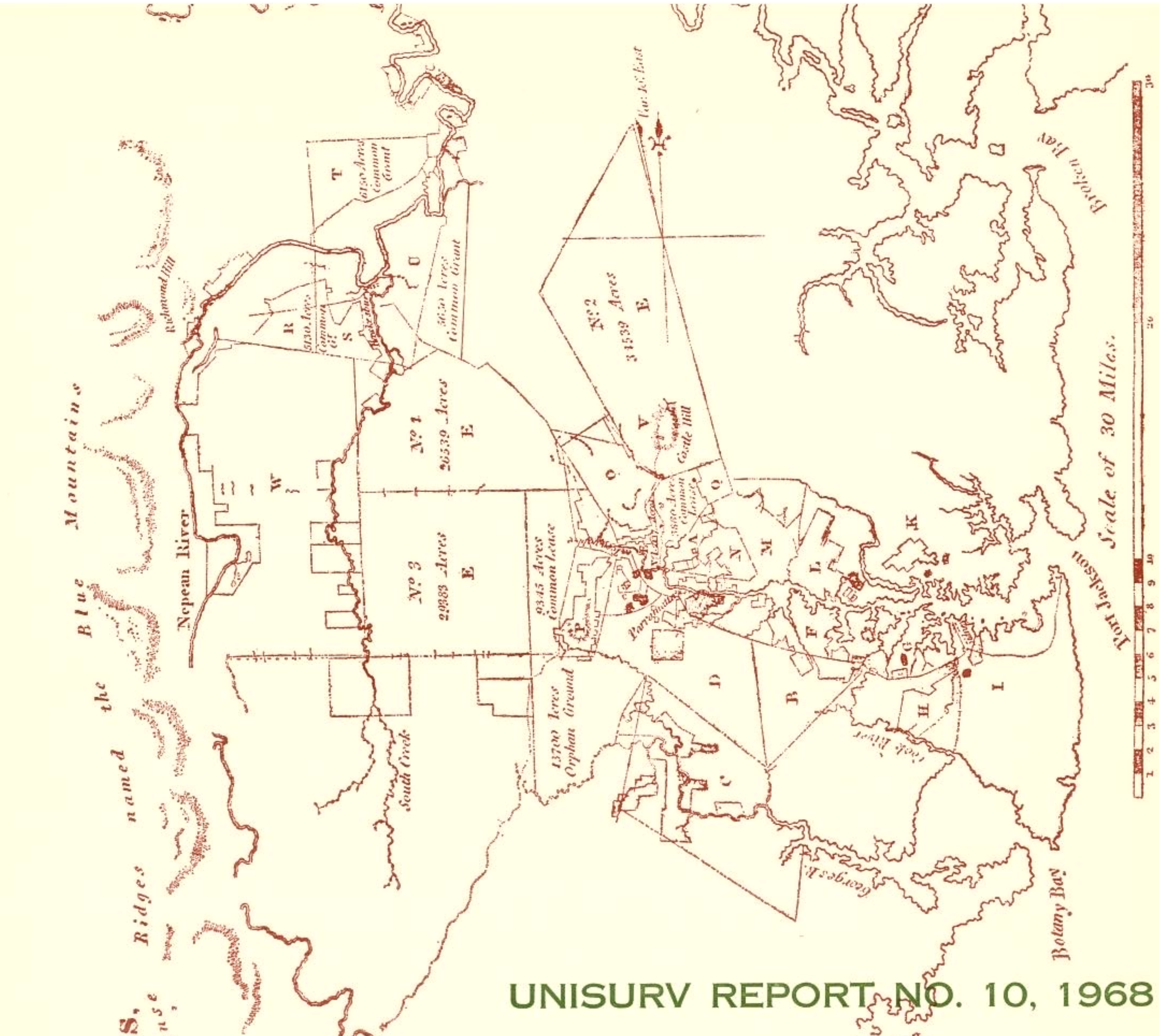

# UNISURV REPORT NO. 10, 1968

# AN EXPERIMENTAL DETERMINATION OF REFRACTION OVER AN ICEFIELD.

## P. V. ANGUS-LEPPAN

UNIVERSITY OF NEW SOUTH WALES
KENSINGTON, N.S.W., AUSTRALIA

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B Liberty Plains
C Banks Town
D Parramatta
EEEE Ground reserved
for Govt. purposes
F Concord
G Petersham
H Bulanaming
I Sydney
K Hunters Hills
L Eastern Farms
M Field of Mars
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O Toongabbey
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R Richmond Hill
S Green Hills
T Phillip
U Nelson
V Castle Hill
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## UNISURV REPORT No. 10.

# AN EXPERIMENTAL DETERMINATION OF REFRACTION OVER AN ICEFIELD.

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Department of Surveying - University of New South Wales

Received 4th March, 1968.

SUMMARY: Refractive index in the atmosphere depends mainly on temperature. Over ice or snow, because of the cold surface, the pattern of temperature variations will differ from that over normal surfaces. In an experiment on the Juneau Icefield, Alaska, air temperatures were measured in the lowest 12 m. above the surface and reciprocal vertical angles were measured in the height range 15-70 m. Results from both types of observation were expressed in terms of the coefficient of refraction K. Remarkably high values for K are indicated, and there is an unexpected absence of the diurnal cycle in the variations.

