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THE EXTENSION OF THE GRAVITY FIELD IN SOUTH AUSTRALIA

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Summary :-

A uniform geodetic gravity network has been compiled from all gravity data available over the state of South Australia. Mean free air anomalies for $1/2^{o} \times 1/2^{o}$, $1^{o} \times 1^{o}$ and $2^{o} \times 2^{o}$ squares are computed in all regions where gravity data is available. The sample is analysed for the mean free air anomaly which best represents each area. The errors of representation computed are in general agreement with Hirvonen's values. The extension of this field to unsurveyed areas is attempted using a least squares fit of a two-dimensional trigonometrical series, which is periodic in character. The accuracy of the values so obtained is studied.

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Introduction

Gravity surveys have been in progress in South Australia for over 20 years (Thyer, 1963). The overall control has been provided by the Authority of the Commonwealth of Australia, the Bureau of Mineral Resources, Geology and Geophysics (B.M.R.) (Dooley et al, 1961: Dooley, 1965). Subsidiary gravity surveys have been carried out by the South Australian Department of Mines, and by various petroleum exploration companies. These surveys, some of which are quite extensive, are often based on arbitrary datums for both height and gravity and considerable time was spent unifying the surveys onto the national datum (Mather, 1966 b). This national datum, established by the Isogal Regional Gravity Survey, was based on the value of 979,979.0 mgal at the National Gravity Base Station at Melbourne.

All available gravity readings in the region bounded by the parallels 26° S and 40° S and the meridians 128° E and 142° E were considered in compiling the sample. The area is reasonably flat with a mean elevation of 120 meters, the maximum and minimum $1^{\circ} \times 1^{\circ}$ square mean elevations being 565 meters and - 275 meters respectively. (The regions beyond the continental shelf area were not considered). The Western half of the state is part of the pre-Cambrian granitic shield, extending further westward and is semi-desert in character. The rock formations are essentially sedimentary, being generally cainozoic with some proterozoic formations in the hilly regions.

The Sample

The representation of the mean anomaly of a square by the mean value of all the observed gravity readings has generally been considered to be unacceptable (e.g. Jeffreys, 1941) as any tendency for the sample to be unevenly distributed in a square could lead to misconceptions as regards the accuracy of the mean anomaly obtained. A more representative sample is obtained by subdividing each square into equi-areal sections, representing the gravity field in each section by a single anomaly and computing the areal mean from these representative anomalies. In the present analysis, the basic square unit considered was the $1/2^{\circ} \times 1/2^{\circ}$ square. $0.1^{\circ} \times 0.1^{\circ}$ squares were chosen as the basic sub-divisional unit within the $1/2^{\circ} \times 1/2^{\circ}$ square as these would be less than 3 times the basic spacing of gravity stations for normal computations of deflections of the vertical (Rice, 1952, 289); (Mather, 1966a, 10).

In computing the geoid - spheroid spearation using Stokes'integral and the deflections (ξ & η) of the vertical using the Vening Meinesz formulae, not only is it necessary to compute the $1/2^{\circ} \times 1/2^{\circ}$ square mean anomalies, but it is equally important to assess the accuracy of the quantity computed. This is dependant on the sample variance (σ^2), the sample size (n) and the distribution of readings over the square. As a result, only about 10 per cent of the available gravity data could be included in the sample. In addition, limited geodetic gravity surveys were also carried out using the South Australian Institute of Technology's Worden Geodesist gravimeter (Mather, 1966 b). The elevations established on this survey by barometric means had a standard error of the order of $\pm$ 3 metres and the resulting error in the $1/2^{\circ} \times 1/2^{\circ}$ mean free air anomaly would be $\pm$ 0.2 mgal if there were no sources of systematic error in the final heights.

In this manner, readings were chosen to represent the corners of $0.1^{\circ} \times 0.1^{\circ}$ squares. When readings did not quite fall on square corners, representation was adopted instead of interpolation (Moritz, 1966, 167). Approximately 4000 stations were incorporated in the analysis, the majority being concentrated along the northern and eastern borders of the state (see fig. 2). The area is essentially a region of negative free air anomalies (see fig. 3), the field being extremely variable in the north - west. (see fig. 4)

Correlation of Mean Free Air Anomalies and Mean Square Elevations

The regional free air anomaly can be represented (Uotila, 1960) over limited extents, by the expression

$$\Delta g_f \doteq C + 0.1118\, h \qquad \ldots\ldots\ldots\ldots\ (1)$$

where h = elevation of gravity station in meters

Δg_f = free air anomaly in mgal

C = a constant over the region.

This expression implies that the Bouguer anomalies are more regional than positional in character. While this expression cannot represent limited extents with any degree of accuracy, the evaluation of a constant for larger areas is of relevance in establishing values for regional mean free air anomalies to be used in low order harmonic analysis of gravity material.

Latitude (Degrees S) \ Longitude (degrees E)	128–132	132–136	136–142
26–30	-64 (415)	-49 (262)	-14 (58)
30–34	-26 (86)	-21 (39)	-19 (175)
34–38		22 (-70)	2 (9)

TABLE 1.

Evaluation of C in mgal. Mean elevations (in parentheses) in meters.

A least squares analysis of the data in 9 sub-divisional areas, using $\frac{1}{2}^{\circ} \times \frac{1}{2}^{\circ}$ means gave values for C as shown in Table 1. The regional character of C is emphasised when these values are compared with the overall mean value of -20 mgal. over the entire area, corresponding to a regional mean elevation of 130 meters.

This relation was not used to evaluate square means as the variation of gravity with height over any limited area in the sample considered was found, in most cases, to be small, compared to the variations with position, independent of height, as represented by Bouguer anomalies.

