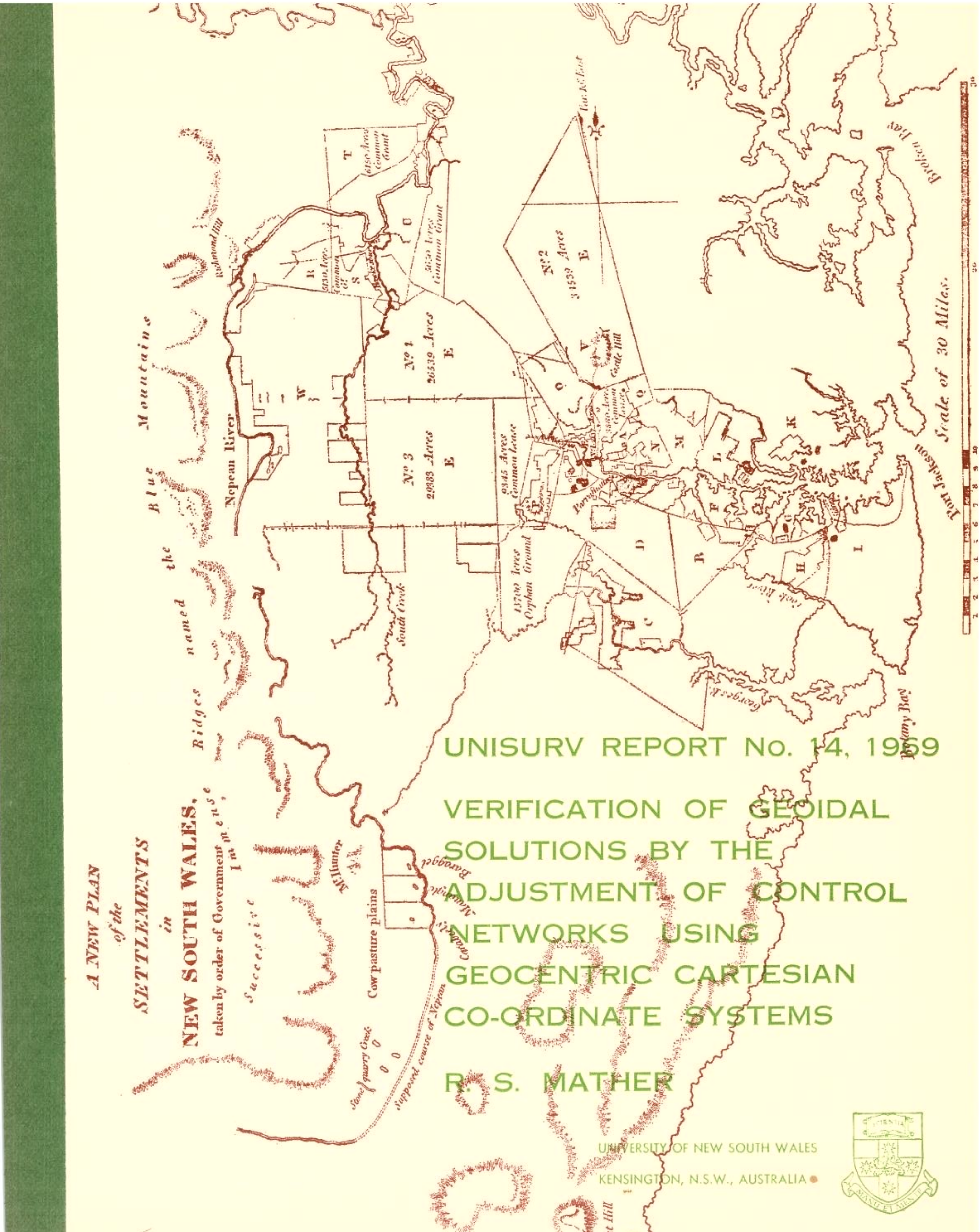

UNISURV REPORT No. 14, 1969

VERIFICATION OF GEOIDAL SOLUTIONS BY THE ADJUSTMENT OF CONTROL NETWORKS USING GEOCENTRIC CARTESIAN CO-ORDINATE SYSTEMS

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Received 14th February 1969

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SUMMARY

The simultaneous adjustment of a horizontal control network, together with astronomical observations and the geoid spheroid separation vector, using a geocentric cartesian system as a reference frame is investigated and formulae are derived for the complete definition of the solution. The precision required for each of the quantities involved in the adjustment is assessed and a relation established between parameters obtained in the adjustment and systematic errors in the geoidal solution. A method is outlined for the study of these position dependent errors in the geoidal solution in which the distant zones are represented by the gravity anomaly values established by a combined solution using satellite data and surface gravimetry.

VERIFICATION OF GEOIDAL SOLUTIONS BY THE ADJUSTMENT OF CONTROL NETWORKS USING GEOCENTRIC CARTESIAN COORDINATE SYSTEMS

by

R. S. Mather

1. INTRODUCTION

The definition of geodetic position using a system of geocentric cartesian coordinates has been dealt with by many geodesists. The use of such a system to afford a reference frame appears to have no obvious advantages over the conventional system in non-polar regions. It has, in fact, the decided disadvantage of requiring a knowledge of the values of the orthometric elevation (h) and the geoid spheroid separation (N) at every control station for the complete definition of position. The relationships between observed quantities and such a cartesian reference frame have also been studied and the possible use of such a system for the adjustment of large scale networks outlined (e.g., Dufour, 1968).

In Australia the current geodetic datum is afforded by Reference Ellipsoid 1967 (I.A.G. Resolutions, 1967, 367) and called the Australian National Spheroid. It is oriented by adopting zero geoid spheroid separation and the mean value of the deflections of the vertical for the continental area from the astrogeodetic data available in 1963 for the Johnston Origin (Bomford, 1967, 56-58). Geodetic position is conventionally determined by relating observed quantities to an arbitrarily oriented spheroidal reference frame, the adjustment, in this context, being carried out in two dimensions only. The quantities involved in the adjustments are angular and linear measurements together with astronomical observations. The adjusted geodetic positions so obtained can be used to determine astrogeodetic deflections of the vertical at those stations where the necessary astronomical observations have been made (e.g., Bomford, 1962,89). The analysis of 600 such astrogeodetic stations on the Australian Geodetic Datum by Fischer and Slutsky (1967) produced a solution for the geoid spheroid separation *on this datum* which has an unknown arbitrary orientation in earth space.

A preliminary determination of the free air geoid for Australia has been completed using a compilation of local gravity data and a composite solution for the distant regions from a combination of satellite data and terrestrial gravimetry (Mather, 1969). This provides an estimate of the geoid spheroid separation which, unlike the astrogeodetic solution described above, is completely independent of any control network measurements. It has generally been held that the accuracy of gravimetric solutions is questionable but the results of the analysis of satellite orbits have, on combination with terrestrial gravity data provided improved solutions for both the terrestrial gravity field and consequently the geoid spheroid separation.

