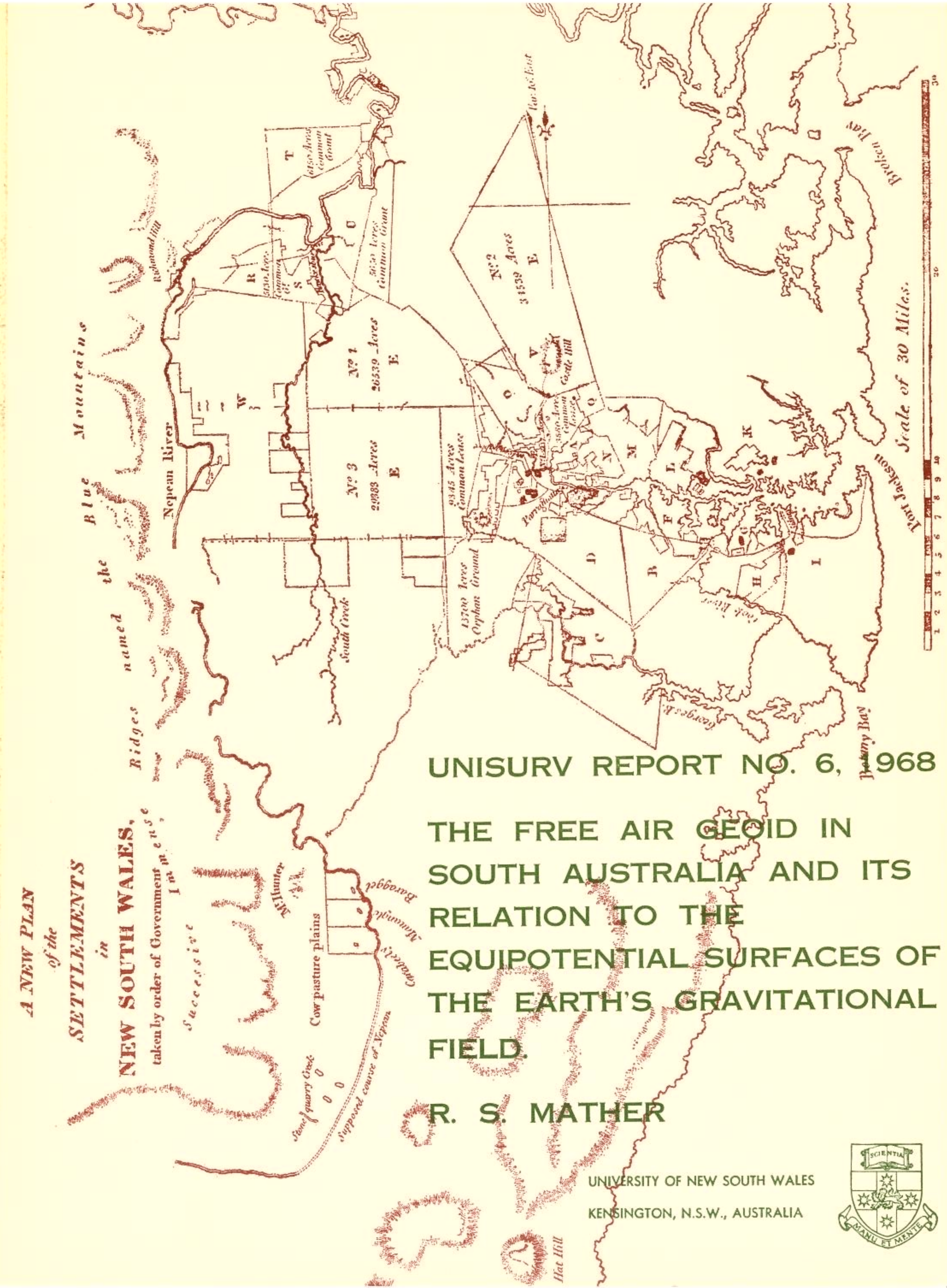

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THE FREE AIR GEOID IN SOUTH AUSTRALIA AND ITS RELATION TO THE EQUIPOTENTIAL SURFACES OF THE EARTH'S GRAVITATIONAL FIELD.

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Reference to Districts.

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B Liberty Plains

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D Parramatta

EEEE Ground reserved
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F Concord

G Petersham

H Bulanaming

I Sydney

K Hunters Hills

L Eastern Farms

M Field of Mars

N Ponds

O Toongabbey

P Prospect

Q

R Richmond Hill

S Green Hills

T Phillip

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W Evan

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13. THE INTERPRETATION OF THE GRAVIMETRIC SOLUTION.

13.1 Introduction.

The gravimetric solution is expressed in the form of three components N (the separation of the two surfaces being mapped), ξ(the deflection of the vertical in the meridian) and η(the component of the deflection of the vertical in the prime vertical). The relation between the deflections of the vertical obtained astro-geodetically and the gravimetric solution has already been discussed in section (11). The derivations and relations given therein presuppose the triangulation spheroid to be correctly orientated in space.

Further, for any comparison to be made, both sets of deflections of the vertical should be computed on the same spheroid. All computations made with the values of gravity anomalies based on the international gravity formula can be expected to refer to the international spheroid, the dimensions and the variations of gravity over which are given in equations (3.29) and (3.30). The parameters assigned to it (Heiskanen and Moritz, 1967, 79) are

$$\gamma_e = 978\ 049.0 \text{ mgal},$$
$$\omega = 0.72921151 \times 10^{-4} \text{ sec}^{-1}$$
and $$k\,M = 3.986\ 329 \times 10^{20} \text{ cm}^3 \text{ sec}^{-2}.$$

The last relation is based on the first, which, in the case of the international gravity formula, (Heiskanen and

Vening Meinesz, 1958, 75) was based on several thousand isostatically reduced gravity stations. The quantities a, f, ω and kM are related by a relatively simple set of equations once the spheroidal model has been adopted with the restriction that its bounding surface is an equipotential. In general these formulae which occur in many sources, for example on pages 45 et seq of the source previously quoted may be written as

$$c = \frac{a^3 \omega^2}{kM} + o\{f^2\} \quad \ldots\ldots\ldots\ldots\ldots(13.1),$$

$$\gamma = \gamma_e(1 + \beta \sin^2\phi + o\{f^2\}) \ldots\ldots\ldots(13.2),$$

where

$$\beta = \frac{5}{2}c - f + o\{f^2\} \quad \ldots\ldots\ldots\ldots\ldots(13.3).$$

Equation (13.3) is known as Clairaut's theorem. The imposition of the condition that the bounding surface is an equipotential, of value U_o gives the latter quantity according to the relation

$$U_o = \frac{kM}{a}\left[1 + \frac{1}{3}f + \frac{1}{3}c + o\{f^2\}\right] \ldots\ldots\ldots(13.4).$$

The value of equatorial gravity is related to the constants by the relation

$$\gamma_e = \frac{kM}{a^2}\left[1 + f - \frac{3}{2}c + o\{f^2\}\right] \ldots\ldots\ldots(13.5).$$

Thus the dimensions of the reference spheroid control the value of γ_e and U_o and/or kM, one of which should also be defined. In putting together the international gravity formula, the values of a and f adopted were those of the international spheroid. The rest of the quantities were bound together by adopting the value of ω for the rotation of the earth as specified earlier, as well as that for γ_e. The value of kM given was then calculated from these quantities, as was the value of U_o, given by (Heiskanen and Moritz, 1967, 80)

$$U_o = 6\,263\,978.7 \text{ kgal metres} \quad \text{.........} \quad (13.5a).$$

The anomalous nature of the earth's interior makes it impractical, at this stage, to assign a value for equatorial gravity from direct observations for g at chosen sites on the equator. In addition, it is not possible to assign a value for U_o by observation unless the shape of the geoid is defined and a value is adopted for kM from an independent method. It is therefore of interest to investigate the nature of the constants used in the computation of normal gravity and assess their reliability. In addition to possible errors in the values of these constants which could contribute to errors in the magnitude of the geoid spheroid separation vector through changes in the value of normal gravity, it is also of interest to consider purely geometrical changes due to changes in the dimensions of the reference spheroid. Thus, the direct gravimetric solution needs interpretation before any comparison can be made with results obtained from other methods. Similarly, astro-geodetic results also require precise definition before attempting comparison with values obtained from gravimetry. These have already been summarised in section (10.2) and will now be dealt with in greater detail.

13.2 The effect of an error in the orientation of a triangulation spheroid on astro-geodetic deflections of the vertical.

The definition of a reference spheroid in space can be subdivided into three distinct operations, involving seven parameters, as follows :-

(i) Determination of the size of the spheroid, specified by a and f.

(ii) The assigning of three parameters fixing the origin of the reference system and a fourth determining the orientation of one of the axes of a three dimensional rectangular cartesian system in space.

(iii) The evaluation of one parameter relating the rotation axis of the spheroid to one of the reference axes.

Conventionally the reference axis system is centred at the earth's centre of mass. In the case of triangulation spheroids, conditions (ii) and (iii) are imposed by fixing the location of the local zenith on the celestial sphere, adopting the celestial pole as fixed in space. Astronomical determinations of latitude give the angular displacement of the pole from the the local zenith along the relevant great circle arc. This arc length is assumed to be the co-latitude of the origin of the triangulation scheme, the longitude being given by the angle, at the celestial pole, between this great circle arc and the equivalent arc which constitutes the Greenwich meridian. The relationship between the quantities involved can be seen in figs (13.1) and (13.2).

In general, three possibilities arise in effecting the

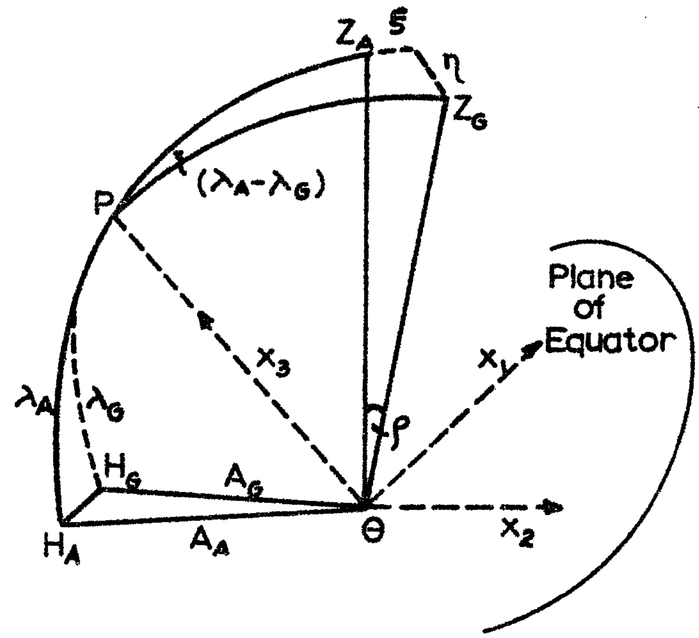


FIG. 13.1

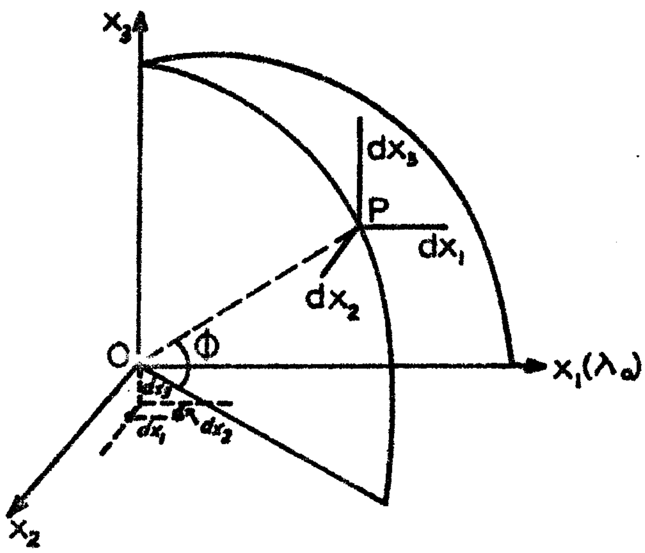


FIG. 13.2

orientation of a triangulation spheroid. They are :-

(i) An absolute orientation is provided using a gravimetric solution combined with astronomical observations.

(ii) The adoption of the astronomical determinations as the geodetic coordinates of the origin.

(iii) The solution at (ii) is adopted as an intermediate. The triangulation, based on these values for the geodetic co-ordinates of the origin, is used, along with further astronomical position determinations to effect an analysis on the basis that the correct geodetic coordinates at the origin will give rise to a set of comparisons of astronomical and geodetic coordinates which form a normally distributed set. Such an analysis will provide a set of corrections to the astronomical values of latitude and longitude at the origin.

All three cases, in essence, reduce to the problem of assigning values for the deflections of the vertical at the origin of the triangulation scheme. In the first, these values are determined gravimetrically and are adopted as the deflections of the vertical at the origin of the scheme. If these values are to be relevant,

(a) the gravimetric deflections must be corrected for the fact that they refer to the international spheroid and not necessarily to the triangulation spheroid ;

(b) the values determined must be those between the surface vertical and the associated spherop, as described in section (11.1). These must be corrected using equation (11.8) to obtain values relevant to the triangulation spheroid.

The adoption of the second alternative is equivalent to forcing coincidence between the astronomical and geodetic zeniths. Such an action can be represented by more than one mathematical model. A reasonable one (Vening Meinesz, 1950) is to represent the distortion vector by three linear components (dx_i, $i=1,3$) along each of the axes (x_i, $i=1,3$), retaining the orientation of the polar axis in space. This same interpretation can be applied to any residual error after the application of the third method.

This is equivalent to forcing the origin to a new position , whose coordinates with respect to the initial and true origin are - $dx_i (i=1,3)$, but without rotation of axes. The resulting changes in the curvilinear coordinates of points on the earth's surface could be expressed as as changes $\Delta\xi$ and $\Delta\eta$ in the components of the deflection of the vertical in the meridian and prime vertical respectively together with a change ΔN along the vertical.

Thus, at any point $P(\phi, \lambda)$ in the triangulation network, based on the origin at $P_o(\phi_o, \lambda_o)$, such that the x_1 axis is the intersection of the equatorial plane and the meridian plane through P_o, the linear changes (δN, δE) in position in the north and east directions due to changes in the deflections of the vertical, assumed positive north and east are given by

$$\delta N = -\rho \, \Delta\xi(1 + \frac{h}{\rho}) \quad \ldots\ldots\ldots\ldots\ldots (13.6)$$

and

$$\delta E = -\nu \, \Delta\eta(1 + \frac{h}{\upsilon}) \quad \ldots\ldots\ldots\ldots\ldots (13.7).$$

The above equations are related to the changes dx_i

($i=1,3$) by the equations

$$-\rho\,\Delta\xi(1+\frac{h}{\rho}) = dx_3\cos\phi - dx_1\sin\phi\cos(\lambda_o-\lambda) - dx_2\sin\phi\sin(\lambda_o-\lambda) \quad \ldots(13.8),$$

$$-\nu\,\Delta\eta(1+\frac{h}{\nu}) = dx_1\sin(\lambda_o-\lambda) - dx_2\cos(\lambda_o-\lambda) \quad \ldots(13.9)$$

and

$$\Delta N = dx_3\sin\phi + dx_1\cos\phi\cos(\lambda_o-\lambda) + dx_2\cos\phi\sin(\lambda_o-\lambda) \quad \ldots\ldots\ldots\ldots\ldots(13.10).$$

The above equations, on evaluation at the origin, give, using the suffix $_o$ to represent the relevant values, give

$$-\rho_o\Delta\xi_o(1+\frac{h_o}{\rho_o}) = dx_3\cos\phi_o - dx_1\sin\phi_o \quad \ldots\ldots(13.11),$$

$$-\nu_o\Delta\eta_o(1+\frac{h_o}{\nu_o}) = -dx_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(13.12)$$

and

$$\Delta N_o = dx_3\sin\phi_o + dx_1\cos\phi_o \quad \ldots\ldots\ldots\ldots\ldots(13.13)$$

Thus,

$$dx_1 = \rho_o\Delta\xi_o(1+\frac{h_o}{\rho_o})\sin\phi_o + \Delta N_o\cos\phi_o \ldots(13.14),$$

$$dx_2 = \nu_o\Delta\eta_o(1+\frac{h_o}{\nu_o}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(13.15)$$

and

$$dx_3 = -\rho_o\Delta\xi_o(1+\frac{h_o}{\rho_o})\cos\phi_o + \Delta N_o\sin\phi_o \ldots(13.16).$$

h, in equations (13.8) to (13.16) refer to the elevation of the earth's surface above the spheroid at P. Substitution for dx_i (i=1,3) in equations (13.8) to (13.10) from equations (13.14) to (13.16) gives

$$-\rho\Delta\xi(1+\frac{h}{\rho}) = -\rho_o\Delta\xi_o(1+\frac{h_o}{\rho_o})\left[\cos\phi_o\cos\phi+\sin\phi_o\sin\phi\cos\Delta\lambda\right]-$$

$$-\nu_o\Delta\eta_o(1+\frac{h_o}{\nu_o})\sin\phi\sin\Delta\lambda +$$

$$+\Delta N_o\left[\sin\phi_o\cos\phi-\cos\phi_o\sin\phi\cos\Delta\lambda\right] \quad \ldots(13.17),$$

$$-\nu\Delta\eta(1+\frac{h}{\nu}) = \rho_o\Delta\xi_o(1+\frac{h_o}{\rho_o})\sin\phi_o\sin\Delta\lambda -$$

$$-\nu_o\Delta\eta_o(1+\frac{h_o}{\nu_o})\cos\Delta\lambda +$$

$$+\Delta N_o\cos\phi_o\sin\Delta\lambda \quad \ldots\ldots\ldots\ldots\ldots(13.18)$$

and

$$\Delta N = \rho_o\Delta\xi_o(1+\frac{h_o}{\rho_o})\left[-\sin\phi\cos\phi_o+\sin\phi_o\cos\phi\cos\Delta\lambda\right]$$

$$+\nu_o\Delta\eta_o(1+\frac{h_o}{\nu_o})\cos\phi\sin\Delta\lambda +$$

$$+\Delta N_o\left[\sin\phi\sin\phi_o+\cos\phi_o\cos\phi\cos\Delta\lambda\right]$$

$$\ldots\ldots\ldots\ldots(13.19),$$

where $\Delta\lambda$, in equations (13.17) to (13.19) is given by

$$\Delta\lambda = \lambda_o - \lambda .$$

As $\Delta\xi_o$, $\Delta\eta_o$ and ΔN_o are unlikely to exceed 1 in 10^5, in all but mountainous regions it would suffice to ignore the effect of the elevation terms which are of order 1 part in 10^3 or less, for most practical purposes. Equations (13.17) to (13.19) in the physical surface of the earth - spheroid separation vector arising from changes in the orientation at the origin of the triangulation spheroid and can be utilised to give the changes in the deflections of the vertical at individual triangulation stations for a specified set of changes in the geodetic coordinates at the origin. Any attempt at comparison of astro-geodetic deflections of the vertical with those obtained gravimetrically will require the introduction of three correction parameters for orientation errors at the origin before any comparisons can be made. Conversely, given a triangulation spheroid whose orientation is arbitrary, observation equations could be set up for the comparison of astro-geodetic deflections of the vertical computed on the erroneously orientated reference figure with those computed gravimetrically, a simple least squares solution of which should give the orientation corrections at the origin.

13.3 Changes in the values of the deflections of the vertical due due to changes in the dimensions of the reference spheroid.

This problem is a purely geometrical one. Given a set of deflections of the vertical on a reference spheroid which has been correctly orientated, if necessary by the application of equations (13.17) to (13.19), it is required to find the relation specified by the heading of this section. If the values of the deflections of the vertical obtained from a network of triangulation and/or geodetic traversing are ξ_i and η_i $(i=1, n)$, the corrected values after allowing for any orientation error (ξ_{t_i}, η_{t_i} (i=1, n) are given by

$$\xi_{t_i} = \xi_i + \Delta\xi_i \,, \; i=1, n \quad \ldots\ldots\ldots\ldots\ldots (13.20)$$

and

$$\eta_{t_i} = \eta_i + \Delta\eta_i \,, \; i=1, n \quad \ldots\ldots\ldots\ldots\ldots (13.21),$$

where ξ_i, η_i (i=1, n) are given by equations (13.17) and (13.18) respectively. Further corrections $\Delta\xi_{d_i}$, $\Delta\eta_{d_i}$ (i=1, n) have to be computed for correcting astro-geodetic deflections from the triangulation spheroid to the international spheroid. The final values of the astro-geodetic deflections of the vertical (ξ_{c_i}, η_{c_i} (i=1, n)) , given by

$$\xi_{c_i} = \xi_{t_i} + \Delta\xi_{d_i} \,, \; i=1, n \quad \ldots\ldots\ldots\ldots (13.22)$$

and

$$\eta_{c_i} = \eta_{t_i} + \Delta\eta_{d_i} \,, \; i=1, n \quad \ldots\ldots\ldots\ldots (13.23),$$

can be compared directly with gravimetric values computed for the physical surface of the earth - telluroid system, after the latter have been corrected for the effect defined in equation (11.8).

If the equatorial radii and flattenings of the triangulation spheroid and the international spheroid are (a_t, f_t) and (a, f) respectively, related by the equations

$$a = a_t + da \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13.24)$$

and

$$f = f_t + df \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13.25),$$

the relation between the points on each spheroid which

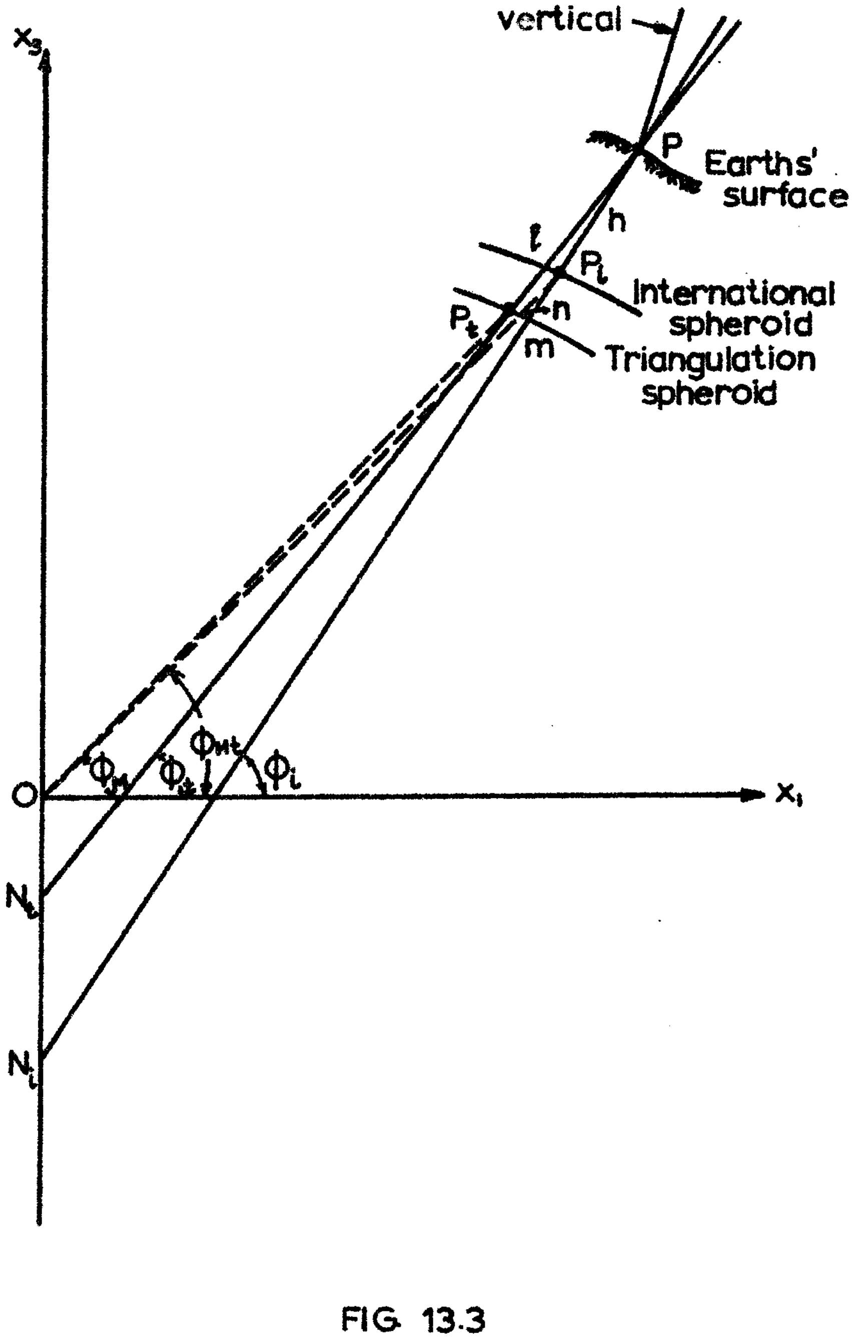

vertical
x_3
P
Earths' surface
h
ℓ
P_i
International spheroid
P_t
n
m
Triangulation spheroid
Φ_{Mt}
Φ_M
Φ_t
Φ_i
O
x_1
N_t
N_i

FIG. 13.3

P_i and P_t which represent the point P on the earth's surface is shown in fig (13.3). P_i and P_t are defined as those points on the international spheroid and the triangulation spheroid, respectively, whose normals pass through P. If ϕ, with the appropriate suffix, refers to the geodetic latitude and the use of the suffix $_m$ with latitude refers to geocentric values, it can be seen that the changes da and df do not affect the component of the deflection of the vertical in the prime vertical so long as the spheroid centres and rotation axes coincide, as changes in the dimensions of the spheroid, which is a solid of revolution, do not bring about a differential positional displacement in the prime vertical as the plane $N_t N_i P$ lies entirely in the meridian at P. Thus

$$\Delta\eta_d = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13.26).$$

The required correction $\Delta\xi_d$ in fig (13.3) is given by

$$\Delta\xi_d = \xi_c - \xi_t = -(\phi_i - \phi_t)$$
$$= -d\phi \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13.27).$$

As $d\phi$ is unlikely to exceed 60", the assumption that the spheroidal arc $P_i \ell$ is parallel to the arc $P_t nm$ is valid to 1 part in 3000.

$P_t m = d\phi \; h_t$ and is unlikely to exceed 3 metres. If h_i is the spheroidal elevation of P with respect to the international spheroid,

$$h_i = h_o + N \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13.28),$$

where h_o is the orthometric elevation and N the geoid - spheroid separation, obtained gravimetrically.

$$P_t m = d\phi\ h_t = P_t n + nm = d\phi\,(h_i + dN)$$

$$= d\phi_m\, OP_t + (\phi_i - \phi_{m_i})\, dN \quad \ldots\ldots\ldots (13.29),$$

where

$$dN = h_t - h_i \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13.30).$$

The geodetic latitude is related to the geocentric value by the relation (Bomford, 1962, 496)

$$\tan\phi_m = (1 - e^2)\tan\phi = (1 - f)^2 \tan\phi.$$

Thus,

$$(\phi - \phi_m) + o\{f^3\} = \tan(\phi - \phi_m) = \frac{\tan\phi - \tan\phi_m}{1 + \tan\phi\tan\phi_m}$$

$$= \sin\phi\cos\phi\, f(2 - f)\,[\,1 - f(2 - f)\sin^2\phi\,]^{-1}$$

$$= f\sin 2\phi - \tfrac{1}{2} f^2 \sin 2\phi\,(1 - 4\sin^2\phi)$$

$$\ldots\ldots\ldots\ldots\ldots (13.31).$$

In fig (13.3), $d\phi_m$ is given by

$$d\phi_m = \phi_{m_t} - \phi_{m_i}.$$

The use of equation (13.31) in this expression gives

$$d\phi_m = \phi_t - \phi_i - f_t \sin 2\phi_t + \tfrac{1}{2} f_t^2 \sin 2\phi_t (1 - 4\sin^2\phi_t) +$$

$$+ (f_t + df)(\sin 2\phi_t + 2\,d\phi\cos 2\phi_t) - \tfrac{1}{2} f_t^2 \left(1 + 2\frac{df}{f_t}\right)(\sin 2\phi_t +$$

$$+ 2\,d\phi\cos 2\phi_t)\left[1 - 4\sin^2\phi_t - 4\,d\phi\sin 2\phi_t\right] + o\{10^{-10}\}$$

Further simplification gives

$$d\phi_m = -\, d\phi + df \sin 2\phi_t + 2 f_t d\phi \cos 2\phi_t - f_t df \sin 2\phi_t -$$

$$- f_t^2 d\phi \cos 2\phi_t + 4 f_t df \sin 2\phi_t \sin^2\phi_t + 2 f_t^2 d\phi \sin^2 2\phi_t$$

$$+ o\{10^{-10}\} \quad \ldots (13.32).$$

Further,

$$OP_t = \nu_t (1 - e_t^2 \sin^2\phi_t)$$

$$\doteq a_t (1 - f_t \sin^2\phi_t) \quad \ldots\ldots\ldots\ldots\ldots\ldots (13.33)$$

Thus

$$d\phi_m \times OP_t = d\phi_m \times a(1 - f_t \sin^2\phi_t + o\{f^2\}) \ldots\ldots (13.34).$$

From fig (13.3), it can be seen that

$$(\nu_t + h_i + dN) \cos\phi_t = (\nu_i + h_i) \cos\phi_i \ldots\ldots\ldots (13.35).$$

As $\nu = a(1 - e^2 \sin^2\phi)^{-\frac{1}{2}}$

$$= a(1 + f \sin^2\phi + o\{f^2\}),$$

$$\nu_i - \nu_t = da + a_t df \sin^2\phi_t + o\{af^2\} \quad \ldots\ldots\ldots\ldots (13.36).$$

Thus

$$(\nu_t + h_i + dN) \cos\phi_t = (\nu_t + da + a_t df \sin^2\phi_t + h_i)(\cos\phi_t -$$

$$- d\phi \sin\phi_t), \text{ from which}$$

$$dN = da + a_t df \sin^2\phi_t - {}_t d\phi \tan\phi_t + o\{af^2\} \ldots.$$

$$\ldots\ldots\ldots (13.37).$$

The use of equations (3.37) and (3.31) gives

$$dN(\phi_i - \phi_{m_i}) = (da + a_t df \sin^2\phi_t - {}_t d\phi \tan\phi_t) [f_t \sin 2\phi_t$$

$$+ df \sin 2\phi_t - \tfrac{1}{2} f_t^2 \sin 2\phi_t (1 - 4 \sin^2\phi_t) + 2 f_t d\phi \cos 2\phi_t$$

$$- 4 f_t df \sin^2\phi_t \sin 2\phi_t] + o\{10^{-9}\}$$

$$dN(\phi_i - \phi_{m_i}) = da\, f_t \sin 2\phi_t + a_t f_t df \sin 2\phi_t \sin^2\phi_t -$$
$$- \nu_t f_t d\phi \tan\phi_t \sin 2\phi_t + o\{af^4\} \quad \ldots .(13.38).$$

Dropping the suffixes and substituting from equations (13.32), (13.33), (13.37) and (13.38) in equation (13.29),

$$d\phi\,(h_o + N + dN) = a\,(1 - f\sin^2\phi)(-d\phi + df\sin 2\phi + 2f\,d\phi\cos 2\phi -$$
$$- f\,df\sin 2\phi + 4f\,df\sin 2\phi\sin^2\phi\,) +$$
$$+ da\,f\sin 2\phi + a\,f\,df\sin 2\phi\sin^2\phi - \nu f\,d\phi \,..$$
$$\tan\phi\sin 2\phi$$

$$d\phi(h_o + N + dN) = -a\,d\phi + a\,df\sin 2\phi + a\,f\,df(-\sin 2\phi -$$
$$- \sin 2\phi\sin^2\phi + 4\sin 2\phi\sin^2\phi + \sin 2\phi\sin^2\phi) +$$
$$+ a\,f\,d\phi\,(2\cos 2\phi + \sin^2\phi - 2\sin^2\phi) +$$
$$+ f\,da\,\sin 2\phi + o\{af^4\}.$$

Re-grouping the terms and simplification gives

$$d\phi(1 + \frac{h}{a} + \frac{N}{a} + \frac{dN}{a} - f(2\cos 2\phi - \sin^2\phi) = df\sin 2\phi +$$
$$+ f\,df\sin 2\phi(4\sin^2\phi - 1) + f\,\frac{da}{a}\sin 2\phi.$$

Thus

$$d\phi = df\sin 2\phi + f\,df(\,4\sin 2\phi\sin^2\phi + 2\cos 2\phi\sin 2\phi - \sin 2\phi -$$
$$- \sin 2\phi\sin^2\phi) - df\,\frac{h}{a}\sin 2\phi + f\,\frac{da}{a}\sin 2\phi + o\;f^4\;.$$
$$= df\sin 2\phi + f\,\frac{da}{a}\sin 2\phi - df\,\frac{h}{a}\sin 2\phi + f\,df\sin 2\phi\,(3\sin^2\phi +$$
$$+ 2\cos 2\phi - 1) + o\{f^4\} \quad \ldots\ldots\ldots\ldots\ldots (13.39).$$

Tp obtain a suitable working relationship, consider likely figures for a_t and f_t. In Australia, the parameters of the triangulation spheroid, called the Australian National Spheroid are (Lambert, 1967, 4)

$$a = 6\ 378\ 160$$

and $$f = 1/298.25 = 0.003353 \qquad \ldots\ldots\ldots\ldots (13.40).$$

The combination of equation (13.40) with equations (13.29), (13.24) and (13.25) gives

$$da = 228 \text{ metres}$$
$$df = 1.41 \times 10^{-5} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (13.41).$$

Thus $d\phi$ is of order 10^{-5} radians which is equivalent to a maximum value of 3" of arc or less. The other terms are two orders smaller but should be considered for the sake of completeness. Thus, the complete expression for $\Delta\xi_d$ is, from equations (13.27) and (13.39)

$$\Delta\xi_d^{(rad)} = -\, df \sin 2\phi - f\frac{da}{a}\sin 2\phi + df\,\frac{h}{a}\sin 2\phi + $$
$$+\ 2\, f\, df \sin 2\phi \cos^2\phi \quad \ldots\ldots (13.42)$$

Thus the final expressions for the deflections of vertical obtained from astro-geodetic methods and comparable with values computed from gravimetry are

$$\xi_c = \xi + \Delta\xi - 0''.00017\, h^{(met)} \sin 2\phi + \Delta\xi_d \qquad \ldots\ldots\ldots (13.43)$$

and $$\eta_c = \eta + \Delta\eta \qquad \ldots\ldots\ldots\ldots\ldots\ldots (13.44),$$

where $\Delta\xi$, $\Delta\eta$ and $\Delta\xi_d$ are given by (13.17), (13.18) and (13.43) respectively.

In the above derivation it has been assumed that the deflections of the vertical computed from gravity anomalies are specific for the chosen reference spheroid used in the computation of normal gravity. This requires a study of the changes in the values of normal gravity and associated gravitational constants due to changes in the basic parameters. Further, it is necessary to study the effect any inaccuracies in the values adopted for the physical constants have on the computed figure.

13.4 The effect of changes in the parameters of the reference spheroid on normal gravity.

The equations which relate the gravitational properties of a rotating spheroid of given parameters a and f, in terms of the physical constants of the earth, are set out, to the order of the square of the flattening, in equations (13.1) to (13.5). Consider the effect of changes da, df, and d (kM) in a, f and k M respectively on these formulae. If the resulting changes in c, β, γ_e and U_o are dc, dβ, dγ_e and dU_o, the quantities defined above are related by the equations

$$dc = c\left[\frac{3\,da}{a} - \frac{d(kM)}{kM}\right] \quad \ldots\ldots\ldots\ldots\ldots(13.45),$$

$$d\beta = \frac{5}{2}\,dc - df \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(13.46),$$

$$d\gamma_e = \gamma_e\left[\frac{d(kM)}{kM} - 2\,\frac{da}{a} + df - \frac{3}{2}\,dc\right] \ldots(13.47),$$

$$d\gamma = \gamma\left[\frac{d\gamma_e}{\gamma_e} + d\beta\sin^2\phi\right] \quad \ldots\ldots\ldots\ldots(13.48)$$

and

$$dU_o = U_o\left[\frac{d(kM)}{kM} - \frac{da}{a} + \frac{1}{3}\,df + \frac{1}{3}\,dc\right] \ldots(13.49).$$

If d(k M) = 0, the above equations will represent changes in the dimensions of the reference spheroid, as expressed in equation (13.41), but with different signs as the new spheroid will be the smaller one. As the magnitude of c (heiskanen and Moritz, 1967, 80) is given by

$$c = 3.45 \times 10^{-3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13.50),$$

$dc \approx -3 \times 10^{-7}$ and can be ignored when considering the magnitude of terms in the evaluation of the other changes. The change in γ_e is of order 60 mgal. The values of normal gravity change from approximately 60 mgal on the equator to 70 mgal at the poles. Thus the change in the dimensions of the reference spheroid produce very definite changes in the values of the gravity anomalies. These changes are systematic in nature and will hence have decided effects on the geoid-spheroid separation as well as on the deflections of the vertical. As such changes are due entirely to the change in the dimensions of the reference spheroid, they can be related directly to changes in N, ξ and η described in section (13.3).

The relevant changes in the gravimetrically determined deflections of the vertical on changing the spheroid of reference from the international spheroid to a chosen triangulation spheroid are given by equations (11.21) through (11.23), where Δg_F is replaced by the value of $- d\gamma$, computed by the use of equation (13.48). The resulting values c_ξ and c_η are primarily given by the use of the appropriate value of $- d\gamma$ in the Vening Meinesz integrals which form the second summation in equation (10.28).

It can be seen from equation (13.49) that a change in the parameters of the reference spheroid as set out in equation (13.41) increases the potential of the bounding equipotential by approximately 200 kgal metres. This draws attention to the fact that the surface being mapped from the reference spheroid is not necessarily defined without ambiguity. Brun's theorem set out in equation (3.13) is based on the potential of the reference surface equalling that of the surface being mapped. As the potential of the geoid is not known, there is no necessity for this value to be equal to that defined for the international spheroid by the adoption of the value for equatorial gravity set out in section (13.1). If, on the other hand, it is not desired to impose the limitation that the potential of the geoid must equal that of the adopted spheroid of reference, it will be necessary to re-examine

(a) the derivation of the expression for N in equation (5.14);

(b) the relation between the height anomaly and the disturbing potential.

for consistency when the potential of the reference surface is not equal to that of the geoid. This is developed in the next section.

13.5 Changes in the physical constants of the earth and their effect on normal gravity.

The two independent physical constants which influence the value of normal gravity are the angular velocity of rotation of the earth (ω) and the constant k M. The former only influences the small order term c and can be considered to be free from error. The value of kM adopted for the international spheroid is given in section (13.1). More recent determinations of this constant, set out in table (13.1), indicate that this value is in error in the fifth significant figure.

Source	Value of k M x 10^{20} cm^3 sec^{-2}
1. International Spheroid	3. 986 329
2. International Astronomical Union (Fricke et al, 1965, 65)	3. 986 03
3. Ranger series lunar probes (Rapp, 1967, 591)	3. 986 013

TABLE (13.1)

Values of the constant k M

The value of kM used in the international gravity formula is a derived quantity, based on isostasy (Heiskanen and Vening Meinesz, 1958, 74). The continuance with the use of the international gravity formula (ibid, 76) is based on the fact that the present world wide gravity network is based on the so called "Potsdam" datum which is generally held to be about 15 mgal too high. The present value of equatorial gravity is also based upon this datum and hence it is accepted that gravity anomalies computed from the international gravity formula are adequate for all geodetic purposes. From equation (13.47) it can be seen that a change in equatorial gravity is tantamount to a change in the value of kM as the effect of a change in dc is about three orders smaller.

It is instructive to study the effect of changes in

(a) the value of kM ;

(b) the dimensions of the reference spheroid and

(c) both the value of kM and the dimensions of the reference spheroid

on normal gravity. Equations (13.45) to (13.49) were programmed using the following changes in the parameters of the reference spheroid and kM from those of the international spheroid, as set out in equation (3.29) and section (13.1).

$$da = - 228 \text{ metres}$$
$$df = - 1.41 \times 10^{-5} \qquad \ldots\ldots\ldots\ldots\ (13.50)$$
$$d(kM) = - 3.14 \times 10^{16} \text{ cm}^3 \text{ sec}^{-2}.$$

These figures are equivalent to changing the reference spheroid to that adopted by the International Astronomical Union and hereafter called the I.A.U. spheroid and accepting $kM = 3.986\ 015 \times 10^{20} \text{ cm}^3 \text{sec}^{-2}$. The results are set out in table (13.2). The first column gives the change in normal gravity if all the above changes were taken into account. This would be very nearly, but not quite offset by the change of nearly - 15 mgal due to all values of observed gravity being in need of a correction for datum error in the value at Potsdam. Estimates of what this change should be are embodied in table (13.3).

Nevertheless, the acceptance of the international gravity formula for geodetic calculations would undoubtedly give rise to a systematic effect in the value of N computed using

Latitude (deg)	Change in normal gravity ($d\gamma$) (mgal)		
		da, df = 0	d(k M) = 0
0	- 20.9	- 77.0	56.0
5	- 20.8	- 77.0	56.2
10	- 20.5	- 77.0	56.6
15	- 20.0	- 77.1	57.1
20	- 19.3	- 77.1	57.8
25	- 18.5	- 77.1	58.6
30	- 17.5	- 77.2	59.7
35	- 16.4	- 77.2	60.8
40	- 15.2	- 77.2	62.0
45	- 14.1	- 77.3	63.2
50	- 12.9	- 77.3	64.4
55	- 11.7	- 77.3	65.6
60	- 10.6	- 77.3	66.7
65	- 9.7	- 77.4	67.7
70	- 8.8	- 77.4	68.6
75	- 8.1	- 77.4	69.3
80	- 7.6	- 77.4	69.8
85	- 7.3	- 77.4	70.1

TABLE (13.2)

Changes in normal gravity due to changes in the parameters and/or the physical constants of the reference spheroid.

Source	Estimated change in Potsdam datum dg_P (mgal).
1. Heiskanen and Vening Meinesz (1958, 152)	from - 10 to - 15
2. Bomford (1962, 337)	- 14
3. Mueller and Rockie (1966, 129)	- 13.7
4. Heiskanen and Moritz (1967, 152)	- 13
5. Rapp (1967b, 21)	- 13.7
6. $(\Sigma d_\gamma \cos \phi)/(\Sigma \cos \phi)$ (col (1), table (13.2))	- 17.1

TABLE (13.3)

Estimates of the correction necessary to the present value of the Potsdam datum.

anomalies computed on such a reference figure. Test calculations carried out for the South Australian region showed that the magnitude of the effect, in the case of the changes described in column (1) of table (13.2) are not negligible. The use of the correction ($c_{\Delta g}$), given by

$$c_{\Delta g} = dg_P - d\gamma \quad \text{.............. (13.51)},$$

where dg_P is the correction necessary to the Potsdam datum and $d\gamma$ is given by equations (13.45) to (13.49) and equation (13.50), will give the correction to the free air geoid obtained with reference to the model provided by the international spheroid and the international gravity formula when used in the Stokes and Vening Meinesz integrals. The resultant free air geoid will be related to the I.A.U. spheroid on the basis that the value of kM adopted by the I.A.U. is correct. Such a computation should give zero correction to η due to the nature of the changes in $d\gamma$ and was used as a test of the formulae used in the computations to ensure that

(a) the programming was free from error and

(b) the programs meshed without serious errors.

The values of the correction (c_η) to η seldom exceeded 0".002 and hence have not been included in table (13.4). The contribution of each of the programs to the effect is shown in the case of three random points in the region. They can be considered to be typical. See table (13.5)

There is little doubt that the international gravity formula and its associated spheroid cannot be used for reliable geodetic calculations. It should be noted any attempts to perform geodetic orientations using gravimetry must be preceded by a conversion of standard gravity anomalies into anomalies on an acceptable spheroid. No large scale revision of gravity data is necessary, however, as the calculation of $d\gamma$ can be performed simultaneously with the standard separation vector calculations. It is also necessary to correct all values of ξ used in the computation of the correction ξ_{c_i} ($i=1,2$) as well as the gravity anomaly used in this expression when evaluating equation (11.22) in determining surface deflections of the vertical.

In preparing table (13.4) and in all subsequent calculations

Latitude (deg N)	Corrections to the free air geoid International spheroid to I.A.U spheroid	
	N (met)	c_ξ (sec)
- 26	12.38	- 2.24
- 27	11.16	- 2.31
- 28	9.91	- 2.36
- 29	8.62	- 2.41
- 30	7.30	- 2.46
- 31	5.98	- 2.52
- 32	4.60	- 2.56
- 33	3.21	- 2.60
- 34	1.80	- 2.64
- 35	0.39	- 2.67
- 36	- 1.11	- 2.71
- 37	- 2.60	- 2.74
- 38	- 4.06	- 2.76
- 39	- 5.51	- 2.78

TABLE (13.4)

Corrections to the free air geoid on changing the parameters of the international spheroid to those of the I.A.U. system and adopting $d\,g_P = -16$ mgal.

the value of the correction to the Potsdam datum (dg_P) was assumed equal to - 16 mgal. This is probably a trifle large and due to the uncertainty regarding the exact

Latitude	-27° 00'			-37° 00'		
Longtiude	131° 00'			138° 00'		
Contributions by program	N (met)	c_{ξ} (sec)	c_{η} (sec)	N (met)	c_{ξ} (sec)	c_{η} (sec)
STOKIN	0.43	- 0.111	- 0.088	0.07	- 0.045	- 0.014
STOKNE	2.07	- 0.060	0.068	0.24	- 0.162	0.018
STOKMD	4.33	- 0.502	0.010	0.91	- 0.550	- 0.001
STOKUT	4.33	- 1.633	0.009	- 3.82	- 1.980	- 0.002
Total	11.16	- 2.306	- 0.001	- 2.60	- 2.737	0.001

TABLE (13.5)

Contributions of the various regions to the corrections to N, ξ and η on conversion of the free air geoid from the international spheroid to the I.A.U. system.

correction to be made, it was decided not to investigate the matter further.

A study of the third column of table (13.2) shows that changes in the dimensions of the spheroid of reference only cause large changes in the values of normal gravity which are definitely systematic in nature. This implies that the choice of a spheroid of reference has a precise meaning in gravimetric geodesy notwithstanding the spherical approximation used in the evaluation of the boundary value problem. Thus equation (13.42) must be equivalent to changes in the quantities computed using equations (11.10) and (11.21) using the correction $c_{\Delta g}$, defined in equation (13.51) with $d(kM) = 0$. The computation of this effect over the South Australian region in the case of N and ξ is set out in columns (1) and (2) of table (13.6). The value of the same correction computed from equation (13.42) is given in the

Latitude (deg N)	Effect of change of spheroid		
	Gravimetry		Geometry
	N (met)	c_ξ (sec)	$\Delta\xi_d$ (sec)
- 26	14.1	- 2.21	- 2.32
- 27	12.9	- 2.24	- 2.38
- 28	11.6	- 2.28	- 2.44
- 29	10.4	- 2.38	- 2.50
- 30	9.1	- 2.42	- 2.55
- 31	7.7	- 2.48	- 2.59
- 32	6.3	- 2.54	- 2.65
- 33	5.0	- 2.59	- 2.69
- 34	3.6	- 2.65	- 2.73
- 35	2.2	- 2.66	- 2.77
- 36	0.7	- 2.68	- 2.80
- 37	- 0.8	- 2.70	- 2.84
- 38	- 2.3	- 2.76	- 2.86

TABLE (13.6)

The comparison of changes in the deflections of the vertical on change of reference spheroid from the international spheroid to the I.A.U. spheroid as obtained from gravimetry and from geometry.

third column of the table. The value of the deflection obtained from gravimetry is that for the free air geoid only, being the result of using the modified value of $c_{\Delta g}$ in the Vening Meinesz integrals. The resulting quantities are systematically smaller than the equivalent quantity computed from equation (13.42) due to the fact that the effect of the changes in the value of the meridian component of the deflection on change of spheroid, on the latter through the term ξ_{c_1}, defined in equation (11.22), has not been considered. Further the magnitude of the contribution of each of the terms participating in the summation, as set out in table (13.7) also accounts for the rather irregular comparison error, the spherical approximation showing in the third significant figure due to the magnitude of the anomaly corrections involved.

Latitude	- 28° 00'			- 34° 00'		
Longitude	131 °00'			140° 00'		
Contributions by program	N (met)	ξ (sec)	η (sec)	N (met)	ξ (sec)	η (sec)
STOKIN	- 13.01	- 2.407	2.688	- 12.76	2.262	2.797
STOKNE	- 65.21	- 1.865	- 0.748	- 62.46	- 0.724	0.560
STOKMD	- 142.67	- 0.341	- 1.001	- 142.79	- 0.762	- 1.814
STOKUT	232.46	- 2.478	- 0.905	221.61	- 3.428	- 1.543
Total	11.57	- 2.277	0.034	3.60	- 2.652	0.000

TABLE (13.7)

Contributions of the various regions to the corrections to the separation vector on change of spheroid considering the free air geoid only

Thus no geometrical consistency is possible from geodetic gravimetry if the international gravity formula is used for providing values of normal gravity. If it is desired to refer the free air geoid to the international spheroid itself, it will be necessary to correct normal gravity by approximately - 77 mgal before computing the relevant gravity anomaly. This is a pointless exercise as all available evidence points to this not being the spheroid of best fit. As pointed out earlier, there is an internal consistency within the international gravity formula model due to compensating errors but these errors do produce a very definite systematic error in any geodetic determinations.

The change in potential of the spheroid of reference due due to the three combinations of changes considered in table (13.2), are set out in table (13.8).

Case	dU_o (kgal met)
1. All variations considered	- 298.9
2. da, df = 0	- 493.5
3. d(kM) = 0	194.5

TABLE (13.8)

Changes in the value of the gravitational potential on the reference spheroid due to changes in definitive parameters.

While the value of the potential on the reference spheroid never appears to come into geodetic calculations, its exact magnitude is of the utmost importance in the derivation of Bruns' theorem, set out in section (3.3), which assumes that the surface to be mapped has the same potential as the reference surface, on a point to point basis. This is an assumption that requires very close study in view of the large changes in the value of the potential of the reference spheroid due to changes in the values adopted for the parameters of even an internally consistent system such as that giving rise to the international gravity formula.

13.5 The zero order term in the separation vector.

Four decisions have to be made in attempting to choose a reference system for the definition of normal gravity (γ) in the evaluation of gravity anomalies.

(i) The choice of a reference model.

A model must be chosen for the stratification of matter within the reference figure. In view of the asymmetry of the existing topography with respect to the ocean areas and the lack of any associated precessionary effect on the earth's axis of rotation, it is reasonable to assume that isostasy prevails in a general sense though there appear to be some systematic departures from the isostatic model (Jeffreys, 1962a, 174). Seismological evidence seems to indicate, to a first order, that any stratification of the earth's crust occurs in accordance with the model postulated by Airy (Heiskanen and Vening Meinesz, 1958, 135). Thus a reference spheroid affords an acceptable figure provided it has the same mass as the existent earth and the same volume as the surface being mapped.

(ii) Choice of the values of a, f for the reference spheroid.

On accepting the model at (i) as resonable, the second and third decisions concern the dimensions of the reference figure. The I.A.U. spheroid was initially postulated by Kaula from an analysis of a combined solution using astro-geodetic, gravimetric and satellite data (Kaula, 1962, 93). The choice of this reference spheroid is discussed further in section (15.4).

(iii) A value for k M.

There are no assumptions involved in the techniques used for the conventional determination of kM as can be seen from McCullagh's theorem (Bomford, 1962, 393). It would appear that any determination of kM on the basis of surface gravimetry, from the present state of incompleteness in the field and lack of detailed stratification studies, is unlikely to be very satisfactory. The extent to which the exact value of kM influences investigations can be gauged from the extent of the changes which occur in converting from the international gravity formula system to the I.A.U. system. While the change in the gravity anomaly is less than 10 mgal, that in the potential of the reference spheroid is approximately - 300 kgal metres.