## 1. Refraction and Temperature.

The refractive index from point to point in the atmosphere is of importance to the surveyor as it represents the effect of the atmosphere on his observations. The numerical value of the index determines the velocity of the electro-magnetic waves, a quantity fundamental to electronic distance measurement, while the rate of change of the index across the path determines the bending of the light ray. For light waves the chief factor in refractive index variations is the temperature, while for radio waves, temperature is still dominant even though the effect of humidity becomes significant.

The pattern of temperature changes for temperate climates and over normal surfaces has been frequently described (for example, Johnson, 1929; Johnson and Heywood, 1938) and has been analysed in some detail (Angus-Leppan, 1967). The characteristic temperature gradient during the day is negative, numerically highest at the ground surface and decreasing upwards. The ground surface is the vital element in receiving heat from the sun's rays and transferring it upwards by convection processes. The lowest air layer, in contact with the surface, is the hottest.

Over névé or ice there is a layer of air which has a temperature of $0^{o}C$ or lower. As long as the air temperatures are higher, the surface acts so as to retain a layer at this fixed temperature. If there is sunshine and little wind, the temperature at one or two metres above the surface is several degrees higher, as evidenced by photographs of bikini-clad skiers. This rapid increase of temperature T with height z indicates a high positive value of the gradient $\frac{dT}{dz}$, known as an inversion, and consequently a high gradient of refraction, causing severe bending of light rays. At night the inversion will remain, as it is characteristic of night conditions everywhere. It is apparent that the surface material of an icefield will cause radical differences in the pattern of diurnal variation of the gradient, its magnitude and the upward extent of the inversion. These quantities are of both scientific and practical interest.

An opportunity arose during the Sixth Summer Institute of Glaciological and Arctic Sciences, 1966, to take measurements of these phenomena over the Juneau Icefield, near Juneau, Alaska. Air temperatures were measured in the lowest 12m. above the surface, and reciprocal vertical angles were measured between stations at each end of a line 5.7km. long (Figure 1). Observations extended over a 24-hour period.

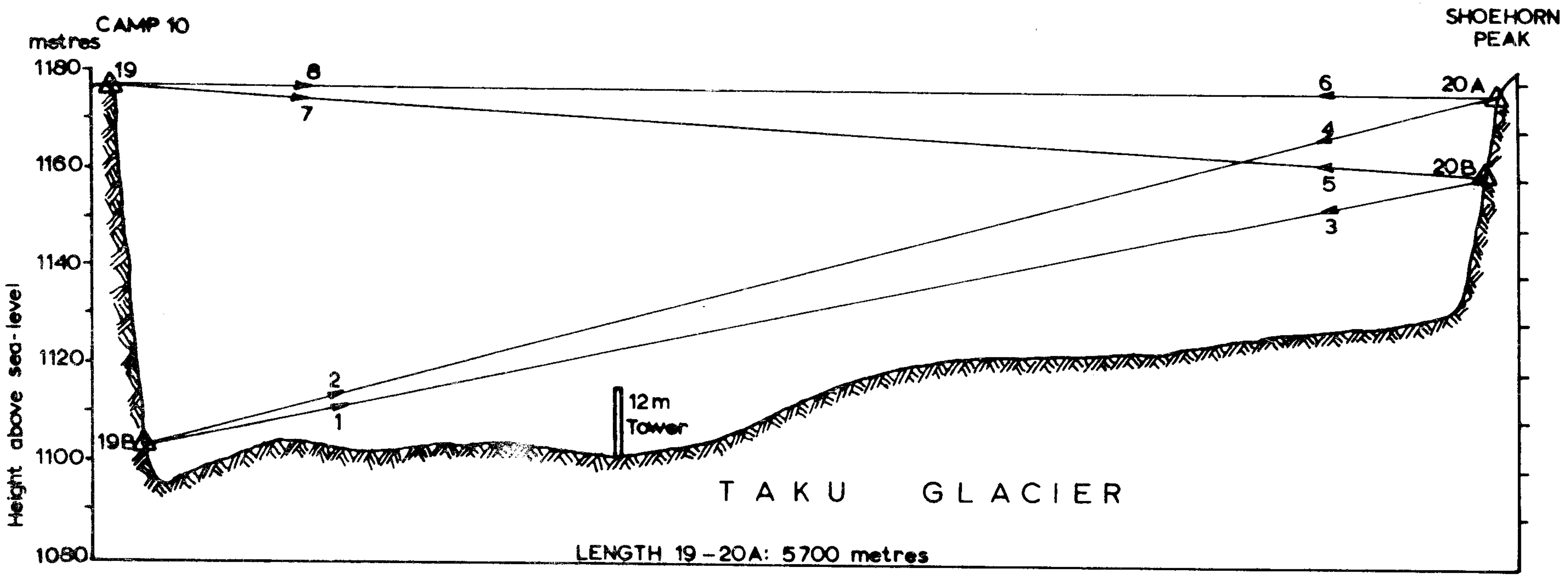


FIG. 1 PROFILE SHOWING THE OBSERVING STATIONS, LINES OF SIGHT, AND TEMPERATURE IN RELATION TO SURFACE.
HORIZONTAL SCALE 1/20000, VERTICAL SCALE 1/1000

## 2. Icefield Experiment.

The site of the experiment was between survey station 19, at Camp 10, and 20A, on a spur of Shoehorn Peak, 5701 metres distant. To cover a wider range of heights an additional station was chosen at each end, giving the four lines shown in Figure 1 which range from 15 to 70m. above the surface. The lines are over a smooth section of the Taku Glacier with a slight downward slope towards the Camp 10 end.

