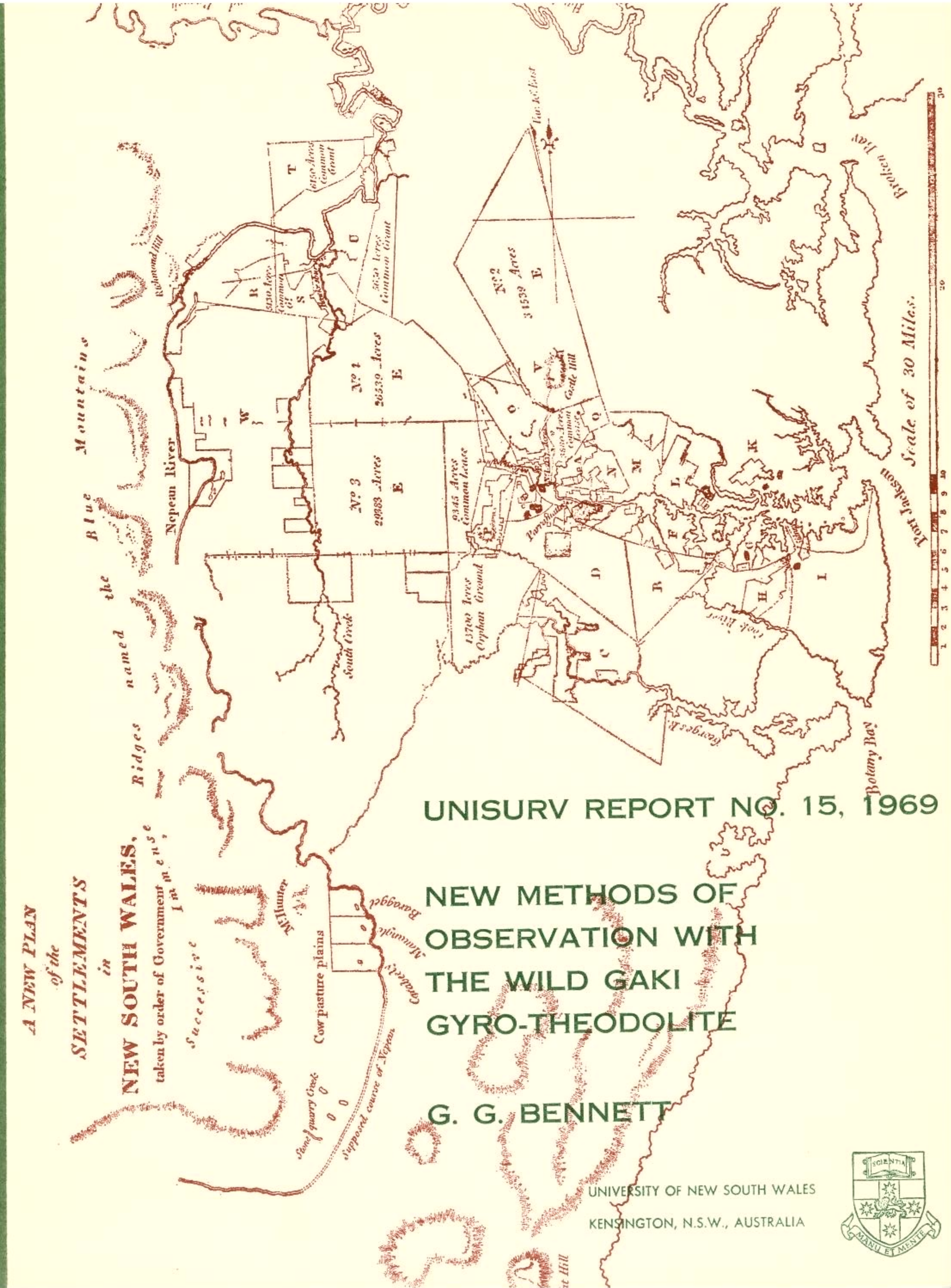

UNISURV REPORT NO. 15, 1969

NEW METHODS OF OBSERVATION WITH THE WILD GAKI GYRO-THEODOLITE

G. G. BENNETT

UNIVERSITY OF NEW SOUTH WALES
KENSINGTON, N.S.W., AUSTRALIA

Reference to Districts.

A Northern Boundaries
B Liberty Plains
C Banks Town
D Parramatta
EEEE Ground reserved
for Govt. purposes
F Concord
G Petersham
H Bulanaming
I Sydney
K Hunters Hills
L Eastern Farms
M Field of Mars
N Ponds
O Toongabbey
P Prospect
Q
R Richmond Hill
S Green Hills
T Phillip
U Nelson
V Castle Hill
W Evan

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UNISURV REPORT NO. 15.

NEW METHODS OF OBSERVATION WITH THE WILD GAKI GYRO-THEODOLITE.

G.G. BENNETT.

Department of Surveying - University of New South Wales

Received March, 1969.

SUMMARY

In early 1967 the Mine Manager's Association of the Broken Hill group of mines donated a Wild GAK 1 gyro-attachment and T16 theodolite to the University of New South Wales. At this time Professor G.B. Lauf of the University of the Witwatersrand visited this University and gave a series of lectures on his experiences with this and other types of gyro-theodolite. It was apparent from Professor Lauf's lectures that the gyro-theodolite was an instrument of great potential value to the mining surveyor. Since then a great deal of experience has been gained in the handling of the instrument and in the interpretation of the results of observations. This report gives the theory and practice of methods which have been developed for the "Transit" and the "Turning Point" methods in order to reduce the total observation time. Worked examples taken from practice illustrate the techniques.

INTRODUCTION

After an initial trial and training period a gyro-theodolite survey was made at the Huntley Colliery, Dapto, on the Sydney coalfield. Access to this mine is by way of two adits into the face of a steep escarpment; a mode of access which seldom presents problems of azimuth transfer. The mine has been worked for many years so that the working faces are remote from the adits, resulting in difficulties in maintaining adequate ventilation. To overcome this latter problem it has been proposed to sink a vertical ventilation shaft from the surface to near the present working area. The present survey connections are in the form of a long closed traverse (about 50 lines) between the adits with a short unclosed spur traverse leading off the closed traverse to the proposed shaft site. The circumstances are further complicated by the fact that many of the traverse lines are short (some are about 50 ft. long). A gyro-theodolite survey was requested for the purpose of strengthening the bearings in the closed traverse and checking the bearings in the spur traverse.

For a relative transfer of azimuth from the surface to underground workings by means of the gyro-theodolite the following sequence of observations is usually made:-

Surface line AB, Underground line PQ.

Sequence A-B, P-Q, Q-P, B-A.

This bracketing procedure provides a check on the stability of the instrument. In addition a regular practice has been made of calibrating the instrument at the University on a reference line of known azimuth before and after each gyro-theodolite survey. If A_{AB} is the azimuth of the line AB etc. and G_{AB} is the measured gyro-azimuth of the same line then

$$A_{PQ} = A_{AB} + \frac{G_{PQ} + G_{QP} \pm 180°}{2} - \frac{G_{AB} + G_{BA} \pm 180°}{2}$$

Applying the law of propagation of variances to this expression we find that the standard deviation of the transferred azimuth A_{PQ} is equal to the standard deviation of a single measured gyro-azimuth provided that the azimuth of the surface line A_{AB} is considered error free and all observations have equal variance and are correlation free.

The results of the gyro-theodolite survey were satisfactory except for one disturbing feature. The difference between the forward and reverse observation on the reference line was large when compared with the differences between observations made at both ends of three other lines, namely 39" as compared with 5", 21", 24". The oscillation graphs of all observations appeared to be normal and no external causes were encountered which could explain this singularly large discrepancy. The reference line was reobserved the next day and the results of the four determinations are as follows:-

Date	Line	Azimuth		v
3/3/67	A-B	109° 50' 16"	Δ=39"	-25"
	B-A	289° 49' 37"		+14"
4/3/67	A-B	109° 49' 44"	Δ=4"	+7"
	B-A	289° 49' 48"		+3"
Mean	A-B	109° 49' 51"		

On later examination it was found that 6" of this 39" difference could be accounted for by applying a more rigorous reduction formula to the observations i.e. instead of taking the simple average of the Schuler Means a weighted average was used as described by this author (1968). The reason for considering the difference of 39" large was that with our experience before this time, differences of this magnitude had never occurred. If the standard deviation of a single gyro determination is $\pm$ s then the standard deviation of the difference between forward and backward azimuths will be $\pm \sqrt{2}s$ and the range of differences at the 5% significance level will be $\pm 1.960 \sqrt{2}s = \pm 2.77s$. If the difference of

33" is attributed to an extreme value at the 5% significance level then the population standard deviation would be ± 12" which is corroborated by the results from experiments and tests which were carried out later and which are described in this report. If an absolute azimuth is required then the uncertainty in the determination of the E value must be included. If the uncertainty in the value of E is of the same magnitude as that of a single gyro determination then the standard deviation of an absolute azimuth would be ±17" which agrees with Schwendener (1966) who reports that "tests by various civilian and military authorities resulted in an absolute mean square error of a measured azimuth of ± 15" to ± 30"."

The method chosen for all of these previous determinations was the turning point method using eight reversal points - a procedure recommended by Professor Lauf. There is little to choose between the turning point and transit methods and their main characteristics are tabulated below.

	Turning Point	Transit
Accuracy of preliminary orientation	± 1°	± 10'
Instrument constants	E	E,c.
Calculations	(Un)weighted Schuler Means	Small Calculation of ΔN
Extra equipment	Extended tangent screw	Split hand timer

For the turning point method a number of independent circle readings are obtained, and it is usual to calculate progressive Schuler Means immediately to check the work. However, with the transit method the observation requires the horizontal circle to be set, close to the reading which corresponds to gyro indicated north and a mistake in this setting may go unnoticed. There is no doubt that the transit method is less tiring for the observer, more so for the reason that the observer must concentrate continuously rather than for the reason that the turning point period is longer than that of the transit method. The transit method has more appeal for those surveyors who have an infrequent need to use the gyro-theodolite, because less skill is required in timing the gyro mark though the vee slot than in keeping the gyro mark centrally placed by means of the tangent screw in the vee slot in the turning point method. The constant manipulation of the instrument in the latter method may also impart small irregularities to the gyroscope's motion: a fact which some manufacturers have recognised by installing automatic following up devices.

From a practical view point we wish to choose an observation technique which is certain, economical, convenient and which will produce acceptable accuracy. Economy of time is certainly not a characteristic possessed by either the turning point or transit methods because a time interval of at least one period must elapse before a single determination of the direction of the meridian can be obtained. It was this aspect which prompted an investigation into the development of techniques which would give a greater number of observations per period and thus a reduction in observation time. Also to guard against uncertainty in the standards by which future observations would be judged it was thought necessary to make a series of observations using different methods on a reference line.

<u>The Modified Transit Method</u>

The transit method as originally devised by Schwendener (1964) and further described by Strasser and

Schwendener (1966) consists of timing the gyro-mark across a central index mark (vee slot) on the auxiliary scale of the autocollimator. The correction to the approximate North setting of the theodolite is given by

$$\Delta N = c \, . \, a \, . \, \Delta t$$

Where ΔN is the correction to the North setting to give the direction of gyro indicated North.

c is an instrument constant

a is the amplitude of oscillation as determined from readings made on the auxiliary scale.

Δt is the second difference of times of transit of the gyro-mark through the vee slot.

The constant c may be evaluated from the relationship

$$c = m \frac{\pi}{2} \frac{T_u^2}{T_D^3}$$

where m is the amplitude of oscillation as determined from readings made on the auxiliary scale,

T_u is the period for the turning point method

T_D is the period for the transit method,

or c may be determined empirically from a number of observations made at settings on either side of the meridian.

Instead of making one timing observation as the gyro-mark passes through the central vee slot, multiple observations can be made by timing the gyro-mark on graduation lines on either side of zero on the auxiliary scale. It is not possible to make a sensitive time estimation when the gyro-mark is centered over one of the graduation lines but if the coincidence of the leading edge with a scale line is observed, a good time estimate of this event can be made. Fig.1 (see appendix A) shows the scheme of observation. It can be proved (see Appendix A) that the first term is dominant in a series expression for ΔN i.e.

$$\Delta N = c\ a\ (1 - \frac{n^2}{a^2})^{\frac{1}{2}}\ \Delta\ t \qquad (1)$$

where n is the number of the graduation line

$$\Delta t = (t_4 - t_1) - (t_6 - t_3)$$

$$\Delta t = (t_3 - t_2) - (t_5 - t_4) \text{ etc.}$$

$$\Delta N = \left(\frac{T_u}{T_D}\right)^2 \Delta y$$

and c, a, have the same meaning as before.

It may be noted that equation (1) is independent of width of the gyro-mark and is a general equation in that it embraces the normal transit method i.e. $n = 0$, $t_1 = t_2$, $t_3 = t_4$, $t_5 = t_6$ etc.

If each timing observation is recorded as though it were observed on the previous graduation line, then for a gyro-mark of normal width (for the GAK1, approximately $2\frac{1}{2}$ divisions) there would be a reduction in the size of the neglected terms in the series. Moreover if the gyro-mark was an integral number of divisions wide, then observations could be simulated as though the mark had zero width. The error caused by the neglect of terms other than the first may be reduced by

(1) Observing only on those scale lines which are close to the zero of the auxiliary scale.

(2) Using a large amplitude.

(3) Keeping ΔN small.

(4) Using a modified auxiliary scale with multiple vee slots. A suggested arrangement is shown in Fig. 2.

It should be noted that the above error can be eliminated by arranging ΔN to be of opposite sign when making reciprocal observations on a survey line.

In Appendix B are given the results of calculation of examples for some selected values of Δy and "a". The proportionality factor "c" is not sensitive to latitude changes (see Strasser and Schwendener (1966)) and therefore these examples illustrate in general the magnitude of errors caused by using the approximate relationship for ΔN. In practice the quantity $K = c\ (a^2 - n^2)^{\frac{1}{2}}$ has been precalculated and tabulated on a digital computer so that field calculations can be kept to a minimum. Then $\Delta N = K. \Delta t$.

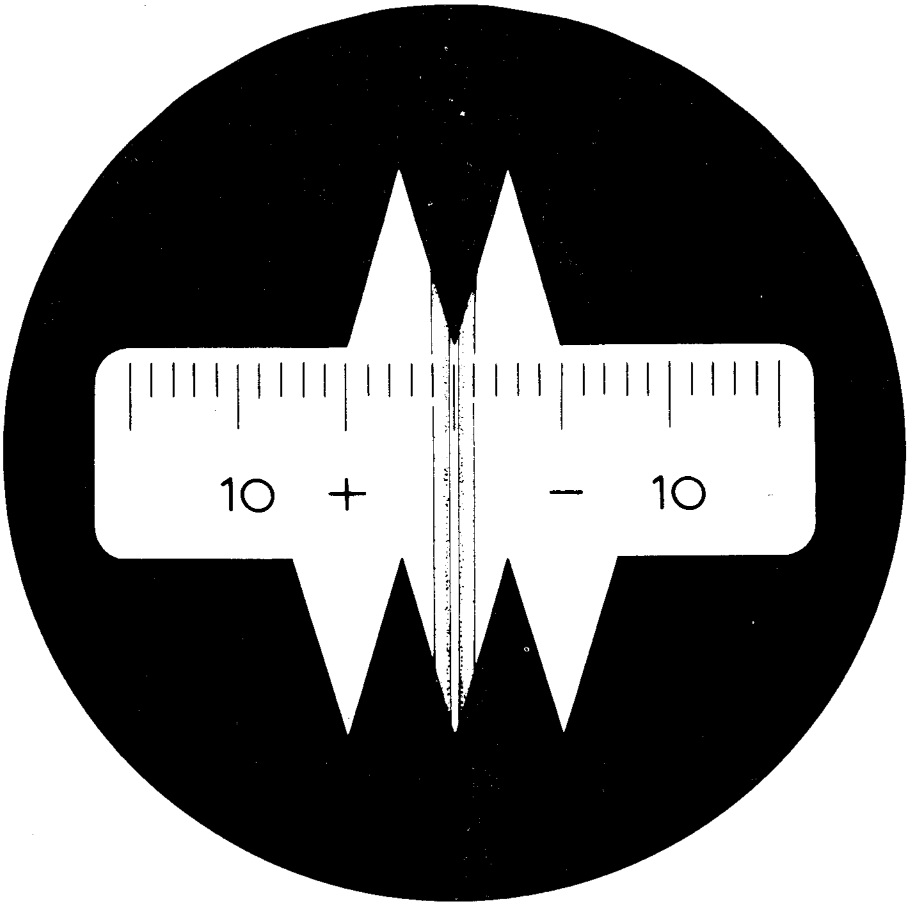

FIG. 2:- MODIFIED AUXILIARY SCALE

The most significant feature of the modified transit method from the user's viewpoint is the increase in the number of timing observations. No difficulty has been found in observing and recording with the GAK 1 where the observations are spaced at intervals of about 5^s in Sydney (ϕ = S34°). The amplitude readings made at the start and finish of the observations provide a useful preliminary check on the calculation of ΔN. It has been found that the value of ΔN calculated purely from amplitude readings is correct to within 1 to 2' (Schwendener (1966) refers to this technique as the "Amplitude Method" under "Quick Methods"). Before moving from a station it is useful to check the consistency of the observations. If the time differences (Column Δ in Appendix B) are calculated then a consistency check can be made by examining the values of Δ in alternate columns to see if they are in reasonable agreement. An example taken from practice is given at the end of Appendix B.

