \in \mathcal{M}$.

2. Players choose whether to buy the add-on or not.

3. Players choose effort.

In the sub-game-perfect Nash Equilibrium, the firm chooses the price and $\mu \in \mathcal{M}$ that maximises firm profits, given the players' equilibrium add-on and effort strategies in the sub-games 2 and 3.

For simplicity of exposition, I identify the sub-game-perfect equilibria in which the two players play symmetric strategies, which I call symmetric equilibria.

Also, I note that players always choose to participate. This is because, while the payoff from not playing the game is zero, by participating and not buying an add-on, the player's payoff would be at least $\varepsilon > 0$.

Section 4
Equilibrium When The Firm is Unconstrained

In this Section I analyse the benchmark case when $\mu \in \mathcal{M}$ only satisfies assumptions $1-3$. This gives the firm a lot of control over how it can choose $\mu \in \mathcal{M}$. It may seem clear that in this case the firm is able to set a very high price for the add-on which extracts almost all the players' surplus. However, the aim is to build the intuition behind how the firm can do this. The firm can always manipulate $\mu \in \mathcal{M}$ such that players' payoff off the equilibrium path is ε. The firm can then maximise players' payoff on the equilibrium path. This allows the firm to extract almost all the players' surplus. This result is robust when I relax the assumption that players have symmetric costs of effort, I relegate this and all proofs to the appendix.

Following the choice the of add-on, there are four relevant sub-games: the sub-games following $\mathbf{a} = \mathbf{a}_{11}, \mathbf{a}_{10}, \mathbf{a}_{01}, \mathbf{a}_{00}$, respectively. However, I restrict attention to the symmetric equilibria, and the firm's objective is to maximize the revenue from add-on sales. Thus, to identify the equilibria of this game, I only need to focus on the respective equilibria of the sub-games following $\mathbf{a}_{11}$ and $\mathbf{a}_{01}$. More specifically, the firm needs to choose p and $\mu \in \mathcal{M}$ such that (i) the equilibrium of sub-game in $\mathbf{a}_{11}$ is either $\mathbf{e}_{00}$ or $\mathbf{e}_{11}$; and (ii) player 1's equilibrium payoff in the sub-game following $\mathbf{a}_{11}$ is higher than the equilibrium payoff in the sub-game following $\mathbf{a}_{01}$ so that player 1 does not deviate by choosing $a_1 = 0$. By symmetry player 2's decisions will follow the same as player 1's decisions.

I start the analysis of the sub-game on the equilibrium path. The normal-form representation of the sub-game following history $\mathbf{a}=\mathbf{a}_{11}$ is[1]:

	$e_2 = 0$	$e_2 = 1$
$e_1 = 0$	$\frac{1}{2} - p$ $\frac{1}{2} - p$	$1 - \mu(\mathbf{e}_{10}\vert\mathbf{a}_{11}) - p$ $\mu(\mathbf{e}_{10}\vert\mathbf{a}_{11}) - p - c$
$e_1 = 1$	$\mu(\mathbf{e}_{10}\vert\mathbf{a}_{11}) - p - c$ $1 - \mu(\mathbf{e}_{10}\vert\mathbf{a}_{11}) - p$	$\frac{1}{2} - p - c$ $\frac{1}{2} - p - c$

(4.1)

Lemma 1. *In the sub-game following* $\mathbf{a}_{11}$, $\mathbf{e}_{00}$ *is an equilibrium if and only if* $\mu(\mathbf{e}_{10}|\mathbf{a}_{11}) \leq 1/2 + c$. *Similarly,* $\mathbf{e}_{11}$ *is an equilibrium if and only if* $\mu(\mathbf{e}_{10}|\mathbf{a}_{11}) \geq 1/2 + c$.

Lemma 1 immediately gives rise to the following Corollary.

Corollary 1. *Define* $\bar{c} \equiv 1/2 - \varepsilon$. *Note that monotonicity implies* $\mu(\mathbf{e}_{10}|\mathbf{a}_{11}) \geq \mu(\mathbf{e}_{00}|\mathbf{a}_{11}) = 1/2$; *and the no-sure-win implies* $\mu(\mathbf{e}|\mathbf{a}) \leq 1 - \varepsilon$. *Thus, if* $c < 0$, *then* $\mathbf{e}_{11}$ *is the unique equilibrium in the sub-game following* $\mathbf{a}_{11}$. *If* $c > \bar{c}$, *then* $\mathbf{e}_{00}$ *is the unique equilibrium in the sub-game following* $\mathbf{a}_{11}$.

The intuition behind Lemma 1 and Corollary 1 is that when a player's cost of effort is sufficiently high, they will never have any incentive to exert effort as the return to effort is just too low, in the sub-game following $\mathbf{a}_{11}$. While if a player has a negative cost of effort then they always have an incentive to exert effort because their return to effort will always be high. There are a range of costs ($c \in [0, \bar{c}]$) that depend on how the firm sets $\mu \in \mathcal{M}$ as this determines their return to effort and thus, they may or may not exert effort.

Definition 2. *Players with cost of effort* $c > \bar{c}$ *are casual players.*
Players with cost of effort $c \in [0, \bar{c}]$ *are regular players.*
Players with cost of effort $c < 0$ *are professional.*

[1] Making use of the fact that $\mu(\mathbf{e}_{01}|\mathbf{a}_{11}) = 1 - \mu(\mathbf{e}_{10}|\mathbf{a}_{11})$

Now I analyse the sub-game following history $\mathbf{a} = \mathbf{a}_{01}$, off the equilibrium path. The normal-form representation of this sub-game is:

	$e_2 = 0$	$e_2 = 1$
$e_1 = 0$	$\mu(\mathbf{e}_{00}\vert\mathbf{a}_{01})$ $1 - \mu(\mathbf{e}_{00}\vert\mathbf{a}_{01}) - p$	$\mu(\mathbf{e}_{01}\vert\mathbf{a}_{01})$ $1 - \mu(\mathbf{e}_{01}\vert\mathbf{a}_{01}) - p - c$
$e_1 = 1$	$\mu(\mathbf{e}_{10}\vert\mathbf{a}_{01}) - c$ $1 - \mu(\mathbf{e}_{10}\vert\mathbf{a}_{01}) - p$	$\mu(\mathbf{e}_{11}\vert\mathbf{a}_{01}) - c$ $1 - \mu(\mathbf{e}_{11}\vert\mathbf{a}_{01}) - p - c$

. (4.2)

From the (weak) monotonicity of $\mu \in \mathcal{M}$ I note the following Remark.

Remark 1. *I note that $\mu \in \mathcal{M}$ only if*

$$\begin{aligned}\mu(\mathbf{e}_{01}|\mathbf{a}_{01}) &\leq \min\{\mu(\mathbf{e}_{00}|\mathbf{a}_{01}), \mu(\mathbf{e}_{11}|\mathbf{a}_{01})\} \\ &\leq \max\{\mu(\mathbf{e}_{00}|\mathbf{a}_{01}), \mu(\mathbf{e}_{11}|\mathbf{a}_{01})\} \leq 1/2;\end{aligned} \tag{4.3}$$

and

$$\max\{\mu(\mathbf{e}_{00}|\mathbf{a}_{01}), \mu(\mathbf{e}_{11}|\mathbf{a}_{01})\} \leq \mu(\mathbf{e}_{10}|\mathbf{a}_{01}). \tag{4.4}$$

Below, I consider three cases: when the players are casual players ($c > \overline{c}$), when players are regular players $c \in [0, \overline{c}]$, and when players are professionals ($c < 0$), respectively

Lemma 2. *Suppose players are casual players i.e. $c > \overline{c}$, then the firm can achieve the profit of $1 - 2\varepsilon$. The firm can do so by choosing $p = 1/2 - \varepsilon$, $\mu \in \mathcal{M}$ such that $\mu(\mathbf{e}_{10}|\mathbf{a}_{01}) < 1/2 - \varepsilon$.*

From Corollary 1 in the sub-game following $\mathbf{a}_{11}$, $\mathbf{e}_{00}$ is the unique equilibrium when players are casual. This sets an upper bound for player 1's payoff to be $1/2 - p$.

Because a player's cost of effort is too large, the firm is unable to design the add-on such that it induces player 2 to exert effort in the sub-game following $\mathbf{a}_{01}$ (when player 2 has the add-on). Consequentially, as the firm aims to minimise player 1's payoff in this sub-game, the firm does not induce player 1 to exert effort either. Hence, the firm can only induce $\mathbf{e}_{00}$ as the equilibrium in the sub-game following $\mathbf{a}_{01}$. This sets a lower bound for player 1's payoff to be ε. This allows the firm to post a higher price for the add-on as the option of not buying punishes player 1 with ε. In doing this the firm extracts almost all the player's surplus. The no-sure-wins assumptions prevents the firm from being able to extract all surplus, because in any equilibrium outcome in any other sub-game, a player must always have at least $\varepsilon > 0$ as an equilibrium payoff. This sets an upper bound for the price the firm can set, because the firm must discount the price by the value of the player's option of not buying. Hence, players must always have at least a payoff of ε if they buy the add-on. I note that ε is very close to zero.

Thus, by setting the price of $p = 1/2 - \varepsilon$ and designing the add-on to punish players who do not buy, the firm maximises its profits. Because the add-on gives both players an incentive to buy.

