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The Role of Direct Capital Cash Transfers Towards Poverty and Extreme Poverty Alleviation - An Omega Risk Process

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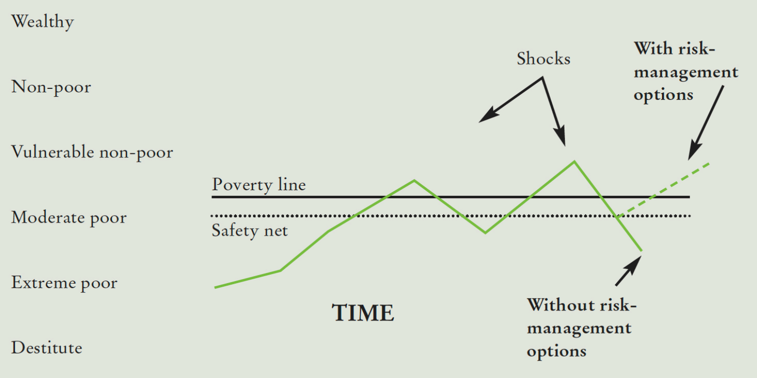

Figure 1: Impact of shocks on households' income and assets. Adapted from McCord (2005).

Introduction

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→ **Cash Transfer (CT) programmes** are one of the main social **protection strategies to reduce poverty**.

→ In their simplest form, these programmes transfer cash, whether in small, regular amounts, or as lump sums, to people living below the poverty line and are generally funded by governments, international organisations, donors or nongovernmental organisations (NGOs) (Garcia and Moore 2012).

→ Adopting a ruin-theoretic approach, we **study the impact of regular CTs on poverty dynamics** and, particularly, their **effectiveness in reducing the likelihood of a household living in poverty and extreme poverty**.

→ Despite the growing interest in studying the impact of CTs on poverty dynamics over the years, **most studies have adopted an empirical approach**.

→ We attempt to **attach a mathematically based theoretical framework** to the vast empirical literature.

- → To do so, we extend the model proposed by Kovacevic and Pflug (2011).
- → Here, a household qualifies to be a beneficiary of the CT programme when its accumulated capital X_t up to time t is below some capital barrier level B.
- → Unlike the original capital process (Kovacevic and Pflug 2011), when the accumulated capital X_t up to time t is below B households will perceive support in the form of CTs.

The Capital Model

→ An individual household's income I_t at time t comprises consumption C_t and savings S_t. Thus, income dynamics are given by,

$$I_t = C_t + S_t.$$

→ Consumption is modeled as an increasing function of income,

$$C_t = \begin{cases} I_t & \textit{if} \quad I_t \leq I^*, \\ I^* + a(I_t - I^*) & \textit{if} \quad I_t > I^*, \end{cases}$$

where I^* represents a critical income level and $0 < a < 1$.

→ X_t will denote the accumulated capital up to time t. The capital grows as follows

$$\frac{dX_t}{dt} = c_S S_t,$$

with $0 < c_S < 1$.

→ On the other hand, income is generated through capital

$$I_t = bX_t,$$

where $0 < b$ holds.

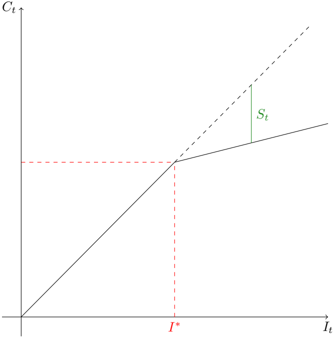


Figure 2: Consumption and savings level.

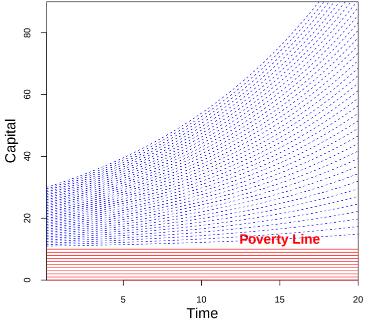


Figure 3: $\frac{dX_t}{dt} = r \cdot [X_t - x^*]^+$.

→ Bringing together the previous equations and defining $x^* = I^*/b > 0$, as a **"critical capital"** level, leads us to the dynamical system for the capital

$$\frac{dX_t}{dt} = r \cdot [X_t - x^*]^+,$$

where $r = (1 - a) \cdot b \cdot c_S$ and $[x]^+ = max(x, 0)$.

