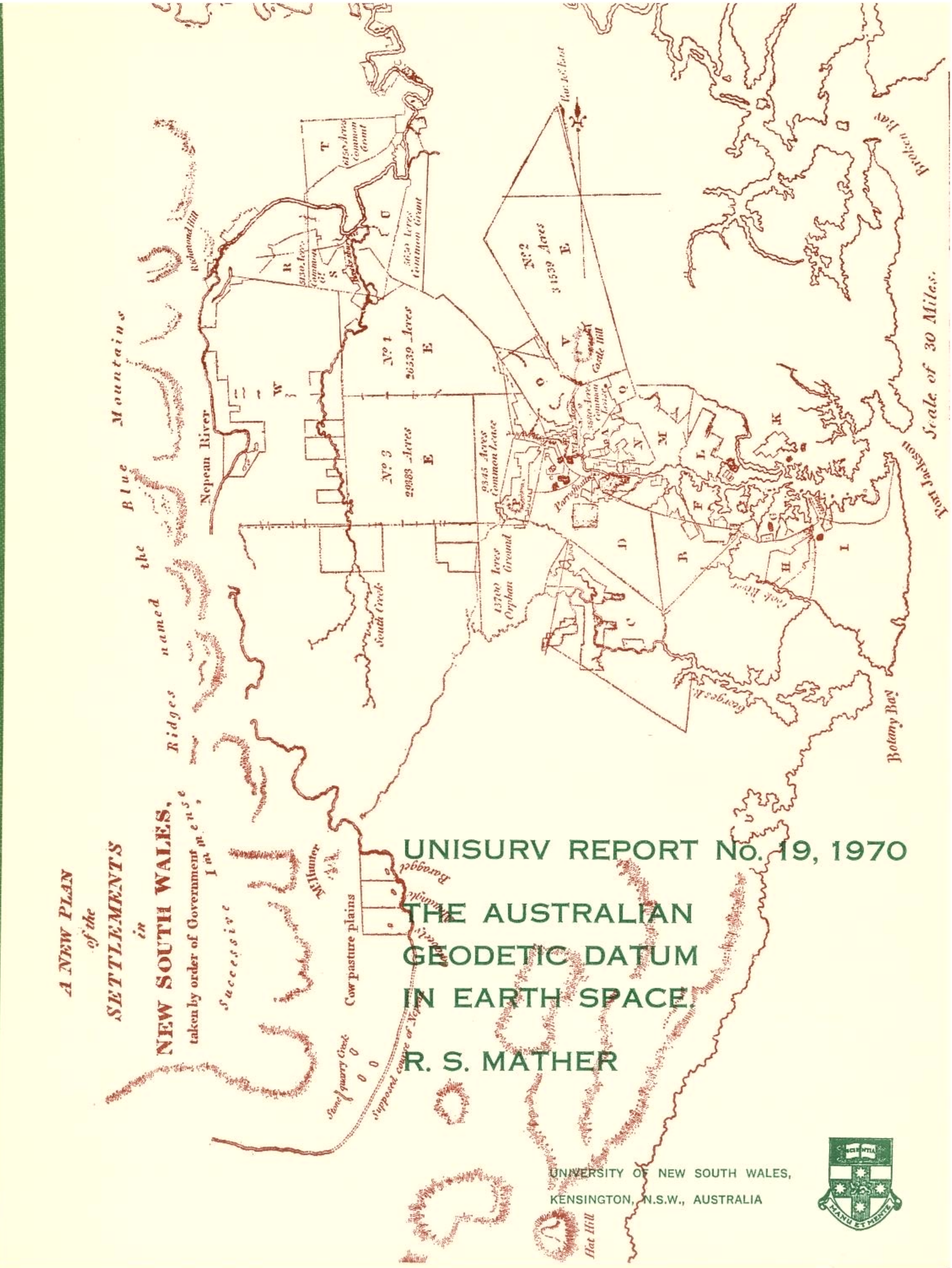

UNISURV REPORT No. 19, 1970

THE AUSTRALIAN GEODETIC DATUM IN EARTH SPACE

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KENSINGTON, N.S.W., AUSTRALIA

Reference to Districts.

A Northern Boundaries
B Liberty Plains
C Banks Town
D Parramatta
EEEE Ground reserved for Govt. purposes
F Concord
G Petersham
H Bulanaming
I Sydney
K Hunters Hills
L Eastern Farms
M Field of Mars
N Ponds
O Toongabbey
P Prospect
Q
R Richmond Hill
S Green Hills
T Phillip
U Nelson
V Castle Hill
W Evan

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U N I S U R V   R E P O R T   NO. 19

THE AUSTRALIAN GEODETIC DATUM IN EARTH SPACE

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Received 3rd April 1970

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SUMMARY

The Australian Geodetic Datum is defined, together with a review of the relationship existing between a complete gravimetric solution and the earth's geocentre. Working formulae are derived for practical computation. It is shown that only the Free Air Geoid need be considered to provide a geocentric orientation of the Australian Geodetic Datum with a precision equivalent to that of the data set currently available. A consistent representation of the gravity anomaly field is used in the computation of the 1970 Free Air Geoid for Australia and the geocentric orientation parameters, obtained either by comparisons at the corners of a one degree grid or by detailed investigations at thirty-eight well-spaced astrogeodetic stations, are in substantial agreement. The required parameters at the Johnston Origin of the Australian Geodetic Datum are

$$\Delta\xi_o = -4.2 \pm 0.2 \text{ sec}$$

$$\Delta\eta_o = -4.5 \pm 0.2 \text{ sec}$$

$$\Delta N_o = -7.2 \pm 0.2 \text{ metres}$$

The error estimate in the last parameter assumes that no significant errors exist in zonal harmonics of degree n and order one ($n < 5$) in the representation of the earth's gravity field which cannot be detected over the 2% of the earth's surface area included in the present study. The consequent error is unlikely to exceed ± 3 metres on current estimates of the accuracy of low degree harmonic coefficients.

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A GUIDE TO NOTATION

1. COMMONLY USED SYMBOLS

a	Equatorial radius of reference spheroid.
A	Azimuth.
C_{nm}	Coefficient of surface harmonic series.
dS	Element of surface area.
$d\sigma$	Element of surface area on unit sphere.
dV	Element of volume
d_i	Components of the separation vector.
$E\{x\}$	Predicted value of X.
f	Flattening of the meridian ellipse.
h	Elevation.
h_o	Orthometric elevation.
h_n	Normal elevation.
k	Gravitational constant.
ℓ_i	Direction cosines.
m	Order of surface harmonic.
m	$a\omega^2/\gamma_e$.
$M\{x\}$	Mean value of X.
n	Degree of a surface harmonic.
N	Geoid/Spheroid separation.
N	Total number of gravity readings for the full representation of an area(Section 3.3).

$\bar{p}_{nm}(\mu)$ $$\left(\frac{(2-\delta_{0m})(n-m)!}{(2n+1)^{-1}(n+m)!}\right)^{\frac{1}{2}} \frac{(1-\mu^2)^{\frac{1}{2}}}{2^n\, n!} \frac{d^{n+m}}{d\mu^{n+m}}\left((\mu^2-1)^n\right) .$$

r — Distance of the general variable point(e.g., dS) from the computation point(e.g., P).

R — Radius of curvature in normal section of reference spheroid; where relevant, the mean radius of the earth.

S_{nm} — Coefficient of surface harmonic series.

U — Potential on the reference system.

V — Potential.

W — Potential of the existent earth (Geopotential).

$x_i\ (i=1,3)$ — A general rectangular Cartesian co-ordinate system in earth space with a local origin.

$X_i\ (i=1,3)$ — As above, but with a geocentric origin.

α — A parameter usually associated with azimuth.

β — Ground slope.

γ — Normal gravity; where relevant, the mean value of normal gravity over the reference spheroid.

Δx — A small change in x.

Δg — Gravity anomaly, which, to the order of the flattening, is the free air anomaly.

$\Delta\xi_{io}\ (i=1,3)$ — Set of curvilinear geocentric orientation parameters at the origin of geodetic datum for conversion to geocentric location. Note that

$$\Delta\xi_{1o} = \Delta\xi_o \ ; \ \Delta\xi_{2o} = \Delta\eta_o \ ; \ \Delta\xi_{3o} = \Delta N_o \ .$$

δ_{ij}	Kronecker delta ($\delta_{ij} = \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}$) .
η	Component of the deflection of the vertical in the prime vertical, positive if the outward vertical is east of the normal; represented as ξ_2 in text.
λ	Longitude, positive east.
ξ	Component of deflection of the vertical in the meridian, positive if outward vertical is north of spheroid normal; represented as ξ_1 in text.
$\xi_i (i=1,3)$	Set of curvilinear parameters defining the separation vector.
ρ	Density.
σ	Surface area on unit sphere.
ϕ	Latitude, positive north.
ψ	Angular distance on unit sphere, usually between the variable point (e.g., dS) and the computation point (e.g., P).
ω	Angular velocity of rotation of the earth.

2. SIGNIFICANCE OF SUBSCRIPTS

Subscripts which are not indices are introduced with the intention of improving the comprehension of concepts by keeping the number of variables down to a minimum and simplifying the written form of the equations.

a	Astronomically determined values ; Astro-geodetic values.
ac	Astro-geodetic values transformed to an equivalent geocentric datum.

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d	Disturbing value; the difference between equivalent values on the true and reference systems.
c	Values at the variable element (e.g., dS).
f	Refers to the Free Air Geoid.
g	Geocentric values; gravimetric values.
G	Values on the geodetic datum; geodetic values.
i	Corrections from Free Air Geoid to height anomaly h_d.
p	Evaluated at the fixed point P.
s	Referred to the spheroid.

3. MISCELLANEOUS POINTS

A.G.D.	Australian Geodetic Datum.
R.S.1967	Reference System 1967.
U.N.S.W.	The gravity anomaly data set prepared from the gravity holdings at the University of New South Wales for the Australian region.
$\mathcal{i}$	Unit vector along the x_i axis.
$\mathcal{N}$	Unit normal vector.
$\mathcal{O}$	Geocentric orientation vector given by the equations

$$\mathcal{O} = \sum_{i=1}^{3} h_{io} \Delta\xi_{io} \mathcal{i}_o = \sum_{i=1}^{3} h_i \Delta\xi_i \mathcal{i} ,$$

where h_i (i=1,3) are the associated linearisation parameters.

THE AUSTRALIAN GEODETIC DATUM IN EARTH SPACE

BY

R. S. MATHER

1. INTRODUCTION

1.1 THE AUSTRALIAN GEODETIC DATUM

The Australian Geodetic Datum (A.G.D.) has been established by the Commonwealth of Australia's Division of National Mapping with the prime purpose of providing a first order geodetic framework for national topographic mapping programs. It is defined by the following parameters :-

(a) The Australian National Spheroid (A.N.S.) whose equatorial radius a and flattening f are given by

$$a = 6{,}378{,}160 \text{ metres}$$
$$f^{-1} = 298.25 \qquad .....(1)$$

and

(b) The geodetic co-ordinates $(\phi_{Go}, \lambda_{Go})$ and the spheroidal elevation (h_{so}) adopted at the Johnston Origin given by (Lambert 1968, p.95)

$$\phi_{Go} = -25^{\circ}\ 56'\ 54.5515''$$
$$\lambda_{Go} = 133^{\circ}\ 12'\ 30.0771'' \qquad .....(2).$$
$$h_{so} = 571.2 \text{ metres}$$

A review of the progressive stages in the definition of this datum is available (Mather & Fryer 1970b), a study of which shows that the geodetic co-ordinates at the origin are based on the mean values of the deflections of the vertical at approximately 150 astro-geodetic stations available in 1963 and well spaced over the six and one half million square kilometre continental area.

The adoption of the numerical mean value of the deflections of the vertical as a datum correction has the effect of fitting the reference spheroid to the *estimate* of the mean geoid slope as defined by the directions of the vertical at the astro-geodetic stations included in the analysis. The matter is discussed further in section 4.2. This procedure is a not uncommon geodetic practice when working over limited areas as it has the considerable advantage of enabling reductions of measurements to the *geoid* to be considered appropriate for the purpose of computations on the spheroid. This is called the *development method* (Molodenskii et al.1962,p.29) and ignores the consequences of the geoid separating from the reference spheroid.

The condition of parallelism between geoid and reference spheroid over limited regions ensures that non-coincidence between the two surfaces produces negligible errors in geodetic co-ordinates. The use of the development method in Australia instead of the rigorous *projection method* (ibid,p.29) cannot be expected to produce accumulations of error greater than 0.3 sec in geodetic co-ordinates even at the peripheries, in the light of the results obtained from the current geoid investigation and must therefore be considered to have negligible magnitude.

A geodetic control network on the A.G.D. can therefore be assumed to meet first order requirements as the system is controlled by over 1000 Laplace stations which have a continent-wide distribution. A preliminary astro-geodetic geoid was prepared by Fischer & Slutsky (1967) and based on 600 astro-geodetic stations, the locations of which are shown in (Bomford 1967,p.56). The Fischer & Slutsky solution, being based on a station density of approximately 1 station per 10,000 km^2 understandably produces an over-smoothened representation of geoidal undu'ations on the A.G.D. and is studied in greater detail in section 4.3. It does, however, show the major features of the geoid with considerable accuracy. This determination is due to be replaced by a much improved solution which is being prepared by the Division of National Mapping and should be available in 1971. In the interim, the determination of Fischer & Slutsky provides an extremely useful astro-geodetic solution for verifying the accuracy of gravimetric solutions for the geoid. These comparisons, in turn, can be used for computing the corrections necessary for determination of the geocentric location of the A.G.D.

1.2 THE 1968 GRAVIMETRIC SOLUTION FOR THE GEOID IN AUSTRALIA

A gravimetric solution for the geoid was initially effected for South Australia in 1967 (Mather 1968a,p.337 et seq.). The determination was extended in 1968 for the entire continent (Mather 1969b,p.499 et seq.) and the resulting solution compared with the astro-geodetic geoid of Fischer & Slutsky after the appropriate geocentric orientation parameters were computed for the latter solution which was based on the A.G.D. These gravimetric determinations were *composite* solutions where the distant zones were represented by $5^{\circ} \times 5^{\circ}$ free air anomaly means obtained by the combination of surface gravimetry and the coefficients defining the surface harmonic representation of the earth's gravity field obtained by the spherical harmonic analysis of the orbital perturbations of near-earth satellites (Kaula 1966b; Rapp 1968).

The validity of such solutions is examined in section

3.3 and its effect on the geocentric orientation parameters is assessed in sections 4.4 and 5.2. The inner zones of the gravity field were represented by surface gravimetry, the details of the representation being set out in the earlier study (Mather 1969b p.501). The gravimetric geoid was represented in these studies by the co-geoid obtained by the use of free air anomalies in the Stokes and Vening Meinesz integrals and called the *Free Air Geoid*. It can be shown (e.g., Mather 1968b) that such a solution is a good approximation to the geoid with an indirect effect less than 2 - 3 metres for the Australian region, much of which is a zero order effect and is currently under investigation.

The salient features of the previous study can be summarised as follows, being based on the comparison of the free air geoid N_f with the determination of Fischer & Slutsky after the translation o. co-ordinate systems (N_{ac}).

(i) The comparisons ΔN, given by

$$\Delta N = N_f - N_{ac}$$

had a root mean square value of ± 5.3 metres over the entire continent about a mean not significantly different from zero.

(ii) The values of ΔN were position dependent and not randomly distributed.

(iii) If restricted regions where large gradients of ΔN occurred and comprising about 20% of the region studies, were excluded, the ΔN values over the balance of the continental area had a root mean square value of ± 3 metres.

(iv) The magnitude of the geocentric orientation parameters required to make the gravimetric and astro-geodetic datums coincident were only marginally dependent on whether the Kaula set of five degree free air anomaly means or the Rapp set were used for the representation of the distant zones in the solution. This was significant as the global comparison between the two sets had a standard deviation of ±12.5 mgal (Mather 1969b, table 3) with no significant correlation.

The following conclusions were therfore drawn from the results of this earlier investigation.

(a) The orientation of the datum was not critically dependent on the nature of the 5^ox 5^o anomaly set used for the representation of the outer zone anomaly field provided the low degree harmonics were reliably assessed. For a further discussion see section 3.3.

(b) Significant inconsistencies existed in the gravity anomaly field used to represent the near zones.

(c) It was therefore necessary to re-compute the geoid for Australia after the re-definition of the gravity field before any serious attempts could be made to specify the location of the A.G.D. in relation to the earth's geocentre.

The significance of the geocentre is reviewed in section 2.6 while the gravity anomaly field is re-defined in accordance with the principles set out in section 3.2.

1.3 THE CURRENT PROBLEM

The purpose of this investigation is to provide the best possible definition for the Australian Geodetic Datum in earth space with reference to acceptable invariants specifying such a space. The term *earth space* is used in this study to refer to the three dimensional Euclidian space which is independent of both the rotation and the galactic motion of the earth. The invariants in earth space to the order of accuracy sought in the current study, are the physical surface of the earth, the earth's gravitational field and its associated inertia tensors (e.g.,Hotine 1969,p.164 et seq.). The physical surface of the earth must, at present, be considered as being incompletely defined though many *relative* partial definitions of first order accuracy are available for limited continental areas.

Present day geodetic practice is to global geodesy as detailed surveys are to a total integrated survey, a *part* in the concept of "surveying from the whole to the part". These local geodetic surveys require a three dimensional fix with reference to earth space invariants before they can be integrated into the global geodetic context. The current aim of geodesy is the mathematical definition of the entire physical surface of the earth in earth space.

Two techniques are available for effecting this global definition which invloves the establishment of a super-control network spanning all local datums which provide the reference framework for first order geodetic surveys.

The *first* is by the use of artificial earth satellites as unoccupied stations in a global triangulation scheme (e.g., Mueller 1964,p.324 et seq). One or more stations in the local geodetic net may be included in the world-wide framework. The comparison of coordinates on the local datum with those obtained from the global adjustment define the geocentric orientation parameters which have exactly the same significance as those formulated in section 2.8.

Alternately, the same geocentric definition of the local datum can be effected by the use of the earth's gravitational field. The technique proposed in this study deals with the definition of a single datum with an accuracy in accord with that of the determination of the parameters of a spheroid of best fit, provided the estimates of errors in the currently adopted values for the low degree harmonics of orders zero and one are representative. The current investigation restricts itself to the complete definition of the A.G.D. which covers approximately 5% of the total *continental* area of the earth. The final quantities sought are the set of parameters for the definition of the datum in relation to the geocentre. The required correction can be uniquely represented by a vector. The numerical parameters defining this vector obviously depend on the reference frame adopted. This unique vector will be called the *geocentric orientation vector* , while the set of parameters at any point on the datum defining the vector in relation to a rectangular Cartesian co-ordinate system in the local Laplacian trihedron (see Dufour 1968,p.127 for a definition) are termed the *geocentric orientation parameters* ($\Delta\xi_i$, i=1,3) which are related to the scalar magnitude (O) of *O* by the relation

$$O^2 = \sum_{i=1}^{3} h_i^2 \Delta\xi_i^2 \qquad \ldots.(3),$$

where h_i (i=1,3) are the associated linearisation parameters and the subscript $_o$, if added to equation 3, refers to values at the origin.

1.4 SCHEME FOR SOLUTION

The problem of geocentric orientation is capable of a unique solution if the earth's gravity field is completely defined over the entire global surface. This condition is only partially satisfied if the determination is restricted to surface gravimetry. It therefore becomes necessary to predict values to represent the unsurveyed regions of the world, the major problem being that of the ocean regions comprising 70% of the surface area and with a current gravity coverage which can only be described as totally inadequate. Resort to prediction has the attendant consequence of errors in all computed quantities from this source, which, in the case of determination prior to about 1960, made any results of academic interest only.

In theory, it is possible, as in the case of geometrical satellite solutions on a global basis, to define the geocentric orientation vector by considering observed quantities at just one point on the datum being investigated. The solution in principle is as follows. Let $\xi_i (i=1,3)$ be the set of parameters which can be defined on both the local astro-geodetic datum (ξ_{ai}) and a ge-centric one, afforded by the same mathematical reference surface, but with its centre at the earth's centre of mass (geocentre), when the parameters take values ξ_{gi}. The latter quantities can be computed either by the use of the global gravity anomaly field or by geometrical satellite geodesy from a world wide coverage. The discrepancies ($\xi_{gi} - \xi_{ai}$) can be used to define the geocentric orientation vector O, which can also be expressed by either the vector equivalent of equation 3 or in terms of changes $\Delta u_i (i=1,3)$ in earth space co-ordinates. Δu_i is also the difference between the co-ordinates u_{ai} on the local datum and the geocentric value u_{gi}. Thus,

$$\Delta u_i = u_{gi} - u_{ai} = \sum_{j=1}^{3} a_{ij} (\xi_{gjo} - \xi_{ajo}) \quad ,i=1,3 \quad \ldots .(4a),$$

where the quantities a_{ij} are the required rotational parameters given by equations of the form

$a_{ij} = a_{ij}(u_{a1}, u_{a2}, u_{a3}, h_j)$, the subscript referring to values at the origin of the local datum. Equation 4a can either be solved for the evaluation of the difference

$(\xi_{gjo} - \xi_{ajo})$ or be used to compute the required changes in co-ordinates at the n other control stations on the datum if this difference is known, there being n equations of the form

$$u_{gik} = u_{aik} + \Delta u_{ik}\ , \quad i=1,3;\ k=1,n, \qquad \ldots.(4b),$$

where

$$\Delta u_{ik} \;=\; \Delta u_{ik}(\{\Delta u_{jo}, u_{jo}, u_{jk}\}, j=1,3).$$

The existence of an incompletely defined gravity field rules out the direct solution of equation 4a as the result will be seriously affected by systematic error. The problem is further complicated by the fact that continental areas have well defined gravity fields while ocean regions are inadequately surveyed for geodetic purposes. In addition, local regions have larger effects on computations than more distant areas. Hence it is preferable to confine investigations to continental extents which are not too close to unsurveyed oceans.

The scheme finally adopted for the determination of the geocentric orientation vector for the A.G.D. using gravity data was influenced by the following conclusions :-

(i) Investigations should not be confined to a single geodetic control station.

(ii) It is desirable to evaluate the parameters $\Delta\xi_{io}$ $(i=1,3)$ defining the geocentric orientation vector through equation 3 and the difference on the right hand side of equation 4a, using comparisons of the type given in equation 4a, at as wide a coverage of astro-geodetic stations on the datum as possible, with the proviso that the near zone gravity field was adequately defined.

The specifications for the digital representation of the gravity anomaly field are given in section 3.1.

The present investigation develops all formulae necessary for defining the quantities $\Delta\xi_{io}$ and the corrections Δu_{ik} to the general space co-ordinates at the k-th astro-geodetic station, as set out in concept in equation 4b. The specifications for the digital representation of the incompletely surveyed gravity are next discussed in section 3. The results obtained are finally analysed for

(a) the best values for $\Delta\xi_{io}$ and hence Δu_{io} at the origin;

(b) estimates of the accuracy of the results so obtained; and

(c) the precision of gravimetric determinations of the geoid and surface deflections of the vertical for the Australian region from the current definition of the gravity field.

2. THEORETICAL CONCEPTS

2.1 SOLUTION OF THE BOUNDARY VALUE PROBLEM FOR THE PHYSICAL SURFACE OF THE EARTH

Numerous solutions, correct to the order of the flattening are available for this problem, one of which is the result of an earlier investigation by the writer (Mather 1968b,p.526 et seq). The formulae in this study are difficult to follow as they also investigate the vertical gradients of normal gravity $\left(\frac{\partial\gamma}{\partial h}\right)$ for those regions where the physical surface of the earth is *below* the reference surface in relation to the outward normal. The adoption of the relation

$$\left|\frac{\partial\gamma}{\partial h}\right| = 2\,\frac{\gamma}{R} \qquad \ldots.(5),$$

where γ is normal gravity, $\left|\frac{\partial\gamma}{\partial h}\right|$ the modulus of the vertical gradient of gravity on the reference system and R the mean radius of the earth, instead of the general relation

$$\left|\frac{\partial\gamma}{\partial h}\right| = 2\,\frac{\gamma}{a} - 4\pi k\rho \qquad \ldots\ldots(6),$$

where a is the equatorial radius of the reference spheroid and ρ is the density of matter in which the vertical gradient is evaluated (e.g.,Heiskanen & Moritz 1967,p.54), has been shown to have a negligible effect on the height anomaly h_d. It is therefore possible to ignore the mass of the reference system locate *outside the physical surface of the earth* and consider equation 5 to apply at all points when defining the height anomaly. In such a case, the linearised form of Green's third identity as applied to the physical surface of the earth S can be written as

$$V_{dp} = \frac{1}{2\pi}\iint\left(V_d\nabla.N\,\frac{1}{r} - \frac{1}{r}\{\nabla.N\;W - \nabla.N\;U\}\right)dS \qquad \ldots\ldots(7),$$

where r is the distance of the element of surface area dS from the computation point P, dS being on the physical surface of the earth, W, U being the geopotential and spheropotential respectively, N the unit normal to the surface element dS , V_d the disturbing potential and

$$\nabla = \sum_{i=1}^{3} \frac{\partial}{\partial x_i}\, i \qquad \ldots\ldots(8),$$

i being the unit vector along the rectangular Cartesian co-ordinate axis x_i (i=1,3).