One may expect that the casual player case is the lowest profit the firm can generate in equilibrium, because casual players value winning less, i.e. their return is lower. This is not the case, and this is the best the firm can do. This will be shown below.

This result is driven by the weak monotonicity assumption on $\mu(\mathbf{e}|\mathbf{a})$. Because of weak monotonicity the firm can set $\mu(\mathbf{e}_{00}|\mathbf{a}_{01}) = \varepsilon$. If the $\mu \in \mathcal{M}$ were strictly increasing in effort then, the firm would have to set $\mu(\mathbf{e}_{00}|\mathbf{a}_{01}) > \varepsilon$. Hence this result is sensitive to the monotonicity assumption over $\mu \in \mathcal{M}$

I now consider the case when two regular players face each other.

Lemma 3. *Suppose players are regular players i.e.* $c \in [0, \bar{c}]$. *Then the firm can achieve profit of* $1 - 2\varepsilon$. *The firm can do so by choosing* $p = 1/2 - \varepsilon$, $\mu \in \mathcal{M}$ *such that* $\mu(\mathbf{e}_{10}|\mathbf{a}_{11}) < 1/2 + c$, $\mu(\mathbf{e}_{00}|\mathbf{a}_{01}) \geq \varepsilon + c$ *and* $\mu(\mathbf{e}_{11}|\mathbf{a}_{01}) \leq \varepsilon + c$

When the firm is faced with regular players, the firm can design the add-on such that in the sub-game following $\mathbf{a}_{01}$ the add-on induces effort in player 2 (with add-on) and deters effort in player 1 (without add-on). By doing so the firm can minimise player 1's equilibrium payoff to ε in the sub-game following $\mathbf{a}_{01}$. This is the lower bound of player 1's payoff. Conversely, the firm designs that add-on such that in the sub-game following $\mathbf{a}_{11}$ the add-on deters effort in both players. The firm does this because if they induced players to exert effort in this sub-game, they would have to discount the price of the add-on by $c \geq 0$. The reason for this is because in the sub-game following $\mathbf{a}_{01}$ player 1 does not exert effort because of the effort deterring effect of the add-on. This means that player 1's equilibrium payoff in the sub-game following $\mathbf{a}_{11}$ is $1/2 - p$. This is the upper bound of player 1's payoff.

Thus, by punishing player 1 for not buying the add-on in the sub-game following $\mathbf{a}_{01}$, the firm can maximise the price it sets for the add-on and sell it to both players. Note that the firm designs the add-on with two features: it induces the holder of the add-on to exert effort; and deters their opponent from exerting effort. We can see these two effects in the sub-game following $\mathbf{a}_{01}$: the add-on induces the holder of the add-on (player 2) to exert effort; and deters their opponent (player 1) from exerting effort.

In the sub-game following $\mathbf{a}_{11}$ the equilibrium outcome is $\mathbf{e}_{00}$. This implies that the firm designs the add-on such that the effort deterring effect is greater than the effort inducing effect. If the effort inducing effect were greater than the deterring effect, then it would be the reverse in the sub-game following $\mathbf{a}_{11}$ and both players would exert effort in equilibrium. However, this is not the case, and the firm designs the add-on such that effort deterring effect is greater than the effort inducing effect. This maximises firm revenue.

The key difference between the casual player and the regular player case, is when faced with regular players the firm induces effort in player 2 in the sub-game following $\mathbf{a}_{01}$. By doing this the firm ensures that player 1 is punished with the lower bound of the probability of winning. This design of the add-on is more robust than the casual player case as it will not change in the absence of weak monotonicity.

I now consider the case when two professionals face each other.

Lemma 4. *Suppose players are professionals i.e.* $c < 0$*, then the firm can achieve the profit of* $1 - 2\varepsilon$*. The firm can do so by choosing* $p = 1/2 - \varepsilon$*,* $\mu \in \mathcal{M}$ *such that* $\mu\left(\mathbf{e}_{11}|\mathbf{a}_{01}\right) = \varepsilon$*.*

Because professionals gain satisfaction from exerting effort ($c < 0$), regardless of how the firm sets the probability of winning, professionals will always exert high effort. By the monotonicity of μ and $c < 0$ in all four relevant sub-games, $\mathbf{e}_{11}$ will be the equilibrium outcome. Thus, in the sub-game following $\mathbf{a}_{11}$, a player's equilibrium payoff is $1/2 - p - c$. To ensure the highest price, the firm sets in sub-game following $\mathbf{a}_{01}$, $\mu(\mathbf{e}_{11}|\mathbf{a}_{01}) = \varepsilon$. This gives player 1 an equilibrium payoff of $\varepsilon - c$ in the sub-game following $\mathbf{a}_{01}$. This allows the firm to set the price $p = 1/2 - \varepsilon$ and induce both players to buy. Note that the firm is not designing the add-on to induce effort in players. This is because players do not need an incentive to exert effort because their cost ($c < 0$) already incentives them to exert effort. Hence, the firm does not need to give an incentive for effort as the players already do it themselves.

The firm is unable to access higher profits from the enjoyment factor that the professionals gain from exerting effort. This is because in the sub-game following $\mathbf{a}_{11}$ the firm's price is bounded from above by $1/2 - c$. Then in the sub-game following $\mathbf{a}_{01}$ both players exert effort hence the firm minimises player 1's payoff to $\varepsilon - c$. The difference in equilibrium payoffs in the sub-games following $\mathbf{a}_{11}$ and $\mathbf{a}_{01}$ is $1/2 - p - \varepsilon$. This bounds from above the price the firm can set and means they cannot increase their profits when players have an added enjoyment from effort. In fact, if it were not for (weak) monotonicity the firm's profits would be lower than profits when faced with regular players. Because in the sub-game following $\mathbf{a}_{01}$ the equilibrium outcome is $\mathbf{e}_{11}$ and if $\mu \in \mathcal{M}$ were strict in player 1's then the firm could not make the equilibrium payoff of player 1 $\varepsilon - c$ in the sub-game following $\mathbf{a}_{01}$.

I note that in the case of professionals the firm is unable to extract almost all the players' surplus, as in the other two cases, because $c < 0$. For the reasons listed above.

We can see that in all three cases that the firm can set the same price and secure the same profit. However, the firm must change the way they set $\mu \in \mathcal{M}$ to induce this price. When faced with players of different costs of effort, the firm must change its strategy on how it induces this price. This leads to the first proposition

Proposition 1. *By selling an add-on the firm can always design the add-on such that it secures a profit of* $1 - 2\varepsilon$ *regardless of the players' cost of effort.*

In each case, the firm can punish those who do not buy the add-on. They give the player the lower bound of $\mu \in \mathcal{M}$ in the sub-game following $\mathbf{a}_{01}$. This is a hallmark of the pay-to-win video-game. If you do not buy the add-on, your chance of winning is significantly low. However, there is a subtle difference across costs of effort cases. The difference that when with casual players the firm makes the add-on more of a substitute to effort. When faced with regular players the firm makes the add-on complimentary to effort as well as deterring effort.

I check this result for robustness when players have asymmetric costs of effort. The result in the symmetric cost of effort case holds when players have asymmetric costs of effort. I have relegated this analysis to the appendix.

4.1 Model Similarities to Real Video-Games

When faced with casual players the firm makes the add-on more of a substitute to effort. We see this type of design in mobile video-games such as *Candy Crush*. A player can buy the firm's add-on but once they have it there is very little return to players exerting effort. Generally, the add-on saves players' time. Essentially by purchasing the add-on the player is substituting the work they would have to do without the add-on. This type of add-on is a hallmark of a pay to win game

On the contrary, when faced with regular players the firm makes the add-on complimentary to effort as well as deterring effort. We see this feature in more serious video-games such as *Star Wars Battlefront*. In *Star Wars Battlefront* players can purchase Heroes such as Darth Vader or Luke Skywalker. These Heroes have extra-ordinary abilities. When a player equips a Hero, they are incredible difficult to defeat. However, the hero's full power is not accessed if the player does not exert effort. Hence, there is a complimentary nature to this add-on. This type of add-on also deters effort the opponent. This add-on can have no counter. When a player sees Darth Vader, there is very little they can do. This effort deterring effect far outweighs the effort inducing effect. I do note that *Star Wars Battlefront* also has an entry fee. This models failure to capture this might be due to some of the restrictions on the model. These limitations include, not capturing any other benefit of the video-game other than winning. Extending the model to include this benefit may yield more empirically accurate results.

Section 5
Equilibrium When The Firm is Constrained

I now impose the technology constraint (assumption 4) on $\mu \in \mathcal{M}$

$$\mu(\mathbf{e}_{00}|\mathbf{a}_{01}) \geq \mu(\mathbf{e}_{01}|\mathbf{a}_{00}) = \overline{\mu}$$

In the benchmark case the firm had almost total control over of its choice of $\mu \in \mathcal{M}$ and this allowed it to extract almost all of players' surplus regardless of their cost of effort. With this added technology constraint, it can be expected that the firm's profit will decrease as it cannot make the add-on's independent effect greater than efforts independent effect. However, this is not the case and if the firm is facing players with a low cost of effort (regular players and professionals) they can extract the same profit as in Section 4. This occurs as the firm can make the add-on have a complimentary effect with effort which in turn punishes those off the equilibrium path (when they don't buy the add-on and their opponent does).