Thus it is necessary to re-examine all derivations to ascertain the effect of not assuming the potential of the geoid (W_o), which is not known, equal to that on the reference spheroid (U_o). By virtue of the defined parameters of the latter, U_o is a known quantity which, in the case of the I.A.U. system, is given by

$$U_o = U_{o(Int.\ Sph)} + dU_o$$

$$= 6,263,978.7 - 298.9 = 6,263,680 \text{ kgal met.} \quad \text{..........(13.52)},$$

where the value for the international spheroid is taken from Heiskanen and Moritz (1967, 80) and the correction dU_o from table (13.8). Re-examination of Bruns' theorem, using equation (3.12), gives

$$V_{D_P} = W_P - U_Q + h_{D_P} \gamma_P \quad \text{..........(13.53)},$$

the points P and Q being defined in fig (3.2). In the earlier expression, Q was defined as the point in space which, on the reference system, " had the same potential as that of the existent system at the point P ". In reality, this definition is unacceptable in an absolute sense as the potential of the geoid (W_o) is not known. The foundation for the acceptance of the statement quoted is that the potential of the reference spheroid is known, along with the difference in potential (ΔW) between sea level and the point P. This estimates the potential of P ($E\{W_P\}$) according to the relation

$$E\{W_P\} = U_o + \Delta W \quad \text{..................(13.54)}.$$

The point Q is assumed to have the same difference in potential with respect to the spheroid as the point P above the geoid. For practical purposes, this is interpreted as Q having the same elevation above the spheroid as P above the geoid. This is not strictly true as no matter is located in the region considered outside the reference spheroid on this system while such matter does exist for the existent earth. There would, in reality be differences of up to 10 metres in the two linear distances involved on the basis that the earth is isostatically compensated, for linear elevations upto 2000 metres above the geoid. By definition,

$$U_Q = U_o + \Delta W \quad \ldots\ldots\ldots\ldots\ldots\ldots(13.55).$$

Conventionally, E W_P is used instead of W_P in the derivation of Bruns' theorem. In reality,

$$W_P = W_o + \Delta W \quad \ldots\ldots\ldots\ldots\ldots\ldots(13.56).$$

Thus, accepting the definition of the telluroid without any change,

$$V_{D_P} = W_o - U_o + h_{D_P} \gamma_P \quad \ldots\ldots\ldots(13.57),$$

irrespective of the location of the point P.

This relation will also hold on the geoid. Thus equation (13.57) relates a point P on either the earth's surface or the geoid to a point Q on the reference system such that both points have the same difference of potential with respect to the related reference surface.

Equation (3.18), relating the gravity anomaly to the disturbing potential, still holds and equation (3.20a) can, in the light of equation (13.57), be re-written as

$$\Delta g_P = -\left[\frac{\partial V_D}{\partial h} + \frac{2\{V_{D_P} - (W_o - U_o)\}}{R} \right] \ldots(13.58),$$

where errors of up to 3 mgal can be introduced into the value of Δg if the anomaly was computed using elevations instead of geopotential in cases where the station elevation could be as large as 2000 metres.

Bruns' theorem and the expression for the gravity anomaly enter into the derivation of the Stokesian term in more ways than one. The chief concern is the existence of a zero order term in both the gravity anomaly and the disturbing potential . On the other hand, by virtue of the assumption that the reference spheroid has the same volume as the geoid, the separation (N) between the geoid and the spheroid must have zero mean on world wide summation. As these conditions are essentially zero order conditions, they do not appear to have any effect on deflections of the vertical and hence, attention will be focused on the expressions for the height anomaly (h_D) and the geoid spheroid separation N.

The first occasion on which equation (3.16) is used is in the evaluation of $\underline{N}.\underline{\nabla}\ V_D$, set out in equations (4.13) to (4.17). The replacement of $\partial V_D/\partial x_3$, using equation (13.58) gives

$$\underline{N}.\underline{\nabla}\ V_D = \cos\beta \left[-\Delta g - \frac{1}{\gamma}\left|\frac{\partial\gamma}{\partial h}\right| V_D - \sum_{i=1}^{2} \frac{\partial V_D}{\partial x_i} \tan\beta_i + \right.$$

$$\left. + \frac{1}{\gamma}\left|\frac{\partial\gamma}{\partial h}\right|(W_o - U_o) \right] \quad \ldots.(13.59).$$

The combination of this result with the replacement of V_D by the use of equation (13.57) in equation (4.21) gives

$$h_{D_P} = \frac{2\phi_{e_P}}{\gamma} - \frac{W_o - U_o}{\gamma} - \frac{1}{2\pi\gamma}\iint_R \frac{\cos\beta}{\gamma}\left|\frac{\partial\gamma}{\partial h}\right| \frac{W_o - U_o}{r}\, dR +$$

$$+ \frac{1}{2\pi\gamma}\iint_R \Bigg[\left\{ \frac{1}{\gamma}\left|\frac{\partial\gamma}{\partial h}\right| \frac{\cos\beta}{r} + \underline{\nabla} \cdot \underline{N}\ \frac{1}{r} \right\} V_D +$$

$$+ \left\{ \Delta g - \gamma(\xi \tan\beta_1 + \eta \tan\beta_2) \right\} \frac{\cos\beta}{r} \Bigg] dR \quad ..(13.60).$$

Adopting the spherical approximation utilised previously in solving the above equation in the case of the geoid - spheroid system, the use of equation (5.9) and the subsequent discussion, in the case of the first integral on the right hand side of equation (13.60) gives

$$\frac{R_m^2}{2\pi\gamma_m} \int_0^{\sigma=4\pi} \frac{1}{\gamma}\left|\frac{\partial\gamma}{\partial h}\right| \frac{W_o - U_o}{r}\, d\sigma$$

$$= \frac{R_m^2}{2\pi\gamma_m} \int_0^{\pi}\int_0^{2\pi} \frac{1}{\gamma}\frac{2\gamma}{R_m} \frac{W_o - U_o}{2R_m \sin\frac{1}{2}\psi} \sin\psi\, d\psi\, d\alpha$$

$$= 4\ \frac{W_o - U_o}{\gamma_m} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (13.61),$$

where $d\alpha$ is the increment in azimuth. Equation (13.60), on lines analagous to the derivation of equation (5.14), reduces to

$$N_P = \frac{2\phi_{e_P}}{\gamma_m} - 5\,\frac{W_o - U_o}{\gamma_m} + \frac{R_m}{4\pi\gamma_m}\int_0^{\sigma=4\pi} (3V_D +$$

$$+ 2R\,\Delta g_o)\,\frac{1}{r}\, d\sigma \quad .....(13.62).$$

The relation between V_D and Δg_o is derived on lines similar to that adopted in section (5.2) with the exception that the expression for the disturbing potential has a zero order term. Equation (5.29) becomes

$$V_D = \frac{A_o}{R} + \sum_{n=2}^{\infty} \frac{A_n}{R^{n+1}} \qquad \text{...........(13.63).}$$

$$\frac{\partial V_D}{\partial h} = -\frac{A_o}{R^2} - \sum_{n=2}^{\infty} (n+1) \frac{A_n}{R^{n+2}} \qquad \text{.......... (13.64).}$$

From equation (13.58),

$$\Delta g_o = -\frac{A_o}{R^2} + \sum_{n=2}^{\infty} \frac{(n-1) A_n}{R^{n+2}} + 2\frac{W_o - U_o}{R}.$$

The revised expression for F, defined in section (5.3), is

$$F = \frac{A_o}{R} + \sum_{n=2}^{\infty} (2n+1) \frac{A_n}{R^{n+1}} + 4(W_o - U_o)$$

$$= -R\, g_o S_o + \sum_{n=2}^{\infty} R \frac{2n+1}{n-1} g_{nm} S_{nm} + 4(W_o - U_o) \text{..(13.65).}$$

In the preceding derivations, the first order terms, by definition, still remain zero for exactly the same reasons as those set out in section (5.2), being functions of the reference coordinate system and permits interpretation as the reference spheroid being centred at the centre of mass of the existent earth. Thus, proceeding on the same lines as adopted in the derivation of equation (5.41),

$$N_P = \frac{2\phi_{e_P}}{\gamma_m} - 5\frac{W_o - U_o}{\gamma_m} + \frac{R_m}{4\pi\gamma_m} \int_0^{\sigma=4\pi} -\left(\frac{R_m}{R}\right) \Delta g_o \, d\sigma +$$

$$+ \frac{R_m}{4\pi\gamma_m} \sum_{n=2}^{\infty} \int_0^{\sigma=4\pi} \left\{ \frac{2n+1}{n-1} \left(\frac{R_m}{R}\right)^{n+1} \Delta g_o p_n(\cos\psi)\, d\sigma + \frac{4(W_o - U_o)}{r} d\sigma \right\}$$

$$\text{.......(13.66).}$$

The first integral on the right hand side of the above equation, on evaluation in the limit, as $R \rightarrow R_m$, gives

$$- \frac{R_m}{\gamma_m} \overline{\Delta g_o} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13.67),$$

where $\overline{\Delta g_o}$ is the mean value of Δg_o over the globe. The new term in the second integral can be treated independently of the rest of this integral, and in the case of the spherical approximation, reduces to

$$\frac{R_m}{4\pi\gamma_m} \int_0^{\pi} \int_0^{2\pi} \frac{4(W_o - U_o)}{2R_m \sin\frac{1}{2}\psi} \sin\psi \, d\psi \, d\alpha$$

$$= 4 \frac{W_o - U_o}{\gamma_m} \quad \ldots\ldots\ldots\ldots\ldots\ldots (13.68).$$

Thus, the fundamental equation for the determination of the separation of the geoid and the spheroid becomes

$$N_P = \frac{2\phi_{e_P}}{\gamma_m} - \frac{W_o - U_o}{\gamma_m} - \frac{R_m}{\gamma_m} \overline{\Delta g_o} + \frac{R_m}{4\pi\gamma_m} \int_0^{\sigma = 4\pi} f(\psi) \, \Delta g_o \, d\sigma \quad \ldots\ldots (13.69).$$

The final solution (N_{P_F}) can be expressed in the form

$$N_{P_F} = N_{P_C} + N_P \quad \ldots\ldots\ldots\ldots\ldots (13.70),$$

where N_P is the quantity defined by equations (5.60) and (5.61). In the case where the calculations are made with respect to the international gravity formula, the corrective term N_{P_C} is made up of two constituents, one of which is a zero order term (N_{P_o}) and has no variation with position. The second term (N_{P_P}) is position dependent, arising from correction terms to normal gravity as computed on the international gravity formula system to obtain the better representation afforded by the I.A.U. system. N_{P_o} is given by

$$N_{P_o} = -\frac{W_o - U_o}{\gamma_m} - \frac{R_m}{\gamma_m}\left\{\overline{\Delta g_o} + dg_P - \overline{d\gamma}\right\} \quad \ldots(13.71),$$

where dg_P is the correction to the Potsdam datum, $\overline{d\gamma}$ is the mean value of $d\gamma$, defined by equations (13.47) and (13.48) for all possible changes, taken on a world wide basis and $\overline{\Delta g_o}$ has the same definition as before. The second term N_{P_P} is given by

$$N_{P_P} = \frac{R_m}{4\pi\gamma_m}\int_0^{\sigma = 4\pi} (dg_P - d\gamma)\, f(\psi)\, d\sigma \quad \ldots\ldots(13.72).$$

The value of U_o used in equation (13.71) will be that for the reference spheroid on the I.A.U. system, given by equation (13.52).

Thus no determination of the geoid spheroid separation can be considered absolute till a value has been established for the potential of the geoid. In the alternative, it would, at a first glance, appear that the surface mapped is one having the same potential as that assigned to the reference spheroid. This, however, is not correct as the interpretation of the gravity anomaly becomes inconsistent with the definition set out in equation (3.17) when practical reductions, using conventional elevations are attempted. Thus, unless a value is assigned for the potential of the existent geoid, no absolute determinations are possible for the separation of the geoid and the spheroid.

It is also interesting to study the effect of introducing the concept of a geoid potential which is not arbitrarily forced to be equal to that of the spheroid of reference, on the height anomaly (h_D) at the telluroid. Equation (13.53) is general and hence the conversion of equation (4.21) to (4.22), in the light of equations (13.58) to (13.69), will become

$$h_{D_P} = -\frac{1}{\gamma_m}\left[W_o - U_o + R_m\overline{\Delta g}\right] + \frac{1}{2\pi\gamma}\iint_R \left[\left\{\frac{1}{\gamma}\left|\frac{\partial\gamma}{\partial h}\right|\frac{\cos\beta}{r} + \underline{\nabla}\cdot\underline{N}\ \frac{1}{r}\right\}V_D + \left\{\Delta g - \gamma(\xi\tan\beta_1 + \eta\tan\beta_2)\right\}\frac{\cos\beta}{r}\right] dR \quad \text{.................(13.70),}$$

where $\overline{\Delta g}$ is the world wide mean of the modified free air anomaly used in the evaluation of the integral. This modified free air anomaly is the difference between observed gravity at the earth's surface and normal gravity on the equipotential surface of the reference system which has the same potential that at the relevant point on the earth's surface. Also see section (14.6). The use of equation (11.10) gives

$$h_{D_P} = -\frac{1}{\gamma_m}\left[W_o - U_o + R_m\overline{\Delta g}\right] + N_F + N_C \quad ,$$

where N_F is obtained using the modified free air anomalies in Stokes' integral.

$$N - h_{D_P} = \frac{2\phi_{e_P}}{\gamma_m} - \frac{R_m}{\gamma_m}(\overline{\Delta g_o} - \overline{\Delta g}) + N_{co} - N_c - N_m \qquad ....(13.71),$$

where N_{co} is the contribution to N by the effect of the differential topographical correction on a world wide basis using Stokes' integral and N_c is as defined in section (11.3). The term N_m is the effect of using the corrections to the free air anomaly in obtaining the modified free air anomaly, in Stokes' integral. N_c is a small term and hence the other terms constitute the major contribution which could amount to hundreds of metres. This may appear ludicrous as the natural expectation is that $N = h_D$ if the point is on the geoid. But this, in fact, is not so if the definitions of each of the two quantities is carefully considered when it is seen that they do not represent the same separation. N is the actual separation between the geoid and the reference spheroid, where the potential of the spheroid on the reference system is not equal to that on the geoid on the existent system. The reference spheroid, in this context, is the figure which best fits the geoid, the governing condition being that it has the property of equal volume.

h_D, on the other hand, is the separation between the surface geop and the spherop which has the same potential as the surface geop. If the term " defined " refers to a location which, by virtue of being located on the earth's surface or on a physical reality like the geoid, is accessible, N is a linear displacement between a defined equipotential and the spheroid of reference. h_D is the displacement between a defined point and a spherop whose position in space is fixed by the postulated condition that its potential on the reference system is equal in magnitude to that of the defined point on the existent system.

13.6 Conclusions.

The changes in the reference spheroid used for the computation of normal gravity, on conversion into effects on the deflections of the vertical by using equations (11.21) and (11.22) have exactly the same magnitude as those obtained in the case where the changes are obtained by direct geometrical considerations, set out in equations (13.26) and (13.39). If it is assumed that the I.A.U. spheroid is one which best fits the geoid according to available astro-geodetic, gravimetric and satellite data, it becomes necessary to abandon the use of the international gravity formula and the associated international spheroid for the computation of normal gravity in connection with geodetic work.

The apparent adequacy of the latter was due to the approximate internal consistency within the adopted model. While this does not introduce serious errors in the values of gravity anomalies in non-geodetic work, due to errors in the parameters being approximately compensated for by the error in the Potsdam datum, its effect on the separation vector in physical geodesy is not negligible, as shown in table (13.4), and hence must be considered for any valid determination.

A consistent reference model is that afforded by the spheroid and value of the constant kM adopted by the International Astronomical Union. When such a spheroid is combined with the correction necessary for the Potsdam datum, it provides a reference system not markedly different from that afforded by the international gravity formula, the maximum discrepancies being of the order of ± 7 mgal.

All computations for the separation of a physical surface, such as the equipotential surface of the earth's gravitational field corresponding to mean sea level, with a definite but unknown potential, from a reference surface to which a potential, by implication, has been assigned, must take into account a zero order term arising from two causes :-

(i) The world wide summation of anomalies on such a surface are not zero ;

(ii) W_o is not equal to U_o.

The evaluation of this zero order term is a pre-requisite for any mapping of the geoid from the spheroid of reference.

The height anomaly (h_D) and its valid first approximation, the Stokes´ integral, excluding the zero order term, are essentially different in character to the geoid - spheroid separation (N) as the former are defined in terms of the separation between two surfaces defined in terms of potential while the latter is the linear separation between two surfaces, one of which is known and the other unknown, which are fixed in a three dimensional linear sense in space, on an earth bound coordinate system.

14. THE RESULTS.

14.1 Introduction.

The results obtained by the use of programs described in section (12) have been expressed in the form of contour maps embodied in figs (14.1) to (14.11), compiled by interpolation from calculations on a grid of points situated at the corners of $1^{\circ} \times 1^{\circ}$ squares. All computations were carried out on the University of N.S.W's I.B.M. 360/50 computer. A pre-requisite for all computations was the institution of checks on the accuracy of all routines used. These checks were carried out in two ways :-

(i) The comparison of the computed values of the Stokes and Vening Meinesz functions against accepted sets of tables. The former were compared with that prepared by the U.S. Coast and Geodetic Survey (Lambert and Darling, 1936, 114 et seq) while the latter were compared with the set prepared by Sollins (1947, 293 et seq).

(ii) The general meshing of any given series of programs to ensure that no areas were excluded in the calculation, was checked by computing differential corrections which were known to have definite values on world wide summation. Two such checks were used. The first was based on the fact that changes in the values of gravity anomalies consequent to a change in the dimensions of the reference spheroid should not produce any change in the component of the deflection in the prime vertical. This property was checked for both the conversion from the international gravity formula to the I.A.U. system, shown in tables (13.4) and (13.5) as well as for the purely geometrical change of the dimensions of the international spheroid to the I.A.U. spheroid. The latter comparisons are shown in tables (13.6) and (13.7). In the former case, as can be seen from table (13.5), the error in the value of the correction to η was of the order of $\pm$ 0.002 sec or less. In the second case, the error in this same correction was $\pm$ 0.04 sec or less. This is due to the fact that the magnitude of the anomalies used to compute the second effect, as can be seen from table (13.2), are of the order of - 75 mgal, while those used in the first case are of the order $\pm$ 5 mgal.

A further check on the accuracy of the formulae was afforded by the computation of the effect described in table (13.6). This had the effect of verifying the accuracy of latitude dependent exclusions. It is therefore estimated that the effect of exclusion and other programming discrepancies in the determination of the free air geoid are less than 2 cm. in height and 0.02 sec in each direction. The effect of these same errors on the determination of the indirect effect arising from a consideration of the differential topographical corrections is expected to be about twice the earlier figures.

14.2 The free air geoid.

The free air geoid has been calculated on both the international spheroid, using the international gravity formula on the Potsdam system as well as using the parameters accepted by the I.A.U. The first set of results are shown in figs (14.1) to (14.3) and the latter set in figs (14.4) and (14.5). There is no map for η on the second system as it is exactly the same as fig (14.3), changes in the reference figure having no effect on the component of the deflection in the prime vertical.

The contribution of the various programs to the relevant quantities on the international spheroid for three randomly chosen points are given in table (14.1). The inner zone was computed from a consideration of equations (12.42), (12.47) and (12.48), the value of r_0 being set equal to the value defined by equation (6.63). It is felt that this procedure is unlikely to be adequate in most cases, but in view of more complete gravity coverage not being available, at the 148 points at which computations were carried out, no alternative was available. The effect of the lack of a closer sampling of the near gravity field is not expected to materially affect the accuracy of the value established for N_F. In the case of deflections, even in cases where the four nearest tenth degree square positions had been sampled, errors of up to $\pm\frac{1}{2}$ sec could be expected. The inner zone estimates of deflection components could have errors

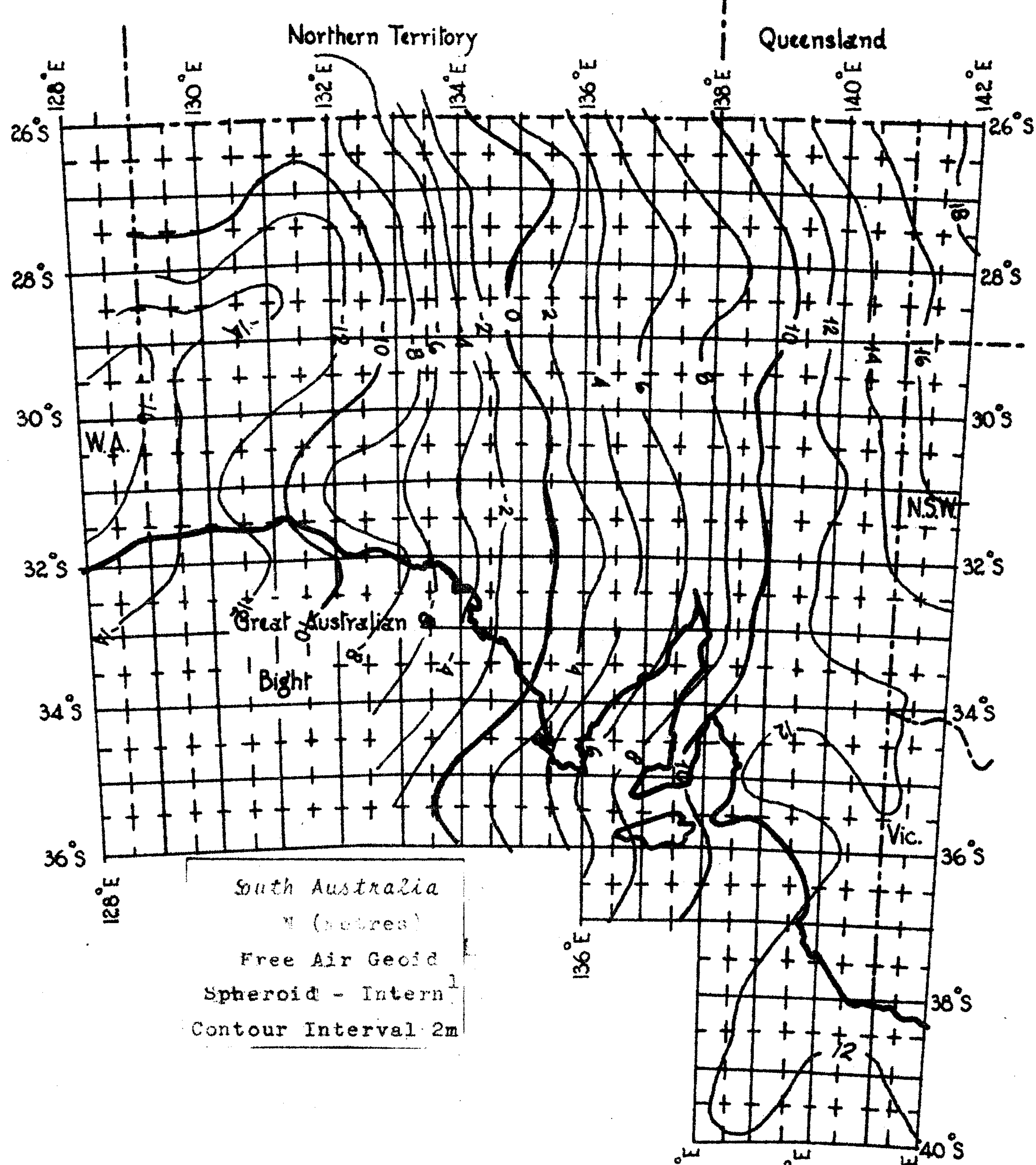

Northern Territory
Queensland
W.A.
N.S.W.
Vic.
Great Australian Bight
128°E
130°E
132°E
134°E
136°E
138°E
140°E
142°E
26°S
28°S
30°S
32°S
34°S
36°S
38°S
40°S
South Australia
Free Air Geoid
Spheroid - Intern[1]
Contour Interval 2m

FIG. 14. 1

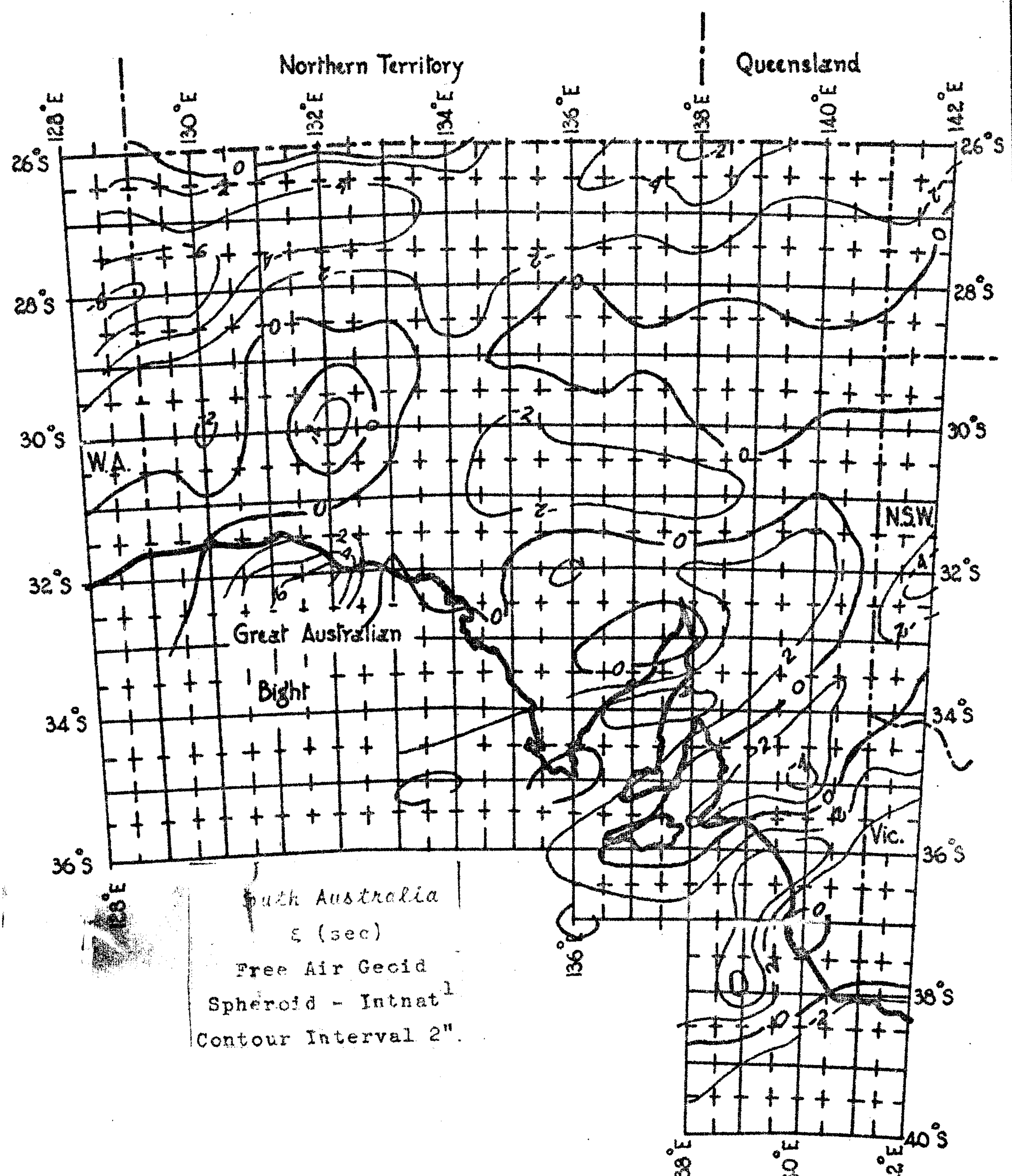
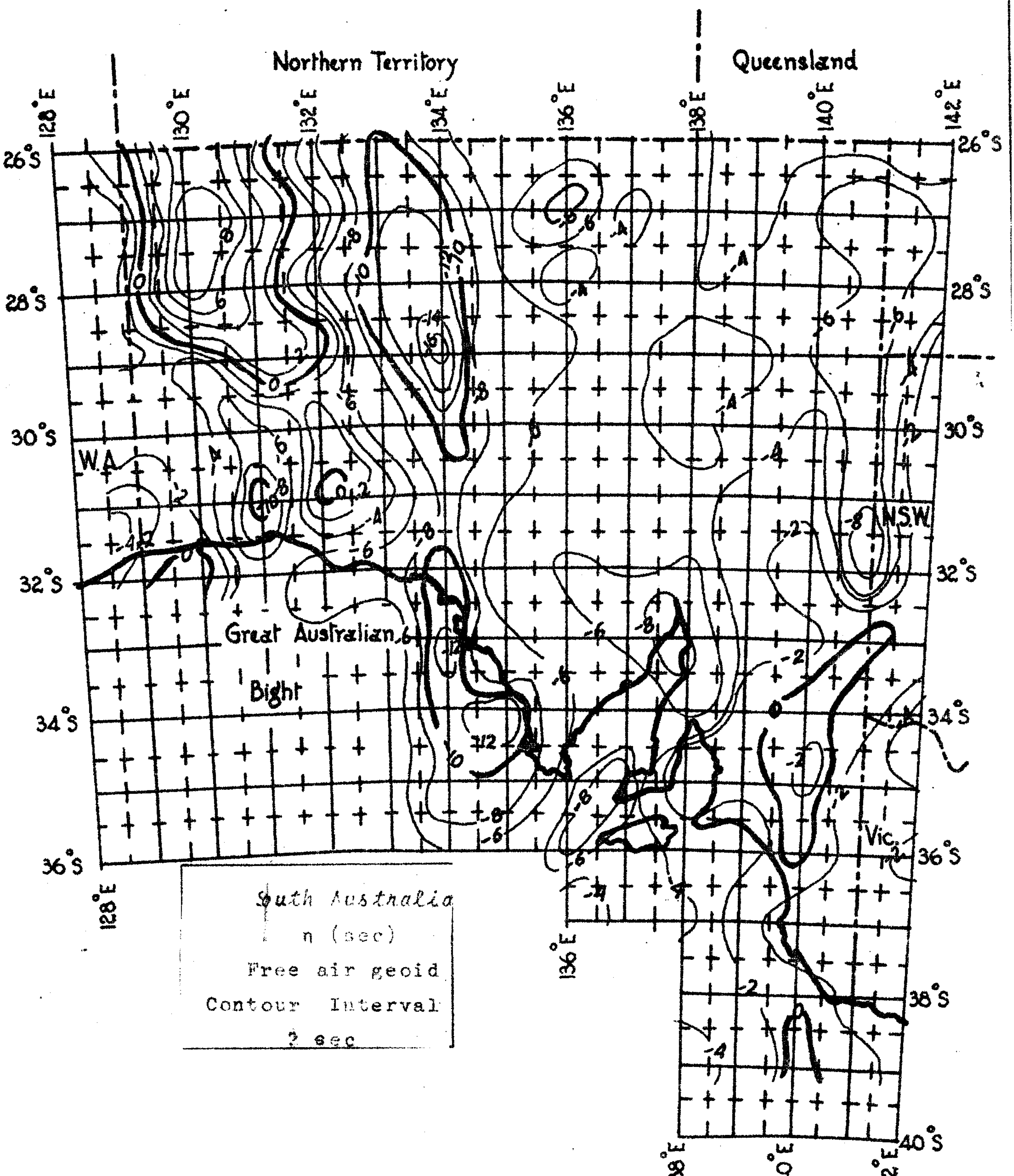

Northern Territory
Queensland
W.A.
N.S.W.
Vic.
Great Australian
Bight
South Australia
ξ (sec)
Free Air Geoid
Spheroid - Intnat[1]
Contour Interval 2".
128°E
130°E
132°E
134°E
136°E
138°E
140°E
142°E
26°S
28°S
30°S
32°S
34°S
36°S
38°S
40°S

FIG. 14.2



Northern Territory
Queensland
W.A.
N.S.W.
Vic.
Great Australian
Bight
South Australia
n (sec)
Free air geoid
Contour Interval
2 sec

FIG. 14.3

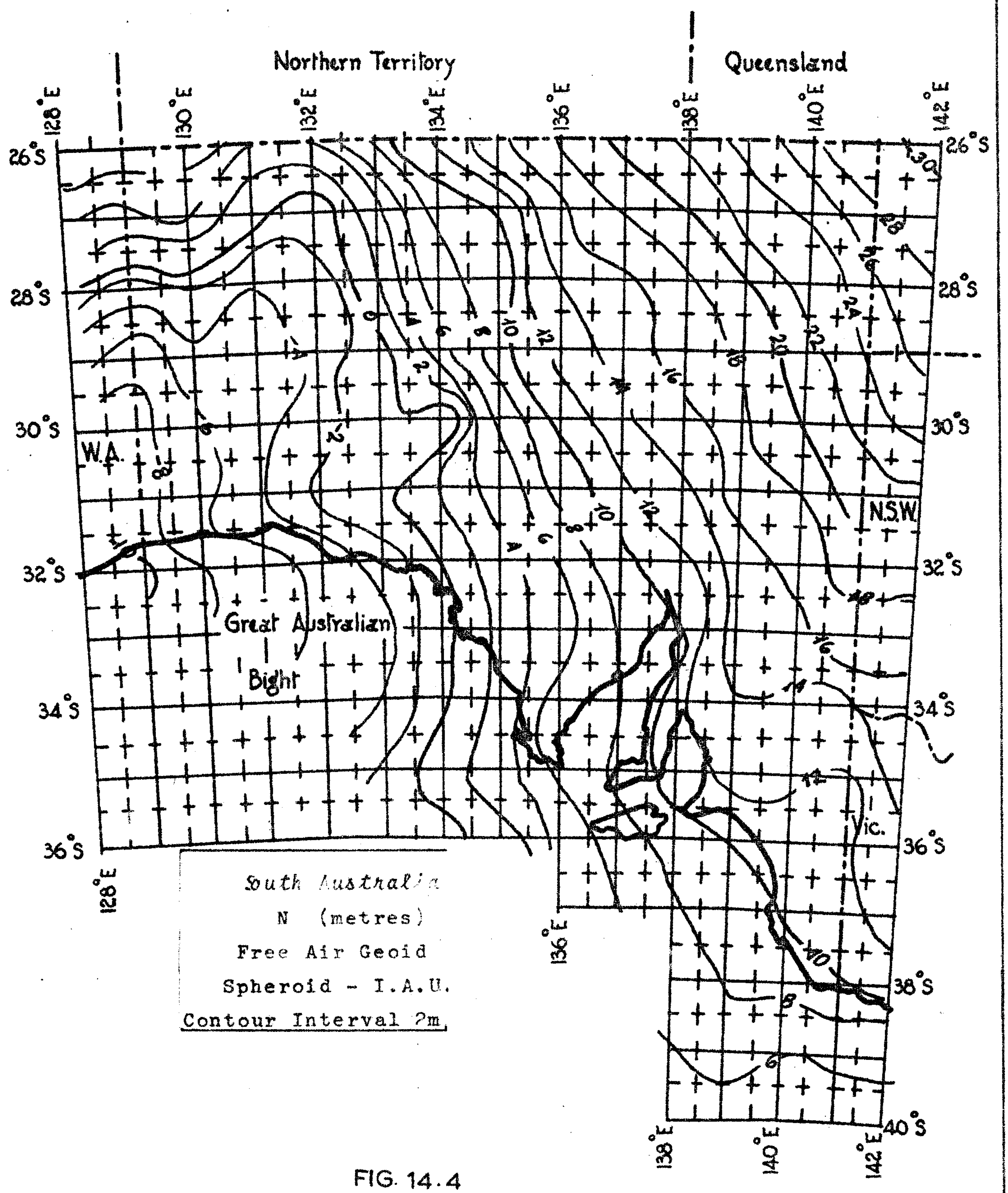

Northern Territory
Queensland
128°E
130°E
132°E
134°E
136°E
138°E
140°E
142°E
26°S
28°S
30°S
32°S
34°S
36°S
38°S
40°S
W.A.
N.S.W.
Vic.
Great Australian
Bight
South Australia
N (metres)
Free Air Geoid
Spheroid - I.A.U.
Contour Interval 2m

FIG. 14.4

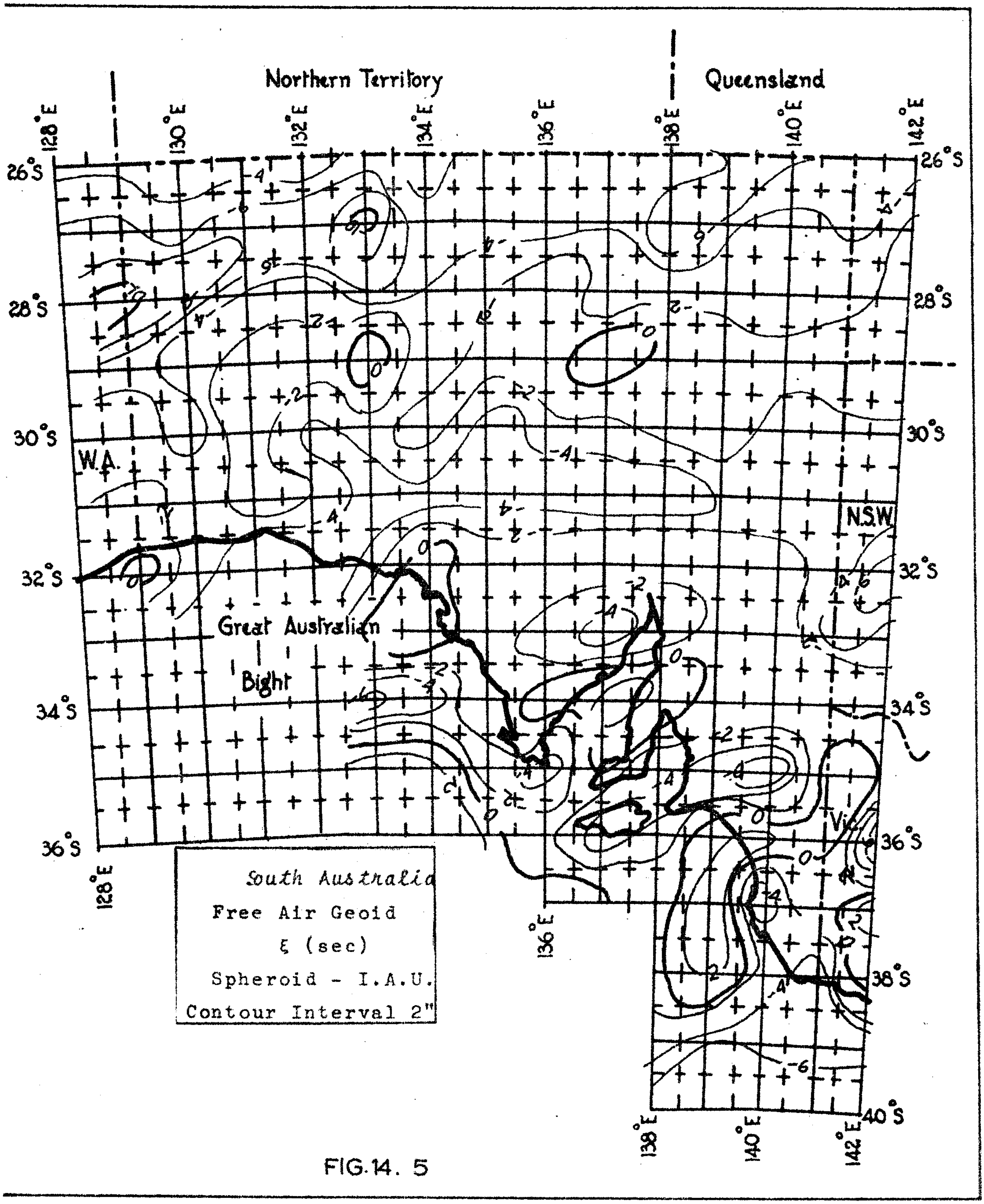

Northern Territory
Queensland
128°E
130°E
132°E
134°E
136°E
138°E
140°E
142°E
26°S
28°S
30°S
32°S
34°S
36°S
38°S
40°S
W.A.
N.S.W.
Vic.
Great Australian
Bight
South Australia
Free Air Geoid
ξ (sec)
Spheroid - I.A.U.
Contour Interval 2"

FIG.14. 5

Latitude	$-26^{\circ}\ 00'$	$-28^{\circ}\ 00'$	$-35^{\circ}\ 00'$
Longitude	$131^{\circ}\ 00'$	$138^{\circ}\ 00'$	$142^{\circ}\ 00'$
Program	Contributions to N (metres)		
Inner Zone	- 0.59	- 0.31	- 0.01
Stokin	- 5.73	- 2.30	- 0.93
Stokne	- 10.38	- 6.05	4.48
Stokmd	5.68	9.40	4.38
Stokut	2.58	6.58	6.91
Totals	- 8.44	7.32	14.83
Program	Contributions to ξ (sec)		
Inner Zone	0.395	- 0.078	0.237
Stokin	4.420	1.494	1.662
Stokne	- 3.433	- 0.265	1.183
Stokmd	- 1.170	- 1.331	- 1.288
Stokut	0.075	- 0.062	0.336
Totals	0.287	- 0.242	2.130
Program	Contributions to η (sec)		
Inner Zone	0.176	0.508	- 0.285
Stokin	4.974	0.484	- 0.127
Stokne	1.687	- 1.563	- 0.265
Stokmd	- 1.353	- 1.871	- 2.042
Stokut	- 1.859	- 1.559	- 1.252
Totals	3.625	- 4.001	- 3.971

TABLE (14.1)

Contributions of the various zones to the free air geoid.

as large as 2 - 3 sec. in cases where the field representation for the inner zones is afforded after extension. Thus the values of the deflections of the vertical can be expected to be reliable only in those regions in which adequate gravity data is available, as shown in fig (9.4). This matter is further discussed in section (14.6).

As can be seen from table (14.1), less than one quarter of the total contribution to any effect is dependent on the Kaula set of data, which is the only part of the data used not based solely on gravimetry. This estimate is not considered to be in any way conclusive as regards the contribution of the outer zone to N_F. The figure, in the case of deflections of the vertical, is expected to be a fair measure of the contribution of the outer zone in view of the nature of the Vening Meinesz integral.

The estimates of the error in the final values computed for the free air geoid depend on the location of the computation point with reference the the gravity data available. In the case of the determination of N, it is estimated that the error in N due to errors in the values adopted for the representation of the gravity field vary from $\pm$ 35 cm to $\pm$ 60 cm. The errors in the deflections are expected to lie in the range $\pm$ 0.5 sec to $\pm$ 3 sec. , the major contributor to the error being the uncertainties in the values used to represent the close field. Also see section (14.7).

The general characteristics of the free air geoid agree with those obtained from the dynamic application of satellite geodesy. Both free air geoids show a tendency towards a north-easterly rise with a range of values between - 10 metres and + 30 metres (e.g., Kaula, 1966a, 30). The component of the deflection of the vertical in the meridian tends to be small, while that in the prime vertical is negative in character. The comparison of the free air geoid with the Australian geodetic control network is dealt with in section (14.6).

14.3 The indirect effect for the free air geoid.

The programs used in the computation of the indirect effect are described in section (12.6). The effect comprises a Stokesian term, in which the differential topographical effect (Δg_{co}) is used in lieu of the gravity anomaly, and the term which evaluates the potential at the geoid due to the matter exterior to it. The model postulated for the topography exterior to the geoid, is that defined by the modified Hunter formula

$$\rho = 2.77 - \frac{h}{21} \qquad \text{for } h < 2.5 \text{ km.}$$

$$\rho = 2.67 \qquad \text{for } h > 2.5 \text{ km} \qquad \ldots\ldots(14.1),$$

where h is the elevation of the topography in km. This model was used both in the calculation of the differential topographical effect as well as in computing the contribution due to the potential of matter exterior to the geoid.

The computations for two points selected at

Latitude	- 26° 00'			- 35° 00'		
Longitude	131° 00'			142° 00'		
Contribution of the exterior potential term						
Program	N (met)	ξ (sec)	η (sec)	N (met)	ξ (sec)	η (sec)
Inner Zone	2.03	-	-	0.24	-	-
Innear	21.91	- 1.844	- 4.835	2.33	0.283	- 0.206
Inmid	96.55	- 3.220	7.135	58.30	-3.585	- 1.661
Inout	487.30	1.466	2.915	493.20	4.179	3.021
Total	607.79	- 3.598	5.215	554.07	0.877	1.154
Contribution of the Stokesian term						
Program	N (met)	ξ (sec)	η (sec)	N (met)	ξ (sec)	η (sec)
Inner Zone	- 1.93	- 1.230	0.213	- 0.22	- 0.172	- 0.316
Stokin	- 22.86	1.702	5.510	- 2.38	0.379	0.45[illegible]
Stokne	- 58.13	1.965	- 5.757	- 24.62	0.281	2.749
Stokmd	- 54.80	1.870	- 1.811	- 36.98	3.930	- 1.670
Stokut	330.30	- 9.793	3.254	236.86	- 17.271	4.422
Total	192.58	- 5.486	1.409	172.66	- 12.853	5.643
Total ind. effect	800.37	- 9.084	6.624	726.73	- 11.976	6.797

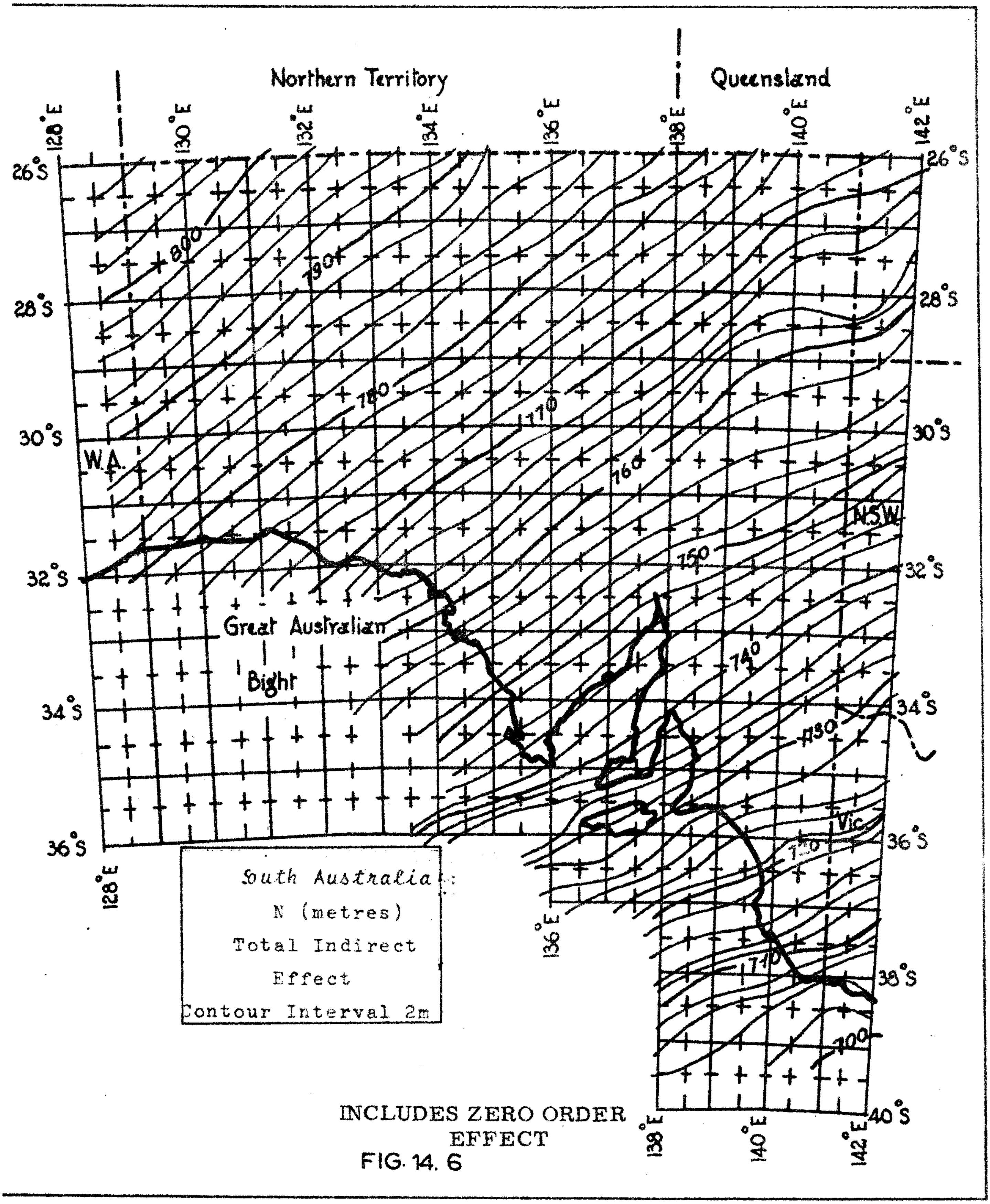
TABLE (14.2)

The contribution of the various zones to the total indirect effect for the free air geoid.

random are set out in table (14.2). The most interesting feature is the magnitude of the contributions of the outer zones to the separation of the geoid and spheroid in every case. This is to be expected in the case of the term arising from the potential of matter exterior to the geoid, as the contribution is always positive. In the case of the Stokesian contribution, the Stokes function admits both positive and negative values, while the sign of the correction Δg_{co} is always negative. As the value of Δg_{co} over ocean regions, which constitute approximately 70 per cent of the earth's surface, is zero, it is anticipated that the magnitude of this term varies through a wide range of values depending of the location of the computation point.

The effect of the closer zones in the region where $\psi < 20^{\circ}$, while individually large in the case of each effect, is small, on summation as the two effects tend to be of opposite magnitude in the case of both N and the deflections. In general, the major term in the indirect effect is due to the distant zones, the values given in table (14.2) being representative of the region. Contour maps showing the three components of the indirect effect are shown in figs (14.6) to (14.8). The indirect effect increases in a north westerly direction, as can be expected from the location of the Eurasian land mass with respect to South Australia. The size of the term arises from the fact that normal gravity is computed with respect to a reference system which best fits gravity as measured at the earth's surface and not at the geoid.