The disturbing potential V_{dp} at P, as shown in figure 1 is given by the relation

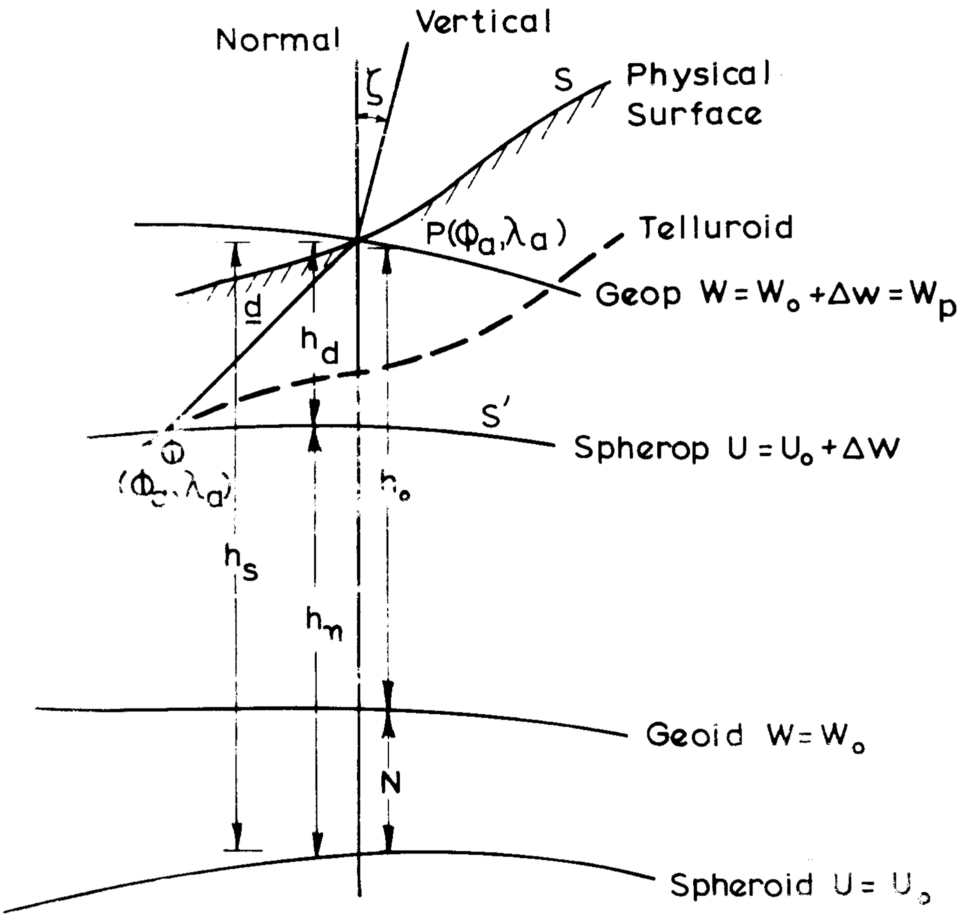

Fig. 1

The telluroid in relation to the physical surface

$$V_d = (W_o - U_o) + \gamma h_d + o\{fV_d\} \qquad \ldots.(10),$$

the subscript $_o$ referring to values at either the geoid or the reference spheroid, and the vertical gradient $(\partial V_d/\partial h)$ by

$$\frac{\partial V_d}{\partial h} = -\left(\Delta g + \left|\frac{\partial \gamma}{\partial h}\right| h_d \right) \qquad \ldots..(11)$$

or

$$\frac{\partial V_d}{\partial h} = -\left(\Delta g + 2\,\frac{V_d}{R} - 2\,\frac{W_o - U_o}{R}\right) \qquad \ldots..(12),$$

where Δg is the *gravity anomaly* given by

$$\Delta g = g_p - \gamma_q,$$

g_p being the value of observed gravity at P and γ_q the value of normal gravity at Q in figure 1. Also see (Mather 1968b,p.526). If the general point P in figure 1 lies on the geop $W = W_p$ and has astronomically determined co-ordinates (ϕ_a, λ_a), where W_p is related to the potential W_o of the geoid by the equation

$$W_p = W_o + \Delta W ,$$

ΔW is related to the results of conventional levelling (dz) over a section where the value of observed gravity is g,by the relation

$$- \Delta W = \int_0^P g \, dz.$$

The displacement $h_n = QQ_o$ in figure 1 is called the normal height and can be accurately computed if ΔW is known, as it is independent of any assumptions about the stratification of matter and can be considered a "free air" difference in potential. h_n is related to ΔW by the equation (e.g.,Heiskanen & Moritz 1967,p.171)

$$h_n = -\frac{\Delta W}{\gamma}\left[1 - (1+f+m-2f\sin^2\phi)\frac{\Delta W}{a\gamma} + o\{f^2\}\right] \qquad \ldots.(13),$$

where the parameters (a,f) define the reference spheroid and m is given by

$$m = \frac{a\ \omega^2}{\gamma_e} + o\{f^2\} \qquad \ldots.(14),$$

ω being the angular velocity of rotation of the earth and γ_e the value of equatorial gravity on the reference spheroid.

In theory it is therefore necessary to calculate normal heights before the gravity anomaly can be defined. For all practical purposes, however, the latter is replaced by the free air anomaly which is conventionally computed using orthometric elevations, as the magnitude of the difference between the two types of elevations is less than the errors in the measurement of most gravity station heights, except those established at bench marks(e.g., see ibid,p.329).

The evaluation of equation 7 requires a knowledge of the direction cosines of the unit normal vector *N* which can be obtained by a consideration of figure 2. Let Q be the point $(\phi_a, \lambda_a, U_o+\Delta W)$ on the reference system which represents the point P at the physical surface of the earth, the locus of Q being called the *telluroid*. The adoption of a local rectangular three dimensional Cartesian co-ordinate system $(x_1 x_2 x_3)$ at Q, specifying the x_3 axis coincident with the normal to the spherop $U = U_o + \Delta W$ will result in the plane containing the $x_1 x_2$ axes being tangential to the spherop at Q. If A_s is the azimuth of the line of greatest slope of the telluroid, which very closely follows the greatest slope β of the

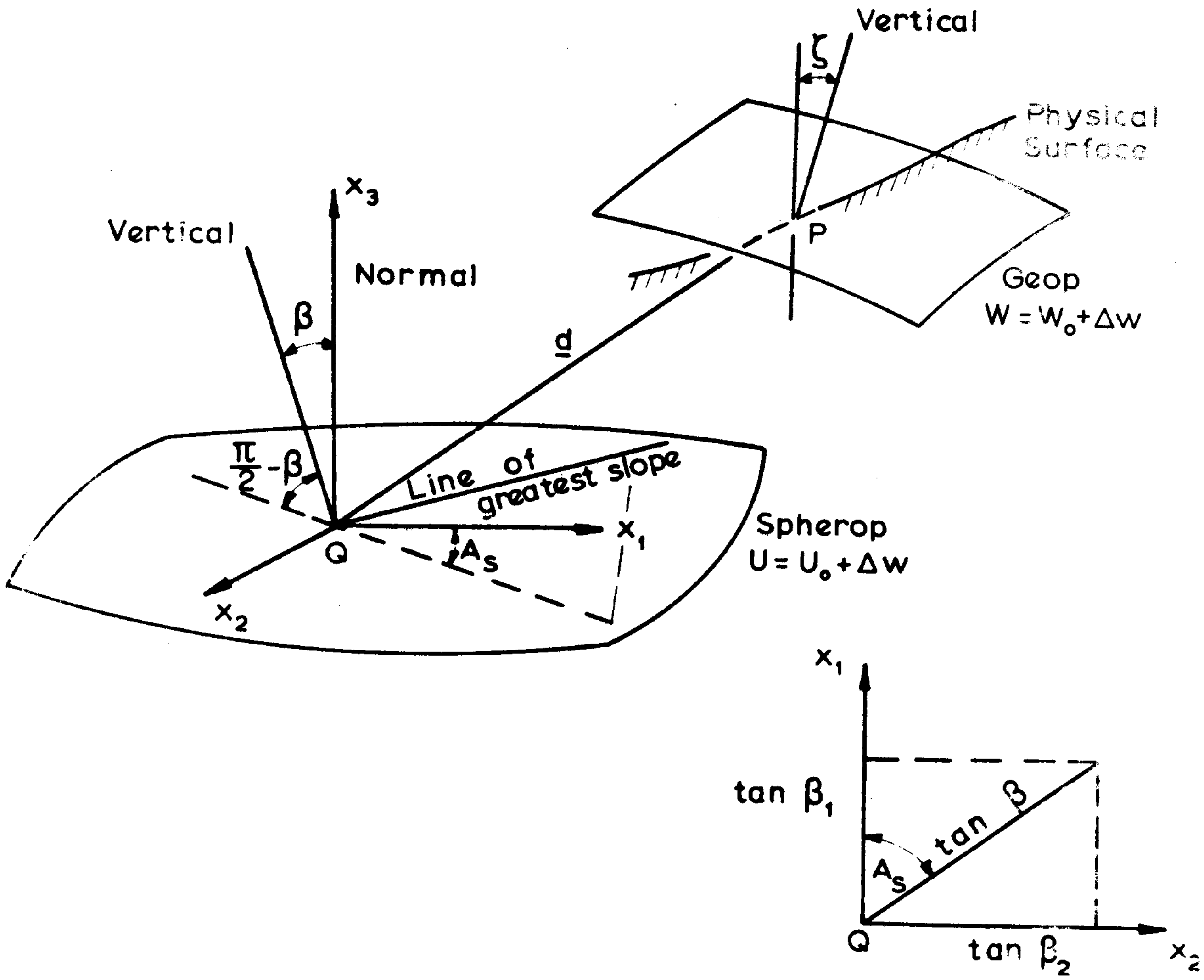

Fig. 2

Topographical gradients in the local cartesian frame at telluroid.

topography at P, the angle between the spherop normal and that to the telluroid is, for all practical purposes, the angle β, being the angle between the local vertical and the normal to the physical surface of the earth. If the ground rises in the positive directions of the x_1 and x_2 axes, oriented north and east respectively, the direction cosines of the telluroid normal are obtained from a direct consideration of figure 2 as

$$\{\sin\beta\cos(\pi+A_s),\ \sin\beta\sin(\pi+A_s),\ \cos\beta\}.$$

The ground slope can also be treated vectorially, when the gradients along the x_1 and x_2 axes are related to that along the line of greatest slope by the relation

$$\tan^2\beta = \tan^2\beta_1 + \tan^2\beta_2.$$

It also follows that

$$\cos\alpha_i = \frac{\tan\beta_i}{\tan\beta}, \quad i=1,2 \quad ,$$

where

$$\alpha_1 = A_s \quad ; \quad \alpha_2 = \tfrac{1}{2}\pi - A_s .$$

Thus

$$\mathcal{N} = \cos\beta\left(\sum_{i=1}^{2} -\tan\beta_i\, \boldsymbol{i} + \boldsymbol{3}\right) \qquad \ldots\ldots(15).$$

The distance r of the element dS from the computation point P can also be related to the local rectangular Cartesian system at the variable point Q by the relation

$$r^2 = \sum_{i=1}^{3} x_i^2 .$$

As

$$\nabla \frac{1}{r} = - \sum_{i=1}^{3} \frac{x_i}{r^3} i \quad ,$$

$$\nabla . N \frac{1}{r} = \cos \beta \left(\sum_{i=1}^{2} \frac{x_i}{r^3} \tan \beta_i - \frac{x_3}{r^3} \right) \quad .....(16).$$

Equations 15 and 16 involve only *slight* approximations in that the irregular physical surface of the earth S has been replaced by the telluroid which is capable of mathematical definition. The resulting differences can be considered to be of no significance for the purpose of the present study. As the x_3 axis coincides with the local spherop normal, the use of equation 15 gives

$$\nabla . N\, W - \nabla . N\, U = \nabla . N\, V_d = - \sum_{i=1}^{2} \frac{\partial '_d}{\partial x_i} \tan \beta_i + \frac{\partial V_d}{\partial h} \quad ...(17),$$

where all relations apply to the telluroid. The deflections of the vertical ξ_i are defined by the equation (e.g.,Moritz 1965,p.15)

$$\xi_i = - \left(\frac{\partial h_d}{\partial x_i} \right)_{W=W_p} \quad , \; i=1,2 \quad ....(18),$$

where ξ_1 and ξ_2 are components of the deflection of the vertical in the meridian and prime vertical respectively. For sign conventions and details of abbreviations used, see section 1 of the *Guide to Notation* . Equation 18 is valid as the deflections are purely a function of the relation in space between the geop $W = W_p$ and its associated spherop $U = U_o + \Delta W$ as defined in figures 1 and 2. Thus,

$$\frac{\partial V_d}{\partial x_i} = \gamma \frac{\partial h_d}{\partial x_i} = - \gamma \xi_i + o\{f \frac{\partial V_d}{\partial x_i}\} \; ,i=1,2 \quad ......(19).$$

The substitution of equations 16, 17 and 19 in equation 7 gives

$$V_{dp} = \frac{1}{2\pi} \iint_S \cos\beta \left[V_d \left(\sum_{i=1}^{2} \frac{x_i}{r^3} \tan\beta_i - \frac{x_i}{r^3} \right) - \frac{1}{r} \left(\gamma \sum_{i=1}^{2} \xi_i \tan\beta_i + \frac{\partial V_d}{\partial h} \right) \right] dS$$

$$= \frac{1}{2\pi} \iint_S \left[\frac{1}{r} \left(\left(-\frac{\partial V_d}{\partial h} - V_d \frac{x_3}{r^2} \right) + \left(V_d \sum_{i=1}^{2} \frac{x_i}{r^2} \tan\beta_i - \gamma \sum_{i=1}^{2} \xi_i \tan\beta_i \right) \right) \right] \cos\beta \, dS \qquad \ldots\ldots(20).$$

If the element of surface area dS on the telluroid is made small enough to be considered as a plane in figure 3, it can be assumed that cross-sections at right angles to the line of greatest slope will be parallel to equivalent lines on the associated spherop. Hence it follows that

$$dS \cos\beta = dS',$$

where dS' is the projection of the element of telluroid surface area on the associated spherop. Thus,

$$dS \cos\beta = (R + h_n)^2 d\sigma ,$$

R being the mean radius of curvature of the equivalent element of surface area on the reference spheroid, h_n the normal height of the associated spherop above the spheroid at Q and $d\sigma$ the spherical representation of dS on unit sphere.

It is necessary to note, when interpreting the surface integral in equation 20, that r is a general distance <u>in space</u>

between the element of surface area dS at Q and the computation point P. The adoption of a spherical approximation for S' will in no way detract from the significance of the expression derived for the *deviation* of r from its equivalent length r_o on the reference surface (sphere or spheroid). Let P_o and Q_o be the points on the reference surface equivalent to P and Q respectively, which are an angular distance ψ apart. Then

$$r_o = P_oQ_o = 2R \sin \tfrac{1}{2} \psi \qquad .....(22).$$

If h, h_p are the normal heights at Q and P respectively,

$$r = PQ = \left((R+h)^2 + (R+h_p)^2 - 2(R+h)(R+h_p)\cos\psi\right)^{\frac{1}{2}}$$

$$= \left(R^2 + 2R(h+h_p) + h^2 + h_p^2 - 2R^2 - 2R(h+h_p) - 2hh_p + 4R^2\sin^2\tfrac{1}{2}\psi + o\{fr_o\}\right)^{\frac{1}{2}}$$

$$= \left(r_o^2 + (h-h_p)^2\right)^{\frac{1}{2}} + o\{r_o f\} \qquad ....(23).$$

Thus,

$$r = r_o + o\{fr_o\}$$

if

$$\frac{1}{2}\left(\frac{h-h_p}{r}\right)^2 = o\{f\} \qquad \text{or} \qquad \left(\frac{h-h_p}{r}\right) \simeq 8 \times 10^{-2}.$$

The direct consideration of figure 4 gives

$$-\frac{x_3}{r^3} = \frac{(R+h_p)\cos\psi - (R+h)}{r_o^3\left[1 + \left(\frac{h-h_p}{r_o}\right)^2\right]^{3/2}} = \frac{h_p - h - 2R\sin^2\tfrac{1}{2}\psi}{r_o^3}\left[1 - \frac{3}{2}\left(\frac{h-h_p}{r_o}\right)^2\right]$$

$$= -\frac{1}{2Rr_o} + \frac{h - h_p}{r_o^3} + o\{f\,\frac{x_3}{r_o^3}\} \quad \text{if} \quad \frac{h-h_p}{r_o} \not> 5\times 10^{-2} \qquad .....(24)$$

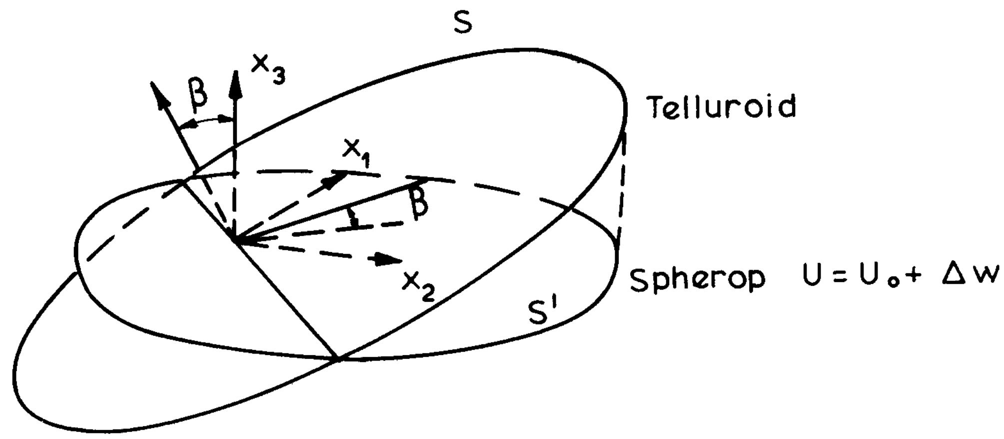

Fig. 3

Projection of telluroid onto the associated spherop

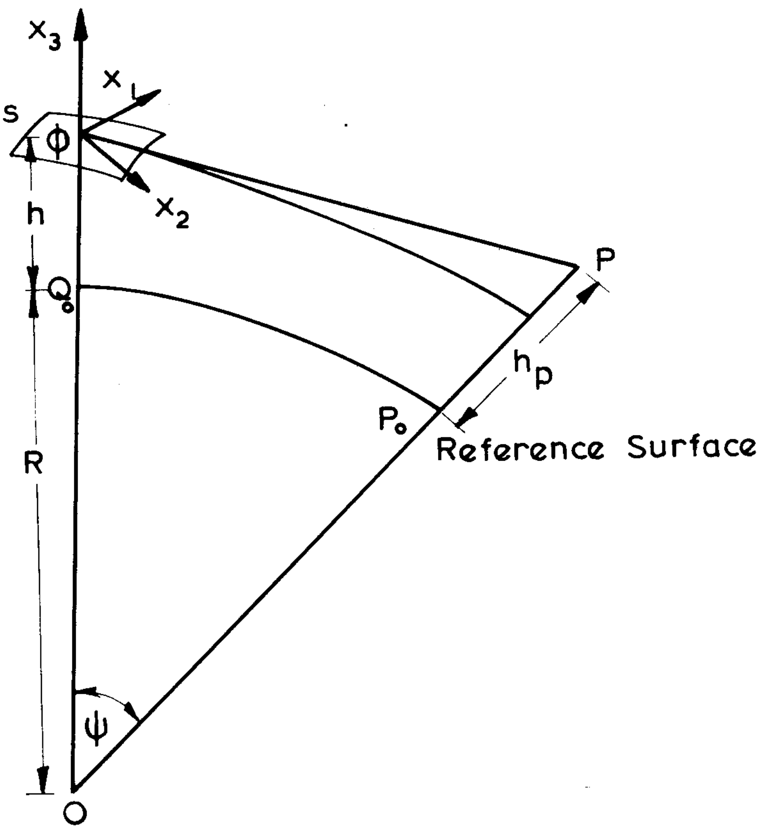

Fig. 4

The topographical effect for a spherical approximation of the earth.

as the term ignored is

$$\frac{3}{4Rr_o}\left(\frac{h-h_p}{r_o}\right)^2$$

and

$$\sum_{i=1}^{2} \frac{x_i}{r^3}\tan\beta_i = \frac{(R+h_p)\sin\psi}{r_o}\left(\cos A_c \tan\beta_1 + \sin A_c \tan\beta_2\right) \times$$

$$\left[1 + \left(\frac{h-h_p}{r_o}\right)^2 + o\left\{\left(\frac{h-h_p}{r_o}\right)^4\right\}\right]^{-\frac{3}{2}},$$

where A_c is the azimuth of the computation point P from the element of surface area dS at Q. The term involving the gradients can be interpreted as follows:-

$$\cos A_c \tan\beta_1 + \sin A_c \tan\beta_2 = \frac{\partial h}{\partial x_1}\frac{dx_1}{dr'} + \frac{\partial h}{\partial x_2}\frac{dx_2}{dr'} = \frac{dh}{dr'},$$

where r' is the projection of r in the horizon plane at Q. Thus

$$\sum_{i=1}^{2} \frac{x_i}{r^3}\tan\beta_i = \frac{(R+h_p)\sin\psi}{r_o^3}\frac{dh}{dr_o}\frac{dr_o}{dr'}\left[1 - \frac{3}{2}\left(\frac{h-h_p}{r_o}\right)^2 + o\left\{\left(\frac{h-h_p}{r}\right)^4\right\}\right] \quad .(25),$$

where

$$\frac{dh}{dr} = \sum_{i=1}^{2} \tan\beta_i \cos\alpha_i \quad ......(26),$$

$$\alpha_1 = A_c \quad ; \quad \alpha_2 = \tfrac{1}{2}\pi - A_c$$

and

$$\frac{dr_o}{dr'} = 1 \quad \text{for all practical purposes.}$$

Thus equation 20 can be visualised as departing from the classical Stokesian problem in three ways:-

(i) The telluroid slopes with respect to the associated spherop.

(ii) The associated spherop is elevated above the reference surface

(iii) The computation point is elevated above the reference surface and the surrounding topography is possibly at a different elevation.

The first deviation is expressed in the second term within the second set of brackets in equation 20 while the second and third are manifest in the departure of r_o from r. The latter effect is only of significance when $r \to 0$, i.e., for near zones and can be neglected in distant zone calculations as shown in equation 24. Equation 20 can therefore be considered to consist of three contributory integrals of the form

$$\frac{1}{2\pi}\iint_{S'} I_i \, dS' , \quad i=1,3 \qquad ......(27),$$

where

$$I_1 = \frac{1}{r_o}\left(- \frac{\partial V_d}{\partial h} - \frac{V_d}{2R}\right) \qquad ......(28)$$

is the classical Stokesian form, I_2 and I_3 being obtained from equations 17, 18 and 19 as

$$I_2 = \sum_{i=1}^{2}\left(\frac{x_i}{r_o^3} V_d - \frac{1}{r_o}\gamma\,\xi_i\right)\tan\beta_i$$

$$= - \sum_{i=1}^{2}\left(V_d \nabla . i \frac{1}{r_o} - \frac{1}{r_o}\nabla . i\, V_d\right) \tan\beta_i$$

$$= V_d \frac{R \sin\psi}{r_o}\frac{dh}{dr_o} - \frac{1}{r_o}\gamma\sum_{i=1}^{2}\xi_i \tan\beta_i \qquad .....(29)$$

and

$$I_3 = \frac{h_p - h}{r_o} V_d + \frac{3}{4Rr_o}\left(\frac{h_p - h}{r_o}\right)^2 V_d - \frac{3R\sin\psi}{2r_o^3}\frac{dh}{dr_o}\left(\frac{h_p - h}{r_o}\right)^2 V_d -$$

$$\tfrac{1}{2} r_o I_1 \left(\frac{h_p - h}{r_o}\right)^2 + o\left\{\left(\frac{h_p - h}{r_o}\right)\right\} \quad .$$

The ratio $(h_p - h)/r_o$ equals unity only for very small values of r_o, being equal to a 45° ground slope. It therefore has a cumulative effect primarily through the first term in the expression for I_3 and at stations situated on the edge of steep scarps with considerable longitudinal extent. Thus I_3 has been adopted as being

$$I_3 = \frac{h_p - h}{r_o^3} V_d \qquad \text{....(30)}$$

for the Australian region. The spherical approximation of equation 27 when $i = 1$ will be shown to give Stokes' integral on development, the relevant gravity anomaly being the free air anomaly for all practical purposes. The terms constituting I_2 in equation 29 are equivalent to the non-Stokesian terms in the earlier solution (Mather 1968b, p.528), allowing for the nature of dS'. The quantity I_3 in equation 30 cannot be of significance except for the near zones when r_o is small. V_{dp} can then be defined by the equation

$$V_{dp} = V_{dfp} + V_{dip} \qquad \text{....(31),}$$

where the Stokesian disturbing potential V_{dfp} is given by

$$V_{dfp} = \frac{1}{2\pi}\iint_{S'} I_1 \, dS' \qquad \text{......(32),}$$

and the small correction V_{dip} to this value being given by

$$V_{dip} = \frac{1}{2\pi}\iint_{S'} \left[\sum_{i=1}^{2} \left(\frac{1}{r_o} \nabla . \; V_d - V_d \nabla . \; \frac{1}{r_o} \right) \tan \beta_i + \frac{h_p - h}{r_o^3} V_d \right] dS'$$

$$= \frac{1}{2\pi}\iint_{S'} \left[\frac{1}{r_o} \left(-\gamma \sum_{i=1}^{2} \xi_i \tan \beta_i \right) + \left(\frac{R \sin \psi}{r_o^3} \frac{dh}{dr} + \frac{h_p - h}{r_o^3} \right) V_d \right] dS' \quad ..(33).$$

2.2 THE SOLUTION FOR THE STOKESIAN COMPONENT OF THE DISTURBING POTENTIAL

For a spherical approximation of the earth,

$$dS' = R^2 \, d\sigma \quad ,$$

where R is the mean radius of the earth and $d\sigma$ the element of surface area on unit sphere. The combination of equations 12, 27 and 28 gives

$$V_{dfp} = \frac{R^2}{2\pi} \iint \frac{1}{r_o} \left(\Delta g + \frac{3V_d}{2R} - 2\frac{W_o - U_o}{R} \right) d\sigma \quad .....(34).$$

The first two terms within the inner bracket constitute the basic components of the classical Stokesian problem, which specifies that zero and first degree harmonics do not exist in any expression for V_d and hence Δg. This restriction is not specified in the generalised equations for the boundary conditions set out in equations 10 to 12. The solution is based on the definitive integral at 34 which is of the type

$$V_{dp} = \iint_S \frac{\Phi}{r} \, dS$$

which can be expressed as a solid harmonic of the type (see Mather

1968b, p.516 for details)

$$V_{dp} = \sum_{n=0}^{\infty} \frac{A_n}{R^{n+1}} \qquad \text{......(35)}.$$

From a study of equations 33 and 34 it can be seen that equation 35 is valid only if all quantities can be expressed as a surface harmonic. This can be proved without too much difficulty. The harmonics (n=0; n=1) are inadmissible in the classical Stokesian problem. The current solution admits $n = 0$ but, as will become apparent, excludes $n = 1$, the manipulation becoming invalid when $n = 1$ as can be seen from equation 38. The significance of the non-existence of these harmonics is examined in section 2.6. The use of equation 35 in equations 10 to 12 gives

$$\Delta g = \sum_{n=0}^{\infty} (n-1) \frac{A_n}{R^{n+2}} + 2 \frac{W_o - U_o}{R} \quad , \; n \neq 1 \qquad \text{.....(36)}.$$

Defining the corrected gravity anomaly Δg_c by the relation

$$\Delta g_c = \sum_{n=0}^{\infty} G_n = \sum_{n=0}^{\infty} (n-1) \frac{A_n}{R^{n+2}} = \Delta g - 2 \frac{W_o - U_o}{R} \quad \text{..(37)},$$

$n \neq 1$. A_n is therefore related to G_n by the relation

$$A_n = R^{n+2} \frac{G_n}{n-1} \quad ,$$

G_n being the n-th degree surface harmonic in the representation of the corrected gravity anomaly, which can be interpreted as the gravity anomaly with a correction for the zero degree effect of the potential at the geoid not being the same as that on the surface of the reference spheroid. The substitution of equations 35 and 37 in 34 gives

$$V_{dfp} = \frac{R^2}{2\pi} \iint \frac{1}{r_o} \sum_{n=0}^{\infty} \frac{2n+1}{2} \frac{A_n}{R^{n+2}} \, d\sigma \qquad , n \neq 1.$$

The use of equation 37 gives

$$V_{dfp} = \frac{R^2}{2\pi} \iint \frac{1}{r_o} \sum_{n=0}^{\infty} \frac{2n+1}{2(n-1)} G_n \, d\sigma \qquad ,n \neq 1$$

$$= \frac{R^2}{2\pi} G_o \iint \frac{1}{r_o} (-\tfrac{1}{2}) d\sigma + \frac{R^2}{2\pi} \iint \frac{1}{r_o} \sum_{n=2}^{\infty} \frac{2n+1}{2(n-1)} G_n \, d\sigma \quad ..(38).$$

The first of the surface integrals is evaluated by putting

$r_o = 2R \sin \tfrac{1}{2}\psi$ and $d\sigma = \sin \psi \, d\psi \, d\alpha$,
$d\alpha$ being the azimuthal increment, when

$$-\frac{R^2}{2\pi} G_o \int_0^{\pi} \int_0^{2\pi} \frac{2 \sin \tfrac{1}{2}\psi \cos \tfrac{1}{2}\psi}{2R \sin \tfrac{1}{2}\psi} \frac{d\psi}{2} d\alpha = -R \, G_o = - R \, M\{\Delta g_c\}$$

$$= - R \, M\{\Delta g\} + 2(W_o - U_o) \qquad .....(39),$$

$M\{\Delta g\}$ being the global mean of the gravity anomaly. The second integral in equation 38 is Stokes' integral expressed in surface harmonics of the gravity anomaly, which can be solved in the standard manner (e.g., Jeffreys 1962,p.142) to give

$$\frac{R^2}{2\pi} \iint \frac{1}{r_o} \sum_{n=2}^{\infty} \frac{2n+1}{2(n-1)} G_n \, d\sigma = \frac{R}{4\pi} \iint \Delta g_c \, f(\psi) \, d\sigma \quad ,$$

where

$$f(\psi) = \operatorname{cosec} \tfrac{1}{2}\psi + 1 - 6 \sin \tfrac{1}{2}\psi - 5 \cos \psi - 3 \cos\psi \, \log\{\sin \tfrac{1}{2}\psi(1+ \sin \tfrac{1}{2}\psi)\} \quad ....(40).$$

The right hand side of the surface integral can be simplified further by the use of equation 37 and the fact that (Lambert & Darling 1936,p.117)

$$\int_0^{\pi} f(\psi) \sin \psi \, d\psi = 0,$$

when

$$\frac{R}{4\pi}\iint \Delta g_c f(\psi)\, d\sigma = \frac{R}{4\pi}\iint \Delta g\, f(\psi)\, d\sigma - \frac{2(W_o - U_o)}{4\pi}\iint f(\psi)\, d\sigma$$

$$= \frac{R}{4\pi}\iint \Delta g\, f(\psi)\, d\sigma \qquad \ldots\ldots\ldots(41).$$

Hence the combination of equations 38, 39 and 41 gives

$$V_{dfp} = 2(W_o - U_o) - R\, M\{\Delta g\} + \frac{R}{4\pi}\iint \Delta g\, f(\psi)\, d\sigma \qquad \ldots\ldots(42),$$

where Δg is the gravity anomaly, which, for all practical purposes, is the free air anomaly, the total error involved in the vertical co-ordinate estimation being about 2 metres in mountainous country.