The Spread of a Sample

The criterion already defined for the spread of a sample is the "Error of Representation" (E_s) (Hirvonen, 1956), given by

$$t\,E_s^2 = \sum_{i=1}^{t} \sum_{j=1}^{n} \frac{(\Delta g_{ij} - \Delta g_{Mi})^2}{n} \quad \ldots\ldots\ldots\ldots\ldots(2)$$

where

Δg_M = mean anomaly for a square

Δg = observed anomaly

n = no. of stations in a square

t = no. of squares considered

As $n < 25$ for $\frac{1}{2}^{\circ} \times \frac{1}{2}^{\circ}$ squares, the denominator used for calculations was $(n - 1)$ instead of n (Spiegel, 1961, 70). The analysis of a high proportion of the South Australian sample shows good agreement with Hirvonen's values except, possibly, in the case of $\frac{1}{2}^{\circ} \times \frac{1}{2}^{\circ}$ square means. This is due to the highly variable field in the North West corner of the state where the free air anomalies approach

-100 mgal and the standard deviation of free air anomalies in a single square could be as large as $\pm$ 80 mgal for a $\frac{1}{2}^{\circ} \times \frac{1}{2}^{\circ}$ square.

The E_s values in Table 2 represent values obtained from the analysis of free air anomalies, and, in view of the nature of the topography, would apply, with relatively small variations, to both Bouguer and Isostatic anomalies. Thus, while the variation of gravity is similar to that of the European gravity field over large extents (Hirvonen, 1956), it can be slightly greater over limited ones.

The extension of the gravity field to unsurveyed areas

The extension of gravity fields from limited gravity data to obtain a continuous field has already been investigated by Jeffreys (1941), Kaula (1959), Uotila (1962) and Moritz (1966). The methods of extension used are fully described by the first two investigators and the last. Jeffreys had much less data available to him than Kaula and sets out his method of computation exhaustively. He worked on a 10° square unit, assuming one value to represent each square and allowing for height correlation. A series of observation equations were then fitted for each parallel and harmonic coefficients were evaluated to the 4th order. Kaula, on the other hand, used Markov theory to interpolate 1° square means from available values. 10° square means were then evaluated using autocorrelation analysis. Thereafter, a set of low order harmonics (to the 8th degree) was fitted by the simple orthogonal method using fully normalised coefficients (Kaula, 1959, 89). Uotila used harmonic analysis by least squares.

The application of spherical harmonics is obviously unsuited for field extension over limited extents and Moritz has used covariance analysis for this purpose. However, both Markovian predictions and covariance analysis, when used under economically feasible conditions, tend to give overly smoothened fields (Kaula, 1965, 4) and such field extensions, under adverse conditions, give results which are hardly better than direct representation.

Square size	Sample Size (n)										Total Sample		Hirvonen's Value
	A		B		C		D		E				
	E_s	t	E_s	t	E_s	t	E_s	t	E_s	t	E_s	t	E_s
$\frac{1}{2}^\circ \times \frac{1}{2}^\circ$	± 11	60	± 11	61	± 11	13	± 9	15	± 10	62	± 10.1	211	± 9.0
$1^\circ \times 1^\circ$	± 12	21	± 16	26	± 13	10	± 13	7	± 9	12	± 13.5	76	± 12.7
$2^\circ \times 2^\circ$	± 21	13	± 15	15	± 14	6	± 14	2	± 11	2	± 17.7	28	± 17.6

TABLE 2

The Error of Representation E_s (mgal)

Key to Classes

Square Size	A	B	C	D	E
$\frac{1}{2}^\circ \times \frac{1}{2}^\circ$	$0 < n \leq 5$	$5 < n \leq 10$	$10 < n \leq 15$	$15 < n \leq 20$	$n > 20$
$1^\circ \times 1^\circ$	$0 < n \leq 10$	$10 < n \leq 30$	$30 < n \leq 50$	$50 < n \leq 75$	$n > 75$
$2^\circ \times 2^\circ$	$0 < n \leq 50$	$50 < n \leq 100$	$100 < n \leq 200$	$200 < n \leq 300$	$n > 300$

In the analysis of the South Australian sample, the field extension was performed in two distinct stages using a two-dimensional trigonometrical series. The first stage was the extension of the gravity anomaly field in a limited $u^o \times u^o$ area in which each single anomaly value represents a $v^o \times v^o$ square. (The total number of readings possible in the area (N) is given by $N = u^2/v^2$). The anomalies used should only have variations dependent on position. Thus Bouguer anomalies were used. If free air anomalies are used, a three-dimensional series should be used with the height variable introduced into the series.

The second stage was the extension of $w^o \times w^o$ square means over a large area to unsurveyed squares.

In the analysis, it was sought to evaluate the coefficients A_i, i = 1,n of a series of the form $\sum_{i=1}^{n} A_i f_i(\phi, \lambda)$ which could be used to predict anomalies using the relation

$$E\left\{\Delta g(\phi, \lambda)\right\} = \sum_{i=1}^{n} A_i f_i(\phi, \lambda) \quad \ldots\ldots\ldots\ldots(3)$$

where $E\left\{\Delta g(\phi, \lambda)\right\}$ is the predicted gravity anomaly at the point whose latitude is ϕ and longitude λ .

The A_i(i=1, n) are determined by setting up observation equations and minimising the sum of the squares of the weighted residuals

$$\sum_{j=1}^{m} w_j r_j^2 = \sum_{j=1}^{m} w_j \left[\Delta g_j - \sum_{i=1}^{n} A_i f_i(\phi_j, \lambda_j)\right]^2 = \text{minimum} \ldots.(4)$$

where w_j(j=1, m) are the weight coefficients, which, in the first case, would be equal to unity.

