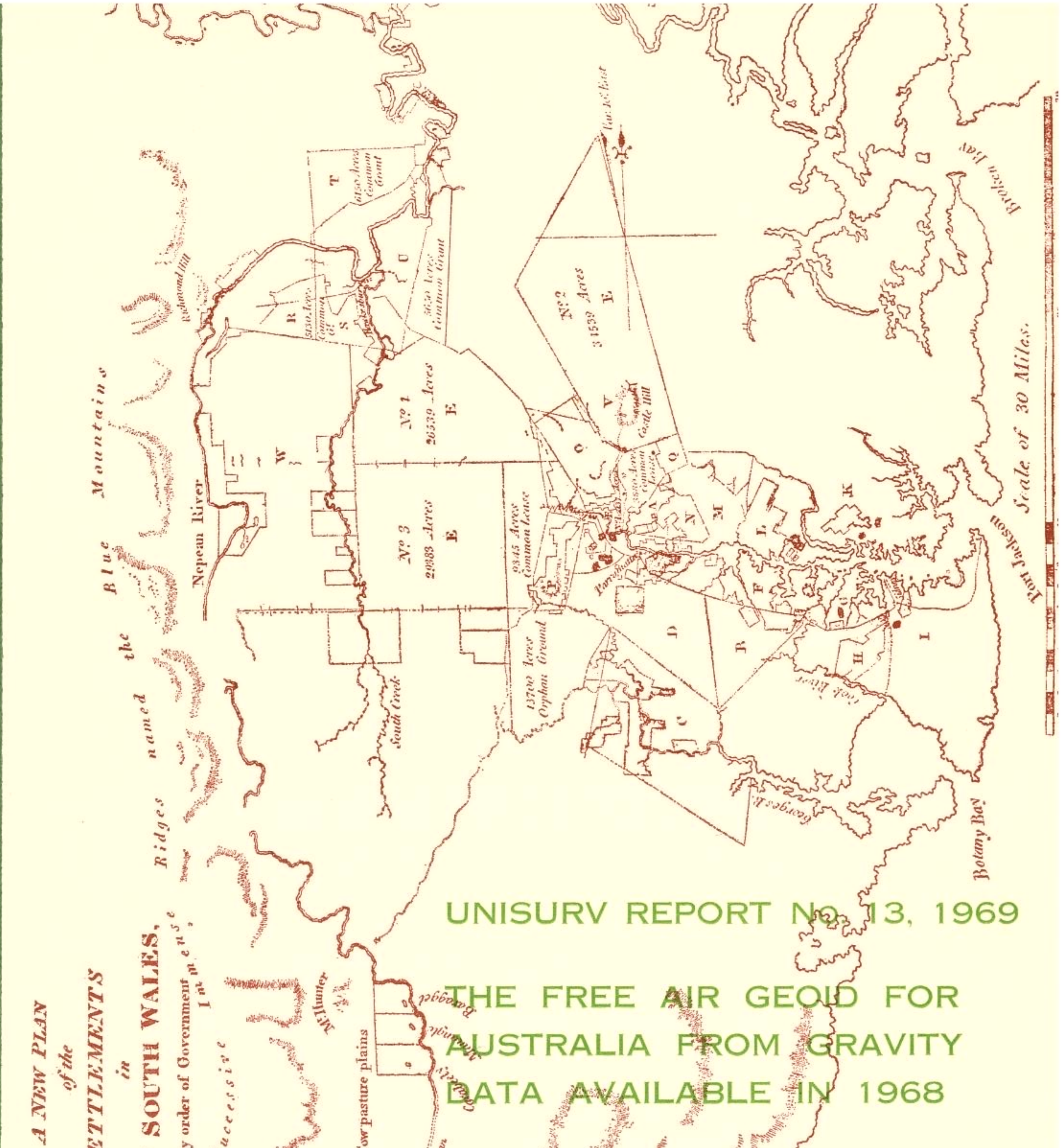

UNISURV REPORT No. 13, 1969

THE FREE AIR GEOID FOR AUSTRALIA FROM GRAVITY DATA AVAILABLE IN 1968

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KENSINGTON, N.S.W., AUSTRALIA

Reference to Districts.

A Northern Boundaries

B Liberty Plains

C Banks Town

D Parramatta

EEEE Ground reserved
for Govt. purposes

F Concord

G Petersham

H Bulanaming

I Sydney

K Hunters Hills

L Eastern Farms

M Field of Mars

N Ponds

O Toongabbey

P Prospect

Q

R Richmond Hill

S Green Hills

T Phillip

U Nelson

V Castle Hill

W Evan

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Received 27th January 1969

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SUMMARY

A free air geoid which is the co-geoid obtained by the use of free air anomalies in Stokes' integral, is computed for Australia from available gravity data. For the outer zones the anomalies had been obtained previously using a combined solution from satellite data and terrestrial gravimetry. The solutions for the free air geoid so obtained are compared with the astrogeodetic determination of the geoid on the Australian Geodetic Datum by Fischer and Slutsky and the accuracy of the comparisons is estimated.

THE FREE AIR GEOID FOR AUSTRALIA FROM GRAVITY DATA AVAILABLE IN 1968

by

R. S. Mather

1. INTRODUCTION

A preliminary attempt at determining a free air geoid for an Australian region was completed in 1967 when a free air geoid map was produced for South Australia (Mather, 1968a, 323-327). Concurrent with the completion of that report, Fischer and Slutsky (1967) produced a preliminary chart of geoidal contours for Australia on the Australian Geodetic Datum (A.G.D.). The geoidal undulations in this solution were compiled from all available astrogeodetic deflections of the vertical on this same datum.

The earlier gravimetric determination was based on a "combined" solution. The term "combined" is used in the sense that all data within 20° of the computation point was based on surface gravimetry only while the free air anomaly field for the regions beyond this inner zone was represented by a set of 5° × 5° area means obtained by the combination of satellite data and available surface gravimetry. The data set used in the 1967 computations was that due to Kaula (1966) and hereafter referred to as the *Kaula Set*. (See Mather, 1968a, 156-172).

Subsequently Rapp (1968) produced a further set of 5° × 5° free air anomaly means by using a slightly different technique for combining the satellite observations with surface gravimetry. This set, referred to as the *Rapp Set*, was also used in the present series of geoidal investigations. Both data sets were used in combination with the surface gravity anomaly set for Australia and its environs, called the *UNSW Set*, to map the free air geoid for this latter region and the solutions compared with the astrogeodetic determination of Fischer and Slutsky.

2. THE DATA

For each computation point the global surface area was divided into five regions as shown in table (1). The gravity anomaly field was represented by smaller subdivisional area means with decrease of ψ, where ψ is the angular distance of the element of surface area being represented from the computation point. For more details see (Mather, 1968a, 156-164).

Range of ψ	Source	Gravity data used
$\psi < 0.1°$	Surface gravity	Individual readings
$0.1° < \psi < 1.5°$	Surface gravity	$0.1° \times 0.1°$ sq. values
$1.5° < \psi < 5°$	Surface gravity	$0.5° \times 0.5°$ sq. means
$5° < \psi < 20°$	Surface gravity	$1° \times 1°$ sq. means
$20° < \psi$	Combined satellite & surface data	$5° \times 5°$ sq. means

TABLE (1)

TYPE OF GRAVITY DATA USED IN CALCULATIONS

Gravity coverage of the Australian mainland region was principally obtained from the records of the Commonwealth of Australia's Bureau of Mineral Resources, Geology & Geophysics. This was supplemented by additional observations by the author, data from the records of the South Australian Department of Mines, St. John's data for New Guinea (1967) and Everingham's data for Western Australia. The general criteria adopted for the inclusion of individual gravity readings in the *UNSW Set* are detailed in an earlier report (Mather, 1966, 2 et seq).

The available gravity field was extended to unsurveyed regions using two dimensional trigonometrical series (Mather, 1967, 132-137) and the extended field so obtained used in the calculations. These extensions, while performed under the conditions of interpolation in continental regions, provided a reasonable representation of the gravity field. On the other hand, extensions in the surrounding ocean areas under conditions of extrapolation cannot be considered to be of equivalent accuracy, except in the immediate vicinity of the continent itself. This did not apply to the areas to the north of Australia where sufficient data was available to allow a reasonable representation of unsurveyed areas.

The gravity field used to represent the Australian region is summarised in figs (1) to (3), the data given being referred to normal gravity computed using the International Gravity Formula. The computations were performed on Reference Ellipsoid 1967 using Gravity Formula 1967 for the computation of normal gravity. The conversion of gravity data from the former system to the latter was effected by the following equations (Mather, 1968c, 343).

$$d\gamma = \gamma \left[\frac{d\gamma_e}{\gamma_e} + d\beta \sin^2\phi \right] \qquad \text{............(1);}$$