The Modified Turning Point Method.

If multiple observations are to be made with this method then the optimum circumstance will occur in the vicinity of the turning point. The observation consists of recording timed horizontal circle readings: a technique which is more suited to optical scale rather than micrometer theodolites. It may be proved (see Appendix C) that the correction to the observed horizontal circle reading required to bring it to the turning point is given by

$$\Delta y = \frac{B\ (-1)^{i-1}(2\pi)^2\ \Delta\tau^2}{2!} - \frac{B\ (-1)^{i-1}(2\pi)^4\ \Delta\tau^4}{4!} \cdots \qquad (2)$$

where B and $\Delta\tau$ are the amplitude and fractional period respectively. This equation may be expressed more simply as

$$\Delta y = \pm \frac{B'\ \pi^2\ \Delta t^2}{T^2} \mp \frac{B'\ \pi^4\ \Delta t^4}{3T^4} \cdots\cdots \qquad (2a)$$

where B', T and Δt are the double amplitude, period, and time interval before or after the turning point respectively.

Equations (2) or (2a) give the first terms in a rapidly converging series for Δy. In Appendix C the effect of neglecting the remaining terms in the series is discussed. The technique of observation and reduction is analogous to the circum-elongation observation for azimuth in field astronomy.

The method requires two observers, one to track the gyro-mark and the other to observe the horizontal circle and record the times. Three techniques of observing have been investigated by trial observation in the following ways

(1) Observing times when the horizontal circle index corresponds with a circle graduation line.

(2) As in (1) with the exception that the <u>same</u> main circle graduations are observed on either side of the turning point.

(3) Making observations at random times and estimating the horizontal circle reading.

Technique (1) is satisfactory with the exception that in the immediate vicinity of the turning point the motion of the gyro-mark is extremely slow and therefore there is a long time interval between readings when the horizontal circle index corresponds with a circle graduation line. Thus we are prevented from taking readings during the optimum observation period.

The second technique suffers from the same defect as the first but the second has the advantage that the quantity Δt can be deduced simply from half the difference of the times of observation made to the same circle graduation line on either side of the turning point. It is also possible to deduce the period of oscillation from these observations made by this technique because the instant of the turning point is the average of these times, and the period will be twice the time intervals between consecutive turning points.

Again there is an analogy between this technique and the equal altitude method of meridian latitude determination in field astronomy. The disadvantage of the limitation in the observing period in techniques (1) and (2) can be avoided by adopting the procedure outlined for the third technique. The presence of the other observer does not distract the observer who is tracking, in fact it is more relaxing for him because he is relieved of the task of taking the circle readings. This procedure could well be adopted in the usual turning point method. It has been found in practice that taking three observations before and three after the turning point and one observation without time at the turning point occupies from between 1 minute and 1 minute 40 seconds. The third technique was found to be satisfactory and was adopted for all subsequent observations with this method.

It is significant that the first and second terms in equations (2) or (2a) are dominant and dependent upon the amplitude, period and time difference between the observation and the turning point and not upon the damping factor, α. These quantities are readily available from the observations. The amplitude may be deduced from the horizontal circle readings taken at the instant of the turning point. Simple general expressions for the least squares estimate of the "middle amplitude" have been given by this author (1968). The period of oscillation can be predetermined in a particular locality or it may be deduced from recording times when the horizontal circle reads zero (the circle is usually oriented to North approximately). The period will be the difference between the alternate values of these time instants. The average of consecutive values of these recorded times will also give the instant of the turning point, and the time interval Δt will be the difference between the instant of observation and the turning point. These aspects are readily seen from an example given at the end of appendix C.

The analysis of the results of the trial observations, apart from gauging the merits or demerits of the three methods of observation referred to previously, disclosed the presence of a systematic error in the observations. On the assumption that the mean of the observations, after reduction to the turning point, represented the most probable value of the turning point, the residuals on one side of the turning point showed a preponderance of the same sign. It was concluded that the observer had a tendency to lag behind in his following of the gyro-mark in the vee slot on both sides of the turning point. This tendency may be natural on the part of the observer, because near the turning point he does not wish to "over run" with the tangent screw and miss the turning point. After the observer was made aware of this effect he paid greater attention to centering the gyro-mark in the vee slot and then the residuals were found to assume a random distribution in sign.

Experimental Series

As indicated in the introduction, there were two reasons for conducting an experimental series of observations, (1) to evaluate new observation methods and (2) to establish standards by which future observations could be judged. The experiments were made in the period between 3rd April and 15th May 1967. The results of these tests were used as a basis for establishing an observational procedure for gyro-theodolite surveys at the Broken Hill Group of mines, which were made in the period between 17th May and 13th July 1967 (including post-calibration observations).

The observations were made with the gryo-theodolite mounted on a bracket fixed to the inside of the East-West wall of the Civil Engineering Building on the University campus. The advantages of this arrangement were that observations could be made in comfort in nearly all weather conditions and errors of centering could be virtually eliminated. The referring mark was a fixed target placed on the Biological Sciences Building at a distance of about 1,000 feet. The azimuth of this line was derived from a small triangulation scheme which had been oriented by astronomical observations. To minimise the correlation which exists between successive observations made in a short period of time, no more than one set of observations was made on any day. Each set consisted of the following observations:-

First Day

(1) 5 auxiliary scale readings of the non-spinning gyro.

(2) 3 pointings in circle left and right on the referring mark.

(3) Turning point method with 8 reversal points.

(4) Repeat (2) after arresting the gyro.

(5) Transit method with 8 transits, observing scale lines 2 and 3 as well as through the vee slot. Scale lines 2 to 6 were observed for the first and last three of the eight transits.

(6) Repeat (2) after arresting the gyro.

(7) Repeat (1).

Second Day

The same sequence of observations as on the first day with (3) and (5) interchanged.

The third day's observations were made in the same sequence as the first day and so on. In all 27 sets of observations were made before the gyro-theodolite surveys were undertaken at Broken Hill.

The experiments were designed to give information about the following aspects:-

(1) The performance of the turning point and transit method and in particular whether there was a significant difference between the means of 4 and 8 observations. Halmos (1967) after extensive tests with the gyro-theodolite MOM-Gi-B1 considers that four reversal points are sufficient because of the likelihood of systematic errors. Halmos also considers that it is advisable to "idle" the motor before beginning the first measurements in order to bring the instrument to an even temperature, the temperature changes being brought about by bearing friction and current supply. Thus the sequence of observations in each set in these experiments were alternated so that half of the observations were made in a "cold" state and the

other half in a "warm" state.

(2) The performance of the modified transit method and in particular whether the increase in the number of observations over the normal method would give a greater precision. An increase in the number of observations can be achieved in two ways (a) over a long period by observing scale lines 2 and 3 for 8 transits or (b) over a short period by observing scale lines 2 to 6 for 3 transits.

The modified turning point method was not developed until after this experimental series.

The results of this experimental series are shown in graphical form in Appendix D. The information from this series was limited when after the fifth set of observations the attachment was lifted from the bridge with the gyro unclamped. The effect of this can be seen in all graphs where there is a sudden change of about 1' in the value of E. In the next nine sets of observations the graphs of E show an upward trend towards, but not quite reaching the original value of E. It is remarkable that the strain on the suspension tape was not removed until after a long set of observations indicating that the stress strain relationship was complex. Probably some fibres were stressed beyond the elastic limit. It was considered that the last 13 sets were free from this effect. To avoid a recurrence of this accident a procedure sheet was drawn up, setting out in detail the sequence of observations required of the observer. The recorder then assumed the responsibility of guiding the observer through the observations. All subsequent observations have been made in this manner and this accident has not been repeated. The manufacturer could give some thought to this aspect which would require a device to prevent the attachment from being lifted from the bridge when the gyro is in a lowered position.

Theoretically it should be possible to account for these changes in E by analysing the values of the mean position of oscillation of the non-spinning gyro readings. A change of 0.1 div. of the non-spinning gyro results in a change of 16" in the value of E for this instrument in this latitude (S34°). The results of all the non-spinning gyro readings are shown in Fig. 3, Appendix D. Mean values for the periods 3rd - 7th April, 10th - 20th April, 24th April - 15th May are shown, which give changes of 35" and 27" for the value of E. These changes agree well with the changes in the average values of E determined by all observation methods. However it will be noted that the non-spinning readings show a large scatter and thus in practice little reliance can be placed on corrections to E deduced from such readings.

Graphs of the individual observations for the turning point and transit method are shown in Figs. 4 and 5 in Appendix D. For the turning point method there is a marked tendency for the first observations in the cold state to lie on the left side of the mean line. When the instrument is warm the observations adopt a random pattern. In the transit method this tendency is not so marked. The range of observations in the turning point method is considerably smaller than that of the transit method and if these ranges are indicative of the external precision then it would be anticipated that the turning point method would give a higher precision. This was not borne out by a later analysis when the reverse appeared to be the case.

The analysis of the results from the last 13 sets of observations are tabulated in Appendix D. The sample size is somewhat reduced because of the need to reject those observations taken at the beginning and those affected by the tape strain. Conclusions drawn from such a sample cannot be definitive but will give some indication of the precision of the various methods and of the presence of systematic effects. It was concluded that:-

(a) The transit method gave better results than the turning point method when 8 observations were taken. (Methods A and B, see Appendix D).

(b) There appeared to be some improvement in precision for both methods when 8 observations were taken rather than 4.

(c) The difference between mean values in the cold and warm states was more pronounced with the turning point method.

(d) Increasing the number of observations by observing scale lines 2 and 3 did not lead to a significant improvement when compared with observations made with the vee slot. This may not be case if a modified auxiliary scale (see Fig. 2) is used where one may expect an improvement in the precision of timing.
(Method C)

(e) Observations made on scale lines 2 to 6 for 3 transits gave results which were significantly better than taking 4 transits through the vee slot. The results from the last 3 transits gave better results than the first three which may be due to the fact that temperature effects at the end of 8 transits have diminished, although there is little difference in the values of the means. (Methods

D1 and D2).

(f) The mean of scale lines 2 to 6 combined for the first and last 3 transits gave excellent results (Standard Deviations of ± 6", 4", 4") indicating that it would be well worthwhile making further tests with this method.

(Method D3)

(g) The mean of all results also gave excellent results.

(Method E)

The small standard deviations achieved with some of the methods are remarkable considering that the least count of the Wild T16 theodolite is 1' and readings can only be estimated to 0.1'.

Broken Hill Gyro-theodolite Survey

For the observations at the Broken Hill group of mines it was decided to adopt the modified transit method, observing scale lines 1 - 5 (excluding the vee slot) for 4 transits with no warm up period. The observing time of about 10 minutes is about one third of that required for the turning point method with 8 reversal points. It was expected that observations made under field conditions would give a lower precision than those taken under laboratory conditions. In the laboratory the observer is working in comfort with a stable instrument support and the centering of instrument and target are under rigid control. It was estimated that the precision of an azimuth determination under laboratory conditions was about ± 10" and under field conditions this would be extended to about ± 15". A precision of this magnitude is quite acceptable for normal mining survey work.

Observations for the transfer of azimuth between surface and underground workings were made at the following mines -

The New Broken Hill Consolidated Mine

The Broken Hill South Mine

The Broken Hill North Mine

These mines are located in North Western New South Wales and their principal products are lead, silver and zinc. Astronomical observations for azimuth were made at each of these mines to provide reference lines of known azimuth for future gyro-theodolite surveys. Each mine uses an independent arbitarily chosen co-ordinate origin for its survey work with co-ordinate axes oriented in the direction of the initial mining development. With the rapid expansion of mining activities in the post war years the mine workings have become quite extensive and a knowledge of the interrelation of the various mines is of prime importance not only from the point of view of economical development but for safe operation.

The results of the 27 gyro-theodolite azimuths made at these mines are given in the following table.

Summary of Broken Hill Gyro-Theodolite Results

Date	Line	Gyro Azimuth			Convergence	Reduced Gyro Azimuth			v
27/5	127-L	323°	48'	52"	-1"	323°	48'	51"	Reject
27/5	L-127	143	47	37	-3	143	47	34	+16"
28/5	127-L	323	48	20	-1	323	48	19	-29
29/5	L-127	143	47	41	-3	143	47	38	+12
27/5	145G-152G	207	15	52	-14	207	15	38	- 7
27/5	152G-145G	27	15	38	-14	27	15	24	+ 7
28/5	52-107	226	43	42	+ 8	226	43	50	- 2
28/5	107-52	46	44	09	-22	46	43	47	+ 1
28/5	F-E	319	03	30	+ 1	319	03	31	+15
28/5	E-F	139	04	02	0	139	04	02	-16
29/5	122-CS32	319	24	31	- 2	319	24	29	-20
29/5	CS32-122	139	23	52	- 3	139	23	49	+20
29/5	523G-530G	228	05	38	-22	228	05	16	-13
29/5	530G-523G	48	05	11	-20	48	04	51	+12

N.B.H.C. Mine

Date	Line	Gyro Azimuth			Convergence	Reduced Gyro Azimuth			v
30/5	A-B	48°	59'	03"	-5"	48°	58'	58"	-5"
30/5	B-A	228	58	51	-3	228	58	48	+5
31/5	B-A	228	58	55	-3	228	58	52	+1
30/5	19-28	34	02	30	-8	34	02	22	-9
30/5	28-19	214	02	11	-7	214	02	04	+9
30/5	22-10	116	04	11	-9	116	04	02	-10
30/5	10-22	296	03	51	-8	296	03	43	+ 9
31/5	39-38	143	07	34	-8	143	07	26	- 7
31/5	38-39	323	07	22	-9	323	07	13	+ 6

South Mine

Date	Line	Gyro Azimuth			Convergence	Reduced Gyro Azimuth			v
1/6	T16-T20	299°	48'	04"	+25"	299°	48'	29"	-5"
1/6	T20-T16	119	48	02	+18	119	48	20	+4
1/6	4-5	70	24	48	+22	70	25	10	-6
1/6	5-4	250	24	35	+23	250	24	58	+6

North Mine

On the whole the results were quite satisfactory, the estimate of the standard deviation of a single determination is ± 15". The observations were made with minimum of delay to mining production. This was partly due to the short observation period of the method. This last aspect is of some importance because delays due to survey operations can inconvenience production personnel and are unpopular with them. No difficulties were found in using the method in the mines. The recorder was able to check for internal consistency quickly (see previously under "The Modified Transit Method") before moving from the station. The final calculations were made later with a slide rule and could be completed in a few minutes.

As with the gyro-theodolite survey at the Huntley Colliery the first observation at Broken Hill gave a value which was considerably different from later determinations made on the same line. Halmos (1967) considers that after long transportation the tape may have minor deformations and recommends "to charge the band with the weight of the oscillation system before the first measurement". In both cases the instrument was used directly after transportation without preliminary observations and the effect as described by Halmos could have been present.

CONCLUSION

The modified methods described above have been used successfully in practice, especially the modified transit method. Their advantage lies in the fact that a greater number of observations per period can be made than with the normal methods. In high latitudes where the period of oscillation is long these methods are of particular value and if the observation errors are of a purely random nature then it may be possible to extend the latitude range of the gyro-theodolite because of the increase in precision of the determination.

