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Honours Thesis

How Much is My Home Worth?

The Value of Online Market Value Estimates

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Declaration

I declare that this thesis is my own work and that, to the best of my knowledge, it contains no material which has been written by another person or persons, except where acknowledgement has been made. This thesis has not been submitted for the award of any degree or diploma at the University of New South Wales Sydney, or at any other institute of higher education.

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Brendan James Wilson
22nd November, 2019

Acknowledgements

It is hidden inside, like words in a pen
It's only a matter of learning to write

Keiichi, thank you for your questions
Relentless as they seemed
Richard, thank you for your guidance
Especially when the ink blotched

Mum and Dad, I know it's been strange
To watch me use the pen
That you gave so differently

But it's not dry yet.

Contents

Abstract

Market value estimates have become a popular tool provided by online real estate listing platforms. The first and most popular being Zillow's Zestimate. Founded in observed behaviour towards such estimates, I assume that home buyers and sellers that meet through a platform treat its market value estimate as a reference. Under this behavioural assumption, I demonstrate how a platform can strategically announce a market value estimate to ensure that desirable trade occurs in equilibrium. I propose that this is one way in which market value estimates can help a platform attract buyers and sellers.

Chapter 1
Introduction

Zillow is an online real estate listing platform with market capitalisation of over US$8 billion.[1] The platform serves as a matchmaker between home buyers and sellers. Its alluring feature— that brought so much traffic that the site crashed shortly after its launch in 2006 (Kaysen, 2018)— is the *Zestimate*. The Zestimate is a tool that estimates the market value of a home— also known as a market value estimate. When a seller lists their home on Zillow, the Zestimate is placed prominently on their listing. When potential buyers view the listing, it is difficult for the Zestimate to go unnoticed. How has the Zestimate helped Zillow achieve success as a platform?

As a matchmaker, Zillow seeks to attract a critical mass of home buyers and sellers. Sellers will use the platform if it can connect them with buyers, and buyers will use it if it can connect them with sellers. The presence of buyers creates value for sellers through an indirect network effect. Similarly, the presence of sellers creates value for buyers. The need for buyers to attract sellers, but also sellers to attract buyers is known as the 'chicken and egg problem' (Evans and Schmalensee, 2016). A successful platform is able to attract enough buyers and sellers and exploit indirect network effects to solve this problem. I will explore how the Zestimate helps solve this problem for Zillow.

The Zestimate could simply be a gimmick. According to *The New York Times* (Wingfield, 2017)

> "It's one of the oldest tricks in an internet company's playbook. Concoct a tool that gives the public new statistics on

[1] https://finance.yahoo.com/quote/Z/

> something— the quality of a restaurant or a toaster, say. Then watch visitors flock."

When it launched, the Zestimate was a new and exciting tool. It certainly gave traction to the platform and may have helped kick-start its success. However, the Zestimate also influences how home buyers and sellers trade through the platform.

Consider a buyer who wishes to purchase a home from a seller. The buyer and seller each have a reservation price: a price above which the buyer is not willing to pay and a price below which the seller is not willing to accept. However, the buyer does not know the seller's reservation price and the seller does not know the buyer's. All that is known is what kind of reservation prices the buyer and seller may have and with what probability a buyer and seller will have a particular reservation price. This is an environment of incomplete information.

What influence can a market value estimate have in such an environment? Suppose that the buyer and seller meet through a platform such as Zillow and commonly observe the platform's market value estimate. Perhaps the estimate can serve to update the buyer's prior belief about the seller's reservation price, and vice versa. However, the platform's estimate is independent of the buyer and seller's reservation prices. It is difficult to see what influence a market value estimate can have in the mind of a rational buyer and seller.

Market value estimates affect buyer and seller behaviour. Home sellers tend to rely on the estimated value as a reference and become attached to it. This attachment has made sellers hesitant to sell their homes below the Zestimate (Kaysen, 2018). Buyers also use it as a reference. One seller filed a lawsuit against Zillow, claiming that her home's low Zestimate was deterring buyers from offering what she thought her home was worth (Harney, 2017).

I consider the behavioural influence of a market value estimate as a reference for home buyers and sellers. Because a market value estimate serves as a reference, I assume that a buyer feels aggrieved if they pay a higher price. Similarly, I assume that a seller feels aggrieved if they receive a lower price. Both the buyer and seller may still be willing to trade at such prices, but one of them may feel shortchanged. With this behaviour in mind, how does a platform's market value estimate affect bilateral trade and can this be beneficial for the platform?

To analyse this, I consider an environment where a buyer and seller meet through a platform. To simplify the analysis, I ignore the presence of real estate agents and the seller's announcement of an listing price. The platform can choose to announce a market value estimate, or not. If it does, the buyer and seller treat the market value estimate as a reference price when they bargain in a double auction. If the platform does not announce a market value estimate, the buyer and seller have no reference when they bargain.

I assume that the platform seeks to ensure that the highest ex ante expected gains from trade are achieved in the double auction equilibrium. This objective allows my model to ignore the platform's revenue and cost structure, and the organisation of the industry in which it operates. More importantly, buyers and sellers are likely to be attracted to a platform that ensures high expected gains from trade. As argued earlier, the number of buyers and sellers is critical to a platform's success. For example, Zillow earns advertising revenue, which is increasing in the number of buyers and sellers.

When the platform refrains from announcing a market value estimate, a number of equilibrium outcomes of the double auction can occur, with varying expected gains from trade. By announcing a market value estimate, the platform can preclude undesirable equilibrium outcomes and ensure that the second-best outcome is the unique equilibrium outcome in which trade occurs. Unfortunately, the platform cannot ensure the first-best equilibrium outcome. This echoes the bilateral trade literature. However, the first-best outcome is not always an equilibrium. When this is the case, the platform's announcement of a reference price is clearly beneficial.

One interpretation of my result is that a platform such as Zillow will fabricate a market value estimate rather than objectively estimating it. However, what defines the market value of a home is difficult. The platform's choice of market value estimate simply prevents trade from occurring at certain prices to preclude undesirable equilibrium outcomes. As a result, trade occurs at prices around the market value estimate. However, the prices that arise could do so in equilibrium when no reference price is announced. In this sense, what is chosen by the platform is not an inaccurate market value estimate.

Chapter 2
Related Literature

To the best of my knowledge, the literature is yet to explore the strategic role of market value estimates. However, the bilateral trade literature is highly relevant to my exploration of this.

A well-known model of bilateral trade is the double auction of Chatterjee and Samuelson (1983). In the double auction, two parties bargain over the price of a good whilst possessing incomplete information with respect to one another's reservation price. The buyer and seller simultaneously make a bid, trade occurs if and only if the buyer makes the highest bid, and the selling price is taken to be a convex combination of the two bids. The buyer and seller have utility for money and the good being bargained for that is additively separable. Incomplete information creates a tradeoff for the buyer and seller. The higher the buyer bids, the higher the probability that trade occurs, but at the cost of a higher price paid if trade occurs. The lower the seller bids, the higher the probability that trade occurs, but at the cost of a lower price received if trade occurs. Because of this tradeoff, the equilibrium bids do not allow all gains from trade to be exploited.

Myerson and Satterthwaite (1983) emphasise this inefficiency by showing that under certain conditions, no equilibrium outcome of the double auction can be ex post Pareto efficient. An outcome is ex post Pareto efficient if gains from trade are always exploited. Using the revelation principle, they show that no individually rational, incentive compatible, and budget balanced mechanism can achieve such efficiency. Keeping the tradeoff faced in the double auction in mind, consider a buyer and seller with high reservation prices and who stand to gain from trade. This buyer may face this seller, or they may face a seller with a lower reservation price. Although trade with the high valuation seller is mutually beneficial, the gains from trade are insufficient to cover the information rent needed for the high valuation buyer to trade with this seller. For the same reason, trade is difficult to achieve between buyers and sellers with low valuations. This is the root of the inefficiency emphasised by Myerson and Satterthwaite (1983).

Farrell and Gibbons (1989) demonstrate one way in which this inefficiency can be mitigated. In their model, the buyer and seller engage in a cheap talk stage prior to the double auction. In equilibrium, a party may be able to sacrifice their bargaining position by revealing information about their reservation price. Relative to the standard double auction, trade can occur in equilibrium for buyers with lower valuations and sellers with higher valuations. This raises the question of how else the game can be changed to improve efficiency. The announcement of a price that buyers and sellers will treat as a reference is one way.

The concept of reference dependence was first proposed by Kahneman and Tversky (1979). With this concept in mind, Hart and Moore (2008) claim that parties to a contract may feel entitled to certain contractual outcomes that can arise. Afflicted with a self-serving bias, parties choose their reference to be the outcome specified in the contract that is best for them. This is similar to how buyers and sellers feel towards market value estimates, although, it is not driven by self-serving bias.

In my model, the buyer and seller treat the market value estimate as a reference in the same way that agents in Hart and Moore (2008) treat a certain contractual outcome as a reference. Just as contractual outcomes occur with some positive probability— otherwise they would not be written in the contract— the market value estimate represents a price that could occur in equilibrium, regardless of whether the platform announces it. This ensures that, like agents in Hart and Moore (2008), the buyer and seller adopt a reasonable reference.