→ We consider the area below the **critical capital** x^* as the **area of poverty**.

- → We now assume that households are susceptible to suffer **heavy capital losses**.
- → Occurrence of these events will follow a Poisson process with intensity λ.
- → The fraction of the capital not destroyed at the *ith* event is described by a random variable $0 \leq Z_i \leq 1$, which is independent of the Poisson process and i.i.d. with common distribution function G_Z.

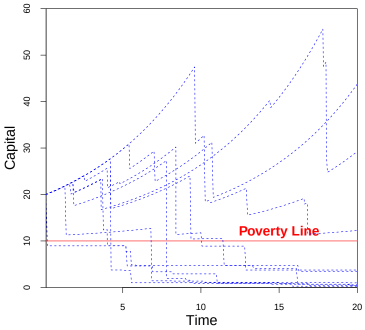


Figure 4: Stochastic process subject to random losses.

The Capital Model

→ The probability of falling (trapping probability) and the moment at which a household falls (trapping time) into the area of poverty have recently attracted the interest of some researchers (see, for example, Kovacevic and Pflug (2011), Azaïs and Genadot (2015), Flores-Contró et al. (2021) and Henshaw et al. (2022)).

→ Here, we assume households may escape from the area of poverty only due to external support received in the form of CTs.

→ To explore these ideas, we consider an **Omega risk process** (Albrecher et al. 2011), which in classical risk theory, **distinguishes between ruin (negative surplus) and bankruptcy (going out of business)**.

→ Thus, here we assume that, even with capital levels below the critical capital x^*, a household's capital process continues until an event of extreme poverty occurs.

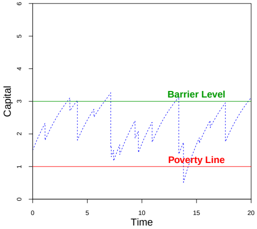
→ An individual household qualifies to be a beneficiary of the CT programme when its accumulated capital X_t up to time t is below some capital barrier level $B \geq x^*$.

→ Thus, the dynamics of the accumulated capital X_t up to time t are now given by

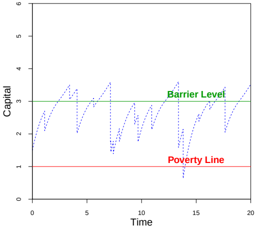
$$\frac{dX_t}{dt} = r \cdot [X_t - x^*]^+ \underbrace{+ c_T \, [B - X_t] \cdot \mathbb{1}_{\{X_t < B\}}}_{\text{Capital Transfer}},$$

with $c_T > 0$.

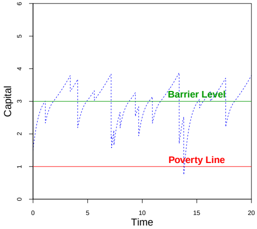
An Alternative Capital Model



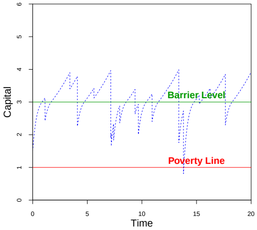
(a) $c_T = 0.5$



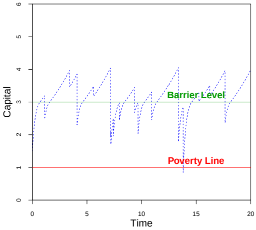
(b) $c_T = 1$



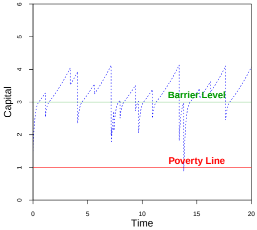
(c) $c_T = 2$



(d) $c_T = 3$



(e) $c_T = 4$



(f) $c_T = 6$

$$m_\delta(x) = \mathbb{E}\left[w(X_{\tau_x^-} - x^*, |X_{\tau_x} - x^*|)e^{-\delta\tau_x}\mathbb{1}_{\{\tau_x<\infty\}}\right]$$

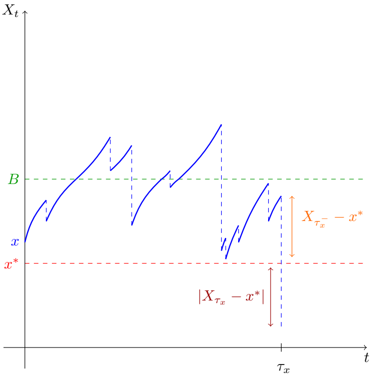


Figure 6: The capital surplus immediately before the trapping time and the capital deficit at the trapping time.