At approximately two hour intervals, from midday 4th August to midday 5th August 1966, vertical angles were measured from each station to the two stations at the far end. They were so arranged as to be simultaneous reciprocals between the two upper stations, then between the two lower stations, at alternate hours. Two Wild T2 theodolites were used for the angle measurements. The sighting targets were kerosene pressure lamps during the night and large orange squares of cotton cloth during the day.

Temperature measurements were made on a 12-metre tower which was situated along the line of sight and roughly midway between the ends. The temperature element was a small rod thermistor of the type used in radiosonde apparatus, coated to eliminate heating by radiation. This was raised and lowered once every hour, pausing at each of six measuring heights for sufficient time for the element to take up the air temperature, and to record the resistance.

The weather during the experiment was stable, being almost entirely cloudless, with light winds.

## 3. Results - Reciprocal Angles.

After applying corrections for target heights, the vertical angles were plotted. (Figure 2) If the temperature gradient were constant with time the angles could be expected to remain unchanged. If it were constant along the line, or varied so as to be symmetrical about the centre, each pair of graphs would match perfectly. Clearly there is variation both in time and in space. The graphs show that at these heights above the icefield surface -

(a) there is no regular diurnal pattern of variation. In normal circumstances there is a definite pattern including a change from positive gradient and large angle of refraction at night to negative gradient and minimum refraction during the day.

(b) the scale of the variations is damped as the height of the line increases.

(c) the reciprocal observations, while exhibiting the same major variations, do not match each other closely at the lower levels. At higher levels they fit more closely.

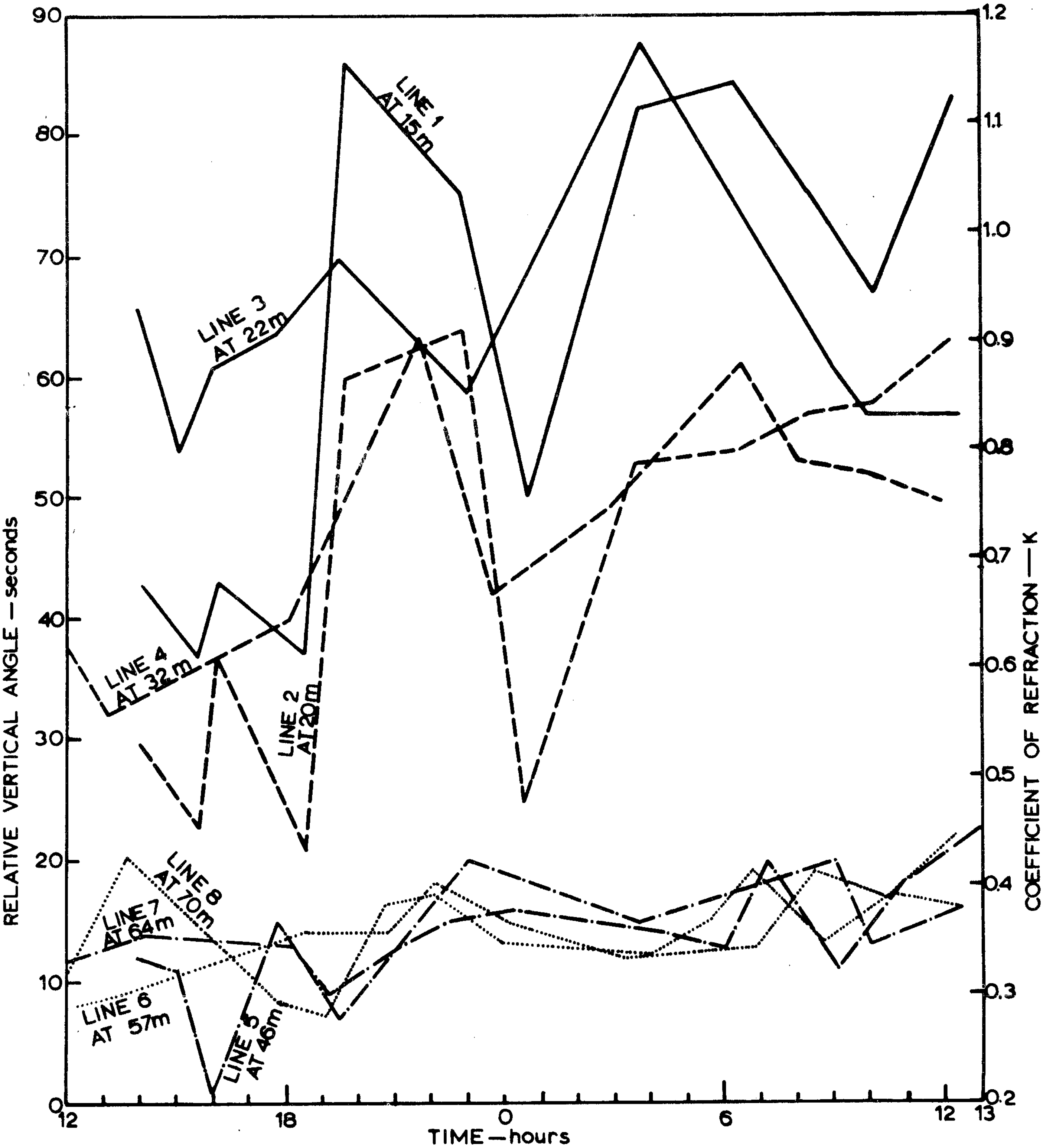


FIG. 2 VARIATION OF RECIPROCALLY OBSERVED VERTICAL ANGLES AND COEFFICIENT OF REFRACTION.