The comparison of this solution with the Fischer Slutsky determination was effected after the latter had been corrected by least squares fitting for arbitrary orientation at the origin. If N and N_f are the values of the geoid spheroid separation for the corrected astrogeodetic and gravimetric solutions respectively, the standard deviation of 557 comparisons $(N-N_f)$ over an area of approximately six million square kilometres, after the exclusion of 20% of the comparisons with comparatively large and explicable discrepancies, was ± 3 metres. The comparison error was not randomly distributed but varied systematically with position (ibid, fig (12)). It was also noticed that the comparison errors had smaller standard deviations if the area of the region considered was reduced. The magnitude

	Region	Area $km^2 \times 10^6$	Corrected Astrogeodetic minus Free Air Geoid $M\{N-N_f\}$ Mean± Std. Dev. (metres)	No. of comparisons
1	Australia	7.6	-1.9±5.2	701
2	Australia* E of 120°E S of -15°N	6.1	-2.1±3.0	667
3	Victoria	0.2	-0.5±2.0	29
4	New South Wales	0.8	-2.0±2.5	68
5	Northern Terr. S of -15°N	1.3	+0.3±1.6	99

TABLE (1)

COMPARISON OF THE CORRECTED FISCHER SLUTSKY ASTROGEODETIC SOLUTION WITH THE 1968 FREE AIR GEOID

Outer zone representation:- Rapp's set of 5°× 5° free air anomalies. Rows (3), (4) and (5) represent the relevant portions of solution in row (1).

** Excludes Officer Basin, South Australia.*

of the standard deviation is however largely dependent on the completeness with which the local gravity field is represented, as can be seen from a study of table (1). In this table, M{ } refers to the mean value, N and N_f being defined earlier. One of the features of the investigation quoted was the consistency with which the value of

-2 metres (±0.2 metres) was obtained for the value of $M\{N-N_f\}$ in the case of continental solutions on fitting the Fischer Slutsky data to the gravimetric determination using the least squares condition. This implied that, on the average the corrected astrogeodetic solution, after adjustment, was two metres smaller than the gravimetric one.

It should be noted that no indirect effect has been considered for the free air geoid (Mather, 1968, equation (51)). Other possible sources for the existence of comparison errors are

(a) errors in the outer zone representation obtained by the combination of satellite data and limited samples of surface gravimetry;

(b) errors in the field extensions for unsurveyed areas of the intermediate gravity field used in computing the free air geoid; and

(c) errors in the astrogeodetic solution used.

While the indirect effect can be computed, the errors arising from source (c) can only be eliminated by effecting comparisons at points on rigorously computed astrogeodetic sections. Errors in (b) can be minimised by restricting comparisons to those regions where intermediate zone field extensions are either not necessary or made from adequate surface gravimetry.

It is therefore of interest to use some independent method to assess the precision of the gravimetric solution for the geoid under conditions free from the influence of interpolation errors in the astrogeodetic solution. Such a method is afforded by the combination of geoidal solutions with horizontal surveys and elevations in a composite adjustment using variations on a geocentric cartesian reference frame. This paper outlines the formulae to be used in such an adjustment and investigates the necessary precision required in the measurement of each of the quantities involved in the calculation. It also studies the nature of the error in the gravimetric solution and means for estimating the magnitude of the contributory effects.

The following system of symbols, subscripts and numbering is adopted to provide a uniform system for the following sections.

2. NOTATION

(i) *Symbols*

a = equatorial radius of meridian ellipse

e = eccentricity of meridian ellipse

(no subscript)

e	=	total error containing both systematic and accidental components *(with subscript)*
f	=	flattening of meridian ellipse
h	=	orthometric elevation
ℓ	=	slope distance
s	=	systematic (or position dependent) error
v	=	accidental (normally distributed) error
x_j	=	geocentric cartesian coordinate
x'_j	=	local laplacian trihedron coordinate
z	=	zenith angle
A	=	azimuth
$M\{X\}$	=	mean value of X
N	=	geoid spheroid separation
R	=	mean radius of the earth
R_m	=	mean radius of curvature in the normal section
α	=	observed horizontal angle
γ	=	global mean value of normal gravity
η	=	component of deflection of the vertical in the prime vertical, positive when outward vertical is east of normal.
λ	=	longitude, positive east
ν	=	radius of curvature on spheroid in prime vertical

ξ = meridian component of deflection of the vertical, positive when outward vertical is north of normal

ρ = radius of curvature in meridian ellipse

ϕ = latitude, positive north

ψ = angular distance on a sphere

ΔX = change in X

(ii) *Subscripts*

$_c$ = computed from provisional coordinates

$_f$ = free air geoid

$_m$ = determined with reference to local vertical *(excluding use with* R *)*

$_u$ = provisional value

The line P_iP_{i+1} is defined by the length ℓ_i, the azimuth A_i and the zenith angle z_i from P_i to P_{i+1}.

The point P_i is defined by its related parameters $x_{ij}(j=1,3)$, N_i, h_i, ϕ_i, λ_i and ν_i.

The position of P_{i+1} relative to P_i is defined by the local laplacian trihedron coordinates $x'_j(j=1,3)$ at P_i.

When the strict use of notation lengthens formulae without any improvement in the clarity of expressions, as in the case of specimen observation equations in section (5), these are derived either for the line P_iP_{i+1} when

$i = 1$ or with the subscripts omitted.

3. COMPUTATION OF PROVISIONAL GEOCENTRIC CARTESIAN COORDINATES FROM OBSERVATIONS

The formulae derived in this section are those required for the computation of provisional coordinates from observed quantities which are all subject to observational error. These values will be used in the next section to set up observation equations from which corrections to the provisional coordinates can be deduced to obtain estimates of the most probable values of these quantities. All observed quantities can be related to a local cartesian system. One such system is the *laplacian trihedron* which is an arbitrary but fixed system with its x_1' and x_2' axes in the local horizon, oriented east and north respectively and the x_3' axis coincident with the local spheroid normal (Dufour, 1968, 128). As the spheroid normal and the horizon plane are defined by the geodetic coordinates of the point, these need not be equivalent to the astronomically determined quantities.

The computation of the provisional geocentric cartesian coordinates for any system of n control points $P_i (i=1,n)$ will have to commence from one point P_1 whose geodetic latitude and longitude are assumed to be known and which, for simplicity, will be called the origin. The computation procedure at the origin is slightly different from that at any other point only if its deflections of the vertical and N are not known. This occurs only in those instances where these quantities have not been determined from gravimetry. The general cases can therefore be subdivided into two categories depending on whether the geoid spheroid separation vector has been defined gravimetrically or not.

