Algorithmic Commercial Risk Retention

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Talk Outline

- An Industry Opportunity
- An Insurance Framework
- ANU Case Study
- A Sketch of Supporting Research
- Summary and Concluding Remarks

Commercial/Business Insurance is Important

- Commercial premiums are approximately 2.3% of world GDP, a significant portion of the world economy
  - From the OECD, insurance premiums are approximately 9.5% of world GPD.
  - From the OECD (2022), non-life premiums account for about 51.7% of insurance premiums (with life insurance taking up the balance).
  - According to the Insurance Information Institute, commercial lines account for 48.0% of non-life insurance premiums.

- Insurance is an important mechanism for transferring risk but others such as self insurance, captives, pools, peer to peer risk exchanges, and so forth are also available.

Firms Face Multiple Risks

- One way to get insights into the multiple risks that a firm faces is to look to concerns expressed by risk managers.
- Consider a survey of global risk managers:

Table. **Top Insurable Risks Facing Firms**

Risk	**Risk**
4. *Business interruption*	44. Directors and Officers personal liability
19. Counter-party credit risk	47. Fraud
21. *Property damage*	52. Theft
22. Environmental risk	55. Terrorism sabotage
23. Weather natural disasters	56. Safety and Pharmacovigilance
24. *Third party liability*	61. Harassment discrimination
28. Injury to workers	66. Kidnap and ransom
40. Product recall	67. Extortion

A risk is **insurable** if it potentially can be transferred to another party for a fee.

2. Insurance Framework for Liability Portfolio Management

For liabilities, we focus on **nonlinear** retention strategies such as

$$S = \min(X_1, u_1) + \cdots + \min(X_p, u_p)$$

with *upper limit* parameters u.

- Mildenhall and Major (2022) refer to this type of contract as the "basic building block of insurance."
- This is contrast to the **linear** investment portfolio $S = c_1 X_1 + \cdots + c_p X_p$.
- The linearity in asset allocations means that investment strategies can be analyzed using linear and convex optimization methods;
  - These methods are readily scalable to large numbers of risks (e.g., in the hundreds).
- The non-linearities in managing insurable risks means that convexity is not available in general.

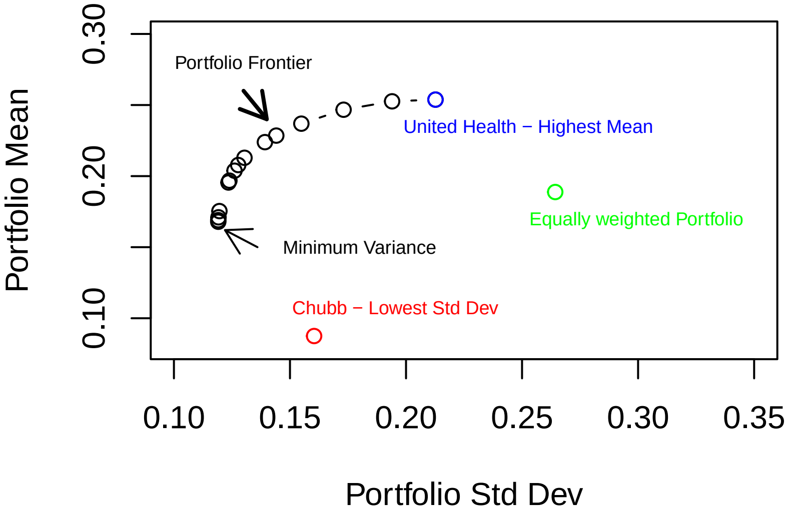
Comparing Investment and Liability Problems

	Investment	**Liability**
Risk Retention Parameters	$c_1, \ldots, c_p$	$u_1, \ldots, u_p$
Retained Risk	$S(\mathbf{c}) = c_1 X_1 + \cdots + c_p X_p$	$S_{XL}(\mathbf{u}) = \min(u_1, X_1) + \cdots + \min(u_p, X_p)$
Objective Function	$\text{minimize}_{\mathbf{c}} \quad RM[S(\mathbf{c})]$	$\text{minimize}_{\mathbf{u}} \quad RM[S_{XL}(\mathbf{u})]$
Constraint	subject to $\mathrm{E}[S(\mathbf{c})] \geq Req_0$	subject to $RTC(\mathbf{u}) \leq RTC_{max}$

- *RM* is a generic risk measure, variance is a possibility, I use expected shortfall
- *RTC* is the *risk transfer cost*
  - The risk transferred is $X_1 + \cdots + X_p - S_{XL}(\mathbf{u})$
  - The associated cost is evaluated using an insurance price principle, starting with an expectation
- Req_0 and RTC_{max} are known constants that trace out the frontier

Example. Portfolio of Insurance Stock Returns

The frontier is traced out by using different values of, e.g., Req_0.



