Static Hedging of Non-Exchange Traded Options in South Africa

Alexis Levendis

University of Pretoria

26 July 2023

Overview

- The risk-neutral measure is often neglected in favour of the real-world measure due to the pricing of contingent claims.
- The real-world measure is extremely useful in risk management applications.
- We consider a static hedging experiment for vanilla European and European spread call options in a South African context.
- The experiment links the risk-neutral and real-world probability measures, which can help inform trading decisions.

Stochastic volatility double jump (SVJJ) model

$$dS(t) = (\mu - \lambda\mu_J)S(t)dt + \sqrt{v(t)}S(t)dW_x(t) + JS(t)dN(t)$$

$$dv(t) = \left(\alpha - \beta v(t)\right) + \sigma_v\sqrt{v(t)}dW_v(t) + ZdN(t)$$

$$dW_x(t)dW_v(t) = \rho_{x,v}dt,$$

where

$$\mu_J = \frac{\exp\left\{\mu_S + \frac{\sigma_S^2}{2}\right\}}{1 - \rho_J\mu_V} - 1,$$

and

$$Z \sim Exponential(\mu_V),$$

$$1 + J \mid Z \sim lognormal(\mu_S + \rho_J Z, \sigma_S^2),$$

SVJJ characteristic function

$$\phi_{SVJJ}(u) = \phi_H(u)\phi_J(u),$$

where

$$\phi_H(u) = e^{iu(x(0)+rT)+C(u,T)\frac{\alpha}{\beta}+D(u,T)v(0)},$$

and

$$C(u,T) = \beta\left[\left(\frac{Q-D_1}{2R}\right)T - \frac{2}{\sigma_v^2}\log\left(\frac{1-Ge^{-D_1T}}{1-G}\right)\right],$$

$$D(u,T) = \frac{Q-D_1}{2R}\left[\frac{1-e^{-D_1T}}{1-Ge^{-D_1T}}\right],$$

with

$$D_1 = \sqrt{Q^2-4PR},$$

$$G = \frac{Q-D_1}{Q+D_1},$$

SVJJ characteristic function (continued)

$$\begin{aligned}
P &= \frac{-u^2 - iu}{2}, \\
Q &= \beta - \rho_{x,v}\sigma_v iu, \\
R &= \frac{1}{2}\sigma_v^2.
\end{aligned}$$

Furthermore,

$$\phi_J(u) = e^{-\lambda T(1+iu\mu_J)+\lambda \exp\left\{iu\mu_S + \frac{\sigma_S^2(iu)^2}{2}\right\}\nu},$$

where

$$\begin{aligned}
\nu = {} & \frac{Q + D_1}{(Q + D_1)c - 2\mu_V P} + \frac{4\mu_V P}{(D_1 c)^2 - (2\mu_V P - Qc)^2} \\
& \times \log\left[1 - \frac{(D_1 - Q)c + 2\mu_V P}{2D_1 c}\left(1 - e^{-D1T}\right)\right], \\
c = {} & 1 - iu\rho_J\mu_V.
\end{aligned}$$

Static hedging approach #1

$$\min_{\boldsymbol{B}} \sum_{i=1}^{n} C(i)B(i),$$

subject to

$$\sum_{i=1}^{n} F(ij)B(i) \geq Y(j), \;\; j = 1, 2, ..., m,$$

where

$i = 1, 2, ..., n$:= the number of instruments in the replicating portfolio;

$j = 1, 2, ..., m$:= the price of the underlying asset at some future time;

$C(i)$:= the current price of the i^{th} instrument;

$B(i)$:= the number of units of the i^{th} instrument;

$F(ij)$:= the future price of the i^{th} instrument in state j; and

$Y(j)$:= the future price of the target option in state j.

Static hedging approach #2

$$\min_{\boldsymbol{B}} \sum_{j=1}^{m} \left(Y(j) - \sum_{i=1}^{n} F(ij)B(i) \right)^2,$$

subject to

$$\sum_{i=1}^{n} C(i)B(i) \leq w,$$

where

$i = 1, 2, ..., n \quad :=$ the number of instruments in the replicating portfolio;

$j = 1, 2, ..., m \quad :=$ the price of the underlying asset at some future time;

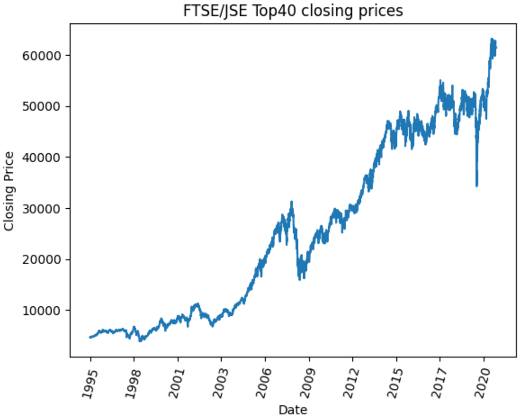
$C(i) :=$ the current price of the i^{th} instrument;

$B(i) :=$ the number of units of the i^{th} instrument;

$F(ij) :=$ the future price of the i^{th} instrument in state j;

$Y(j) :=$ the future price of the target option in state j.