(i) *For the origin and cases where the geoid spheroid separation vector has been determined from gravimetry.*

Case (a) :- At the origin, where the astronomical values of the latitude (ϕ_{m1}) and longitude (λ_{m1}) are known together with the orthometric elevation (h_1). The provisional geocentric coordinates of P_1 are given from figures 1(a) to 1(c) by (Bomford, 1962, 186)

$$\begin{aligned} x_{11} &= [\nu_1 + h_1 + N_1] \cos \phi_1 \cos \lambda_1 \\ x_{12} &= [\nu_1 + h_1 + N_1] \cos \phi_1 \sin \lambda_1 \qquad \text{......(1),} \\ x_{13} &= [\nu_1(1 - e^2) + h_1 + N_1] \sin \phi_1 \end{aligned}$$

where

$$\phi_1 = \phi_{m1} - \xi_1 \qquad \text{....... (2)}$$

and

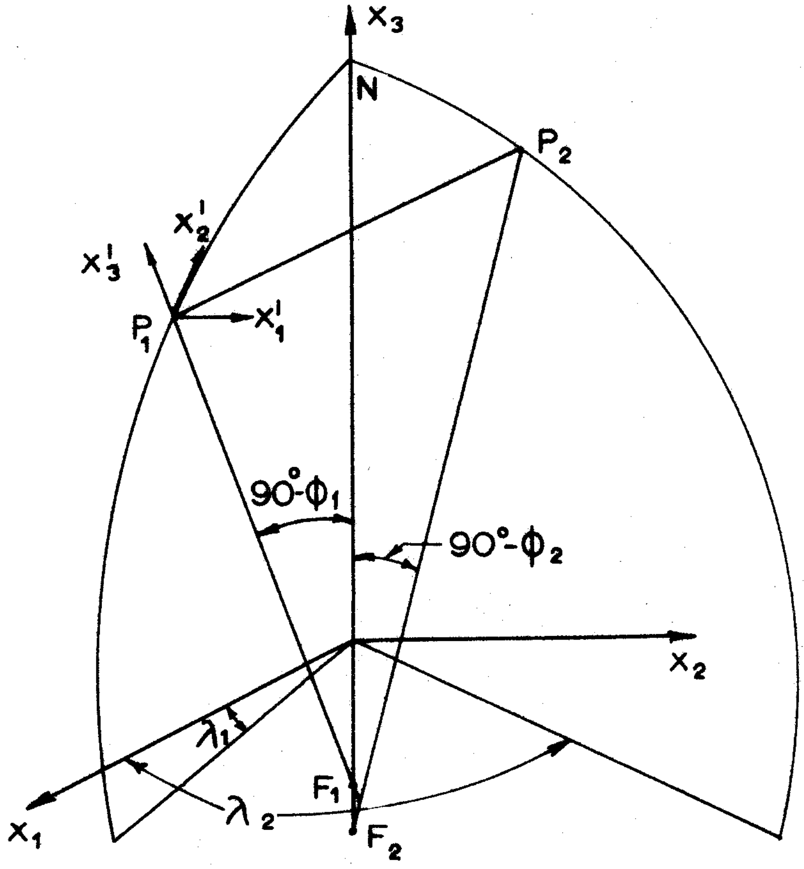

FIG. 1(a)

Relation between the geocentric cartesian system and laplacian trihedron.

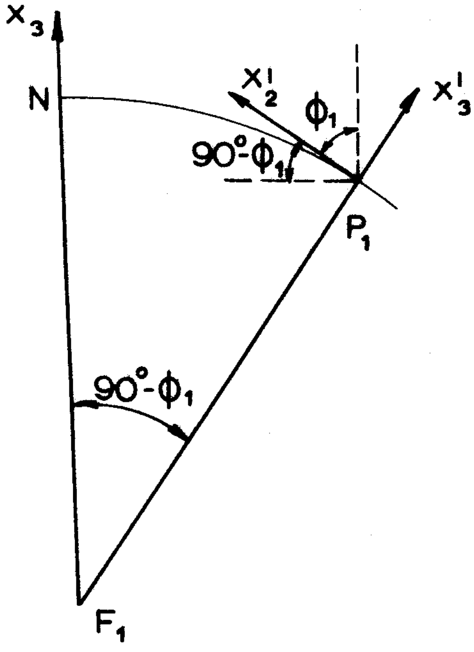

FIG. 1(b)

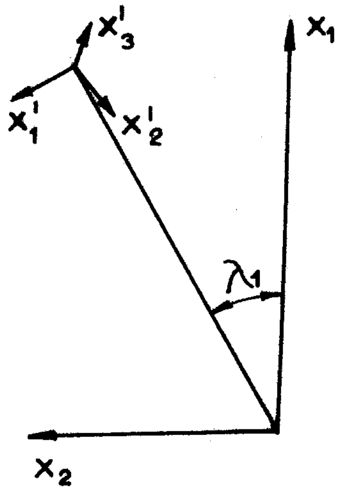

FIG. 1(c)

$$\lambda_1 = \lambda_{m1} - \eta_1 \sec \phi_1 \qquad \text{..........(3).}$$

All quantities in equations (1) to (3) are defined in section (2). These formulae define the geocentric cartesian coordinates at the origin.

<u>Case (b)</u> :- At other points the computations are performed in two stages.

<u>Stage (1)</u> :- *The conversion of observations to laplacian trihedron coordinates.*

The provisional geocentric cartesian coordinates of a point P_2 are obtained from those of P_1 and the following possible observed quantities.

(i) The length (ℓ) either measured or implied from horizontal observations as in the case of a triangulation network.

(ii) The zenith angle (z) either measured with respect to the local vertical (z_m) or implied from elevation measurement.

(iii) The azimuth (A_m) with respect to the astronomical meridian which is defined by the plane containing the astronomical zenith and the celestial pole.

Let the angle between the local vertical and the spheroid normal be ζ in the plane containing these two lines. This angle can be resolved into meridian and prime vertical components ξ and η as defined in section (2), the x_3' axis in the local laplacian trihedron coinciding with the spheroid normal. The non-coincidence of the vertical and the x_3' axis has to be taken into account when relating observed quantities to the laplacian trihedron whose orientation is completely defined by the local *geodetic* coordinates.

In fig (2) if z is the zenith angle with respect to the x_3' axis, z is given by

$$z = z_m + \xi \cos A + \eta \sin A \qquad \ldots\ldots(4).$$

If z_m is not an observed quantity but the differences in the orthometric elevation (Δh) and/or the geoid spheroid separation (ΔN) are known, it follows from fig (3) that

$$\cos z_1 = \frac{2R_m[N_2 - N_1 + h_2 - h_1] + (N_2 + h_2)^2 - (N_1 + h_1)^2 - \ell_1^2}{2R_m \ell_1 \left(1 + \frac{h_1}{R_m} + \frac{N_1}{R_m}\right)}$$

$$= \left\{\frac{\Delta h}{\ell_1} + \frac{\Delta N}{\ell_1} - \frac{\ell_1}{2R_m} + \frac{[h_1 + h_2 + N_1 + N_2](\Delta h + \Delta N)}{2R_m \ell_1}\right\} \left(1 + \frac{h_1}{R_m} + \frac{N_1}{R_m}\right)^{-1}$$

where R_m is the value of the quantity defined in section (2) for the line P_1P_2. As z can only be observed with a precision of 1 part in 10^6 under ideal conditions, the above expression can be reduced to

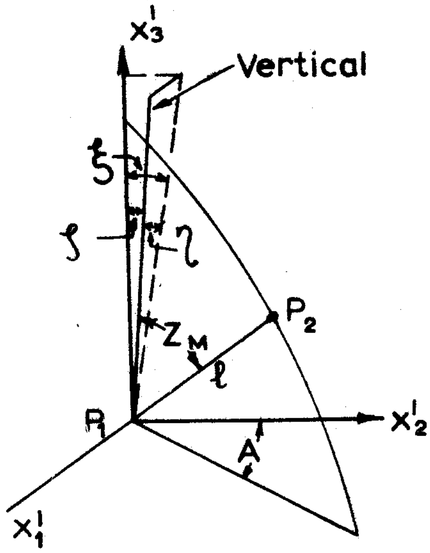

FIG. 2(a)

