Exploring the Impact of Quality of Care in a Multi-State Long Term Care Model

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Presentation to AFRIC 2023
Wednesday, July 26, 2023
Victoria Falls, ZIMBABWE

What is Long Term Care (LTC)?

1. When you cannot perform activities (ADLs) on your own
    1. LTC needed to meet your health/personal care needs
    2. LTC needed to help you live independently and safely
2. Long term care is NOT medical care
3. LTC can be at home or in a facility such as a nursing home
4. LTC most often provided at home by unpaid family and friends

Activities of Daily Living (ADLs)

1. Walking: getting around the home or outside
2. Feeding: being able to feed oneself
3. Dressing and grooming: choosing your own clothes, putting them on, and grooming yourself
4. Toileting: moving to and from the toilet, using the toilet, and cleaning yourself
5. Bathing: washing your face and body in a bath or shower
6. Transferring: being able to move from a bed to a chair, or into a wheelchair

Instrumental ADLs (IADLs)

1. Managing finances: e.g., paying bills and managing financial assets
2. Managing transportation: e.g., driving yourself or organizing other means of transport
3. Shopping and meal preparation
4. House cleaning and home maintenance: cleaning up after eating, home reasonably clean and tidy, and keeping up with home maintenance
5. Managing communication: e.g., telephone, mail, email, social media
6. Managing medications: e.g., getting medications and taking them as directed

Who May Need LTC?

1. Age: LTC risk tends to increase with age
2. Gender: Women are at higher risk than men because they tend to outlive their partners
3. Marital status: Single people are more likely than married people to need PAID care
4. Lifestyle. Poor diet and poor exercise habits can increase LTC risk
5. Health and family history also affect LTC risk

US Seniors (Age 65+) and LTC

1. 70% of seniors may need LTC in their lifetime
2. 35% of seniors may enter a nursing home at least once in their lifetime
3. A senior couple's lifetime LTC costs from US$265,000 to US$575,000
4. 2021 median US nursing home costs US$94,900 (semi-private room) and US$108,405 (private room)

Model Assumptions: Quality of Care

1. LTC providers offer services of differing quality
2. Quality is captured by a publicly known parameter α
3. Higher values of α denote better "quality" of service
4. A 5-star rating system (like hotels) exists, i.e., $1 \leq \alpha \leq 5$
5. Average quality is $\bar{\alpha} = 3$
6. Retirees stick with a type-α LTC care provider in all states
7. Higher quality $\Leftrightarrow$ higher costs
8. No moral hazard issues

Types of Retirees

1. Retirees range from very healthy to very sick
2. Each retiree has a hidden health parameter, $\theta > 0$
3. High risk retirees have low values of θ and are in good health
4. Low risk retirees have high values of θ and are in poor health
5. Retirees experience random health shocks
6. Retirees morbidity and mortality follow multi-state Markov process

Multi-State Model: Levels of Care

1. State 1: retiree is well;
2. State 2: retiree performs all ADLs but NOT all IADLs;
3. State 3: retiree not cognitively impaired & cannot perform 1 ADL
4. State 4: retiree not cognitively impaired & cannot perform 2 ADLs
5. State 5: retiree not cognitively impaired & cannot perform 3+ ADLs
6. State 6: retiree is cognitively impaired & cannot perform ≤ 1 ADLs
7. State 7: retiree is cognitively impaired & cannot perform 2+ ADLs
8. State 8: retiree is dead

Multi-State Markov Model Details

1. Assume retiree type and quality of care affect transition intensities.
2. $\bar{\mu}^{ij}_{[x]+t}$ = standard force of transition from state i to state j
3. Suppose we are given a fixed quality of care α in LTC market, then retirees with higher levels of θ (i.e., less healthy individuals):
   1. More likely to transition a higher level of care (to get sicker); and
   2. Less likely to transition a lower level of care (i.e., to get better)

   than retirees with lower levels of θ (i.e., healthier individuals), i.e.,

$$\frac{\partial}{\partial\theta}\bar{\mu}^{ij}_{[x]+t}(\theta,\alpha) \begin{cases} \geq 0 & \text{for } i = 1, 2, \dots, 7 \text{ and } j \geq i+1 \\ \leq 0 & \text{for } i = 2, \dots, 7 \text{ and } j \leq i-1 \\ \text{undefined} & \text{for } i = j, i = 1, 2, \dots, 7 \end{cases} \tag{1}$$