FTSE/JSE Top40 index



SVJJ calibration to FTSE/JSE Top40 ($\mathbb{P}$-measure)

Parameter	Estimate
μ	0.1180
α	0.2888
β	6.0176
σ_v	0.4543
$\rho_{x,v}$	-0.9374
λ	4.7284
σ_S	0.0137
μ_V	0.0077
ρ_J	-0.3052
$T\hat{\theta}$	5.1022
$\chi^2_{0.05}$	5.9915

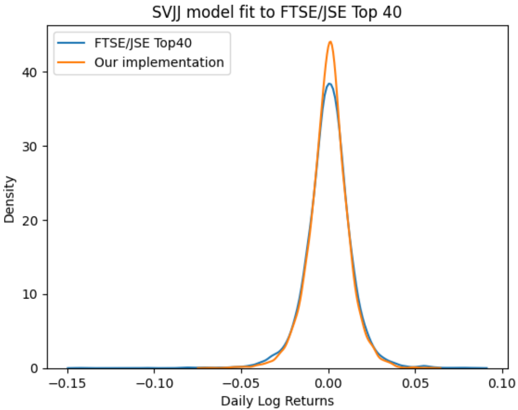
Table: SVJJ $\mathbb{P}$-parameters for FTSE/JSE Top40

SVJJ daily statistics versus FTSE/JSE Top40

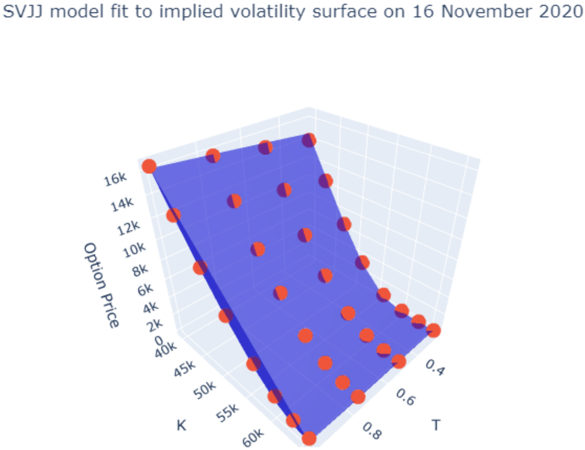
Statistic	FTSE/JSE Top40 index	SVJJ model
Mean	0.0385%	0.0406%
Std dev	1.3290%	1.1410%
Skewness	-0.4369	-0.2418
Kurtosis	9.4344	5.0463
Minimum	-0.1429	-0.0695
Maximum	0.0845	0.0592

Table: SVJJ model daily statistics for the FTSE/JSE Top40

SVJJ fit to FTSE/JSE Top40



SVJJ calibration to Top40 option price surface



SVJJ parameters ($\mathbb{Q}$-measure)

Parameter	**Estimate**
r	0.0700
$\tilde{\alpha}$	0.0333
$\tilde{\beta}$	0.9995
$\tilde{\sigma_v}$	0.3827
$\tilde{\rho_{x,v}}$	-0.9205
$\tilde{\lambda}$	0.0583
$\tilde{\sigma_S}$	0.0058
$\tilde{\mu_V}$	0.0058
$\tilde{\rho_J}$	0.0097

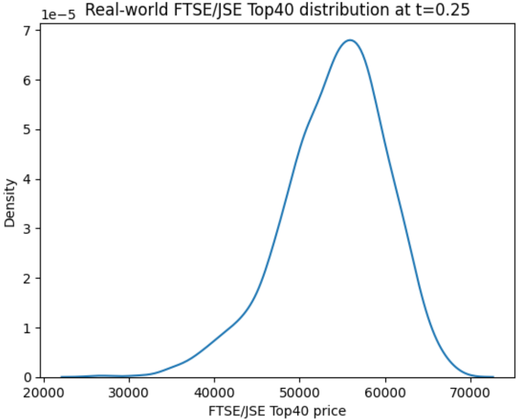
Table: SVJJ $\mathbb{Q}$-parameters for FTSE/JSE Top40