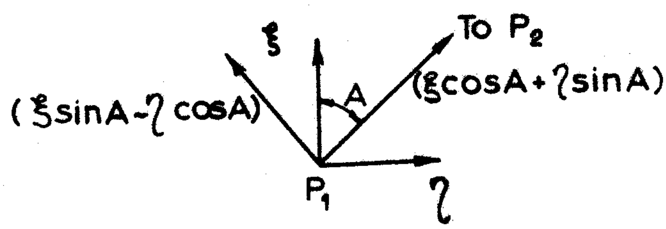

FIG. 2(b)

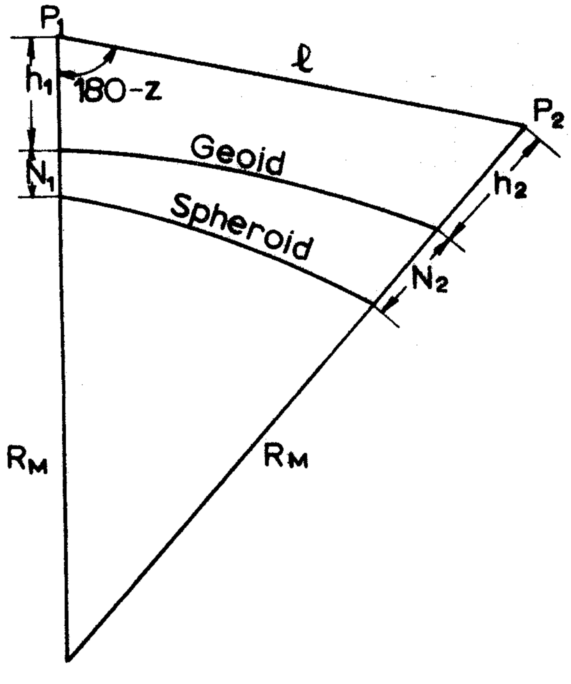

FIG. 3

$$\cos z_1 = \frac{\Delta h}{\ell_1} - \frac{\ell_1}{2R_m} + \frac{\Delta N}{\ell_1} + \frac{h_1 \ell_1}{2R_m^2} + o\{10^{-7}\} \quad \ldots\ldots\ldots(5).$$

All quantities in the above equation are defined in terms of the last two lines of section (2).

The observed astronomical meridian differs from the plane $x_2' x_3'$ for two reasons.

(i) The effect of the non-coincidence of the astronomical and geodetic zeniths which is expressed mathematically by the Laplace equation

$$A_1 = A_{m1} - [\lambda_{m1} - \lambda_1] \sin \phi_1 \quad \ldots\ldots\ldots(6),$$

where the subscript $_m$ refers to quantities determined astronomically. The combination of equations (3) and (6) gives

$$A_1 = A_{m1} - \eta_1 \tan \phi_1 \quad \ldots\ldots\ldots\ldots\ldots(7).$$

(ii) Non coincidence of the vertical and the x_3' axis makes the transference of angles measured with respect to the former, to the latter axis without distortion, conditional on the zenith angle being $\pi/2$. In all other circumstances, as can be seen from figures 2(a) and 2(b), the measured horizontal circle reading is too great by

$$[\xi \sin A - \eta \cos A] \cot z \; .$$

T hus the final expression for geodetic azimuth

(A), which is the angle between the x_2' axis and the projection of the line P_1P_2 in the $x_1'x_2'$ plane is

$$A_1 = A_{m1} - \eta_1 \tan \phi_1 - [\xi_1 \sin A_1 - \eta_1 \cos A_1]\cot z_1 \quad ..(8).$$

If, on the other hand, the azimuth of the line is not observed astronomically but deduced from a reverse bearing (A_r) and a measured angle α,

$$A = A_r + [\xi \sin A_r - \eta \cos A_r]\cot z_r + \alpha - [\xi \sin A - \eta \cos A]\cot z \quad ....(9).$$

No complications arise in the interpretation of ℓ which, if computed from conventional triangulation, is the geoidal distance. In such a case, equation (5) will be used with both h_1 and Δh put equal to zero.

The laplacian trihedron coordinates of P_2 are given by

$$\begin{aligned} x_1' &= \ell_1 \sin z_1 \sin A_1 \\ x_2' &= \ell_1 \sin z_1 \cos A_1 \\ x_3' &= \ell_1 \cos z_1 \end{aligned} \quad ....(10).$$

Conversely,

$$A_1 = \tan^{-1}\left(\frac{x_1'}{x_2'}\right) \quad ....(11),$$

$$z_1 = \tan^{-1}\left(\frac{x_2' \sec A_1}{x_3'}\right) = \tan^{-1}\left(\frac{x_1' \operatorname{cosec} A_1}{x_3'}\right) \quad ..(12)$$

and

$$\ell = x_1' \text{ cosec } A_1 \text{ cosec } z_1 = x_2' \sec A_1 \text{ cosec } z_1$$

$$= x_3' \sec z_1 \qquad \ldots\ldots..(13).$$

<u>Stage (2)</u> :- *Conversion of laplacian trihedron coordinates to differences in geocentric cartesian coordinates.*

Direct consideration of figures 1(a) to 1(c) shows that the coordinates x_j' (j=1,3) defined in equation (10) can be converted to differences in geocentric cartesian coordinates Δx_j (j=1,3) by the relation

$$\Delta X = R X' \qquad \ldots\ldots\ldots.(14),$$

where

$$\Delta X = \begin{vmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \end{vmatrix} \quad ; \quad X' = \begin{vmatrix} x_1' \\ x_2' \\ x_3' \end{vmatrix}$$

and

$$R = \begin{vmatrix} -\sin\lambda_1 & -\sin\phi_1\cos\lambda_1 & \cos\phi_1\cos\lambda_1 \\ \cos\lambda_1 & -\sin\phi_1\sin\lambda_1 & \cos\phi_1\sin\lambda_1 \\ 0 & \cos\phi_1 & \sin\phi_1 \end{vmatrix}$$

$$\ldots\ldots\ldots\ldots(15).$$

Thus in the case of a network where gravimetrically determined values of the geoid spheroid separation vector and orthometric elevations are available at every point, equations (1) to (3) define the evaluation of the geocentric cartesian coordinates of the origin P_1 at which astronomically determined ϕ_m, λ_m and A_m are available. The provisional geocentric coordinates of all other points P_i (i=2,n) are obtained from the observed quantities, using equations (4) to (15), by conversion in the first instance to local laplacian trihedron coordinates and then to differences in geocentric cartesian coordinates.

