Dependence modeling in General Insurance using LGC and HMMs

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Dependence modeling-Motivation

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- LGC and HMMs presented as alternative approach

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- Linear to non-linear ρ interpretation

HMMs

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- Handling Varying Patterns and Regimes.

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HMMs

- Capture complex temporal dependencies
- Handling Varying Patterns and Regimes.
- Assumes that data-generating process corresponds to a time-dependent **mixture** of conditional distributions driven by **Hidden states**

LGC basic notation

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- The local population parameters $\lambda(x) = (\mu(x), \Sigma(x))$ can be defined by minimizing a penalty function q measuring the difference between f and ψ

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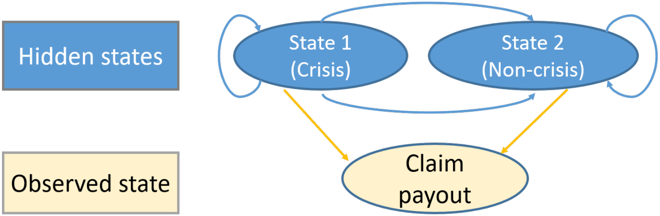
- Used TMB



QUESTION?

What is the probability of an observed claim sequence given the parameters of the HMM?
i.e. Transition Probability Matrix, Emission Probability Matrix and the stationary Distribution

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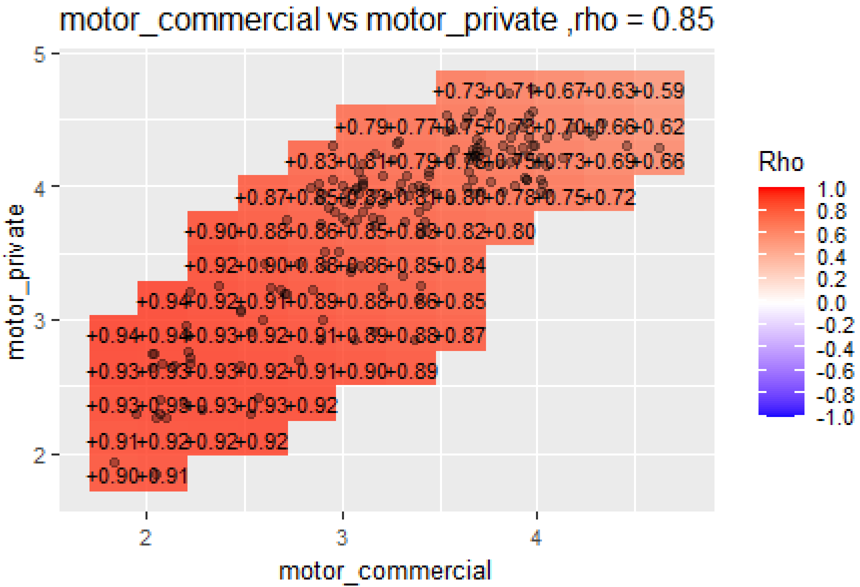
- For homeowners-7 years from October 2012 to November 2018 giving 314 weekly records

Table: Pair-wise correlation with p-values of ordinary correlation test and LGC test for independence

Variables Pairs	Pearson's ρ on original scale	Pearson's ρ on Log scale	p-value Pearson's ρ test on original scale	p-value Pearson's ρ test on log scale	p-value LGC'test
engineering vs fire-industrial	0.15	0.32	0.05*	0.00***	0.00***
engineering vs liabilities	-0.03	0.015	0.69	0.84	0.23
engineering vs motor-commercial	0.08	0.26	0.32	0.00***	0.00***
engineering vs motor-private	0.17	0.36	0.03*	0.00***	0.00***
engineering vs workers-compensation	0.01	0,20	0.92	0.01**	0.00***
engineering vs personal accident	-0.02	0.18	0.83	0.017*	0.00***
fire-industrial vs liabilities	0.02	0.12	0.75	0.11	0.00***
fire-industrial vs motor-commercial	0.15	0.44	0.05*	0.00***	0.00***
fire-industrial vs motor-private	0.24	0.51	0.00***	0.00***	0.00***
fire-industrial vs workers-compensation	0.07	0.37	0.37	0.00***	0.00***
fire-industrial vs personal-accidents	-0.02	0.21	0.74	0.01**	0.00***
liabilities vs motor-commercial	0.01	0.09	0.85	0.246	0.00***
liabilities vs motor-private	0.04	0.14	0.57	0.07*	0.00***
liabilities vs worker-compensation	0.05	0.09	0.65	0.25	0.00***
liabilities vs personal-accidents	-0.01	0.05	0.88	0.5114	0.01**
motor-commercial vs motor-private	0.75	0.85	0.00***	0.00***	0.00***
motor-commercial vs workers-compensation	0.32	0.65	0.00***	0.00***	0.00***
motor-commercial vs personal-accidents	0.10	0.42	0.17	0.00***	0.00***
motor-private vs workers-compensation	0.32	0.67	0.00***	0.00***	0.00***
motor-private vs personal-accidents	0.21	0.51	0.00***	0.00***	0.00***
workers-compensation vs personal-accidents	0.01	0.29	0.87	0.00***	0.00***


motor_commercial vs motor_private ,rho = 0.85
motor_private
motor_commercial
Rho
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0

