GEODETIC SURVEYING

1. INTRODUCTION

1.1 Objects of Geodetic Surveys

The objects of a Geodetic Control Surveys are

(a) to establish a framework on which less precise observations may be based. The latter may in turn form a framework for topographical, cadastral and engineering surveys and maps.

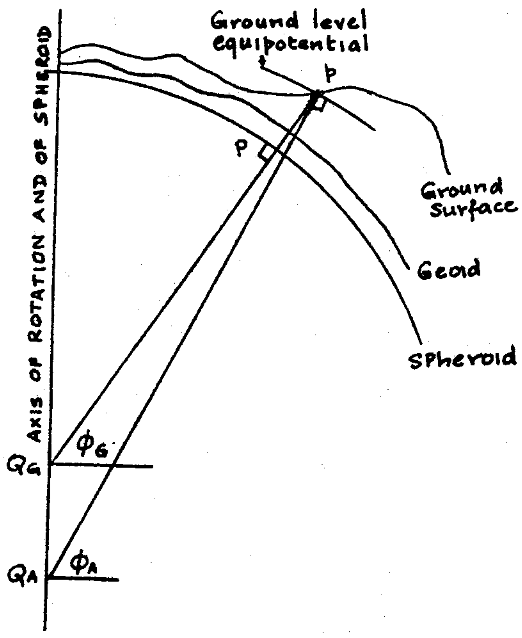
(b) to assist, in combination with astro and satellite etc. observations and gravity determinations, in determining the size, shape and density distribution of the earth, and the investigation of the nature of phenomena on earths surface such as crustal movement etc.

1.2 The Geoid and the Spheroid

Geoid - is the equipotential surface of the earth at mean sea level (level which oceans find at state of equilibrium). It is not perfectly regular due to local anomalies in gravitational field caused by non homogeneous nature of sub strata etc.

The axis of the bubble tube of a levelled theodolite at sea level will be parallel to the Geoid and hence the rotation axis of the theodolite will be normal to the geoid; this direction is called "the vertical" or "plumbline".

Spheroid - is the mathematical surface which most closely approximates the geoid. The surface adopted for this purpose is an "oblate spheroid" which is the surface obtained by rotating an ellipse about its minor axis (sometimes known as ellipsoid of revolution).
A line perpendicular to the surface of the spheroid at a point on it is called the "normal" at that point.



The Geodetic Latitude: The Geodetic latitude of a point is the angle between the normal to the spheroid through the point and the equitorial plane of the spheroid.

The Geodetic Longitude of a point is the angle between the meridian plane through the point and an arbitrarily defined zero meridian plane. This zero meridian plane is usually chosen as the meridian plane passing through Greenwich.

Since Astronomical observations depend on the direction of gravity at the point, the Astronomical latitude and longitude are defined as follows:-

Astronomical Latitude of a point is the angle between the meridianal component of the direction of gravity (or Vertical) at that point and the equitorial plane of the Earth.

Astronomical Longitude. The plane containing the plumb line at a point P and parallel to the rotation axis of the earth is the astronomic meridian plane of P.

The angle between some designated zero (Greenwich) astronomic meridian plane and that of the point P is the astronomic longitude of P.

For computation purposes the surface used is a spheroid. If a spheroid could be chosen which had parameters (semi-major axis, semi-minor axis, directions of axis in space etc.) which give the best mean fit to the world wide geoid, then the undulations of the geoid would be about this best fit spheroid. However it is only recently, since satellite work and continental connections, that this could even be envisaged. The Normal practice was to adopt the ϕ_A , λ_A of a point (P) known as the "origin" of surveys as ϕ_G λ_G , (thus forcing the normal of the spheroid to be along the vertical) and to start calculating from there.

On calculating through the survey one found a ϕ_G , λ_G (calculated) for a second point (Q) at which ϕ_A , λ_A were observed.

The descrepancy between these two set of values would be due to

(1) parameters choosen for the best fit spheroid
(2) poor orientation of spheroid at origin
(3) the fact that geoid surface is not parallel to spheroid surface at all points which gives rise to "deflection of the Vertical".

Deflection of the Vertical of a point is the angle between the normal to the spheroid through the point and the direction of the vertical at that point. This deflection can be resolved into two components, one along the meridian section and the other along the prime vertical section.

The deviation in the meridianal section gives rise to differences in ϕ_G & ϕ_A and deviation in the prime vertical section gives rise to λ_G & λ_A.

NOTE: The deflection (deviation) of the vertical is only an absolute fixed quantity if the geodetic calculations are made on one spheroid. Values of the deviation of the Vertical may be expected to vary between a few seconds of arc in plains to 5 or 10" in hilly areas. In mountainous country higher values may be expected and an extreme value of 71" was recorded in 1952, 30 miles south of Mount Everest.

1.3 Types of Survey used for Horizontal Control of Geodetic Standards

(a) Precision Astronomy

$\sigma \simeq \pm 0.3''$ in ϕ & λ $1'' \simeq 30.9$ metres

(b) Triangulation: Prior to about 1950, the main framework of a geodetic survey was almost always a triangulation network. This was replaced by traverse only if the topography was such as to render triangulation impossible. Modern triangulation is really a combination of classical triangulation and trilateration, i.e. with sides measured directly by e.d.m.

(c) Traverse: Traverse using flexible measuring apparatus (tapes, wires etc) was used only rarely prior to 1950. The invention of the Geodimeter and later, the Tellurometer introduced the more frequent use of traversing in topographically suitable areas, such as Central Australia.

(d) Trilateration: The advent of e.d.m. also introduced the possibility of trilateration, using triangles of about the same size as those used in a triangulation net. There seems little advantage in this, and the only trilateration which appears practicable is the measurement by radar methods, of very large triangles to enable quick connections between areas separated by terrain of little topographical interest.

(e) Triangulation and Trilateration combined - Presently known as triangulation - This strengthens the triangulation network and reduces accumulation of error in scale.

2. CLASSIFICATION AND ACCURACIES OF GEODETIC SURVEYS

2.1 Classification

(a) Triangulation: There are various calssifications of triangulation schemes. The following classification is into Primary, Secondary, and Tertiary (sometimes called First, Second and Third order)

Order	Triangle Sides	Triangle Miscloses	STD Error Computed Length	Base Length	Base Frequency	Astro azimuth frequency
Primary	15-150 km	Av. 1" Max. 3"	1/40,000 to 1/150,000	5-30 km	About 400 km apart	About 250 km apart
Secondary	10-40 km	Max. 5"	1/15,000 to 1/35,000	Based on primary		
Tertiary	1 -10 km	Max. 15"	1/3,000 to 1/15,000	"		

Normally, Secondary and Tertiary triangulations are based on sides in the Primary net, so that no additional bases or azimuths are required. However, in a small country, the most accurate survey required may be only of Secondary standard. In such a case a proprotionally shorter baseline, measured to less rigourour standards than first order, could be used.

NB. Of these classifications only Primary is properly considered a Geodetic Survey.

(b) Traverse

A traverse is only considered to be of Primary standard if the traverse distances can all be measured with standard errors of between 1/40,000 and 1/150,000 and if the azimuths of the traverse lines are controled so as to have standard errors no larger than those found in a properly adjusted Primary triangulation. This requires very frequent Laplace azimuth stations. If lengths are being measured by e.d.m. (Tellurometer) and are of about 30-40 km, then Laplace azimuths at alternate stations are desirable.

(c) Trilateration

Long-line radar trilateration is of geodetic accuracy only if lines are long enough i.e. 300-700 km. With shorter lines the uncertainities of measurement due to large wave lengths used, are greater than the permissible relative errors in primary work.

3. PRIMARY TRIANGULATION

3.1 What is Triangulation ?

Triangulation as the name implies, is a method of fixing relative positions of points by measuring the angles of a series of triangles, each of which has at least one side which is common to at least one other triangle. Provided the length of any side in the scheme is known, the length of any other side may be computed. This initial measured length is called a baseline.

Triangulation schemes may be of two main types -

a) Continuous network covering the whole area to be controlled - (Bomford 3rd Ed. Fig. 1.1)

b) A series of interlocking chains of triangles - (Bomford 3rd Ed. Fig. 1.2)

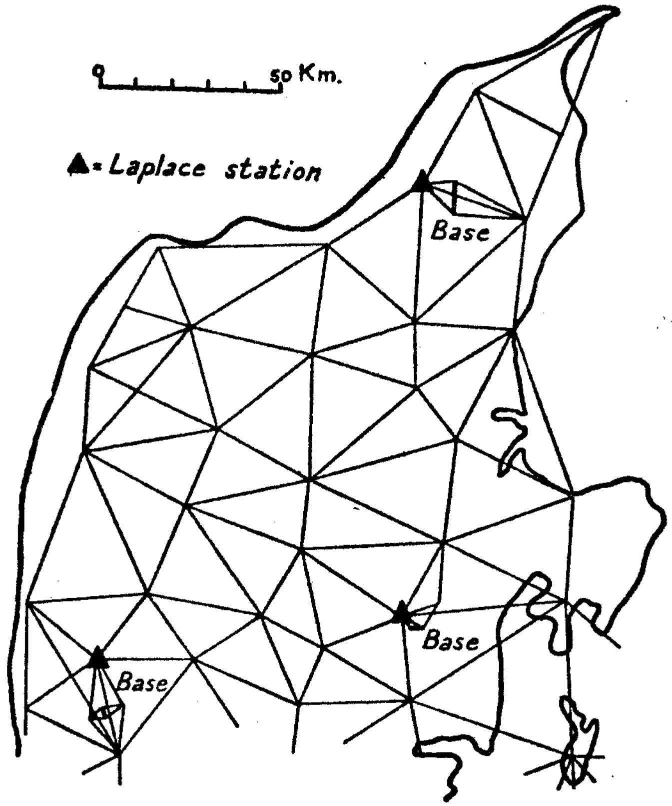


Fig. 2. Part of the triangulation of Denmark.