The values of geodetic coordinates for use in equation (15) at all points other than the origin can be computed from equation (1) by the following set of formulae which can be iterated if necessary.

$$\lambda_i = \tan^{-1}\left(\frac{x_{i2}}{x_{i1}}\right) \qquad .... (16).$$

$$\Delta\phi \doteq \frac{\ell_j}{\rho_j} \cos A_j \ ,$$

$$\Delta\nu = \frac{a\ e^2}{2[1 - e^2\sin^2\phi_j]^{3/2}} \sin 2\phi_j \ \ \Delta\phi$$

and

$$\nu_i = \nu_j + \frac{e^2}{2[1 - e^2]} \ell_j \sin 2\phi_j \cos A_j + o\{2 - 3 \text{ met.}\}$$

$$...........(17),$$

where

j = i - 1. If N_i is not available from gravimetry,

$$N_i = N_j - \ell_j[\xi_m \cos A_j + \eta_m \sin A_j\ ], \quad j=i-1 \ ....(18),$$

where ξ_m and η_m are the mean values of the deflections of the vertical at the two terminals,being taken equal to ξ_{i-1}, η_{i-1} in the first iteration. Then

$$\phi_i = \tan^{-1}\left[\frac{x_{i3}\left(1 + \frac{h_i + N_i}{\nu_i}\right)}{x_{i2}\text{cosec }\lambda_i\left(1 - e^2 + \frac{h_i + N_i}{\nu_i}\right)}\right]$$

$$= \tan^{-1}\left(\frac{x_{i3}}{x_{i2}\text{cosec }\lambda_i} f(\phi,h,N)\right)$$

$$= \tan^{-1}\left(\frac{x_{i3}}{x_{i1}\sec \lambda_i} f(\phi,h,N)\right) \text{..........(19)},$$

where

$$f(\phi,h,N) = 1 + e^2[1 - \frac{h_i + N_i}{\nu_i} + \left(\frac{h_i + N_i}{\nu_i}\right)^2]$$

$$+ e^4[1 - 2\frac{h_i + N_i}{\nu_i}] + e^6 + o\{10^{-7}\}\text{..(20)}.$$

(ii) *Cases where N, ξ and η are not available from gravimetry.*

In such instances the origin at P_1 will once again be completely defined as in (i), case (a). The following procedure will apply at subsequent points. The geocentric cartesian coordinates of P_2, based on values of the geoid spheroid separation vector at P_1 can be computed by means of equations (3) to (15). The problem in the general case can therefore be stated as follows. Given the geocentric cartesian coordinates x_{ij} (j=1,3) of the point P_i whose astronomically determined geographical coordinates are ϕ_{mi} and λ_{mi}, the latter can be converted to pseudo geocentric cartesian coordinates x_{mij} (j=1,3) by the relations

$$x_{mi1} = [\nu_{mi} + h_i] \cos\phi_{mi} \cos\lambda_{mi}$$
$$x_{mi2} = [\nu_{mi} + h_i] \cos\phi_{mi} \sin\lambda_{mi} \quad \ldots\ldots (21),$$
$$x_{mi3} = [\nu_{mi}(1 - e^2) + h_i] \sin\phi_{mi}$$

where ν_{mi} is the value of ν corresponding to the latitude ϕ_{mi}. The relation between these pseudo coordinates and the true geocentric cartesians (x_{ij}, j=1,3) of P_i can be conceptually expressed in one of two ways.

(a) The above coordinates refer to two points P_i and P' on the same geocentric cartesian reference system such that P' whose coordinates are (ϕ_{mi}, λ_{mi}, h_i, 0) has, for all practical purposes, the following displacements from $P_i(\phi_i, \lambda_i, h_i, N_i)$ in the local laplacian trihedron at P_i.

$$x'_1 = [\rho_i + h_i + N_i](\phi_{mi} - \phi_i) = [\rho_i + h_i + N_i]\xi_i$$
$$x'_2 = [\nu_i + h_i + N_i](\lambda_{mi} - \lambda_i)\cos\phi_i = [\nu_i + h_i + N_i]\eta_i \ldots (22)$$
$$x'_3 = -N_i$$

(b) If the true coordinates of P_i, referred to an origin whose coordinates are (0, 0, 0) and the pseudo coordinates represent the same point in space, the use of the latter system infers that its origin has coordinates (Δx_{ij}, j=1,3) on the former system, given by

$$\Delta x_{ij} = x_{mij} - x_{ij},\ j=1,3 \quad ....(23).$$

As equations (22) and (23) define the same set of changes and the relationship between the quantities is shown in figures 1(a) to 1(c),

$$\xi_i = \frac{1}{\rho_i + h_i + N_i}[-\Delta x_{i1}\cos\lambda_i\sin\phi_i - \Delta x_{i2}\sin\lambda_i\sin\phi_i + \Delta x_{i3}\cos\phi_i] \quad ....(24),$$

$$\eta_i = \frac{1}{\nu_i + h_i + N_i}[-\Delta x_{i1}\sin\lambda_i + \Delta x_{i2}\cos\lambda_i] \quad .....(25)$$

and

$$-N_i = \Delta x_{i1}\cos\lambda_i\cos\phi_i + \Delta x_{i2}\sin\lambda_i\cos\phi_i + \Delta x_{i3}\sin\phi_i \quad ..(26)$$

Only four significant figure accuracy is sought from equations (24) to (26) and hence no problem arises in evaluating ϕ_i and λ_i. Further computations are effected using equations (4) to (15). The values of ϕ and λ for use in equation (15) are obtained from equations (2) and (3).

4. THE ADJUSTMENT OF OBSERVATIONS

The procedures set out in section (3) enable provisional values of the geocentric cartesian coordinates to be computed for all points in the control network. This reference frame is an arbitrary one in that its centre, in practice, cannot be expected to coincide with the earth's centre of mass (geocentre). It should be noted that the adjustment of a world wide network of points could be used to obtain an estimate of the position of the geocentre. Ideally, if the earth's gravity field were completely defined, the combination of gravimetrically determined values of the geoid spheroid separation vector with astronomical observations over any reasonable extent of the earth's surface should provide a good estimate of the location of the geocentre. The orientation of the reference frame is further discussed in section (6).

The resulting quantities defined are the provisional coordinates x_{uij} (j=1,3) for the n points P_i (i=1,n) in the scheme, along with the associated provisional latitudes (ϕ_{ui}) and longitudes (λ_{ui}). These provisional values can be used to linearise the system by using the conventional technique (e.g., Schmid & Schmid, 1965, 27 et seq.). Two groups of quantities comprise the system:-

(a) the observed quantities $O_i (i=1,m)$, e.g., measured lengths, N values, observed directions, etc. ; and (b) the parameters $P_i (i=1,p)$, e.g., station coordinates, systematic errors in station orientation, N values, etc. .

These two sets of quantities are related by some mathematical model of the type

$$F(O,P) = 0 \qquad \text{.............} \; (27).$$

This equation is linearised by introducing approximate values (P_u) where necessary for the parameters P when

$$\frac{\partial F}{\partial O} V_O + \frac{\partial F}{\partial P} S_P + F(O,P_u) = 0 \qquad \text{...........}(28),$$

where V_O is the matrix of the residuals of the observations and

$$S_P = P - P_u \; .$$

If W_O is the matrix of weight coefficients of the residuals V_O and as the quantities in the matrix S_P are not normally distributed, the least squares condition to be satisfied is

$$\tfrac{1}{2} V_O^T W_O V_O = \text{minimum} \qquad \text{.........}(29).$$

In addition there is the possibility that certain condition equations have also to be simultaneously satisfied. For example, over a closed loop,

$$\sum_{i=1}^{n} \Delta x_{i1} = \sum_{i=1}^{n} \Delta x_{i2} = \sum_{i=1}^{n} \Delta x_{i3} = 0 \ .$$

