29.803/~~1973~~ Shaft levelling SL

I Definition

SL uses ~~level~~ instruments and (vertically) suspended tapes in lieu of levelling staffs in order to determine height differences between points.

Purpose

SL is used for control surveys in mining surveys and construction surveys, settlement surveys being included in the latter category.

Bibliography

1. Rack, Längung freihängender Messbänder durch Eigengewicht. (Lengthening of freely hanging survey tapes owing to their own weight) *5 German words = 11 English words*, Mitteilungen aus dem Markscheidewesen (News from Mines.) Heft 3, 1956, 103-108
2. M. Pearce Corrections for Tension in a vertically suspended tape. Aust. Surveyor June 1959, No 6, 364-66.
3. M. Pearce Private correspondence on 2 with H.W. 1971-2, copies distributed to 3rd F/T 1972 & 1973. & 1974 & Staff 1972
4. E.A. Brady, Master's thesis - The determination of Subsidence, Shrinkage & Deformation of a High Rise Tower Block, UNSW, Feb. 1973. B. quoted valuable bibliography.

II Notation

This varies from that used by Rack & Pearce.

x - unextended length at zero pull on flat.

x' - extended length at zero pull, hanging freely.

dx - unextended small tape interval

dx' - extended " " "

$(W) \equiv P_f$ - tension ≡ Pull at field work [newton]

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notation ...

ω = N/m weight of tape per metre length

L = length at zero pull (P_0) on interval AB (total length of graduation or freely hanging part of tape)

L' = extended L (owing to own weight)

A_i; B X_i – points along tape

a_i' = extended lengths of tape counted from B below upwards (x' being counted in the opposite direction i.e. downward from A)

L_f, x_f = length at pull P_f

P_{ST} – standard pull (tension)