3. Australian National University (ANU) Case

- To illustrate its size, in 2020 it enjoyed an asset base of about 4.6 billion AUD
- To illustrate its complexity, its financial statements summarize many risks including property damage, general liability, cyber security, and so forth.
  - There are 15 risks in total, the "usual suspects."
  - It pays about 25 million AUD per year in insurance premiums.
- In the absence of risk transfer agreements, ANU has responsibility for

$$S = X_1 + \cdots + X_{15}.$$

- In 2020 ANU suffered a major property loss (250 million AUD) meaning that its risk appetite has been subject to renewed scrutiny
  - This suggests that we will need to employ models that allow for long-tail distributions that permit very high values of outcomes with not insignificant probabilities.

Predictive Models of Risks

- Providing predictive models of risks are doable but challenging
  - Empirical historical data are available for high frequency risk types, but. . .
  - Most risk types are low frequency lines - little data available.
- Focus - sensitivity of optimal risk retentions to changes in assumptions.

Class of Insurance	Insurer	Deductible	Limit	Premium
Property	London Syndicate and Others	5,000,000	1,000,000,000	23,564,759
General and (G & P) Products Liability	Newline	100,000	20,000,000	340,000
G & P Umbrella Liability	Liberty	20,000,000	50,000,000	27,500
G & P 1st Exess Liability	QBE	50,000,000	100,000,000	27,500
G & P 2nd Excess Liability	Chubb	100,000,000	150,000,000	17,500
G & P 3rd Excess Liability	CGU	150,000,000	200,000,000	16,000
G & P 4th Excess Liability	Zurich	200,000,000	250,000,000	20,000
Cyber	London	250,000	2,000,000	75,721
Crime	AIG	100,000	20,000,000	100,000
Employment Practices Liability	AIG	100,000	2,000,000	84,000
Expatriate	Chubb			11,676
Group Personal Accident	Chubb	Various	As per schedule	104,920
Corporate Travel	Chubb	Various	As per schedule	75,000
Professional Indemnity	Newline	100,000	\$20m / \$40m	130,000
Medical Malpractice	Newline	100,000	\$20m / \$40m	
Clinical Trial	Newline	2,500	\$20m	
Statutory Liability	Berkley Insurance Australia (SUA)	\$1,000 / \$15,000	1,000,000	8,360
Motor Vehicle	Vero	1,000	As per schedule	84,700
Marine Cargo	Richard Oliver (QBE)	5,000	5,000,000	6,127
Marine Hull	Richard Oliver (QBE)	150	5,000,000	11,552
TOTAL				24,407,255

Optimizing the ANU Risk Portfolio

- In investment applications, the size of a risk is not an issue
- Because of its size, the algorithm performs poorly when including the Property Risk
  - Instead, assume that the risk owner transfers all property risks in excess of 5000, in thousands of AUD.
  - The portfolio of ANU's retained risks is

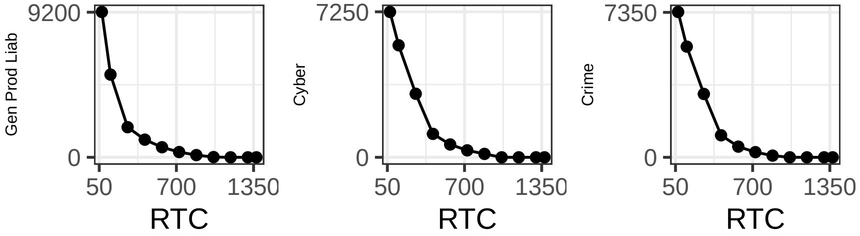
$$S_{Prop}(u_2, \ldots, u_{15}) = \min(X_1, 5000) + \min(X_2, u_2) + \cdots + \min(X_{15}, u_{15}).$$

  - The goal is to determine the optimal values of the upper limits $\{u_2, \ldots, u_{15}\}$.
  - Property is still included in the risk portfolio but not part of the potential risk transfer costs.
- Motivation: this treaty will be negotiated separately from others

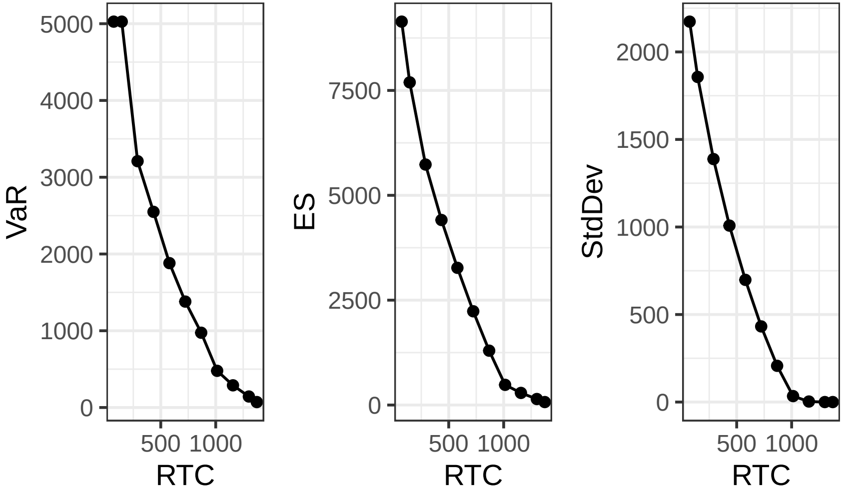
Table 1: **ANU Optimization over Levels of Maximal Risk Transfer**