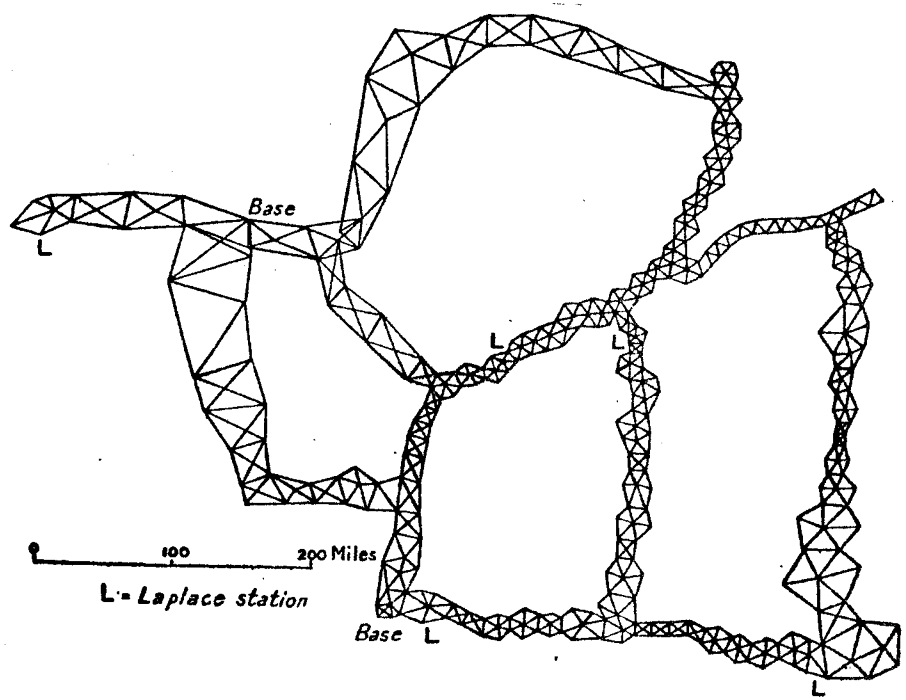


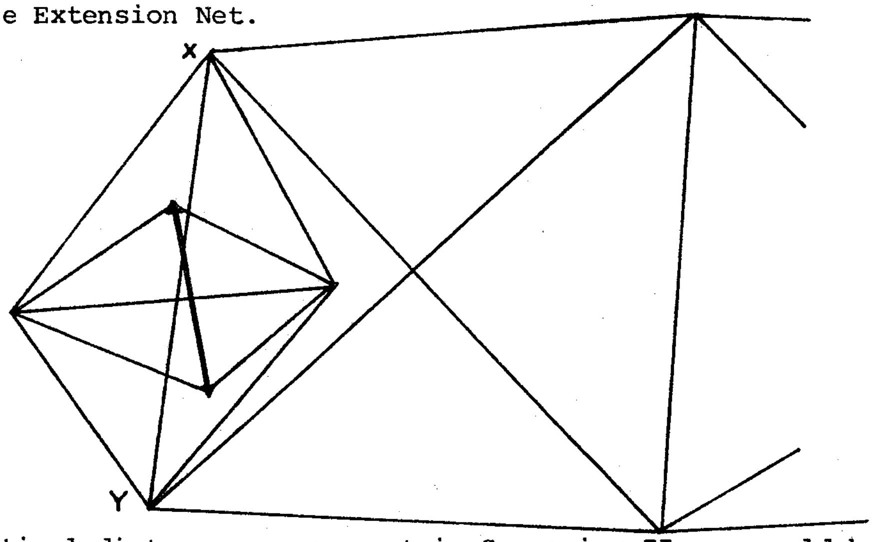
Fig. 4. Part of the triangulation of India.

In pre-computer era, the second was used more than the first because the adjustment was more easily carried out. This is no longer necessarily so.

3.2 Baseline and Base Extension net

Prior to the advent of EDM, base lines were measured to an accuracy of 1/500 000 - 1/1000 000 by means of tapes and wires. This was an extremely tedious, painstaking and highly expensive undertaking. For this reason, base-lines measured prior to about 1950 were generally considerably shorter than the primary triangulation side. A very careful triangulation network was employed which worked from the baseline through a series of triangles graded in size until a length approaching that of a primary side was reached. This side was then used in the first figure of the triangulation chain proper. This is called a Base Extension Net.

Typical Base Extension Net



When dealing with optical distance measurement in Surveying II you would have come across an exact analogy of this process in the methods used to reduce the errors in measuring a fairly long line by means of a subtense bar. You would probably have been given an analysis of the improvement in error propagation which is derived from using a method such as that shown above and that analysis of course holds for this situation which is exactly the same except for the matter of scale.

The Geodimeter is capable of measuring directly distances comparable to primary triangle sides, to accuracies approaching that required of a primary base-line. Certainly, the length XY could be measured at least as accurately, by Geodimeter, as it could be computed through the base extension net from a primary base-line as shown. Where triangulation is still used, the need for short base-lines allied with extension nets has therefore vanished.

3.3 Laplace Stations for External Control

It is sometimes said that triangulation based on a short base line violates the important principle of working from the whole to the part. However, we should properly consider the widely spaced Laplace astro stations as being the major control system, the triangulation being a method of linking them up.

At Laplace stations, very accurate astro determinations are made of latitude and longitude. While these are not of sufficient accuracy to provide reasonable relative accuracy at geodetic triangle size, over long distances the relative errors are smaller than those obtained by triangulation.

Example:- High order astro obs. for latitude and longitude are generally regarded as having standard errors of about ±0.3". This represents a distance error of about 10 metres for each component.

$(0.3 \times \frac{1}{2\times10^5} \times 6 \times 10^6 \approx 10$ metres)

$\therefore$ The actual error in position $\approx \sqrt{10^2 + 10^2} \approx 15$ metres

Consider two such points P and Q.

Error in computed distance PQ $\simeq \sqrt{15^2 + 15^2} \simeq 20$ metres

If PQ $\simeq$ 50 km, $\dfrac{\text{error}}{\text{PQ}} = \dfrac{20}{50\text{x}10^3} = 1/2500$

If PQ $\simeq$ 2000 km, $\dfrac{\text{error}}{\text{PQ}} = \dfrac{20}{20\text{x}10^5} = 1/100\ 000$

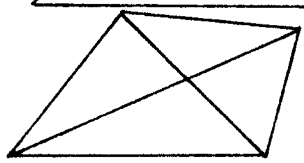
The second case approaches the accuracy of an adjusted triangulation. It seems a reasonable approach, in a large area, to have precisely determined astro. latitudes and longitudes at intervals of 2000 - 3000 km and adjust the triangulation network to fit onto these. This would, of course, be working from the whole to the part as required by good survey practice.

3.4 Triangulation Figures

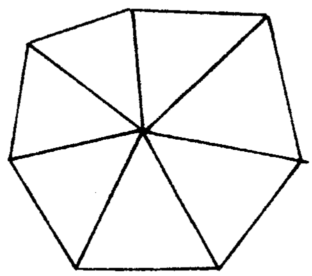
A triangulation figure is a combination of triangles, which may be adjoining or interlocking. They are often used as units in setting up an adjustment process for a chain or network. The usual figures for triangulation are

1. Simple triangles

2. Braced quadrilateral



3. Centred polygons



We shall now consider the relative merits of these three alternatives, in terms of speed, accuracy, and area covered.

A fair criterian of speed will be the number of stations required for a certain length of chain. The controlling factor is likely to be the length of line that can be observed, and this will be assumed a constant length L. The accuracy of a layout will depend on the shape of the figures, and the number of conditions that must be satisfied.

Fig. 4(a) shows a system of simple triangles, equilateral, with side length L. Every new station after the first adds a length L/2 to the chain; if there are S stations, the length of chain is then (S-1)L/2, and there are (S-2) triangles. The number of conditions to be satisfied is one angle condition per triangle; there are then (S-2) conditions. The area of each triangle is $\frac{\sqrt{3}}{4} L^2$; the total area covered is then $0.43\ L^2\ (S-2)$.

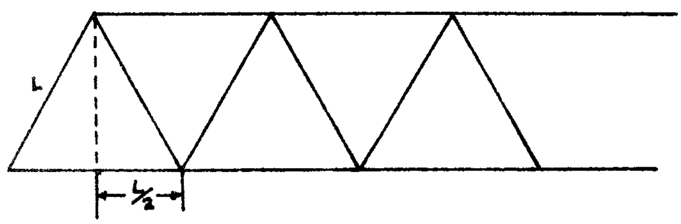


Fig. 4(a)

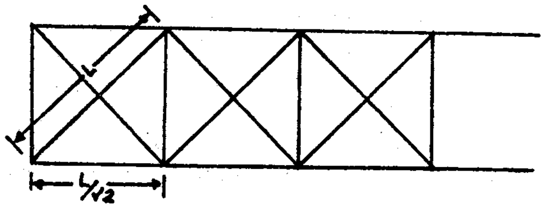


Fig. 4(b)

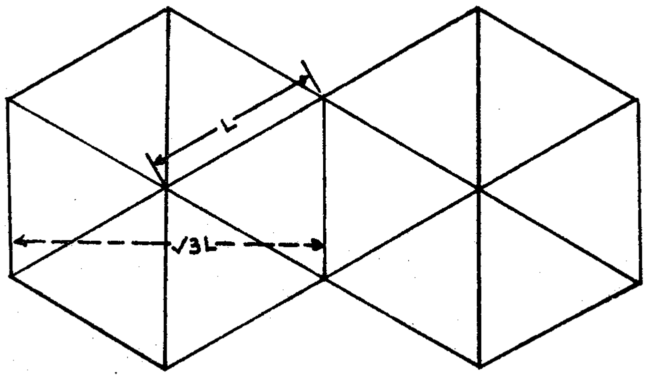


Fig. 4(c)

Fig 4(b) shows a regular system of squares, with diagonals observed. The limiting factor is the length of the diagonals; if the diagonal is L, the side length is $L/\sqrt{2}$; every pair of stations after the first 2 gives a new square, of which there will be $\frac{S-2}{2}$, of total area $\frac{S-2}{2} \times \frac{L^2}{2}$. The number of conditions will be one side condition and three angle conditions per square; (three, not four, angle conditions, for only three are independent; if three are correct, the fourth follows automatically). The total conditions will be $4 \times \frac{S-2}{2}$, or $2(S-2)$.

Fig 4(c) shows regular hexagons; every five stations after the first pair give a new figure, of which there will be $\frac{S-2}{5}$; the length of each figure in the direction of the chain is $\sqrt{3}$ L , so the distance covered is $\sqrt{3}\ L \times \frac{(S-2)}{5}$. The number of conditions per figure is six independent angle conditions, and one side condition; the condition that the angles at the centre of the polygon add to 360° is not strictly an angle condition, but a 'local condition'; it must be satisfied if the round of angles taken at the control point closes, and need not be considered. The area covered by each figure is six times the area of one triangle, or $6 \times \frac{\sqrt{3}}{4}\ L^2$; the total area is then $\frac{3\sqrt{3}}{10}\ L^2(S-2)$.