However, when players have a high cost of effort (casual players) this complimentary effect of the add-on with effort is still not enough to induce player 2 to exert effort, in the sub-game following $\mathbf{a}_{01}$. This causes firm profits to fall when faced with players who have a high cost of effort (casual players).

Again there are four relevant sub-games $\mathbf{a} = \mathbf{a}_{11}, \mathbf{a}_{10}, \mathbf{a}_{01}, \mathbf{a}_{00}$, respectively. However, I restrict attention to the symmetric equilibria and the firm's objective to maximise revenue from add-on sales. Thus, to identify the equilibria of this game, I only need to focus the respective equilibria of the sub-games following $\mathbf{a}_{11}$ and $\mathbf{a}_{01}$. More specifically, the firm needs to choose p and $\mu \in \mathcal{M}$ such that (i) the equilibrium of sub-game in $\mathbf{a}_{11}$ is either $\mathbf{e}_{00}$ or $\mathbf{e}_{11}$; and (ii) player 1's equilibrium payoff in the sub-game following $\mathbf{a}_{11}$

is higher than the equilibrium payoff in the sub-game following $\mathbf{a}_{01}$ so that player 1 does not deviate by choosing $a_1 = 0$.

In the sub-game following history, $\mathbf{a}= \mathbf{a}_{11}$ the analysis is unchanged from Section 4 and results are the same as started in Lemma 1 and Corollary 1.

Now I analyse the sub-game following history $\mathbf{a}= \mathbf{a}_{01}$. The normal-form representation of this sub-game is:

	$e_2 = 0$	$e_2 = 1$
$e_1 = 0$	$\mu(\mathbf{e}_{00}\|\mathbf{a}_{01})$ $1 - \mu(\mathbf{e}_{00}\|\mathbf{a}_{01}) - p$	$\mu(\mathbf{e}_{01}\|\mathbf{a}_{01})$ $1 - \mu(\mathbf{e}_{01}\|\mathbf{a}_{01}) - p - c$
$e_1 = 1$	$\mu(\mathbf{e}_{10}\|\mathbf{a}_{01}) - c$ $1 - \mu(\mathbf{e}_{10}\|\mathbf{a}_{01}) - p$	$\mu(\mathbf{e}_{11}\|\mathbf{a}_{01}) - c$ $1 - \mu(\mathbf{e}_{11}\|\mathbf{a}_{01}) - p - c$

. (5.1)

Remark 1 applies here as well as the technology constraint.

In the discussion below I consider three cases: when the players are casual ($c > \overline{c}$), when players are regular ($c \in [0, \overline{c}]$), and when players are professionals ($c < 0$), respectively

Lemma 5. *Suppose players are casual i.e. $c > \overline{c}$, then the firm can achieve the profit of $1 - 2\overline{\mu}$. The firm can do so by choosing $p = 1/2 - \overline{\mu}$, $\mu \in \mathcal{M}$ such that $\mu(\mathbf{e}_{10}|\mathbf{a}_{01}) < 1/2 - \varepsilon + \overline{\mu}$.*

As with the unconstrained case, the casual players' cost of effort is too large and so the firm is unable to design the add-on such that it induces effort in player 2 in the sub-games following $\mathbf{a}_{01}$ and $\mathbf{a}_{10}$. Hence, the firm aims to induce $\mathbf{e}_{00}$ as the equilibrium in the sub-game following $\mathbf{a}_{01}$. They do this to minimise player 1's equilibrium payoff in the sub-game following $\mathbf{a}_{01}$. This allows the firm set a lower bound for player 1's payoff to be $\overline{\mu}$. Because of the technology constraint, this means the firm is required to increase player 1's equilibrium payoff in the sub-game following $\mathbf{a}_{01}$ from ε to $\overline{\mu}$. This in turn decreases the possible price that the firm can charge for the add-on, compared to the unconstrained case.

By Corollary 1 when players are casual the unique equilibrium in the sub-game following $\mathbf{a}_{11}$ is $\mathbf{e}_{00}$. This sets an upper bound for player 1's equilibrium payoff to be $1/2 - p$ (i.e. the same as the unconstrained case). So, the firm is unable to increase the upper bound of player 1's payoff on the equilibrium path.

The firm must increase players' lower bound payoff off the equilibrium path because of the technology constraint. However, the firm cannot increase players' upper bound payoff on the equilibrium path. This in turn causes the price of the add-on to decrease from the price in the unconstrained case.

Notice that the firm can set the price of the add-on equal to players' return to effort. The interpretation of this is that because casual players' cost of effort is to large the firm replaces effort for the add-on and players pay exactly the benefit they receive from the add-on.

I now consider the case when two regular players face each other.

Lemma 6. *Suppose players are regular players $c \in [0, \overline{c}]$, then the firm can achieve the same profits as in the unconstrained case $1 - 2\varepsilon$. The firm can do so by choosing $p = 1/2 - \varepsilon$, $\mu \in \mathcal{M}$ such that $\mu(\mathbf{e}_{10}|\mathbf{a}_{11}) < 1/2 + c$, $\mu(\mathbf{e}_{00}|\mathbf{a}_{01}) \geq \varepsilon + c$ and $\mu(\mathbf{e}_{11}|\mathbf{a}_{01}) \leq \varepsilon + c$*

When faced with regular players the firm designs the add-on such that, in the sub-game following $\mathbf{a}_{01}$, the add-on induces effort in player 2 (with add-on) and deters effort in player 1 (without add-on). By doing so the firm can minimise player 1's equilibrium payoff to ε in the sub-game following $\mathbf{a}_{01}$. The same way as in the unconstrained case. By designing the add-on to be complimentary to effort, giving the add-on some effort inducing power the firm can escape the technology constraint that is placed on $\mu \in \mathcal{M}$. From the complimentary nature of the add-on the firm can capture extra value in the add-on.

In the sub-game following $\mathbf{a}_{11}$ the firm deters effort in both players. This maximises the upper bound of the equilibrium payoff on the equilibrium path to be $1/2 - p$. Hence the firm designs the add-on such that the effort deterring effect is greater than the complimentary effort inducing effect, as in the unconstrained case.

The key difference between the casual players and regular players in the unconstrained case is that in the regular player case the firm is able to escape the technology constraint by making the add-on complimentary to effort for the holder on the add-on in the sub-games following $\mathbf{a}_{01}$ and $\mathbf{a}_{10}$. By doing this, the firm maintains the unconstrained lower/ upper bound payoff for players off/on the equilibrium path and captures the same profit.

I now consider the case when two professionals face each other.

Lemma 7. *Suppose players are professionals i.e. $c < 0$, then the firm can achieve the profit of $1 - 2\varepsilon$. The firm can do so by choosing $p = 1/2 - \varepsilon$, $\mu \in \mathcal{M}$ such that $\mu(\mathbf{e}_{11}|\mathbf{a}_{01}) = \varepsilon$.*

Here it is important to notice that the firm is making the add-on complimentary to effort for the holder of the add-on (player 2) in the sub-game following $\mathbf{a}_{01}$. The technology constraint forces the firm to do this. The firm rewards player 2 for exerting effort in this sub-game following $\mathbf{a}_{01}$. By doing this the firm can get around the constraint that is placed on the firm. The firm does not reward player 1 for exerting effort in the sub-game following $\mathbf{a}_{01}$. Because the firm aims to minimises player 1's payoff in the sub-game following $\mathbf{a}_{01}$. However, player 1 still exerts effort because they enjoy effort. This makes the equilibrium outcome in the sub-game following $\mathbf{a}_{01}$, $\mathbf{e}_{11}$, this sets a lower bound for player 1's payoff to be $\varepsilon - c$. The key difference to the unconstrained case is the firm makes the add-on complimentary to effort for the holder of the add-on in the sub-game following $\mathbf{a}_{01}$.

Now in the sub-game following $\mathbf{a}_{11}$ the firm can set $\mu \in \mathcal{M}$ however it wants and the equilibrium outcome in the sub-game following $\mathbf{a}_{11}$ will be $\mathbf{e}_{11}$, as per Corollary 1. This sets an upper bound for player 1's payoff to be $1/2 - c - p$. This allows the firm to recapture the profits it gained in the unconstrained case.

Here the firm can cash in on the players gaining enjoyment from effort. The enjoyment of effort professional's gain is what allows the firm to always escape the constraint which has hindered the firm profits when faced with casual players. Note this result is sensitive to the weak monotonicity assumption.

This gives rise the next proposition.

Proposition 2. *The firm can achieve the same profit as the unconstrained case* $(1 - 2\varepsilon)$ *if the firm faces regular players or professionals.*

When firms must satisfy the technology constraint, they are more hindered by players with high cost of effort. This result can give some intuition for external firms offering help to players in a contest. When those they seek to help have a high cost of effort the external firm can price their add-on no more than the return to effort. However, when players have a lower cost of effort the firm can make the add-on complimentary and create extra value for the add-on through effort.

