On the relationship between insurers insureds and intermediaries: a Cooperative Game Theory model approach

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Roadmap

Background

Determine the conditions for the optimal presence of insurance brokers in their relations with insurers (Insurance companies).

Existing contribution

Eckardt M. (2007) Insurance intermediation : An Economic Analysis of the Information Services Market, *In Contributions to Economics Series 22, Springer, Physica-Verlag*.

1. Analyzed the question with one insurer, an insured and an intermediary.
2. Propose a condition "IE" under which it is beneficial for the insurer to cooperate with Intermediary in signing a contract.

Drawback of existing contribution

1. No formal underlying model or theory in her framework.
2. What become her results when we have many insurers

Idea and history

Idea and two observations

1. Use Cooperative Game Theory
2. Insurer can form a coalition with an insured (contract managed between insurer and insured without intermediary) and the same insurer can form a coalition with another insured and the intermediary.
3. In the classical theory of cooperative games, a player can only participate in one and only one coalition.

History of Game Theory in Actuarial or Insurance :
Lemaire (1991), Dutang et al. (2013) and Asimit and Boonen (2018).

Objective

Formalize and study the interactions between insurer, insured and intermediary through cooperative games with multiple cooperation.

Specific objectives

- Extend first notions on CG to have first notions on CG with multiple Cooperation
- Study stability of the new game
- Application of obtained results to formalize relationships in Insurance

Definition : Game, payoff and Core

Definition

1. A cooperative game (shortly CG) : (N, v)
   N : set of players (actors, economic agents)
   v : mapping from $\mathcal{P}$(N) to $\mathbb{R}$ satisfying $v(\emptyset) = 0$.
2. A payoff (allocation) of (N, v) : $x = (x_1, x_2, \cdots, x_n) \in \mathbb{R}^N$ satisfying efficiency, that is, $x(N) = \sum_{i \in N} x_i = v(N)$.
3. A payoff $x = (x_1, x_2, \cdots, x_n)$ is :
   - individually rational if $\forall i \in N,\ x_i \geqslant v(i)$,
   - collectively rational if $\forall S \in 2^N,\ x(S) = \sum_{i \in S} x_i \geqslant v(S)$.
4. $\chi(N, v)$: the set of all payoffs of the game (N, v).

Definition

Dominance between payoffs

Let (N, υ) be a CG, $S \in 2^N$ and $x, y \in \chi(N, \upsilon)$.

1. y dominates x through S if $\forall i \in S,\ y_i > x_i$ and $y(S) \leqslant \upsilon(S)$.
2. x is dominated if $\exists y \in \chi(N, v), \exists S \in 2^N$ such that y dominates x through S.

Core of the CG (N, υ) : definition and known result

1. Core of the CG (N, υ) : the subset of $\chi(N, \upsilon)$ made up of all non-dominated payoffs.
2. (N, υ) is stable if $\mathcal{C}(N, \upsilon) \neq \varnothing$.

Known results on characterization of core elements :
$\mathcal{C}(N, \upsilon) = \{x \in \chi(N, \upsilon) : \forall S \in 2^N, x(S) \geqslant v(S)\}$.

Non emptiness of the core of the CG (N, v)

Definition : balancedness coefficients

1. A nonempty family of coalitions ζ of N is balanced if there exists a sequence $\gamma = (\gamma_T)_{T \in \zeta}$ of positive reals numbers satisfying : $\forall i \in N, \sum_{T \in \zeta,\ i \in T} \gamma_T = 1.$
2. (N, v) is balanced if for all balanced family of coalitions ζ of N with balancedness coefficients $(\gamma_T)_{T \in \zeta}$, we have : $\sum_{T \in \zeta} \gamma_T v(T) \leqslant v(N)$.

$(\gamma_T)_{T \in \zeta}$: balancedness coefficients of ζ.

NSC for non-emptiness of the core or stability of CG (N, v)

Theorem (Bondareva 1963 and Shapley 1967)

A CG (N, v) is stable if and only if it is balanced.