It should be noted that the equipotential surface mapped



INCLUDES ZERO ORDER EFFECT

FIG. 14. 6

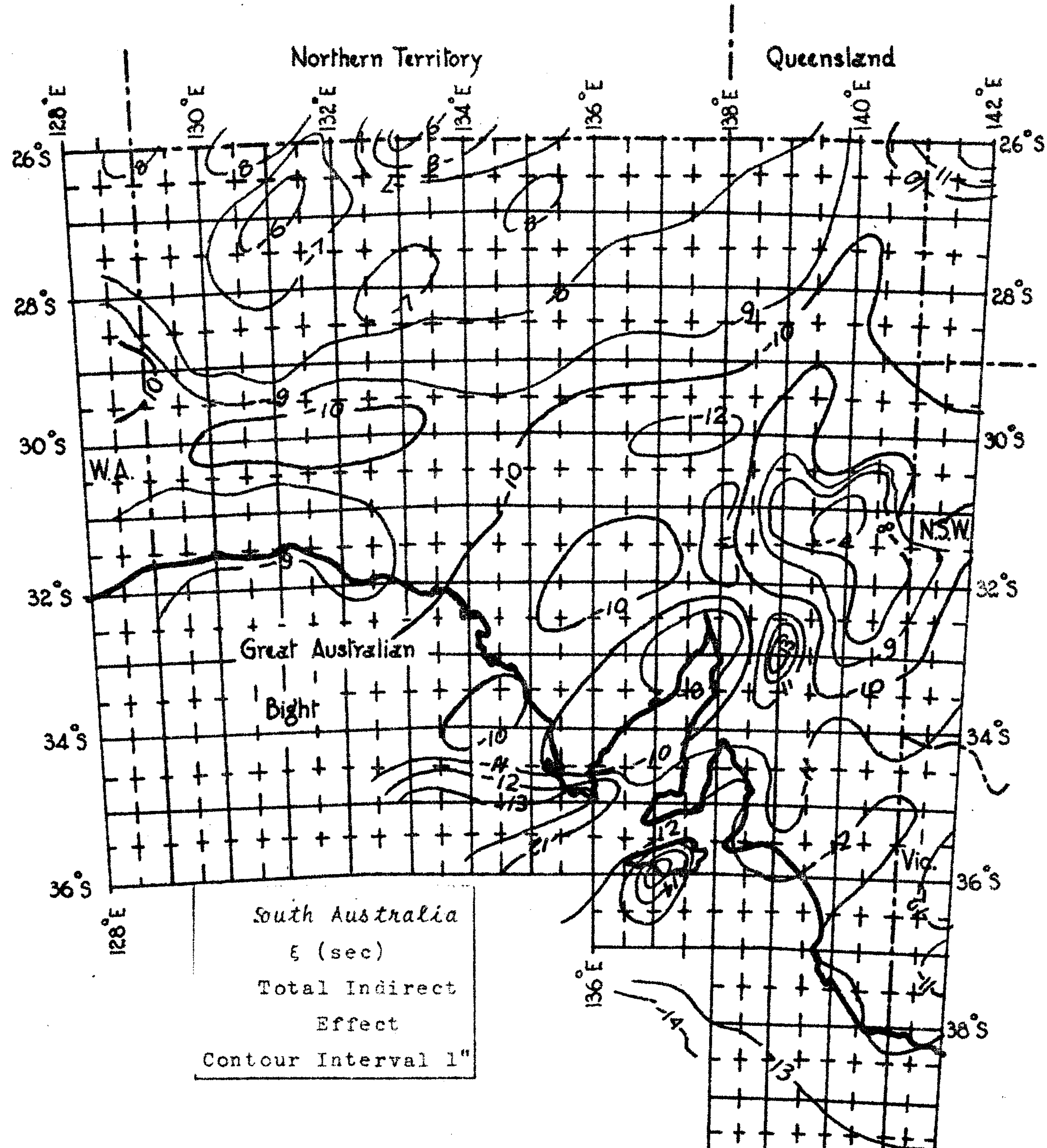
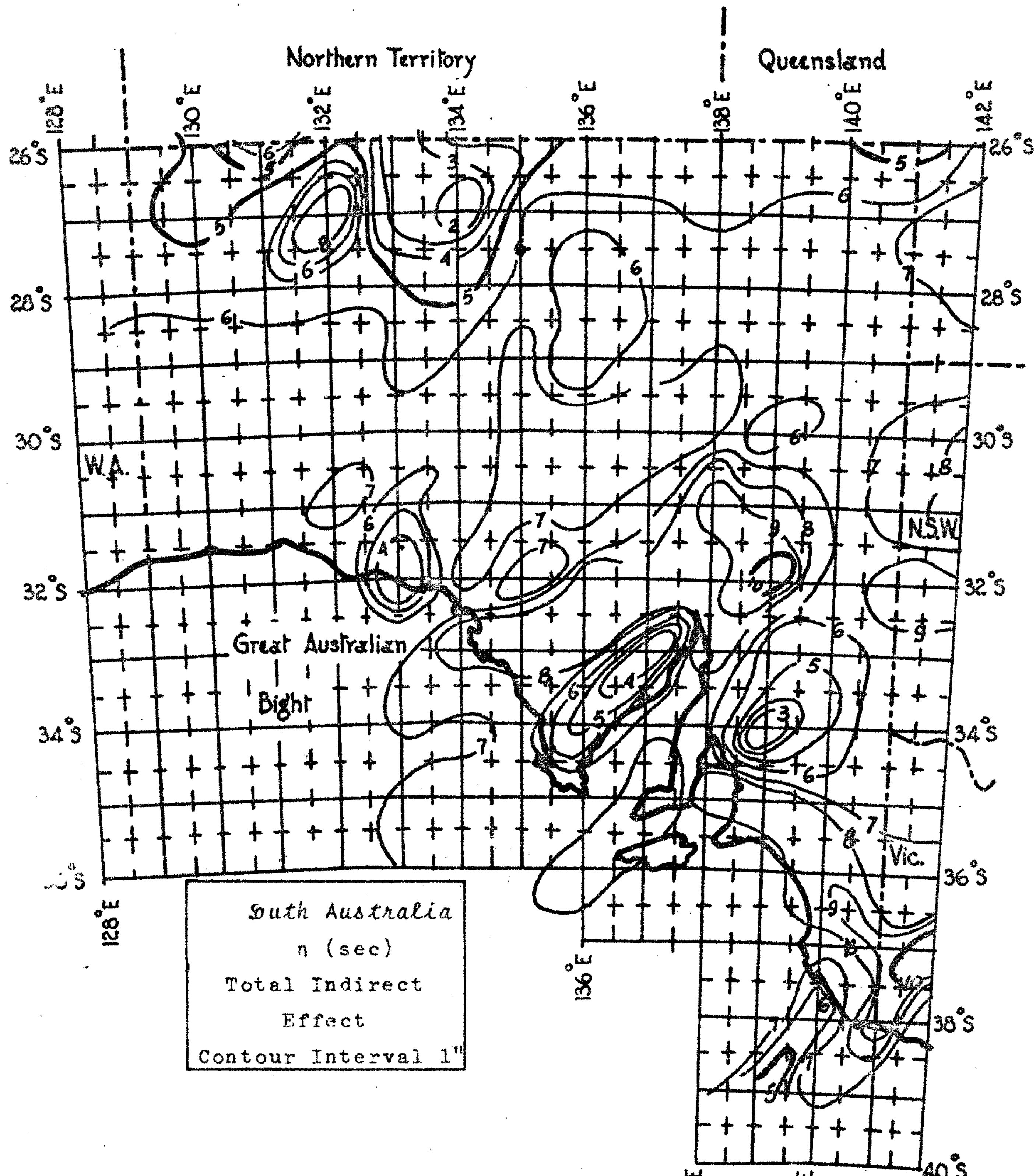

Northern Territory
Queensland
128°E
130°E
132°E
134°E
136°E
138°E
140°E
142°E
26°S
28°S
30°S
32°S
34°S
36°S
38°S
40°S
W.A.
N.S.W.
Vic.
Great Australian
Bight
South Australia
ξ (sec)
Total Indirect
Effect
Contour Interval 1"

FIG. 14.7


Northern Territory
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128°E
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138°E
140°E
142°E
26°S
28°S
30°S
32°S
34°S
36°S
38°S
40°S
W.A.
N.S.W.
Vic.
Great Australian
Bight
South Australia
η (sec)
Total Indirect
Effect
Contour Interval 1"

FIG. 14.8

on combining the results embodied in figs (14.4) and (14.6) is one which has the same potential as the reference spheroid, once the correction term for the zero order effect of the gravity anomaly has been taken into account. This surface lies consistently outside the reference spheroid, even in the case where the reference spheroid is the I.A.U. spheroid. This surface is not the geoid which has been defined as being referred to the I.A.U. spheroid which has the same volume as the geoid. If this assumption is accepted, it would imply that the potential of the geoid itself, being closer to the earth's centre of mass than the other equipotential, is greater than that of the reference spheroid, which is equal to that of the outer surface mapped. This seems to be in agreement with the general principle of an Airy type system of isostatic compensation.

Any determination of the separation of the geoid and the spheroid will therefore have to be preceded by a determination of the potential of the geoid.

14.4 Determination of the potential of the geoid.

If the geoid has the same volume as the spheroid of reference, which, in the case of the current investigation, is the I.A.U. Spheroid, the values of N must have a mean value of zero over the earth. From the discussions in sections (13.5) and (14.4), it can be assumed in the case of the geoid spheroid separation, for the I.A.U. spheroid,

(a) N is distributed over the earth with zero mean ;

(b) the inner zone effects ($\psi < 20^{\circ}$) for both the IN series of programs and the STOK series of programs using values of Δg_{co} as the anomaly, tend to be of equal magnitude and of opposite sign ;

(c) the effects computed from INOUT and STOKUT, in the case of the indirect effect, tend to be large positive quantities.

Thus, from a consideration of equation (13.81), the following relation is valid.

$$M\{N_i\} - \frac{W_o - U_o}{\gamma_m} - \frac{R_m}{\gamma_m}\overline{\Delta g_o} = 0 \quad \ldots(14.2),$$

where $M\{N_i\}$ is the mean value of the sum total of the outer zone effects computed using the programs INOUT and STOKUT, with Δg_{co} as the anomaly, on a world wide basis. Computations for the magnitude of this effect at 20° intervals over the earth's surface indicate that

$$M\{N_i\} = +\,653 \text{ metres} \quad \ldots\ldots\ldots\ldots(14.3).$$

Hence,

$$W_o = U_o + \gamma_m M\{N_i\} - R_m\overline{\Delta g_o} \quad \ldots\ldots(14.4).$$

To obtain the mean values which, for convenience of computation were at specified intervals of latitude and longitude, care had to be taken that the world wide means were reliable, as numerical means would bias the means towards polar values. Hence all means were computed using the relation

$$4 \pi \bar{f} = \frac{\pi^2 n^2}{180^2} \Sigma f \cos \phi \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (14.5)$$

where $\bar{f}$ is the mean value of the function f over the earth, the values of f representing $n^{o} \times n^{o}$ squares. The mean value of the free air anomaly obtained from the Kaula set, which in itself had zero mean, after correction to the I.A.U. spheroid was

$$\overline{\Delta g_F} = 1.1 \text{ mgal} \quad \ldots\ldots\ldots\ldots\ldots (14.6).$$

The mean value of the differential topographical correction over the earth is given by

$$\overline{\Delta g_{co}} = -49.5 \text{ mgal} \quad \ldots\ldots\ldots\ldots (14.7).$$

Thus

$$\overline{\Delta g_o} = \overline{\Delta g_F} + \overline{\Delta g_{co}} = -48.4 \text{ mgal} \ldots\ldots (14.8).$$

As the value of the potential on the I.A.U. spheroid is 6, 263, 680 kgal metres, the potential of the geoid is 6, 264, 628 kgal metres. Consequently, the correction N_{P_0}, defined by equation (13.71) is given by

$$N_{P_o} = -653 \text{ metres} \quad \ldots\ldots\ldots\ldots\ldots\ldots (14.9).$$

14.5 The geoid - spheroid separation.

The separation vector of the geoid - spheroid system, as represented by the conventional components N, and after allowing for the zero order effects are shown in figs (14.9) to (14.11) for the South Australian region. As can readily be seen, the separation of the non-regularised geoid

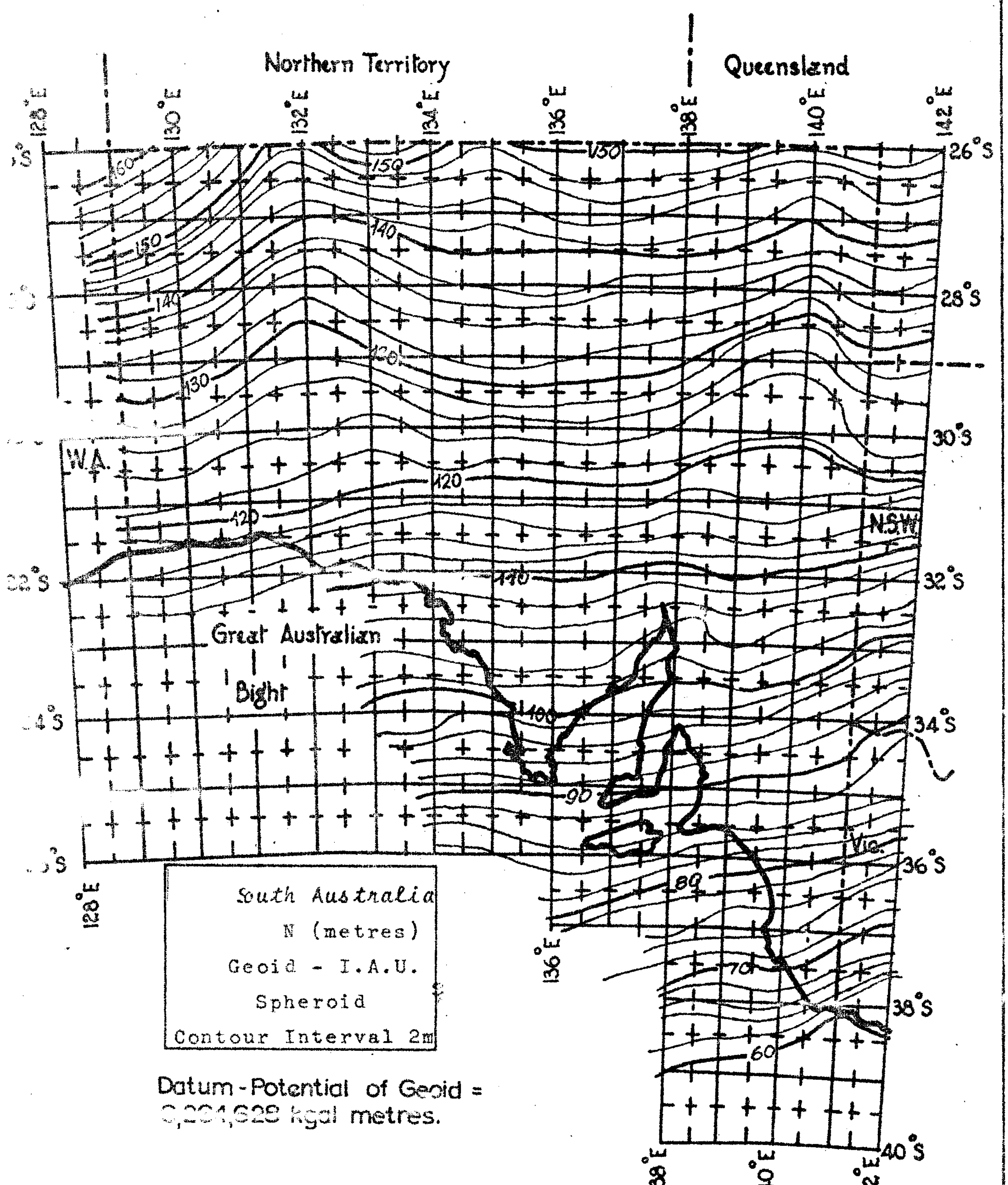

Northern Territory
Queensland
W.A.
N.S.W.
Vic.
Great Australian
Bight
128°E
130°E
132°E
134°E
136°E
138°E
140°E
142°E
26°S
28°S
30°S
32°S
34°S
36°S
38°S
40°S
160
150
140
130
120
110
100
90
80
70
60
South Australia
N (metres)
Geoid - I.A.U.
Spheroid
Contour Interval 2m
Datum - Potential of Geoid =
kgal metres.

FIG. 14.9

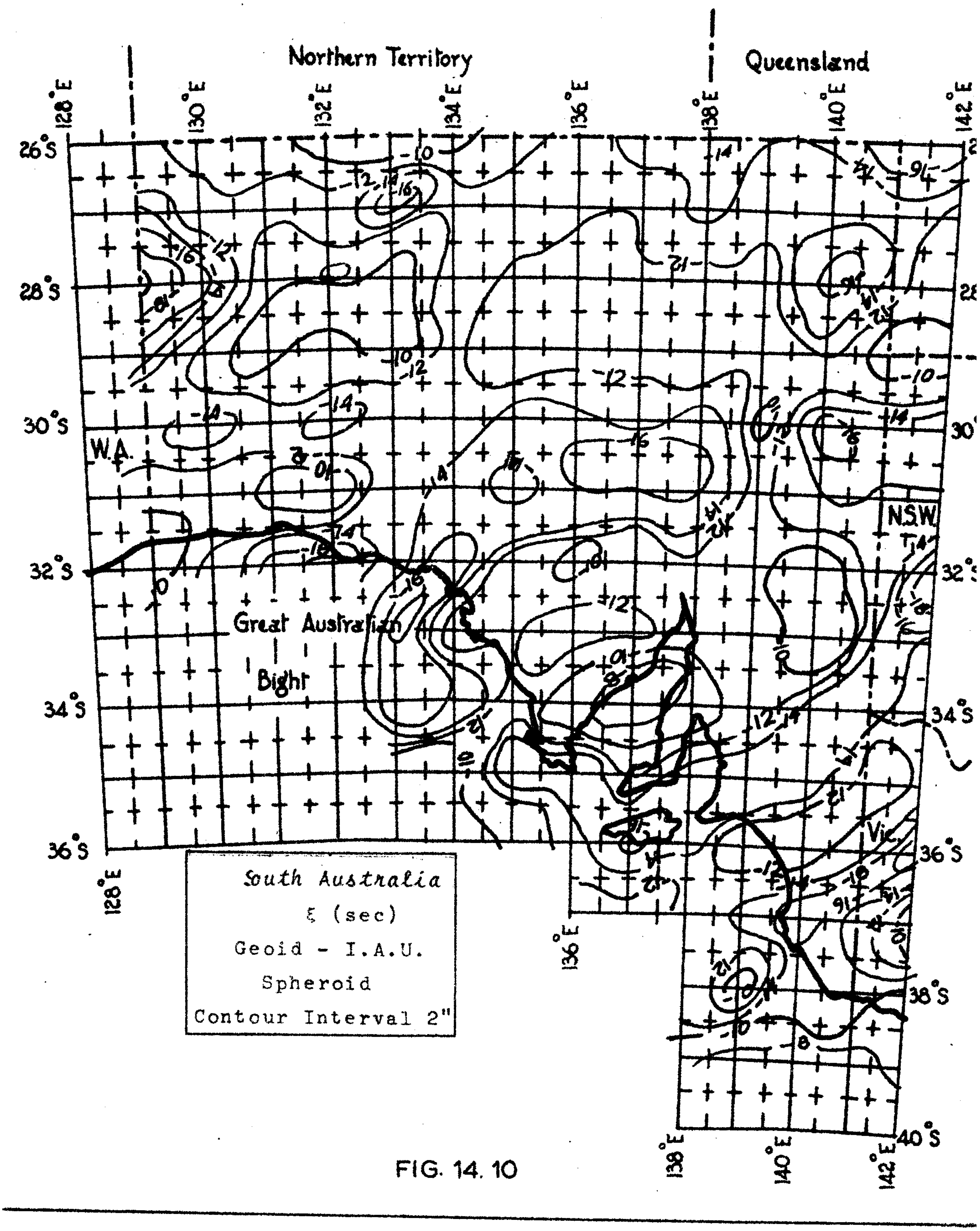
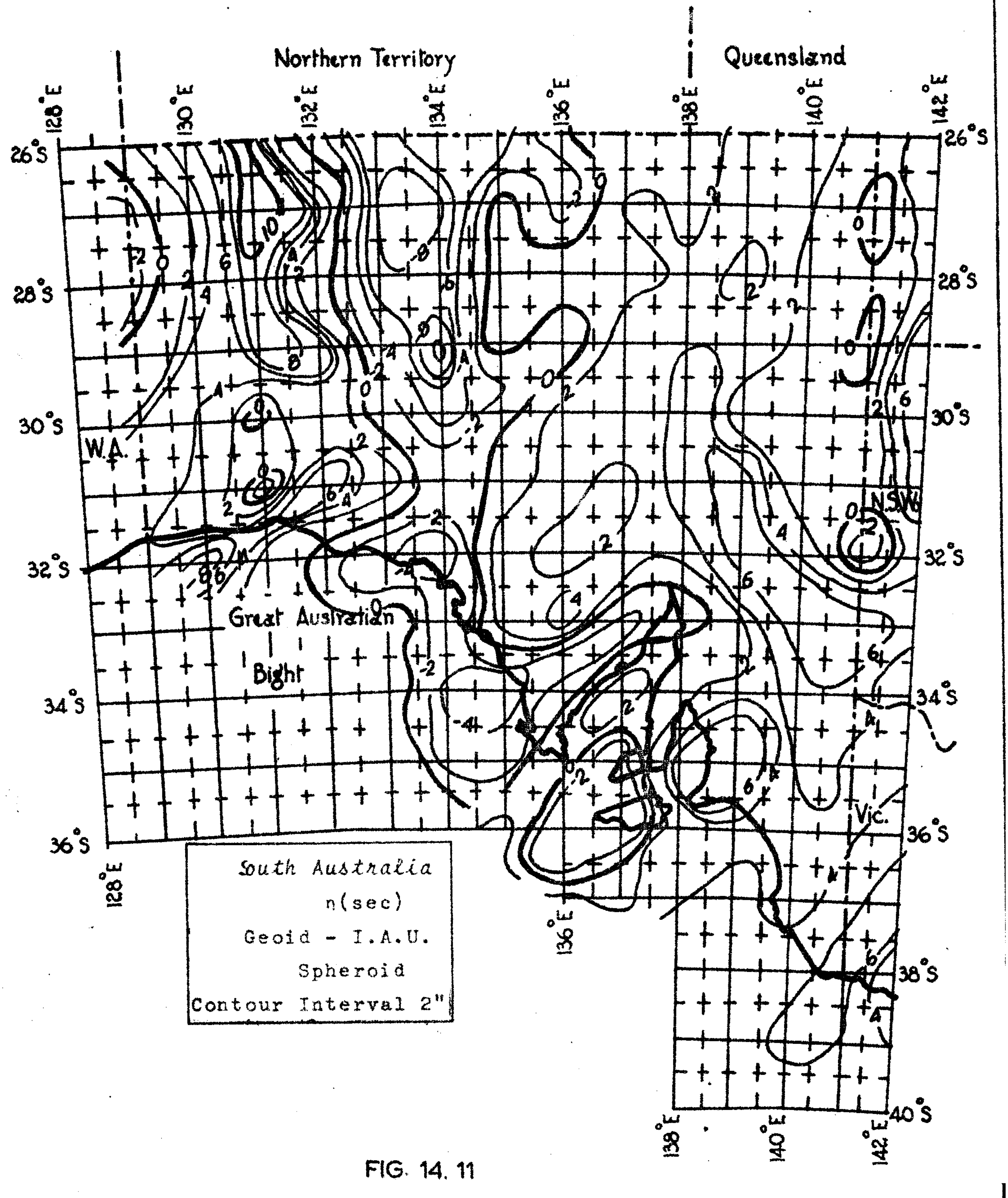

Northern Territory
Queensland
W.A.
N.S.W.
Vic.
Great Australian Bight
128°E
130°E
132°E
134°E
136°E
138°E
140°E
142°E
26°S
28°S
30°S
32°S
34°S
36°S
38°S
40°S
South Australia
ξ (sec)
Geoid - I.A.U.
Spheroid
Contour Interval 2"

FIG. 14. 10



Northern Territory
Queensland
W.A.
N.S.W.
Vic.
Great Australian Bight
South Australia
n(sec)
Geoid - I.A.U.
Spheroid
Contour Interval 2"

FIG. 14. 11

has very little resemblance to the free air geoid, which nevertheless contributes significantly to it, and has a considerable gradient in the north-south direction over South Australia. The indirect effect, and hence the external topography of the earth, appears to be a dominant feature contributing to the slope of the geoid and hence the north south direction of slope is not considered to be of any significance.

There is little doubt that the non-regularised geoid varies much more rapidly with position in relation to the reference spheroid than the free air geoid, having lows which are generally associated with ocean regions and highs in the neighbourhood of continental areas. Preliminary studies seem to indicate that the non-regularised geoid has variations which are up to twice as great as those of the free air geoid, though there is unlikely to be any correlation between the highs and lows of each system which is solely position dependent.

It should be borne in mind that all these deductions are based on the acceptance of the modified Hunter formula, specified by equation (14.1), as an adequate mathematical model for the topography exterior to the geoid. The geoid, as determined in figs (14.9) to (14.11) can be expected to have errors of $\pm$ 10 metres arising from errors in the assumed model and hence the results have significance only in relation to the model assumed for the topography exterior to the geoid. This figure is based on current estimates of the upper layers of the earth's crust and is dependent on errors in the crustal density not exceeding five per cent of the adopted values. The existence of apparent ambiguities in this model are discussed in section (15.4).

The separation N in fig (14.9) has been further qualified by the value adopted for the potential of the geoid. A further critical factor in assessing the absoluteness of the determination is the correctness of the assumption that the I.A.U. spheroid has the same volume as the geoid. This assumption is considered in further detail in section (15.4).

Apart from these sources of uncertainty, the error in the computed values of the non-regularised geoid, due to errors in the computed values of the indirect effect are of the order of ± 1.2 metres in the value of N and ± 0.3 sec in the value of each of the components of the deflection.

14.6 The free air geoid and the Australian control network.

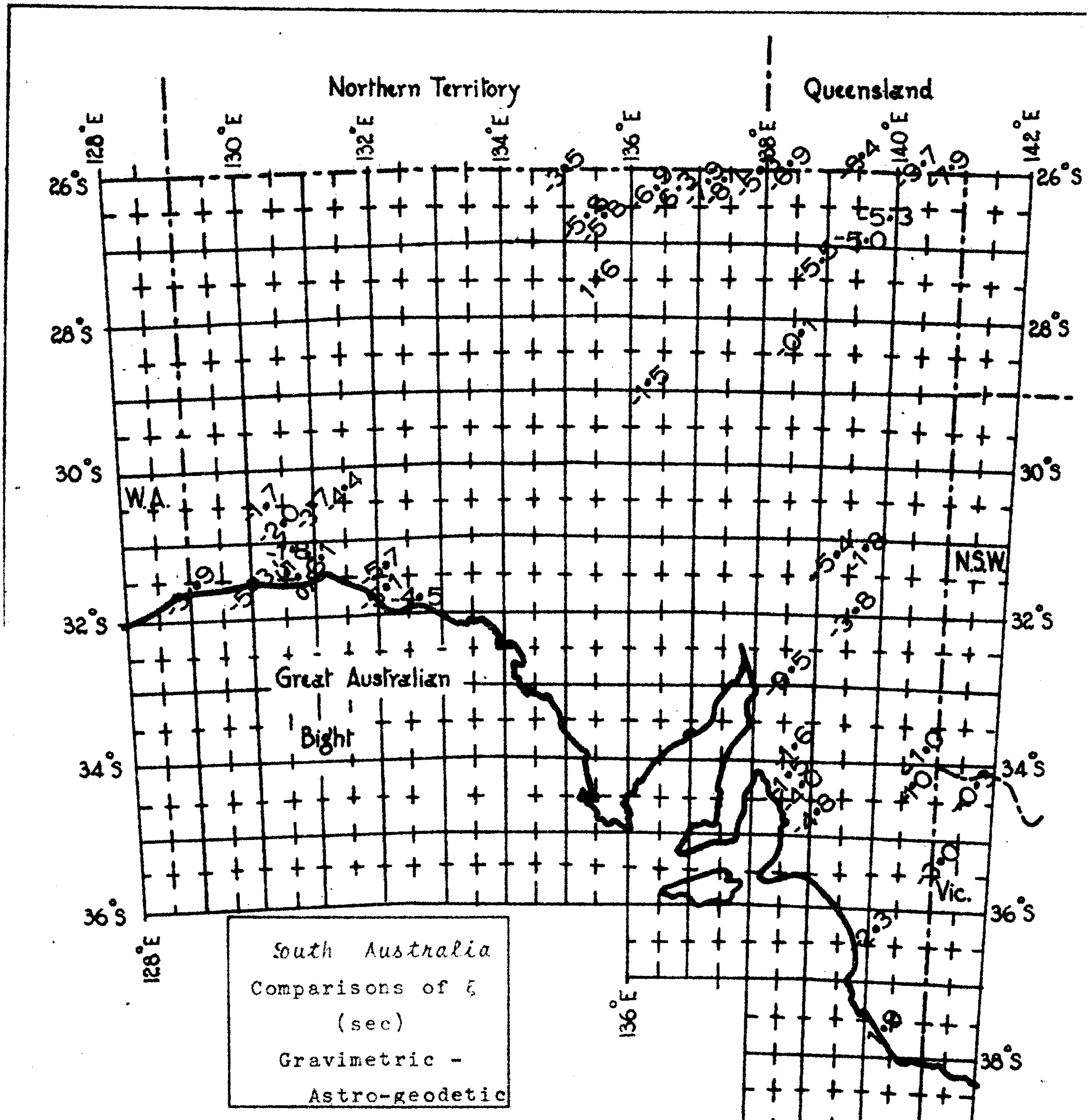
The adoption of the proposition that the free air geoid is a good first approximation to the telluroid requires re-examination in view of the potential of the reference spheroid being less than that of the existent geoid. This is developed in section (15.2)where it is shown that the telluroid, in addition to having a zero order term when the chosen spheroid is the I.A.U. spheroid, is also subject to slowly changing systematic effects which are probably slow enough to be considered as being uniform over continental extents. This should be borne in mind when considering the significance of the following comparisons. Any consistency in the comparisons will therefore be a reflection on the accuracy of estimation of the constancy in the rate of change of the systematic effect over the area considered.

Approximately 100 Laplace stations have been observed in South Australia, at which deflections have been determined astro-geodetically on the Australian National Spheroid (A.N.S.), which has the same dimensions as the I.A.U. spheroid, but is arbitrarily orientated in space, using a technique similar in nature to that set out in equation (10.17). The resulting values , after adjustment, of the astro-geodetic deflections of the vertical (ξ_A, η_A) had means, based on a continental coverage, of - 0.12 sec in the meridian and - 0.33 sec in the prime vertical (Bomford, 1967, 58). It should be noted that the sign convention adopted by the Division of National Mapping for prime vertical deflections is opposite in sign to that used in this investigation as , in the former case, longitude has been considered positive in the westerly direction.

It can be concluded that the A.N.S. has been orientated in space to fit the average slope, over Australia, of planes perpendicular to local verticals. In the absolute sense it it could

(a) be either systematically above or below a truly centred spheroid of the same dimensions ;

(b) be sloping systematically with respect to such a figure.



VALUES	REF. SPHEROID
Gravimetric	I.A.U
Astro Geodetic	A.N.S.

FIG. 14.12

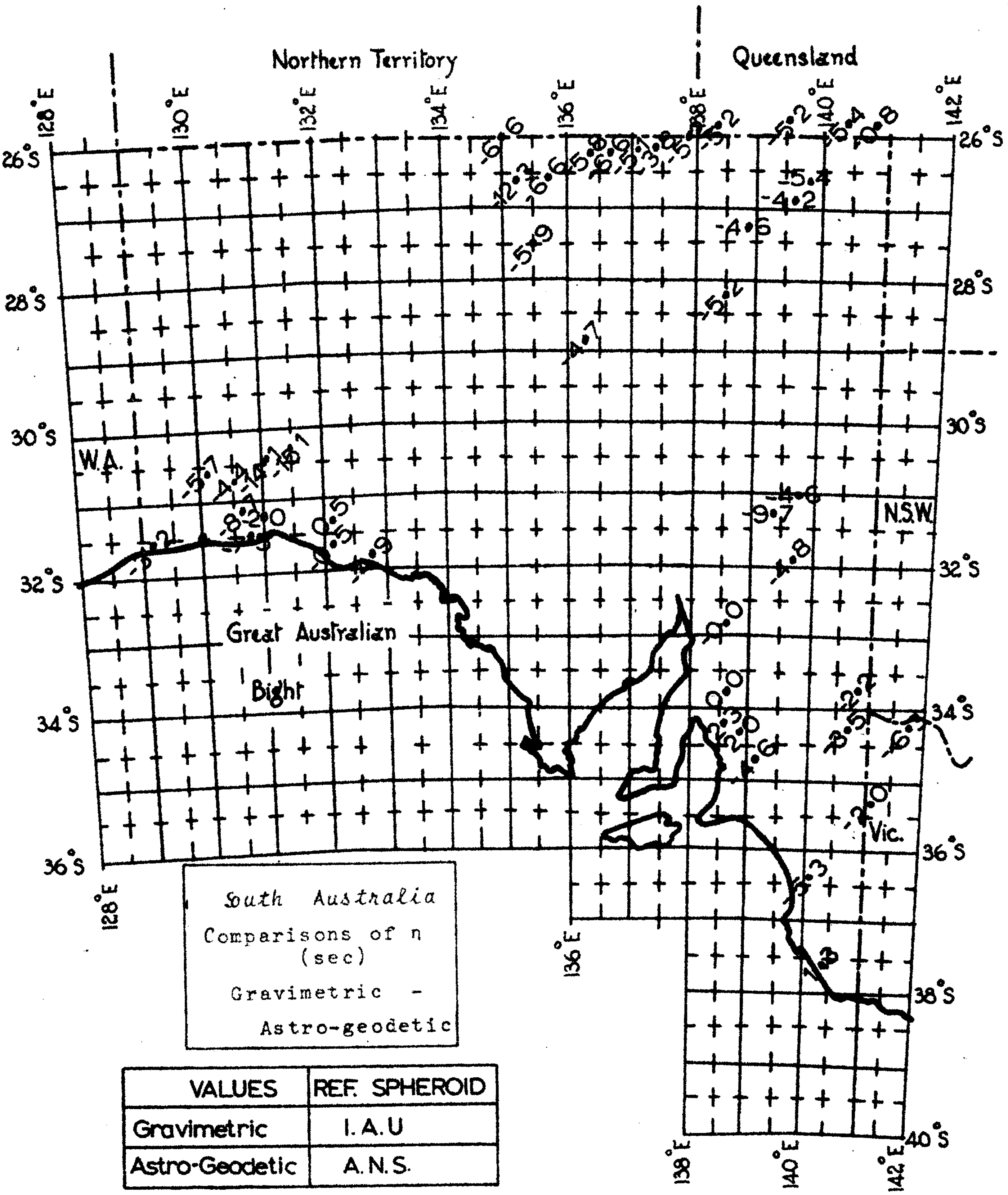

Northern Territory
Queensland
W.A.
N.S.W.
Vic.
Great Australian
Bight
128°E
130°E
132°E
134°E
136°E
138°E
140°E
142°E
26°S
28°S
30°S
32°S
34°S
36°S
38°S
40°S
South Australia
Comparisons of n (sec)
Gravimetric - Astro-geodetic
VALUES | REF. SPHEROID
Gravimetric | I. A. U
Astro-Geodetic | A. N. S.

FIG. 14.13

This is discussed in greater detail in section (15.3). In the case of South Australia, a large number of the available Laplace stations were clustered along two east - west traverses situated in regions for which the gravity coverage was not very satisfactory. Tables (14.3) and (14.4) summarise the comparisons of the astro-geodetic deflections of the vertical (ξ_A, η_A) with those obtained from the use of free air anomalies in the Vening Meinesz integrals, after conversion to the I.A.U. system (ξ_{G_F}, η_{G_F}). There is very little doubt that the gravimetric values are systematically more negative, in both cases, than the astro-geodetic ones.

Type of data	Meridian (sec)		Prime Vert. (sec)		
	Mean	St. Dev	Mean	St. Dev	No
Astro-geodetic	- 0.6	± 3.4	+ 0.55	± 3.8	94
Gravimetric - Astro-geodetic (all comparisons)	- 2.8	± 3.8	- 3.4	± 4.6	94
Gravimetric - Astro-geodetic (selected areas only)	- 3.9	± 2.5	- 4.3	± 3.7	42

TABLE (14.3)

The comparison of gravimetric values with those obtained astro-geodetically from the Australian control network in South Australia

In the case of deflections of the vertical in the meridian, table (14.3) shows that the inclusion of regions where the close gravity field is inadequately represented does not produce

Range of values v (sec)	Percentage of sample					
	ξ_A	$\xi_{G_F} - \xi_A$		η_A	$\eta_{G_F} - \eta_A$	
		T	R		T	R
$v < -10.5$	-	2	2	-	6	7
$-10.5 < v < -9.5$	2	-	-	1	-	-
$-9.5 < v < -8.5$	2	1	-	-	6	-
$-8.5 < v < -7.5$	-	5	9	1	2	-
$-7.5 < v < -6.5$	1	2	5	2	7	9
$-6.5 < v < -5.5$	1	11	17	1	8	12
$-5.5 < v < -4.5$	3	11	17	2	19	29
$-4.5 < v < -3.5$	4	11	12	3	6	12
$-3.5 < v < -2.5$	8	5	2	2	7	5
$-2.5 < v < -1.5$	10	16	17	7	10	10
$-1.5 < v < -0.5$	13	14	7	9	4	2
$-0.5 < v < -0.5$	21	7	10	10	10	7
$0.5 < v < 1.5$	11	4	-	24	4	2
$1.5 < v < 2.5$	13	6	2	15	2	5
$2.5 < v < 3.5$	7	2	-	5	4	-
$3.5 < v < 4.5$	2	2	-	7	-	-
$4.5 < v < 5.5$	-	-	-	2	2	-
$5.5 < v < 6.5$	-	1	-	3	2	-
$6.5 < v < 7.5$	-	-	-	2	-	-
$7.5 < v < 8.5$	-	-	-	1	1	-
$8.5 < v < 9.5$	1	-	-	1	-	-
$9.5 < v < 10.5$	-	-	-	2	-	-
$v > 10.5$	1	-	-	-	-	-

TABLE (14.4)

Comparisons of deflections of the vertical. Astro-geodetic values on the A.N.S. compared with gravimetric values for the free air geoid (I.A.U. system).

T = Total sample R = Restricted sample

any marked difference in the standard deviation of the sample ($\xi_{G_F} - \xi_A$) from that of the astro-geodetic deflections themselves. The standard deviation of the first set of quantities should be zero if the free air geoid exactly fitted the surface to which the triangulation spheroid had been adjusted. Any departures from this would be due to

(a) inaccurate evaluations of the near field ;

(b) slow systematic effects which cause the free air geoid to deviate from the telluroid and

(c) the correction term ξ_{c_1}, defined in equation (11.22).

If the standard deviation of the comparison sample did not exceed that of the sample of astro-geodetic deflections, it can be assumed that the free air geoid as computed, without actual gravity survey of the inner zones, gave the general trend of the free air geoid and the comparisons were a measure of the contribution of the free air geoid to the correction necessary for the orientation of the A.N.S. to make its centre coincide with the centre of gravity of the existent earth. Thus ($\xi_{G_F} - \xi_A$) is an estimate towards the magnitude of defined in equation (13.17).

The restriction of the comparison to regions where the gravity field is adequately represented shows a marked improvement in the standard deviation of the comparisons in both cases. The prime vertical comparisons appear to be weaker than those in the meridian. From the lay out of the 42 stations used in the restricted set, shown in figs (14.12) and (14.13), it is expected that this is due to the inadequacy of representation of the disturbed field in the north west of the state. While it is difficult to reach any definite conclusions, it would appear that

(a) the free air geoid as mapped does represent all the changes of the telluroid except those due to the inner zones and the correction term ξ_{c_i} (i=1, 2) ;

(b) the correction term ξ_{c_i} (i=1, 2) is not of the same order as the contribution by the free air geoid ;

(c) the estimates made for the errors in the deflections of the vertical for the free air geoid in section (14.2) are reasonable;

(d) any slow changing systematic effect over the area seems to have a constant magnitude. This is to be expected from the geoid itself as mapped in fig (14.9).

The mean comparison error in the case of ξ, which is approximately + 4 sec, agrees with Bomford's estimate (1967, 57) of the correction necessary to the orientation of the A.N.S., based on the free air geoid obtained from satellite data. The comparison error in the case of η is + 4", which differs from Bomford's estimate by about 1". It is interesting to note that deflections of the vertical on the old Clarke 1858 figure in the South Australian region (Bomford, 1963, 9) are approximately $\frac{1}{2}$ sec. larger in ξ and 7 - 8 sec. smaller in η than equivalent values on the A.N.S.

The gravimetric solution was also tested against the Division of National Mapping's Woomera Geoid survey, based on astro-geodetic data alone (ibid, 1963). Zero "geoid" elevation was adopted at one of the Laplace stations in the Woomera area and relative astro-geodetic levelling performed in the area of the survey. The quantities so established are approximately differences in the height anomaly if the spheroid used in computing the astro-geodetic values was properly orientated in space. The quantities so obtained, if corrected as detailed in section (11), are directly comparable with surface deflections of the vertical.

In the present case, the values of dh_D, obtained by astro-geodesy, without the application of equation (11.8) and computed on a spheroid which was not properly orientated, were compared with the values of N_F computed for the free air geoid. The survey, which covers an area approximately $1\frac{1}{2}^o \times 2^o$ around the tracking station complex, on comparison with the data used in preparing fig (14.4) gave the standard deviation of individual differences ($N_F - dN_A$) for a set of 15 comparisons, shown in table (14.5), as $\pm$ 1.8 metres.

In assessing the significance of this figure it is necessary to bear the following points in mind.

(i) Astro-geodetic results are computed on an arbitrarily orientated spheroid. In this particular case where the A.N.S. has been orientated so that the values of the deflections of the vertical will be a minimum, any astro-geodetic levelling will only be sensitive to changes with respect to a plane which is generally normal to all the local surface verticals and , in no sense do they give geoidal undulations.

Longitude / Latitude	$134\frac{1}{2}^{\circ}$	135°	$135\frac{1}{2}^{\circ}$	136°	$136\frac{1}{2}^{\circ}$	137°
-30°		8.5	9.4	11.7	-	-
$-30\frac{1}{2}^{\circ}$	6.6	7.6	9.1	10.6	6.9	-
-31°	5.7	5.7	7.0	8.9	10.0	11.2
$-31\frac{1}{2}^{\circ}$	-	-	-	-	-	9.4

TABLE (14.5)

Comparison of the astro-geodetic Woomera Geoid Survey with the free air geoid referred to the I.A.U. spheroid.

Tabulated quantities are (N_F - dN_A) in metres.

(ii) Earlier investigations seem to indicate that the the contribution of the free air geoid to the correction for the orientation of the A.N.S. is of the order of - 4 sec in the meridian and - 4.3 sec in the prime vertical. As a rough guide this means that the astro-geodetic data has to be corrected by + 1 metre for every half degree change in position in the north and east directions. If the astro-geodetic data were corrected in this manner , the values in the above table would lie in the range $0.9 < (N_F - dN_A) < 5.6$ with a standard deviation for the sample of ± 1 metre. If the reading at ($-30\frac{1}{2}^{\circ}$, $136\frac{1}{2}^{\circ}$) was excluded from the comparison set, the range of values would lie in the range $3.7 < (N_F - N_A) < 5.6$, with a sample standard deviation of ± 40 cm.

As can be seen from figure (9.4), this is an acid test for the field extensions performed in attempting to to provide a continuous gravity coverage for South Australia as the Woomera area is very poorly covered by gravity stations. If the one exception is excluded, the accuracy obtained after allowing for the errors in the astro-geodetic data due to the incorrect orientation of the A.N.S., is as predicted by the error functions in section (14.2). This would imply that the contribution by the free air geoid to the geoid-spheroid separation is unlikely to have errors in excess of 1 metre as a result of errors in the representation of the near field. The test cannot be considered conclusive for the reliability of the Kaula set, though there is no reason to doubt the validity of this data.

An important observation from this simple comparison both before and after correcting for the orientation error in the spheroid used in the astro-geodetic calculations is the assessment of the real significance of astro-geodetic levelling in determining the undulations of the geoid. Astro-geodetic levelling is carried out presumably for obtaining an estimate of the change in the geoid-spheroid separation between adjacent astro-geodetic stations and hence providing a better basis for reducing measured lengths to the spheroid, instead of to the geoid, as is done when orthometric elevations are used.

It is obvious that astro-geodetic levelling on an arbitrarily orientated spheroid hardly determines this quantity or even changes in it and hence its use in such circumstances is scarcely warranted. The results of astro-geodetic levelling should be very carefully examined before any significance is attached to the "geoid" so obtained. Also see section (15.1).

14.7 Assessment of the accuracy of the final results.

(i) The free air geoid.

In the determination of the free air geoid, where considerable field extension had to be performed prior to computation, a serious attempt was made to assess the accuracy of the final result, allowing for correlation between values predicted to represent consecutive unobserved stations. On combining the results of all zones, it was found that uncertainties in inner zone values overshadowed those estimated for the outer extended field. The statistical assessment of the standard error of determination of N_F varied from $\pm$ 30 cm to $\pm$ 60 cm, the errors in program meshing being small in comparison to representation errors. On the other hand, it is not impossible that distant zone effects due to the $1^{\circ} \times 1^{\circ}$ data and the Kaula set all contribute errors that are larger than these estimates and test indicate. This is especially so in the Southern ocean region where a vast area is completely unrepresented by observed gravity, the entire representation being carried by satellite data. On reflection, it seems wishful to expect to represent very large areas in which no observed quantities are available.

There is little doubt that deflections of the vertical are more critically affected by correlation characteristics in the error of extension, especially if extension is necessary for near zones, than values of N_F. In such complex cases, there is a limit to the accuracy of the error estimate due to the unknowns involved and comparisons of computed quantities against external conditions appear. to provide better estimates in the error of the former.

Deflections of the vertical for the free air geoid, being less dependent on the distant zones, are considered to have a standard error of ± 0.5 sec. to ± 3 sec depending on

(a) the available gravity coverage in the near zones

and (b) the nature of the gravity variations in the mid zones.

It is estimated that a more accuracte sampling of the inner zones, together with the evaluation of ξ_{c_i} (i=1, 2) given by equation (11.22) should enable the precision of determination of surface deflections of the vertical to approach those of extended astronomical observations. The Kaula set appears to provide adequate representation of the distant zones for such work.

(ii) Separation of the geoid and reference spheroid.

The accuracy of such determinations can be discussed at two distinct levels.

(a) The absolute accuracy of the determination depends on a number of factors. These are

(i) the accuracy of the assumption that the I.A.U. spheroid has the same volume as the geoid. See section (15.4).

(ii) the accuracy with which the modified Hunter formula represents the topography exterior to the geoid. As discussed in section (6.5), the mere change from one model to another, involving changes of approximately 5 per cent in the density, can cause changes of up to 7 metres in the value of N. On the other hand, a considerable proportion of this can be interpreted as being a variation in the estimated value of the potential of the geoid. This latter value cannot be determined with a precision much in excess of 1 part in 300,000 , i.e., to about 20 kgal metres.

(b) On adoption of a model for the topography exterior to the geoid, the errors that occur in the determination of N_i and ζ_i arise from weaknesses in the elevations used in the calculations. The resulting estimate of the standard error in the value of N_i so determined was $\pm$ 1.2 metres and of ζ_i was $\pm$ 0.5 sec. These values do not take into account any correlation between adjacent elevations in each set in view of the fact that no significant correlation was detected in comparing consecutive values of the U.N.S.W. set and the U.C.L.A. 1S set, as discussed in section (12.4).

The resultant error of determination of the geoid is obtained by the combination of the estimates of errors in the indirect effect with those arising from the determination of the free air geoid.

15. CONCLUSIONS.

15.1 The gravimetric solution in practical geodesy.

The rigorous application of the gravimetric solution to practical geodesy has been the subject of much theoretical discussion without serious attention being paid to the geometrical rigidity of such solutions. This is a deficiency which has to be put right if the science is not to be relegated to the status of being merely a field for intellectual exercise. This entire investigation is a pilot project for verifying the precision of the theoretical foundations of gravimetric geodesy so that the complete gravimetric solution can be applied to the Australian control network in order that the absolute orientation of the Australian National Spheroid may be effected.

The loose use of the word "geoid" and the variety of solutions,provided by so-called "independent" techniques, which are not in very good agreement with one another, unless they happen to use the same theoretical approach and similar observational data, has led to a "credibility gap" between physical geodesists and researchers in allied fields. Thus it is paramount to correct certain concepts which are prevalent today about the science and put all derived quantities into perspective.

The complete set of conditions relating the spheroid of reference in gravimetry with that used in astro-geodesy has

has been developed in sections (11.1), (13.2) and (13.3). From discussions in the previous section it can be seen that the geoid has to be defined, irrespective of the nature of the solution attempted in gravimetric geodesy, before an absolute value is determined for the separation of the geoid and the spheroid. The following quantities, obtained from gravimetric geodesy, are essential before an absolute orientation can be provided for a triangulation spheroid.

(i) The potential of the geoid.

This quantity has to be evaluated in determining any zero order condition for the value of N. This is analagous to defining a datum for elevations if the latter are to have any relevance in fixing position on a three dimensional cartesian system, or on any other such system, for that matter.

(ii) The geoid - spheroid separation at every control station in the network.

Unless the elevation of the geoid above the spheroid has been determined at every control station where scale checks are performed, which would apply to every station in a network established by traversing, scale errors as large as 2 parts in 100,000 could be introduced into relative position determination. In standard theory, it has been accepted that the gravimetric method alone can give an absolute solution, but with a rather low absolute accuracy. Thus conceptually, if gravimetry could afford an absolute value for the separation at one point in the network, astro-geodesy could determine differences in N with greater precision.

Astro-geodetic levelling (Molodenskii, 1962, 24) is based on the relation linking the change in spheroidal height (dh_s) to changes in the orthometric height (dh_o) by the equation

$$dh_s = dh_o - \int \zeta_G d\ell \qquad \ldots\ldots\ldots(15.1),$$

where ζ_G is given by equation (10.4). As the correct value of ζ_G used in equation (15.1) must be based on a properly orientated spheroid, any error in orientation, on the lines described in section (14.6) will consistently ignore a large and nearly constant correction term in the value used for ζ_G and hence the results obtained in such instances would virtually be high order changes in h_D about the surface which is normal to local verticals, in the case of the A.N.S. Astro-geodetic levelling, on arbitrarily orientated spheroids, are subject to corrections for both the error in the geoid spheroid separation at the origin and also for the corrections $\Delta\xi_o$ and $\Delta\eta_o$, defined in equations (13.17) and (13.18) at the origin. It is necessary to adopt the following procedure if astro-geodetic levelling is to give acceptable values for estimating the magnitude of the geoid spheroid separation.

(a) Determine the geoid spheroid separation (ΔN_o) and the corrections $\Delta\xi_o$ and $\Delta\eta_o$ to the deflections of the vertical at the origin of the control network from gravimetry.

(b) Using these values, determine the corrections ($\Delta\xi$, $\Delta\eta$) to the deflections and Δ N to N at each control station, by the use of equations (13.17) to (13.19).

(c) The complete geoid spheroid separation at the n-th station in a chain of control stations is

$$N_n = \Delta N_n - \sum_{i=1}^{n} (\zeta_{G_{m_i}} + \zeta_{Gc_{m_i}}) \, d\ell_i \quad ..(15.2),$$

where ζ_{G_m} and $d\ell$ are defined by equations (10.4) to (10.6) and

$$\zeta_{G_c} = \Delta\xi \cos A + \Delta\eta \sin A \quad ...... (15.3),$$

where A is the azimuth of the line joining the terminal control stations. The suffix $_m$ refers to mean values along the line.

One further assumption is inherent in the above derivation if the quantity N_n is to be the true geoid spheroid separation at the point n. The entire derivation supposes that the vertical at the surface is parallel to that at the geoid. Thus, in addition to the consideration of (11.8), it is also necessary to use equation (11.22). Thus, astro-geodesy by itself, cannot provide a means for removing the scale errors introduced into control networks due to that fact that measured distances are reduced to an irregular geoid, using orthometric heights, instead of the spheroid of reference used in calculations. As can be seen from loop closures obtained on the Australian geodetic network (Bomford, 1967, 61), these scale errors do not have any effect on closed circuits. This is in agreement with the general nature of the geoid slope which is extremely uniform and the regular nature of the traverses. Also see section (15.3).

It is therefore unlikely that the absolute positional accuracy obtainable from control networks can ever exceed 2 parts per million as this level of uncertainty will always exist due to errors in the model adopted for the topography exterior to the geoid. If the geoid-spheroid separation is not considered, the accuracy of positional determinations in an absolute sense can hardly exceed 1 part in 50,000. This has no effect on loop closures.

This very same limitation will also apply to the telluroid and its related constituent, the free air geoid, which, due to the potential of the geoid not being equal to that of the spheroidal model, has a considerable zero order term in the height anomaly. Like the astro-geodetic levelling process, the conventional determination of height anomaly does provide a means of determining the true difference in height anomaly, which, if properly reduced, taking into account the factors listed in section (14.6), will give the geoid - spheroid separation.

(iii) Corrections to the orientation of the spheroid normal at the origin.

The deflections of the vertical obtained gravimetrically from equation (11.21) and associates equations, after the application of corrections , if necessary, for

(a) the gravity datum ;

(b) errors in the parameters of the reference spheroid and

(c) the effect of the term $(W_o - U_o)$ on the location of associated spherops with respect to the gravitating reference spheroid, can be compared with astro-geodetic values, corrected by equation (11.8). The combination of these equations with equations (13.43) and (13.44) provide a set of observation equations on introduction into equations (13.17) and (13.18), from which the corrections $\Delta\xi_o$ and $\Delta\eta_o$ to the orientation at the origin can be determined.

It is essential that the spheroid chosen for the computation of normal gravity be a relevant model, which is not merely internally consistent as the changes in normal gravity, due to changes in the parameters of the reference system, when used appropriately in the Vening Meinesz integrals, give mathematically exact changes in the deflections of the vertical which are identical with those obtained geometrically. The I.A.U. system provides such a model (Fricke et al, 1965), while the use of the international gravity formula and its associated spheroid can give rise to systematic errors of significant magnitude, as shown in tables (13.4) and (13.5).