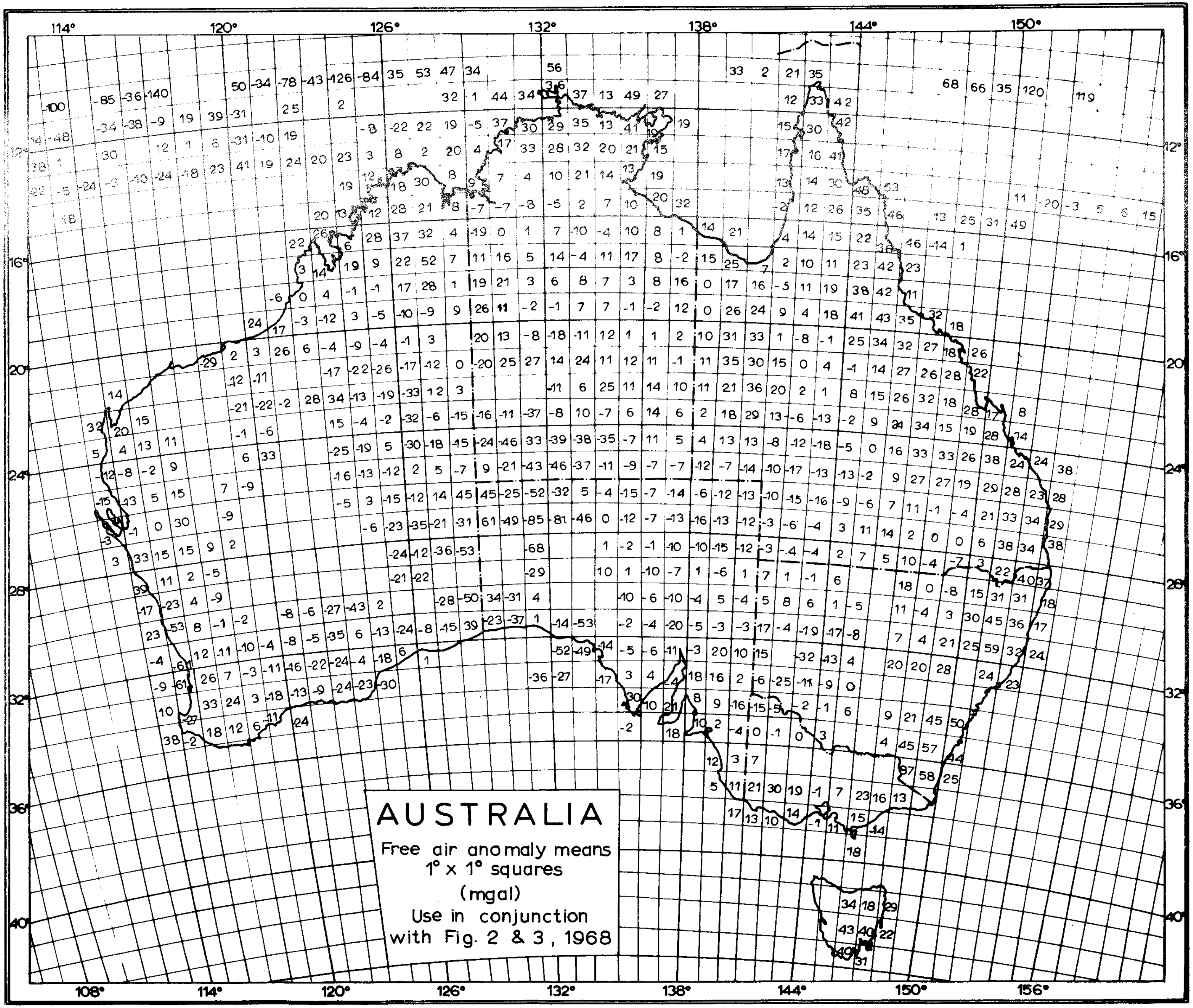
where

$$d\gamma_e = \gamma_e \left[\frac{d(kM)}{kM} - 2\frac{da}{a} + df - \frac{3}{2}dm \right] \qquad \text{........ (2),}$$

$$d\beta = \frac{5}{2}\, dm - df \qquad \text{............ (3),}$$

$$dm = m\left[\frac{3\, da}{a} - \frac{d(kM)}{kM} \right] \qquad \text{............(4)}$$

and



Anomalies: referred to International Gravity Formula FIG. 1

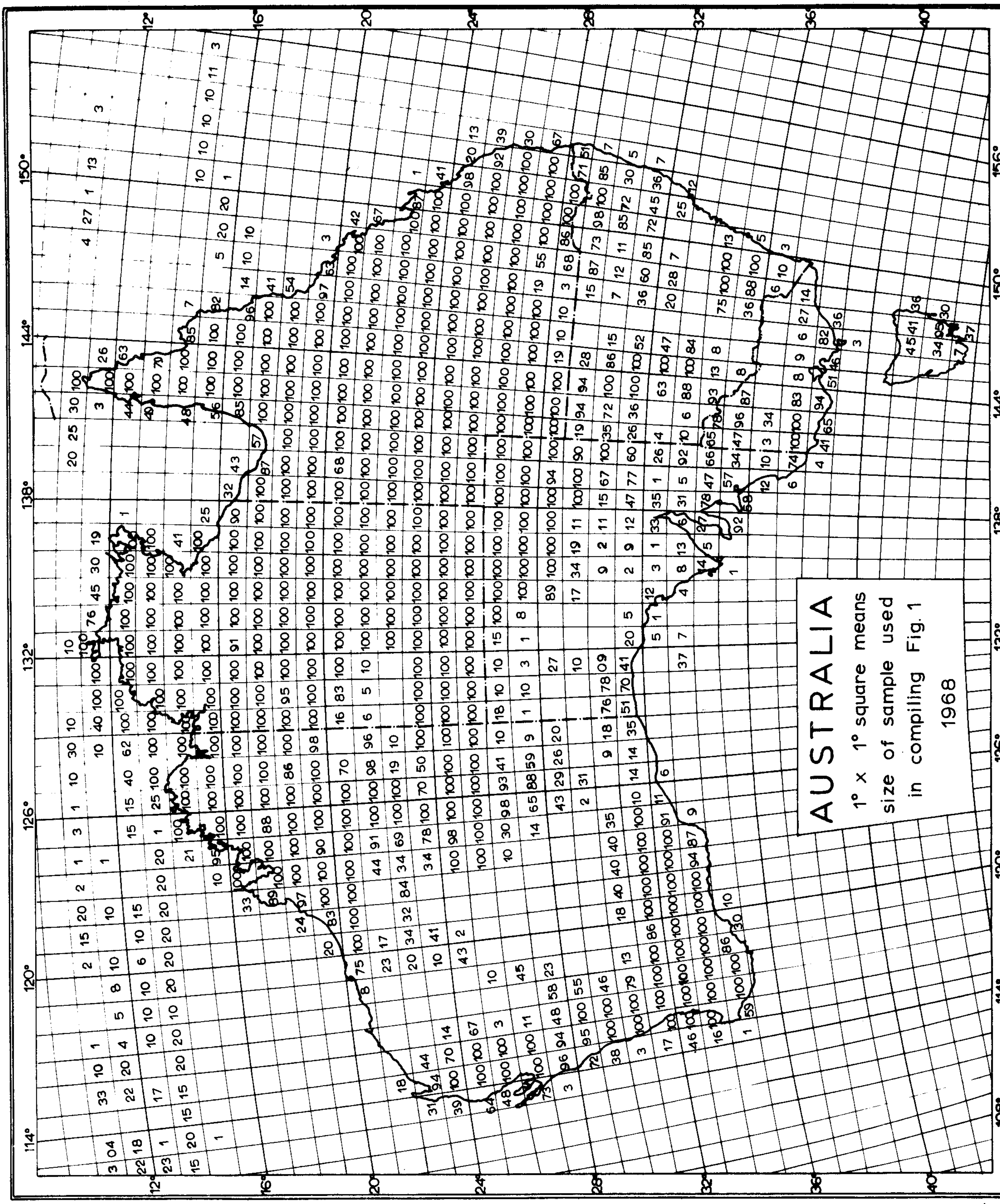

AUSTRALIA
1° x 1° square means
size of sample used
in compiling Fig. 1
1968

FIG. 2

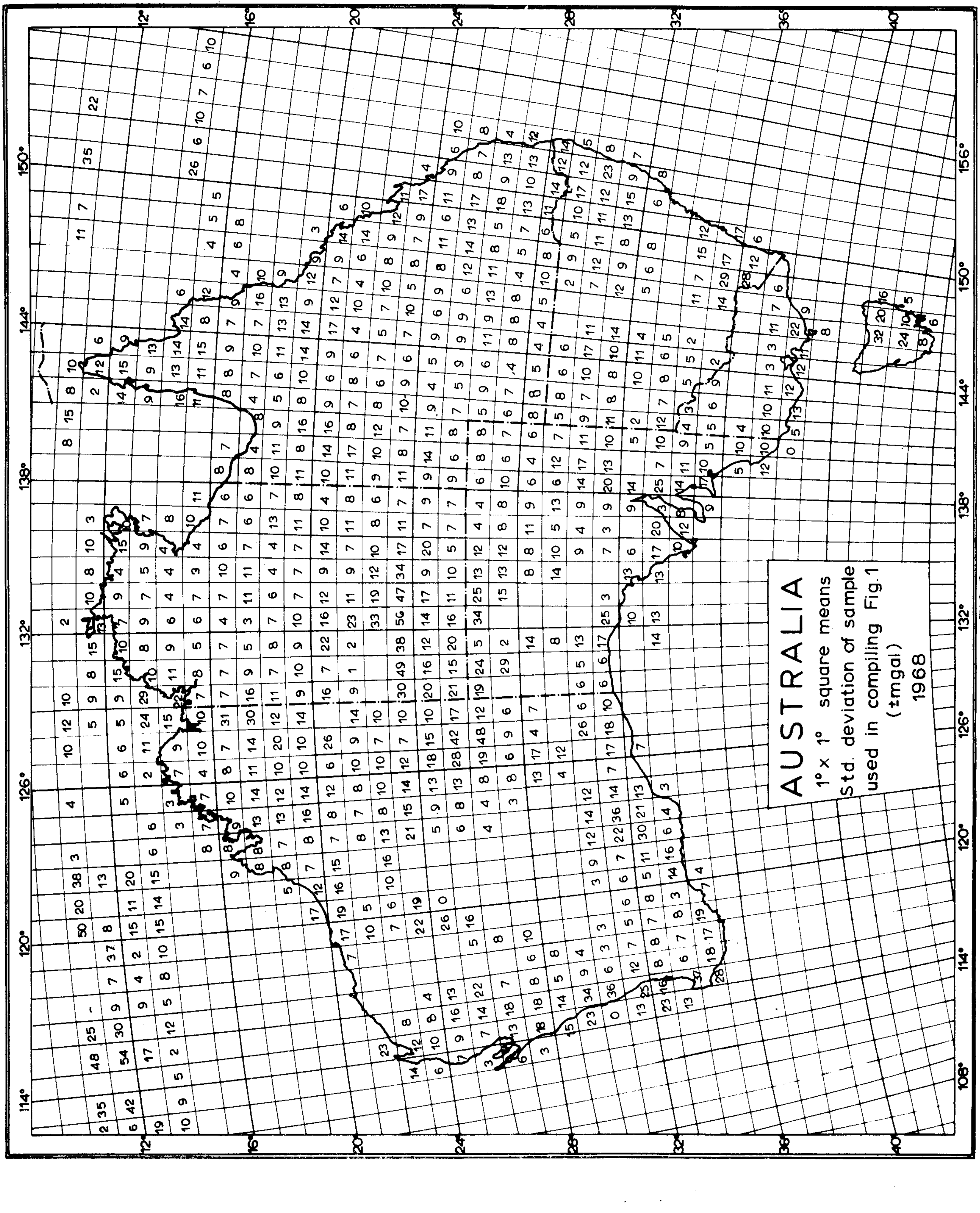

AUSTRALIA
1° x 1° square means
Std. deviation of sample
used in compiling Fig.1
(±mgal)
1968