The free air geoid is defined by N_f which is the contribution of the term V_{dfp} to the height anomaly h_d through equation 10 when

$$N_{fp} = \frac{V_{dfp}}{\gamma} - \frac{W_o - U_o}{\gamma}$$

$$= \frac{W_o - U_o}{\gamma} - R\,\frac{M\{\Delta g\}}{\gamma} + \frac{R}{4\pi\gamma}\iint \Delta g\, f(\psi)\, d\sigma \qquad \ldots\ldots(43),$$

where the gravity anomaly can be expressed by a surface harmonic series of the form

$$\Delta g = \sum_{n=0}^{\infty} G_n \quad , \; n \neq 1 \qquad \ldots\ldots\ldots(44),$$

equation 43 having been derived on the basis that no first degree harmonic exists in the expansion for Δg. Conversely, the use of Stokes' integral as set out in equation 43 for the determination of the height anomaly requires that the gravity anomaly be defined with respect to a reference system such that its magnitude has no first degree harmonic on global analysis. This a perfectly valid assumption, irrespective of the interpretation of the first degree harmonic as *gravity anomalies are not invariants in earth space.* To emphasise the point, observed gravity and geopotential are invariants while the magnitude of the gravity anomaly is dependent not only on the parameters of the reference spheroid but also on its location in earth space. The significance of assigning the value zero for the coefficients of certain chosen harmonics is discussed in section 2.6.

2.3 THE CORRECTION TO THE FREE AIR GEOID

The correction arisies solely as a consequence of the existence of topography exterior to the geoid. The height anomaly h_d is related to the free air geoid separation N_f by the relation

$$h_d = N_f + N_i \qquad ......(45).$$

From equations 10, 31 and 43,

$$h_d = \frac{V_d}{\gamma} = N_f + \frac{V_{di}}{\gamma} ,$$

The correction N to the free air geoid is given by

$$N_i = \frac{V_{di}}{\gamma} \qquad .....(46).$$

The use of equation 33 gives

$$N_{ip} = \frac{1}{2\pi\gamma}\iint \left[\left(\frac{R \sin\psi}{r_o^3} \frac{dh}{dr_o} + \frac{h_p - h}{r_o^3} \right) V_d - \frac{\gamma}{r_o} \sum_{i=1}^{2} \xi_i \tan\beta_i \right] dS \quad .(47),$$

where

$$\frac{dh}{dr} = \sum_{i=1}^{2} \tan\beta_i \cos\alpha_i$$ as defined

previously in equation 26. The final expression for the height anomaly is obtained by combining equations 43, 45 and 47 when

$$h_{dp} = \frac{W_o - U_o}{\gamma} - R\frac{M\{\Delta g\}}{\gamma} + \frac{R}{4\pi\gamma}\iint \Delta g\ f(\psi)\ d\sigma \quad +$$

$$\frac{R^2}{2\pi\gamma}\iint \left[\left(\frac{R \sin\psi}{r_o^3}\frac{dh}{dr_o} + \frac{h_p - h}{r_o^3} \right) V_d - \frac{\gamma}{r_o} \sum_{i=1}^{2} \xi_i \tan\beta_i \right] d\sigma \quad ...(48).$$

The correctness of the derivation was checked with Moritz' comprehensive classification of solutions of the Molodenskii problem (Moritz 1966,p.91-92) from whence it can be seen that the solution given in equation 48 has the greatest similarity with the "Arnold type" solution. Equality holds if

$$-\frac{R}{4\pi}\iint \sum_{i=1}^{2} \xi_i \tan\beta_i f(\psi)\ d\sigma = \frac{R^2}{2\pi}\iint \frac{1}{r_o}\sum_{i=1}^{2} - \xi_i \tan\beta_i\ d\sigma$$

on allowing for the fact that Moritz has not considered zero degree terms in his study. The expression on the left has been obtained by the inclusion of the term

$$p = -\sum_{i=1}^{2} \xi_i \tan\beta_i$$

along with the gravity anomaly as the modified anomaly Δg_m used in obtaining Stokes' integral, i.e.,

$$\Delta g_m = \Delta g + p \qquad (49).$$

Moritz goes on to prove that the effect of doing this has a negligible consequence on the final result. The solutions discussed by him assume that the modified anomaly Δg_m defined in equation (49) has no zero or first degree terms on global surface harmonic analysis. The existence of any zero degree term in Δg_m poses no problem and can be taken into account by equation 48. The effect of the neglect of the first degree term is discussed in section 2.6.

2.4 SURFACE DEFLECTIONS OF THE VERTICAL

The basic principles are clearly outlined in Heiskanen & Moritz(1967,p.312) following Molodenskii. The deflections of the vertical in the meridian (ξ_1) and prime vertical (ξ_2), positive if the outward vertical is north or east of the normal, are the components of the angle between the surface vertical, which is the normal to the geop, and the normal to the associated spherop. These components will be designated the subscript $_g$ when determined from gravimetry as they are not directly equivalent to astro-geodetic deflections of the vertical, irrespective of considerations of the reference spheroid, as the latter quantity, referred to by the subscript $_a$, is the angle between the *spheroid* normal and the local vertical at the earth's surface.

The difference between ξ_{ai} and ξ_{gi} is a function of the convergence of the equipotential surfaces of the gravitating spheroid (i.e., spherops) towards the pole (e.g.,ibid,p.49). Consequently the point P_o on the reference spheroid in figure 5, whose normal passes through the surface point P has a co-latitude which is greater

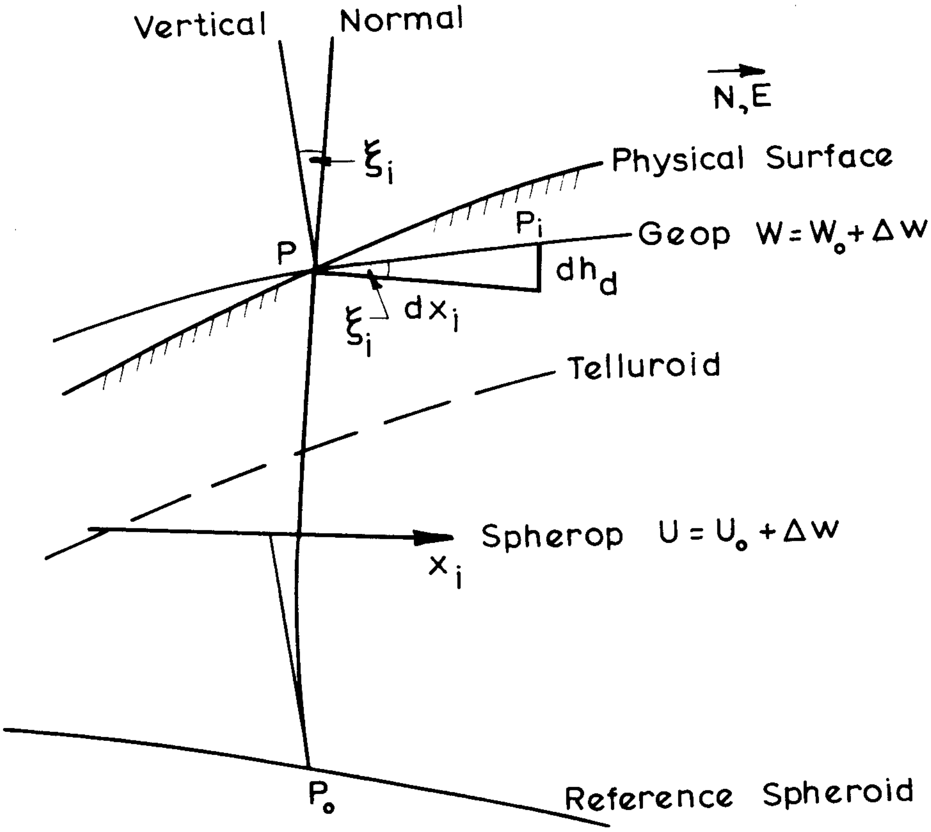

Fig. 5

Surface deflections of the vertical

than that of the equivalent point P on the associated spherop. The magnitude $c_{\xi 1}$ of the difference in the meridian is given by

$$c_{\xi 1} = \xi_{a1} - \xi_{g1} .$$

$c_{\xi 1}$ is the well known correction for the normal plumb line (e.g., ibid,p.196), given for all practical purposes by $0.17\ h^{(km)} \sin 2\phi$. Thus

$$\xi_{a1}^{(sec)} = \xi_{g1}^{(sec)} + 0.17\ h^{(km)} \sin 2\phi \quad \ldots\ldots (50)$$

where h is the elevation of P in km and ϕ is its latitude. As the spherops are a family of rotation spheroids, the equivalent term $c_{\xi 2}$ in the prime vertical is zero. Hence

$$\xi_{a2} = \xi_{g2} \quad \ldots(51).$$

The quantities ξ_{gi} are therefore given by the following equations from a consideration of figure 5.

$$\xi_{gi} = - \left(\frac{\partial h_d}{\partial x_i}\right)_{U=U_o+\Delta W} , \ i=1,2 \quad \ldots\ldots(52),$$

being a function of the variation of the separation h_d between the geop $W = W_p$ and the associated spherop $U = U_o + \Delta W$, the x_i (i=1,2) axes having the same significance as in the discussion preceding equation 15 in defining this variation with position, with the proviso that P has the same definition as in equation 48 and is *not* the variable element dS as in equation 15. This problem could be solved directly if the values of all the gravity anomalies on the geop $W = W_p$ were known, when the problem becomes Stokesian, provided these anomalies do not have a first degree harmonic on global analysis.

Such a solution, derived from the Molodenskii formulation, is comprehensively dealt with by Moritz (1966,p.98). It can also be obtained from rationalisation from equation 48 for the geop $W = W_p$ when the corrected gravity anomaly Δg_m is given by

$$\Delta g_m = \Delta g - \frac{\partial \Delta g}{\partial h} (h - h_p) \qquad \ldots \quad (53),$$

where $(\partial\Delta g/\partial h)$ is the vertical gradient of the gravity anomaly. All topography dependent terms will have no effect, as the surface being mapped is an equipotential and the use of equation 48, without consideration of zero order terms, which have no effect on deflections of the vertical, gives

$$h_{d_{W+W_p}} = \frac{R}{4\pi\gamma} \iint \Delta g_m \, f(\psi) \, d\sigma \qquad \ldots \quad (54).$$

The non-regularised geoid would therefore be a particular case of equation 54 when the concepts outlined in (Mather 1968b, p.522 et seq.) will apply, with the proviso that ϕ_e now refers to the potential of matter exterior to $W = W_p$.

The vertical gradient of the gravity anomaly is a quantity capable of measurement in theory but is unlikely to be obtained as an observation in the foreseeable future. Alternately, the vertical gradient of the gravity anomaly may be computed from the spherical formula (Heiskanen & Moritz 1967,p.115)

$$\left(\frac{\partial \Delta g}{\partial h}\right)_p = \frac{R^2}{2\pi} \iint \frac{\Delta g - \Delta g_p}{r_0^3} \, d\sigma - \frac{2}{R} \Delta g_p \qquad \ldots. \quad (55).$$

The surface integral in equation 55 need not be evaluated beyond an inner spherical cap with dimensions similar to the Hayford zone O (e.g.,Heiskanen & Vening Meinesz 1958,p.161). On the other hand, it is critically dependent upon the accuracy with which the gravity anomaly field is known in the immediate vicinity of the computation point. This is further exacerbated by the function $1/r_o^3$ which takes extremely large values for small values of r_o. This, unfortunately, is a characteristic of all solutions which stem from Green's third identity. The gradient solution is therefore not a simple one from a practical point of view in that

(i) a knowledge of the gradients of gravity is required all over the world;

and (ii) this, in turn, requires a knowledge of either the global gravity anomaly field or the topography with a precision much greater than that required in the computation of Stokes' integral.

The determination of deflections of the vertical from equations of the form set out in in 48 is not desirable as near zone computations which are differences of two large quantities may give rise to substantial errors even if small errors occur in any of the computed quantities. However, it was decided to investigate the resulting formulae for reasons given in section 2.7 for use in the Australian computation as only one station had an elevation in excess of 1 km in the entire scheme.

As h_d is related to the gravity anomaly field in equation 48 through the telluroid, which departs from a spherop over the length increment dx_i in the horizon plane of the latter, ξ_{gi} can also be expressed for computational purposes as

$$\xi_{gi} = -\left(\frac{\partial h_d}{\partial x_i}\right)_{Tell} + \frac{\partial h_d}{\partial x_3} \cdot \frac{dx_3}{dx_i}, \qquad i=1,2,$$

where the second term takes into account the variations of h_d with elevation as the telluroid refers to different spherops with change of horizontal position, the topographical gradients being given by

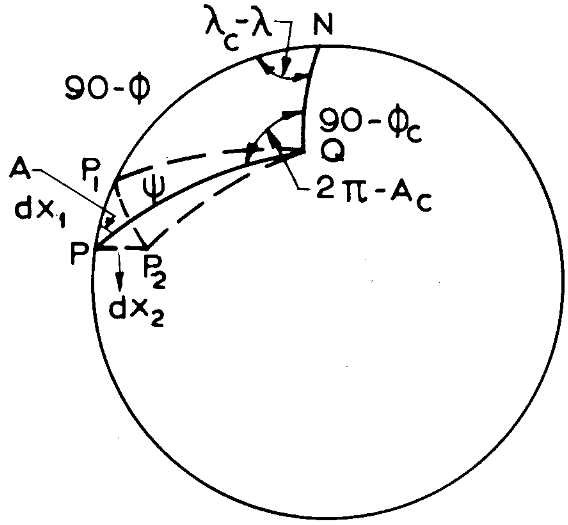

Fig. 6

Spherical relations for computations of deflections of the vertical

$$\frac{\partial x_3}{\partial x_i} = \tan \beta_i \qquad ,i=1,2.$$

The first partial derivatives of the second term of the earlier equation are obtained from equations 10 and 12 as

$$\frac{\partial h_d}{\partial x_3} + \frac{\partial \gamma}{\partial x_3} h_d = \frac{\partial V_d}{\partial h} = - \Delta g - 2 \frac{V_d}{R} + 2 \frac{W_o - U_o}{R} \quad .$$

Thus

$$\frac{\partial h_d}{\partial x_3} = \frac{1}{\gamma}\left[- \frac{\partial \gamma}{\partial x_3} h_d - \Delta g - 2 \frac{V_d}{R} + 2\frac{W_o - U_o}{R} \right]$$

$$= \frac{1}{\gamma}\left[\frac{2}{R}(V_d - 2(W_o - U_o)) - \Delta g - 2\frac{V_o}{R} + 2\frac{W_o - U_o}{R} \right] = - \frac{\Delta g}{\gamma}.$$

on using equation 5. Therefore,

$$\xi_{gi} = - \left(\frac{\partial h_d}{\partial x_i}\right)_{Tell} - \frac{\Delta g}{\gamma} \tan \beta_i \ , \ i=1,2 \qquad \ldots \quad (56),$$

which is the standard solution of Molodenskii(e.g.,Heiskanen & Moritz 1967,p.313) as expected, expressions for the deflections of the vertical being unaffected by zero degree consoderations.

Deflections of the vertical ξ_{ai} comparable with astro-geodetic deflections are obtained by the combination of equations 50 and 56 as

$$\xi_{ai} = - \left(\frac{\partial h_d}{\partial x_i}\right)_{Tell} - \frac{\Delta g}{\gamma} \tan \beta_i + c_{\xi i} \quad ,i=1,2 \qquad \ldots(57),$$

where

$$c_{\xi 1} = 0.17\ h^{(km)} \sin 2\phi \ \sec \qquad \text{and} \qquad c_{\xi 2} = 0.$$

From the conventional treatment of deflections of the vertical (e.g.,ibid,p.112), the analytic expression for $(\partial h_d/\partial x_i)_{Tell}$

is given by

$$-\left(\frac{\partial h_d}{\partial x_i}\right)_{Tell} = -\frac{1}{h_i}\frac{\partial h_i}{\partial u_i} \quad ,i=1,2 \ ,$$

where u_1 and u_2 are a set of curvilinear surface parameters (ϕ,λ) and h_1, h_2 are the associated linearisation parameters give, for a spherical approximation of the earth by

$$h_1 = R \quad ; \quad h_2 = R \cos\phi \qquad \ldots \quad (58).$$

An examination of equation 48 in the light of equation 26 and figure 6 shows that

$$h_d = h_d(\phi,\lambda) = h\,(\psi, A_c, h_p)$$

when considering the differential changes in h_d from surface integrals. Hence,

$$-\left(\frac{\partial h_d}{\partial x_i}\right)_{Tell} = -\frac{1}{h_i}\left[\frac{\partial h_d}{\partial\psi}\cdot\frac{d\psi}{du_i} + \frac{\partial h_d}{\partial A_c}\frac{dA_c}{du_i}\right] - \frac{\partial h_d}{\partial x_3}\frac{dx_3}{dx_i} \ , \ i=1,2 \quad (59),$$

the last term being of equal magnitude but with opposite sign to the second term in equation 57. This was noticed by Moritz and attributed to an inadequacy of the planar approximation (Moritz 1968,p.23) when deriving the basic equation 27. The differential relations between the two sets of surface parameters can be established from a consideration of the spherical trigonometry of the Δ NPQ in figure 6 (e.g.,Heiskanen & Moritz 1967,p.113)when

$$\frac{d\psi}{d\phi} = -\cos A \quad ; \quad \frac{d\psi}{d\lambda} = -\cos\phi \sin A \qquad \ldots \quad (60).$$

The application of sine formula to this triangle gives

$$\sin(2\pi - A_c) = \frac{\cos\phi \sin\Delta\lambda}{\sin\psi} \ ,$$

where

$$\Delta\lambda = \lambda_c - \lambda ,$$

which, on differentiation gives

$$\cos A_c \frac{dA_c}{d\phi} = \frac{\sin\phi \sin\Delta\lambda}{\sin\psi} + \frac{\cos\phi \sin\Delta\lambda}{\sin^2\psi}\cos\psi \frac{d\psi}{d\phi}$$

The use of the five part formula and equation 60 gives

$$\frac{dA_c}{d\phi} = \frac{\sin\Delta\lambda}{\cos A_c \sin\psi}\left(\frac{\sin\phi\sin\psi - \cos\phi\cos\psi\cos A}{\sin\psi}\right)$$

$$= \frac{\sin\Delta\lambda}{\sin\psi}\left(\frac{\sin(\tfrac{1}{2}\pi - \phi_c)\cos(2\pi - A_c)}{\sin\psi\cos A_c}\right) = \frac{\sin A}{\sin\psi} \qquad \ldots(61).$$

Also,

$$\cos A_c \frac{dA_c}{d\lambda} = \frac{\cos\phi}{\sin\psi}\left(\cos\Delta\lambda + \frac{\cos\psi}{\sin\psi}\sin\Delta\lambda\frac{d\psi}{d\lambda}\right) .$$

The use of equation 60 and manipulation gives

$$\frac{dA_c}{d\lambda} = \frac{\cos\phi}{\sin\psi}\left[-\frac{\cos(2\pi - A_c)\cos A}{\cos A_c}\right] = -\frac{\cos A\cos\phi}{\sin\psi} \qquad \ldots(62).$$

The use of equations 60 to 62 in equation 59 gives

$$-\left(\frac{\partial h_d}{\partial\lambda}\right)_{i\,\mathrm{Tell}} = \frac{1}{R}\left[\left(\frac{\partial h_d}{\partial\psi}\right)_{\mathrm{Tell}}\cos\alpha_i + (-1)^i\left(\frac{\partial h_d}{\partial A}\right)_{\mathrm{Tell}}\frac{\sin\alpha_i}{\sin\psi}\right] - \frac{\partial h_d}{\partial x_3}\frac{dx_3}{dx_i}, i=1,2 \qquad \ldots(63),$$

where

$$\alpha_1 = A \qquad ; \qquad \alpha_2 = \tfrac{1}{2}\pi - A.$$

As $r_o = 2R\sin\tfrac{1}{2}\psi + o\{fr_o\}$,

$$\frac{\partial}{\partial\psi}\left(\frac{1}{r_o}\right) = -\frac{1}{R}\frac{\cos\tfrac{1}{2}\psi}{4\sin^2\tfrac{1}{2}\psi} .$$

The combination of equations 48, 57 and 63, bearing in mind that zero degree terms do not contribute to deflections of the vertical, gives

$$\xi_{ai} = \frac{1}{4\pi\gamma}\iint \Delta g \frac{\partial}{\partial\psi}\{f(\psi)\} \cos \alpha_i \, d\sigma \quad +$$
$$\frac{R}{2\pi\gamma}\iint\Bigg((\{R \sin \psi \frac{dh}{dr_o} + h_p - h\} \frac{-3 \cos \frac{1}{2}\psi}{16R^3 \sin^4 \frac{1}{2}\psi} +$$
$$\frac{R \cos \psi}{8R^3 \sin^3 \frac{1}{2}\psi} \frac{dh}{dr_o}) V_d + \frac{\gamma \cos \frac{1}{2}\psi}{4R \sin^2 \frac{1}{2}\psi} \sum_{j=1}^{2} \xi_j \tan \beta_j\Bigg) \cos\alpha_i \, d\sigma \quad +$$
$$(-1)^i \frac{R}{2\pi\gamma}\iint \frac{R \sin \psi}{8R^3 \sin^3 \frac{1}{2}\psi} \frac{\partial}{\partial A_c}\left(\frac{dh}{dr_o}\right) V_d \frac{\sin \alpha_i}{\sin \psi} d\sigma + c_{\xi i}, i=1,2 \qquad (64),$$

where (e.g., ibid, p.114),

$$\frac{\partial}{\partial\psi}\{f(\psi)\} = -\tfrac{1}{2} \cos \tfrac{1}{2}\psi \operatorname{cosec}^2 \tfrac{1}{2}\psi - 6 \cos \tfrac{1}{2}\psi + 8 \sin \psi - 3\frac{1 - \sin \frac{1}{2}\psi}{\sin \psi} +$$
$$3 \sin \psi \log\{\sin \tfrac{1}{2}\psi(1 + \sin \tfrac{1}{2}\psi)\} \quad \ldots \quad (65),$$

$$\frac{dh}{dr_o} = \sum_{j=1}^{2} \cos \alpha_{cj} \tan \beta_j \quad \ldots \quad (66),$$

and

$$\frac{\partial}{\partial A_c}\left(\frac{dh}{dr_o}\right) = \sum_{j=1}^{2} (-1)^j \sin\alpha_{cj} \tan \beta_j \quad \ldots \quad (67),$$

the angles α_j being given by

$$\alpha_1 = A \quad ; \quad \alpha_2 = \tfrac{1}{2}\pi - A \quad \ldots \quad (68),$$

and noting that the subscript $_c$ refers to evaluation of quantities at the variable element of surface area $d\sigma$. The first row of equation 64 is the Vening Meinesz integral. The second and third rows are topography dependent, being, with one exception, zero for ocean areas. As all terms dependent on the topography are functions of $1/r_o^3$, distant zones should have even smaller effects on the deflection of the vertical than on h_d values. Hence expressions of the deflection of the vertical in the form at 64 has the advantage that no computation of either a vertical gradient of gravity or a terrain correction is necessary prior to the estimation of the topographical effects, this factor being a decided advantage at the present time. Thus ξ_{ai} is given by the equation

$$\xi_{ai} = \xi_{fi} + \xi_{ci} + c_{\xi i} \quad ,i=1,2 \qquad \ldots \quad (69),$$

where $\xi_{fi}(i=1,2)$ are the deflections of the vertical for the free air geoid, given by

$$\xi_{fi} = \frac{1}{4\pi\gamma} \iint \Delta g \frac{\partial}{\partial\psi}\{f(\psi)\} \cos \alpha_i \; d\sigma \quad ,i=1,2 \quad \ldots \quad (70),$$

ξ_{ci} being the topographical correction for surface deflections of the vertical which need only be evaluated over a spherical cap with a limiting distance ψ_o from the computation point. If $\psi_o = 3^0$,

$$\sin \psi = \psi + o\{10^{-4}\} \; ; \quad \cos \psi = 1 + o\{f\} \; .$$

can be be expressed by the equation

$$\xi_{ci} = \frac{1}{2\pi\gamma R}\iint \Big(\big(\frac{R\gamma}{\psi^2} \sum_{j=1}^{2} \xi_j \tan \beta_j - \{2 \frac{dh}{dr_o} + 3 \frac{h_p - h}{R\psi}\} \frac{V_d}{\psi^3} \big) \cos \alpha_i + (-1)^i \frac{\partial}{\partial A_c} \big(\frac{dh}{dr_o}\big) \frac{V_d}{\psi^3} \sin \alpha_i \Big) d\sigma \quad ,i=1,2 \quad \ldots\ldots(71).$$

The final formulae considered are listed in section 2.7.