A Key Innovation: Quality of Care

1. Given a retiree type θ
2. Clients of LTC providers with higher levels of α (i.e., better quality):
    1. Are less likely to transition a higher level of care (to get sicker); and
    2. More likely to transition a lower level of care (i.e., to get better)

   than clients of LTC providers with lower levels of α (i.e., lower quality)

$$\frac{\partial}{\partial \alpha}\bar{\mu}^{ij}_{[x]+t}(\theta,\alpha) \quad \begin{cases} \leq 0 & \text{for } i = 1,2,\ldots,7 \text{ and } j \geq i+1 \\ \geq 0 & \text{for } i = 2,\ldots,7 \text{ and } j \leq i-1 \\ \text{undefined} & \text{for } i = j, i = 1,2,\ldots,7 \end{cases} \tag{2}$$

1. Define $\theta^{\max}(\alpha) = g^{(0)} + g^{(1)}(\alpha - \bar{\alpha})$ for possible rehabilitation
2. Our model is: $\mu^{ij}_{[x]+t}(\theta, \alpha) = \varphi_{ij}(\theta, \alpha)\bar{\mu}^{ij}_{[x]+t}$ where

$$\varphi_{ij}(\theta, \alpha) = \begin{cases} \theta^{g^{(4)}} & i = 1; j = i+1, \ldots, 8 \\ \theta \mathrm{e}^{-g^{(2)}(\alpha - \bar{\alpha})} & i = 2, \ldots, 7; j = i+1, \ldots, 8 \\ h(\theta, \alpha) \exp\left(g^{(3)}(\alpha - \bar{\alpha})\right) & i = 2, \ldots, 7; j = 1, \ldots, i-1 \\ -\mu^{i\bullet}_{[x]+t}(\theta, \alpha)/\bar{\mu}^{i\bullet}_{[x]+t} & i = j; j = 1, 2, \ldots, 7 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

$$h(\theta, \alpha) = \left(\max\left\{0, \frac{\theta^{\max}(\alpha) - \theta}{(\theta^{\max}(\alpha) - 1)\theta}\right\}\right)^{g^{(4)}} \tag{4}$$

3. We assume $g^{(0)} = 7$, $g^{(1)} = 2$, $g^{(2)} = 0.3$, $g^{(3)} = 0.1$, and $g^{(4)} = 1$

1. Chapman-Kolmogorov differential equations are rewritten as:

$$\frac{\mathrm{d}}{\mathrm{d}t}\, {}_t\mathbf{p}_{[x]}(\theta,\alpha) = \mathbf{B}(t,\theta,\alpha)\, {}_t\mathbf{p}_{[x]}(\theta,\alpha) \tag{5}$$

2. If we assume constant intensities, i.e., $\mu^{ij}_{[x]+t}(\theta,\alpha) = \mu^{ij}(\theta,\alpha)$

$${}_t\mathbf{p}_{[x]}(\theta,\alpha) = \mathrm{e}^{t\mathbf{B}(\theta,\alpha)}\, {}_0\mathbf{p}_{[x]}(\theta,\alpha) \tag{6}$$

where ${}_0\mathbf{p}_{[x]}(\theta,\alpha)$ denotes initial conditions. For square matrix $\mathbf{B}$,

$$\mathrm{e}^{t\mathbf{B}} = I + t\mathbf{B} + \frac{t^2\mathbf{B}^2}{2!} + \frac{t^3\mathbf{B}^3}{3!} + \cdots + \frac{t^k\mathbf{B}^k}{k!} + \cdots. \tag{7}$$

3. For calculating a matrix exponential see Moler, C. and C. Van Loan. 2003. Nineteen Dubious Ways to Compute the Exponential of a Matrix, Twenty-Five Years Later. *SIAM Review* 45, 1: 3–49.