Tabulating these results, we have

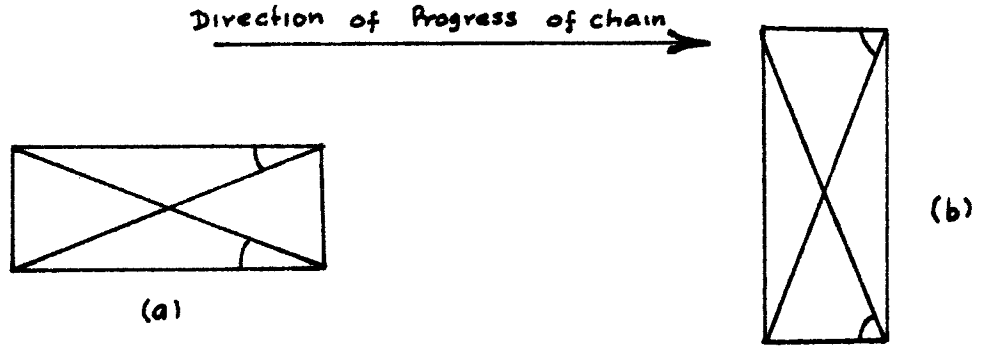
	Distance	Conditions	Area
Triangles	$(S-1)L/2$	$(S-2)$	$.43\ L^2(S-2)$
Squares	$.35\ (S-2)L$	$2(S-2)$	$.25\ L^2(S-2)$
Hexagons	$.35\ (S-2)L$	$1.4(S-2)$	$.52\ L^2(S-2)$

For a chain of 22 stations, this given

	Distance	Conditions	Area
Triangles	10.5 L	20	$8.6\ L^2$
Squares	7 L	40	$5.0\ L^2$
Hexagons	7 L	28	$10.4\ L^2$

The deduction therefore is that triangles give most rapid progress; that squares with diagonals give best accuracy, and that hexagons cover the greatest area. It is generally preferred in a geodetic chain to use a system of squares or hexagons, so that computation is possible by two independent routes. In hilly country, the visibility will not restrict the observation of the diagonals of a square, and this layout may then be preferred; in flat country, hexagons would be better.

The choice of a figure depends not only on the type of figure but also on the shapes of the triangles composing it. We refer to triangles being "well-conditioned" if they are of a shape which gives only a small increase in error of a computed side. In general this requires the angles used in side computation across the figure to be no smaller than 30° and no larger than 150°, and the closer they can be made to 90° the better.



For the given direction of progress of the chain, the error propagation across (b) will be slower than across (a) because of the better sized angles used in the length transfer across the figure. Even though the triangles used in both are the same shape, in (a) they are not as well-conditioned as they are in (b), because of their orientation with respect to the chain.

A more detailed treatment of error propagation in triangulation figures will probably be part of Geodesy II

3.5 Steps in Triangulation

1. Reconnaissance
2. Station building
3. Base Measurement
4. Angle measurement
5. Laplace observations
6. Computation through triangles

3.5-1 Reconnaissance

Bomford mentions three forms which he implies are alternatives. These consist of:-

a) Examination of existing maps (if any)

This may enable a "paper" layout of a possible chain, or a number of alternative chains. In suitable country (i.e. very hilly or moderately mountainous) he claims a scheme may be set out with 95% certainty that it will prove practicable. Personal experience, admittedly in relatively poor triangulation country, indicates that the paper layout is only a first try, and requires careful examination on the ground.

b) Aerial Reconnaissance - has been practised in Canada. Helicopters were used for similar purposes in Australia in the middle 1950's and proved extremely useful. A special plane table is mounted in the aircraft which is flown on a straight line marked on the table. The position of the aricraft is estimated from the map and rays drawn by alidade to hills thought to be possible stations. Intervisibility is checked by low level runs.

c) Ground Reconnaissance - this is the most tedious method, but probably the one with the highest chance of being 100% correct. All prospective stations are visited, and intervisibility physically proven. It has the added advantage that feasibility of routes to prospective stations, location of water, firewood and sources of supply of food and other requisites are actually known prior to the actual measuring phase of the survey commencing.

The objectives of the reconnaissance are to choose suitable stations forming a well conditioned triangulation chain, such that it is physically possible to make the required measurements (i.e. that stations are intervisible either from ground level or by using suitable observing towers). It includes the choice of bases and extension nets (although these will almost certainly not be required in modern triangulation).

It seems that a ground reconnaissance should supplement the "paper" or aerial reconnaissance, at least for doubtful stations or lines. If helicopters are used, they may be able to land personnel at such stations and combine aerial and ground reconnaissance in one operation.

A useful formula in determining intervisibility is

$$h = h_A + (h_B - h_A) \frac{d_A}{d_A + d_B} - 0.066\, d_A d_B \quad \text{metres}$$

where h_A, h_B are the heights of the two stations, A + B in metres

h is the height of the ray of light between them at distance d_A km from A, d_B km from B.

e.g. Reduced Level of Inst. Axis at A is 500 metres
" " of Beacon at B is 400 metres
AB is 30 kilometres
C is 10 kilometres from A, and its R.L. is 460 metres

(a) Can the line AB be observed directly ?

$$h = 500 - 100 \times \frac{10}{30} - .066 \times 10 \times 20$$

$$= 500 - 33.3 - 13.2$$

$$= 453.5$$

<u>Answer</u>: NO !

(b) To what R.L. must the beacon at B be lifted so that the line of sight from A will clear C by 3 metres.

This is done by direct proportion.

i.e. 10 km from A, line of sight is to be lifted 9.5 metres.
30 km " A, " " " " 28.5 metres.

∴ Required R.L. of Beacon at B will be 428.5 metres.

(This can be checked by substituting back into the former equation seeing that the new value of h is in fact 463 m).

3.5-2 Station Building

This term includes the marking and permanent beaconing of triangulation stations.

The essentials are

a) A distinctive, and as far as possible, indestructible, mark at a below ground level. It may be a brass mark driven into rock, or set in concrete poured "in site". A mark cut on rock is sometimes a suitable substitute.

b) Provision of two or three reference, or witness marks. These should be of a suitably durable nature. The bearings and distances of each from the station mark, and from each other, and their relative heights, should be carefully measured.

c) If required, a beacon should be erected. This may be a metal or wooden pole carrying Vanes, which is guyed to concrete anchor blocks. For stations which will be infrequently occupied, the pole may be supported by a cairn of rocks. In such cases, the observations are often made from satellite station, usually one of the witness marks.

For stations which will be frequently occupied (those in areas of future closer development) special beacon consisting of a short pole and vanes supported by a tripod or quadripod structure may be used. A theodolite may then be set up under the beacon.

3.5-3 Base Measurement

Traditional methods of measuring bases by rigid bars or flexible tapes are now of only historical interest. An excellent account of the methods is given in "Plane and Geodetic Surveying",Vol.II,Clark. Modern baselines would be of full triangle side lengths measured by Geodimeters. The Geodimeter will be dealt with in the subject "Surveying III" which forms part of your course.

3.5-4 Angle Measurement

Angle measurements in Primary Geodetic Surveys were formerly made using large double centre or double axis theodolites (9" - 12" circles) with 2 or more micrometers mounted around the circle and reading directly from it. Individual angles, and various combinations of angles and/or their explements were measured by allowing the angle to "build up" a number of times on the circle, using what many authors call the "Repetition" method of Angle Measurement.

In modern Primary Geodetic Surveys this is invariably done by the Direction Method, using glass arc theodolites such as the Wild T3, or the Geodetic Tavistock, with 5½" and 5" circles respectively. Many of these instruments, which are often called Direction Theodolites, are constructed without a lower clamp and slow motion screw. The Repetition method of angle measurement therefore cannot be used with them. These theodolites are known as single centre or single-axis theodolite. Angular measurement is now done by measuring the individual directions of lines using this type of theodolite, and we tend to deal in differences between directions rather than in angles.

Measurement of Directions

Directions are measured using the general observing method which you have been taught under the name of the Direction Method, or possibly, the Reiteration Method. In Geodetic Surveying, we are seeking to obtain the utmost accuracy, so we must carefully attempt to analyse the systematic errors which are known to affect the method, and to postulate any possible additional sources of systematic error which may be present. We must then design a method of observing which will eliminate, or reduce the effects of, as many as possible of these errors.

Systematic Errors

The systematic errors which may be encountered in the Direction Method, and methods of countering them are:-

a) Circle Drag

i) Definition:- Shift of absolute orientation of the circle in the direction of rotation.
ii) Remedy. Circle Drag in the originally postulated form is no longer believed to exist. It is not difficult to set up an experiment to test for it and no tests have shown conclusively that it exists.

However, there is some evidence to indicate that a condition somewhat akin to it exists in some theodolites, particularly those that have seen considerable use. As wear develops in the mounting, say in the threads of the foot-screws etc. a certain amount of "sloppiness" appears. When the theodolite is rotated in a particular direction, there is a twisting of the mounting until all the slack is taken up. When the direction of rotation is reversed, the twist is also reversed, until all the slack is again taken up. Between these two extreme positions there may be several seconds of arc change of orientation of the circle.

The condition for this movement to have no effect on the final results is that there shall be no such movement during the observation of any semi-arc. (NOTE. We will use Richardus, "Project Surveying", terminology, in which a semi-arc consists of taking readings on all targets whilst rotating telescope in one direction. An "arc" consists of two "semi-arcs" taken in opposite directions).

This can only be ensured by making certain that the total change has occurred prior to any readings being taken in that semi-arc. This is done by rotating the instrument through several complete revolutions in the direction of each semi-arc, before the semi-arc itself is commenced.

b) Twist

i) Definition

A change in the absolute orientation of the circle, independent of amount or direction of rotation of the theodolite.

ii) Remedy

For this to be of a serious nature, it would need to be a continuous twist in one direction for some period of time. If the direction and rate of the twist were varying rapidly with time, the effects would tend to become random.

If we take as a model for the twist a movement which goes on at a steady rate in the same direction for both semi-arcs of a particular arc, then by measuring one semi-arc swinging in one direction, and the other swinging in the opposite direction, the effect on the differences of mean directions for the two semi-arcs will be zero.

Consider a twist at rate r in an anticlockwise direction (i.e. observed directions to a given target is increasing)

Let the time taken between pointing at target 1 and pointing at target 2 be Δt_{12},
the time taken between pointing at target 2 and pointing at target 3 be Δt_{23}
the time taken between pointing at target 3 and pointing at target 4 be Δt_{34}.
Let the times taken in the reverse direction by the same.
Let the true directions to 1, 2, 3, 4 be d_1, d_2, d_3, d_4
Let the time taken between pointing to target 4 at the end of the first semi-arc and pointing to the same target at the beginning of the second semi-arc be Δt_{44}.