ACKNOWLEDGEMENTS

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APPENDIX A

PROOF OF THE MODIFIED TRANSIT METHOD

Lauf (1963) has shown that the oscillation of a pendulous gyroscope is of the form -

$$y = m\,a\,e^{\frac{-\alpha(t-t_o)}{T}} \cos\left\{\frac{2\pi(t-t_o)}{T} + \gamma\right\}$$

where y is the displacement in arc.

m is the arc value of each scale division

a is the amplitude in scale divisions at time $t = t_o$

T is the period of oscillation

t_o is the initial time instant

t is any time instant

α is the damping constant

γ is an arbitrary phase angle

Consider observations to be made when the leading edge of the gyro mark is coincident with a graduation line on the auxiliary scale. Then for the first half period

$$\Delta y = y_{t_i} - \frac{mw}{2} - nm = m\,a\,e^{\frac{-\alpha(t_i-t_o)}{T}} \cos\left\{\frac{2\pi(t_i-t_o)}{T} + \gamma\right\} - \frac{mw}{2} - nm \quad (1)$$

for n = 5, i = 1 . 4, 2 . 3, 3 ....0, 6....... -5, 11.

and for the second half period

$$\Delta y = y_{t_i} + \frac{mw}{2} - nm = m\,a\,e^{\frac{-\alpha(t_i-t_o)}{T}} \cos\left\{\frac{2\pi(t_i-t_o)}{T} + \gamma\right\} + \frac{mw}{2} - nm \quad (2)$$

for n = -5, i = 12 . -4, 13 . -3, 14 .... 0, 17 .....5, 22 where Δy is the displacement of the zero graduation line from the axis of mean oscillation.

y_{t_i} is the displacement of the centre of the gyro mark at time t_i.

w is the width of the gyro mark in scale divisions.

n is the number of the scale graduation on either side of the zero graduation line.

For convenience put $t_o = 0$ and $y = 0$. For simplicity in the following derivation we will consider observations made to a single scale division on either side of the zero graduation line with the timing sequence renumbered accordingly, see Fig. 1.

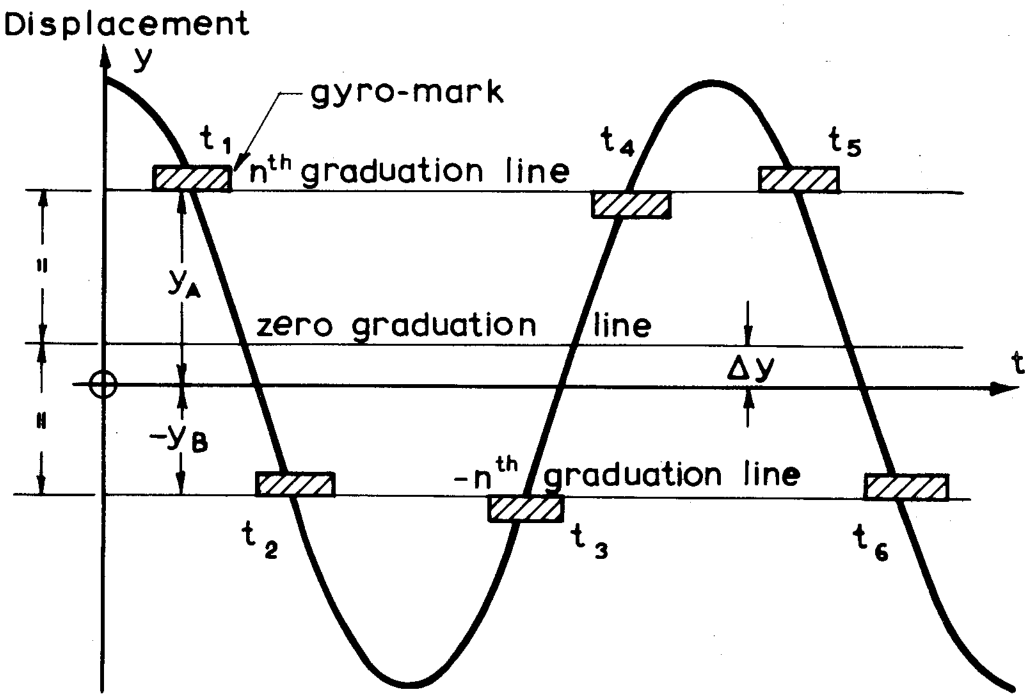


FIG. 1

From (1) we may write after expanding the exponential coefficient

$$y_A = ma\ (1-\alpha\tau_1 + \frac{\alpha^2\tau_1^2}{2!}\ \ldots\ldots)\cos 2\pi\ \tau_1 - \frac{mw}{2} \qquad (3)$$

$$y_B = ma\ (1-\alpha\tau_2 + \frac{\alpha^2\tau_2^2}{2!}\ \ldots\ldots)\cos 2\pi\ \tau_2 - \frac{mw}{2} \qquad (4)$$

$$y_A = ma\ (1-\alpha\tau_5 + \frac{\alpha^2\tau_5^2}{2!}\ \ldots\ldots)\cos 2\pi\ \tau_5 - \frac{mw}{2} \qquad (5)$$

$$y_B = ma\ (1-\alpha\tau_6 + \frac{\alpha^2\tau_6^2}{2!}\ \ldots\ldots)\cos 2\pi\ \tau_6 - \frac{mw}{2} \qquad (6)$$

and from (2)

$$y_B = ma(1-\alpha\tau_3 + \frac{\alpha^2\tau_3^2}{2!}\ \ldots\ldots)\cos 2\pi\ \tau_3 + \frac{mw}{2} \qquad (7)$$

$$y_A = ma(1-\alpha\tau_4 + \frac{\alpha^2\tau_4^2}{2!}\ \ldots\ldots)\cos 2\pi\ \tau_4 + \frac{mw}{2} \qquad (8)$$

after replacing $\frac{t_1}{T}$, $\frac{t_2}{T}$ ..... by τ_1 τ_2 .....

The mean of (4) and (7) gives

$$y_B = \frac{ma}{2}(\cos 2\pi\tau_2 + \cos 2\pi\tau_3) - \frac{ma\alpha}{2}(\tau_2 \cos 2\pi\tau_2 + \tau_3 \cos 2\pi\tau_3)$$
$$+ \frac{ma\alpha^2}{4}(\tau_2^2 \cos 2\pi\tau_2 + \tau_3^2 \cos 2\pi\tau_3) \ \ldots\ldots \ (9)$$

$$y_B = m\ a \cos 2\pi\ (\frac{\tau_2+\tau_3}{2})\cos 2\pi\ (\frac{\tau_2-\tau_3}{2}) - \frac{ma\alpha}{2}(\tau_2\cos 2\pi\tau_2 + \tau_3\cos 2\pi\tau_3)$$
$$\ldots\ldots \ (10)$$

put $\frac{\tau_2 + \tau_3}{2} = \frac{1}{2} + d\tau_{23}$ (10a)

$\therefore$ $\cos 2\pi \left(\frac{T_2 + \tau_3}{2}\right) = \cos (\pi + 2\pi d\tau_{23})$ and a Taylor expansion of the right hand side gives

$$\cos \pi - 2\pi d\tau_{23} \sin \pi - \frac{4\pi^2 d\tau_{23}^2}{2!} \cos \pi \text{ ....}$$

$$= -1 + \frac{4\pi^2 d\tau^2_{23}}{2} \text{ ....}$$

$\therefore$ (10) becomes

$$y_B = -ma \cos 2\pi \left(\frac{\tau_2 - \tau_3}{2}\right) + \frac{ma\ 4\pi^2 d\tau_{23}^2}{2} \cos 2\pi \left(\frac{\tau_2 - \tau_3}{2}\right)$$

$$-\frac{ma\alpha}{2} (\tau_2 \cos 2\pi\tau_2 + \tau_3 \cos 2\pi\tau_3) \text{ ....(12)}$$

The mean of (5) and (8) gives

$$y_A = \frac{ma}{2} (\cos 2\pi\tau_4 + \cos 2\pi\tau_5) - \frac{ma\alpha}{2}(\tau_4 \cos 2\pi\tau_4 + \tau_5 \cos 2\pi\tau_5)$$

$$+ \frac{ma\alpha^2}{4} (\tau_4^2 \cos 2\pi\tau_4 + \tau_5^2 \cos 2\pi\tau_5) \text{....(13)}$$

$$y_A = ma \cos 2\pi\left(\frac{\tau_4 + \tau_5}{2}\right)\cos 2\pi\left(\frac{\tau_4 - \tau_5}{2}\right) - \frac{ma\alpha}{2}(\tau_4 \cos 2\pi\tau_4 + \tau_5 \cos 2\pi\tau_5)$$

.... (14)

put $\frac{\tau_4 + \tau_5}{2} = 1 + d\tau_{45}$ (14a)

$\therefore$ $\cos 2\pi\left(\frac{\tau_4 + \tau_5}{2}\right) = \cos (2\pi + 2\pi d\tau_{45})$ and a Taylor expansion of the right hand side gives

$$\cos 2\pi - 2\pi d\tau_{45} \sin 2\pi - \frac{4\pi^2}{2!} d\tau_{45}^2 \cos 2\pi \text{ ......}$$

$$= 1 - \frac{4\pi^2 d\tau^2_{45}}{2} \text{ ....}$$

∴ (14) becomes

$$y_A = ma \cos 2\pi \left(\frac{\tau_4 - \tau_5}{2}\right) - \frac{ma\, 4\pi^2\, \tau_{45}^2}{2} \cos 2\pi \left(\frac{\tau_4 - \tau_5}{2}\right)$$

$$- \frac{ma\alpha}{2} (\tau_4 \cos 2\pi\tau_4 + \tau_5 \cos 2\pi\tau_5) \ldots (15)$$

From Fig. 1. $y_A - \Delta y = - y_B + \Delta y$

$$\therefore \quad \Delta y = \frac{y_A + y_B}{2}$$

and substituting (12) and (15) in this equation gives

$$\Delta y = \frac{ma}{2} \left\{ \cos 2\Pi \left(\frac{\tau_4 - \tau_5}{2}\right) - \cos 2\Pi \left(\frac{\tau_2 - \tau_3}{2}\right) \right\}$$

$$+ ma\, \Pi^2 \left\{ d\tau_{23}^2 \cos 2\Pi \left(\frac{\tau_2 - \tau_3}{2}\right) - d\tau_{45}^2 \cos 2\Pi \left(\frac{\tau_4 - \tau_5}{2}\right) \right\}$$

$$- \frac{ma\alpha}{4} (\tau_2 \cos 2\Pi\tau_2 + \tau_3 \cos 2\Pi\tau_3 + \tau_4 \cos 2\Pi\tau_4 + \tau_5 \cos 2\Pi\tau_5) \ldots (16)$$

The first term of (16) $= -ma \sin 2\Pi\left(\frac{\tau_2 - \tau_3 + \tau_4 - \tau_5}{4}\right) \sin 2\Pi\left(\frac{-\tau_2 + \tau_3 + \tau_4 - \tau_5}{4}\right)$

(17)

Taking the difference between (10a) and (14a) gives

$$\tau_2 + \tau_3 - \tau_4 - \tau_5 = -1 + 2(d\tau_{23} - d\tau_{45})$$

$$\tau_2 - \tau_3 + \tau_4 - \tau_5 = -1 + 2(\tau_4 - \tau_3) + 2\,(d\tau_{23} - d\tau_{45})$$

and substituting in (17) gives

$$-ma \sin 2\Pi \left\{ \frac{-1 + 2(\tau_4 - \tau_3) + 2(d\tau_{23} - d\tau_{45})}{4} \right\} \sin 2\Pi \frac{(-\tau_2 + \tau_3 + \tau_4 - \tau_5)}{4}$$

$$= - ma \sin \left\{ \frac{2\pi(\tau_4-\tau_3)}{2} - \frac{\pi}{2} + 2\pi \frac{(d\tau_{23}-d\tau_{45})}{2} \right\} \sin 2\pi \frac{(-\tau_2+\tau_3+\tau_4-\tau_5)}{4}$$

A Taylor expansion of the first term gives

$$-ma \sin \left\{ 2\pi \frac{(\tau_4-\tau_3)}{2} - \frac{\pi}{2} \right\} \sin 2\pi \frac{(-\tau_2+\tau_3+\tau_4-\tau_5)}{4}$$

$$-ma\, 2\pi \frac{(d\tau_{23}-d\tau_{45})}{2} \cos\left\{ 2\pi \frac{(\tau_4-\tau_3)}{2} - \frac{\pi}{2} \right\} \sin 2\pi \frac{(-\tau_2+\tau_3+\tau_4-\tau_5)}{4}$$

$$+\frac{ma4\pi^2}{2} \frac{(d\tau_{23}-d\tau_{45})^2}{4} \sin \left\{ 2\pi \frac{(\tau_4-\tau_3)}{2} - \frac{\pi}{2} \right\} \sin 2\pi \frac{(-\tau_2+\tau_3+\tau_4-\tau_5)}{4} \cdots\cdots$$

$$= ma \cos 2\pi \frac{(\tau_4-\tau_3)}{2} \sin 2\pi \frac{(-\tau_2+\tau_3+\tau_4-\tau_5)}{4}$$

$$-ma\, 2\pi \frac{(d\tau_{23}-d\tau_{45})}{2} \sin 2\pi \frac{(\tau_4-\tau_3)}{2} \sin 2\pi \frac{(-\tau_2+\tau_3+\tau_4-\tau_5)}{4}$$

$$- \frac{ma\pi^2}{2} (d\tau_{23}-d\tau_{45})^2 \cos 2\pi \frac{(\tau_4-\tau_3)}{2} \sin 2\pi \frac{(-\tau_2+\tau_3+\tau_4-\tau_5)}{4} \cdots\cdots$$

Resubstituting in (16)

$$\Delta y = ma \cos 2\pi \frac{(\tau_4-\tau_3)}{2} \sin 2\pi \frac{(-\tau_2+\tau_3+\tau_4-\tau_5)}{4}$$

$$-ma\, 2\pi \frac{(d\tau_{23}-d\tau_{45})}{2} \sin 2\pi \frac{(\tau_4-\tau_3)}{2} \sin 2\pi \frac{(-\tau_2+\tau_3+\tau_4-\tau_5)}{4}$$

$$- \frac{ma\pi^2}{2} (d\tau_{23}-d\tau_{45})^2 \cos 2\pi \frac{(\tau_4-\tau_3)}{2} \sin 2\pi \frac{(-\tau_2+\tau_3+\tau_4-\tau_5)}{4}$$

$$+ ma\pi^2 \left\{ d\tau_{23}^2 \cos 2\pi \frac{(\tau_2-\tau_3)}{2} - d\tau_{45}^2 \cos 2\pi \frac{(\tau_4-\tau_5)}{2} \right\}$$

$$- \frac{ma\alpha}{4} (\tau_2 \cos 2\pi\tau_2 + \tau_3 \cos 2\pi\tau_3 + \tau_4 \cos 2\pi\tau_4 + \tau_5 \cos 2\pi\tau_5) \cdots\cdots$$

put $(t_3-t_2) - (t_5-t_4) = \Delta t$

then $$\frac{2\Pi(-\tau_2+\tau_3+\tau_4-\tau_5)}{4} = \frac{\Pi\Delta t}{2T}$$

$$\therefore \Delta y = ma \cos 2\Pi \frac{(\tau_4-\tau_3)}{2} \sin \frac{\Pi\Delta t}{2T}$$

$$-ma\Pi\ (d\tau_{23}-d\tau_{45}) \sin 2\Pi \frac{(\tau_4-\tau_3)}{2} \sin \frac{\Pi\Delta t}{2T}$$

$$-\frac{ma\Pi^2}{2}(d\tau_{23}-d\tau_{45})^2 \cos 2\Pi \frac{(\tau_4-\tau_3)}{2} \sin \frac{\Pi\Delta t}{2T}$$

$$+ ma\Pi^2 \left\{ d\tau_{23}^2 \cos 2\Pi \frac{(\tau_2-\tau_3)}{2} - d\tau_{45}^2 \cos 2\Pi \frac{(\tau_4-\tau_5)}{2} \right\}$$

$$-\frac{ma\alpha}{4}(\tau_2 \cos 2\Pi\tau_2+\tau_3 \cos 2\Pi\tau_3+\tau_4 \cos 2\Pi\tau_4+\tau_5 \cos 2\Pi\tau_5)$$

..... (18)