The variations discussed so far are only relative. In order to deal with absolute angles the relative elevations at the opposite ends of the lines are needed. The only practicable way of determining these is by using vertical angles. Since not one, but four lines of reciprocal observations are available, with observations spread throughout the day and night, the determination should be considerably more reliable than that obtained from a single observation.

The coefficient of refraction K is used by surveyors as a practical measure of the curvature of the line of sight. This coefficient is the ratio of the radius of curvature of the earth to the radius of curvature of the line of sight.

$$K = \frac{\text{radius of earth}}{\text{radius of line of sight}} = \frac{R}{R'} \quad \ldots\ldots\ldots\ldots (1)$$

Generally a value between K = + 0.12 and K = + 0.16 is adopted. The vertical correction for curvature of the earth and of the line of sight on a line of length S, is given by

$$\text{Correction} = \frac{(1-K)S^2}{2R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

where R is the radius of the earth's curvature in the same units as S. Although adoption of the coefficient implies the questionable assumption that the curvature of the line of sight is constant, it is often sufficiently accurate for practical purposes.

For the calculation of the refractive index we have the vertical angles $\alpha_1$, $\alpha_2$ observed reciprocally; (Figure 3) using the usual sign convention (positive for elevation, negative for depression)

$$K = \frac{R}{R'}$$

$$= \frac{R}{\dfrac{S\rho}{\theta + \alpha_1 + \alpha_2}} \qquad \text{where } \theta = \frac{S\rho}{R}$$

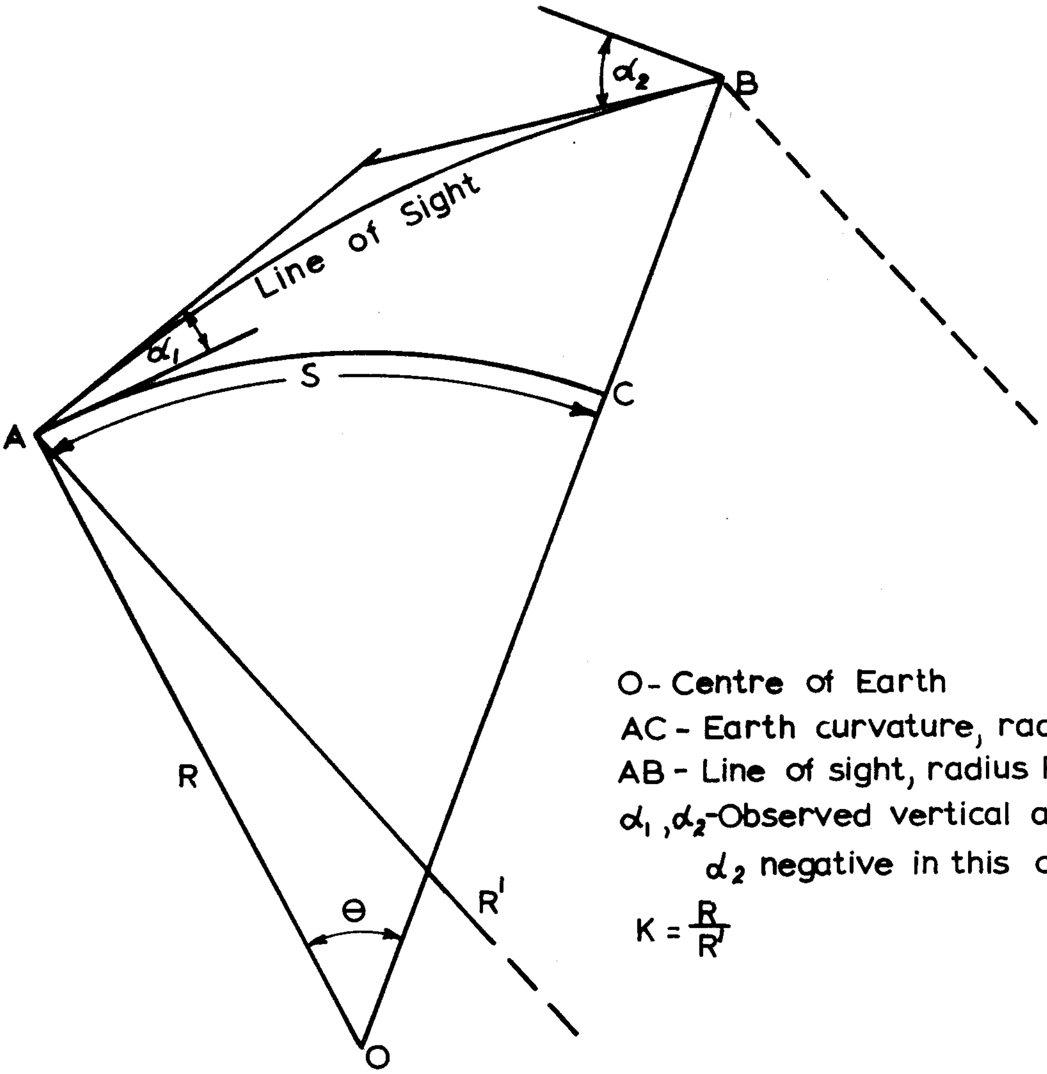
$$= 1 + \frac{R}{S\rho} (\alpha_1 + \alpha_2) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

where $\alpha_1$, $\alpha_2$ are in seconds; $\rho =$ 206000 seconds.