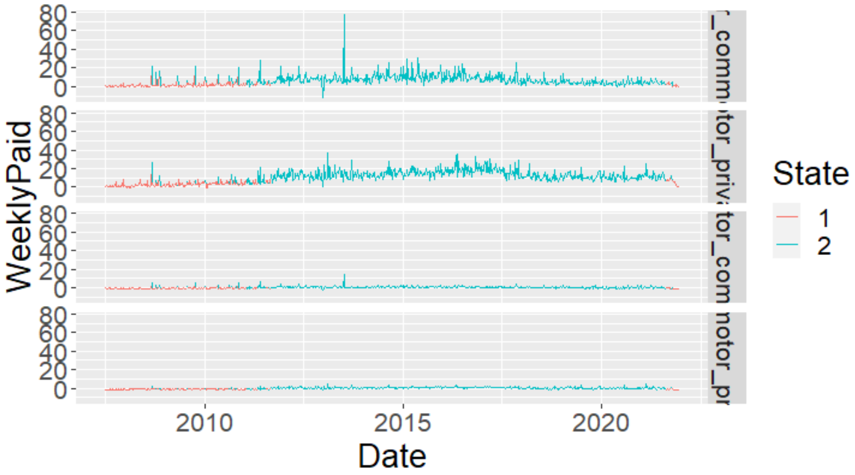
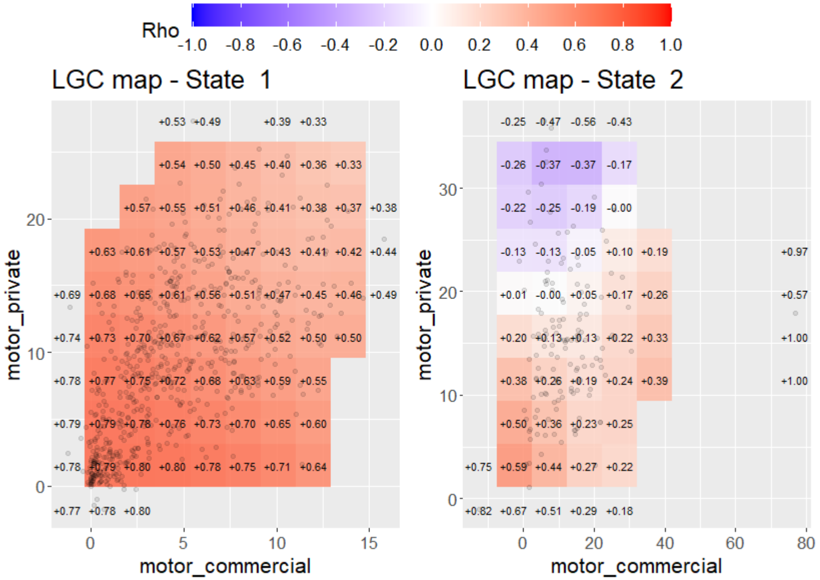

WeeklyPaid
80
60
40
20
0
r_comm
tor_priv
tor_com
notor_pr
2010
2015
2020
Date
State
1
2