FIG. 3

$$m = \frac{a^3\omega^2}{kM} \quad \ldots\ldots\ldots\ldots\ldots(5).$$

In equations (1) to (5),

γ represents normal gravity,

γ_e is equatorial gravity,

a the equatorial radius and

f the flattening of the spheroid of reference,

k the gravitational constant,

M the mass of the earth and

ω the angular velocity of rotation of the earth. The prefix d represents changes in the quantities defined. The introduction of these changes, together with a correction of -14 mgal. to the Potsdam datum gave the final anomalies for use in the computations (I.A.G. Resolutions, 1967, 383).

The 5°× 5° data sets obtained from a combination of satellite data and surface gravimetry are considered in section (3).

3. THE COMPUTATIONS

The free air geoid (N_f) is the separation between the co-geoid obtained by the use of free air anomalies in Stokes' integral and the spheroid of reference used in the computation of normal gravity. N_f is given by

$$N_f = \frac{R_m}{4\pi\gamma_m} \int_0^{\sigma=4\pi} f(\psi)\ \Delta g_f\ d\sigma \quad \ldots\ldots\ldots\ldots\ldots\ldots(6),$$

where R_m is the mean radius of the earth,

γ_m the mean value of normal gravity,

Δg_f the free air anomaly representing the element of surface area $d\sigma$ on unit sphere which is at an angular distance ψ from the computation point; and

$$f(\psi) = \operatorname{cosec} \tfrac{1}{2}\psi + 1 - 5 \cos \psi - 6 \sin \tfrac{1}{2}\psi - 3 \cos \psi \log\{\sin \tfrac{1}{2}\psi(1 + \sin \tfrac{1}{2}\psi)\} \quad \ldots\ldots\ldots\ldots\ldots\ldots(7).$$

The free air geoid defined in equation (6) assumes that no zero or first order terms exist in the solution. The first is equivalent to adopting zero value for the global mean ($M\{\Delta g_f\}$) of the free air anomaly and the second to setting the centre of the reference spheroid at the centre of mass of the earth. If the former condition is not satisfied, a more complete expression for the free air geoid (Mather, 1968b, 25) is

$$N_f = \frac{W_o - U_o}{\gamma_m} - R_m \frac{M\{\Delta g_f\}}{\gamma_m} + \frac{R_m}{4\pi\gamma_m} \int_0^{\sigma=4\pi} f(\psi)\ \Delta g_f\ d\sigma \quad \ldots\ldots (8),$$

where W_o is the potential of the geoid and U_o that of the reference spheroid used in computing normal gravity (Lambert, 1961, 13). The assumption of zero value for the first term in equation (8) is equivalent to assigning the value of the potential on the reference spheroid to the geoid.

If ξ and η are the components of the deflection of the vertical in the meridian and prime vertical respectively, being positive if the outward vertical is either north or east of the normal, as the case may be, ξ and η are given by the Vening Meinesz formulae

$$\xi_i^{(sec)} = \frac{206265}{4\pi\gamma_m} \int^{\sigma=4\pi} \frac{\partial}{\partial\psi}\{f(\psi)\}\cos\alpha_i \; \Delta g_f \; d\sigma \; , i=1,2 \; \ldots(9),$$

where

$$\xi_1 = \xi \quad ; \quad \xi_2 = \eta$$
$$\alpha_1 = A \quad ; \quad \alpha_2 = \frac{\pi}{2} - A \qquad \ldots\ldots.(10)$$

and A is the azimuth of the element of surface area $d\sigma$ from the computation point.

The surface integrals in both equations (6) and (9) are indeterminate at $\psi = 0$. The contribution (N_i) of the innermost zone to N_f can be evaluated by the expression (e.g., Mather, 1968a, 264)

$$N_i = \frac{\Delta g_f}{\gamma_m} r_o\left(1 + \frac{r_o}{R_m}\right) \qquad \ldots\ldots\ldots\ldots(11),$$

where r_o is the radius of the inner zone assumed circular and Δg_f is the value of the local free air anomaly. The contributions to ξ_i of these inner zones (ξ_{in_i}, i=1,2) are given by Sollins' formulae (Sollins, 1947, 282)

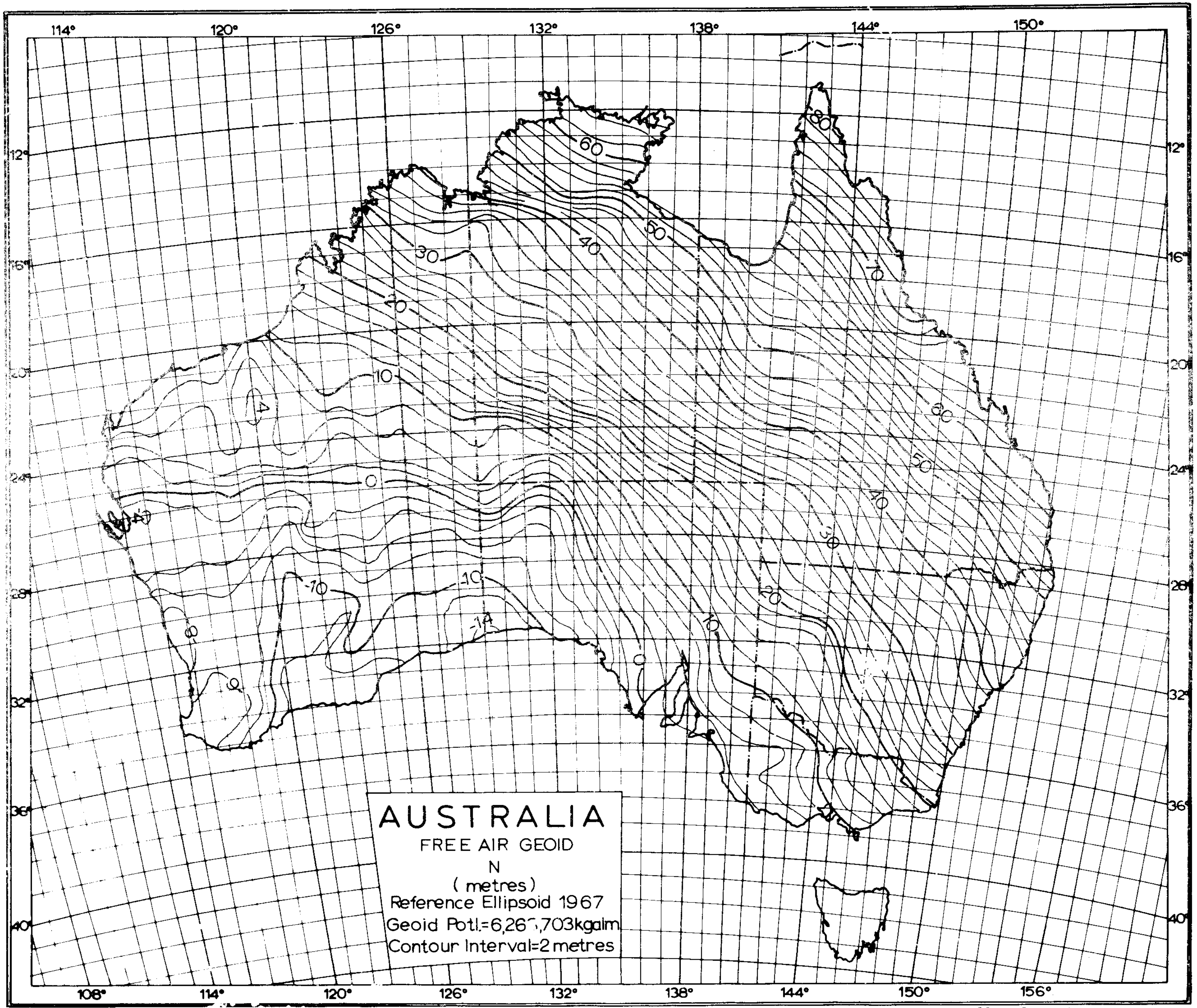
$$\xi_{in_i}^{(sec)} = -\frac{206265}{2\gamma_m} r_o \frac{\partial\Delta g_f}{\partial x_i}\left(1 + \frac{3r_o}{4R_m}\right) \; , \; i=1,2 \; \ldots\ldots(12),$$

where the x_1 and x_2 axes are oriented north and east respectively in the local horizon.