FFT implementation test

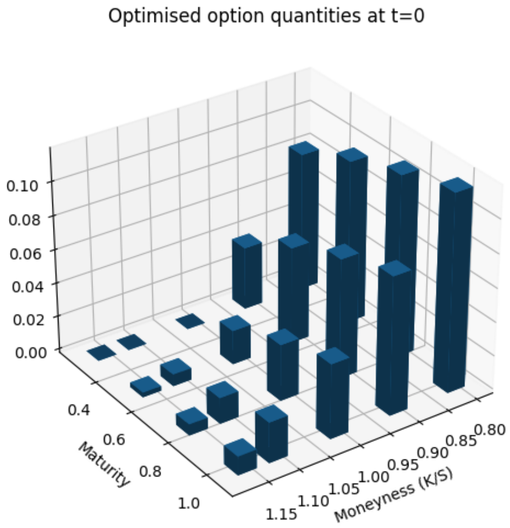
Table: MC and FFT European call option prices under SVJJ model with $S(0) = 100,\ r = 0.1,\ v(0) = 0.04,\ \beta = 1,\ \alpha = 0.04,\ \sigma_v = 0.05,\ \rho_{x,v} = -0.5,\ \lambda = 5,\ \mu_S = 0,\ \sigma_S = 0.01,\ \rho_J = -0.3,\ \mu_V = 0.02,\ N = 256,\ \bar{u} = 40,\ \epsilon_1 = -3,\ \epsilon_2 = 1,\ T = 1.$

K	MC Price	FFT Price	Absolute Difference
20	81.906164	81.903234	0.002930
30	72.872779	72.855205	0.017574
40	63.773907	63.811614	0.037707
50	54.788865	54.795772	0.006907
60	45.908156	45.885217	0.022939
70	37.259369	37.253208	0.006161
80	29.160481	29.176272	0.015791
90	21.995527	21.975674	0.019853

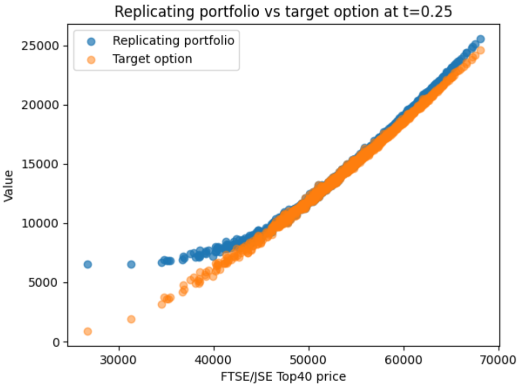
Real-world FTSE/JSE Top40 distribution at $t = 0.25$



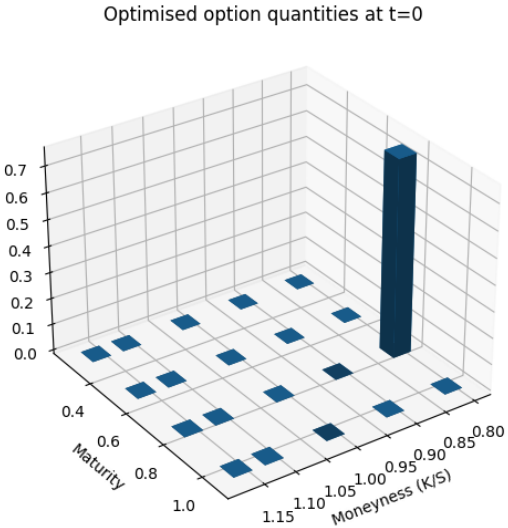
Option quantities based on static hedge #1



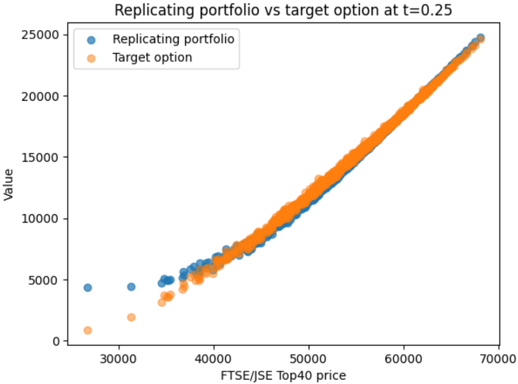
Static hedge #1 performance for European call option