A video-game which exhibits this characteristic includes *FIFA*; the biggest footballing video-game on the market. Within *FIFA*, the skill of real-life players such as Cristiano Ronaldo and Lionel Messi are reflected in-game (henceforth known as 'characters'). With certain characters receiving better character ratings than others. *FIFA* customers can acquire these add-on characters throughout the game by the means of in-game purchases, earning the chance to play as characters with stronger abilities than their opponents. The firm is unable to make these characters level increase in players' probability of winning greater than the return of effort alone. The characters gains most of their value through their complimentary nature with effort. These players are good, however if you are playing an opponent without such players and you don't exert effort then you may not feel the full force of the add-on powers. This reduces the effect the add-on has.

However, these characters have an effort deterring effect. If one player has Cristiano Ronaldo while the other has Daivd de Gea then even though both players have these characters, it deters effort. As even Cristiano Ronaldo may not be able to get many chances past David de Gea. As such, this form of add-on works to ensure the opponent has no form of counter, thus reducing the effect exerting higher effort has on the match. This is to say that the marginal benefit of effort for player 1 is decreasing in a_2. However, *FIFA* has an entry fee as well as an add-on. The model's failure to capture this may be due to some of the restrictions on the model. These include that the model does not capture any other benefit of the video-game other than winning. Extending the model to include this benefit may yield more empirically accurate results. It may also be that the firm is averse to complete change their pricing mechanism to no entry fee.

Games that are like *FIFA* in this way are the *NHL franchise*, *Madden* (King et al., 2019) and the *NFL franchise*.

5.1 Add-on Versus Entry Fee

Thus far, I have solely analysed the add-on as a pricing mechanism. We now have a good understanding of the features that make an add-on desirable to firms and how they will implement them if this were their only pricing instrument. However, an add-on is not the only pricing mechanism that the firm can implement. The firm could simply charge an entry fee that is equal to players' willingness to pay for the video-game with no add-on.

The add-on is more profitable than an entry fee when return to effort is high. When return to effort is high an entry fee limits the firm's ability to extract surplus from players if $c > 0$. When return to effort is high, an entry fee limits the firm's ability to extract more surplus from players; when return to effort is high, players are more likely to exert effort, thereby incurring the cost of effort and reducing their willingness to pay for the video-game if $c > 0$. This may explain why firms such as *Blizzard* (Riekki, 2016) and *MindArk* (Olsson and Sidenblom, 2010) have moved to the pricing mechanism of an add-on and no entry fee.

Suppose that the firm is no longer selling an add-on but charging an entry fee $E \in \mathbb{R}_+$. Following the choice of entry from players, there is only one relevant sub-game: the sub-game identical to that following $\mathbf{a} = \mathbf{a}_{00}$. Thus, in order to identify the equilibria of this game, I only need to focus the equilibrium of the sub-game following $\mathbf{a}_{00}$. More specifically, the firm needs to choose E such that player 1's equilibrium payoff in the sub-game following $\mathbf{a}_{00}$ less the entry fee E is higher than their outside option of playing the video-game that is normalised to 0.

Now I analyse the sub-game following history $\mathbf{a}= \mathbf{a}_{00}$, on the equilibrium path. The normal-form representation of this sub-game is:

	$e_2 = 0$	$e_2 = 1$
$e_1 = 0$	$\frac{1}{2}, \frac{1}{2}$	$\overline{\mu}, 1 - \overline{\mu} - c$
$e_1 = 1$	$1 - \overline{\mu} - c, \overline{\mu}$	$\frac{1}{2} - c, \frac{1}{2} - c$

. (5.2)

Lemma 8. *In the sub-game following $\mathbf{a}_{00}$, $\mathbf{e}_{11}$ is an equilibrium if an only if $\overline{\mu} \leq 1/2 - c$. Similarly, $\mathbf{e}_{00}$ is an equilibrium if and only if $\overline{\mu} \geq 1/2 - c$.*

Lemma 8 immediately gives rise to Corollary 2.

Corollary 2. *If $c > \overline{c}$ then e_{00} is the unique equilibrium in the sub-game following $\mathbf{a}_{00}$.*

From Corollary 2 the equilibrium payoff for casual players in the sub-game following $\mathbf{a}_{00}$ is 1/2. Because casual players' equilibrium outcome in the sub-game following $\mathbf{a}_{00}$ will be $\mathbf{e}_{00}$. Thus, by charging an entry fee to casual players of 1/2 the firm can best the profit of the add-on and gain a profit of 1. I show this below.

Below, I consider two cases: when the players are casual players ($c > \overline{c}$) and when players are regular players $c \in [0, \overline{c}]$

Lemma 9. *If the firm faces casual players, then the firm can achieve the profit of* 1*. The firm can do so by setting an entry fee of* $E = 1/2$ *and not selling an add-on.*

The main intuition behind this result is, when facing casual players, introduction of an add-on creates an upper bound on the price the firm can charge (because of no-sure-win). The add-on does not change the equilibrium outcome of the game. By charging an entry fee the firm can capture all the players' surplus as they do not exert effort in equilibrium.

Now I analyse the case when players are regular players.

Lemma 10. *If players are regular players* $c \in [0, \overline{c}]$ *then there exists some* $\overline{\mu} \in \mathcal{M}$ *such that* $\overline{\mu} \leq 1/2 - c$ *and* $\mathbf{e}_{11}$ *is an equilibrium in the sub-game following* $\mathbf{a}_{00}$*.*

This means when players are regular players, it is plausible that $\mathbf{e}_{11}$ is the equilibrium outcome in the in the sub-game following $\mathbf{a}_{00}$. Notice that when the return to effort is larger there exist more $c \in [0, \overline{c}]$ such that $\mathbf{e}_{11}$ is the equilibrium in the sub-game following $\mathbf{a}_{00}$. This is a key detail. If the video-game is structured such that it offers high return to effort then it seems plausible that this will hinder the entry fee as a profit maximising pricing mechanism.

Lemma 11. *If the firm faces regular players and* $\overline{\mu} \leq 1/2 - c$ *then the firm can achieve the profit of* $1 - 2c$*. The firm can do so by setting an entry fee of* $E = 1/2 - c$ *and not selling an add-on.*

When it is an equilibrium for both players to exert effort in the sub-game following $\mathbf{a}_{00}$, then the firm is not able to charge the same entry fee it did to the casual players, it must discount the cost of effort incurred. This is what reduces the firms profits and creates a weakness in the entry fee as a pricing mechanism. This creates the following proposition.

Proposition 3. *If* $c \in [\varepsilon, 1/2 - \overline{\mu}]$*, then the firm maximises its profits by selling an add-on to the players. The firm can do so by choosing* $p = 1/2 - \varepsilon$*,* $\mu \in \mathcal{M}$ *such that* $\mu(\mathbf{e}_{10}|\mathbf{a}_{11}) < 1/2 + c$*,* $\mu(\mathbf{e}_{00}|\mathbf{a}_{01}) \geq \varepsilon + c$ *and* $\mu(\mathbf{e}_{11}|\mathbf{a}_{01}) \leq \varepsilon + c$

Note that this can only be the case if the firm is facing regular players ($c \in [0, \bar{c}]$). Also, as the return to effort increases it becomes more likely that an add-on is the profit maximising pricing mechanism. The intuition behind this result is when return to effort is high players are more likely to exert effort. This reduces the entry fee the firm can charge because players incur the cost of effort. However, the firm can design an add-on such that effort is deterred in the opponent. This gives incentive for both players to buy putting them on the equilibrium path of $\mathbf{a}_{11}$. This makes the equilibrium outcome $\mathbf{e}_{00}$ in the sub-game following $\mathbf{a}_{11}$. Thus, by selling an add-on the firm changes the equilibrium outcome on the equilibrium path from $\mathbf{e}_{11}$ to $\mathbf{e}_{00}$. This allows them to capture more surplus from the players and maximise its profits.

I note that proposition 3 implies, when faced with professionals it is more profitable for the firm to charge an entry fee than to sell an add-on. This is because professionals' cost of effort is negative, and the add-on cannot capture this. However, and entry fee can capture this.

5.2 Video-Games with an Add-on and No Entry Fee

There are empirical observations that are consistent with my model's projection. The video-game *Hearthstone* is a good example of a game which exhibits the add-on pricing mechanism (Riekki, 2016). *Hearthstone* has no entry fee. Players each have a deck of cards and they battle each other with this deck of cards in a strategic way. Each card has its own special abilities. Because at the base level of the video-game when neither player has the add-on there is a high return to effort. However, players can purchase cards that significantly strengthen their deck. This in turn deters effort in players.

Another video-game which exhibits the model's projection is *Project Entropia* (Olsson and Sidenblom, 2010). The video-game is free to play and has a high return to effort when neither player has the add-on. *Project Entropia* is a large scale, online multiplayer video-game. There are many different objectives and missions. The goal is to generate profits from completing these objectives and missions. There is a high return to effort when players do not have access to the add-on. This game can only be won if players invest their own money in the game purely because everyone else has purchased an add-on.

Section 6
Conclusion

I have analysed a binary choice effort contest in which two players prior to the contest are offered an advantage in said contest by the firm designing the contest. When the firm has a lot of control over players' probability of winning, the firm can extract almost all the players' surplus when their cost of effort is $c \geq 0$. I show when the firm becomes more constrained over the impact of the add-on on contest outcomes, they can still ensure the same profit if players' cost of effort is sufficiently low. I compare the add-on business model to the old business model of a simple entry fee to the contest. I have shown that when the game is structured such that the players' return to effort is high and $c > 0$ then the add-on business model maximises firm's profits. It does so by altering the equilibrium path such that neither player exerts effort. This allows the firm to capture more of the players' surplus from a win. Firms may have noticed this and begun reducing their use of the entry fee business model and started increasing their use of add-ons as a business model.

