Linking Annuity Benefits to Financial and Longevity Experience: A Joint Pricing Framework

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Outline

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Background

- The demand for longevity guarantees remains low due to high costs.
- Alternative solutions: longevity-linked products with flexible guarantees.
- The benefit amount is updated to the mortality (longevity) experience.
- Also common: financial-linked products, the benefit amount is updated to the realized investment experience.
- Allow sharing of losses, and possibly profits between the provider and annuitants.

Previous Literature

- Insurance products: adaptive algorithmic annuities [Luthy et al., 2001], longevity-indexed life annuities [Denuit et al., 2011], longevity-contingent life annuities [Denuit et al., 2015], participating longevity-linked life annuities [Bravo, 2022], etc.
- Also a common practice to link the annuity benefits to financial experience: with-profit annuities or PLAs [Maurer et al., 2013].
- Forms of participation also present in risk sharing products: GSAs, pooled annuities and tontines [Piggott et al., 2005], [Qiao and Sherris, 2013], [Milevsky and Salisbury, 2015], [Donnelly et al., 2013], [Donnelly et al., 2014], [Chen et al., 2019] no explicit guarantees.

Motivation

Literature Gaps

- Little previous work on linked annuity arrangements that include both financial and (partial) longevity participation, possibly including guarantees.

Motivation

Research Goals

- Investigate the joint presence of financial and longevity participation.
- The benefits are updated based on longevity and financial experience, including (partial) guarantees.
- Devise the realistic price of the risk retained by the provider (prices (fees) of guarantees under uncertain mortality and interest rates).
- Explore trade-offs between the retained risk amount and the guarantee cost from the individual and provider perspectives.

Methodology

- We follow the general linking mechanism proposed in [Olivieri and Pitacco, 2020].
- We estimate interest and mortality using an affine term structure model, the AFNS independent factor model [Christensen et al., 2011, Huang et al., 2019].
- We use a periodic fee structure adopted in variable annuities [Bacinello et al., 2011, Olivieri, 2021].

Outline

The Linking Framework

- Financial Linking

$$B_t = B_{t-1} \times \begin{cases} 1 + \vartheta_t \tilde{r}_t, & \text{if } \vartheta_t \tilde{r}_t \geq r_{\min}, \\ 1 + r_{\min}, & \text{otherwise}, \end{cases} \qquad (1)$$
$$= B_{t-1} \times \left(1 + \max\left(r_{\min}, \vartheta_t \tilde{r}_t\right)\right),$$

- $\vartheta_t \in [0, 1]$ is the part of realized interest rate provided to the policyholder and $1 - \vartheta_t$ is the remaining part retained by the insurer, given a minimum guaranteed rate of $r_{\min}$.

The Linking Framework

- Longevity linking

$$B_t = B_{t-1} \times \left(\frac{p^0_{x+t-1}}{\tilde{p}_{x+t-1}} \right), \text{ for } 1 - \psi_t \le \frac{p^0_{x+t-1}}{\tilde{p}_{x+t-1}} \le 1 + \psi_t \tag{2}$$

$$B_t = B_{x_{\max}-x}, \text{ for } t > x_{\max} - x, \tag{3}$$

- $\psi_t \in [0, 0.5]$ is part of realized longevity experience shared with the policyholder, the remaining amount $1 - \psi_t$ is retained by the insurer,
- p^0_{x+t-1} is the best estimate one year survival probability for an individual aged $x + t - 1$ at time $t - 1$ and $\tilde{p}_{x+t-1}$ is the simulated one year survival probability.

The Linking Framework

- Financial and Longevity linking

$$B_t = B_{t-1} \times \max\left(1-\psi_t, \min\left(\frac{p^0_{x+t-1}}{\tilde{p}_{x+t-1}}, 1+\psi_t\right)\right) \times (1+\max(r_{\min}, \vartheta_t \tilde{r}_t)), \tag{4}$$

- where $\psi_t \in [0, 0.5]$ is the longevity participation proportion and $\vartheta_t \in [0, 1]$ is the financial participation proportion.
- We determine the price of financial and longevity participation.

The Pricing Framework

- The metric used to determine the required fees is the business value for the provider.
- Defined as the present value of future profits net of the cost of capital.
- We examine the structure of periodic fees (the corresponding discount factor) under different assumptions.

The Pricing Framework

The dynamics of the policy fund is then described by the following equation:

$$A_t \cdot N_{x+t} = A_{t-1} \cdot N_{x+t-1} \cdot (1-\zeta) \cdot (1+\tilde{r}_t) - B_t \cdot N_{x+t}, \tag{5}$$

where ζ is the proportional premium loading (or fee) that is charged to each policy fund.