From Fig. 1. $\frac{y_A - y_B}{2} = mn$

and substituting from (7) and (8)

$$\frac{y_A-y_B}{2} = \frac{ma}{2}(\cos 2\Pi\tau_4-\cos 2\Pi\tau_3) - \frac{ma\alpha}{2}(\tau_4 \cos 2\Pi\tau_4-\tau_3 \cos 2\Pi\tau_3)....$$

$$= -\ ma \sin 2\Pi \frac{(\tau_4+\tau_3)}{2} \sin 2\Pi \frac{(\tau_4-\tau_3)}{2} - \frac{ma\alpha}{2}(\tau_4 \cos 2\Pi\tau_4-\tau_3 \cos 2\Pi\tau_3)..$$

put $\frac{\tau_3 + \tau_4}{2} = \frac{3}{4} + d\tau_{34}$ and substitute in the first term

$$\frac{y_A-y_B}{2} = -\ ma \sin 2\Pi(\tfrac{3}{4}+d\tau_{34})\sin 2\Pi \frac{(\tau_4-\tau_3)}{2} - \frac{ma\alpha}{2}(\tau_4 \cos 2\Pi\tau_4$$

$$-\tau_3 \cos 2\Pi\tau_3) \ ....$$

but $\sin 2\Pi\ (\frac{3}{4} + d\tau_{34}) = \sin\ (\frac{3\Pi}{2} + 2\Pi d\tau_{34})$

$$= \sin\frac{3\Pi}{2} + 2\Pi d\tau_{34}\cos\frac{3\Pi}{2} - 4\Pi^2\frac{d\tau_{34}^2}{2}\sin\frac{3\Pi}{2}\ \ldots\ldots$$

$$= -1 + 4\Pi^2\frac{d\tau_{34}^2}{2}\ \ldots\ldots\ldots\ldots$$

and $\therefore$ $mn = \frac{y_A+y_B}{2} = ma\sin 2\Pi\frac{(\tau_4-\tau_3)}{2} - 4\Pi^2 ma\frac{d\tau_{34}^2}{2}\sin 2\Pi\frac{(\tau_4-\tau_3)}{2}$

$$- \frac{ma\alpha}{2}(\tau_4\cos 2\Pi\tau_4 - \tau_3\cos 2\Pi\tau_3)\ \ldots\ldots\ldots$$

$$\sin 2\Pi\frac{(\tau_4-\tau_3)}{2} = (1-2\Pi^2 d\tau_{34}^2\ \ldots.)^{-1}\left\{\frac{n}{a} + \frac{\alpha}{2}(\tau_4\cos 2\Pi\tau_4 - \tau_3\cos 2\Pi\tau_3)\ \ldots.\right\}$$

$$= \frac{n}{a} + \frac{\alpha}{2}(\tau_4\cos 2\Pi\tau_4 - \tau_3\cos 2\Pi\tau_3) + \frac{2n\Pi^2 d\tau_{34}^2}{a}\ \ldots\ldots$$

and $\cos 2\Pi\frac{(\tau_4-\tau_3)}{2} = \left\{1 - \frac{n^2}{a^2} - \frac{n\alpha}{a}(\tau_4\cos 2\Pi\tau_4 - \tau_3\cos 2\Pi\tau_3) - \frac{4n^2\Pi^2 d\tau_{34}^2}{a^2}\ \ldots\right\}^{\frac{1}{2}}$

$$= \left(1 - \frac{n^2}{a^2}\right)^{\frac{1}{2}} - \frac{1}{2}\left(1 - \frac{n^2}{a^2}\right)^{-\frac{1}{2}}\left\{\frac{n\alpha}{a}(\tau_4\cos 2\Pi\tau_4 - \tau_3\cos 2\Pi\tau_3) + \frac{4n^2\Pi^2 d\tau_{34}^2}{a^2}\ \ldots.\right\}\ldots.$$

substituting in (18) and also putting $\sin\frac{\Pi\Delta t}{2T}$,

which is a very small quantity $= \frac{\Pi\Delta t}{2T}$ we have

$$\Delta y = \frac{ma\,\Pi\Delta t}{2T}\left(1 - \frac{n^2}{a^2}\right)^{\frac{1}{2}} \qquad (i)$$

$$- \frac{ma\,\Pi\,\Delta t}{4T}\left(1 - \frac{n^2}{a^2}\right)^{-\frac{1}{2}}\left\{\frac{n\alpha}{a}(\tau_4\cos 2\Pi\tau_4 - \tau_3\cos 2\Pi\tau_3) + 4n^2\Pi^2 d\tau_{34}^2\ \ldots.\right\} \qquad (ii)$$

$$- \frac{ma\Pi^2 \Delta t}{2T} (d\tau_{23} - d\tau_{45}) \{ \frac{n}{a} + \frac{\alpha}{2} (\tau_4 \cos 2\Pi\tau_4 - \tau_3 \cos 2\Pi\tau_3) + \frac{2n\Pi^2 d\tau_{34}^2}{a} .. \} \quad \text{(iii)}$$

$$- \frac{ma\Pi^3 \Delta t}{4T} (d\tau_{23} - d\tau_{45})^2 (1 - \frac{n^2}{a^2})^{\frac{1}{2}} \quad \text{(iv)}$$

$$+ ma\, \Pi^2 \{ d\tau_{23}^2 \cos 2\Pi \frac{(\tau_2 - \tau_3)}{2} - d\tau_{45}^2 \cos 2\Pi \frac{(\tau_4 - \tau_5)}{2} \} \quad \text{(v)}$$

$$- \frac{ma\ \alpha}{4} (\tau_2 \cos 2\Pi\tau_2 + \tau_3 \cos 2\Pi\tau_3 + \tau_4 \cos 2\Pi\tau_4 + \tau_5 \cos 2\Pi\tau_5) .. \text{(vi)} \quad (19)$$

where $\Delta t = (t_3 - t_2) - (t_5 - t_4)$

$$d\tau_{23} = \frac{\tau_2 + \tau_3}{2} - \frac{1}{2}$$

$$d\tau_{34} = \frac{\tau_3 + \tau_4}{2} - \frac{3}{4}$$

$$d\tau_{45} = \frac{\tau_4 + \tau_5}{2} - 1$$

It may also be proved that

$$\Delta y = \frac{ma\Pi\Delta t'}{2T} (1 - \frac{n^2}{a^2})^{\frac{1}{2}} \quad \text{(i)}$$

$$- \frac{ma\Pi\Delta t'}{4T} (1 - \frac{n^2}{a^2})^{-\frac{1}{2}} \{ \frac{n\alpha}{a} (\tau_4 \cos 2\Pi\tau_4 - \tau_3 \cos 2\Pi\tau_3) + \frac{4n^2 \Pi^2 d\tau_{3.4}^2}{a^2} .. \} \quad \text{(ii)}$$

$$+ \frac{ma\Pi^2 \Delta t'}{2T} (d\tau_{14} - d\tau_{36}) \{ \frac{n}{a} + \frac{\alpha}{2} (\tau_4 \cos 2\Pi\tau_4 - \tau_3 \cos 2\Pi\tau_3) + \frac{2n\Pi^2 d\tau_{34}^2}{a} ... \} \quad \text{(iii)}$$

$$- \frac{ma\Pi^3 \Delta t'}{4T} (d\tau_{14} - d\tau_{36})^2 (1 - \frac{n^2}{a^2})^{\frac{1}{2}} \quad \text{(iv)}$$

$$+ ma\Pi^2 \{ d\tau_{14}^2 \cos 2\Pi \frac{(\tau_1 - \tau_4)}{2} - d\tau_{36}^2 \cos 2\Pi \frac{(\tau_3 - \tau_6)}{2} \} \quad \text{(v)}$$

$$- \frac{ma\alpha}{4} (\tau_1 \cos 2\Pi\tau_1 + \tau_3 \cos 2\Pi\tau_3 + \tau_4 \cos 2\Pi\tau_4 + \tau_6 \cos 2\Pi\tau_6) \ldots (vi)$$

(20)

where $\Delta t' \ (t_4 - t_1) - (t_6 - t_3)$

$$d\tau_{14} = \frac{\tau_1 + \tau_4}{2} - \frac{1}{2}$$

$$d\tau_{36} = \frac{\tau_3 + \tau_6}{2} - 1$$

Example to assess the magnitude of each term in (19) and (20)

$\Delta y = +10'$ a = 10 div. $T = 390^s$ $m = 12'/\text{div}$

$\alpha = \frac{1}{150}$ $w = 2\frac{1}{2}$ div. $\Delta t = 24.92^s$ $\Delta t' = 24.37^s$

$t_1 = 48.59^s$ $\tau_1 = 0.1246$ $\tau_1 \cos 2\Pi\tau_1 = 0.0883$

$t_2 = 115.92$ $\tau_2 = 0.2972$ $\tau_2 \cos 2\Pi\tau_2 = -0.0867$

$t_3 = 256.79$ $\tau_3 = 0.6584$ $\tau_3 \cos 2\Pi\tau_3 = -0.3567$

$t_4 = 322.22$ $\tau_4 = 0.8262$ $\tau_4 \cos 2\Pi\tau_4 = 0.3786$

$t_5 = 438.17$ $\tau_5 = 1.1235$ $\tau_5 \cos 2\Pi\tau_5 = 0.7083$

$t_6 = 506.05$ $\tau_6 = 1.2976$ $\tau_6 \cos 2\Pi\tau_6 = -0.3785$

$d\tau_{23} = -0.0222$ $\cos 2\Pi \frac{(\tau_2-\tau_3)}{2} = 0.4223$

$d\tau_{34} = -0.0077$ $\cos 2\Pi \frac{(\tau_4-\tau_5)}{2} = 0.5946$

$d\tau_{45} = -0.0251$ $\cos 2\Pi \frac{(\tau_1-\tau_4)}{2} = -0.5918$

$d\tau_{14} = -0.0246$ $\cos 2\Pi \frac{(\tau_3-\tau_6)}{2} = -0.4237$

$d\tau_{36} = -0.0220$

Equation (19)

Term (i)	+10.43'
(ii)	- 0.02
(iii)	- 0.06
(iv)	0.00
(v)	- 0.20
(vi)	- 0.15
Σ	+10.00

Equation (20)

Term	(i)	+10.19'
	(ii)	− 0.01
	(iii)	− 0.05
	(iv)	0.00
	(v)	− 0.18
	(vi)	+ 0.05
	Σ	+10.00

Example of the Modified Transit Method

Broken Hill South Mine, 2,400 ft. level. Line 28 to 19.

Date: 30th May 1967

Scale	t	Δ	t	Δ	t	Δ	t
+5	$0^m\ 00.0^s$	$-4^m 00.4^s$	$4^m 00.4^s$	$+2^m 26.8^s$	$6^m 27.2^s$	$4^m 01.0^s$	$10^m 28.2$
+4	0 05.7	-3 49.6	3 55.3	+2 37.5	6 32.8	3 49.9	10 22.7
+3	0 11.0	-3 39.3	3 50.3	+2 47.7	6 38.0	3 39.8	10 17.8
+2	0 15.7	-3 30.2	3 45.9	+2 56.8	6 42.7	3 30.5	10 13.2
+1	0 20.6	-3 19.9	3 40.5	+3 07.2	6 47.7	3 20.4	10 08.1
0	0 25.4	-3 10.7	3 36.1	+3 16.9	6 53.0	3 10.3	10 03.3
-1	0 30.2	-3 01.0	3 31.2	+3 26.1	6 57.3	3 01.0	9 58.3
-2	0 35.4	-2 50.6	3 26.0	+3 36.6	7 02.6	2 50.4	9 53.0
-3	0 40.1	-2 40.7	3 20.8	+3 46.6	7 07.4	2 40.6	9 48.0
-4	0 45.0	-2 30.6	3 15.6	+3 56.8	7 12.4	2 30.3	9 42.7
-5	0 50.3	-2 19.5	3 09.8	+4 07.8	7 17.6	2 19.4	9 37.0

Scale	K*	Δt	ΔN	Δt	ΔN
5	0.590	$+7.4^s$	+4.36'	$+7.4^s$	+4.36'
4	0.609	+7.2	+4.38	+7.2	+4.38
3	0.624	+7.3	+4.55	+7.1	+4.42
2	0.634	+6.4	+4.06	+6.4	+4.06
1	0.641	+6.2	+3.97	+6.2	+3.97
0	0.643	+6.2	+3.98	+6.6	+4.24
1	0.641	+6.2	+3.97	+5.7	+3.65
2	0.634	+6.2	+3.93	+6.1	+3.87
3	0.624	+7.0	+4.37	+6.8	+4.24
4	0.609	+6.9	+4.20	+6.9	+4.20
5	0.590	+7.3	+4.30	+6.8	+4.01

$*K = c(a^2 - n^2)^{\frac{1}{2}}$

c = 0.051 amplitude a = 12.6 div.

Mean ΔN		+4.16'
Circle Setting	0°	16.00
Circle Reading of G.I.N.	0	20.16
Mean R.O. Circle Reading	214	36.93
Gyro Azimuth	214	16.77
E		-13.59
Azimuth of R.O.	214	03.18

APPENDIX B

Sample Calculations

The following calculations are based on the formula

$$y = m\,a\,e^{-\frac{\alpha t}{T}} \cos \frac{2\pi t}{T} \pm \frac{mw}{2}$$

with $m = 12'/\text{div.}$ $T = 6^m\ 30^s$

$\alpha = \frac{1}{150}$ $w = 2\frac{1}{2}$ div.

and $\Delta y = K \Delta t$

where $K = \frac{ma\pi}{2T}\left(1 - \frac{n^2}{a^2}\right)^{\frac{1}{2}} = c(a^2 - n^2)^{\frac{1}{2}}$

Two methods of carrying out these sample calculations have been made in order to assess the effect of having a gyro-mark of smaller width than that of the GAKI, which is $2\frac{1}{2}$ divisions wide. In method A the observations are recorded against the observed scale graduation number. In method B the observations are recorded as though they were observed against the previous graduation line. The effect of this latter variation is to simulate observations with a gyro-mark reduced in width by two scale divisions.

Method A

a = 15 div $\qquad$ $\Delta y = +10'$

Scale	Time	Δ	Time	Δ	Time	Δ	Time	Δ	Time	Δ	Time
5	66.^{s}93	244.^{s}96	311.^{s}89	144^s.83	456^s.72	245^s.30	702.^{s}02	144.^{s}49	846.^{s}51	245^s.64	1092^s.15
4	71. 55	236. 00	307. 55	153. 82	461. 37	236. 28	697. 65	153. 54	851. 19	236 .56	1087 .75
3	76. 00	227. 31	303. 31	162. 54	465. 85	227. 53	693. 38	162. 32	855. 70	227 .75	1083 .45
2	80. 36	218. 75	299. 11	171. 13	470. 24	218. 91	689. 15	170. 97	860. 12	219 .07	1079 .19
1	84. 62	210. 31	294. 93	179. 60	474. 53	210. 42	684. 95	179. 49	864. 44	210 .53	1074 .97
0	88. 84	201. 92	290. 76	188. 02	478. 78	201. 97	680. 75	187. 97	868. 72	202 .02	1070 .74
-1	93. 02	193. 58	286. 60	196. 39	482. 99	193. 57	676. 56	196. 40	872. 96	193 .56	1066 .52
-2	97. 15	185. 26	282. 41	204. 74	487. 15	185. 19	672. 34	204. 81	877. 15	185 .12	1062 .27
-3	101. 30	176. 86	278. 16	213. 17	491. 33	176. 73	668. 06	213. 30	881. 36	176 .60	1057 .96
-4	105. 47	168. 39	273. 86	221. 66	495. 52	168. 21	663. 73	221. 84	885. 57	168 .03	1053 .60
-5	109. 67	159. 79	269. 46	230. 29	499. 75	159. 55	659. 30	230. 53	889. 83	159 .31	1049 .14
	K		Δt	Δy	Δt	Δy	Δt	Δy	Δt	Δy	
5	0.6835		+14.^{s}67	+10'.03	+14.^{s}72	+10'.06	+14.^{s}77	+10'.10	+14.^{s}82	+10' .13	
4	0.6987		14. 34	10. 02	14. 39	10. 05	14. 44	10. 09	14. 49	10 .12	
3	0.7103		14. 14	10. 04	14. 19	10. 08	14. 23	10. 11	14. 28	10 .14	
2	0.7185		14. 01	10. 07	14. 06	10. 10	14. 10	10. 13	14. 15	10 .17	
1	0.7233		13. 92	10. 07	13. 97	10. 10	14. 02	10. 14	14. 07	10 .18	
0	0.7250		13. 90	10. 08	13. 95	10. 11	14. 00	10. 15	14. 05	10 .19	
-1	0.7233		13. 98	10. 11	14. 03	10. 15	14. 08	10. 18	14. 13	10 .22	
-2	0.7185		14. 13	10. 15	14. 17	10. 18	14. 22	10. 22	14. 26	10 .25	
-3	0.7103		14. 32	10. 17	14. 36	10. 20	14. 41	10. 24	14. 45	10 .26	
-4	0.6987		14. 57	10. 18	14. 62	10. 21	14. 67	10. 25	14. 72	10 .28	
-5	0.6835		14. 96	10. 23	15. 02	10. 26	15. 06	10. 29	15. 11	10 .33	
			Mean	+10. 10		+10. 14		+10. 17		+10 .21	
			Error	0. 10		0. 14		0. 17		0 .21	