The idea of reference dependence in real estate markets is not new. Genesove and Mayer (2001) empirically find that home owners are averse to selling their homes for less than what they paid for it. In my model, I assume that the only reference that buyers and sellers have is the market value estimate. This simplifies the analysis and allows me to refrain from making further questionable behavioural assumptions about what else buyers and sellers may treat as a reference.

Rosenkranz and Schmitz (2007) consider first- and second-price auctions attended by bidders with reference dependent utility. In their model, a bidder adopts a reference price that depends on the seller's publicly announced reserve price. Their assumption that the seller's reserve price can affect a bidder's reference is similar to my assumption that the platform's market value estimate can serve as a reference for buyers and sellers.

Benkert (2016) explores reference dependence in the context of bilateral trade. In their model, agents form their own reference point. Following Kőszegi and Rabin (2006, 2007), agents evaluate a bilateral trade outcome relative to a reference point formed in rational expectation over the outcome. In contrast, my model considers agents that adopt a reference price that is chosen by the platform. Divergent from this literature, this assumption is made with the goal of exploring one channel through which the platform benefits from announcing a market value estimate.

Chapter 3
Statement of the Problem

There exists a buyer B, a seller S, and a platform. The seller wishes to sell their home and meets the buyer through the platform. Each $i \in \{B, S\}$ has a privately known and independent valuation for the object, v_i, that is either low or high: $v_i \in \{v_{Li}, v_{Hi}\} \subset \mathbb{R}$. The buyer and seller's possible valuations are commonly known and ordered such that $v_{LS} < v_{LB} < v_{HS} < v_{HB}$, ensuring that each type of buyer and seller has a profitable potential trading partner. It is also commonly known that the buyer and seller's valuations are high with probability $\Pr\{v_i = v_{Hi}\} = \mu_i \in (0, 1)$. This is summarised in Figure 3.1 (note that the valuations need not be equally spaced).

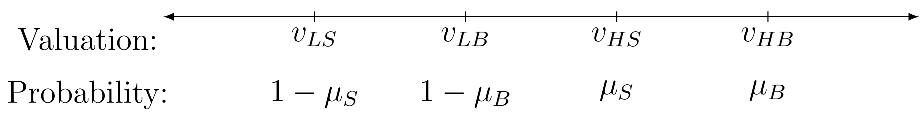

Figure 3.1: Buyer and seller valuations

The platform chooses to publicly announce a reference price $r \in [v_{LS}, v_{HB}]$ or to not announce a reference price. Its choice becomes common knowledge. If a platform announces a reference price, it is constrained not to be less than the lowest price a seller would accept nor greater than the highest price a buyer would pay. After all, the announced reference price is supposed to be a market value estimate. As such, it should assume the value of a price at which a buyer and seller would be willing to trade.

The buyer makes a bid $b \in \mathbb{R}$ and the seller makes a bid $s \in \mathbb{R}$. The bids made determine the probability of trade:

$$t(b,s) = \begin{cases} 1 & \text{if } b \geq s \\ 0 & \text{otherwise} \end{cases}$$

and the selling price:

$$p(b,s) = \frac{b+s}{2}$$

Trade occurs if and only if the buyer makes a bid that is greater than or equal to the seller's bid. If trade occurs, the selling price is the midpoint of the two bids. A buyer and seller's payoff depends on their valuation, the bids made, the reference price announced by the platform, and how strongly they treat this as a reference. The buyer's payoff is:

$$U_B(v_B; b, s|r) = t(b,s) \times u_B(v_B; b, s|r)$$

where

$$u_B(v_B; b, s|r) = \begin{cases} v_B - p(b,s) - \psi \max\{p(b,s) - r, 0\} & \text{if } r \text{ is announced} \\ v_B - p(b,s) & \text{otherwise} \end{cases}$$

and the seller's payoff is:

$$U_S(v_S; b, s|r) = t(b,s) \times u_S(v_S; b, s|r)$$

where

$$u_S(v_S; b, s|r) = \begin{cases} p(b,s) - v_S - \psi \max\{r - p(b,s), 0\} & \text{if } r \text{ is announced} \\ p(b,s) - v_S & \text{otherwise} \end{cases}$$

The buyer and seller each earn a payoff of zero if trade does not occur. If the platform does not announce a reference price and trade occurs, the buyer and seller each earn a payoff related to the difference between the selling price and their valuation. If the platform announces a reference price and trade occurs, the buyer and seller may also incur a psychic cost.

The parameter $\psi \geq 0$ represents the degree of this psychic ("ψ-chic") cost.

If $\psi = 0$, the buyer and seller are rational and the announced reference price never affects their payoff. If $\psi > 0$, the buyer and seller are behavioural and treat the price announced by the platform as a reference. Reference dependence pervades through a feeling of aggrievement if the price faced is less favourable than the reference price. If the buyer faces a price that is higher than r, they feel aggrieved and incur a psychic cost. If the seller faces a price that is lower than r, they feel aggrieved and incur a psychic cost. An unfavourable difference between the selling price and reference price of \$1 induces a psychic cost of $\$\psi$.

I have assumed that the buyer and seller have utility that depends on the reference price announced by the platform. Returning to the leading example of Zillow, this assumption is intended to capture the idea that buyers and sellers in the real estate market treat an announced market value estimate such as the Zestimate as a reference price. I could have instead assumed that buyers and sellers treat the market value estimate as their valuation. However, this is a stronger assumption and fails to reflect the fact that homes do sell for prices above and below their market value estimate.

I have also assumed that if the platform refrains from announcing a reference price, the buyer and seller have no reference. This is a simplifying assumption. I could have instead assumed that the buyer and seller are endowed with their own exogenous reference price, just as they are endowed with a valuation. In this case, if the platform announces a reference price, the buyer and seller's reference could be a convex combination of their endowed reference and the announced reference. This would be similar to how the seller's announced reserve price influences the reference of bidders in Rosenkranz and Schmitz (2007). However, this would complicate the analysis and still requires the assumption that the buyer and seller treat the platform's announcement as a reference. Furthermore, homes are heterogeneous and buyers and sellers in the real estate market are typically inexperienced. Despite being afflicted with reference dependence, without the platform's announcement the buyer and seller simply do not know what to treat as a reference.

The buyer and seller's problem is to choose a bidding strategy that maximises their expected payoff over the other party's possible valuations. A bidding strategy is defined as a pair of bids that are prescribed if the buyer or seller's valuation is low or high, respectively: (b_L, b_H) for the buyer and (s_L, s_H) for the seller. Given that the seller plays the strategy (s_L, s_H), the buyer's expected payoff over the seller's possible valuations is:

$$\mathbb{E}_{v_S}\left[U_B\right] = \mu_S \times U_B(v_B; b, s_H|r) + (1-\mu_S) \times U_B(v_B; b, s_L|r)$$

Given that the buyer plays the strategy (b_L, b_H), the seller's expected payoff over the buyer's possible valuations is:

$$\mathbb{E}_{v_B}\left[U_S\right] = \mu_B \times U_S(v_S; b_H, s|r) + (1-\mu_B) \times U_S(v_S; b_L, s|r)$$

The platform's problem is to choose to announce a reference price, or not, to ensure that the equilibrium in which ex ante expected gains from trade are highest is achieved. For an equilibrium bidding strategy profile $((b_L^*, b_H^*), (s_L^*, s_H^*))$, ex ante expected gains from trade are given by:

$$\begin{aligned}\Pi\left((b_L^*, b_H^*), (s_L^*, s_H^*)\right) = {} & \mu_B \times \mu_S \times t(b_H^*, s_H^*) \times (v_{HB} - v_{HS}) \\ & + \mu_B \times (1-\mu_S) \times t(b_H^*, s_L^*) \times (v_{HB} - v_{LS}) \\ & + (1-\mu_B) \times \mu_S \times t(b_L^*, s_H^*) \times (v_{LB} - v_{HS}) \\ & + (1-\mu_B) \times (1-\mu_S) \times t(b_L^*, s_L^*) \times (v_{LB} - v_{LS})\end{aligned}$$

The timing is as follows:

Stage 1: The platform chooses to publicly announce a reference price or not.

Stage 2: The buyer and seller privately realise their valuations, observe the platform's announcement, and engage in a double auction where they simultaneously make bids. Once bids are made, payoffs are realized.

The equilibrium concept I use for Stage 2 is Bayesian Nash equilibrium. A bidding strategy profile $((b_L, b_H), (s_L, s_H))$ constitutes a Bayesian Nash equilibrium $((b_L^*, b_H^*), (s_L^*, s_H^*))$ if each party's expected payoff is maximised by playing their equilibrium strategy, given that the other party is playing their equilibrium strategy.