For the current set of computations, values of N_f were calculated at the corners of one degree squares over the Australian mainland. The inner zone was considered to be the four tenth degree squares comprising the area around the computation point. The values of Δg_f and $\frac{\partial \Delta g_f}{\partial x_i}$ for use in equations (11) and (12) were computed from the four values of the free air anomaly representing these squares. N_i seldom exceeded 20 cm but the values of ξ_{in} could be as large as 1 sec. under not uncommon circumstances. Thus the magnitude of the deflections of the vertical can be significantly affected by the approximate technique used for the evaluation of the inner zone in this set of computations.

The programs used in the computations were versions of those described in the report on the South Australian investigation (Mather, 1968a, 232 et seq.). The modifications completely automated all computations and the resulting free air geoids are shown in figs (4) to (9). Figs (4) and (5) give the free air geoid based on a geoid potential of 6,263,703 kgal met. This is equivalent to adopting Gravity Formula 1967 for normal gravity and assuming that W_o is equal to U_o.

It should be noted that these two maps of the free air geoid do *not* include the zero order term (N_o) which is set



Outer zone:- Rapp's O.S.U. set

FIG. 4

km= $3.98603 \times 10^{20} cm^3 sec^{-2}$

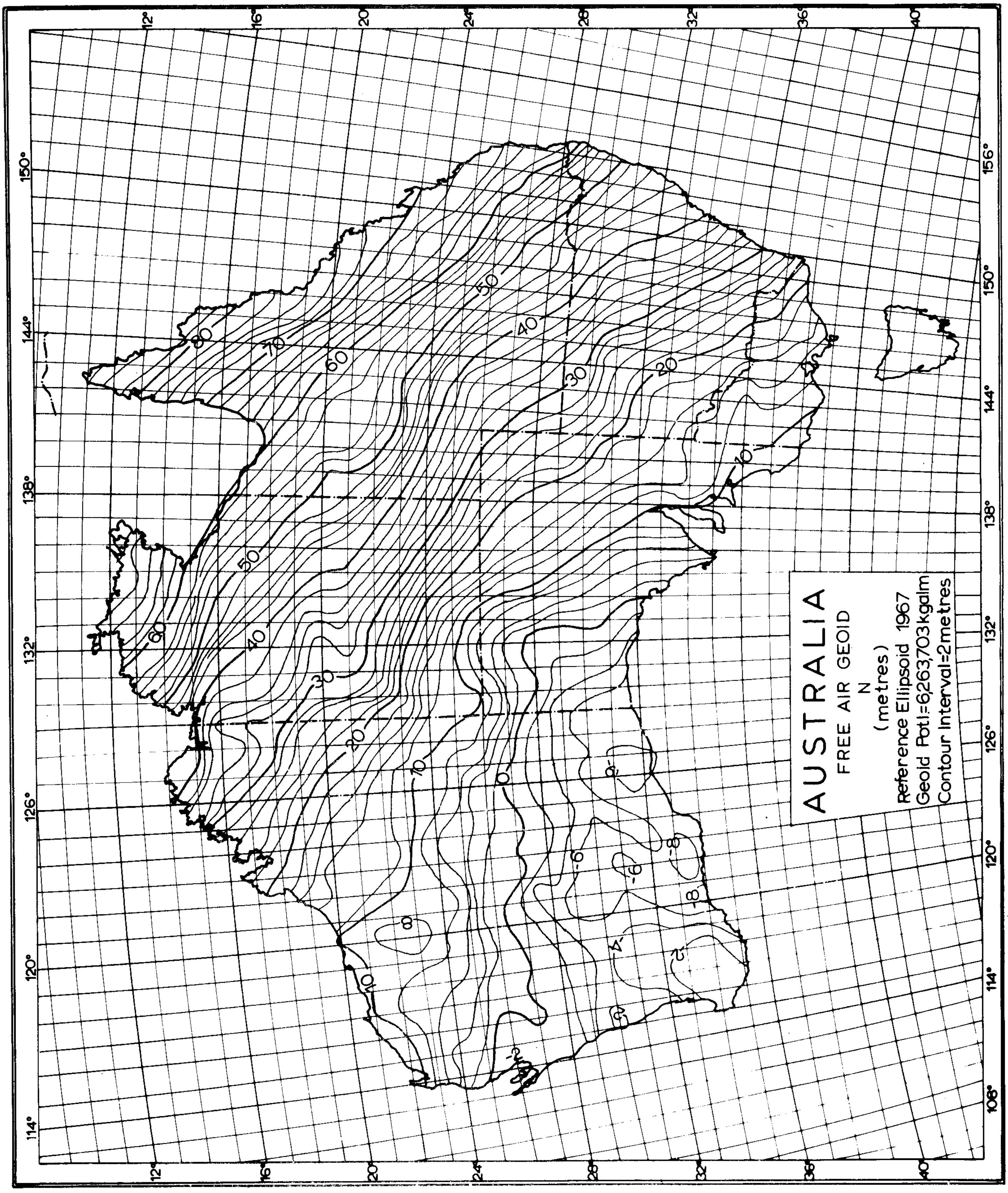


FIG. 5

Outer zone:- Kaula's UCLA set

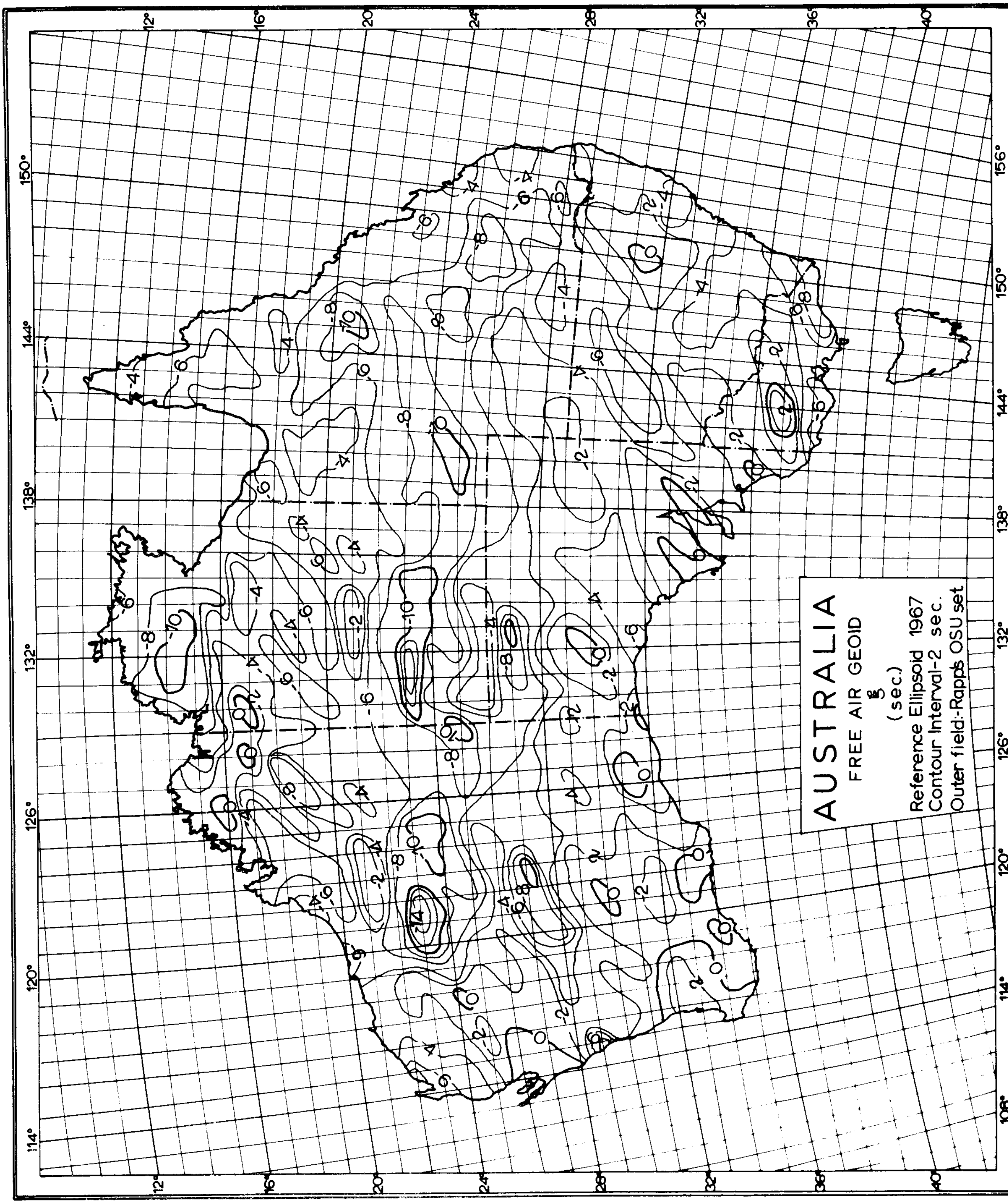

AUSTRALIA
FREE AIR GEOID
ξ
(sec.)
Reference Ellipsoid 1967
Contour Interval-2 sec.
Outer field:-Rapp's OSU set