RTCmax	RTC	VaR	ES	Std Dev	u2	u3	u4
1374	1374	5069	5069	2030	0	0	0
1302	1302	5141	5141	2030	0	0	0
1157	1157	5287	5287	2031	0	0	0
1013	1013	5476	5478	2038	15	0	0
868	868	5870	6286	2124	140	170	86
723	723	6172	7195	2233	334	347	262
579	579	6532	8169	2372	640	639	542
434	434	6825	9205	2554	1119	1167	1117
289	289	7332	10369	2811	1910	3166	3213
145	145	7507	11717	3139	5244	5582	5614
72	72	8089	12799	3384	9217	7242	7372

ANU Optimization - First Three Retention Levels



ANU Optimization Results - Efficient Frontier



4. A Sketch of Supporting Research

Multivariate Excess of Loss Algorithm

- We now consider p risks $X_1, \ldots, X_p$ with upper limits $\mathbf{u} = (u_1, \ldots, u_p)$.
- The retained risk is

$$S(\mathbf{u}) = X_1 \wedge u_1 + \cdots + X_p \wedge u_p.$$

- The empirical (simulation) version of the expected shortfall excess of loss retention problem is:

$$\begin{array}{ll} \text{minimize}_{x,\mathbf{u}} & ES1_R(x,\mathbf{u}) = x + \frac{1}{R}\sum_{r=1}^{R}[S_r(\mathbf{u}) - x]_+ \\ \text{subject to} & RTC_R(\mathbf{u}) \leq RTC_{max} \\ & u_1 \geq 0, \ldots, u_p \geq 0. \end{array}$$

- For risk transfer costs, we use $RTC(\mathbf{u}) = RC_1(u_1) + \cdots + RC_p(u_p)$

More on the Supporting Research

- Documented the extent that **deterministic calculations** could be used and described simulation as a handy alternative
- Established the **non-convexity** of the problem, mostly through illustrative examples
- Introduced a method for computing **starting values**, an important topic in non-convex optimization.
  - Showed how to compute starting values quickly by using the variance as an approximation to the risk measure of interest, the expected shortfall.
- **Stress testing** - one examines the impact on risk modeling results based on changes made to underlying assumptions.
  - How important is the level of confidence α?
  - Which risk measure (*VaR* or *ES*) to use? I use the range value at risk *RVaR* and *GlueVaR* as intermediate measures
- **Sensitivity** and robustness results are developed using an extension of the Envelope Theorem from economics.
- Used the Karush-Kuhn-Tucker (*KKT*) conditions to provide interesting aspects of the parameters and the **sensitivities at the optimum**.

Supporting Research

- In addition, there is a broad academic literature of *optimal design of insurance contracts*, both based on utility-based preferences and risk measures.
  - Selected references include Schlesinger (2013), Cai and Tan (2007), Cai et al. (2008), Lo (2017), Cheung, Chong, and Lo (2019), Gollier (2013), Tan and Weng (2014), Asimit et al. (2018), and Cai and Chi (2020).
  - These are very useful for helping to think about the form of an insurance contract.
- In contrast, work presented here takes the form as given and seeks to obtain optimal contract parameters.
- By working on this much smaller class of problems, I am able to consider **multivariate** risks unlike almost all of the above literature.

5. Summary and Concluding Remarks

- Introduced an algorithm designed to assist risk managers in recommending the appropriate amount of retention, and transfer, for a set of risks.
- Presented the algorithm outputs as an efficient frontier with optimal retention levels
  - It is likely that many risk advisors are familiar with this type of output from Markowitz investment applications.
  - Suggests that “robo-advising”, incorporating client-specific preferences, will be the next stage of development.
- For a commercial insurer, risk broker, or reinsurer
  - This concept will aid the acquisition and underwriting of commercial insurance.
  - Will likely be utilized as an app or web portal.

Next Steps

- I seek **business partners** in which to enable these ideas in industrial applications.
  - I believe many in academia will find the framework presented here to be compelling in its own right.
  - It will be much more compelling if used in industry.
  - Suggestions welcome!
- If you are interested in learning more:
  - Check out my website that contains these overheads and references.
  - I am writing a book on this topic. I seek chapter reviewers.
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Thank you for your time and attention!

Selected References

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