Chapter 4
Equilibrium Analysis

4.1 Characterisation of Stage 2 Equilibria

I now characterise the equilibria of the double auction in Stage 2. In equilibrium, the buyer and seller's strategies must be individually rational and incentive compatible. For all $j \in \{L, H\}$ the buyer's equilibrium individual rationality (IR) and incentive compatibility (IC) constraints are:

$$\mu_S \times U_B(v_{jB}; b_j^*, s_H^*|r) + (1 - \mu_S) \times U_B(v_{jB}; b_j^*, s_L^*|r) \geq 0 \qquad \text{(Buyer IR)}$$

and for all $b \in \mathbb{R}$:

$$\begin{aligned}&\mu_S \times U_B(v_{jB}; b_j^*, s_H^*|r) + (1 - \mu_S) \times U_B(v_{jB}; b_j^*, s_L^*|r) \geq \\ &\mu_S \times U_B(v_{jB}; b, s_H^*|r) + (1 - \mu_S) \times U_B(v_{jB}; b, s_L^*|r)\end{aligned} \qquad \text{(Buyer IC)}$$

For all $j \in \{L, H\}$ the seller's equilibrium IR and IC constraints are:

$$\mu_B \times U_S(v_{jS}; b_H^*, s_j^*|r) + (1 - \mu_B) \times U_S(v_{jS}; b_L^*, s_j^*|r) \geq 0 \qquad \text{(Seller IR)}$$

and for all $s \in \mathbb{R}$:

$$\begin{aligned}&\mu_B \times U_S(v_{jS}; b_H^*, s_j^*|r) + (1 - \mu_B) \times U_S(v_{jS}; b_L^*, s_j^*|r) \geq \\ &\mu_B \times U_S(v_{jS}; b_H^*, s|r) + (1 - \mu_B) \times U_S(v_{jS}; b_L^*, s|r)\end{aligned} \qquad \text{(Seller IC)}$$

Buyer and seller incentive compatibility allows me to establish Lemma 1. All proofs are relegated to the Appendix.

Lemma 1. *In equilibrium, the probability that a buyer (seller) trades with a given type of seller (buyer) is monotonically increasing (decreasing) in the buyer's (seller's) valuation.*

Without loss of generality, consider the buyer. The high valuation buyer

(HB) can always mimic the strategy of the low valuation buyer (LB) and earn a weakly greater payoff than LB. This is because HB values the home more. HB will only follow a different strategy if the payoff is weakly greater than what can be earned by mimicking LB. This can only happen if HB trades with more types of sellers than LB. Thus, in equilibrium, the probability that a buyer trades with a given type of seller must be monotonically increasing in the buyer's valuation. Lemma 1 leads to the following corollary:

Corollary 1. *The equilibrium bid made by a given player is monotonically increasing in the player's valuation.*

Corollary 1 implies that $b_L^* \leq b_H^*$ and $s_L^* \leq s_H^*$. Individual rationality allows me to establish the following lemma:

Lemma 2. *The low valuation buyer does not trade with the high valuation seller in equilibrium.*

Since $v_{LB} < v_{HS}$, trade between LB and the high valuation seller (HS) can never be mutually individually rational, whether either party is aggrieved or not.

It is never incentive compatible for a seller to bid less than the lowest valuation buyer with which they trade in equilibrium. Otherwise, the seller would be leaving money on the table. This is similar for the buyer, who will never bid more than the highest valuation seller with which they trade in equilibrium. I establish this in the following lemma:

Lemma 3. *In equilibrium, the buyer never bids more than the highest valuation seller with which they trade and the seller never bids less than the lowest valuation buyer with which they trade.*

Lemmas 1-2 reveal which types of buyer and seller will trade with one another in equilibrium. Lemma 3 clarifies how the buyer and seller will bid in equilibrium. Using lemmas 1-3, I characterise the double auction equilibria in Proposition 1.

Proposition 1. *When the platform announces a reference price, there exist five possible equilibrium outcomes:*

1. *No trade.*
2. *Only the high valuation buyer and the low valuation seller trade.*
3. *Both sellers only trade with the high valuation buyer.*
4. *Both buyers only trade with the low valuation seller.*
5. *All pairs except the low valuation buyer and high valuation seller pair trade.*

When the platform does not announce a reference price, the first four equilibrium outcomes always exist and the fifth equilibrium outcome exists if and only if

$$\mu_S \geq \frac{v_{HS} - v_{LB}}{2v_{HB} - v_{HS} - v_{LB}} \equiv \underline{\mu}_S$$

and

$$\mu_B \leq \frac{2(v_{LB} - v_{LS})}{v_{HS} + v_{LB} - 2v_{LS}} \equiv \bar{\mu}_B$$

First, I explore what can occur in equilibrium when the platform refrains from announcing a reference price. This serves as the benchmark case.

4.2 Equilibria When No Reference Price is Announced

Let us establish some intuition as to why the first four equilibrium outcomes always exist when no reference price is announced. Consider the no trade equilibrium outcome. Both buyers can believe that the price needed to trade with any seller is too high and both sellers can believe that the price needed to trade with any buyer is too low. These beliefs are sustained if the buyers bid below any seller's valuation and the sellers bid above any buyer's valuation.

Consider the second equilibrium outcome listed in Proposition 1. LB can believe that the price needed to trade with any seller is too high and HS can believe that the price needed to trade with any buyer is too low. These beliefs are sustained if HB and the low valuation seller (LS) trade at a price that is too high for LB and too low for HS.

The intuition behind the third and fourth equilibrium outcomes is similar. Consider the third equilibrium outcome. LB can believe that the price needed to trade with any seller is too high. Such a belief is sustained if both sellers trade with HB at a price too high for LB. Consider the fourth equilibrium outcome. HS can believe that the price needed to trade with any buyer is too low. Such a belief is sustained if both buyers trade with LS at a price too low for HS.

The fifth equilibrium outcome is not sustained so easily. Figure 4.1 depicts trade in this equilibrium outcome. In this equilibrium, LB and LS trade at a price between v_{LS} and v_{LB}, HB and HS trade at a price between v_{HS} and v_{HB}, and HB and LS trade at the midpoint of these two prices. However, notice that it is also individually rational for HB to bid b_L^* and for LS to bid s_H^*.

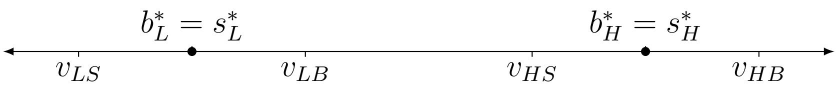

Figure 4.1: Trade in the efficient equilibrium

The existence of this equilibrium outcome hinges on the incentive compatibility of HB trading with HS and LS trading with LB. This is because it is also individually rational for HB to mimic LB's bid or LS to mimic HS's bid. This incentive compatibility boils down to a simple tradeoff. For HB, the marginal cost of trading with HS is the expected value of the increase in price paid if LS is faced. The marginal benefit is the expected surplus earned from trading with HS. Thus, incentive compatibility for HB to trade with HS requires:

$$\mu_S \overbrace{(v_{HB} - s_H^*)}^{\text{Surplus from trading with } HS} \geq (1 - \mu_S) \underbrace{\left(\frac{s_H^* - s_L^*}{2} \right)}_{\text{Price increase when } LS \text{ is faced}}$$

If this inequality fails to hold for even the most favourable price at which HB could trade with HS in equilibrium (v_{HS}), then the fifth equilibrium outcome does not exist. This is true if and only if the prior probability that HS is faced (μ_S) is sufficiently low: $\mu_S < \underline{\mu}_S$.

For LS, the marginal cost of trading with LB is the expected value of the decrease in the price received if HB is faced. The marginal benefit is the expected surplus earned from trading with LB. Thus, incentive compatibility for LS to trade with LB requires:

$$(1-\mu_B)\overbrace{(b_L^* - v_{LS})}^{\text{Surplus from trading with } LB} \geq \mu_B \underbrace{\left(\frac{b_H^* - b_L^*}{2}\right)}_{\text{Price decrease when } HB \text{ is faced}}$$

If this inequality fails to hold for even the most favourable price at which LS could trade with LB in equilibrium (v_{LB}), then the fifth equilibrium outcome does not exist. This is true if and only if the prior probability that LB is faced ($1-\mu_B$) is sufficiently low, or equivalently, μ_B is sufficiently high: $\mu_B > \bar{\mu}_B$. The shaded region in Figure 4.2 includes the pairs of prior probabilities for which the fifth equilibrium outcome exists.

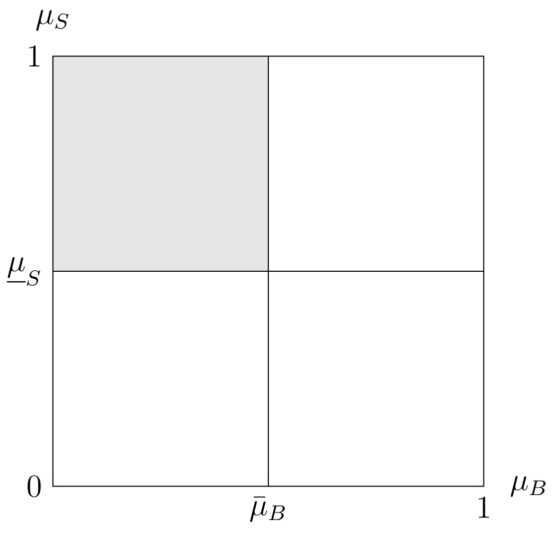

Figure 4.2: Existence of efficient equilibria when r is not announced

The fifth equilibrium outcome is different from the other equilibrium outcomes: all gains from trade are exploited in this equilibrium outcome.

For this reason, it is ex post Pareto efficient. Myerson and Satterthwaite (1983) establish the impossibility of an individually rational, incentive compatible, and budget balanced bilateral trading mechanism that achieves such efficiency. However, this impossibility is predicated on the valuations being distributed with positive probability density over an interval. Because I have treated the valuations as binary, this condition is violated. Thus, the efficient equilibrium can exist in my model. However, the difficulty in sustaining it echoes the result of Myerson and Satterthwaite (1983).