FIG. 6

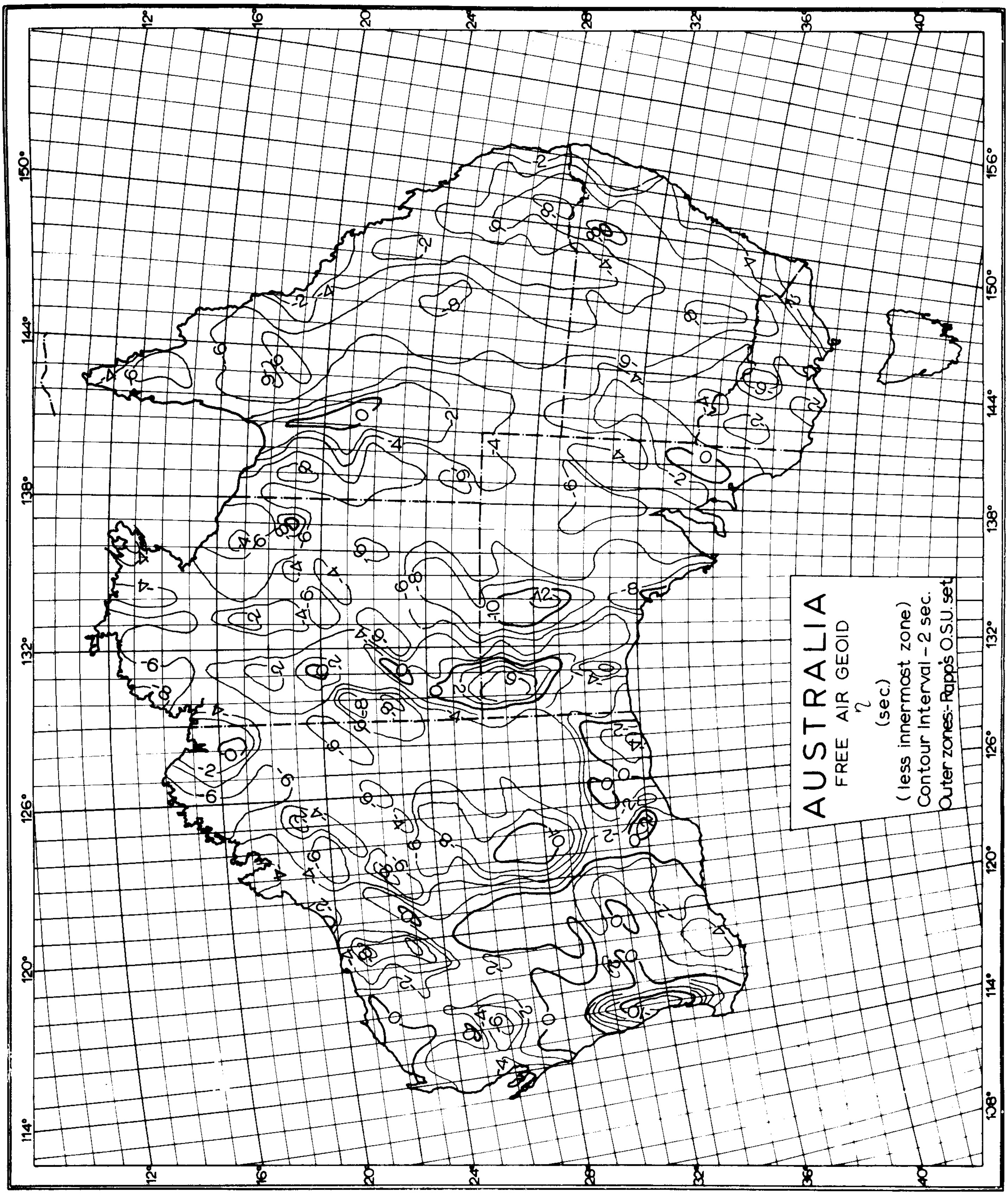

AUSTRALIA
FREE AIR GEOID
η
(sec.)
(less innermost zone)
Contour interval - 2 sec.
Outer zones:- Rapp's O.S.U. set

FIG. 7

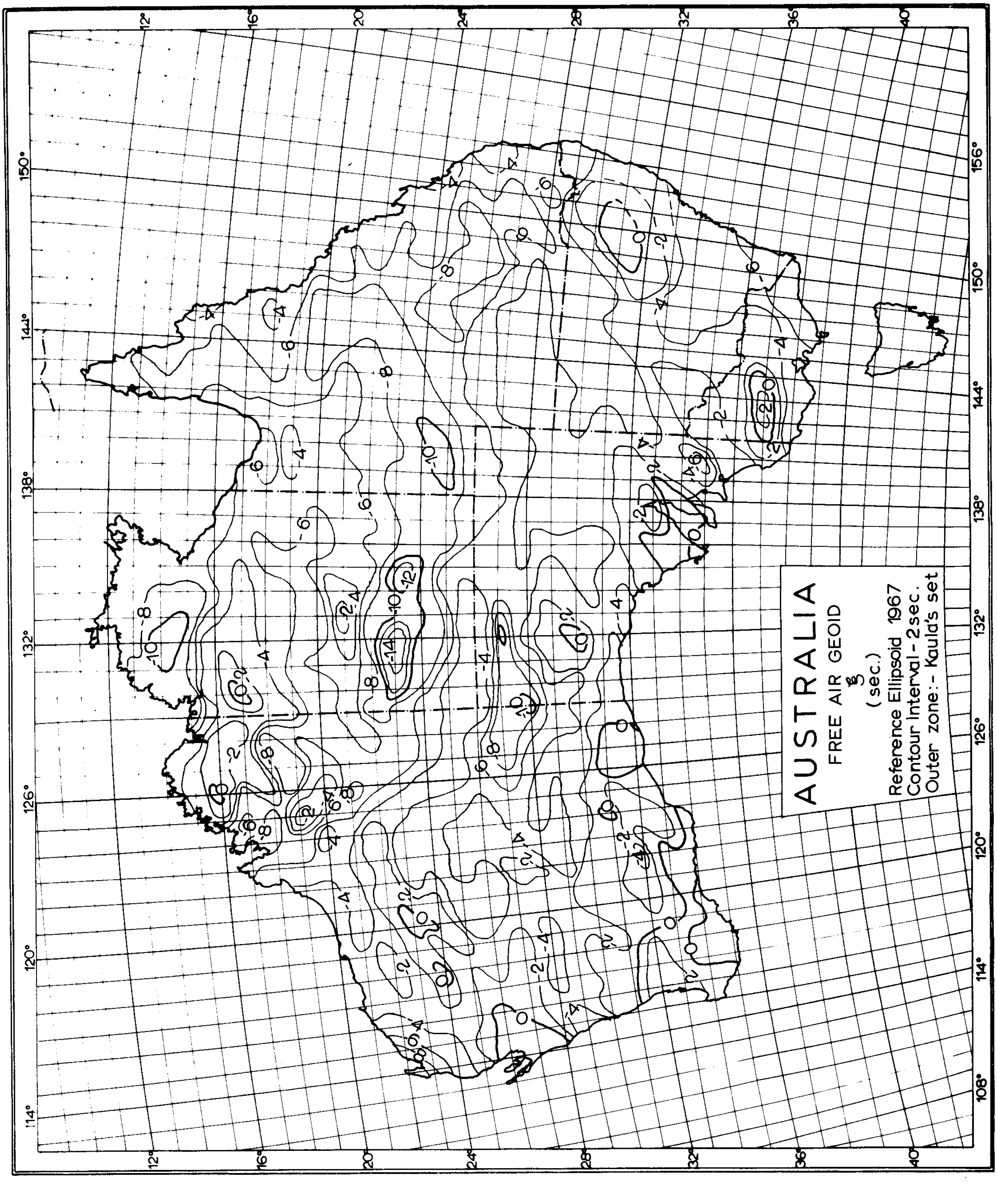
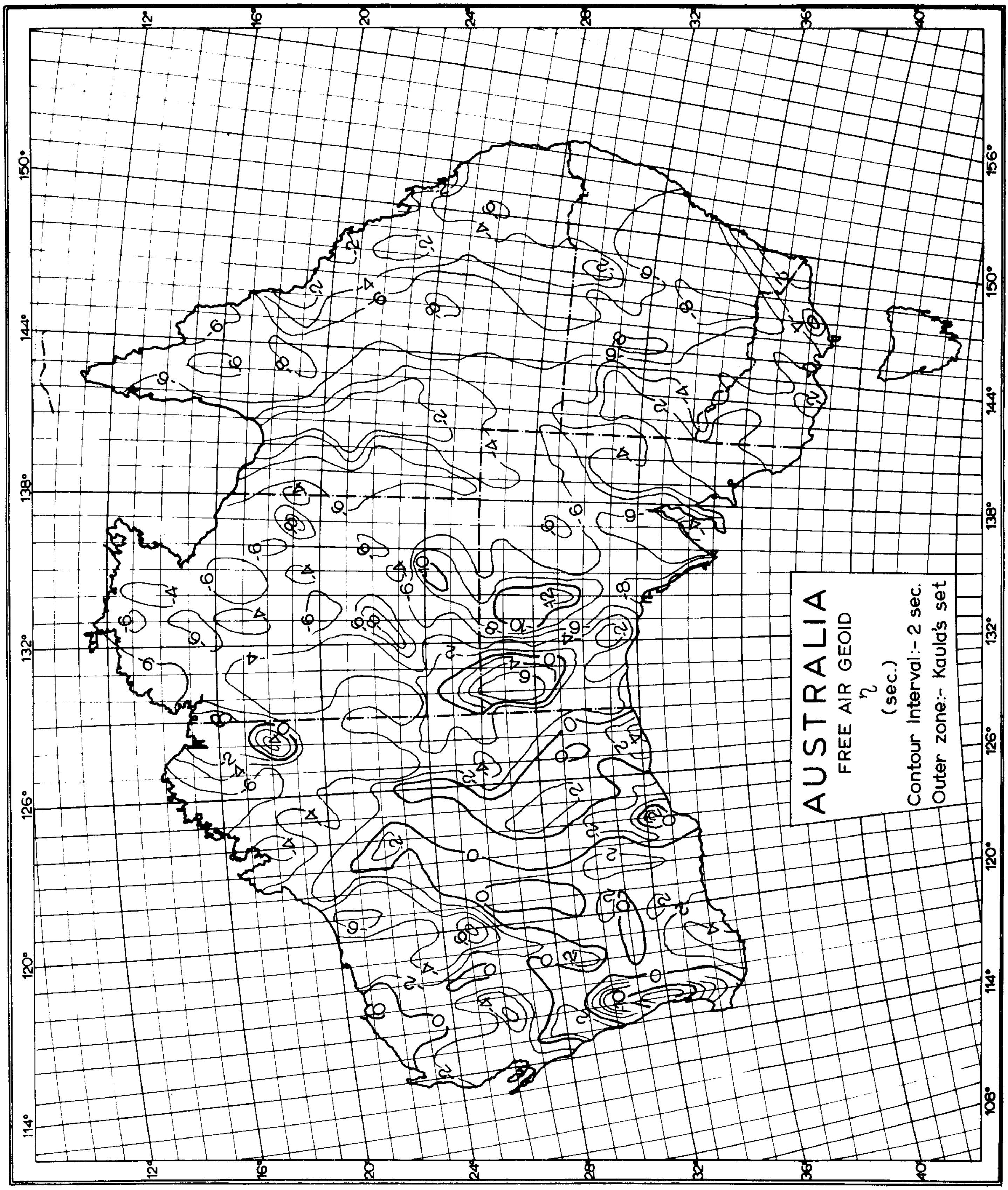