2.5 THE INNERMOST ZONE

A study of equation 48 together with equations 40 and 64 as well as 65 shows that the surface integrals for both h_d and ξ_{ai} are indeterminate at $\psi = 0$. Further, The effect of the innermost zone is always of the form

$$I = K\int_0^{r_i}\int_0^{2\pi} \frac{f(r,A)}{r^n} \; g(A) \; r \; dr \; dA \qquad \ldots \quad (72),$$

where g(A) is a set of trigonometrical functions of the azimuth A, r being the distance between the element of surface area r dr dA and the computation point. It is assumed that the bounding radius r_i of the inner zone, assumed circular, is small enough to warrant the treatment of the variations in f(r,A) as linear over the region. In such a case,

$$f(r,A) = f(0,0) + r \sum_{j=1}^{2} \cos \alpha_j \frac{\partial f}{\partial x_j} \qquad \ldots \quad (73),$$

where $\partial f/\partial x_j$ is the rate of change of f(r,A) with distance, the x_j axes having the same significance as in equation 52. The substitution of equation 73 in 72 gives

$$I = K\left(f(0,0)\int_0^{r_i}\int_0^{2\pi} g(A) r^{1-n} dr \; dA + \int_0^{r_i}\int_0^{2\pi} \sum_{j=1}^{2} \cos \alpha_j g(A) \frac{\partial f}{\partial x_j} \; r^{2-n} dr \; dA\right) \quad ..(74)$$

The integral in equation 74 is soluble if

(i) $$\int_0^{2\pi} g(A) \; dA = \int_0^{2\pi} g(A) \cos \alpha_j \; dA = 0 \; ;$$

and/or

(ii) $$1 - n \geq 0 \; ;$$

(iii) $$\int_0^{2\pi} g(A) \; dA = 0 \qquad\qquad 1 - n < 0.$$

In cases where these conditions are not satisfied, solutions can be obtained by rationalisation. This occurs when

$$2 - n < 0 \; ; \text{ e.g., when } n > 3.$$

The following assumptions are made in the case of all evaluations.

(a) Any gravity anomaly in this innermost region can be represented in terms of the value Δg_p at the computation point by an equation of the type at 73.

(b) The physical surface of the earth is a plane over the inner zone. It would, in fact, suffice if the topographical variations were symmetrical about the computation point if this condition could not be met. Thus, tan β_j will be constant over the region.

(c) The height anomaly has linear variations over this innermost zone. For all practical purposes, this is equivalent to the deflections of the vertical being constant over the region as

$$\frac{\partial h_d}{\partial x_j} = - \xi_j \qquad ,j=1,2.$$

The assumption at (b) is probably the most questionable unless r_i is reduced to meet the requirement. The solutions for the Stokes and Vening Meinesz integrals are well known (e.g.,Mather 1969b,p504), being given by

$$N_{fin} = \frac{\Delta g_p}{\gamma} r_i \left(1 + \frac{r_i}{4R}\right) \qquad \ldots \quad (75)$$

and

$$\xi_{finj} = - \frac{1}{\gamma} r_i \left(\frac{\partial \Delta g}{\partial x_j}\right)\left(1 + \frac{3r_i}{4R}\right) \qquad ,j=1,2 \quad \ldots \quad (76).$$

The effects of the innermost zone on N_i (N_{iin}) are evaluated after allowing for the fact that dh/dr_o and ξ_{fj} as well as tan β_j are constants, when

$$N_{iin} = \frac{1}{2\pi\gamma}\int_0^{r_i}\int_0^{2\pi}\left[\left(\frac{1}{r^2}\frac{dh}{dr_0} - \frac{1}{r^2}\sum_{k=1}^{2}\frac{dh}{dx_k}\cos\alpha_k\right)V_d - \frac{\gamma}{r}\sum_{k=1}^{2}\xi_k \tan\beta_k\right] r\,dr\,dA$$

$$= - r_i\left(\sum_{k=1}^{2}\xi_k \tan\beta_k\right)_{r=0} \qquad \ldots \quad (77).$$

The expressions involved in the evaluation of ξ_{cinj} appear to be more complex. The process of evaluation is considerably simplified when it is remembered that zero order changes do not contribute to deflections of the vertical and it is possible to express V_d on a revised datum as

$$\frac{V_d}{\gamma} = 0 + \sum_{k=1}^{2} r \frac{\partial h_d}{\partial x_k} \cos \alpha_k = - r \sum_{k=1}^{2} \xi_k \cos \alpha_k$$

without affecting the contributions to the deflections (e.g.,de Graaff-Hunter 1950,p.4). The following simplifications are possible over limited planar regions.

$$A_c = \pi + A \; ; \qquad R\psi = r \; .$$

Thus equations at 71 become

$$\xi_{c\,inj} = \frac{1}{2\pi} \int_0^{r_i} \int_0^{2\pi} \frac{1}{r^2} \sum_{k=1}^{2} \xi_k \tan \beta_k \cos \alpha_j r \, dr \, dA \; -$$

$$\int_0^{r_i} \int_0^{2\pi} \left(\left(2\frac{dh}{dr_o} - 3\frac{h_p - h}{r}\right) \cos \alpha_j + (-1)^j \frac{\partial}{\partial A_c} \left(\frac{dh}{dr_o}\right) \sin \alpha_j \right) \times$$

$$\left(- \frac{r \sum_{k=1}^{2} \xi_k \cos \alpha_k}{r^3} \right) r \, dr \, dA, \quad j=1,2$$

As

$$\frac{dh}{dr_o} = - \sum_{k=1}^{2} \cos \alpha_k \tan \beta_k ; \quad \frac{\partial}{\partial A_c} \left(\frac{dh}{dr_o}\right) = \sum_{k=1}^{2} (-1)^k \sin \alpha_k \tan \beta_k$$

and

$$h_p - h = - r \sum_{k=1}^{2} \tan \beta_k \cos \alpha_k ,$$

$$\xi_{c\,inj} = 0 - \frac{1}{2\pi} \int_0^{r_i} \int_0^{2\pi} \sum_{k=1}^{2} \Big(\cos \alpha_k \tan \beta_k (2-3) \cos \alpha_j \; +$$

$$(-1)^{j+k} \sin \alpha_k \tan \beta_k \sin \alpha_j \Big) \frac{\sum_{k=1}^{2} \xi_k \cos \alpha_k}{r^2} r \, dr \, dA, \; j=1,2,$$

which, in all instances integrates to zero, as it is composed of a series of integrals of the form

$$\xi_k \tan \beta_k \int_0^{2\pi} \cos^r A \sin^s A \, dA$$

where $r + s = 3$, $0 < r \leq 3$;

as $\alpha_1 = A$; $\alpha_2 = \frac{1}{2}\pi - A$.

These integrals can be converted to the form

$$\int_0^{2\pi} \tfrac{1}{2}(\cos \alpha_k \pm \cos \alpha_k \cos 2A) \, dA, \qquad k=1,2$$

which always integrates between the limits to zero. Hence,

$$\xi_{cinj} = 0, \qquad j=1,2 \qquad \ldots \quad (75)$$

if the limiting radius r_i is small enough to warrant the following assumptions:-

(i) The topographical surface does not depart significantly from a plane

(ii) The local geop is planar over the region.

It is interesting to note a relationship given by Moritz (1966, p. 74- eq. 220) which implies the above possibility.

2.6 A REVIEW OF THE CONDITIONS FOR GEOCENTRICITY

The formula for the height anomaly set out in equation 48 takes into consideration the possibility of the existence of zero degree terms in the disturbing potential V_d which is defined as the difference

$$V_{dp} = W_p - U_p$$

at a point P on the physical surface of the earth, where U_p is the potential of the spherop and W_p that of the geop passing through P. The rotational terms can be ignored if the definition is confined to earth space, the geopotential in such a space being expressed by

$$W_p = k\iiint \frac{dm}{r} \qquad \ldots \quad (76),$$

where r is the distance of the element of mass dm at Q in figure 7 from P. If R is the distance of $P(X_i, i=1,3)$ from the centre of mass C of the earth, which is chosen as the origin of the Cartesian co-ordinate system $X_1X_2X_3$ and if P is assumed to be external to the earth, the choice of axes being purely a matter of convenience,

$$X_i = R\,\ell_i \quad ,i=1,3$$

and $\ldots \quad (77),$

$$X_{ci} = R_c \ell_{ci} \quad ,i=1,3$$

where X_{ci} are the co-ordinates of the element of mass dm (= ρ dV) at Q and R_c is the distance of Q from C. ℓ_i with the appropriate subscripts, refer to the direction cosines of the lines concerned. W_p can then be expressed without approximation as

$$W_p = \frac{k}{R} \sum_{n=0}^{\infty} \frac{1}{R^{n+1}} \iiint R_c^n \; p_{no}'\cos\psi)\; dV \qquad \ldots \quad (78),$$

where r^{-1} has been expressed by the standard zonal harmonic expansion (e.g.,Jeffreys & Jeffreys 1962,p.634) and $\rho$is the density of matter in the element of volume dV. The geopotential can also be expressed by the spherical harmonic series

$$W_p = \sum_{n=0}^{\infty} \frac{1}{R^{n+1}} \sum_{m=0}^{n} p_{nm}(\sin\phi)\left(C_{nm}\cos m\lambda + S_{nm}\sin m\lambda \right) \qquad (79).$$

If equations 79 and 80 are identical, the equation of the contributions of harmonics of the same degree in the case of low degree harmonics gives the following results.

When $n = 0$,

$$C_{oo} = k\iiint \rho\; dV = kM,$$

where M is the zero order inertia tensor or mass of the earth.

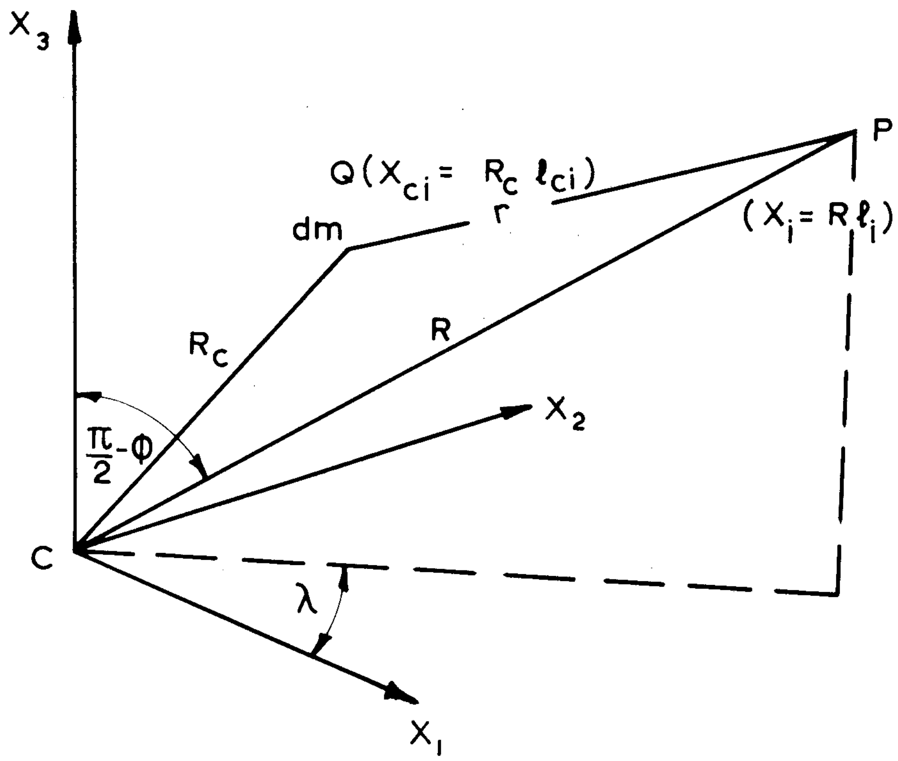

Fig. 7

The geocentre and associated coordinates

When $n = 1$,

$$p_{10}(\cos\psi) = \cos\psi = \sum_{i=1}^{3} \ell_i \ell_{ci} .$$

The appropriate term from equation 78 can be written as

$$k \iiint R_c \cos\psi\, \rho\, dV = k \sum_{i=1}^{3} \ell_i \iiint X_{ci}\, \rho\, dV$$

$$= k \sum_{i=1}^{3} \ell_i\, M\, \overline{X}_i \qquad = 0$$

where $\overline{X}_i\,(i=1,3)$ are the co-ordinates of the centre of mass of the earth which, by definition, are zero, the second equality following from the definition of the first order inertia tensor (e.g., Hotine 1969, p.156). Hence the coefficients of all terms comprising the first degree harmonic in equation 80 must be zero;, i.e.,

$$C_{10} = C_{11} = S_{11} = 0.$$

When $n = 2$.

The kernel of the second degree term in equation 78 is

$$R_c^2 \; p_{20}(\cos\psi) = R_c^2\left(\frac{3}{2}\cos^2\psi - \frac{1}{2}\right)$$

$$= \tfrac{1}{2} R_c^2 \left(3\left(\sum_{i=1}^{3} \ell_i \ell_{ci}\right)^2 - 1\right) = \tfrac{1}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} \left(3X_{ci}X_{cj} - \delta_{ij}R_c^2\right) \quad ..(80),$$

where δ_{ij} is the Kronecker delta. The resulting second degree term from equation 78 can be expressed as

$$\tfrac{1}{2} k \sum_{i=1}^{3} \sum_{j=1}^{3} \ell_i \ell_j \iiint (3X_{ci}X_{cj} - \delta_{ij}R_c^2)\rho\, dV = -\frac{3}{2} k \sum_{i=1}^{3} \sum_{j=1}^{3} \ell_i \ell_j \times$$

$$\iiint (\delta_{ij}R_c^2 - X_{ci}X_{cj})\rho\, dV + k \sum_{i=1}^{3} \sum_{j=1}^{3} \ell_i \ell_j \iiint \delta_{ij} R_c^2\, \rho\, dV.$$

The second integral on the right hand side of the equality is the scalar form of the second order inertia tensor I_s, given by (e.g., ibid, p. 165)

$$I_s = \iiint \sum_{i=1}^{3} X_{ci}^2 \rho \; dV = \tfrac{1}{2}(I_1 + I_2 + I_3) \qquad \ldots (81),$$

where I_i $(i=1,3)$ are the moments of inertia of the earth about the axes X_1, X_2 and X_3 respectively, given by

$$I_i = \iiint (X_{i+1}^2 + X_{i+2}^2)\rho \; dV, \qquad \text{if subscript} > 3, - 3.$$

The following relation is obtained on equating second degree terms between equations 78 and 79, on expressing the relation preceding equation 81 in matrix notation.

$$\sum_{m=0}^{2} P_{2m}(\sin\phi)(C_{2m}\cos m\lambda + S_{2m}\sin m\lambda) = -\frac{3}{2}\,k\{L^T I\, L\} + kI_s \quad (82),$$

where

$$I = \begin{vmatrix} \iiint (R_c^2 - X_{c1}^2)\rho \; dV & -\iiint X_{c1}X_{c2}\,\rho \; dV & -\iiint X_{c1}X_{c3}\rho \; dV \\ -\iiint X_{c2}X_{c1}\,\rho \; dV & \iiint (R_c^2 - X_{c2}^2)\rho \; dV & -\iiint X_{c2}X_{c3}\,\rho \; dV \\ -\iiint X_{c3}X_{c1}\,\rho \; dV & -\iiint X_{c3}X_{c2}\rho \; dV & \iiint (R_c^2 - X_{c3}^2)\rho dV \end{vmatrix} \quad (83),$$

and

$$L = \begin{vmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \end{vmatrix} = \begin{vmatrix} \cos\phi\cos\lambda \\ \cos\phi\sin\lambda \\ \sin\phi \end{vmatrix} \qquad \ldots \quad (84),$$

the elements of L being the components of the unit vector in the direction CP. If the non-diagonal elements of the array I were zero,

$$I = I_s$$

as the elements I_{ii} along the diagonal are related to the quantities defined in equation 81 by the expression

$$I_{ii} = I_i, \qquad i=1,3$$

As the array I is symmetrical,

$$L^T I L = I_1\cos^2\phi\cos^2\lambda + I_2\cos^2\phi\sin^2\lambda + I_3\sin^2\phi - 2I_{12}\cos^2\phi\sin\lambda\cos\lambda - 2I_{13}\sin\phi\cos\phi\cos\lambda - 2I_{23}\sin\phi\cos\phi\sin\lambda \;.$$

$$\sum_{m=0}^{2} p_{2m}(\sin\phi)\left(C_{2m}\cos m\lambda + S_{2m}\sin m\lambda\right) = -k\Big(I_1\left(\tfrac{3}{2}\cos^2\phi\cos^2\lambda - \tfrac{1}{2}\right) + I_2\left(\tfrac{3}{2}\cos^2\phi\sin^2\lambda - \tfrac{1}{2}\right) + I_3\left(\tfrac{3}{2}\sin^2\phi - \tfrac{1}{2}\right) - 3I_{12}\cos^2\phi\sin\lambda\cos\lambda - 3I_{13}\sin\phi\cos\phi\cos\lambda - 3I_{23}\sin\phi\cos\phi\sin\lambda\Big)$$

$$= -k\Big[I_1\left(\tfrac{3}{4}(1-\sin^2\phi)(1+\cos 2\lambda) - \tfrac{1}{2}\right) + I_2\left(\tfrac{3}{4}(1-\sin^2\phi)(1-\cos 2\lambda) - \tfrac{1}{2}\right) + I_3\, p_{20}(\sin\phi) - \tfrac{1}{2}\, p_{22}(\sin\phi)\sin\lambda - p_{21}(\sin\phi)(I_{13}\cos\lambda + I_{23}\sin\lambda)\Big]$$

$$= k\Big\{p_{20}(\sin\phi)\left(\tfrac{1}{2}(I_1 + I_2) - I_3\right) + p_{21}(\sin\phi)\left(I_{13}\cos\lambda + I_{23}\sin\lambda\right) + p_{22}(\sin\phi)\left(\tfrac{1}{2} I_{12}\sin 2\lambda + \tfrac{1}{4}(I_2 - I_1)\cos 2\lambda\right)\Big\} \;,$$

where (e.g.,Heiskanen & Moritz 1967,p.23),

$$p_{21}(\sin\phi) = 3\cos\phi\sin\phi \qquad\qquad p_{22}(\sin\phi) = 3\cos^2\phi \;.$$

Thus,

$$C_{20} = k\left(\tfrac{1}{2}(I_1 + I_2) - I_3\right)$$

$$C_{21} = k\, I_{13} \quad ; \quad S_{21} = k\, I_{23} \quad \ldots \quad (84).$$

$$C_{22} = \tfrac{1}{2}\, k\, I_{12} \quad ; \quad S_{22} = \tfrac{1}{4}\, k(I_2 - I_1)$$

From a study of equations 83 and 84, it can be seen that $L^T I L$ can, in effect, be diagonalised if

$$\phi = \tfrac{1}{2}\pi$$

or the direction CP coincides with the X_3 axis. Such an axis is callec a principle axis of inertia. It can be shown (e.g.,Hotine 1969, p.166) that products of inertia using any pair of mutually perpendicular axes which are also orthogonal to the X_3 axis, is zero. Thus, if the X_3 axis is a principal axis of inertia,

$$I_{13} = I_{23} = 0$$

This condition is satisfied in earth space if the axis is the rotation axis. The departures of the existent earth from perfect rigidity give rise to an Eulerian or free nutation. This results in variations of the position of the instantaneous pole in earth space which can be as large as 0.2 sec (e.g.,Jordan-Eggert 1962, p.535). Non-coincidence of the axis of rotation and the principle axis of greatest moment of inertia will give rise to a wobble of the instantaneous axis of rotation with respect to a geodetic reference frame in earth space. The existence of any such effects which produce changes in latitude in excess of 0.5 sec have never been recorded. Such limits are equivalent to the poles of the instantaneous axis of rotation describing an elliptical path with a mean diameter of approximately 20 ft (Munk & MacDonald 1960,p.5) about the pole of geodetic reference. Such magnitudes are disregarded in the present study.

It can therefore be concluded that, for all practical purposes :-

(i) The centre of mass of the earth lies on the rotation axis, which is a principle axis of inertia.

(ii) If the x_3 axis is made coincident with this axis,

$$C_{21} = S_{21} = 0$$

(iii) Coincidence of the centre of mass and the origin of the rectangular Cartesian co-ordinate system imposes the condition

$$C_{10} = C_{11} = S_{11}.$$

The other dynamic conclusions which can be drawn from equation 85 are not of direct geometrical significance to the current problem

The reference system

The potential U_p due to the reference system is a consequence of adopting the oblate spheroid as the reference figure. The gravitational field of such a spheroid at exterior points can be expressed by a spherical harmonic series of the form (e.g.,Heiskanen & Moritz 1967,p.73)

$$U_p = \sum_{s=0}^{\infty} \frac{1}{r^{2s+1}} P_{2s\,0}(\sin\phi)\, C'_{2s\,0} \quad \ldots \quad (86),$$

which is independent of longitude dependent terms if the mass distribution of the spheroid is symmetrical. In addition, no odd degree zonal harmonics are permissible as symmetry prevails about the equatorial plane. Such a series converges rapidly and an accuracy of 2 parts in 10^7 is achieved by considering only the first three terms in equation 86.

The disturbing potential is obtained by embedding the spheroid in earth space, with the centre of the spheroid at the centre of mass of the earth and its minor axis coincident with the rotation axis of the earth. Consequently r and ϕ have the same significance in both equations 79 and 86. This results in the linearisation of the geopotential into the disturbing potential V_d by the equation

$$V_d = W_p - U_p = \frac{kM}{r} - \frac{C'_{oo}}{r} + \frac{P_{20}(\sin\phi)}{r^3}\left[k\left(\tfrac{1}{2}(I_1 + I_2) - I_3\right) - C'_{20} \right] + \frac{1}{r^3}P_{22}(\sin\phi)\, k\left(\tfrac{1}{4}(I_2 - I_1)\cos 2\lambda + \tfrac{1}{2}I_{12}\sin 2\lambda\right) + \sum_{n=3}^{\infty} \frac{1}{r^{n+1}} \times \sum_{m=0}^{n} P_{nm}(\sin\phi)\left(C_{nm}\cos m\lambda + S_{nm}\sin m\lambda\right) - \frac{1}{r^5}C'_{40}P_{40}(\sin\phi) \quad ..(87).$$

Note that the adoption of the equivalence for ϕ which is implied in the formation of equation 87, merely establishes the condition of equality of direction cosines for the rotation axis of the earth and the minor axis of the oblate spheroid, the former being the X_3 axis and is the principle axis of greatest moment of inertia of the existent earth. The absence of significant wobble implies that the centre of mass of the earth lies on the rotation axis and, consequently, coincidence between the minor axis of the oblate spheroid and the rotation axis of the earth.

The equivalence of r places the centre of the reference spheroid at the centre of mass of the earth, called the geocentre in this study. Such definition is, as implied earlier, subject to the limitations imposed by any uncertainties introduced by the Chandler wobble , which, however, is of the order of observational accuracy attainable from astro-geodetic techniques and hence neglected in the present study.

Under these conditions, V_d can be represented by a spherical harmonic series of the form

$$V_d = C_{oo} + \frac{kM}{r} \sum_{n=2}^{\infty} \left(\frac{a}{r}\right)^n \sum_{m=0}^{n} \overline{P}_{nm}(\sin\phi)\left(\overline{C}_{nm}\cos m\lambda + \overline{S}_{nm}\sin m\lambda\right) + o\{fV_d\} \quad (88),$$

the overbar referring to normalised harmonics and their coefficients (see both the guide to notation and Heiskanen & Moritz 1967,p.31 for definition) , which, for degree greater than 4 will be the same as those in equation 87. In addition,

$$C_{oo} = 0$$

if the parameters of the reference spheroid have been chosen to match the zero order inertia tensor of the existent earth. Further,

$$\overline{C}_{21} = \overline{S}_{21} = 0 .$$

The gravity anomaly at the earth's surface is obtained by the use of equation 36, together with the series expression of the disturbing potential given in equation 88 when $r \to a$. Thus

$$\Delta g = \gamma \sum_{n=2}^{\infty} (n-1) \sum_{m=0}^{n} \bar{P}_{nm}(\sin \phi)(\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda) + O\{f\ \Delta g\} \quad \ldots \quad (89),$$

where γ is the mean value of normal gravity. Thus, unambiguous representation of height anomalies and deflections of the vertical is dependent on the correct assessment of the dynamic implications of this section. The following conclusions can be drawn.

(i) The disturbing potential and the gravity anomaly have no first degree harmonics on global analysis if the centre of the reference spheroid is situated at the geocentre. Height anomalies and deflections of the vertical computed from a gravity anomaly set which has no first degree harmonic are referred to a spheroid which has its centre at the geocentre.

(ii) The coefficients C_{21}, S_{21} of the global gravity anomaly data set are zero if the rotation axis of the spheroid coincides with the principal axis of greatest moment of inertia of the earth, which, for all practical purposes, is the mean rotation axis of the earth.

Note that these conclusions are relevant for V_d and Δg only but not for any other function. The imposition of zero coefficients for first degree terms in the expansion of any other quantity on global analysis does not imply either of the conditions stated above. In addition, Stokes' integral is valid only if no first degree harmonic terms are present in the surface harmonic representation of both the disturbing potential and the gravity anomaly. Hence the use of any other anomaly, as pointed out by several writers (e.g., Jeffreys 1953, p.333; Moritz 1966, p.107) must ensure that the first degree harmonic of its global representation is zero as a precondition for its use in Stokes' integral.

Any corrected anomaly for use in surface solutions is is essentially the gravity anomaly corrected for a terrain correction (Moritz 1968) and hence the assumption of zero values for the coefficients of first degree harmonic terms in the anomaly $(\Delta g + G')$, where G' is the terrain correction, should not upset conditions (i) & (ii) by more than the order of f h_d (i.e., 30 cm or less). The Molodenskii-Moritz type equations have been avoided in the current solution *not* as a result of possible difficulties in interpretation, which are negligible, but for practical reasons set out in the next section.

2.7 THE EQUATIONS USED FOR PRACTICAL SOLUTION

The equations used in the current solution are based on previous experience in determining the free air geoid for Australia (Mather 1969b) and comparing the solution, after the introduction of the geocentric orientation vector, with the astro-geodetic solution of Fischer & Slutsky on the A.G.D. It was obvious that errors in the representation of the earth's gravity field were large enough to mask the correction terms for the free air geoid. This was further confirmed on comparing deflections of the vertical obtained by rather crude interpolation from gravimetry (Mather & Fryer 1970a, Table 2).

