Optimal Savings and Portfolio Choice with Risky Labor Income and Reference-Dependent Preferences

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based on joint work with

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1. Introduction

- How to optimally save and invest total wealth over the life cycle has been studied extensively.
- Human capital constitutes the largest part of total wealth.
  - For average US household, it is 90% of total wealth.
- Impact of risk-free and tradable labor income on optimal choice is well understood.
  - Bond-like human wealth diversifies stock return risk.
- Some authors explore the impact of non-tradable risky labor income but assume traditional preferences.
  - CRRA or Epstein-Zin.
- We explore joint impact of reference-dependent preferences and non-tradable risky labor income on optimal savings and portfolio decisions.

Risky Labor Income and Reference-Dependent Preferences

- Labor income is not risk-less as has been vividly illustrated by the recent Covid-19 crisis.
  - US unemployment rates rise from 3.5% to 14.7%.
- A large experimental and empirical literature has shown substantial deviations from traditional preferences.
  - Reference-dependence is one of the strongest empirical phenomena in decision under risk.
- To understand how risky human wealth affects optimal savings and portfolio decisions is of great importance.
- This paper analyzes this question for an individual with reference-dependent preferences.

Three Main Findings

1. Impact of a labor income shock on the optimal savings rate and the optimal portfolio share is more pronounced under reference-dependent preferences than under CRRA preferences.
    - Excess sensitivity of optimal savings rate and optimal portfolio share.
    - Protect current consumption and postpone painful reductions.
    - In a wide range of scenarios, the individual already withdraws pension wealth before retirement.
    - An institutional setting in which individuals cannot easily unlock pension wealth before retirement can be quite costly in welfare terms.

Three Main Findings

2. Optimal response of the savings rate and portfolio share to a labor income shock is highly heterogeneous and varies heavily with the ratio of consumption to the reference level (proxy for income).
    - Low ratio ⇒ more sensitive; high ratio ⇒ less sensitive.
    - Under CRRA, the optimal responses are independent of this ratio.

Three Main Findings

3. Optimal investment strategy is more conservative compared to the case with risk-less labor income and CRRA preferences.
    - Non-tradable risky labor income causes the optimal share invested in the risky stock to decrease.
    - An endogenous reference level has two additional counteracting effects on the optimal portfolio share.
    - For a typical range of parameter values, we find the net effect yields a reduction in the optimal share invested in the risky stock.

Methodological Contribution

- As our model involves market incompleteness and behavioral preferences, we cannot use standard solution methods.
- We develop a non-trivial solution technique to determine the optimal policies and the shadow price of labor income risk.
- Solution procedure is a methodological contribution of interest in its own right.
- We can determine optimal choices in a setting where not only past own consumption but also consumption of neighbors or individual labor income may affect reference level.
- One may even incorporate loss aversion into our setting.

Relating Our Findings to Empirical Analysis

- Our analysis generates several testable implications.
- We briefly explore how our main findings relate to real monthly savings data.
- Using monthly data on total expenditures and incomes, we test the excess sensitivity of the optimal savings rate and the heterogeneous response of optimal savings rate.
- Consistent with our main findings, we find excess sensitivity of the optimal savings rate; and find that the optimal savings rate of a low-income individual exhibits higher degree of excess sensitivity than that of a high-income individual.

Outline

1. Introduction
2. Model
3. Solution Method
4. Main Findings
5. Conclusion

2. Model

Preferences:

- Denote by $c(t)$ and $h(t)$ the individual's consumption level and reference level at time t.
- Expected lifetime utility is given by

$$U = \mathbb{E}_0 \left[\int_0^{T_D} e^{-\delta t} u \left(c(t) - h(t) \right) \mathrm{d}t \right],$$

  with $\delta > 0$ time preference rate and T_D the date of death.
- We impose weak assumptions on the utility function u.
- In the base model, we assume that the reference level satisfies

$$\mathrm{d}h(t) = \left(\beta c(t) - \alpha(t) h(t) \right) \mathrm{d}t,$$

  where the depreciation rate α is allowed to be time-dependent.
- In a more general specification, the reference level is allowed to depend not only on own past consumption but also on past consumption of the individual's neighbors and individual past labor income.

Model

State Variables, Individual Labor Income, and Financial Market:

- We consider an economy with two state variables: non-tradable risky labor income $Y(t)$ and the risky stock price $S(t)$.
- We assume generic dynamics of individual labor income, driven by a Brownian motion $Z_Y(t)$.
- We assume the following dynamics for the stock price $S(t)$ and the price of a risk-less asset $B(t)$:

$$\begin{aligned} \mathrm{d}S(t) &= (r + \lambda_S \sigma_S)\, S(t)\mathrm{d}t + \sigma_S S(t)\mathrm{d}Z_S(t), \\ \mathrm{d}B(t) &= rB(t)\mathrm{d}t, \end{aligned}$$

  where $\lambda_S \in \mathbb{R}$ denotes the market price of stock return risk, $\sigma_S > 0$ models the stock return volatility, $Z_S(t)$ is a Brownian motion, and $r \in \mathbb{R}$ denotes the risk-less interest rate.
- We allow $Z_S(t)$ and $Z_Y(t)$ to be correlated, and denote their correlation coefficient by $\rho_{SY} \in [-1, +1]$.