AUSTRALIA
FREE AIR GEOID
ξ
(sec.)
Reference Ellipsoid 1967
Contour Interval - 2sec.
Outer zone:- Kaula's set

FIG. 8



AUSTRALIA
FREE AIR GEOID
η
(sec.)
Contour Interval:- 2 sec.
Outer zone:- Kaula's set

FIG. 9

Data set	M{Δg}(mgal)	N_o(met)
Kaula	- 1.3	8.4
Rapp	+ 0.5	-3.2

TABLE (2)

ZERO ORDER TERM IN STOKES' INTEGRAL

$$N_o = - R_m \frac{M\{\Delta g_f\}}{\gamma_m}$$

Normal gravity based on Gravity Formula 1967

out in table (2). $M\{\Delta g_f\}$ in each case was computed using

$$M\{\Delta g_f\} = \frac{\sum_{i=1}^{2592} \Delta g_{f_i} \cos \phi_i}{\sum_{i=1}^{2592} \cos \phi_i} \qquad \text{...........(13)},$$

where Δg_{f_i} is the 5°× 5° free air anomaly mean representing the square whose centre is at latitude ϕ_i.

Due to the approximations introduced in evaluating the inner zones in the current determination and the relative variability of deflections of the vertical over limited regions, it was felt that no useful purpose would be served by producing maps of total deflections of the vertical as these quantities do not interpolate well over angular distances of one degree. Instead, more representative maps of the deflections were produced by omitting the inner zone effects. Figs (6) to (9) show deflections of the vertical without inner zone effects.

True deflections of the vertical could be obtained by sampling the inner zone gravity field, which, on substitution in either equation (12) or a combination of equations (9) and (12) evaluates the inner zone effect. The combination of this quantity with the values read off the map will give the total deflection of the vertical.

The two solutions for the free air geoid obtained by the representation of the outer zones by the Kaula and Rapp sets of 5° × 5° free air anomalies are not identical. Differences between the two sets of anomalies were studied on a global basis and the comparisons set out in table (3). It is interesting to note that the spread of the comparisons is not correlated

Range of ϕ °N	Range of λ °E	Mean Values (mgal)			Std. Dev. K'la minus Rapp ± mgal
		Kaula set	Rapp set	Kaula - Rapp	
$0 < \phi < 90$	$-180 < \lambda < 0$	-2.4	0.0	-2.4	11.4
$0 < \phi < 90$	$0 < \lambda < 180$	-1.2	0.2	-1.4	12.3
$-90 < \phi < 0$	$-180 < \lambda < 0$	0.0	1.8	-1.8	12.0
$-90 < \phi < 0$	$0 < \lambda < 180$	-1.6	0.0	-1.6	14.2
Global Mean		-1.3	0.5	-1.8	12.5

TABLE (3)

COMPARISON OF *KAULA* AND *RAPP* SETS OF 5° × 5° FREE AIR ANOMALY MEANS globally, the standard deviation for each quarter surface area being of similar magnitude. The value for the standard deviation of the global sample (±12.5 mgal) is about half the magnitude of Hirvonen's estimate of the error of representation (E_s) for a 5° × 5° square which is ±23.1 mgal (Hirvonen, 1956, 3). For other studies on this subject see (Rapp, 1967, 7).

An assessment is also necessary as to whether each or either of the sets of five degree area means provides an adequate representation of the exterior gravity field. This is of great importance as combined satellite and gravimetric solutions afford the only means of providing an acceptable representation of the exterior gravity field in the foreseeable future. The combination of satellite data and surface gravimetry can be effected by least squares adjustment of hybridised data either in terms of corrections to harmonic coefficients, as used by Kaula, or as corrections to the mean area gravity anomalies. The latter procedure was adopted by Rapp who has studied both methods comparatively and concludes that the latter approach matches terrestrial gravity best (Rapp, 1968, 2-5).

There are two methods available for verifying whether combined satellite gravimetric solutions adequately represent the terrestrial gravity field for geodetic calculations.

(i) *Comparison of each of the sets of data with available terrestrial samples.*

No. of readings terrestrial samp. n	No. of Comparisons	$M\{\Delta g_f - \Delta g_c\}$ mgal.	
		Rapp set	Kaula set
$50 < n \leqq 2500$	40	$+ 1.7 \pm 10.2$	$+ 0.2 \pm 10.9$
$500 < n \leqq 2500$	29	$+ 1.1 \pm 9.1$	$- 1.5 \pm 9.7$
$1000 < n \leqq 2500$	19	$+ 0.9 \pm 8.0$	$- 2.7 \pm 6.4$
$2000 < n \leqq 2500$	8	-1.6 ± 5.2	$- 1.4 \pm 5.2$

TABLE (4)

COMPARISON OF COMBINED SOLUTION VALUES WITH TERRESTRIAL SAMPLES OF 5° × 5° MEAN FREE AIR ANOMALIES.

Δg_f = *Area Mean from Terrestrial Gravity.*

Δg_c = *Area Mean from Combined Solution.*

Such comparisons were carried out for the Australian mainland and the results are set out in table (4). In assessing these comparisons it should be noted that only one gravity value represents a single tenth degree square and therefore the maximum sample which could be used in the evaluation of a 5° × 5° area mean was 2500. The standard deviations of comparisons of the combined solution with terrestrial gravity samples with less than 50 readings were found to be greater than the figures quoted above. Allowing for the limited sample, there is little to choose between the two sets of data for closeness of agreement with terrestrial gravity.

There is however a significant correlation, irrespective of the set used, between the standard deviation of comparisons and the size of the terrestrial sample in the data listed in table (4). If the combined solution *were*, in fact, a true representation of the earth's gravity field, it could be expected that small samples would have greater deviations from the true value than larger ones.

(ii) *Comparison of the N values determined using each of the sets of anomalies in turn.*

Table (2) shows that the use of the Kaula set for geoidal determinations will require a positive zero order correction to N while the use of the Rapp set necessitates a negative one. This is to be expected as the Kaula set was obtained by imposing the condition of no zero order term when referred to the International Gravity Formula while the Rapp solution using the same reference system is made to take the value of +1.9 mgal for $M\{\Delta g\}$ (Rapp, 1968,11).

These zero order terms are of no significance in considering systematic variations, being akin to datum corrections. Further, the interpretation of the zero order term is, at the present time, ambiguous (Mather, 1968b, 41).

The two solutions obtained using the Kaula and Rapp sets of data were compared at one degree intervals and the regional means for 100 comparisons together with the standard deviation are shown in table (5). Systematic differences,apart from zero order effects, are seen to exist

Longitude (°E)

Latitude °N	110–120	120–130	130–140	140–150
-10 to -20		+2.5 ±1.4	+5.0 ±0.6	+4.9 ±0.4
-20 to -30	+1.4 ±0.7	+2.8 ±0.7	+5.4 ±0.6	+5.6 ±0.5
-30 to -40	+3.9 ±0.8	+5.4 ±0.9	+6.7 ±0.7	+5.1 ±0.3

TABLE (5)

COMPARISONS KAULA OUTER ZONES MINUS RAPP OUTER ZONES.

Means and Standard Deviations for Ten Degree Squares

Units metres.

even though the variation over limited regions is small. The mean of the comparisons for the Australian region was +4.4 metres. The significance of these systematic variations can only be assessed by comparison with astrogeodetic solutions. This is considered in the next section.

4. TESTS OF THE FREE AIR GEOID WITH THE FISCHER SLUTSKY ASTROGEODETIC SOLUTION.

The free air geoid can be directly compared with the astrogeodetic solution if

(a) the reference spheroid used in both solutions has the same equatorial radius (a) and flattening (f); and

(b) both spheroids have coincident locations in earth space.