Hence it was decided to compute the free air geoid as the first approximation to h_d and ξ_{gi} using equations 43 and 70, along with equations 75 and 76 for the evaluation of the innermost zone, assuming that no zero degree term existed in the solution. This was equivalent to adopting the relation

$$W_o - U_o - R\, M\{\Delta g\} = 0.$$

This subject is discussed further in section 5.2. The corrections to the free air geoid were treated as expressed in equations 47 and 71 instead of the standard iterative approach of Molodenskii (Moritz 1966, pp.90-92 & 98) as the effect of N_{ip} and ξ_{ci} are essentially local in character, the distant zones having negligible effects on ξ_{ci} , while

N_{ip}, though converging less rapidly, was a small quantity. Heiskanen & Moritz (1967, p.329) quote Arnold's correction for Mount Blanc as being -0.2 metres.

From a practical point of view, significant terrain corrections occur in regions of rugged topography, not being a function of h but of tan β. Consequently significant corrections are confined to a few select regions of the world like the Himalayas, the Andes and the Rocky Mountains. In Australia itself, only the Snowy Mountains in the south east can give rise to any significant effect. Thus it was decided to restrict computations of corrections to a region within 2^{o} of the computation point, if such corrections were warranted after the study of results obtained by the use of the free air geoid. Such a procedure would completely define ξ_{ci}. The final formulae adopted were

$$N_{fp} = \frac{R}{4\pi\gamma}\int_{\psi_o}^{\pi}\int_{0}^{2\pi} \Delta g \; f(\psi) \; d\sigma \; + \; \frac{\Delta g_p}{\gamma} \; r_i\left(1 + \frac{r_i}{4R}\right) \qquad \ldots(90);$$

$$\xi_{fpj} = \frac{1}{4\pi\gamma}\int_{\psi_o}^{\pi}\int_{0}^{2\pi} \Delta g \; \frac{\partial}{\partial\psi}\{f(\psi)\}\cos\alpha_j d\sigma \; - \; \frac{1}{2\gamma} r_i \left(\frac{\partial \Delta g}{\partial x_j}\right)\left(1 + \frac{3r_i}{4R}\right), j=1,2 \qquad \ldots \quad (91);$$

$$N_{ip} = \frac{1}{2\pi}\int_{\psi_o}^{\psi'}\int_{0}^{2\pi}\left(\frac{1}{\psi^2}\left(\frac{dh}{dr_o} + \frac{h_p - h}{R}\right)N_f - \frac{R}{\psi}\sum_{k=1}^{2}\xi_{fk}\tan\beta_k\right)d\sigma \; - \; r_i\left(\sum_{k=1}^{2}\xi_{fk}\tan\beta_k\right)_p \qquad \ldots \quad (92)$$

$$\xi_{cpj} = \frac{1}{2\pi}\int_{\psi_o}^{\psi'}\int_{o}^{2\pi}\left\{\frac{1}{\psi^2}\sum_{k=1}^{2}\xi_{fk}\tan\beta_k - \left[\left(2\frac{dh}{dr_o} + 3\frac{h_p - h}{R}\right)\cos\alpha_j + (-1)^j\frac{\partial}{\partial A_c}\left(\frac{dh}{dr_o}\right)\sin\alpha_j\right]\frac{(N_f - N_{fp})}{\psi^3}\right\} d\sigma, j=1,2 \qquad \ldots \quad (93),$$

where

$$\psi' \simeq 2^{\circ} \quad ; \quad d\sigma = \sin\psi \, d\psi \quad dA = \cos\phi \, d\phi \, d\lambda;$$

$$\psi_o \simeq 0.01^{\circ} \quad ; \quad r_i = R\psi \quad ;$$

$$\frac{dh}{dr_o} = \cos A_c \tan \beta_1 + \sin A_c \tan \beta_2 \quad ;$$

$$\frac{\partial}{\partial A_c}\left(\frac{dh}{dr_o}\right) = -\sin A_c \tan \beta_1 + \cos A_c \tan \beta_2 \quad ;$$

$$\alpha_1 = A \quad ; \quad \alpha_2 = \tfrac{1}{2}\pi - A \, ,$$

A being the azimuth of $d\sigma$ from P and A_c the reverse azimuth. The use of $(N_f - N_{fp})$ instead of N_f in equation 93 is equivalent to a zero degree datum shiit which does not affect either the deflections of the vertical or the validity of the integral.

The above set of formulae are practically feasible in that all data necessary for computations are readily available. The use of the Molodenskii type anomaly $(\Delta g + G')$ in the Stokes and Vening Meinesz integrals (Moritz 1968,p.1) requires, as a pre-requisite, the computation of the terrain correction G', given by

$$G' = \frac{R^2}{4\pi}\iint \frac{(h - h_p)(\Delta g - \Delta g_p)}{r_o^3} \, d\sigma = \tfrac{1}{2} k \rho R^2 \iint \frac{(h - h_p)^2}{r_o^3} \, d\sigma \quad ..(94)$$

on a world wide basis. The integral, being a function of r_o^{-3} , is critically dependent on the accuracy with which near zone elevations have been digitised for computations and therefore must be considered impractical for world wide assessment at this stage, though local computations are a distinct possibility.

The probable existence of a first degree harmonic in the anomaly $(\Delta g + G')$ can be interpreted as providing a solution with

Stokes' integral where the centre of the spheroid and its associated spherops, to which the values of h_d and ξ_{gi} are referred, are no longer situated at the geocentre. Any other expectation, in view of the asymmetric distribution of the topography which has a significant first degree harmonic on global analysis (Lee & Kaula 1967,p.754), and in the absence of computational evidence to the contrary, must be considered an unlikely possibility. In practical terms, as pointed out earlier, the resulting effect is of no relevance as it would produce no significant effect on the determination of the geocentric orientation vector until a distinct improvement occurs in the representation available for the gravity anomaly field.

Satisfactory solutions will therefore be obtained for the parameters defining the separation vector either by the use of equations 90 to 93 or by the use of the expressions of Molodenskii and Moritz, adopting Moritz' interpretation of G', the anomaly (Δg + G') being used in the Vening Meinesz integrals for the inner zones ($\psi < 3^{o}$) only, the rest of the global field being represented by the free air geoid. The latter procedure is more tedious than the former, requiring an extra stage in the computation, and has the same disadvantage in computing deflections of the vertical of being highly unstable close to the computation point. Thus, no solution for surface deflections of the vertical can be wholly satisfactory until a combination of accurate gravity surveys and heighting is available in the near vicinity of the point being investigated.

The distant zone effects on h_d, through the expression for the correction N_i to N_f, do pose a problem as the solution does not converge rapidly and the height data available for the distant regions is inadequate. Any significant errors will be due to low degree harmonics in N_i, which, as argued by Jeffreys (1962,p.192), are due to the departure of the earth from a sphere and cannot therefore contribute in excess of f h_d to the final value for h_d. This, however, is the order of accuracy attainable from the adoption of the Stokes and

Vening Meinesz integrals.

The problem can therefore be summarised as follows :-

h_d and ξ_{gi} at the surface of the earth are influenced by two causes, to the order of the flattening, these being

(i) near zone effects over a region where which have planar characteristics, with magnitudes dependent on the departure of the earth's surface from a *level plane* and not on elevation alone;

and (ii) outer zone effects, which to the order of the flattening have spherical characteristics and are completely represented by the Stokes and Vening Meinesz integrals.

Solutions which are practical possibilities at the present time, correct to the order of the flattening , can be obtained by *either*

the use of equations 90 to 93

or

the following adaptation of the equations of Molodenskii as modified by Moritz (1968,p.1)

$$h_d = \frac{R}{4\pi\gamma} \iint \Delta g \ f(\psi) \ d\sigma + \frac{R}{4\pi\gamma} \int_0^{\psi_o} \int_0^{2\pi} G' \ f(\psi) \sin\psi \ d\psi \ dA \quad \ldots(95),$$

$$\xi_{gi} = \frac{1}{4\pi\gamma} \iint \Delta g \ \frac{\partial}{\partial\psi}\{f(\psi)\} \cos\alpha_i \, d\sigma + \frac{1}{4\pi\gamma} \int_0^{\psi_o} \int_0^{2\pi} G' \ \frac{\partial}{\partial\psi}\{f(\psi)\} \cos\alpha_i \sin\psi \ d\psi \times dA, \ i=1,2 \quad \ldots\ldots(96),$$

where

$$G' = \tfrac{1}{2} k \rho R^2 \int_0^{\psi} \int_0^{2\pi} \frac{(h - h_p)^2}{r_o^3} \sin\psi \ d\psi \ dA$$

and $\psi_o \leqq 3$, its actual magnitude being defined from equation 23 as the minimum value of ψ at which

$$r_o = R\psi + o\{f\, r_o\} \qquad ; \text{ i.e., when } \left(\frac{h - h_p}{r_o}\right)^2 \simeq f.$$

Conclusions:-

(i) Correction terms for the free air geoid are of significance only at those points where the near zone 'topography has' elevations considerably different from that of the computation point.

(ii) There is no elevation dependence as such. Corrections to the free air geoid are not significant in relatively high plateaus of great extent. This is of significance in Australia where considerable expanses of the continent have elevations in the vicinity of 500 metres but are relatively flat (Mather 1968a, app.4).

(iii) Large corrections to the deflections as computed for the free air geoid using the Vening Meinesz integrals, occur only in regions where the surrounding topography is at a considerably different elevation from that of the computation point and the general topographical features are asymmetrically situated with respect to it.

Thus it would appear that larger corrections could be expected at those astro-geodetic stations situated on the peripheries of mountain ranges than at those in the interior, though local effects may mask the drawing of such simplistic conclusions. Also see the comments in section 4.2.

2.8 THE COMPARISON OF GRAVIMETRIC AND ASTRO-GEODETIC SOLUTIONS

Two problems arise in the comparison of astro-geodetic and gravimetric determinations of the separation vector.

(a) The solutions may be referred to spheroids of different dimensions.

(b) The centres of the spheroids used in the gravimetric and astro-geodetic solutions may have different locations in earth space.

(a) Change of spheroid

This is a classic geodetic problem of direct relevance as all gravity anomalies are referred to the International Gravity Formula (I.G.F.) which is based on a spheroid of flattening of $(297.0)^{-1}$ and a value of equatorial gravity (γ_e) equal to 978.0490 gal (e.g., Heiskanen & Moritz 1967, p. 79 & 80). Gravimetric geodesists prefer to define the reference system in terms of f and γ_e, but it is also possible to do so using the four parameters (a, f) defining the reference spheroid, kM and ω (e.g., Lambert 1961, pp. 13-18), all of which are capable of numerical estimation independent of normal gravity.

Reference system 1967 (R.S.1967) is defined in terms of the following three parameters, the value of ω being assumed to be a known quantity (I.A.G. Resolutions 1967, p. 367)

$$a = 6,378,160 \text{ metres}$$

$$C_{20} = -1082.7 \times 10^{-6} \qquad \ldots (97),$$

and $$kM = 3.986\ 03 \times 10^{20} \text{cm}^3\text{sec}^{-2}$$

the value of C_{20} being related to the value of the flattening through the equation (e.g., Heiskanen & Moritz 1967, p.78)

$$C_{20} = -\frac{2}{3} f + \frac{1}{3} m + \frac{1}{3} f^2 - \frac{2}{21} mf \quad \ldots \quad (98),$$

where m is given by equation 14. The value -1082.7×10^{-6} is equivalent to

$$f = \frac{1}{298.25} .$$

The spheroid of reference so defined is called *Reference Ellipsoid 1967* and hence the datum provided by R.S.1967 is based on a spheroid with the same dimensions as the Australian National Spheroid, defined in equatioı 1. Hence h_d and ξ_{gi} values computed using R.S. 1967 require no correction for change in dimensions of the reference spheroid.

The changes from the datum afforded by the I.G.F. to that provided by R.S.1967 can be performed by one of two methods.

(i) Compute h_d and ξ_{gi} using anomalies referred to the I.G.F. and apply a correction to the computed values using the relations

$$\Delta\xi_{s1} = - df \sin 2\phi \qquad + o\{f^3\}$$

$$\Delta\xi_{s2} = 0 \qquad \ldots (99),$$

$$\Delta\xi_{s3} = da + R(df \sin^2\phi + \Delta\xi_{s1} \tan \phi) + o\{af^3\}$$

where $\Delta\xi_{si}$ (i=1,3) are the changes to the curvilinear parameters ξ_i (i=1,3) defining the components of the separation vector *d*.

Alternately,

(ii) convert the free air anomalies from the I.G.F. reference system to R.S.1967 and allow for the Potsdam datum correction (Mather 1968c).

The second method has been adopted, even though more expensive to compute, as it has the advantage of affording a check on the correct usage of computer routines.

(b) The corrections for coincidence of spheroids used in the gravimetric and astro-geodetic solutions.

The spheroid used in the gravimetric solutuon, obtained from equations 90 to 93, satisfies the two conditions following equation 89 in section 2.6. Therefore the gravimetric spheroid has its centre at the centre of mass of the earth and its minor axis coincident with the earth's axis of rotation as this is the principal axis of greatest moment of inertia. The second condition holds only if

$$C_{21} = S_{21} = 0$$

in the surface harmonic representation of the disturbing potential and hence, the gravity anomaly, as given in equation 89.

The spheroid used in the astro-geodetic solution has a location in earth space defined by the geodetic co-ordinates adopted at the origin. These values are assigned in practice from either the astronomical values at the origin or as a function of the magnitudes of these quantities at points covering the region for which the datum is being defined.

Astronomical co-ordinates (ϕ_a, λ_a) are totally dependent on the earth space location of the earth's rotation axis. The longitude (λ_a) can be defined as the angle between two planes containing the rotation axis, one of which passes through a reference point (e.g., the concept of Greenwich). The Latitude (ϕ_a) is the complement of the angle between the direction of the local zenith, with direction cosines (ℓ_i, i=1,3) and the rotation axis whose direction cosines are ℓ_{3i}(i=1,3)when

$$\sum_{j=1}^{3} \ell_j \ell_{3j} = \sin \phi_a .$$

If the rotation axis coincides with the X_3 axis of a rectangular Cartesian system with the X_1X_2 plane coinciding with the earth's equator, the X_1 axis lying in the meridian of reference for longitudes, as shown in figure 8, and if the direction cosines of the X_i axis are ℓ_{ij}(j=1,3) while those of the projection of the local vertical on the X_1X_2 plane are ℓ_{pj}(j=1,3), then

$$\sum_{j=1}^{3} \ell_{1j}\ell_{pj} = \cos \lambda_a .$$

The 15 direction cosines defining the five directions are linked by basic equations of the form

$$\sum_{j=1}^{3} \ell_j^2 = 1 ,$$

while those of the rectangular Cartesian axes are related by two non-redundant equations of the type

$$\sum_{j=1}^{3} \ell_{ij} \ell_{kj} = 0.$$

Thus the adoption of the astronomical co-ordinates at the origin affords a system of 9 equations with 15 unknowns and no linear parameter to scale the system. The 6 parameters arbitrarily assigned can be considered to be :

(i) The direction cosines of the rotation axis, which are made equal to those of the minor axis of the spheroid of reference. This merely ensures that these two lines are parallel in earth space.

(ii) The direction cosines of the vertical at the origin of the geodetic datum as defined by the astronomical observations, are made equal to those of the normal to the spheroid at the point (ϕ_a, λ_a). The centre of the reference spheroid, whose adoption scales the system, can be expected to be located at some point C' in earth space, which does not, in general, coincide with the geocentre C as shown in figure 8.

The correction in earth space between the astro-geodetic spheroid centred at C' and the gravimetric spheroid centred at the geocentre C can be expressed by the *geocentric orientation vector (O)* given by

$$O = \vec{C'C} = \sum_{i=1}^{3} \Delta x_{io}\, i_o = \sum_{i=.}^{3} h_i\, \Delta\xi_{io}\, i_o \quad \ldots \quad (100),$$

where Δx_{io} (i=1,3) are the components of the geocentric orientation vector on the local Laplacian trihedron co-ordinate system (for definition see Dufour 1968,p.128), with unit vectors i_o, at the origin of the datum. The second equality in equation 100 holds through equation 3, where the geocentric orientation parameters $\Delta\xi_i$ and their associated linearisation parameters h_i at the geodetic origin are given by

$$\begin{aligned} \Delta\xi_{1o} &= \Delta\xi_o \quad ; \quad & h_1 &= -(\rho_o + h_{so}) \\ \Delta\xi_{2o} &= \Delta\eta_o \quad ; \quad & h_2 &= -(\nu_o + h_{so}) \quad \ldots \quad (101), \\ \Delta\xi_{3o} &= h_{do} \quad ; \quad & h_3 &= 1 \end{aligned}$$

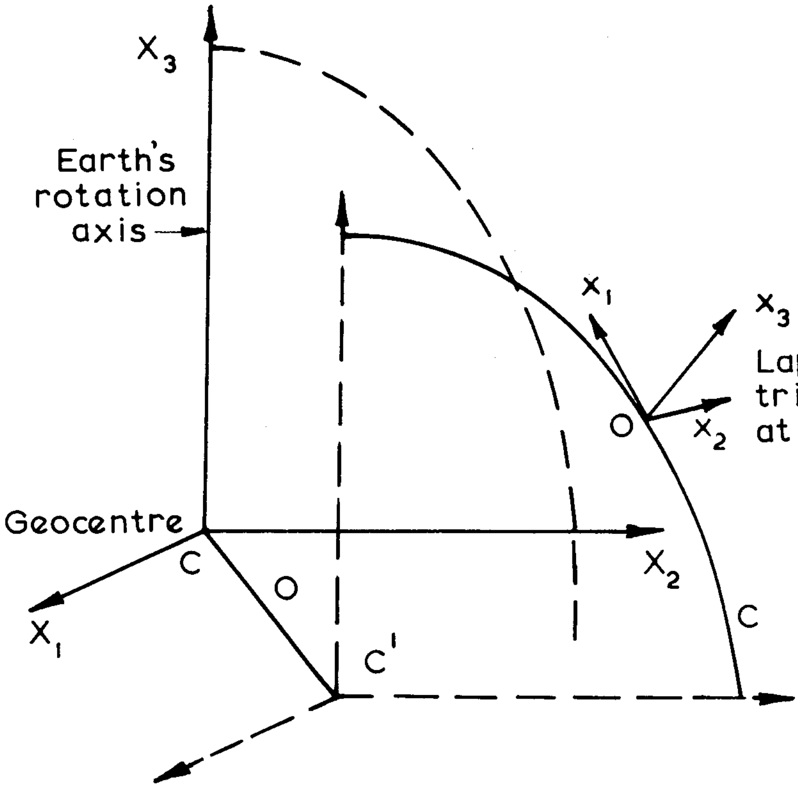

Fig. 8

The geocentre orientation vector

ρ and ν being the radii of curvature in the meridian and prime vertical normal sections while h_s is the spheroidal elevation given by

$$h_s = h_n + h_d \qquad ...(102),$$

h_n being the normal height defined in equation 13. The subscript $_o$ refers to values at the origin. The astro-geodetic deflections of the vertical ξ_{ai} (i=1,2) are given by the standard formulae (e.g.,Bomford 1962,p.89)

$$\xi_{a1} = (\phi_a - \phi_G) \quad ; \quad \xi_{a2} = (\lambda_a - \lambda_G) \cos \phi \qquad ... (102).$$

The geocentric orientation vector O can also be represented by components Δx_i with respect to the local Laplacian trihedron coordinate system at the general point P on the datum, the unit vectors along whose axes are i (i=1,3) when

$$O = \sum_{i=1}^{3} \Delta x_{io}\, i_o = \sum_{i=1}^{3} \Delta x_i\, i = \sum_{i=1}^{3} h_i\, \Delta\xi_i i \qquad ...(103).$$

In equations 100 to 103, the $x_1 x_2 x_3$ rectangular Cartesian coordinate system defining the local Laplacian trihedron is such that the x_1 axis is oriented north in the geodetic horizon, x_2 axis similarly situated in the geodetic prime vertical and the x_3 axis along the local normal as shown in figures 8 and 9. The required parameters ξ_{gi} (i=1,3) defining the separation vector d on a geocentric spheroid are related to those on the astro-geodetic datum ξ_{ai} by the relations

$$\xi_{gi} = \xi_{ai} + \Delta\xi_i \quad , i=1,3 \qquad ... (104),$$

where $\xi_3 = h_d$.

The relation between the various components can be expressed in matrix form by the equation

$$DX = A\,DX_o \qquad \ldots(105),$$

where the column matrices

$$DX = \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \end{bmatrix} \qquad DX_o = \begin{bmatrix} \Delta x_{1o} \\ \Delta x_{2o} \\ \Delta x_{3o} \end{bmatrix}$$

define the components of the geocentric orientation vector in the Laplacian trihedron at the general point P and the origin O respectively and the array A, given by

$$A = \begin{bmatrix} \cos\phi_o\cos\phi + \sin\phi_o\sin\phi\cos\Delta\lambda & \sin\phi\sin\Delta\lambda & \sin\phi_o\cos\phi - \cos\phi_o\sin\phi\cos\Delta\lambda \\ -\sin\phi_o\sin\Delta\lambda & \cos\Delta\lambda & 0 \\ \sin\phi\cos\phi_o - \sin\phi_o\cos\phi\cos\Delta\lambda & -\cos\phi\sin\Delta\lambda & \sin\phi\sin\phi_o + \cos\phi_o\cos\phi\cos\Delta\lambda \end{bmatrix} \qquad \ldots(106),$$

where $\Delta\lambda = \lambda_o \quad \lambda$ ...(107),

is the matrix defining the earth space transformation between the vector triads i_o and i. The use of equation 104 gives the corrected values of the appropriate parameters ξ_{gi} $(i=1,3)$ defining the separation vector d as referred to the geocentric spheroid by the equations

$$\xi_{gi} = \xi_{ai} + \frac{1}{h_i}\sum_{j=1}^{3} A_{ij}\, h_{jo}\Delta\xi_{jo} \quad ,i=1,3 \quad \ldots(108),$$

where $\xi_3 = h_d$ and ξ_i $(i=1,2)$ are the components of the deflections of the vertical in the meridian and prime vertical respectively, A_{ij} being the appropriate element in the array A. Expanded versions of these equations are given in (Mather & Fryer 1970a, equations 8 to 11).

The quantity ξ_{gi} can be directly compared with gravimetric values obtained by the use of equations 90 to 93, if the results of astro-geodetic levelling are assumed to be equal to differences in the height anomaly, which isn't strictly so though considered valid for the present study for reasons given in the paragraph following equation 14, when

$$\xi_{gi} + v_{g\xi i} = \xi_{ai} + \Delta\xi_i + v_{a\xi i} \quad ,i=1,3 \qquad \ldots(109),$$

where ξ_{gi} now refer to gravimetric values, $v_{\xi i}$ being the corrections to be made to the observed values to obtain the true magnitudes of ξ_i, the subscript $_a$ referring to astro-geodetic values and $_g$ to gravimetric ones. If

$$v_{\xi i} = v_{g\xi i} - v_{a\xi i} \quad ,i=1,3 \qquad \ldots(110),$$

observation equations can be formed by comparing astro-geodetic and gravimetric values of ξ_i using equation 109, which can be expressed in matrix notation as

$$V = C\,X + K \qquad \ldots(111),$$

where, in the most general case, the element C_{ij} in the array C is related to $\{A_{rj}\}_m$ of the array $\{A_\ell\}_m$ at the m-th station by the relation

$$C_{ij} = \frac{h_{jo}}{\{h_r\}_m}\{A_{rj}\}_m \qquad \ldots(112),$$

where $\quad i = \ell(m-1) + r \qquad \ldots(113)$

and the element k_i in the array K is given by

$$k_i = \{\xi_{ar} - \xi_{gr}\}_m \; , \quad r=1,\ell \; ; \ell \leq 3 \qquad \ldots (114),$$

ℓ being the number of parameters ξ_i being compared. Thus, if only h_d values are compared, $\ell = 1$; if both h_d values and deflections are compared, $\ell = 3$. The array V is defined by the quantities in equation 110. The solution of equation 111 using the principle of least squares for the elements of the array X, given by

$$X^T = \left(\Delta\xi_{o1} \quad \Delta\xi_{02} \quad \Delta\xi_{03} \right) ,$$

is

$$X = -(C^T W C)^{-1} C^T W K \qquad \ldots (115),$$

where W is the matrix of weight coefficients of the quantities defined in equation 110. This equation supposes that no systematic errors exist in any of either ξ_{ai} or ξ_{gi}. Such errors should be allowed for, if necessary, when setting up equation 111. For an example of the considerations involved see (Mather 1969a, p.31 et seq.). The final quantities determined are the geocentric orientation parameters

$$\Delta\xi_{1o} = \Delta\xi_o \; ; \quad \Delta\xi_{2o} = \Delta\eta_o \; ; \; \Delta\xi_{3o} = h_{do} \; ,$$

which, as defined earlier, are the unknown elements in the components of the geocentric orientation vector an the Laplacian trihedron at the origin.

3. DATA SETS

3.1 INTRODUCTION

The precision of any solution for the geoid is primarily dependent on the adequacy of the representation adopted for the earth's gravitational field. The final result sought is the determination of the geocentric orientation parameters $\Delta\xi_{1o}$, $\Delta\xi_{2o}$ and Δh_{do} at the Johnston Origin of the Australian Geodetic Datum through equation 115. The accuracy of such a solution is dependent on the correctness of the magnitudes of the elements in the array X and hence on the gravimetric and astro-geodetic values of ξ_1, ξ_2 and h_d on their respective datums. The precision of the latter were beyond the control of the investigator, the astro-geodetic deflections of the vertical being established under the control of the Division of National Mapping while the astro-geodetic solution for the geoid currently available was deduced from the deflections by Fischer and Slutsky (1967).

A study of the nature of Stokes' function $f(\psi)$ and the Vening Meinesz function $\partial\{f(\psi)\}/\partial\psi$ (e.g.,Heiskanen & Vening Meinesz 1958,p.81) as used in the gravimetric solution defined in equations 90 to 93 indicate that ξ_1 and ξ_2 are critically dependent on the accuracy with which the near zone gravity fields have been established, while the determination of h_d is less dependent on the precision with which the field in these regions have been defined. It therefore becomes necessary to attempt the accurate representation of the near zone gravity field if all three types of observation equations comprising equation 111 are to be used.

It should also be remembered that astro-geodetic determinations of the geoid are deduced while the deflections of the verticals are observed quantities, as the contributions of errors in the geodetic co-ordinates to those in ξ_{ai} must be considered to be negligible. Consequently it is undesirable to avoid observation equations involving deflections of the vertical, which, in turn calls for field surveys to be carried out in the proximity of any station investigated. It would suffice, in theory, if the solution was effected from error-free observations at a single point on the datum, preferably the origin itself. Previous expreience in geoid solutions for the Australian region using gravity data currently available indicated the existence of considerable systematic error in the field representation adopted for unsurveyed areas (Mather 1969b,p.513).

The current study was therefore planned to include investigations at 38 astro-geodetic stations on the A.G.D. which were evenly spaced over the total extent of the datum but avoiding regions where the field representation was inadequate. For a further discussion see section 4.3. The nature of the data sets used in the present solution was essentially the same as those used in the 1968 determination (ibid,p.501). In addition to the sets of $5^{\circ}\times 5^{\circ}$, $1^{\circ}\times 1^{\circ}$, $\frac{1}{2}^{\circ}\times \frac{1}{2}^{\circ}$ free air anomaly area means and the set of readings representing the corners of a $0.1^{\circ}\times 0.1^{\circ}$ grid covering the Australian region, inner zone fields were established around the chosen astro-geodetic stations included in the present study.

