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Asset Pricing When Trading is Entertainment

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We analyze a model where agents derive direct utility from the act of trading. The resulting equilibrium shows that trading for entertainment creates an excessive tendency to buy low and sell high, and causes attenuation of covariance risk pricing and return volatility. The model also implies a negative relation between volume and future returns, consistent with the empirical evidence. Further, if agents derive greater utility from trading more volatile stocks, our model is consistent with the "volatility anomaly" wherein more volatile stocks earn lower average returns in the cross-section, although the risk premium on the market portfolio is positive. Agents who derive greater utility from trading trade more aggressively on private information and raise the informational efficiency of prices. If agents' utility from trading increases when they make positive profits in earlier rounds, this leads to "bubbles," i.e., disproportionate jumps in asset returns as a function of past prices.

1 Introduction

"The game of investing is intolerably boring and over-exacting to any one who is entirely exempt from the gambling instinct; whilst he who has it must pay to this propensity the appropriate toll."

–Keynes (1936)

Why do agents trade? In the neoclassical paradigm, trade occurs to rebalance portfolios according to the risk-return tradeoff, either upon the change of market values of securities, or due to a change in preferences, or due to the receipt of new information.[1] However, volume in financial markets appears to be too large to be explained by such considerations alone. Milgrom and Stokey (1982) imply that there should be no trade among investors with only speculative motives for trading. Barber and Odean (2000), however, report extremely high levels of trading; specifically, they document an average annual turnover of 75% by customers with accounts at a large discount brokerage firm. According to the NYSE website,[2] annual turnover on the NYSE has ranged between 60% and 100% of shares outstanding over the past ten years. De Bondt and Thaler (1995) (p. 392) note that "the high trading volume observed in financial markets is perhaps the single most embarrassing fact to the standard finance paradigm." Indeed, Tkac (1999) shows that real-world volume exceeds that indicated by rational portfolio-rebalancing for a vast majority of traded stocks. Motivated by the generally high levels of volume in financial markets, Black (1986) (p. 531) mentions the need to "introduce direct utility of trading" to explain volume.

[1] See Grossman and Stiglitz (1980), Kyle (1985) Grundy and McNichols (1989), Foster and Viswanathan (1993), and Wang (1994). Trading is induced by differences of opinion in Harrison and Kreps (1978), Varian (1985), Harris and Raviv (1993), and Kandel and Pearson (1995).

[2] See the NYSE fact Books at http://www.nyxdata.com/nysedata/asp/factbook/main.asp.

The goal of our paper is to consider a financial market equilibrium where agents derive utility from the act of trading. In much of our work, information is symmetric and there are multiple assets, each traded by agents who possess the standard exponential-normal utility function over wealth but some of whom get additional utility from trading. We do not model the origins of this utility; it could possibly emanate from the thrill of seeing position values fluctuate. The notion that agents may gamble for pleasure is well-established in literature (see, for example, Coventry and Brown (1993) or Kuley and Jacobs (1988)), and Markiewicz and Weber (2013) show that gambling tendencies also create excessive stock trading. Motivated by these observations, we simply consider the nature of equilibrium in financial markets when trading is a consumption good.[3] To attain tractability, we assume that the direct utility is convex (quadratic) in the amount of the traded asset.

We find that when agents derive utility from trading, they have an excessive tendency to "buy low and sell high." The intuition is that the normal levels of trade to capture risk premia are magnified owing to the additional utility derived from trading. Interestingly, such agents result in *lower* asset volatility. This is because they act as *de facto* liquidity providers in equilibrium.[4] We also find that beta pricing is obscured when agents derive direct utility from trading. Specifically, expected returns are linear in beta, but the coefficient on beta attenuates as the direct utility from trading increases. This result accords with the notion that it is generally hard to find evidence of covariance risk pricing in equity markets (see, for example, Fama and French (1992) and Haugen and Baker (1996)). The intuition in our setting is that the compensation for risk demanded in

[3] An issue is whether the mass of traders who trade for entertainment is sufficiently large to affect prices. While this is an empirical questions, Barber, Odean, and Zhu (2009) show that retail traders, (who are more likely to trade for enjoyment than professionals) do have an impact on financial market prices.

[4] Barrot, Kaniel, and Sraer (2016) and Kaniel, Saar, and Titman (2008) demonstrate that individual investors do act as liquidity providers in financial markets.

equilibrium declines as the direct utility from trading rises.

It is reasonable to suppose that agents may derive greater utility from some stocks relative to others. The two main characteristics proposed by Kumar (2009) for stocks that are attractive to individual investors are high (positive) skewness, and high volatility. Since our model has normally distributed payoffs it unfortunately cannot speak to skewness preference. We instead consider the assumption that agents obtain more utility from stocks with more volatile payoffs. We find evidence that under reasonable conditions, such stocks get "overvalued" and earn negative future returns on a risk-adjusted basis. Since high payoff volatility, under reasonable conditions, also corresponds to high idiosyncratic volatility (as we show), our analysis is consistent with the empirical result of Ang, Hodrick, Xing, and Zhang (2006) that idiosyncratic volatility negatively forecasts asset returns.

An interesting set of stylized facts in finance is that while volatility is negatively priced in the cross-section, on aggregate, the risk premium on equities is positive (Haugen and Baker (2010), Mehra and Prescott (1985)). We show that our analysis accords with the "low volatility" anomaly wherein low risk stocks earn higher average returns than high risk stocks (as shown empirically in Baker and Haugen (2012)). At the same time, however, the market portfolio commands a positive risk premium because our agents are risk averse. Thus, our analysis is simultaneously consistent with positive risk pricing in the aggregate, but negative pricing of volatility in the cross-section.

When agents derive direct utility from trading, they create additional volume, which accords with levels of volume greater than normally expected levels from neoclassical models (Tkac (1999)).[5] Further, when trading volume is high in a stock, it is associated

[5]In a complementary and important view Odean (1998) and Statman, Thorley, and Vorkink (2006) consider the notion that overconfidence creates excessive trading volume. A distinguishing feature of our setting and that of these papers is that overconfidence increases price volatility whereas in our setting

with agents desiring greater utility from trading that stock. However, stocks from which agents derive greater utility from trading also tend to become more overpriced. Thus, our model is consistent with the negative cross-sectional relation between volume and returns documented in Datar, Naik, and Radcliffe (1998) or Brennan, Chordia, and Subrahmanyam (1998).

In an extension of our basic setting, we consider how price informativeness is affected by traders who derive direct utility from trading. We find that such traders, when they can acquire private information, trade more aggressively on their information when their direct utility from trading is high, thus raising pricing efficiency in equilibrium.

Previous literature argues that the pleasure from gambling rises when the outcome of a previous gamble is positive (e.g., Coventry and Constable (1999); Thaler and Johnson (1990)). Accordingly, we extend our model to a dynamic setting where agents derive more utility from trading if previous rounds of trade have been profitable. We find that the impact of a modestly positive piece of news can be greatly magnified. Specifically, as the news crosses a threshold, it causes a discontinuous jump in the mass of agents who trade for entertainment purposes, which results, in turn, in a greatly enhanced response of asset prices to the news. This suggests that relatively modest favorable price moves can create "bubbles" in asset prices, wherein later price moves represent substantial overreactions to the initial price move, followed by subsequent corrections. Thus, in our setting, it is the "emotional excitement" caused by the initial price move which results in the bubble in asset prices, which accords with the experimental observation that excitement fuels bubbles (Andrade, Odean, and Lin (2015)).

Our analysis suggests untested implications. Specifically, we argue that stocks that are heavily traded and held by retail investors (who are more likely to trade for entertainment)

agents who derive utility from trading act as de facto market makers and thus reduce volatility.

should exhibit greater trading volume and lower volatility, and less evidence of covariance risk pricing. These stocks should also exhibit nonlinear responses to positive news. Our analysis also suggests that covariance risk pricing should be less visible in countries or economies where retail investors form a bigger fraction of the trading population.

The idea that agents may trade for purposes of deriving enjoyment from trading is not new; but explicit theoretical modeling of this notion does not yet appear in the literature. For example, Dorn and Sengmueller (2009) argue that stock market trading provides direct utility to agents in the form of entertainment. Specifically, they show that agents who state that they derive "enjoyment" from trading turn over their portfolios to a greater degree than other investors. In a survey of retail investors, Dhar and Goetzmann (2006) state that more than 25% of investors view stock market investing as a hobby. Grinblatt and Keloharju (2009) argue that sensation seeking personalities may obtain a thrill from the act of trading but again, do not explicitly model such investors. Gao and Lin (2015) show that equity trading volume in Taiwan decreases as the total jackpot of a major statewide lottery increases, indicating that stock trading acts as an alternative outlet for gambling beyond lotteries. Barberis and Xiong (2012) model the important insight that realizing gains conveys pleasure to agents, but they do not consider a setting where agents derive utility from trading.[6] Our paper fills this void in the literature by explicitly considering agents for whom trading is a consumption good.

This paper is organized as follows. Section 2 presents the model. Section 4 examines

[6] Our paper is complementary to Friedman and Heinle (2016) and Luo and Subrahmanyam (2016), which consider a setting where agents derive direct utility or disutility from owning certain types of stocks. The utility there emanates from the *signed* position; for example, an environmentally conscious agent derives disutility from owning stocks (and utility from shorting stocks) in firms that heavily use coal. In contrast, direct utility in our model emanates from the unsigned quantity of trade. Further, Doran, Jiang, and Peterson (2012), Shefrin and Statman (2000), and Brunnermeier, Gollier, and Parker (2007) consider the notion that agents may express an affinity towards gambling by preferring high skewness assets. We instead consider a complementary setting where agents have standard risk averse preferences but derive additional utility from the act of trading an asset (which is normally distributed).

how agents who derive direct utility from trading affect price informativeness. Section 5 presents a dynamic extension, and Section 6 concludes. All proofs of propositions and corollaries, unless otherwise stated, appear in Appendix A, while Appendix B presents some ancillary derivations.

2 The Model

There are two dates, 0 and 1, and $K + N$ risky securities. At date 1, these securities pay liquidating dividends of $V = (V_1, ..., V_{K+N})'$, which follows a multivariate normal distribution. At date 0, investors trade these securities. The per capita supplies of these securities also follow a multivariate normal distribution. The prices of these securities are indicated by $P = (P_1, ..., P_{K+N})'$ and are determined in equilibrium. There is also a riskless asset, the price and return of which are normalized to unity.

There are two types of agents. First, there is a mass ρ of regular utility-maximizing agents. Second, there is a mass $1-\rho$ of agents we call "G traders;" these achieve direct utility from trading risky securities. One can view the former class of agents as sophisticated investors (possibly institutions), and at least part of the latter class as unsophisticated (e.g, individual) investors.

The i'th trader is endowed with $\bar{X}_i = (\bar{X}_{i1}, ..., \bar{X}_{i,K+N})'$ units of risky securities and $\bar{M}_i$ units of risk free asset. His wealth levels at t dates 0 and 1 are given by

$$\begin{aligned} W_{i0} &= \bar{M}_i + \bar{X}_i' P, \\ W_{i1} &= W_{i0} + X_i'(V - P), \end{aligned}$$

where $X_i = (X_{i1}, ..., X_{i,K+N})'$ is the quantity of risky securities he holds after the trading is complete.

The utility function of the i'th regular (non-G) trader is the standard exponential one:

$$U(W_{i1}) = -\exp(-\gamma W_{i1}),$$

with $\gamma > 0$. Based on the normality assumption of our model, he chooses X_i to maximize

$$E\Big[U(W_{i1})\Big] = -\exp\Big[-\gamma W_{i0} - \gamma\Big[X_i' E(V-P) - 0.5\gamma X_i' \mathrm{Var}(V) X_i\Big]\Big].$$

The first order condition (f.o.c.) with respect to (w.r.t.) X_i implies that his demand can be expressed as:

$$X_{NG}(P) = \frac{1}{\gamma}\mathrm{Var}(V)^{-1}(E(V) - P).$$

However, the i'th G trader has the utility function:

$$U_G(W_{i1}, X_i) = -\exp(-\gamma W_{i1})\exp(-0.5 X_i' G X_i),$$

where G is a diagonal, positive definite matrix. The above utility function captures the notion that the bigger the quantity of trade (in absolute terms), the bigger is the utility derived from trading. It may be argued that increasing the scale of the transaction increases the "thrill" derived from this larger scale of "gambling" in financial markets (Dorn and Sengmueller (2009)). The G trader chooses X_i to maximize

$$E\Big[U_G(W_{i1}, X_i)\Big] = -\exp\Big[-\gamma W_{i0} - \gamma\Big[X_i' E(V-P) - 0.5\gamma X_i' \mathrm{Var}(V) X_i + 0.5 X_i' G X_i/\gamma\Big]\Big].$$

The f.o.c. w.r.t. X_i implies that his demand can be expressed as:

$$X_G(P) = \frac{1}{\gamma}(\mathrm{Var}(V) - G/\gamma^2)^{-1}(E(V) - P).$$

The s.o.c. holds under the assumption that $\mathrm{Var}(V) - G/\gamma^2$ is a positive definite matrix.

As can be seen, the "numerator" in the above demand is the same as for non-G traders. However the position is larger per unit gain relative to non-G traders, i.e., the G traders

take more aggressive positions relative to traditional utility maximizers. As the utility of trading increases, the position vector explodes, and beyond a certain level of G, there is no interior optimum. The scale of the position taken per unit expected price appreciation increases in G, which governs how much additional utility is derived from trading.

2.1 Risky Security Payoffs–The Factor Structure

We now explicitly model security payoffs as a factor structure to analyze how volume, volatility, and the pricing of covariance risk are affected by the presence of G traders. In what follows, unless otherwise specified, a generic random variable, $\tilde{\eta}$, follows a normal distribution with mean zero and variance ν_η.

The payoff of the j'th risky security takes a factor expression:

$$V_j = \bar{V}_j + \sum_{k=1}^{K} (\beta_{jk} \tilde{f}_k) + \tilde{\epsilon}_j \tag{1}$$

All $\tilde{f}$'s and $\tilde{\epsilon}$'s follow independent normal distributions.

2.2 An Equivalent Maximization Problem

As in Daniel, Hirshleifer, and Subrahmanyam (2001), we use the risky securities to construct portfolios mimicking the K factors and N residuals. We refer to these portfolios as the basic securities. Use $\tilde{\theta}_j$, $j = 1, ..., K+N$, to denote the payoffs of the basic securities. Specifically, the first K $\tilde{\theta}$'s indicate the payoffs of the K factors, that is, $\tilde{\theta}_j = \tilde{f}_j$ for $j = 1, ..., K$. The next N $\tilde{\theta}$'s indicate the payoffs of the N residuals, that is, $\tilde{\theta}_{K+j} = \tilde{\epsilon}_j$ for $j = 1, ..., N$.

The per capita supply of the j'th basic security is indicated by $\bar{\xi}_j + \tilde{z}_j$, where $\bar{\xi}_j$ is a constant. Henceforth, we will assume that the mean supply of securities is positive (i.e. $\bar{\xi}_j > 0 \, \forall j$). The supply noise is not necessary for most of our main results (except those on

price volatility). To simplify our analysis, we assume that the variance ν_{z_j} is sufficiently small. Specifically, we assume that

$$\nu_{z_j} < \frac{1}{\gamma^2 \nu_{\theta_j}} \min(1/4, \rho/2). \tag{2}$$

This condition facilitates the derivation of the results because it ensures that the G traders' penchant for aggressively "buying low and selling high," which drives many of our results, is not too adversely affected by excessive supply noise. The assumption is reasonable because we would not expect uncertainty in stock issuance and buyback activity to be unduly large in general.

We denote the utility from trading the j'th basic security as G_j. This implies that the G traders' utility from trading is not identical across stocks, which captures the notion that there are many stock-specific properties that may appeal to a G trader; for example, brand appeal, volatility, industry sector, and so on. Rather than model these attributes, we simply let the direct utility from trading vary in the cross-section. We do, however, consider a scenario in Section 2.6 where G_j depends positively on the volatility of the stock.