Target	Face Left	Face Right	Mean	Reduced Mean
1	d_1	$d_1+r(2\Delta t_{12}+2\Delta t_{23}+2\Delta t_{34}+\Delta t_{44})$	$d_1+r(\Delta t_{12}+\Delta t_{23}+\Delta t_{34}+\frac{1}{2}\Delta t_{44})$	0
2	$d_2+r\cdot\Delta t_{12}$	$d_2+r(\Delta t_{12}+2\Delta t_{23}+2\Delta t_{34}+\Delta t_{44})$	d_2+ "	d_2-d_1
3	$d_3+r(\Delta t_{12}+\Delta t_{23})$	$d_3+r(\Delta t_{12}+\Delta t_{23}+2\Delta t_{34}+\Delta t_{44})$	d_3+ "	d_3-d_1
4	$d_4+r(\Delta t_{12}+\Delta t_{23}+\Delta t_{34})$	$d_4+r(\Delta t_{12}+\Delta t_{23}+\Delta t_{34}+\Delta t_{44})$	d_4+ "	d_4-d_1

Of course, if a number of arcs are observed, there may be one arc in which the twist is reversed.

Even if this goes undetected, its effect on the means derived from a number of arcs is going to be small.

Fluctuations in r may be regarded as introducing random errors.

c) Systematic Errors in Circle Graduation

i) Definition

The Dividing Engines used for graduating theodolite circles contain a large number of gears, cogs, etc. Small deviations from exact circularity, eccentricity of axes etc. of these components introduce a cyclic systematic error into the graduations. This will have basically the same pattern for all circles graduated on a particular engine, although there may be variations over a long term due to wear in mechanical parts.

The systematic error in the position of a division line on the graduated circle is given by the Fourier series

$$t = a' \sin(\phi - A') + a \sin(2\phi + A) + b' \sin(3\phi + B') + b \sin(4\phi + B)$$

$$+ c' \sin(5\phi + C') + c \sin(6\phi + C) + \ldots$$

where A', B', C', A, B, C, a', b', c', a, b, and c are constants and ϕ the angle represented by the division line (See Richardus p 162)

ii) Remedy

It can be shown mathematically that (1) reading each direction at diametrically opposite points on the circle and meaning eliminates half the terms in the fourier series by which the cyclic error of graduation may be represented (2) such pairs of readings taken at a minimum of 4 equally distributed points on the circle and meaned will eliminate the first 4 terms of those remaining, and the ones after these are too small to be significant.

The first part of the remedy is built into the instruments used, and in most of them the reading is observed by bringing into coincidence the images of two diametrically opposite parts of the circle. The resulting reading is the mean of readings taken at the two opposite points on the circle.

The second part of the remedy is to change the circle reading on the R.O. (Referring Object) by $180^{\circ}/n$ for each of the n arcs to be measured. In Geodetic Surveys, n is made a multiple of 4 so that the cyclic error will thus be reduced to infinitesmal proportions.

N.B. For less accurate surveys, perhaps only two arcs will be observed. Theoritically, it would be better to change the circle orientation after each semi-arc in this case. However the increase in accuracy is unlikely to be worth worrying about in this type of survey. The maximum cyclic error in a good theodolite is probably less than 0.5". (Treated in Surveying III).

d) Systematic Error in the Micrometer Graduation

i) Definition

The micrometer reading in a glass arc theodolite is usually made by measuring an apparent shift of the circle image relative to an index mark. This apparent shift is caused by rotating a knob which activates a "parallel plate" or a"wedge"in the optical train. The micrometer graduations are linear, disregarding the fact that the function governing the operation of a parallel plate is a sine function.

ii) Remedy

The effect is minimised in the ultimate mean of say n arcs, if the micrometer reading on the R.O. is made to vary by $\frac{r}{n}$ for each new arc, where r is the range of the micrometer.

Another source of systematic error due to the micrometer is"micrometer run."

This is the case where the movement of the micrometer through its whole range results in the image of the circle being moved through more or less than the micrometer's range.

Consider the case where a 10' micrometer gives a 10'10" shift of the circle. i.e. a reading of 1.0' on the micrometer should be 1'01" etc.

	Target A	Target B	Angle
Observed	0° 01' 00".0	60° 07' 00".0	60° 06' 00".0
True	0° 01' 01".0	60° 07' 07".0	60° 06' 06".0
Observed	90° 06' 00".0	150° 12' 09".8	60° 06' 09".8
True	90° 06' 06".0	150° 12' 12".0	60° 06' 06".0

$\therefore$ True angle = 60° 06' 06".0 Mean Observed Angle = 60° 06' 04".9

Here the taking of two observations on different parts of the micrometer results in a reduction of the error from the source from 6" to 1.1". Further reduction can be expected by further evenly distributed readings. So the remedy for the linear division of the parallel plate micrometer also results in a reduction of the error due to "micrometer run". (Of course, an instrument with this much "micrometer run" should be returned to the makers for adjustment).

(N.B. A detailed treatment of Circle Graduation errors, Micrometer Run, and the theory of the Parallel Plate, will probably form part of Surveying III).

e) Residual Instrumental Errors

The effects of all residual instrumental errors ~~is~~ are eliminated from the mean directions by ensuring that equal numbers of readings of each direction are taken in Face Left and Face Right, EXCEPT FOR that component of the dislevelment of the trunnion axis which is due to non-verticality of the vertical axis. The effect due to this source on the observed direction is α tan h, where α is that component of the inclination of the transit axis due to non-verticality of the vertical axis and h is the vertical angle to the target concerned. For Geodetic Surveys in all but very mountainous terrain, the vertical angles between targets are usually small (because of long distances). Errors due to this source can therefore be kept small by simply keeping the Plate Level in good adjustment and being very meticulous about levelling the theodolite.

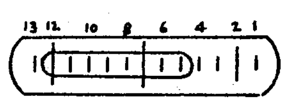
Where large differences of height, and hence large values of h, are encountered, it may be advisable to use a striding level, if the instrument is designed to accept one. This is a level which is supported parallel to the horizontal axis on journals ground onto the latter. Whilst the telescope is directed towards the distant target, the graduations on the level tube at each end of the bubble are read. The Level is then turned end for end, and the bubble extremities again observed.

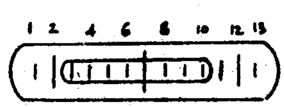
Consider a dislevelment of 5", (say left side higher than right side) to be determined by a striding level which is out of adjustment by 2". (1 graduation $\equiv$ d" arc).

Let the readings at the right hand end be R_1 and R_2 before and after reversal respectively and that at the left hand end be L_1 and L_2 before and after reversal respectively.

Two cases have to be considered

Case 1 Bubble tube graduated continuously from one end. In this case let the reading at the centre of the bubble tube be M.





In the first position of the striding level, the bubble is out of centre by

$$5 + 2'' = \pm\left(\frac{L_1 + R_1}{2} - M\right) d''$$

In the reversed position of the striding level, the bubble is out of centre by

$$5 - 2'' = \pm\left(M - \frac{L_2 + R_2}{2}\right) d''$$

(because the direction of the level error is opposite, but that of dislevelment remains same)

$\therefore$ Taking mean $$5'' = \pm\left(\frac{L_1 - L_2 + R_1 - R_2}{4}\right) d''$$

<u>Case 11</u> Bubble tube graduated outward from centre

Here $$5 + 2 = \frac{L_1 - R_1}{2} \cdot d''$$

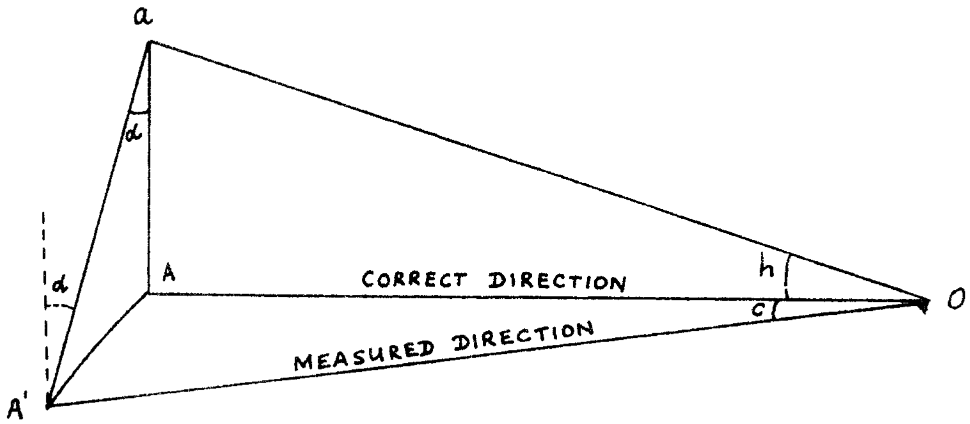
and $$5 - 2 = \frac{L_2 - R_2}{2} \cdot d''$$

$\therefore$ Taking mean $$5'' = \left(\frac{L_1 + L_2 - R_1 - R_2}{4}\right) d''$$

If a sign convention is adopted where readings increasing from left to right (clock wise) are considered positive and vice versa, then the dislevelment angle α can be expressed, irrespective of the manner in which the level tube is graduated, by

$$\alpha = -\left(\frac{\Sigma L + \Sigma R}{n}\right) \cdot d''$$ where n is the number of

bubble readings. This corresponds to a +ve α when left end is higher than the right end i.e. the vertical axis is tilted towards the right.



In the figure, OAA' represents a horizontal plane through the instrument axis at O.

a is the target, A its Vertical projection on the horizontal plane and A' its projection on the horizontal plane along the direction of the vertical axis, which is inclined to the right by the angle α. h is the vertical angle to a.

$$c = \frac{A'A}{OA} \qquad \text{(c in radians, a small angle)}$$

$$A'A = \alpha .aA \qquad (\alpha \text{ in radians, a small angle})$$

$$aA = OA \tan h$$

$$A'A = \alpha .\ OA \tan h$$

$$\therefore \quad c = \frac{\alpha .OA \tan h}{OA} = \alpha \tan h$$ and multiplying both sides

by $\rho = 206265''$, we get

$$\text{Correction } c'' = \alpha'' \tan h = -\left(\frac{\Sigma L + \Sigma R}{n}\right) .\ d'' \tan h$$

and the correct sign is automatically given by strict algebraic calculation (i.e. if a is below the horizon, the sign of the correction reverses and also if $\Sigma L + \Sigma R$ is negative. If both these occur together the sign remains positive.

3.5 - 4(a) Observing Routine.

From the considerations of the systematic errors in angle measurement, we can now design a routine suitable for Geodetic Direction measurement.

We must first of all decide on the number of arcs (n) to be measured. This we can do by studying the random errors of pointing, reading and circle graduation (as distinct from cyclic errors of graduation).

(a) Point to Target 1 in F.L. Set micrometer reading to about zero and set circle reading to about $0^\circ 00'$.

(b) Unclamp and rotate telescope through 360° in clockwise direction. Sight to target 1, make final pointing with tangent screw approaching from left, read and record .

(c) Swing clockwise to target 2, intersect approaching always from left, read and record.

Continue swinging clockwise through full set of targets until last target has been sighted, read and record.