15.2 The geoid and the reference spheroid.

A problem that has worried geodesists over the years has been the lack of precision with which the geoid can be defined mathematically in continental areas. This has led to the introduction of the concept of geopotential as a means for defining the third dimension in lieu of elevations. While the difference in potential is a measurable quantity, it hardly provides any improvement in evaluating the third dimension, which , in a linear sense, is still subject to the same precision limitations as orthometric height. Further, as the potential of the geoid is not known with a degree of precision in excess of that for determining orthometric elevations, it cannot be said that the value of the potential at a point on the earth's surface is known any better, even after gravimetric determinations, than its distance from the centre of the reference system. If the geoid-spheroid separation approximated to the current definition of the height anomaly, as given by equation (4.22), even to the nearest 10 metres, it could still be considered justifiable to drop the geoid completely from serious consideration, as the telluroid has the advantage of having a direct relation to the surface vertical.

At a first glance, it would appear that the free air geoid should approximate to the geoid over ocean areas. A study of figs (14.4) and (14.9) shows that this is not so. Two reasons exist for this discrepancy.

(i) The surfaces of reference are not identical.

The telluroid is mapped from the associated spherop. As the spheroid does not have the same potential as the geoid, the reference surface from which the value of h_D is determined in ocean areas is not the spheroid itself but a spherop which has the same potential as the geoid. As shown in section (14.4), on assumption of the modified Hunter formula for the topography exterior to the geoid and the I.A.U. spheroid as having the same volume as the geoid, the potential of the geoid is approximately 1000 kgal metres greater than that of the reference spheroid, which has the same mass as the earth. Thus, the surface of reference for the geoid and low lying continental areas is the related associated spherop which lies <u>within</u> the reference spheroid. This poses the second reason for the discrepancy between the telluroid and the geoid over ocean areas, on the current understanding of the former,

(ii) <u>Normal gravity for the anomaly is the value at the associated spherop</u>.

The gravity anomaly is defined in equation (3.17) as the difference between observed gravity at the surface being mapped and normal gravity at the associated spherop. If the latter lies within the reference spheroid, the anomaly to be used in equation (4.22) and its revised form set out in equation (13.70) is no longer the free air anomaly as was previously held. The disturbing potential (V_D) remains the same as defined in equation (3.12) and modified by equation (13. 7). The differential expression for V_D with respect to height is given by

$$\frac{\partial V_D}{\partial h} = \frac{\partial}{\partial h}(W_P - U_P) = \frac{\partial W_P}{\partial h} - \frac{\partial}{\partial h}[U_o - \Delta W + + (W_o - U_o)] \quad \ldots .(15.4).$$

The first term, as before, is observed gravity at P, with the reversed sign and the latter term is a modified form of equation (13.58). No other interpretation of the telluroid is possible without requiring a fresh derivation of all constituent boundary value definitions and relations. If the surface solutions are to be meaningful, in addition to zero order terms arising from the nature of the differential topographical corrections over ocean areas, which will be constant in magnitude over all oceans, there are other varying effects which must be taken into account before a complete solution is obtained. As all these corrections are governed by properties of the model, they should not provide any real computational difficulties.

Thus the height anomaly itself is not of great significance in geodetic work. On the other hand, the deflections of the vertical computed from the surface solution are of great intrinsic value as they relate the surface vertical to the associated spherop normal. In the light of equations (13.75) to (13.79), constant world wide changes in the gravity anomaly do not produce any change in ξ or η as determined from the solution of the boundary value problem. Thus it is only the regularly changing constant effects which affect the computed values of ξ and η . Such effects tend to be undetected when comparing incomplete gravimetric results with astro-geodetic data over limited regions.

It is interesting to see how Molodenskii, who is the prime mover behind the solution at the physical surface of the earth, deals with the mapping of the non-regularised geoid and the zero order term in the disturbing potential. In the former case, he summarises the different methods for solution. That due to Malkin (Molodenskii, 1962, 65) approximates very closely to equation (5.14), the only difference being in the definition of the anomaly. Malkin uses the Prey anomaly (Heiskanen and Moritz, 1967, 64), which approximates to Δg_0, as the differential topographical correction approximates to the Prey reduction.

In deriving his expression for the equivalent of the height anomaly, Molodenskii considers the term $\frac{W_o - U_o}{\gamma}$ as a problem in the choice of the reference surface , after the acceptance of a value for both the flattening and equatorial gravity. Admitting the term containing $(W_o - U_o)$, he gives the complete solution for the height anomaly h_{D_f} (ibid, 103) as

$$h_{D_f} = h_D + Z - \frac{W_o - U_o}{\gamma} \qquad \text{...........(15.5)},$$

where Z is defined as a " first order spherical function".

Molodenskii's deductions are in agreement with the contention that determinations of the telluroid give deflections of the the vertical which difference consistently from astro-geodetic values on the basis of equations (13.17) to (13.19). Neither is there any theoretical inconsistency in the actual geoid having no obvious relation to the free air geoid, even though the latter contributes significantly to its character. Molodenskii's reasons for ignoring all terms except h_D on the right hand side of equation (15.5) for the solution of the boundary value problem are as follows :-

(i) The zero order term is a function dependent on the choice of the reference spheroid (ibid, 79). He argues that as equatorial gravity and the flattening can be determined independently, a can be altered till $W_o = U_o$.

(ii) The first order term is a function of the separation of the centre of gravity of the earth and that of the spheroid of reference(ibid, 104).

He then concluded that if these two conditions are satisfied, the determination of h_D is absolute.

The telluroid determination must be considered from two distinct angles.

(a) The determination of h_D itself would have to take into account the fact that the potential of the geoid is not equal to that of the spheroid and cannot be made to satisfy this condition except at the cost of ignoring independent evidence as to the spheroid which appears to have the same volume as the geoid and the value of kM. Any attempt to force either of these conditions invalidates one of two conditions

(i) that of both systems having the same mass and centre of gravity ;

(ii) the spheroid being that which best fits the geoid.

As such, h_D is a very large value and not of much significance, as it does not approximate to the separation of the geoid and the spheroid.

(b) Determinations of the deflections will be of importance as they are directly comparable with astro-geodetic values. But the anomaly to be used is not the free air anomaly but must take into account the location of the associated spherop in relation to the spheroid of reference.

As such, Molodenskii's derivation also requires revision for any zero order term in the gravity anomaly.

15.3 The gravimetric solution and the orientation of spheroids used in astro-geodesy.

If the spheroid of reference used for the computation of gravity anomalies is the same as that used for control network calculations, the quantities $\Delta\xi_o$, $\Delta\eta_o$ and ΔN_o, which appear in equations (13.17) to (13.19) constitute the required orientation corrections. These three terms make up a mixed set as $\Delta\xi_o$ and $\Delta\eta_o$ are defined with respect to the surface vertical while the separation ΔN_o refers to a correction for rectifying the error introduced in assuming the geoid to be coincident with the spheroid at the origin. This quantity can be computed at any point, including the origin, with an accuracy approaching $\pm$ 10 metres on adopting a reasonable model for the topography, based on current estimates for crustal density.

As pointed out in section (14.5), the conventional value adopted by geodesists for the density of the upper crust is 2.67 $gm.cm^{-3}$ (Heiskanen and Vening Meinesz, 1958, 7). Hunter's analysis, discussed in section (1.4), gave rise to the Hunter formula used in the current investigation. Geophysicists use much lower values, based on profile investigations using the Nettleton technique, for crustal density. The density values given by the modified Hunter formula, set out in equation (12.7) are higher than those used in geophysical prospection. As the latter are based on profiles which seldom exceed 1 - 2 miles, it is felt that these densities are probably based on surface sedimentary rocks and are probably too small. In view of the importance of the geoid to all geodetic calculations, it is of the utmost importance that more detailed studies are made of the nature of the density distribution in the upper crust.

The free air geoid itself is not identical with the telluroid, even as a first approximation, and deflections of the vertical on the former differ from equivalent values on the latter by amounts which can be considered to have two contributory terms, one of which is constant over large areas and the other which varies slowly. Thus, unless surface integrals are calculated using all relevant corrections, the results will not provide an absolute set of values for $\Delta\xi_o$ and $\Delta\eta_o$ for use in equations (13.17) to (13.19).

The solution for the geoid can, of course, be used to establish absolute values for the deflections of the vertical at coastal astro-geodetic stations from which values can be calculated for the corrections to the deflections of the vertical at the origin of coordinates. This method will be difficult to apply in Australia due to the weak gravity field available at sea.

The Johnston Origin of the Australian control network (Bomford, 1967, 57) has coordinates

$$\phi = -25^{\circ} 56' 54''.6 \qquad \lambda = 133^{\circ} 12' 30''.1 \quad ..(15.6),$$

which locates it just over the northern border of South Australia. Its location gives it an estimated N value from fig (14.9) of approximately 155 metres. The ratio

$$\frac{\Delta N_o}{R_m} \times 206265 \quad \approx \quad 5 \text{ sec} \quad \ldots\ldots\ldots\ldots (15.7).$$

A study of equations (13.17) to (13.19) shows that the effect of this term for a country the size of Australia, situated in a region of mid-latitude, varies as follows :-

(i) ranges from + 1.4 sec to - 1.1 sec as the latitude of the computation point varies from - 10° to - 40° on the meridian through the origin.

(ii) ranges from - 1.5 sec to + 1.5 sec as the computation point varies from the western seaboard to the eastern one.

If the estimate quoted is a true **representation** of the geoid-spheroid separation at the Johnston Origin, and if the control network, were evenly distributed about the origin, the division of the network into four quarters about the origin should give residuals of the following signs in the deflections of the vertical on a spheroid which has the geoid-spheroid separation at the origin arbitrarily set to zero.

	Meridian	Prime Vertical
NE	- ve	- ve
SE	+ ve	- ve
NW	- ve	+ ve
SW	+ ve	+ ve

The prime vertical values being based on the convention that longitude is positive east. An analysis made by Bomford (1967, 57) of the astro-geodetic data in flat regions to determine the mean deflection components in each quarter, agrees with the above prediction, with a single exception. The flat areas sample was used as this would have masking effects reduced to a minimum. Thus the nature of the mean deflections of the vertical in each quarter seems to confirm the existence of a considerable elevation of the geoid above the spheroid at the origin. (Note :- The signs of the prime vertical component of the deflection should be reversed when converting to the sign convention adopted for longitude in this investigation).

Thus all evidence points to the consistency of the Australian control network which needs a correction of approximately + 155 metres at the Johnston origin for the separation of the geoid and the spheroid and a set of corrections for the deflections of the vertical at the origin, the contributions to which by the free air geoid are approximately - 4 sec in the meridian and - 4.5 sec in the prime vertical.

15.4 The parameters for a reference model of the earth.

All theoretical inconsistencies, to date, have arisen from one fact, i.e., the imposition of a value to a physical reality for which there is no justification. This commenced with the assigning of a value for equatorial gravity using a misleading reference model for the earth. This resulted in the system of reference afforded by the international gravity formula. Molodenskii, as already discussed, has , in his derivation, assumed that both f for the best fitting spheroid and γ_e are known quantities, the value for a being assigned so that U_o becomes equal to W_o. This is easier said than done, as, on his system, there is no means available for assigning a value for W_o. As can be seen, the only manner in which a value can be assigned for the potential of the geoid is by assuming as the spheroid of reference , that figure which , on the basis of independent determinations, appears to best fit the geoid. It is critical that the spheroid of reference have the same volume as the geoid as the value deduced for W_o is dependent on the assumption that the sum total of all values of N, taken over the earth's surface, is zero.

If the reference spheroid satisfies this condition, the potential of the geoid can then be obtained from equation (14.2), provided the quantity $M\{N_i\}$ has been adequately determined. In view of the solution being heavily dependent on the accuracy of the assumption that the geoid and the reference spheroid have the same volume, it is necessary to examine the I.A.U. spheroid for accuracy of fit. The value adopted was first suggested by Kaula (1961, 1807) and is based on a set of 196 " observations" , 54 per cent of which were based on astro-geodetic data, 42 per cent on gravimetric data and 4 per cent on satellite data. Only about 20 per cent of the earth was represented in the analysis with a bias towards land based observations.

If there is a tendency for the geoid to lie systematically above the best fitting spheroid over continental areas and below it in ocean regions, it can be expected that the spheroid obtained by such an analysis is one of best fit to the geoid over land areas. From the calculations carried out to determine $M\{N_i\}$, as described in section (14.4), which certainly cannot be described as being final by any means, except for the purpose for which it was made, seems to indicate a low over the Pacific region which passes into a high as the ocean gives way to the continents on either side.

If the effect described above does exist in the equatorial radius of the reference spheroid, there is the possibility that the spheroid of best fit has a radius which is approximately 100 metres less than that of the I.A.U. spheroid. The resulting changes in U_o and γ_e, from equations (13.47) and (13.49) are + 98.5 kgal metres and + 30 mgal respectively. As there is no change in either the value of the flattening or kM, there is no differential variation in normal gravity of any consequence over the reference spheroid and hence there are no changes of significant magnitude in the values of the deflections. If the magnitude of the zero order term is to remain zero, the potential of the geoid must be 6,264,542 kgal metres, the value of $W_o - U_o$ reducing to 764 kgal metres.

Thus the maximum change indicated in the value of the potential of the geoid due to the I.A.U. spheroid being approximately 100 metres larger than the geoid, is of the order of + 100 kgal metres. This value of W_o has no real relevance in geodesy except in the determination of the separation of the geoid and the spheroid. Its value has no bearing on the value of kM, and is, in fact, dependent on the value of kM adopted for the reference model.

15.5 Conclusion.

The complete solution for the determination of the separation between the adopted spheroid of reference and the equipotential surfaces of the earth's gravitational field requires the determination of the potential of the existent geoid. Such a value is obtainable if it is possible to define a spheroid of reference by an independent method, having exactly the same volume as the geoid. If this spheroid is the I.A.U. spheroid (a = 6,378,160 met, f = 1/298.25), the potential of the geoid, excluding the effect described in appendix (18), is 6,264,628 kgal. met.

The model adopted for the topography exterior to the geoid in computing this potential is a modified form of the Hunter formula, set out in equation (12.8). The geoid in South Australia, based on this value of the geoid potential, is given in fig (14.9). Any bias which may have occured in the evaluation of the parameters of the I.A.U. spheroid would be due to the observational data used in this determination being continent oriented and lying in areas where the geoid is systematically higher than the spheroid. It is estimated that the maximum error in the geoid potential due to this cause is approximately - 100 kgal. metres.

The geoid spheroid separation of the Johnston Origin of the Australian control network, on the basis of the I.A.U. spheroid - modified Hunter model is estimated to be nearly 160 metres. This result is not inconsistent with the nature of the deflections of the vertical in the Australian control network.

The free air geoid contributes significantly to the separation of the geoid and the spheroid, but neither the magnitude of the separation nor the deflections of the free air geoid are any reliable guide to the equivalent components of the geoid-spheroid separation vector. As the potential of the geoid is not equal to that of the reference spheroid, the free air geoid is not a first approximation of the telluroid, as is commonly held.

For any attempts to orientate astro-geodetic reference spheroids with respect to the earth's centre and axis of rotation, it is necessary to consider the geoid spheroid system to obtain the separation N_o at the origin. The corrections $\Delta \xi_o$ and $\Delta \eta_o$, defined in equations (13.17) to (13.19), have to be obtained from a solrtion at the physical surface of the earth, to which the Stokesian term makes the major contribution, the anomaly used being a modified form of the free air anomaly, after allowing for the fact that the potential of the existent geoid is not equal to that of the chosen spheroid of reference.

Astro-geodetic deflections of the vertical, when used to afford astro-geodetic levelling, contribute toward the magnitude of the geoid-spheroid separation, but give results which ignore orientation errors in the triangulation spheroid. As these quantities are much larger than the quantities determined by astro-geodetic levelling, the results are of limited relevance in attempting to remove scale errors introduced in control networks due to the geoid-spheroid separation being unknown.

If all these considerations, including the ones listed in appendix (18), are taken into account, the gravimetric solution is exactly equivalent to a purely geometrical one.

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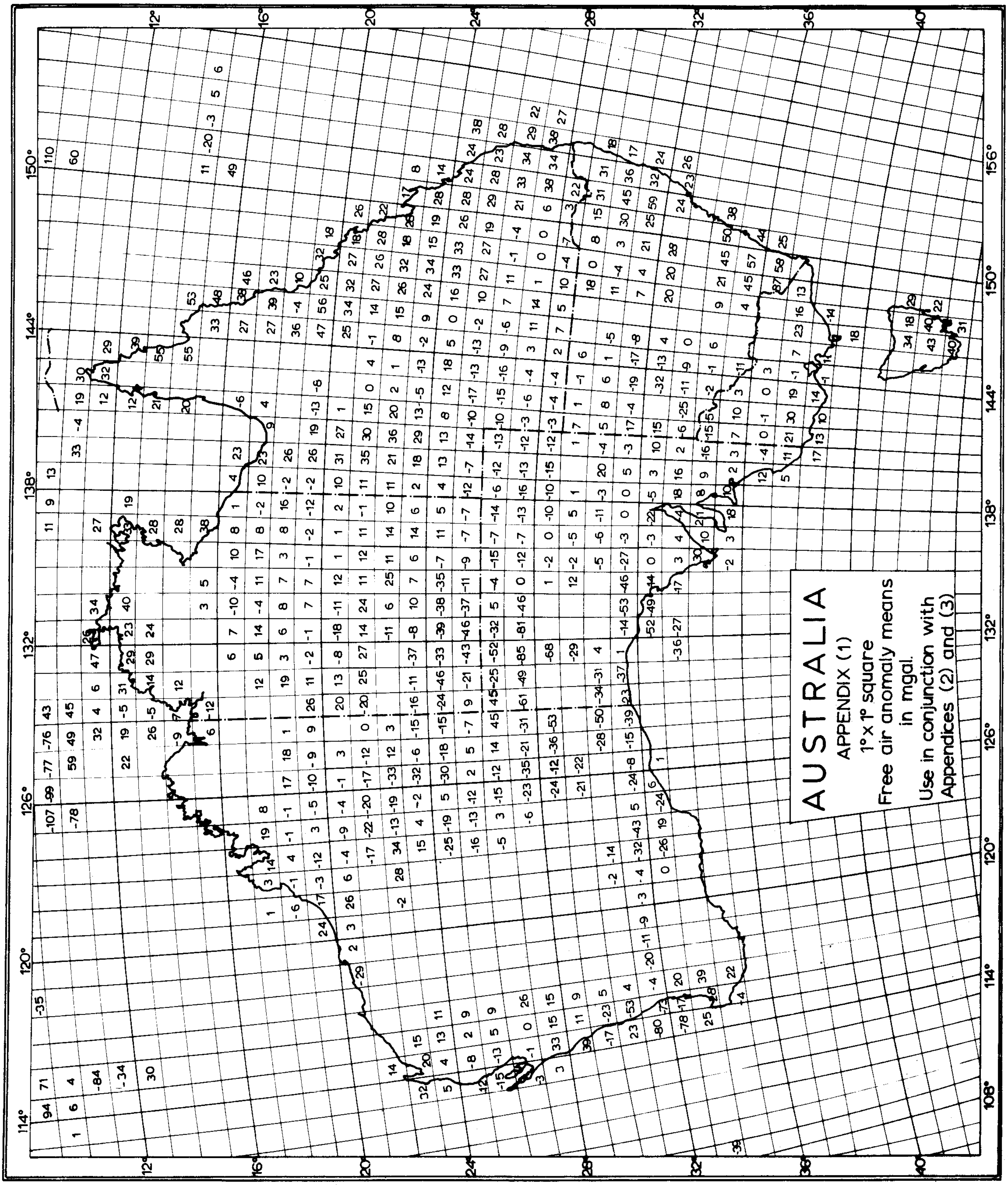
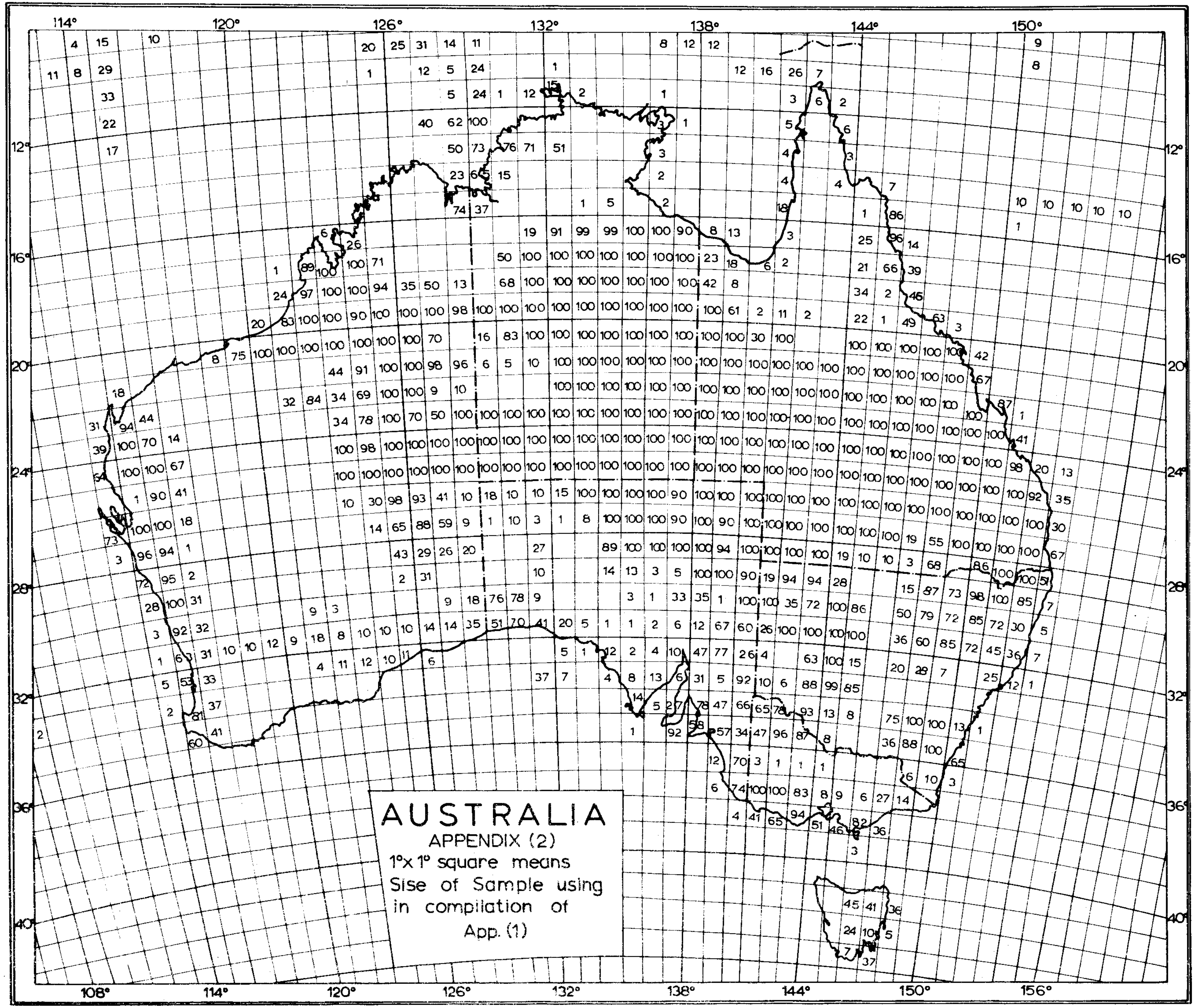
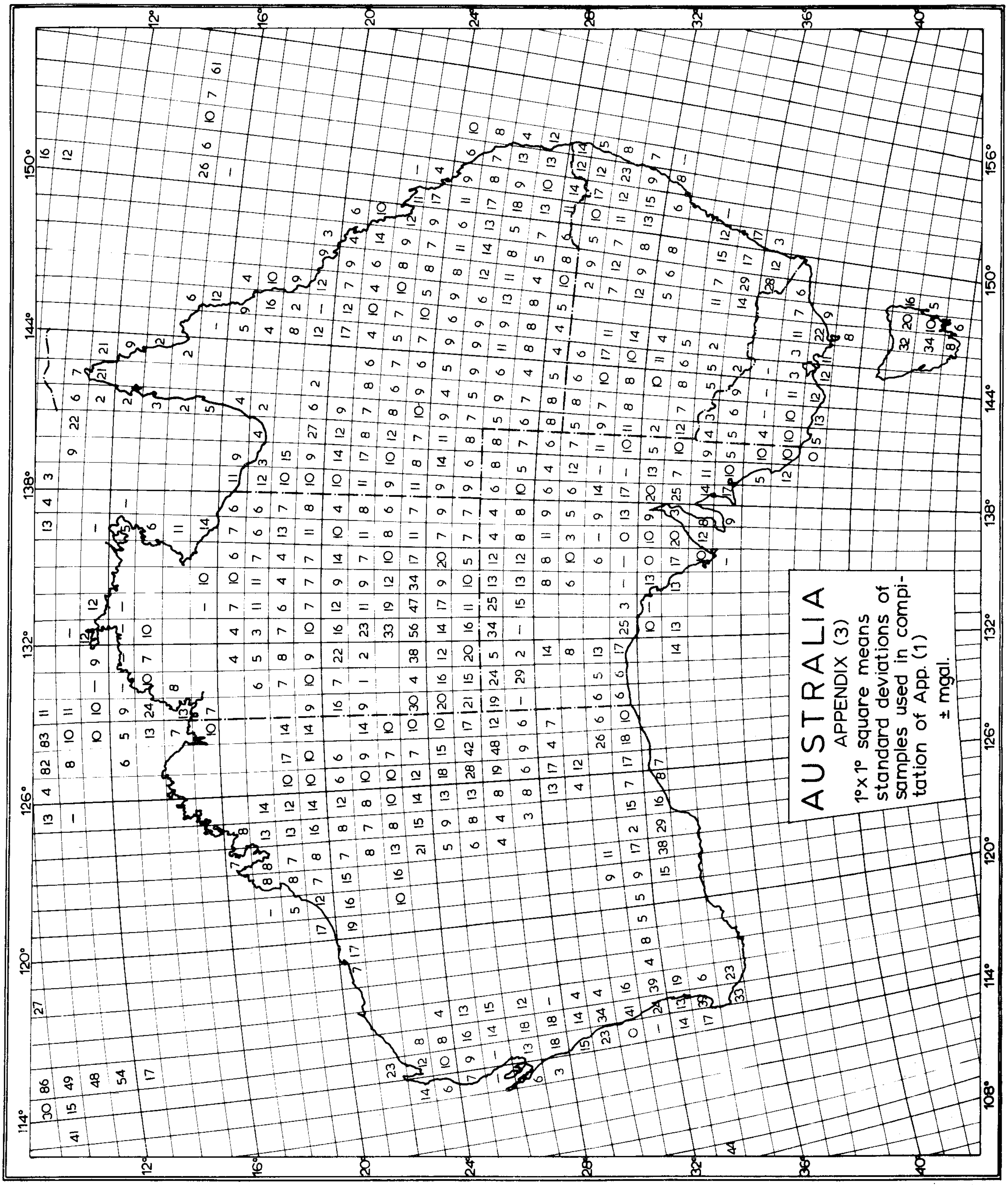
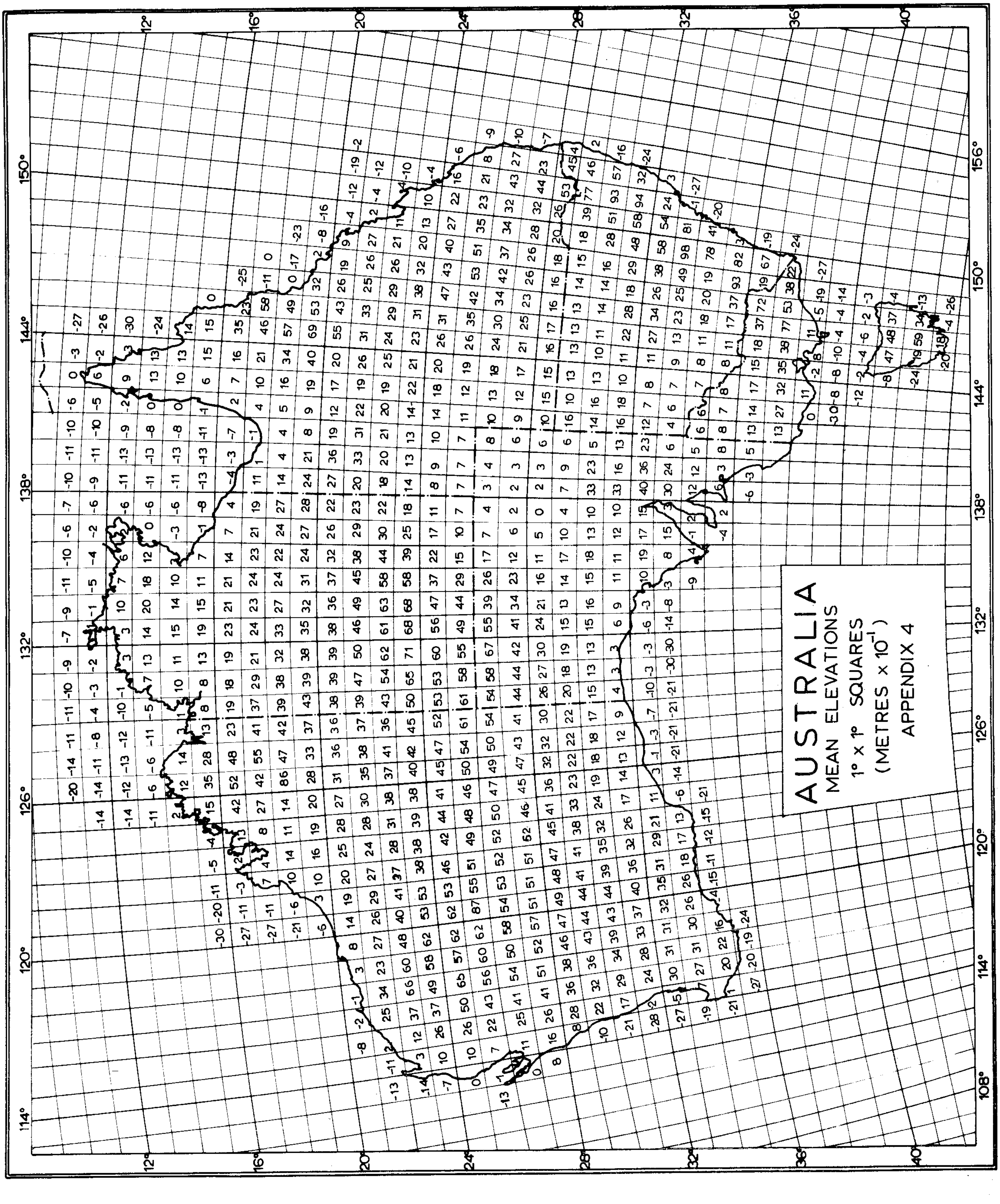

AUSTRALIA
APPENDIX (1)
1°×1° square
Free air anomaly means
in mgal.
Use in conjunction with
Appendices (2) and (3)


AUSTRALIA
APPENDIX (2)
1°x 1° square means
Sise of Sample using
in compilation of
App. (1)


AUSTRALIA
APPENDIX (3)
1°x1° square means
standard deviations of
samples used in compi-
tation of App. (1)
± mgal.


AUSTRALIA
MEAN ELEVATIONS
1° x 1° SQUARES
(METRES x 10^{-1})
APPENDIX 4

APPENDIX (5)

3200 FORTRAN (2,0) 24/12/65

```
      PROGRAM LS
      REAL IGVAL,ICBG,ISTG,ICOR
      COMMON IOBG,IVLAT,IHT,IA,LHT,IFA,JGAN,A
      DIMENSION LAD(2),LAM(2),LAS(2),LOD(2),LOM(2),LOS(2),IGVAL(2),JHT(2
     1),ICO(2),IOBG(20,20),IVLAT(20,20),IHT(20,20),IA(20,20),A(29,30),BA
     2(30),KHT(10),LHT(20,20),IFA(20,20),JGAN(20,20),ISTG(5),ISLV(5),ISH
     3(5)
C     GEOGRAPHICAL COORDS OF NW CORNER-OF NW  TWO DEGREE AREA   IN
C     DEGREES LAT POSITIVE SOUTH LONG POSITIVE EAST
      READ(60,100)LAOF,LOOF
  100 FORMAT(2I5)
      REWIND 55
      LL1=2
      LL2=12
      LL3=14
      LL4=2
      LL=20
      DO 210 IIK=LL4,LL3,LL1
      LAO=LAOF+IIK-2
      DO 207 JJK=LL1,LL3,LL1
      LOO=LOOF+JJK-2
      READ(60,240)ICH
  240 FORMAT(I5)
      IF(ICH)101,101,241
  101 DO 102 I=1,LL
      DO 102 J=1,LL
      IOBG(I,J)=0
      IVLAT(I,J)=0
      IHT(I,J)=0
      LHT(I,J)=0
```

```
      IFA(I,J)=0
      JGAN(I,J)=0
  102 IA(I,J)=0
      DO 103 I=1,29
      DO 103 J=1,30
  103 A(I,J)=0
      WRITE(61,216)
  216 FORMAT(//60H COMPUTATION OF BOUGUER ANOMALY MEANS IN A  TWO  DEGRE
     1E AREA//)
      WRITE(61,37)LAO,LOO
   37 FORMAT(27H NW CORNER LATITUDE SOUTH  ,I5,27H DEGREES   LONGITUDE E
     1AST  ,I5,9H DEGREES //)
C     READ MEAN HEIGHTS OF TENTH DEGREE SQUARES IN FEET LAT LONG OF
C     FIRST SQUARE IN HUNDREDTHS OF DEGREES
  175 READ(60,176)ITE,LAH,LOH,(KHT(I),I=1,10)
  176 FORMAT(I1,2I5,10I6)
C     TEST IF LAST CARD IN BATCH
      IF(ITE-1)177,180,179
  177 LAOH=LAO*100+5
      II=(LAH-LAOH)/10.0+1
      IF(II)220,220,221
  221 IF(II-LL-1)222,220,220
  222 LOOH=LOO*100+5
      DO 178 IM=1,10
      JJ=(LOH-LOOH)/10.0+IM
      IF(JJ)178,178,223
  223 IF(JJ-LL-1)233,178,178
  233 LHT(II,JJ)=KHT(IM)*0.304799
  178 CONTINUE
C     STORED IN METRES
  220 IF(ITE-1)175,180,179
```

```
179 WRITE(60,181)
181 FORMAT(34H HEIGHT INDEX PUNCHED IN ERROR     )
180 DO 301 I=1,LL
    DO 301 J=1,LL
    IF(I-1)188,182,185
182 IF(J-1)188,183,184
183 IHT(I,J)=LHT(I,J)
    GO TO 190
184 IHT(I,J)=(LHT(I,J)+LHT(I,J-1))/2.0
    GO TO 190
185 IF(J-1)188,186,187
186 IHT(I,J)=(LHT(I,J)+LHT(I-1,J))/2.0
    GO TO 190
187 IHT(I,J)=(LHT(I,J)+LHT(I,J-1)+LHT(I-1,J)+LHT(I-1,J-1))/4.0
    GO TO 190
188 WRITE(61,189)
189 FORMAT(54H  I J LESS THAN ONE ERROR IN DO LOOP CIRCUITRY IN HTS )
190 IF(IHT(I,J))301,300,301
300 IHT(I,J)=6000
301 CONTINUE
    DO 191 I=1,LL
    DO 191 J=1,LL
191 LHT(I,J)=0
104 READ(60,105)((LAD(I),LAM(I),LAS(I),LOD(I),LOM(I),LOS(I),IGVAL(I),J
   1HT(I)),I=1,2),IT,(ICO(I),I=1,2)
105 FORMAT(2(I5,2I3,I5,2I3,F10,I5),I1,1X,2I2)
C   GRAVITY IN TENTH MILLIGALS HEIGHTS IN METRES
    IF(IT-1)212,158,156
212 DO 155 I=1,2
C   NINE HUNDRED ELEMENTS TO BE STORED INCL ROW COL THRO NW CORNER
```

```
      WRITE(61,20)
      WRITE(61,70)
      WRITE(61,67)LAON,LOON,MAN1F,SERF1,INO1,MAN1,SER1,MAN1I,SER1I,MAN1J
     1,SER1J,IN1,MHT1,SDHT1
   67 FORMAT(3I10,F10.2,2I10,F10.2,I10,F10.2,I10,F10.2,2I5,F10.2/////)
   68 CONTINUE
      IF(INO3-1)74,72,73
   72 MAN3=IX3
      SER3=0
      MAN3J=IX3J
      SER3J=0
      MAN3F=IF3
      SERF3=0
      GO TO 74
   73 MAN3=IX3/INO3
      SER3=SQRT(ABS(IXSQ3/(INO3-1.0)-INO3*MAN3*MAN3/(INO3-1.0)))
      MAN3F=IF3/INO3
      SERF3=SQRT(ABS((IFSQ3-INO3*MAN3F*MAN3F)/(INO3-1.0)))
      MAN3J=IX3J/INO3
      SER3J=SQRT(ABS(IXSQ3J/(INO3-1.0)-INO3*MAN3J*MAN3J/(INO3-1.0)))
   74 IJS=LL*LL
      MAN3I=IX3I/IJS
      SER3I=SQRT(ABS((IXSQ3I-IJS*MAN3I*MAN3I)/(IJS-1.0)))
      IF(IN3-1)309,309,310
  310 MHT3=NHT3/IN3
      SDHT3=SQRT(ABS((SHT3-IN3*MHT3*MHT3)/(IN3-1.0)))
  309 WRITE(61,75)
   75 FORMAT(/////130H  LAT NW     LONG NW       MEAN  ANOMALY  PARTICULARS
     1S  IN  TENTH  MILLIGALS  FOR  TWO  DEGREE     SQUARE          MEAN
     2 HT  STD DEV  )
```

Appendix (5) ctd.

```
      WRITE(61,20)
      WRITE(61,70)
      WRITE(61,67)LAO,LOO,MAN3F,SERF3,INO3,MAN3,SER3,MAN3I,SER3I,MAN3J,S
     1ER3J,IN3,MHT3,SDHT3
   47 CONTINUE
      WRITE(61,79)LAO,LOO
   79 FORMAT(//2I10//)
      DO 81   I3=1,LL
      DO 81   J3=1,LL
      IF(IFA(I3,J3)-3000)76,77,76
   76 IFAN=IFA(I3,J3)/10
      GO TO 78
   77 IFAN=300
   78 IFA(I3,J3)=IFAN
   81 CONTINUE
      WRITE(61,80)((IFA(I,J),J=1,LL),I=1,LL)
   80 FORMAT(20I6/)
      WRITE(61,79)LAO,LOO
      DO 86 I3=1,LL
      DO 86 J3=1,LL
      IF(IA(I3,J3)-3000)82,83,82
   82 IBAN=IA(I3,J3)/10
      GO TO 84
   83 IBAN=300
   84 IA(I3,J3)=IBAN
   86 CONTINUE
      WRITE(61,85)((IA(I,J),J=1,LL),I=1,LL)
   85 FORMAT(20I6/)
      DO 350 I3=1,LL
      DO 350 J3=1,LL
      IIAN=JGAN(I3,J3)/10
      JGAN(I3,J3)=IIAN
```

```
  350 CONTINUE
      WRITE(61,351)((JGAN(I,J),J=1,LL),I=1,LL)
  351 FORMAT(20I6/)
C     STORE GRAVITY VALUES ON TAPE
      WRITE(55,201)LAO,LOO
  201 FORMAT(2I10)
      DO 206 I1=1,2
      DO 206 J1=1,2
      DO 205 IT=1,10
C     STORE LATITUDE IN TENTH DEGREES
  230 ILAT=LAO*10+(I1-1)*10+IT-1
      DO 204 JH=1,2
C     STORE LONGITUDE OF EAST OF FIVE ELEMENTS IN TENTH DEGREES
  231 ILONG=LOO*10+(J1-1)*10+(JH-1)*5
      DO 202 JT=1,5
      IE=(I1-1)*10+IT
      JE=(J1-1)*10+(JH-1)*5+JT
      ISTG(JT)=IOBG(IE,JE)
      ISLV(JT)=IVLAT(IE,JE)
      ISH(JT)=IHT(IE,JE)
  202 CONTINUE
      II=0
      WRITE(55,203)II,ILAT,ILONG,((ISTG(I),ISLV(I),ISH(I)),I=1,5)
  203 FORMAT(I1,2I5,5(F7,I3,I4))
  204 CONTINUE
  205 CONTINUE
  206 CONTINUE
      IK=1
      WRITE(55,211)IK
  211 FORMAT(I1)
      END FILE 55
```

```
     G) TO 207
241  W?ITE(55,201)L/ ),LOO
      K=1
      RITE(55,242)I
242   ORMAT(I1)
     ?ITE(61,245   K,JJ ,LL1, L2
245  F RMAT(//4I4
207  C NTINUE
     E D FILE 55
210  (ONTINUE
     IK=2
     WRITE(55,24 )IK
243  FORMAT(I1)
     REWIND 55
2 9  READ(55,20  II,ILAT, LON ,((IS  (I),ISLV(I),  SH(I)),I=1,5)
208  FORMAT(I1, I5,5(F7, 3,I4))
     WRITE(61,   )II, L T, LONG,((I  TG(I),ISLV(I  ,ISH(I)),I=1,5)
     IF(II-1)2 9,246,  4
246  CALL SKI (55)
     GO TO 2 9
244  CONTIN E
     STOP
     END

      3200 FO TRAN DIAGNOSTIC RE ULTS - FOR   LS
```

APPENDIX (6)

```
AN_COMP:PROCEDURE OPTIONS(MAIN) ;


AN_COMP:PROCEDURE OPTIONS(MAIN) ;
    DECLARE KHT(10) DEC (6),HT_INT FIXED (5,2),LAT DEC (6),LONG DEC (6),
         LATOP FIXED DEC (10) , LONGOP FIXED DEC (10),STD FIXED DEC (5),
         HAN_M(10,10) FIXED (10) , HHT_M(10,10) FIXED (10) ,
    KEY_UNI  DEC (2),KEY DEC (1),LAT_OR DEC (10),LONG_OR DEC (10),IHT(50,50)
    DEC(10),IFA_AN(50,50) FIXED DEC (10),DENSITY FIXED (3,2),AN_INT FIXED(3,1),
    KTOTH DEC (10), KTOTO DEC (10), KTOTF DEC (10),
     MEAN_H(2,2) DEC (10,1),STD_ERH(2,2) FIXED (10), NOH(2,2) FIXED DEC (5),
    RLATH(2,2) FIXED (9,2), RLONGH(2,2) FIXED (9,2),MEAN_O(5,6) DEC (10,1),
   STD_ERO(5,6) FIXED (9),NOO(5,6) FIXED DEC (5),RLATO(5,6) FIXED (9,1),
     RLONGO(5,6) FIXED(9,1),MEAN_F DEC(10,1),STD_ERF DEC (9),NOF FIXED DEC (5),
     RLATF FIXED (9,1),
    TOTSQH DEC (10), TOTSQO DEC (10), TOTSQF DEC (10),
    RLONGF FIXED (9,1),HTH(2,2)  FIXED (9)  ,HSTDERH(2,2) FIXED (9),
    HTO(5,6) FIXED (9)  ,HSTDERO(5,6) FIXED (9)  ,HTF FIXED (9)  ,HSTDERF
    FIXED (9);
    PUT LIST('           COMPUTATION OF FREE AIR ANOMALY DATA ');
 START: GET LIST(LAT_OR,LONG_OR);/*LATITUDE AND LONGITUDE OF SW CORNER.VALUES
    REFER TO CENTRE OF SQUARE */
    IHT(*,*)=100000;                   /* VALUE 100000 STORED WHERE NO READINGS
   AVAILABLE */
   RD_FHT      :GET LIST(KEY,KEY_UNI ,LAT,LONG,HT_INT,(KHT(I) DO I=1 TO 10)) ;
    PUT EDIT(KEY,KEY_UNI,LAT,LONG,HT_INT,(KHT(I) DO I=1 TO 10))(F(2), F(3),
     2F(7),F(5,1),10F(7)) SKIP(1) ;
    IF KEY=1 THEN GO TO READ_HT ;
```

```
STR_HT: DO I=1 TO 10;
 IN=(LAT-LAT_OR)/(HT_INT*100.0)+1 ;  JN=(LONG-LONG_OR)/(HT_INT*100.0)+I ;
 IF IN<=0 THEN GO TO RD_FHT         ; IF IN > 50 THEN GO TO RD_FHT       ;
IF JN <=0 THEN GO TO RD_FHT         ; IF JN>50 THEN GO TO RD_FHT  ;
                                      IHT(IN,JN)=KHT(I)*(1-KEY_UNI*0.6952);
  END ;    GO TO RD_FHT ;
READ_HT      :GET EDIT(KEY,KEY_UNI ,LAT,LONG,HT_INT,(KHT(I) DO I=1 TO 10))(
  F(1),F(2),F(6),F(6),F(5,2),10F(6)) ;
  PUT EDIT(KEY,KEY_UNI,LAT,LONG,HT_INT,(KHT(I) DO I=1 TO 10))(F(2), F(3),
   2F(7),F(5,1),10F(7)) SKIP(1) ;
/* KEY=1,THEN THIS IS LAST CARD;ELSE,KEY=0;IF KEY_UNIT=0,HT IN MEYERS;=1,HT IN
  FT;LAT,LONG IN 10**-2 DEGREES;OF CENTRE OF W SQ.HT_INT= INTERVAL BETWEEN
  SUCCESSIVE  SQUARES IN DEGREES */
  IF KEY=1 THEN GO TO CYC_AN;
STO_HT: DO I=1 TO 10;
 IN=(LAT-LAT_OR)/(HT_INT*100.0)+1 ;  JN=(LONG-LONG_OR)/(HT_INT*100.0)+I ;
 IF IN<=0 THEN GO TO READ_HT        ; IF IN > 50 THEN GO TO READ_HT      ;
IF JN <=0 THEN GO TO READ_HT        ; IF JN>50 THEN GO TO READ_HT;
                                      IHT(IN,JN)=KHT(I)*(1-KEY_UNI*0.6952);
END; /* HEIGHTS STORES IN METERS */
 GO TO READ_HT;
  CYC_AN: IFA_AN(*,*)=5000;
   READ_AN:GET LIST(KEY,AN_INT,LAT,LONG,DENSITY,(KHT(I) DO I=1 TO 10));
 PUT EDIT(' ',KEY,AN_INT,LAT,LONG,DENSITY,(KHT(I) DO I=1 TO 10))(A,F(2),
  F(5,1),2F(7),F(6,2),10F(7)) SKIP(1) ;
/* KEY=1 WHEN LAST CARD;LAT,LONG AS FOR HEIGHTS;AN_INT=INT BETWEEN SUCCESSIVE
SQUARES IN DEGREES.DENSITY IN G/CC;BOUGUER ANOMALIES IN MGAL X 10**-1 */
 IF KEY=1 THEN GO TO MN_CLCS ;
   ELV=0;
   STO_AN:DO I=1 TO 10 ;
       IN=(LAT-LAT_OR)/(AN_INT*100.0)+1; JN=(LONG-LONG_OR)/(AN_INT*100.0)+1;
  IF IN<=0 THEN GO TO READ_AN        ;IF IN>50 THEN GO TO READ_AN;
```

```
AN_COMP:PROCEDURE OPTIONS(MAIN) ;

         IF JN<=0 THEN GO TO READ_AN        ;  IF JN>50 THEN GO TO READ_AN;
            IF KHT(I) = 5000 THEN GO TO L_END  ; ELSE
          IF IN=50 THEN IF JN=1 THEN MHT=IHT(IN,JN) ;
             ELSE MHT=(IHT(IN,JN)+IHT( IN,JN-1))/2.0 ;       ELSE
         IF JN=1 THEN MHT=(IHT(IN,JN)+IHT(IN+1,JN))/2.0  ; ELSE
         MHT=(IHT(IN,JN)+IHT(IN,JN-1)+IHT(IN+1,JN)+IHT(IN+1,JN-1))/4.0;
                                 IF MHT >= 0 THEN
          IFA_AN(IN,JN)=KHT(I)+0.41870*DENSITY*(1-ELV*MHT/(21000*DENSITY))*MHT;
            ELSE IFA_AN(IN,JN)=KHT(I)+0.41870*(DENSITY-1.03)*(1-ELV*MHT/(21000*
          DENSITY))*MHT ;
        L_END: END;    GO TO READ_AN;
          MN_CLCS: PUT LIST(        '    LAT       LONG   SAMP. F.A.ANOM  STD.DEV.
HEIGHT  STD.ERR.   LAT       LONG   SAMP. F.A.ANOM  STD.DEV.   HEIGHT  HT.STD.ER
R.') SKIP(3);   PUT SKIP(3);
        PUT LIST('                       NO.    MGAL.     MGAL.    METERS    METERS
                   NO.    MGAL.     MGAL.    METERS    METERS'); PUT SKIP(3);
        PUT LIST(         '                                 HALF DEGREE MEANS')SKIP(2);
          PUT SKIP(2);
        TOTF,TOTSQF,KTOTF,HTRF,HTSQRF,HTNOF=0;
        DO I=1 TO 5 ;   DO J = 1 TO 5 ;
         TOTO,TOTSQO,KTOTO,HTRO,HTSQRO,HTNOO=0;
           DO II=1 TO 2 ;   DO JJ =1 TO 2;
           TOTH,TOTSQH,KTOTH,HTRH,HTSQRH,HTNOH=0;
            DO III=1 TO 5  ;  DO JJJ =1 TO 5 ;
        IN=(I-1)*10+(II-1)*5+III  ; JN=(J-1)*10+(JJ-1)*5+JJJ;
```

```
IF IFA_AN(IN,JN)¬=5000 THEN DO;
   TOTH=TOTH+IFA_AN(IN,JN); TOTSQH=TOTSQH+IFA_AN(IN,JN)*IFA_AN(IN,JN);
   KTOTH=KTOTH+1;  END; IF IHT(IN,JN) ¬= 100000 THEN DO ; HTNOH=HTNOH+1;
     HTRH=HTRH+IHT(IN,JN);HTSQRH=HTSQRH+IHT(IN,JN)*IHT(IN,JN);
 END ;
         END;END;
    IF KTOTH¬=0 THEN MEAN_H(II,JJ)=TOTH/KTOTH;ELSE MEAN_H(II,JJ)=0;
    IF KTOTH > 1 THEN STD_ERH(II,JJ)=SQRT(ABS((TOTSQH-KTOTH*MEAN_H(II,JJ)*
    MEAN_H(II,JJ))/(KTOTH-1))); ELSE STD_ERH(II,JJ)=0;
  NOH(II,JJ)=KTOTH;RLATH(II,JJ)=LAT_OR/100.0+I-1+(II-1)*0.5+0.15;
   RLONGH(II,JJ)=LONG_OR/100.0+J-1+(JJ-1)*0.5+0.25;
   IF HTNOH ¬=0 THEN HTH(II,JJ)=HTRH/HTNOH; ELSE HTH(II,JJ)=0;
   IF HTNOH > 1 THEN HSTDERH(II,JJ)=SQRT(ABS((HTSQRH-HTNOH*HTH(II,JJ)*
   HTH(II,JJ))/(HTNOH-1))); ELSE HSTDERH(II,JJ)=0;
   TOTO=TOTO+TOTH; TOTSQO=TOTSQO+TOTSQH;KTOTO=KTOTO+KTOTH;
   HTRO=HTRO+HTRH; HTSQRO=HTSQRO+HTSQRH; HTNOO=HTNOO+HTNOH;
    INH=(I-1)*2+II ;    JNH=(J-1)*2+JJ ;
 STD=STD_ERH(II,JJ)/10.0 ;
    IF MEAN_H(II,JJ) < 0 THEN DO ;
    HAN_M(INH,JNH)=MEAN_H(II,JJ)-10000*NOH(II,JJ)-1000000*STD ;
    END ;   ELSE DO ;
    HAN_M(INH,JNH)= MEAN_H(II,JJ)+10000*NOH(II,JJ)+1000000*STD ;
    END ;
    HHT_M(INH,JNH) = HTH(II,JJ) ;
 END;          END;
 IF KTOTO ¬=0 THEN MEAN_O(I,J)=TOTO/KTOTO; ELSE MEAN_O(I,J)=0;
 IF KTOTO > 1 THEN STD_ERO(I,J)=SQRT(ABS((TOTSQO-KTOTO*MEAN_O(I,J)
 *MEAN_O(I,J))/(KTOTO-1))); ELSE STD_ERO(I,J)=0;  NOO(I,J)=KTOTO ;
 RLATO(I,J)=LAT_OR/100.0+I-1+0.40; RLONGO(I,J)=LONG_OR/100.0+J-1+0.50;
IF HTNOO ¬=0 THEN HTO(I,J)=HTRO/HTNOO; ELSE HTO(I,J)=0;
 IF HTNOO > 1 THEN HSTDERO(I,J)=SQRT(ABS((HTSQRO-HTO(I,J)*HTO(I,J)*HTNOO)
 /(HTNOO-1))); ELSE HSTDERO(I,J)=0;
```

```
AN_COMP:PROCEDURE OPTIONS(MAIN) ;

      DO IK=1 TO 2 ;          DO JK=1 TO 2 ;
     PUT EDIT(RLATH(IK,JK),RLONGH(IK,JK),NOH(IK,JK),MEAN_H(IK,JK),
      STD_ERH(IK,JK),HTH(IK,JK),HSTDERH(IK,JK))(2F(9,2),F(5),F(9,1),3F(9));
     END ; PUT SKIP(1) ;   END ;
      TOTF=TOTF+TOTO;TOTSQF=TOTSQF+TOTSQO;KTOTF=KTOTF+KTOTO;
      HTRF=HTRF+HTRO;  HTSQRF=HTSQRF+HTSQRO; HTNOF=HTNOF+HTNOO;
      END;     END;
     PUT LIST(        '                      ONE DEGREE SQUARE MEANS')
 SKIP(3); PUT SKIP(3);
    DO KI=1 TO 5 ;
    RLATO(KI,6),RLONGO(KI,6),NOO(KI,6),MEAN_O(KI,6),STD_ERO(KI,6),HTO(KI,6),
    HSTDERO(KI,6)=0;          END;
    DO IK= 1 TO 5  ;    DO JK = 1 TO 3 ;  DO KI=1 TO 2 ;
            JJK=(JK-1)*2+KI      ;
    PUT EDIT(RLATO(IK,JJK),RLONGO( IK,JJK),NOO(IK,JJK),MEAN_O(IK,JJK),
       STD_ERO(IK,JJK),HTO(IK,JJK),HSTDERO( IK,JJK))(2F(9,1),F(5),F(9,1),3F(9));
   END; PUT SKIP(1) ;  END ; END ;
     IF KTOTF ¬=0 THEN MEAN_F=TOTF/KTOTF; ELSE MEAN_F=0;
     IF KTOTF ¬=1 THEN STD_ERF=SQRT(ABS((TOTSQF-KTOTF*MEAN_F*MEAN_F)/(KTOTF-1))
     )  ;     ELSE STD_ERF=0   ;        NOF=KTOTF  ;
    IF HTNOF ¬=0  THEN HTF=HTRF/HTNOF;ELSE HTF=0;
    IF HTNOF ¬=1 THEN HSTDERF=SQRT(ABS((HTSQRF-HTF*HTF*HTNOF)/(HTNOF-1)));
    ELSE HSTDERF=0; RLATF=LAT_OR/100.0+2.50; RLONGF=LONG_OR/100.0+2.50;
    PUT LIST(       '                      FIVE DEGREE SQUARE MEANS')
  SKIP(3);   PUT SKIP(3) ;
```

```
   PUT EDIT(RLATF,RLONGF,NOF,MEAN_F,STD_ERF,HTF,HSTDERF)(2F(9,1),F(5),F(9,1),
   3F(9));
   PUT LIST(        '                               PRINT OUT OF FREE AIR ANOMALIES')
SKIP(3);   PUT SKIP(2);
   DO I=1 TO 5;   DO J=1 TO 5;
     LAT=LAT_OR/100.0+I - 1  ;        LONG = LONG_OR/100.0 + J - 1 ;
 PUT LIST(' LATITUDE AND LONGITUDE OF  OF SW CORNER TENTH DEGREE SQUARE:',
   LAT,' N AND',LONG,' E RESPECTIVELY') SKIP(2) ; PUT SKIP(3) ;
   DO II=1 TO 10 ; IN =(I-1)*10+II ;  JN =(J-1)*10 ;
    PUT EDIT((IFA_AN(IN,JN+JJ) DO JJ=1 TO 10))(10F(10)) SKIP(2) ;
   END;     END;    END;
     PUT LIST ( '            LISTING OF CARDS PUNCHED ') SKIP(3) ;
     PUT SKIP (4) ;
   PUT LIST ('           HALF DEGREE FREE AIR ANOMALY MEANS ') SKIP(1) ;
         PUT SKIP(1) ;
     DO I=1 TO 10 ;    DO J=1 TO 2 ;
     LATOP=LAT_OR+(I-1)*50+15 ; LONGOP=LONG_OR+(J-1)*250+25 ;
     JJ=(J-1)*5 ;
             PUT EDIT(' 0', LATOP,LONGOP,(HAN_M(I,JJ+JK) DO JK=1 TO 5))(A(2),
    7F(10)) SKIP(1);
 PUT FILE(PUNCH) EDIT(' 0', LATOP,LONGOP,(HAN_M(I,JJ+JK) DO JK=1 TO 5),
     '           ')(A(2),7F(10),A(8)) ;    END ;   END ;
  PUT LIST('         ONE DEGREE FREE AIR ANOMALY MEANS ')SKIP(4) ;
 DO I=1 TO 5 ;    DO J=1 TO 5 ;
   STD=STD_ERO(I,J)/10.0 ;
    IF NOO(I,J)=100 THEN NOO(I,J)=99 ; ELSE ;
  IF MEAN_O(I,J) < 0 THEN DO;
 MEAN_O(I,J)=MEAN_O(I,J)-10000*NOO(I,J)-1000000*STD ;
 END ;
ELSE DO ;   MEAN_O(I,J)=MEAN_O(I,J)+10000*NOO(I,J)+1000000*STD ;
END ;
 END ;   END ;
```