There are a few limitations with this model. Firstly, the choice of effort is not continuous; a continuous effort choice would likely enrich the model. I also assume complete information. There are several extensions of this model where the firm does not know players' cost of effort and that players do not know each other's cost of effort. This may change the way players interpret players choice over add-on and could subsequently change the way players choose effort. This is an area of future research. This model also assumes perfect information. If the firm were not to disclose players' choice of add-on prior to the contest this may change the results. However, for the purposes of this exposition perfect information makes sense as players of a video-game know if their opponent has an add-on when they are choosing what effort to exert. Another extension of this game is to make players value defeating an opponent who exerts high effort and has the add-on. Players may gain extra satisfaction from defeating an opponent who has the add-on, they may also gain satisfaction from defeating an opponent who exerts more effort. This is an area to be explored in future research. Another limitation is that I only analyse a one-shot contest, while in reality, it is a repeated game, and this may hinder the results I obtain. My results lay the groundwork for the analyses of these extensions.

Section A
Proofs

Lemma 1. *In the sub-game following* $\mathbf{a}_{11}$*,* $\mathbf{e}_{00}$ *is an equilibrium if and only if* $\mu(\mathbf{e}_{10}|\mathbf{a}_{11}) \leq 1/2 + c$*. Similarly,* $\mathbf{e}_{11}$ *is an equilibrium if and only if* $\mu(\mathbf{e}_{10}|\mathbf{a}_{11}) \geq 1/2 + c$*.*

Proof. The necessary and sufficient condition for $\mathbf{e}_{00}$ to be an equilibrium is:

$$\begin{aligned}\frac{1}{2} - p &\geq \mu(\mathbf{e}_{10}|\mathbf{a}_{11}) - p - c \\ \Leftrightarrow \frac{1}{2} &\geq \mu(\mathbf{e}_{10}|\mathbf{a}_{11}) - c;\end{aligned}$$

and

$$\begin{aligned}\frac{1}{2} - p &\geq 1 - \mu(\mathbf{e}_{01}|\mathbf{a}_{11}) - p - c \\ \Leftrightarrow \frac{1}{2} - p &\geq 1 - (1 - \mu(\mathbf{e}_{10}|\mathbf{a}_{11})) - p - c \\ \Leftrightarrow \frac{1}{2} &\geq \mu(\mathbf{e}_{10}|\mathbf{a}_{11}) - c.\end{aligned}$$

Similarly, the necessary and sufficient condition for $\mathbf{e}_{11}$ to be an equilibrium is:

$$\begin{aligned}\frac{1}{2} - p - c &\geq \mu(\mathbf{e}_{01}|\mathbf{a}_{11}) - p \\ \Leftrightarrow \frac{1}{2} - c &\geq \mu(\mathbf{e}_{01}|\mathbf{a}_{11}) \\ \Leftrightarrow \mu(\mathbf{e}_{10}|\mathbf{a}_{11}) - c &\geq \frac{1}{2}\end{aligned}$$

and

$$\frac{1}{2} - p - c \geq 1 - \mu\left(\mathbf{e}_{10}|\mathbf{a}_{11}\right) - p$$
$$\Leftrightarrow \mu\left(\mathbf{e}_{10}|\mathbf{a}_{11}\right) - c \geq \frac{1}{2}.$$

■

Corollary 1. *Define* $\overline{c} \equiv \frac{1}{2} - \varepsilon$. *Note that monotonicity implies* $\mu\left(\mathbf{e}_{10}|\mathbf{a}_{11}\right) \geq \mu\left(\mathbf{e}_{00}|\mathbf{a}_{11}\right) = 1/2$; *and the no-sure-win implies* $\mu\left(\mathbf{e}|\mathbf{a}\right) \leq 1 - \varepsilon$. *Thus, if* $c < 0$, *then* $\mathbf{e}_{11}$ *is the unique equilibrium in the sub-game following* $\mathbf{a}_{11}$. *If* $c > \overline{c}$, *then* $\mathbf{e}_{00}$ *is the unique equilibrium in the sub-game following* $\mathbf{a}_{11}$.

Proof. Suppose that $c < 0$, by the monotonicity of $\mu(\mathbf{e}, \mathbf{a})$

$$\mu(\mathbf{e}_{10}|\mathbf{a}_{11}) \geq \frac{1}{2} > \frac{1}{2} + c$$

By lemma 1 if $c < 0$ then $\mathbf{e}_{11}$ is the unique equilibrium in the sub-game following $\mathbf{a}_{11}$
Now suppose that $c > \overline{c}$, by no-sure-win

$$\mu(\mathbf{e}_{10}|\mathbf{a}_{11}) \leq 1 - \varepsilon < \frac{1}{2} + c$$

By lemma 1 if $c > \overline{c}$ then $\mathbf{e}_{00}$ is the unique equilibrium in the sub-game following $\mathbf{a}_{11}$ ■

Lemma 2 *Suppose players are casual players i.e.* $c > \overline{c}$, *then the firm can achieve the profit of* $1 - 2\varepsilon$. *The firm can do so by choosing* $p = 1/2 - \varepsilon$, $\mu \in \mathcal{M}$ *such that* $\mu\left(\mathbf{e}_{10}|\mathbf{a}_{01}\right) < 1/2 - \varepsilon$.

Proof. As $c > \overline{c}$ by Corollary 1 $\mathbf{e_{00}}$ is the unique equilibrium in the sub-game following $\mathbf{a}_{11}$.
Thus, the firm's equilibrium profit is positive if and only if player 1's equilibrium payoff in the sub-game following $\mathbf{a}_{01}$ is bounding from above by $1/2 - p$
The firm can achieve this goal by setting.

$$\mu\left(\mathbf{e}_{10}|\mathbf{a}_{01}\right) < 1/2 - \varepsilon.$$

and

$$\mu\left(\mathbf{e}_{00}|\mathbf{a}_{01}\right)=\varepsilon.$$

This makes $\mathbf{e}_{00}$ the unique equilibrium in the sub-game following $\mathbf{a}_{01}$.
Furthermore, by symmetry, Player 1 and 2 buys the add-on in $t=2$ if and only if

$$\frac{1}{2}-p\geq\varepsilon$$
$$\frac{1}{2}-\varepsilon\geq p$$

Thus, the firm maximises its profit by setting $p=\frac{1}{2}-\varepsilon$. By symmetry, both players buy, and the firm achieve a profit of $1-2\varepsilon$.

■

Lemma 3. *Suppose players are regular players i.e.* $c\in[0,\overline{c}]$. *Then the firm can achieve profit of* $1-2\varepsilon$. *The firm can do so by choosing* $p=1/2-\varepsilon$, $\mu\in\mathcal{M}$ *such that* $\mu(\mathbf{e}_{10}|\mathbf{a}_{11})<1/2+c$, $\mu(\mathbf{e}_{00}|\mathbf{a}_{01})\geq\varepsilon+c$ *and* $\mu(\mathbf{e}_{11}|\mathbf{a}_{01})\leq\varepsilon+c$

Proof. If the firm set $\mu(\mathbf{e}_{10}|\mathbf{a}_{11})<1/2+c$, then by lemma 1 $\mathbf{e}_{00}$ is the unique equilibrium in the sub-game following $\mathbf{a}_{11}$.
Thus, the firm's equilibrium profit is positive if and only if player 1's equilibrium payoff in the sub-game following $\mathbf{a}_{01}$ is bounding from above by $1/2-p$
The firm can achieve this goal by setting.

$$\mu\left(\mathbf{e}_{11}|\mathbf{a}_{01}\right)\leq\varepsilon+c.$$

$$\mu(\mathbf{e}_{00}|\mathbf{a}_{01})\geq\varepsilon+c$$

and

$$\mu(\mathbf{e}_{01}|\mathbf{a}_{01})=\varepsilon$$

This makes $\mathbf{e}_{01}$ the unique equilibrium in the sub-game following $\mathbf{a}_{01}$.
Furthermore, by symmetry Player, 1 and 2 buys the add-on in $t=2$ if and

only if

$$\frac{1}{2} - p \geq \varepsilon$$
$$\frac{1}{2} - \varepsilon \geq p$$

Thus, the firm maximises its profit by setting $p = \frac{1}{2} - \varepsilon$. By symmetry, both players buy, and the firm achieve a profit of $1 - 2\varepsilon$. ■

Lemma 4. *Suppose players are professionals i.e. $c < 0$, then the firm can achieve the profit of $1 - 2\varepsilon$. The firm can do so by choosing $p = 1/2 - \varepsilon$, $\mu \in \mathcal{M}$ such that $\mu(\mathbf{e}_{11}|\mathbf{a}_{01}) = \varepsilon$.*