4.3 Equilibria When Reference Price is Announced

By Proposition 1, the set of possible equilibrium outcomes is the same as when no reference price is announced. However, depending on the value of the reference price, the necessary and sufficient conditions for each of the equilibrium outcomes are different. The equilibrium outcome in which no trade occurs always exists. Buyers can still believe that the price needed to trade with any seller is too high and bid below any seller's valuation. Sellers can still believe that the price needed to trade with any buyer is too low and bid above any buyer's valuation.

However, the other four equilibrium outcomes do not always exist. The reference price introduces a potential psychic cost for the buyer and seller. First, I will consider how this makes it more difficult to sustain the efficient equilibrium outcome than when no reference price is announced.

First consider HB. Trade with HS must be incentive compatible: HB must earn a higher payoff from trading with HS than from trading with only LS. The marginal cost of trading with HS is the expected value of the increase in price paid if LS is faced. But, if the price at which HB will trade with LS is above the reference price, then HB faces an even greater effective price increase when LS is faced. The marginal benefit is the expected surplus earned from trading with HS. But, if the price needed to trade with HS is above the reference price, HB earns less surplus from this trade relative to when no reference price is announced. Thus, incentive compatibility for HB to trade with HS requires:

$$\mu_S \overbrace{\left(v_{HB} - s_H^* - \psi \max\left\{s_H^* - r, 0\right\}\right)}^{\text{Net surplus from trading with } HS} \geq$$
$$(1-\mu_S) \underbrace{\left(\frac{s_H^* - s_L^*}{2}\right)}_{\text{Price increase when } LS \text{ is faced}} + (1-\mu_S) \underbrace{\psi\left[\max\{p(s_H^*, s_L^*) - r, 0\} - \max\{s_L^* - r, 0\}\right]}_{\text{Increased psychic cost when } LS \text{ is faced}}$$

If $r < s_H^*$, the marginal benefit of trading with HS is strictly lower and the marginal cost is weakly higher, relative to when r is not announced. In this case, the above inequality holds for fewer values of μ_S relative to the benchmark case.

Consider LS. Trade with LB must be incentive compatible: LS must earn a higher payoff from trading with LB than from trading with only HB. The marginal cost of trading with LB is the expected value of the decrease in price received if HB is faced. But, if the price at which LS will trade with HB is below the reference price, then LS faces an even greater effective price decrease when HB is faced. The marginal benefit is the expected surplus earned from trading with LB. But, if the price needed to trade with LB is below the reference price, LS earns less surplus from this trade relative to when no reference price is announced. Thus, incentive compatibility for LS to trade with LB requires:

$$(1-\mu_B) \overbrace{\left(b_L^* - v_{LS} - \psi \max\left\{r - b_L^*, 0\right\}\right)}^{\text{Net surplus from trading with } LB} \geq$$
$$\mu_B \underbrace{\left(\frac{b_H^* - b_L^*}{2}\right)}_{\text{Price decrease when } HB \text{ is faced}} + \mu_B \underbrace{\psi\left[\max\{r - p(b_H^*, b_L^*), 0\} - \max\{r - b_H^*, 0\}\right]}_{\substack{\text{Increased psychic cost} \\ \text{when } HB \text{ is faced}}}$$

If $r > b_L^*$, the marginal benefit of trading with LB is strictly lower and the marginal cost is weakly higher, relative to when r is not announced. In this case, the above inequality holds for fewer values of μ_B relative to the benchmark case.

Recall Figure 4.1 that depicted the prices at which buyer-seller pairs trade in the efficient equilibrium. If the platform announces a reference price,

HB or LS will feel aggrieved when they face at least one type of the other party. Therefore, a psychic cost will always enter the incentive compatibility constraint of HB or LS. Recall Figure 4.2 that depicted the pairs of priors for which the efficient equilibrium outcome exists when no reference price is announced. I have argued that when r is announced, the efficient equilibrium outcome exists for fewer pairs of prior probabilities. If HB feels aggrieved, μ_S needs to be higher to ensure that the efficient equilibrium outcome exists. If LS feels aggrieved, μ_B needs to be lower to ensure that this is the case.

This is depicted in Figure 4.3. In general, the announcement of a reference price shrinks the region of priors for which the efficient equilibrium outcome exists. Furthermore, the greater the degree of the psychic cost, the smaller this region is. This is because the prior probability that HS or LB is faced must be increased to offset to increased psychic cost incurred by HB trading with HS or LS trading with LB.

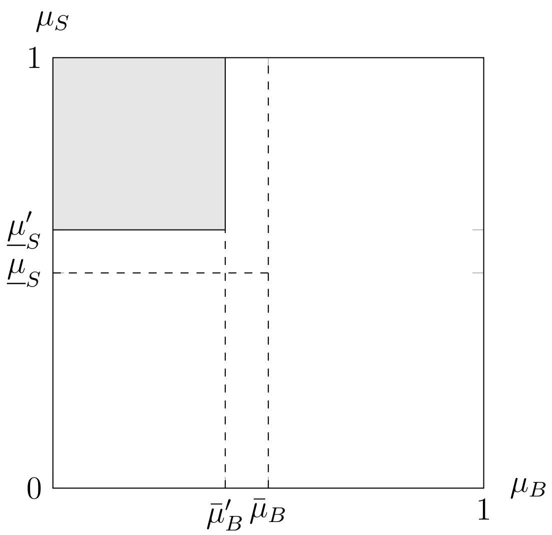

Figure 4.3: Existence of efficient equilibria when r is announced

Reference dependence makes it harder for the efficient equilibrium to exist. This is consistent with Benkert (2016) who finds that trade is less efficient when agents are afflicted with loss aversion to a reference. So far, I have shown how the psychic cost affects incentive compatibility and through this the existence of the efficient equilibrium outcome. In Chapter 5, I demonstrate how this cost affects individual rationality and can preclude equilibrium outcomes that are undesirable to the platform.

Chapter 5
The Platform

5.1 Ex Ante Expected Gains from Trade

When the platform refrains from announcing a reference price, there are five possible equilibrium outcomes: the efficient equilibrium outcome only exists for certain pairs of prior probabilities, but the remaining four always exist. The existence of multiple equilibria is a problem for the platform that cares about ex ante expected gains from trade. This is because expected gains from trade vary across the possible equilibrium outcomes. In this chapter, I demonstrate when and how a platform can choose a reference price to ensure that the second-best equilibrium outcome is the unique equilibrium outcome in which trade occurs. This is clearly desirable for the platform if the given priors do not support the existence of the efficient equilibrium outcome when r is not announced.

In terms of expected gains from trade, the efficient equilibrium outcome is the first-best equilibrium outcome. However, what constitutes the second-best equilibrium outcome depends on the values of the prior probabilities. This is clarified in Proposition 2.

Proposition 2. *The equilibrium outcome in which both sellers trade only with the high valuation buyer is second-best in terms of ex ante expected gains from trade if and only if*

$$\mu_S \geq \frac{(1-\mu_B)(v_{LB}-v_{LS})}{\mu_B(v_{HB}-v_{HS})+(1-\mu_B)(v_{LB}-v_{LS})}$$

The equilibrium outcome in which both buyers trade only with the low valuation seller is second-best if and only if the direction of the above inequality is reversed.

For some pairs of prior probabilities, the equilibrium outcome in which both sellers trade with HB is second-best. For other pairs, the equilibrium outcome in which both buyers trade with LS is second-best. When $v_{LB} - v_{LS} = v_{HB} - v_{HS}$, these pairs are separated by the line $\mu_S = 1 - \mu_B$. Figure 5.1 depicts the pairs of prior probabilities for which each of the equilibrium outcomes described in Proposition 2 are second-best when $v_{LB} - v_{LS} = v_{HB} - v_{HS}$.

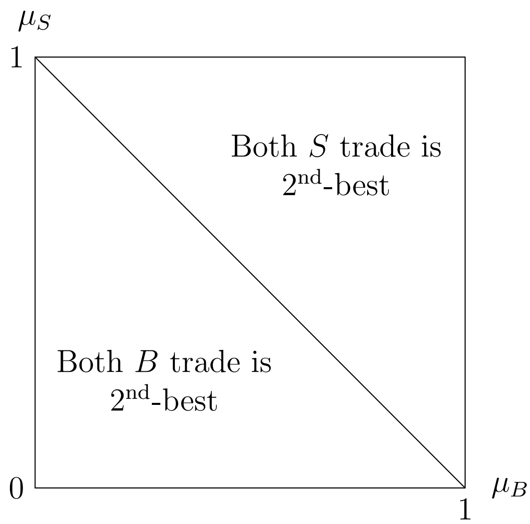

Figure 5.1: Second best equilibrium outcome

The equilibrium outcome in which both sellers trade with HB is second-best for the pairs of priors that lie above the curve in Figure 5.1. The equilibrium outcome in which both buyers trade with LS is second-best for the pairs of priors that lie below the curve. When the mass of low valuation buyers and low valuation sellers is high, the equilibrium outcome where LB and LS trade is second-best. When the mass of high valuation buyers and high valuation sellers is high, the equilibrium outcome where HB and HS trades is second-best.

5.2 Equilibrium Selection with a Reference Price

Now, I demonstrate how the platform's choice of reference price can ensure that when trade occurs, it occurs in the second-best equilibrium outcome. Consider the equilibrium outcomes in which trade occurs that are not ex post Pareto efficient.