In figure 2, values of K can be read off the right-hand vertical scale. It is obvious that K decreases with increasing height. There is considerable random variation which also decreases with increasing height. Most remarkable however are the very high numerical values. The significance of a value of K of 1.0 is that the curvature of the line of sight matches earth curvature. A normal value for K is 0.14, but this value is not approached over the icefield, even at considerable heights above the surface. A value of 0.35 appears to be more appropriate for lines which are more than 50m. above the surface.

## 4. Temperature Results.

The temperature readings on the tower represent conditions in



O- Centre of Earth
AC - Earth curvature, radius R
AB - Line of sight, radius $R'$
$\alpha_1$ ,$\alpha_2$-Observed vertical angles,
$\alpha_2$ negative in this case
$K = \frac{R}{R'}$

FIG. 3 OBSERVED VERTICAL ANGLES AND THE COEFFICIENT OF REFRACTION.

the lowest layers 1m. to 12m. above the surface. Temperature in the atmosphere is a quantity which is undergoing fluctuations at high and low frequencies with time and similar fluctuations with position. If refraction is computed from temperatures and from angles, the results will differ in the sensitivity to short-term fluctuations, because of differences in the instruments and techniques of observation and also because temperature is measured at a point whereas the angle measures an integrated effect over a line.

The temperatures are shown in Figure 4. There is a marked diurnal variation which can, at lower levels, be well approximated by a sine wave. At higher levels the amplitude decreases and at the same time irregular variations become larger and tend to obscure the diurnal pattern. These variations are due to higher wind velocities, bringing in pockets of air from considerable distances. The regular curves shown correspond to a function T(z, t) for the temperature in the form:

$$T(z,t) = 0.36z + (1.8 - .055z) \sin (t - 8\tfrac{1}{3}) \quad \ldots\ldots\ldots\ldots\ldots (6)$$

where z is the height above the surface and t is the time in hours after midnight. (1 hour = $\frac{\pi}{12}$ radians). The corresponding temperature gradient is :

$$\frac{dT}{dz} = .36 - .055 \sin (t-8\tfrac{1}{3}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

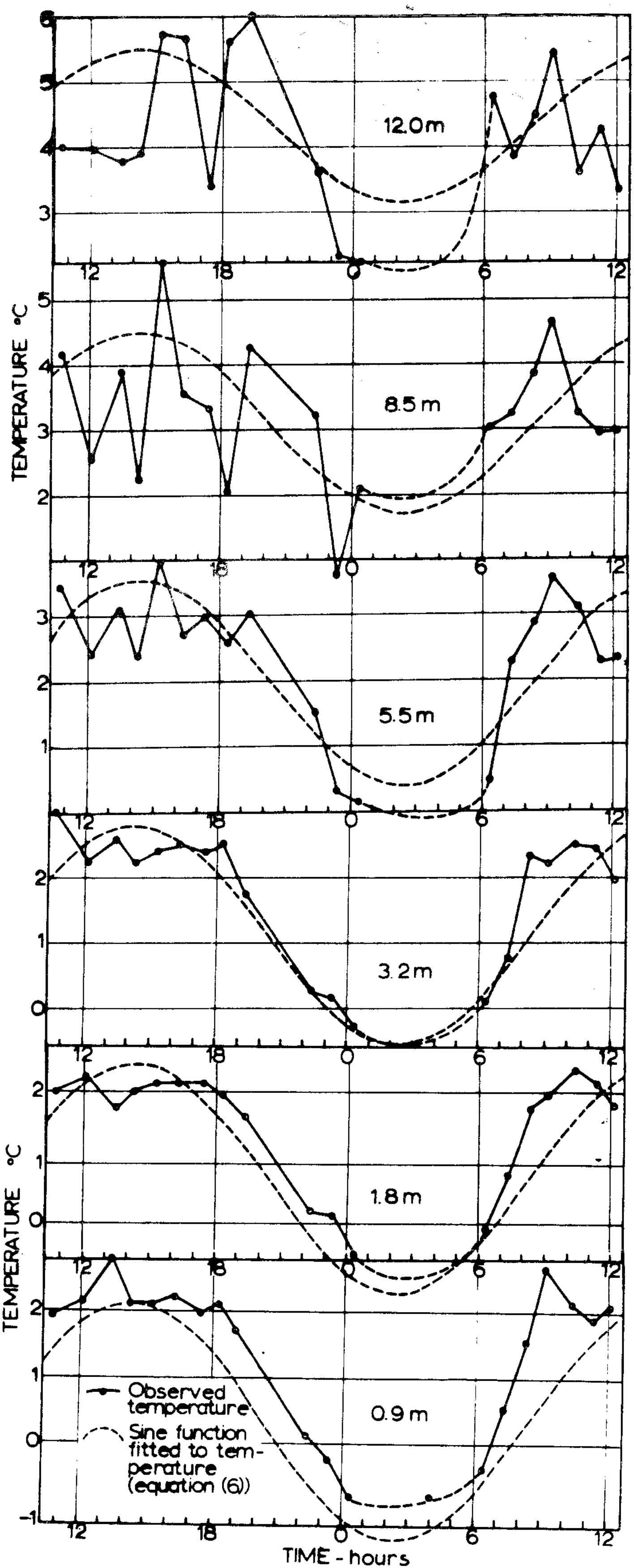


FIG. 4 OBSERVED TEMPERATURES AND APPROXIMATING FUNCTION.