Dynamic Budget Constraint

- Denote by $\omega(t)$ the share of pension wealth $F(t)$ invested in the risky stock at adult age t.
- The individual's dynamic budget constraint is given by

$$\begin{aligned}\mathrm{d}F(t) = &\left(r + \omega(t)\lambda_S\sigma_S\right) F(t)\mathrm{d}t + \omega(t)\sigma_S F(t)\mathrm{d}Z_S(t) \\ &+ \left(Y(t) - c(t)\right)\mathrm{d}t.\end{aligned}$$

- Pension wealth grows because of two reasons:
  - (i) investment results;
  - (ii) new savings $Y(t) - c(t)$.

Dynamic Optimization Problem

- The individual faces the following dynamic maximization problem:

$$\max_{c(t),\omega(t)} \quad \mathbb{E}_0\left[\int_0^{T_D} e^{-\delta t} u\left(c(t)-h(t)\right)\mathrm{d}t\right]$$

$$\mathrm{d}h(t) = \left(\beta c(t) - \alpha(t)h(t)\right)\mathrm{d}t$$

$$\mathrm{d}F(t) = \left(r + \omega(t)\lambda_S\sigma_S\right)F(t)\mathrm{d}t + \omega(t)\sigma_S F(t)\mathrm{d}Z_S(t) + (Y(t)-c(t))\mathrm{d}t.$$

- Maximize expected lifetime utility subject to the reference level dynamics and the dynamic budget constraint.

Shadow Price of Labor Income Risk

- Solving the dynamic optimization problem using martingale methods requires a stochastic discount factor, denoted by $m(t)$.
- One can show that $m(t)$ satisfies the following dynamics:

$$\mathrm{d}m(t) = -rm(t)\mathrm{d}t + \phi^{\top}(t)m(t)\mathrm{d}Z(t),$$

  where $Z(t) \equiv (Z_S(t), Z_Y(t))$ and $\phi(t)$ is a vector of factor loadings.
- One can determine $\phi(t)$ from the vector of prices of risk $\lambda(t) \equiv (\lambda_S, \lambda_Y(t))$, where $\lambda_Y(t)$ denotes the shadow price of labor income risk.
  - By the principle of no arbitrage, $\lambda_S = (\mu_S - r)/\sigma_S$.
- However, as labor income risk is non-tradable, the principle of no arbitrage does not uniquely determine $\lambda_Y(t)$.

3. Solution Method

1. Given a price of labor income risk $\lambda_Y(t)$, we determine the optimal consumption policy.
   a. Transform individual's dynamic optimization problem into a static variational problem.
   b. Transform static variational problem into a dual maximization problem.
   c. Determine optimal dual consumption using standard techniques; and transform back into optimal (primal) consumption.
2. Endogenously determine $\lambda_Y(t)$ and $\omega(t)$ such that changes in the value of future optimal consumption match changes in (tradable) total wealth.
   - $\lambda_Y(t)$ is chosen such that the demand for consumption plans that are not marketed is zero.
3. Substitute $\lambda_Y^*(t)$ in the optimal (primal) consumption policy.

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Inspired by

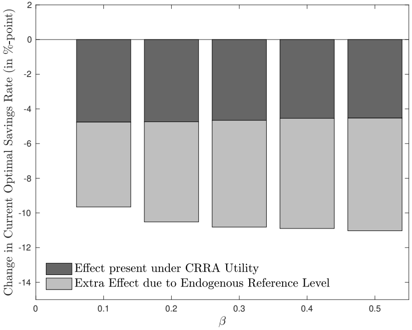
- Pliska (1986), Karatzas, Lehoczky, and Shreve (1987), and Cox and Huang (1989, 1991) (Step 1.a);
- Schroder and Skiadas (2002) and Van Bilsen, Laeven and Nijman (2020) (Steps 1.b–c.);
- He and Pearson (1991) and Sangvinatsos and Wachter (2005) (Steps 2–3).

4. Main Findings

- For the illustrations that follow, we rely where possible on parameter values from the existing literature.
- Our main implications remain qualitatively unchanged if we vary the values of the parameters within reasonable limits.