Note that $\tilde{\theta}_j \sim N(0, \nu_{\theta_j})$. Each non-$G$ and G trader's demands for the j'th basic security are given by

$$X_{NG,j}(P_j) = \frac{E(\tilde{\theta}_j) - P_j}{\gamma \mathrm{Var}(\tilde{\theta}_j)} = \frac{-P_j}{\gamma \nu_{\theta_j}}, \tag{3}$$

$$X_{G,j}(P_j) = \frac{E(\tilde{\theta}_j) - P_j}{\gamma(\mathrm{Var}(\tilde{\theta}_j) - G_j/\gamma^2)} = \frac{-P_j}{\gamma(\nu_{\theta_j} - G_j/\gamma^2)}. \tag{4}$$

The denominator in Eq. (4) needs to be positive for the second-order condition of the G traders to be satisfied. The intuition is that the effect of G_j alone is to induce G-traders to want to trade infinite quantities. This tendency, however, is tempered by risk aversion and the volatility of the asset's final payoff. Henceforth, we will assume that the condition for an interior optimum is indeed satisfied, i.e., that $G_j < \gamma^2 \nu_{\theta_j}$.

The market clearing condition requires

$$\bar{\xi}_j + \tilde{z}_j \;=\; \rho X_{NG,j}(P_j) + (1-\rho) X_{G,j}(P_j),$$

from which we derive the prices and returns (i.e., price changes) as presented in the following proposition.

Proposition 1 *The price and the return of the j'th basic security are given by*

$$\begin{aligned} P_j &= -\gamma a_j(G_j)(\bar{\xi}_j + \tilde{z}_j), \\ \tilde{R}_j &= \tilde{\theta}_j - P_j = \tilde{\theta}_j + \gamma a_j(G_j)(\bar{\xi}_j + \tilde{z}_j), \end{aligned}$$

where $a_j(G_j) = \dfrac{1}{\dfrac{\rho}{\nu_{\theta_j}} + \dfrac{1-\rho}{\nu_{\theta_j} - G_j/\gamma^2}}$.

The term $a_j(G_j)$ represents the effect of G traders on the price and required return. By being willing to trade more stock for a given market price, the G traders assist the non-G traders in absorbing the supply of the risky asset.

From the above proposition, we can derive the corollary below:

Corollary 1 *(i) $a_j(G_j)$ decreases in G_j, with $a_j(0) = \nu_{\theta_j}$.*

(ii) $a_j(G_j)$ increases in ρ.

Thus, the price of the j'th basic security is a risk premium, and this premium decreases in G_j, the direct utility of trading the j'th basic security. As the utility from trading grows without bound, the risk premium on the security decreases. Hence, agents who derive greater utility from trading reduce risk premia and required returns on the security.

It is instructive to calculate the expected profits of the G traders. From Eqs. (3) and (4) and Proposition 1, the expected profit earned by a non-G or G trader from the j'th

basic security is given by

$$\begin{aligned} E\Pi_{NG,j} &= E\left[X_{NG,j}(P_j)\tilde{R}_j\right] = E\left[\frac{\gamma a_j(G_j)(\bar{\xi}_j+\tilde{z}_j)}{\gamma\nu_{\theta_j}}(\tilde{\theta}_j+\gamma a_j(G_j)(\bar{\xi}_j+\tilde{z}_j))\right] \\ &= \frac{\gamma a_j(G_j)^2}{\nu_{\theta_j}}(\bar{\xi}_j^2+\nu_{z_j}), \\ E\Pi_{G,j} &= E\left[X_{G,j}(P_j)\tilde{R}_j\right] = E\left[\frac{\gamma a_j(G_j)(\bar{\xi}_j+\tilde{z}_j)}{\gamma(\nu_{\theta_j}-G_j/\gamma^2)}(\tilde{\theta}_j+\gamma a_j(G_j)(\bar{\xi}_j+\tilde{z}_j))\right] \\ &= \frac{\gamma a_j(G_j)^2}{\nu_{\theta_j}-G_j/\gamma^2}(\bar{\xi}_j^2+\nu_{z_j}). \end{aligned} \tag{5}$$

Note that $E\Pi_{G,j} > E\Pi_{NG,j}$. Thus, G traders earn a *greater* expected profit from trading the basic security than do non-G traders. This simply emanates from the notion that, in effect, they trade more aggressively to capture the risk premium.

Of course, it follows that G traders bear more risk than non-G traders; that is, it is easy to show that the total volatility of the positions taken by G traders is higher than that of non-G traders. Our result is similar to that of Kyle and Wang (1997) who show that overconfidence acts as a commitment to trade aggressively in a strategic environment and thus results in greater expected profit for overconfident agents relative to that for rational agents. In our setting, agents who obtain direct utility from trading obtain a greater expected profit than neoclassical agents because the increased position size of the former class of agents increases their average gain, relative to that for the latter class. Like De Long, Shleifer, Summers, and Waldmann (1991), who obtain a similar result on the expected profits of overconfident vs. rational traders, our result also challenges the notion that G traders would not exist because they would persistently lose money in financial markets.

2.3 Volatility

It follows from Proposition 1 that the price and return volatilities of the j'th basic security are

$$\begin{aligned} \text{Var}(P_j) &= (\gamma a_j(G_j))^2 \nu_{z_j}, \\ \text{Var}(\tilde{R}_j) &= \nu_{\theta_j} + (\gamma a_j(G_j))^2 \nu_{z_j}. \end{aligned}$$

Express the return of the market portfolio as $\tilde{R}_M = \sum_{j=1}^{K+N} (\bar{\xi}_j + \tilde{z}_j)\tilde{R}_j$. We can compute the return volatility of the market portfolio. (The computation of this variance, which is tedious, is provided in Appendix B.)

$$\text{Var}(\tilde{R}_M) = \sum_{j=1}^{K+N} \left[(\nu_{\theta_j} + 4\gamma^2 a_j(G_j)^2 \nu_{z_j}) \bar{\xi}_j^2 + \nu_{\theta_j} \nu_{z_j} + 2\gamma^2 a_j(G_j)^2 \nu_{z_j}^2 \right]. \tag{6}$$

Corollary 1 then implies the following results on the volatilities.

Corollary 2 *(i) The individual security's volatilities,* $\text{Var}(P_j)$ *or* $\text{Var}(\tilde{R}_j)$*, decrease in* G_j *and increase in* ρ*.*

(ii) The aggregate volatility of the market portfolio, $\text{Var}(\tilde{R}_M)$*, decreases in* G_j $\forall j$ *and increases in* ρ*.*

These volatilities decrease in $1-\rho$, the mass of G traders (and, of course, decrease in G_j). This is because G traders behave like *de facto* liquidity providers.[7] When prices increase (drop), they sell (buy) more relative to the non-G traders.

An empirical proxy of ρ is the percentage holdings of institutional investors. The time trend in past decades is that institutional holdings have increased (e.g., Chordia, Roll, and Subrahmanyam (2011)). At the same time, there is evidence that firms' individual

[7]Kaniel, Saar, and Titman (2008) provide evidence that individual investors provide liquidity to institutions.

volatilities have increased (Campbell, Lettau, Malkiel, and Xu (2001)). Our evidence accords with these stylized facts.

Note that the results in Corollary 2 hold only if $G_j > 0$. If $\forall j\ G_j = 0$, then $a_j(G_j) = \nu_{\theta_j}$ (see Corollary 1). The volatilities become $\mathrm{Var}(P_j) = (\gamma\nu_{\theta_j})^2\nu_{z_j}$, $\mathrm{Var}(\tilde{R}_j) = \nu_{\theta_j} + (\gamma\nu_{\theta_j})^2\nu_{z_j}$, and

$$\mathrm{Var}(\tilde{R}_M) \;=\; \sum_{j=1}^{K+N}\Big[(\nu_{\theta_j} + 4\gamma^2\nu_{\theta_j}^2\nu_{z_j})\bar{\xi}_j^2 + \nu_{\theta_j}\nu_{z_j} + 2\gamma^2\nu_{\theta_j}^2\nu_{z_j}^2\Big],$$

where the last equality follows from Eq. (6). These volatilities are independent of ρ.

2.4 The Pricing of Covariance Risk

We now turn to how the G traders affect beta pricing. For this purpose, we fix $\rho > 0$ and vary G_j. We can compute the covariance between the returns of the j'th basic security and the market portfolio (the computation, which is tedious, is provided in Appendix B).

$$\begin{aligned}\mathrm{Cov}(\tilde{R}_j, \tilde{R}_M) &= \mathrm{Cov}(\tilde{R}_j, \sum_{j=1}^{K+N}(\bar{\xi}_j + \tilde{z}_j)\tilde{R}_j) \\ &= \mathrm{Cov}(\tilde{R}_j, (\bar{\xi}_j + \tilde{z}_j)\tilde{R}_j) = \nu_{\theta_j}\bar{\xi}_j + 2\gamma^2 a_j(G_j)^2\nu_{z_j}\bar{\xi}_j. \qquad (7)\end{aligned}$$

Proposition 1 and Eq. (7) indicate that the expected return of the basic security can be expressed as:

$$E(\tilde{R}_j) \;=\; \gamma a_j(G_j)\bar{\xi}_j = \frac{\gamma a_j(G_j)\mathrm{Var}(\tilde{R}_M)}{\nu_{\theta_j} + 2\gamma^2 a_j(G_j)^2\nu_{z_j}}\beta_{jM}.$$

where $\beta_{jM} = \dfrac{\mathrm{Cov}(\tilde{R}_j, \tilde{R}_M)}{\mathrm{Var}(\tilde{R}_M)}$.

Let $\lambda_j \equiv \dfrac{\gamma a_j(G_j)\mathrm{Var}(\tilde{R}_M)}{\nu_{\theta_j} + 2\gamma^2 a_j(G_j)^2\nu_{z_j}}$ denote the slope of the relation between $E(\tilde{R}_j)$ and β_{jM}. The following proposition describes the comparative static of λ_j with respect to G_j.

Proposition 2 *(i) Consider two basic securities, j and j', with $\nu_{\theta_j} = \nu_{\theta_{j'}}$, $\nu_{z_j} = \nu_{z_{j'}}$, but $G_j > G_{j'}$. Then, $\lambda_j < \lambda_{j'}$.*

(ii) The basic security with very large G_j (i.e., $G_j/\gamma^2 \nearrow \nu_{\theta_j}$) has $\lambda_j \searrow 0$.

This proposition suggests that high G_j can lead to low λ_j and, therefore, attenuate the predictive power of β's. Particularly, $\lambda_j \searrow 0$ for stocks with very large G_j (i.e., $G_j/\gamma^2 \nearrow \nu_{\theta_j}$). In this extreme case, β's lose power in explaining stock return completely. The basic intuition is that G-traders, via their tendency to "buy low and sell high" attenuate the pricing of risk (they reduce the equilibrium risk premium demanded by the non-G traders).

We now use Proposition 1 and Eq. (7) to express the j'th basic security's expected return as

$$\begin{aligned} E(\tilde{R}_j) &= \gamma a_j(G_j)\bar{\xi}_j = \gamma \mathrm{Cov}(\tilde{R}_j, \tilde{R}_M) - \gamma\Big[\nu_{\theta_j} + 2\gamma^2 a_j(G_j)^2 \nu_{z_j} - a_j(G_j)\Big]\bar{\xi}_j \\ &= \gamma \mathrm{Var}(\tilde{R}_M)\beta_{jM} - \gamma\Big[\nu_{\theta_j} + 2\gamma^2 a_j(G_j)^2 \nu_{z_j} - a_j(G_j)\Big]\bar{\xi}_j, \end{aligned} \tag{8}$$

where the last item, $-\gamma\Big[\nu_{\theta_j} + 2\gamma^2 a_j(G_j)^2 \nu_{z_j} - a_j(G_j)\Big]\bar{\xi}_j$, is referred to as the β-adjusted expected return.

Proposition 3 *(i) The β-adjusted expected return of the j'th basic security, $-\gamma\Big[\nu_{\theta_j} + 2\gamma^2 a_j(G_j)^2 \nu_{z_j} - a_j(G_j)\Big]\bar{\xi}_j$, is negative.*

(ii) Consider two basic securities, j and j', with $\nu_{\theta_j} = \nu_{\theta'_j}$, $\nu_{z_j} = \nu_{z'_j}$, $\bar{\xi}_j = \bar{\xi}_{j'}$, but $G_j > G_{j'}$. Then, the β-adjusted expected return of the j'th basic security is lower than that of the j''th basic security.

The above proposition indicates that G-traders cause securities to on average be "over-priced" on a risk-adjusted basis. To see the intuition, first note that with positive average

net supply, a high average price denotes a high risk premium on average, and prices on average decrease as they converge to fundamental values. Thus "overpricing" is a natural feature of markets with positive net supply even without G-traders. However, since G traders get direct utility from trading, when they absorb risky supplies, they are willing on average to pay more than rational investors for absorbing a given amount of supply, leading to greater "overpricing" than that naturally induced by risk premia.

2.5 Trading Volume

We now examine trading volume within our model. We aim to ascertain how trading volume is influenced by the presence of agents who derive direct utility from trading, and to investigate how volume might be associated with required returns on risky assets.

Let us assume that the initial endowment of the j'th basic security possessed by each agent equals the per capita mean supply $\bar{\xi}_j + \tilde{z}_j$. It follows from Eq. (3) and Proposition 1 that the i'th non-G trader's trade equals

$$X_{NG,j}(P_j) - (\bar{\xi}_j + \tilde{z}_j) \quad \sim \quad N\left[(\frac{a_j(G_j)}{\nu_{\theta_j}} - 1)\bar{\xi}_j, (\frac{a_j(G_j)}{\nu_{\theta_j}} - 1)^2 \nu_{z_j}\right]. \tag{9}$$

From Corollary 1, $\left[\frac{a_j(G_j)}{\nu_{\theta_j}} - 1\right]\bar{\xi}_j < 0$. Therefore, non-$G$ traders on average take a short position in the j'th basic security.

It follows from Eq. (4) and Proposition 1 that the i'th G trader's trade equals

$$X_{G,j}(P_j) - (\bar{\xi}_j + \tilde{z}_j) \quad \sim \quad N((\frac{a_j(G_j)}{\nu_{\theta_j} - G_j/\gamma^2} - 1)\bar{\xi}_j, (\frac{a_j(G_j)}{\nu_{\theta_j} - G_j/\gamma^2} - 1)^2 \nu_{z_j}). \tag{10}$$

It is straightforward to show that $\left[\frac{a_j(G_j)}{\nu_{\theta_j} - G_j/\gamma^2} - 1\right]\bar{\xi}_j > 0$. Therefore, G traders on average take a long position in the j'th security.

The expected trading volume is given by half the sums of the expected absolute changes in each type of agent's position via trading in the market for the j'th basic security. Using

Eqs. (9) and (10), we can express the total expected trading volume in the basic security as

$$T_j \equiv 0.5\rho E\Big[|X_{NG,j}(P_j) - (\bar{\xi}_j + \tilde{z}_j)|\Big] + 0.5(1-\rho)E\Big[|X_{G,j}(P_j) - (\bar{\xi}_j + \tilde{z}_j)|\Big]. \quad (11)$$

We then have the following result.

Corollary 3 *The expected trading volume, T_j, increases in G_j.*

Corollary 3 indicates that stocks in which agents have a greater level of utility from trading exhibit greater trading volume, which is an intuitive result. Since the β-adjusted expected return is more negative, the greater is G_j (Proposition 3), our analysis indicates that, *ceteris paribus*, stocks with high volume (i.e., high G_j stocks) will earn low average returns on a risk-adjusted basis. This is consistent with the negative relation between trading volume and required returns documented, for example, in Datar, Naik, and Radcliffe (1998) and Brennan, Chordia, and Subrahmanyam (1998).[8] Based on Merton (1987) who argues that some (possibly, retail) investors might invest only in the most visible stocks, visibility (as measured by analyst following and brand visibility) might be a reasonable proxy for G_j. Our analysis suggests that such proxies will be associated with high volume and low average returns. In the next subsection, we consider another proxy for G_j, the volatility of the underlying asset's cash flows.

2.6 Idiosyncratic Volatility and Expected Returns

We now consider a situation where the utility from trading an asset depends on its volatility. This assumption is motivated from Kumar (2009), who shows that retail investors are

[8] In a complementary explanation, Baker and Stein (2004) argue that high volume implies high sentiment, and, under short-selling constraints, extreme optimism, that is reversed out the following month. In contrast, our rationale for the link between expected returns and volume does not rely on short-selling constraints.

more attracted to volatile companies.[9] We thus assume that $G_j = \mu\nu_{\theta_j}$ where $\mu < \gamma^2$ (the assumption on μ is needed to obtain an interior optimum). We show below that under reasonable conditions, our analysis accords with Ang, Hodrick, Xing, and Zhang (2006), who demonstrate a negative cross-sectional relation between idiosyncratic volatility and average returns.