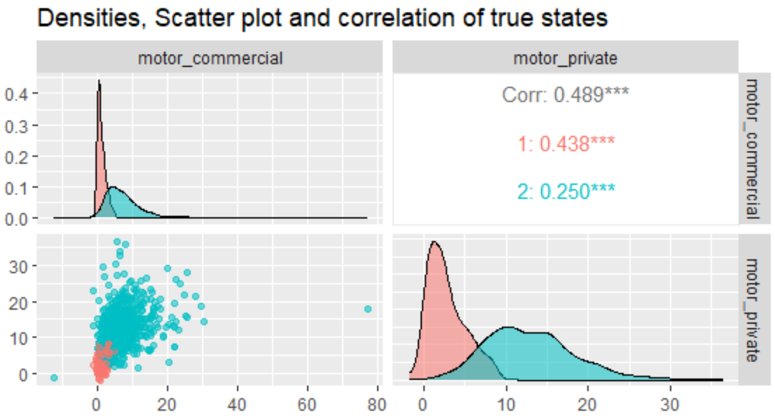
Table: Table central measures

Descriptive measures										
LoB	States	n	mean	sd	median	min	max	Skew	Kurtosis	se
Motor-commercial	state 1	615	4.41	3.53	3.74	-1.23	15.789	0.69	-0.33	0.14
	state 2	141	11.65	9.14	10.53	-12.59	77.03	2.62	16.79	0.77
	All states	756	5.76	5.79	4.44	-12.59	77.03	3.42	30.96	0.21
Motor-private	state 1	615	9.09	5.94	8.94	-1.74	27.33	0.3	-0.69	0.24
	state 2	141	15.34	7.11	15.29	-1.04	36.52	0.54	0.27	0.6
	All states	756	10.25	6.64	9.99	-1.74	36.52	0.52	0.2	0.24
Workers-Compen	state 1	615	1.30	1.00	1.05	-1.04	4.62	0.82	0.07	0.04
	state 2	141	5.91	11.93	4.92	-0.83	139.48	9.98	108.55	1.00
	All states	756	2.16	5.5.2	1.3	-1.04	139.48	20.66	506.02	0.2

Table: HMM parameter Estimates

Estimated parameters								
Parameter	Estimate	Std.Error	Parameter	Estimate	Std.Error	Parameter	Estimate	Std.Error
$\mu_{1,1}$	1.1713	0.127	$\sigma_{1,21}$	1.2101	0.2852	γ_{11}	0.9099	0.0229
$\mu_{1,2}$	2.7979	0.2097	$\sigma_{1,22}$	5.36397	0.7005	γ_{12}	0.0902	0.0229
$\mu_{2,1}$	7.4306	0.2599	$\sigma_{2,11}$	34.7537	2.100	γ_{21}	0.0350	0.0087
$\mu_{2,2}$	12.9586	0.2624	$\sigma_{2,12}$	8.2245	1.4764	γ_{22}	0.9650	0.0087
$\sigma_{1,11}$	1.3655	0,2657	$\sigma_{2,21}$	8.2245	1.4764	δ_1	0.2797	0.0641
$\sigma_{1,12}$	1.2101	0.2852	$\sigma_{2,22}$	30.5270	1.9031	δ_2	0.7203	0.0641


Rho
-1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8 1.0
LGC map - State 1
motor_private
motor_commercial
LGC map - State 2
motor_private
motor_commercial


Densities, Scatter plot and correlation of true states
motor_commercial
motor_private
Corr: 0.489***
1: 0.438***
2: 0.250***
motor_commercial
motor_private
0.4
0.3
0.2
0.1
0.0
30
20
10
0
0
20
40
60
80
0
10
20
30

Test of asymmetric dependence between states

Hypothesis:

$$H_0: \quad \rho_1(x_i, y_j) = \rho_2(x_i, y_j) \quad \text{for} \quad i, j = 1, \cdots, n$$
$$H_1: \quad \rho_1(x_i, y_j) \neq \rho_2(x_i, y_j) \quad \text{for} \quad i, j = 1, \cdots, n$$

- 1000 bootstrap replicates carried out

- p-value =0.000

- Statistically significant differences between the dependence structures of the two states

- Comparing with historical events in Kenya, it appears that the HMM can identify crisis periods

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- Model found time carrying dependency-financial crisis
- Dependency structure differs
- This analysis enables insurers to better assess and manage their overall risk exposure across different lines of business especially during economic,political crisis periods

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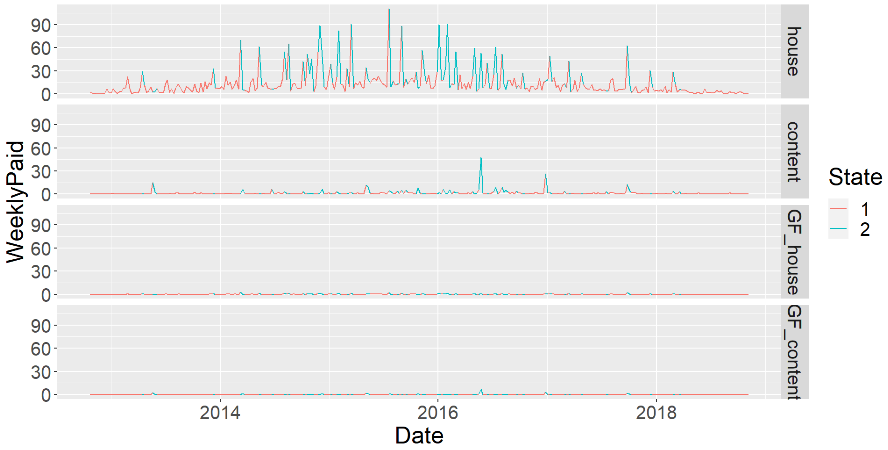
- Apply to Reserving, pricing,reinsurance arrangements?



WeeklyPaid
90
60
30
0
house
content
GF_house
GF_content
2014
2016
2018
Date
State
1
2