Main Finding I: Excess Sensitivity of Optimal Savings Rate

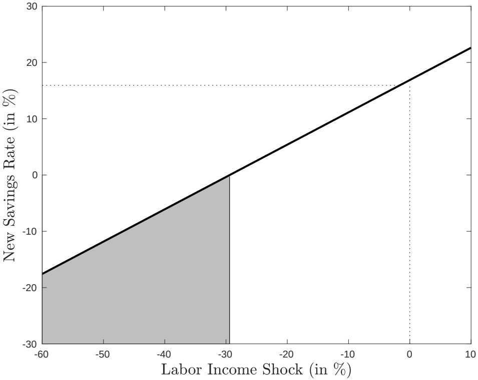
- After a permanent drop in current labor income, the current optimal savings rate decreases.
- We can decompose the optimal response into two parts.
  - The first part is due to a preference for consumption smoothing.
  - The second part is due to the endogeneity of the reference level: Excessive sensitivity!



Decomposition of Optimal Response

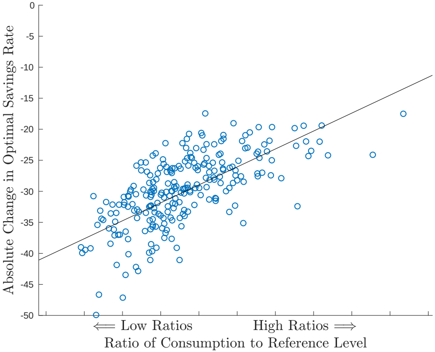
Main Finding I: Withdrawing Pension Wealth

- In a wide range of economic scenarios, i.e., the grey area, the individual does not save at all and withdraws pension wealth already before retirement.
- Excessive sensitivity!



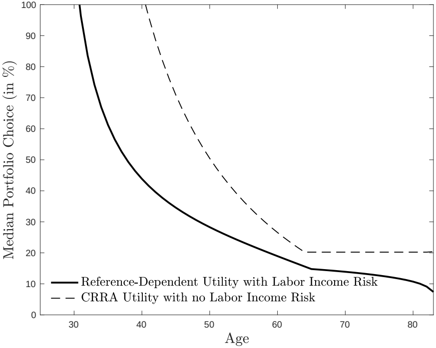
Main Finding II: Heterogeneity in Optimal Response

- The optimal response of the current savings rate heavily varies with the ratio of consumption to the reference level.
  - The ratio of consumption to the reference level can be seen as a proxy for income.
- If the ratio of consumption to the reference level is small, the optimal savings rate is heavily reduced after a permanent drop in labor income.



Main Finding III: Conservatism

- Non-tradable labor income risk and reference-dependent preferences lead to a conservative optimal portfolio strategy.



Welfare Costs

A strategy in which the savings rate does not respond excessively sensitive to a labor income shock can be quite costly in welfare terms.

true parameters α and β		
α	β	minimum welfare loss (in %)
0.05	0.1	38.04
0.1	0.2	35.08
0.2	0.3	30.13
0.3	0.4	26.04
0.4	0.5	23.52

Welfare costs are measured in terms of the relative decline in certainty equivalent consumption.

Different Types of Labor Income Shocks

- We also explore the robustness of our main findings for the case in which labor income shocks are not permanent.
- We analyze a continuous-time labor income process with temporary (non-permanent, partially transitory) labor income shocks.
- Our main findings remain intact.
- After a labor income shock, the optimal policies converge back to their levels before the shock, due to the gradual absorption of shocks and the temporary nature of the income shocks.

Testable Implications

- Our model generates a number of implications that can be tested using data.
- We briefly explore monthly savings data.
- We test the excess sensitivity (over-responsiveness) of the savings rate, which is the counterpart of excess smoothness (under-responsiveness) of consumption.
- We also analyze heterogeneity in the response of the savings rate.

Data

- We obtain data from the U.S. Bureau of Labor Statistics (Consumer Expenditure Survey).
- We use monthly data on labor income and total expenditures.
- Our dataset runs from January 2020 to August 2021 (20 periods).
- Our dataset includes 15,381 unique individuals.

Heterogenous Response of Expenditures to Income Shocks

- Regression model:

$$\Delta \log c(t) = \beta \Delta \log Y(t-1) + \epsilon(t).$$

- We divide the data into 3 income groups:
  - Low monthly gross incomes
  - Middle monthly gross incomes
  - High monthly gross incomes
- Coefficient estimates (all statistically significant):
  - $\hat{\beta}_{Low} = 0.0270$
  - $\hat{\beta}_{Middle} = 0.1200$
  - $\hat{\beta}_{High} = 0.2159$
- Heterogeneous excess sensitivity.
- CRRA preferences do not predict any of this.

5. Conclusion

- We have explored the joint impact of reference-dependent preferences and non-tradable labor income risk.
- Three key findings:
  1. Excess sensitivity of optimal consumption and portfolio share to labor income shocks. Withdrawing pension wealth in a wide range of economic scenarios.
  2. Response is heterogeneous and heavily varies with ratio of consumption to reference level.
  3. Conservative consumption and investment strategies.
- Welfare losses can be as large as 35%.
- Findings remain intact in the case in which labor income shocks are not permanent.
- Findings are consistent with patterns in monthly savings data.
- To analyze the optimal policies and to determine the shadow price of labor income risk, we have developed a non-trivial solution procedure.

Thank you for your attention!