A Cash Flow Approach to Expected Credit Loss Modelling

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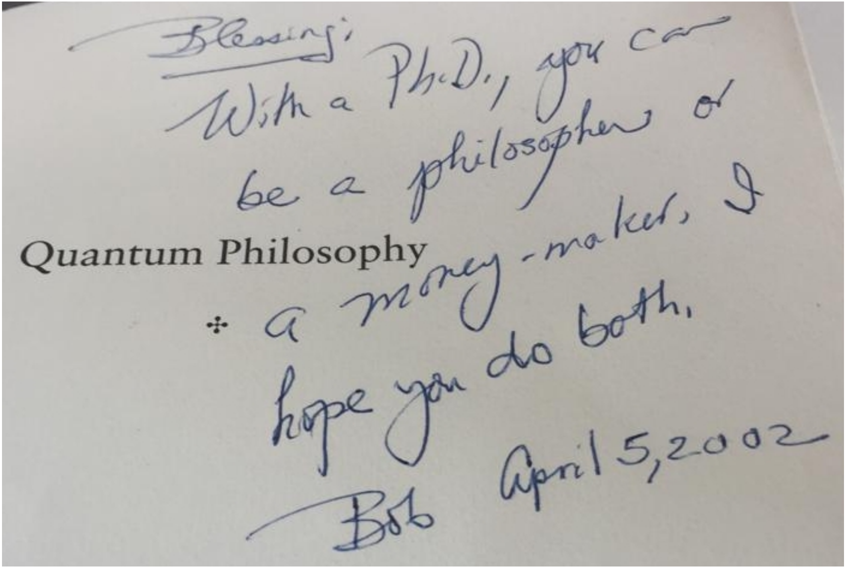
Collaboration between Industry and Academia



International Accounting Standards Board (IASB) requirements for IFRS9 and CECL

- Credit losses are the present value of all cash shortfalls over the expected life of the financial instrument. A cash shortfall is the difference between the cash flows that are due to an entity in accordance with the contract and the cash flows that the entity expects to receive. Both amount and timing of payments should be considered. [B5.5.28]; [B5.5.29–B5.5.35]

- Expected credit losses of a financial instrument shall reflect an unbiased and probability weighted amount that is determined by evaluating a range of possible outcomes. [5.5.17, B5.5.41 – B5.5.43]

IASB ...Continued

- Expected credit losses shall reflect the time value of money. In particular, they shall be discounted to the reporting date using the effective interest rate (EIR), except for purchased or originated credit-impaired financial assets, in which case the credit-adjusted EIR is applied. [5.5.17, B5.5.44 – B5.5.48]

Standard ECL Calculation Framework

The commonly used Expected Credit Loss (ECL) calculation framework [KPMG 2017, etc.,] is given by

$$ECL = \sum_{j=1}^{m} \Delta PD_j \times LGD_j \times EAD_j \times D_j$$

where ΔPD is the marginal probability of default, LGD is the loss given default, EAD is the exposure at default and D is a discount factor.

Thus, ECL is the discounted probability-weighted cash shortfalls discounted using the effective interest rate

Towards an IASB Compliant Approach

The expected credit loss is computed as follows:

$$\xi = V - (V_S + V_D)$$

where V is the PV of contractual cash flows under certainty, V_S is the PV of cash flows expected on default not occurring (survival) and V_D is the PV of recovery cash flows on default occurring.

$$V = \sum_{j=1}^{m} \frac{CF_j}{(1+\widehat{r})^j}, \quad V_S = \sum_{j=1}^{m} \omega_j^0 \frac{CF_j}{(1+\widehat{r})^j} \quad \text{and} \quad V_D = \sum_{j=1}^{m} \omega_j^1 \frac{R_j}{(1+\widehat{r})^j}$$

ω_j^0 be the probability equal to no default occurring up to time j, ω_j^1 be the probability equal to default occurring between time $j-1$ and $j$and R_j is the recovery cash flow at time j