Method B

a = 15 div Δy = +10'

Scale	Time	Δ	Time	Δ	Time	Δ	Time	Δ	Time	Δ	Time
6	66^{s}.93				456^{s}.72				846^{s}.51		
5	71.55				461.37				851.19		
4	76.00	235^{s}.89	311^{s}.89	153^{s}.96	465.85	236^{s}.17	702^{s}.02	153^{s}.68	855.70	236^{s}.45	1092^{s}.15
3	80.36	227.19	307.55	162.69	470.24	227.41	697.65	162.47	860.12	227.63	1087.75
2	84.62	218.69	303.31	171.22	474.53	218.85	693.38	171.06	864.44	219.01	1083.45
1	88.84	210.27	299.11	179.67	478.78	210.37	689.15	179.57	868.72	210.47	1079.19
0	93.02	201.91	294.93	188.06	482.99	201.96	684.95	188.01	872.96	202.01	1074.97
-1	97.15	193.61	290.76	196.39	487.15	193.60	680.75	196.40	877.15	193.59	1070.74
-2	101.30	185.30	286.60	204.73	491.33	185.23	676.56	204.80	881.36	185.16	1066.52
-3	105.47	176.94	282.41	213.11	495.52	176.82	672.34	213.23	885.57	176.70	1062.27
-4	109.67	168.49	278.16	221.59	499.75	168.31	668.06	221.77	889.83	168.13	1057.96
-5			273.86				663.73				1053.60
-6			269.46				659.30				1049.14
	K		Δt	Δy	Δt	Δy	Δt	Δy	Δt	Δy	
4	0.6987		+14^{s}.30	+ 9'.99	+14^{s}.35	+10'.03	+14^{s}.40	+10'.06	+14^{s}.45	+10'.10	
3	0.7103		14.08	10.00	14.13	10.04	14.18	10.07	14.23	10.11	
2	0.7185		13.96	10.03	14.01	10.07	14.05	10.09	14.10	10.13	
1	0.7233		13.88	10.04	13.93	10.08	13.97	10.10	14.02	10.14	
0	0.7250		13.85	10.04	13.90	10.08	13.95	10.11	14.00	10.15	
-1	0.7233		13.94	10.08	13.98	10.11	14.03	10.15	14.07	10.18	
-2	0.7185		14.08	10.12	14.12	10.15	14.17	10.18	14.21	10.21	
-3	0.7103		14.25	10.12	14.30	10.16	14.35	10.19	14.40	10.23	
-4	0.6987		14.53	10.15	14.58	10.19	14.63	10.22	14.68	10.23	
		Mean		+10.06		+10.10		+10.13		+10.17	
		Error		0.06		0.10		0.13		0.17	

Method A

a = 10 div Δy = +10'

Scale	Time	Δ	Time	Δ	Time	Δ	Time	Δ	Time	Δ	Time
5	48^{s}.59	273^{s}.63	322^{s}.22	115^{s}.95	438^{s}.17	274^{s}.26	712^{s}.43	115^{s}.32	827^{s}.75	274^{s}.89	1102^{s}.64
4	56.86	258.52	315.38	131.16	446.54	259.00	705.54	130.68	836.22	259.48	1095.70
3	64.36	244.45	308.81	145.31	454.12	244.80	698.92	144.96	843.88	245.15	1089.03
2	71.36	231.05	302.41	158.77	461.18	231.30	692.48	158.52	851.00	231.55	1082.55
1	78.02	218.13	296.15	171.74	467.89	218.28	686.17	171.59	857.76	218.43	1076.19
0	84.45	205.46	289.91	184.45	474.36	205.53	679.89	184.38	864.27	205.60	1069.87
−1	90.75	192.88	283.63	197.07	480.70	192.87	673.57	197.08	870.65	192.86	1063.51
−2	96.98	180.30	277.28	209.70	486.98	180.20	667.18	209.80	876.98	180.10	1057.08
−3	103.22	167.53	270.75	222.51	493.26	167.34	660.60	222.70	883.30	167.15	1050.45
−4	109.49	154.46	263.95	235.62	499.57	154.18	653.75	235.90	889.65	153.90	1043.55
−5	115.92	140.87	256.79	249.26	506.05	140.47	646.52	249.66	896.18	140.07	1036.25
	K		Δt	Δy	Δt	Δy	Δt	Δy	Δt	Δy	
5	0.4185		+24^{s}.37	+10'.19	+24^{s}.52	+10'.26	+24^{s}.60	+10'.30	+24^{s}.75	+10'.36	
4	0.4430		22.90	10.14	23.02	10.20	23.10	10.23	23.22	10.29	
3	0.4610		21.94	10.11	22.03	10.16	22.10	10.19	22.19	10.23	
2	0.4735		21.35	10.11	21.43	10.15	21.50	10.18	21.58	10.22	
1	0.4809		21.06	10.13	21.13	10.16	21.20	10.20	21.27	10.23	
0	0.4833		21.01	10.15	21.08	10.19	21.15	10.22	21.22	10.26	
−1	0.4809		21.14	10.17	21.21	10.20	21.28	10.23	21.35	10.27	
−2	0.4735		21.53	10.19	21.60	10.23	21.68	10.27	21.75	10.30	
−3	0.4610		22.22	10.24	22.29	10.28	22.38	10.32	22.45	10.35	
−4	0.4430		23.30	10.32	23.38	10.36	23.50	10.41	23.58	10.45	
−5	0.4185		24.92	10.43	25.00	10.46	25.15	10.53	25.23	10.56	
		Mean		+10.20		+10.24		+10.28		+10.32	
		Error		0.20		0.24		0.28		0.32	

Method B

a = 10 div $\Delta y = 10'$

Scale	Time	Δ	Time	Δ	Time	Δ	Time	Δ	Time	Δ	Time
6	$48^{s}.59$				$438^{s}.17$				$827^{s}.75$		
5	56.86				446.54				836.22		
4	64.36	$257^{s}.86$	$322^{s}.22$	$131^{s}.90$	454.12	$258^{s}.31$	$712^{s}.43$	$131^{s}.45$	843.88	$258^{s}.76$	$1102^{s}.64$
3	71.36	244.02	315.38	145.80	461.18	244.36	705.54	145.46	851.00	244.70	1095.70
2	78.02	230.79	308.81	159.08	467.89	231.03	698.92	158.84	857.76	231.27	1089.03
1	84.45	217.96	302.41	171.95	474.36	218.12	692.48	171.79	864.27	218.28	1082.55
0	90.75	205.40	296.15	184.55	480.70	205.47	686.17	184.48	870.65	205.54	1076.19
-1	96.98	192.93	289.91	197.07	486.98	192.91	679.89	197.09	876.98	192.89	1069.87
-2	103.22	180.41	283.63	209.63	493.26	180.31	673.57	209.73	883.30	180.21	1063.51
-3	109.49	167.79	277.28	222.29	499.57	167.61	667.18	222.47	889.65	167.43	1057.08
-4	115.92	154.83	270.75	235.30	506.05	154.55	660.60	235.58	896.18	154.27	1050.45
-5			263.95				653.75				1043.55
-6			256.79				646.52				1036.25
	K		Δt	Δy	Δt	Δy	Δt	Δy	Δt	Δy	
4	0.4430		$+22^{s}.56$	$+9'.99$	$+22^{s}.65$	$+10'.03$	$+22^{s}.73$	$+10'.07$	$+22^{s}.82$	$+10'.11$	
3	0.4610		21.73	10.02	21.81	10.05	21.89	10.09	21.97	10.13	
2	0.4735		21.16	10.02	21.23	10.05	21.30	10.09	21.37	10.12	
1	0.4809		20.89	10.05	20.96	10.08	21.03	10.11	21.10	10.15	
0	0.4833		20.85	10.08	20.92	10.11	20.99	10.14	21.06	10.18	
-1	0.4809		20.98	10.09	21.05	10.12	21.12	10.16	21.19	10.19	
-2	0.4735		21.33	10.10	21.45	10.16	21.47	10.17	21.54	10.20	
-3	0.4610		21.99	10.14	22.07	10.17	22.15	10.21	22.23	10.25	
-4	0.4430		22.93	10.16	23.01	10.19	23.10	10.23	23.18	10.27	
		Mean		+10.08		+10.11		+10.14		+10.18	
		Error		0.08		0.11		0.14		0.18	

<u>Method A</u>

<u>a = 10 div</u> <u>Δy = 0'</u>

Scale	Time	Δ	Time	Δ	Time	Δ	Time	Δ	Time	Δ	Time
5	$55^{s}.55$	$260^{s}.94$	$316^{s}.49$	$128^{s}.73$	$445^{s}.22$	$261^{s}.44$	$706^{s}.66$	$128^{s}.23$	$834^{s}.89$	$261^{s}.94$	$1096^{s}.83$
4	63.15	246.74	309.89	143.00	452.89	247.12	700.01	142.62	842.63	247.50	1090.13
3	70.21	233.27	303.48	156.49	459.97	233.58	693.55	156.18	849.73	233.89	1083.62
2	76.92	220.26	297.18	169.60	466.78	220.43	687.21	169.43	856.64	220.60	1077.24
1	84.40	207.54	290.94	182.36	473.30	207.63	680.93	182.27	863.20	207.72	1070.92
0	89.71	194.97	284.68	194.98	479.66	194.97	674.63	194.98	869.61	194.97	1064.58
-1	95.95	182.40	278.35	207.59	485.94	182.31	668.25	207.68	875.93	182.22	1058.15
-2	102.17	169.68	271.85	220.35	492.20	169.51	661.71	220.52	882.23	169.34	1051.57
-3	108.44	156.65	265.09	233.42	498.51	156.34	654.85	233.73	888.58	156.03	1044.61
-4	114.83	143.19	258.02	246.93	504.95	142.81	647.76	247.31	895.07	142.43	1037.50
-5	121.40	128.99	250.39	261.18	511.57	128.49	640.06	261.68	901.74	127.99	1029.73

	K		Δt	Δy	Δt	Δy	Δt	Δy	Δt	Δy'
5	0.4185		$-0^{s}.24$	$-0'.10$	$-0^{s}.24$	$-0'.10$	$-0^{s}.24$	$-0'.10$	$-0^{s}.24$	$-0'.10$
4	0.4430		-0.19	-0.08	-0.19	-0.08	-0.19	-0.08	-0.19	-0.08
3	0.4610		-0.15	-0.07	-0.15	-0.07	-0.15	-0.07	-0.15	-0.07
2	0.4735		-0.09	-0.04	-0.09	-0.04	-0.09	-0.04	-0.09	-0.04
1	0.4809		-0.05	-0.02	-0.05	-0.02	-0.05	-0.02	-0.05	-0.02
0	0.4833		-0.01	0	-0.01	0	-0.01	0	-0.01	0
-1	0.4809		+0.04	+0.02	+0.04	+0.02	+0.04	+0.02	+0.04	+0.02
-2	0.4735		+0.08	+0.04	+0.08	+0.04	+0.08	+0.04	+0.08	+0.04
-3	0.4610		+0.16	+0.07	+0.16	+0.07	+0.16	+0.07	+0.16	+0.07
-4	0.4430		+0.19	+0.08	+0.19	+0.08	+0.19	+0.08	+0.19	+0.08
-5	0.4185		+0.26	+0.11	+0.26	+0.11	+0.26	+0.11	+0.26	+0.11
		Mean		0.00		0.00		0.00		0.00
		Error		0		0		0		0

Method B

a = 10 div　　　　　　　　　　$\Delta y = 0'$

Scale	Time	Δ	Time	Δ	Time	Δ	Time	Δ	Time	Δ	Time
6	$55^{s}.55$				$445^{s}.22$				$834^{s}.89$		
5	63.15				452.89				842.63		
4	70.21	$246^{s}.28$	$316^{s}.49$	$143^{s}.48$	459.97	$246^{s}.69$	$706^{s}.66$	$143^{s}.07$	849.73	$247^{s}.10$	$1096^{s}.83$
3	76.92	232.97	309.89	156.89	466.78	233.23	700.01	156.63	856.64	233.49	1090.13
2	83.40	220.08	303.48	169.82	473.30	220.25	693.55	169.65	863.20	220.42	1083.62
1	89.71	207.47	297.18	182.48	479.66	207.55	687.21	182.40	869.61	207.63	1077.24
0	95.95	194.99	290.94	195.00	485.94	194.99	680.93	195.00	875.93	194.99	1070.92
−1	102.17	182.51	284.68	207.52	492.20	182.43	674.63	207.60	882.23	182.35	1064.58
−2	108.44	169.91	278.35	220.16	498.51	169.74	668.25	220.33	888.58	169.57	1058.15
−3	114.83	157.02	271.85	233.10	504.95	156.76	661.71	233.36	895.07	156.50	1051.57
−4	121.40	143.69	265.09	246.48	511.57	143.28	654.85	246.89	901.74	142.87	1044.61
−5			258.02				647.76				1037.50
−6			250.39				640.06				1029.73
	K		Δt	Δy	Δt	Δy	Δt	Δy	Δt	Δy	
4	0.4430		$-0^{s}.20$	$-0'.09$	$-0^{s}.20$	$-0'.09$	$-0^{s}.20$	$-0'.09$	$-0^{s}.20$	$-0'.09$	
3	0.4610		−0.13	−0.06	−0.13	−0.06	−0.13	−0.06	−0.13	−0.06	
2	0.4735		−0.08	−0.04	−0.08	−0.04	−0.08	−0.04	−0.08	−0.04	
1	0.4809		−0.05	−0.02	−0.05	−0.02	−0.05	−0.02	−0.05	−0.02	
0	0.4833		−0.01	0	−0.01	0	−0.01	0	−0.01	0	
− 1	0.4809		+0.03	+0.01	+0.03	+0.01	+0.03	+0.01	+0.03	+0.01	
− 2	0.4735		+0.09	+0.04	+0.09	+0.04	+0.09	+0.04	+0.09	+0.04	
− 3	0.4610		+0.13	+0.06	+0.13	+0.06	+0.13	+0.06	+0.13	+0.06	
− 4	0.4430		+0.21	+0.09	+0.21	+0.09	+0.21	+0.09	+0.21	+0.09	
		Mean		0.00		0.00		0.00		0.00	
		Error		0		0		0		0	

Method A

a = 15 div   Δy = 0'

Scale	Time	Δ	Time	Δ	Time	Δ	Time	Δ	Time	Δ	Time
5	$70^s.79$	$237^s.48$	$308^s.27$	$152^s.33$	$460^s.60$	$237^s.78$	$698^s.38$	$152^s.03$	$850^s.41$	$238^s.08$	$1088^s.49$
4	75.28	228.73	304.01	161.12	465.13	228.96	694.09	160.89	854.98	229.19	1084.17
3	79.63	220.17	299.80	169.71	469.51	220.34	689.85	169.54	859.39	220.51	1079.90
2	83.92	211.71	295.63	178.20	473.83	211.82	685.65	178.09	863.74	211.93	1075.67
1	88.14	203.32	291.46	186.62	478.08	203.37	681.45	186.57	868.02	203.42	1071.44
0	92.31	194.98	287.29	194.99	482.28	194.98	677.26	194.99	872.25	194.98	1067.23
-1	96.47	186.63	283.10	203.36	486.46	186.58	673.04	203.41	876.45	186.53	1062.98
-2	100.62	178.25	278.87	211.77	490.64	178.14	668.78	211.88	880.66	178.03	1058.69
-3	104.77	169.81	274.58	220.24	494.82	169.64	664.46	220.41	884.87	169.47	1054.34
-4	108.97	161.23	270.20	228.85	499.05	161.00	660.05	229.08	889.13	160.77	1049.90
-5	113.22	152.47	265.69	237.64	503.33	152.17	655.50	237.94	893.44	151.87	1045.31