Model Details: More Transition Probabilities

1. Occupancy probabilities ${}_tp^{\overline{ii}}_{[x]+s}(\theta,\alpha)$ are given by:

$$ {}_tp^{\overline{ii}}_{[x]+s}(\theta,\alpha) = \exp\left(-\int_0^t \mu^{i\cdot}_{[x]+s+r}(\theta,\alpha)\mathrm{d}r\right) \tag{8} $$

$$ \frac{\mathrm{d}}{\mathrm{d}t}\,{}_tp^{ij}_{[x]+s}(\theta,\alpha) = \sum_{\substack{k=1\\k\neq j}}^{8} {}_tp^{ij}_{[x]+s}(\theta,\alpha)\mu^{ij}_{[x]+t+s}(\theta,\alpha) - {}_tp^{ij}_{[x]+s}(\theta,\alpha)\mu^{j\cdot}_{[x]+t+s}(\theta,\alpha) \tag{9} $$

for $i, j \in \{1,2,3,4,5,6,7,8\}$.

Key Equations for Health and Longevity

$$\psi_{[x]}^{ikj}(s,t,\theta,\alpha) = {}_{t-s}p_{[x]}^{ik}(\theta,\alpha)\mu_{[x]+t-s}^{kj}(\theta,\alpha)\,{}_{s}p_{[x]+t-s}^{\overline{jj}}(\theta,\alpha) \quad (10)$$

$$\mathring{e}_i^{(j)}(x,\theta,\alpha) = \int_0^\infty {}_{t}p_{[x]}^{ij}(\theta,\alpha)\mathrm{d}t \quad (11)$$

$$\mathring{e}_i^{(j8)}(x,\theta,\alpha) = \int_0^\infty \int_0^\infty \sum_{\substack{k=1 \\ k\neq j}}^{7} s\,\psi_{[x]}^{ikj}(s,t,\theta,\alpha)\mu_{[x]+t}^{j8}(\theta,\alpha)\mathrm{d}s\,\mathrm{d}t \quad (12)$$

$$p_{[x]}^{ij8}(\theta,\alpha) = \int_0^\infty {}_{t}p_{[x]}^{ij}(\theta,\alpha)\mu_{[x]+t}^{j8}(\theta,\alpha)\mathrm{d}t. \quad (13)$$

1. $\mathring{e}_i^{(j)}$ = Expected time spent in state j, $\mathring{e}^{(j8)}$ = Expected time spent in state j just before death, and $p_{[x]}^{ij8}$ = Probability of dying in state j.

Transition Intensities (Rates) Used

1. Robinson (1996, Table 3) rates are for females age 75-85.
2. Robinson's rates used to define $\bar{\mu}^{ij}_{[65]+t}$ in Table 1 below:

Table 1: Constant Transition Intensities (Rates) $\bar{\mu}^{ij}_{[65]+t}$ for $t \geq 0$

	State j							
i	1	2	3	4	5	6	7	8
1		0.075	0.004	0.000	0.005	0.011	0.000	0.021
2	0.050		0.146	0.023	0.007	0.063	0.018	0.082
3	0.021	0.665		0.116	0.140	0.114	0.000	0.142
4	0.000	0.038	0.364		0.314	0.031	0.134	0.098
5	0.000	0.006	0.014	0.091		0.000	0.140	0.228
6	0.046	0.106	0.142	0.044	0.030		0.250	0.046
7	0.010	0.000	0.009	0.000	0.328	0.049		0.249

3. Robinson, J. 1996. A Long-Term Care Status Transition Model. In *Proceedings of The Old-Age Crisis–Actuarial Opportunities: The 1996 Bowles Symposium*, Atlanta, GA: Georgia State University, pp. 72–79.