Let this set of q conditions be represented by

$$G(P) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (30).$$

The introduction of provisional values for the parameters gives

$$G(P_u) + \left(\frac{\partial G}{\partial P}\right) S_P = 0 \quad .$$

The least squares condition becomes

$$\Phi = \tfrac{1}{2} V_O^T W_O V_O - L^T \left[G(P_u) + \left(\frac{\partial G}{\partial P}\right) S_P\right] = \text{minimum} \quad \ldots.(31),$$

where L is the matrix of lagrangian multipliers. Equations (28) and (31) are combined by the substitution for V_O in the latter equation when

$$\Phi = \tfrac{1}{2} (A\ S_P + K)^T W_O (A\ S_P + K) - L^T (A_C\ S_P + K_C) = \text{minimum},$$

where

$$A = -\frac{\left(\frac{\partial F}{\partial P}\right)}{\left(\frac{\partial F}{\partial O}\right)} \ ; \quad K = -\frac{F(O, P_u)}{\left(\frac{\partial F}{\partial O}\right)} \ ; \quad A_C = \left(\frac{\partial G}{\partial P}\right); \ K_C = G(P_u)$$

$$\ldots\ldots\ldots\ldots\ldots (32).$$

Such a condition is satisfied by assigning values for the corrections S_P to the provisional parameters according to

$$\frac{\partial \Phi}{\partial S_P} = 0 \ ;$$

i.e.,

$$A^T W_O A S_P + A^T W_O K - A_C^T L = 0 \quad \ldots\ldots\ldots (33),$$

whence substitution for S_P from equation (33) in equation (30) gives

$$S_P = (A^T W_O A)^{-1} (A_C^T L - A^T W_O K) \quad \ldots\ldots (34)$$

and

$$L = [A_C (A^T W_O A)^{-1} A_C^T]^{-1} (A_C (A^T W_O A)^{-1} A^T W_O K - K_C) \quad \ldots\ldots (35).$$

The values of the corrections to the assumed parameters (S_P) are obtained from the values of L by the use of equation (34).

5. THE OBSERVATION EQUATIONS

The changes Δx_{ij} (j=1,3) in the geocentric cartesian coordinates over the line $P_i P_{i+1}$ are equivalent to changes x'_j (j=1,3) in the local laplacian trihedron coordinates at P_i, given from a study of figures 1(a) to 1(c) by the following equations.

$$x_1' = -\ \Delta x_{i1}\sin\lambda_i + \Delta x_{i2}\cos\lambda_i$$

$$x_2' = -\ \Delta x_{i1}\cos\lambda_i\sin\phi_i - \Delta x_{i2}\sin\lambda_i\sin\phi_i + \Delta x_{i3}\cos\phi_i \quad ..(36)$$

$$x_3' = \Delta x_{i1}\cos\lambda_i\cos\phi_i + \Delta x_{i2}\sin\lambda_i\cos\phi_i + \Delta x_{i3}\sin\phi_i$$

The subscripts used in the following sub-sections are as set out in the last sentence of section (2).

(i) *The length observation equation.*

The length ℓ_1 is related to the geocentric cartesian coordinates of the terminals P_1 and P_2 by the relation

$$\ell_1^2 = \sum_{j=1}^{3}[x_{2j} - x_{1j}]^2 .$$

The change $d\ell_1$ in ℓ_1 produced by changes dx_{1j} and dx_{2j} $(j=1,3)$ in the coordinates of P_1 and P_2 is related to the latter by the relation

$$\ell_1 d\ell_1 = \sum_{j=1}^{3}[x_{2j} - x_{1j}](dx_{2j} - dx_{1j}).$$

If ℓ_{c1} is the length of the line P_1P_2 as computed from the provisional coordinates of P_1 and P_2, ℓ_1 the observed length of the line and $v_{\ell 1}$ the error in the observed length,

$$v_{\ell 1} = \ell_{c1} - \ell_1 + d\ell_1$$

$$= \ell_{c1} - \ell_1 + \sum_{j=1}^{3}\frac{x_{2j} - x_{1j}}{\ell_1}[dx_{2j} - dx_{1j}] \quad ....(37).$$

All quantities requiring evaluation in equation (37) are computed from the provisional coordinates, ℓ_c being computed with the same precision as the observed quantity ℓ.

(ii) *The azimuth observation equation.*

The azimuth of a line can be computed by the use of equation (10), the relationship between the differences in provisional geocentric coordinates and the local laplacian trihedron coordinates being given by equation (36).

$$\tan A_1 = \frac{x_1'}{x_2'} .$$

Changes in A_1 produced by changes in the provisional coordinates of the terminal points are related by the following set of equations.

$$\sec^2 A_1 \, dA_1 = \frac{dx_1'}{x_2'} - \frac{x_1'}{(x_2')^2} dx_2' .$$

Consideration of equation (36) gives

$$dx_1' = - d\Delta x_{11} \sin \lambda_1 + d\Delta x_{12} \cos \lambda_1$$

and

$$dx_2' = - d\Delta x_{11} \sin \phi_1 \cos \lambda_1 - d\Delta x_{12} \sin \phi_1 \sin \lambda_1 + d\Delta x_{13} \cos\phi_1$$

where

$$d_\Delta x_{1j} = dx_{2j} - dx_{1j} , \; j=1,3 \quad \ldots\ldots\ldots\ldots (38).$$

The combination of the above equations and some manipulation gives

$$dA_1 = \sin A_1 \cos A_1 [- d\Delta x_{11}(\frac{\sin \lambda_1}{x_1'} - \frac{\sin \phi_1 \cos \lambda_1}{x_2'})$$

$$+ d\Delta x_{12}(\frac{\cos \lambda_1}{x_1'} + \frac{\sin \phi_1 \sin \lambda_1}{x_2'}) - d\Delta x_{13}\frac{\cos \phi_1}{x_2'}] \quad ..(39).$$

The observation equation for azimuth is

$$A_1 + s_{A1} + v_{A1} = A_{c1} + dA_1 \; ,$$

where A_1 is the observed azimuth of the line P_1P_2 deduced from the unadjusted observations by the use of either equation (8) and/or equation (9) and s_{A1} is the correction for unknown station orientation error. This is further discussed in section (6). v_{A1} is the local observational error which is part of a normally distributed population. Thus,

$$v_{A1} = [A_{c1} - A_1] - s_{A1} + \sin A_1 \cos A_1 [d\Delta x_{11}(\frac{\sin \phi_1 \cos \lambda_1}{x_2'}$$

$$- \frac{\sin \lambda_1}{x_1'}) + d\Delta x_{12}(\frac{\cos \lambda_1}{x_1'} + \frac{\sin \phi_1 \sin \lambda_1}{x_2'})$$

$$- d\Delta x_{13}\frac{\cos \phi_1}{x_2'}] \quad ....(40).$$

The value of A_{c1} is computed to the same order of accuracy as A_1 from equations (36) and (10) using the provisional coordinates. Other values can be computed to a lower order of precision provided that the provisional coordinates have been adequately established.