Scale	K		Δt	Δy	Δt	Δy	Δt	Δy	Δt	Δy
5	0.6835		$-0^s.16$	$-0'.11$	$-0^s.16$	$-0'.11$	$-0^s.16$	$-0'.11$	$-0^s.16$	$-0'.11$
4	0.6987		-0.12	-0.08	-0.12	-0.08	-0.12	-0.08	-0.12	-0.08
3	0.7103		-0.07	-0.05	-0.07	-0.05	-0.07	-0.05	-0.07	-0.05
2	0.7185		-0.06	-0.04	-0.06	-0.04	-0.06	-0.04	-0.06	-0.04
1	0.7233		-0.04	-0.03	-0.04	-0.03	-0.04	-0.03	-0.04	-0.03
0	0.7250		-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
-1	0.7233		+0.01	+0.01	+0.01	+0.01	+0.01	+0.01	+0.01	+0.01
-2	0.7185		+0.05	+0.04	+0.05	+0.04	+0.05	+0.04	+0.05	+0.04
-3	0.7103		+0.10	+0.07	+0.10	+0.07	+0.10	+0.07	+0.10	+0.07
-4	0.6987		+0.11	+0.08	+0.11	+0.08	+0.11	+0.08	+0.11	+0.08
-5	0.6835		+0.14	+0.10	+0.14	+0.10	+0.14	+0.10	+0.14	+0.10
		Mean		0.00		0.00		0.00		0.00
		Error		0		0		0		0

Method B

a = 15 div $\qquad$ Δy = 0'

Scale	Time	Δ	Time	Δ	Time	Δ	Time	Δ	Time	Δ	Time
6	$70^{s}.79$				$460^{s}.60$				$850^{s}.41$		
5	75.28				465.13				854.98		
4	79.63	$228^{s}.64$	$308^{s}.27$	$161^{s}.24$	469.51	$228^{s}.87$	$698^{s}.38$	$161^{s}.01$	859.39	$229^{s}.10$	$1088^{s}.49$
3	83.92	220.09	304.01	169.82	473.83	220.26	694.09	169.65	863.74	220.43	1084.17
2	88.14	211.66	299.80	178.28	478.08	211.77	689.85	178.17	868.02	211.88	1079.90
1	92.31	203.32	295.63	186.65	482.28	203.37	685.65	186.60	872.25	203.42	1075.67
0	96.47	194.99	291.46	195.00	486.46	194.99	681.45	195.00	876.45	194.99	1071.44
-1	100.62	186.67	287.29	203.35	490.64	186.62	677.26	203.40	880.66	186.57	1067.23
-2	104.77	178.33	283.10	211.72	494.82	178.22	673.04	211.83	884.87	178.11	1062.98
-3	108.97	169.90	278.87	220.18	499.05	169.73	668.78	220.35	889.13	169.56	1058.69
-4	113.22	161.36	274.58	228.75	503.33	161.13	664.46	228.98	893.44	160.90	1054.34
-5			270.20				660.05				1049.90
-6			265.69				655.50				1045.31
	K		Δt	Δy	Δt	Δy	Δt	Δy	Δt	Δy	
4	0.6987		$-0^{s}.11$	-0'.08	$-0^{s}.11$	-0'.08	$-0^{s}.11$	-0'.08	$-0^{s}.11$	-0'.08	
3	0.7103		-0.09	-0.06	-0.09	-0.06	-0.09	-0.06	-0.09	-0.06	
2	0.7185		-0.06	-0.04	-0.06	-0.04	-0.06	-0.04	-0.06	-0.04	
1	0.7233		-0.03	-0.02	-0.03	-0.02	-0.03	-0.02	-0.03	-0.02	
0	0.7250		-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	
-1	0.733		+0.02	+0.01	+0.02	+0.01	+0.02	+0.01	+0.02	+0.01	
-2	0.7185		+0.05	+0.04	+0.05	+0.04	+0.05	+0.04	+0.05	+0.04	
-3	0.7103		+0.08	+0.06	+0.08	+0.06	+0.08	+0.06	+0.08	+0.06	
-4	0.6987		+0.12	+0.08	+0.12	+0.08	+0.12	+0.08	+0.12	+0.08	
		Mean		0.00		0.00		0.00		0.00	
		Error		0		0		0		0	

The error in the means derived from each of the previous calculations are shown below in the form of a summary in which the columns headed 1,2,3,4 refer to successive values of the errors as the observations progress.

Summary of errors (unit 0.01')

	Method A								Method B							
	a=10				a=15				a=10				a=15			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Δy=0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Δy=10'	20	24	28	32	10	14	17	21	8	11	14	18	6	10	13	17

It should be noted that the mathematical model assumes "a" to be the amplitude at t=0. In practice amplitude readings are made on the auxiliary scale at the beginning and end of each determination and the mean value is used in the calculation. The effect of this approximation would be to smooth the errors slightly.