Appendix (6) concluded.

```
AN_COMP:PROCEDURE OPTIONS(MAIN) ;

     DO I=1 TO 5 ;
    LATOP =LAT_OR+(I-1)*100+40 ;    LONGOP = LONG_OR + 40 ;
   PUT FILE(PUNCH) EDIT(' 0', LATOP,LONGOP,(MEAN_O(I,J) DO J=1 TO 5),
       '         ')(A(2),7F(10),A(8))             ;
                PUT EDIT(' 0', LATOP,LONGOP,(MEAN_O(I,J) DO J=1 TO 5))(A(2),
    7F(10)) SKIP(1) ;
   END ;
  PUT LIST('           TENTH DEGREE FREE AIR ANOMALY MEANS')SKIP(4);
    DO I=1 TO 5 ;     DO J=1 TO 5 ;
     DO II=1 TO 10 ;     DO JJ= 1 TO 2 ;    IK=(I-1)*10+II ;
      JK=(J-1)*10+(JJ-1)*5  ;  LATOP=LAT_OR-5+(I-1)*100+(II-1)*10 ;
      LONGOP=LONG_OR+5+JK*10 ;
   PUT FILE(PUNCH) EDIT(' 0', LATOP,LONGOP,(IFA_AN(IK,JK+JL) DO JL=1 TO 5),
       '         ')(A(2),7F(10),A(8)) ;
                PUT EDIT(' 0', LATOP,LONGOP,(IFA_AN(IK,JK+JL) DO JL=1 TO 5))(
      A(2),7F(10)) SKIP(1) ;              END;    END;
                                          END;    END;
  PUT LIST ('           HALF DEGREE MEAN HEIGHTS ')SKIP(6) ;
     PUT SKIP(1);
  DO I=1 TO 10 ;    DO  J=1 TO 2 ;
      LATOP=LAT_OR+(I-1)*50+15 ;  LONGOP=LONG_OR+(J-1)*250+25 ;
    JJ=(J-1)*5 ;
  PUT FILE(PUNCH) EDIT(' 0', LATOP,LONGOP,(HHT_M(I,JJ+JK) DO JK=1 TO 5),
       '         ')(A(2),7F(10),A(8))           ;
               PUT EDIT(' 0', LATOP,LONGOP,(HHT_M(I,JJ+JK) DO JK=1 TO 5))(A(2),
    7F(10)) SKIP (1) ;        END  ;   END ;
   PUT LIST ('            ONE DEGREE HEIGHT MEANS ')SKIP(4) ;
    DO I=1 TO 5 ;  LATOP=LAT_OR+(I-1)*100+40; LONGOP=LONG_OR ;
   PUT FILE(PUNCH) EDIT(' 0',  LATOP,LONGOP,(HTO(I,J) DO J=1 TO 5),'         ')
         (A(2),7F(10),A(8)) ;
                PUT EDIT(' 0',  LATOP,LONGOP,(HTO(I,J) DO J=1 TO 5))(A(2),
    7F(10)) SKIP(1) ;     END ;
    GO TO START;
 END AN_COMP;
```

APPENDIX (7)

```
L: 1JUL66                                IBM OS/360 BASIC FORTRAN IV (E) COMPILATION

      C     PROGRAM TOPNE
      C     CALCULATES DIFFERENTIAL ATTRACTION OF TOPOGRAPHY AT GEOID LEVEL
      C     ADOPTING A POSTULATED MODEL CONSIDERING THE EFFECT OF TOPOGRAPHY
      C     UPTO PSI GREATER THAN 1.5 DEGREES,EXCLUDING THE INNER FOUR TENTH
      C     DEGREE SQUARES.ELEVATIONS IN FEET.COMPUTATION AT TENTH DEGREE
      C     INTERVALS.OUTPUT FOR 1 X 1 DEGREE AREAS
            DIMENSION KHT(10),IHT(81,81),TOPEFF(12,12),CLAT(10),CLONG(10),
           1ITOT(11,11),ERF(11,11),IPVAL(10)
            IHV=1
            ICAP=81
            INT=1
            DO 1 I=1,ICAP
            DO 1 J=1,ICAP
          1 IHT(I,J)=-1000
      C     READ LAT,LONG OF CENTRE OF NW TENTH DEGREE SQUARE.+VE S,E
         90 READ(1,2)LAC,LOC
          2 FORMAT(2I10)
      C     READ HEIGHTS OF TENTH DEGREE SQ.MEANS IN FT,WITH LAT,LONG OF
      C     MOST WESTERLY SQUARE
          3 READ(1,4)ITE,KEY,LAH,LOH,RINT,(KHT(I),I=1,10)
          4 FORMAT(I2,I2,2I6,F4.1,10I6)
            WRITE(3,100)ITE,LAH,LOH,(KHT(I),I=1,10)
        100 FORMAT(' ',I1,2I10,10I6)
            IF(ITE-1)5,105,15
          5 DO 14 I=1,10
            IN=(LAH-LAO)/10.0+1
            JN=(LOH-LOO)/10.0+I
            IF(IN)12,12,6
          6 IF(IN-ICAP)7,7,12
          7 IF(JN)12,12,8
          8 IF(JN-ICAP)9,9,12
          9 IF(KHT(I))10,10,11
         10 IHT(IN,JN)=100000
            GO TO 14
         11 IHT(IN,JN)=KHT(I)
            GO TO 14
         12 WRITE(3,13)IN,JN
         13 FORMAT(44H HEIGHT READING OUTSIDE AREA,POSITION BEING ,2I10/)
         14 CONTINUE
            IF(ITE-1)3,105,15
         15 WRITE(3,16)LAH,LOH
         16 FORMAT(15H CARD WITH LAT ,I5,17H S AND LONGITUDE ,I5,29H E HAS BEE
           1N PUNCHED IN ERROR )
            GO TO 3
      C     END OF READING HEIGHTS.STORED IN FEET
        105 READ(1,106)ITE,KEY,LAH,LOH,RINT,(KHT(I),I=1,10)
        106 FORMAT(I1,I2,2I6,F5.2,10I6)
            WRITE(3,100)ITE,LAH,LOH,(KHT(I),I=1,10)
            IF(ITE-1)107,17,116
        107 DO 115 I=1,10
            IN=(LAH-LAO)/10.0+1
            JN=(LOH-LOO)/10.0+I
            IF(IN)114,114,108
```

```
    1C8 IF(IN-ICAP)109,109,114
    109 IF(JN)114,114,110
    110 IF(JN-ICAP)111,111,114
    111 IF(KHT(I))112,112,113
    112 IHT(IN,JN)=100000
        GC TC 115
    113 IHT(IN,JN)=KHT(I)
        GC TO 115
    114 WRITE(3,13)IN,JN
    115 CCNTINUE
        IF(ITE-1)105,17,116
    116 WRITE(3,16)LAH,LOH
        GO TO 1C5
     17 REAC(1,61)ITE,KEY,LAH,LOH,RINT,(KHT(I),I=1,10)
     61 FORMAT(I1,I2,2I6,F4.1,10I6)
        WRITE(3,100)ITE,LAH,LCH,(KHT(I),I=1,10)
        IF(ITE-1)62,72,71
     62 DC 7C I=1,10
        IN=(LAH-LAC)/10.0+1
        JN=(LCH-LOO)/10.0+I
        IF(IN)69,69,63
     63 IF(IN-ICAP)64,64,69
     64 IF(JN)69,69,65
     65 IF(JN-ICAP)66,66,69
     66 IF(KHT(I))67,67,68
     67 IHT(IN,JN)=100000
        GC TO 70
     68 IHT(IN,JN)=KHT(I)
        GC TO 70
     69 WRITE(3,13)IN,JN
     7C CCNTINUE
        IF(ITE-1)17,72,71
     71 WRITE(3,16)LAH,LOH
        GO TC 17
     72 LIM=26
        ISPA=2
        WRITE(3,18)
     18 FCRMAT(109H COMPUTATION OF DIFFERENTIAL TCPOGRAPHICAL EFFECT FOR O
       1UTER ZONES FOR PSI LESS THAN 1.5.INNER ZONE EXCLUDED  ///)
        IF(IHV-1)5C,48,48
     48 WRITE(3,49)
     49 FORMAT(/86H              THE HUNTER FORMULA,               RHO=(2.77-H/21)
       1,              H IN KM.BEING USED  ///)
     50 READ(1,47)CON,RHO,PI,RADM,CON2,CON1
     47 FORMAT(E10.3,F10.2,F10.5,E10.4,E1C.2,E10.1)
        CCCN=(CCN*RHO*PI*PI*CON2)/(4*180*180*RADM)
        CCCN1=-4*PI*CCN*RHO*CCN1
        WRITE(3,73)CON,RHO,PI,RADM,CON2,CCON,CCON1
     73 FORMAT(' CON= ',E10.3,' RHO= ',F10.2,' PI= ',F10.5,' RADM= ',
       1E10.4,' CCN2= ',E10.2,' CCON= ',E10.5,' CCON1= ',E10.5///)
        NC=1
        NCL=1
        DC 36 I=16,LIM,INT
        IND=(I-16.0)/INT+1
        CLAT(IND)=LAO/100.0+(NU-1)*5.0+(I-16)*0.5+1.5
```

```
      RADF=12.546*1000*SQRT(COS(CLAT(IND)*0.0174533))
      DC 35 J=16,LIM,INT
      JND=(J-16.0)/INT+1
      CLONG(JND)=LCO/100.0+(NOL-1)*5.0+(J-16)*0.5+1.5
      ACC=0
      KTCT=C
      ACCER=0
      DC 31 II=1,30
      IH=(I-16)*5+(NO-1)*50+II
      QLAT=LAC/100.0+IH/10.0-0.1
      CQLAT=CCS(QLAT*0.0174533)
      DC 30 JJ=1,30
      IF(II-15)22,2C,19
   19 IF(II-16)30,20,22
   2C IF(JJ-15)22,30,21
   21 IF(JJ-16)30,30,22
   22 JH=(NCL-1)*50+(J-16)*5+JJ
      IF(IHT(IH,JH))30,32,23
   23 IF(IHT(IH,JH)-1C0000)24,30,28
   24 QLCNG=LCO/100.C+JH/10.0-0.1
      DLAT=(QLAT-CLAT(IND))*0.0174533
      DLCNG=(QLCNG-CLCNG(JND))*CQLAT*0.0174533
      PSSQ=DLAT*DLAT+DLCNG*DLONG
      PSI=SQRT(PSSQ)
      IF(PSI)26,26,25
   25 FPSI=-4.0*(1-15*PSSQ/24.0)/(PSSQ*PSI)
      RHT=C.304799*IHT(IH,JH)
      VAR=FPSI*CQLAT*(1-IHV*RHT/(21000*RHO))
      ACC=ACC+VAR*RHT
      ACCER=ACCER+VAR*VAR
      KTCT=KTCT+1
      GC TC 30
   26 WRITE(3,27)PSI
   27 FORMAT(41H ERROR IN SINE PSI. VALUE OBTAINED BEING ,F20.8/)
      GC TC 30
   28 WRITE(3,29)IH,JH,IHT(IH,JH)
   29 FCRMAT(41H ERROR IN LOADING HEIGHT FOR ELEMENT NO.(,2I10,28H ),THE
     1 VALUE CBTAINED BEING ,I10//)
   3C CCNTINUE
   31 CCNTINUE
      GC TC 34
   32 WRITE(3,33)CLAT(IND),CLONG(JND)
   33 FORMAT(43H INSUFFICIENT HEIGHTS TC COMPUTE EFFECT FOR,F10.2,14H DE
     1GREES SOUTH,F10.2,14H DEGREES EAST //)
      TCPEFF(INC,JNC)=0
      ERF(IND,JND)=0
      ITCT(IND,JND)=0
      GC TC 35
   34 LA=(NC-1)*50+(I-16)*5+16
      LC=(NCL-1)*50+(J-16)*5+16
      IF(IHT(LA-1,LC-1)-100000)75,74,86
   74 IT1=C
      GC TC 76
   75 IT1=IHT(LA-1,LO-1)
   76 IF(IHT(LA-1,LO)-100000)78,77,86
```

```
   77 IT2=C
      GO TO 79
   78 IT2=IHT(LA-1,LO)
   79 IF(IHT(LA,LC-1)-100000)81,80,86
   80 IT3=0
      GC TC 82
   81 IT3=IHT(LA,LC-1)
   82 IF(IHT(LA,LO)-100000)84,83,86
   83 IT4=C
      GO TO 85
   84 IT4=IHT(LA,LC)
   85 JHT=(IT1+IT2+IT3+IT4)*0.304799/4.0
      GO TC 88
   86 JHT=1
      WRITE(3,87)CLAT(IND),CLONG(JND)
   87 FORMAT(' ERROR IN LOADING HEIGHTS FUR LAT= ',F10.2,' N;LONG= ',
     1F10.2,' E.MULTIPLY RESULT BY TRUE HT FOR ANS. ')
   88 IF(JHT)153,153,154
  153 TCPIN=0
      TCPUT=C
      GC TC 155
  154 RATIO=JHT/RADF
      TOPIN=CCON1*JHT*(1-0.5*RATIO+RATIO*RATIO*RATIO/8.0)*(1-IHV*JHT/(
     121000*RHO))
      TOPUT=CCCN*JHT*ACC
  155 TOPEFF(INC,JND)=TCPIN+TOPUT
      WRITE(3,89)CLAT(IND),CLONG(JND),JHT,TOPIN,TOPUT
   89 FCRMAT(' LAT= ',F10.2,' N.LCNG=',F10.2,' E.MN ELEV= ',I5,' MET.IN.
     1CCNT=',F10.2,' MGAL.CUTER= ',F10.2,' MGAL.')
      ITOT(INC,JND)=KTOT
      ERF(INC,JND)=CCCN*30*SQRT(ACCER)
   35 CONTINUE
   36 CONTINUE
      WRITE(3,37)
   37 FORMAT(//120H TOP.COR.  TOP.COR.  TCP.COR.  TOP.COR.  TOP.COR.  TO
     1P.COR.  TOP.COR.  TCP.COR.  TOP.COR.  TCP.COR.          LATITUDE
     2 /)
      WRITE(3,38)
   38 FORMAT(120H   MGAL.     MGAL.     MGAL.     MGAL.     MGAL.     MG
     1AL.     MGAL.     MGAL.     MGAL.     MGAL.          DEGREES S /
     2//)
      DC 150 I=1,10
      ST=TCPEFF(I,11)
      TOPEFF(I,11)=CLAT(I)
      WRITE(3,39)(TCPEFF(I,J),J=1,11)
   39 FORMAT(10F10.2,F15.2/)
      TCPEFF(I,11)=ST
  15C CCNTINUE
      WRITE(3,40)(CLONG(J),J=1,10)
   40 FORMAT(///10F10.2/)
      WRITE(3,41)
   41 FORMAT(72H                                  L O N G I T U D E   I N   D
     1E G R E E S   E ///)
      WRITE(3,54)
   54 FORMAT(///58H                          ESTIMATES OF ERRORS IN COMPUTED V
```

```
     1ALUES//)
      WRITE(3,37)
      WRITE(3,38)
      DC 151 I=1,10
      ERF(I,11)=CLAT(I)
      WRITE(3,51)(ERF(I,J),J=1,11)
   51 FCRMAT(10F10.5,F15.2/)
  151 CCNTINUE
      WRITE(3,40)(CLONG(J),J=1,10)
      WRITE(3,41)
      WRITE(3,42)
   42 FCRMAT(120H NC. USED  NO. USED  NC. USED  NO. USED  NO. USED  NO.
     1USED  NO. USED  NO. USED  NO. USED  NO. USED            LATITUDE  /
     2)
      WRITE(3,43)
   43 FCRMAT(120H IN CCMP.  IN COMP.  IN COMP.  IN COMP.  IN COMP.  IN C
     1CMP.  IN COMP.  IN COMP.  IN COMP.  IN COMP.            DEGREES S /
     2//)
      DC 152 I=1,1C
      ITOT(I,11)=CLAT(I)
      WRITE(3,44)(ITCT(I,J),J=1,11)
   44 FCRMAT(10I10,I15/)
  152 CONTINUE
      WRITE(3,40)(CLCNG(J),J=1,10)
      WRITE(3,41)
      DC 117 I=1,80
      DC 117 J=1,80
  117 IHT(I,J)=0
      DC 128 I=1,10
      DC 127 J=1,1C
      DC 126 II=1,5
      DC 125 JJ=1,5
      IN=(I-1)*5+II
      JN=(J-1)*5+JJ
      IF(II-1)123,118,121
  118 IF(JJ-1)123,119,120
  119 IHT(IN,JN)=TOPEFF(I,J)*10
C       STCRED IN TENTH MILLIGAL
      GC TO 125
  120 IHT(IN,JN)=((5-JJ)*TCPEFF(I,J)+JJ*TOPEFF(I,J+1))*10/5.0
      GC TC 125
  121 IF(JJ-1)123,122,125
  122 IHT(IN,JN)=((5-II)*TOPEFF(I,J)+II*TCPEFF(I+1,J))*10/5.0
      GC TC 125
  123 WRITE(3,124)II,JJ,I,J,IN,JN
  124 FORMAT(' ERROR IN INDEX CALCS.II= ',I5,' JJ= ',I5,' I= ',I5,' J= '
     1,I5,' IN= ',I5,' JN= ',I5)
  125 CCNTINUE
  126 CCNTINUE
  127 CCNTINUE
  128 CCNTINUE
      DC 135 I=1,1C
      DC 134 J=1,10
      DC 133 II=1,5
      DG 132 JJ=1,5
```

```
      INN=(I-1)*5+1
      JNN=(J-1)*5+1
      IN=(I-1)*5+II
      JN=(J-1)*5+JJ
      IF(II-1)131,132,129
  129 IF(JJ-1)131,132,130
  130 IHT(IN,JN)=((5-II)*IHT(INN,JN)+II*IHT(INN+5,JN)+(5-JJ)*IHT(IN,JNN)
     1+JJ*IHT(IN,JNN+5))/10.0
      GC TC 132
  131 WRITE(3,124)II,JJ,I,J,IN,JN
  132 CONTINUE
  133 CCNTINUE
  134 CCNTINUE
  135 CONTINUE
      DC 139 I=1,5
      DC 138 J=1,5
      DC 137 II=1,10
      LAT=LAC+(I-1)*100+(II-1)*10+150
      LCNG=LCC+(J-1)*100+150
      IND=(I-1)*10+II
      JND=(J-1)*10
      DC 145 JJ=1,10
      JJND=JND+JJ
      IPVAL(JJ)=IHT(IND,JJND)
  145 CCNTINUE
      WRITE(2,136)LAT,LCNG,(IPVAL(JJ),JJ=1,10)
      WRITE(3,136)LAT,LONG,(IPVAL(JJ),JJ=1,10)
  136 FCRMAT(' 0',2I6,10I6)
  137 CCNTINUE
  138 CCNTINUE
  139 CCNTINUE
      WRITE(3,140)
  140 FCRMAT(//'    PRINT CUT OF INTERPOLATED VALUES IN TENTH MGAL '//)
      DC 144 I=1,5
      DC 143 J=1,5
      DC 142 II=1,10
      IND=(I-1)*10+II
      JND=(J-1)*10
      DC 146 JJ=1,10
      JJND=JND+JJ
      IPVAL(JJ)=IHT(IND,JJND)
  146 CCNTINUE
      WRITE(3,141)IND,JND,(IPVAL(JJ),JJ=1,10)
  141 FCRMAT(12I9)
  142 CCNTINUE
  143 CCNTINUE
  144 CCNTINUE
   45 CONTINUE
   46 CCNTINUE
      STCP
      END

      SIZE CF COMMON   0C0CC0    PROGRAM   041018

                 END OF COMPILATION   MAIN
```

APPENDIX (8)