Now, for the j'th basic security, if one runs a time series regression of $\tilde{R}_j$ against $\tilde{R}_M$, then the variance of the residual, which we refer to as the square idiosyncratic volatility (or simply IVOL), equals

$$\text{IVOL}_j \quad = \quad \text{Var}(\tilde{R}_j) - \frac{\text{Cov}(\tilde{R}_j, \tilde{R}_M)^2}{\text{Var}(\tilde{R}_M)}. \tag{12}$$

We let ν_{θ_j} vary while holding other exogenous parameters constant. Intuitively, the return IVOL should be positively related with the cash flow volatility measured by ν_{θ_j}. It turns out in our model that this is true of "typical securities," i.e., those with small $\text{Var}(\tilde{R}_j)/\text{Var}(\tilde{R}_M)$. This implies that their value variation is relatively low compared to the value variation of the market portfolio (in a sense formalized in Appendix A). The following lemma formalizes this observation:

Lemma 1 *Consider two typical basic securities, j and j', with $\nu_{z_j} = \nu_{z_{j'}}$, $\bar{\xi}_j = \bar{\xi}_{j'}$, but $\nu_{\theta_j} > \nu_{\theta_{j'}}$. Then, the idiosyncratic volatility* $\text{IVOL}_j > \text{IVOL}_{j'}$.

We then have the following proposition:

Proposition 4 *If $G_j = \mu\nu_{\theta_j}$ where $\mu < \gamma^2$ is a positive constant, then the β-adjusted expected return, $-\gamma\Big[\nu_{\theta_j} + 2\gamma^2 a_j(G_j)^2\nu_{z_j} - a_j(G_j)\Big]\bar{\xi}_j$, decreases in ν_{θ_j}.*

[9] The analysis of Kumar (2009), in turn, is derived from the notion that unsophisticated agents are more attracted to lotteries (Rubenstein, Scafidi, and Rubinstein (2002)), and lotteries demonstrate extremely high variance and high skewness. As our model assumes normal distributions, it cannot speak to skewness, of course, and considers volatility instead.

Lemma 1 and Proposition 4 imply that for typical basic securities, there is a negative relation (induced by ν_{θ_j}) between IVOL and the β-adjusted expected return. This is broadly consistent with Ang, Hodrick, Xing, and Zhang (2006), where stocks with high idiosyncratic volatility earn lower average returns. Proxying for total volatility by ν_{θ_j}, our analysis also accords with Baker and Haugen (2012) who show that low risk stocks outperform high risk stocks in the vast majority of international equity markets.

The above analysis indicates that total volatility is negatively priced in the cross-section. However, in aggregate, risk is positively priced. To see this, note from Proposition 1 that the return of the market portfolio (over the risk free interest rate which is normalized to be zero) is given by

$$\tilde{R}_M = \sum_{j=1}^{K+N} (\bar{\xi}_j + \tilde{z}_j)\tilde{R}_j = \sum_{j=1}^{K+N} \Big[(\bar{\xi}_j + \tilde{z}_j)(\tilde{\theta}_j + \gamma a_j(G_j)(\bar{\xi}_j + \tilde{z}_j))\Big]. \tag{13}$$

The following proposition can readily be derived.

Proposition 5 *The market risk premium, $E(\tilde{R}_M)$, is positive.*

Thus, our model is consistent with the negative pricing of volatility in the cross-section, but a positive pricing of risk in the aggregate (Haugen and Baker (2010), Ang, Hodrick, Xing, and Zhang (2006), and Mehra and Prescott (1985)).

2.7 Back to the Original Securities

The previous analysis focused on the basic securities for tractability. We now show that our main results carry over to the original securities. We can use Eq. (1) to reconstruct the original risky assets using the basic securities. Specifically, consider the j'th original risky asset as a portfolio of $\bar{V}_j$ units of the risk free asset, β_{jk} $(k = 1, ..., K)$ units of the k'th basic security, and one unit of the j'th basic security. Note from Proposition 1 that

the return of the original risky asset can be expressed as:

$$\tilde{R}_j \;=\; \sum_{k=1}^{K} \beta_{jk}\Big[\tilde{\theta}_k + \gamma a_k(G_k)(\bar{\xi}_k + \tilde{z}_k)\Big] + \Big[\tilde{\theta}_j + \gamma a_j(G_j)(\bar{\xi}_j + \tilde{z}_j)\Big]. \tag{14}$$

The expected return of the j'th original risky asset is given by

$$E(\tilde{R}_j) \;=\; \sum_{k=1}^{K} \beta_{jk}\Big[\gamma a_k(G_k)\bar{\xi}_k\Big] + \Big[\gamma a_j(G_j)\bar{\xi}_j\Big]. \tag{15}$$

The expected return in our model takes a similar form as multi-factor models such as ICAPM (Merton (1973)) and APT (Ross (1976)). It follows from Corollary 1 that this expected return decreases in G_k $\forall k = 1, ..., K$ and G_j, and increases in ρ (so long as $\beta_{jk} > 0$ $\forall k = 1, ..., K$).

The volatility of the j'th original risky asset is given by

$$\mathrm{Var}(\tilde{R}_j) \;=\; \sum_{k=1}^{K} \beta_{jk}^2\Big[\nu_{\theta_k} + (\gamma a_k(G_k))^2\nu_{z_k}\Big] + \Big[\nu_{\theta_j} + (\gamma a_j(G_j))^2\nu_{z_j}\Big].$$

It follows from Corollary 2 that this volatility decreases in G_k $\forall k = 1, ..., K$ and G_j, and increases in ρ.

Factor loadings can predict expected returns because the factor premium, $\gamma a_k(G_k)\bar{\xi}_k$, is identical across all assets. Can β_{jM} (i.e., the beta with respect to the market return) also predict returns? One can estimate β_{jM} by regressing $\tilde{R}_j$ against the returns on factor mimicking portfolio and the market portfolio. From Eq. (15), it also follows that after adjusting for factor returns, the j'th original risky asset's return is identical to the j'th basic security's return. Thus, the market beta of a j'th original security is identical to that of the j'th basic security given in Eq. (7):

$$\beta_{jM} = \frac{\nu_{\theta_j}\bar{\xi}_j + 2\gamma^2 a_j(G_j)^2\nu_{z_j}\bar{\xi}_j}{\mathrm{Var}(\tilde{R}_M)}. \tag{16}$$

Plugging into Eq. (15) yields

$$E(\tilde{R}_j) \;=\; \sum_{k=1}^{K} \beta_{jk}\Big[\gamma a_k(G_k)\bar{\xi}_k\Big] + \frac{\gamma a_j(G_j)\mathrm{Var}(\tilde{R}_M)}{\nu_{\theta_j} + 2\gamma^2 a_j(G_j)^2\nu_{z_j}}\beta_{jM}.$$

Proposition 2 indicates that $\lambda_j = \dfrac{\gamma a_j(G_j)\mathrm{Var}(\tilde{R}_M)}{\nu_{\theta_j} + 2\gamma^2 a_j(G_j)^2 \nu_{z_j}}$, the slope of the relation between $E(\tilde{R}_j)$ and β_{jM}, decreases in G_j and can be as low as zero.

One can estimate IVOL_j by regressing $\tilde{R}_j$ on factor mimicking portfolios' and the market portfolio's returns. From Eq. (14) we see that after adjustment for the factor returns, this IVOL_j is identical to the IVOL of the j'th basic security given in Eq. (12), which as argued in Lemma 1, increases in ν_{θ_j}. Use Eqs. (15) and (16) to write

$$E(\tilde{R}_j) \;=\; \sum_{k=1}^{K} \beta_{jk}\Big[\gamma a_k(G_k)\bar{\xi}_k\Big] + \gamma \mathrm{Var}(\tilde{R}_M)\beta_{jM} - \gamma\Big[\nu_{\theta_j} + 2\gamma^2 a_j(G_j)^2\nu_{z_j} - a_j(G_j)\Big]\bar{\xi}_j,$$

where the last term, $-\gamma\Big[\nu_{\theta_j}+2\gamma^2 a_j(G_j)^2\nu_{z_j}-a_j(G_j)\Big]\bar{\xi}_j$, represents the β-adjusted expected return. It follows from Proposition 4 that if $G_j = \mu\nu_{\theta_j}$ where $\mu < \gamma^2$ is a positive constant, then the β-adjusted expected return, $-\gamma\Big[\nu_{\theta_j} + 2\gamma^2 a_j(G_j)^2\nu_{z_j} - a_j(G_j)\Big]\bar{\xi}_j$, decreases in ν_{θ_j}. Thus, there is a negative relation (induced by ν_{θ_j}) between IVOL and the β-adjusted expected return. Note that our result in Proposition 5 on the market risk premium being positive continues to hold for the original securities. The reason is that the market portfolio of the original securities is just a reshuffle of the basic securities, and is therefore identical to the market portfolio of the basic securities.

All of the preceding analysis indicates that results for the mimicking portfolios carry over to the original securities. Our work suggests the following untested empirical implications, which rely on the premise that stocks with more retail traders are likely to also have more G traders. We predict that *ceteris paribus*, stocks with proportionally more retail traders are less volatile and more actively traded. The cross-sectional (negative) relation between volatility and future returns is also likely to be more evident in stocks actively traded and held by retail investors.

3 Comparing to the Economy With No or Partial Presence of G Traders

We now compare the equilibria with (i) complete absence of the G traders and (ii) presence of the G traders in some, but not all, securities. For simplicity, the analysis in this section is focused on the basic securities.

3.1 Comparing to the Economy With No G Traders

Consider two economies. In the first economy, all agents are non-G traders, while in the second, all are G traders. For a variable η in the basic economy, we use $\eta^{A,G}$ and $\eta^{A,NG}$ to indicate its counterpart in the all-G and all-non-G economies.

The first economy, the all-non-G economy, is equivalent to the basic economy with $\rho = 1$ and $a_j(G_j) = \nu_{\theta_j}$. Using a similar derivation as that for Proposition 1, we can show that the price and return of the j'th basic security are given by

$$\begin{aligned} P_j^{A,NG} &= -\gamma\nu_{\theta_j}(\bar{\xi}_j + \tilde{z}_j), \\ \tilde{R}_j^{A,NG} &= \tilde{\theta}_j + \gamma\nu_{\theta_j}(\bar{\xi}_j + \tilde{z}_j). \end{aligned} \tag{17}$$

The return of the market portfolio is $\tilde{R}_M^{A,NG} = \sum_{j=1}^{K+N} (\bar{\xi}_j + \tilde{z}_j)\tilde{R}_j^{A,NG}$.

Similar to Eq. (7), the covariance between the returns of the basic security and the market portfolio is given by

$$\mathrm{Cov}(\tilde{R}_j^{A,NG}, \tilde{R}_M^{A,NG}) = \mathrm{Cov}(\tilde{R}_j^{A,NG}, (\bar{\xi}_j + \tilde{z}_j)\tilde{R}_j^{A,NG}) = \nu_{\theta_j}\bar{\xi}_j + 2\gamma^2\nu_{\theta_j}^2\nu_{z_j}\bar{\xi}_j.$$

Then, the expected return of the basic security can be expressed as:

$$E(\tilde{R}_j^{A,NG}) = \gamma\nu_{\theta_j}\bar{\xi}_j = \frac{\gamma\nu_{\theta_j}\mathrm{Var}(\tilde{R}_M^{A,NG})}{\nu_{\theta_j} + 2\gamma^2\nu_{\theta_j}^2\nu_{z_j}}\beta_{jM}^{A,NG},$$

where $\beta_{jM}^{A,NG} = \dfrac{\text{Cov}(\tilde{R}_j^{A,NG}, \tilde{R}_M^{A,NG})}{\text{Var}(\tilde{R}_M^{A,NG})}$.

Let $\lambda_j^{A,NG} \equiv \dfrac{\gamma \nu_{\theta_j} \text{Var}(\tilde{R}_M^{A,NG})}{\nu_{\theta_j} + 2\gamma^2 \nu_{\theta_j}^2 \nu_{z_j}}$ denote the slope of the relation between $E(\tilde{R}_j^{A,NG})$ and $\beta_{jM}^{A,NG}$. An obvious observation is that $\lambda_j^{A,NG} > 0$. Therefore, β's still have power to predict stock returns. If $\nu_{z_j} = 0$, then $\lambda_j^{A,NG} = \gamma \text{Var}(\tilde{R}_M^{A,NG})$ is identical across all assets. In this case, β's are the only predictive variable for expected returns.

The second economy, the all-G economy, is equivalent to the basic economy with $\rho = 0$ and $a_j(G_j) = \nu_{\theta_j} - G_j/\gamma^2$. Using a similar derivation as that for Proposition 1, we can show that the price and return of the j'th basic security is given by

$$\begin{aligned} P_j^{A,G} &= -\gamma(\nu_{\theta_j} - G_j/\gamma^2)(\bar{\xi}_j + \tilde{z}_j), \\ \tilde{R}_j^{A,G} &= \tilde{\theta}_j + \gamma(\nu_{\theta_j} - G_j/\gamma^2)(\bar{\xi}_j + \tilde{z}_j). \end{aligned} \tag{18}$$

The return of the market portfolio is $\tilde{R}_M^{A,G} = \sum_{j=1}^{K+N} (\bar{\xi}_j + \tilde{z}_j) \tilde{R}_j^{A,G}$.

Similar to Eq. (7), the covariance between the returns of the basic security and the market portfolio is given by

$$\text{Cov}(\tilde{R}_j^{A,G}, \tilde{R}_M^{A,G}) = \text{Cov}(\tilde{R}_j^{A,G}, (\bar{\xi}_j + \tilde{z}_j)\tilde{R}_j^{A,G}) = \nu_{\theta_j} \bar{\xi}_j + 2\gamma^2 (\nu_{\theta_j} - G_j/\gamma^2)^2 \nu_{z_j} \bar{\xi}_j.$$

Then, the expected return of the basic security can be expressed as:

$$E(\tilde{R}_j^{A,G}) = \gamma(\nu_{\theta_j} - G_j/\gamma^2)\bar{\xi}_j = \frac{\gamma(\nu_{\theta_j} - G_j/\gamma^2)\text{Var}(\tilde{R}_M^{A,G})}{\nu_{\theta_j} + 2\gamma^2(\nu_{\theta_j} - G_j/\gamma^2)^2 \nu_{z_j}} \beta_{jM}^{A,G},$$

where $\beta_{jM}^{A,G} = \dfrac{\text{Cov}(\tilde{R}_j^{A,G}, \tilde{R}_M^{A,G})}{\text{Var}(\tilde{R}_M^{A,G})}$.

We compare the two economies in the following proposition.

Proposition 6 *(i)* $E(\tilde{R}_j^{A,NG}) > E(\tilde{R}_j^{A,G})$ *and* $\text{Var}(\tilde{R}_j^{A,NG}) > \text{Var}(\tilde{R}_j^{A,G})$. *Thus, the j'th basic security has higher expected return and volatility in the all-non-G economy than in the all-G economy.*

(ii) $E(\tilde{R}_M^{A,NG}) > E(\tilde{R}_M^{A,G})$ *and* $\mathrm{Var}(\tilde{R}_M^{A,NG}) > \mathrm{Var}(\tilde{R}_M^{A,G})$. *Thus, the market portfolio has higher expected return and volatility in the all-non-G economy than in the all-G economy.*

(iii) $\lambda_j^{A,NG} > \lambda_j^{A,G}$. *Thus,* β*'s have more predictive power in the all-non-G economy than in the all-G economy.*

In general, within the all-G economy, risk premiums (and volatilities) are attenuated because of the G-traders' penchant to buy low and sell high, which, in turn, attenuates the pricing of risk. This suggests the testable implication that economies which are dominated by retail investors should exhibit lower volatility and less evidence of covariance risk pricing. Indeed, under the assumption that retail investors are more likely to participate during periods of positive sentiment (Grinblatt and Keloharju (2001) and Lamont and Thaler (2003)), and less likely to participate during periods of negative sentiment, our analysis accords with Antoniou, Doukas, and Subrahmanyam (2016) who show that covariance risk pricing is more prevalent during periods of low sentiment and vice versa.

3.2 Comparing to the Economy With Partial Presence of G Traders

Consider a hybrid case in which G traders are present in the trading of the basic securities mimicking the K factors and the first N_1 residuals. They are not present in the remaining $N - N_1$ basic securities. Like the basic economy, there are a mass ρ of non-G traders and a mass $1 - \rho$ of G traders.

We continue to write the return of the j'th basic security as $\tilde{R}_j = \tilde{\theta}_j - P_j$. (For convenience, we use the same notation for prices in all securities, i.e., P_j for the j'th security, even though prices of securities without G traders can take a different form from

that in Subsection 2.4.) The market portfolio has a return $\tilde{R}_M = \sum_{j=1}^{K+N} (\bar{\xi}_j + \tilde{z}_j)\tilde{R}_j$.