Cooperation in Insurance with Multi-cooperation : An Example

- $N = \{A, a_1, a_2, I\}$: set of four players with A : an insurer, a_1 and a_2 : two insureds and I : intermediary.
- Some possible cooperations (recoverings on N) :
  1. $\mathcal{S}_1 = \{\{A, a_1, a_2, I\}\} = \{N\}$
  2. $\mathcal{S}_2 = \{\{A, a_1, a_2\}; \{I\}\}$
  3. $\mathcal{S}_3 = \{\{A, a_1, I\}; \{A, a_2\}\}$
  4. $\mathcal{S}_4 = \{\{A, a_1, I\}; \{A, a_2, I\}\}$
  5. $\mathcal{S}_5 = \{\{A, a_1\}; \{A, a_2, \}; \{I\}\}$

Analysis of Example

1. $\mathcal{S}_3$, $\mathcal{S}_4$ and $\mathcal{S}_5$: the multiple cooperations with the presence of insurer (A) in each coalition,
   Recoverings : Union of their subsets is N.
2. Particular recovering : $\mathcal{S}_1$

Cooperative Game with Multi-cooperation

Definition of SCG and SCGMC

1. A structured cooperative game (shortly SCG) : $(N, v, \mathcal{S})$ where (N, v) is a CG and $\mathcal{S}$ is a recovering of N.
2. Cooperative game with multiple cooperations (shortly SCGMC) : The SCG $(N, v, \mathcal{S})$ where
   some elements of $\mathcal{S}$ are not pairwise disjoints, that is, $\mathcal{S}$ is not a partition of N.

Payoff of a Cooperative Game with Multi-cooperation

Let $(N, v, \mathcal{S})$ be a SCG with $\mathcal{S} = \{R_1, ..., R_m\}$.
Payoff of $(N, v, \mathcal{S})$: a sequence $x = \{x_{i,k}\}_{i \in N,\ 1 \leqslant k \leqslant m}$ of real numbers satisfying efficiency on coalitions of the recovering $\mathcal{S}$, that is,

$$\forall k \in \{1, 2, \cdots, m\},\ x(R_k) = \sum_{i \in R_k} x_{i,k} = v(R_k). \quad (1)$$

Dominance between payoffs in a SCG $(N, v, \mathcal{S})$ with $\mathcal{S} = \{R_1, ..., R_m\}$.

Two payoffs of $(N, v, \mathcal{S})$: $x = \{x_{i,k}\}_{i \in N,\ 1 \leqslant k \leqslant m}$ and $y = \{y_{i,k}\}_{i \in N,\ 1 \leqslant k \leqslant m}$.

1. y dominates x if

$$\exists\ k_0 \in \{1, 2, \cdots, m\},\quad \exists\ S \subseteq R_{k_0} \quad \text{such that} \sum_{i \in S} x_{i,k_0} < \sum_{i \in S} y_{i,k_0} \leqslant v(S).$$

2. x is dominated in the SCG $(N, v, \mathcal{S})$ if there exists a payoff z of $(N, v, \mathcal{S})$ that dominates x.

Core of a SCG $(N, v, \mathcal{S})$ with $\mathcal{S} = \{R_1, ..., R_m\}$: $\mathcal{C}(N, v, \mathcal{S})$.

$\mathcal{C}(N, v, \mathcal{S})$: the set of non-dominated payoffs.

Cooperative Game with Multi-cooperation : 1st main result

Characterization of elements of $\mathcal{C}(N, v, \mathcal{S})$

Let $x = \{x_{i,k}\}_{i \in N,\ 1 \leqslant k \leqslant m}$ be a payoff of $(N, v, \mathcal{S})$ be a SCG with $\mathcal{S} = \{R_1, ..., R_m\}$.
$x \in \mathcal{C}(N, v, \mathcal{S})$ if and only if

$$\forall k \in \{1, 2, \cdots, m\},\ \forall S \subseteq R_k, \sum_{i \in S} x_{i,k} \geqslant v(S).$$

Cooperative Game with Multi-cooperation : On the path of stability

Definition

Let $(N, v, \mathcal{S})$ be a SCG with $\mathcal{S} = \{R_1, ..., R_m\}$.