UL66

IBM OS/360 BASIC FORTRAN IV (E) COMPILATION

```
C       PROGRAM TOPMNS
C       CALCULATES THE DIFFERENTIAL ATTRACTION OF TOPOGRAPHY AT GEOID LEVEL
C       ADOPTING A POSTULATED MODEL CONSIDERING EFFECT OF TOPOGRAPHY UP TO
C       PSI EQUALS 1.5 DEGREES .THIS IS VIRTUALLY THE DIFFERENTIAL ATTRACTION
C       CYLINDER.
        DIMENSION KHT(5),IHT(80,80),TOPEFF(11,11),CLAT(10),CLONG(10),
       1ITOT(10),IER(10),ITOP(5)
        IHV=1
        KEY=3
        IF(KEY-2)1,2,3
      1 ICAP=50
        JCAP=80
        RNT=0.5
        GO TO 4
      2 ICAP=64
        JCAP=64
        RNT=1.0
        GO TO 4
      3 ICAP=36
        JCAP=72
        RNT=5.0
      4 DO 5 I=1,ICAP
        DO 5 J=1,JCAP
      5 IHT(I,J)=-1000
C       READ LAT,LONG OF CENTRE OF SW SQUARE IN TENTH DEGREES,+VEN,E
        READ(1,6)LAO,LOO
      6 FORMAT(2I10)
C       READ MEAN HEIGHTS OF SQUARES IN FT.,WITH LAT,LONG OF MOST WESTERLY SQ
        IF(KEY-2)7,7,7
    118 READ(5,119)LA,IHN,(KHT(I),I=1,15)
    119 FORMAT(I3,I2,15I5)
        IF(LA+8)120,121,125
    120 IF(LA[illegible])125,121,121
    121 IF(IHN-8)125,123,122
    122 IF(IHN-11)123,123,125
    123 DO 124 I=1,15
        LAH=(LA-0.5)*10
        LOH=((15*(IHN-1)+(I-1)+0.5)*10)
        IN=(LAH-LAO)/(RNT*10)+1
        IF(KHT(I))126,126,127
    126 IHT(IN,JN)=100000
        GO TO 124
    127 IHT(IN,JN)=KHT(I)
    124 CONTINUE
    125 [illegible])21,21,118
      7 READ(1,8)ITE,[illegible]LOH,(KHT(I),I=1,5)
      8 FORMAT(I1,2I10,5I10)
        IF(ITE-1)9,21,19
      9 DO 18 I=1,5
        IN=(LAH-LAO)/50+1
        JN=(LOH-LOO)/50+I
        IF(IN)16,16,10
     10 IF(IN-ICAP)11,11,16
```

```
   11 IF(JN)16,16,12
   12 IF(JN-JCAP)13,13,16
   13 IF(KHT(I))14,14,15
   14 IHT(IN,JN)=100000
      GO TO 18
   15 IHT(IN,JN)=KHT(I)
   18 CONTINUE
      WRITE(3,132)ITE,LAH,LOH,(KHT(I),I=1,5),IN,JN
  132 FORMAT(' ',I5,7I10,' IN =',I5,' JN =',I5)
      GO TO 137
   16 RRLOH=LOH+(I-1)*100*RNT
      WRITE(3,17)IN,JN,LAH,RRLOH
   17 FORMAT(/' HEIGHT READING OUTSIDE AREA,POSN =',2I10,' AND LAT',I10,
     1' ,LONG ',F10.1)
  137 IF(ITE-1)7,21,19
   19 WRITE(3,20)LAH,LOH
   20 FORMAT(15H CARD WITH LAT ,I10,15H AND LONGITUDE ,I10,27H HAS INDEX
     1 PUNCHED IN ERROR/)
      GO TO 7
C     END OF READING HEIGHTS.STORED IN FT.
   21 IF(KEY-2)22,23,24
   22 IBASE=4
      LIM=13
      INT=1
      ISPA=6
      JSPA=7
      GO TO 25
   23 IBASE=3
      LIM=12
      INT=1
      ISPA=6
      JSPA=6
      GO TO 25
   24 IBASE=2
      LIM=11
      INT=1
      JSPA=8
      ISPA=4
   25 WRITE(3,26)RNT,RNT
   26 FORMAT(53H COMPUTATION OF DIFFERENTIAL TOPOGRAPHICAL EFFECT FOR,F1
     10.1,10H DEGREE X ,F10.1,20H DEGREE SQUARE MEANS///)
      WRITE(3,310)LAO,LOO
  310 FORMAT(/' GEOGRAPHICAL COORDINATES OF SW CORNER ARE:- LAT =',I10,
     1' N; LONG =',I10,' E')
      IF(IHV-1)130,128,102
  128 WRITE(3,129)
  129 FORMAT(/86H            THE HUNTER FORMULA,            RHO=(2.77-H/21)
     1,            H IN KM.BEING USED  ///)
  130 READ(1,27)CON,RHO,PI,CON2
   27 FORMAT(E10.3,F10.2,F10.5,E10.2)
      READ(1,62)(IER(I),I=1,3)
   62 FORMAT(3I10)
      CCON=-4*PI*CON*RHO*CON2
      RAD=166700.0
      WRITE(3,311)CON,RHO,PI,CON2,CCON,RAD
```

```
     1E VALUE OBTAINED BEING ,I10//)
   48 CONTINUE
   49 CONTINUE
   52 IF(KTOT)53,303,55
   53 WRITE(3,54)RNT,CLAT(IND),CLONG(JND)
   54 FORMAT(/36H ERROR IN COMP. OF MEAN HEIGHTS FOR ,F10.2,22H DEGREE S
     1QUARE AT LAT ,F10.2,9H AND LONG,F10.2)
      GO TO 92
  303 MHT=0
      KTOT=0
      GO TO 305
   55 MHT=ACC/KTOT
  305 WRITE(3,140)CLAT(IND),CLONG(JND),MHT,IHT(INDHH-3,JNDHH-3),KTOT
  140 FORMAT(' AT LAT= ',F10.1,' AND LONG =',F10.1,' E,ELEVATION=',I10,
     1' MET.WEIGHTED ELEV =',I10,' MET.SUM OF WTS.= ',I10)
      GO TO 93
   56 ACC=0
      KTOT=0
      WE1=0.0
      WE2=0.0
      WE3=1.0
      DO 73 II=1,5
      INDHH=INDH+II-3
      DO 72 JJ=1,5
      JNDHH=JNDH+JJ-3
      IF(II-2)64,58,57
   57 IF(II-4)60,58,64
   58 IF(JJ-2)64,65,59
   59 IF(JJ-4)65,65,64
   60 IF(JJ-2)64,65,61
   61 IF(JJ-4)66,65,64
   64 IWE=WE1
      GO TO 67
   65 IWE=WE2
      GO TO 67
   66 IWE=WE3
   67 IF(IHT(INDHH,JNDHH))72,74,68
   68 IF(IHT(INDHH,JNDHH)-100000)70,72,71
   70 ACC=ACC+IHT(INDHH,JNDHH)*IWE
      KTOT=KTOT+IWE
      GO TO 72
   71 WRITE(3,47)INDHH,JNDHH,IHT(INDHH,JNDHH)
   72 CONTINUE
   73 CONTINUE
      GO TO 75
   74 WRITE(3,51)CLAT(IND),CLONG(JND)
   51 FORMAT(' ERROR IN LOADING AT LAT=',F10.1,' AND LONG=',F10.1)
   75 IF(KTOT)76,308,77
   76 WRITE(3,54)RNT,CLAT(IND),CLONG(JND)
      GO TO 92
  308 MHT=0
      KTOT=0
      GO TO 309
   77 MHT=ACC/KTOT
  309 WRITE(3,140)CLAT(IND),CLONG(JND),MHT,IHT(INDHH-2,JNDHH-2),KTOT
```

```
      GO TO 93
   78 IF(IHT(INDH,JNDH))92,92,79
   79 IF(IHT(INDH,JNDH)-100000)80,92,81
   80 MHT=IHT(INDH,JNDH)
      GO TO 93
   81 WRITE(3,82)CLAT(IND),CLONG(JND)
   82 FORMAT(' ERROR IN LOADING HEIGHT AT LAT =',F10.2,' LONG =',F10.2)
   92 TOPEFF(IND,JND)=0
      WRITE(3,84)INDH,JNDH,CLAT(IND),CLONG(JND),IHT(INDH,JNDH),
     1TOPEFF(IND,JND)
   84 FORMAT(' INDEX =',2I5,' LAT =',F10.2,' LONG =',F10.2,' ELEVATION =
     1',I10,' TOP COR =',F10.2)
      GO TO 94
   93 RATIO=MHT/RAD
      RATIOS=RATIO*RATIO
      IF(MHT-2500)300,300,301
  300 IHV=1
      COON=CCON
      GO TO 302
  301 IHV=0
      COON=(CCON*2.67)/2.77
  302 TOPEFF(IND,JND)=COON*MHT*(1-0.5*RATIO+RATIO*RATIOS/8.0-RATIOS*
     1RATIOS*RATIO/16.0)*(1-IHV*MHT/(21000*RHO))
      WRITE(3,84)INDH,JNDH,CLAT(IND),CLONG(JND),IHT(INDH,JNDH),
     1TOPEFF(IND,JND)
   94 CONTINUE
   95 CONTINUE
      WRITE(3,96)
   96 FORMAT(//'1   TOP.COR.  TOP.COR.  TOP.COR.  TOP.COR.  TOP.COR.  TO
     1P.COR.  TOP.COR.  TOP.COR.  TOP.COR.  TOP.COR.          LATITUDE
     2'/)
      WRITE(3,97)
   97 FORMAT(120H   MGAL.     MGAL.     MGAL.     MGAL.     MGAL.     MG
     1AL.     MGAL.     MGAL.     MGAL.     MGAL.           DEGREES N /
     2//)
      DO 131 I=1,10
      TOPEFF(I,11)=CLAT(I)
      WRITE(3,98)(TOPEFF(I,J),J=1,11)
   98 FORMAT(10F10.2,F15.2/)
  131 CONTINUE
      WRITE(3,99)(CLONG(J),J=1,10)
   99 FORMAT(///10F10.2/)
      WRITE(3,100)
  100 FORMAT(/72H                          L O N G I T U D E   I N   D
     1 E G R E E S   E ///)
      WRITE(3,90)
   90 FORMAT('1            LISTING OF CARDS PUNCHED '///)
      DO 136 I=1,10
      DO 135 JJ=1,2
      JK=(JJ-1)*5
      LATP=CLAT(I)*100
      LONGP=CLONG(JK+1)*100
      DO 133 K=1,5
      JJK=JK+K
      ITOP(K)=TOPEFF(I,JJK)*10
```

```
      133 CONTINUE
          WRITE(2,134)LATP,LONGP,(ITOP(K),K=1,5)
          WRITE(3,134)LATP,LONGP,(ITOP(K),K=1,5)
      134 FORMAT(' 0',7I10)
      135 CONTINUE
      136 CONTINUE
      101 CONTINUE
      102 CONTINUE
          ERF=CCON*IER(KEY)/RAD
          WRITE(3,63)ERF
       63 FORMAT(//47H ERROR IN FINAL RESULT ESTIMATED AS                ,F10.1,
         116H            MGAL. ///)
          STOP
          END

             SIZE OF COMMON  000000   PROGRAM  036460

                              END OF COMPILATION  MAIN
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1 JUL66 APPENDIX (9)

IBM OS/360 BASIC FORTRAN IV (E) COMPILATION

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C        PROGRAM STOKUT
C        CALCULATES THE COMPONENTS OF STOKESIAN HEIGHT AND DEFLECTIONS OF THE
C        VERTICAL FOR AN AREA PSI> 20,THE INTERVAL OF COMPUTATION BEING 1.BASI
C        PROGRAM COMPUTES EFFECT OVER 10 X 10 AREA,THE ANOMALIES  REPRESENTING
C        5 SQUARES.THE EFFECT US CONSIDERED FOR FOR WORLD WIDE EFFECT AROUND T
C        X 10  SQUARE OF COMPUTATION.SYMMETRIC WITH RESPECT TO THE SW CORNER
C        AN INNER AREA OF45 X 45  AROUND THE COMPUTATION POINT,SYMMETRIC W.R.T
C        TO IT  IS OMITTED.LAT,LONG +VE N,E RESPECTIVELY.PROGRAM HANDLES A
C        AREA WHOSE SIZE IS CONTROLLED BY IBA AND ISPA.ALL UNITS ARE DEGREES
         DIMENSION KAN(5),IAN(36,72),COMPN(11,11),COMPDM(11,11),
        1COMPDP(11,11),CLAT(10),CLONG(10),ITOT(11,11),PRECN(11,11),
        2PRECDM(11,11),PRECDP(11,11),INTER(3,6),ISIS(3,6),CORGCN(11,11),
        3CORGCM(11,11),CORGCP(11,11),CORSN(11,11),CORSM(11,11)
         DIMENSION CORSP(11,11),CORTN(11,11),CORTM(11,11),CORTP(11,11),
        1SISN(11,11),SISM(11,11),SISP(11,11)
         DIMENSION RAN(9),IHT(36,72)
         ICAP=36
         JCAP=72
         INT=1
         DO 1 I=1,ICAP
         DO 1 J=1,JCAP
         IHT(I,J)=0
       1 IAN(I,J)=0
C        READ LAT,LONG OF CENTRE OF SW FIVE  DEGREE SQUARE IN TENTH DEGREES.
         READ(1,2)LAC,LOC
       2 FORMAT(2I10)
C        READ GRAVITY ANOMALIES REPRESENTING  FIVE DEGREE SQUARE MEANS,IN TENT
C        MILLIGAL,WITH LAT,LONG OF MOST WESTERLY SQ.IN  TENTH DEGREES.
         READ(1,300)FLAT,DFLAT,RAD,DRAD,GRCON,DGRCON,POTCOR,OMSQ
     300 FORMAT(8E10.4)
         WRITE(3,323)FLAT,RAD
     323 FORMAT(' FLATTENING =',E10.5,' RADIUS =',E10.5)
         BETA=5.2884E-3
         EQGR=9.78049E6
         SEE=RAD*100.0*OMSQ/(EQGR*1.0E-4)
         RATRAD=DRAD/RAD
         RATGRC=DGRCON/GRCON
         DSEEGC=-SEE*RATGRC
         DSEES=SEE*3*RATRAD
         DSEET=SEE*(3*RATRAD-RATGRC)
         DBTAGC=DSEEGC*5/2.0
         DBTAS=DSEES*5/2.0-DFLAT
         DBTAT=DSEET*5/2.0-DFLAT
         DEQGC=EQGR*(RATGRC-DSEEGC*5/2.0)
         DEQGS=EQGR*(-2*RATRAD+DFLAT-DSEES*5/2.0)
         DEQGT=EQGR*(RATGRC-2*RATRAD+DFLAT-DSEET*5/2.0)
       3 READ(1,4)(RAN(J),J=1,9),RLAH,RLOH
       4 FORMAT(11F6.1)
         DO 12 I=1,9
         IN=(RLAH+90)/5.0-I+1
         LLCH=RLOH
         IF(LLCH-180)92,91,91
      91 LLCH=LLCH-360
```

```
   92 JN=(LLCH+180)/5+1
      IF(IN)10,10,6
    6 IF(IN-ICAP)7,7,10
    7 IF(JN)10,10,8
    8 IF(JN-JCAP)9,9,10
    9 IAN(IN,JN)=RAN(I)*10.0
   12 CONTINUE
      WRITE(3,70)RLAH,RLOH,(RAN(J),J=1,9),IN,JN
   70 FORMAT(' ',11F10.1,' IN=',I5,' JN=',I5)
      IF(RLAH+45.0)13,138,3
  138 IF(RLOH-355.0)3,15,13
   10 WRITE(3,11)IN,JN,RLAH,RLOH
   11 FORMAT(' ANOMALY WITH INDEX VALUES (',2I10,' )OUTSIDE LIMITS.
     1LATITUDE ON CARD IS',F10.1,' AND LONG', F10.1)
      GO TO 3
   13 WRITE(3,14)RLAH,RLOH
   14 FORMAT('  CARD WITH LAT =',F10.1,' AND LONG =',F10.1,'  HAS INDEX
     1 PUNCHED IN ERROR')
C     END OF READING ANOMALIES.STORED IN TENTH MILLIGAL.
C     IAN IS STRUCTURED   AS FOLLOWS:-COUNTING DIGITS FROM RIGHT,1-4 GIVE
C     MEAN ANOMALY IN TENTH MILLIGAL,,5-6 GIVE NO. OF READINGS IN SQUARE,
C     7 TO 8 GIVES KEY TO THE PROCESS:- 0 = DIRECT ; 1 = INTERPOLATION
C     2 = INTERPOLATION/EXTRAPOLATION ; 3 = EXTRAPOLATION
   15 READ(1,5)ITE,LAH,LOH,(KAN(I),I=1,5)
    5 FORMAT(I1,7I10)
      IF(ITE-1)93,117,115
   93 DO 97 I=1,5
      IN=(LAH+875)/50.0+1
      JN=(LOH+1775)/50.0+1
      IF(IN)99,99,94
   94 IF(IN-ICAP)95,95,99
   95 IF(JN)99,99,96
   96 IF(JN-JCAP)97,97,99
   97 IHT(IN,JN)=KAN(I)
      WRITE(3,98)LAH,LOH,(KAN(J),J=1,5),IN,JN
   98 FORMAT(' ',7I10,' IN =',I5,' JN =',I5)
      GO TO 114
   99 WRITE(3,139)IN,JN,LAH,LOH
  139 FORMAT(' ANOMALY WITH INDEX VALUES(',2I10,' ) OUTSIDE LIMITS. LATI
     1TUDE ON CARD IS ',I10,' LONG =',I10)
  114 IF(ITE-1)15,117,115
  115 WRITE(3,116)LAH,LOH
  116 FORMAT(' CARD WITH LAT =',I10,' AND LONG =',I10,' HAS INDEX PUNCHED
     1D IN ERROR ')
      GO TO 15
  117 IBA=40
      ISPA=5
      LIM=IBA+9
      READ(1,16)CON,RHO,PI,IARC,TGRAV,CON1,RAD
   16 FORMAT(E10.3,F10.2,F10.5,I10,F10.2,E10.2,E10.4)
      CCONN=(RAD*CON1*PI)/(4*TGRAV*180*180)
      CCOND=(PI*IARC*CON1)/(4*TGRAV*180*180)
      WRITE(3,17)CCONN,CCOND
   17 FORMAT(/47H COEFFICIENTS FOR COMPUTATION OF SEPARATION IS ,E20.5,
     1'  AND OF DEFLECTIONS IS  ',E20.5)
```

```
      READ(1,110)SER,NUM
  110 FORMAT(F10.1,I10)
      NO=1
      NCL=1
      DO 46 I=1,10
      IND=I
      CLAT(IND)=LAO/10.0-2.5+(NO-1)*10.0+I-1
      COL=CLAT(IND)*0.0174533
      SCLAT=SIN(COL)
      CCLAT=COS(COL)
      CLAT5=CLAT(IND)/5.0
      LAT5=CLAT5
      DEC5=(CLAT5-LAT5)*5
      IF(DEC5)75,76,76
   75 DEC5=DEC5+5.00
   76 AEC5=ABS(DEC5)
      IF(AEC5)19,21,18
   18 IF(AEC5-5)21,19,19
   19 WRITE(3,20)CLAT(IND)
   20 FORMAT(41H ERROR IN CALCULATION OF DEC AT LATITUDE ,F20.2/)
      GO TO 46
   21 JBA=100
      LIMJ=JBA+9
      DO 45 J=1,10
      JND=J
      CLONG(JND)=LCO/10.0-2.5+(NCL-1)*10+J-1
      CLONG5=CLONG(JND)/5.0
      LONG5=CLONG5
      DECL5=(CLONG5-LONG5)*5
      AECL5=ABS(DECL5)
      IF(AECL5)23,25,22
   22 IF(AECL5-5)25,23,23
   23 WRITE(3,24)CLAT(IND),CLONG(JND)
   24 FORMAT(/45H ERROR IN THE CALCULATION OF DECL AT LATITUDE,F20.2
     1 AND LONGITUDE ,F20.2/)
      GO TO 45
   25 ACC=0
      KTOT=0
      ACCM=0
      ACCP=0
      ACCER=0
      ACCPER=0
      ACCMER=0
      ACCN=0.0
      AGCM=0.0
      AGCP=0.0
      ASN=0.0
      ASM=0.0
      ASP=0.0
      ATN=0.0
      ATM=0.0
      ATP=0.0
      SISTON=0.0
      SISTOM=0.0
      SISTOP=0.0
```

```
      WRITE(3,71)CLAT(IND),CLONG(JND),DEC5,DECL5
   71 FORMAT(' LAT=',F10.2,' N LONG =',F10.2,' E DEC5=',F10.3,' DEG.D
     15=',F10.3)
      DO 41 II=1,36
      IH=II
      PLAT=((IH-1)*5+2.5-90)
      QLAT=PLAT*0.0174533
      SQLAT=SIN(QLAT)
      CQLAT=COS(QLAT)
      SSQLA=SQLAT*SQLAT
      TER=1.0+BETA*SSQLA
      TER1=EQGR*SSQLA
      ANGC=-(DEQGGC*TER+DBTACC*TER1)+PCTCOR
      ANS=-(DEQGS*TER+DBTAS*TER1)+PCTCOR
      ANT=-(DEQGT*TER+DBTAT*TER1)+PCTCOR
      DO 40 JJ=1,72
      JH=JJ
   35 QLONG=(JH-1)*5+2.5-180
      QLON=QLONG
      IF(CLAT(IND)-20-DEC5-PLAT)26,27,29
   26 IF(CLAT(IND)+25-DEC5-PLAT)29,27,27
   27 IF(CLONG(JND)-20-DECL5-QLON)28,72,29
   28 IF(CLONG(JND)+25-DECL5-QLON)29,72,72
   29 JAN=IAN(IH,JH)
      IF(IHT(IH,JH))119,118,128
  118 SIS=0.0
      WE=14.0*14.0
      GO TO 137
  119 IF(IHT(IH,JH)+1000)121,120,120
  120 SIS=1.0
      WE=16.0*16.0
      GO TO 137
  121 IF(IHT(IH,JH)+2000)123,122,122
  122 SIS=4.0
      WE=21.0*21.0
      GO TO 137
  123 IF(IHT(IH,JH)+3000)125,124,124
  124 SIS=5.0
      WE=24.0*24.0
      GO TO 137
  125 IF(IHT(IH,JH)+4000)127,126,126
  126 SIS=6.0
      WE=26.0*26.0
      GO TO 137
  127 SIS=7.0
      WE=28.0*28.0
      GO TO 137
  128 IF(IHT(IH,JH)-1000)129,129,130
  129 SIS=-1.4
      WE=16.0*16.0
      GO TO 137
  130 IF(IHT(IH,JH)-2000)131,131,132
  131 SIS=-5.0
      WE=21.0*21.0
      GO TO 137
```

```
  132 IF(IHT(IH,JH)-3000)133,133,134
  133 SIS=-6.0
      WE=24.0*24.0
      GO TO 137
  134 IF(IHT(IH,JH)-4000)135,135,136
  135 SIS=-7.0
      WE=26.0*26.0
      GO TO 137
  136 SIS=-8.0
      WE=28.0*28.0
  137 CONTINUE
      DLONG=(QLONG-CLONG(JND))*0.0174533
      CPSI=SCLAT*SQLAT+CCLAT*CQLAT*COS(DLONG)
      SPSI=SQRT(1-CPSI*CPSI)
      SPSI2=SQRT(ABS(0.5*(1-CPSI)))
      IF(SPSI2)38,38,36
   36 CPSI2=SQRT(ABS(0.5*(1+CPSI)))
      RFPSI=ALOG(SPSI2*(1+SPSI2))
      SINAZ=CQLAT*SIN(DLONG)/SPSI
      IF(CCLAT)37,40,37
   37 COSAZ=(SQLAT-CPSI*SCLAT)/(SPSI*CCLAT)
      STCF=CQLAT*(1+1/SPSI2-6*SPSI2-5*CPSI-3*CPSI*RFPSI)
      ACC=ACC+STCF*JAN
      FDEF=CQLAT*(-0.5*CPSI2/(SPSI2*SPSI2)-3*CPSI2+5*SPSI+3*SPSI*RFPSI-
     11.5*(1+2*SPSI2)*CPSI2*CPSI/((1+SPSI2)*SPSI2))
      FDEFM=FDEF*COSAZ
      FDEFP=FDEF*SINAZ
      ACCM=ACCM+FDEFM*JAN
      ACCP=ACCP+FDEFP*JAN
      KTCT=KTCT+1
      ACCER=ACCER+WE*STCF*STCF
      ACCMER=ACCMER+WE*FDEFM*FDEFM
      ACCPER=ACCPER+WE*FDEFP*FDEFP
      AGCN=AGCN+STCF*ANGC
      ASN=ASN+STCF*ANS
      ATN=ATN+STCF*ANT
      AGCM=AGCM+FDEFM*ANGC
      ASM=ASM+FDEFM*ANS
      ATM=ATM+FDEFM*ANT
      AGCP=AGCP+FDEFP*ANGC
      ASP=ASP+FDEFP*ANS
      ATP=ATP+FDEFP*ANT
      SISTCN=SISTCN+SIS*STCF
      SISTCM=SISTCM+SIS*FDEFM
      SISTCP=SISTOP+SIS*FDEFP
      IF(I-1)40,77,40
   77 IF(J-1)40,78,40
   78 IF(II-1)40,87,80
   80 IF(II-5)40,87,81
   81 IF(II-10)40,87,82
   82 IF(II-15)40,87,83
   83 IF(II-20)40,87,84
   84 IF(II-25)40,87,85
   85 IF(II-30)40,87,86
   86 IF(II-35)40,87,40
```

```
   87 PSI=ATAN(SPSI/CPSI)/0.0174533
      AZ=ATAN(SINAZ/COSAZ)/0.0174533
      STO=STOF/CQLAT
      FDE=FDEF/CQLAT
      WRITE(3,88)PLAT,QLON,IH,JH,PSI,STO,FDE,AZ
   88 FORMAT('  LAT =',F6.1,' LONG =',F6.1,' IH=',I3,' JH =',I3,' PSI=',
     1E10.3,' FPSI =',E10.3,' DFPSI =',E10.3,' AZ =',F6.2)
      WRITE(3,204)SPSI,CPSI,SPSI2,CPSI2,RFPSI
  204 FORMAT(' SIN PSI =',E10.3,' COS PSI =',E10.3,' SIN 0.5 PSI =',
     1E10.3,' COS 0.5 PSI =',E10.3,' LFPSI =',E10.3)
      WRITE(3,89)FDEF,FDEFM,FDEFP
   89 FORMAT('  FDEF =',E10.3,' FDEFM =',E10.3,' FDEFP =',E10.3)
      GO TO 40
   38 WRITE(3,39)SPSI2
   39 FORMAT(/42H ERROR IN SINE PSI/2.VALUE OBTAINED BEING ,F20.8)
      GO TO 40
   72 IF(I-2)73,73,40
   73 WRITE(3,74)IH,JH,IAN(IH,JH)
   74 FORMAT('  ELEMENT OMITTED HAS IH=',I5,' JH=',I5,' VALUE =',I10)
   40 CONTINUE
   41 CONTINUE

      [illegible]TE(3,43)CLAT(IND),CLONG(JND)
     [illegible]MAT(/55H INSUFFICIENT GRAVITY VALUES TO COMPUTE EFFECT FOR
     1,F10.2,14H DEGREES NORTH,F10.2,14H DEGREES EAST )
      COMPN(IND,JND)=0
      COMPDM(IND,JND)=0
      COMPDP(IND,JND)=0
      ITOT(IND,JND)=0
      PRECN(IND,JND)=0
      PRECDM(IND,JND)=0
      PRECDP(IND,JND)=0
      GO TO 45
   44 COMPN(IND,JND)=CCONN*ACC
      COMPDM(IND,JND)=CCOND*ACCM
      COMPDP(IND,JND)=CCOND*ACCP
      ITOT(IND,JND)=KTOT
      PRECN(IND,JND)=SQRT(ACCER)*CCONN/10.0
      [illegible]JND)=SQRT(ACCMER)*CCOND/10.0
      [illegible]IND,JND)=SQRT(ACCPER)*CCOND/10.0
      CORGCN(IND,JND)=CCONN*AGCN
      CORSN(IND,JND)=CCONN*ASN
      CORTN(IND,JND)=CCONN*ATN
      CORGCM(IND,JND)=CCOND*AGCM
      CORSM(IND,JND)=CCOND*ASM
      CORTM(IND,JND)=CCOND*ATM
      CORGCP(IND,JND)=CCOND*AGCP
      CORSP(IND,JND)=CCOND*ASP
      CORTP(IND,JND)=CCOND*ATP
      SISN(IND,JND)=CCONN*SISTON/10.0
      SISM(IND,JND)=CCOND*SISTOM/10.0
      SISP(IND,JND)=CCOND*SISTOP/10.0
   45 CONTINUE
   46 CONTINUE
      WRITE(3,47)
```

```
      47 FORMAT('1      CALCULATION OF GEOID SPHEROID SEPARATION  N  ,ANOM
        1IES USED ARE'//)
         WRITE(3,48)
      48 FORMAT(34H    FREE AIR ANOMALIES               //)
         WRITE(3,49)
      49 FORMAT(//120H      N          N          N          N          N
        1    N          N          N          N          N              LATITU
        2 /)
         WRITE(3,50)
      50 FORMAT(120H    CM.        CM.        CM.        CM.        CM.
        1M.        CM.        CM.        CM.        CM.            DEGREES N
        2//)
         DO 151 I=1,10
         COMPN(I,11)=CLAT(I)
         WRITE(3,51)(COMPN(I,J),J=1,11)
      51 FORMAT(/10F10.2,F15.2)
     151 CONTINUE
         WRITE(3,52)(CLONG(J),J=1,10)
      52 FORMAT(///10F10.2/)
         WRITE(3,53)
      53 FORMAT(72H                                    L O N G I T U D E   I N  
        1E G R E E S   E ///)
         WRITE(3,113)
     113 FORMAT('1      ESTIMATES OF ERROR IN THE FINAL RESULT '////)
         WRITE(3,49)
         WRITE(3,50)
         DO 152 I=1,10
         PRECN(I,11)=CLAT(I)
         WRITE(3,51)(PRECN(I,J),J=1,11)
         WRITE(3,321)(SISN(I,J),J=1,10)
     321 FORMAT(10F10.2)
     152 CONTINUE
         WRITE(3,52)(CLONG(J),J=1,10)
         WRITE(3,53)
         WRITE(3,54)
      54 FORMAT('1      CALCULATION OF DEFLECTIONS OF THE VERTICAL,ANOMALIE
        1 USED ARE '//)
         WRITE(3,48)
         WRITE(3,55)
      55 FORMAT(//120H MERIDIAN  MERIDIAN  MERIDIAN  MERIDIAN  MERIDIAN  M
        1RIDIAN  MERIDIAN  MERIDIAN  MERIDIAN  MERIDIAN            LATITUDE
        2 /)
         WRITE(3,56)
      56 FORMAT(120H   SEC.       SEC.       SEC.       SEC.       SEC.      S
        1C.       SEC.       SEC.       SEC.       SEC.            DEGREES N
        2//)
         DO 153 I=1,10
         COMPDM(I,11)=CLAT(I)
         WRITE(3,57)(COMPDM(I,J),J=1,11)
      57 FORMAT(/10F10.4,F15.2)
     153 CONTINUE
         WRITE(3,52)(CLONG(J),J=1,10)
         WRITE(3,53)
         WRITE(3,113)
         WRITE(3,55)
```

```
      WRITE(3,56)
      DO 154 I=1,10
      PRECOM(I,11)=CLAT(I)
      WRITE(3,57)(PRECOM(I,J),J=1,11)
      WRITE(3,322)(SISM(I,J),J=1,10)
  322 FORMAT(10F10.4)
  154 CONTINUE
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,58)
   58 FORMAT(//120H PRI.VERT. PRI.VERT. PRI.VERT. PRI.VERT. PRI.VERT. PR
     1I.VERT. PRI.VERT. PRI.VERT. PRI.VERT. PRI.VERT.          LATITUDE
     2 /)
      WRITE(3,56)
      DO 155 I=1,10
      COMPDP(I,11)=CLAT(I)
      WRITE(3,57)(COMPDP(I,J),J=1,11)
  155 CONTINUE
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,113)
      WRITE(3,58)
      WRITE(3,56)
      DO 156 I=1,10
      PRECDP(I,11)=CLAT(I)
      WRITE(3,57)(PRECDP(I,J),J=1,11)
      WRITE(3,322)(SISP(I,J),J=1,10)
  156 CONTINUE
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,59)
   59 FORMAT(///'1    NUMBER    NUMBER    NUMBER    NUMBER    NUMBER
     1NUMBER    NUMBER    NUMBER    NUMBER    NUMBER          LATITUDE
     2 '/)
      WRITE(3,60)
   60 FORMAT(120H IN COMP.  IN COMP.  IN COMP.  IN COMP.  IN COMP.  IN C
     1OMP.  IN COMP.  IN COMP.  IN COMP.  IN COMP.         DEGREES N A
     2/)
      DO 157 I=1,10
      ITOT(I,11)=CLAT(I)
      WRITE(3,61)(ITOT(I,J),J=1,11)
   61 FORMAT(10I10,I15  /)
  157 CONTINUE
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
   62 CONTINUE
   63 CONTINUE
      WRITE(3,301)
  301 FORMAT('1  CHANGES IN  N  DUE TO THE FOLLOWING CHANGES IN THE PARA
     1METERS OF THE INTERNATIONAL SPHEROID')
      WRITE(3,302)DGRCON,RATGRC
  302 FORMAT(//' CHANGE IN K*M =',E10.4,'  CM***3  SEC**-2;  RATIO WITH
     1REFERENCE TO K*M =',E10.4)
      WRITE(3,303)PCTCOR
  303 FORMAT('  CHANGE IN POTSDAM DATUM =',F10.2,' TENTH MGAL')
```

```
      WRITE(3,49)
      WRITE(3,50)
      DC 304 I=1,10
      CORGCN(I,11)=CLAT(I)
  304 WRITE(3,51)(CORGCN(I,J),J=1,11)
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,305)
  305 FORMAT('1  CHANGES IN  XI  DUE TO THE FOLLOWING CHANGES IN THE PAR
     1AMETERS OF THE INTERNATIONAL SPHEROID')
      WRITE(3,302)DGRCON,RATGRC
      WRITE(3,303)PCTCOR
      WRITE(3,55)
      WRITE(3,56)
      DC 306 I=1,10
      CORGCM(I,11)=CLAT(I)
  306 WRITE(3,57)(CORGCM(I,J),J=1,11)
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,307)
  307 FORMAT('1   CHANGES IN  ETA  DUE TO THE FOLLOWING CHANGES IN THE P
     1ARAMETERS OF THE INTERNATIONAL SPHEROID')
      WRITE(3,302)DGRCON,RATGRC
      WRITE(3,303)PCTCOR
      WRITE(3,58)
      WRITE(3,56)
      DO 308 I=1,10
      CORGCP(I,11)=CLAT(I)
  308 WRITE(3,57)(CORGCP(I,J),J=1,11)
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,301)
      WRITE(3,309)DRAD,RATRAD,DFLAT
  309 FORMAT('  CHANGE IN EQ. RADIUS =',F10.1,' METRES.  DRAD/RADIUS=',
     1E10.4,' DFLATTENING =',E10.4)
      WRITE(3,303)PCTCOR
      WRITE(3,49)
      WRITE(3,50)
      DC 310 I=1,10
      CORSN(I,11)=CLAT(I)
  310 WRITE(3,51)(CORSN(I,J),J=1,11)
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,305)
      WRITE(3,309)DRAD,RATRAD,DFLAT
      WRITE(3,303)PCTCOR
      WRITE(3,55)
      WRITE(3,56)
      DO 311 I=1,10
      CORSM(I,11)=CLAT(I)
  311 WRITE(3,57)(CORSM(I,J),J=1,11)
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,307)
      WRITE(3,309)DRAD,RATRAD,DFLAT
```

```
      WRITE(3,3C3)PCTCCR
      WRITE(3,58)
      WRITE(3,56)
      DC 312 I=1,10
      CCRSP(I,11)=CLAT(I)
  312 WRITE(3,57)(CCRSP(I,J),J=1,11)
      WRITE(3,52)(CLCNG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,3C1)
      WRITE(3,302)DGRCCN,RATGRC
      WRITE(3,309)DRAD,RATRAC,DFLAT
      WRITE(3,303)PCTCCR
      WRITE(3,49)
      WRITE(3,50)
      DC 313 I=1,10
      CCRTN(I,11)=CLAT(I)
  313 WRITE(3,51)(CCRTN(I,J),J=1,11)
      WRITE(3,52)(CLCNG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,305)
      WRITE(3,302)DGRCCN,RATGRC
      WRITE(3,309)CRAC,RATRAC,DFLAT
      WRITE(3,303)PCTCCR
      WRITE(3,55)
      WRITE(3,56)
      DO 314 I=1,10
      CORTM(I,11)=CLAT(I)
  314 WRITE(3,57)(CCRTM(I,J),J=1,11)
      WRITE(3,52)(CLCNG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,307)
      WRITE(3,302)DGRCCN,RATGRC
      WRITE(3,309)DRAD,RATRAC,DFLAT
      WRITE(3,3C3)PCTCCR
      WRITE(3,58)
      WRITE(3,56)
      DC 315 I=1,10
      CCRTP(I,11)=CLAT(I)
  315 WRITE(3,57)(CCRTP(I,J),J=1,11)
      WRITE(3,52)(CLCNG(J),J=1,10)
      WRITE(3,53)
      STCP
      END

              SIZE CF COMMON  000CCO   PROGRAM  051174

                              END CF COMPILATICN  MAIN
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```
C       PROGRAM STCKMD
C       CALCULATES THE COMPONENTS OF STOKESIAN HEIGHT AND DEFLECTIONS OF THE
C       VERTICAL FOR AN AREA 5<PSI<15,INTERVAL OF COMPUTATION BEING 1.BASIC
C       PROGRAM COMPUTES EFFECT OVER 10 X 10 AREA,THE ANOMALIES  REPRESENTING
C       1 SQUARES.THE EFFECT US CONSIDERED FOR A 45 X 45 AREA AROUND THE
C       10 X 10 SQUARE OF COMPUTATION. SYMMETRIC WITH RESPECT TO THE SW CORNE
C       AN INNER AREA OF 15 X 15 ROUND THE COMPUTATION POINT,SYMMETRICAL
C       WITH RESPECT TO IT IS OMITTED.LAT,LONG +VE N,E RESPECTIVELY.PROGRAM
C       HANDLES A 100 X 100 AREA.ALL UNITS ARE DEGREES
        DIMENSION KAN(10),IAN(60,60),COMPN(11,11),COMPDM(11,11),
       1COMPDP(11,11),CLAT(10),CLONG(10),ITGT(11,11),PRECN(11,11),
       2PRECDM(11,11),PRECDP(11,11),INTER(3,6),ISIS(3,6),CORGCN(11,11),
       3CORGCM(11,11),CORGCP(11,11),CORSN(11,11),CORSM(11,11)
        DIMENSION CORSP(11,11),CORTN(11,11),CORTM(11,11),CORTP(11,11),
       1SISN(11,11),SISM(11,11),SISP(11,11)
        LOCCR=10
        ICAP=60
        JCAP=60
        INT=1
        DO 1 I=1,ICAP
        DO 1 J=1,JCAP
      1 IAN(I,J)=0
C       READ LAT,LONG OF CENTRE OF SW  ONE  DEGREE SQUARE IN TENTH DEGREES.
        READ(1,2)LAC,LOC
      2 FORMAT(2I10)
        WRITE(3,88)LAC,LOC
     88 FORMAT('1     COMPUTATIONS USING ONE DEGREE SQUARE MEANS.S.W.CORNER
       1 LAT=',I10,' DEG N; LONG = ',I10,' DEG E'///)
        READ(1,300)FLAT,DFLAT,RAD,DRAD,GRCON,DGRCON,POTCOR,OMSQ
    300 FORMAT(8E10.4)
        WRITE(3,323)FLAT,RAD
    323 FORMAT(' FLATTENING =',E10.5,' RADIUS =',E10.5)
        BETA=5.2884E-3
        EQGR=9.78049E6
        SEE=RAD*100.0*OMSQ/(EQGR*1.0E-4)
        RATRAD=DRAD/RAD
        RATGRC=DGRCON/GRCON
        DSEEGC=-SEE*RATGRC
        DSEES=SEE*3*RATRAD
        DSEET=SEE*(3*RATRAD-RATGRC)
        DBTAGC=DSEEGC*5/2.0
        DBTAS=DSEES*5/2.0-DFLAT
        DBTAT=DSEET*5/2.0-DFLAT
        DEQGGC=EQGR*(RATGRC-DSEEGC*5/2.0)
        DEQGS=EQGR*(-2*RATRAD+DFLAT-DSEES*5/2.0)
        DEQGT=EQGR*(RATGRC-2*RATRAD+DFLAT-DSEET*5/2.0)
        WRITE(3,64)
     64 FORMAT('    COMMENCE READING OF FREE AIR ANOMALIES'//)
C       READ GRAVITY ANOMALIES REPRESENTING   ONE DEGREE SQUARE MEANS,IN TENT
C       MILLIGAL,WITH LAT,LONG OF MOST WESTERLY SQ.IN  TENTH DEGREES.
      3 READ(1,4)ITE,LAH,LOH,(KAN(I),I=1,5)
      4 FORMAT(I2,2I10,5I10)
        IF(ITE-1)5,15,13
```

```
    5 DO 12 I=1,5
      IN=(LAH-LAO)/100.+1
      JN=(LCH-LOO)/100.+1
      IF(IN)10,10,6
    6 IF(IN-ICAP)7,7,10
    7 IF(JN)10,10,8
    8 IF(JN-JCAP)9,9,10
    9 IAN(IN,JN)=KAN(I)
   12 CONTINUE
      WRITE(3,87)ITE,LAH,LCH,(KAN(I),I=1,5),IN,JN
   87 FORMAT(' ',I2,7I10,' IN =',I5,' JN =',I5)
      GO TO 86
   10 WRITE(3,11)IN,JN,LAH,LCH
   11 FORMAT(/28H ANOMALY WITH INDEX VALUES (,2I10,38H ) OUTSIDE LIMITS.
     1LATITUDE ON CARD IS ,I10,15H AND LONGITUDE ,I10)
   86 IF(ITE-1)3,15,13
   13 WRITE(3,14)LAH,LOH
   14 FORMAT(/15H CARD WITH LAT ,I10,17H N AND LONGITUDE ,I10,29H E HAS
     1INDEX PUNCHED IN ERROR)
      GO TO 3
C     END OF READING ANOMALIES.STORED IN TENTH MILLIGAL.
C     IAN IS STRUCTURED   AS FOLLOWS:-COUNTING DIGITS FROM RIGHT,1-4 GIVE
C     MEAN ANOMALY IN TENTH MILLIGAL,,5-6 GIVE NO. OF READINGS IN SQUARE,
C     7-, THE STAD DEVIATION OF SAMPLE,9 GIVES KEY TO PROCESS,0=DIRECT;1=
C     INTERPOLATION;2 = INTERPOLATION/EXTRAPOLATION; 3 = EXTRAPOLATION. S
C      INTERVAL FOR CASES  1 - 3 GIVEN IN COLUMNS 5 - 6.
   15 LIM=30
      ISPA=1
      IBA=21
      READ(1,16)CON,RHO,PI,IARC,TGRAV,CON1,RAD
   16 FORMAT(E10.3,F10.2,F10.5,I10,F10.2,E10.2,E10.4)
      CCONN=(RAD*CON1*PI)/(4*TGRAV*180*180)
      CCOND=(PI*IARC*CON1)/(4*TGRAV*180*180)
      WRITE(3,17)CCONN,CCOND
   17 FORMAT(/47H COEFFICIENTS FOR COMPUTATION OF SEPARATION IS ,E20.5,
     1'  AND OF DEFLECTIONS IS  ',E20.5)
      READ(1,110)SER,NUM
  110 FORMAT(F10.1,I10)
      DO 112 I=1,3
      READ(1,111)(INTER(I,J),J=1,6)
  111 FORMAT(6I10)
  112 CONTINUE
      DO 63 NO=1,ISPA
      DO 317 I=1,3
  317 READ(1,111)(ISIS(I,J),J=1,6)
      DO 62 NOL=1,ISPA
      DO 46 I=IBA,30,INT
      IND=(I-21.0)/INT+1
      CLAT(IND)=LAO/100.-0.5+(NO-1)*(LIM-IBA+INT)/INT+I
      COL=CLAT(IND)*0.0174533
      SCLAT=SIN(COL)
      CCLAT=COS(COL)
      CLAT5=CLAT(IND)/5.0
      LAT5=CLAT5
      DEC5=(CLAT5-LAT5)*5
```

```
      IF(DEC5)69,70,70
   69 DEC5=DEC5+5
   70 IF(DEC5)19,21,18
   18 IF(DEC5-5)21,19,19
   19 WRITE(3,20)CLAT(IND)
   20 FORMAT(41H ERROR IN CALCULATION OF DEC AT LATITUDE ,F20.2/)
      GO TO 46
   21 LL=45
      DO 45 J=IBA,30,INT
      JND=(J-21.0)/INT+1
      CLONG(JND)=(LCO+LOCOR)/100.0-0.5+(NOL-1)*(LIM-IBA+INT)/INT+J+3
      CLONG5=CLONG(JND)/5.0
      LONG5=CLONG5
      DECL5=(CLONG5-LONG5)*5
      IF(DECL5)23,25,22
   22 IF(DECL5-5)25,23,23
   23 WRITE(3,24)CLAT(IND),CLONG(JND)
   24 FORMAT(/45H ERROR IN THE CALCULATION OF DECL AT LATITUDE,F20.2,15H
     1 AND LONGITUDE ,F20.2/)
      GO TO 45
   25 ACC=0
      KTCT=0
      ACCM=0
      ACCP=0
      ACCER=0
      ACCPER=0
      ACCMER=0
      AGCN=0.0
      AGCM=0.0
      AGCP=0.0
      ASN=0.0
      ASM=0.0
      ASP=0.0
      ATN=0.0
      ATM=0.0
      ATP=0.0
      SISTON=0.0
      SISTOM=0.0
      SISTOP=0.0
      WRITE(3,65)CLAT(IND),CLONG(JND),DEC5,DECL5
   65 FORMAT('  AT LAT =',F10.2,' AND LONG =',F10.2,' DEC5=',F10.2,
     1' DECL5 =',F10.2)
      DO 41 II=1,LL
      IEC5=DEC5
      IH=(NO-1)*(LIM-IBA+INT)/INT+I-IBA+II-IEC5+1
      QLAT=(IH+LAC/100.-1)*0.0174533
      SQLAT=SIN(QLAT)
      CQLAT=COS(QLAT)
      SSQLA=SQLAT*SQLAT
      TER=1.0+BETA*SSQLA
      TER1=EOGR*SSQLA
      ANGC=-(DEQGGC*TER+DBTAGC*TER1)+POTCOR
      ANS=-(DEQGS*TER+DBTAS*TER1)+POTCOR
      ANT=-(DEQGT*TER+DBTAT*TER1)+POTCOR
      DO 40 JJ=1,LL
```

```
      JH=(NCL-1)*(LIM-IBA+INT)/INT+J-IBA+JJ-DECL5+1+3
      IF(II-16)29,27,26
   26 IF(II-30)27,27,29
   27 IF(JJ-16)29,66,28
   28 IF(JJ-30)66,66,29
   29 IF(IAN(IH,JH))31,42,30
   30 ID1=IAN(IH,JH)/10000
      ID2=ID1/100
      IINT=ID2/100
      ISDEV=ID2-IINT*100
      NS=ID1-(IINT*100+ISDEV)*100
      JAN=IAN(IH,JH)-((IINT*100+ISDEV)*100+NS)*10000
      GO TO 90
   31 ID1=-IAN(IH,JH)/10000
      ID2=ID1/100
      IINT=ID2/100
      ISDEV=ID2-IINT*100
      NS=ID1-(IINT*100+ISDEV)*100
      JAN=IAN(IH,JH)+((IINT*100+ISDEV)*100+NS)*10000
   90 IF(IINT)102,101,100
  100 IF(IINT-4)320,319,319
  319 IINT=3
      NS=6
  320 IF(NS-6)325,325,71
  325 WE=SER*SER+INTER(IINT,NS)*INTER(IINT,NS)
      SIS=ISIS(IINT,NS)
      GO TO 35
  101 IF(NS)102,71,33
   33 WE=(ISDEV*ISDEV+(NUM-NS)*(NUM-NS)*SER*SER/((NUM-1)*(NUM-1)))/NS
      SIS=0.0
      GO TO 35
  102 WRITE(3,103)CLAT(IND),CLONG(JND),II,JJ,IH,JH
  103 FORMAT(' CMP.ERR.AT( ',F10.2,' S',F10.2,' E ).ARRAY REF.IS ',2I1
     1'MATRIX REF.IS',2I10/)
   71 WE=SER*SER
      SIS=0.0
   35 QLONG=JH+(LCO+LOCOR)/100.0-1
      SDEV=ISDEV
      DLONG=(QLONG-CLONG(JND))*0.0174533
      CPSI=SCLAT*SQLAT+CCLAT*CQLAT*COS(DLONG)
      SPSI=SQRT(1-CPSI*CPSI)
      SPSI2=SQRT(ABS(0.5*(1-CPSI)))
      IF(SPSI2)38,38,36
   36 CPSI2=SQRT(ABS(0.5*(1+CPSI)))
      RFPSI=ALOG(SPSI2*(1+SPSI2))
      SINAZ=CQLAT*SIN(DLONG)/SPSI
      IF(CCLAT)37,40,37
   37 COSAZ=(SQLAT-CPSI*SCLAT)/(SPSI*CCLAT)
      STOF=CQLAT*(1+1/SPSI2-6*SPSI2-5*CPSI-3*CPSI*RFPSI)
      ACC=ACC+STOF*JAN
      FDEF=CQLAT*(-0.5*CPSI2/(SPSI2*SPSI2)-3*CPSI2+5*SPSI+3*SPSI*RFPSI-
     11.5*(1+2*SPSI2)*CPSI2*CPSI/((1+SPSI2)*SPSI2))
      FDEFM=FDEF*COSAZ
      FDEFP=FDEF*SINAZ
      ACCM=ACCM+FDEFM*JAN
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```
      ACCP=ACCP+FDEFP*JAN
      KTOT=KTOT+1
      ACCER=ACCER+WE*STOF*STOF
      ACCMER=ACCMER+WE*FDEFM*FDEFM
      ACCPER=ACCPER+WE*FDEFP*FDEFP
      AGCN=AGCN+STOF*ANGC
      ASN=ASN+STOF*ANS
      ATN=ATN+STOF*ANT
      AGCM=AGCM+FDEFM*ANGC
      ASM=ASM+FDEFM*ANS
      ATM=ATM+FDEFM*ANT
      AGCP=AGCP+FDEFP*ANGC
      ASP=ASP+FDEFP*ANS
      ATP=ATP+FDEFP*ANT
      SISTON=SISTON+SIS*STOF
      SISTOM=SISTOM+SIS*FDEFM
      SISTOP=SISTOP+SIS*FDEFP
      GO TO 114
   38 WRITE(3,39)SPSI2
   39 FORMAT(/42H ERROR IN SINE PSI/2.VALUE OBTAINED BEING ,F20.8)
      GO TO 40
  114 CONTINUE
      GO TO 40
   66 CONTINUE
   42 CONTINUE
   40 CONTINUE
   41 CONTINUE
   44 COMPN(IND,JND)=CCONN*ACC
      COMPDM(IND,JND)=CCOND*ACCM
      COMPDP(IND,JND)=CCOND*ACCP
      ITOT(IND,JND)=KTOT
      PRECN(IND,JND)=SQRT(ABS(ACCER))*CCONN/10.0
      PRECDP(IND,JND)=SQRT(ABS(ACCPER))*CCOND/10.0
      PRECDM(IND,JND)=SQRT(ABS(ACCMER))*CCOND/10.0
      CORGCN(IND,JND)=CCONN*AGCN
      CORSN(IND,JND)=CCONN*ASN
      CORTN(IND,JND)=CCONN*ATN
      CORGCM(IND,JND)=CCOND*AGCM
      CORSM(IND,JND)=CCOND*ASM
      CORTM(IND,JND)=CCOND*ATM
      CORGCP(IND,JND)=CCOND*AGCP
      CORSP(IND,JND)=CCOND*ASP
      CORTP(IND,JND)=CCOND*ATP
      SISN(IND,JND)=CCONN*SISTON/10.0
      SISM(IND,JND)=CCOND*SISTOM/10.0
      SISP(IND,JND)=CCOND*SISTOP/10.0
      WRITE(3,94)COMPN(IND,JND),PRECN(IND,JND),COMPDM(IND,JND),
     1PRECDM(IND,JND),COMPDP(IND,JND),PRECDP(IND,JND)
   94 FORMAT(/' N =',F10.2,' +/-',F10.2,' XI =',F10.4,' +/-',F10.4,' E
     1 =',F10.4,' +/-',F10.4)
      WRITE(3,95)SISN(IND,JND),SISM(IND,JND),SISP(IND,JND)
   95 FORMAT(' SYSTEMATIC ERROR ',F10.2,17X,F10.4,18X,F10.4)
      WRITE(3,96)CORGCN(IND,JND),CORGCM(IND,JND),CORGCP(IND,JND)
   96 FORMAT(' WHEN GRAV CON IS CHANGED, D N =',F10.2,' D XI =',F10.4,
     1' D ETA =',F10.4)
```

```
      WRITE(3,97)CORSN(IND,JND),CORSM(IND,JND),CORSP(IND,JND)
   97 FORMAT(' WHEN ONLY SPHEROID IS CHANGED, D N =',F10.2,' D XI =',
     1F10.4,' D ETA =',F10.4)
      WRITE(3,98)CORTN(IND,JND),CORTM(IND,JND),CORTP(IND,JND)
   98 FORMAT(' WHEN BOTH GR CON AND SPHEROID ARE CHANGED, D N =',F10.2
     1' D XI =',F10.4,' D ETA =',F10.4)
   45 CONTINUE
   46 CONTINUE
      WRITE(3,47)
   47 FORMAT(///'1  CALCULATION OF GEOID SPHEROID SEPARATION  N  ,ANOM
     1IES USED ARE'///)
      WRITE(3,48)
   48 FORMAT('    FREE AIR ANOMALIES'//)
      WRITE(3,49)
   49 FORMAT(//120H      N          N          N          N          N
     1   N          N          N          N          N          LATITU
     2 /)
      WRITE(3,50)
   50 FORMAT(120H    CM.        CM.        CM.        CM.       CM.
     1M.        CM.        CM.        CM.        CM.        DEGREES N
     2//)
      DO 151 I=1,10
      COMPN(I,11)=CLAT(I)
      WRITE(3,51)(COMPN(I,J),J=1,11)
   51 FORMAT(/10F10.2,F15.2)
  151 CONTINUE
      WRITE(3,52)(CLONG(J),J=1,10)
   52 FORMAT(///10F10.2/)
      WRITE(3,53)
   53 FORMAT(72H                              L O N G I T U D E   I N
     1E G R E E S   E ///)
      WRITE(3,113)
  113 FORMAT('1       ESTIMATES OF ERROR IN THE FINAL RESULT'///)
      WRITE(3,89)SER
   89 FORMAT('    ESTIMATED ERROR  OF REPRESENTATION =',F10.2,' MGAL.'/
      WRITE(3,49)
      WRITE(3,50)
      DO 152 I=1,10
      PRECN(I,11)=CLAT(I)
      WRITE(3,51)(PRECN(I,J),J=1,11)
      WRITE(3,321)(SISN(I,J),J=1,10)
  321 FORMAT(10F10.2)
  152 CONTINUE
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,54)
   54 FORMAT(///'1  CALCULATION OF DEFLECTIONS OF THE VERTICAL,ANOMALI
     1 USED ARE'//)
      WRITE(3,48)
      WRITE(3,55)
   55 FORMAT(//120H MERIDIAN  MERIDIAN  MERIDIAN  MERIDIAN  MERIDIAN
     1RIDIAN  MERIDIAN  MERIDIAN  MERIDIAN  MERIDIAN          LATITUD
     2 /)
      WRITE(3,56)
   56 FORMAT(120H    SEC.       SEC.       SEC.       SEC.       SEC.
```

```
     1C.         SEC.         SEC.         SEC.         SEC.                    DEGREES N
     2//)
      DO 153 I=1,10
      COMPDM(I,11)=CLAT(I)
      WRITE(3,57)(COMPDM(I,J),J=1,11)
  57  FORMAT(/10F10.4,F15.2)
 153  CONTINUE
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,113)
      WRITE(3,89)SER
      WRITE(3,55)
      WRITE(3,56)
      DO 154 I=1,10
      PRECOM(I,11)=CLAT(I)
      WRITE(3,57)(PRECOM(I,J),J=1,11)
      WRITE(3,322)(SISM(I,J),J=1,10)
 322  FORMAT(10F10.4)
 154  CONTINUE
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,58)
  58  FORMAT(//120H PRI.VERT. PRI.VERT. PRI.VERT. PRI.VERT. PRI.VERT.
     1I.VERT. PRI.VERT. PRI.VERT. PRI.VERT. PRI.VERT.            LATITUE
     2 /)
      WRITE(3,56)
      DO 155 I=1,10
      COMPDP(I,11)=CLAT(I)
      WRITE(3,57)(COMPDP(I,J),J=1,11)
 155  CONTINUE
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,113)
      WRITE(3,89)SER
      WRITE(3,58)
      WRITE(3,56)
      DO 156 I=1,10
      PRECDP(I,11)=CLAT(I)
      WRITE(3,57)(PRECDP(I,J),J=1,11)
      WRITE(3,322)(SISP(I,J),J=1,10)
 156  CONTINUE
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,59)
  59  FORMAT(///'1    NUMBER     NUMBER     NUMBER     NUMBER     NUMBER
     1NUMBER     NUMBER     NUMBER     NUMBER     NUMBER             LATITI
     2 '/)
      WRITE(3,60)
  60  FORMAT(120H IN COMP.   IN COMP.   IN COMP.   IN COMP.   IN CCMP.   I
     1OMP.  IN COMP.  IN COMP.  IN COMP.  IN COMP.              DEGREES
     2/)
      DO 157 I=1,10
      ITOT(I,11)=CLAT(I)
      WRITE(3,61)(ITOT(I,J),J=1,11)
 157  CONTINUE
```

```
 61 FORMAT(10I10,I15/)
    WRITE(3,52)(CLONG(J),J=1,10)
    WRITE(3,53)
    WRITE(3,301)
301 FORMAT('1  CHANGES IN  N  DUE TO THE FOLLOWING CHANGES IN THE PA
   1METERS OF THE INTERNATIONAL SPHEROID')
    WRITE(3,302)DGRCON,RATGRC
302 FORMAT(//' CHANGE IN K*M =',E10.4,'  CM***3  SEC**-2;  RATIO WIT
   1REFERENCE TO K*M =',E10.4)
    WRITE(3,303)PCTCOR
303 FORMAT('  CHANGE IN POTSDAM DATUM =',F10.2,' TENTH MGAL')
    WRITE(3,49)
    WRITE(3,50)
    DO 304 I=1,10
    CORGCN(I,11)=CLAT(I)
304 WRITE(3,51)(CORGCN(I,J),J=1,11)
    WRITE(3,52)(CLONG(J),J=1,10)
    WRITE(3,53)
    WRITE(3,305)
305 FORMAT('1  CHANGES IN  XI  DUE TO THE FOLLOWING CHANGES IN THE PA
   1AMETERS OF THE INTERNATIONAL SPHEROID')
    WRITE(3,302)DGRCON,RATGRC
    WRITE(3,303)PCTCOR
    WRITE(3,55)
    WRITE(3,56)
    DO 306 I=1,10
    CORGCM(I,11)=CLAT(I)
306 WRITE(3,57)(CORGCM(I,J),J=1,11)
    WRITE(3,52)(CLONG(J),J=1,10)
    WRITE(3,53)
    WRITE(3,307)
307 FORMAT('1   CHANGES IN  ETA  DUE TO THE FOLLOWING CHANGES IN THE
   1ARAMETERS OF THE INTERNATIONAL SPHEROID')
    WRITE(3,302)DGRCON,RATGRC
    WRITE(3,303)PCTCOR
    WRITE(3,58)
    WRITE(3,56)
    DO 308 I=1,10
    CORGCP(I,11)=CLAT(I)
308 WRITE(3,57)(CORGCP(I,J),J=1,11)
    WRITE(3,52)(CLONG(J),J=1,10)
    WRITE(3,53)
    WRITE(3,301)
    WRITE(3,309)DRAD,RATRAD,DFLAT
309 FORMAT('  CHANGE IN EQ. RADIUS =',F10.1,' METRES.  DRAD/RADIUS=',
   1E10.4,' DFLATTENING =',E10.4)
    WRITE(3,303)PCTCOR
    WRITE(3,49)
    WRITE(3,50)
    DO 310 I=1,10
    CORSN(I,11)=CLAT(I)
310 WRITE(3,51)(CORSN(I,J),J=1,11)
    WRITE(3,52)(CLONG(J),J=1,10)
    WRITE(3,53)
    WRITE(3,305)
```

```
      WRITE(3,309)DRAD,RATRAD,DFLAT
      WRITE(3,303)PCTCOR
      WRITE(3,55)
      WRITE(3,56)
      DO 311 I=1,10
      CORSM(I,11)=CLAT(I)
  311 WRITE(3,57)(CORSM(I,J),J=1,11)
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,307)
      WRITE(3,309)DRAD,RATRAD,DFLAT
      WRITE(3,303)PCTCOR
      WRITE(3,58)
      WRITE(3,56)
      DO 312 I=1,10
      CORSP(I,11)=CLAT(I)
  312 WRITE(3,57)(CORSP(I,J),J=1,11)
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,301)
      WRITE(3,302)DGRCON,RATGRC
      WRITE(3,309)DRAD,RATRAD,DFLAT
      WRITE(3,303)PCTCOR
      WRITE(3,49)
      WRITE(3,50)
      DO 313 I=1,10
      CORTN(I,11)=CLAT(I)
  313 WRITE(3,51)(CORTN(I,J),J=1,11)
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,305)
      WRITE(3,302)DGRCON,RATGRC
      WRITE(3,309)DRAD,RATRAD,DFLAT
      WRITE(3,303)PCTCOR
      WRITE(3,55)
      WRITE(3,56)
      DO 314 I=1,10
      CORTM(I,11)=CLAT(I)
  314 WRITE(3,57)(CORTM(I,J),J=1,11)
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,307)
      WRITE(3,302)DGRCON,RATGRC
      WRITE(3,309)DRAD,RATRAD,DFLAT
      WRITE(3,303)PCTCOR
      WRITE(3,58)
      WRITE(3,56)
      DO 315 I=1,10
      CORTP(I,11)=CLAT(I)
  315 WRITE(3,57)(CORTP(I,J),J=1,11)
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
   62 CONTINUE
   63 CONTINUE
      STOP
```

APPENDIX (11)


```
      PROGRAM WEIGHTED LEAST SQUARES
      DIMENSION KHT(10),IFA(40,40),IHT(40,40),BA(42),IOP(20)
      COMMON A(41,42)
      READ GEOGRAPHICAL COORDINATES OF SE CORNER IN 10**-2 DEG. +VE N,E
    1 READ(1,2)LAO,LOO
    2 FORMAT(2I10)
      WRITE(3,16)
   16 FORMAT(///'  INTERPOLATION OF FREE AIR ANOMALY MEANS - HALF DEGREE
     1 ANOMALIES'//)
      EREP=3.0
      ICAP=40
      LOCOR=0
      JCAP=40
      NO=1
      IFAC=10
      WRITE(3,17)ICAP,JCAP,LAO,LOO
   17 FORMAT(//' AREA COVERED IS',I5,' X ',I5,' AREA NW OF LAT =',I8,' N
     1. LONG=',I8,' E'///)
      WRITE(3,18)EREP
   18 FORMAT(' ERROR OF REPRESENTATION CHOSEN AS',F10.1,' MGAL.'//)
      DO 3 I=1,ICAP
      DO 3 J=1,JCAP
      IFA(I,J)=0
      IHT(I,J)=0
    3 CONTINUE
      WRITE(3,15)
   15 FORMAT(//'1            READING OF HEIGHT MEANS COMMENCES'///)
      IF(IFAC-10)4,112,112
    4 READ(1,5)KEY,LAT,LONG,(KHT(I),I=1,5)
    5 FORMAT(' ',I1,7I10)
      IF(KEY-1)6,21,19
    6 DO 11 I=1,5
      IN=(LAT-LAO)*IFAC/100.0+1
      JN=(LONG-LOO+LOCOR)*IFAC/100.0+I
      IF(IN)13,13,7
    7 IF(IN-ICAP)8,8,13
    8 IF(JN)13,13,9
    9 IF(JN-JCAP)10,10,13
   10 IHT(IN,JN)=KHT(I)
   11 CONTINUE
      WRITE(3,12)KEY,LAT,LONG,(KHT(I),I=1,5),IN,JN
   12 FORMAT(' ',I1,7I10,' IN=',I5,' JN=',I5)
      GO TO 4
   13 WRITE(3,14)LAT,LONG,IN,JN
   14 FORMAT(' CARD WITH LAT =',I10,' N AND LONG =',I10,' E OUT OF AREA.
     1IN =',I5,' JN =',I5)
      GO TO 4
   19 WRITE(3,20)LAT,LONG
   20 FORMAT(' CARD WITH LAT =',I10,' N,LONG =',I10,' E HAS INDEX PUNCHED
     1D IN ERROR')
      GO TO 4
  112 READ(1,113)ITE,KEY,LAT,LONG,RINT,(KHT(I),I=1,10)
  113 FORMAT(2I2,2I6,F4.1,10I6)
```

```
      IF(ITE-1)114,125,123
  114 DO 121 I=1,10
      IN=(LAT-5-LAO)/10.0+1
      JN=(LONG+5-LOO)/10.0+1
      IF(IN)121,121,115
  115 IF(IN-ICAP)116,116,121
  116 IF(JN)121,121,117
  117 IF(JN-JCAP)118,118,121
  118 IF(KHT(I))119,119,120
  119 IHT(IN,JN)=KHT(I)
      GO TO 121
  120 IHT(IN,JN)=KHT(I)
  121 CONTINUE
      WRITE(3,122)ITE,LAT,LONG,(KHT(I),I=1,10),IN,JN
  122 FORMAT(' ',I1,2I10,10I6,' IN=',I5,' JN=',I5)
      GO TO 112
  123 WRITE(3,124)LAT,LONG
  124 FORMAT(' CARD WITH LAT ',I5,' S AND LONG. ',I5,' E HAS BEEN PUNCH
     1ED IN ERROR')
      GO TO 112
  125 READ(1,126)ITE,KEY,LAT,LONG,RINT,(KHT(I),I=1,10)
  126 FORMAT(I1,I2,2I6,F5.2,10I6)
      IF(ITE-1)127,136,135
  127 DO 134 I=1,10
      IN=(LAT-5-LAO)/10.0+1
      JN=(LONG+5-LOO)/10.0+1
      IF(IN)134,134,128
  128 IF(IN-ICAP)129,129,134
  129 IF(JN)134,134,130
  130 IF(JN-JCAP)131,131,134
  131 IF(KHT(I))132,132,133
  132 IHT(IN,JN)=KHT(I)
      GO TO 134
  133 IHT(IN,JN)=KHT(I)
  134 CONTINUE
      WRITE(3,122)ITE,LAT,LONG,(KHT(I),I=1,10),IN,JN
      GO TO 125
  135 WRITE(3,124)LAT,LONG
      GO TO 125
  136 READ(1,137)ITE,KEY,LAT,LONG,RINT,(KHT(I),I=1,10)
  137 FORMAT(I1,I2,2I6,F4.1,10I6)
      IF(ITE-1)138,21,146
  138 DO 145 I=1,10
      IN=(LAT-5-LAO)/10.0+1
      JN=(LONG+5-LOO)/10.0+1
      IF(IN)145,145,139
  139 IF(IN-ICAP)140,140,145
  140 IF(JN)145,145,141
  141 IF(JN-JCAP)142,142,145
  142 IF(KHT(I))143,143,144
  143 IHT(IN,JN)=KHT(I)
      GO TO 145
  144 IHT(IN,JN)=KHT(I)
  145 CONTINUE
      WRITE(3,122)ITE,LAT,LONG,(KHT(I),I=1,10),IN,JN
```

```
      GC TO 136
  146 WRITE(3,124)LAT,LONG
      GO TO 136
   21 WRITE(3,22)
   22 FORMAT('1            READING OF FREE AIR ANOMALIES COMMENCES'///)
      KTCT=0
   23 READ(1,5)KEY,LAT,LONG,(KHT(I),I=1,5)
      IF(KEY-1)24,34,33
   24 DC 31 I=1,5
      IN=(LAT-LAC)*IFAC/100.0+1
      JN=(LCNG-LCO)*IFAC/100.0+1
      IF(IN)32,32,25
   25 IF(IN-ICAP)26,26,32
   26 IF(JN)32,32,27
   27 IF(JN-JCAP)28,28,32
   28 IF(KHT(I))30,29,30
   29 IFA(IN,JN)=5000
      GC TO 31
   3C IFA(IN,JN)=KHT(I)
      KTCT=KTOT + 1
   31 CCNTINUE
      WRITE(3,12)KEY,LAT,LONG,(KHT(I),I=1,5),IN,JN
      GC TO 23
   32 WRITE(3,14)LAT,LONG,IN,JN
      GC TO 23
   33 WRITE(3,20)LAT,LONG
      GC TC 23
   34 WRITE(3,35)
   35 FCRMAT('1            COMMENCEMENT OF CALCULATIONS'///)
      WRITE(3,190)KTOT
  190 FCRMAT(//'  TOTAL NUMBER OF GRAVITY READINGS IN SAMPLE =',I10//)
      K=1
      IF(KTCT-100)183,185,185
  183 IF(KTOT-10)186,184,184
  184 ICAL=KTCT/10
      IND=41-(10-ICAL)*4
      JND=42-(10-ICAL)*4
      GC TC 187
  185 IND=41
      JND=42
      GC TO 187
  186 IND=5
      JND=6
  187 WRITE(3,196)IND,JND
  196 FORMAT('  VALUES OF IND =',I10,' AND JND =',I10)
      IF(K-2)42,44,41
   41 IF(K-3)44,46,48
   42 WRITE(3,43)
   43 FORMAT(//'     UNIFORM WEIGHTS ASSUMED. W = 1,0 '///)
      GC TO 50
   44 WRITE(3,45)
   45 FCRMAT(//'    WTS PROPORTIONAL TO SAMPLE NO. W = NS/(0.5*N) '///)
      GO TO 50
   46 WRITE(3,47)
   47 FORMAT(//'    WTS PROPORTIONAL TO I/(STD.DEV)**2.  W =(EREP/SIGMA)
```

```
   1*2 '///)
     GO TO 50
  48 WRITE(3,49)
  49 FORMAT(//'     WTS. PROP. TO NS/((STD.DEV)**2+((N-NS)EREP/(N-1))**2
   1  .  W = (NS/N)*(EREP**2/((STD DEV)**2+((N-NS*EREP/(N-1)**2)'//)
  50 DO 36 I=1,IND
     DO 36 J=1,JND
  36 A(I,J)=0
     IQ=(IND-1)/4
     JQ=(JND-2)/4
     INCAC=IQ+1
     INCASL=IQ+2
     INCAS=IQ*2+1
     INCOCL=IQ*2+2
     INCOC=IQ*3+1
     INCOSL=IQ*3+2
     INCOS=IQ*4+1
     DO 70 MM=1,ICAP
     DO 70 NN=1,JCAP
     IF(IFA(MM,NN)-5000)37,147,37
  37 IF(IFA(MM,NN))39,147,38
  38 ID1=IFA(MM,NN)/10000
     ID2=ID1/100
     IINT=ID2/100
     ISDEV=ID2-IINT*100
     NS=ID1-(IINT*100+ISDEV)*100
     JAN=IFA(MM,NN)-((IINT*100+ISDEV)*100+NS)*10000
     GO TO 40
  39 ID1=-IFA(MM,NN)/10000
     ID2=ID1/100
     IINT=ID2/100
     ISDEV=ID2-IINT*100
     NS=ID1-(IINT*100+ISDEV)*100
     JAN=IFA(MM,NN)+((IINT*100+ISDEV)*100+NS)*10000
  40 IF(K-2)52,53,51
  51 IF(K-3)53,54,57
  52 WE=1.0
     GO TO 60
  53 WE=NS/(NO*0.5)
     GO TO 60
  54 IF(ISDEV)55,55,56
  55 WE=1.0
     GO TO 60
  56 WE=(EREP*EREP)/(ISDEV*ISDEV)
     GO TO 60
  57 IF(IINT)58,89,58
  89 VAL=(ISDEV*ISDEV+(NO-NS)*(NO-NS)*EREP*EREP/((NO-1)*(NO-1)))
     IF(VAL)58,58,59
  58 WE=1/(EREP*EREP)
     GO TO 60
  59 WE=NS/VAL
  60 DO 61 II=1,JND
  61 BA(II)=0
     REC=NN*6.283185/JCAP
     REP=MM*6.283185/ICAP
```

```
      DO 62 II=1,INCAC
   62 BA(II)=COS(REC*(II-1))
      DO 63 II=INCASL,INCAS
   63 BA(II)=SIN(REC*(II-INCAC))
      DO 64 II=INCOCL,INCOC
   64 BA(II)=COS(REP*(II-INCAS))
      DO 65 II=INCOSL,INCOS
   65 BA(II)=SIN(REP*(II-INCOC))
      IF(IHT(MM,NN))66,104,104
  104 IBAN=JAN-1.118*IHT(MM,NN)
      GO TO 67
   66 IBAN=JAN-0.6852*IHT(MM,NN)
   67 BA(JND)=IBAN
      DO 68 I=1,IND
      DO 68 J=1,JND
      A(I,J)=A(I,J)+BA(I)*BA(J)*WE
   68 CONTINUE
      GO TO 148
  147 WE=0.0
      JAN=0
      IBAN=0
      IINT=0
      ISDEV=0
      NS=0
  148 CONTINUE
      IFA(MM,NN)=JAN
   70 CONTINUE
      CALL SIMLEQ(IND,JND,IRR)
      WRITE(3,71)
   71 FORMAT(//' COEFFS OF SOLUTION ARE '//)
      WRITE(3,72)(A(I,JND),I=1,IND)
   72 FORMAT(5(5X,F10.1)/)
      WRITE(3,81)
   81 FORMAT('1          COMPARISON OF TRUE VS INTERPOLATED'//)
      WRITE(3,82)
   82 FORMAT(/' LATITUDE  LONGITUDE   OBSD. F.A.A. INTERP.F.A.A.   IN
     1.-OBSD.')
      WRITE(3,83)
   83 FORMAT('  DEG. N.    DEG. E.    TENTH MGAL.     TENTH MGAL.     TE
     1MGAL.'///)
      KTOT=0
      ACC=0
      DO 86 MM=1,ICAP
      DO 85 NN=1,JCAP
      REC=NN*6.283185/JCAP
      REP=MM*6.283185/ICAP
      DO 73 II=1,INCAC
   73 BA(II)=COS(REC*(II-1))
      DO 74 II=INCASL,INCAS
   74 BA(II)=SIN(REC*(II-INCAC))
      DO 75 II=INCOCL,INCOC
   75 BA(II)=COS(REP*(II-INCAS))
      DO 76 II=INCOSL,INCOS
   76 BA(II)=SIN(REP*(II-INCOC))
      IBAN=0
```

```
      DO 77 II=1,IND
      IBAN=IBAN+A(II,JND)*BA(II)
   77 CONTINUE
      IF(IHT(MM,NN))78,79,79
   78 IFAN=IBAN+0.6852*IHT(MM,NN)
      GO TO 80
   79 IFAN=IBAN+1.118*IHT(MM,NN)
   80 IHT(MM,NN)=IFAN
      LAT=LAC+(MM-1)*100/IFAC
      LONG=LOD+LOCOR/4+(NN-1)*100/IFAC
      IDIFF=IHT(MM,NN)-IFA(MM,NN)
      WRITE(3,84)LAT,LONG,IFA(MM,NN),IHT(MM,NN),IDIFF
   84 FORMAT(' ',2I10,3I15)
      IF(IFA(MM,NN))195,85,85
  195 KTOT=KTOT+1
      ACC=ACC+IDIFF*IDIFF
   85 CONTINUE
   86 CONTINUE
      EPRED=SQRT(ACC/KTOT)
      WRITE(3,105)KTOT,EPRED
  105 FORMAT(///' FOR  ',I8,' COMPARISONS,THE ERROR OF PREDICTION IS ',
     1F10.1,' TENTH MGAL.')
      JJND=JCAP/20
      DO 161 J=1,JJND
      WRITE(3,153)
  153 FORMAT(///'1                              PRINT OUT OF AVAILABLE FREE AIR ANO
     1MALIES'///)
      IF(J-2)154,156,158
  154 WRITE(3,155)
  155 FORMAT(' IN   JN 1      2      3      4      5      6      7      8      9
     1  10     11     12     13     14     15     16     17     18     19     20'
     2///)
      GO TO 160
  156 WRITE(3,157)
  157 FORMAT(' IN   JN 21     22     23     24     25     26     27     28     29
     1  30     31     32     33     34     35     36     37     38     39     40
     2'///)
      GO TO 160
  158 WRITE(3,159)
  159 FORMAT(' IN   JN 41     42     43     44     45     46     47     48     49
     1  50     51     52     53     54     55     56     57     58     59     60
     2'///)
  160 DO 152 I=1,ICAP
      DO 151 JJ=1,20
      JIND=(J-1)*20+JJ
      IF(JIND-JCAP)149,149,150
  149 IF(IHT(I,JIND))193,194,194
  193 IOP(JJ)=IFA(I,JIND)/10.0-0.5
      GO TO 151
  194 IOP(JJ)=IFA(I,JIND)/10.0+0.5
      GO TO 151
  150 IOP(JJ)=0
  151 CONTINUE
      WRITE(3,100)I,(IOP(JJ),JJ=1,20)
  100 FORMAT(' ',I4,20I6/)
```

```
152 CONTINUE
161 CONTINUE
    JJND=JCAP/20
    DO 171 J=1,JJND
    WRITE(3,162)
162 FORMAT(///'1                              PRINT OUT OF INTERPOLATED FREE AIR
   1 ANOMALIES'///)
    IF(J-2)163,164,165
163 WRITE(3,155)
    GO TO 166
164 WRITE(3,157)
    GO TO 166
165 WRITE(3,159)
166 DO 170 I=1,ICAP
    DO 169 JJ=1,20
    JIND=(J-1)*20+JJ
    IF(JIND-JCAP)167,167,168
167 IF(IHT(I,JIND))191,192,192
191 IOP(JJ)=IHT(I,JIND)/10.0-0.5
    GO TO 169
192 IOP(JJ)=IHT(I,JIND)/10.0+0.5
    GO TO 169
168 IOP(JJ)=0
169 CONTINUE
    WRITE(3,100)I,(IOP(JJ),JJ=1,20)
170 CONTINUE
171 CONTINUE
    JJND=JCAP/20
    DO 182 J=1,JJND
    WRITE(3,172)
172 FORMAT(///'1                              PRINT OUT OF ERROR OF PREDICTION'/
   1/)
    IF(J-2)173,174,175
173 WRITE(3,155)
    GO TO 176
174 WRITE(3,157)
    GO TO 176
175 WRITE(3,159)
176 DO 181 I=1,ICAP
    DO 180 JJ=1,20
    JIND=(J-1)*20+JJ
    A(I,JIND)=(IHT(I,JIND)-IFA(I,JIND))/10.0
    IF(IFA(I,JIND))189,179,189
189 IF(A(I,JIND))188,177,177
179 IOP(JJ)=0
    GO TO 180
188 IOP(JJ)=A(I,JIND)-0.5
    GO TO 180
177 IOP(JJ)=A(I,JIND)+0.5
180 CONTINUE
    WRITE(3,100)I,(IOP(JJ),JJ=1,20)
181 CONTINUE
182 CONTINUE
    STOP
    END
```