For the first $K+N_1$ basic securities, i.e., $\forall j \leq K+N_1$, our analysis in the subsection 2.4 still holds. Particularly, the expected return of the j'th basic security takes the form

$$E(\tilde{R}_j) = \lambda_j \beta_{jM}.$$

where $\lambda_j = \dfrac{\gamma a_j(G_j)\mathrm{Var}(\tilde{R}_M)}{\nu_{\theta_j} + 2\gamma^2 a_j(G_j)^2 \nu_{z_j}}$ denote the slope of the relation between $E(\tilde{R}_j)$ and β_{jM}.

For the remaining $N - N_1$ basic securities, i.e., $\forall j > K + N_1$, there is only a mass ρ of non-G traders to clear the market. Using a similar analysis as in Subsection 2.4, we can show that for these securities,

$$\begin{aligned} P_j^o &= -(\gamma/\rho)\nu_{\theta_j}(\bar{\xi}_j + \tilde{z}_j), \\ \tilde{R}_j^o &= \tilde{\theta}_j + (\gamma/\rho)\nu_{\theta_j}(\bar{\xi}_j + \tilde{z}_j). \end{aligned}$$

[Here, we use the superscript-^{o} to indicate these securities.] Similar to Eq. (7), the covariance between the returns of the basic security and the market portfolio is given by

$$\mathrm{Cov}(\tilde{R}_j^o, \tilde{R}_M) = \nu_{\theta_j}\bar{\xi}_j + 2(\gamma/\rho)^2 \nu_{\theta_j}^2 \nu_{z_j} \bar{\xi}_j.$$

Then, the expected return of the basic security can be expressed as:

$$E(\tilde{R}_j^o) = (\gamma/\rho)\nu_{\theta_j}\bar{\xi}_j = \frac{(\gamma/\rho)\nu_{\theta_j}\mathrm{Var}(\tilde{R}_M)}{\nu_{\theta_j} + 2(\gamma/\rho)^2\nu_{\theta_j}^2\nu_{z_j}} \beta_{jM}^o = \lambda_j^o \beta_{jM}^o,$$

where $\beta_{jM}^o = \dfrac{\mathrm{Cov}(\tilde{R}_j^o, \tilde{R}_M)}{\mathrm{Var}(\tilde{R}_M)}$, and $\lambda_j^o = \dfrac{(\gamma/\rho)\nu_{\theta_j}\mathrm{Var}(\tilde{R}_M)}{\nu_{\theta_j} + 2(\gamma/\rho)^2\nu_{\theta_j}^2\nu_{z_j}}$ denotes the slope of the relation between $E(\tilde{R}_j^o)$ and β_{jM}^o. We then have the following proposition.

Proposition 7 *Consider two basic securities, j and j', with $\nu_{\theta_j} = \nu_{\theta_{j'}}$, $\nu_{z_j} = \nu_{z_{j'}}$, $\bar{\xi}_j = \bar{\xi}_{j'}$, but the j'th (j''th) security is (is not) traded by G traders. Then, $\lambda_j < \lambda_{j'}^o$. Thus, the presence of G traders reduces the predictive power of β's.*

The above proposition implies that beta pricing will be less evident in securities that are traded relatively more by G traders. Again, the notion is simply that G traders, via their more aggressive trading in securities where they are present, attenuate the pricing of risk. The above proposition indicates cross-sectional variation in risk pricing according to whether G traders are more or less likely to be present. Thus, if retail investors are more likely to be present in visible, brand name stocks (Frieder and Subrahmanyam (2005)), then covariance risk pricing will be less evident in these stocks.

4 G Traders and the Informational Efficiency of Stock Prices

We now consider a model with information asymmetry which allows us to examine how G traders affect the extent to which prices reveal information. Thus, consider a Grossman and Stiglitz (1980)-type modification of our model in which, for simplicity, a single stock is traded at Date 0.[10] At Date 1, it pays off a liquidation dividend

$$V = \bar{V} + \tilde{\theta} + \tilde{\epsilon}.$$

$\bar{V}$ is a positive constant, and $\tilde{\theta}$ and $\tilde{\epsilon}$ are zero mean, and are mutually independent and normally distributed. The additional term ϵ is added to create additional risk which bounds the position of informed agents. The per capita supply of the stock is indicated by $\bar{\xi} + \tilde{z}$, where $\bar{\xi}$ is a constant. The stock price is indicated by P and is determined in equilibrium. There is also a riskless asset, the price and return of which are normalized to unity. Again, a generic random variable, $\tilde{\eta}$, follows a normal distribution with mean zero and variance ν_η.

As before, there are masses ρ and $1 - \rho$ of non-G and G traders, respectively. The

[10] An extension to multiple stocks is possible, but notationally complex, and does not yield substantive intuition beyond that presented in earlier sections.

endowments of riskfree assets and preferences are unchanged relative to the basic model and the direct utility parameter is denoted by G (without the subscript to denote the single asset). We modify the basic model by postulating that each G trader can observe $\tilde{\theta}$ by spending a positive and constant cost c. In equilibrium, a mass $(1-\rho)\tau$ of G traders choose to become informed by paying the cost c; a mass $(1-\rho)(1-\tau)$ of G traders choose to remain uninformed. $\tau \in [0,1]$ is determined in equilibrium. The following proposition describes the pricing function in this setting:

Proposition 8 *In equilibrium, the price function takes a linear form*

$$P = \bar{V} - a + b\omega(\tilde{\theta}, \tilde{z}), \tag{19}$$

where $\omega(\tilde{\theta}, \tilde{z}) = \tilde{\theta} - f\tilde{z}$ *(or simply* ω*) has a variance* $\nu_\omega = \nu_\theta + f^2\nu_z$*. The parameters,* a*,* b*, and* f*, are given by*

$$\begin{aligned} a &= \frac{\bar{\xi}}{N_1 + N_2 + N_3}, \\ b &= \frac{N_1 + N_2 + N_3\dfrac{\nu_\theta}{\nu_\omega}}{N_1 + N_2 + N_3}, \\ f &= \frac{1}{N_1 + N_2}, \end{aligned}$$

where

$$\begin{aligned} N_1 &\equiv \frac{\rho}{\gamma\nu_\epsilon}, \\ N_2 &\equiv \frac{(1-\rho)\tau}{\gamma(\nu_\epsilon - G/\gamma^2)}, \\ N_3 &\equiv \frac{(1-\rho)(1-\tau)}{\gamma(\nu_\theta(1 - \dfrac{\nu_\theta}{\nu_\omega}) + \nu_\epsilon - G/\gamma^2)}. \end{aligned}$$

Using standard Grossman and Stiglitz (1980)-type arguments, we can derive the following lemma:

Lemma 2 *If* $\psi(\tau) \equiv \exp(2\gamma c) \cdot \dfrac{\mathrm{Var}(V|\tilde{\theta}) - G/\gamma^2}{\mathrm{Var}(V|\omega) - G/\gamma^2} - 1$ *is negative (positive), then the* G *trader prefers to become informed by spending* c *(remain uninformed). If* $\psi(\tau) = 0$*, then he is indifferent between becoming informed and remaining uninformed.*

In the above lemma, $\psi(\tau)$ is a function of τ because according to Proposition 8, f and therefore $\mathrm{Var}(V|\omega)$ are functions of τ.

Proposition 9 $\tau \in [0,1]$ *is uniquely determined by the function* $\psi(\tau)$ *as follows:*

- *If* $\psi(0) \geq 0$*, then* $\tau = 0$*.*
- *If* $\psi(0) < 0$ *and* $\psi(1) > 0$*, then an interior* $\tau \in (0,1)$ *is given by* $\psi(\tau) = 0$*.*
- *If* $\psi(1) \leq 0$*, then* $\tau = 1$*.*

Consider an interior $\tau \in (0,1)$, which according to Lemma 2 and Proposition 9, is given by

$$\psi(\tau) \equiv \exp(2\gamma c) \cdot \frac{\mathrm{Var}(V|\tilde{\theta}) - G/\gamma^2}{\mathrm{Var}(V|\omega) - G/\gamma^2} - 1 = 0. \tag{20}$$

We then have the following corollary.

Corollary 4 *If* τ *is interior (with an equilibrium specified by Eq. 20), then*

(i) $\mathrm{Var}(V|\omega)$ *decreases in* G*;*

(ii) $\mathrm{Var}(V|\omega)$ *does not depend on* ρ*.*

In fact, Part (i) of the corollary above holds even for corner solutions where $\tau = \{0,1\}$, as given in Proposition 9. The intuition for this part is as follows. G has two effects on $\mathrm{Var}(V|\omega)$. First, given τ, G decreases $f = \dfrac{1}{\dfrac{\rho}{\gamma\nu_\epsilon} + \dfrac{(1-\rho)\tau}{\gamma(\nu_\epsilon - G^2/\gamma)}}$ (this expression of f is

obtained from Proposition 8) and therefore the informativeness of $\omega = \tilde{\theta} - f\tilde{z}$, $\text{Var}(V|\omega)$. Second, G may increase or decrease τ, the mass of G traders who choose to become informed, and therefore increase or decrease the informativeness of $\omega = \tilde{\theta} - f\tilde{z}$, $\text{Var}(V|\omega)$. The appendix shows that the first effect dominates. Therefore, taken together, $\text{Var}(V|\omega)$ decreases in G.

Part (ii) states that $\text{Var}(V|\omega)$ does not depend on ρ. The reason for this is that as Eq. (20) indicates, an increase in ρ, the mass of informed non-G traders, reduces $(1-\rho)\tau$, the mass of informed G traders. Thus, $f = \dfrac{1}{\dfrac{\rho}{\gamma\nu_\epsilon} + \dfrac{(1-\rho)\tau}{\gamma(\nu_\epsilon - G^2/\gamma)}}$ and therefore the informativeness of $\omega = \tilde{\theta} - f\tilde{z}$, $\text{Var}(V|\omega)$, remain unchanged.

Overall, we find an increase in utility derived from trading leads to increased price informativeness in equilibrium but the mass of G traders does not affect this informativeness.

5 A Dynamic Extension: Equilibrium Where Trading Value Depends on Past Market Outcomes

We now consider a dynamic extension of our setting where the utility from trading depends on past profits. Specifically, we model the notion that if an agent earns positive profits, he may derive greater utility from gambling in the stock market (see, for example, Thaler and Johnson (1990) or Coventry and Constable (1999)). We show that the "excitement" created by positive profits can lead to an overreaction to mildly positive information and thus cause a "bubble" in stock prices. Our rationale for this bubble is consistent with experimental arguments that emotional excitement can cause bubbles (Bellotti, Taffler, and Tian (2010) and Andrade, Odean, and Lin (2015)); in our model, the pathway is that positive profits increase the "excitement" or utility from additional trading and thus

cause an overreaction to mildly positive information.

We assume that a single risky security is traded at Dates $t = 0$, 1, 2, and 3 and revert to the case of symmetric information.[11] For convenience, as in the previous section, we suppress j subscripts. At Date 3, the security pays off a liquidation dividend

$$V = \bar{V} + \tilde{\theta}_1 + \tilde{\theta}_2 + \tilde{\theta}_3.$$

$\bar{V}$ is a positive constant, which represents the expected dividend. The variables $\tilde{\theta}_t$, $t = 1$, 2, and 3, represent exogenous cash flow shocks, which are mutually independent and multivariate normally distributed with mean zero. $\tilde{\theta}_t$'s are public signals released at Dates $t = 1, 2$. The supply of the risky security is a positive constant, $\bar{\xi}$.[12] Its prices are P_t at Dates $t = 0$, 1, and 2.

There is a mass unity of identical agents who trade the risky security. At Date 0, they all hold an identical long position, in aggregate $\bar{\xi}$, to clear the market. At Date 1, if $P_1 > P_0$, they make money at Date 1; otherwise, they do not. If $P_1 > P_0$, so that they make money at Date 1, then after Date 1 and before Date 2, with probability ρ, an agent becomes a non-G trader; with probability $1 - \rho$, he becomes a G trader. If $P_1 \leq P_0$ and the agents do not make money at date 1, then all agents remain non-G traders with a probability of unity.

The i'th non-G trader's utility function is the standard exponential:

$$U(W_{i3}) = -\exp(-\gamma W_{i3}),$$

where W_{i3} is his final wealth, and γ is a positive constant representing the absolute risk aversion coefficient. The i'th G trader's utility function takes the form

$$U_G(W_{i3}, X_{i2}) = -\exp(-\gamma W_{i3})\exp(-0.5GX_{i2}^2),$$

[11] An extension to multiple assets and asymmetric information does not convey any additional intuition.
[12] Assuming random supply will complicate the analysis but convey no additional intuition.

where X_{i2} is the quantity of risky security he has bought at date 2 and continues to hold until the end of the game, and G is a positive constant.

The i'th trader is endowed with $\bar{X}_{i0}$ units of risky securities. For convenience, we let $\tilde{\theta}_t$'s, $t = 1$, 2, and 3, have the same variance ν_θ. Let the price and return of the risk free asset be 1. We then have the following result:

Proposition 10 *There is an equilibrium characterized by the following prices:*

- P_0 *is given by*

$$P_0 = \bar{V} + H_\theta - 2\gamma\nu_\theta\bar{\xi},$$

where $a(G) = \dfrac{1}{\dfrac{\rho}{\nu_\theta} + \dfrac{1-\rho}{\nu_\theta - G/\gamma^2}}$. *The variable* H_θ *is uniquely determined by*

$$\begin{aligned} 0 = & \int_{H_\theta}^{\infty} \frac{\tilde{\theta}_1 - H_\theta + \gamma(\nu_\theta - a(G))\bar{\xi}}{\exp(\gamma\bar{\xi}\tilde{\theta}_1)} \\ & \cdot\Big[\rho\exp(-0.5\frac{(\gamma a(G)\bar{\xi})^2}{\nu_\theta}) + (1-\rho)\exp(-0.5\frac{(\gamma a(G)\bar{\xi})^2}{\nu_\theta - G/\gamma^2})\Big]d\Phi(\frac{\tilde{\theta}_1}{\sqrt{\nu_\theta}}) \\ & + \int_{-\infty}^{H_\theta} \frac{\tilde{\theta}_1 - H_\theta}{\exp\Big[\gamma\bar{\xi}(\tilde{\theta}_1 - \gamma(\nu_\theta - a(G))\bar{\xi})\Big]}\exp(-0.5\frac{(\gamma\nu_\theta\bar{\xi})^2}{\nu_\theta})d\Phi(\frac{\tilde{\theta}_1}{\sqrt{\nu_\theta}}). \end{aligned}$$

and $\Phi(.)$ *is the cumulative density function of standard normal distribution.*

- *If* $\tilde{\theta}_1 > H_\theta$, *then*

$$\begin{aligned} P_1 &= \bar{V} + \tilde{\theta}_1 - \gamma\nu_\theta\bar{\xi} - \gamma a(G)\bar{\xi}, \\ P_2 &= \bar{V} + \tilde{\theta}_1 + \tilde{\theta}_2 - \gamma a(G)\bar{\xi}. \end{aligned}$$

Because $P_1 > P_0$, *a mass* $1-\rho$ *of traders convert to* G *traders immediately following Date 1.*

- *If* $\tilde{\theta}_1 \leq H_\theta$*, then*

$$\begin{aligned} P_1 &= \bar{V} + \tilde{\theta}_1 - 2\gamma\nu_\theta\bar{\xi}, \\ P_2 &= \bar{V} + \tilde{\theta}_1 + \tilde{\theta}_2 - \gamma\nu_\theta\bar{\xi}. \end{aligned}$$

Because $P_1 \leq P_0$*, all traders remain non-*G *traders throughout the timeline.*

Here is a sketch of the proof of this proposition (the formal proof is in the Appendix). We use backward induction. There are three steps. In the first step, we study the equilibrium demands and prices at Dates 1 and 2 conditional on the event that at Date 1, traders make money because $\tilde{\theta}_1 > H_\theta$ so $P_1 > P_0$ (call this Regime 1). Note that in this regime, at Date 2, there is a mass ρ $(1-\rho)$ of non-G traders (G traders). At Date 1, an agent knows that he will be a non-G trader (G trader) with probability ρ $(1-\rho)$. In the second step, we study the equilibrium demands and prices at dates 1 and 2 conditional on the event that at Date 1, traders do not make money because $\tilde{\theta}_1 \leq H_\theta$ so $P_1 \leq P_0$ (call this Regime 2). This step is simpler than the first step because all traders are non-G traders. In the third step, we focus on Date 0, and derive the expressions for P_0 and the threshold H_θ.