(d) Change face, turn theodolite anticlockwise through about 540°, point to last target approaching from right, read and record.

(e) Swing anticlockwise to second last target, approaching from right, read and record. Continue anticlockwise through targets till first target has been sighted, read and record.

(f) Change face, with telescope pointed to first target, increase micrometer reading by about $\frac{r}{n}$, and increase circle reading by $\frac{180^\circ}{n}$, where r is the range (run) of the micrometer and n the number of arcs.

(g) Repeat setps (b) to (f) until the n arcs have been read.

If any steep sights are included, the striding level may be used. Observations will include striding level readings.

3.5 - 4(b) Choice of n for 1st Order Surveys - simplified analysis

One of the cirteria for 1st Order Triangulation is that errors in triangle angle sums should average < 1", with a maximum of 3".

Rainsford has shown that the errors in triangle angle sums, for a large number of triangles, approximates fairly well the normal distribution. If we accept this, it seems reasonable, since errors in triangle closures of greater than 3" are rarely encountered, to assume 3" to represent the 99% Confidence Interval, i.e. that 3" = 3 σ_T where σ_T is the Standard Error of a triangle closure.

$$\text{i.e.} \quad \sigma_T = 1''$$

This means that, of a large number of triangles, 68% would have errors of 1" or less, which would seem to be reasonable if the mean misclose is not to exceed 1" (it can be shown that this should give a mean misclose of about 0.8").

Each triangle contains 3 angles, each of which is defined by the difference between mean of two observed directions.

$$\therefore \quad \sigma_T^2 = 6\,\sigma_m^2$$

where σm is the standard error of the mean of 2n observations of a direction (n arcs, 2 semi-arcs in each).

$$\text{i.e.} \quad \sigma_m^2 = \frac{\sigma_T^2}{6} = \frac{1}{6}$$

Let σ_D be the Standard Error of a single observation of a direction (estimated by Richardus, "Project Surveying" as 1.5" for a Wild T3 or similar instrument).

Then

$$\sigma_D^2 = 2n.\ \sigma_m^2$$

$$2.25 = 2n.\ \frac{1}{6}$$

$$\therefore \quad n = 6.75$$

The most convenient value is probably 8. This is a widely used value. Many survey department at least in Australia, have for many years made a practice of observing 32-36 arcs, usually spread over two observing sessions. This seems unreasonably high, particularly when we consider that our estimate of σ_T = 1" is probably low by about 25%, and Richardus" estimate of σ_D is considered by many authorities to be very conservative.

(σ_T = 1.25 , σ_D = 1" leads to n = 2!!!)

However we need n $\nless$ 4 for reduction of cyclic graduation error).

3.5-4(c) Recording, Reducing & Testing Precision

Richardus, "Project Surveying", p167 has a table for recording and reducing 4 arcs (eight semi-arcs). We use this for a class example by taking only every second semi-arc from that table.

To make it easier to follow Richardus' example, we will adopt his symbols

s = number of semi-arcs observed
n = number of targets (not number of arcs)

Richardus proves that the following procedure gives the least squares solution to the adjustment of the observed directions, and hence the sum of squares of the residuals, $[\varepsilon\varepsilon]$, is a minimum. These ε values can then be used in a calculation of the standard error of an observed direction in the set, and compared with the estimated standard error of directions observed in this manner. This comparison will indicate whether the observations are of an acceptable standard.

In general, let there be s semi arcs on n targets. Let p_{ij}, be the ith observed direction in the j^{th} semi arc. Let p'_{ij} be the i^{th} reduced direction in the j^{th} semi arc. The steps to be followed are

(a) Reduce the observed directions to zero on the R.O.

1	2	3	4	5	6	7
Observed Stations	Face	Observed Dir p_{ij} ° ' "	Reduced to R.O. p'_{ij} ° ' "	q "	$\varepsilon = q - \frac{[q]}{n}$	$\varepsilon\varepsilon$
1	L	0 00 08.0	0 00 00.0	00.0	-1.0	1.00
2		26 28 56.7	26 28 48.7	+ 1.4	+0.4	0.16
3		54 43 22.3	54 43 14.3	+ 1.9	+0.8	0.64
4		65 52 34.5	65 52 26.5	+ 0.9	-0.2	0.04
			[q]	+ 4.2	Σ =0	
1	R	225 00 39.9	00 00.0	0.0	-0.1	.01
2		251 29 28.0	28 48.1	+ 0.8	+0.7	.49
3		279 43 53.7	43 13.8	+ 1.4	+1.3	1.69
4		290 53 03.7	52 23.8	- 1.8	-1.9	3.61
			[q]	+ 0.4	Σ =0	
1	L	90 01 11.2	00.0	0.0	+2.2	4.84
2		116 29 56.3	28 45.1	- 2.2	0.0	0.00
3		144 44 19.8	43 08.6	- 3.8	-1.6	2.56
4		155 53 34.1	52 22.9	- 2.7	-0.5	.25
			[q]	- 8.7	Σ+0.1	
1	R	315 00 42.8	00.0	0.0	-1.0	1.00
2		341 29 30.1	28 47.3	0.0	-1.0	1.00
3		9 44 55.9	43 13.1	+ 0.7	-0.4	0.16
4		20 53 11.9	52 29.1	+ 3.5	+2.4	5.76
			[q]	+ 4.2	Σ = 0	
1		Final Mean (P_i)	0 00 00.0	s^2=	$\frac{[\varepsilon\varepsilon] = 23.21}{3 \times 3}$ =	23.21
2			26 28 47.3			2.58
3			54 43 12.4			
4			65 52 25.6			

i.e. $p'_{ij} = p_{ij} - p_{1j}$, $i = 1, n; \quad j = 1, s$

(b) Calculate the mean reduced directions P_i

$$P_i = \frac{[p'_{ij}]^{s}_{j=1}}{s} \quad , \quad i = 1, n; \qquad \text{(These are least square adjusted values)}$$

(c) Calculate $q_{ij} = p'_{ij} - P_i$

(d) Calculate $\varepsilon_{ij} = q_{ij} - \frac{[q_{ij}]^{n}_{i=1}}{n} \quad , \quad j = 1, s$

[Many observers use q_{ij} as a measure of the reliability of their observations. However, this implies that observations on the R.O. are completely free of error which is obvious nonsense.] A quick check is to consider the ε_{ij}. Since we believe the standard error for a T3 to be ±1.5", any ε_{ij} which exceeded 4.5" would be looked at with considerable suspicion.

However, the best method to test reliability is as follows -

(e) Calculate $s^2 = \frac{[\varepsilon\varepsilon]}{(s-1)(n-1)}$, s being called the variance factor.

Richardus sets $\sigma^2 = 2.25$ for a T3 (formerly we called this σ^2_D).

Our estimate of the variance factor s^2 can be tested against this, using the right hand tail region of the F-distribution.

In our example $s^2 = 2.58$.

Variance ratio $\frac{s^2}{\sigma^2} = \frac{2.58}{2.25} = 1.15$

$F_{0.95, q, \infty} = 1.88$

↑

((s-1)(n-1) degrees of freedom = number of redundant observations for s^2. The number of degrees of freedom for σ^2 is infinite, since it it assumed to be the variance of an infinitely large population).

Since 1.15 < 1.88 the observations may be assumed to be of precision consistent with the method and instrument being used.

N.B. Students should read Richardus, "Project Surveying", Sections 14.1 and 14.2 for relevant proofs.

3.5-4(d) Corrections to Mean Directions

The following corrections may need to be applied to the Mean Observed Directions (i.e. the P_i of the previous section).

(i) Correction for Phase of Signal

Most Primary observations are made at night using electric lamps as signals. Observing to cylindrical poles in daylight introduces an error known as phase error.

If the signal has a highly reflective surface, sunlight will be reflected towards the observer from a single vertical line, which will appear to be very bright. The observer will sight to this bright line. The correction to be applied is -

$$c'' = \frac{r \cos \frac{1}{2} a}{D} \cdot \rho'' \quad \text{where} \qquad \ldots(1)$$

r = radius of cylindrical signal
D = distance from observer to signal
a = angle measured clockwise from the sun to the signal, at the observer's position
ρ" = 206265 (no. of seconds/radians)

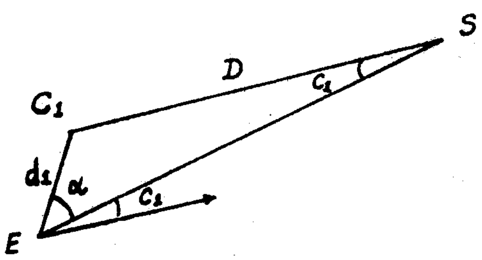
If the cylinder has a matt finish, part of the signal will be illuminated and part in shadow. The observer will bisect the illuminated section. In this case, the correction will be

$$c'' = \frac{r \cos^2 \frac{1}{2} a}{D} \quad \rho'' \qquad \ldots(2)$$

N.B. Proofs of (1) and (2) may be found in Clark, "Plane & Geodetic Surveying", Vol. II.

(ii) Eccentricity

(a) Eccentricity of Instrument. Let E be the Eccentric station
C_1 be the centre
S be the Distant Signal



Let α be the angle measured clockwise from EC_1 to ES
D = distance C_1S
d_1 = distance EC_1 (<< CS)

c_1 is defined by , c_1 = direction C_1S - direction ES

By sine rule,

$$\frac{\sin c_1}{d_1} = \frac{\sin \alpha}{D}$$

$$\sin c_1 = \frac{d_1}{D} \sin \alpha$$

$$\max c_1 \simeq \frac{d_1}{D} \simeq \frac{50 \text{ metres}}{50 \text{ km}} \simeq .001 \text{ radians}$$

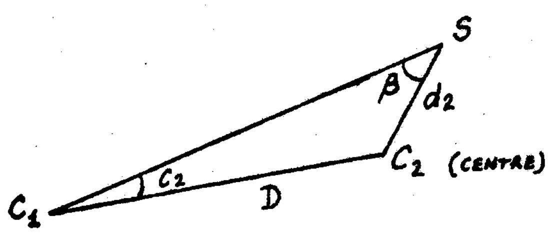
The difference between c_1 and $\sin c_1$ is therefore $\simeq 2 \times 10^{-10}$ radians $\simeq 4 \times 10^{-5}$ secs.