L66 IBM OS/360 BASIC FORTRAN IV (E) COMPILATION

```
      SUBROUTINE SIMLEQ(N,M,IRR)
      COMMON A(41,42)
C     A IS AN NXN EQUATION MATRIX, PLUS M-N RHS VECTORS.
C     THE X SOLUTIONS REPLACE THE RHS'S.
C     METHOD OF GAUSSIAN ELIMINATION WITH ROW PIVOTING AND BACK
C     SUBSTITUTION
      EPS=1.0E-60
      NN=N-1
      IF(NN)  80,130,10
C     FOR EACH COLUMN FIND LARGEST ELEMENT IN LOWER TRIANGLE FOR PIVOT
   10 DO 120  L=1,NN
      KK=L
      DO  40 K=L,N
      IF(ABS (A(K,L))-ABS (A(KK,L)))  40,40,30
   30 KK=K
   40 CONTINUE
C     MAKE PIVOT A DIAGONAL ELEMENT BY ROW INTERCHANGE
      IF(KK-L)  70,70,50
   50 DO  60  J=L,M
      B=A(L,J)
      A(L,J)=A(KK,J)
   60 A(KK,J)=B
C     TEST FOR MATRIX SINGULAR (PIVOT TOO SMALL)
   70 IF( ABS(A(L,L))-EPS     ) 80,80,90
   80 IRR=1
      RETURN
C     ELIMINATE COLUMN BELOW DIAGONAL BY ROW SUBTRACTION
   90 KK=L+1
      DO 120 K=KK,N
      IF(A(K,L))  100,120,100
  100 DO 110 J=KK,M
  110 A(K,J)=A(K,J)-A(K,L)/A(L,L)*A(L,J)
  120 CONTINUE
C     SOLVE FOR X BY BACK SUBSTITUTION
  130 IF ( ABS(A(N,N))- EPS     ) 80,80,140
  140 NN=N+1
      DO 170  II=1,N
      I=N-II+1
      KK=I+1
      DO 170 J=NN,M
      B=0.0
      IF(I-N) 150,170,150
  150 DO    160  K=KK,N
  160 B=B+A(I,K)*A(K,J)
  170 A(I,J)=(A(I,J)-B)/A(I,I)
      IRR=0
      RETURN
      END

        SIZE OF COMMON  006888   PROGRAM  001462

                              END OF COMPILATION  SIMLEQ
```

```
C       PROGRAM STCKNE
C       CALCULATES THE COMPONENTS OF STOKESIAN HEIGHT AND DEFLECTIONS OF THE
C       VERTICAL FOR AN AREA 1<PSI<5,THE INTERVAL OF COMPUTATION BEING 1.BASI
C       PROGRAM COMPUTES EFFECT OVER 5  X 5  AREA,THE ANOMALIES  REPRESENTING
C       0.5 SQUARES.THE EFFECT IS CONSIDERED FOR A 15 X 15 AREA AROUND THE
C       5 X 5 SQUARE OF COMPUTATION.SYMMETRIC WITH RESPECT TO THE SW CORNER
C       AN INNER AREA OF 3 X 3 ROUND COMPUTATION POINT,SYMMETRICAL WITH RESPE
C       TO IT  IS OMITTED.LAT,LONG +VE N,E RESPECTIVELY.PROGRAM HANDLES A
C       50 X 50 AREA.ALL UNITS ARE DEGREES.
        DIMENSION KAN(10),IAN(60,60),COMPN(11,11),COMPDM(11,11),
       1COMPDP(11,11),CLAT(10),CLONG(10),ITOT(11,11),PRECN(11,11),
       2PRECDM(11,11),PRECDP(11,11),INTER(3,6),ISIS(3,6),CORGCN(11,11),
       3CORGCM(11,11),CORGCP(11,11),CORSN(11,11),CORSM(11,11)
        DIMENSION CORSP(11,11),CORTN(11,11),CORTM(11,11),CORTP(11,11),
       1SISN(11,11),SISM(11,11),SISP(11,11)
        ICAP=60
        JCAP=60
        INT=1
        DO 1 I=1,ICAP
        DO 1 J=1,JCAP
      1 IAN(I,J)=0
C       READ LAT,LONG OF CENTRE OF SW  HALF DEGREE SQUARE IN HUNDREDTH DEGREE
        READ(1,2)LAO,LOO
      2 FORMAT(2I10)
C       READ GRAVITY ANOMALIES REPRESENTING  HALF DEGREE SQUARE MEANS,IN TEN
C       MILLIGAL,WITH LAT,LONG OF MOST WESTERLY SQ.IN  HUNDREDTH DEGREES
        READ(1,300)FLAT,DFLAT,RAD,DRAD,GRCON,DGRCON,POTCOR,OMSQ
    300 FORMAT(8F10.4)
        WRITE(3,323)FLAT,RAD
    323 FORMAT(' FLATTENING =',E10.5,' RADIUS =',E10.5)
        BETA=5.2884E-3
        EQGR=9.78049E6
        SEE=RAD*100.0*OMSQ/(EQGR*1.0E-4)
        RATRAD=DRAD/RAD
        RATGRC=DGRCON/GRCON
        DSEEGC=-SEE*RATGRC
        DSEES=SEE*3*RATRAD
        DSEET=SEE*(3*RATRAD-RATGRC)
        DBTAGC=DSEEGC*5/2.0
        DBTAS=DSEES*5/2.0-DFLAT
        DBTAT=DSEET*5/2.0-DFLAT
        DEQGGC=EQGR*(RATGRC-DSEEGC*5/2.0)
        DEQGS=EQGR*(-2*RATRAD+DFLAT-DSEES*5/2.0)
        DEQGT=EQGR*(RATGRC-2*RATRAD+DFLAT-DSEET*5/2.0)
      3 READ(1,4)ITE,LAH,LOH,(KAN(I),I=1,5)
      4 FORMAT(I2,2I10,5I10)
        IF(ITE-1)5,15,13
      5 DO 12 I=1,5
        IN=(LAH-LAO)/50.0+1
        JN=(LOH-LOO)/50.0+I
        IF(IN)10,10,6
      6 IF(IN-ICAP)7,7,10
      7 IF(JN)10,10,8
```

```
  8 IF(JN-JCAP)9,9,10
  9 IAN(IN,JN)=KAN(I)
 12 CONTINUE
    GC TC 86
 10 WRITE(3,11)IN,JN,LAH,LOH
 11 FORMAT(/28H ANOMALY WITH INDEX VALUES (,2I10,38H ) OUTSIDE LIMITS.
   1LATITUDE ON CARD IS ,I1C,15H AND LONGITUDE ,I10)
 86 WRITE(3,92)ITE,LAH,LOH,(KAN(I),I=1,5),IN,JN
 92 FORMAT(' ',I2,7I10,' IN =',I5,' JN =',I5)
    IF(ITE-1)3,15,13
 13 WRITE(3,14)LAH,LOH
 14 FORMAT(/15H CARD WITH LAT ,I10,17H N AND LONGITUDE ,I10,29H E HAS
   1INDEX PUNCHED IN ERROR)
    GC TO 3
 15 READ(1,70)ITE,LAH,LOH,(KAN(I),I=1,5)
 70 FORMAT(I2,2I10,5I10)
    WRITE(3,4)ITE,LAH,LOH,(KAN(I),I=1,5)
    IF(ITE-1)71,79,78
 71 DO 77 I=1,5
    IN=(LAH-LAO)/50.0+1
    JN=(LOH-LOO)/50.0+I
    IF(IN)76,76,72
 72 IF(IN-ICAP)73,73,76
 73 IF(JN)76,76,74
 74 IF(JN-JCAP)75,75,76
 75 IAN(IN,JN)=KAN(I)
 77 CONTINUE
    GC TC 85
 76 CONTINUE
 85 IF(ITE-1)15,79,78
 78 WRITE(3,14)LAH,LOH
    GC TO 15
C     END OF READING ANOMALIES.STORED IN TENTH MILLIGAL.
C     DATA HAS BEEN LOADED WITH THE NUMBER OF READINGS IN SAMPLE IN THE
C     5TH AND 6TH PLACES FROM THE RIGHT.7-8TH PLACES REPRESENT STD.DEV. (
C     KEY USED IN COL 9 :- 0 = DIRECT REPRESENTATION ; 1 = INTERPOLATION
C     2 = INTERPOLATION/EXTRAPOLATION ; 3 = EXTRAPOLATION. SIGN IN 10TH
C     PLACE.
 79 LIM=20
    ISPA=1
    IBA=11
    READ(1,16)CCN,RHO,PI,IARC,TGRAV,CON1,RAD
 16 FORMAT(E10.3,F10.2,F10.5,I10,F10.2,E10.2,E10.4)
    CCONN=(RAD*CCN1*PI)/(4*TGRAV*180*180)
    CCOND=(PI*IARC*CON1)/(4*TGRAV*180*180)
    WRITE(3,17)CCCNN,CCOND
 17 FORMAT(/47H COEFFICIENTS FOR COMPUTATION OF SEPARATION IS ,E20.5,
   1'H AND OF DEFLECTIONS IS  ',E20.5)
    READ(1,110)SER,NUM
110 FORMAT(F10.1,I10)
    DO 112 I=1,3
    READ(1,111)(INTER(I,J),J=1,6)
111 FORMAT(6I10)
112 CCNTINUE
    DO 317 I=1,3
```

```
317 READ(1,111)(ISIS(I,J),J=1,6)
    DO 63 NO=1,ISPA
    DO 62 NOL=1,ISPA
    DO 46 I=IBA,20,INT
    IND=(I-11.0)/INT+1
    CLAT(IND)=(NO-1)*5.0+(I-IBA)*1.0-34.00
    COL=CLAT(IND)*0.0174533
    SCLAT=SIN(COL)
    CCLAT=COS(CCL)
    LAT=CLAT(IND)
    CLAT5=CLAT(IND)/5.0
    LAT5=CLAT5
    DEC=(CLAT(IND)-LAT)*2.0
    DEC5=(CLAT5-LAT5)*10
    IF(DEC5)89,90,90
 89 DEC5=10.0+DEC5
    IEC5=DEC5
    IEC=DEC
 90 AEC=ABS(DEC)
    IF(AEC)19,21,18
 18 IF(AEC-2)21,19,19
 19 WRITE(3,20)CLAT(IND)
 20 FORMAT(41H ERROR IN CALCULATION OF DEC AT LATITUDE ,F20.2/)
    GO TO 46
 21 LL=30
    DO 45 J=IBA,20,INT
    JND=(J-11.0)/INT+1
    CLONG(JND)=(NOL-1)*5.0+(J-IBA)*1.0+138.00
    LONG=CLONG(JND)
    CLONG5=CLONG(JND)/5.0
    LONG5=CLONG5
    DECL=(CLONG(JND)-LONG)*2.0
    DECL5=(CLONG5-LONG5+0.01)*10.0
    AECL=ABS(DECL)
    IECL=DECL
    IF(AECL)23,25,22
 22 IF(AECL-2)25,23,23
 23 WRITE(3,24)CLAT(IND),CLONG(JND)
 24 FORMAT(/45H ERROR IN THE CALCULATION OF DECL AT LATITUDE,F20.2,15H
   1 AND LONGITUDE ,F20.2/)
    GO TO 45
 25 ACC=0
    IECL5=DECL5
    WRITE(3,80)CLAT(IND),CLONG(JND),IEC,IEC5,IECL,IECL5
 80 FORMAT(' LAT=',F10.1,'LONG=',F10.1,' IEC=',I10,' IEC5 =',I10,' IEC
   1L=',I10,' IECL5 =',I10)
    KTCT=0
    ACCM=0
    ACCP=0
    ACCEP=0
    ACCPER=0
    ACCMER=0
    AGCN=0.0
    AGCM=0.0
    AGCP=0.0
```

```
      ASN=0.0
      ASM=0.0
      ASP=0.0
      ATN=0.0
      ATM=0.0
      ATP=0.0
      SISTCN=0.0
      SISTCM=0.0
      SISTCP=0.0
      DO 41 II=1,LL
      IH=(NO-1)*(LIM-IBA+INT)/INT+(I-IBA)*2+II-IEC5+2
      QLAT=(IH/2.0+LAO/100.0-0.5)*0.0174533
      SQLAT=SIN(QLAT)
      CQLAT=COS(QLAT)
      SSQLA=SQLAT*SQLAT
      TER=1.0+BETA*SSQLA
      TER1=EQGR*SSQLA
      ANGC=-(DEQGGC*TER+DBTAGC*TER1)+POTCOR
      ANS=-(DEQGS*TER+DBTAS*TER1)+POTCOR
      ANT=-(DEQGT*TER+DBTAT*TER1)+POTCOR
      DO 40 JJ=1,LL
      JH=(NCL-1)*(LIM-IBA+INT)/INT+(J-IBA)*2-IECL5+6+JJ
   84 QLONG=JH/2.0+LOO/100.0-0.5
      IF(II-IEC5-IEC-9)29,27,26
   26 IF(II-IEC5-IEC-14)27,27,29
   27 IF(JJ-IECL5+IECL-9)29,87,28
   28 IF(JJ-IECL5+IECL-14)87,87,29
   29 IF(IAN(IH,JH))32,42,30
   30 ID1=IAN(IH,JH)/10000
      ID2=ID1/100
      IINT=ID2/100
      ISDEV=ID2-IINT*100
      NS=ID1-(IINT*100+ISDEV)*100
      JAN=IAN(IH,JH)-((IINT*100+ISDEV)*100+NS)*10000
      GO TO 93
   32 ID1=-IAN(IH,JH)/10000
      ID2=ID1/100
      IINT=ID2/100
      ISDEV=ID2-IINT*100
      NS=ID1-(IINT*100+ISDEV)*100
      JAN=IAN(IH,JH)+((IINT*100+ISDEV)*100+NS)*10000
   93 IF(IINT)102,101,100
  100 IF(IINT-4)320,319,319
  319 IINT=3
      NS=6
  320 WE=SER*SER+INTER(IINT,NS)*INTER(IINT,NS)
      SIS=ISIS(IINT,NS)
      GO TO 35
  101 IF(NS)102,91,33
   33 WE=(ISDEV*ISDEV+(NUM-NS)*(NUM-NS)*SER*SER/((NUM-1)*(NUM-1)))/NS
      SIS=0.0
      GO TO 35
  102 WRITE(3,103)CLAT(IND),CLONG(JND),II,JJ,IH,JH
  103 FORMAT(' COMP ERROR AT (',F10.2,' S',F10.2,' E).ARRAY REF IS',
     12I10,' MATRIX REF IS',2I10)
```

```
 91 WE=SER*SER
    SIS=0.0
 35 CONTINUE
    DLONG=(QLONG-CLONG(JND))*0.0174533
    CPSI=SCLAT*SQLAT+CCLAT*CQLAT*COS(DLONG)
    SPSI=SQRT(1-CPSI*CPSI)
    SPSI2=SQRT(ABS(0.5*(1-CPSI)))
    IF(SPSI2)38,38,36
 36 CPSI2=SQRT(ABS(0.5*(1+CPSI)))
    RFPSI=ALOG(SPSI2*(1+SPSI2))
    SINAZ=CQLAT*SIN(DLONG)/SPSI
    IF(CCLAT)37,40,37
 37 COSAZ=(SQLAT-CPSI*SCLAT)/(SPSI*CCLAT)
    STOF=CQLAT*(1+1/SPSI2-6*SPSI2-5*CPSI-3*CPSI*RFPSI)
    ACC=ACC+STOF*JAN
    FDEF=CQLAT*(-0.5*CPSI2/(SPSI2*SPSI2)-3*CPSI2+5*SPSI+3*SPSI*RFPSI-
   11.5*(1+2*SPSI2)*CPSI2*CPSI/((1+SPSI2)*SPSI2))
    FDEFM=FDEF*COSAZ
    FDEFP=FDEF*SINAZ
    ACCM=ACCM+FDEFM*JAN
    ACCP=ACCP+FDEFP*JAN
    KTOT=KTOT+1
    ACCER=ACCER+WE*STOF*STOF
    ACCMER=ACCMER+WE*FDEFM*FDEFM
    ACCPER=ACCPER+WE*FDEFP*FDEFP
    AGCN=AGCN+STOF*ANGC
    ASN=ASN+STOF*ANS
    ATN=ATN+STOF*ANT
    AGCM=AGCM+FDEFM*ANGC
    ASM=ASM+FDEFM*ANS
    ATM=ATM+FDEFM*ANT
    AGCP=AGCP+FDEFP*ANGC
    ASP=ASP+FDEFP*ANS
    ATP=ATP+FDEFP*ANT
    SISTON=SISTON+SIS*STOF
    SISTOM=SISTOM+SIS*FDEFM
    SISTOP=SISTOP+SIS*FDEFP
105 IF(I-IBA)46,106,40
106 IF(J-IBA)45,107,40
107 IF(II-1)40,108,40
108 PSI=ATAN(SPSI/CPSI)/0.0174533
    STO=STOF/CQLAT
    SDEV=ISDEV
    WRITE(3,109)IH,JH,IINT,SDEV,NS,JAN,PSI,STO
109 FORMAT(' LOC = (',2I5,' ) INT KEY =',I5,' STD ERR =',F10.2,' NO=',
   1I5,' ANOM =',I7,' PSI =',F10.3,' DEG.FPSI =',E10.4)
    GO TO 40
 38 WRITE(3,39)SPSI2
 39 FORMAT(/42H ERROR IN SINE PSI/2.VALUE OBTAINED BEING ,F20.8)
    GO TO 40
 87 QQLAT=QLAT/0.0174533
    WRITE(3,88)QQLAT,QLONG,II,IH,JJ,JH,IAN(IH,JH)
 88 FORMAT(' ELEMENT EXCLUDED LAT =',F10.2,' LONG =',F10.2,' II =',I5,
   1' IH =',I5,' JJ =',I5,' JH =',I5,' ANOM =',I10)
    GO TO 40
```

```
42 CONTINUE
40 CONTINUE
41 CONTINUE
44 COMPN(IND,JND)=CCONN*ACC
   COMPDM(IND,JND)=CCOND*ACCM
   COMPDP(IND,JND)=CCOND*ACCP
   ITOT(IND,JND)=KTOT
   PRECN(IND,JND)=SQRT(ACCER)*CCONN/10.0
   PRECDM(IND,JND)=SQRT(ACCMER)*CCOND/10.0
   PRECDP(IND,JND)=SQRT(ACCPER)*CCOND/10.0
   CORGCN(IND,JND)=CCONN*AGCN
   CORSN(IND,JND)=CCONN*ASN
   CORTN(IND,JND)=CCONN*ATN
   CORGCM(IND,JND)=CCOND*AGCM
   CORSM(IND,JND)=CCOND*ASM
   CORTM(IND,JND)=CCOND*ATM
   CORGCP(IND,JND)=CCOND*AGCP
   CORSP(IND,JND)=CCOND*ASP
   CORTP(IND,JND)=CCOND*ATP
   SISN(IND,JND)=CCONN*SISTON/10.0
   SISM(IND,JND)=CCOND*SISTOM/10.0
   SISP(IND,JND)=CCOND*SISTOP/10.0
   WRITE(3,94)COMPN(IND,JND),PRECN(IND,JND),COMPDM(IND,JND),
  1PRECDM(IND,JND),COMPDP(IND,JND),PRECDP(IND,JND)
94 FORMAT(/' N =',F10.2,' +/-',F10.2,' XI =',F10.4,' +/-',F10.4,' ETA
  1 =',F10.4,' +/-',F10.4)
   WRITE(3,95)SISN(IND,JND),SISM(IND,JND),SISP(IND,JND)
95 FORMAT(' SYSTEMATIC ERROR ',F10.2,17X,F10.4,18X,F10.4)
   WRITE(3,96)CORGCN(IND,JND),CORGCM(IND,JND),CORGCP(IND,JND)
96 FORMAT(' WHEN GRAV CON IS CHANGED, D N =',F10.2,' D XI =',F10.4,
  1' D ETA =',F10.4)
   WRITE(3,97)CORSN(IND,JND),CORSM(IND,JND),CORSP(IND,JND)
97 FORMAT(' WHEN ONLY SPHEROID IS CHANGED, D N =',F10.2,' D XI =',
  1F10.4,' D ETA =',F10.4)
   WRITE(3,98)CORTN(IND,JND),CORTM(IND,JND),CORTP(IND,JND)
98 FORMAT(' WHEN BOTH GR CON AND SPHEROID ARE CHANGED, D N =',F10.2,
  1' D XI =',F10.4,' D ETA =',F10.4)
45 CONTINUE
46 CONTINUE
   WRITE(3,47)
47 FORMAT('1     CALCULATION OF GEOID SPHEROID SEPARATION  N  ,ANOMAL
  1IES USED ARE'//)
   WRITE(3,48)
48 FORMAT('   TOPOGRAPHICAL CORRECTIONS USING HUNTER FORMULA FOR H <
  1 2500 .   FOR H > 2500, RHO = 2.67'///)
   WRITE(3,49)
49 FORMAT(//120H      N          N          N          N          N
  1   N          N          N          N          N          LATITUD
  2 /)
   WRITE(3,50)
50 FORMAT(120H    CM.        CM.        CM.        CM.        CM.
  1M.        CM.        CM.        CM.        CM.        DEGREES N
  2//)
   DO 151 I=1,10
   COMPN(I,11)=CLAT(I)
```

```
      WRITE(3,51)(COMPN(I,J),J=1,11)
   51 FORMAT(/10F10.2,F15.2)
  151 CONTINUE
      WRITE(3,52)(CLONG(J),J=1,10)
   52 FORMAT(///' ',10F10.2/)
      WRITE(3,53)
   53 FORMAT(72H                                  L O N G I T U D E   I N   D
     1E G R E E S   E ///)
      WRITE(3,113)
  113 FORMAT('1                           ESTIMATES OF ERROR IN THE FINAL RESU
     1T'///)
      WRITE(3,49)
      WRITE(3,50)
      DO 152 I=1,10
      PRECN(I,11)=CLAT(I)
      WRITE(3,51)(PRECN(I,J),J=1,11)
      WRITE(3,321)(SISN(I,J),J=1,10)
  321 FORMAT(10F10.2)
  152 CONTINUE
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,54)
   54 FORMAT(///'1  CALCULATION OF DEFLECTIONS OF THE VERTICAL,ANOMALIE
     1 USED ARE'//)
      WRITE(3,48)
      WRITE(3,55)
   55 FORMAT(//120H MERIDIAN  MERIDIAN  MERIDIAN  MERIDIAN  MERIDIAN  M
     1RIDIAN  MERIDIAN  MERIDIAN  MERIDIAN  MERIDIAN           LATITUDE
     2 /)
      WRITE(3,56)
   56 FORMAT(120H   SEC.      SEC.      SEC.      SEC.      SEC.      S
     1C.      SEC.      SEC.      SEC.      SEC.           DEGREES N
     2//)
      DO 153 I=1,10
      COMPDM(I,11)=CLAT(I)
      WRITE(3,57)(COMPDM(I,J),J=1,11)
   57 FORMAT(/10F10.4,F15.2)
  153 CONTINUE
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,113)
      WRITE(3,55)
      WRITE(3,56)
      DO 154 I=1,10
      PRECDM(I,11)=CLAT(I)
      WRITE(3,57)(PRECDM(I,J),J=1,11)
      WRITE(3,322)(SISM(I,J),J=1,10)
  322 FORMAT(10F10.4)
  154 CONTINUE
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,54)
      WRITE(3,58)
   58 FORMAT(//120H PRI.VERT. PRI.VERT. PRI.VERT. PRI.VERT. PRI.VERT. P
     1I.VERT. PRI.VERT. PRI.VERT. PRI.VERT. PRI.VERT.          LATITUDE
```

```
      2 /)
       WRITE(3,56)
       DO 155 I=1,10
       COMPDP(I,11)=CLAT(I)
       WRITE(3,57)(COMPDP(I,J),J=1,11)
  155 CONTINUE
       WRITE(3,52)(CLONG(J),J=1,10)
       WRITE(3,53)
       WRITE(3,113)
       WRITE(3,58)
       WRITE(3,56)
       DO 156 I=1,10
       PRECDP(I,11)=CLAT(I)
       WRITE(3,57)(PRECDP(I,J),J=1,11)
       WRITE(3,322)(SISP(I,J),J=1,10)
  156 CONTINUE
       WRITE(3,52)(CLONG(J),J=1,10)
       WRITE(3,53)
       WRITE(3,59)
   59 FORMAT(///'1    NUMBER    NUMBER    NUMBER    NUMBER    NUMBER
     1NUMBER    NUMBER    NUMBER    NUMBER    NUMBER              LATITUDI
     2 '/)
       WRITE(3,60)
   60 FORMAT(120H IN COMP.  IN COMP.  IN COMP.  IN COMP.  IN COMP.  IN '
     1OMP.  IN COMP.  IN COMP.  IN COMP.  IN COMP.           DEGREES N
     2/)
       DO 157 I=1,10
       ITOT(I,11)=CLAT(I)
       WRITE(3,61)(ITOT(I,J),J=1,11)
   61 FORMAT(' ',10I10,I15/)
  157 CONTINUE
       WRITE(3,52)(CLONG(J),J=1,10)
       WRITE(3,53)
       WRITE(3,301)
  301 FORMAT('1  CHANGES IN  N  DUE TO THE FOLLOWING CHANGES IN THE PAR
     1METERS OF THE INTERNATIONAL SPHEROID')
       WRITE(3,302)DGRCON,RATGRC
  302 FORMAT(//' CHANGE IN K*M =',E10.4,'  CM***3  SEC**-2;  RATIO WITH
     1REFERENCE TO K*M =',E10.4)
       WRITE(3,303)POTCOR
  303 FORMAT('  CHANGE IN POTSDAM DATUM =',F10.2,' TENTH MGAL')
       WRITE(3,49)
       WRITE(3,50)
       DO 304 I=1,10
       CORGCN(I,11)=CLAT(I)
  304 WRITE(3,51)(CORGCN(I,J),J=1,11)
       WRITE(3,52)(CLONG(J),J=1,10)
       WRITE(3,53)
       WRITE(3,305)
  305 FORMAT('1  CHANGES IN  XI  DUE TO THE FOLLOWING CHANGES IN THE PA
     1AMETERS OF THE INTERNATIONAL SPHEROID')
       WRITE(3,302)DGRCON,RATGRC
       WRITE(3,303)POTCOR
       WRITE(3,55)
       WRITE(3,56)
```

```
      DO 306 I=1,10
      CORGCM(I,11)=CLAT(I)
  306 WRITE(3,57)(CORGCM(I,J),J=1,11)
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,307)
  307 FORMAT('1   CHANGES IN  ETA  DUE TO THE FOLLOWING CHANGES IN THE
     1ARAMETERS OF THE INTERNATIONAL SPHEROID')
      WRITE(3,302)DGRCON,RATGRC
      WRITE(3,303)POTCOR
      WRITE(3,58)
      WRITE(3,56)
      DO 308 I=1,10
      CORGCP(I,11)=CLAT(I)
  308 WRITE(3,57)(CORGCP(I,J),J=1,11)
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,301)
      WRITE(3,309)DRAD,RATRAD,DFLAT
  309 FORMAT('  CHANGE IN EQ. RADIUS =',F10.1,' METRES.  DRAD/RADIUS='
     1E10.4,' DFLATTENING =',E10.4)
      WRITE(3,303)PCTCOR
      WRITE(3,49)
      WRITE(3,50)
      DO 310 I=1,10
      CORSN(I,11)=CLAT(I)
  310 WRITE(3,51)(CORSN(I,J),J=1,11)
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,305)
      WRITE(3,309)DRAD,RATRAD,DFLAT
      WRITE(3,303)PCTCOR
      WRITE(3,55)
      WRITE(3,56)
      DO 311 I=1,10
      CORSM(I,11)=CLAT(I)
  311 WRITE(3,57)(CORSM(I,J),J=1,11)
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,307)
      WRITE(3,309)DRAD,RATRAD,DFLAT
      WRITE(3,303)PCTCOR
      WRITE(3,58)
      WRITE(3,56)
      DO 312 I=1,10
      CORSP(I,11)=CLAT(I)
  312 WRITE(3,57)(CORSP(I,J),J=1,11)
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,301)
      WRITE(3,302)DGRCON,RATGRC
      WRITE(3,309)DRAD,RATRAD,DFLAT
      WRITE(3,303)PCTCOR
      WRITE(3,49)
      WRITE(3,50)
```

Appendix (12) concluded.

```
    DO 313 I=1,10
    CORTN(I,11)=CLAT(I)
313 WRITE(3,51)(CORTN(I,J),J=1,11)
    WRITE(3,52)(CLONG(J),J=1,10)
    WRITE(3,53)
    WRITE(3,305)
    WRITE(3,302)DGRCON,RATGRC
    WRITE(3,309)DRAD,RATRAD,DFLAT
    WRITE(3,303)PCTCOR
    WRITE(3,55)
    WRITE(3,56)
    DO 314 I=1,10
    CORTM(I,11)=CLAT(I)
314 WRITE(3,57)(CORTM(I,J),J=1,11)
    WRITE(3,52)(CLONG(J),J=1,10)
    WRITE(3,53)
    WRITE(3,307)
    WRITE(3,302)DGRCON,RATGRC
    WRITE(3,309)DRAD,RATRAD,DFLAT
    WRITE(3,303)PCTCOR
    WRITE(3,58)
    WRITE(3,56)
    DO 315 I=1,10
    CORTP(I,11)=CLAT(I)
315 WRITE(3,57)(CORTP(I,J),J=1,11)
    WRITE(3,52)(CLONG(J),J=1,10)
    WRITE(3,53)
 62 CONTINUE
 63 CONTINUE
    STOP
    END

      SIZE OF COMMON  000000   PROGRAM  045062

                    END OF COMPILATION  MAIN
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166 IBM OS/360 BASIC FORTRAN IV (E) COMPILATION

```
C     PROGRAM STOKIN
C     CALCULATES THE COMPONENTS OF STOKESIAN HEIGHT AND DEFLECTIONS OF THE
C     VERTICAL FOR AN AREA 0.1< PSI< 1,THE INTERVAL OF COMPUTATION BEING 0.1
C     BASIC PROGRAM COMPUTES EFFECTS OVER 1 X 1 AREA,THE ANOMALIES REPRESENT
C     INGO.1 SQUARES.THE EFFECT IS CONSIDERED FOR A 3 X 3 AREA AROUND THE
C     1 X 1 SQUARE OF COMPUTATION,SYMMETRIC WITH RESPECT TO THE SW CORNER.
C     AN INNER AREA OF 4 0.1 X 0.1 SQUARES ROUND COMPUTATION POINT IS OMITTE
C     LAT,LONG +VE N,E RESPECTIVELY.PROGRAM HANDLES A 10 X 10 AREA.ALL UNITS
C     ARE DEGREES.
      DIMENSION KAN(10),IAN(80,80),COMPN(11,11),COMPDM(11,11),
     1COMPDP(11,11),CLAT(10),CLONG(10),ITOT(11,11),PRECN(11,11),
     2PRECDM(11,11),PRECDP(11,11),INTER(3,6),ISIS(3,6),CORGCN(11,11),
     3CORGCM(11,11),CORGCP(11,11),CORSN(11,11),CORSM(11,11)
      DIMENSION CORSP(11,11),CORTN(11,11),CORTM(11,11),CORTP(11,11),
     1SISN(11,11),SISM(11,11),SISP(11,11)
      ICAP=80
      JCAP=80
      INT=1
      DO 1 I=1,ICAP
      DO 1 J=1,JCAP
    1 IAN(I,J)=0
C     READ LAT,LONG OF CENTRE OF SW TENTH DEGREE SQUARE IN HUNDREDTH DEGREES
      READ(1,2)LAC,LOO
    2 FORMAT(2I10)
      WRITE(3,70)LAC,LOO
   70 FORMAT('1    CALCULATICN OF N,XI & ETA.INDEX LAT =',I8,' N.INDEX L
     1ONG =',I8,' E'//)
      LOCO=0
      LACO=0
C     READ GRAVITY ANOMALIES REPRESENTING TENTH DEGREE SQUARE MEANS,IN TENTI
C     MILLIGAL,WITH LAT,LONG OF MOST WESTERLY SQ.IN  HUNDREDTH DEGREES
    3 READ(1,4)ITE,LAH,LOH,(KAN(I),I=1,10)
    4 FORMAT(I2,12I6)
      IF(ITE-1)5,73,13
    5 DO 12 I=1,10
      IN=(LAH-LAO)/10.0+1
      JN=(LOH-LOO)/10.0+I
      IF(IN)10,10,6
    6 IF(IN-ICAP)7,7,10
    7 IF(JN)10,10,8
    8 IF(JN-JCAP)9,9,10
    9 IAN(IN,JN)=KAN(I)
   12 CONTINUE
       GO TO 71
   10 GO TO 3
   71 WRITE(3,72)ITE,LAH,LOH,(KAN(I),I=1,10),IN,JN
   72 FORMAT(' ',I1,12I8,' IN=',I5,' JN =',I5)
      IF(ITE-1)3,73,13
   13 WRITE(3,14)LAH,LOH
   14 FORMAT(/15H CARD WITH LAT ,I10,17H N AND LONGITUDE ,I10,29H E HAS
     1INDEX PUNCHED IN ERROR)
      GO TO 3
   73 CONTINUE
```

```
C     END OF READING ANOMALIES.STORED IN TENTH MILLIGAL.
C     VALUES IN IAN CONSTRUCTED AS FOLLOWS :-NUMBERING COLUMNS FROM TH
C     RIGHT, COL NOS 1 - 4 HAVE GRAVITY ANOMALY IN TENTH MILLIGAL;
C     IN COL 6,1 =  INTERPOLATED  ANOMALY,2 = INTERP/EXTRAPOLATED ANOM
C     3 REPRESENTS EXTRAPOLATED ANOMALY.
      LIM=20
      IBA=11
      ISPA=1
      READ(1,16)CCN,RHO,PI,IARC,TGRAV,CON1,RAD
   16 FORMAT(E10.3,F10.2,F10.5,I10,F10.2,E10.2,E10.4)
      CCONN=(RAD*CON1*PI)/(4*TGRAV*180*180)
      CCOND=(PI*IARC*CON1)/(4*TGRAV*180*180)
      WRITE(3,17)CCONN,CCOND
   17 FORMAT(/47H COEFFICIENTS FOR COMPUTATION OF SEPARATION IS ,E20.5
     1'  AND OF DEFLECTIONS IS  ',E20.5)
      READ(1,110)SER,NUM
  110 FORMAT(F10.1,I10)
      DO 112 I=1,3
      READ(1,111)(INTER(I,J),J=1,6)
  111 FORMAT(6I10)
  112 CONTINUE
      DC 317 I=1,3
  317 READ(1,111)(ISIS(I,J),J=1,6)
      DO 63 NO=1,ISPA
      DO 62 NOL=1,ISPA
      DO 46 I=IBA,20,INT
      IND=(I-11.0)/INT+1
      CLAT(IND)=(LAO+LACO)/100.0+(NO-1)*5.0+(I-IBA)*0.5+0.95
      COL=CLAT(IND)*0.0174533
      SCLAT=SIN(CCL)
      CCLAT=COS(CCL)
      LAT=CLAT(INC)
      DEC=(CLAT(IND)-LAT)*10+0.499
      AEC=ABS(DEC)
      IF(AEC)19,21,18
   18 IF(AEC-10)21,19,19
   19 WRITE(3,20)CLAT(IND)
   20 FCRMAT(41H ERROR IN CALCULATION OF DEC AT LATITUDE ,F20.2/)
      GO TO 46
   21 LL=30
      IEC=DEC
      DC 45 J=IBA,20,INT
      JND=(J-11.0)/INT+1
      CLONG(JND)=(LOO+LOCO)/100.0+(NOL-1)*5.0+(J-IBA)*0.5+0.95
      LONG=CLONG(JND)
      DECL=(CLONG(JND)-LONG)*10+0.5001
   25 ACC=0
      KTCT=0
      ACCM=0
      ACCP=0
      ACCER=0
      ACCPER=0
      ACCMER=0
      WRITE(3,87)CLAT(IND),CLONG(JND),DEC,DECL
   87 FORMAT(' COMP PT. LAT =',F10.2,' LONG =',F10.2,' DEC =',F10.2,
```

```
   1'DECL =',F10.2)
    DO 41 II=1,LL
    IH=(NO-1)*(LIM-IBA+INT)/INT+(I-IBA)*5+II+IEC
    QLAT=(IH/10.0+(LAO+LACO)/100.0-0.1)*0.0174533
    SQLAT=SIN(QLAT)
    CQLAT=COS(QLAT)
    DO 40 JJ=1,LL
    IECL=DECL
    IF(IECL-10)93,92,92
 92 IECL=0
 93 CONTINUE
    JH=(NOL-1)*(LIM-IBA+INT)/INT+(J-IBA)*5+JJ-IECL
    IF(II+IEC-10)29,27,26
 26 IF(II+IEC-11)27,27,29
 27 IF(JJ-IECL-10)29,88,28
 28 IF(JJ-IECL-11)88,88,29
 29 IF(IAN(IH,JH))32,42,30
 30 ID1=IAN(IH,JH)/10000
    IINT=ID1/10
    NS=ID1-IINT*10
    JAN=IAN(IH,JH)-(NS +IINT*10)*10000
    GO TO 33
 32 ID1=-IAN(IH,JH)/10000
    IINT=ID1/10
    NS=ID1-IINT*10
    JAN=IAN(IH,JH)+(NS +IINT*10)*10000
 33 IF(IINT)102,101,100
100 IF(IINT-4)320,319,319
319 IINT=3
    NS=6
320 WE=SER*SER+INTER(IINT,NS)*INTER(IINT,NS)
    SIS=ISIS(IINT,NS)
    GO TO 35
101 WE=SER*SER
    SIS=0.0
    GO TO 35
102 WRITE(3,103)CLAT(IND),CLONG(JND),II,JJ,IH,JH
103 FORMAT('COMP.ERR.AT( ',F10.2,' S',F10.2,' E ).ARRAY REF.IS ',2I1
   1'MATRIX REF.IS',2I10/)
    WE=SER*SER
    SIS=0.0
 35 QLONG=JH/10.0+(LOO+LOCO)/100.0-0.1
    DLONG=(QLONG-CLONG(JND))*0.0174533
    CPSI=SCLAT*SQLAT+CCLAT*CQLAT*COS(DLONG)
    SPSI=SQRT(1-CPSI*CPSI)
    SPSI2=SQRT(ABS(0.5*(1-CPSI)))
    IF(SPSI2)38,38,36
 36 CPSI2=SQRT(ABS(0.5*(1+CPSI)))
    RFPSI=ALOG(SPSI2*(1+SPSI2))
    SINAZ=CQLAT*SIN(DLONG)/SPSI
    IF(CCLAT)37,40,37
 37 COSAZ=(SQLAT-CPSI*SCLAT)/(SPSI*CCLAT)
    STOF=CQLAT*(1+1/SPSI2-6*SPSI2-5*CPSI-3*CPSI*RFPSI)
    ACC=ACC+STOF*JAN
    FDEF=CQLAT*(-0.5*CPSI2/(SPSI2*SPSI2)-3*CPSI2+5*SPSI+3*SPSI*RFPSI-
```

```
    11.5*(1+2*SPSI2)*CPSI2*CPSI/((1+SPSI2)*SPSI2))
     FDEFM=FDEF*COSAZ
     FDEFP=FDEF*SINAZ
     ACCM=ACCM+FDEFM*JAN
     ACCP=ACCP+FDEFP*JAN
     KTCT=KTOT+1
     ACCER=ACCER+WE*STOF*STOF
     ACCMER=ACCMER+WE*FDEFM*FDEFM
     ACCPER=ACCPER+WE*FDEFP*FDEFP
105  IF(I-IBA)46,106,40
106  IF(J-IBA)45,107,40
107  IF(II-1)40,108,40
108  STO=STOF/CQLAT
     PSI=ATAN(SPSI/CPSI)/0.0174533
     WRITE(3,109)IH,JH,IINT,NS,JAN,STO,PSI
109  FORMAT(' LOC =(',2I5,' ) INT KEY =',I5,' NO =',I5,' ANOM =',I7,
    1' FPSI =',E10.4,' PSI =',F10.2,'DEG')
     WRITE(3,324)WE
324  FORMAT('  WEIGHT =',F12.4)
     GO TO 40
 38  WRITE(3,39)SPSI2
 39  FORMAT(/42H ERROR IN SINE PSI/2.VALUE OBTAINED BEING ,F20.8)
     GO TO 40
 88  QLA=QLAT/0.0174533
     WRITE(3,89)QLA,QLONG,IH,JH,II,JJ
 89  FORMAT(' COMP. ELEM. EXCLUDED LAT =',F10.2,' LONG =',F10.2,' IH ='
    1,I5,' JH =',I5,' II =',I5,' JJ =',I5)
 42  CONTINUE
 40  CONTINUE
 41  CONTINUE
 44  COMPN(IND,JND)=CCONN*ACC
     COMPDM(IND,JND)=CCOND*ACCM
     COMPDP(IND,JND)=CCOND*ACCP
     ITOT(IND,JND)=KTOT
     PRECN(IND,JND)=SQRT(ABS(ACCER))*CCONN/10.0
     PRECDM(IND,JND)=SQRT(ABS(ACCMER))*CCOND/10.0
     PRECDP(IND,JND)=SQRT(ABS(ACCPER))*CCOND/10.0
     WRITE(3,90)IND,JND,CLAT(IND),CLONG(JND),COMPN(IND,JND),
    1PRECN(IND,JND)
 90  FORMAT(/' ',2I3,' LAT =',F10.2,' LONG =',F10.2,' N =',F10.2,' +/-'
    1,F10.2)
     WRITE(3,91)COMPDM(IND,JND),PRECDM(IND,JND),COMPDP(IND,JND),
    1PRECDP(IND,JND)
 91  FORMAT(' DEFLECTIONS OF THE VERTICAL:  XI =',E10.5,' +/-',E10.5,
    1' ETA =',F10.4,' +/-',F10.4)
 45  CONTINUE
 46  CONTINUE
     WRITE(3,47)
 47  FORMAT(///'1  CALCULATION OF GEOID SPHEROID SEPARATION  N  ,ANOMAL
    1IES USED ARE'///)
     WRITE(3,48)
 48  FORMAT(34H          TOPOGRAPHICAL CORRECTIONS///)
     WRITE(3,49)
 49  FORMAT(//120H     N         N         N         N         N
    1  N         N         N         N         N          LATITUDE
```

```
     2 /)
      WRITE(3,50)
   50 FORMAT(120H    CM.        CM.        CM.        CM.        CM.        C
     1M.        CM.        CM.        CM.        CM.               DEGREES N /
     2//)
      DO 151 I=1,10
      COMPN(I,11)=CLAT(I)
      WRITE(3,51)(COMPN(I,J),J=1,11)
   51 FORMAT(/10F10.2,F15.2)
  151 CONTINUE
      WRITE(3,52)(CLONG(J),J=1,10)
   52 FORMAT(///10F10.2/)
      WRITE(3,53)
   53 FORMAT(72H                              L O N G I T U D E   I N   D
     1E G R E E S   E ///)
      WRITE(3,113)
  113 FORMAT('1      ESTIMATES OF ERROR IN THE FINAL RESULT'////)
      WRITE(3,49)
      WRITE(3,50)
      DO 152 I=1,10
      PRECN(I,11)=CLAT(I)
      WRITE(3,51)(PRECN(I,J),J=1,11)
  152 CONTINUE
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,54)
   54 FORMAT(///'1  CALCULATION OF DEFLECTIONS OF THE VERTICAL,ANOMALIES
     1 USED ARE'//)
      WRITE(3,48)
      WRITE(3,55)
   55 FORMAT(//120H MERIDIAN  MERIDIAN  MERIDIAN  MERIDIAN  MERIDIAN  ME
     1RIDIAN  MERIDIAN  MERIDIAN  MERIDIAN  MERIDIAN           LATITUDE
     2 /)
      WRITE(3,56)
   56 FORMAT(120H   SEC.       SEC.       SEC.       SEC.       SEC.       SE
     1C.       SEC.       SEC.       SEC.       SEC.              DEGREES N /
     2//)
      DO 153 I=1,10
      COMPDM(I,11)=CLAT(I)
      WRITE(3,57)(COMPDM(I,J),J=1,11)
   57 FORMAT(/10F10.4,F15.2)
  153 CONTINUE
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,113)
      WRITE(3,55)
      WRITE(3,56)
      DO 154 I=1,10
      PRECDM(I,11)=CLAT(I)
      WRITE(3,57)(PRECDM(I,J),J=1,11)
  154 CONTINUE
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,58)
   58 FORMAT(//120H PRI.VERT. PRI.VERT. PRI.VERT. PRI.VERT. PRI.VERT. PR
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```
     1I.VERT. PRI.VERT. PRI.VERT. PRI.VERT. PRI.VERT.          LATITUDE
     2 /)
      WRITE(3,56)
      DO 155 I=1,10
      COMPDP(I,11)=CLAT(I)
      WRITE(3,57)(COMPDP(I,J),J=1,11)
  155 CONTINUE
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,113)
      WRITE(3,58)
      WRITE(3,56)
      DO 156 I=1,10
      PRECDP(I,11)=CLAT(I)
      WRITE(3,57)(PRECDP(I,J),J=1,11)
  156 CONTINUE
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
      WRITE(3,59)
   59 FORMAT(///'1     NUMBER     NUMBER     NUMBER     NUMBER     NUMBER
     1NUMBER     NUMBER     NUMBER     NUMBER     NUMBER          LATITUDE
     2 '/)
      WRITE(3,60)
   60 FORMAT(120H IN COMP.  IN COMP.  IN COMP.  IN COMP.  IN COMP.  IN C
     1OMP.  IN COMP.  IN COMP.  IN COMP.  IN COMP.          DEGREES N /
     2/)
      DO 157 I=1,10
      ITOT(I,11)=CLAT(I)
      WRITE(3,61)(ITOT(I,J),J=1,11)
   61 FORMAT(10I10,I15//)
  157 CONTINUE
      WRITE(3,52)(CLONG(J),J=1,10)
      WRITE(3,53)
   62 CONTINUE
   63 CONTINUE
      STOP
      END