Since this is independent of height, the coefficient K will also be independent. The process of fitting temperatures to a simple function has led to a loss of many features of the temperature variation.

In spite of the fact that the most rapid fluctuations are damped out by the mass of the thermistor element, the remaining random variations are so large that they obscure the pattern. As a result some smoothing technique is necessary. A method using running means was adopted. The smoothed temperatures were used to calculate refraction for each height range and time of observation.

Curvature of a line of sight at a point (Bomford, 1962, 212) is given by

$$F = 16.5 \frac{P}{T^2} \left( .0334 + \frac{dT}{dz} \right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

where F is in seconds per metre,

P is atmospheric pressure in millibars and

T is in $^{o}$C Abs.

In (1), making use of the relation $F = \frac{\rho}{R'}$, and substituting (4) with appropriate values for P and T:

$$K = \frac{RF}{\rho} = .205 + 6.15 \frac{dT}{dz} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Results are shown in Figure 5 and will be discussed below.

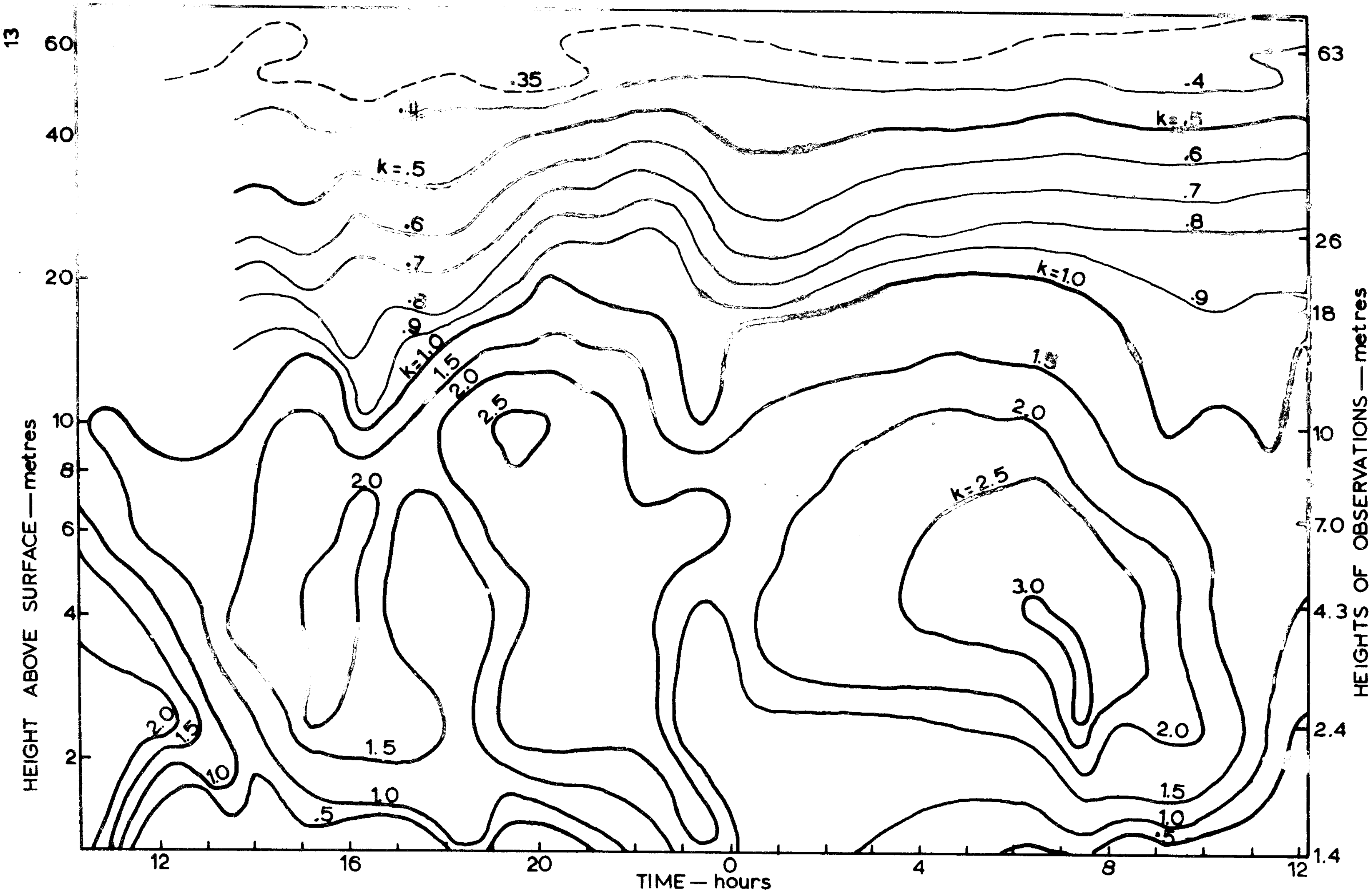


FIG. 5 COEFFICIENT OF REFRACTION DETERMINED FROM VERTICAL ANGLE AND TEMPERATURE OBSERVATIONS. VARIATION WITH HEIGHT AND TIME INDICATED.

As a check against possible distortions due to the method of reduction, mean day and night temperatures were calculated for each height, taking the periods 9.30 - 18.00 hours and 22.30 - 7.00 hours to represent day and night conditions respectively. It was noted that corresponding temperatures for the two days involved, agreed within 0.2°C in every case. The coefficient K calculated from these temperatures is consistent with the values in the appropriate areas of Figure 5.