There are two interesting results. The first relates to the price reaction for $\tilde{\theta}_1$ around the threshold H_θ:

$$\begin{aligned} P_1(\tilde{\theta}_1 \searrow H_\theta) &= \bar{V} + \tilde{\theta}_1 - \gamma\nu_\theta\bar{\xi} - \gamma a(G)\bar{\xi}, \\ P_1(\tilde{\theta}_1 \nearrow H_\theta) &= \bar{V} + \tilde{\theta}_1 - 2\gamma\nu_\theta\bar{\xi}. \end{aligned}$$

It is easy to show that

$$P_1(\tilde{\theta}_1 \searrow H_\theta) - P_1(\tilde{\theta}_1 \nearrow H_\theta) = \gamma\nu_\theta\bar{\xi} - \gamma a(G)\bar{\xi} > 0,$$

because $a(G) < a(0) = \nu_\theta$. This suggests a small $\tilde{\theta}_1$ (e.g., earnings) can induce a significant price movement. Another interpretation of this observation is that a relatively minor piece of news can cause substantial moves in prices.

The second result relates to long-run performance. If $\tilde{\theta}_1 > H_\theta$, then the subsequent returns are

$$P_2 - P_1 = \tilde{\theta}_2 + \gamma\nu_\theta\bar{\xi}, \text{ and } V - P_2 = \tilde{\theta}_3 + \gamma a(G)\bar{\xi}.$$

If $\tilde{\theta}_1 \leq H_\theta$, then the subsequent returns are

$$P_2 - P_1 = \tilde{\theta}_2 + \gamma\nu_\theta\bar{\xi}, \text{ and } V - P_2 = \tilde{\theta}_3 + \gamma\nu_\theta\bar{\xi}.$$

A comparison between these two cases suggests that if $\tilde{\theta}_1 > H_\theta$, then there is a long-run underperformance because $a(G) < a(0) = \nu_\theta$. Thus, a minor piece of good news can cause securities to become dramatically overpriced and thus exhibit subpar returns in the long run.

Figure 1 plots the price paths conditional on the public announcement $\tilde{\theta}_1$. We assume the parameter values $\bar{V} = 5$, $\nu_\theta = 1$, $\bar{\xi} = 1$, $\gamma = 0.5$, $\rho = 0.5$, and $G = 0.2$. The realizations of $\tilde{\theta}_2$ and $\tilde{\theta}_3$ are assumed to be zero, i.e., their mean. This implies that the threshold $H_\theta = -0.388$. Moving from the bottom to the top, each path in the figure represents a realization of $\tilde{\theta}_1$ from -1 to 0 (step size=0.025). $\tilde{\theta}_1 \leq H_\theta$ for the paths indicated by $\triangle$'s. $\tilde{\theta}_1 > H_\theta$ for the paths indicated by *'s. We see that if $\tilde{\theta}_1$ is below the threshold $H_\theta = -0.388$, the price reaction to $\tilde{\theta}_1$ is non-positive. Once $\tilde{\theta}_1$ has surpassed the threshold $H_\theta = -0.388$, the price reaction becomes positive.

Particularly, look at the two paths bordering the hollow area. The south path is for $\tilde{\theta} = -0.4$. The north path is for $\theta = -0.375$. Although $\tilde{\theta}_1$ differs by only 0.025 across the two path groups, the price reactions are very different. On both paths, $P_0 = 3.612$. However, on the south path, $P_1 = 3.6$ so $P_1 - P_0 = -0.012$; on the north path, $P_1 = 3.9583$ so $P_1 - P_0 = 0.3463$. The difference in the price reaction, $P_1 - P_0$, equals 0.3583, which is more than fourteen times the difference in $\tilde{\theta}_1$ (0.025).

The immediate return subsequent to the release of $\tilde{\theta}_1$, $P_2 - P_1 = 0.5$, is identical

across all $\tilde{\theta}_1$ paths. But the long-run performance for the paths with $\tilde{\theta}_1 > H_\theta$ indicated by *'s, $V - P_2 = 0.1667$, is lower than that for the paths with $\tilde{\theta}_1 \leq H_\theta$ indicated by $\triangle$'s, $V - P_2 = 0.5$. This indicates long-run underperformance following a good public announcement. The underperformance is characteristic of bubble-like episodes in the stock market (such as the technology bubble of the 1990s, viz. Brunnermeier and Nagel (2004)), whereas the positive event (that creates the bubble) could be something as simple as good initial sales or earnings figures for the relevant sector.

More generally, the preceding analysis suggests a testable implication. Specifically, for stocks that are popular amongst retail investors, we predict a nonlinear response to positive news, that is a small reaction to modest news announcement, but a disproportionately larger reaction to major (positive) announcements. Following the large positive announcements, these stocks should exhibit long-run reversals (conditional on the news).

6 Conclusion

In this paper, we present a model where agents derive direct utility from trading. We show that the presence of such agents causes assets to be overpriced, attenuates beta pricing and volatility, and raises trading volume in financial markets. Assets with high trading volume earn lower expected returns. Assuming that agents derive greater utility from trading more volatile stocks, our model accords with a set of intriguing empirical findings: Volatility is priced negatively in the cross-section, but positively in the aggregate (viz. Haugen and Baker (2010)). Agents with greater utility from trading exploit private information more aggressively, thus raising pricing efficiency. Further, the presence of agents who receive direct utility from trading causes "bubbles," i.e., overreactions in asset prices if agents' utility from trading depends on past profit outcomes in financial markets.

Untested implications of our analysis are that stocks that are popular amongst retail investors should exhibit weaker evidence of covariance risk pricing and lower volatility, with greater trading activity. These stocks should also exhibit disproportionate price reactions to moderately positive news announcements. The analysis, under reasonable additional assumptions, also accords with a variety of documented stylized facts: the negative relation between average returns and volume as well as idiosyncratic (or total) volatility (Datar, Naik, and Radcliffe (1998), Ang, Hodrick, Xing, and Zhang (2006), Baker and Haugen (2012)), the lack of evidence consistent with covariance risk pricing (Fama and French (1992)), the pricing of covariance risk conditional on sentiment (Antoniou, Doukas, and Subrahmanyam (2016)), and the rise of volatility in conjunction with the rise in institutional holdings (Campbell, Lettau, Malkiel, and Xu (2001) and Malkiel and Xu (1999)).

Our work raises many issues. First, it would be interesting to examine a fully dynamic model with exits and entry by such agents. Second, it may be interesting to combine trading for entertainment and other investor biases, such as representativeness and overconfidence, and to examine the market equilibrium that results. Finally, the specific factors that influence how much utility is derived per unit trade (such as age, personality attributes) need to be considered in more depth within a theoretical setting). These and other issues are left for future research.

Appendix A

Proof of Corollary 1: From Proposition 1, $a_j(G_j) = \dfrac{1}{\dfrac{\rho}{\nu_{\theta_j}} + \dfrac{1-\rho}{\nu_{\theta_j} - G_j/\gamma^2}}$. It is easy to show after taking derivatives that $a_j(G_j)$ decreases in G_j, and increases in ρ. Finally, $a_j(0) = \dfrac{1}{\dfrac{\rho}{\nu_{\theta_j}} + \dfrac{1-\rho}{\nu_{\theta_j}}} = \nu_{\theta_j}$.

Q.E.D.

Proof of Proposition 2: (i) From Corollary 1, $a_j(G_j) < a_j(G_{j'})$ because $G_j > G_{j'}$. Note that $\lambda_j = \dfrac{\gamma a_j(G_j)\mathrm{Var}(\tilde{R}_M)}{\nu_{\theta_j} + 2\gamma^2 a_j(G_j)^2 \nu_{z_j}}$. For $\lambda_j < \lambda_{j'}$, it suffices that

$$
\begin{aligned}
& \frac{a_j(G_j)}{\nu_{\theta_j} + 2\gamma^2 a_j(G_j)^2 \nu_{z_j}} - \frac{a_j(G_{j'})}{\nu_{\theta_j} + 2\gamma^2 a_j(G_{j'})^2 \nu_{z_j}} \\
\propto\ & a_j(G_j)\Big[\nu_{\theta_j} + 2\gamma^2 a_j(G_{j'})^2 \nu_{z_j}\Big] - a_j(G_{j'})\Big[\nu_{\theta_j} + 2\gamma^2 a_j(G_j)^2 \nu_{z_j}\Big] \\
=\ & \Big[a_j(G_j) - a_j(G_{j'})\Big]\Big[\nu_{\theta_j} - 2\gamma^2 a_j(G_{j'}) a_j(G_j) \nu_{z_j}\Big] \\
\propto\ & 2\gamma^2 a_j(G_{j'}) a_j(G_j) \nu_{z_j} - \nu_{\theta_j} \\
<\ & 2\gamma^2 \nu_{\theta_j}^2 \nu_{z_j} - \nu_{\theta_j} \\
<\ & 0,
\end{aligned}
$$

where the second "$\propto$" follows from $a_j(G_j) < a_j(G_{j'})$, the first inequality follows from $a_j(G_j), a_j(G_{j'}) < \nu_{\theta_j}$ (see Corollary 1), and the last inequality obtains under the assumption $\nu_{z_j} < \dfrac{1}{\gamma^2 \nu_{\theta_j}} \min(1/4, \rho/2)$ (see Condition (2)).

(ii) If $G_j/\gamma^2 \nearrow \nu_{\theta_j}$, then $a_j(G_j) = \dfrac{1}{\dfrac{\rho}{\nu_{\theta_j}} + \dfrac{1-\rho}{\nu_{\theta_j} - G_j/\gamma^2}} \searrow 0$ so that $\lambda_j \searrow 0$.

Q.E.D.

Proof of Proposition 3: (i) From Corollary 1, $a_j(G_j) < \nu_{\theta_j}$. Therefore, the β-adjusted expected return of the j'th basic security $-\gamma\Big[\nu_{\theta_j} + 2\gamma^2 a_j(G_j)^2 \nu_{z_j} - a_j(G_j)\Big]\bar{\xi}_j < 0$.

(ii) From Corollary 1, $a_j(G_j) < a_j(G_{j'})$ because $G_j > G_{j'}$. The difference between the β-adjusted expected returns of the j'th and j''th basic securities is

$$\begin{aligned}
& -\gamma\Big[\nu_{\theta_j} + 2\gamma^2 a_j(G_j)^2\nu_{z_j} - a_j(G_j)\Big]\bar{\xi}_j + \gamma\Big[\nu_{\theta_j} + 2\gamma^2 a_j(G_{j'})^2\nu_{z_j} - a_j(G_{j'})\Big]\bar{\xi}_j \\
= \;& \gamma\Big[a_j(G_j) - a_j(G_{j'}) - 2\gamma^2(a_j(G_j)^2 - a_j(G_{j'})^2)\nu_{z_j}\Big]\bar{\xi}_j \\
\propto \;& 2\gamma^2(a_j(G_j) + a_j(G_{j'}))\nu_{z_j} - 1 \\
< \;& 4\gamma^2\nu_{\theta_j}\nu_{z_j} - 1 \\
< \;& 0,
\end{aligned}$$

where the "$\propto$" follows from $a_j(G_j) < a_j(G_{j'})$, the first inequality follows from $a_j(G_j), a_j(G_{j'}) < \nu_{\theta_j}$ (see Corollary 1), and the last inequality obtains under the assumption $\nu_{z_j} < \dfrac{1}{\gamma^2\nu_{\theta_j}}\min(1/4, \rho/2)$ (see Condition (2)).

Q.E.D

Proof of Corollary 3: Write Eqs. (9) and (10) as

$$\begin{aligned}
X_{NG,j}(P_j) - (\bar{\xi}_j + \tilde{z}_j) \;&\sim\; N(A_{NG}\bar{\xi}_j, A_{NG}^2\nu_{z_j}), \\
X_{G,j}(P_j) - (\bar{\xi}_j + \tilde{z}_j) \;&\sim\; N(A_G\bar{\xi}_j, A_G^2\nu_{z_j}),
\end{aligned}$$

where

$$A_{NG} \equiv \frac{a_j(G_j)}{\nu_{\theta_j}} - 1, \quad \text{and} \quad A_G \equiv \frac{a_j(G_j)}{\nu_{\theta_j} - G_j/\gamma^2} - 1.$$

Here are some intermediate results we will use in the proof of this corollary. First, $A_{NG} < 0$ and decreases in G_j from Corollary 1. Second, $A_G = \dfrac{a_j(G_j)}{\nu_{\theta_j} - G_j/\gamma^2} - 1 = \dfrac{1}{\frac{\rho}{\nu_{\theta_j}}(\nu_{\theta_j} - G_j/\gamma^2) + 1 - \rho} - 1 > 0$, and increases in G_j.

Using the fact listed in Footnote 13 and the above new expressions for the distributions

of $X_{NG,j}(P_j)-(\bar{\xi}_j+\tilde{z}_j)$ and $X_{G,j}(P_j)-(\bar{\xi}_j+\tilde{z}_j)$,[13] we can express the total expected trading volume in the j'th basic security (Eq. (11)) as

$$\begin{aligned} T_j &= 0.5\rho E\Big[|X_{NG,j}(P_j)-(\bar{\xi}_j+\tilde{z}_j)|\Big]+0.5(1-\rho)E\Big[|X_{G,j}(P_j)-(\bar{\xi}_j+\tilde{z}_j)|\Big] \\ &= 0.5\rho(-A_{NG}\sqrt{\nu_{z_j}})\Big[2\phi(-\frac{\bar{\xi}_j}{\sqrt{\nu_{z_j}}})-\frac{\bar{\xi}_j}{\sqrt{\nu_{z_j}}}(1-2\Phi(\frac{\bar{\xi}_j}{\sqrt{\nu_{z_j}}}))\Big] \\ &\quad +0.5(1-\rho)A_G\sqrt{\nu_{z_j}}\Big[2\phi(\frac{\bar{\xi}_j}{\sqrt{\nu_{z_j}}})+\frac{\bar{\xi}_j}{\sqrt{\nu_{z_j}}}(1-2\Phi(-\frac{\bar{\xi}_j}{\sqrt{\nu_{z_j}}}))\Big]. \end{aligned}$$

Footnote 13 indicates that the values in the brackets are positive. From the above analysis, A_{NG} decreases in G_j, and A_G increases in G_j. Therefore, T_j increases in G_j.

Q.E.D.

Proof of Lemma 1: From Proposition 1, $\dfrac{\mathrm{Var}(\tilde{R}_j)}{\mathrm{Var}(\tilde{R}_M)}=\dfrac{\nu_{\theta_j}+\gamma^2a_j(G_j)^2\nu_{z_j}}{\mathrm{Var}(\tilde{R}_M)}$. This implies that typical basic securities with small $\dfrac{\mathrm{Var}(\tilde{R}_j)}{\mathrm{Var}(\tilde{R}_M)}$ also have small $\dfrac{\nu_{\theta_j}}{\mathrm{Var}(\tilde{R}_M)}$ and $\dfrac{\gamma^2a_j(G_j)^2\nu_{z_j}}{\mathrm{Var}(\tilde{R}_M)}$. We will use this property in the proof of this Lemma.