→ Let

$$\tau_x := \inf\left\{t \geq 0 : X_t < x^* \mid X_0 = x\right\}$$

denote the time when a household falls into the **area of poverty**.

→ $m_\delta(x)$ denotes the **expected discounted penalty function at the trapping time (or ruin)** for an initial capital $x \geq x^*$.

→ Similarly, $\psi(x) \equiv \mathbb{P}(\tau_x < \infty)$ is the **infinite-time trapping probability** for an initial capital $x \geq x^*$.

→ Note that, with $w(x_1, x_2) = 1$, $m_\delta(x)$ is the **Laplace transform of the trapping time**. Moreover, we have $\lim_{\delta \downarrow 0} m_\delta(x) = \psi(x)$.

Theorem. When $x \geq B$, we have

$$m_\delta^+(x) = \frac{\lambda}{r}(x - x^*)^{\frac{\lambda+\delta}{r}} \int_x^\infty \frac{1}{(u - x^*)^{\frac{\lambda+\delta}{r}+1}} \left[\int_{B/u}^{1} m_\delta^+(u \cdot z) dG_Z(z) + \int_{x^*/u}^{B/u} m_\delta^-(u \cdot z) dG_Z(z) + A(u) \right] du,$$

and when $x^* \leq x < B$, we have

$$m_\delta^-(x) = \frac{\lambda}{r - c_T}(x + x^{**})^{\frac{\lambda+\delta}{r-c_T}} \int_x^B \frac{1}{(u + x^{**})^{\frac{\lambda+\delta}{r-c_T}+1}} \left[\int_{x^*/u}^{1} m_\delta^-(u \cdot z) dG_Z(z) + A(u) \right] du$$

$$+ \frac{\lambda}{r}(B - x^*)^{\frac{\lambda+\delta}{r}} \left(\frac{x + x^{**}}{B + x^{**}} \right)^{\frac{\lambda+\delta}{r-c_T}} \int_B^\infty \frac{1}{(v - x^*)^{\frac{\lambda+\delta}{r}+1}} \left[\int_{B/v}^{1} m_\delta^+(v \cdot z) dG_Z(z) \right.$$

$$\left. + \int_{x^*/v}^{B/v} m_\delta^-(v \cdot z) dG_Z(z) + A(v) \right] dv,$$

where the function $A(u) := \int_0^{x^*/u} w(u - x^*, x^* - u \cdot z) dG_Z(z)$ and $x^{**} = \frac{c_T B - r x^*}{r - c_T}$.

Theorem. When $x \geq B$, we have

$$r(x - x^*)m'^+_\delta(x) - (\lambda + \delta)m^+_\delta(x) + \lambda\left[\int_{B/x}^{1} m^+_\delta(x \cdot z)dG_Z(z) + \int_{x^*/x}^{B/x} m^-_\delta(x \cdot z)dG_Z(z) + A(x)\right] = 0,$$

and when $x^* \leq x < B$, we have

$$(r - c_\tau)(x + x^{**})m'^-_\delta(x) - (\lambda + \delta)m^-_\delta(x) + \lambda\left[\int_{x^*/x}^{1} m^-_\delta(x \cdot z)dG_Z(z) + A(x)\right] = 0.$$

In addition, the boundary conditions for $m^+_\delta(x)$ and $m^-_\delta(x)$ are given by $\lim_{x\to\infty} m^+_\delta(x) = 0$, $m^+_\delta(B) = m^-_\delta(B)$ and $m^-_\delta(x^*) = \frac{1}{\lambda+\delta}\left[c_\tau(B - x^*)m'^-_\delta(x^*) + \lambda A(x^*)\right]$.

Question: Is it possible to obtain closed-form solutions for the expected discounted penalty function at the trapping time? Yes, it turns out that for the particular case in which $Z_i \sim Beta(\alpha, 1)$, we can obtain analytical expressions.