(iii) *The zenith angle observation equation.*

The zenith angle z_1 is given from equation (10) by

$$\tan z_1 = \frac{x'_2 \sec A_1}{x'_3} = \frac{[(x'_1)^2 + (x'_2)^2]^{\frac{1}{2}}}{x'_3} .$$

Differentiation and rearrangement of terms gives

$$dz_1 = \cos^2 z_1 \left(\frac{x'_1\, dx'_1 + x'_2\, dx'_2}{(x'_1)^2 + (x'_2)^2} \tan z_1 - \tan z_1 \frac{dx'_3}{x'_3} \right).$$

The changes in dx'_j (j=1,3) are converted into changes in dx_{1j}, dx_{2j} in x_{1j}, x_{2j} (j=1,3) by the use of equations (36) and (38) when

$$\begin{aligned} dz_1 = \cos z_1 \sin z_1 \Big(&\frac{1}{(x'_1)^2 + (x'_2)^2} [x'_1(- d\Delta x_{11} \sin \lambda_1 \\ &+ d\Delta x_{12} \cos \lambda_1) + x'_2(- d\Delta x_{11} \sin \phi_1 \cos \lambda_1 \\ &- d\Delta x_{12} \sin \phi_1 \sin \lambda_1 + d\Delta x_{13} \cos \phi_1)] \\ &- \frac{1}{x'_3} [d\Delta x_{11} \cos \phi_1 \cos \lambda_1 + d\Delta x_{12} \cos \phi_1 \sin \lambda_1 \\ &+ d\Delta x_{13} \sin \phi_1] \Big) . \end{aligned}$$

The observation equation for the zenith angle is

$$z_1 + s_{z1} + v_{z1} = z_{c1} + dz_1 ,$$

where z_1 is the observed value of the zenith angle and s_{z1} is the position dependent error in z which is discussed in section (6). Thus

$$v_{z1} = z_{c1} - z_1 - s_{z1} + \sin z_1 \cos z_1 [-d\Delta x_{11}(\frac{\cos \phi_1 \cos \lambda_1}{x_3'}$$

$$+ \frac{x_1' \sin \lambda_1 + x_2' \sin \phi_1 \cos \lambda_1}{(x_1')^2 + (x_2')^2}) - d\Delta x_{12}(\frac{\cos \phi_1 \sin \lambda_1}{x_3'}$$

$$- \frac{x_1' \cos \lambda_1 - x_2' \sin \phi_1 \sin \lambda_1}{(x_1')^2 + (x_2')^2}) - d\Delta x_{13}(\frac{\sin \phi_1}{x_3'}$$

$$- \frac{x_2' \cos \phi_1}{(x_1')^2 + (x_2')^2})] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (41),$$

where $x_j'(j=1,3)$ are given by equation (36) and $d\Delta x_{1j}(j=1,3)$ by equation (38).

6. THE NATURE OF THE SOLUTION

The nature of the solution is best investigated after estimates are made of the precision of the observed quantities. Observations are made at the origin for ϕ_{m1}, λ_{m1}, A_{m1}, h_{m1}, ξ_1, η_1 and N_1. The estimates of error in ϕ_m and λ_m for observations of geodetic precision are well established as being in the range ± 0.3 to ± 0.5

sec. If the geoid spheroid separation vector is defined gravimetrically at the origin, the error estimates e_{N_1}, e_{ξ_1} and e_{η_1} in N_1, ξ_1 and η_1 are not, in the strictest sense, independent of each other. e_{N_1} is given by the equation

$$e_{N_1} = \frac{1}{4\pi\gamma R}\iint e_{\Delta g}\ f(\psi)\ dS \quad \text{.................(42)},$$

where $e_{\Delta g}$ is the error in the gravity anomaly representing the element of surface area dS at an angular distance ψ from the computation point. The other symbols in this equation, which is Stokes' integral, are defined in section (2) with the exception of $f(\psi)$ which is Stokes' function (Heiskanen & Moritz, 1967, 94). Let the error (e_{N_i}) in the value of N at any point P_i be represented by an equation of the form

$$e_{N_i} = e_{N_1} + s_{N_i} + v_{N_i} \quad \text{..........(43)},$$

where v_{N_i} is a normally distributed quantity which will arise from errors in sampling the near zone gravity field and s_{N_i} is the position dependent (or systematic) error given by

$$s_N = s_N(\phi,\lambda).$$

The error s_{N_i} will give rise to position dependent errors s_{ξ_i} and s_{η_i} in the deflections of the vertical given by

$$s_{\xi_i} = - \frac{1}{R}\left(\frac{\partial s_N}{\partial \phi}\right)_i$$

$$s_{\eta_i} = - \frac{1}{R \cos \phi_i}\left(\frac{\partial s_N}{\partial \lambda}\right)_i \quad \ldots\ldots\ldots\ldots\ldots(44).$$

These equations will also apply at the origin. Any reasonable two dimensional series will afford an adequate mathematical representation of s_N. In Australia, the comparison of astrogeodetic deflections of the vertical with gravimetric values *after* the former have been corrected for the error in the orientation of the Australian Geodetic Datum, will provide an estimate of the coefficients of such a series.

Recent investigations indicate that neither e_{ξ_1} nor e_{η_1}, allowing for current uncertainties in the definition of the global gravity field arelikely to exceed 0.5 sec if established in such a manner (Mather, 1969, 29). The effect of $e_{\xi 1}$ and $e_{\eta 1}$ on the errors e_{ϕ_1} and e_{λ_1} in ϕ_1 and λ_1 can be conservatively estimated as being of the order of ±0.7 sec. If e_{h_1} is the error in the orthometric elevation and $e_{\nu 1}$ that in ν_1 due to e_{ϕ_1}, the error e_{x11} is given by

$$e_{x_{11}} = \pm x_{11}\left[\left(\frac{e_{\nu_1}}{\nu_1 + h_1 + N_1} + \tan\phi_1 e_{\phi_1}\right)^2 + \frac{(e_{h_1})^2 + (e_{N_1})^2}{[\nu_1 + h_1 + N_1]^2} + (\tan\lambda_1 e_{\lambda_1})^2\right]^{\frac{1}{2}} \qquad \ldots\ldots\ldots(45).$$

Similar expressions hold for the errors $e_{x_{1j}}$ in $x_{1j}(j=2,3)$ and are based on the errors e_{ϕ_1}, e_{λ_1} and e_{N_1} being non-correlated. This is not strictly so but, from a statistical point of view, such an assumption may be accepted as being valid for the present study.

The uncertainties in the values adopted for ϕ_1 and λ_1 give rise to errors of 3-4 p.p.m. in each of $x_{1j}(j=1,3)$. This figure would not be materially affected by errors in h_1 and N_1 if these, on combination, do not exceed 12 metres. The resulting uncertainty of approximately 20-25 metres in the position of the origin can be interpreted as being an error in the location of the centre of the cartesian system in relation to the true geocentre. If, on the other hand, an ideal gravity field were available and the deflections of the vertical at the origin were computed from an analysis of comparisons over a region, the uncertainty of location of the origin can be reduced by a factor of ten. In this context errors arising from coordinates adopted at the origin will not be considered further in this study.