The required set of equations for solution are

$$\sum_{j=1}^{m} w_j \left[\Delta g_j - \sum_{i=1}^{n} A_i f_i(\phi_j, \lambda_j)\right] f_k(\phi_j, \lambda_j) = 0, \; k = 1, n \ldots..(5)$$

These equations, in matrix notation, will be of the form

$$F.A \quad = \quad G \qquad \text{.............(6)}$$

where

$$F = \begin{bmatrix} f_{11} & f_{12} & f_{13} \cdots\cdots & f_{1n} \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ f_{m1} & f_{m2} & f_{m3} \cdots\cdots & f_{mn} \end{bmatrix} \qquad A = \begin{bmatrix} A_1 \\ A_2 \\ \cdot \\ \cdot \\ A_n \end{bmatrix} \qquad G = \begin{bmatrix} \Delta g_1 \\ \Delta g_2 \\ \cdot \\ \cdot \\ \Delta g_m \end{bmatrix}$$

and

$$f_{ij} = f_j(\phi_i, \lambda_i)$$

The matrix of residuals (R) is given by

$$R \quad = \quad FA - G \qquad \text{..............(7)}$$

For a least squares solution

$$\tfrac{1}{2} R^T WR \quad = \quad \text{minimum} \qquad \text{..............(8)}$$

where W is the matrix of weight coefficients, given by

$$W = \begin{bmatrix} w_{11} & O & O & O \cdots\cdots & O \\ O & w_{22} & O & O \cdots\cdots & O \\ O & O & w_{33} & O \cdots\cdots & O \\ . & O & O & O \cdots\cdots & w_{mm} \end{bmatrix}$$

Substituting for R from (7) in (8), after expansion and differentiation partially with reference to A, (8) reduces to

$$F^T \; WFA - F^T \; WG = O \qquad \text{.............(9)}$$

the solution of which is

$$A = (F^T W F)^{-1} F^T W G \qquad \ldots\ldots\ldots\ldots\ldots (10)$$

The extension of gravity fields in $2^o \times 2^o$ areas

The choice of a suitable trigonometrical series hardly depends on the available computer storage but, paradoxically, on the available gravity field. The field extensions obtained from limited amounts of data using functions with a high degree of resolution are generally unreliable. The computations in the case of the South Australian data were carried out on a C.D.C. 3200 computer and from the point of view of convenience in handling data, u was chosen as equal to two degrees. v, as explained earlier was chosen to be 0.1^o. A number of series were experimented with and a general series which gave adequate results was

$$E\left\{\Delta g(\phi, \lambda)\right\} = \sum_{i=O}^{a} A_i \cos\left[\pi(\phi - \phi_o)i\right] + \sum_{i=(a+1)}^{2a} A_i \sin\left[\pi(\phi - \phi_o)(i-a)\right]$$

$$+ \sum_{i=(2a+1)}^{3a} A_i \cos\left[\pi(\lambda - \lambda_o)(i-2a)\right]$$

$$+ \sum_{i=(3a+1)}^{4a} A_i \sin\left[\pi(\lambda - \lambda_o)(i-3a)\right] \qquad \ldots\ldots\ldots\ldots (11)$$

where ϕ_o, λ_o are the co-ordinates of the SW corner of the $2^o \times 2^o$ area, and ϕ, λ are the geographical co-ordinates of the $0.1^o \times 0.1^o$ square corner which is represented by the gravity anomaly $\Delta g(\phi, \lambda)$.

Repeated application to varying sets of data showed that the minimum conditions for a non-trivial solution are :-

(i) at least 5 readings should be available in every constituent $\frac{1}{2}^{o}$ x $\frac{1}{2}^{o}$ square.

(ii) at least 1 reading should be available in every row and column of the 20 x 20 array.

While lesser data have provided seemingly acceptable solutions, the results are subject to fortuitous circumstances. Thus 80 well distributed observations can give estimates of the balance 320 values, i.e., a 1 to 5 extension.

The value of a in equation (11) governs the degree of resolution of the function and while the maximum value is controlled by the available storage, the actual value used would be influenced by the amount of gravity data available. In a pilot investigation, it was found that decreasing the ratio a : u/v below $\frac{1}{2}$, while requiring much more computer time, did not materially improve the accuracy of extensions. Reasonably adequate representation was obtained by setting a : u/v = 1/3 in the case of well surveyed fields. In regions with inadequate data, better results were obtained by reducing a in equation (11) in the range of values $7 \leq a \leq O$ progressively as m in equation (5) reduces through the range $80 \leq m \leq 1$, the extreme case being one of direct representation.

These conclusions were used to predict values of gravity anomalies to represent unsurveyed areas, using Bouguer anomalies. The Bouguer anomaly so predicted was then corrected for the height term (Heiskanen and Vening Meinesz, 1958, 153) using the estimated elevation of the 0.1^{o} square corner. An attempt was made to check the accuracy of field extension by studying comparisons between predicted values in areas satisfying conditions (i) and (ii) for non-trivial solutions, with gravity data which was available subsequent to the computations. 154 comparisons were made in 6 different $2^o \times 2^o$ areas and the differences (Predicted - Observed) were found to be normally distributed with a standard deviation of $\pm$ 7.2 mgal.

In fitting the two-dimensional series defined in equation (11) to a field with a variable number (n) of gravity stations in it, the error of prediction (e_p), given by

$$e_p = E\{\Delta g(\phi, \lambda)\} - \Delta g(\phi, \lambda) \qquad \ldots\ldots\ldots\ldots\ldots (12)$$

was found to increase with n for a fixed value of a. Table 3 sets out values of $M\{e_p\}$ for a = 7, where $M\{e_p\}$ is the mean error of prediction.