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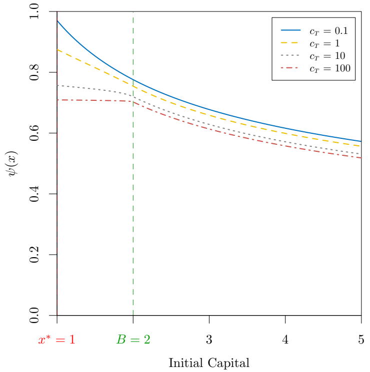

Figure 7: **Trapping probability $\psi(x)$ when $Z_i \sim Beta(1.25, 1)$, $a = 0.1$, $b = 3$, $c_s = 0.4$, $B = 2$, $\lambda = 1$ and $x^* = 1$ for $c_T = 0.1, 1, 10, 100$.**

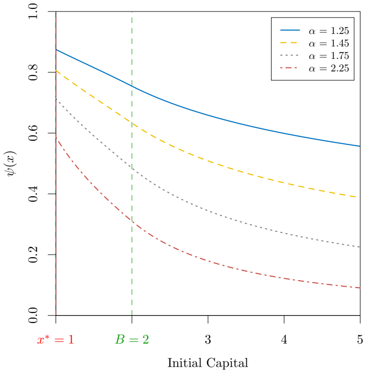

Figure 8: **Trapping probability $\psi(x)$ when $Z_i \sim Beta(\alpha, 1)$, $a = 0.1$, $b = 3$, $c_s = 0.4$, $c_T = 1$, $B = 2$, $\lambda = 1$ and $x^* = 1$ for $\alpha = 1.25, 1.45, 1.75, 2.25$.**

- We define the random variable τ_x^ω for $x \in (0, \infty)$ as the **time of extreme poverty** i.e. the moment at which a household with initial capital x becomes extremely poor and $\psi^\omega(x) = \mathbb{P}(\tau_x^\omega < \infty)$ as the **probability of extreme poverty**.
- The probability of extreme poverty is quantified by an **extreme poverty rate function** $\omega(x)$, where x represents the value of capital below the critical capital x^*.
- The extreme poverty rate function is defined in such a way in which **the probability of extreme poverty increases when the capital deficit** $|X_s - x^*|$ **grows**.
- Namely, for some capital $X_s < x^*$ and no prior extreme poverty event, the probability of extreme poverty on the time interval $[s, s + dt)$ is given by $\omega(X_s)\, dt$.

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1. Constant extreme poverty rate functions.

Let $\omega_1(x) \equiv \omega_c \cdot \mathbb{1}_{\{x<x^*\}}$ with $\omega_c > 0$. This is the simplest choice of the extreme poverty rate function and here, for example, the event of extreme poverty can be explained in the framework of randomised observation periods (Albrecher et al. 2013).

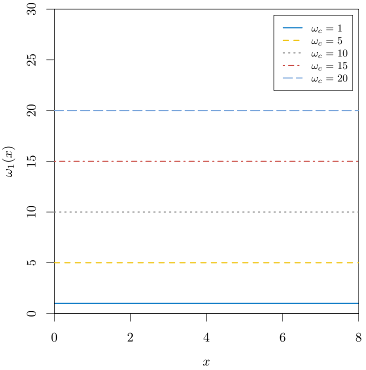

Figure 9: Constant extreme poverty rate function $\omega_1(x) \equiv \omega_c$ for $\omega_c = 1, 5, 10, 15, 20$.

Examples of Extreme Poverty Rate Functions

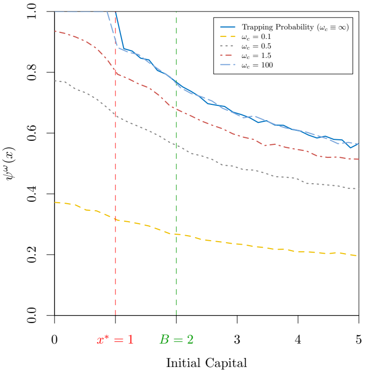

Figure 10: Probability of extreme poverty $\psi^{\omega}(x)$ when $Z_i \sim Beta(1.25, 1)$, $a = 0.1$, $b = 3$, $c_s = 0.4$, $c_\tau = 1$, $B = 2$, $\lambda = 1$, $x^* = 1$ for constant extreme poverty rate function $\omega_c = 0.1, 0.5, 1.5, 100$.