The remaining considerations are those of point to point computation with the coordinates of the origin $P_1(x_{1j}, j=1,3)$ held fixed. Such computations require, in addition to quantities determined either astronomically and/or from surface gravity, the determination of A, z and ℓ. Both A and ℓ can be determined with a measuring accuracy of 2-3 p.p.m. An equivalent accuracy in z would require that e_z not exceed 0.6 sec. This accuracy cannot be obtained by the measurement of vertical angles over geodetic distances. If z is deduced from equation (5),

$$-\sin z\, e_z = \pm\left[\left(\frac{e_{\Delta h}}{\ell}\right)^2 + \left|\left(\frac{\Delta h + \Delta N}{\ell^2} - \frac{1}{2R_m}\right)v_\ell\right|^2 + \left(\frac{e_{\Delta N}}{\ell}\right)^2\right]^{\frac{1}{2}} \quad ..(46),$$

where $e_{\Delta h}$ and $e_{\Delta N}$ are errors in Δh and ΔN, all other quantities being defined in section (2). A study of equation (46) for lines of length 50 km. shows that both $e_{\Delta h}$ and $e_{\Delta N}$ should be of the order of ±15 cm if the accuracy of the deduced value of z is on par with those of A and ℓ. This can be achieved if the orthometric elevation is established by conventional levelling and ΔN is computed from astronomical observations using equations (24) to (26). If N is computed gravimetrically, e_N is a correlated quantity given by equation (43) and the error in z due to the differential value of s_N has to be taken into account. Australian studies (ibid, fig(12)) indicate that over a normal geodetic line in a region where the close gravity field is well represented, the change in e_N (de_N) can be expected to be of the order of 50 cm. If the resulting error s_Z in z is separated when setting up the observation equation as shown in equation (41), the resulting contribution to v_z is normally distributed and of the same magnitude as $e_{\Delta h}$.

If A is computed from astronomical observations and gravimetric deflections using equation (8), a study of equation (40) shows that the term s_A will account for the contribution of the position dependent term s_η defined in equation (44), as the errors in the deflections contribute negligibly to s_A through the term multiplied by cot z. Thus, for the line P_iP_{i+1}, it can be seen from equations (5), (8), (40), (41), (43) and (44) that

$$s_{z_i} = - \frac{s_{N_{i+1}} - s_{N_i}}{\ell_i} \qquad \ldots\ldots\ldots\ldots\ldots \text{(47)}$$

$$s_{A_i} = \frac{1}{R \cos \phi_i} \left(\frac{\partial s_N}{\partial \lambda}\right)_i \tan \phi_i \qquad \ldots\ldots\ldots \text{(48).}$$

It should be noted that equation (47) assumes that no significant systematic error exists in the orthometric elevations. This can however be assumed to be an order smaller than that in N.

The values of s_{z_i} and s_{A_i} (i=1,n) can then be analysed by a two dimensional series of the type

$$s_N(\phi,\lambda) = \sum_{i=0}^{a} C_i \cos[\pi\, i\Delta\phi] + \sum_{i=a+1}^{2a} C_i \sin[\pi(i-a)\Delta\phi] + \sum_{i=2a+1}^{3a} C_i \cos[\pi(i-2a)\Delta\lambda] + \sum_{i=3a+1}^{4a} C_i \sin[\pi(i-3a)\Delta\lambda \quad \ldots\ldots\ldots\ldots\ldots\ldots (49)$$

and

$$\left(\frac{\partial s_N}{\partial\lambda}\right)(\phi,\lambda) = -\sum_{i=2a+1}^{3a} C_i (i-2a)\pi \sin[\pi(i-2a)\Delta\lambda] + \sum_{i=3a+1}^{4a} C_i (i-3a)\pi \cos[\pi(i-3a)\Delta\lambda] \quad \ldots.(50),$$

where C_i (i=0,4a) are the associated coefficients and a will be limited by the storage available in the computer being used. $\Delta\phi$ and $\Delta\lambda$ in equations (49) and (50) are the differences of geographical coordinates with respect to the origin. The properties of this type of series have been studied in (Mather, 1967, 132 et seq.). If the coefficient C_0 is set equal to zero it is assumed that

$$M\{s_N\} = 0$$

over the region.

The trigonometrical functions with low values in the term containing i (i.e., i, i-a, etc.) are similar in nature to low order harmonics and represent changes with longer period. The evaluation of the C_i's using a least squares technique has been dealt with in the reference quoted. It must be emphasised that this method of analysis fails under conditions of extrapolation and is most satisfactory when the values of s_z and s_A are evaluated at points which are evenly distributed over the area being studied.

7. CONCLUSIONS

A normal horizontal control network can be used for the determination of position on a geocentric cartesian system provided the geoid spheroid separation vector and the orthometric elevation are known at every point. Any positional error which occurs in the definition of the coordinates of the origin which, at present, is unlikely to exceed 20 metres, can be interpreted as an error in the location of the cartesian system. Such a consideration obviously requires a modified interpretation in the case of a world wide adjustment.

The zenith angle as determined by trigonometrical levelling is of inadequate precision to warrant inclusion in the adjustment. It can instead be computed from the orthometric elevation and the geoid spheroid separation using equation (5). An analysis of equation (46) shows that the orthometric elevation of points in the scheme has to be established to ±15 cm if the computed zenith angle is to have the same precision as the azimuth.

The geoid spheroid separation vector can be evaluated either gravimetrically or by the combination of astronomical observations and the results of horizontal surveys using equations (24) to (26). *In the latter case* it is preferable that the values of A, ϕ and λ determined astronomically be established at every point in the scheme. If this is not possible sections between adjacent astronomical stations will have to be computed by projecting the observed quantities onto the line joining the terminals. Alternatively, a gravimetric solution, if available, could be used to define the geoid spheroid separation vector. Neither of these methods can be considered satisfactory if the nature of the propagation of error is to be studied.

If the values of N, ξ and η are determined gravimetrically under conditions where the inner zone gravity field around every point in the scheme has been adequately sampled, the solutions obtained can be used to study the nature of the position dependent (or systematic) errors in N. Such errors can be due to one of the following reasons.

(i) Errors arising from the representation of the gravity field of distant areas by the solutions obtained by a combination of satellite data and gravimetry.

(ii) Errors due to the omission of the indirect effect in cases where the gravimetric solution is merely the free air geoid.

(iii) Errors in estimating values for unsampled local fields.

The systematic error (s_N) in N is defined by equation (43) and the resulting error in the deflections is given by equation (44). These quantities are related to the terms s_A and s_z , obtained as a by-product of the adjustment of the network, through equations (47) and (48). The analysis of these position dependent quantities using a two dimensional trigonometrical series will yield, through equations (49) and (50), estimates of low and high order area harmonics in s_N, the former representing variations which are more likely to be due to cause (i) above while the latter will be more dependent on cause (iii).

The indirect effect, which is not expected to have a variation in excess of 3 metres over Australia can also be studied using this form of analysis on a comparative basis. In such an investigation, solutions in which the indirect effect has been computed should have significantly smaller s_N values provided the indirect effect is of magnitude comparable with those of other sources of systematic error. This provides a means of verifying the adequacy of the formulae used in computing the indirect effect.

Thus the three dimensional adjustment of a network of control points at which orthometric elevations and values of the geoid spheroid separation vector have been established, afford not only a means of verifying expressions used to evaluate the indirect effect but also a technique for assessing the adequacy with which distant gravity fields are represented by combined solutions from satellite data and surface gravimetry.

February 1969

Sydney

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ACKNOWLEDGEMENTS

Mr. F.L.Clarke verified the derivation of formulae set out in this paper and suggested certain improvements.

The text also incorporates clarification of points raised by Professor P.V. Angus-Leppan who was kind enough to critically examine a draft of the paper.

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BIOGRAPHICAL NOTES

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