Table 2: Complete Expectation of Life Starting in State 1, $\mathring{e}_1(65, \alpha, \theta)$ for Different Values of θ and α

α	$\theta = 1$	$\theta = 2$	$\theta = 3$	$\theta = 4$	$\theta = 5$	$\theta = 6$
1.0	18.806	8.554	5.630	4.222	3.378	2.815
2.0	21.239	9.336	6.100	4.548	3.630	3.025
3.0	24.804	10.394	6.724	4.996	3.981	3.311
4.0	29.841	11.880	7.580	5.605	4.456	3.702
5.0	36.443	13.994	8.767	6.439	5.103	4.233
Δ Quality	17.637	5.440	3.137	2.217	1.725	1.418
% Change	0.938	0.636	0.557	0.525	0.511	0.504

Notes: $\Delta\text{Quality} = \mathring{e}_1(65, 5, \theta) - \mathring{e}_1(65, 1, \theta)$

Notes: $\%\Delta\text{Quality} = \Delta\text{Quality}/\mathring{e}_1(65, 1, \theta)$

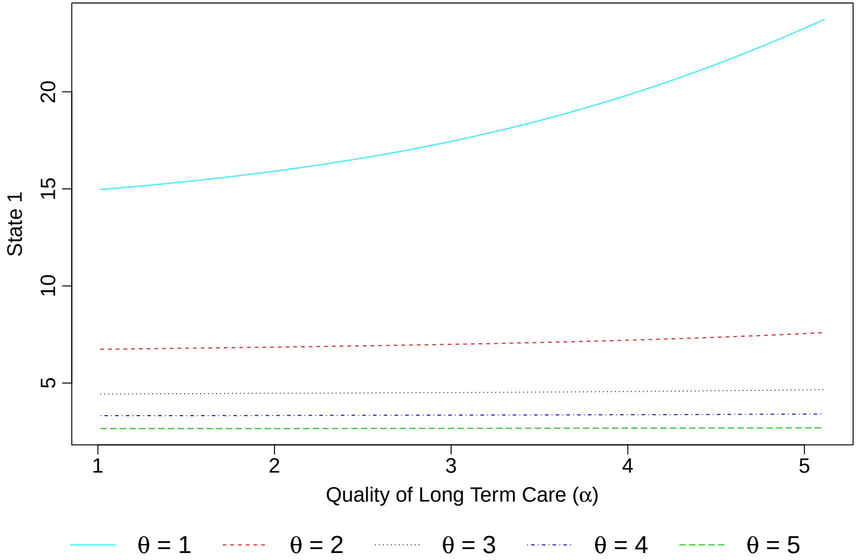

Figure: Life Expectancies in Years, $\mathring{e}_1(65, \theta, \alpha)$

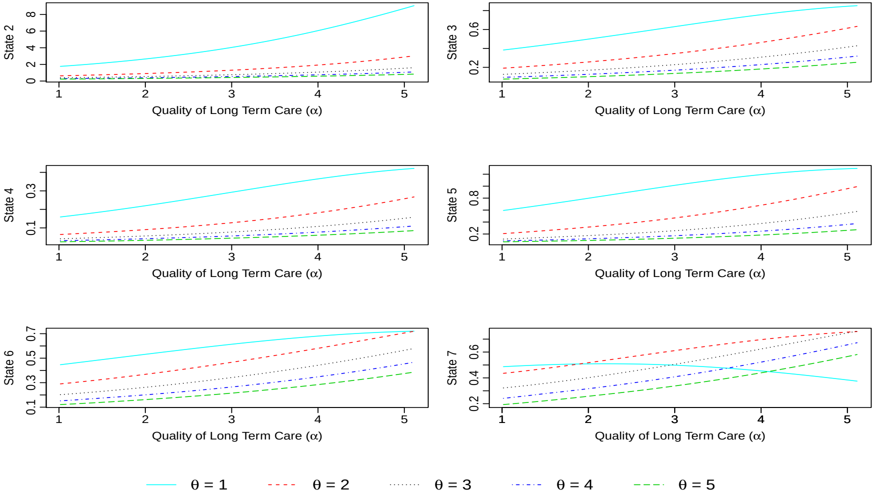

Figure: Life Expectancies in Years, $\mathring{e}_1(65, \theta, \alpha)$

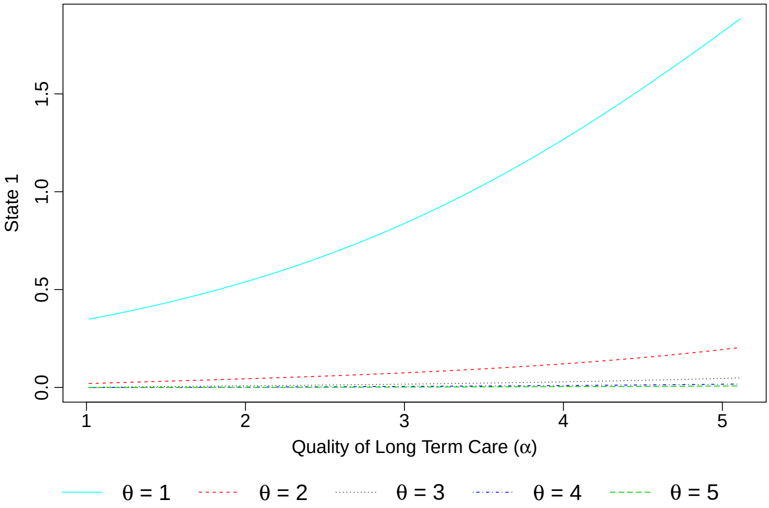

Figure: Expected Number of Years Spent Continuously in State 1 Immediately before Death

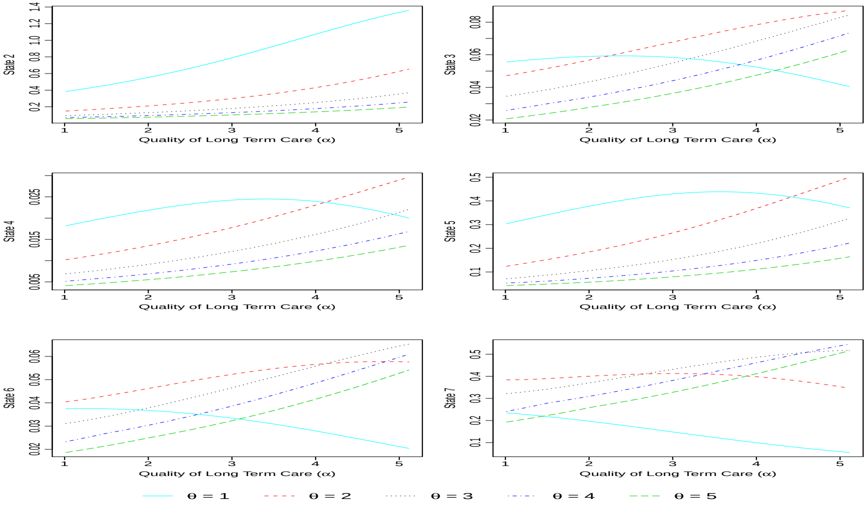

Figure: Expected Number of Years Spent Continuously in State j immediately before death, $\mathring{e}_1^{(j8)}(65, \theta, \alpha)$