3.2 THE U.N.S.W. DATA SET

The sets of $\frac{1}{2}^{\circ}\times \frac{1}{2}^{\circ}$, $1^{\circ}\times 1^{\circ}$ area means and the individual readings on a $0.1^{\circ} \times 0.1^{\circ}$ grid comprise the U.N.S.W. data set. This representation of the gravity field in the Australian region is based on the national Australian Isogal datum (Barlow 1967; Mather 1969b, p.499), the values adopted for the control stations being termed *May 1965 Isogal values* by the Commonwealth of Australia's Bureau of Mineral Resources, Geology & Geophysics. The distribution of data used in this region is shown in figure 10, the resulting area mean being given in figure 11. The representation was more comprehensive that the 1968 data set (ibid, p.500), whose variability can be seen from (ibid, p.503). Additional information was included in the coverage of the following regions:-

(i) Gaps in the representation of the Northern Territory which is now completely represented.

(ii) Helicopter gravity surveys of portions of Western Australia and northern New South Wales.

(iii) Gravity surveys carried out by the investigator in the Snowy Mountains and Victoria.

(iv) Marine gravity surveys of the continental shelf regions to the north and north-west of Australia.

(v) Gravity holdings of the Aeronautical Chart and Information Center, St. Louis, Mo. for regions within 25 degrees of the continental margins.

(vi) Gravity holdings of the D.S.I.R., Wellington, N.Z. for the New Zealand region.

(vii) Area means for the Indian Ocean produced by Le Pichon and Talwani (1969).

(viii) Gravity surveys in the Coral Sea (Falvey & Talwani 1969).

(ix) Detailed gravity surveys around the selected astro-readings on a $0.1^{\circ} \times 0.1^{\circ}$ grid comprise the U.N.S.W. data set. This representation of the gravity field in the Australian region is based on the national Australian Isogal datum (Barlow 1967; Mather 1969b,p.499), the values adopted for the control stations being termed *May 1965 Isogal values* by the Commonwealth of Australia's Bureau of Mineral Resources, Geology & Geophysics. The distribution of data used in this region is shown in figure 10, the resulting area mean being given in figure 11. The representation was more comprehensive that the 1968 data set (ibid,p.500), whose variability can be seen from (ibid,p.503). Additional information was included in the coverage of the following regions:-

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(vi) Gravity holdings of the D.S.I.R., Wellington, N.Z. for the New Zealand region.

(vii) Area means for the Indian Ocean produced by Le Pichon and Talwani (1969).

(viii) Gravity surveys in the Coral Sea (Falvey & Talwani 1969).

(ix) Detailed gravity surveys around the selected astro-geodetic stations by the investigator (see acknowledgments).

The data sets used in the 1968 solution were independent of one another. This did not constitute a problem in either fully represented regions or in totally unsurveyed areas where all means were compatible. In partially represented areas however, the values adopted for the area means of larger blocks were based on observed readings only. Consequently discrepancies existed between representations of the same region by different data sets.

Thus, if n is the number of readings of the gravity anomaly Δg and if $n < N$, where N is the number of readings in a fully represented square, the area mean Δg_m in the 1968 solution was obtained by the equation

$$\Delta g_m = \frac{1}{n} \sum_{i=1}^{n} \Delta g_i \qquad \ldots (116)$$

and held invariant in the subsequent computations, irrespective of the magnitude of n. In computations however, the $0.1^{\circ} \times 0.1^{\circ}$ representation of this region had n readings and $(N-n)$ predicted values $E\{\Delta g\}$ which gave a representation equivalent to an area mean $E\{\Delta g_m\}$ given by

$$E\{\Delta g_m\} = \frac{\sum_{i=1}^{n} \Delta g_i + \sum_{i=1}^{N-n} E\{\Delta g_i\}}{N} \qquad \ldots (117)$$

The use of $\frac{1}{2}^{\circ} \times \frac{1}{2}^{\circ}$ and $1^{\circ} \times 1^{\circ}$ data sets based on equation 116 in the 1968 solution gave rise to steep gradients in the comparisons between the astro-geodetic geoid and the gravimetric solution when passing across meridians defining the boundaries of those larger units of area situated on the peripheries of the continent where the partially represented areas are primarily concentrated. The use of equation 116 when $n \to 0$ gave values of Δg when the "noise" could completely mask the "signal" (see Moritz 1969, p.1

for an explanation of terminology) which, in this case is the true area mean $M\{\Delta g\}$.

This was clearly illustrated in a study of the comparisons of local $5^{\circ} \times 5^{\circ}$ area means from gravimetry with those obtained by the combination of satellite data and surface gravimetry when area means computed from smaller samples exhibited significantly larger deviations from the satellite means than those from more comprehensive ones (Mather 1969b, p. 509). A study of the estimated value of area means computed at various stages in the compilation of the U.N.S.W. data set indicated that a 40% representation was the minimum sample size (n_{min}) which would provide an acceptable area mean. This is equivalent to adopting the relation

$$M\{\Delta g\} = \Delta g_m \qquad \text{if} \qquad n > n_{min} (=0.4\ N).$$

In regions where $n > n$, the following technique was used to predict values to represent the gravity anomaly field with a spacing of 0.1°. A prediction function of the form

$$E\{\Delta g_i\} = \sum_{j=1}^{k} C_j \left(f(\phi,\lambda)\right)_{ij} \quad , \ i=1, (N-n)$$

was adopted, where $\left(f(\phi,\lambda)\right)$ is of the form used in an earlier investigation (Mather 1967, p. 133). Let the predicted values $E\{\Delta g_i\}$ of the gravity anomaly have an error of prediction which, for the present is assumed to be normally distributed. If this is not so, v_i should be replaced by a normally distributed component v_i' and a systematic component v_{si} which becomes a parameter in the subsequent solution. The true value of the gravity anomaly Δg_i is given by

$$\Delta g_i = E\{\Delta g_i\} + v_i = \sum_{j=1}^{k} C_j \left[f(\phi,\lambda)\right)_{ij} + v_i \ , \ i=1, (N-n) \ .$$

The coefficients C_j are determined by the least squares analysis of the n observed gravity anomalies in the area (Δg_i, i=1,n) when observations equations of the form

$$\Delta g_i \quad - \quad \sum_{j=1}^{k} C_j \left(f(\phi,\lambda)\right)_{ij} \quad = v_i \quad ,i=1,n \qquad \ldots (118).$$

are obtained. In addition, it may be desirable to preserve a given value for the area mean while making these predictions. For instance, in cases where $n > n_{min}$, the area mean should be retained at Δg_m as defined in equation 116. In such a case, the predicted values should combine with the observed values to satisfy the condition equation

$$\sum_{i=1}^{N-n} E\{\Delta g_i\} \quad + \quad \sum_{i=1}^{n} \Delta g_i \quad = N \quad \Delta g_m \qquad \ldots(119).$$

Thus equation 119 can also be written as

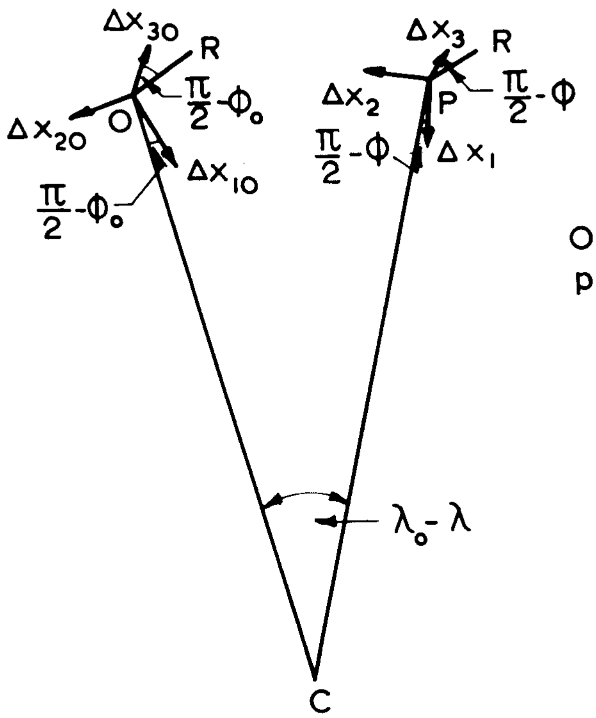
$$\sum_{i=1}^{N-n} \quad \sum_{j=1}^{k} C_j \left(f(\phi,\lambda)\right)_{ij} \quad = N \quad \Delta g_m - \quad \sum_{i=1}^{n} \quad \Delta g_i \ldots(120)$$

The composite block of equations can be solved by minimising

$$\sum_{i=1}^{n} \quad w_i \quad v_i^2 \qquad \ldots (121),$$

where w^{-1} was chosen as 9 $mgal^2$ for an observed reading in the analysis of $0.1^\circ \times 0.1^\circ$ data. The value assigned for w_i in the case of predictions was based on the interval on the grid from the nearest available gravity station and should also be dependent on whether prediction was effected by interpolation, extrpolation or a mixture of both techniques. More complex weighting functions were found to have marginal effects only on the predictions. To avoid undue biases, the simple formula

$$w^{-1} \quad = (\ell + 3)^2 \ mgal^2 \quad , \quad \ell \nleq 6,$$



OR, PR normal plane of paper

Fig. 9

Geocentric orientation parameters at origin and the general point was adopted, where ℓ is the interval from the nearest gravity station. This probably overestimates the accuracy of predictions (e.g.,see Rapp 1964,pp.85-88) especially when predictions are obtained by extrapolation (Mather 1967,p.136). The lest squares condition for one such area mean can be expressed in matrix notation as

$$\Phi = V^T W V - L^T (A_c' - \Delta g_r) = \text{minimum} \qquad (122),$$

where L is a single element in this case, being the Lagrangian multiplier of the signle condition equation expressed at 120. In many regions however, n is less than n_{min} or equal to zero. Let the estimate of the area mean from available samples in such cases be $\Delta g'$. Let v_m be the quantity given by

$$v_m = M\{\Delta g\} - \Delta g_m' .$$

In such a case, equation 120 can be written as

$$\frac{1}{N} \sum_{i=1}^{N-n} \left(\sum_{i=1}^{k} C_j \left(f(\phi,\lambda)\right)_{ij} \right) = \Delta g_m' + v_m - \frac{1}{N} \sum_{i=1}^{n} \Delta g_i \quad \ldots (123).$$

In addition, a second type of condition equation has to be satisfied where the regional mean over a $y^o \times y^o$ block, obtained from the area means defined in either equations 120 or 123 for a smaller $x^o \times x^o$ block, must satisfy an external constraint. The regional mean can either be obtained from the combined solutions of either Rapp or Kaula, described in section 3.3 or, in cases where the number of readings in the region satisfy the required condition, the value obtained from equation 116 can be used in preference.

If

$$s = \frac{y^2}{x^2} ,$$

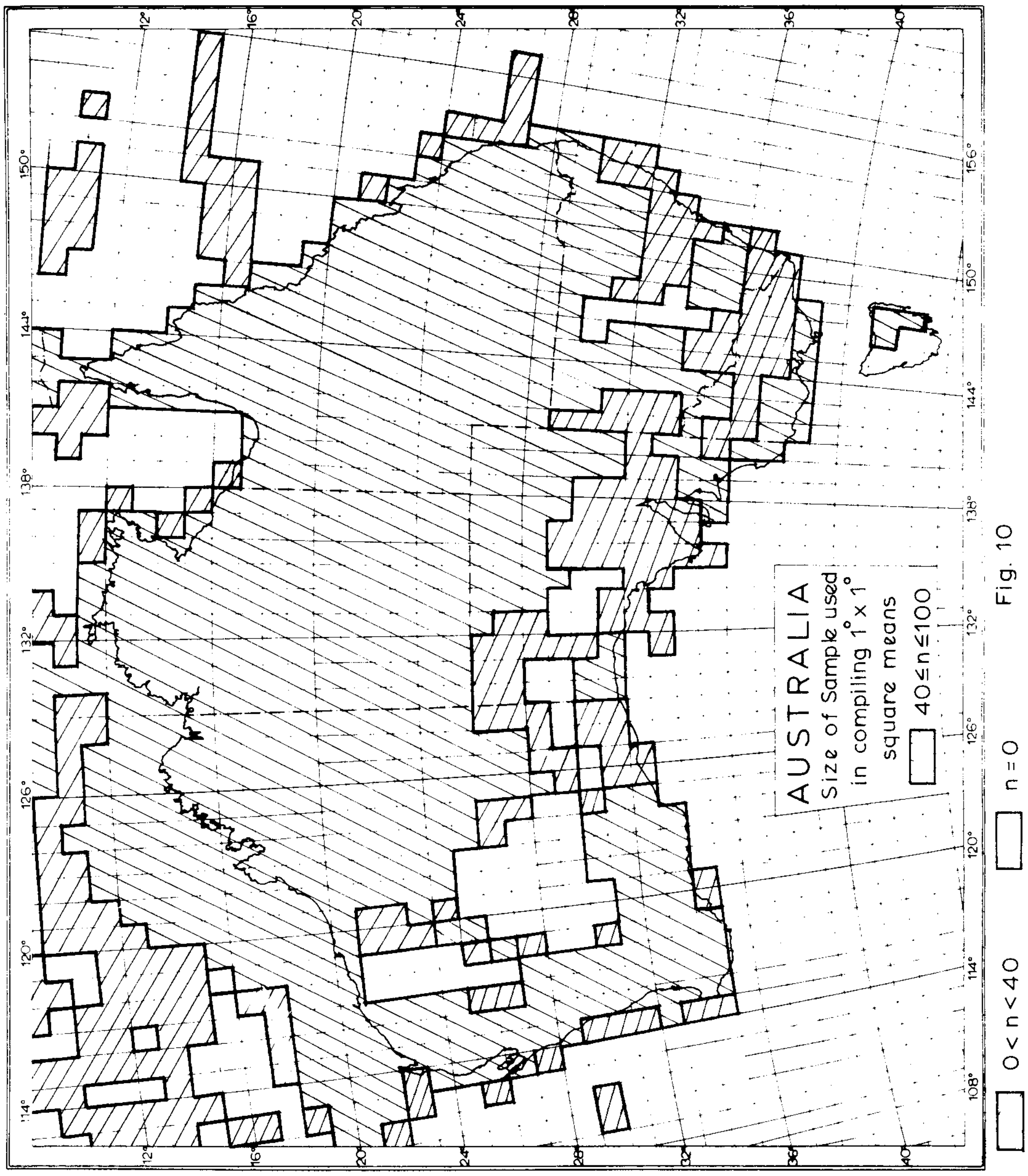

AUSTRALIA
Size of Sample used in compiling 1° x 1° square means
40 ≤ n ≤ 100
n = 0
0 < n < 40

Fig. 10

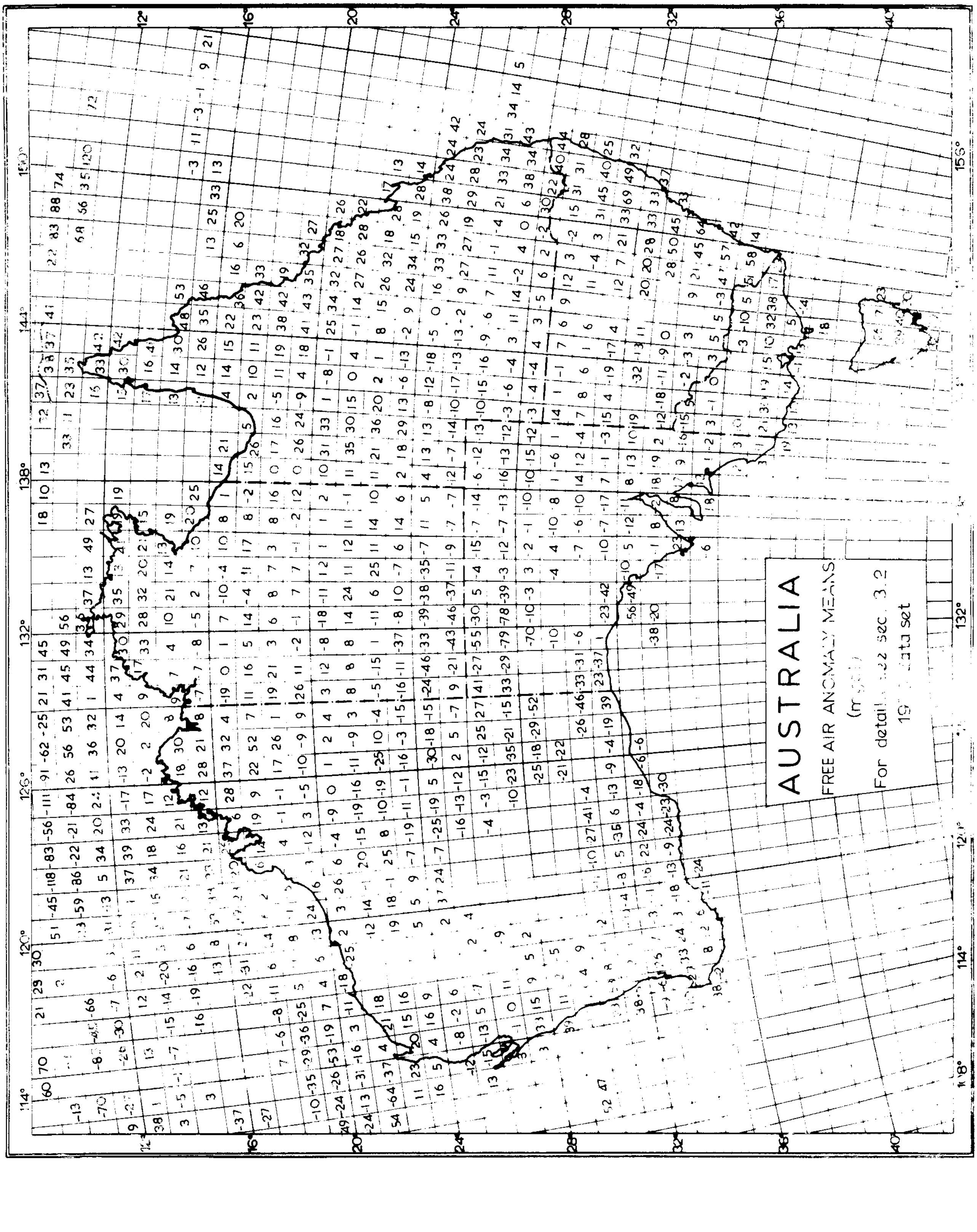


Fig. 11

the following condition equation must also be satisfied.

$$\sum_{r=1}^{t} (\Delta g'_{mr} + v_{mr}) \quad + \quad \sum_{i=1}^{s-t} \Delta g_{mi} \quad - s\ \overline{\Delta g} = 0 \quad \ldots (124),$$

where $\overline{\Delta g}$ is the regional mean anomaly obtained either from surface gravity in instances when the number of individual gravity readings n exceeds n_{min} or, if such a condition cannot be satisfied, from the combined solution from satellite data and surface gravimetry.

The end product of any solution is the determination of the coefficients C_j, together with the quantities v_{mr} which correct the relatively weak estimates $\Delta g'_{mr}$ of the area means of those smaller squares where the number of readings is less than n_{min}. For reasons of practical expediency four sets of C_j values were used to represent a region which was a $5^\circ \times 5^\circ$ square in the current solution. Thus each set of C_j values represented a $2\frac{1}{2}^\circ \times 2\frac{1}{2}^\circ$ region. The number of coefficients used (k) was dependent on the total number of observed readings available in a square as described in the earlier study (ibid,p.133), being

$$41 \leq k \leq 1 \qquad \text{as} \qquad n_{lim} \leq n \leq 0,$$

where n_{lim} is the minimum number of readings which enabled a full set of coefficients to be used. For a $2\frac{1}{2}^\circ \times 2\frac{1}{2}^\circ$ square,

$$n_{lim} = 250.$$

The solution was effected as follows for the four sets of C_j values and the t terms v_{mr} in each $5^\circ \times 5^\circ$ square which was not fully represented. Consequently, the same set of C_j values could possibly occur in 25 equations of the type at either 123 or 120. The resulting set of equations of the type at 123 are treated as observation equations which can be expressed in matrix form by the relation

$$A\,C + K = V_m \qquad ..(125)$$

where the general element A_{rs} in the array A is given by

$$A_{rs} = \frac{1}{N}\sum_{i=1}^{N-n_r} \left(f(\phi,\lambda)\right)_{irs} \qquad ...\ (126),$$

the equivalent element k_r in K being

$$k_r = -\frac{1}{n_r}\sum_{i=1}^{n_r} \Delta g_{ir} + \frac{1}{N}\sum_{i=1}^{n_r} \Delta g_{ir},\ n_r < n_{min} \qquad ..(127),$$

where, in general,

$$\frac{1}{n_r}\sum_{i=1}^{n_r} \Delta g_{ir} = \begin{cases} 0 & \text{if } n = 0 \\ \Delta g'_{mr} & \text{if } n < n_{min} \\ \Delta g_{mr} & \text{if } n > n_{min} \end{cases} \qquad ...\ (128).$$

V_m is the array of the corrections v_{mr} to $\Delta g'_{mr}$. In cases where $n_r > n_{min}$, Δg_{mr} was held fixed in the adjustment, the modified form of equation 123 is treated as a condition equation, the resulting relations being expressed in matrix notation as

$$A_c\ C + K_c = 0 \qquad ...\ (129),$$

the coefficients A_{crs} being similar in form to A_{rs} given in equation 126 and the general element k_{cr} is given by

$$k_{cr} = \frac{1}{N}\sum_{i=1}^{n_r} \Delta g_{ir} - \Delta g_{mr},\quad n > n_{min} \qquad ...\ (130).$$

The general least squares condition can be expressed as

$$\Phi = V_m^T\, W\, V_m - L^T(A_c\ C + K_c) = \text{minimum} \qquad ...\ (131),$$

where L is the column matrix of Lagrangian multipliers and

W the matrix of weight coefficients. Equation 124 is introduced into the solution as a condition equation which can be expressed in matrix notation by the form

$$U^T(A\ C\ + K') + K'_c = 0 \qquad \ldots (132)$$

where U is unit column matrix of order t and the general element k'_r of the array K' is given by

$$k'_r = \frac{1}{N} \sum_{i=1}^{n_r} \Delta g_i \quad ;$$

the (1,1) array K' being

$$K' = \sum_{i=1}^{s-t} \Delta g_{mi} - s\ \overline{\Delta g}$$

Equation 132 can be added to the second group of terms producting the transpose of the array of Lagrangian multipliers without loss of generality. Equation 131 is the standard type of least squares condition combining observation equations involving the parameters C_j which also have to satisfy certain condition equations. The resulting solution (e.g.,Mather 1969a,p.25) is

$$C = (A^T\ W\ A)^{-1}\ (A_c^T L - A^T\ W\ K) \qquad \ldots (133),$$

where

$$L = \left\{A_c\ (A^T\ W\ A)^{-1}\ A_c^T\right\}^{-1}\left\{A_c(A^T\ W\ A)^{-1}\ A^T\ W\ K - K_c\right\} \qquad (134)$$

The procedure adopted for the field extensions can therefore be summarised as follows.

(i) Each $5^{\circ}\times 5^{\circ}$ block was divided into four $2\frac{1}{2}^{\circ}\times 2\frac{1}{2}^{\circ}$ blocks, each of which was analysed using separate two dimensional trigonometrical series.

(ii) All free air anomalies on a 0.1° degree grid were converted to regional Bouguer anomalies using tenth degree square height means and these were used in setting up observation equations for half degree squares using either equations of the type at 123 or 120

(iii) If the total number of readings in the general $\frac{1}{2}^{\circ} \times \frac{1}{2}^{\circ}$ square (n) exceeded a limiting number n_{min} ($\simeq$ 0.4 N), where N is the total number of readings possible in the half degree square (N = 25 in this case), the area means obtained by the use of equation 116 was held fixed in the ensuing solution. Consequently, V_m in equation 123 becomes zero and the resulting equation can be treated as a condition equation.

(iv) If the number of gravity readings available in a $5^{\circ} \times 5^{\circ}$ square exceeded 1000 (i.e., 0.4 × 2500), the resulting area mean from surface gravimetry was held fixed in the adjustment. Else the value from the combined satellite and surface gravimetry set was considered to be invariant in the adjustment. In either case, a further condition equation related the field extension coefficients C_j to the $5^{\circ} \times 5^{\circ}$ mean.

(v) The weight coefficients used were based on equation 119 and table 4 of an earlier study (Mather 1967, p.135-6).

The result of carrying out such an analysis was the establishment of consistent sets of $0.1^{\circ} \times 0.1^{\circ}$, $\frac{1}{2}^{\circ} \times \frac{1}{2}^{\circ}$, $1^{\circ} \times 1^{\circ}$ and $5^{\circ} \times 5^{\circ}$ area means for the free air anomaly field in the Australian region, removing one major source of error in the earlier solution for the free air geoid, when each of the above data sets were independent of the others.

An important corollary is that area means adopted in the representation are not necessarily the numerical means of all the available readings within the region, as they also include predicted values when forming the mean in those areas where the number of gravity readings available was less than the lower limit n_{min} described in (iii) above. In this manner, the problem of "noise" which almost totally drowned out the "signal" in sparsely surveyed regions, was minimised and the obvious defects in the gravity field representation used in the 1968 solution were largely rectified.

3.3 THE DATA SET USED TO REPRESENT THE OUTER ZONES

The $5^{\circ}\times 5^{\circ}$ sets of free air anomalies used in the 1968 solution were thos produced by Kaula (1966b) and Rapp (1968). These sets could not be used without some amendment if condition (iv) in section 3.2 is to be satisfied for regions well represented by surface gravity. Earlier computations indicated that the values calculated for the geocentric orientation parameters were only marginally affected by the type of data set used to represent the outer zone (Mather 1969b, p.513). It was therefore decided to represent these regions with the Rapp set after appropriate amendment as described at (iv) in section 3.2.

The adoption of revised values for the free air anomaly means representing $5^{\circ}\times 5^{\circ}$ squares affects the zero degree term in the surface harmonic representation for Δg though not the implicit zero magnitudes of the coefficients C_{10}, C_{11}, S_{11}, C_{21} and S_{21}. The differences in the zero degree terms between the original and amended Rapp data sets are shown in table 2.

The combination of the U.N.S.W. data set and the amended Rapp set based on both satellite data and surface gravimetry affords a means of solving the surface integrals of Stokes and Vening Meinesz, the latter set being one method for representing the effect of the outer zones, a subject that has been of considerable interest to geodesists in the recent past. This problem has been interpreted

Rapp Data Set	Year	$M\{\Delta g\}$ (mgal)	$N_o = -\frac{R\ M\{\Delta g\}}{\gamma}$ (met)
Original	1968	+ 0.5	- 3.2
Amended	1970	+ 0.4	- 2.8

Table 1
Zero Degree Term in N
Reference System 1967 ; $W_o = U_o$
$M\{\ \}$ = Global mean value

as one of integration of an inner spherical cap at a limiting angular distance ψ_o from the computation point while the distant zone effects are represented by low degree spherical harmonic analysis (Cook 1950 ; Cook 1951). The contribution of the distant zones are well known to have a significantly larger effect on the separation N_f than on the deflections ξ_{fi}, the effect increasing as ψ_o decreases.