1. A $\mathcal{S}$-family is a sequence $\mathcal{F} = (\mathcal{F}_k)_{1 \leqslant k \leqslant m}$ satisfying $\forall k \in \{1, ..., m\}, \mathcal{F}_k$ is a family of non empty subsets of R_k.
2. A $\mathcal{S}$-family $\mathcal{F} = (\mathcal{F}_k)_{1 \leqslant k \leqslant m}$ is balanced if for all $k \in \{1, 2, \cdots, m\}$, there exists a family of positive real numbers $(\lambda_{\zeta_k})_{\zeta_k \in \mathcal{F}_k}$ satisfying :

$$\forall i \in N, \forall k \in \{1, 2, \cdots, m\}, \quad \sum_{i \in \zeta_k} \lambda_{\zeta_k} = 1.$$

3. The SCG $(N, v, \mathcal{S})$ is balanced if for all balanced $\mathcal{S}$-family $\mathcal{F} = (\mathcal{F}_k)_{1 \leqslant k \leqslant m}$, with balancedness coefficients $(\lambda_{\zeta_k})_{\zeta_k \in \mathcal{F}_k}$, we have :

$$\forall k \in \{1, 2, \cdots, m\}, \sum_{\zeta_k \in \mathcal{F}_k} \lambda_{\zeta_k} v(\zeta_k) \leqslant v(R_k).$$

Cooperative Game with Multi-cooperation : 2nd main result

Theorem : about stability of a SGC $(N, v, \mathcal{S})$ or non-emptiness of its core

$(N, v, \mathcal{S})$ is stable if and only if it is balanced.

Insurance market model : set of players and some hypothesis

Three types of players in an insurance market

- insurer : A; n insureds $(n \in \mathbb{N}^*)$: $a_1,...,a_n$; and the intermediary : I.
- Set of players :

$$N = \{A, a_1 \cdots , a_n, I\}. \quad (2)$$

$N_a = \{a_1 \cdots , a_n\}$ the subset of N made up of insureds. We have : $N = N_a \cup \{A, I\}$ and 2^{N_a} is the set of all coalitions of insureds.

Two other hypotheses about recovering of N in an insurance market

Assumption H_3 : insurance is compulsory and therefore, any coalition which contains at least one insured must contain the insurer.
Assumption H_4 : only the insurer can simultaneous participate to direct and intermediary contracts.

Insurance market model : Coalitions gains (inspired from Eckardt, 2007)

Gross gain of in exchange between the insurer and the insured a_i and the characteristic function

- Direct cooperation : $V_{a_i}^D - C_{a_i}^D - (T_{insured}^D + T_{insurer}^D)$.
- Intermediary exchange : $V_{a_i}^I - C_{a_i}^I - (T_{insured}^I + T_{insurer}^I)$.

$v(K)$ is given by :

$$\begin{cases} \sum_{a \in K \cap N_a} (V_a^D - C_a^D) - (T_{K \cap N_a}^D + T_{ins, K \cap N_a\}}^D) & \text{if } A \in K, \exists a_i \in K \text{ and } I \notin K, \\ \sum_{a \in K \cap N_a} (V_a^I - C_a^I) - (T_{K \cap N_a}^I + T_{ins, K \cap N_a}^I) & \text{if } A \in K, \exists a_i \in K \text{ and } I \in K, \\ 0 & \text{otherwise.} \end{cases} \quad (3)$$

Insurance market model : First contribution with Game Theory approach

Proposition : Third main result

Let $(N, v, \mathcal{S})$ be a ISCG with $\mathcal{S} = \{\{A, a_1, I\}; \cdots; \{A, a_n, I\}\}$. The following assertions are equivalent.

1. IE_1 is satisfied

$$\forall a_i \in N_a, V_{a_i}^D - C_{a_i}^D - (T_{\{a_i\}}^D + T_{ins,\{a_i\}}^D) \leqslant V_{a_i}^I - C_{a_i}^I - (T_{\{a_i\}}^I + T_{ins,\{a_i\}}^I).$$

2. $\mathcal{C}(N, v, \mathcal{S}) \neq \varnothing$.