From Eq. (12), Proposition 1, and our computation of $\mathrm{Cov}(\tilde{R}_j,\tilde{R}_M)$ in Appendix B,

$$\begin{aligned} \mathrm{IVOL}_j &= \mathrm{Var}(\tilde{R}_j)-\frac{\mathrm{Cov}(\tilde{R}_j,\tilde{R}_M)^2}{\mathrm{Var}(\tilde{R}_M)} \\ &= \nu_{\theta_j}+\gamma^2a_j(G_j)^2\nu_{z_j}-\frac{(\nu_{\theta_j}+2\gamma^2a_j(G_j)^2\nu_{z_j})^2\bar{\xi}_j^2}{\mathrm{Var}(\tilde{R}_M)}. \end{aligned}$$

Denote $G(\nu_{\theta_j})=\mu\nu_{\theta_j}$. It follows that for the j'th and j''th typical basic securities,

$$\begin{aligned} \mathrm{IVOL}_j-\mathrm{IVOL}_{j'} &= \Big[\nu_{\theta_j}+\gamma^2a_j(G(\nu_{\theta_j}))^2\nu_{z_j}\Big]-\Big[\nu_{\theta_{j'}}+\gamma^2a_j(G(\nu_{\theta_{j'}}))^2\nu_{z_j}\Big] \\ &\quad -\frac{\nu_{\theta_j}+2\gamma^2a_j(G(\nu_{\theta_j}))^2\nu_{z_j}+\nu_{\theta_{j'}}+2\gamma^2a_j(G(\nu_{\theta'_j}))^2\nu_{z_j}}{\mathrm{Var}(\tilde{R}_M)}\bar{\xi}_j^2 \\ &\quad \Big[\Big[\nu_{\theta_j}+2\gamma^2a_j(G(\nu_{\theta_j}))^2\nu_{z_j}\Big]-\Big[\nu_{\theta_{j'}}+2\gamma^2a_j(G(\nu_{\theta_{j'}}))^2\nu_{z_j}\Big]\Big] \end{aligned}$$

[13] If $y\sim N(\bar{y},\nu)$, then $E[|y|]=\sqrt{\nu}\Big[2\phi(\frac{\bar{y}}{\sqrt{\nu}})+\frac{\bar{y}}{\sqrt{\nu}}(1-2\Phi(\frac{-\bar{y}}{\sqrt{\nu}}))\Big]$, where $\phi(.)$ and $\Phi(.)$ denote the probability density function (p.d.f.) and cumulative density function (c.d.f.) of standard normal distribution.

$$
\begin{aligned}
&= \Big[\nu_{\theta_j} + \gamma^2 a_j(G(\nu_{\theta_j}))^2\nu_{z_j}\Big] - \Big[\nu_{\theta_{j'}} + \gamma^2 a_j(G(\nu_{\theta_{j'}}))^2\nu_{z_j}\Big] \\
&\quad -\delta\Big[\Big[\nu_{\theta_j} + 2\gamma^2 a_j(G(\nu_{\theta_j}))^2\nu_{z_j}\Big] - \Big[\nu_{\theta_{j'}} + 2\gamma^2 a_j(G(\nu_{\theta_{j'}}))^2\nu_{z_j}\Big]\Big] \\
&= (1-\delta)(\nu_{\theta_j} - \nu_{\theta_{j'}}) + (1-2\delta)\gamma^2\nu_{z_j}\Big[a_j(G(\nu_{\theta_j}))^2 - a_j(G(\nu_{\theta_{j'}}))^2\Big],
\end{aligned}
$$

where $\delta = \dfrac{\nu_{\theta_j} + 2\gamma^2 a_j(G(\nu_{\theta_j}))^2\nu_{z_j} + \nu_{\theta_{j'}} + 2\gamma^2 a_j(G(\nu_{\theta'_j}))^2\nu_{z_j}}{\mathrm{Var}(\tilde{R}_M)}\bar{\xi}_j^2$. It follows from the above derived property that for the j'th and j''th typical basic securities, δ must be small. Thus, $1-\delta, 1-2\delta > 0$. Note that $\nu_{\theta_j} > \nu_{\theta_{j'}}$. For $\mathrm{IVOL}_j > \mathrm{IVOL}_{j'}$, it suffices that $a_j(G(\nu_{\theta_j})) > a_j(G(\nu_{\theta_{j'}}))$, which holds because

$$
\frac{da_j(G(\nu_{\theta_j}))}{d\nu_{\theta_j}} = a_j(G(\nu_{\theta_j}))^2\Big[\frac{\rho}{\nu_{\theta_j}^2} + \frac{1-\rho}{(\nu_{\theta_j} - \mu\nu_{\theta_j}/\gamma^2)^2}(1-\mu/\gamma^2)\Big] > 0.
$$

Q.E.D.

Proof of Proposition 4: Given $G_j = \mu\nu_{\theta_j}$ where $\mu < \gamma^2$, and taking derivatives of the β-adjusted expected return, $-\gamma(\nu_{\theta_j} + 2\gamma^2 a_j(G_j)^2\nu_{z_j} - a_j(G_j))\bar{\xi}_j$, w.r.t. ν_{θ_j} yields

$$
\begin{aligned}
&\frac{d\Big[-\gamma(\nu_{\theta_j} + 2\gamma^2 a_j(G_j)^2\nu_{z_j} - a_j(G_j))\bar{\xi}_j\Big]}{d\nu_{\theta_j}} \\
\propto\ & -1 + \Big[1 - 4\gamma^2 a_j(G_j)\nu_{z_j}\Big]a_j(G_j)^2\Big[\frac{\rho}{\nu_{\theta_j}^2} + \frac{1-\rho}{(\nu_{\theta_j} - G_j/\gamma^2)^2}(1-\mu/\gamma^2)\Big] \\
<\ & -1 + a_j(G_j)^2\Big[\frac{\rho}{\nu_{\theta_j}^2} + \frac{1-\rho}{(\nu_{\theta_j} - G_j/\gamma^2)^2}(1-\mu/\gamma^2)\Big] \\
\propto\ & -(\frac{\rho}{\nu_{\theta_j}} + \frac{1-\rho}{\nu_{\theta_j} - \mu\nu_{\theta_j}/\gamma^2})^2 + \frac{\rho}{\nu_{\theta_j}^2} + \frac{1-\rho}{(\nu_{\theta_j} - \mu\nu_{\theta_j}/\gamma^2)^2}(1-\mu/\gamma^2) \\
\propto\ & -(\rho + \frac{1-\rho}{1-\mu/\gamma^2})^2 + \rho + \frac{1-\rho}{1-\mu/\gamma^2} \\
\propto\ & -(\rho + \frac{1-\rho}{1-\mu/\gamma^2}) + 1 \\
<\ & 0.
\end{aligned}
$$

Q.E.D.

Proof of Proposition 5: It follows from Eq. (13) that

$$\begin{aligned} E(\tilde{R}_M) &= \sum_{j=1}^{K+N} E\Big[\bar{\xi}_j\tilde{\theta}_j + \tilde{z}_j\tilde{\theta}_j + \gamma a_j(G_j)(\bar{\xi}_j^2 + 2\bar{\xi}_j\tilde{z}_j + \tilde{z}_j^2)\Big] \\ &= \sum_{j=1}^{K+N}\Big[\gamma a_j(G_j)(\bar{\xi}_j^2 + \nu_{z_j})\Big] \\ &> 0. \end{aligned}$$

Q.E.D.

Proof of Proposition 6: (i) From Eqs. (17) and (18),

$$\begin{aligned} E(\tilde{R}_j^{A,NG}) = \gamma\nu_{\theta_j}\bar{\xi}_j &> E(\tilde{R}_j^{A,G}) = \gamma(\nu_{\theta_j} - G_j/\gamma^2)\bar{\xi}_j, \\ \mathrm{Var}(\tilde{R}_j^{A,NG}) = \nu_{\theta_j} + \gamma^2\nu_{\theta_j}^2\nu_{z_j} &> \mathrm{Var}(\tilde{R}_j^{A,G}) = \nu_{\theta_j} + \gamma^2(\nu_{\theta_j} - G_j/\gamma^2)^2\nu_{z_j}. \end{aligned}$$

(ii) From Eq. (17), it follows that the return of the market portfolio in the all-non-G economy is given by

$$\tilde{R}_M^{A,NG} = \sum_{j=1}^{K+N}(\bar{\xi}_j + \tilde{z}_j)\tilde{R}_j^{A,NG} = \sum_{j=1}^{K+N}\Big[(\bar{\xi}_j + \tilde{z}_j)(\tilde{\theta}_j + \gamma\nu_{\theta_j}(\bar{\xi}_j + \tilde{z}_j))\Big].$$

It follows immediately that

$$\begin{aligned} E(\tilde{R}_M^{A,NG}) &= \sum_{j=1}^{K+N} E\Big[(\bar{\xi}_j + \tilde{z}_j)(\tilde{\theta}_j + \gamma\nu_{\theta_j}(\bar{\xi}_j + \tilde{z}_j))\Big] = \sum_{j=1}^{K+N}\Big[\gamma\nu_{\theta_j}(\bar{\xi}_j^2 + \nu_{z_j})\Big], \\ \mathrm{Var}(\tilde{R}_M^{A,NG}) &= \sum_{j=1}^{K+N}\Big[(\nu_{\theta_j} + 4\gamma^2\nu_{\theta_j}^2\nu_{z_j})\bar{\xi}_j^2 + \nu_{\theta_j}\nu_{z_j} + 2\gamma^2\nu_{\theta_j}^2\nu_{z_j}^2\Big], \end{aligned}$$

where the last equality follows from Eq. (6) because the all-non-G economy can be viewed as the case in which $a(G_j) = \nu_{\theta_j}$.

From Eq. (18), it follows that the return of the market portfolio in the all-G economy is given by

$$\tilde{R}_M^{A,G} = \sum_{j=1}^{K+N}(\bar{\xi}_j + \tilde{z}_j)\tilde{R}_j^{A,G} = \sum_{j=1}^{K+N}\Big[(\bar{\xi}_j + \tilde{z}_j)(\tilde{\theta}_j + \gamma(\nu_{\theta_j} - G_j/\gamma^2)(\bar{\xi}_j + \tilde{z}_j))\Big].$$

It follows immediately that

$$\begin{aligned} E(\tilde{R}_M^{A,G}) &= \sum_{j=1}^{K+N} E\Big[(\bar{\xi}_j + \tilde{z}_j)(\tilde{\theta}_j + \gamma(\nu_{\theta_j} - G_j/\gamma^2)(\bar{\xi}_j + \tilde{z}_j))\Big] = \sum_{j=1}^{K+N} \Big[\gamma(\nu_{\theta_j} - G_j/\gamma^2)(\bar{\xi}_j^2 + \nu_{z_j})\Big], \\ \mathrm{Var}(\tilde{R}_M^{A,G}) &= \sum_{j=1}^{K+N} \Big[(\nu_{\theta_j} + 4\gamma^2(\nu_{\theta_j} - G_j/\gamma^2)^2\nu_{z_j})\bar{\xi}_j^2 + \nu_{\theta_j}\nu_{z_j} + 2\gamma^2(\nu_{\theta_j} - G_j/\gamma^2)^2\nu_{z_j}^2\Big], \end{aligned}$$

where the last equality follows from Eq. (6) because the all-G economy is the case in which $a(G_j) = \nu_{\theta_j} - G_j/\gamma^2$.

A direct comparison indicates that $E(\tilde{R}_M^{A,NG}) > E(\tilde{R}_M^{A,G})$ and $\mathrm{Var}(\tilde{R}_M^{A,NG}) > \mathrm{Var}(\tilde{R}_M^{A,G})$.

(iii) Note that $\lambda_j^{A,NG} = \dfrac{\gamma\nu_{\theta_j}\mathrm{Var}(\tilde{R}_M^{A,NG})}{\nu_{\theta_j} + 2\gamma^2\nu_{\theta_j}^2\nu_{z_j}}$ and $\lambda_j^{A,G} = \dfrac{\gamma(\nu_{\theta_j} - G_j/\gamma^2)\mathrm{Var}(\tilde{R}_M^{A,G})}{\nu_{\theta_j} + 2\gamma^2(\nu_{\theta_j} - G_j/\gamma^2)^2\nu_{z_j}}$. From part (ii), $\mathrm{Var}(\tilde{R}_M^{A,NG}) > \mathrm{Var}(\tilde{R}_M^{A,G})$. Thus, for $\lambda_j^{A,NG} > \lambda_j^{A,G}$, it suffices that

$$\begin{aligned} & \frac{\nu_{\theta_j}}{\nu_{\theta_j} + 2\gamma^2\nu_{\theta_j}^2\nu_{z_j}} - \frac{\nu_{\theta_j} - G_j/\gamma^2}{\nu_{\theta_j} + 2\gamma^2(\nu_{\theta_j} - G_j/\gamma^2)^2\nu_{z_j}} \\ \propto\ & \nu_{\theta_j}\Big[\nu_{\theta_j} + 2\gamma^2(\nu_{\theta_j} - G_j/\gamma^2)^2\nu_{z_j}\Big] - (\nu_{\theta_j} - G_j/\gamma^2)(\nu_{\theta_j} + 2\gamma^2\nu_{\theta_j}^2\nu_{z_j}) \\ \propto\ & \nu_{\theta_j} - 2\gamma^2(\nu_{\theta_j} - G_j/\gamma^2)\nu_{\theta_j}\nu_{z_j} \\ >\ & \nu_{\theta_j} - 2\gamma^2\nu_{\theta_j}^2\nu_{z_j} \\ >\ & 0, \end{aligned}$$

where the last inequality obtains under the assumption $\nu_{z_j} < \dfrac{1}{\gamma^2\nu_{\theta_j}}\min(1/4, \rho/2)$ (see Condition (2)).

Q.E.D.

Proof of Proposition 7: Note that $\lambda_j = \dfrac{\gamma a_j(G_j)\mathrm{Var}(\tilde{R}_M)}{\nu_{\theta_j} + 2\gamma^2 a_j(G_j)^2\nu_{z_j}}$ and $\lambda_{j'}^o = \dfrac{(\gamma/\rho)\nu_{\theta_j}\mathrm{Var}(\tilde{R}_M)}{\nu_{\theta_j} + 2(\gamma/\rho)^2\nu_{\theta_j}^2\nu_{z_j}}$. For $\lambda_j < \lambda_{j'}^o$, it suffices to show that

$$\begin{aligned} & \frac{\gamma a_j(G_j)}{\nu_{\theta_j} + 2\gamma^2 a_j(G_j)^2\nu_{z_j}} - \frac{(\gamma/\rho)\nu_{\theta_j}}{\nu_{\theta_j} + 2(\gamma/\rho)^2\nu_{\theta_j}^2\nu_{z_j}} \\ \propto\ & \gamma a_j(G_j)\Big[\nu_{\theta_j} + 2(\gamma/\rho)^2\nu_{\theta_j}^2\nu_{z_j}\Big] - (\gamma/\rho)\nu_{\theta_j}\Big[\nu_{\theta_j} + 2\gamma^2 a_j(G_j)^2\nu_{z_j}\Big] \\ =\ & \Big[\gamma a_j(G_j) - (\gamma/\rho)\nu_{\theta_j}\Big]\Big[\nu_{\theta_j} - 2\gamma a_j(G_j)(\gamma/\rho)\nu_{\theta_j}\nu_{z_j}\Big] \end{aligned}$$

$$
\begin{aligned}
&\propto \quad 2\gamma^2 a_j(G_j)\nu_{z_j} - \rho \\
&< \quad 2\gamma^2 \nu_{\theta_j}\nu_{z_j} - \rho \\
&< \quad 0,
\end{aligned}
$$

where the second "$\propto$" and the first inequality follow from $a_j(G_j) < \nu_{\theta_j}$ (see Corollary 1), and the last inequality obtains under the assumption $\nu_{z_j} < \frac{1}{\gamma^2 \nu_{\theta_j}} \min(1/4, \rho/2)$ (see Condition (2)).

Q.E.D.