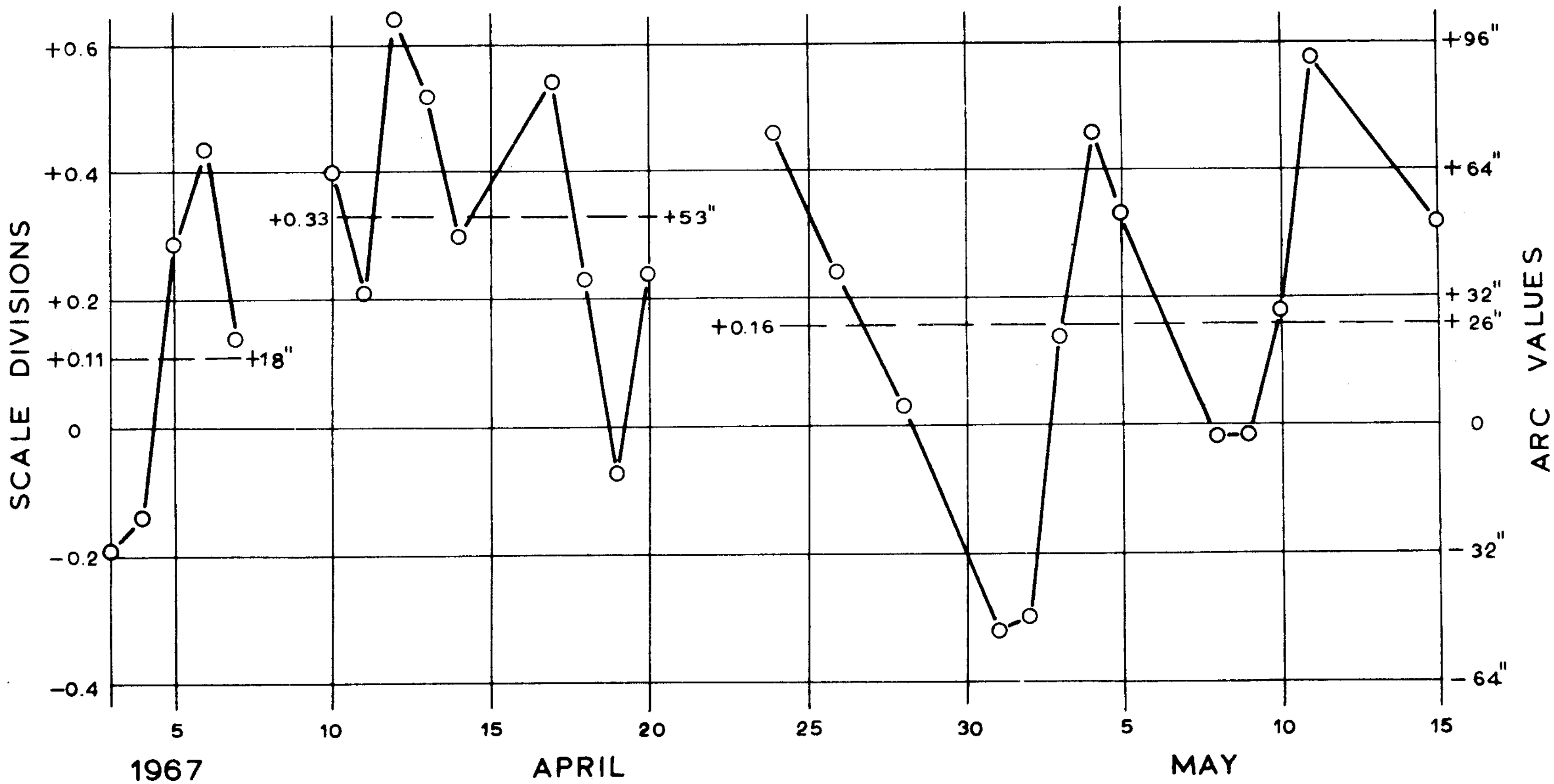


FIG. 3:- NON-SPINNING GYRO READINGS

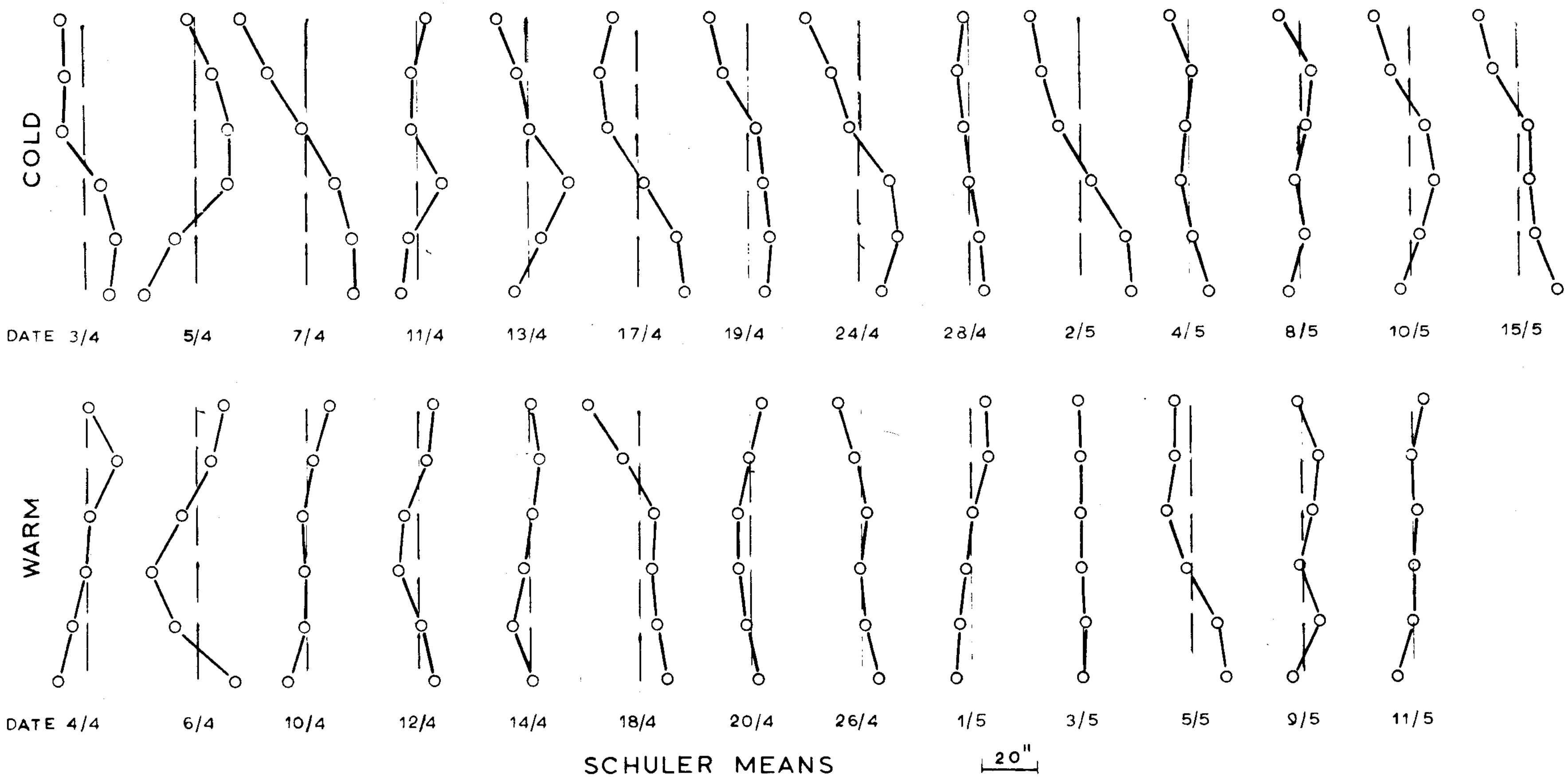


FIG. 4:- TURNING POINT METHOD

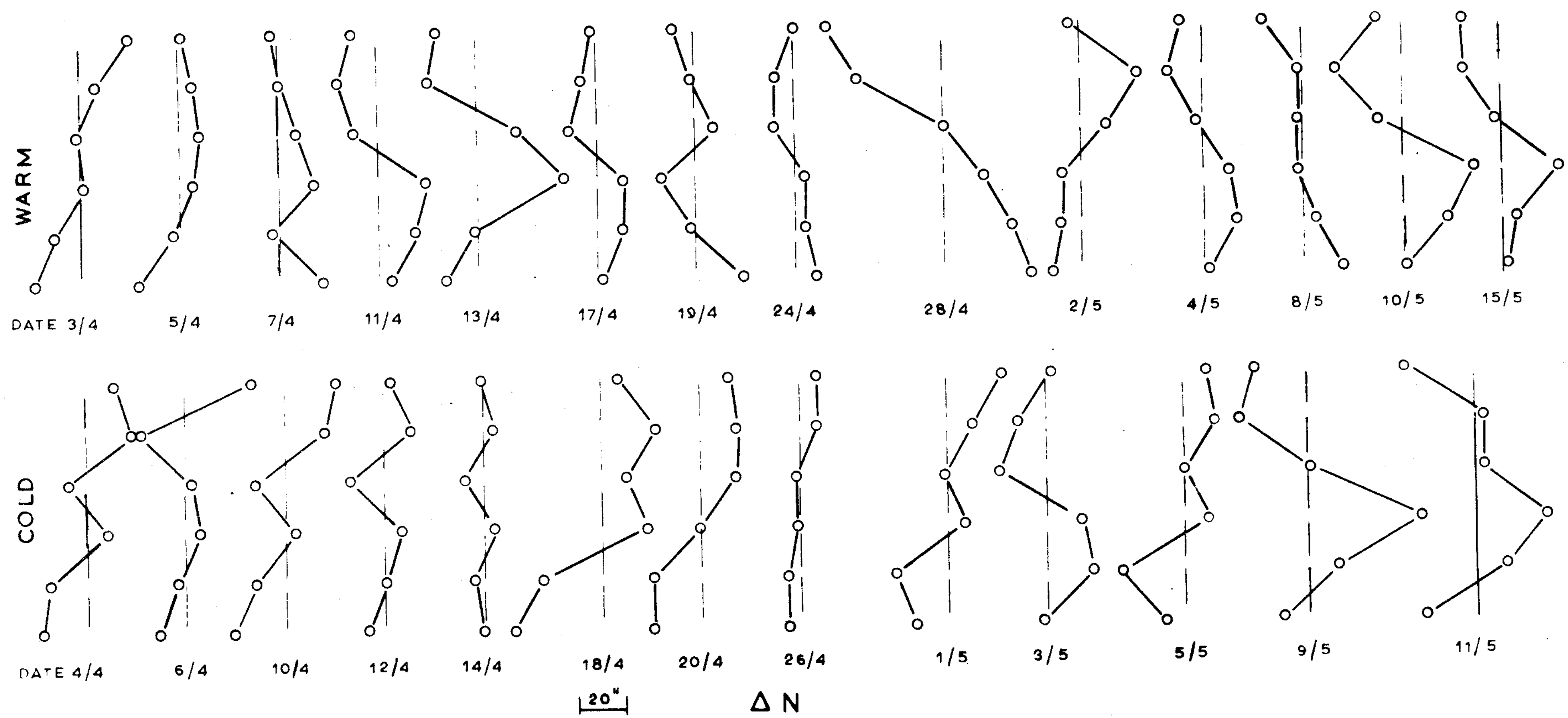
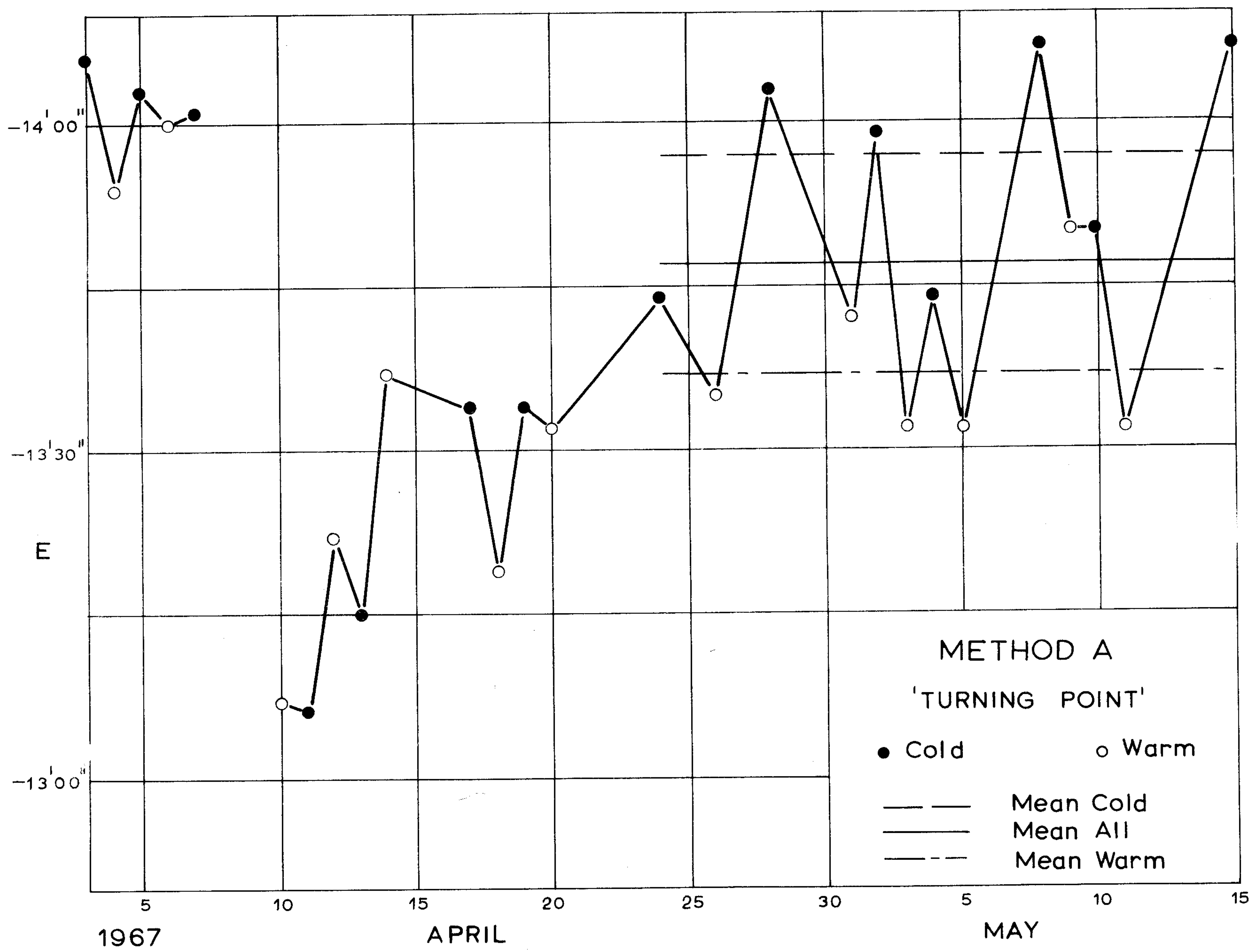
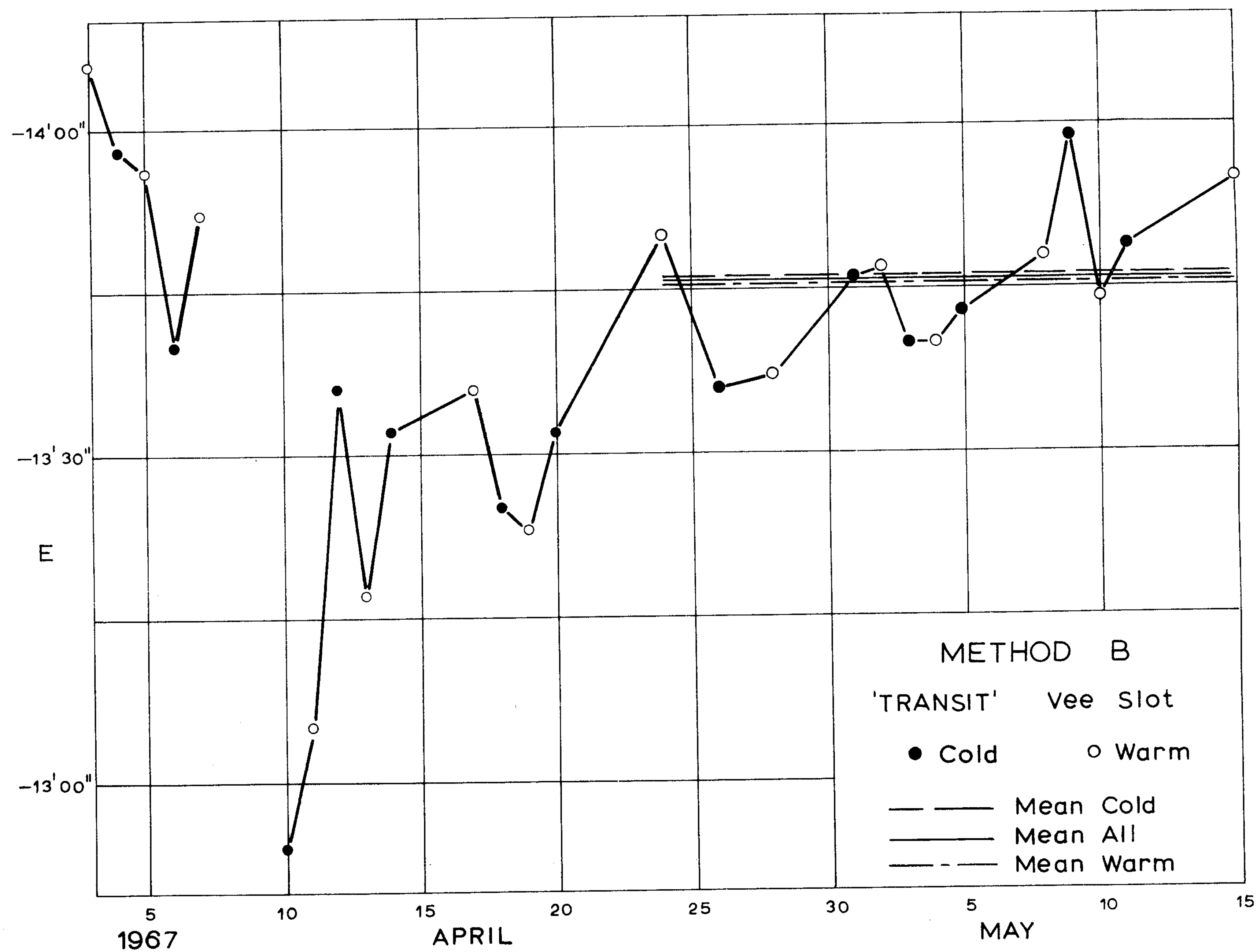


FIG. 5 TRANSIT METHOD



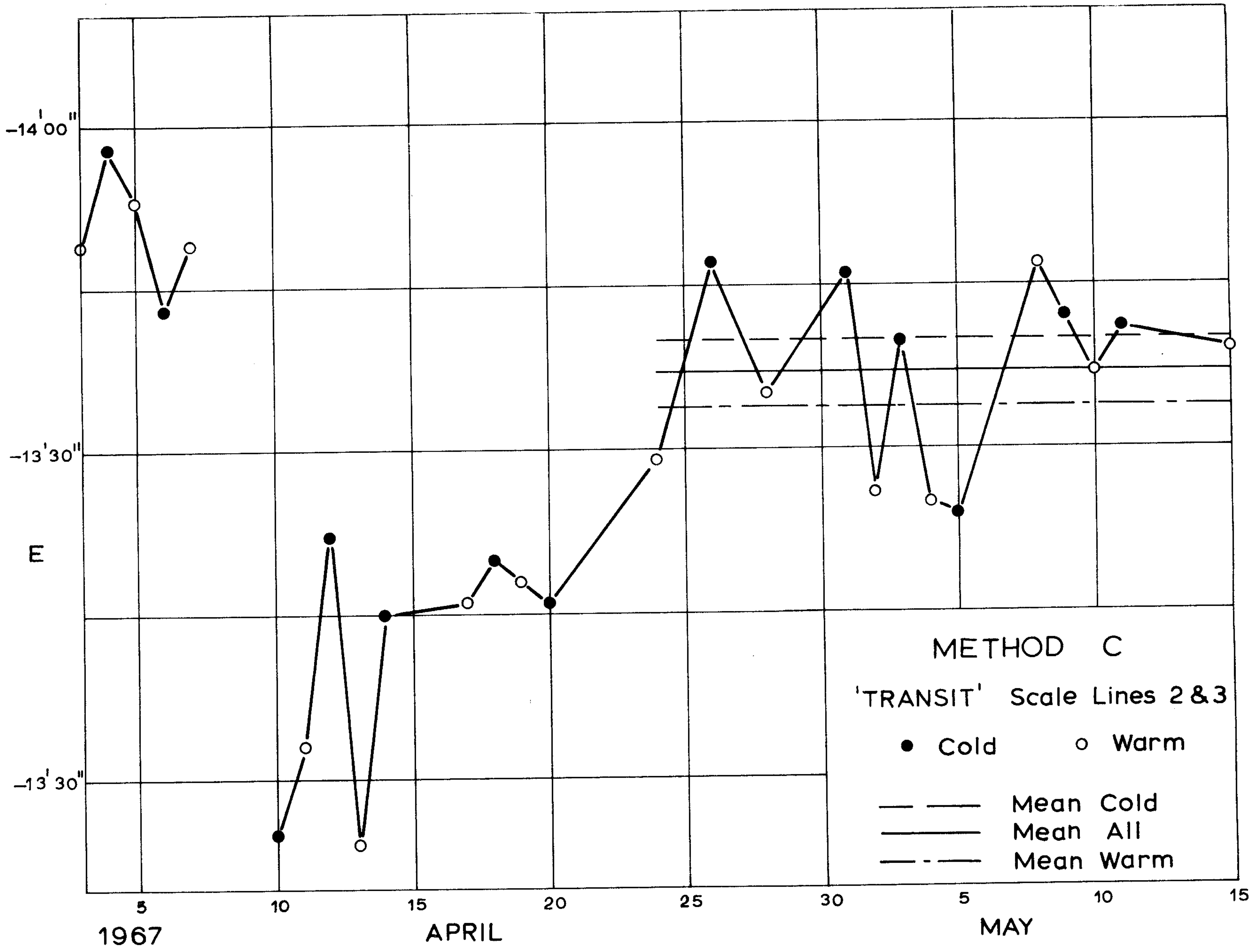
METHOD A
'TURNING POINT'
● Cold
○ Warm
Mean Cold
Mean All
Mean Warm
−14' 00"
−13' 30"
−13' 00"
E
5
10
15
20
25
30
5
10
15
1967
APRIL
MAY

FIG. 6



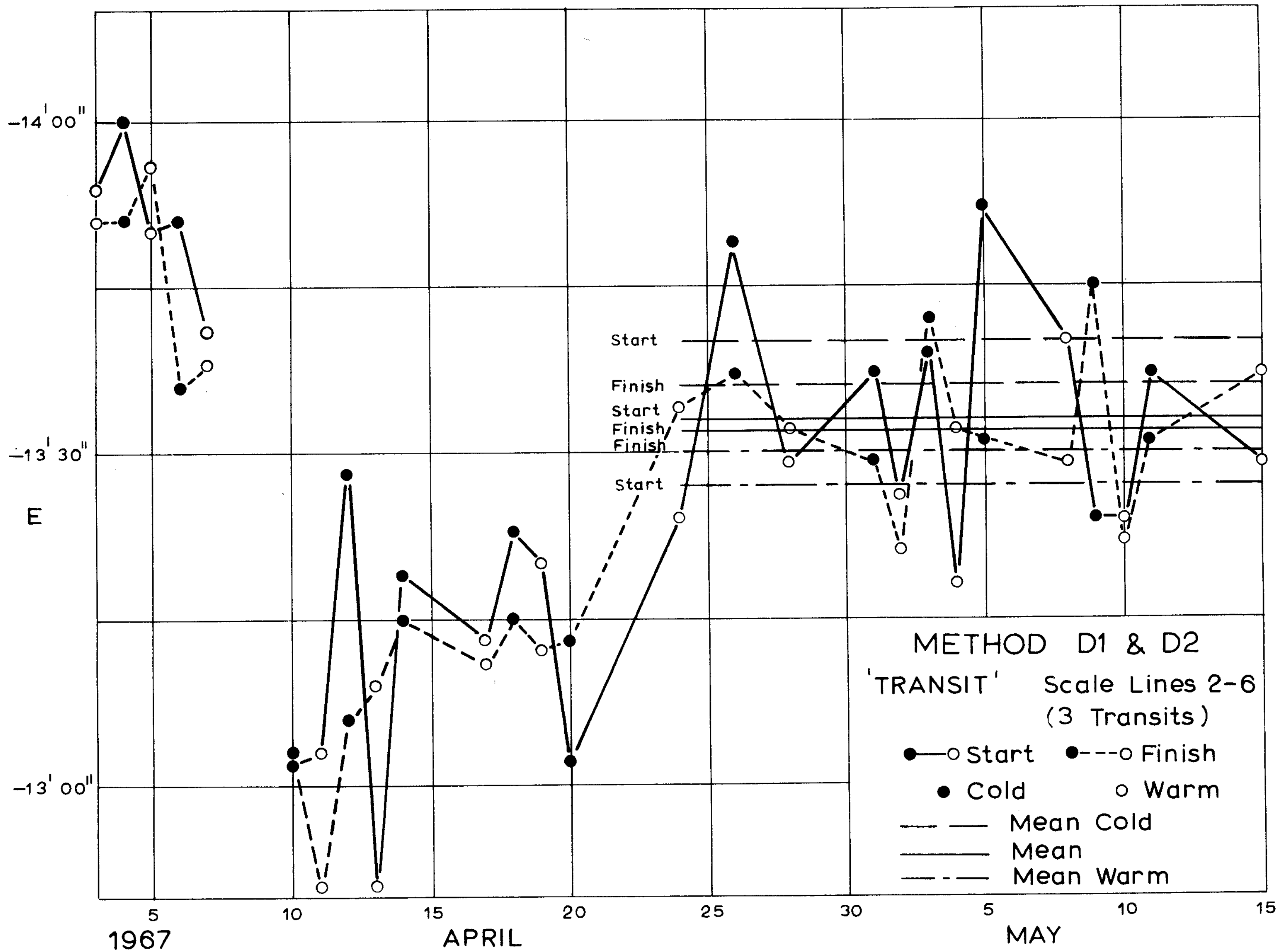
METHOD B
'TRANSIT' Vee Slot
Cold
Warm
Mean Cold
Mean All
Mean Warm
−14' 00"
−13' 30"
−13' 00"
E
5
10
15
20
25
30
5
10
15
1967
APRIL
MAY