Three studies in particular warrant attention when considering the use of such composite solutions for the definition of the separation vector. The first, in chronological order, is that of Molodenskii, who estimates from the use of Zhongolovich's solution for the geoid (Molodenskii et al. 1962,p.164) that the effect of harmonics of degree greater than 8 in the representation of the zones exterior to a cap with a limiting radius $\psi_o = 23^o$ would give rise to errors not exceeding ± 1.9 metres in N_f and ± 1.1 sec in ξ_{fi}.

The second study is that of Cook who used Jeffreys' solution for the geoid to represent the second and third degree harmonics and estimated the balance effects of the distant zones, after a consideration of these harmonics for $\psi_o = 20^o$ at ± 5.1 metres in N_f (Cook 1951,p.136) and ± 0.7 sec in ξ_{fi} (Cook 1950,p.383). A review of the earlier studies is given by de Witte (1967) who investigates the truncation effects to higher degree harmonic values using Molodenskii's formulae (Molodenskii et al. 1962,p.147), given by

$$N_{out} = \frac{R}{2\gamma} \sum_{n=0}^{\infty} q_n Q_n \qquad \ldots (134),$$

where

$$Q_n = -4 \int_0^{\sin \frac{1}{2}\psi_o} p_{no}(\cos \psi)\ f(\psi)\ \sin \tfrac{1}{2}\psi \ d(\sin \tfrac{1}{2}\psi)$$

and Δg_n is the n-th degree surface harmonic in the representation of the gravity anomaly, defined by

$$\Delta g_n = \sum_{m=0}^{n} \bar{p}_{nm}(\sin \phi)\left(\bar{g}_{1nm}\cos m\lambda + \bar{g}_{2nm}\sin m\lambda\right),$$

to estimate the distant zone effect (N_{out}) on N_f. de Witte uses Cook's expressions for the evaluation of the outer region effect on ξ_{fi}, given by the equations (de Witte 1967,p.455)

$$\xi_{out_i} = \frac{1}{2} \sum_{n=0}^{\infty} (n-1)\ c_{nli}\ q_n \quad , i=1,2 \qquad \ldots (135),$$

where

$$c_{nli} = \frac{1}{4(n-1)\gamma} \iint \Delta g_n\ p_{nl}(\cos \psi)\ \cos \alpha_i \ d\sigma \ ,i=1,2$$

α_i having the same significance as in the note to equation 59. Cook's truncation function q_n (Cook 1950,p.377) is similar in structure to Molodenskii's and is given by

$$q_n = \int_1^{\cos \psi_o} \frac{\partial}{\partial \psi} \{f(\psi)\} p_{nl}(\cos \psi)\ d(\cos \psi).$$

de Witte recommends spherical cap calculations to $\psi_o = 39^o$ which approximates to a zero point of Stokes' function and at which value the Molodenskii truncation function Q_n, excluding Q_o, Q_1 and Q_2 have minimum values (de Witte 1967, pp.456-8). Such an extension of the bounds of the U.N.S.W. data set was considered to be of no great value in view of the lack of adequate representation of the gravity field over the ocean areas to the south-east, south and west of Australia.

The use of these truncation functions to represent the distant zone effects of the earth's gravitational field, together with a spherical cap representation of the local region, is a technique well worth detailed investigation as test calculations show that computations of the separation vector using the truncation functions for the representation of the distant zones up to degree 8 is approximately 100 times faster on an electronic computer than using the Stokes and Vening Meinesz formulae and the standard anomaly representation.

The use of the functions defined in equations 134 and 135 does not necessarily mean that *relative* errors in a regional study of the type undertaken will be as large as the figures given by Molodenskii and Cook. This has been demonstrated analytically by Cook (ibid, p.374) and also been confirmed by detailed computations in the course of regional geodal studies in Australia (Mather & Fryer 1970a, table 2). The prime danger in using the results of any low degree harmonic analysis of gravity data for the determination of what is, in effect, the mean slope of equipotential surfaces across a region whose area is approximately $40^o \times 30^o$ (or 2% of the earth's surface), is the existence of errors in the magnitude of the coefficients of those harmonics which

either retain their magnitude over the region

or else have variations, the magnitudes of which cannot be detected over the region on the comparison of geoidal solutions. Zonal harmonics of degree n will have n zeros between the poles, while tesseral harmonics of order m change sign m times over 180 degrees in longitude. The danger of undetected systematic errors in a determination for Australia will therefore arise primarily from harmonics of degree less than six and order either zero or one.

A further possible complication is the fact that the magnitude of higher degree harmonics have not been considered in the analysis. Such a procedure may significantly affect the values obtained in the solution (Cook 1965, p.181 et seq). The Rapp set of data is prepared by the use of harmonics determined by satellite orbital analysis for what is, in effect, *the prediction of gravity anomalies in unsurveyed areas*. The technique adopted is described in (Rapp 1968) and is a variation on a technique initially used by Kaula (1966b, p5310).

If the gravity anomaly is expressed by a set of surface harmonics, given in equation 89, it can also be expressed by the general surface harmonic series

$$\Delta g_n = \sum_{n=2}^{\infty} G_n = \sum_{n=2}^{\infty} \sum_{m=0}^{n} \overline{P}_{nm}(\sin\phi)(\overline{g}_{1nm}\cos m\lambda + \overline{g}_{2nm}\sin m\lambda) \quad \ldots(136),$$

where the coefficients $\overline{C}_{nm}$, $\overline{S}_{nm}$ in equation 89 are related to those in equation 136 by the relation

$$\begin{Bmatrix} \overline{C}_{nm} \\ \overline{S}_{nm} \end{Bmatrix} = \frac{1}{\gamma(n-1)} \begin{Bmatrix} \overline{g}_{1nm} \\ \overline{g}_{2nm} \end{Bmatrix} \quad \ldots(137)$$

The coefficients $\overline{g}_{1nm}$, $\overline{g}_{2nm}$ can be obtained by the standard technique of multiplying each surface integral by the appropriate term

$$\overline{P}_{nm}(\sin\phi)\begin{Bmatrix} \cos m\lambda \\ \sin m\lambda \end{Bmatrix}$$

when

$$\iint G_n \; \overline{P}_{nm}(\sin\phi) \begin{Bmatrix} \cos m\lambda \\ \sin m\lambda \end{Bmatrix} d\sigma = 4\pi \begin{Bmatrix} \overline{C}_{nm} \\ \overline{S}_{nm} \end{Bmatrix} \quad \frac{4\pi}{\gamma(n-1)} \begin{Bmatrix} \overline{g}_{2nm} \\ \overline{g}_{2nm} \end{Bmatrix} .$$

The use of the orthogonal property of spherical harmonics on equation 136 gives

$$\iint \Delta g \; \overline{P}_{nm}(\sin\phi) \begin{Bmatrix} \cos m\lambda \\ \sin m\lambda \end{Bmatrix} d\sigma = \sum_{n=2}^{\infty} \sum_{m=0}^{n} \overline{P}_{nm}(\sin\phi)\,(\overline{g}_{1nm}\cos m\lambda +$$
$$\overline{g}_{2nm}\sin m\lambda) \;\; \overline{P}_{nm}(\sin\phi) \begin{Bmatrix} \cos m\lambda \\ \sin m\lambda \end{Bmatrix} d\sigma = 4\pi \begin{Bmatrix} \overline{g}_{1nm} \\ \overline{g}_{2nm} \end{Bmatrix} = 4\pi(n-1)\gamma \begin{Bmatrix} \overline{C}_{nm} \\ \overline{S}_{nm} \end{Bmatrix} .$$

Hence observation equations of the type

$$\gamma(n-1) \begin{Bmatrix} \overline{C}_{nm} + \overline{dC}_{nm} \\ \overline{S}_{nm} + \overline{dS}_{nm} \end{Bmatrix} - \frac{1}{4\pi} \iint (\Delta g + v_{\Delta g})\overline{P}_{nm}(\sin\phi) \begin{Bmatrix} \cos m\lambda \\ \sin m\lambda \end{Bmatrix} d\sigma = 0 \qquad \ldots(138)$$

can be set up and solved for both $\overline{dC}_{nm}$, $\overline{dS}_{nm}$ and $v_{\Delta g}$, the integral being replaced in digital evaluation by a series. This method was initially used by Kaula when he adopted an equal area series of 300 nautical mile means (Kaula et al.1966,IID,pp.9 et sq). It could be applied to $5^{\circ}\times 5^{\circ}$ means with appropriate precautions.

Rapp's solution is obtained by setting up observation equations of the form

$$\Delta g(\phi,\lambda) + v_{\Delta g} = \gamma \sum_{n=2}^{\infty} (n-1) \sum_{m=0}^{n} \overline{P}_{nm}(\sin\phi)\big((\overline{C}_{nm} + d\overline{C}_{nm})\cos m\lambda +$$
$$(\overline{S}_{nm} + d\overline{S}_{nm})\sin m\lambda\big) \qquad \ldots (139)$$

which can be solved in a similar manner (Rapp 1968,p.7).

The validity of such techniques of solution to provide the parameters of the earth's gravitational field can be verified by a study of the cross-covariances $M\{\Delta g_s \; \Delta g\}$ between the area means as determined from satellite data (Δg_s) and surface gravimetry (Δg)

(Kaula 1966b, p.5308). The surface mean Δg can be given by an expression of the form

$$\Delta g = \Delta g_{so} + \delta g + v \qquad \ldots (140),$$

where Δg_{so} is the contribution to Δg by the numerical values of the coefficients adopted for the representation of Δg_s by the use of equation 89. δg is the equivalent of near zone gravitational effects and v is the error in Δg. Similarly,

$$\Delta g_s = \Delta g_{so} + v_s \qquad \ldots (141),$$

where v_s is the error in the value of Δg_s defined by the adopted values of the coefficients for the surface harmonic series. The variability of the area means expressed by surface harmonic series can be expressed as degree variances when it can easily be shown, as a consequence of the orthogonality relationship which exists between harmonics of different degrees that (e.g., Heiskanen & Moritz 1967, p.259)

$$M\{\Delta g_n^2\} = \sum_{m=0}^{n} \left(\bar{g}_{1nm}^2 + \bar{g}_{2nm}^2 \right) \qquad \ldots (142),$$

where the n-th degree surface harmonic of the gravity anomaly is given by equation 136. Equation 136 readily gives the auto-covariance of Δg_{so} (Kaula 1966b, p.5309). While $M\{\Delta g^2\}$ can be expected to be larger than $M\{\Delta g_{so}^2\}$, the latter should have a magnitude similar to the cross-covariance $M\{\Delta g\ \Delta g_{so}\}$ if no significant discrepancy exists between satellite and surface representations, as near zone effects should be randomly positive and negative, $M\{\Delta g\}$ being expected to be near zero.

These expectations are borne out by Kaula's tests given in the reference quoted and hence it can be concluded that any undesirable systematic error is unlikely to exist in low degree harmonics of orders zero and one which cannot be detected by an Australia-wide analysis. This tentative conclusion is examined further in section 4.4.

3.4 THE INNER ZONE DATA SETS

The four innermost tenth degree squares comprise the inner zone around each of the 38 astro-geodetic stations included in the study, the locations of which are shown in figure 12. The points can be seen to be evenly spaced over the continental area in accordance with the criteria laid down in section 1.4. The gravity anomaly field of this inner zone, while not contributing significantly to the magnitude of N_f and hence h_d, nevertheless has a critical effect on the computation of ξ_{fi} as has been experienced by many investigators (e.g, de Graaff Hunter 1935, p.422; Rice 1952; Szabo 1962). A value of 4.32 km has been suggested by Rice for the radius r_i of the innermost zone, which is evaluated using equations 75 and 76, in regions where normal horizontal gravity gradients occur. Other comprehensive studies on the nature of the inner zone field have been made by Cook (1950) and Shimbirev in (Brovar et al. 1964, p.290).

In view of the limitations imposed by time and finance, it was decided, for reasons given in section 1.4, to cover as many stations as possible. This was in accordance with conclusions drawn from previous studies that it was more important to obtain a representative coverage of the entire datum than to impose rigid criteria for inner zone field representation, which nevertheless, had to be verified so that gross errors were avoided. The technique adopted was as follows. An initial compilation was made of all gravity readings in the region which was approximately 20 km^2, many of these being helicopter gravity stations. These were supplemented by additional readings as planned by the investigator.

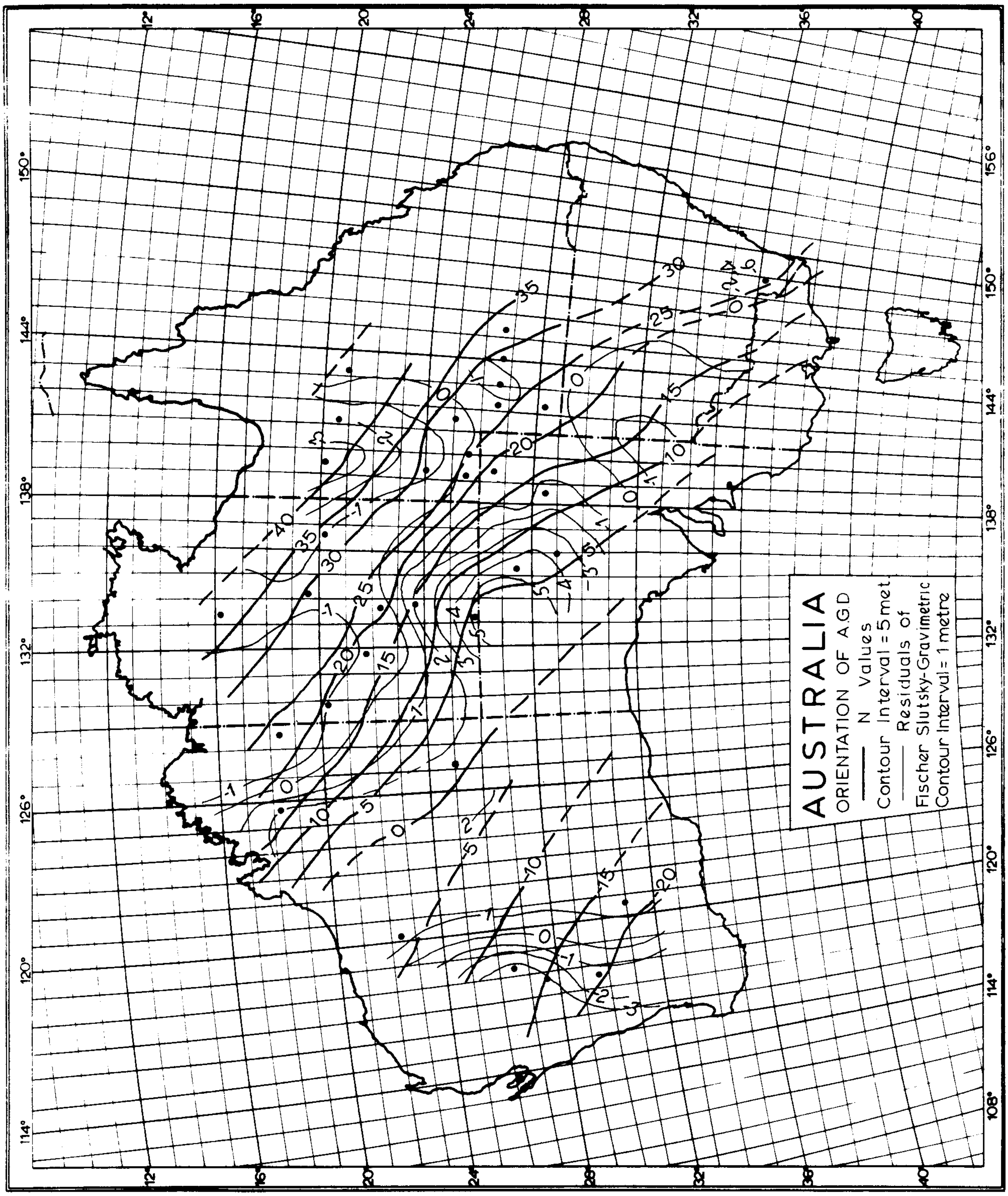


Fig 12

• Astro geodetic stations

The normal field program involved the establishment of the gravity value at the astro-geodetic station itself together with north-south and east-west gradients. The value of $\partial \Delta g / \partial x$ was accurately established in this manner over the inner zone regions which comprised the area within 3 km of the computation point and all quantities required for the evaluation of equations 75 and 76 were determined. These gravity stations were tied to the national Isogal network by means of traverses between Isogal stations which were assigned the relevant May 1965 Isogal values when allowing for drift.

The elevations (h) of the gravity stations were obtained on a differential basis with respect to that of the astro-geodetic station (h_o) which had previously been established by trigonometrical levelling. The difference ($h-h_o$) was not expected to be in excess of ±30 cm if the distance of the gravity station from the astro-geodetic control point did not exceed 3 km. The errors in the elevations of stations established over greater distances by altimeter are not expected to exceed ±2 metres.

The gravity anomaly gradient over the inner zone, *if linear*, was therefore not expected to cause any significant error in the computation of ξ_{fini}. Errors in N_{fin} were estimated as being more significant as the elevations of the astro-geodetic stations could have regional-type systematic effects as large as ± 10 metres with consequent effects of the order of ±3 mgal in Δg_f and hence approximately ±4 cm in N_f. In addition, a regional warping could occur in the deduced geoid whose magnitude could be as large as ± 60 cm if the systematic error in elevation retained a magnitude of ±10 metres over a three degree area about the computation point. The deflection of the vertical, being free from zero degree considerations, is less significantly affected by such errors.

The average gravity station distribution for a single astro-geodetic station is given in table 2. Also given are Shimbirev's estimates of a station distribution which restricts the interpolation error in ξ_i **for** the inner regions to 0.15 sec (ibid,p.320). In this connection, the interested reader is also referred to (Molodenskii et al.1962,p.178).

Distance from computation point (km)	No. of stations required	
	Shimbirev's estimate	Average no. of stations used
0	1	1
1.5	5	2
3.0	7	6
7.5	9	10
15	not available	18

Table 2

Number of gravity stations used in the definition of the inner zone field

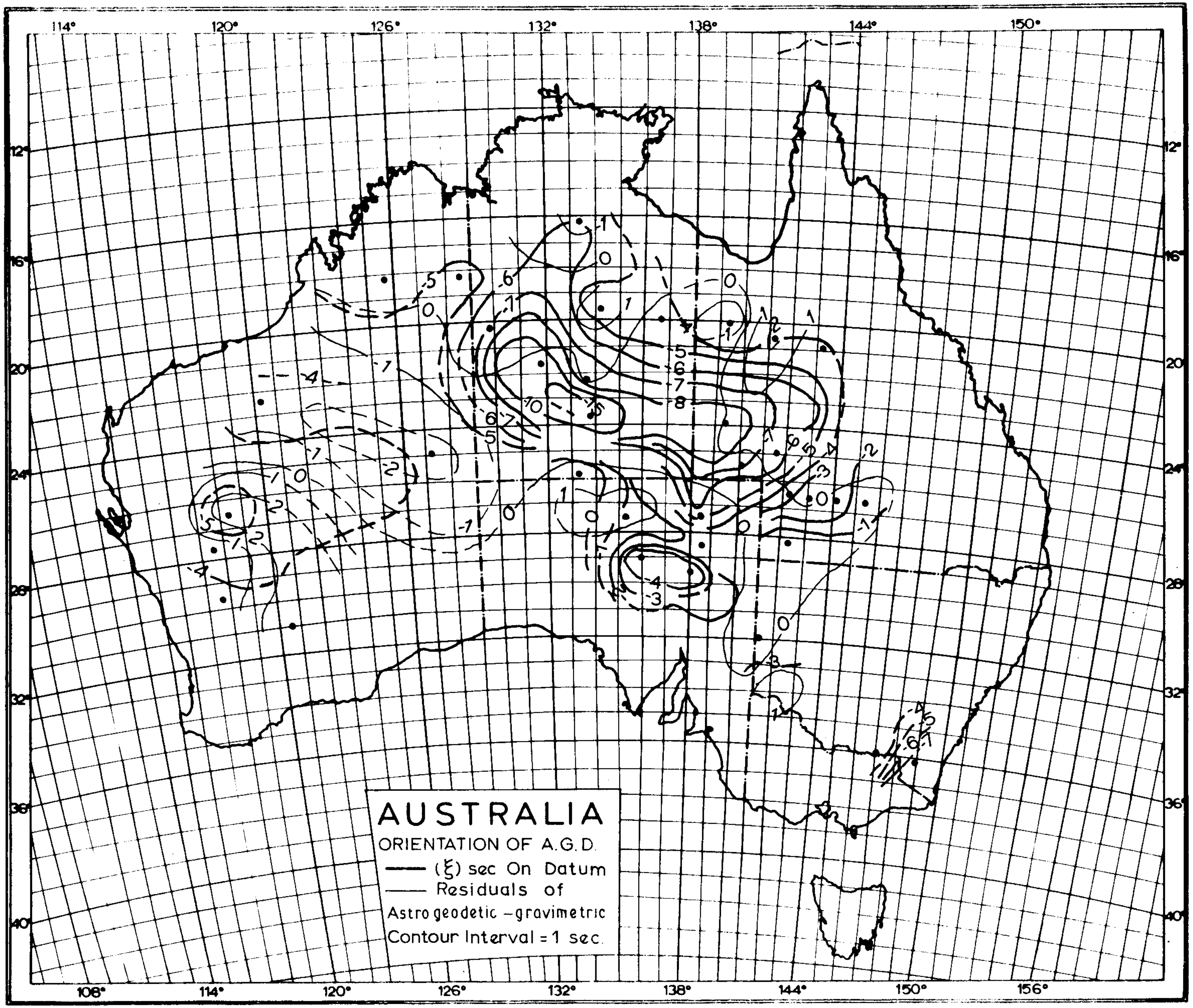
The criteria for the validity of the inner zone representation always require re-interpretation at every station visited. Most astro-geodetic points included in the present study are situated on isolated hilltops or rises on vast plains. Little error is caused in such cases by totally ignoring the existence of the hill in calculating $\partial\Delta g/\partial x_i$ if the case can be considered one where a conical hill is situated on an infinite plain. N_f values however are affected as the free air anomaly is significantly correlated with elevations, even in the case of these isolated hills. More complex topographical forms cause concern and the simple 5 station evaluation of the innermost zone has to be replaced by a nine station grid from which the weighted mean of three gradients is taken as representative of the inner zone (Rice 1952,p.290).

These ideal requirements were not always complied with due to access difficulties and lack of time on field expeditions. The inner zone gravity field was digitised in the form of a uniform $0.01^{\circ} \times 0.01^{\circ}$ grid established by prediction from the available gravity stations of the equivalent Bouguer anomalies using observations equations of the type at 118 and the technique set out in (Mather 1967). In view of the fact that the observed gravity anomalies did not always meet the criteria of 9 stations in the inner zone, this region was represented by the 9 innermost $0.01^{\circ} \times 0.01^{\circ}$ square values, symmetrical with respect to the square containing the astro-geodetic station. The effect of this region is evaluated using equations 75 and 76. The contribution of the other 391 squares comprising the rest of the zone were computed using equations 43 and 70. Consistency of the U.N.S.W. data set was maintained by replacing the $0.1^{\circ} \times 0.1^{\circ}$ square values by the 100 relevant values on the $0.01^{\circ} \times 0.01^{\circ}$ grid within the area.

4. THE RESULTS

4.1 THE SOLUTION FROM COMPARISONS AT 38 ASTRO-GEODETIC STATIONS

The location of the 38 stations chosen to define the Australian Geodetic Datum (A.G.D.) in the present study are shown in figure 12. The astro-geodetic deflections of the vertical are "true" values in that they are free from any effects except



• Astro geodetic stations

Fig. 13

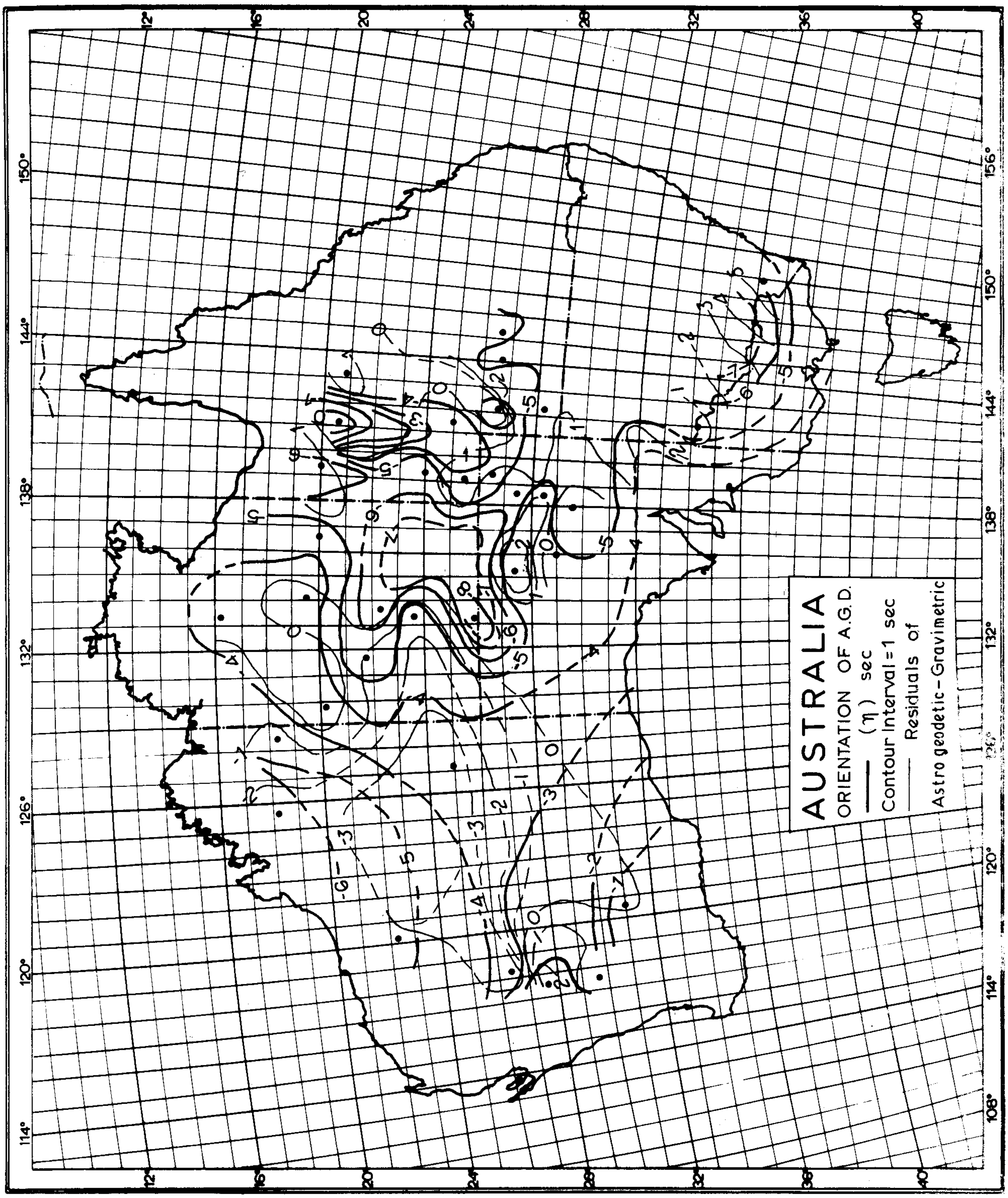


Fig. 14

• Astro geodetic stations

those arising from the relevant observations, while the current set of astro-geodetic separation values (N_a) are *deduced* from approximately 600 control stations over the six and one half square kilometre area over which the A.G.D. extends (Fischer & Slutsky 1967). The possibility of errors other than those due to observation may therefore be involved in the assessment of their precision. The principal contributor to the quantities required in the evaluation of the gravimetric values for use in equations 105 to 115 is the free air geoid defined in equations 43, 70,75 and 76 . This is obtained by the use of the data sets described in sections 3.2 and 3.3 using the sub-divisions given in (Mather 1969b,p.501). The four inner tenth degree squares were represented by the inner zone data sets described in section 3.4, equations 75 and 76 applying to the 9 innermost $0.01^\circ \times 0.01^\circ$ squares.