ECL Cash Flow Approach -1

We can write the expected recoveries cash flow as;

$$V_D = \sum_{j=1}^{m} \omega_j^1 (1 - \Lambda_j) \kappa^j \sum_{i=j}^{m} \frac{CF_i}{(1+r)^i}$$

where $\kappa = (1+r)/(1+\widehat{r})$. Λ_j is the loss given default applicable at time j.
Thus, the ECL can be written as

$$\xi = \sum_{j=1}^{m} \frac{CF_j}{(1+\widehat{r})^j} - \sum_{j=1}^{m} \omega_j^0 \frac{CF_j}{(1+\widehat{r})^j} - \sum_{j=1}^{m} \omega_j^1 (1 - \Lambda_j) \kappa^j \sum_{i=j}^{m} \frac{CF_i}{(1+r)^i} \quad (1)$$

ECL Cash Flow Approach -2

Given that:

$$V_D = \sum_{j=1}^{m} \omega_j^1 \kappa^j \sum_{i=j}^{m} \frac{CF_i}{(1+r)^i} - \sum_{j=1}^{m} \omega_j^1 \Lambda_j \kappa^j \sum_{i=j}^{m} \frac{CF_i}{(1+r)^i}$$

It is possible to re-arrange (1) such that:

$$\xi = \sum_{j=1}^{m} \omega_j^1 \Lambda_j \kappa^j \sum_{i=j}^{m} \frac{CF_i}{(1+r)^i} + \sum_{j=1}^{m} (1-\omega_j^0) \frac{CF_j}{(1+\widehat{r})^j} - \sum_{j=1}^{m} \omega_j^1 \kappa^j \sum_{i=j}^{m} \frac{CF_i}{(1+r)^i} \tag{2}$$

ECL Cash Flow Approach - 3

Let the survival probability $\omega_j^0 = q_j$, and so $\omega_j^1 = (q_{j-1} - q_j)$ From (2) the expected credit loss ξ is given by:

$$\xi = \sum_{j=1}^{m}(q_{j-1} - q_j) \times \Lambda_j \times \kappa^j \sum_{i=j}^{m} \frac{CF_i}{(1+r)^i} + \zeta \tag{3}$$

where

$$\zeta = \sum_{j=1}^{m} \frac{CF_j}{(1+\widehat{r})^j} - \sum_{j=1}^{m} q_j \frac{CF_j}{(1+\ \widehat{r})^j} - \sum_{j=1}^{m}(q_{j-1} - q_j)\kappa^j \sum_{i=j}^{m} \frac{CF_i}{(1+r)^i} \tag{4}$$

The first part of the ECL in (3) is the commonly used ECL valuation formula. We think of ζ as a correction term

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The correction term, ζ, can be written as

$$\zeta = \sum_{j=1}^{m} \frac{CF_j}{(1+\widehat{r})^j} - \kappa \sum_{j=1}^{m} \frac{CF_j}{(1+r)^j} - (1-\kappa) \sum_{j=1}^{m} q_j \frac{CF_j}{(1+\widehat{r})^j} + (1-\kappa) \sum_{j=1}^{m} q_j \kappa^j \sum_{i=j}^{m} \frac{CF_i}{(1+r)^i}$$

In the one period case ($m = 1$), $\zeta = 0$. When the effective interest rate (or reference rate) is equal to the contractual rate then $\zeta = 0$, thus the commonly used ECL formula is accurate.

It can also be shown that when $\widehat{r} > r$, as is commonly the case, that $\zeta < 0$. Therefore in most applications the commonly used ECL formula is a conservative estimate of ECL.

Example A: bullet loan

A fixed rate bullet bond with interest rate paid at maturity has cash flows given by $CF_i = 0$ for $i < m$ and $CF_m = (1+r)^m N$. The expected credit loss (ECL) is given by

$$\xi = N \sum_{j=1}^{m} (q_{j-1} - q_j) \Lambda_j \kappa^j + \zeta$$

where

$$\zeta = N \left(-\kappa + \kappa^m - (1-\kappa) q_m \kappa^m + (1-\kappa) \sum_{j=1}^{m} q_j \kappa^j \right)$$