Thus, if the available computer storage limits the maximum possible value of a, it is necessary to "normalise" predicted values prior to use, due to the magnitude of prediction errors in well represented fields. This can be effected either manually using a graphical extension technique or by the use of Markov theory (Bartlett, 1960, 24 et seq), as the accuracy of the predicted value obtained is dependent not only on the error of prediction e_p at adjacent stations but also on the average gravity anomaly gradient (G), where

$$|G| = \left| \frac{d\,\Delta g}{dl} \right| \qquad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

where $d\Delta g$ is the change in gravity anomaly over a distance dl.

$\frac{1}{2}^{o}$ x $\frac{1}{2}^{o}$ squares		2^{o} x 2^{o} squares	
Sample size (n)	$M\{e_p\}$ (mgal)	Sample size (n)	$M\{e_p\}$ (mgal)
$0 < n \leq 5$	± 4.4	$0 < n \leq 20$	± 1.8
$5 < n \leq 10$	± 6.0	$20 < n \leq 50$	± 2.3
$10 < n \leq 15$	± 6.4	$50 < n \leq 100$	± 4.7
$15 < n \leq 20$	± 7.3	$100 < n \leq 200$	± 6.7
$20 < n \leq 25$	± 6.5	$200 < n \leq 300$	± 8.3
		$300 < n \leq 400$	± 8.3
Total sample	± 6.5	Total sample	± 8.3

TABLE 3

Classification of Errors of Prediction.

Following a procedure similar to that adopted by Kaula (1959, 9) let the couplet c_{iu} be given by

$$c_{iu} = \begin{bmatrix} e_{p_i} \\ G_u \end{bmatrix} \qquad \ldots\ldots\ldots\ldots\ldots(14)$$

The expected error o prediction ($E\{e_{p_i}\}$) is given by

$$E\{e_{p_i}\} = \frac{c_i \, p_{jv}^{iu}(\Delta 1)}{p_{jv}^{u}(\Delta 1)} \qquad \ldots\ldots\ldots\ldots\ldots(15)$$

where p_{jv}^{iu} (Δ l) is the probability of the couplet c_{iu} occurring a distance Δl away from couplet c_{jv} ; and suppression of an index denotes summation with respect to that index. The mean value of $|G|$ was 8.9 mgal /50 km, with maximum, modal and minimum values of 48 6 and 0 respectively.

The mean comparison error (e_c), $M\{e_c\}$, given by

$$\left[M\{e_c\}\right]^2 = \frac{\sum_{i=1}^{n}\left[E\,\Delta g(\phi,\lambda) - E\{e_{p_i}\} - \Delta g_i(\phi,\lambda)\right]^2}{n} \qquad \ldots\ldots\ldots\ldots (16)$$

was $\pm$ 3.2 mgal for the 154 comparisons.

Field extensions under the above conditions can be expected to have an estimated error of $\pm$ 3 mgal, which is of an accuracy comparable with representation of a single tenth degree square by a single reading (Hirvonen, 1956, 2). Relaxation of criteria at (i) to 3 stations within each of the constituent $\frac{1}{2}^{\circ} \times \frac{1}{2}^{\circ}$ squares and maintaining those at (ii) gave estimated comparison errors of $\pm$ 8 mgal, which, on normalisation reduced to $\pm$ 6 mgal.

If these minimum conditions are not satisfied, the error of field extension becomes much larger and, unless a in equation (11) is reduced proportionately, the functional representation becomes erratic. This, in effect, reduces the resolution of the trigonometrical series, which, in the limit, becomes a case of direct representation.

The extension of the field to $\frac{1}{2}^{\circ} \times \frac{1}{2}^{\circ}$ square means over a $14^{\circ} \times 14^{\circ}$ area

A field extension, similar to the above, can be performed from the $\frac{1}{2}^{\circ} \times \frac{1}{2}^{\circ}$ square means obtained from the gravity data available to evaluate estimates of the $\frac{1}{2}^{\circ} \times \frac{1}{2}^{\circ}$ means of areas in which no readings occur. The field extensions are made from data not of equal weight (w) as

(i) the number (n) of readings used to evaluate the mean

(ii) the standard deviation (σ) of each sample

vary from square to square.

While n is dependent on the available gravity field, σ is a function of the variability of the latter, which is not dependent on topography alone. The weight coefficient

$$w = f(n, 1/\sigma^2)$$

In the evaluation of $f(n, 1/\sigma^2)$, it should be borne in mind that the final weight coefficient must reflect the distribution of the sample within the area represented (i.e., a high sample density in a restricted area may give a value for σ which is not a true representation of the variability of the gravity field within the square considered). Further, the expression should reduce to

$$w = 1/E_s^2 \qquad \text{.............(17)}$$

when $n = 1$ and $\sigma = 0$

and

$$w \rightarrow \frac{N}{\sigma^2} \qquad \text{...............(18)}$$

where N is the maximum number of readings possible, as $n \rightarrow N$.

In general, the weight coefficient should be inversely proportional to the variance of the sample mean. However, in squares where the sample covers only a small fraction of the total area, the use of σ^2/n tends to over-estimate the weight coefficient. An expression for the latter, which satisfies not only the limiting conditions, but also the general requirements is

$$w = \frac{n}{\sigma^2} \left\{ 1 + \frac{(N - n)^2 e_s^2}{(N-1)^2 \sigma^2} \right\}^{-1} \quad \ldots\ldots\ldots\ldots\ldots(19)$$

as

$$\frac{1}{w} = \frac{1}{n} \left\{ \sigma^2 + \left[\frac{N - n}{N - 1}\right]^2 e_s^2 \right\} \quad \ldots\ldots\ldots\ldots\ldots(20)$$

The individual weight coefficients for each $\frac{1}{2}^{\circ} \times \frac{1}{2}^{\circ}$ square were incorporated in Equation (5), which was expressed in the form set out in Equation (11) prior to solution. In this manner the field was extended to the unsurveyed areas.