x_t – length at pull P_{ST}

c_p – amount of lengthening for an interval X_2, X_1, i.e. $x_2' - x_1'$

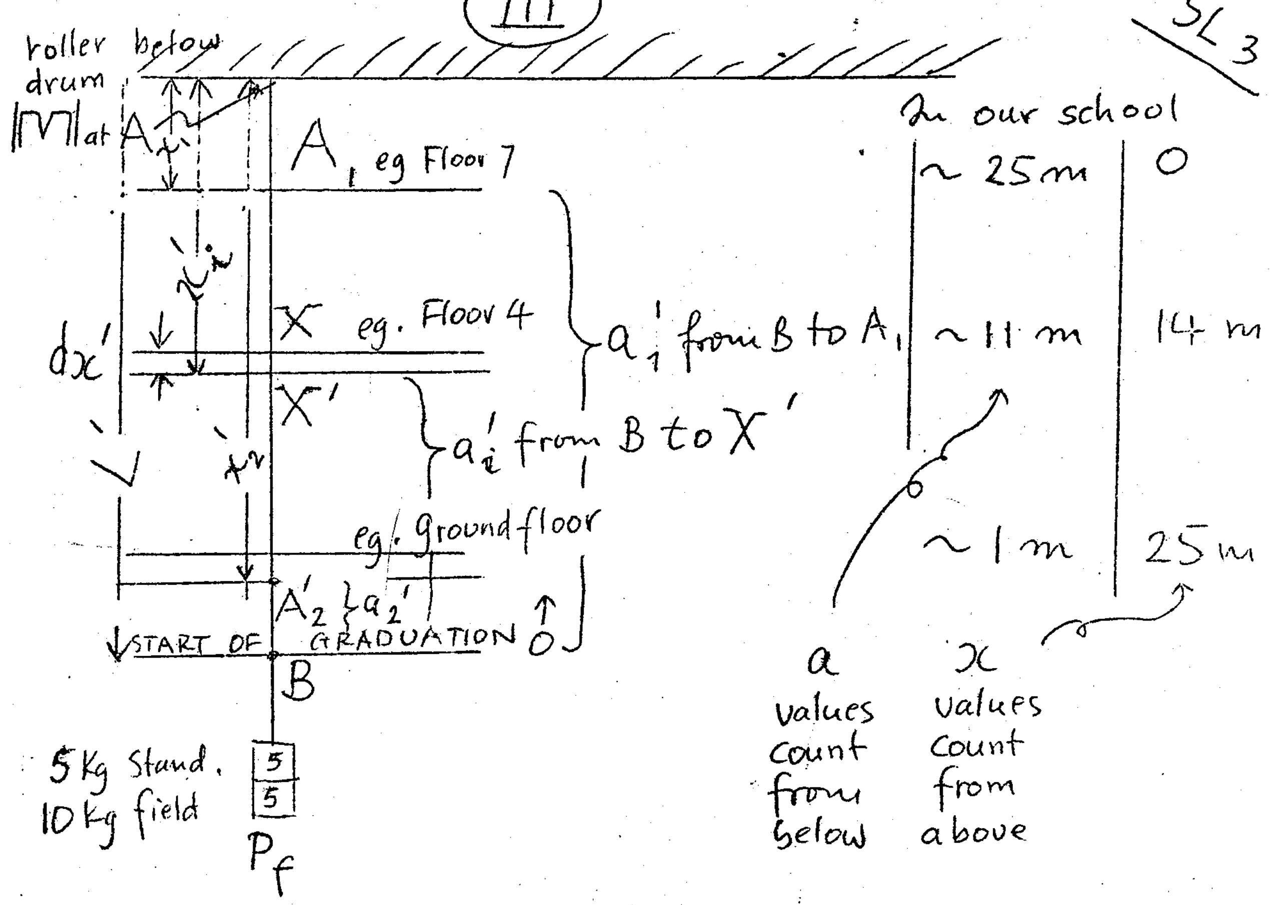
c_t – as c_p but at standard pull

α – coefficient of thermal expansion of steel $\sim 11 \times 10^{-6} / C^\circ$

A – cross sectional area of steel tape (UNSW 0.5×3 mm^2 or 0.5×10 mm^2 for the white tape rolled on canvass winder marked SL)

E – Young's modulus of elasticity of steel 20.14×10^{10} N/m^2
(2.05×10^{10} Kgf/m^2)
(2.05×10^{6} Kgf/cm^{-2})

γ – density of steel (German formulae)
7.8×10^3 Kg/m^3 (7.8 g/cm^3 : 7.8 [illegible]



According to Pearce (2) – at any point X
the pull $P_x = P_f + w(L - x)$ (01)

$$dx' = dx\left(1 + \frac{P_x}{AE}\right) \quad (02)$$

(01) in (02) $$dx' = dx\left(1 + \frac{P_f + w(L-x)}{AE}\right) \quad (03)$$

then the extended length x' from A to X

$$x' = \int_0^x \left[1 + \frac{P_f + w(L-x)}{AE}\right] dx \quad (04)$$

$$\therefore \quad x' = x\left[1 + \frac{P_f + wL - w^{x}/_2}{AE}\right] \quad (1)$$

$$x' - x = \frac{1}{AE}\left[P_f + \tfrac{1}{2}w(2Lx - x^2)\right] \quad (2)$$

The length x occurs at pull 0 and is usually unknown, we know the length of the tape on a flat surface, fully supported at a pull P_f; x_f means x at pull P_f

$$x_f = x\left(1 + \frac{P_f}{AE}\right) \quad (3)$$

or for $L = AB \rightarrow L_f = L\left(1 + \frac{P_f}{AE}\right)$ (4)

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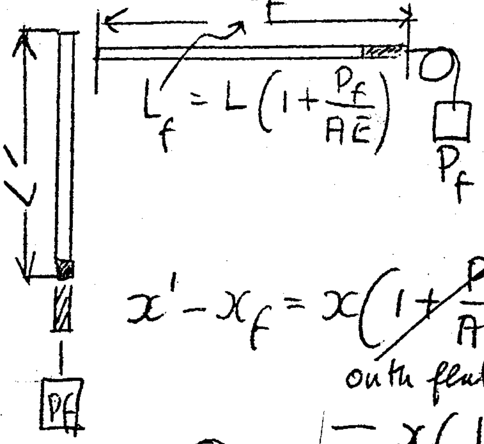
What is $x' - x_f$? → the difference between x' (own weight) which can be computed and x_f which can be observed. (or $L' - L_f$)



Re-arranging ①

$$x' = x\Big(1 + \underbrace{\frac{P_f}{AE}}_{\text{on the flat}}\Big) + \frac{wx}{AE}\underbrace{\Big(L - \frac{x}{2}\Big)}_{\text{hanging}}$$

$$x' - x_f = \underbrace{x\Big(1 \cancel{+ \frac{P_f}{AE}}\Big)}_{\text{on the flat}} - \underbrace{x\Big(1 \cancel{+ \frac{P_f}{AE}}\Big)}_{\text{hanging}} + \underbrace{\frac{wx}{AE}\Big(L - \frac{x}{2}\Big)}_{\text{hanging}} \quad (5.1)$$