For an equilibrium to exist in which HS never trades, LS must trade at a price less than the v_{HS}. To preclude such equilibria, a reference price can be introduced such that LS is too aggrieved to trade at a price less than v_{HS}. For an equilibrium to exist in which LB never trades, HB must trade at a price greater than v_{LB}. To preclude such equilibria, a reference price can be introduced such that HB is too aggrieved to trade at a price greater than v_{LB}. Using this idea, I establish the following proposition:

Proposition 3. *There exists a reference price such that the equilibrium outcome in which both sellers trade only with the high valuation buyer can be sustained as the unique equilibrium outcome in which trade occurs if and only if*

$$\psi > \frac{v_{HS} - v_{LS}}{v_{HB} - v_{HS}} \equiv \psi_1$$

Similarly, there exists a reference price such that the equilibrium outcome in which both buyers trade only with the low valuation seller can be sustained as the unique equilibrium outcome in which trade occurs if and only if

$$\psi > \frac{v_{HB} - v_{LB}}{v_{LB} - v_{LS}} \equiv \psi_2$$

A reference price can be used to direct trade into the equilibrium outcome where both sellers trade only with HB if and only if the degree of psychic cost is sufficiently high. This is because the reference price needs to induce a high enough psychic cost such that it is not individually rational for LS to trade at a price less than v_{HS}.

However, ψ_1 depends on the valuations of the types that trade in the equilibrium outcome in which both sellers trade only with HB. The reference price must lie between v_{HS} and v_{HB} to ensure that trading at a price less than v_{HS} is not individually rational for LS. If trade at the price of v_{HS} yields a negative payoff for LS, so will any lower price. The threshold ψ_1 decreases as the difference between v_{HB} and v_{HS} increases. This is because the larger this difference is, the higher a reference price can be chosen relative to v_{HS}. The higher the reference price is, the lower the degree of psychic cost needs to be, holding LS's payoff from trading at the price of v_{HS} constant. The numerator of ψ_1 measures LS's gross surplus from trading at a price of v_{HS}. The higher this gross surplus is, the higher the degree of psychic cost needs to be, ceteris paribus.

A reference price can also be used to direct trade into the equilibrium outcome where both buyers trade only with LS if and only if the degree of psychic cost is sufficiently high. This is because the reference price needs to induce a high enough psychic cost such that it is not individually rational for HB to trade at a price greater than v_{LB}. Given the valuations, this can only be achieved if the degree of psychic cost is sufficiently high.

However, ψ_2 depends on the valuations of the types that trade in the equilibrium outcome in which both buyers trade only with LS. The reference price must lie between v_{LS} and v_{LB} to ensure that trading at a price greater than v_{LB} is not individually rational for HB. If trade at the price of v_{LB} yields a negative payoff for HB, so will any greater price. The threshold ψ_2 decreases as the difference between v_{LB} and v_{LS} increases. This is because the larger this difference is, the lower a reference price can be chosen relative to v_{LB}. The lower the reference price is, the lower the degree of psychic cost needs to be, holding HB's payoff from trading at the price of v_{LB} constant. The numerator of ψ_2 measures HB's gross surplus from trading at a price of v_{LB}. The higher this gross surplus is, the higher the degree of psychic cost needs to be, ceteris paribus.

Given a sufficiently high degree of psychic cost, the platform is able to choose a reference price that ensures the second-best equilibrium outcome is the unique equilibrium outcome in which trade occurs. This is clearly beneficial if the given pair of priors does not support the existence of the efficient equilibrium outcome when no reference price is announced. If the efficient equilibrium outcome does exist when no reference price is announced, then whether or not this is beneficial is unclear.

5.3 Numerical Example

For example, suppose that the buyer and seller's valuations are given by $v_{LS} = 0$, $v_{LB} = 2$, $v_{HS} = 8$, and $v_{HB} = 10$. Furthermore, suppose that the prior probabilities are given by $\mu_B = \mu_S = 0.5$.

From Proposition 1, we know that when no reference price is announced, the efficient equilibrium does not exist: $\bar{\mu}_B = 0.4 < \mu_B = 0.5$ and $\mu_S = 0.5 < \underline{\mu}_S = 0.6$. By Proposition 2, the equilibrium outcomes in which both sellers trade only with HB and in which both buyers only trade with LS yield identical ex ante expected gains from trade and are both second-best. By Proposition 3, the platform can ensure that one of these equilibrium outcomes is the unique equilibrium outcome in which trade occurs if and only if $\psi > \psi_1 = \psi_2 = 4$.

Suppose $\psi = 5$. The platform can announce $r = 1$ and ensure that when trade occurs in equilibrium, only LB, HB, and LS will trade. In one set of equilibria, trade will occur at the reference price: $b_L^* = b_H^* = s_L^* = 1$ and $s_H^* \geq 2$. However, consistent with the observation that homes do sell at prices above and below their market value estimate, trade can also occur in equilibrium at prices of 1.1 and 0.9, for example. Alternatively, the platform can announce $r = 9$ and ensure that when trade occurs in equilibrium, only HB, LS, and HS will trade. Similarly, trade can occur in equilibrium at, above, and below this reference price.

Chapter 6
Conclusion

I have explored one channel through which the announcement of a market value estimate can be beneficial for a real estate listing platform. By announcing a market value estimate, a platform is able to ensure that the second-best equilibrium outcome in terms of ex ante expected gains from trade is achieved. This is clearly beneficial for a platform that faces a population of buyers and sellers that cannot trade efficiently in equilibrium. By ensuring high equilibrium expected gains from trade, a platform is likely to attract buyers and sellers. This is one way in which the Zestimate may have helped Zillow achieve success as a platform.

My result hinges on the critical assumption that buyers and sellers treat a market value estimate as a reference. Furthermore, buyers and sellers must feel sufficiently aggrieved if a price less favourable than the market value estimate is faced. In future work, it would be interesting to see if the behavioural assumptions that I have made hold in an experimental setting.

If the Zestimate does ensure high expected gains from trade and attracts buyers and sellers to the platform, one would expect competing platforms to publish their own market value estimates. Since Zillow introduced the Zestimate, market value estimates have become a common feature of real estate listing platforms. In the US, Redfin and Trulia now publish estimates similar to Zillow's Zestimate.

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Appendix A
Proofs

Lemma 1. *In equilibrium, the probability that a buyer (seller) trades with a given type of seller (buyer) is monotonically increasing (decreasing) in the buyer's (seller's) valuation.*

Proof. Consider the proof for the buyer. LB IC implies

$$\mu_S \times U_B(v_{LB}; b_L^*, s_H^*|r) + (1-\mu_S) \times U_B(v_{LB}; b_L^*, s_L^*|r) \geq$$
$$\mu_S \times U_B(v_{LB}; b_H^*, s_H^*|r) + (1-\mu_S) \times U_B(v_{LB}; b_H^*, s_L^*|r)$$

HB IC implies

$$\mu_S \times U_B(v_{HB}; b_H^*, s_H^*|r) + (1-\mu_S) \times U_B(v_{HB}; b_H^*, s_L^*|r) \geq$$
$$\mu_S \times U_B(v_{HB}; b_L^*, s_H^*|r) + (1-\mu_S) \times U_B(v_{HB}; b_L^*, s_L^*|r)$$

Summing over these two inequalities yields

$$\mu_S \left[t(b_H^*, s_H^*) - t(b_L^*, s_H^*)\right] + (1-\mu_S)\left[t(b_H^*, s_L^*) - t(b_L^*, s_L^*)\right] \geq 0 \qquad \text{(A.1)}$$

(A.1) is satisfied iff $t(b_H^*, s_H^*) \geq t(b_L^*, s_H^*)$ and $t(b_H^*, s_L^*) \geq t(b_L^*, s_L^*)$. To see this, suppose $t(b_H^*, s_H^*) < t(b_L^*, s_H^*)$, which implies $b_H^* < b_L^*$. But then for (A.1) to hold we need $t(b_H^*, s_L^*) > t(b_L^*, s_L^*)$, which implies $b_L^* < b_H^*$— a contradiction. If we instead suppose $t(b_H^*, s_L^*) < t(b_L^*, s_L^*)$, we arrive at the same contradiction. Therefore, the probability that a buyer trades with a given type of seller is monotonically increasing in the buyer's valuation.