The use of the weight coefficients in the analysis of a given field with considerable local variation was found to give rise to a smoothened field, when compared with a similar extension, but with the weight coefficients set at unity for all values. The values of $E\{\Delta g(\Phi,\lambda)\}$ so obtained in the weighted solution were normalised as explained earlier , using Equations (12) to (15) and the final extended value accepted was

$$\Delta g(\Phi,\lambda) = E\{\Delta g(\Phi,\lambda) - E\{e_p(\Phi,\lambda)\} \quad \ldots\ldots\ldots\ldots(21)$$

The assumption that the Bouguer anomalies used in the extension were free from height correlation is justifiable as the maximum mean $\frac{1}{2}^{\circ} \times \frac{1}{2}^{\circ}$ square elevation in the region considered was 803 meters.

The extended free air anomaly means were then obtained by allowing for the mean Bouguer reduction, using the mean square elevation. These extended values were used to supplement the observed values in compiling Figure (3).

The accuracy of the field extension

The accuracy of the field extension can be checked in one of two ways. Firstly, the extended values can be compared with actual values. An alternative method would be the comparison of the extended values as obtained from field extensions carried out by more than a single acceptable method.

Let the predictions be required in certain positions of a m x n array of Δg where certain values of Δg are available. Three distinct cases of prediction of the anomaly $\Delta g(i,u)$ are possible

(i) Interpolation:-

In this case, the readings $\Delta g(h,u)$, $\Delta g(j,u)$, $\Delta g(i,t)$ and $\Delta g(i,v)$ are available; $h < i < j$; $t < u < v$.

(ii) Interpolation/extrapolation:-

The circumstance in which only one of h, j, t, v is zero, on adopting the convention that $\Delta g(0,u) = \Delta g(i,0)$ = no reading available.

(iii) Extrapolation:-

At least one each of (h, j) and (t, v) is zero.

A preliminary study showed that the reliability of the predictions were strongly affected by the variability of the gravity field and all predictions were normalised in terms of the average gravity gradient in the area using the relation

$$N\{C\} = \frac{E\{C\}}{E\{G\}} M\{G\} \qquad \text{................(22)}$$

where $N\{C\}$ = the normalised prediction

$E\{C\}$ = the predicted value

$E\{G\}$ = the predicted gravity anomaly gradient

$M\{G\}$ = the mean gravity anomaly gradient.

The normalised predictions so obtained were classified according to

(a) the minimum interval (I_i) (j - h) or (v - t), as the case may be for interpolations and interpolation/extrapolations.

(b) the minimum interval (I_E) $|i - h(\text{or } j)|$ or $|u - v(\text{or } t)|$ in the case of extrapolation.

In all, four methods of prediction were used:-

(a) graphical

(b) Markov theory

(c) trigonometrical series with weighting

(d) trigonometrical series without weighting.

If the standard deviations of the comparisons between method (c) and the others were σ_{c_i} (i=1, 3), the reliability limits were set to the values of $I_{i(E)}$ accepted on the basis that

$$(\sigma_{c_i} - \sigma_{c_{i+1}}) \not> K.M\{G\} \qquad \text{............(23)}$$

$$i=1,3 \ (\text{if } i > 3, \text{ then } i=1)$$

where K is a comparison factor.

For interpolations, in cases where $I_i \leq 6$, good agreement was obtained between the σ_{c_i} for each of the different methods of extension. For $I_i > 6$, however, the comparison between the values for σ_{c_i} was erratic (K was generally greater than 0.2).

In case (ii), no comparison of $\sigma_{c_i} - \sigma_{c_{i+1}}$ could be considered acceptable unless the comparison factor was increased to K = 0.4.

In case (iii) too, the value 0.4 was adopted for K.

The results in Table 4 summarise the accuracy of comparisons in one instance, after discarding those whose I_i values are large enough to make the resulting extensions unreliable, by the standards defined in expression (23), with K = 0.4.

	Standard deviations of discrepancies in field extension (± mgal) (Numbers in brackets represent sample size)					
$I_{i(E)}$	1	2	3	4	5	6
Interpolation	2 (15)	5 (22)	5 (22)	5 (45)	5 (61)	6 (71)
Interpolation/ Extrapolation	6 (2)	5 (7)	11 (20)	18 (37)	10 (41)	9 (44)
Extrapolation	10 (29)	12 (46)	14 (56)	15 (61)	15 (63)	---

T A B L E 4

Field Extension discrepancies for $\frac{1}{2}^{o} \times \frac{1}{2}^{o}$ square means; Case (i) / Case (iii)

While the samples are too small for definite conclusions to be drawn, it would appear that, as a general rule, the weighting of extensions according to

Interpolations: Interpolations/Extrapolations: Extrapolations

$$= \frac{1}{1^2} : \frac{1}{2^2} : \frac{1}{3^2} = 1 : 0.25 : 0.11$$

is indicated.

The factors to be considered in estimating the error of the predicted value are

(a) the nature of the extension, i.e., interpolation, interpolation/ extrapolation or extrapolation,

(b) the interval from the most reliable value,

(c) the accuracy of this value.

If the error of representation of the nearest basic square mean is e_{ref} and the estimated error in the extension is e_{ext}, the estimated error of prediction ($E\{e_p\}$) is given by

$$\left[E\{e_p\}\right]^2 = e_{ref}^2 + e_{ext}^2 \qquad \text{.................(24)}$$

The value for e_{ref} is obtained from Equation (20)

Conclusion

The extension of gravity fields using mathematical functions, unless carried out under carefully controlled conditions, could produce results of questionable accuracy. The use of the two dimensional trigonometrical series described in Equation (11) is quite satisfactory for limited extents, provided that the degree of resolution of the function is adequately reduced when a paucity of data occurs. In the case where field extension is effected using area means which could be of differing reliability, the extension should be performed after adequate weighting. The accuracy of the values so predicted is dependent on whether the extension performed was an interpolation, interpolation/extrapolation or an extrapolation. Field extensions over intervals more than 6 positions from the nearest available value (i.e., $I_{i,E} > 6$) were found to be unreliable. For $I_{i,E} \leq 6$, the ratios of the accuracies of

interpolation: interpolation/extrapolation: extrapolation = 1 : 2 : 3 .