The current free air geoid is computed on Reference Ellipsoid 1967 which has the same dimensions as the Australian National Spheroid (Bomford, 1967, 56). This spheroid was fitted to an estimate of the mean geoid slope across Australia made in 1963 and provides what is termed the Australian Geodetic Datum (A.G.D.). The N value at the Johnston Origin, whose coordinates are

-25° 56' 55" N; 133° 12' 30" E,

was assumed to be zero. The Fischer Slutsky astrogeodetic geoid was based on 600 astrogeodetic stations on the A.G.D. (Fisher & Slutsky, 1967, 327). The pattern of distribution of these stations can be obtained from a study of fig (1) in Bomford's paper which shows the stations to be spaced along the perimeters of approximately 85 loops which cover all but the extreme north east of the continent. The mean length of these loops is approximately 1400 km. The Fischer Slutsky astrogeodetic determination, while lacking the precision of a rigorous solution, is nevertheless acceptable as

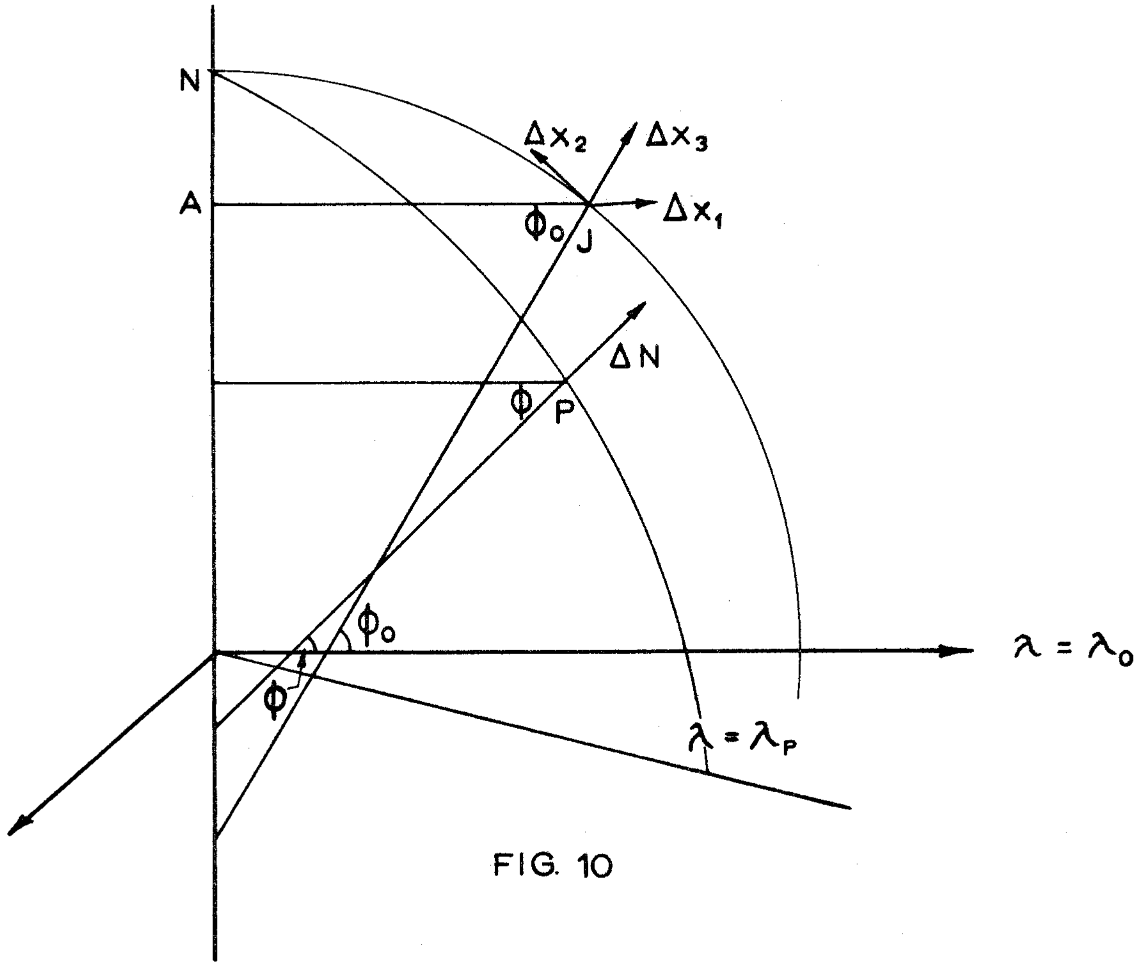


FIG. 10

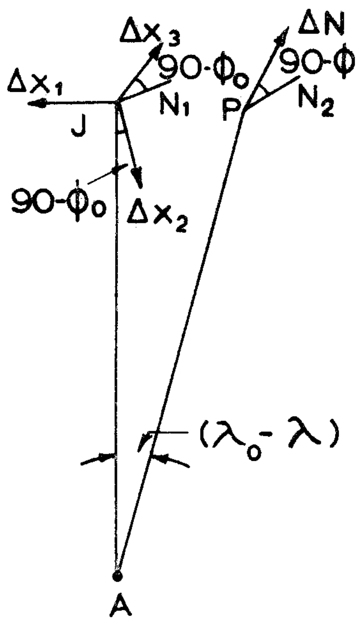


FIG. 11

JN_1, PN_2 // NA

NA ⊥r plane of paper

an *independent* means of adequate accuracy for investigating the precision of the free air geoid. The latter is referred to Reference Ellipsoid 1967 which has its minor axis coincident with the mean axis of rotation of the earth and centre at the centre of mass of the earth. The minor axis coincidence condition is only implicitly involved in the solution. The A.G.D. can be converted to the datum of the free air geoid by means of three correction parameters to the coordinates of the origin of the A.G.D.. Let these corrections be ΔN_o to the geoid spheroid separation (N_o), $\Delta\xi_o$ to the meridian component ξ_o and $\Delta\eta_o$ to the prime vertical component η_o of the deflection of the vertical, all at the Johnston Origin on the A.G.D. (J in figs (10) and (11)). These can be considered as three corrections $\Delta x_i (i=1,3)$ in the local laplacian trihedron at J (Dufour, 1968, 128), where

$$\Delta x_1 = -(\nu + N_o + h)\Delta\eta_o$$
$$\Delta x_2 = -(\rho + N_o + h)\Delta\xi_o \qquad \dots\dots\dots\dots\dots\dots(14),$$
$$\Delta x_3 = \Delta N_o$$

where ρ, ν are the radii of curvature in the meridian and prime vertical and h the orthometric elevation at J. From a study of figs (10) and (11) it can be seen that the introduction of corrections to the orientation of the A.G.D., defined by equation (14), will change the astrogeodetically obtained value of the geoid spheroid separation (N_a) by an amount ΔN to obtain the true geoid spheroid separation (N) according to

$$N = N_a + \Delta N \quad \text{.................(15),}$$

where resolution into components gives

$$\Delta N = -\Delta x_1 \sin \Delta\lambda \cos \phi + \Delta x_2[-\sin \phi_o \cos \Delta\lambda \cos\phi + \cos \phi_o \sin \phi]$$
$$+ \Delta x_3[\sin \phi_o \sin \phi + \cos \phi_o \cos \phi \cos\Delta\lambda] \quad \text{....(16),}$$

$\Delta\lambda$ being given by

$$\Delta\lambda = \lambda_o - \lambda \quad .$$

The use of equations (14), (15) and (16) gives

$$N = N_a + \Delta\xi_o(\rho + h + N_o)[-\sin \phi \cos \phi_o + \cos \phi \sin \phi_o \cos \Delta\lambda]$$
$$+ \Delta\eta_o(\upsilon + h + N_o)\cos \phi \sin \Delta\lambda$$
$$+ \Delta N_o[\sin \phi \sin \phi_o + \cos \phi \cos \phi_o \cos \Delta\lambda] \quad \text{.......(17)}$$

Values of $\Delta\xi_o$, $\Delta\eta_o$ and ΔN_o can be obtained by setting up observation equations of the form

$$N + v_a = N_f + v_f \quad \text{............(18),}$$

where v_a and v_f are errors in the values of the corrected astrogeodetic solution and the free air geoid respectively. If

$$v = v_f - v_a \quad \text{............(19),}$$

$$v = N_a - N_f + \Delta\xi_o(\rho + h + N_o)[-\sin \phi \cos \phi_o + \cos \phi \sin \phi_o \cos \Delta\lambda]$$
$$+ \Delta\eta_o(\upsilon + h + N_o)\cos \phi \sin \Delta\lambda$$
$$+ \Delta N_o[\sin \phi \sin \phi_o + \cos \phi \cos \phi_o \cos \Delta\lambda] \quad \text{...(20)}$$

which is of the form

$$AX + K = V,$$

where

$$X = \begin{vmatrix} \Delta\xi_o \\ \Delta\eta_o \\ \Delta N_o \end{vmatrix} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (21).$$

If values of N_f and N_a are available at m points, the m observation equations obtained by the use of equation (20) can be solved by the method of least squares when

$$X = -(A^TWA)^{-1}A^TWK \quad \ldots\ldots\ldots\ldots\ldots (22)$$

(see, e.g., Mather, 1967, 132). Such comparisons of solutions were effected between the free air geoids obtained by the use of the Rapp and Kaula sets of data for the outer zones and the Fischer Slutsky astrogeodetic solution. The orientation correction parameters so obtained are listed in table (6). Four different types of comparisons were effected in all on both free air geoids.

In the first type,

all data was considered and comparisons made for all points on the Australian mainland at the corners of a one degree grid. A plot of the magnitudes of the differences $(N-N_f)$ between the corrected Fischer Slutsky and the free air geoid in the case where the outer zones were represented by the Rapp set is given in fig (12). The nature of the comparisons obtained

Outer set used	Key to soln type	Corrns. to parameters at origin				No. of Comparisons
		$\Delta\xi_o$ sec.	$\Delta\eta_o$ sec.	ΔN_o met.	$M\{N-N_f\}$ met.	
Rapp	1	-4.61	-3.97	15.9	-1.9±5.2	701
Rapp	2	-4.69	-4.17	6.0	-11.7±5.3	701
Rapp	3	-4.69	-4.46	14.1	-2.1±3.0	566
Rapp	4	-5.03	-2.88	16.4	-1.5±7.5	701
Kaula	1	-4.54	-4.19	20.4	-2.0±5.2	690
Kaula	2	-4.65	-4.33	11.5	-10.8±5.3	690
Kaula	3	-4.61	-4.69	18.8	-2.2±3.3	557
Kaula	4	-5.03	-2.88	16.4	-5.8±8.2	690

TABLE (6)

COMPARISON OF FREE AIR GEOIDS WITH THE CORRECTED ASTRO-GEODETIC SOLUTION

Key to col.2

Type 1 :- All data considered.