Consider the proof for the seller. HS IC implies

$$\mu_B \times U_S(v_{HS}; b_H^*, s_H^*|r) + (1-\mu_B) \times U_S(v_{HS}; b_L^*, s_H^*|r) \geq$$
$$\mu_B \times U_S(v_{HS}; b_H^*, s_L^*|r) + (1-\mu_B) \times U_S(v_{HS}; b_L^*, s_L^*|r)$$

LS IC implies

$$\mu_B \times U_S(v_{LS}; b_H^*, s_L^*|r) + (1-\mu_B) \times U_S(v_{LS}; b_L^*, s_L^*|r) \geq$$
$$\mu_B \times U_S(v_{LS}; b_H^*, s_H^*|r) + (1-\mu_B) \times U_S(v_{LS}; b_L^*, s_H^*|r)$$

Summing over these two inequalities yields

$$\mu_B \left[t(b_H^*, s_L^*) - t(b_H^*, s_H^*)\right] + (1-\mu_B)\left[t(b_L^*, s_L^*) - t(b_L^*, s_H^*)\right] \geq 0 \qquad \text{(A.2)}$$

(A.2) is satisfied iff $t(b_H^*, s_L^*) \geq t(b_H^*, s_H^*)$ and $t(b_L^*, s_L^*) \geq t(b_L^*, s_H^*)$. To see this, suppose $t(b_H^*, s_L^*) < t(b_H^*, s_H^*)$, which implies $s_H^* < s_L^*$. But then for (A.2) to hold we need $t(b_L^*, s_L^*) > t(b_L^*, s_H^*)$, which implies $s_L^* < s_H^*$— a contradiction. If we instead suppose $t(b_L^*, s_L^*) < t(b_L^*, s_H^*)$, we arrive at the same contradiction. Therefore, the probability that a seller trades with a given type of buyer is monotonically decreasing in the seller's valuation. ■

Corollary 1. *The equilibrium bid made by a given player is monotonically increasing in the player's valuation.*

Proof. Consider the buyer. By Lemma 1 we have

$$t(b_H^*, s_H^*) \geq t(b_L^*, s_H^*) \text{ and}$$
$$t(b_H^*, s_L^*) \geq t(b_L^*, s_L^*)$$

Recall that

$$t(b,s) = \begin{cases} 1 & \text{if } b \geq s \\ 0 & \text{otherwise} \end{cases}$$

Thus, we have the above iff $b_H^* \geq b_L^*$. Consider the seller. By Lemma 1 we have

$$t(b_H^*, s_L^*) \geq t(b_H^*, s_H^*) \text{ and}$$
$$t(b_L^*, s_L^*) \geq t(b_L^*, s_H^*)$$

Similarly, we have this iff $s_H^* \geq s_L^*$. ■

Lemma 2. *The low valuation buyer does not trade with the high valuation seller in equilibrium.*

Proof. Assume for the sake of contradiction that LB and HS trade in equilibrium at the price $p(b_L^*, s_H^*)$. LB IR implies

$$v_{LB} - p(b_L^*, s_H^*) - \psi \max\{p(b_L^*, s_H^*) - r, 0\} \geq 0$$

and HS IR implies

$$p(b_L^*, s_H^*) - v_{HS} - \psi \max\{r - p(b_L^*, s_H^*), 0\} \geq 0$$

Summing over these two inequalities yields

$$v_{LB} - v_{HS} - \psi \max\{p(b_L^*, s_H^*) - r, 0\} - \psi \max\{r - p(b_L^*, s_H^*), 0\} \geq 0$$

Since $v_{LB} < v_{HS} \iff v_{LB} - v_{HS} < 0$, this implies

$$-\psi \max\{p(b_L^*, s_H^*) - r, 0\} - \psi \max\{r - p(b_L^*, s_H^*), 0\} > 0$$

But, this is a contradiction since the left hand side is always less than or equal to zero. Therefore, LB and HS never trade in equilibrium since it will never be mutually individually rational for them to do so. ■

Lemma 3. *In equilibrium, the buyer never bids more than the highest valuation seller with which they trade and the seller never bids less than the lowest valuation buyer with which they trade.*

Proof. Consider the buyer. Suppose that in equilibrium, a given type of buyer bids b^* and the highest valuation seller with which this buyer trades bids s^*. For trade to occur $b^* \geq s^*$. Suppose for the sake of contradiction that $b^* > s^*$. The buyer can decrease their bid by a small $\varepsilon > 0$ such that $t(b^* - \varepsilon, s^*) = t(b^*, s^*) = 1$ and $u_B(v_B; b^* - \varepsilon, s^*) > u_B(v_B; b^*, s^*)$. This contradicts the supposition that $b^* > s^*$ is the buyer's equilibrium bid. Therefore, $b^* = s^*$ and the bid made by a given type of buyer is equal to the bid of the highest valuation seller with which that type of buyer will trade.

Consider the seller. Suppose that in equilibrium, a given type of seller bids s^* and the lowest valuation buyer with which that seller trades bids b^*. For trade to occur $s^* \leq b^*$. Suppose for the sake of contradiction that

$s^* < b^*$. The seller can increase their bid by a small $\varepsilon > 0$ such that $t(b^*, s^* + \varepsilon) = t(b^*, s^*) = 1$ and $u_S(v_S; b^*, s^* + \varepsilon) > u_S(v_S; b^*, s^*)$. This contradicts the supposition that $s^* < b^*$ is the seller's equilibrium bid. Therefore, $s^* = b^*$ and the bid made by a given type of seller is equal to the bid of the lowest valuation buyer with which that type of seller will trade. ■

Proposition 1. *When the platform announces a reference price, there exist five possible equilibrium outcomes:*

1. *No trade.*
2. *Only the high valuation buyer and the low valuation seller trade.*
3. *Both sellers only trade with the high valuation buyer.*
4. *Both buyers only trade with the low valuation seller.*
5. *All pairs except the low valuation buyer and high valuation seller pair trade.*

When the platform does not announce a reference price, the first four equilibrium outcomes always exist and the fifth equilibrium outcome exists iff

$$\mu_S \geq \frac{v_{HS} - v_{LB}}{2v_{HB} - v_{HS} - v_{LB}} \equiv \underline{\mu}_S$$

and

$$\mu_B \leq \frac{2(v_{LB} - v_{LS})}{v_{HS} + v_{LB} - 2v_{LS}} \equiv \bar{\mu}_B$$

Proof. An equilibrium outcome can be represented by a quadruple of the equilibrium probabilities of trade: $(t(b_H^*, s_H^*), t(b_H^*, s_L^*), t(b_L^*, s_H^*), t(b_L^*, s_L^*))$. There exist $2^4 = 16$ possible quadruples. By Lemmas 1 and 2, only five of these can be equilibrium outcomes: (0,0,0,0), (0,1,0,0), (1,1,0,0), (0,1,0,1), and (1,1,0,1). In equilibrium, a player's bid must be individually rational and incentive compatible.

Consider equilibria in which $(t(b_H^*, s_H^*), t(b_H^*, s_L^*), t(b_L^*, s_H^*), t(b_L^*, s_L^*)) = (0, 0, 0, 0)$. IR is satisfied for each type of buyer and seller since $t(b_H^*, s_H^*) =$

$t(b^*_H, s^*_L) = t(b^*_L, s^*_H) = t(b^*_L, s^*_L) = 0$. Consider HB IC. Since $t(b^*_H, s^*_H) = t(b^*_H, s^*_L) = 0$, $b^*_H < \min\{s^*_L, s^*_H\}$ and HB earns a zero payoff. If

$$v_{HB} - \min\{s^*_L, s^*_H\} - \psi \max\{\min\{s^*_L, s^*_H\} - r, 0\} \leq 0$$

then HB IC is satisfied since otherwise HB could earn a higher payoff by bidding $b = \min\{s^*_L, s^*_H\}$. This implies that LB IC is satisfied since $v_{LB} < v_{HB}$. Consider LS IC. Since $t(b^*_H, s^*_L) = t(b^*_L, s^*_L) = 0$, $s^*_L > \max\{b^*_L, b^*_H\}$ and LS earns a zero payoff. If

$$\max\{b^*_L, b^*_H\} - v_{LS} - \psi \max\left\{r - \max\{b^*_L, b^*_H\}, 0\right\} \leq 0$$

then LS IC is satisfied since otherwise LS can earn a higher payoff by bidding $s = \max\{b^*_L, b^*_H\}$. This implies that HS IC is satisfied since $v_{LS} < v_{HS}$. Notice that $\forall r \in [v_{LS}, v_{HB}]$ the IC constraints are satisfied if $b^*_L, b^*_H \leq v_{LS}$ and $s^*_L, s^*_H \geq v_{HB}$. Therefore, this equilibrium outcome exists for all reference prices and prior probabilities.

Consider equilibria in which $(t(b^*_H, s^*_H), t(b^*_H, s^*_L), t(b^*_L, s^*_H), t(b^*_L, s^*_L)) = (0, 1, 0, 0)$. By Lemma 3, $b^*_L < b^*_H = s^*_L < s^*_H$. LB and HS IR are satisfied since $t(b^*_H, s^*_H) = t(b^*_L, s^*_H) = t(b^*_L, s^*_L) = 0$. HB IR is satisfied iff

$$v_{HB} - b^*_H - \psi \max\{b^*_H - r, 0\} \geq 0$$

LS IR is satisfied iff

$$s^*_L - v_{LS} - \psi \max\{r - s^*_L, 0\} \geq 0$$

Consider HB IC. By HB IR, bidding $b < s^*_L$ is not a profitable deviation. Bidding $b = s^*_H$ and trading with HS is a profitable deviation iff

$$\mu_S \left(v_{HB} - s^*_H - \psi \max\left\{s^*_H - r, 0\right\}\right) > \tag{A.3}$$
$$(1 - \mu_S)\left(\frac{s^*_H - s^*_L}{2}\right) + (1 - \mu_S)\psi\left[\max\{p(s^*_H, s^*_L) - r, 0\} - \max\{s^*_L - r, 0\}\right]$$

Thus, HB IC holds iff the negation of (A.3) holds. As in the no trade equilibria, LB IC is satisfied iff

$$v_{LB} - \min\{s_L^*, s_H^*\} - \psi \max\{\min\{s_L^*, s_H^*\} - r, 0\} \leq 0$$