Within this range, the error of the predicted value would not be materially larger than the error of representation of the nearest value used in the extension.

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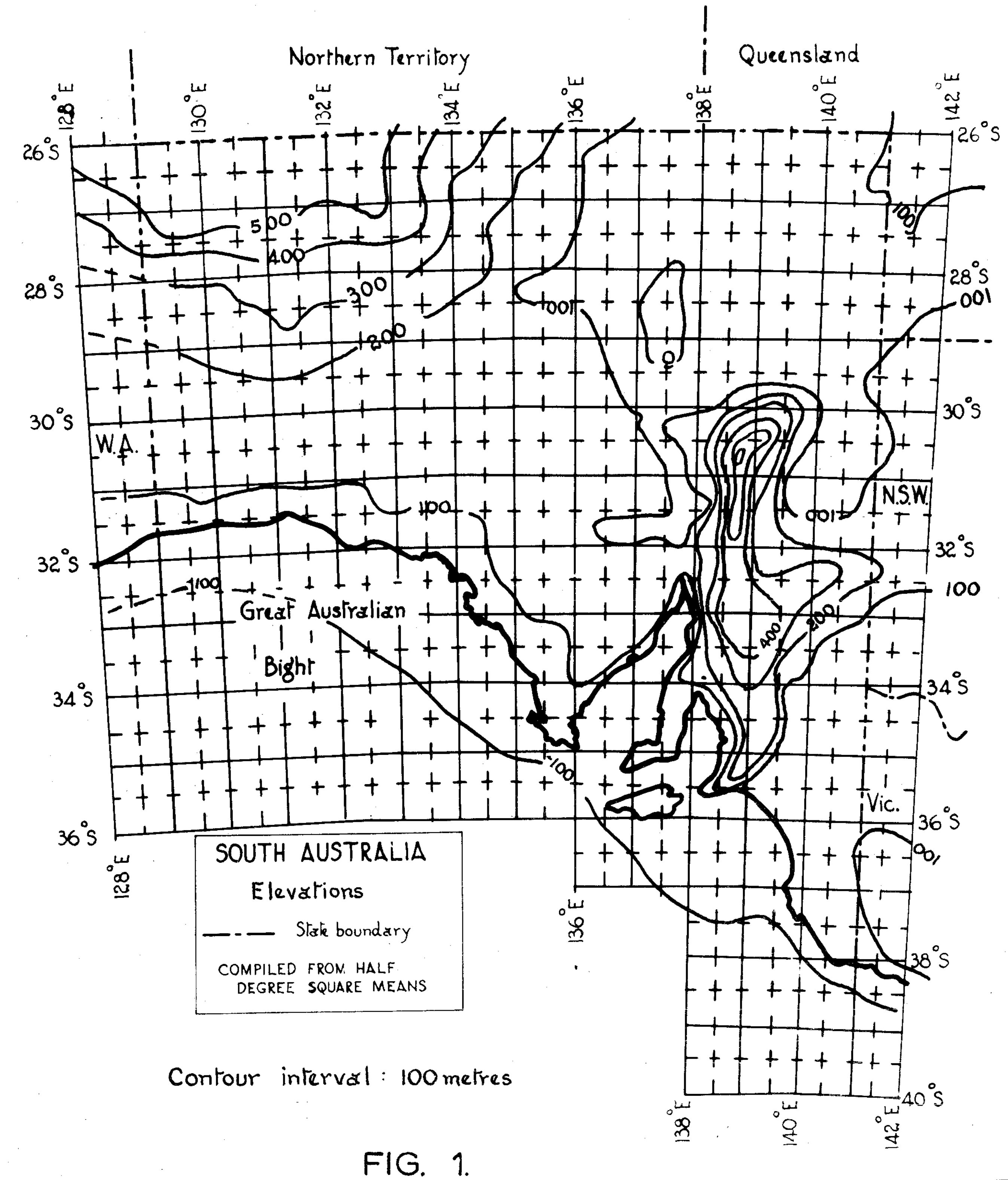


Northern Territory
Queensland
W.A.
N.S.W.
Vic.
Great Australian Bight
SOUTH AUSTRALIA
Elevations
State boundary
COMPILED FROM HALF DEGREE SQUARE MEANS
Contour interval : 100 metres

FIG. 1.