∴ we can write

$$c_1 = \frac{d_1 \sin \alpha}{D}$$

$$\text{or} \quad c_1'' = \frac{d_1 \sin \alpha}{D} \quad \rho''$$

(b) Eccentricity of Signal



By similar reasoning, if β is the angle measured clockwise from SC_2 to SC_1

$$c_2'' = \frac{d_2 \sin\beta}{D} \cdot \rho''$$

If both singal and instrument are eccentric, then the total correction is

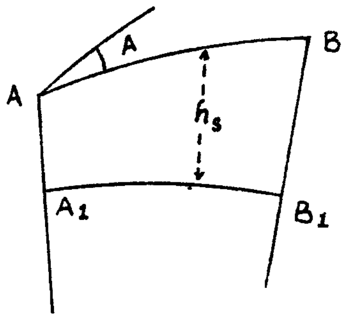
$$c'' = \frac{\rho''}{D} (d_1 \sin \alpha + d_2 \sin\beta)$$

(iii) Reduction of Directions to the Spheroid

This is required only in the most refined Primary Triangulation at considerable elevations above the spheroid (h_s) , especially in low latitudes.

In general the normals to the spheroid passing through points A and B on the topographical surface are not coplanar (due to reference surface not being spherical). The projections of A and B along the normals onto the spheroid are A_1 and B_1 . The plane AA_1B, which contains the observed direction AB, and the plane AA_1B_1 , which contains its projection on the spheroid are not the same. The observed direction AB should be corrected to yield the spheroidal direction A_1B_1 .

The correction is given by



$$c'' = \frac{e^2 h_s}{2a} \sin 2A \cos^2 \phi \, \rho''$$

where $c'' = (A_1B_1 - AB)''$
e = eccentricity of spheroid
h_s = mean spheroidal height of A and B
a = semi-major axis of spheroid
A = azimuth of AB
ϕ = the mid-latitude of line AB

We can see that sin 2A has max value = 1 when $A = 45^\circ$

$$\cos^2 \phi = 1 \quad \text{when } \phi = 0$$

Consider a case where $h_s \simeq 3$ km

$$c'' \simeq \frac{7 \times 10^{-3} \times 3 \times 10^3}{2 \times 6 \times 10^6} \times 2 \times 10^5$$

$$\simeq 3.5 \times 10^{-1}$$

In Australia, $\cos^2 \phi$ max is about .93

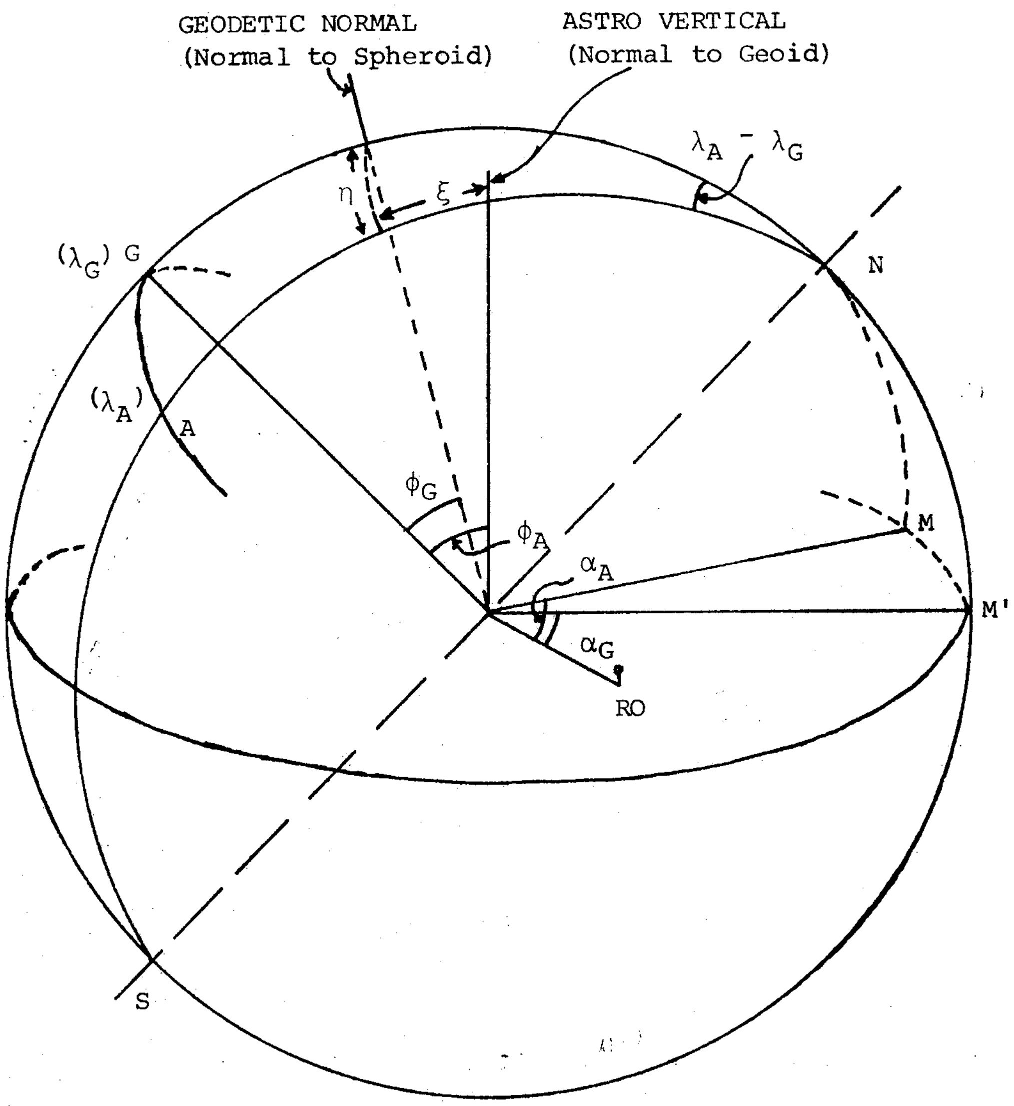
h_s max is about 2 km

$\therefore$ $c'' < 0.2''$ anywhere in Australia.

3.5-5 Laplace Observations

The main purpose of Laplace observations is to control the accumulation of error in the azimuths of sides in the triangulation. Because the deflection of the local vertical from the normal to the spheroid causes the astronomical azimuth to be different to the geodetic azimuth (i.e. azimuth on the reference spheroid), the deflection must be determined. This is done by accurate observation of the latitude and longitude at the Laplace station. Comparison with the geodetic values (i.e. those computed by applying the geodetic measurements to the origin, calculations being done on the reference spheroid) yields a value for the deflection which can be used to correct the astro. azimuth onto the spheroid.

3.5 -5(a) Derivation of Laplace Equation (a simple treatment)



For small ξ , η $\ngtr$ 20''

On the celestial sphere,

(1) $\xi'' = (\phi_A - \phi_G)''$ Deflection of the Vertical in the **meridian**

(2) $\eta'' = (\lambda_A - \lambda_G)'' \cos\phi$ Deflection of the vertical **in prime vertical**

or $\lambda_A - \lambda_G = \eta \sec\phi$

Geodetic Azimuth of R.O. α_G = M' → R.O. (Clockwise)

Astronomic Azimuth of R.O. α_A = M → R.O. (Clockwise)

M'M = $(\lambda_A - \lambda_G) \sin\phi = \alpha_A - \alpha_G$

or $\alpha_G = \alpha_A - (\lambda_A - \lambda_G) \sin\phi$ - This is Laplace Equation

Substituting for $\lambda_A - \lambda_G$

$$\alpha_G = \alpha_A - \eta \sec\phi \sin\phi$$

This derived geodetic azimuth is known as Laplace azimuth.

3.5-5(b) Example on the Use of Laplace Stations in Controlling Azimuth

Calculated geodetic longitude for (say) Woodford $\lambda_G = +151^\circ\ 01'\ 49''.0$
observed Astro longitude for Woodford $\lambda_A = +151^\circ 01'\ 48''.30$

$$\lambda_A - \lambda_G = -0.70$$

$$(\lambda_A - \lambda_G)\sin\phi = +0.44 \qquad (\phi = -33^\circ)$$

Astro azimuth, Woodford to Razorback $\alpha_A = 143^\circ\ 16'\ 13''.55$

$\therefore$ Laplace azimuth $\alpha_G = \alpha_A - (\lambda_A - \lambda_G)\sin\phi = 143^\circ\ 16'\ 13''.11$

Calculated Geodetic Azimuth (as obtained through network) $= 143^\circ\ 16'\ 15''.64$

So misclose in calculated geodetic azimuth $+2''.53$

$\therefore$ Correction to be distributed through scheme $-2''.53$

3.5-5(c) Effect of Errors in λ on Azimuth Correction

The Geodetic and Astronomic λ in Australia are subject to an observational σ of less than $1''.0$. Astronomic azimuths are determined with about the same precision. Geodetic Azimuths, since they are carried through the triangulation, are subject to an error about 10x larger. The trinagulation network may therefore be greatly strengthened by correcting the geodetic azimuths at Laplace stations by means of the Laplace Equation.

In previous years, the Laplace Azimuth was held fixed in an adjustment. Modern techniques allow one to supply a variance factor to this value and "float" in the adjustment (see Comps II).

3.5-6 Computation through triangles

This involves the following four steps.

1) The observed values are adjusted either by

   a) approximate adjustment between Laplace stations or by
   b) using Least squares by the condition or parametric method (Comps II & Geodesy II)

Note - When a condition other than the least squares is applied, it is called approximate adjustment when least squares is applied it is called rigorous adjustment.

2) Computation of triangle sides
   Triangles are reduced to plane triangles using Legendre's theorem and solved using plane triangle sine formula.

3) Computation of Azimuths are done by applying the adjusted spheroidal angles to known reverse azimuths which are obtained by the application of convergence to foreward Azimuths.

4) Computation of Geographical and/or projection Co-ords
   This is covered fully in Geodesy I (Part A) and under map projections.

4. Trilateration

4.1 Long sides using aircraft and shoran type equipment

(a) Uses and Applications:- (see Bomford, pp 94-102)

Provides net of lines 300-700 km in length, used mainly for either

(i) Connection between two continental or islands triangulation systems - now largely superseded by satellite triangulation.

e.g. Europe - Iceland - Greenland - Canada
Crete - North Africa
Florida - Trinidad in West Indies

(ii) Cover of large uninhabited areas, e.g. Australia, Nth. Canada, with the basic framework. This will need to be broken down by triangulation or traverse, but makes it possible to start breaking down anywhere in the network as and when required for topo - etc. and puts all surveys on same origin.

(b) Method of Measurement

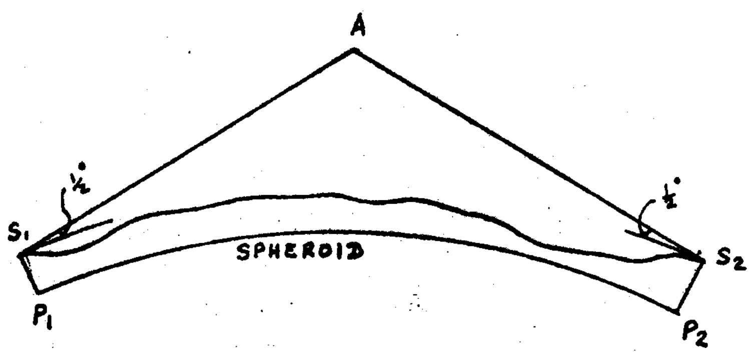
Operation of Shoran/Hiran equipment will be given in more detail in Geodesy II.