Similarly, HS IC is satisfied iff

$$\max\{b_L^*, b_H^*\} - v_{HS} - \psi \max\{r - \max\{b_L^*, b_H^*\}, 0\} \leq 0$$

Consider LS IC. By LS IR, bidding $s > b_H^*$ is not a profitable deviation. Bidding $s = b_L^*$ and trading with LB is a profitable deviation iff

$$\begin{aligned} &(1-\mu_B)\left(b_L^* - v_{LS} - \psi \max\{r - b_L^*, 0\}\right) > \\ &\mu_B \left(\frac{b_H^* - b_L^*}{2}\right) + \mu_B \psi \left[\max\{r - p(b_H^*, b_L^*), 0\} - \max\{r - b_H^*, 0\}\right] \end{aligned} \tag{A.4}$$

Thus, LS IC holds iff the negation of (A.4) holds. Notice that the IR and IC constraints are satisfied if

(1) $\max\left\{v_{LB}, \frac{v_{LS}+\psi r}{1+\psi}\right\} \leq b_H^* \leq \min\left\{\frac{v_{HB}+\psi r}{1+\psi}, v_{HS}\right\}$

(2) $b_L^* \leq \max\left\{v_{LS}, \frac{v_{LS}+\psi r}{1+\psi}\right\}$

(3) $s_H^* \geq \min\left\{v_{HB}, \frac{v_{HB}+\psi r}{1+\psi}\right\}$

(4) $\max\left\{v_{LS}, \frac{v_{LS}+\psi r}{1+\psi}\right\} \leq s_L^* \leq \min\left\{v_{HB}, \frac{v_{HB}+\psi r}{1+\psi}\right\}$

(5) $b_H^* = s_L^*$

When no reference price is announced (which is equivalent to $\psi = 0$), this equilibrium outcome exists for all pairs of prior probabilities.

Consider equilibria in which $(t(b_H^*, s_H^*), t(b_H^*, s_L^*), t(b_L^*, s_H^*), t(b_L^*, s_L^*)) = (1, 1, 0, 0)$. By Lemma 3, $b_L^* < b_H^* = s_L^* = s_H^*$. LB IR is satisfied since $t(b_L^*, s_H^*) = t(b_L^*, s_L^*) = 0$. HB IR is satisfied iff

$$v_{HB} - b_H^* - \psi \max\{b_H^* - r, 0\} \geq 0$$

HS IR is satisfied iff

$$s_H^* - v_{HS} - \psi \max\{r - s_H^*, 0\} \geq 0$$

LS IR is satisfied iff

$$s_L^* - v_{LS} - \psi \max\{r - s_L^*, 0\} \geq 0$$

Consider HB IC. By HB IR, bidding $b < b_H^*$ is not a profitable deviation, so HB IC always holds. As in the no trade equilibria, LB IC is satisfied iff

$$v_{LB} - \min\{s_L^*, s_H^*\} - \psi \max\left\{\min\{s_L^*, s_H^*\} - r, 0\right\} \leq 0$$

Consider HS IC. By HS IR, bidding $s > b_H^*$ is not a profitable deviation. Bidding $s = b_L^*$ and trading with LB is a profitable deviation iff

$$\begin{aligned}&(1-\mu_B)\left(b_L^* - v_{HS} - \psi \max\left\{r - b_L^*, 0\right\}\right) > \\ &\mu_B \left(\frac{b_H^* - b_L^*}{2}\right) + \mu_B \psi \left[\max\{r - p(b_H^*, b_L^*), 0\} - \max\{r - b_H^*, 0\}\right]\end{aligned} \tag{A.5}$$

Thus, HS IC holds iff the negation of (A.5) holds. Similarly, LS IC holds iff the negation of (A.4) holds. Notice that the IR and IC constraints are satisfied if

(1) $\max\left\{v_{HS}, \frac{v_{HS}+\psi r}{1+\psi}\right\} \leq b_H^* \leq \min\left\{v_{HB}, \frac{v_{HB}+\psi r}{1+\psi}\right\}$

(2) $b_L^* \leq \max\left\{v_{LS}, \frac{v_{LS}+\psi r}{1+\psi}\right\}$

(3) $\max\left\{v_{HS}, \frac{v_{HS}+\psi r}{1+\psi}\right\} \leq s_H^* \leq \min\left\{v_{HB}, \frac{v_{HB}+\psi r}{1+\psi}\right\}$

(4) $\max\left\{v_{HS}, \frac{v_{HS}+\psi r}{1+\psi}\right\} \leq s_L^* \leq \min\left\{v_{HB}, \frac{v_{HB}+\psi r}{1+\psi}\right\}$

(5) $b_H^* = s_L^* = s_H^*$

When no reference price is announced (which is equivalent to $\psi = 0$), this equilibrium outcome exists for all pairs of prior probabilities.

Consider equilibria in which $(t(b_H^*, s_H^*), t(b_H^*, s_L^*), t(b_L^*, s_H^*), t(b_L^*, s_L^*)) = (0, 1, 0, 1)$. By Lemma 3, $b_L^* = b_H^* = s_L^* < s_H^*$. HS IR is satisfied since $t(b_H^*, s_H^*) = t(b_L^*, s_H^*) = 0$. HB IR is satisfied iff

$$v_{HB} - b_H^* - \psi \max\{b_H^* - r, 0\} \geq 0$$

LB IR is satisfied iff

$$v_{LB} - b_L^* - \psi \max\{b_L^* - r, 0\} \geq 0$$

LS IR is satisfied iff

$$s_L^* - v_{LS} - \psi \max\{r - s_L^*, 0\} \geq 0$$

As in the equilibria where only HB-LS trade, HB IC holds iff the negation of (A.3) holds. Since $v_{LB} < v_{HB}$, if HB IC holds, then LB IC holds. As in the no trade equilibria, HS IC is satisfied iff

$$\max\{b_L^*, b_H^*\} - v_{HS} - \psi \max\left\{r - \max\{b_L^*, b_H^*\}, 0\right\} \leq 0$$

Consider LS IC. By LS IR, bidding $s > b_L^* = b_H^*$ is not a profitable deviation, so LS IC always holds. Notice that the IR and IC constraints are satisfied if

(1) $\max\left\{v_{LS}, \frac{v_{LS}+\psi r}{1+\psi}\right\} \leq b_H^* \leq \min\left\{v_{LB}, \frac{v_{LB}+\psi r}{1+\psi}\right\}$

(2) $\max\left\{v_{LS}, \frac{v_{LS}+\psi r}{1+\psi}\right\} \leq b_L^* \leq \min\left\{v_{LB}, \frac{v_{LB}+\psi r}{1+\psi}\right\}$

(3) $s_H^* \geq \min\left\{v_{HB}, \frac{v_{HB}+\psi r}{1+\psi}\right\}$

(4) $\max\left\{v_{LS}, \frac{v_{LS}+\psi r}{1+\psi}\right\} \leq s_L^* \leq \min\left\{v_{LB}, \frac{v_{LB}+\psi r}{1+\psi}\right\}$

(5) $b_L^* = b_H^* = s_L^*$

When no reference price is announced (which is equivalent to $\psi = 0$), this equilibrium outcome exists for all pairs of prior probabilities.

Consider equilibria in which $(t(b_H^*, s_H^*), t(b_H^*, s_L^*), t(b_L^*, s_H^*), t(b_L^*, s_L^*)) = (1, 1, 0, 1)$. By Lemma 3, $b_L^* = s_L^* < b_H^* = s_H^*$. HB IR is satisfied iff

$$v_{HB} - b_H^* - \psi \max\{b_H^* - r, 0\} \geq 0$$

LB IR is satisfied iff

$$v_{LB} - b_L^* - \psi \max\{b_L^* - r, 0\} \geq 0$$

HS IR is satisfied iff

$$s_H^* - v_{HS} - \psi \max\{r - s_H^*, 0\} \geq 0$$

LS IR is satisfied iff

$$s_L^* - v_{LS} - \psi \max\{r - s_L^*, 0\} \geq 0$$

Consider HB IC. By HB IR, bidding $b < s_L^*$ is not a profitable deviation. Bidding $b = s_L^*$ and trading only with LS is a profitable deviation iff

$$\begin{aligned} &\mu_S \left(v_{HB} - s_H^* - \psi \max\left\{s_H^* - r, 0\right\}\right) < \\ &(1-\mu_S)\left(\frac{s_H^* - s_L^*}{2}\right) + (1-\mu_S)\psi\left[\max\{p(s_H^*, s_L^*) - r, 0\} - \max\{s_L^* - r, 0\}\right] \end{aligned} \tag{A.6}$$

Thus, HB IC holds iff the negation of (A.6) holds. Consider LB IC. By LB IR, bidding $b < s_L^*$ is not a profitable deviation. By Lemma 2, bidding $b = s_H^*$ is not a profitable deviation. Thus, LB IC always holds. Consider HS IC. By HS IR bidding $s > b_H^*$ is not a profitable deviation. By Lemma 2, bidding $s = b_L^*$ is not a profitable deviation. Thus, HS IC always holds. Consider LS IC. By LS IR, bidding $s > b_H^*$ is not a profitable deviation. Bidding $s = b_H^*$ and trading only with HB is a profitable deviation iff

$$\begin{aligned} &(1-\mu_B)\left(b_L^* - v_{LS} - \psi \max\left\{r - b_L^*, 0\right\}\right) < \\ &\mu_B \left(\frac{b_H^* - b_L^*}{2}\right) + \mu_B\psi\left[\max\{r - p(b_H^*, b_L^*), 0\} - \max\{r - b_H^*, 0\}\right] \end{aligned} \tag{A.7}$$