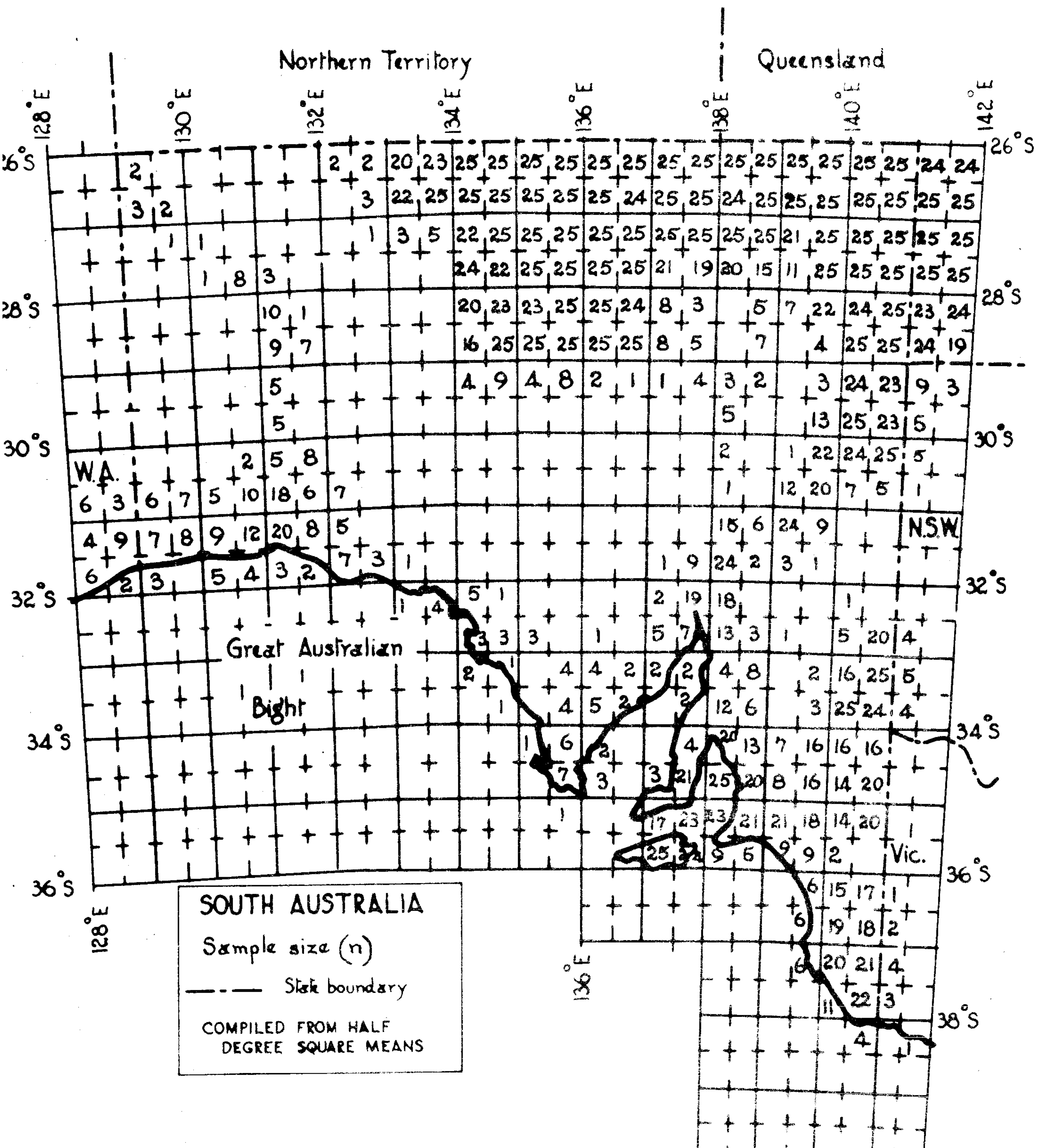

Northern Territory
Queensland
W.A.
N.S.W.
Vic.
Great Australian Bight
SOUTH AUSTRALIA
Sample size (n)
State boundary
COMPILED FROM HALF DEGREE SQUARE MEANS