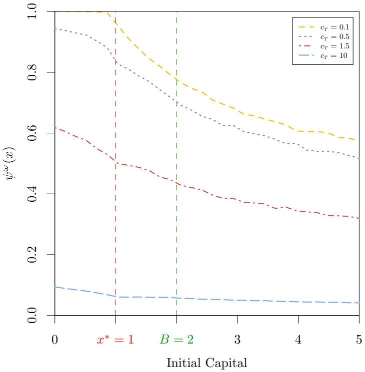

Figure 11: Probability of extreme poverty $\psi^{\omega}(x)$ with constant extreme poverty rate function $\omega_c = 0.5$ when $Z_i \sim Beta(1.25, 1)$, $a = 0.1$, $b = 3$, $c_s = 0.4$, $B = 2$, $\lambda = 1$, $x^* = 1$ for $c_\tau = 0.1, 0.5, 1.5, 10$.

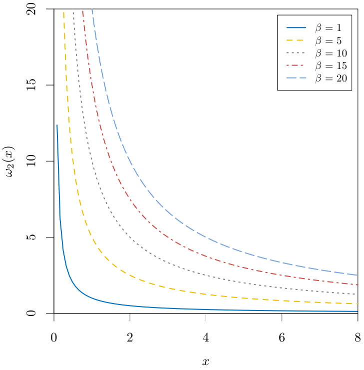

Figure 12: **Exponential extreme poverty rate function $\omega_2(x) = \frac{\beta}{x}$ for $\beta = 1, 5, 10, 15, 20$.**

2. *Exponential extreme poverty rate functions.*

Let now $\omega_2(x) = \frac{\beta}{x} \cdot \mathbb{1}_{\{x<x^*\}}$, for some $\beta > 0$. In this case, the extreme poverty rates take fairly flat values for lower deficit levels and reach higher values when the capital level gets close to zero.

Examples of Extreme Poverty Rate Functions

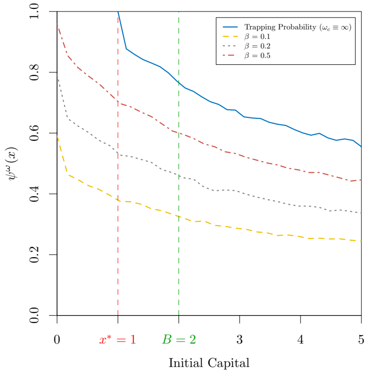

Figure 13: Probability of extreme poverty $\psi^{\omega}(x)$ when $Z_i \sim Beta(1.25, 1)$, $a = 0.1$, $b = 3$, $c_s = 0.4$, $c_T = 1$, $B = 2$, $\lambda = 1$, $x^* = 1$, $\omega_2(x) = \frac{\beta}{x}$ for $\beta = 0.1, 0.2, 0.5$.

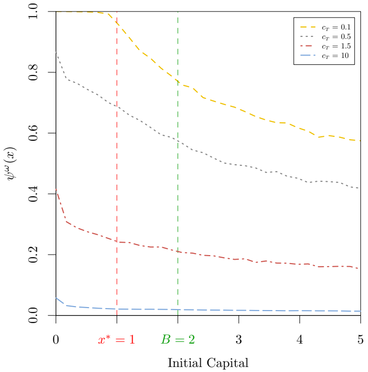

Figure 14: Probability of extreme poverty $\psi^{\omega}(x)$ when $Z_i \sim Beta(1.25, 1)$, $a = 0.1$, $b = 3$, $c_s = 0.4$, $B = 2$, $\omega_2(x) = \frac{0.1}{x}$, $\lambda = 1$, $x^* = 1$ for $c_T = 0.1, 0.5, 1.5, 10$.

Conclusion and Future Research

- **This article extends the model first introduced in** Kovacevic and Pflug (2011) by allowing households to have the possibility of escaping from the area of poverty due to CTs provided by donors or organisations.
- We **generalise the trapping concept to the extreme poverty one** by defining an extreme poverty rate as a function of the poverty gap, which designates a higher probability of becoming extremely poor to those households with greater deficit.
- Furthermore, for certain types of extreme poverty rate functions, we illustrate how the probability of extreme poverty can be simulated. We are still looking to derive **closed-form expressions for the probability of extreme poverty**.
- The numerical examples presented suggest that **periodic capital cash transfers could be an appropriate social protection strategy to reduce both the probability of a household becoming poor and extremely poor**.

Thank You For Your Attention

Any questions?

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References I

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