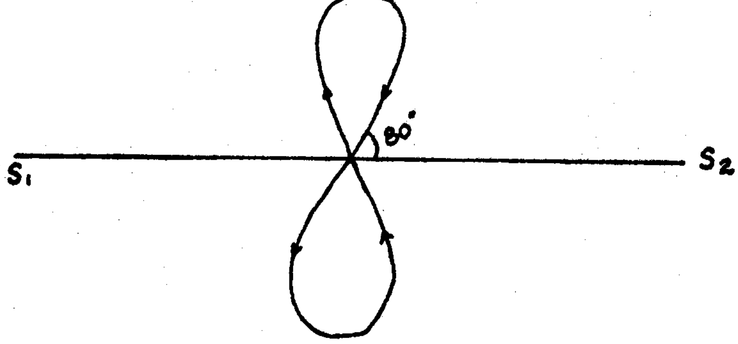
The measurement technique may be as follows

(i) Fly figure-of-eight pattern across line, constantly monitoring sum of distances $AS_1 + AS_2$. Three 8s giving six crossings per day for two days is regarded in Canadian practice as a minimum requirement (If measured lengths differ by $> 1:10^5$, a 3rd set is observed).

(ii) The sum of $AS_1 + AS_2$ (or return transmission times) is recorded at intervals of 150 m along line of flight (≃ perpendicular to measured line) and for 1½ - 3 km either side of S_1S_2. Plotting $AS_1 + AS_2$ vs time gives a parabola. The minimum of the parabola gives the required distance, $AS_1 + AS_2$ when the aircraft was over the line S_1S_2.

Given heights above spheroid of S_1 & S_2 and the aircraft, calibration constants of the equipment, meteorological data (or good estimate) all along lines AS_1, AS_2, and the velocity of transmission, the spheroidal Geodetic distance P_1P_2 may be computed.





(c) **Station** Siting and Aircraft height

(i) Stations should be on hills with a clear slope in the direction of far terminals of lines, and line should not graze ground or sea. Want lines to avoid abnormal & vertical refraction. If a microwave passes horizontally into a temperature inversion, its curvature may become equal to that of the earth. For some distance it will be trapped at that level and its path will depart seriously from normal curve. This risk can be avoided if the lines leaves the lower station at an elevation of at least $\frac{1}{2}^{o}$.

Therefore angular elevation of aircraft should be at least $\frac{1}{2}^{o}$ above horizon.

(ii) From point of view of refraction and velocity of transmission, observations are probably best taken between noon and 4pm, but this will have less importance on a line with good ground clearance, or one across the sea. For these reasons flying height should be as great as possible, but if obtained by barometric methods it will be subject to large errors, and theoretical considerations show a strong demand for minimum flying height, especially for shorter lines.

For ground stations at sea level minimum flying height is taken as $1000(AS/125)^2$ metres, where AS is in km. To this must be added the height of s above sea level, and 9 x AS metres if minimum elevation of $\frac{1}{2}^{o}$ above horizon is to be maintained.

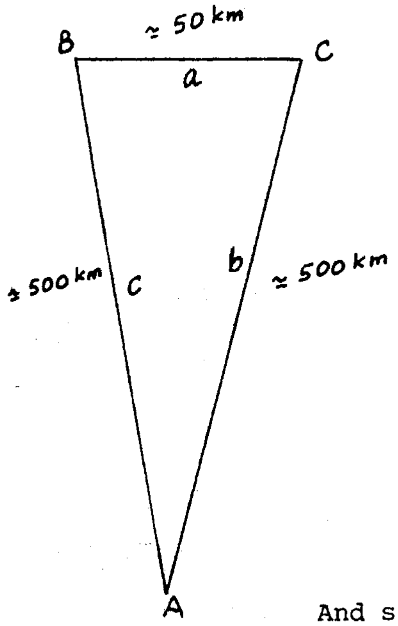
3. Main Systems

	SHORAN	HIRAN	AERODIST
Tech. Details	Uses metre waves Signals in pulses of 0.8 s every 930 s Returned by Transponders at ground stations One operator at aircraft and two at each ground stns.	Same as for Shoran	Similar to Tellurometer system. Continuous signal 1200-1470 MHz (20 cm) modulated to 1.5 MHz. Ranges simultaneously recorded by pen recorder in aircraft one operator in aircraft and one at each ground stn.
Weight of Equipment	Airborne 340 kg Transponders 680 kg each	Same as for Shoran	Master 15 kg
Atmospheric Condition	Works in cloud or mist. Lines must be clear of surface trees and buildings.	Same as for Shoran	
Range	Up to 700 km	Up to 700 km	Up to 400 km
Sensitivity	8 m range, error varies with variation of signal strength.	3.5 m due to modification to keep signal strength continuously controlled to a constant.	$\pm 1\ m + \frac{1}{100,000}$

(d) Laplace Azimuth Control in long line Trilateration

Certain problems arise in attempting to control azimuth in a Shoran/Hiran trilateration. These problems are particularly acute when the trilateration figures form a long narrow chain, rather than a continuous network over a more or less square area.

In order to include a Laplace azimuth, it is necessary to build in a side of length short enough for astronomical observation i.e. about 50 km. Extension of an azimuth from such a line to control a trilateration with sides of 500 km is bound to introduce considerable weaknesses, as the angles used are deduced, not measured. Care similar to that required of a classical base-line extension net is needed.



Azimuth of BC is observed

$BC \simeq 50$ km $\quad AB \simeq AC \simeq 500$ km

$$b^2 = c^2 + a^2 - 2ac \cos B$$

Differentiating $= 2b\,db = 2c\,dc + 2a\,da -$
$2a \cos B.dc - 2c \cos B.da + 2ac \sin B.\partial B$

If $B \simeq 90^\circ \quad \cos B \simeq 0, \quad \sin B \simeq 1$

$$dB = \frac{2b\,db - 2a\,da - 2c\,dc}{2ac}$$

And since $b \simeq c \qquad dB = \dfrac{db}{a} - \dfrac{da}{c} - \dfrac{dc}{a}$

For Shoran/Hiran Std error/distance is generally $>$ 1:200,000

$$\therefore \quad \frac{db}{a} \simeq \frac{dc}{a} \simeq \frac{1}{200{,}000} \times 500 \times \frac{1}{50} = \frac{10}{200{,}000} \simeq 10'' \text{ arc}$$

$$\frac{da}{c} = \frac{1}{200{,}000} \times 50 \times \frac{1}{500} \simeq 0.1'' \quad \text{(may be ignored)}$$

Since db, dc may be either +ive or -ive, we can say

$$(dB)^2 \simeq \left(\frac{db}{a}\right)^2 + \left(\frac{dc}{a}\right)^2 = 100 + 100$$
$$dB \simeq 14''$$

The effect of an error of 14" in the bearing of BA on the position of A is

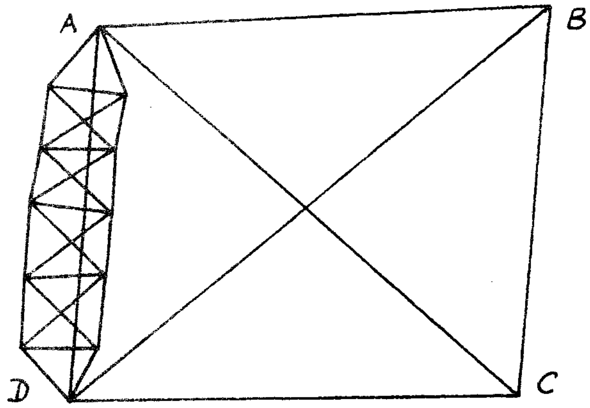
$$\frac{500000 \times 14}{206265} \simeq 35 \text{ metres}$$

The error in position of A due to the error in length of BA is only

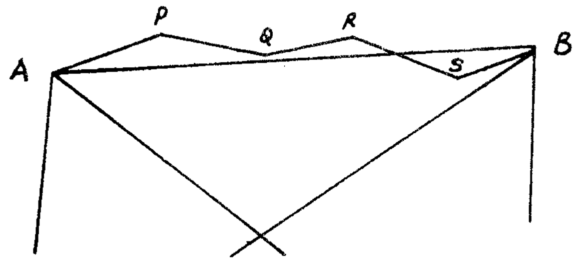
$$\frac{500000}{200000} \simeq 2.5 \text{ metres}$$

Four possible methods of improving the accuracy of azimuth transfer have been proposed -

(i) Run a chain of geodetic triangulation along a trilaterated side of normal (≃ 500 km) length with adequate Laplace control. This is accurate but fairly expensive.



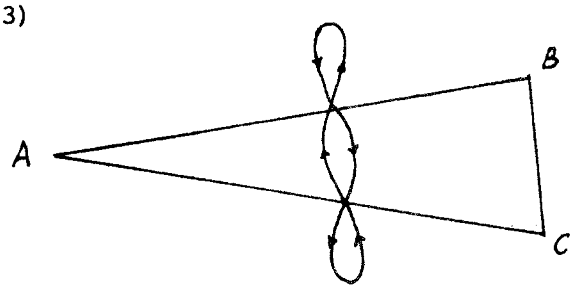
(2) Run a traverse along a normal trilaterated side.



(Azimuths controlled by Laplace stations, and distances measured by edm)

This is also accurate but expensive.

(3)



Looking back at the equation obtained above

$$dB = \frac{db}{a} - \frac{dc}{a} - \frac{da}{c}$$

where last term is 2 orders smaller than the others.

If we were reasonably certain that db and dc are of about the same magnitude and have the same algebraic sign, then $dB \to 0$

If both lines AB and AC were measured at the same time (i.e. in same runs by aircraft) which may not be too inconvenient, their meteorological and instrumental errors are likely to be similar, and $\frac{db}{a}$, $\frac{dc}{a}$ are likely to be of about the same magnitude and algebraic sign and hence will tend to cancel in their effects on $\hat{B}$.