Example - A bond loan

A fixed rate bullet loan with period interest payments is defined by cash flows given by $CF_i = rN$ for $i < m$ and $CF_m = (1+r)N$. The ECL is;

$$\xi = (1+r)N\sum_{j=1}^{m}\frac{(q_{j-1}-q_j)\Lambda_j}{(1+\widehat{r})^j} + \zeta$$

where

$$\begin{aligned}
\zeta = \quad & N \quad \Bigg[\left(\frac{r}{\widehat{r}}\right)\left(\frac{(1+\widehat{r})^{m-1}-1}{(1+\widehat{r})^{m-1}}\right) + \frac{\kappa}{(1+\widehat{r})^{m-1}} - \kappa\left(\frac{(1+r)^{m-1}-1}{(1+r)^{m-1}}\right) \\
& - \quad \frac{\kappa}{(1+r)^{m-1}} - (1-\kappa)r\sum_{j=1}^{m-1}\frac{q_j}{(1+\widehat{r})^j} - (1-\kappa)\frac{\kappa q_m}{(1+\widehat{r})^{m-1}} \\
& + \quad \frac{(1-\kappa)}{(1+r)^{m-1}}\sum_{j=1}^{m} q_j\kappa^j(1+r)^{m-j}\Bigg]
\end{aligned}$$

Example C - amortizing loan

A credit facility offered to a borrower for which there is a payment of both principal and interest. r, are made with each payment. The repayment amount is $P = r(1+r)^m N/((1+r)^m - 1)$, the ECL is;

$$\xi = \frac{N}{(1+r)^m - 1} \sum_{j=1}^{m} (q_{j-1} - q_j) \Lambda_j \kappa^j \left((1+r)^{m-j+1} - 1 \right) + \zeta$$

where

$$\begin{aligned} \zeta = \quad & N \quad \left[\left(\frac{r}{\widehat{r}} \right) \left(\frac{(1+\widehat{r})^m - 1}{(1+r)^m - 1} \right) \kappa^m - \kappa - (1-\kappa) \frac{r(1+r)^m}{(1+r)^m - 1} \sum_{j=1}^{m} \frac{q_j}{(1+\widehat{r})^j} \right. \\ & + \quad \left. (1-\kappa) \frac{1}{(1+r)^m - 1} \sum_{j=1}^{m} q_j \kappa^j \left((1+r)^{m-j+1} - 1 \right) \right] \end{aligned}$$

Replication Theory and Expected Credit Losses

The ECL for an amortizing loan with periodic repayments, P, can be expressed as ECLs of a fixed paying loan and a bullet loan with notionals $\widehat{N}$ and $\widetilde{N}$, respectively. where $\widehat{N} = \gamma N$, $\widetilde{N} = (1-\gamma)N$ and $\gamma = (1+r)^m/((1+r)^m - 1)$

The ECL for an amortizing loan is thus given by

$$\xi = (1+r)\widehat{N}\sum_{j=1}^{m}\frac{(q_{j-1}-q_j)\Lambda_j}{(1+\widehat{r})^j} + \widetilde{N}\sum_{j=1}^{m}(q_{j-1}-q_j)\Lambda_j\kappa^j + (\zeta_1+\zeta_2)$$

The above ECL can be written as

$$\xi = \frac{N}{(1+r)^m - 1}\sum_{j=1}^{m}(q_{j-1}-q_j)\Lambda_j\kappa^j\left((1+r)^{m-j+1}-1\right) + \zeta$$

where $\zeta = \zeta_1 + \zeta_2$.

Numerical Examples

Product	ECL	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Bullet	ECL(1 yr)	24.72	24.23	23.52	22.39
Loan	ECL(5 yrs)	117.62	106.52	91.84	71.78
	ECL(10 yrs)	221.40	181.60	134.98	82.46
Coupon	ECL(1 yr)	23.63	23.17	22.51	21.44
Bond	ECL(5 yrs)	93.51	85.42	74.66	59.81
	ECL(10 yrs)	143.66	122.06	96.14	65.66
Amortising	ECL(1 yr)	13.24	13.06	12.79	12.37
Loan	ECL(5 yrs)	56.16	52.72	48.03	41.26
	ECL(10 yrs)	98.12	87.18	73.39	55.82

Table 1: Expected Credit Loss for three loan products with, 1,000 notional, 5% default probability, a contractual rate of 10% under the four discount (effective interest) rate scenarios, 10%; 12%; 15% and 20%; respectively.

Conclusion

- We show that the formula most commonly applied in the literature for calculating lifetime expected credit loss is inconsistent with measuring expected loss based on expected discounted cash flows.
- Valuation framework presented is flexible and can be used for loans with floating interest rates using forward rates
- In this framework we can easily incorporate prepayment options
- We can use replication theory to compute Expected Credit Losses for loans with complex cash flows