IE_1 : extension of condition IE (Eckardt, 2007) :

$$(V^D - C^D) - (T_{insured}^D + T_{insurer}^D) < (V^I - C^I) - (T_{insured}^I + T_{insurer}^I).$$

Insurance market model : Impact of the structure of coalitions on stability through an example

Set of players and various amounts

$N = \{A, a_1, a_2, I\}$

1) $V^D_{a_1} = V^D_{a_2} = 15$ and $V^I_{a_1} = V^I_{a_2} = 14.5$.
2) $C^D_{a_1} = C^D_{a_2} = 9$ and $C^I_{a_1} = C^I_{a_2} = 8$.
3) $T^D_{\{a_1\}} = T^D_{\{a_1\}} = 3$, $T^D_{\{a_1,a_2\}} = 4$, $T^I_{\{a_1\}} = T^I_{\{a_1\}} = 2$ and $T^I_{\{a_1,a_2\}} = 6$.
4) $T^D_{ins,\{a_1\}} = T^D_{ins,\{a_1\}} = 1$, $T^D_{ins,\{a_1,a_2\}} = 1$, $T^I_{ins,\{a_1\}} = T^I_{ins,\{a_1\}} = 1$, and $T^I_{ins,\{a_1,a_2\}} = 1$.

Characteristic function v

$\{A, a_1, a_2, I\}$	$\{A, a_1, a_2\}$	$\{A, a_i, I\}\quad_{i=1,2}$	$\{A, a_i\}\quad_{i=1,2}$	others
6	7	3.5	2	0

Insurance market model : Impact of the structure of coalitions on stability through an example

Impact of the structure of coalitions on stability

- For $\mathcal{S} = \{\{A, a_1, I\}; \{A, a_2, I\}\}$:
  $(N, v, \mathcal{S})$ satisfies condition IE_1. Thus $\mathcal{C}(N, v, \mathcal{S}) \neq \emptyset$.
- For $\mathcal{S} = \{N\}$: $\mathcal{C}(N, v, \{N\}) = \mathcal{C}(N, v) = \emptyset$.

Insurance market model : second contribution with Game Theory approach

ISGC with intermediary exchange

The ISGC $(N, \upsilon, \mathcal{S})$ is in an intermediary exchange if $I \in \underset{K \in \mathcal{S}}{\cap} K$.

Proposition : Fourth main result

Let $(N, \upsilon, \mathcal{S})$ be a ISCG with an intermediary exchange.
The following assertions are equivalent :

1. Condition IE_2 is satisfied and the total transaction cost mapping T is sub-additive.
2. $\mathcal{C}(N, \upsilon, \mathcal{S}) \neq \varnothing$.

$T : 2^{N_a} \to \mathbb{R}^+$ where $\forall F \in 2^{N_a}$, $T(F) = T_F^I + T_{ins,F}^I$.

T is sub-additive, that is,
$\forall F, G \in 2^{N_a}, F \cap G = \varnothing \implies T(F \cup G) \leqslant T(F) + T(G)$.

Condition IE_2 : second extension of condition IE

$\forall K \subseteq N$ with $A \in K$,

$$\left(\sum_{a \in K \cap \{a_1, \cdots, a_n\}} (V_a^D - C_a^D) \right) - (T^D_{K \cap \{a_1, \cdots, a_n\}} + T^D_{ins, K \cap \{a_1, \cdots, a_n\}}) \leqslant \left(\sum_{a \in K \cap \{a_1, \cdots, a_n\}} (V_a^I - C_a^I) \right) - (T^I_{K \cap \{a_1, \cdots, a_n\}} + T^I_{ins, K \cap \{a_1, \cdots, a_n\}}). \quad (4)$$

Summary

- Introduce the first concepts of a cooperative game with multiple cooperations : recovery of all players, payoff, dominance between payoffs, core of the game and game stability.
- Characterize the elements of the classical core of the new game, Establish stability of cooperative game with multiple cooperations.
- Establish the conditions under which it would be in the insurer's interest to draw up each contract with a policyholder (individual contract) through an intermediary.

Open question

Study an insurance market with several insurance companies and several intermediaries.

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