The Pricing Framework

The premium amount paid to the provider is determined by solving backwards Equation (5) (note that $A_{\omega-x} = 0$), we find as follow:

$$A_0 = \sum_{s=1}^{\omega-(x)} B_s \cdot ((1-\zeta)\cdot(1+r))^{-s} \cdot \frac{N_{x+s}}{n_x},$$
$$A_0 = S.$$

Outline

Numerical Illustration

Assumptions

The proposed products are issued to Italian males aged 65 in 2021.

The maximum attainable age is assumed to be 100.

The annuity payments are made at the end of the year.

For financial linking: $\vartheta_t = 1$ and $r_{\text{min}} = -0.1\%$.

For longevity linking: $\psi_t = 0.1$ so that $0.9 \leq \frac{p^0_{x+t-1}}{\tilde{p}_{x+t-1}} \leq 1.1$.

The initial benefits: $B_0 = 1$ and the maximum age to stop linking $x_{\text{max}} = 95$.

Main Results

Table 1: The linked annuity benefits from financial linking, longevity linking, and financial and longevity linking.

Base case scenario									
	Financial linking			Longevity linking			Financial and longevity linking		
Statistic	5%	Mean	95%	5%	Mean	95%	5%	Mean	95%
Time 10 (Age 75)	1.0871	1.1168	1.1471	0.9789	1.0000	1.0213	1.0789	1.1168	1.1556
Time 15 (Age 80)	1.1565	1.1967	1.2389	0.9635	1.0001	1.0377	1.1378	1.1969	1.2590
Time 31 (Age 96)	1.4186	1.4942	1.5716	0.8181	1.0075	1.2299	1.1954	1.4837	1.8220
Time 35 (Age 100)	1.4941	1.5796	1.6674	0.8181	1.0075	1.2299	1.1954	1.4837	1.8220

Main Results

Now assume initial premium S=1000 in monetary units then the initial benefit B0 =60.07

Table 2: Periodic fee to be charged each year to the policy fund value and the adjusted initial benefit amount given the initial premium of S = 1,000.

Arrangement	Periodic fee	Adjusted initial benefit
Financial linking	2.14%	46.06
Longevity linking	0.1%	59.39
Financial and longevity linking	1.99%	47.03

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Conclusion

- The joint presence of financial and longevity participation has compensation effects as well as risk-return trade-offs for the provider and policyholders.
- Financial and longevity-linked annuity benefits are slightly lower on average, but there is compensation in the form of a higher upside of the realized investment return.
- Compensation effects from financial participation, reducing the participation fee or the retained cost of the guarantees.

Thanks! Questions/comments?

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Appendix

Table 3: AFNS Interest Rate Model Estimated Parameters

k^P_{11}	k^P_{22}	k^P_{33}	σ_{11}	σ_{22}	σ_{33}
0.4983	1.0128	1.0529	0.0250	0.0208	0.0370
θ^P_1	θ^P_2	θ^P_3	δ		
0.0803	-0.0695	-0.0182	0.1277		

Appendix

Table 4: AFNS Interest Model Goodness of Fit.

Log likelihood	RMSE	No. of parameters	No. of observations	AIC	BIC
-23739.63	0.0035	33	3840	47545.26	47751.62

- We have used the guidelines provided in [Christensen et al., 2011] for calibrating the AFNS independent factor interest rate model and the codes available at `https://cepr.org/event/1854/Codes_slides`.

Appendix

Table 5: AFNS Mortality Model Estimated Parameters

$k^P_{\mu,11}$	$k^P_{\mu,22}$	$k^P_{\mu,33}$	s_{11}	s_{22}	s_{33}
0.0665	0.0151	0.0192	0.0011	0.0004	0.0001
r_1	r_2	r_c	δ_μ		
1.5502e-16	0.7777	1.6548e-06	-0.1127		

Appendix

Table 6: AFNS Mortality Model Estimated Parameters

$k^{P}_{\mu,11}$	$k^{P}_{\mu,22}$	$k^{P}_{\mu,33}$	s_{11}	s_{22}	s_{33}
0.0665	0.0151	0.0192	0.0011	0.0004	0.0001
r_1	r_2	r_c	δ_μ		
1.5502e-16	0.7777	1.6548e-06	-0.1127		

- We have used the guidelines provided in [Ungolo et al., 2021] for calibrating affine mortality model and the codes available in the Github repository `https://github.com/ungolof/affine_mortality`.