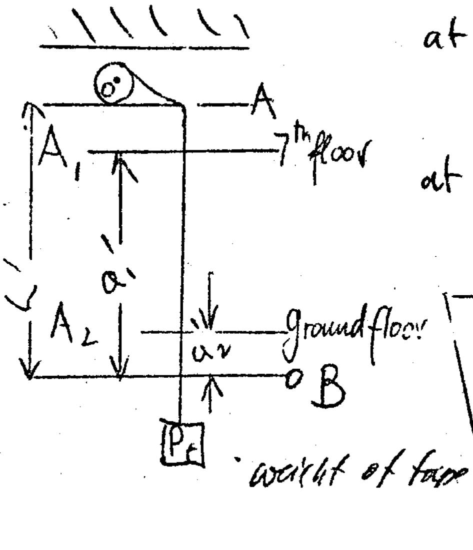
Using ③

$$x' - x_f = \frac{w x_f}{AE\left(1+\frac{P_f}{AE}\right)}\left[\frac{L_f}{\left(1+\frac{P_f}{AE}\right)} - \frac{x_f}{2\left(1+\frac{P_f}{AE}\right)}\right] \quad (5.2)$$

$$\boxed{x' - x_f \simeq \frac{\frac{w x_f}{AE}\left(L_f - \frac{x_f}{2}\right)}{\underbrace{\left(1 + \frac{P_f}{AE}\right)^2}_{\approx 1}}} = \quad (5.3)$$

and if $x = L$ $\quad \boxed{\therefore\ x' - x_f \simeq \frac{w L_f^2}{2AE}} \quad (6)$

IV Now, if an interval $X_1 X_2$ is of interest such as the extension between $A_1 = X_1$ (floor 7) and $A_2 = X_2$ (groundfloor, not B), we have in accordance with equation ① and fig. 2



at A_1 (Wallmark 7)

$$x_1' = X_1\left(1+\frac{P_f}{AE}\right) + \frac{X_1 w}{AE}\left(L - \frac{X_1}{2}\right)$$

at A_2

$$x_2' = X_2\left(1+\frac{P_f}{AE}\right) + \frac{X_2 w}{AE}\left(L - \frac{X_2}{2}\right)$$

pull P_f

$$\boxed{x_2 - x_1 = (X_2 - X_1)\left(1 + \frac{P_f}{AE}\right) + \frac{w}{AE}\left[X_2\left(L - \frac{X_2}{2}\right) - X_1\left(L - \frac{X_1}{2}\right)\right]} \quad (7)$$

V If we now introduce a pull differing from that of the standard, say 10 kg × 9.80665 = 98 N instead of the standard pull of 5 kg × g m/s² = 49 N and call P_f = field pull
P_{ST} = standard pull

we obtain → $$\left.\begin{aligned}(x_2-x_1)_f &= (x_2-x_1)\left(1+\frac{P_f}{AE}\right)\\ (x_2-x_1)_{st} &= (x_1-x_1)\left(1+\frac{P_{ST}}{AE}\right)\end{aligned}\right\}\text{divide}$$

$$(x_2-x_1)_f = (x_2-x_1)_{st}\cdot\frac{(1+P_f/AE)}{(1+P_{ST}/AE)}\;; \qquad (8.1)$$

because of $\frac{1}{1+x}\sim 1-x$ and $\frac{P_f}{AE}$ 2×10^{-4}, $\frac{P_{ST}}{AE}$ 1×10^{-4}

we can form $C_p = (x_2-x_1)_f - (x_2-x_1)_{ST}$
because (8.1) reads now

$$(x_2-x_1)_f = (x_2-x_1)_{ST}\left(1+\frac{P_f-P_{ST}}{AE}\right) \qquad (8.2)$$

and this makes

$$\boxed{C_p = (x_2-x_1)_{ST}\left(\frac{P_f-P_{ST}}{AE}\right)} \qquad (8.3)$$

For our white tape $\cancel{8}^{10}\times 0.5\,mm^2 = 5\times10^{-6}m^2$ and E $= 2.039\times10^{10}$ (Kgf/m²), $\frac{P_f-P_{ST}}{AE}$ becomes $\frac{10-5}{AE} = 4.9\times10^{-5}$

VI Temperature correction

$$C_t = (x_2-x_1)_{f\,or\,ST}\times\alpha\times(T_f-20°C)$$

where $\alpha = 11\times10^{-6}/°C$
and $T_f = \frac{TA_1+TA_2}{2}$

VII link between Pearce (2) and Rack (1) – term 2 of Equation 7 refers to lengthening caused by the tapes own weight and the correction C_w would be

$$C_w = \frac{w}{AE}\left[x_2\left(L-\frac{x_2}{2}\right)-x_1\left(L-\frac{x_1}{2}\right)\right] \qquad (9.1)$$

$$= \frac{w}{AE}\left[(x_2-x_1)L-\frac{x_2^2-x_1^2}{2}\right]$$

$$C_\omega = \frac{\omega}{AE}\left[(x_2-x_1)L - \frac{(x_2-x_1)(x_2+x_1)}{2}\right]$$

$$= \frac{\omega}{AE}(x_1-x_1)\left(L-\frac{x_2+x_1}{2}\right)$$

$$= \frac{\omega}{AE}(x_2-x_1)\left[\underbrace{\frac{(L-x_2)+(L-x_1)}{2}}\right];$$