Proof of Proposition 8: Note that

$$V|\tilde{\theta} \sim N(\bar{V} + \tilde{\theta}, \nu_\epsilon).$$

An i'th non-G trader, who observes $\tilde{\theta}$, has the following expected utility conditional on $\tilde{\theta}$:

$$E\Big[U(W_{i1})|\tilde{\theta}\Big] = -\exp\Big[-\gamma\Big[W_{i0} + X_i(E(V|\tilde{\theta}) - P) - 0.5\gamma X_i^2 \mathrm{Var}(V|\tilde{\theta})\Big]\Big].$$

The f.o.c. w.r.t. X_i implies that his demand can be expressed as:

$$X_{NG}(P, \tilde{\theta}) = \frac{E(V|\tilde{\theta}) - P}{\gamma \mathrm{Var}(V|\tilde{\theta})} = \frac{\bar{V} + \tilde{\theta} - P}{\gamma \nu_\epsilon}. \tag{21}$$

An i'th G trader, who chooses to spend c to observe $\tilde{\theta}$, has the following expected utility conditional on $\tilde{\theta}$:

$$
\begin{aligned}
E\Big[U_{G,I}(W_{i1})|\tilde{\theta}\Big] \quad = \quad & -\exp\Big[-\gamma\Big[(W_{i0} - c) \\
& + X_i(E(V|\tilde{\theta}) - P) - 0.5\gamma X_i^2(\mathrm{Var}(V|\tilde{\theta}) - G/\gamma^2)\Big]\Big].
\end{aligned} \tag{22}
$$

The f.o.c. w.r.t. X_i implies that his demand can be expressed as:

$$X_{G,I}(P, \tilde{\theta}) = \frac{E(V|\tilde{\theta}) - P}{\gamma(\mathrm{Var}(V|\tilde{\theta}) - G/\gamma^2)} = \frac{\bar{V} + \tilde{\theta} - P}{\gamma(\nu_\epsilon - G/\gamma^2)}. \tag{23}$$

Conjecture that the stock price takes the linear form given in Eq. (19). An i'th G trader, who chooses to remain uninformed, can infer ω from the stock price. Note that

$$V|\omega \sim N(\bar{V} + \frac{\nu_\theta}{\nu_\omega}\omega, \nu_\theta(1 - \frac{\nu_\theta}{\nu_\omega}) + \nu_\epsilon).$$

The uninformed trader has the following expected utility conditional on ω:

$$\begin{aligned} E\Big[U_{G,U}(W_{i1})|\omega\Big] &= -\exp\Big[-\gamma\Big[W_{i0} \\ &\quad +X_i(E(V|\omega) - P) - 0.5\gamma X_i^2(\mathrm{Var}(V|\omega) - G/\gamma^2)\Big]\Big]. \end{aligned} \tag{24}$$

The f.o.c. w.r.t. X_i implies that his demand can be expressed as:

$$X_{G,U}(P,\omega) = \frac{E(V|\omega) - P}{\gamma(\mathrm{Var}(V|\omega) - G/\gamma^2)} = \frac{\bar{V} + \frac{\nu_\theta}{\nu_\omega}\omega - P}{\gamma(\nu_\theta(1 - \frac{\nu_\theta}{\nu_\omega}) + \nu_\epsilon - G/\gamma^2)}. \tag{25}$$

The market clearing condition requires that

$$\bar{\xi} + \tilde{z} = \rho \cdot X_{NG}(P,\tilde{\theta}) + (1-\rho)\tau \cdot X_{G,I}(P,\tilde{\theta}) + (1-\rho)(1-\tau) \cdot X_{G,U}(P,\omega).$$

Plugging in the expressions for $X_{NG}(P,\tilde{\theta})$, $X_{G,I}(P,\tilde{\theta})$ and $X_{G,U}(P,\omega)$ into Eqs. (21), (23), and (25), and using the conjectured expression of P in Eq. (19) yields

$$\begin{aligned} \bar{\xi} + \tilde{z} &= N_1 \cdot (\bar{V} + \tilde{\theta} - P) + N_2 \cdot (\bar{V} + \tilde{\theta} - P) + N_3 \cdot (\bar{V} + \frac{\nu_\theta}{\nu_\omega}\omega - P) \\ &= (N_1 + N_2) \cdot \tilde{\theta} + N_3 \cdot \frac{\nu_\theta}{\nu_\omega}\omega - (N_1 + N_2 + N_3) \cdot b\omega \\ &\quad +(N_1 + N_2 + N_3) \cdot a, \end{aligned}$$

where N_1, N_2, and N_3 are defined in Proposition 8. It is straightforward to verify that this market clearing requirement is ensured by the parameters, a, b, and f, given in Proposition 8.

Q.E.D.

Proof of Lemma 2: Consider an informed G trader's expected utility, given by Eq. (22). Plugging in the optimal demand for the stock in Eq. (23) yields

$$\begin{aligned} E\Big[U_{G,I}(W_{i1})|\tilde{\theta}\Big] &= -\exp\Big[-\gamma(W_{i0}-c)\Big]\exp\Big[-0.5\frac{\Big[E(V|\tilde{\theta})-P\Big]^2}{\mathrm{Var}(V|\tilde{\theta})-G/\gamma^2}\Big] \\ &= -\exp\Big[-\gamma(W_{i0}-c)\Big]\exp\Big[-0.5\frac{(\bar{V}+\tilde{\theta}-P)^2}{\nu_\epsilon-G/\gamma^2}\Big] \\ &= -\exp\Big[-\gamma(W_{i0}-c)\Big]\exp\Big[-0.5\frac{\mathrm{Var}(\tilde{\theta}|\omega)}{\nu_\epsilon-G/\gamma^2}Y^2\Big], \end{aligned}$$

where $Y \equiv \dfrac{\bar{V}+\tilde{\theta}-P}{\sqrt{\mathrm{Var}(\tilde{\theta}|\omega)}}$ and $Y|\omega \sim N(\dfrac{\bar{V}+E(\tilde{\theta}|\omega)-P}{\sqrt{\mathrm{Var}(\tilde{\theta}|\omega)}},1)$. Thus,

$$\begin{aligned} E\Big[U_{G,I}(W_{i1})|\omega\Big] &= E\Big[E\Big[U_{G,I}(W_{i1})|\tilde{\theta}\Big]|\omega\Big] \\ &= -\exp\Big[-\gamma(W_{i0}-c)\Big]E\Big[\exp\Big[-0.5\frac{\mathrm{Var}(\tilde{\theta}|\omega)}{\nu_\epsilon-G/\gamma^2}Y^2\Big]|\omega\Big] \\ &= -\frac{\exp\Big[-\gamma(W_{i0}-c)\Big]}{\sqrt{1+\dfrac{\mathrm{Var}(\tilde{\theta}|\omega)}{\nu_\epsilon-G/\gamma^2}}}\exp\Big[-0.5\frac{\dfrac{\mathrm{Var}(\tilde{\theta}|\omega)}{\nu_\epsilon-G/\gamma^2}\Big[\dfrac{\bar{V}+E(\tilde{\theta}|\omega)-P}{\sqrt{\mathrm{Var}(\tilde{\theta}|\omega)}}\Big]^2}{1+\dfrac{\mathrm{Var}(\tilde{\theta}|\omega)}{\nu_\epsilon-G/\gamma^2}}\Big] \\ &= -\exp\Big[-\gamma(W_{i0}-c)\Big]\sqrt{\frac{\mathrm{Var}(V|\tilde{\theta})-G/\gamma^2}{\mathrm{Var}(V|\omega)-G/\gamma^2}}\exp\Big[-0.5\frac{\Big[E(V|\omega)-P\Big]^2}{\mathrm{Var}(V|\omega)-G/\gamma^2}\Big]. \end{aligned}$$

Here, we use the fact in Footnote 14 and the facts $\mathrm{Var}(V|\tilde{\theta}) = \nu_\epsilon$ and $\mathrm{Var}(V|\omega) = \nu_\epsilon + \mathrm{Var}(\tilde{\theta}|\omega)$.[14]

Now consider an uninformed trader's expected utility, given by Eq. (24). Plugging in the optimal demand for the stock from Eq. (25) yields

$$E\Big[U_{G,U}(W_{i1})|\omega\Big] = -\exp(-\gamma W_{i0})\exp\Big[-0.5\frac{\Big[E(V|\omega)-P\Big]^2}{\mathrm{Var}(V|\omega)-G/\gamma^2}\Big].$$

[14] If $y \sim N(\bar{y},1)$, then $E(\exp(-ty^2)) = \dfrac{1}{\sqrt{1+2t}}\exp(-\dfrac{t\bar{y}^2}{1+2t})$.

It follows immediately that

$$\begin{aligned} & E\Big[U_{G,I}(W_{i1})|\omega\Big] - E\Big[U_{G,U}(W_{i1})|\omega\Big] \\ = \quad & \Big[\exp(\gamma c)\sqrt{\frac{\text{Var}(V|\tilde{\theta}) - G/\gamma^2}{\text{Var}(V|\omega) - G/\gamma^2}} - 1\Big] E\Big[U_{G,U}(W_{i1})|\omega\Big]. \end{aligned}$$

Taking the ex ante expectation yields

$$E\Big[U_{G,I}(W_{i1})\Big] - E\Big[U_{G,U}(W_{i1})\Big] = \Big[\exp(\gamma c)\sqrt{\frac{\text{Var}(V|\tilde{\theta}) - G/\gamma^2}{\text{Var}(V|\omega) - G/\gamma^2}} - 1\Big] E\Big[U_{G,U}(W_{i1})\Big].$$

Denote $\psi(\tau) \equiv \exp(2\gamma c)\dfrac{\text{Var}(V|\tilde{\theta}) - G/\gamma^2}{\text{Var}(V|\omega) - G/\gamma^2} - 1$. $\psi(\tau)$ is a function of τ because according to Proposition 8, f and therefore $\text{Var}(V|\omega)$ are functions of τ. If $\psi(\tau)$ is negative (positive), then the above difference in the ex ante utility is positive (negative) because $E\Big[U_{G,U}(W_{i1})\Big]$ is negative, and the G trader prefers to become informed by spending c (remain uninformed). If $\psi(\tau) = 0$, then he is indifferent between becoming informed and remaining uninformed.

Q.E.D.

Proof of Proposition 9: Lemma 2 uses the function $\psi(\tau)$ to describe the G traders' decision to become informed (by spending c) and remain uninformed. It follows that to prove this proposition, it suffices to show that $\psi(\tau)$ increases in $\tau \in [0, 1]$. We show this monotonic property in what follows.

Proposition 8 implies that f and, therefore, $\nu_\omega = \nu_\theta + f^2\nu_z$ decrease in τ. It follows that $\text{Var}(V|\omega) = \nu_\theta(1 - \dfrac{\nu_\theta}{\nu_\omega}) + \nu_\epsilon$ decreases in τ. Note that $\psi(\tau)$ (from its expression in Lemma 2) decreases in $\text{Var}(V|\omega)$. It follows immediately that $\psi(\tau)$ increases in τ.

Q.E.D.

Proof of Corollary 4: (i) Consider Eq. (20), which specifies the interior equilibrium for τ. It is straightforward to show that $\psi(\tau)$ decreases in both $\text{Var}(V|\omega)$ and G. Therefore,

$\text{Var}(V|\omega)$ decreases in G.

(ii) Consider Eq. (20) again. It is obvious that $\text{Var}(V|\omega)$ depends on variables such as c, γ, G, and $\text{Var}(V|\tilde{\theta})$, which do not involve ρ.

Q.E.D.

Proof of Proposition 10: We solve for the equilibrium and prove the proposition using backward induction, in three steps.

Step 1: In this step, suppose $\tilde{\theta}_1 > H_\theta$ and therefore $P_1 > P_0$ (which we will show below). An agent remains a non-G trader (becomes a G trader) with probability $\rho\ (1-\rho)$. Thus, there is a mass ρ of non-G traders and a mass $1-\rho$ of G traders at Date 2.

Focus on Date 2 for the moment. Write an i'th non-G trader's wealth at Date 3 as $W_{i3} = W_{i2} + X_{i2}(V - P_2)$. His expected utility at Date 2 can be expressed as:

$$E\Big[U_{NG}(W_{i3})|\tilde{\theta}_1, \tilde{\theta}_2\Big] = -\exp\Big[-\gamma W_{i2} \\ -\gamma\Big[X_{i2}(E(V|\tilde{\theta}_1, \tilde{\theta}_2) - P_2) - 0.5\gamma X_{i2}^2 \text{Var}(V|\tilde{\theta}_1, \tilde{\theta}_2)\Big]\Big]. \quad (26)$$

He needs to choose X_{i2} to maximize this expected utility. The f.o.c. implies that his demand can be expressed as:

$$X_{NG,2}(\tilde{\theta}_1, \tilde{\theta}_2, P_2) = \frac{E(V|\tilde{\theta}_1, \tilde{\theta}_2) - P_2}{\gamma \text{Var}(V|\tilde{\theta}_1, \tilde{\theta}_2)} = \frac{\bar{V} + \tilde{\theta}_1 + \tilde{\theta}_2 - P_2}{\gamma \nu_\theta}. \quad (27)$$

An i'th G trader's expected utility at Date 2 can be expressed as:

$$E\Big[U_G(W_{i3}, X_{i2}))|\tilde{\theta}_1, \tilde{\theta}_2\Big] = -\exp\Big[-\gamma W_{i2} \\ -\gamma\Big[X_{i2}(E(V|\tilde{\theta}_1, \tilde{\theta}_2) - P_2) - 0.5\gamma X_{i2}^2 \text{Var}(V|\tilde{\theta}_1, \tilde{\theta}_2) + 0.5 G X_{i2}^2/\gamma\Big]\Big]. \quad (28)$$

He needs to choose X_{i2} to maximize this expected utility. The f.o.c. implies that his demand can be expressed as:

$$X_{G2}(\tilde{\theta}_1, \tilde{\theta}_2, P_2) = \frac{E(V|\tilde{\theta}_1, \tilde{\theta}_2) - P_2}{\gamma \text{Var}(V|\tilde{\theta}_1, \tilde{\theta}_2) - G/\gamma} = \frac{\bar{V} + \tilde{\theta}_1 + \tilde{\theta}_2 - P_2}{\gamma(\nu_\theta - G/\gamma^2)}. \quad (29)$$

It follows from Eqs. (27) and (29) that the market clearing condition requires

$$\begin{aligned}\bar{\xi} &= \rho \cdot X_{NG,2}(\tilde{\theta}_1, \tilde{\theta}_2, P_2) + (1-\rho) \cdot X_{G2}(\tilde{\theta}_1, \tilde{\theta}_2, P_2) \\ &= \frac{\rho}{\gamma \nu_\theta}(\bar{V} + \tilde{\theta}_1 + \tilde{\theta}_2 - P_2) + \frac{1-\rho}{\gamma(\nu_\theta - G/\gamma^2)}(\bar{V} + \tilde{\theta}_1 + \tilde{\theta}_2 - P_2).\end{aligned}$$

Therefore,

$$P_2 = \bar{V} + \tilde{\theta}_1 + \tilde{\theta}_2 - \gamma a(G)\bar{\xi},$$

where $a(G) = \dfrac{1}{\dfrac{\rho}{\nu_\theta} + \dfrac{1-\rho}{\nu_\theta - G/\gamma^2}}$.

Consider a non-G trader's expected utility at Date 2 in Eq. (26). Plugging in the optimal demand for the risky security from Eq. (27) yields

$$\begin{aligned}E\Big[U_{NG}(W_{i3})|\tilde{\theta}_1, \tilde{\theta}_2\Big] &= -\exp\Big[-\gamma W_{i2} - 0.5\frac{(\bar{V} + \tilde{\theta}_1 + \tilde{\theta}_2 - P_2)^2}{\nu_\theta}\Big] \\ &= -\exp\Big[-\gamma W_{i2} - 0.5\frac{(\gamma a(G)\bar{\xi})^2}{\nu_\theta}\Big]. \qquad (30)\end{aligned}$$

Consider a G trader's expected utility at Date 2 in Eq. (28). Plugging in the optimal demand for the risky security from Eq. (29) yields

$$\begin{aligned}E\Big[U_G(W_{i3}, X_{i2})|\tilde{\theta}_1, \tilde{\theta}_2\Big] &= -\exp\Big[-\gamma W_{i2} - 0.5\frac{(\bar{V} + \tilde{\theta}_1 + \tilde{\theta}_2 - P_2)^2}{\nu_\theta - G/\gamma^2}\Big] \\ &= -\exp\Big[-\gamma W_{i2} - 0.5\frac{(\gamma a(G)\bar{\xi})^2}{\nu_\theta - G/\gamma^2}\Big]. \qquad (31)\end{aligned}$$

Now focus on Date 1. As argued above, an agent knows that he may or may not become a G trader. Write an i'th trader's wealth at Date 2 as $W_{i2} = W_{i1} + X_{i1}(P_2 - P_1)$. It follows from Eqs. (30) and (31) that his expected utility at Date 1 can be expressed as:

$$\begin{aligned}E\Big[U(W_{i3})|\tilde{\theta}_1\Big] &= \rho E\Big[U_{NG}(W_{i3})|\tilde{\theta}_1\Big] + (1-\rho)E\Big[U_G(W_{i3}, X_{i2})|\tilde{\theta}_1\Big] \\ &= -\exp\Big[-\gamma W_{i1} - \gamma\Big[X_{i1}(E(P_2|\tilde{\theta}_1) - P_1) - 0.5\gamma X_{i1}^2 \mathrm{Var}(P_2|\tilde{\theta}_1)\Big]\Big] \\ &\quad \cdot\Big[\rho\ \exp(-0.5\frac{(\gamma a(G)\bar{\xi})^2}{\nu_\theta}) + (1-\rho)\exp(-0.5\frac{(\gamma a(G)\bar{\xi})^2}{\nu_\theta - G/\gamma^2})\Big]. \qquad (32)\end{aligned}$$

He needs to choose X_{i1} to maximize this expected utility. The f.o.c. implies that his demand can be expressed as:

$$X_1(\tilde{\theta}_1, P_1) = \frac{E(P_2|\tilde{\theta}_1) - P_1}{\gamma \text{Var}(P_2|\tilde{\theta}_1)} = \frac{\bar{V} + \tilde{\theta}_1 - \gamma a(G)\bar{\xi} - P_1}{\gamma \nu_\theta}.$$

The market clearing condition, $X_1(\tilde{\theta}_1, P_1) = \bar{\xi}$, implies

$$P_1 = \bar{V} + \tilde{\theta}_1 - \gamma \nu_\theta \bar{\xi} - \gamma a(G)\bar{\xi}. \tag{33}$$

Plugging the derived $X_1(P_1, \tilde{\theta}_1)$ and P_1 back into his expected utility at Date 1 using Eq. (32) yields

$$\begin{aligned} E\Big[U(W_{i3})|\tilde{\theta}_1\Big] &= -\exp\Big[-\gamma W_{i1} - 0.5\frac{(\gamma\nu_\theta\bar{\xi})^2}{\nu_\theta}\Big] \\ &\quad \cdot\Big[\rho\ \exp(-0.5\frac{(\gamma a(G)\bar{\xi})^2}{\nu_\theta}) + (1-\rho)\exp(-0.5\frac{(\gamma a(G)\bar{\xi})^2}{\nu_\theta - G/\gamma^2})\Big]. \end{aligned} \tag{34}$$

Step 2: In this step, suppose $\tilde{\theta}_1 \leq H_\theta$ and therefore $P_1 \leq P_0$ (which we will show). An agent remains a non-G trader. We can use a similar derivation as in Step 1, except that we impose $G = 0$ (note $a(0) = \nu_\theta$), to show

$$\begin{aligned} P_2 &= \bar{V} + \tilde{\theta}_1 + \tilde{\theta}_2 - \gamma\nu_\theta\bar{\xi}, \\ P_1 &= \bar{V} + \tilde{\theta}_1 - 2\gamma\nu_\theta\bar{\xi}. \end{aligned} \tag{35}$$

Moreover, the agent's expected utility at date 1 is given by

$$EU(W_{i3}|\tilde{\theta}_1) = -\exp\Big[-\gamma W_{i1} - \frac{(\gamma\nu_\theta\bar{\xi})^2}{\nu_\theta}\Big]. \tag{36}$$

Step 3: We now focus on date 0. Since it is before $\tilde{\theta}_1$ is released and P_1 is formed, all agents have identical preferences and beliefs and hold the same long position of the risky asset to clear the market.