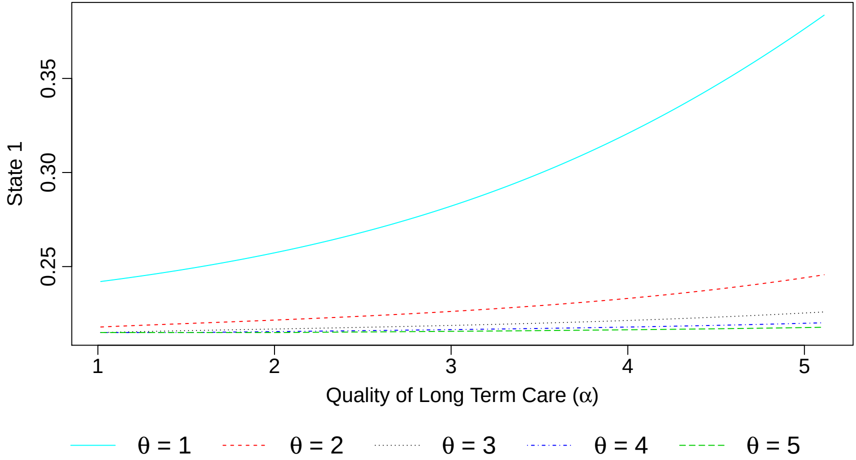

Figure: Probability of Dying in State 1, $p^{18}_{[65]}(\theta, \alpha)$

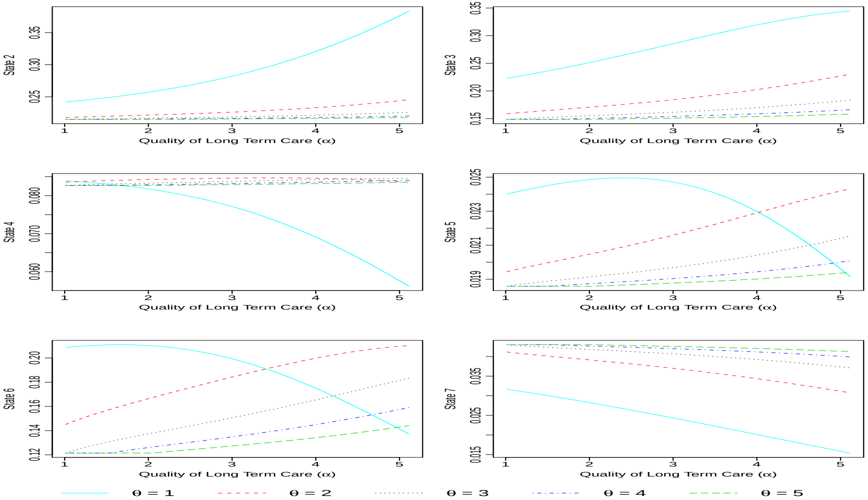

Figure: Probability of Dying in State j, $p^{j8}_{[65]}(\theta, \alpha)$

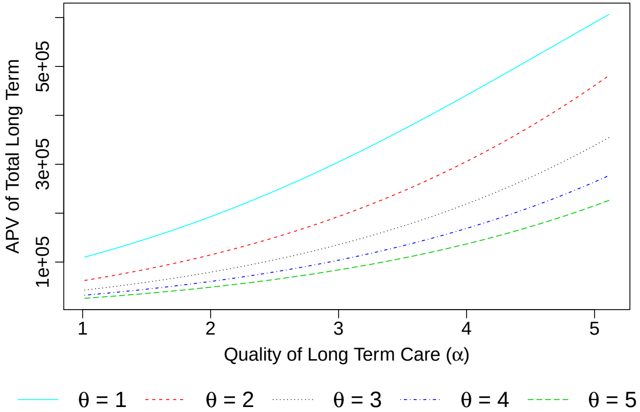

Figure: APV of Lifetime Long Term Care Costs $(1e{+}05 = \$100,000)$ under the Multi-State Model

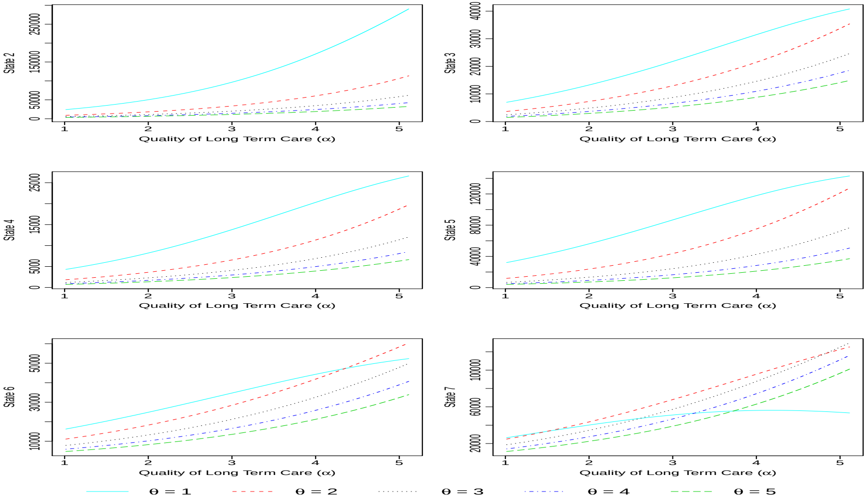

Figure: APV of Lifetime Long Term Care Costs (in $s) in State j

1. Increasing quality of care does the following:
   1. Increases total lifetime LTC costs
   2. Increases quality of life;
   3. Increases life expectancy in all health states;
   4. Decreases time spent in sicker states; and
   5. Decreases probability of dying in sicker states.
2. We know of no actuarial model that uses quality of LTC.

Thank You!

Any Questions?