Proof. As $c < 0$, by corollary 1 $\mathbf{e}_{11}$ is the unique equilibrium in the sub-game following $\mathbf{a}_{11}$.
$\mathbf{e}_{11}$ is an equilibrium in the sub-game following $\mathbf{a}_{01}$ as $c < 0$ and $\mu \in \mathcal{M}$ is monotonically increasing in player 1's effort.
Thus, if the firm sets

$$\mu(\mathbf{e}_{11}|\mathbf{a}_{01}) = \varepsilon$$

This makes $\mathbf{e}_{11}$ the unique equilibrium in the sub-game following $\mathbf{a}_{01}$.
Furthermore, by symmetry, player 1 and 2 buys the add-on in $t = 2$ if and only if

$$\frac{1}{2} - p - c \geq \varepsilon - c$$
$$\frac{1}{2} - \varepsilon \geq p$$

Thus, the firm maximises its profit by setting $p = \frac{1}{2} - \varepsilon$. By symmetry, both players buy, and the firm achieve a profit of $1 - 2\varepsilon$. ■

Proposition 1 *By selling an add-on the firm can always design the add-on such that it secures a profit of $1 - 2\varepsilon$ regardless of the players cost of effort.*

Proof. By Lemma 2, 3, 4, the firm can always secure a profit of $1 - 2\varepsilon$ regardless of players' cost of effort ■

Lemma 5. *Suppose players are casual i.e. $c > \overline{c}$, then the firm can achieve the profit of $1 - 2\overline{\mu}$. The firm can do so by choosing $p = 1/2 - \overline{\mu}$, $\mu \in \mathcal{M}$*

such that $\mu(\mathbf{e}_{10}|\mathbf{a}_{01}) < 1/2 - \varepsilon + \overline{\mu}$.

Proof. As $c > \overline{c}$ by Corollary 1 $\mathbf{e_{00}}$ is the unique equilibrium in the sub-game following $\mathbf{a}_{11}$.
Thus, the firm's equilibrium profit is positive if and only if player 1's equilibrium payoff in the sub-game following $\mathbf{a}_{01}$ is bounding from above by $1/2 - p$
The firm can achieve this goal by setting.

$$\mu(\mathbf{e}_{10}|\mathbf{a}_{01}) < 1/2 - \varepsilon + \overline{\mu}.$$

and

$$\mu(\mathbf{e}_{00}|\mathbf{a}_{01}) = \overline{\mu}.$$

This makes $\mathbf{e}_{00}$ the unique equilibrium in the sub-game following $\mathbf{a}_{01}$.
Furthermore, by symmetry, player 1 and 2 buys the add-on in $t = 2$ if and only if

$$\frac{1}{2} - p \geq \overline{\mu}$$
$$\frac{1}{2} - \overline{\mu} \geq p$$

Thus, the firm maximises its profit by setting $p = \frac{1}{2} - \overline{\mu}$. By symmetry, both players buy, and the firm achieve a profit of $1 - 2\overline{\mu}$.

■

lemma 6. *Suppose players are regular players* $c \in [0, \overline{c}]$*, then the firm can achieve the same profits as in the unconstrained case* $1 - 2\varepsilon$*. The firm can do so by choosing* $p = 1/2 - \varepsilon$*,* $\mu \in \mathcal{M}$ *such that* $\mu(\mathbf{e}_{10}|\mathbf{a}_{11}) < 1/2 + c$*,* $\mu(\mathbf{e}_{00}|\mathbf{a}_{01}) \geq \varepsilon + c$ *and* $\mu(\mathbf{e}_{11}|\mathbf{a}_{01}) \leq \varepsilon + c$

Proof. The proof follows the same as Lemma 3. ■

Lemma 7. *Suppose players are professionals i.e.* $c < 0$*, then the firm can achieve the profit of* $1 - 2\varepsilon$*. The firm can do so by choosing* $p = 1/2 - \varepsilon$*,* $\mu \in \mathcal{M}$ *such that* $\mu(\mathbf{e}_{11}|\mathbf{a}_{01}) = \varepsilon$*.*

Proof. As $c < 0$, by Corollary 1 $\mathbf{e}_{11}$ is the unique equilibrium in the sub-game following $\mathbf{a}_{11}$.
Thus, the firm's equilibrium profit is positive if and only if player 1's equilibrium payoff in the sub-game following $\mathbf{a}_{01}$ is bounding from above by $1/2 - p - c$.
In the sub-game following $\mathbf{a}_{01}$ the firm must set

$$\mu(\mathbf{e}_{00}|\mathbf{a}_{01}) \geq \mu(\mathbf{e}_{01}|\mathbf{a}_{00})$$

The firm can get around this by setting

$$\mu(\mathbf{e}_{11}|\mathbf{a}_{01}) = \varepsilon$$

This makes $\mathbf{e}_{11}$ the unique equilibrium in the sub-game following $\mathbf{a}_{01}$.
Furthermore, by symmetry player 1 and 2 buys the add-on in $t = 2$ if and only if

$$\frac{1}{2} - p - c \geq \varepsilon - c$$
$$\frac{1}{2} - \varepsilon \geq p$$

Thus, the firm maximises its profit by setting $p = \frac{1}{2} - \varepsilon$. Hence, the firm achieve a profit of $1 - 2\varepsilon$. ■

Proposition 2. *The firm can achieve the same profit as the unconstrained case* $(1 - 2\varepsilon)$ *if the firm faces regular players or professionals.*

Proof. By Lemma 5, 6, 7, if the firm faces regular players or professionals then the firm can always secure a profit of $1 - 2\varepsilon$ ■

Lemma 8. *In the sub-game following* $\mathbf{a}_{00}$*,* $\mathbf{e}_{11}$ *is an equilibrium if an only if* $\overline{\mu} \leq 1/2 - c$*. Similarly,* $\mathbf{e}_{00}$ *is an equilibrium if and only if* $\overline{\mu} \geq 1/2 - c$*.*

Proof. The necessary and sufficient condition for $\mathbf{e}_{00}$ to be an equilibrium is:

$$\frac{1}{2} \geq 1 - \overline{\mu} - c$$
$$\Leftrightarrow \overline{\mu} \geq \frac{1}{2} - c;$$

Similarly, the necessary and sufficient condition for $\mathbf{e}_{11}$ to be an equilibrium is:

$$\frac{1}{2} - c \geq \overline{\mu}$$
$$\Leftrightarrow \frac{1}{2} - c \geq \overline{\mu}$$

■

Corollary 2. If $c > \overline{c}$ then e_{00} is the unique equilibrium in the sub-game following $\mathbf{a}_{00}$

Proof. By no-sure-wins if $c > \overline{c}$ then e_{00}

$$\overline{\mu} \geq \varepsilon > \frac{1}{2} - c$$

Thus, by Lemma 8 if $c > \overline{c}$ then e_{00} then $\mathbf{e}_{00}$ is the unique equilibrium in the sub-game following $\mathbf{a}_{00}$ ■

Lemma 9. *If the firm faces casual players, then the firm can achieve the profit of* 1*. The firm can do so by setting an entry fee of* 1/2 *and not selling an add-on.*

Proof. By Corollary 2 if players are casual players then $\mathbf{e}_{00}$ is they unique equilibrium in the sub-game following $\mathbf{a}_{00}$.
Thus, casual players' equilibrium payoff in the sub-game following $\mathbf{a}_{00}$ is 1/2
Thus, players will enter the game if and only if

$$E \leq \frac{1}{2}$$

Thus, by charging entry fee of $E = 1/2$ the firm can obtain profits of 1. By Lemma 2 firms profits from selling an add-on to casual players was $1 - 2\varepsilon$. Thus, the firm if the firm is facing casual players to firm can maximise its profits by charging an entry fee of 1/2. ■

Lemma 10. *If players are regular players* $c \in [0, \overline{c}]$ *then there exists some* $\overline{\mu} \in \mathcal{M}$ *such that* $\overline{\mu} \leq 1/2 - c$ *and* $\mathbf{e}_{11}$ *is an equilibrium in the sub-game following* $\mathbf{a}_{00}$.

Proof. Take the lower bound of $c = 0$
By the upper bound on $\overline{\mu}$

$$\overline{\mu} \leq \frac{1}{2} = \frac{1}{2} - c$$

Thus, there exists a $\overline{\mu} \in \mathcal{M}$ such that $\overline{\mu} \leq 1/2 - c$ for the lower bound of c. Hence, there exists a $\overline{\mu} \in \mathcal{M}$ such that $\mathbf{e}_{11}$ is an equilibrium in the sub-game following $\mathbf{a}_{00}$. Take the upper bound of $c = \overline{c}$

$$\overline{\mu} \leq \frac{1}{2} - \left(\frac{1}{2} - \varepsilon\right)$$

Thus, by no sure wins there exist only one $\overline{\mu} \in \mathcal{M}$ such that $\overline{\mu} \leq 1/2 - c$ for the upper bound of c. Hence, there exists a $\overline{\mu} \in \mathcal{M}$ such that $\mathbf{e}_{11}$ is an equilibrium in the sub-game following $\mathbf{a}_{00}$.

More over for every $c \in [0, \overline{c}]$ there exist a $\overline{\mu} \in \mathcal{M}$ such that $\overline{\mu} \leq 1/2 - c$ ■

Lemma 11. If the firm faces regular players and $\overline{\mu} \leq 1/2 - c$ then the firm can achieve the profit of $1 - 2c$. The firm can do so by setting an entry fee of $1/2 - c$ and not selling an add-on.