        SIZE OF COMMON  000000   PROGRAM  047858

                     END OF COMPILATION  MAIN
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1JUL66 IBM OS/360 BASIC FORTRAN IV (E) COMPILATION

```
C        PROGRAM INCUT
C        CALCULATES THE INDIRECT EFFECT AND THE RESULTING CORRECTION TO THE
C        DEFLECTIONS OF THE VERTICAL IN THE RANGE GREATER THAN PSI= 20 DEGREES
C        COMPUTATION IS MADE OVER A TEN DEGREE SQUARE,COMPUTATION POINTS BEING
C        AT ONE DEGREE INTERVALS..NO LIMIT OVER THE AREA IN WHICH THIS EFFECT
C        CAN BE COMPUTED. IN A SINGLE RUN
C        CAN BE COMPUTED IN A SINGLE RUN.THE COMPUTATION INTERVAL IS ONE
C        DEGREE,THE PRINT OUT BEING BY 10 X 10 ARRAYS,LONGITUDE BEING INCREMEN
C        IN THE FIRST INSTANCE.LAT,LONG OF SW CORNER REQUIRE DEFINITION IN
C        TENTH DEGREES;+VE N,E RESPECTIVELY.
         DIMENSION KHT(10),IHT(80,80),RINEFF(11,11),CLAT(10),CLONG(10),
        1ITOT(11,11),DEFM(11,11),DEFP(11,11),ERN(11,11),ERM(11,11),
        2ERP(11,11)
         DO 1 I=1,36
         DO 1 J=1,72
       1 IHT(I,J)=0
C        READ LAT,LONG OF CENTRE OF SW DEGREE SQ.IN TENTH DEGREES
         READ(1,2)LAO,LCO
       2 FORMAT(2I10)
C        READ HTS OF 5 DEG. SQ. MEANS IN MET. LAT,LONG,OF W SQ. IN TENTH DEG
       3 READ(1,4)ITE,LAH,LCH,(KHT(I),I=1,5)
       4 FORMAT(I1,2I10,5I10)
         IF(ITE-1)5,100,15
       5 DO 14 I=1,5
         IN=(LAH-LAO)/50.0+1
         JN=(LCH-LCO)/50.0+I
         IF(IN)12,12,6
       6 IF(IN-36)7,7,12
       7 IF(JN)12,12,8
       8 IF(JN-72)9,9,12
       9 IF(KHT(I))10,10,11
      10 IHT(IN,JN)=100000
         GO TO 14
      11 IHT(IN,JN)=KHT(I)
      14 CONTINUE
         WRITE(3,28)ITE,LAH,LCH,(KHT(I),I=1,5),IN,JN
      28 FORMAT(' ',I1,7I10,' IN =',I5,' JN =',I5)
         GO TO 17
      12 LLCH=LCH+(I-1)*50
         WRITE(3,13)IN,JN,LAH,LLCH
      13 FORMAT(/44H HEIGHT READING OUTSIDE AREA,POSITION BEING ,2I10,9H AN
        1D LAT ,I10,6H LONG ,I10)
      17 IF(ITE-1)3,100,15
      15 WRITE(3,16)LAH,LCH
      16 FORMAT(15H CARD WITH LAT ,I10,15H AND LONGITUDE ,I10,27H HAS INDEX
        1 PUNCHED IN ERROR/)
         GO TO 3
C        END OF READING HEIGHTS.STORED IN METRES.
     100 READ(1,101)LAT,LONG
     101 FORMAT(2I10)
         LIM=30
         INT=1
C        COMMENCE COMPUTATION OVER 10 DEGREE X 10 DEGREE AREAS.
```

```
      READ(1,150)CON,RHO,PI,IARC,TGRAV,CON1,RAD
  150 FORMAT(E10.3,F10.2,F10.5,I10,F10.2,E10.2,E10.4)
      WRITE(3,18)
   18 FORMAT('1            COMPUTATION OF INDIRECT EFFECT.DENSITY USED IS'
     1//)
      DIG=RHO-0.10
      WRITE(3,19)RHO
   19 FORMAT(//'  RHO =',F5.2,'  - H / 21 , H IN KM FOR H < 2.5 KM.')
      WRITE(3,20)DIG
   20 FORMAT(/' AND RHO = ',F5.2,' FOR H > 2.5 KM')
      CCON=(CON*RHO*PI*PI*RAD*CON1)/(TGRAV*180*180)
      CCOND=(-CON*RHO*PI*PI*IARC*CON1)/(2*TGRAV*180*180)
      WRITE(3,151)CCON,CCOND
  151 FORMAT(53H COEFFICIENTS FOR COMPUTATION OF INDIRECT EFFECT IS   ,E2
     10.5,25H AND OF DEFLECTIONS IS    ,E20.5//)
      ISPA=1
      DO 139 NO=1,ISPA
      DO 138 NOL=1,ISPA
      DO 129 I=1,10
      IND=I
      CLAT(IND)=LAT/10.0+(NO-1)*10+(I-1)*9
      VAL=CLAT(IND)*0.0174533
      SCLAT=SIN(VAL)
      CCLAT=COS(VAL)
      CONP=(0.33333-SCLAT*SCLAT)/297.0
      CLAT5=CLAT(IND)/5.0
      LAT5=CLAT5
      DEC=(CLAT5-LAT5)*5
      IF(DEC)31,30,31
   30 LAT5=LAT5+1
   31 AEC=ABS(DEC)
      IF(AEC)127,103,102
  102 IF(AEC-5)103,127,127
  103 DO 126 J=1,10
      JND=J
      CLONG(JND)=LONG/10.0+(NOL-1)*10+(J-1)*9
      CLONG5=CLONG(JND)/5.0
      LONG5=CLONG5
      DECL=(CLONG5-LONG5)*5
      AECL=ABS(DECL)
      WRITE(3,27)CLAT(IND),CLONG(JND),DEC,DECL
   27 FORMAT('   COMPS AT LAT =',F10.2,' LONG =',F10.2,' DEC =',F10.2,
     1' DECL=',F10.2)
      IF(AECL)124,105,104
  104 IF(AECL-5)105,124,124
  105 ACC=0
      ACCM=0
      ACCP=0
      KTOT=0
      ACCER=0
      ACCERM=0
      ACCERP=0
```

```
      SQLAT=SIN(QLAT)
      CQLAT=COS(QLAT)
      CCNQ=(0.33333-SQLAT*SQLAT)/297.0
      DC 119 JJ=1,72
      RJH=LCC/1C.0+(JJ-1)*5
      IF(LAT5*5-25-RIH)106,117,109
  106 IF(LAT5*5+20-RIH)109,117,107
  107 IF(LCNG5*5-20-RJH)108,117,109
  108 IF(LCNG5*5+25-RJH)109,117,24
  109 IF(IHT(II,JJ))115,121,110
  110 IF(IHT(II,JJ)-100000)111,119,115
  111 DLONG=(RJH-CLCNG(JND))*0.0174533
      CPSI=SCLAT*SQLAT+CCLAT*CQLAT*COS(DLCNG)
      SPSI=SQRT(1-CPSI*CPSI)
      SPSI2=SQRT(ABS(C.5*(1-CPSI)))
      CPSI2=SQRT(ABS(C.5*(1+CPSI)))
      CCN3=IHT(II,JJ)/RAD
      IF(SPSI2)113,113,112
  112 SINAZ=SIN(DLONG)*CQLAT/SPSI
      CCSAZ=(SQLAT-SCLAT*CPSI)/(SPSI*CCLAT)
      IF(IHT(II,JJ)-2500)21,21,22
   21 ELV=1
      SF=1
      GC TC 23
   22 ELV=0
      SF=2.67/RHC
   23 TV=CQLAT*(1-0.5*(CONP-3*CONQ)+0.75*CON3)*(1-ELV*IHT(II,JJ)/(21000*RH
     1RHC))*SF/SPSI2
      ACC=ACC+TV*IHT(II,JJ)
      ACCER=ACCER+TV*TV
      VALM=TV*CPSI2*CCSAZ/SPSI2
      ACCM=ACCM+IHT(II,JJ)*VALM
      VALP=TV*CPSI2*SINAZ/SPSI2
      ACCP=ACCP+IHT(II,JJ)*VALP
      ACCERM=ACCERM+VALM*VALM
      ACCERP=ACCERP+VALP*VALP
      KTCT=KTCT+1
      GC TC 119
  113 WRITE(3,114)SPSI2
  114 FCRMAT(42H ERROR IN SINE PSI/2.VALUE CBTAINED BEING ,F20.8/)
      GC TO 119
  115 WRITE(3,116)II,JJ,IHT(II,JJ)
  116 FCRMAT(41H ERROR IN LOADING HEIGHT FOR ELEMENT NO.(,2I10,28H ),THE
     1 VALUE CBTAINED BEING ,I10//)
      GC TC 119
  117 WRITE(3,118)RIH,RJH,CLAT(IND),CLONG(JND)
  118 FCRMAT(/'ERROR DETECTED IN CCMP.CF LAT,LONG AT',2F10.1,' FOR EFFEC
     1T AT ',2F10.2/)
      GC TO 119
   24 IF(I-2)119,25,119
   25 IF(J-2)29,29,119
   29 WRITE(3,26)II,JJ,RIH,RJH,IHT(II,JJ)
   26 FORMAT(' ELEMENT OMITTED HAS II=',I5,' JJ =',I5,' LAT=',F10.2,
     1' LCNG=',F10.2,' HT =',I10)
  119 CCNTINUE
```

```
120 CCNTINUE
    GC TC 123
121 WRITE(3,122)CLAT(IND),CLONG(JND)
122 FCRMAT(43H INSUFFICIENT HEIGHTS TC COMPUTE EFFECT FOR,F10.2,14H DEG
   1GREES SOUTH,F10.2,14H DEGREES EAST //)
    DEFP(INC,JND)=0
    DEFM(INC,JND)=0
    RINEFF(INC,JND)=0
    ERN(INC,JND)=0
    ERM(INC,JND)=0
    ERP(INC,JNC)=0
    GC TO 126
123 RINEFF(INC,JND)=CCON*ACC
    ITCT(INC,JND)=KTCT
    DEFM(IND,JND)=CCCND*ACCM
    DEFP(INC,JND)=CCCND*ACCP
    ERN(INC,JND)=CCCN*SQRT(ACCER)*50
    ERM(INC,JND)=CCCND*SQRT(ACCERM)*50
    ERP(IND,JND)=CCCND*SQRT(ACCERP)*50
    GC TO 126
124 WRITE(3,125)
125 FCRMAT(/59H ERRCR IN EVALUATION OF FRACTICN OF FIVE DEGREE SQUARE.
   1LCNG/)
126 CCNTINUE
    GC TC 129
127 WRITE(3,128)
128 FCRMAT(58H ERROR IN EVALUATION OF FRACTION OF FIVE DEGREE SQUARE.L
   1AT/)
129 CCNTINUE
    WRITE(3,157)
157 FCRMAT(//'1  COMPUTATICNS OF INDIRECT EFFECT'///)
    WRITE(3,130)
130 FORMAT(//120H IND.EFF.  IND.EFF.  IND.EFF.  IND.EFF.  IND.EFF.  IN
   1D.EFF.  IND.EFF.  IND.EFF.  IND.EFF.  IND.EFF.           LATITUDE
   2 /)
    WRITE(3,131)
131 FCRMAT(120H    CM.         CM.         CM.         CM.        CM.       C
   1M.         CM.         CM.         CM.         CM.               DEGREES N /
   2//)
    DC 200 I=1,10
    RINEFF(I,11)=CLAT(I)
    WRITE(3,132)(RINEFF(I,J),J=1,11)
132 FCRMAT(10F10.2,F15.2/)
200 CCNTINUE
    WRITE(3,133)(CLCNG(J),J=1,10)
133 FCRMAT(///10F10.2/)
    WRITE(3,134)
134 FCRMAT(72H                              L O N G I T U D E   I N   D E (
   1E G R E E S    E ///)
    WRITE(3,159)
159 FCRMAT(//'1                        COMPUTATION OF ESTIMATES CF ERRORS
   1 IN CCMPUTED RESULTS'//)
    WRITE(3,130)
    WRITE(3,131)
    DC 201 I=1,10
```

```
      ERN(I,11)=CLAT(I)
      WRITE(3,132)(ERN(I,J),J=1,11)
  201 CONTINUE
      WRITE(3,133)(CLONG(J),J=1,10)
      WRITE(3,134)
      WRITE(3,152)
  152 FORMAT(//'1  COMPUTATION OF CORRECTIONS TO DEFLECTIONS OF THE VERT
     1ICAL DUE TO INDIRECT EFFECT'///)
      WRITE(3,153)
  153 FORMAT(//120H MERIDIAN  MERIDIAN  MERIDIAN  MERIDIAN  MERIDIAN  ME
     1RIDIAN  MERIDIAN  MERIDIAN  MERIDIAN  MERIDIAN          LATITUDE
     2 /)
      WRITE(3,154)
  154 FORMAT(120H   SEC.      SEC.      SEC.      SEC.      SEC.      SE
     1C.      SEC.      SEC.      SEC.      SEC.          DEGREES N /
     2//)
      DO 202 I=1,10
      DEFM(I,11)=CLAT(I)
      WRITE(3,155)(DEFM(I,J),J=1,11)
  155 FORMAT(10F10.4,F15.2/)
  202 CONTINUE
      WRITE(3,133)(CLONG(J),J=1,10)
      WRITE(3,134)
      WRITE(3,159)
      WRITE(3,153)
      WRITE(3,154)
      DO 203 I=1,10
      WRITE(3,155)(ERM(I,J),J=1,11)
      ERM(I,11)=CLAT(I)
  203 CONTINUE
      WRITE(3,133)(CLONG(J),J=1,10)
      WRITE(3,134)
      WRITE(3,152)
      WRITE(3,156)
  156 FORMAT(//120H PRI VERT  PRI VERT  PRI VERT  PRI VERT  PRI VERT  PR
     1I VERT  PRI VERT  PRI VERT  PRI VERT  PRI VERT          LATITUDE
     2 /)
      WRITE(3,154)
      DO 204 I=1,10
      DEFP(I,11)=CLAT(I)
      WRITE(3,155)(DEFP(I,J),J=1,11)
  204 CONTINUE
      WRITE(3,133)(CLONG(J),J=1,10)
      WRITE(3,134)
      WRITE(3,159)
      WRITE(3,156)
      WRITE(3,154)
      DO 205 I=1,10
      ERP(I,11)=CLAT(I)
      WRITE(3,155)(ERP(I,J),J=1,11)
  205 CONTINUE
      WRITE(3,133)(CLONG(J),J=1,10)
      WRITE(3,134)
      WRITE(3,135)
  135 FORMAT(//'1    NUMBER    NUMBER    NUMBER    NUMBER    NUMBER    N
```

```
   1UMBER     NUMBER     NUMBER     NUMBER     NUMBER              LATITUDE
   2'/)
    WRITE(3,136)
136 FORMAT(//120H IN COMP    IN COMP    IN COMP    IN COMP    IN COMP    IN
   1 COMP    IN COMP    IN COMP    IN COMP    IN COMP              DEGREES S
   2 //)
    DC 206 I=1,10
    ITCT(I,11)=CLAT(I)
    WRITE(3,137)(ITCT(I,J),J=1,11)
137 FCRMAT(1CI10,I15///)
2C6 CCNTINUE
    WRITE(3,133)(CLCNG(J),J=1,10)
    WRITE(3,134)
    DC 2C9 I=1,10
    CC 212 J=1,10
212 RINEFF(I,J)=RINEFF(I,J)/100.0
    WRITE(2,207)CLAT(I),CLCNG(1),(RINEFF(I,J),J=1,10)
207 FCRMAT(' 0',2F7.2,10F6.1)
2C9 WRITE(3,207)CLAT(I),CLCNG(1),(RINEFF(I,J),J=1,10)
    DC 210 I=1,10
    WRITE(2,208)CLAT(I),CLCNG(1),(DEFM(I,J),J=1,10)
208 FCRMAT(' 0',2F7.2,10F6.2)
210 WRITE(3,208)CLAT(I),CLCNG(1),(DEFM(I,J),J=1,10)
    DC 211 I=1,10
    WRITE(2,208)CLAT(I),CLCNG(1),(DEFP(I,J),J=1,10)
211 WRITE(3,2C8)CLAT(I),CLCNG(1),(DEFP(I,J),J=1,10)
138 CCNTINUE
139 CCNTINUE
    STOP
    END

     SIZE CF CCMMON  0C0000   PROGRAM  041158

                    END CF COMPILATICN  MAIN
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L66 IBM OS/360 BASIC FORTRAN IV (E) COMPILATION

```
C      PROGRAM INMIL
C      CALCULATES THE INDIRECT EFFECT AND THE RESULTING CORRECTION TO THE
C      DEFLECTIONS OF THE VERTICAL IN THE RANGE PSI=2 TO PSI = 20 DEGREES.
C      COMPUTATION IS MADE OVER A TEN DEGREE SQUARE,COMPUTATION POINTS BEING
C      AT ONE DEGREE INTERVALS.64 SUCH BASIC AREAS CAN BE CONSIDERED IN A
C      SINGLE RUN.IF LESS,AMEND DIMENSION STATEMENT.THE INTERVAL OF COMPUTATIO
C      IS DEFINED BY INT IN TENTH DEGREES AND REQUIRES DEFINITION.PRINT-OUT
C      BY 10 X 10 ARRAYS,LONGITUDE BEING INCREMENTED IN THE FIRST INSTANCE
C      LAT,LONG OF SW CORNER REQUIRE DEFINITION IN TENTH DEGREES,+VE N,E
       DIMENSION KHT(10),IHT(80,80),RINEFF(11,11),CLAT(10),CLONG(10),
      1ITOT(11,11),DEFM(11,11),DEFP(11,11),ERN(11,11),ERM(11,11),
      2ERP(11,11)
       ICAP=80
       INT=1
       DO 1 I=1,ICAP
       DO 1 J=1,ICAP
     1 IHT(I,J)=-1000
       READ(1,2)LAC,LCC
     2 FORMAT(2I10)
C      READ HEIGHTS OF DEGREE SQ.MEANS IN METRES.LAT,LONG OF W SQ. IN TENTH DE
     3 READ(1,4)ITE,LAH,LOH,(KHT(I),I=1,5)
     4 FORMAT(I2,2I10,5I10)
       IF(ITE-1)5,100,15
     5 DO 14 I=1,5
       IN=(LAH-LAC)/100.+1
       JN=(LCH-LCC)/100.+I
       IF(IN)12,12,6
     6 IF(IN-ICAP)7,7,12
     7 IF(JN)12,12,8
     8 IF(JN-ICAP)9,9,12
     9 IF(KHT(I))10,10,11
    10 IHT(IN,JN)=10000
       GO TO 14
    11 IHT(IN,JN)=KHT(I)
       GO TO 14
    12 LLCH=LCH+(I-1)*100
       WRITE(3,13)IN,JN,LAH,LLCH
    13 FORMAT(/44H HEIGHT READING OUTSIDE AREA,POSITION BEING ,2I10,9H AN
      1D LAT ,I10,6H LONG ,I10)
    14 CONTINUE
       WRITE(3,26)ITE,LAH,LCH,(KHT(I),I=1,5),IN,JN
    26 FORMAT(' ',I1,7I10,' IN=',I5,' JN=',I5)
       IF(ITE-1)3,100,15
    15 WRITE(3,16)LAH,LOH
    16 FORMAT(15H CARD WITH LAT ,I10,15H AND LONGITUDE ,I10,27H HAS INDEX
      1 PUNCHED IN ERROR/)
       GO TO 3
   100 IF(INT-10)102,101,101
   101 LIM=300
       GO TO 103
   102 LIM=210+INT*9
C      CALCULATES AT DEGREE INTERVALS
   103 SPA=(ICAP-40.0)/10
```

```
      ISPA=2
      WRITE(3,17)
   17 FORMAT(//'1           COMPUTATION OF INDIRECT EFFECT FOR 2 < PSI <
     12C '//)
      READ(1,15C)CCN,RHC,PI,IARC,TGRAV,CON1,RAD
  150 FCRMAT(E1C.3,F1C.2,F10.5,I10,F10.2,E10.2,E10.4)
      IF(ELV-1)20,18,18
   18 WRITE(3,19)RHC
   19 FCRMAT(' DENSITY CALCULATED USING HUNTER FORMULA. RHO= ',F5.2,' -
     1 H/21 , H IN KM.'///)
      WRITE(3,35)
   35 FORMAT('  IF ELEVATION > 2.5 KM., RHO PUT EQUAL TO 2.67')
   2C WRITE(3,31)LAC,LOG
   31 FCRMAT(//' GEOGRAPHICAL CCORDS OF SE CORNER OF DATA BLOCK:- LAT=',
     111C,'  LONG=',I10//)
C     COMMENCE COMPUTATION OF 10 DEGREE X 10 DEGREE BLOCK.
      CCCN=(CCN*RHC*PI*PI*RAD*CCN1)/(TGRAV*180*180)
      CCOND=(-CCN*RHO*PI*PI*IARC*CCN1)/(2*TGRAV*180*180)
      WRITE(3,151)CCCN,CCOND
  151 FCRMAT(53H COEFFICIENTS FCR COMPUTATION CF INDIRECT EFFECT IS  ,E2
     10.5,25H AND OF DEFLECTIONS IS    ,E20.5//)
      DC 135 NC=1,ISPA
      DC 134 NCL=1,ISPA
      DC 125 I=210,22C,INT
      IND=(I-210)+1
      CLAT(IND)=LAO/1CC.0-0.50+(NO-1)*10+I-210+20.0
      VAL=CLAT(IND)*0.0174533
      SCLAT=SIN(VAL)
      CCLAT=CCS(VAL)
      CCNP=(0.33333-SCLAT*SCLAT)/297.0
      CLAT5=CLAT(IND)/5.0
      LAT5=CLAT5
      DEC=(CLAT5-LAT5)*50
      IF(DEC)36,37,37
   36 DEC=DEC+51
   37 AEC=ABS(DEC)
      IF(AEC)125,1C5,1C4
  104 IF(AEC-50)105,21,125
   21 WRITE(3,22)CLAT(IND)
   22 FCRMAT(' FCR LAT = ',F10.1,' DEC = 50,NOW PUT EQUAL TO 0')
      DEC=C
  1C5 LL=45
      DC 124 J=210,22C,INT
      JND=(J-210)+1
      CLONG(JND)=LCC/10C.0+(NOL-1)*10+J-210+20.0
      CLCNG5=CLONG(JND)/5.0+0.01
      LCNG5=CLCNG5
      DECL=(CLCNG5-LCNG5)*5C
      AECL=ABS(DECL)
      IF(AECL)124,1C7,1C6
  106 IF(AECL-5C)107,23,124
   23 WRITE(3,24)CLAT(IND),CLONG(JND)
   24 FCRMAT(' FCR LAT= ',F10.1,' AND LCNG = ',F10.1,' DECL= 50,NCW PUT
     1EQUAL TO 0')
      DECL=0
```

```
  107 MM=45
      ACC=0
      ACCM=0
      ACCP=0
      KTOT=0
      ACCER=0
      ACCERM=0
      ACCERP=0
      JDDECL=DECL/10.0
      IDDEC=DEC/10.0+0.01
      WRITE(3,25)CLAT(IND),CLONG(JND),IDDEC,JDDECL
   25 FORMAT(' LAT= ',F10.1,' LONG=',F10.1,' IDDEC=',I10,' JDDECL=',I10)
      DO 120 II=1,LL
      IH=(NC-1)*10+(I-210)+II-IDDEC
      QLAT=(IH+LAO/100.0-1.0)*0.0174533
      SQLAT=SIN(QLAT)
      CQLAT=COS(QLAT)
      CONQ=(0.33333-SQLAT*SQLAT)/297.0
      DO 119 JJ=1,MM
      JH=(NCL-1)*10+(J-210)+JJ-JDDECL
      IF(20+IDDEC-II)108,109,111
  108 IF(22+IDDEC-II)111,109,109
  109 IF(20+JDDECL-JJ)110,29,111
  110 IF(22+JDDECL-JJ)111,29,29
  111 IF(IHT(IH,JH))117,121,112
  112 IF(IHT(IH,JH)-100000)113,119,117
  113 QLONG=JH+(LOO+50)/100.0-1.0
      DLONG=(QLONG-CLONG(JND))*0.0174533
      CPSI=SCLAT*SQLAT+CCLAT*CQLAT*COS(DLONG)
      SPSI=SQRT(1-CPSI*CPSI)
      SPSI2=SQRT(ABS(0.5*(1-CPSI)))
      CPSI2=SQRT(ABS(0.5*(1+CPSI)))
      CON3=IHT(IH,JH)*100/RAD
      IF(SPSI2)115,115,114
  114 SINAZ=SIN(DLONG)*CQLAT/SPSI
      COSAZ=(SQLAT-SCLAT*CPSI)/(SPSI*CCLAT)
      IF(IHT(IH,JH)-2500)32,32,33
   32 ELV=1
      CF=1
      GO TO 34
   33 ELV=0
      CF=2.67/RHO
   34 TV=CQLAT*(1-0.5*(CONP-3*CONQ)+0.75*CON3)*(1-ELV*IHT(IH,JH)/(21000*
     1RHO))*CF/SPSI2
      ACC=ACC+TV*IHT(IH,JH)
      ACCER=ACCER+TV*TV
      VALM=TV*CPSI2*COSAZ/SPSI2
      ACCM=ACCM+IHT(IH,JH)*VALM
      VALP=TV*CPSI2*SINAZ/SPSI2
      ACCP=ACCP+IHT(IH,JH)*VALP
      KTOT=KTOT+1
      ACCERM=ACCERM+VALM*VALM
      ACCERP=ACCERP+VALP*VALP
      GO TO 119
  115 WRITE(3,116)SPSI2
```

```
    116 FORMAT(42H ERROR IN SINE PSI/2.VALUE OBTAINED BEING ,F20.8/)
        GO TO 119
    117 CONTINUE
        GO TO 119
    121 IF(IND-1)119,27,119
     27 IF(JND-1)119,28,119
     28 CONTINUE
        GO TO 119
     29 WRITE(3,30)IH,JH
     30 FORMAT(' THE ELEMENT IH = ',I5,' , JH = ',I5,' HAS BEEN EXCLUDED')
    119 CONTINUE
    120 CONTINUE
    123 RINEFF(IND,JND)=CCON*ACC
        ITOT(IND,JND)=KTOT
        DEFM(IND,JND)=CCOND*ACCM
        DEFP(IND,JND)=CCOND*ACCP
        ERN(IND,JND)=CCON*SQRT(ACCER)*250
        ERM(IND,JND)=CCOND*SQRT(ACCERM)*250
        ERP(IND,JND)=CCOND*SQRT(ACCERP)*250
    124 CONTINUE
    125 CONTINUE
        WRITE(3,157)
    157 FORMAT(//'1  COMPUTATIONS OF INDIRECT EFFECT'///)
        WRITE(3,126)
    126 FORMAT(//120H IND.EFF.  IND.EFF.  IND.EFF.  IND.EFF.  IND.EFF.  IN
       1D.EFF.  IND.EFF.  IND.EFF.  IND.EFF.  IND.EFF.          LATITUDE
       2 /)
        WRITE(3,127)
    127 FORMAT(120H    CM.        CM.        CM.        CM.        CM.        C
       1M.        CM.        CM.        CM.        CM.            DEGREES N /
       2//)
        DO 200 I=1,10
        RINEFF(I,11)=CLAT(I)
        WRITE(3,128)(RINEFF(I,J),J=1,11)
    128 FORMAT(10F10.2,F15.2/)
    200 CONTINUE
        WRITE(3,129)(CLONG(J),J=1,10)
    129 FORMAT(///10F10.2/)
        WRITE(3,130)
    130 FORMAT(72H                                  L O N G I T U D E   I N   D
       1E G R E E S   E ///)
        WRITE(3,199)
    199 FORMAT(//'1                              COMPUTATION OF ESTIMATES OF ERRORS
       1 IN THE COMPUTED RESULT'//)
        WRITE(3,126)
        WRITE(3,127)
        DO 201 I=1,10
        ERN(I,11)=CLAT(I)
        WRITE(3,128)(ERN(I,J),J=1,11)
    201 CONTINUE
        WRITE(3,129)(CLONG(J),J=1,10)
        WRITE(3,130)
        WRITE(3,152)
    152 FORMAT(//'1  COMPUTATION OF CORRECTIONS TO DEFLECTIONS OF THE VER
       1TICAL DUE TO INDIRECT EFFECT'///)
```

```
      WRITE(3,153)
  153 FORMAT(//120H MERIDIAN  MERIDIAN  MERIDIAN  MERIDIAN  MERIDIAN  ME
     1RIDIAN  MERIDIAN  MERIDIAN  MERIDIAN  MERIDIAN          LATITUDE
     2 /)
      WRITE(3,154)
  154 FORMAT(120H   SEC.      SEC.      SEC.      SEC.      SEC.      SE
     1C.      SEC.      SEC.      SEC.      SEC.             DEGREES N /
     2//)
      DO 202 I=1,10
      DEFM(I,11)=CLAT(I)
      WRITE(3,155)(DEFM(I,J),J=1,11)
  155 FORMAT(10F10.4,F15.2/)
  202 CONTINUE
      WRITE(3,129)(CLONG(J),J=1,10)
      WRITE(3,130)
      WRITE(3,199)
      WRITE(3,153)
      WRITE(3,154)
      DO 203 I=1,10
      ERM(I,11)=CLAT(I)
      WRITE(3,155)(ERM(I,J),J=1,11)
  203 CONTINUE
      WRITE(3,129)(CLONG(J),J=1,10)
      WRITE(3,130)
      WRITE(3,152)
      WRITE(3,156)
  156 FORMAT(//120H PRI VERT  PRI VERT  PRI VERT  PRI VERT  PRI VERT  PR
     1I VERT  PRI VERT  PRI VERT  PRI VERT  PRI VERT          LATITUDE
     2 /)
      WRITE(3,154)
      DO 204 I=1,10
      DEFP(I,11)=CLAT(I)
      WRITE(3,155)(DEFP(I,J),J=1,11)
  204 CONTINUE
      WRITE(3,129)(CLONG(J),J=1,10)
      WRITE(3,130)
      WRITE(3,199)
      WRITE(3,156)
      WRITE(3,154)
      DO 205 I=1,10
      ERP(I,11)=CLAT(I)
      WRITE(3,155)(ERP(I,J),J=1,11)
  205 CONTINUE
      WRITE(3,129)(CLONG(J),J=1,10)
      WRITE(3,130)
      WRITE(3,131)
  131 FORMAT(//'1    NUMBER    NUMBER    NUMBER    NUMBER    NUMBER    N
     1UMBER    NUMBER    NUMBER    NUMBER    NUMBER           LATITUDE
     2 '/)
      WRITE(3,132)
  132 FORMAT(//120H IN COMP   IN COMP   IN COMP   IN COMP   IN COMP   IN
     1 COMP   IN COMP   IN COMP   IN COMP   IN COMP           DEGREES S
     2 ///)
      DO 206 I=1,10
      ITOT(I,11)=CLAT(I)*10
```

Appendix (15) concluded.

```
      WRITE(3,133)(ITCT(I,J),J=1,11)
  133 FORMAT(' ',11I10/)
  206 CONTINUE
      WRITE(3,129)(CLCNG(J),J=1,10)
      WRITE(3,130)
  134 CONTINUE
  135 CONTINUE
      STOP
      END

        SIZE OF COMMON  000000   PROGRAM  039974

                              END OF COMPILATION  MAIN
```

JUL66 IBM OS/360 BASIC FORTRAN IV (E) COMPILATION

```
C      PROGRAM.INNEAR
C      CALCULATES INDIRECT EFFECT FOR AREAS IN RANGE OF PSI 0.1 TO ONE DEGREE
C      BASIC AREA CONSIDERED IS 10 DEGREES SQUARE.LESSER AREAS CAN BE CONSIDE
C      DIMENSION STATEMENT NEEDS AMENDMENT.INTERVAL OF COMPUTATION IS DEFINED
C      INT IN TENTH DEGREES AND REQUIRES DEFINITION.PRINT OUT IS BY 10 X 10 A
C      S,LONGITUDE INCREMENTED IN FIRST INSTANCE.LATITUDE,LONGITUDE OF NW COR
C      REQUIRE DEFINITION-POSITIVE S,E RESPECTIVELY
       DIMENSION KHT(10),IHT(80,80),RINEFF(11,11),CLAT(10),CLONG(10),
      1ITOT(11,11),DEFM(11,11),DEFP(11,11),ERN(11,11),ERM(11,11),
      2ERP(11,11)
       ELV=1.0
       ICAP=80
       INT=1
   207 DO 1 I=1,ICAP
       DO 1 J=1,ICAP
     1 IHT(I,J)=-1000
C      READ LAT,LONG OF CENTRE OF NW TENTH DEGREE SQUARE.+VE S,E
       READ(1,2)LAC,LOO
     2 FORMAT(2I10)
C      READ HEIGHTS OF TENTH DEGREE SQ.MEANS IN FT,WITH LAT,LONG OF
C      MOST WESTERLY SQUARE
     3 READ(1,4)ITE,KEY,LAH,LOH,RINT,(KHT(I),I=1,10)
     4 FORMAT(I2,I2,2I6,F4.1,10I6)
       IF(ITE-1)5,105,15
     5 DO 14 I=1,10
       IN=(LAH-LAO)/10.0+1
       JN=(LCH-LOO)/10.0+I
       IF(IN)12,12,6
     6 IF(IN-ICAP)7,7,12
     7 IF(JN)12,12,8
     8 IF(JN-ICAP)9,9,12
     9 IF(KHT(I))10,10,11
    10 IHT(IN,JN)=100000
       GC TO 14
    11 IHT(IN,JN)=KHT(I)
       GC TO 14
    14 CONTINUE
       WRITE(3,50)ITE,LAH,LOH,(KHT(I),I=1,10),IN,JN
    50 FORMAT(' ',I1,2I7,10I8,' IN =',I5,' JN =',I5)
    12 CONTINUE
       IF(ITE-1)3,105,15
    15 WRITE(3,16)LAH,LOH
    16 FORMAT(15H CARD WITH LAT ,I5,17H S AND LONGITUDE ,I5,29H E HAS BEE
      1N PUNCHED IN ERROR )
       GO TC 3
C      END OF READING HEIGHTS.STORED IN FEET
   105 READ(1,106)ITE,KEY,LAH,LOH,RINT,(KHT(I),I=1,10)
   106 FORMAT(I1,I2,2I6,F5.2,10I6)
       IF(ITE-1)107,17,116
   107 DO 115 I=1,10
       IN=(LAH-LAO)/10.0+1
       JN=(LCH-LOO)/10.0+I
       IF(IN)114,114,108
```

```
  108 IF(IN-ICAP)109,109,114
  109 IF(JN)114,114,110
  110 IF(JN-ICAP)111,111,114
  111 IF(KHT(I))112,112,113
  112 IHT(IN,JN)=100000
      GO TO 115
  113 IHT(IN,JN)=KHT(I)
      GO TO 115
  114 CONTINUE
  115 CONTINUE
      WRITE(3, 50)ITE,LAH,LOH,(KHT(I),I=1,10),IN,JN
      IF(ITE-1)105,17,116
  116 WRITE(3,16)LAH,LOH
      GO TO 105
   17 READ(1,61)ITE,KEY,LAH,LOH,RINT,(KHT(I),I=1,10)
   61 FORMAT(I1,I2,2I6,F4.1,10I6)
      IF(ITE-1)62,72,71
   62 DO 70 I=1,10
      IN=(LAH-LAO)/10.0+1
      JN=(LCH-LCO)/10.0+I
      IF(IN)69,69,63
   63 IF(IN-ICAP)64,64,69
   64 IF(JN)69,69,65
   65 IF(JN-ICAP)66,66,69
   66 IF(KHT(I))67,67,68
   67 IHT(IN,JN)=100000
      GO TO 70
   68 IHT(IN,JN)=KHT(I)
      GO TO 70
   69 CONTINUE
   70 CONTINUE
      WRITE(3, 50)ITE,LAH,LOH,(KHT(I),I=1,10),IN,JN
      IF(ITE-1)17,72,71
   71 WRITE(3,16)LAH,LOH
      GO TO 17
   72 IF(INT-10)19,18,18
   18 LIM=19
      ISPA=1
      GO TO 20
   19 LIM=INT*9+10
      SPA=(ICAP-20.0)/(INT*10)
      ISPA=1
   20 READ(1,150)CON,RHO,PI,IARC,TGRAV,CON1,RAD
      WRITE(3,73)CON,RHO,PI,IARC,TGRAV,CON1,RAD
   73 FORMAT(' CON= ',E10.4,' RHO= ',F10.2,' PI= ',F10.5,' IARC= ',I10,
     1' TGRAV= ',F10.2,' CON1= ',E10.2,' RAD= ',E10.4)
  150 FORMAT(E10.3,F10.2,F10.5,I10,F10.2,E10.2,E10.4)
      CCON=(CON*RHO*PI*PI*RAD*CON1)/(TGRAV*180*180)
      CCOND=(-CON*RHO*PI*PI*IARC*CON1)/(2*TGRAV*180*180)
      WRITE(3,151)CCON,CCOND
  151 FORMAT(53H COEFFICIENTS FOR COMPUTATION OF INDIRECT EFFECT IS  ,E20.5,
     10.5,25H AND OF DEFLECTIONS IS    ,E20.5//)
      DO 45 NO=1,ISPA
      DO 44 NOL=1,ISPA
      DO 38 I=10,19,INT
```

```
      IND=(I-10.0)/INT+1
      CLAT(IND)=LAO/100.0+0.50+(NO-1)*(LIM-10.0+INT)/2.00+(I-10)*0.5+1
      VAL=CLAT(IND)*0.0174533
      SCLAT=SIN(VAL)
      CCLAT=COS(VAL)
      CONP=(0.33333-SCLAT*SCLAT)/297.0
      LAT=CLAT(IND)
      DEC=(CLAT(IND)-LAT)*10
      AEC=ABS(DEC)
      IF(AEC)38,101,100
  100 IF(AEC-10)101,52,51
   51 IF(AEC-10.5)52,52,38
   52 DEC=0
  101 LL=30
      IEC=DEC
      DO 37 J=10,19,INT
      JND=(J-10.0)/INT+1
      CLONG(JND)=LOO/100.0+0.50+(NOL-1)*(LIM-10.0+INT)/2.0+(J-10)*0.5+1
      LONG=CLONG(JND)
      DECL=(CLONG(JND)-LONG)*10
      IF(DECL)37,104,103
  103 IF(DECL-10)104,54,53
   53 IF(DECL-10.5)54,54,37
   54 DECL=0
  104 WRITE(3,49)CLAT(IND),CLONG(JND),DEC,DECL
   49 FORMAT(' VALUES OF DEC,DECL AT',F10.1,' S AND',F10.1,' E ARE',
     1F10.2,'AND',F10.2,' RESPECTIVELY')
      IECL=DECL
      ACCM=0
      ACCP=0
      ACC=0
      ACCER=0
      ACCERM=0
      ACCERP=0
      KTOT=0
      DO 33 II=1,LL
      IH=(NO-1)*(LIM-10.0+INT)*5+(I-10)*5+IEC+II+6
      DLAT=(IH/10.0+LAO/100.0-0.15)
      QLAT=DLAT*0.0174533
      SQLAT=SIN(QLAT)
      CQLAT=COS(QLAT)
      CONQ=(0.33333-SQLAT*SQLAT)/297.0
      DO 32 JJ=1,LL
      JH=(NOL-1)*(LIM-10.0+INT)*5+(J-10)*5+JJ-IECL+5
      QLONG=(JH/10.0+LOO/100.0-0.05)
      IF(II+IEC-10)24,22,21
   21 IF(II+IEC-11)32,22,24
   22 IF(JJ-IECL-10)24,117,23
   23 IF(JJ-IECL-11)32,117,24
   24 IF(IHT(IH,JH))32,34,25
   25 IF(IHT(IH,JH)-100000)26,32,30
   26 IF(I-10)32,55,58
   55 IF(J-10)32,56,58
   56 RQLAT=QLAT/0.0174533
      WRITE(3,57)RQLAT,QLONG,IH,JH
```

```
      57 FORMAT(' COMP. ELEMENT AT LAT.',F10.2,' AND LONG ',F10.2,' MATRIX
        1POSN. BEING ',I10,',',I10)
      58 DLONG=(QLONG-CLONG(JND))*0.0174533
         SPSI=SQRT(DLONG*DLONG*CQLAT*CQLAT+(QLAT-VAL)*(QLAT-VAL))
         CPSI=1-SPSI*SPSI*0.5
         SPSI2=SPSI/2
         CPSI2=SQRT(ABS(0.5*(1+CPSI)))
         HTM=IHT(IH,JH)*0.304799
         CON3=HTM*100/RAD
         IF(SPSI2)28,28,27
      27 SINAZ=SIN(DLONG)*CQLAT/SPSI
         COSAZ=(SQLAT-SCLAT*CPSI)/(SPSI*CCLAT)
         TV=CQLAT*(1-0.5*(CONP-3*CONQ)+0.75*CON3)*(1-ELV*HTM/(21000*RHO))/
        1SPSI2
         ACC=ACC+TV*HTM
         ACCER=ACCER+TV*TV
         VALM=TV*CPSI2*COSAZ/SPSI2
         ACCM=ACCM+VALM*HTM
         VALP=TV*CPSI2*SINAZ/SPSI2
         ACCP=ACCP+VALP*HTM
         ACCERM=ACCERM+VALM*VALM
         ACCERP=ACCERP+VALP*VALP
         KTOT=KTOT+1
         GO TO 32
      28 WRITE(3,29)SPSI2
      29 FORMAT(42H ERROR IN SINE PSI/2.VALUE OBTAINED BEING ,F20.8)
         GO TO 32
      30 WRITE(3,31)IH,JH,IHT(IH,JH)
      31 FORMAT(41H ERROR IN LOADING HEIGHT FOR ELEMENT NO.(,2I10,28H ),THE
        1 VALUE OBTAINED BEING ,I10//)
         GO TO 32
     117 WRITE(3,118)DLAT,QLONG,IH,JH,IHT(IH,JH)
     118 FORMAT(' ELEMENT OMITTED HAS LAT =',F10.2,' LONG =',F10.2,' IH =',
        1I5,' JH=',I5,' ELEVATION =',I8)
      32 CONTINUE
      33 CONTINUE
         GO TO 36
      34 WRITE(3,35)CLAT(IND),CLONG(JND)
      35 FORMAT(43H INSUFFICIENT HEIGHTS TO COMPUTE EFFECT FOR,F10.2,14H DE
        1GREES SOUTH,F10.2,14H DEGREES EAST //)
         RINEFF(IND,JND)=0
         DEFM(IND,JND)=0
         DEFP(IND,JND)=0
         ERN(IND,JND)=0
         ERM(IND,JND)=0
         ERP(IND,JND)=0
         GO TO 37
      36 RINEFF(IND,JND)=CCON*ACC
         DEFM(IND,JND)=CCOND*ACCM
         DEFP(IND,JND)=CCOND*ACCP
         ITOT(IND,JND)=KTOT
         ERN(IND,JND)=CCON*SQRT(ACCER)*30
         ERM(IND,JND)=CCOND*SQRT(ACCERM)*30
         ERP(IND,JND)=CCOND*SQRT(ACCERP)*30
      37 CONTINUE
```

```
 38 CONTINUE
    WRITE(3,39)
 39 FORMAT(//'1   IND.EFF.  IND.EFF.  IND.EFF.  IND.EFF.  IND.EFF.  IN
   1D.EFF.  IND.EFF.  IND.EFF.  IND.EFF.  IND.EFF.          LATITUDE
   2'/)
    WRITE(3,40)
 40 FORMAT(120H    CM.       CM.       CM.       CM.       CM.       C
   1M.       CM.       CM.       CM.       CM.                DEGREES S /
   2//)
    DO 200 I=1,10
    RINEFF(I,11)=CLAT(I)
    WRITE(3,41)(RINEFF(I,J),J=1,11)
 41 FORMAT(10F10.2,F15.2/)
200 CONTINUE
    WRITE(3,42)(CLONG(J),J=1,10)
 42 FORMAT(///10F10.2/)
    WRITE(3,43)
 43 FORMAT(72H                             L O N G I T U D E   I N   D
   1E G R E E S   E ///)
    WRITE(3,157)
157 FORMAT(//'1                          COMPUTATION OF ESTIMATES OF ERRORS
   1 IN THE COMPUTED RESULT'//)
    WRITE(3,39)
    WRITE(3,40)
    DO 201 I=1,10
    ERN(I,11)=CLAT(I)
    WRITE(3,41)(ERN(I,J),J=1,11)
201 CONTINUE
    WRITE(3,42)(CLONG(J),J=1,10)
    WRITE(3,43)
    WRITE(3,152)
152 FORMAT(//'1  COMPUTATION OF CORRECTIONS TO DEFLECTIONS OF THE VERT
   1ICAL DUE TO INDIRECT EFFECT'///)
    WRITE(3,153)
153 FORMAT(//120H MERIDIAN  MERIDIAN  MERIDIAN  MERIDIAN  MERIDIAN  ME
   1RIDIAN  MERIDIAN  MERIDIAN  MERIDIAN  MERIDIAN          LATITUDE
   2 /)
    WRITE(3,154)
154 FORMAT(120H   SEC.      SEC.      SEC.      SEC.      SEC.      SE
   1C.      SEC.      SEC.      SEC.      SEC.                DEGREES S /
   2//)
    DO 202 I=1,10
    DEFM(I,11)=CLAT(I)
    WRITE(3,155)(DEFM(I,J),J=1,11)
155 FORMAT(10F10.4,F15.2/)
202 CONTINUE
    WRITE(3,42)(CLONG(J),J=1,10)
    WRITE(3,43)
    WRITE(3,157)
    WRITE(3,153)
    WRITE(3,154)
    DO 203 I=1,10
    ERM(I,11)=CLAT(I)
    WRITE(3,155)(ERM(I,J),J=1,11)
203 CONTINUE
```

```
      WRITE(3,42)(CLONG(J),J=1,10)
      WRITE(3,43)
      WRITE(3,156)
  156 FORMAT(//120H PRI.VERT. PRI.VERT. PRI.VERT. PRI.VERT. PRI.VERT. PR
     1I.VERT. PRI.VERT. PRI.VERT. PRI.VERT. PRI.VERT.          LATITUDE
     2 /)
      WRITE(3,154)
      DO 204 I=1,10
      DEFP(I,11)=CLAT(I)
      WRITE(3,155)(DEFP(I,J),J=1,11)
  204 CONTINUE
      WRITE(3,42)(CLONG(J),J=1,10)
      WRITE(3,43)
      WRITE(3,157)
      WRITE(3,156)
      WRITE(3,154)
      DO 205 I=1,10
      ERP(I,11)=CLAT(I)
      WRITE(3,155)(ERP(I,J),J=1,11)
  205 CONTINUE
      WRITE(3,42)(CLONG(J),J=1,10)
      WRITE(3,43)
      WRITE(3,46)
   46 FORMAT(120H NO. USED  NO. USED  NO. USED  NO. USED  NO. USED  NO.
     1USED  NO. USED  NO. USED  NO. USED  NO. USED           LATITUDE  /
     2)
      WRITE(3,47)
   47 FORMAT(120H IN COMP.  IN COMP.  IN COMP.  IN COMP.  IN COMP.  IN C
     1OMP.  IN COMP.  IN COMP.  IN COMP.  IN COMP.          DEGREES S /
     2//)
      DO 206 I=1,10
      ITOT(I,11)=CLAT(I)
      WRITE(3,48)(ITOT(I,J),J=1,11)
   48 FORMAT(10I10,I15///)
  206 CONTINUE
      WRITE(3,42)(CLONG(J),J=1,10)
      WRITE(3,43)
   44 CONTINUE
   45 CONTINUE
      READ(1,208)LKEY
  208 FORMAT(I3)
      IF(LKEY-1)207,209,209
  209 CONTINUE
      STOP
      END

        SIZE OF COMMON  000000   PROGRAM  042454

                             END OF COMPILATION  MAIN
```

APPENDIX (17)


```
C     PROGRAM ININ
C     COMPUTES THE INDIRECT EFFECT FOR POSTULATED MODEL OF THE GEOID,DUE TO
C     INNER ZONE COMPOSED OF FOUR TENTH DEGREE SQUARES,ADJACENT TO THE COMPU
C     ON POINT,FOR 10 METRE INCREMENTS IN ELEVATIONS FROM 10 TO 10000 METRE
C     AT LATITUDES FROM 5 TO 85 DEGREES
      DIMENSION ENINT(1000),DIFF(1000),EINT(10),EDIFF(5)
      ELV=1
      READ(1,1)LAT,LIM,INM
    1 FORMAT(3I10)
      READ(1,2)MINEL,MAXEL,MHINC
    2 FORMAT(3I10)
      READ(1,3)CON,PI,RHO,RADF,CON1,TGRAV
    3 FORMAT(E10.3,F10.5,F10.2,F10.3,E10.2,F10.2)
      CCON=4*PI*CON*RHO*RADF*CON1/TGRAV
      WRITE(3,4)RHO,RADF
    4 FORMAT(//77H COMPUTATION OF INDIRECT EFFECT TO CO-GEOID ON POSTULA
     1TED MODEL,DENSITY BEING,F10.2,31H FOR INNER ZONE ONLY OF RADIUS ,F
     210.5,4H  KM///)
      IF(ELV-1)25,23,25
   23 WRITE(3,24)
   24 FORMAT(/86H              THE HUNTER FORMULA,             RHO=(2.77-H/21)
     1,            H IN KM.BEING USED  ///)
   25 DO 22 I=LAT,LIM,INM
      SCI=SQRT(COS(I*0.0174533))
      RAD=SCI*RADF*1000
      WRITE(3,5)I
    5 FORMAT(///27H COMPUTATIONS FOR LATITUDE ,I10,9H DEGREES ///)
      DO 12 J=MINEL,MAXEL,MHINC
      RATIO=J/RAD
      INC=1.0*(J-MINEL)/MHINC+1
      IF(J-5000)6,7,7
    6 ENINT(INC)=CCON*J*SCI*(1-0.5*RATIO)*(1-ELV*J/(21000*RHO))
      GO TO 8
    7 SRATIO=RATIO*RATIO
      ENINT(INC)=CCON*J*SCI*(1-0.5*RATIO+SRATIO/6.0-SRATIO*SRATIO/40.0)*
     1(1-ELV*J/(21000*RHO))
    8 IF(INC-1)10,12,9
    9 DIFF(INC-1)=ENINT(INC)-ENINT(INC-1)
      GO TO 12
   10 WRITE(3,11)I,J
   11 FORMAT(39H ERROR IN COMPUTING INDEX FOR LATITUDE ,I10,16H  AND ELE
     1VATION ,I10//)
   12 CONTINUE
      DO 21 J=1,20
      WRITE(3,13)
   13 FORMAT(//127H   HEIGHT  IND. EFF.  DIFF. HEIGHT  IND. EFF.  DIFF.
     1HEIGHT  IND. EFF.  DIFF. HEIGHT  IND. EFF.  DIFF. HEIGHT  IND. EFF
     2.  DIFF.)
      WRITE(3,14)
   14 FORMAT(127H    MET.       CM.       CM.  MET.       CM.       CM.
     1 MET.       CM.       CM.  MET.       CM.       CM.  MET.       CM.
     2  CM. ///)
      DO 20 K=1,10
```

```
      IND=(J-1)*50+K
      DC 17 L=1,5
      LIND=IND+(L-1)*10
      EINT(2*L-1)=IND*MHINC+(L-1)*100.0
      EINT(2*L)=ENINT(LIND)
      IF(IND+(L-1)*10-999)16,16,15
   15 EDIFF(L)=0
      GC TC 17
   16 EDIFF(L)=DIFF(LIND)
   17 CCNTINUE
      WRITE(3,18)(EINT(M),M=1,10)
   18 FCRMAT(5(F8.0,F10.2,7X))
      WRITE(3,19)(EDIFF(M),M=1,5)
   19 FCRMAT(5(18X,F7.3))
   20 CONTINUE
   21 CONTINUE
      ERF=SCI*CCON*RADF
      WRITE(3,26)ERF
   26 FORMAT(//'  OBTAIN ESTIMATES ERROR IN N BY MULTIPLYING EST. HT. ER
     1ROR BY ',E20.3, 'RESULT IN CM. FOR HT. IN METRES.'//)
   22 CCNTINUE
      STCP
      END

        SIZE CF COMMON  00000     PRCGRAM  11086

                          END OF COMPILATICN  MAIN
```

APPENDIX (18)

The interpretation of $\left|\frac{\partial \gamma}{\partial h}\right|$ in the fundamental theorem.

In compiling the final expression for the separation N of the geoid and the spheroid, given on equation (13.69), the gradient of normal gravity was evaluated, on page 64, as $2\gamma/R$, which is the free air reduction. This expression is correct if the geoid is above the spheroid, but is inconsistent when it lies below the surface of the latter. In such a case, re-investigation shows that the source of the term $\left|\frac{\partial \gamma}{\partial h}\right|$ is the evaluation of the expression $\underline{\nabla} \cdot \underline{N}\ V_D$, which, in the case of the geoid spheroid system, can be expressed as

$$\underline{\nabla} \cdot \underline{N}\ V_D = \frac{\partial V_D}{\partial h} = \frac{\partial W_P}{\partial h} - \frac{\partial U_P}{\partial h}$$

$$= - g_o + \gamma_P$$

$$= - g_o + \gamma_o - N\left|\frac{\partial \gamma}{\partial h}\right| + c\, N\left|\frac{\partial \gamma}{\partial h}\right|_c$$

$$= \Delta g_o - N\left|\frac{\partial \gamma}{\partial h}\right| + c\, N\left|\frac{\partial \gamma}{\partial h}\right|_c \quad \ldots . \text{(A18.1)},$$

where c is a constant such that $c = 0$ when $N > 0$ and $c = 1$ when $N < 0$. $\left|\frac{\partial \gamma}{\partial h}\right|$ has exactly the same interpretation as before, being equal to the free air reduction term. $\left|\frac{\partial \gamma}{\partial h}\right|_c$ is the constituent term of the differential topographical correction, which, by equation (6.43), is approximately $4\pi k \rho$.

The extra term introduced into equation (13.69) is

$$N_{ci} = - \frac{1}{2\pi\gamma} \iint_R c\, N\left|\frac{\partial \gamma}{\partial h}\right|_c \frac{1}{r}\ dR \quad \ldots\ldots\ldots . \text{(A18.2)}$$

The magnitude of this effect can be estimated by assuming negative values of N to have a mean value of approximately - 75 metres and noting the two following points.

(i) Negative N values occur over approximately half the earth, if the spheroid of reference has the same volume as the geoid.

(ii) Such values are likely to occur only in the distant zones for continental regions.

The use of the spherical approximation and evaluation on summation by quadratures using $n^{\circ} \times n^{\circ}$ squares,

$$N_{ci} = \frac{k \rho \pi^2 R_m}{180^2 \gamma_m} \sum_i n_i^2 \sum_j \partial \rho_{ij} c_{ij} N_{ij} \cos \phi_{ij} \operatorname{cosec} \tfrac{1}{2} \psi_{ij} \qquad \text{(A18.3)}$$

where ρ, $\partial \rho$ have the same definition as in equation (12.7). As N values are not available for an actual evaluation of N_{ci}, its effect has not been included in sections (14.4) and (14.5). As its structure is similar to that for N_{E_P}, given in equation (12.52), the magnitude of the term N_{ci}, estimated at being approximately 130 metres, is expected to be reasonable constant over the entire earth, with variations from the mean value of the order of 10 metres or less.

For the same reasons as set out in section (14.4), assuming the I.A.U. spheroid to have the same volume as the geoid, the potential of the geoid is more likely to be approximately 130 kgal metres greater than the value given in the section quoted. This will also give rise to a small but slowly changing effect on the deflection of the vertical.

BIOGRAPHICAL NOTES

RON MATHER was educated at the University of Ceylon, Christs' College, University of Cambridge and the University of New South Wales. After graduating with a Bachelor of Science degree in 1955, he joined the Ceylon Survey Department as an Assistant Superintendent of Surveys where his duties included the supervision of the Survey Training School. After a spell as lecturer in the South Australian Institute of Technology from 1962, he joined the University of New South Wales in the same capacity in 1966.

Dr. Mather has published papers on the propagation of errors, extension of gravity fields and aspects of physical geodesy. He is at present preparing a map of the non-regularised geoid for Australia and working on providing an orientation for the Australian Geodetic Datum from gravity data.

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