(4) Technique described in a paper "A Long Line Azimuth Technique complements Hiran" H.R. Kahler and Owen W. Williams, I.A.G. Helsinki, 1960. Used to determine azimuth of long Hiran line, as follows -

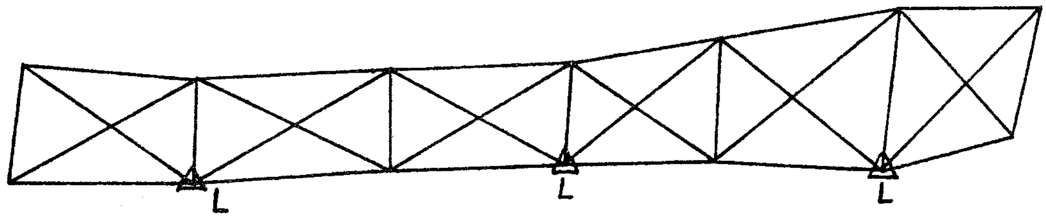
An aircraft, flashing a light, flies across the line at such an altitude that it is visible from both ends. A series of flashes is simultaneously photographed against the stellar background by 1000 mm focal length cameras installed at each end of the line. (The light is made to flash so that simultaneous images are automatically secured and it is only necessary to identify corresponding images on the two plates by (for example) having every tenth flash omitted). The azimuth of the ground line joining the cameras may be computed from information derived from the photos.

4.2 Trilateration with sides of normal geodetic length

It is obviously possible to measure all the sides of ordinary geodetic triangles using the Tellurometer, instead of measuring all the angles by theodolite.

There is an obvious advantage that there is no accumulation of scale error such as that occurs in triangulation, and which may still be present, midway between baselines even after adjustment.

However, error in azimuth will accumulate at least as rapidly as and probably more rapidly than, it does in triangulation of similar quality. The benefit of accurate scale is therefore lost unless Laplace azimuths are included in at least half the stations along one flank, or other continuous line of stations.



This is the method recommended by Bomford. It seems however that if one were going to occupy every second station with theodolite one might as well read rounds of angles whilst there. This would give almost Laplace azimuths for many lines in the network and would seem to almost overcontrol it for azimuth if such is possible. It would seem more economical, and probably as effective, to just turn the flank into a traverse, with Laplace stations every second station.

Another disadvantage is that trilateration provides no simple check comparable to the summation of angles in a triangle (angles derived by solution of a triangle from the measured sides will always add up to 180°).

In some types of country, a chain of geodetic triangles suitable for trilateration may be difficult to find. It appears that traversing would be a suitable alternative for such places.

5. Primary Traverse

5.1 Traverse vs Triangulation

Traverse was for long regarded as an inferior substitute for triangulation since

i) It was generally held to be of lower accuracy
ii) It provides fewer control stations than triangulation.

However the accuracy of the traverse can be increased to that of Primary triangulation or even better by

(1) Measuring each traverse line to Primary triangulation base line accuracy.
(2) Measuring traverse angles by procedures similar to that of Primary triangulation angles.
(3) Having Laplace azimuths at each stn.

To obtain Primary triangulation accuracy, the accuracy of linear measurement and frequency of Laplace azimuths may be relaxed. The amount of relaxation has to be found.

Primary triangulation usually has closing errors on base lines of $< 1:10^5$ and scale error remaining after adjustment between bases should be less than $1:2 \times 10^5$. Overall accuracy between points separated by continental distances should be $\approx 1:5 \times 10^5$.

Primary traverse must be of similar accuracy in order to supersede triangulation as the main control framework.

In suitable country primary traverse can be measured using invar tapes in catinary, but the method in general use in Australia employs the Tellurometer and Geodimeter for distance measurement.

Advantages of e.d.m. traverse compared to triangulation

(a) Ease with which a single line can be selected compared to a chain of triangles.
(b) Ease of organising observations to only 2 stns at a time compared to several scattered stations.
(c) Radio - telephone is part of the equipment.
(d) There is no accumulation of scale error.
(e) A system of traverse is easier to adjust than a system of triangulation.

5.2 Azimuth Control in traverses

Aims to determine the frequency of Laplace Azimuths needed to match directional accuracy of any single traverse line to that of its distance accuracy.

Procedure:
The accuracy of tellurometer measurement of length over reasonable lines

$$\frac{\sigma_D}{D} \quad \pm 1{:}200{,}000$$

This is roughly comparable to 1" of arc in direction. To increase the possibility of getting this accuracy in direction take 87% confidence interval i.e. 87% of all directions are within ± 1" from mean.

$$\therefore \quad \sigma_\theta = \frac{1''\!.0}{1.5} \qquad \text{(since 87\% confidence = 1.5 } \sigma\text{)}$$

$$\sigma_\theta = 0.67 \text{ is aimed for.}$$

1) Now, say $\sigma_{\text{single obs}}$ for T3 is $\approx +0''\!.6$ (RICHARDUS gives 1.5)
σ_{angle} " " " $\approx \pm\sqrt{.36 + .36} = \pm .85$

and so σ if mean of 16 (16 semi arcs or 8 arcs) measures of an angle

$$\simeq \pm 0''\!.21 \quad \left(\frac{.85}{\sqrt{16}}\right)$$

2) The σ_α for Laplace Determination $\simeq$ $\pm 0''\!.6$ (conservative).

3) $\therefore$ The σ of the derived azimuth thro' a traverse after n angles (each measured by 8 arcs) would be

$$\sigma_\alpha \simeq \pm\sqrt{0.6^2 + n(.21)^2} = \pm\sqrt{0.36 + 0.45\,n}$$

So equating this to our desired accuracy for $\sigma_\theta = \pm 0.67$

$$\pm 0.67 = \pm\sqrt{0.36 + .045\,n}$$

$$\therefore \quad n < 2$$

i.e. Laplace azimuths at at least every second station are needed to meet criteria.

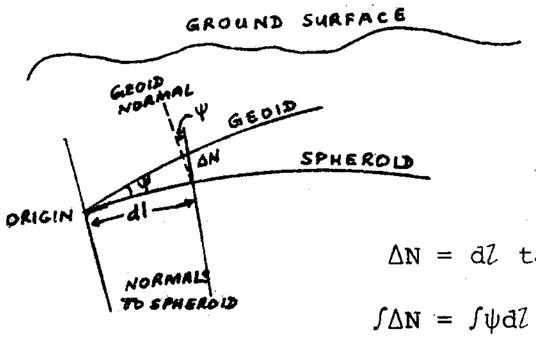
6. Height Control for Reduction of Distances to Reference Surface

When heights are determined by spirit or trig levelling, the value stated is the height above the geoid, denoted by h_o. However, distance measurements are to be reduced to the spheroid for computation, so it is necessary to determine the separation existing along the normal at a point between the geoid and the spheroid.

6.1 The Geoid-Spheroid Separation (N)

This is the distance (measured radially from centre of earth) between the geoid and spheroid. It can be determined by means of astro-geodetic levelling. A geoid'sections'is determined along a 1st order control survey, in which precise ϕ_A & λ_A are measured at frequent intervals ($\simeq$ 20 km). Comparison with ϕ_G, λ_G yields deflections of the vertical at the stations, and, since these define the slope of the geoid with respect to the spheroid, and also because distance between successive "levelling" stations are known, the variation in N is determined.

An initial value for N is adopted at the Origin of the Survey (at Johnston origin in Australia $N = 0$, $h_o = h_s = 571.2$ m), and successive N values computed by accumulation along the geoid profile.



N is ht of Geoid above spheroid

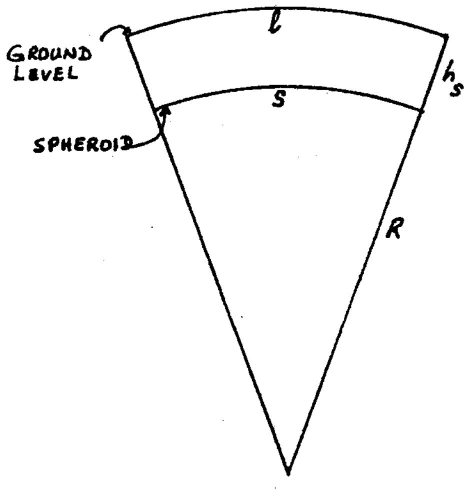
ψ = deviation of Vertical in the plane of the profile

$$\Delta N = dl \tan\psi = \psi dl$$

$$\int \Delta N = \int \psi dl \qquad N_B - N_A = \int_A^B \psi dl$$

N values are found at each astro geodetic station, and can be found at the intermediate stations by interpolation. In fact, contours of the whole continent can be drawn up. Ref. "The Australian Geoid" J.G. Tryer, A.S. Dec. 72, pp203-214. "A Prelim Geoid Chart for Australia" Fischer & Slutsky, AS, Dec. 67, 327-311. The N values can also be determined by combining gravimetric surveys with Laplace obs. taken at greater intervals along the profile. Here the gravity readings, taken along a wide belt along the survey route, are used to determine the slope of the geoid with respect to the spheroid, with the ξ, η values providing the general framework.

6.2 Specifications of accuracy for h_o, N



- s = spheroidal distance
- l = ground level distance
- h_s = mean spheroidal height
- R = Mean radius

From the figure $\dfrac{l}{s} = \dfrac{R+h_s}{R}$

$$= 1 + \frac{h_s}{R}$$

$$\therefore \quad s = l\left(1 + \frac{h_s}{R}\right)^{-1} = l\left(1 - \frac{h_s}{R} + \ldots\right)$$

Differenciating with respect to h_s

$$\frac{ds}{dh_s} = -\frac{l}{R}$$

$$\frac{ds}{s} = -\frac{l}{s} \cdot \frac{dh_s}{R} \simeq -\frac{dh_s}{R} \quad \text{since} \quad \frac{l}{s} \text{ is nearly equal to 1.}$$

$\frac{ds}{s}$ is the fractional error in s due to reduction of ground distance to spheroidal distance and we want this fractional error not to exceed $1:10^6$.

$$\text{i.e.} \quad \left| \frac{ds}{s} \right| \leq \frac{1}{10^6} \quad \text{and therefore} \quad \frac{dh_s}{R} \leq \frac{1}{10^6}$$

$$\text{or} \quad dh_s \leq \frac{R}{10^6} = 6 \text{ metres.} \qquad (R \simeq 6 \times 10^6 m)$$

$$\text{i.e.} \quad \sigma_{h_s} = \pm 6 \text{ m}$$

$$\text{but} \quad h_s = h_o + N$$

$$\text{i.e.} \quad \sigma_{h_s}^2 = \sigma_{h_o}^2 + \sigma_N^2$$

So if we assume $\sigma_{h_o} = \sigma_N$

then $\sigma_{h_o} = \sigma_N = \pm 4.2$ m

i.e. In order to achieve $1:10^6$ accuracy in our reduction of distance to sea level, we must know N and h_o to better than σ of ± 4.2.

In practice h_o will be known to a higher accuracy than this; but until the total geodetic survey and adjustment has been carried out, N will only be approximate, final values of ϕ_G & λ_G (and hence of ξ & η) will of course depend on the determination of N, which is in turn dependent on ϕ_G, λ_G - Iterative Process.