Thus, LS IC holds iff the negation of (A.7) holds. Notice that the IR constraints are satisfied if

(1) $\max\left\{v_{HS}, \frac{v_{HS}+\psi r}{1+\psi}\right\} \leq b_H^* \leq \min\left\{v_{HB}, \frac{v_{HB}+\psi r}{1+\psi}\right\}$

(2) $\max\left\{v_{LS}, \frac{v_{LS}+\psi r}{1+\psi}\right\} \leq b_L^* \leq \min\left\{v_{LB}, \frac{v_{LB}+\psi r}{1+\psi}\right\}$

(3) $\max\left\{v_{HS}, \frac{v_{HS}+\psi r}{1+\psi}\right\} \leq s_H^* \leq \min\left\{v_{HB}, \frac{v_{HB}+\psi r}{1+\psi}\right\}$

(4) $\max\left\{v_{LS}, \frac{v_{LS}+\psi r}{1+\psi}\right\} \leq s_L^* \leq \min\left\{v_{LB}, \frac{v_{LB}+\psi r}{1+\psi}\right\}$

(5) $b_H^* = s_H^*$

(6) $b_L^* = s_L^*$

However, it is always individually rational for HB to mimic LB's bid and for LS to mimic HS's bid. Thus, the IC constraints for HB and LS are sufficient conditions for this equilibrium outcome. When no reference price is announced (which is equivalent to $\psi = 0$) these sufficient conditions are

$$\mu_S(v_{HB} - s_H^*) \geq (1 - \mu_S)\left(\frac{s_H^* - s_L^*}{2}\right) \tag{A.8}$$

and

$$\mu_B\left(\frac{b_H^* - b_L^*}{2}\right) \leq (1 - \mu_B)(b_L^* - v_{LS}) \tag{A.9}$$

(A.8) is violated for all $s_H^* \in [v_{HS}, v_{HB}]$ and $s_L^* \in [v_{LS}, v_{LB}]$ iff it is violated for the lower bound of s_H^* and the upper bound of s_L^*. This is true iff

$$\mu_S(v_{HB} - v_{HS}) < (1 - \mu_S)\left(\frac{v_{HS} - v_{LB}}{2}\right)$$
$$\iff \mu_S < \frac{v_{HS} - v_{LB}}{2v_{HB} - v_{HS} - v_{LB}} \equiv \underline{\mu}_S$$

(A.9) is violated for all $b_H^* \in [v_{HS}, v_{HB}]$ and $b_L^* \in [v_{LS}, v_{LB}]$ iff it is violated for the lower bound of b_H^* and the upper bound of b_L^*. This is true iff

$$\mu_B\left(\frac{v_{HS} - v_{LB}}{2}\right) \leq (1 - \mu_B)(v_{LB} - v_{LS})$$
$$\iff \mu_B > \frac{2(v_{LB} - v_{LS})}{v_{HS} + v_{LB} - 2v_{LS}} \equiv \bar{\mu}_B$$

Therefore, when no reference price is announced, this equilibrium outcome exists iff $\mu_S \geq \underline{\mu}_S$ and $\mu_B \leq \bar{\mu}_B$ ■

Proposition 2. *The equilibrium outcome in which both sellers trade only with the high valuation buyer is second-best in terms of ex ante expected gains from trade if and only if*

$$\mu_S \geq \frac{(1 - \mu_B)(v_{LB} - v_{LS})}{\mu_B(v_{HB} - v_{HS}) + (1 - \mu_B)(v_{LB} - v_{LS})}$$

The equilibrium outcome in which both buyers trade only with the low valuation seller is second-best if and only if the direction of the above *inequality is reversed.*

Proof. In the no trade equilibrium outcome, ex ante expected gains from trade are given by

$$\Pi_1^* = 0$$

In the equilibrium outcome in which HB trades only with LS, the ex ante expected gains from trade are given by

$$\Pi_2^* = \mu_B(1-\mu_S)(v_{HB}-v_{LS})]$$

In the equilibrium outcome in which both sellers trade only with HB, the ex ante expected gains from trade are given by

$$\Pi_3^* = \mu_B\left[\mu_S(v_{HB}-v_{HS}) + (1-\mu_S)(v_{HB}-v_{LS})\right]$$

In the equilibrium outcome in which both buyers trade only with LS, the ex ante expected gains from trade are given by

$$\Pi_4^* = (1-\mu_S)\left[\mu_B(v_{HB}-v_{LS}) + (1-\mu_B)(v_{LB}-v_{LS})\right]$$

In the efficient equilibrium outcome, the ex ante expected gains from trade are given by

$$\begin{aligned}\Pi_5^* &= \mu_B\left[\mu_S(v_{HB}-v_{HS}) + (1-\mu_S)(v_{HB}-v_{LS})\right]\\ &+ (1-\mu_B)\left[(1-\mu_S)(v_{LB}-v_{LS})\right]\end{aligned}$$

We have $\Pi_1^* < \Pi_2^* < \Pi_3^*, \Pi_4^* < \Pi_5^*$. We have $\Pi_3^* \geq \Pi_4^*$ iff

$$\begin{aligned}\mu_B &\geq \frac{(1-\mu_S)(v_{LB}-v_{LS})}{\mu_S(v_{HB}-v_{HS}) + (1-\mu_S)(v_{LB}-v_{LS})}\\ \iff \mu_S &\geq \frac{(1-\mu_B)(v_{LB}-v_{LS})}{\mu_B(v_{HB}-v_{HS}) + (1-\mu_B)(v_{LB}-v_{LS})}\end{aligned}$$

and $\Pi_3^* \leq \Pi_4^*$ iff the direction of this inequality is reversed. ■

Proposition 3. *There exists a reference price such that the equilibrium outcome in which both sellers trade only with the high valuation buyer can be sustained as the unique equilibrium outcome in which trade occurs if and* *only if*

$$\psi > \frac{v_{HS} - v_{LS}}{v_{HB} - v_{HS}} \equiv \psi_1$$

Similarly, there exists a reference price such that the equilibrium outcome in which both buyers trade only with the low valuation seller can be sustained as the unique equilibrium outcome in which trade occurs if and only if

$$\psi > \frac{v_{HB} - v_{LB}}{v_{LB} - v_{LS}} \equiv \psi_2$$

Proof. To preclude equilibria in which only HB-LS trade and in which both buyers trade only with LS, trade at any price $\leq v_{HS}$ must not be IR for LS. Since the seller's payoff is increasing in price, this is true iff LS's payoff is negative at the price of v_{HS}. This is true iff

$$v_{HS} - v_{LS} - \psi \max\{r - v_{HS}, 0\} < 0$$

Since $v_{LS} < v_{HS} \iff v_{HS} - v_{LS} > 0$, it must be that $r - v_{HS} > 0$. Thus

$$v_{HS} - v_{LS} - \psi(r - v_{HS}) < 0$$
$$\iff r > \frac{(1+\psi)v_{HS} - v_{LS}}{\psi}$$

We must have $r \leq v_{HB}$. For both of these inequalities to hold we must have

$$\frac{(1+\psi)v_{HS} - v_{LS}}{\psi} < v_{HB}$$
$$\iff \psi > \frac{v_{HS} - v_{LS}}{v_{HB} - v_{HS}} \equiv \psi_1$$

If this is true, $\exists r \in (v_{HS}, v_{HB}]$ such that LS's payoff is negative at the price of v_{HS}. Trade at a price equal to the reference price is IR for LS, HS, and HB since $r \leq v_{HB}$ and $r > v_{HS}$. Thus, at least one equilibrium exists in which LS, HS, and HB trade.

To preclude equilibria in which only HB-LS trade and in which both sellers trade only with HB, trade at any price $\geq v_{LB}$ must not be IR for HB. Since the buyer's payoff is decreasing in price, this is true iff HB's payoff is negative at the price of v_{LB}. This is true iff

$$v_{HB} - v_{LB} - \psi \max\{v_{LB} - r, 0\} < 0$$

Since $v_{LB} < v_{HB} \iff v_{HB} - v_{LB} > 0$, it must be that $v_{LB} - r > 0$. Thus

$$v_{HB} - v_{LB} - \psi(v_{LB} - r) < 0$$
$$\iff r < \frac{(1+\psi)v_{LB} - v_{HB}}{\psi}$$

We must have $r \geq v_{LS}$. For both of these inequalities to hold we must have

$$v_{LS} < \frac{(1+\psi)v_{LB} - v_{HB}}{\psi}$$
$$\iff \psi > \frac{v_{HB} - v_{LB}}{v_{LB} - v_{LS}} \equiv \psi_2$$

If this is true, $\exists r \in [v_{LS}, v_{LB})$ such that HB's payoff is negative at the price of v_{LB}. Trade at a price equal to the reference price is IR for LB, HB, and LS since $r < v_{LB}$ and $r \geq v_{LS}$. Thus, at least one equilibrium exists in which LB, HB, and LS trade. ■