FIG. 2

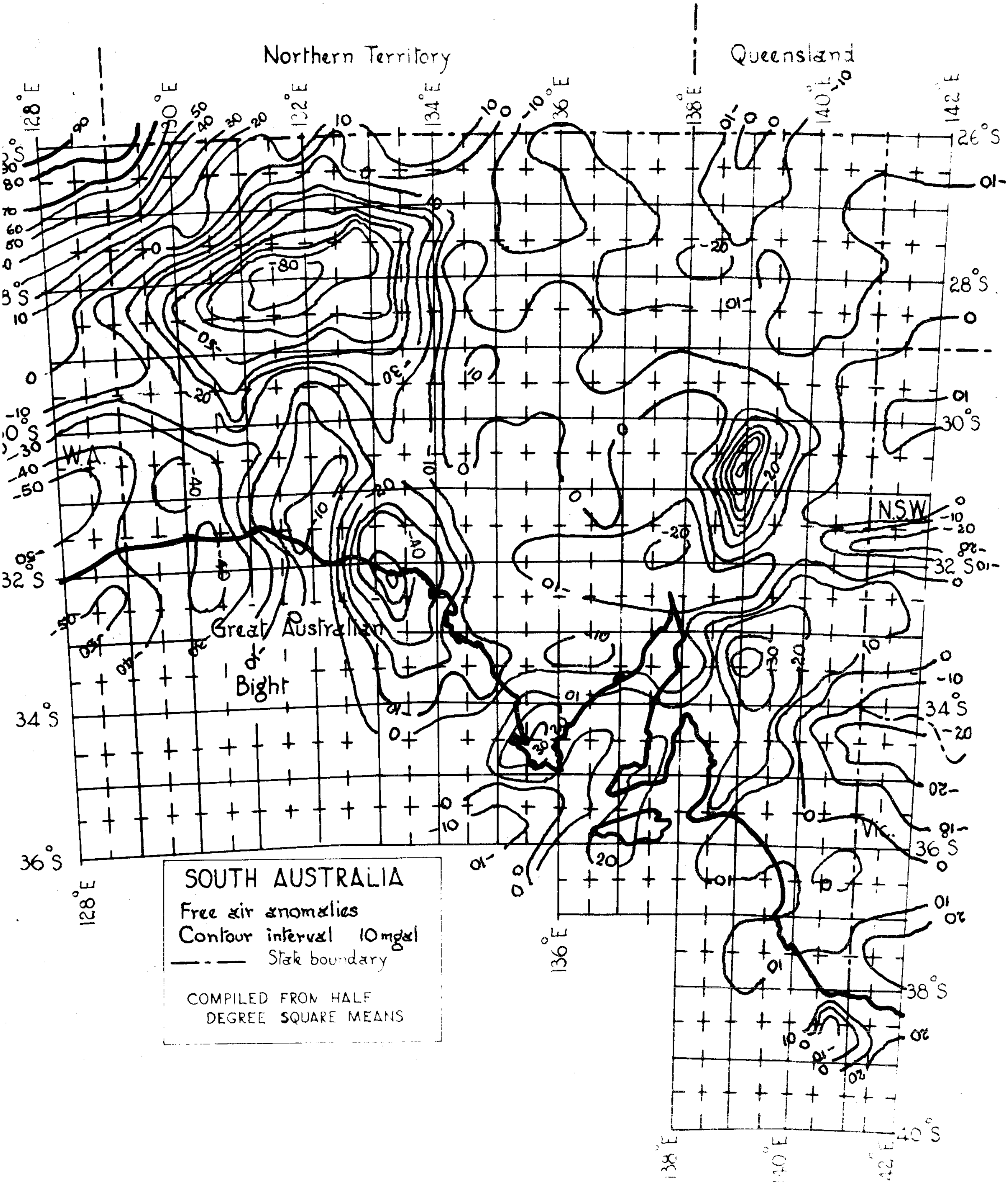
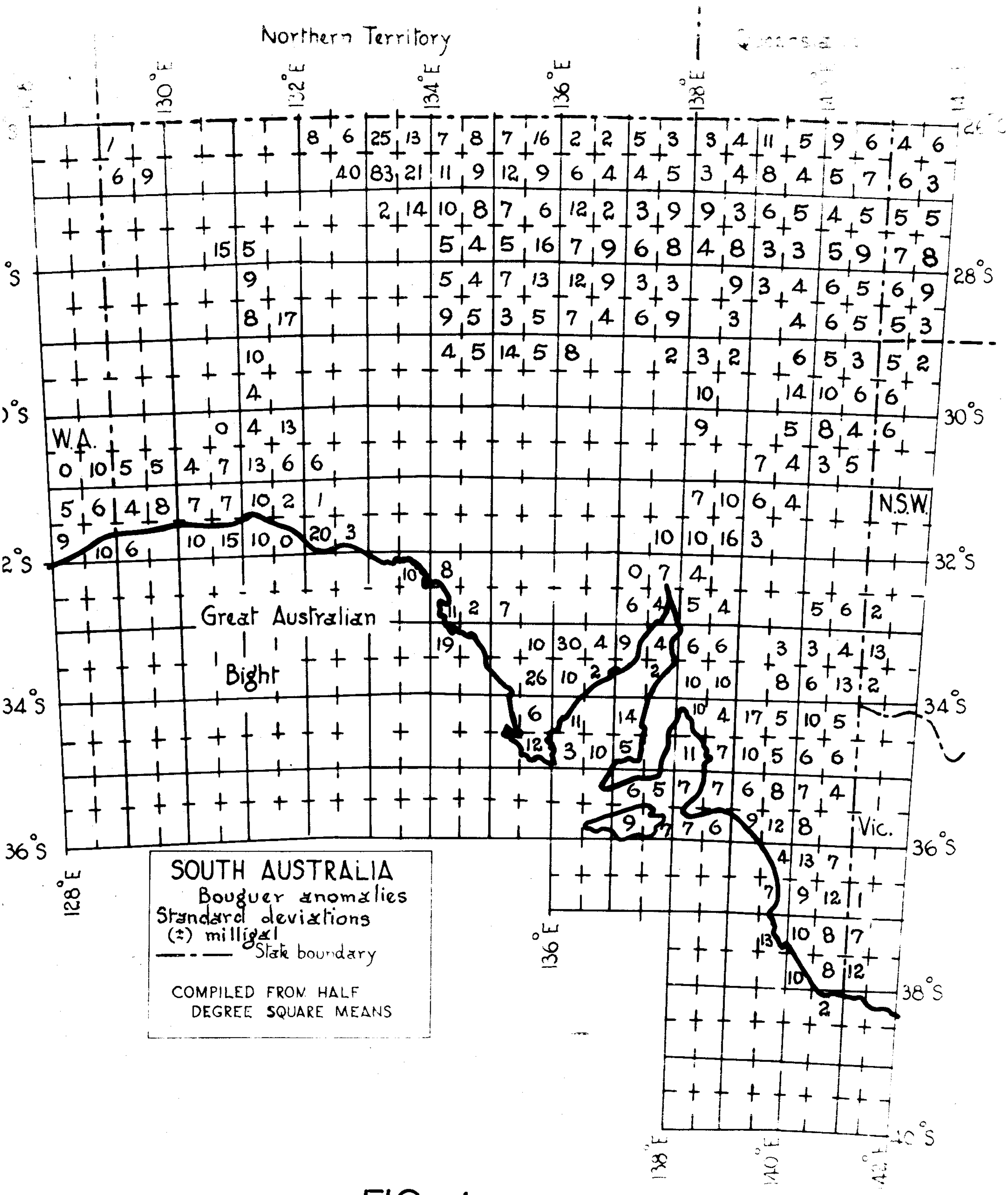

Northern Territory
Queensland
W.A.
N.S.W.
Vic.
Great Australian Bight
SOUTH AUSTRALIA
Free air anomalies
Contour interval 10 mgal
State boundary
COMPILED FROM HALF DEGREE SQUARE MEANS

FIG. 3.



Northern Territory
W.A.
N.S.W.
Vic.
Great Australian
Bight
128°E
130°E
132°E
134°E
136°E
138°E
140°E
28°S
30°S
32°S
34°S
36°S
38°S
SOUTH AUSTRALIA
Bouguer anomalies
Standard deviations
(±) milligal
State boundary
COMPILED FROM HALF
DEGREE SQUARE MEANS

FIG. 4.