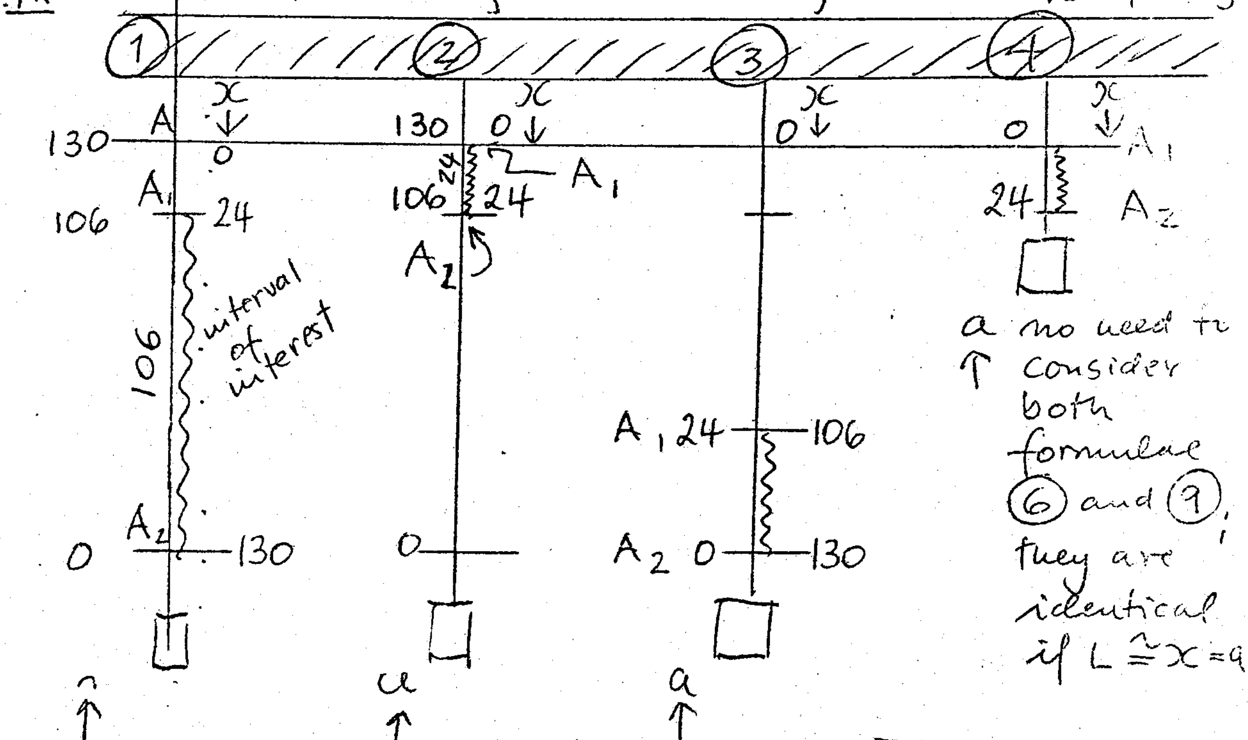
now, $x_2-x_1 = a_1-a_2$ and $[\ \ ] = a_1+a_2$

$$\boxed{\therefore C_\omega = \frac{\omega}{2AE}\left(a_1^2-a_2^2\right)} \qquad (9)$$

formulae like this are very useful; it make no practical difference (< measuring precision) whether corrections mentioned earlier are made or not as far as our building is concerned. Reading Mr. Pearce's letter to HW regarding a student report on shaft levelling (S. DIXON) will give better understanding.

VIII An example (Dixon 1972) see (3) page 7.

IX TUTORIAL, 4 cases, numbers are tape reads



a no need to consider both formulae (6) and (9), they are identical if $L \cong x = a$

(5.3) $x'-x_f = \frac{\omega x_f}{AE}\left(L_f - \frac{x_f}{2}\right)$ (6) $x'-x_f = \frac{\omega L_f^2}{2AE}$ (9) $C_\omega = \frac{\omega}{2AE}(a_1^2 - a_2^2)$