TABLE I.

| Height metres | Coefficient of Refraction K | |
|---|---|---|
| | Day | Night |
| 0.9 - 3.0 | 1.5 | 1.2 |
| 3.0 - 8.5 | 1.5 | 2.6 |
| 8.5 - 12.0 | 1.5 | 1.7 |

## 5. Combined Results.

In Figure 5, values of the coefficient of refraction derived from both methods are shown. The relationships between K and the height and time are shown in the form of contours of equal coefficient of refraction K. Tracing along any horizontal line will give the time variation of K at that level, while values along any vertical line show the variation with height at the particular time.

The upper half of the figure, above 14m., is derived from angle measurements and confirms that there is no marked diurnal pattern at these levels. Above 40m. the variations are comparatively smooth and regular, while below this level there are more marked irregularities. Even at these elevations above the surface, values of K are very much higher than the standard value of 0.14.

In the lower half of the figure, even allowing for the differences in measurement discussed above, it is clear that the pattern is different. The substantially horizontal trend of isolines gives way to a vertical trend with globules of very high refractive index in the layer 2 - 10m., falling off again below 2m. to lower values. Extraordinarily high values of K up to 3.0, are found.

The lower level results, derived from temperatures, provide important and independent confirmation of the higher results. The two sets give numerical values which are consistent along the boundary level of 14m. In addition it can be noted that each increase or decrease along the 18m. level has a corresponding, though magnified, variation at the 10m. level.

## 6. Discussion.

The adiabatic temperature gradient of approximately $-0.01^{\circ}C/m.$ is a critical value. For smaller gradients air at lower levels is warm, of lower density and will have a tendency to rise. This convection has the effect of mixing air from various levels and producing a regular gradient. For larger gradients there is vertical stability, with the heaviest air at the base and a decrease in density in the successively higher layers. There is no tendency for mixing and there is a possibility of discontinuities in the gradient. These conditions prevail over an icefield, where mixing will occur through wind and the associated turbulence. Even over a large icefield wind can move warmer air masses in from outside, and cause apparent temperature anomalies. It should also be noted that because of friction at the boundary layer, and between successive layers, the typical profile of wind velocities starts at zero at ground level and increases upwards in successively higher layers.

The temperature gradients measured and deduced indicate high gradients at lower levels and a general, fairly regular, decrease upwards. The surface provides a reference temperature which is lower than the general atmospheric temperature. Warmer air is moved in by the wind. Higher velocities at upper levels cause more efficient mixing and hence lower gradients. At some still higher level the general trend in the upper atmosphere will take over, the gradient will decrease beyond zero and steady down to a value close to $-0.006^{\circ}C/m$. However this level is much higher than the 70m. reached in the present investigation.

In the layer 2-10m. there is an anomaly, as high gradients are built up, persist for a period, then break down again. Below 2m. the gradients remain comparatively low. It appears that in this lowest layer temperatures are controlled by the cold surface. Through the small amount of air movement, and actual conduction, a fairly uniform temperature, and hence low gradient, is maintained. Above 10m. warmer air from a distance is being moved in by wind, which causes mixing and again a comparatively low gradient. Between 2 and 10m. there is an intermediate layer which is not directly affected by the surface and in which the winds are often absent. Sandwiched between the cold air below and the warm air above, a high gradient is found, for example in Figure 5, 15.00 - 23.00 hours. At other times the winds stir up the intermediate layers, mixing in the warmer air and decreasing the gradient. (Figure 5, at about 14.00 hours; 11.00 - 12.00 hours).

Normally the coefficient of refraction is close to the value + 0.14. This means that the radius of line of sight is about seven times the earth's radius and objects viewed along the tangent to the line of sight, appear elevated slightly above their actual positions. (Figure 3) If the surface temperature is very high, the gradient is negative and numerically high and K changes sign according to Equation (5). The curvature becomes concave upwards, as shown in Figure 6A. If there is a shallow layer of very high gradient, it causes the familiar heat mirage, frequently seen above the black surface of a road. Because of the sharp bending at the layer a portion of the sky appears on the roadway below its true position.

Values of K above 0.5 are unusual and only to be expected close to the surface. Values as high as 1.0 have been observed previously (Angus-Leppan, 1962, 31). The value of unity leads to interesting if impractical speculations as at this value the earth is effectively flattened for the observer, and if it persisted over the entire surface, he would see the back of his own head on the distant horizon, in whatever direction he looked.

When K reaches values as high as 2.0 and 3.0 as in the present study, the observer, looking in a horizontal direction, sees parts of the surface which are in fact well below the horizontal, making a flat plain appear bowl-shaped. (Figure 6B) For example over a flat icefield where K = 3 an observer with his eye 2m. above

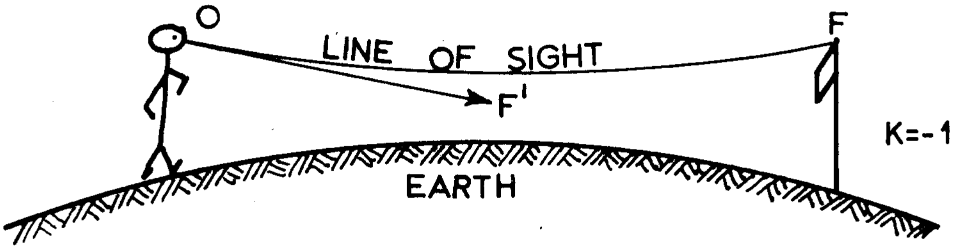


FIG. 6A HIGH SURFACE TEMPERATURES CAUSE HIGH NEGATIVE GRADIENTS AND REFRACTION. OBJECT AT F APPEARS IN DIRECTION F.'