Proof. From Lemma 5 if $\overline{\mu} \leq 1/2 - c$ then $\mathbf{e}_{11}$ is an equilibrium in the sub-game following $\mathbf{a}_{00}$. Thus, regular players equilibrium payoff in the sub-game following $\mathbf{a}_{00}$ is $1/2 - c$
Thus, players will enter the game if and only if

$$E \leq \frac{1}{2} - c$$

Thus, by charging entry fee of $E = 1/2 - c$ the firm can obtain profits of $1 - 2c$. ■

Proposition 3 *If $\overline{\mu} \leq 1/2 - c$ and $c \geq \varepsilon$, then the firm maximises its profits by selling an add-on to the players. The firm can do so by choosing $p = 1/2 - \varepsilon$, $\mu \in \mathcal{M}$ such that $\mu(\mathbf{e}_{10}|\mathbf{a}_{11}) < 1/2 + c$, $\mu(\mathbf{e}_{00}|\mathbf{a}_{01}) \geq \varepsilon + c$ and $\mu(\mathbf{e}_{10}|\mathbf{a}_{01}) \leq 1/2 - \varepsilon$.*

Proof. If the firm is facing regular players from Lemma 3 the firm can design an add-on and obtain a profit of $1 - 2\varepsilon$.

From Lemma 11 the firm can obtain a profit of $1 - 2c$ by charging an entry fee.
Thus, the firm maximises its profits by offering an add-on if and only if

$$\begin{aligned} &1 - 2\varepsilon \geq 1 - 2c \\ \iff &c \geq \varepsilon \end{aligned}$$

■

A.1 Asymmetric Cost of Effort

I now relax the assumption of symmetric costs of effort. The result in the symmetric cost (firm can achieve a profit of $1/2 - \varepsilon$) case holds when players have asymmetric costs.

Suppose that player 1 has a cost of effort c_1 and player 2 has a cost of effort c_2 such that $c_1 < c_2$. These costs are still commonly known. I note that players may no longer play symmetric strategies however I still identify the sub-game-perfect equilibria. As in the symmetric case, following the choice of add-ons, there are four relevant sub-games following $\mathbf{a} = \mathbf{a}_{11}, \mathbf{a}_{10}, \mathbf{a}_{01}, \mathbf{a}_{00}$. However, I restrict attention to the firm's objective to maximise revenue from add-on sales.

Below I analyses three cases: when a regular player ($c_1 \in [0, \bar{c}]$) plays a casual player ($c_2 > \bar{c}$); when a professional ($c_1 < 0$) is playing a casual player ($c_2 > \bar{c}$); and when a professional ($c_1 < 0$) plays a regular player ($c_2 \in [0, \bar{c}]$) .

With asymmetric costs it seems plausible that the firm may favour one player over the other and only sell to one of the players to maximise its revenue. Because in the sub-game following $\mathbf{a}_{00}$, one player may have a very low probability of winning, as they have a high cost of effort and may not exert effort while their opponent does exert effort. By the monotonicity of μ this leads to them have a lower chance of winning. Hence by selling the add-on to the higher cost player the firm can increase their probability by a substantial amount and capture that as the price. However it turns out that the firm can actually do best by setting $\mu \in \mathcal{M}$ such that when both players buy the add-on, the firm equalises players' probability of winning, and the firm is able to recapture the same profit as if it was facing two players with symmetric costs.

I first analyses the case where a regular player ($c_1 \in [0, \overline{c}]$) faces a casual player ($c_2 > \overline{c}$).

Lemma 12. *Suppose the first player is a regular player ($c_1 \in [0, \overline{c}]$) and the second player is a casual player ($c_2 > \overline{c}$). The firm can achieve a profit of $1-2\varepsilon$ by selling to both players. The firm can do so by choosing $p = 1/2-\varepsilon$, and $\mu \in \mathcal{M}$ such that $\mu(\mathbf{e}_{10}|\mathbf{a}_{11}) < 1/2 + c_1$ and $\mu(\mathbf{e}_{10}|\mathbf{a}_{01}) < 1/2 - \varepsilon$*

Proof. When the firm sets $\mu(\mathbf{e}_{10}|\mathbf{a}_{11}) < 1/2 + c_1$ by Lemma 1 the regular player exerts low effort in the sub-game following $\mathbf{a}_{11}$.
By Corollary 1 the casual player exerting low effort is a strictly dominate strategy.
Hence if the firm sets $\mu(\mathbf{e}_{10}|\mathbf{a}_{11}) < 1/2 + c_1$, then $\mathbf{e}_{00}$ is the unique equilibrium outcome following sub-game $\mathbf{a}_{11}$
Thus, the firm's equilibrium profit is positive if the regular and casual players' equilibrium payoff in the sub-game following $\mathbf{a}_{01}$ is bounded from above by $1/2 - p$.
The firm can achieve this goal by setting

$$\mu(\mathbf{e}_{10}|\mathbf{a}_{01}) < 1/2 - \varepsilon$$

and

$$\mu(\mathbf{e}_{00}|\mathbf{a}_{01}) = \varepsilon$$

This makes $\mathbf{e}_{00}$ the unique equilibrium in the sub-game following $\mathbf{a}_{01}$ and $\mathbf{a}_{10}$.
Furthermore, the regular and casual player buy if and only if

$$\frac{1}{2} - p \geq \varepsilon$$
$$p \leq \frac{1}{2} - \varepsilon$$

Thus, the firm maximises its profits by setting $p = 1/2 - \varepsilon$. Hence, the firm can achieve a profit of $1 - 2\varepsilon$ by selling to both players. ∎

When the firm is faced with two players of asymmetric costs such that one is a regular player and one is a casual player, the firm designs the add-on such that it deters effort in all players even the holder of the add-on across all sub-games where a player has the add-on. There is no complimentary effect of effort with the add-on like in the regular v regular case. This is because if the firm wanted to design the add-on such that it was complimentary to effort the firm would have to discount the price by player 1's cost of effort. This is because even when the add-on is complimentary to effort, casual players will not exert effort even in the sub-game following $\mathbf{a}_{01}$ (casual player has add-on). As the casual player's cost of effort is just too high. As such the firm aims to induce $\mathbf{e}_{00}$ as the equilibrium outcome in the sub-game following $\mathbf{a}_{01}$ to minimise the regular player's payoff in this sub-game to ε. If the add-on were complimentary to effort the regular player's equilibrium payoff in the sub-game following $\mathbf{a}_{01}$ would be $\varepsilon + c_1$. This comes from the anonymity assumption.

Hence by designing the add-on such that it does not induces effort in the regular player when they have the add-on, the firm can make the regular player's payoff in the sub-game following $\mathbf{a}_{01}$ ε and by the anonymity assumption the casual player's equilibrium payoff in the sub-game following $\mathbf{a}_{10}$ (when the casual player does not have the add-on.) is the same (ε).

Moreover, in the sub-games following $\mathbf{a}_{01}$ and $\mathbf{a}_{10}$ the firm designs the add-on such that it deterred effort in the players. As a result, the firm will want the equilibrium outcome in the sub-game following $\mathbf{a}_{11}$ to be neither player exerts effort. To maximise both player payoff on the equilibrium path. This is no problem for the firm when it comes to the casual player. By Corollary 1 we know the casual player will never exert effort in the sub-game following $\mathbf{a}_{11}$. However, the firm is required to deter effort in the regular player in the sub-game following $\mathbf{a}_{11}$. If the firm does not, then they will be required to discount the price by the regular player's cost of effort in order to incentives the regular player to buy the add-on.

I now analyse the case where a professional ($c_1 < 0$) faces a casual player ($c_2 > \overline{c}$).

Lemma 13. *Suppose the first player is a professional ($c_1 < 0$) and the second player is a casual player ($c_2 > \overline{c}$). The firm can achieve a profit of $1-2\varepsilon$ by selling to both players. The firm can do so by choosing $p = 1/2-\varepsilon$, $\mu \in \mathcal{M}$ such that $\mu\left(\mathbf{e}_{10}|\mathbf{a}_{01}\right) = \varepsilon$*

Proof. By Corollary 1 it is a strictly dominant strategy for the professional to play $e_1 = 1$ in the sub-game following $\mathbf{a}_{11}$ and it is a strictly dominant strategy for the casual player to play $e_2 = 0$ in the sub-game following $\mathbf{a}_{11}$. Thus, in the sub-game following $\mathbf{a}_{11}$, $\mathbf{e}_{10}$ is the unique equilibrium.
Thus, the firm's equilibrium profit is positive if the professionals equilibrium payoff in the sub-game following $\mathbf{a}_{01}$ is bounded from above by $\mu(\mathbf{e}_{10}|\mathbf{a}_{11}) - p - c_1$ and the casual player's equilibrium payoff in the sub-game following $\mathbf{a}_{10}$ is bounded from above by $\mu(\mathbf{e}_{10}|\mathbf{a}_{11}) - p$.
The firm can achieve this by setting

$$\mu(\mathbf{e}_{10}|\mathbf{a}_{01}) = \varepsilon$$

This makes $\mathbf{e}_{10}$ the unique equilibrium in the sub-game following $\mathbf{a}_{01}$ and $\mathbf{a}_{10}$.
Thus, the professional buys the add-on if and only if

$$\begin{aligned}\mu(\mathbf{e}_{10}|\mathbf{a}_{11}) - p - c_1 &\geq \varepsilon - c_1 \\ p &\leq \mu(\mathbf{e}_{10}|\mathbf{a}_{11}) - \varepsilon\end{aligned}$$