Write an i'th trader's wealth at date 1 as $W_{i1} = W_{i0} + X_{i0}(P_1 - P_0)$. If $\tilde{\theta}_1 > H_\theta$ so $P_1 > P_0$ and he makes money, he may become a G trader. He will be in the regime

characterized by Eqs. (33) and (34). If $\tilde{\theta}_1 \leq H_\theta$ so $P_1 \leq P_0$ and he does not make money, he will remain a non-G trader for sure. He will be in the regime characterized by Eqs. (35), and (36). Accounting for both cases, we can write his expected utility at date 0 as

$$\begin{aligned} E\Big[U(W_{i3})\Big] &= -\int_{H_\theta}^{\infty} \exp\Big[-\gamma W_{i0} - \gamma X_{i0}(\bar{V} + \tilde{\theta}_1 - \gamma\nu_\theta\bar{\xi} - \gamma a(G)\bar{\xi} - P_0) - 0.5\frac{(\gamma\nu_\theta\bar{\xi})^2}{\nu_\theta}\Big] \\ &\quad \cdot\Big[\rho\exp(-0.5\frac{(\gamma a(G)\bar{\xi})^2}{\nu_\theta}) + (1-\rho)\exp(-0.5\frac{(\gamma a(G)\bar{\xi})^2}{\nu_\theta - G/\gamma^2})\Big]d\Phi(\frac{\tilde{\theta}_1}{\sqrt{\nu_\theta}}) \\ &\quad -\int_{-\infty}^{H_\theta} \exp\Big[-\gamma W_{i0} - \gamma X_{i0}(\bar{V} + \tilde{\theta}_1 - 2\gamma\nu_\theta\bar{\xi} - P_0) - \frac{(\gamma\nu_\theta\bar{\xi})^2}{\nu_\theta}\Big]d\Phi(\frac{\tilde{\theta}_1}{\sqrt{\nu_\theta}}), \end{aligned}$$

where $\Phi(.)$ is the cumulative density function of standard normal distribution. The agent needs to choose X_{i0} to maximize this expected utility. Taking the f.o.c. w.r.t. X_{i0}, imposing the market clearing condition $X_{i0} = \bar{\xi}$, and simplifying items yields

$$\begin{aligned} 0 &= \int_{H_\theta}^{\infty} \frac{\bar{V} + \tilde{\theta}_1 - \gamma\nu_\theta\bar{\xi} - \gamma a(G)\bar{\xi} - P_0}{\exp(\gamma\bar{\xi}\tilde{\theta}_1)} \\ &\quad \cdot\Big[\rho\exp(-0.5\frac{(\gamma a(G)\bar{\xi})^2}{\nu_\theta}) + (1-\rho)\exp(-0.5\frac{(\gamma a(G)\bar{\xi})^2}{\nu_\theta - G/\gamma^2})\Big]d\Phi(\frac{\tilde{\theta}_1}{\sqrt{\nu_\theta}}) \\ &\quad +\int_{-\infty}^{H_\theta} \frac{\bar{V} + \tilde{\theta}_1 - 2\gamma\nu_\theta\bar{\xi} - P_0}{\exp\Big[\gamma\bar{\xi}(\tilde{\theta}_1 - \gamma(\nu_\theta - a(G))\bar{\xi})\Big]}\exp(-0.5\frac{(\gamma\nu_\theta\bar{\xi})^2}{\nu_\theta})d\Phi(\frac{\tilde{\theta}_1}{\sqrt{\nu_\theta}}). \end{aligned} \tag{37}$$

If $\tilde{\theta}_1 = H_\theta$, then $P_1 = P_0$ where P_1 is given by Eq. (35). This implies

$$P_0 = \bar{V} + H_\theta - 2\gamma\nu_\theta\bar{\xi}.$$

Plugging this expression for P_0 back into Eq. (37) yields

$$\begin{aligned} 0 &= \int_{H_\theta}^{\infty} \frac{\tilde{\theta}_1 - H_\theta + \gamma(\nu_\theta - a(G))\bar{\xi}}{\exp(\gamma\bar{\xi}\tilde{\theta}_1)} \\ &\quad \cdot\Big[\rho\exp(-0.5\frac{(\gamma a(G)\bar{\xi})^2}{\nu_\theta}) + (1-\rho)\exp(-0.5\frac{(\gamma a(G)\bar{\xi})^2}{\nu_\theta - G/\gamma^2})\Big]d\Phi(\frac{\tilde{\theta}_1}{\sqrt{\nu_\theta}}) \\ &\quad +\int_{-\infty}^{H_\theta} \frac{\tilde{\theta}_1 - H_\theta}{\exp\Big[\gamma\bar{\xi}(\tilde{\theta}_1 - \gamma(\nu_\theta - a(G))\bar{\xi})\Big]}\exp(-0.5\frac{(\gamma\nu_\theta\bar{\xi})^2}{\nu_\theta})d\Phi(\frac{\tilde{\theta}_1}{\sqrt{\nu_\theta}}). \end{aligned}$$

It is straightforward to show that the right hand side of this equation decreases in H_θ, and is positive (negative) if $H_\theta \searrow -\infty$ ($H_\theta \nearrow \infty$). Therefore, H_θ is uniquely determined by this equation.

Q.E.D.

Appendix B

[Computation of Market Portfolio's Return Volatility From the Definition in Section 2.3)]

From Proposition 1, the returns of the j'th basic security and the market portfolio are given by

$$\begin{aligned}\tilde{R}_j &= \tilde{\theta}_j - P_j = \tilde{\theta}_j + \gamma a_j(G_j)(\bar{\xi}_j + \tilde{z}_j), \\ \tilde{R}_M &= \sum_{j=1}^{K+N} (\bar{\xi}_j + \tilde{z}_j)\tilde{R}_j = \sum_{j=1}^{K+N} \Big[(\bar{\xi}_j + \tilde{z}_j)(\tilde{\theta}_j + \gamma a_j(G_j)(\bar{\xi}_j + \tilde{z}_j))\Big].\end{aligned}$$

It follows that

$$\begin{aligned}\mathrm{Var}(\tilde{R}_M) &= \sum_{j=1}^{K+N} \mathrm{Var}\Big[(\bar{\xi}_j + \tilde{z}_j)(\tilde{\theta}_j + \gamma a_j(G_j)(\bar{\xi}_j + \tilde{z}_j))\Big] \\ &= \sum_{j=1}^{K+N} \mathrm{Var}\Big[\bar{\xi}_j\tilde{\theta}_j + 2\bar{\xi}_j\gamma a_j(G_j)\tilde{z}_j + \tilde{z}_j\tilde{\theta}_j + \gamma a_j(G_j)\tilde{z}_j^2\Big] \\ &= \sum_{j=1}^{K+N} \Big[E\Big[(\bar{\xi}_j\tilde{\theta}_j + 2\bar{\xi}_j\gamma a_j(G_j)\tilde{z}_j + \tilde{z}_j\tilde{\theta}_j + \gamma a_j(G_j)\tilde{z}_j^2)^2\Big] \\ &\quad -\Big[E(\bar{\xi}_j\tilde{\theta}_j + 2\bar{\xi}_j\gamma a_j(G_j)\tilde{z}_j + \tilde{z}_j\tilde{\theta}_j + \gamma a_j(G_j)\tilde{z}_j^2)\Big]^2\Big] \\ &= \sum_{j=1}^{K+N} \Big[(\nu_{\theta_j} + 4\gamma^2 a_j(G_j)^2\nu_{z_j})\bar{\xi}_j^2 + \nu_{\theta_j}\nu_{z_j} + 3\gamma^2 a_j(G_j)^2\nu_{z_j}^2 - \gamma^2 a_j(G_j)^2\nu_{z_j}^2\Big] \\ &= \sum_{j=1}^{K+N} \Big[(\nu_{\theta_j} + 4\gamma^2 a_j(G_j)^2\nu_{z_j})\bar{\xi}_j^2 + \nu_{\theta_j}\nu_{z_j} + 2\gamma^2 a_j(G_j)^2\nu_{z_j}^2\Big].\end{aligned}$$

Note that in the third equality, all cross-products of items in the first bracket have mean zero. In the fourth equality, $E(\tilde{z}_j^4) = 3\nu_{z_j}^2$.

Q.E.D.

[Computation of the Covariance between the j'th Basic Security's and Market Portfolio's Returns From the Definition in Section 2.4)]

From the expressions of $\tilde{R}_j$ and $\tilde{R}_M$,

$$\begin{aligned}\mathrm{Cov}(\tilde{R}_j, \tilde{R}_M) &= \mathrm{Cov}(\tilde{R}_j, \sum_{j=1}^{K+N} (\bar{\xi}_j + \tilde{z}_j)\tilde{R}_j) \\ &= \mathrm{Cov}(\tilde{R}_j, (\bar{\xi}_j + \tilde{z}_j)\tilde{R}_j) = \mathrm{Cov}(\tilde{R}_j, \bar{\xi}_j\tilde{R}_j) + \mathrm{Cov}(\tilde{R}_j, \tilde{z}_j\tilde{R}_j),\end{aligned}$$

where the second equality obtains because basic securities have independent random supplies and payoffs.

$$\begin{aligned}\mathrm{Cov}(\tilde{R}_j, \bar{\xi}_j\tilde{R}_j) &= \bar{\xi}_j \mathrm{Var}(\tilde{R}_j) = \nu_{\theta_j}\bar{\xi}_j + \gamma^2 a_j(G_j)^2\nu_{z_j}\bar{\xi}_j, \\ \mathrm{Cov}(\tilde{R}_j, \tilde{z}_j\tilde{R}_j) &= E(\tilde{z}_j\tilde{R}_j^2) - E(\tilde{R}_j)E(\tilde{z}_j\tilde{R}_j) \\ &= E\Big[\tilde{z}_j(\tilde{\theta}_j + \gamma a_j(G_j)(\bar{\xi}_j + \tilde{z}_j))^2\Big] \\ &\quad -E\Big[\tilde{\theta}_j + \gamma a_j(G_j)(\bar{\xi}_j + \tilde{z}_j)\Big]E\Big[\tilde{z}_j(\tilde{\theta}_j + \gamma a_j(G_j)(\bar{\xi}_j + \tilde{z}_j))\Big] \\ &= 2\gamma^2 a_j(G_j)^2\nu_{z_j}\bar{\xi}_j - \gamma^2 a_j(G_j)^2\nu_{z_j}\bar{\xi}_j \\ &= \gamma^2 a_j(G_j)^2\nu_{z_j}\bar{\xi}_j.\end{aligned}$$

Therefore, $\mathrm{Cov}(\tilde{R}_j, \tilde{R}_M) = \nu_{\theta_j}\bar{\xi}_j + 2\gamma^2 a_j(G_j)^2\nu_{z_j}\bar{\xi}_j$.

Q.E.D.

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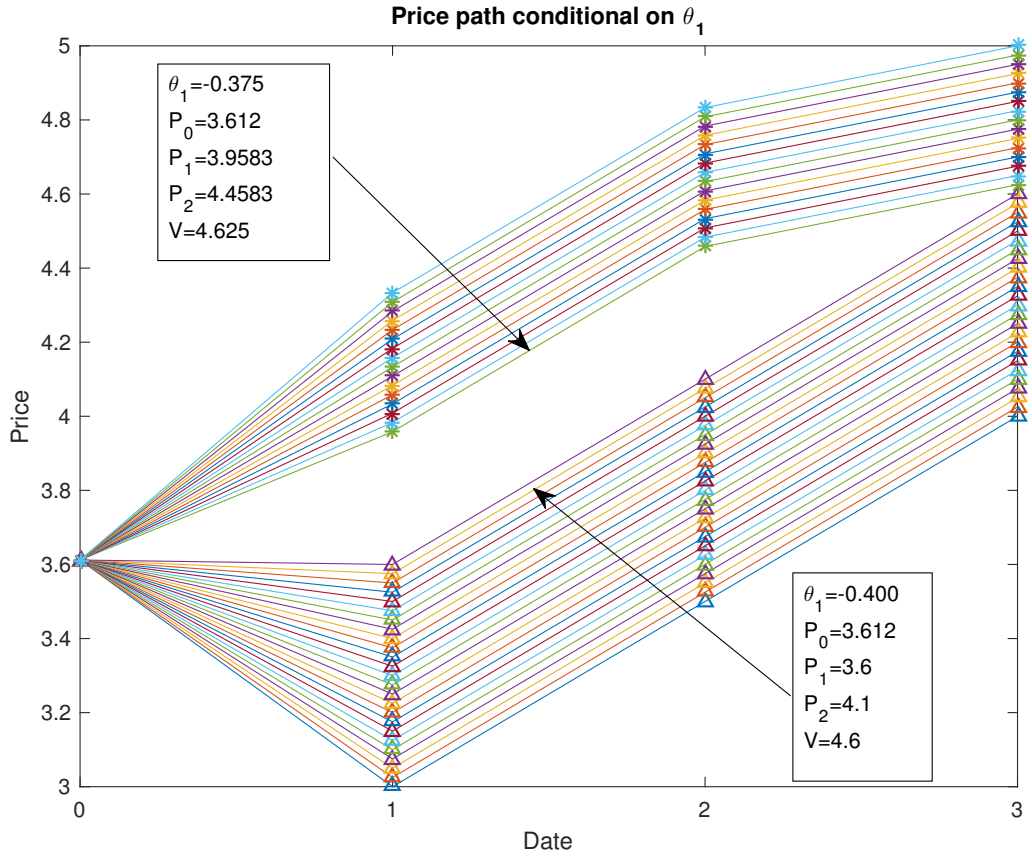
Figure 1: Price Reaction and Long-Run Performance

This graph plots the price paths conditional on the public announcement $\tilde{\theta}_1$. We assume the parameter values $\bar{V} = 5$, $\nu_\theta = 1$, $\bar{\xi} = 1$, $\gamma = 0.5$, $\rho = 0.5$, and $G = 0.2$. This implies that the threshold for the probabilistic conversion to a G trader is $H_\theta = -0.388$. The realizations of $\tilde{\theta}_2$ and $\tilde{\theta}_3$ are assumed to be zero, i.e., their mean. From the bottom to the top, each path represents a realization of $\tilde{\theta}_1$ from -1 to 0 (step size=0.025). $\tilde{\theta}_1 \leq H_\theta$ for the paths indicated by $\triangle$'s. $\tilde{\theta}_1 > H_\theta$ for the paths indicated by *'s.