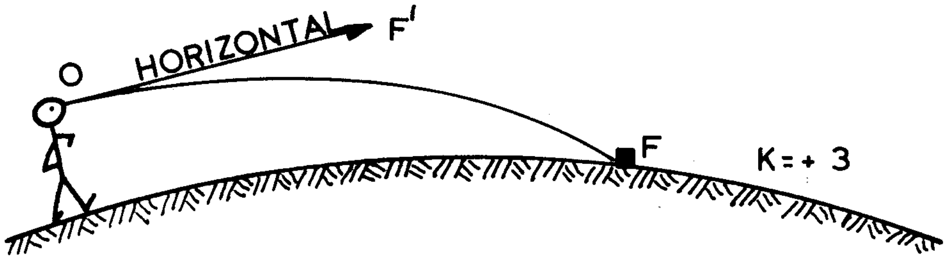


FIG. 6B LOW SURFACE TEMPERATURE AND INVERSION CAUSE HIGH REFRACTION COEFFICIENT.

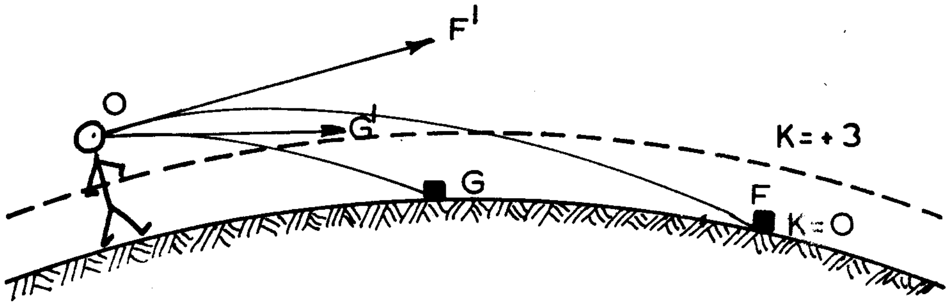


FIG. 6C INVERSIONS AND STRATIFIED GRADIENTS. APPARENT ANGLE F'OG' BETWEEN OBJECTS GREATER THAN TRUE ANGLE FOG.

FIG. 6 OPTICAL EFFECTS OF REFRACTION.

the surface will see, along a horizontal direction, an object on the surface which is $3\frac{1}{2}$km. distant. It is actually 3m. below the horizontal.

Figure 6C illustrates a simplified case of the type appearing in Figure 5 where there is a high value of K at the level of the observer's eye and lower values below. It is assumed for simplicity that K = 0 up to 1m. and K = 3.0 above 1m. Rays which reach the observer in a horizontal direction have been bent by travelling a further distance through the high refraction layer than rays below the horizontal. The result is an increased angular separation or magnification. The vertical looming of distant coastlines across the sea, or vertical exaggeration of insignificant rock cliffs across an icefield have been noted on numerous occasions.

## 7. Conclusions.

The measurements show that, as expected, the refraction above a frozen surface is markedly different from refraction over a normal surface. The values of the coefficient of refraction K are remarkably high. In general the higher values are found near the surface and there is a decrease upwards. However even at the highest levels of measurement the values of K do not reach the normal values around 0.12 - 0.14. It appears rather that a value of 0.35 is appropriate for lines which do not graze the surface.

The general trend of decreasing K with height does not hold for the layer 2-10m. where the highest values, up to 3.0, were measured. This is a transition zone between the lowest layer in contact with the cold surface, and higher warmer layers where winds perform more efficient mixing, and in which high gradients are encountered.

Another remarkable feature is the lack of any discernible diurnal cycle in the observations. This might be expected at higher layers, but even at the lowest levels it was absent. The reason appears to be the constant high gradient and absence of convection even during the day, again a consequence of the constantly cold surface.

Although the measurements taken provide a detailed picture of certain aspects it should be stressed that they were limited in time and confined to a single locality. It is not at all certain how far they can be generalised. Further investigations will be necessary and they should extend over a longer period, a greater height range for both temperature and angle measurement, and should include more comprehensive micrometeorological measurements, such as for example the vertical wind profile.

Acknowledgements.

The author wishes to express his thanks to staff and students of the Institute of Glaciological and Arctic Sciences for assistance in carrying out the programme of observations, and in particular Dr. Maynard M. Miller, the Director; Dr. Adam Chrzanowski, who took all the observations at the remote end of the line; and the graduate students who manned the tower.

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# BIOGRAPHICAL NOTES.

PETER ANGUS-LEPPAN graduated from the University of the Witwatersrand, South Africa in 1951 and received his doctorate in 1959 from the University of Natal. He also holds a Diploma in Town Planning from the latter University. Dr. Angus-Leppan held academic appointments from 1954-1962 at the University of Natal and since then at the University of New South Wales where he was appointed in 1963 to the first full Professorial position in Surveying in Australia. At present he is Head of the Department of Surveying in the University of New South Wales. Dr. Angus-Leppan has published papers on atmospheric refraction, surveying education and cadastral survey systems. His main research interests continue to be atmospheric refraction for which he was awarded a Canadian Commonwealth Research Fellowship in 1966.

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