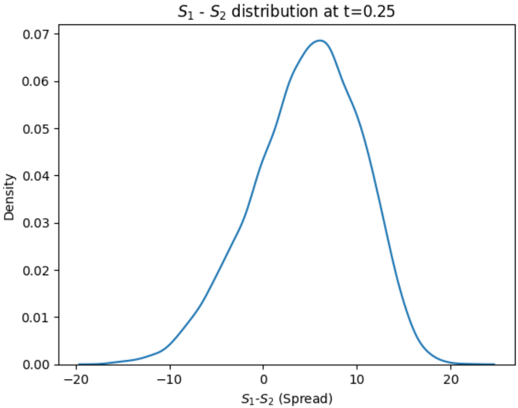
Option quantities based on static hedge #2



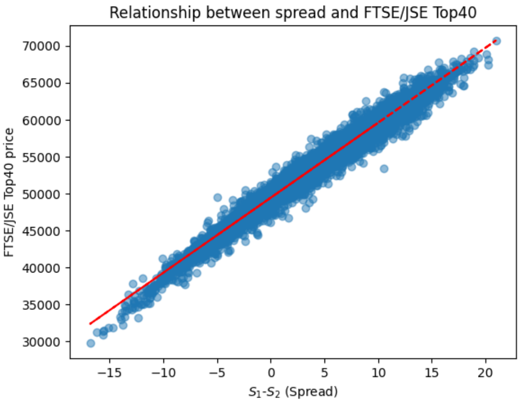
Static hedge #2 performance for European call option



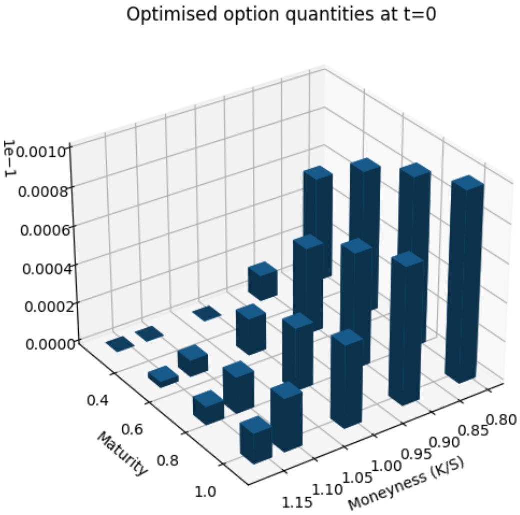
Real-world spread distribution at $t = 0.25$



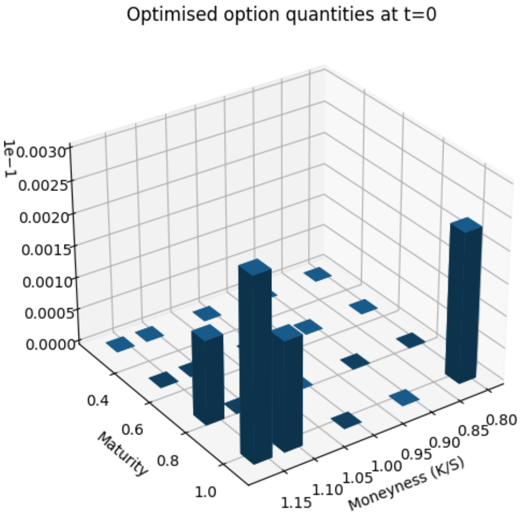
Relationship between spread and FTSE/JSE Top40 price



Option quantities based on static hedge #1

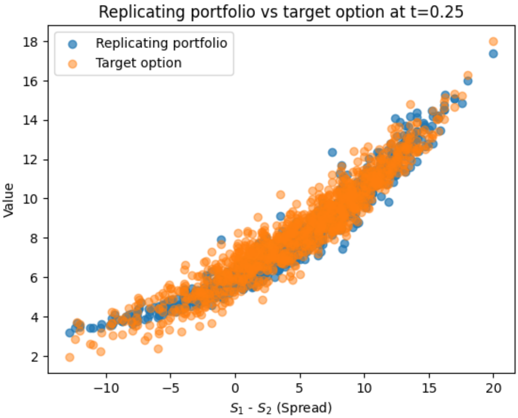


Option quantities based on static hedge #2



Static hedge #1 performance for European spread call option

Static hedge #2 performance for European spread call option



Conclusion

- The hypothesis that the observed data are observed from the SVJJ model is not rejected at a 5% level of significance.
- For a vanilla European call option, static hedging gives a simple and effective way to replicate the written option.
- For European spread call options, static hedging is only useful if the there is a linear relationship between the spread and price of the underlying hedging instrument.

Thank you