The casual player buys if and only if

$$\begin{aligned}\mu(\mathbf{e}_{10}|\mathbf{a}_{11}) - p &\geq \varepsilon \\ p &\leq \mu(\mathbf{e}_{10}|\mathbf{a}_{11}) - \varepsilon\end{aligned}$$

By weak monotonicity the firm can set

$$\mu(\mathbf{e}_{10}|\mathbf{a}_{11}) = \frac{1}{2}$$

and maximise the price it sells to both players and set a price of

$$p \leq \frac{1}{2} - \varepsilon$$

Thus, the firm maximises its profits by setting $p = 1/2 - \varepsilon$. Hence, the firm can achieve a profit of $1 - 2\varepsilon$ by selling to both players. ■

In all relevant sub-games, the professional will exert effort because $c_1 < 0$ and $\mu \in \mathcal{M}$ is monotonically increasing in effort. Thus, the professional's payoff is always greatest when they exert effort regardless of what the casual players action is in any relevant sub-game. Also, from Corollary 1 we know the casual player will never exert effort in the sub-game following $\mathbf{a}_{11}$ as their cost of effort is just too high. Hence the equilibrium outcome in the sub-game following $\mathbf{a}_{11}$ is $\mathbf{e}_{10}$. The professional exerts effort and the casual player does not exert effort. This bounds from above the professional's payoff to be $\mu(\mathbf{e}_{10}|\mathbf{a}_{11}) - p - c$ and the casual player's payoff to be $1 - \mu(\mathbf{e}_{10}|\mathbf{a}_{11}) - p$.

I now focus attention on the sub-game following $\mathbf{a}_{10}$ and the effort choice of the casual player. Again, the professional will exert effort, because $\mu(\mathbf{e}|\mathbf{a})$ is monotonically increasing in a_1 and $c_1 < 0$. The firm aims to induce $\mathbf{e}_{10}$ to minimise the casual player's payoff in the sub-game following $\mathbf{a}_{10}$, which is of the equilibrium path for the casual player. Hence the firm minimises the casual player's equilibrium payoff in the sub-game following $\mathbf{a}_{10}$ to be ε. This bounds the casual player's payoff from below to ε.

A consequence of the firm not being able to induce the casual player to exert effort in the sub-game following $\mathbf{a}_{01}$ is that the equilibrium outcome will be $\mathbf{e}_{10}$. Thus, in order for the firm to minimise the professional's equilibrium payoff in the sub-game following $\mathbf{a}_{01}$ (off the equilibrium path) the firm sets $\mu(\mathbf{e}_{10}, \mathbf{a}_{01}) = \epsilon$. This gives the professional the lower bound payoff of $\epsilon - c$.

Now, in order for the firm to maximise the price that it can sell to both players, the firm needs to bring down the professionals probability of winning in the sub-game following $\mathbf{a}_{11}$ and equalise it, by doing this the firm increases the casual player's willingness to pay for the add-on. As they are charging both players the same price for the add-on, by taking any excess from the professional and transferring it to the casual player's chance of winning in the sub-game following $\mathbf{a}_{11}$, the firm increases their price.

I now analyses the case where a professional ($c_1 < 0$) faces a regular player ($c_2 \in [0, \bar{c}]$).

Lemma 14. *Suppose the first player is a professional* ($c_1 < 0$) *and the second player is a regular player* ($c_2 \in [0, \bar{c}]$). *The firm can achieve a profit of* $1 - 2\varepsilon$ *by selling to both players. The firm can do so by choosing* $p = 1/2 - \varepsilon$, *and* $\mu \in \mathcal{M}$ *such that* $\mu(\mathbf{e}_{10}|\mathbf{a}_{11}) < 1/2 + c_2$, $\mu(\mathbf{e}_{00}, \mathbf{a}_{01}) \geq \varepsilon + c_2$ *and* $\mu(\mathbf{e}_{11}, \mathbf{a}_{10}) = 1 - \varepsilon$.

Proof. By Corollary 1 it is a strictly dominant strategy for the professional to play $e_1 = 1$ in the sub-game following $\mathbf{a}_{11}$.
If the firm sets

$$\mu(\mathbf{e}_{10}|\mathbf{a}_{11}) < 1/2 + c_2$$

then, by Lemma 1 the regular player exerts low effort in the sub-game following $\mathbf{a}_{11}$.
Thus, $\mathbf{e}_{10}$ is the unique equilibrium in the sub-game following $\mathbf{a}_{11}$ if and only if

$$\mu(\mathbf{e}_{10}|\mathbf{a}_{11}) < 1/2 + c_2$$

Thus, the firm's equilibrium profit is positive if the professionals equilibrium payoff in the sub-game following $\mathbf{a}_{01}$ is bounded from above by $\mu(\mathbf{e}_{10}|\mathbf{a}_{11}) - p - c_1$ and the regular player's equilibrium payoff in the sub-game following $\mathbf{a}_{10}$ is bounded from above by $\mu(\mathbf{e}_{10}|\mathbf{a}_{11}) - p$. The firm can achieve this by setting

$$\mu(\mathbf{e}_{00}, \mathbf{a}_{01}) \geq \varepsilon + c_2$$
$$\mu(\mathbf{e}_{11}, \mathbf{a}_{10}) = 1 - \varepsilon$$

This makes the equilibrium outcome in the sub-game following $\mathbf{a}_{01}$, $\mathbf{e}_{11}$ giving the professional and equilibrium payoff in the sub-game following $\mathbf{a}_{01}$ of $\varepsilon - c_1$

This also makes the equilibrium outcome in the sub-game following $\mathbf{a}_{10}$, $\mathbf{e}_{10}$ giving the regular player and equilibrium payoff in the sub-game following $\mathbf{a}_{01}$ of ε

Thus, the professional buys the add-on if and only if

$$\begin{aligned} \mu(\mathbf{e}_{10}|\mathbf{a}_{11}) - p - c_1 &\geq \varepsilon - c_1 \\ p &\leq \mu(\mathbf{e}_{10}|\mathbf{a}_{11}) - \varepsilon \end{aligned}$$

The regular player buys if and only if

$$\begin{aligned} \mu(\mathbf{e}_{10}|\mathbf{a}_{11}) - p &\geq \varepsilon \\ p &\leq \mu(\mathbf{e}_{10}|\mathbf{a}_{11}) - \varepsilon \end{aligned}$$

By weak monotonicity the firm can set

$$\mu(\mathbf{e}_{10}|\mathbf{a}_{11}) = \frac{1}{2}$$

and maximise the price it sells to both players and set a price of

$$p \leq \frac{1}{2} - \varepsilon$$

Thus, the firm maximises its profits by setting $p = 1/2 - \varepsilon$. Hence, the firm can achieve a profit of $1 - 2\varepsilon$ by selling to both players. ■

In all relevant sub-games, the professional will exert effort because $c_1 < 0$ and $\mu \in \mathcal{M}$ is monotonically increasing in effort. Thus, the professional's payoff is always greatest when they exert effort regardless of what the regular players action is in an relevant sub-game. This narrows the problem the firm has. The firm must now ask, given that the professional will always exert effort how should the firm design the add-ons to affect the regular players effort choice?

The firm can maximise the bound from above of the regular player by ensuring that in the sub-game following $\mathbf{a}_{11}$ the regular does not exert effort. This maximises the upper bound on the regular player's payoff on the equilibrium path to be $1 - \mu(\mathbf{e}_{10}|\mathbf{a}_{11}) - p$. Hence the firm designs the add-on such that it deters effort in the regular player in the sub-game following $\mathbf{a}_{11}$.

I focus attention on the sub-games following $\mathbf{a}_{10}$ and $\mathbf{a}_{01}$, to understand whether the firm should design the add-on to induce effort in the regular player when they have the add-on. By making the add-on complimentary to effort the firm can induce the regular player to exert effort in the sub-game following $\mathbf{a}_{01}$ this minimises the professional payoff to be $\varepsilon - c_1$. However, the add-on also deters effort in the regular player when they don't have the add-on. This makes the equilibrium outcome in the sub-game following $\mathbf{a}_{10}$ to be $\mathbf{e}_{10}$ and this minimises the regular player's payoff to be ε.

Furthermore, the firm designs the add-on such that it deters effort in the regular player in the sub-game following $\mathbf{a}_{11}$ and induces effort in the regular player when they have the add-on; and deters effort in the regular player when their opponent has the add-on. By doing this the firm bounds from below the regular player to be ε and professional's payoff to be $\varepsilon - c_1$. Now, in order for the firm to maximise the price that it can sell to both players, the firm needs to bring down the professionals probability of winning in the sub-game following $\mathbf{a}_{11}$ and equalise it, by doing this the firm increases the regular player's willingness to pay for the add-on. As they are charging both players the same price for the add-on, by taking any excess from the professional and transferring it to the regular player's chance of winning in the sub-game following $\mathbf{a}_{11}$, the firm increases their price.

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