FIG. 7



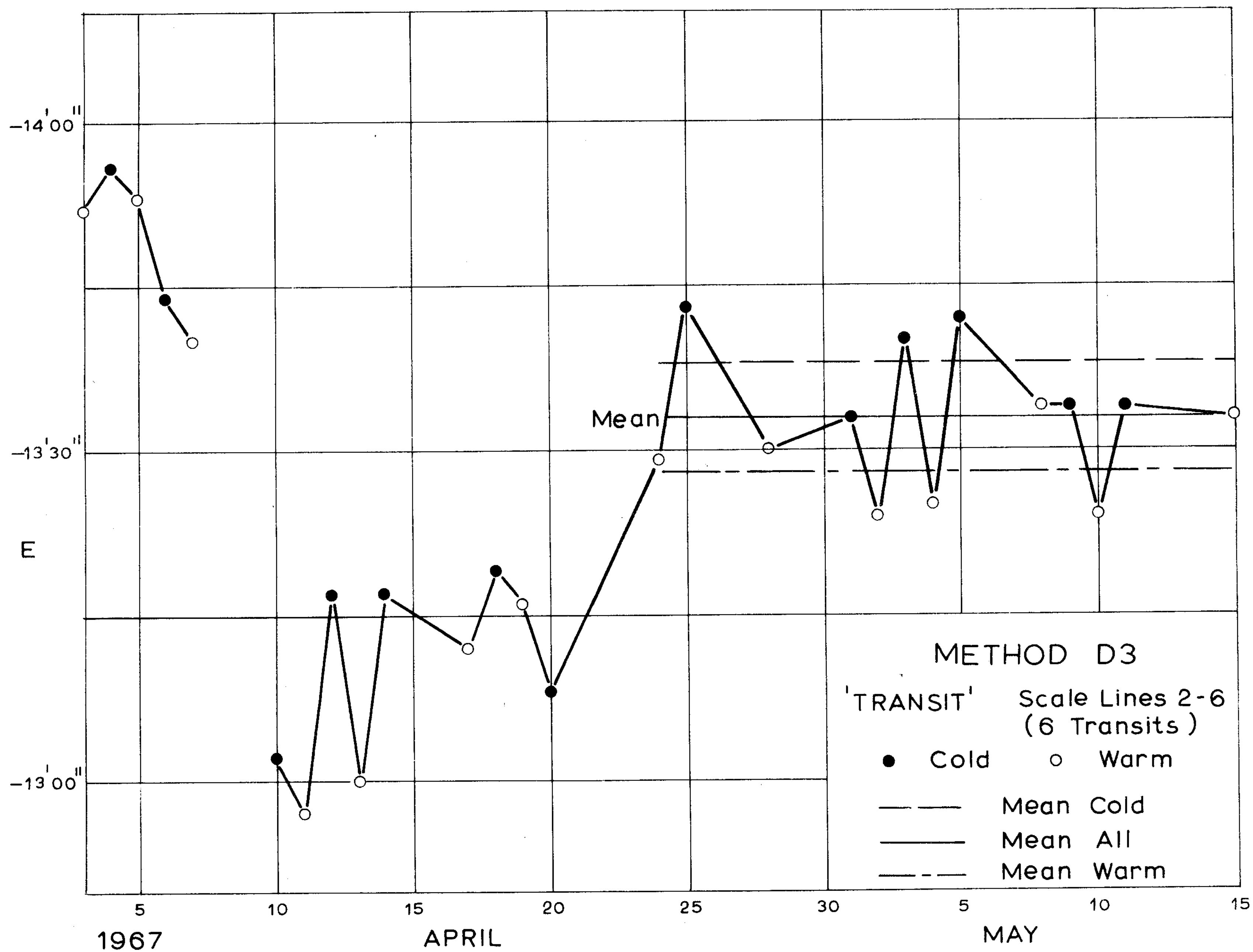
-14'00"
-13'30"
-13'30"
E
METHOD C
'TRANSIT' Scale Lines 2 & 3
Cold
Warm
Mean Cold
Mean All
Mean Warm
5
10
15
20
25
30
5
10
15
1967
APRIL
MAY

FIG. 8



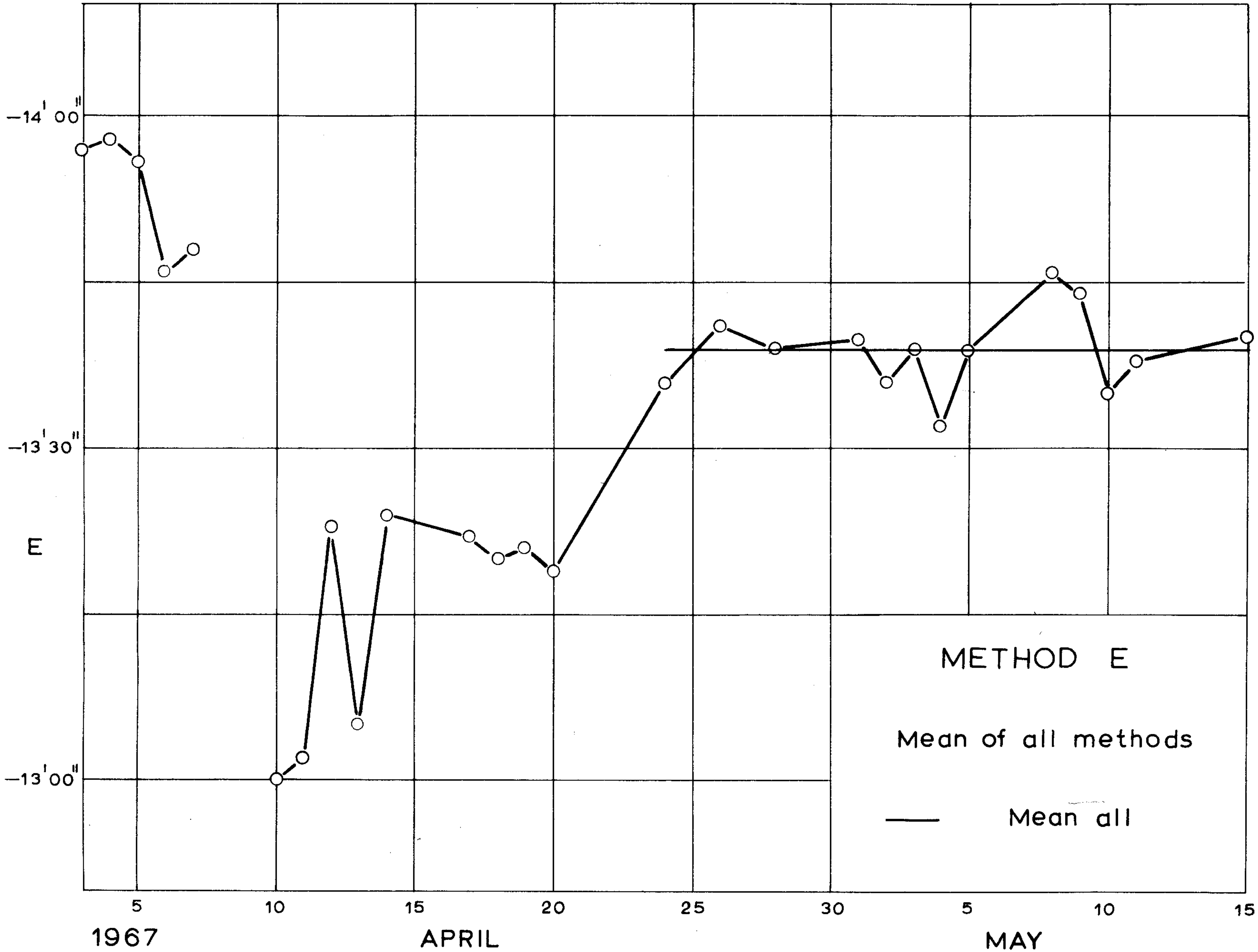
METHOD D1 & D2
'TRANSIT' Scale Lines 2-6
(3 Transits)
Start
Finish
Cold
Warm
Mean Cold
Mean
Mean Warm
Start
Finish
Start
Finish
Finish
Start
-14' 00"
-13' 30"
-13' 00"
E
5
10
15
20
25
30
5
10
15
1967
APRIL
MAY

FIG. 9



METHOD D3
'TRANSIT' Scale Lines 2-6
(6 Transits)
Cold
Warm
Mean Cold
Mean All
Mean Warm
Mean
−14'00"
−13'30"
−13'00"
E
5
10
15
20
25
30
5
10
15
1967
APRIL
MAY

FIG. 10



−14′ 00″
−13′ 30″
−13′ 00″
E
5
10
15
20
25
30
5
10
15
1967
APRIL
MAY
METHOD E
Mean of all methods
Mean all

FIG. 11

APPENDIX C

THE MODIFIED TURNING POINT METHOD

Adopting a similar mathematical model to that used for the proof of the modified transit method, i.e.

$$y = Be^{-\alpha\tau} \cos 2\pi\tau \qquad (1)$$

where B = ma, then "turning points" y_i will occur when $\tau = 0, \frac{1}{2}, 1 \ldots$ etc.

At the "turning point"

$$y_i = Be^{-\alpha\left(\frac{i-1}{2}\right)} (-1)^{i-1} \qquad (2)$$

where i is a positive integer and corresponds to the number of the "turning point".

Let $\tau = \frac{i-1}{2} + \Delta\tau$, and substituting this in (1) gives

$$y = B(-1)^{i-1} e^{-\alpha\left(\frac{i-1}{2}\right)} e^{-\alpha\Delta\tau} \cos 2\pi\Delta\tau$$

since $\sin \pi(i-1) = 0$ and $\cos (i-1) = (-1)^{i-1}$

A series expansion of the exponential and cosine quantities gives

$$y = B(-1)^{i-1}\left(1 - \alpha\left(\frac{i-1}{2}\right) + \frac{\alpha^2 (i-1)^2}{2 ! \, 4} \ldots\right)\left(1 - \alpha\Delta\tau + \frac{\alpha^2\Delta\tau^2}{2!} \ldots\right)$$
$$\left(1 - \frac{(2\pi)^2 \Delta\tau^2}{2 !} + \frac{(2\pi)^4 \Delta\tau^4}{4 !} \ldots\right) \qquad (3)$$

Put $\Delta y = y_i - y$, and from (2) and (3),

$$\Delta y = \frac{B(-1)^{i-1} (2\pi)^2 \Delta\tau^2}{2 !} - \frac{B(-1)^{i-1} (2\pi)^4 \Delta\tau^4}{4 !} \ldots\ldots\ldots\ldots$$

$$+ B(-1)^{i-1}\,\alpha\Delta\tau\left(1 - \frac{(2\pi)^2\Delta\tau^2}{2\,!} + \frac{(2\pi)^4\Delta\tau^4}{4\,!}\,\ldots\ldots\right)$$

$$- \frac{B(-1)^{i-1}\alpha(i-1)}{2}\left(\frac{(2\pi)^2\Delta\tau^2}{2\,!} - \frac{(2\pi)^4\Delta\tau^4}{4\,!}\,\ldots\ldots\right)$$

$$- \frac{B(-1)^{i-1}\alpha^2(i-1)\Delta\tau}{2}\left(1 - \frac{(2\pi)^2\Delta\tau^2}{2\,!} + \frac{(2\pi)^4\Delta\tau^4}{4\,!}\,\ldots\right) \qquad (4)$$

In order to estimate the number of terms required for a practical solution, each term in (4) has been evaluated using conservative values of the parameters B, α and $\Delta\tau$, i.e. $B = 180'$, $\alpha = \frac{1}{150}$, $\Delta\tau = 0.1$.

Term	i=1	i=2	i=3	i=4
1	+35.53'	-35.53'	+35.53'	-35.53'
2	- 1.17	+ 1.17	- 1.17	+ 1.17
3	+ 0.10	- 0.10	+ 0.10	- 0.10
4	0	+ 0.11	- 0.23	+ 0.34
5	0	0	0	0
$\Sigma = \Delta y$	+34.46	-34.35	+34.23	-34.12

In practice it would be convenient to use term 1 only. The remaining terms could be reduced in magnitude by using a small amplitude and/or a small time difference. The neglect of terms other than the first may be significant, leading to a distortion of the value of the turning point which is derived from the mean of the corrected observations. This distortion will be such as to displace each mean turning point away from the line of mean oscillation by a constant amount if the observations are distributed in time in the same manner about each turning point. However, there will be negligible effect on the resulting Schuler Means. The previous numerical example will serve to illustrate this aspect where each observation is made at a time instant $\Delta\tau = 0.1$ before or after the turning point.

	Observation	Correction(1st term)	Corrected observ. L	Corrected observ. R	Schuler Mean
y_1	+145.54'	+35.53'	+181.07'		
y_2	-145.05	-35.53	(+180.58)	-180.58'	0.00'
y_3	+144.57	+35.53	+180.10	(-180.10)	0.00
y_4	-144.09	-35.53		-179.62	

Even though the effect on the final values of the Schuler Means of neglecting all terms other than the first will be minute, it must be recognised that each observation after reduction will suffer from a systematic effect and thus the error distribution will be distorted.

EXAMPLE OF THE MODIFIED TURNING POINT METHOD.

Byrd Station, Antarctica. Line Jamesway to New Byrd Astro (Ecce).

Date: 2nd December 1968.

Hor. Circle		t		Δt	AΔt²	(AΔt²)²/3B'	T.P.
0°	00'	0ᵐ	00ˢ	to			
356	52.0	3	11.9	39.3ˢ	-6.98'	+0.04'	45.06'
	48.0	3	24.8	26.4	3.15	0.01	44.86
	45.0	3	41.8	9.4	0.40	0	44.60
	44.8	(3	51.2)				44.80
	47.5	4	15.7	24.5	2.71	0.01	44.80
	53.0	4	33.5	42.3	8.09	0.06	44.97
356	60.0	4	49.7	58.5	-15.47	+0.21	44.74
0	00	7	42.4	t_1		Mean	(44.83)
3	04.0	10	48.0	43.6	+ 8.59	-0.06	12.53
	10.0	11	07.3	24.3	2.67	0.01	12.66
	13.0	11	26.5	5.1	0.12	0	13.12
	13.3	(11	31.6)				13.30
	09.5	11	59.9	28.3	3.62	0.01	13.11
3	03.0	12	18.3	46.7	9.86	0.08	12.78
2	56.0	12	33.7	62.1	+17.43	-0.26	13.17
0	00	15	20.7	t_2		Mean	(12.95)
356	55.0	18	28.8	42.2	- 8.05	+0.06	47.01
	49.0	18	49.7	21.3	2.05	0	46.95
	47.0	19	07.3	03.7	0.06	0	46.94
	46.8	(19	11.0)				46.80
	50.0	19	37.7	26.7	3.22	0.01	46.79
	54.3	19	51.3	40.3	7.34	0.05	47.01
356	64.0	20	13.5	62.5	-17.66	+0.27	46.61
0	00	23	01.3	t_3		Mean	(46.87)

Period $T = t_2 - t_o = 15^m \; 20.7^s$

$t_3 - t_1 = 15 \; 18.9$

Mean $15 \; 19.8 \quad (919.8^s)$

L		R		
356°	44.8'			
(356	45.8)	3° 13.3'	B' = difference	= 6°27.5'
356	46.8			= 387.5'
			3B'	= 1162'

$$A = \frac{B'\pi^2}{T^2} = 0.00452$$

L		R	Schuler Mean
356°	44.83'		
(356	45.85)	3° 12.95'	359° 59.40'
356	46.87		

Mean R.O. Circle Reading	26°	50.03'
Circle Reading of G.I.N.	359	59.40
Gyro Azimuth	26	50.63
E		-12.93
Azimuth R.O.	26	37.70

APPENDIX D

Analysis of results of the observations made in the period 24th April to 15th May 1967.

	Method	State	n	Range	S.D.	Mean	Difference Cold - Warm
A	Turning Point	All	13	35"	±13"	47"	
	(8 T.P.'s)	Cold	7	23	10	57	+20"*
		Warm	6	18	7	37	
	Turning Point	All	13	49	17	52	
	(4 T.P.'s)	Cold	7	32	12	65	+27 *
		Warm	6	17	6	38	
B	Transit (Vee Slot)	All	13	23	7	46	
	(8 Transits)	Cold	6	23	8	46	0
		Warm	7	18	6	46	
	Transit (Vee Slot)	All	13	36	12	44	
	(4 Transits)	Cold	6	28	11	44	0
		Warm	7	36	13	44	
C	Transit	All	13	23	8	37	
	(8 Transits)	Cold	6	23	8	40	+6
	(Scale lines 2 & 3)	Warm	7	21	8	34	
D1	Transit	All	13	28	10	33	
	(Scale lines 2 to 6)	Cold	6	28	10	40	+13*
	First 3 transits)	Warm	7	22	7	27	
D2	Transit	All	13	24	7	32	
	(Scale lines 2 to 6)	Cold	6	16	7	36	+6
	(Last 3 Transits)	Warm	7	16	6	30	
D3	Transit	All	13	20	6	33	
	(Scale lines 2 to 6)	Cold	6	10	4	38	+10
	(Mean of First & Last 3 Transits)	Warm	7	11	4	28	
E	Mean of all		13	14	4	39	

* Those differences marked with an asterisk are significant at the 5% significance level with the "t" test.

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BIOGRAPHICAL NOTES.

G.G. BENNETT at present holds the appointment of Senior Lecturer in the Department of Surveying, University of New South Wales, to which he was appointed in 1962. He received a First Class Honours degree in Surveying from the University of Melbourne in 1954. After graduation, Mr. Bennett worked for the Snowy Mountains Hydro-Electric Authority from 1954 to 1959 where he specialised in Geodetic Astronomy. He joined the University of New South Wales in 1959 as a lecturer and completed his Master of Surveying degree at the University of Melbourne in 1962. In 1965 he spent a year as research officer with the Geodetic Surveys Section of the Department of Mines and Technical Surveys, Canada.

Mr. Bennett has published papers on both Geodetic Astronomy and the adjustment of control networks. His current research interests include in addition to the above topics, gyro-theodolites and their applications, as well as all aspects of error theory.

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