Type 2 :- All data considered. ΔN_o read from figs. (4) and (5) and held fixed in solution.

Type 3 :- Excluding regions west of meridian 120° E north of parallel -15°N and the Officer Basin, South Australia.

Type 4 :- Astrogeodetic solution corrected using parameters obtained from Tranet solution.

by the use of the Kaula set is similar. The following points should be taken into account when studying fig.(12).

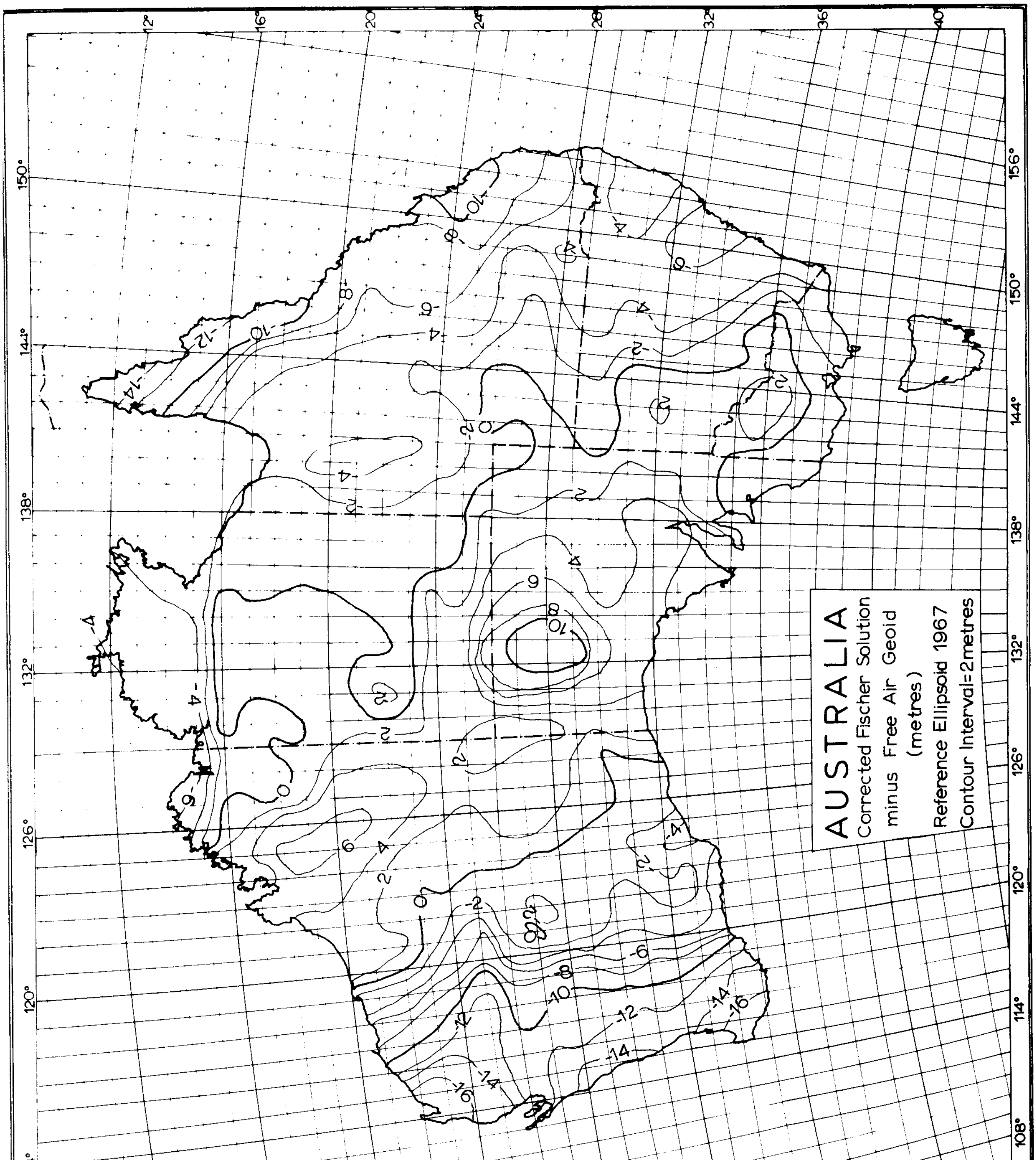

AUSTRALIA
Corrected Fischer Solution
minus Free Air Geoid
(metres)
Reference Ellipsoid 1967
Contour Interval=2metres

FIG. 12

(i) An error of 1 sec. in astronomical determinations will give rise to an error of half a metre in the geoid height over 100 km.

(ii) An area of over six and a half million square kilometres is represented by 600 astrogeodetic stations, many of which are clustered along two traverses (Bomford, 1967, fig (1)). It is therefore vital that each astrogeodetic station should be representative of the surrounding region which is in excess of 12,000 sq.km., if the astrogeodetic chart is of acceptable accuracy. The current astrogeodetic chart cannot therefore be considered to be error free.

Consequently it is reasonable to draw the following conclusions.

(a) Variations in $(N-N_f)$ of the order of 1 metre over 100 km. can be considered to occur in regions where an adequate astrogeodetic solution has been compared with a free air geoid whose intermediate zone representation is satisfactory.

(b) Variations of the order of 1 - 2 metres per 100 km. in $(N-N_f)$ can be considered instances where an adequate astro-geodetic geoid has been compared with a free air geoid whose local field has been suitably represented but with intermediate zone extensions which are not entirely satisfactory.

A study of fig (12) shows that there are three regions in which variations in excess of these limits occur:-

(a) along the meridian 120°E ;

(b) along the parallel -15°N ; and

(c) in north western South Australia.

The variation at (a) is greater than that at (b) but it is likely that both are due to inadequate representation of intermediate zones for unsurveyed and poorly represented ocean areas, the large change occurring on moving from one 5 degree block to another especially in the west. The rapid change in $(N-N_f)$ in case (c) occurs in a region of large and rapidly changing deflections of the vertical on the A.G.D. In addition to the consequent uncertainty in the astrogeodetic solution, this is also a region of rapidly varying gravity which has been inadequately represented in the current solution, thus giving rise to uncertainties in the value of the inner zone determinations for the free air geoid.

Thus under suitable conditions which occur over 80% of the continent, the agreement between the two solutions can be considered to be satisfactory.

In the second type of solution,
the value of ΔN_o was held fixed at the value read off figs (4) and (5). As to be expected there were significant differences in the value of $M\{N-N_f\}$ as obtained from type (1) and type (2) solutions, being approximately equal in magnitude to the difference in the ΔN_o values in each solution. The changes in $\Delta\xi_o$ and $\Delta\eta_o$ which occurred as a consequence were less than 0.2 sec. even when different sets of outer zone data were used.

In the third type of solution, the areas of large discrepancies listed above were excluded. This procedure would not only provide the most reliable orientation corrections for the A.G.D. (which, incidentally, is not the best technique for obtaining these parameters), but would also provide an assessment of the extent of agreement between the two solutions.

The correction parameters applied to the Fischer Slutsky astrogeodetic data in the *fourth type of solution* were the mean of the corrections to the geographical coordinates of the nine Tranet stations in Australia, as established on the A.G.D., after a world wide adjustment (solution NWL-8D-4 of November 1967). The comparisons obtained were inferior to all other types of solution.

5. CONCLUSIONS

In summarising the conclusions which can be drawn from this investigation it should be borne in mind that the free air geoid is not exactly equivalent to the geoid itself. Current *unconfirmed* research seems to indicate that the indirect effect has, in addition to zero order implications, a maximum variation over the Australian continent of about 3 metres. In this context it can be concluded that current combined solutions for the global free air anomaly field from satellite data and gravimetry give, so far as the Australian region is concerned, an acceptable representation of the outer zones in a solution for the free air geoid.

Two such solutions used in this investigation were independently prepared using slightly different techniques and were found to have a global comparison error of ±12.5 mgal. This is approximately half the error of representation of a five degree square by a single reading. The use of each of these data sets in turn gave similar solutions for the Australian Geodetic Datum orientation correction parameters. The resultant corrections place the datum at approximately 9 metres above the current geoid solution height at the Johnston Origin with corrections to the deflections of approximately -4.6 sec. to ξ_o and -4.2 sec. to η_o. The correction to N_o is due to the weakness in the local gravity field in north western South Australia, as can be seen from fig (12).

While areas west of the meridian 120°E are in fair agreement with the corrected Fischer Slutsky solution, the variation in comparisons across this meridian shows weaknesses in the extended values for 1° × 1° free air anomaly field in the Indian and Southern Oceans west and south of the continent. If the regions west of meridian 120°E, north of parallel -15°N and the poorly represented Officer Basin in South Australia are excluded, the comparisons between the corrected Fischer Slutsky solution and the free air geoid have a standard deviation of ±3 metres. The present solution can be improved by strengthening the gravity field in the oceans south and west of Australia and in the north west of South Australia.

Further research is necessary to establish the nature and magnitude of the zero order term and the indirect effect for the free air geoid. It is of relevance not only to assess the contribution of each of these effects to the final geoid spheroid separation but also to confirm the validity of the representation of the outer zones by current combined solutions.

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BIOGRAPHICAL NOTES

RON MATHER was educated at the University of Ceylon, Christ's College, Cambridge and the University of New South Wales. On graduating with a Bachelor of Science degree in 1955 he joined the Ceylon Survey Department in which he served till 1962. During this period he was for a while in charge of the Ceylon Survey Training School. After a spell as a lecturer at the South Australian Institute of Technology, he joined the University of New South Wales in 1966 where he is at present a senior lecturer.

Dr. Mather has published papers on the propagation of errors, extension of gravity fields and other aspects of physical geodesy. He is currently working on the orientation of the Australian Geodetic Datum using gravity data and other problems connected with improving geoidal solutions from gravimetry.