The accuracy of the astro-geodetic deflections of the vertical is almost totally dependent on that of the astronomically determined values of latitude and longitude. The root mean square errors of the procedures adopted by the Division of National Mapping has previously been estimated at ±0.6 sec in ξ_{a1} and ±1.0 sec in ξ_{a2} (Mather & Fryer 1970a,section 4). From the discussion in section 2.7, it can be seen that any non-Stokesian effects are essentially local in character and the use of a maximum coverage of the geodetic datum ensures that the exclusion of the topographical terms can be treated as local errors which will not significantly affect the geocentric orientation parameters. Further, the weaknesses in the elevation of the gravity stations make it a fruitless task, at the present stage, to distinguish between normal and orthometric elevations, as astro-geodetic levelling gives the true spheroid elevation h_{sp} according to the relation (Molodenskii et al. 1962,p.24)

$$h_{sp} = \int_0^P dz - \int_0^P \sum_{i=1}^{2} \xi_i \cos \alpha_i d\ell \qquad ..(143),$$

where ξ_i, in this case, are the usual components of the angle between the vertical and *the spheroid normal*, dz being the difference in orthometric elevation over the distance $d\ell$, taken over the route of the levelling. Thus the second term is not strictly the height anomaly neither is it the geoid/spheroid separation. Arnold's calculations for Mt. Blanc quoted by Heiskanen & Moritz (1967,p.329) estimate the difference between these two quantities as approximately 2 metres, a magnitude much larger than any likely result in Australia. It can therefore be safely assumed that the results of astro-geodetic levelling can be directly compared with gravimetric values as represented by the free air geoid N_f over 90% of the Australian region without taking into account any discrepancies arising from differences in definition. The resulting errors are unlikely to exceed ±50 cm and are most probably regional in character. Further the free air geoid has been adopted as the gravimetric solution and is known to be a good approximation to both the geoid/spheroid separation and the height anomaly (Mather 1968b).

Consequently astro-geodetic deflections of the vertical (ξ_{a1}, ξ_{a2}) and the results of astro-geodetic levelling ($\xi_{a3} = N_a$) as computed by Fischer and Slutsky (1967) were compared with the deflections of the vertical (ξ_{f1}, ξ_{f2}) and the separation ($N_f = \xi_{f3}$) of the free air geoid using equation 105 to 115 at the 38 stations mentioned above. The resulting geocentric orientation parameters ($\Delta\xi_{1o} = \Delta\xi_o; \Delta\xi_{2o} = \Delta\eta_o; \Delta\xi_{3o} = \Delta N_o$) were used to compute the effect of the geocentric orientation vector at the 38 stations using equations 106 to 109. The residuals which resulted, as defined in equation 110, were computed and the results are set out in tables 3 and 4. The astro-geodetic deflections of the vertical were based on values available before 1968 Some of these stations have since been re-observed but the differences were not significant enough to affect the orientation parameters. The station residuals are shown for a typical solution in table 4.

Code	Description	Type	Class	ξ_3(met)		ξ_1(sec)		ξ_2(sec)	
				$\Delta\xi_{3o}$	$M\{\sigma_{\xi 3}\}$	$\Delta\xi_{1o}$	$M\{\sigma_{\xi 1}\}$	$\Delta\xi_{2o}$	$M\{\sigma_{\xi 2}\}$
1	Planar approximation	1	A	0.0	-0.5±3.9	-0.5	-0.7±2.6	-3.5	0.0±2.7
2	do	2	A	-	-	0.5	0.0±3.8	0.0	0.0±2.6
3	38 stations	1	A	9.8	0.0±2.2	-4.0	0.3±1.0	-4.3	0.4±1.8
4	do	1	B	9.8	-0.0±2.3	-4.1	0.2±1.0	-4.3	0.4±1.8
5	do	1	C	10.7	0.8±2.2	-3.9	0.4±1.0	-4.3	0.5±1.8
6	do	2	A	-	-	-4.2	0.0±1.2	-4.5	0.0±1.6
7	do	2	B	-	-	-4.2	0.0±1.2	-4.5	0.0±1.6
8	do	2	C	-	-	-3.7	0.5±1.2	-4.4	0.0±1.5
9	do	3	A	9.8	0.0±2.2	-4.0	0.3±1.0	-4.3	0.4±1.8
10	do	3	B	9.8	0.0±2.3	-4.1	0.2±1.0	-4.3	0.4±1.8
11	do	3	C	10.7	0.8±2.2	-3.9	0.4±1.0	-4.3	0.5±1.8
12	At 693 pts on 1^ogrid	4	-	10.1	0.0±2.5	-4.2		-4.4	
13	Code 12 at 38 stns	3	-	10.1	0.5±2.3	-4.2	0.2±1.0	-4.4	0.3±1.8
14	Composite-codes 3 & 6	3	-	9.8	0.2±2.4	-4.2	0.1±1.1	-4.5	0.3±1.8
15	Composite	3	-	9.8	0.4±2.6	-4.3	0.0±1.1	-4.7	0.1±1.8
16	Code 9 on 1^o grid	4	-	9.8	-0.4±2.7	-4.0		-4.3	
17	Code 14 on 1^o grid	4	-	9.8	-0.3±2.5	-4.2		-4.5	
18	Code 15 on 1^o grid	4	-	9.8	-0.2±2.7	-4.3		-4.6	
19	Excl. inner cap $\psi<1\frac{1}{2}$	1	A	10.0	0.0±1.6	-3.8	0.4±3.3	-4.2	0.2±2.5
20	do	2	A	-	-	-4.2	0.0±3.4	-4.2	0.0±2.4
21	Excl.outer zone $\psi>20^o$	1	A	1.3	0.0±2.8	-3.3	-0.4±1.5	-3.7	-0.6±2.3
22	do	2	A	-	-	-2.7	0.0±1.3	-2.8	0.0±1.7

Table 3

Solutions for the geocentric orientation parameters

Key to table 3

Solution types :-

1 = Comparison of $\xi_3(N_f)$ values only at 38 astro-geodetic stations.

2 = Comparison of ξ_1 and ξ_2 values only at 38 stations.

3 = Comparison of all three parameters - for 38 stations.

4 = comparison of $\xi_3$values only at 693 points on 1^o continent wide grid.

Class of weight coefficient :-

A ≡ $\{w_1 = 1.5\ ;\ \ w_2 = 1.0\ ;\ \ w_3 = 3.0\}$

B ≡ $\{w_i = \left(M\{\sigma^2_{\xi i}\}\right)^{-1}\ ,\ i=1,3\}$

C ≡ $\{w_{ir} = \left(\sigma_{\xi ir} - M\{\sigma_{\xi i}\}\right)^{-2}$, where $\sigma_{\xi ir} = \xi_{ai} + \Delta\xi_{ri} - \xi_{gi}$,i=1,3(equation 144).}

The quantities $\sigma_{\xi i}$ in tables 3 and 4 are given by the relation

$$\sigma_{\xi i} = \xi_{ai} + \Delta\xi_i - \xi_{gi} \quad ,i=1,3 \qquad \ldots(144),$$

where all quantities have the same significance as in equation 109, the free air geoid value ξ_{fi} representing the gravimetric solution ξ_{gi}, $M\{\xi_i\}$ being mean values for the set.

Four different types of solutions are used to obtain the estimates of the geocentric orientation parameters and called *types* 1 to 4 in table 3.

In type 1 solutions, only ξ_3 values (i.e., separation) were used in setting up the observation equations at the 38 astro-geodetic stations.

In solutions of type 2, only values of the deflections of the vertical (ξ_1,ξ_2) were used, again at 38 stations.

Type 3 solutions involve all three parameters (ξ_1,ξ_2,ξ_3) defining the separation vector in the determination of the geocentric orientation vector.

Type 4 solutions are described in section 4.3.

Three *classes* of weight coefficients w_i for the observation equations in $\xi_i(i=1,3)$ were used in the current solutions. The first (*class* A) is the uniform set

$$w_1 = 1.5 \; ; \quad w_2 = 1.0 \; ; \quad w_3 = 3.0 \qquad \ldots(145),$$

based on the conclusions drawn from a previous study (Mather & Fryer 1970a, section 4). The second (*class* B in table 3) is the set

$$w_i^{-1} = M\{\sigma^2_{\xi i}\} \quad ,i=1,3 \qquad \ldots(146),$$

where $M\{\sigma^2_{\xi i}\}$ is the mean square residual of the comparisons between the gravimetric and astro-geodetic solutions on the common geocentric datum. In the current investigation, values of class B weight coefficients were obtained from the residuals after a solution using class A solution but of the same *type*. For example, the weight coefficients used in table 3, code 4, which is a type 1 solution, comprise a class B set and have numerical values

$$w_1 = 0.75 \quad ; \quad w_2 = 0.28 \quad ; \quad w_3 = 0.20.$$

It is interesting to note that ξ_3 was given twice the weight of ξ_1 in the type 1 solution using class A w_i values but one-third the weight when adopting class B coefficients. The resulting change in the numerical values of the parameters defining the geocentric orientation vector are however, extremely small, as can be seen by an examination of table 3, codes 3 & 4.

The third class (C) of weight coefficients was based on the relations

$$w_i^{-1} = (\sigma_{\xi i} - M\{\sigma_{\xi i}\})^2 \quad , \; i=1,3$$

the values of $\sigma_{\xi i}$ and $M\{\sigma_{\xi i}\}$ being based on a previous solution using class B weight coefficients, each observation equation having a different w_i value.

Not surprisingly, the results obtained from class C weight coefficients do not give the smallest residuals, as those points at which comparatively large discrepancies occur have a tendency to be weighted out of the solution. As these mainly occur on the peripheries of the datum due to the weaknesses in the representation adopted for the gravity field in these regions, the solution is consequently biased towards comparisons at stations in the centre of the continent. Such an orientation is not desirable in view of the existence of systematic errors in the gravity field representation as discussed in section 1.4 and in a previous study (ibid, section 4). Thus solutions using class C weight coefficients, while apparently desirable from a theoretical point of view, do

No	Name	Position		Deflections				N (met)		N_i
		ϕ	λ	North (sec)		South (sec)				
		°N	°E	ξ_{1a}	$\sigma_{\xi 1}$	ξ_{2a}	$\sigma_{\xi 2}$	ξ_{3a}	$\sigma_{\xi 3}$	
1	Quilberry	-26.66	145.36	2.2	-0.8	1.2	1.6	1.3	1.3	18
2	Bulloo NM/B/176	-26.68	144.16	1.7	-0.5	-0.2	1.5	0.0	1.2	15
3	Durrie T1/443	-25.64	140.24	-4.0	-0.9	1.4	0.9	-1.4	0.6	18
4	D'tina T1/482	-26.57	139.58	-0.4	0.0	0.7	0.7	-1.9	0.9	14
5	Gason T2/510	-27.36	138.70	-1.2	0.2	-0.9	0.1	-2.1	0.8	21
6	Mulka T1/550	-28.34	138.66	2.2	-0.1	0.3	0.9	-1.4	1.1	24
7	Bransby NM/B/ 27	-28.18	142.24	5.1	0.9	-0.5	1.0	0.3	1.1	12
8	Howitt NM/B/ 34	-26.54	142.38	0.7	-0.4	-3.3	-0.4	-0.7	1.6	18
9	Belalie NM/B/180	-26.63	143.18	2.4	0.1	2.5	2.0	-0.1	1.8	18
10	Palp'ra NM/B/ 42	-24.96	141.57	-1.5	0.3	0.8	0.3	-0.9	0.4	16
11	Brds'vl T1/441	-25.55	139.41	-4.0	-0.4	1.1	1.6	-0.8	0.8	14
12	Bedourie T1/382	-24.04	139.55	-4.0	-0.2	-1.1	0.5	1.9	-0.3	19
13	Low Cliff	-29.01	135.04	-0.2	0.0	-0.9	0.1	0.9	3.7	29
14	Attraction	-29.59	138.05	0.1	0.0	-3.1	-1.0	0.1	1.2	24
15	Bishop Ck A 427	-20.79	140.71	2.5	2.2	2.6	-0.7	4.4	-0.7	23
16	Mt.Isa A 419	-20.70	139.55	0.4	-1.5	-1.5	1.8	4.4	-2.0	18
17	Soudan A 443	-20.09	136.84	-1.2	-0.6	0.6	0.6	3.4	-0.2	17
18	Shamrock NM/G/ 2	-19.62	134.19	1.0	1.3	0.3	-0.2	4.1	0.0	18
19	Daly Wrs NM/G/94	-16.20	133.42	-3.1	-1.3	0.7	-0.1	5.0	0.6	17
20	Anzac	-23.70	133.88	-15.7	-0.7	-2.4	-1.8	-0.2	2.6	20
21	O'Halloran	-27.51	135.44	4.7	1.5	1.7	2.1	0.6	4.6	24
22	RawlinsonNM/E/34	-25.00	128.31	-2.5	-2.7	-2.6	-3.7	1.8	2.1	14
23	Frankenia Rise	-20.16	129.87	-1.5	1.5	1.6	0.3	5.0	-0.6	15
24	Campbell NM/G/48	-21.79	131.85	-9.1	-1.1	-2.3	-1.0	1.5	-0.3	22
25	Solitary	-22.19	133.65	-3.5	-0.3	-2.1	0.6	2.3	-0.1	22
26	Johnston Origin	-25.95	133.21	7.5	0.4	-4.3	0.6	0.0	5.7	31
27	Wambrook	-36.19	148.88	-2.8	0.2	6.2	5.1	5.6	-6.6	20
28	Black Range	-36.02	146.96	1.5	0.2	0.3	3.7	6.2	-0.2	17
29	Yelta	-34.13	142.00	2.5	1.1	-1.4	0.4	1.7	-0.9	20
30	Sundown	-31.90	141.45	1.1	-0.9	1.2	2.6	1.5	-1.5	9
31	Hilltop A 411	-20.91	143.38	0.5	0.2	-2.3	-1.4	4.2	2.3	17
32	Windoo NM/F/ 4	-18.38	128.63	-0.3	0.3	-1.2	-1.3	5.2	-2.4	17
33	Go Go R 009	-18.29	125.59	-1.4	0.8	-4.2	-2.2	11.3	2.3	14
34	Rat Hill M 49	-22.45	120.34	-1.7	-1.9	-3.3	-3.3	9.8	0.5	8
35	Meekatharra MC15	-26.37	118.59	0.6	2.4	-3.5	-4.7	7.6	-4.7	17
36	Mt.Magnet MC 17	-27.76	117.90	-0.6	0.4	5.7	1.7	7.7	-5.4	16
37	Jaclean K 85	-29.78	117.70	0.6	0.3	3.1	0.7	9.6	-3.1	18
38	Bullabulling	-31.00	120.84	3.0	-0.4	3.8	-0.4	8.2	-0.5	9

Table 4

Residuals on the A.G.D. after application of orientation parameters
$O = \{\Delta\xi_{1o} = -4.22$ sec; $\Delta\xi_{2o} = -4.46$ sec; $\Delta\xi_{3o} = 9.82$ metres $\}$

N_i = Number of stations available in innermost zone ($\psi < 0.1^o$)

not give the best solution for the geocentric orientation parameters and are shown for reference only.

It should be emphasised that, under normal circumstances, the use of class C weight coefficients should provide the best solution but the weakness of the gravity field on the peripheries of the continental region vitiates such a syllogism. The typical distribution of ξ_{ai} and $\sigma_{\xi i}$ are shown in figures 12 to 14 in the case where the geocentric orientation parameters are based on table 3, code 9. It can be seen that any errors which are systematic over the entire region are obscured by the "noise" generated due to errors in both the prediction of the local gravity field and the astronomical observations. Also see section 4.3.

4.2 THE EFFECT OF THE INNER AND OUTER ZONES ON THE EVALUATION OF GEOCENTRIC ORIENTATION PARAMETERS

The stability of solutions for the geocentric orientation parameters depend on two factors :-

(i) the lack of any systematic error in the representation of the distant zones as discussed in section 3.3 ; and

(ii) the minimal dependence of any solution on the accuracy of local field determinations.

The 9 solutions discussed in section 4.1 involving all three classes of weight coefficients and solution types and given in table 3, codes 3 to 11, provided very similar solutions for $\Delta\xi_{io}$ which are discussed further in section 4.4. Tests for the stability of the solution were carried out by determining the orientation parameters by excluding the inner zone contributions, the resulting determination being listed in table 3, codes 19 and 20, the former being a type 1 and the latter a type 2 solution. In this case, the gravimetric representation excluded the effects of all regions within 1.5 degrees of the computation point.

Two interesting observations were made from this solution.

(a) The orientation parameters obtained were only marginally greater than those from a full solution (e.g., table 3, code 9). It could therefore be concluded that computation of non-Stokesian terms in the separation vector are not necessary for the determination of geocentric orientation parameters as they can be treated as purely local errors. Such a conclusion must however, be accepted with reservation especially if there is a systematic variation in the nature of the topography across the region being investigated.

(b) The root mean square residual of comparisons with the astro-geodetic geoid was significantly smaller, while those of comparisons between deflections of the vertical increased in magnitude. This indicates that the astro-geodetic geoid is more smoothered than the total gravimetric solution.

A similar procedure was carried out with a gravimetric solution restricted to a spherical cap of radius 20 degrees about each computation point and the results are shown in table 3, codes 21 & 22. The exclusion of the outer zone is seen to give a poorer fit in all of ξ_1, ξ_2 and ξ_3, a comparison with the determination at code 3 shows that the root mean square residual in ξ_3 increases from ±2.25 metres to ±2.83 metres, the outer zone contributing about ±1.7 metres through the root mean square residual to improving the match between the gravimetric and astro-geodetic' solutions.

No definite conclusions can be drawn from the above tests until some assessment is made of the current location of the A.G.D. in earth space. This may be obtained by effecting the solution defined by equations 105 to 115 with all gravimetric values held constant at, for example, zero. The resulting solution, shown at table 3, codes 1 and 2, gives orientation parameters necessary to correct the existing datum to the mean geoid slope across it *as represented by the 38 stations included in the study*. Table 3, code 1 is a type 1 class A solution while code 2 is of type 2 class A and hence independent of the astro-geodetic geoid. Code 1 can be interpreted as indicating that a correction of -3.45 metres to the astro-geodetic geoid height at the Johnston origin (presently assigned the value zero), together with corrections of 0.00 sec to ξ_{10} and -0.50 sec to ξ_{20}, would transform the A.G.D. to lie in the mean geoid slope across the datum. These figures are in close agreement with the estimate made by Fischer and Slutsky (1967,p.331).

The code 2 solution which is based only on the deflections of the vertical however indicates that the corrections are 0.48 sec for ξ_{10} and -0.01 sec for ξ_{20} if the astro-geodetic geoid is not considered in the determination. These figures, which could have been predicted from the mean residuals in the previous solution, will not agree with Bomford's mean deflections of the vertical (Bomford 1967,p.57-8) as the former are the means after allowing for the translation of the local trihedrons to the Laplacian trihedron at the origin while the latter are straight numerical means.

It can therefore be concluded that

(i) the adequate representation of the outer zone gravity field is critical in the determination of the geocentric orientation parameters; but

(ii) it would suffice if only reasonable representation of the inner zone gravity fields were available around selected stations in obtaining geocentric orientation parameters with adequate precision.

It was therefore decided that the use of the free air geoid alone would provide the required parameters with a precision approaching ±0.2 sec, which cannot be exceeded in view of the limitations inherent in the representation of the gravity field currently available. The non-Stokesian effects due to the topographic terms would be included in the residuals obtained by the use of equation 144.

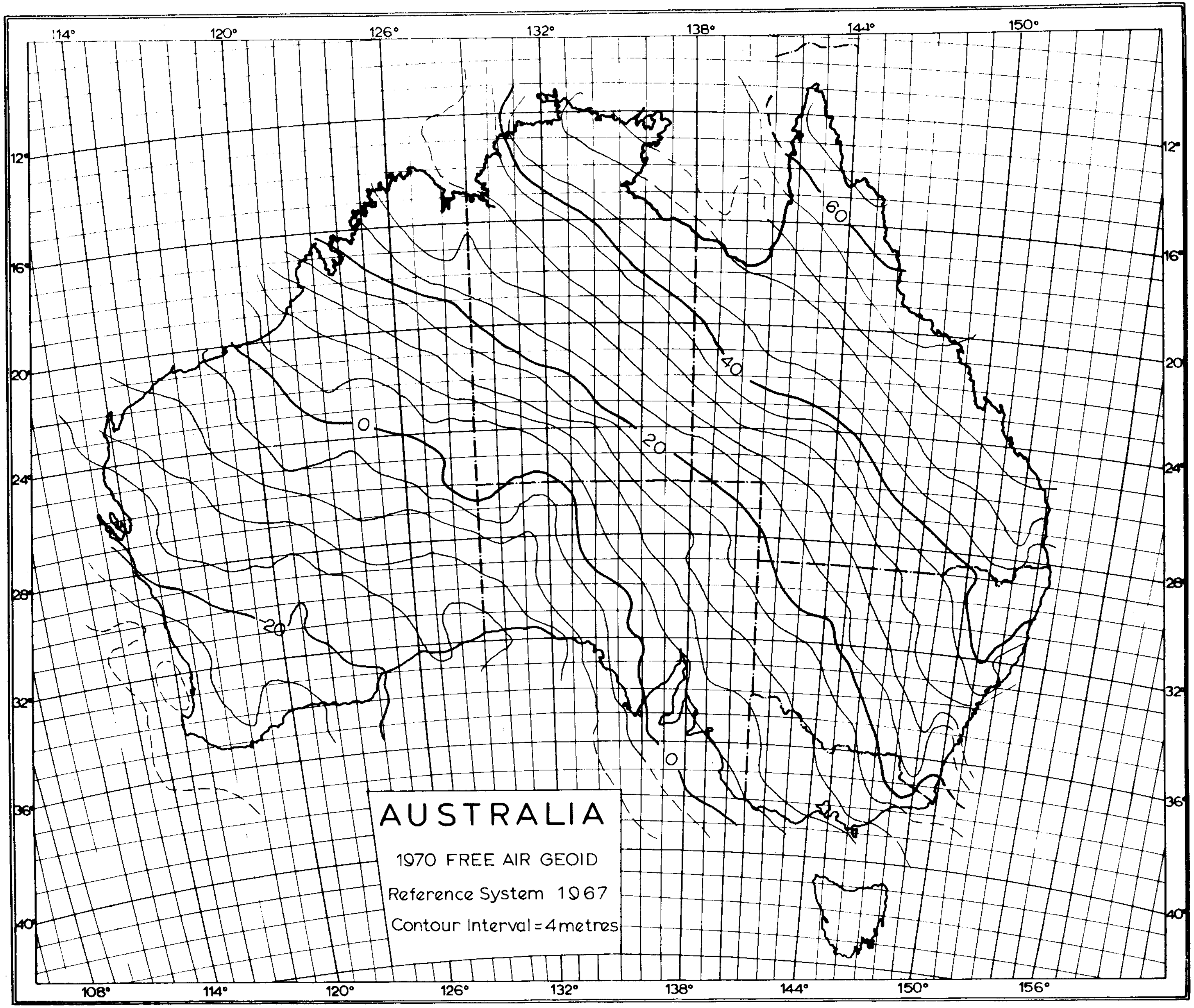
The study of such residuals could well become an integral part of physical geodesy as they are a consequence of comparisons between the results obtained from *two completely independent methods* and, provided the astronomical observations are of adequate precision, afford models of little ambiguity for the study of a variety of problems. The principal difficulty at the present time is the necessity for large scale prediction and the undoubted existence of correlation effects in predicted fields. The efficacy of computation routines for the topography dependent terms can be properly assessed only if the residuals are not masked by errors arising from these correlation effects in the field extensions. This may be more closely analysed by a procedure suggested in (Mather 1969a).

4.3 THE COMPARISON OF ASTRO-GEODETIC AND GRAVIMETRIC QUANTITIES

A study of table 3 shows that orientation parameters obtained for ξ_1 and ξ_2, on comparing deflections only (type 2 solutions) are slightly different from the results of types 1 and 3 in which the results of the astro-geodetic geoid are included. The solution of Fischer and Slutsky (1967) was based on approximately 600 stations, many of which were situated on two closely spaced traverses which provided the framework for the computation. The analysis at table 3, code 1 *which is independent of the gravimetric solution*, indicates that the geoid in the vicinity of the Johnston origin is depressed with respect to the rest of the datum if a mean fit is adopted over the 38 stations included in the study.

This figure is also borne out by solutions obtained by the comparison of ξ_3 values only between the astro-geodetic geoid and the free air geoid at the corners of a one degree grid at 693 points covering the entire Australian mainland, on allowing for the free air geoid elevation at the Johnston origin (row 26 in table 4). The gravimetric determination used in this calculation and called the *1970 free air geoid for Australia* is shown in figure 15 and the resulting orientation parameters listed in table 3, code 12, the technique being identified as *solution type 4*. The residuals in ξ_3 are shown in figure 16 which is equivalent to figure 10 in (Mather 1969b). A comparative assessment of these figures shows that the major inconsistencies which existed between the astro-geodetic geoid and the 1968 solution have been eliminated by the adoption of the technique outlined in section 3 for effecting the necessary predictions and used in the preparation of the 1970 free air geoid. The root mean square residual $\{\sigma^2_{\xi_3}\}$ from the 1970 solution is ±2.5 metres over the entire continent (table 3, code 12), whereas it was ± 3.0 metres over only 80% of the continent in the 1968 solution and ±5.3 over the entire region (ibid,p.513). It is also apparent from figure 16 that systematic discrepancies still exist between the two solutions, but over limited extents and, with one exception, on the peripheries.

The exception is the geoidal low over the Officer Basin in South Australia which shows up clearly on both the astro-geodetic determination of Fischer and Slutsky as well as on the 1970 free air geoid when the latter is converted to the A.G.D., as can be seen from figure 17. However, the minimum as obtained from gravimetry is about 7 metres in excess of that predicted from astro-geodesy. Another point of interest is the markedly greater geoidal high obtained in the Snowy Mountains area from the 1970 free air geoid than from the solution of Fischer and Slutsky.



Note: Less Zero Degree Term

Fig. 15

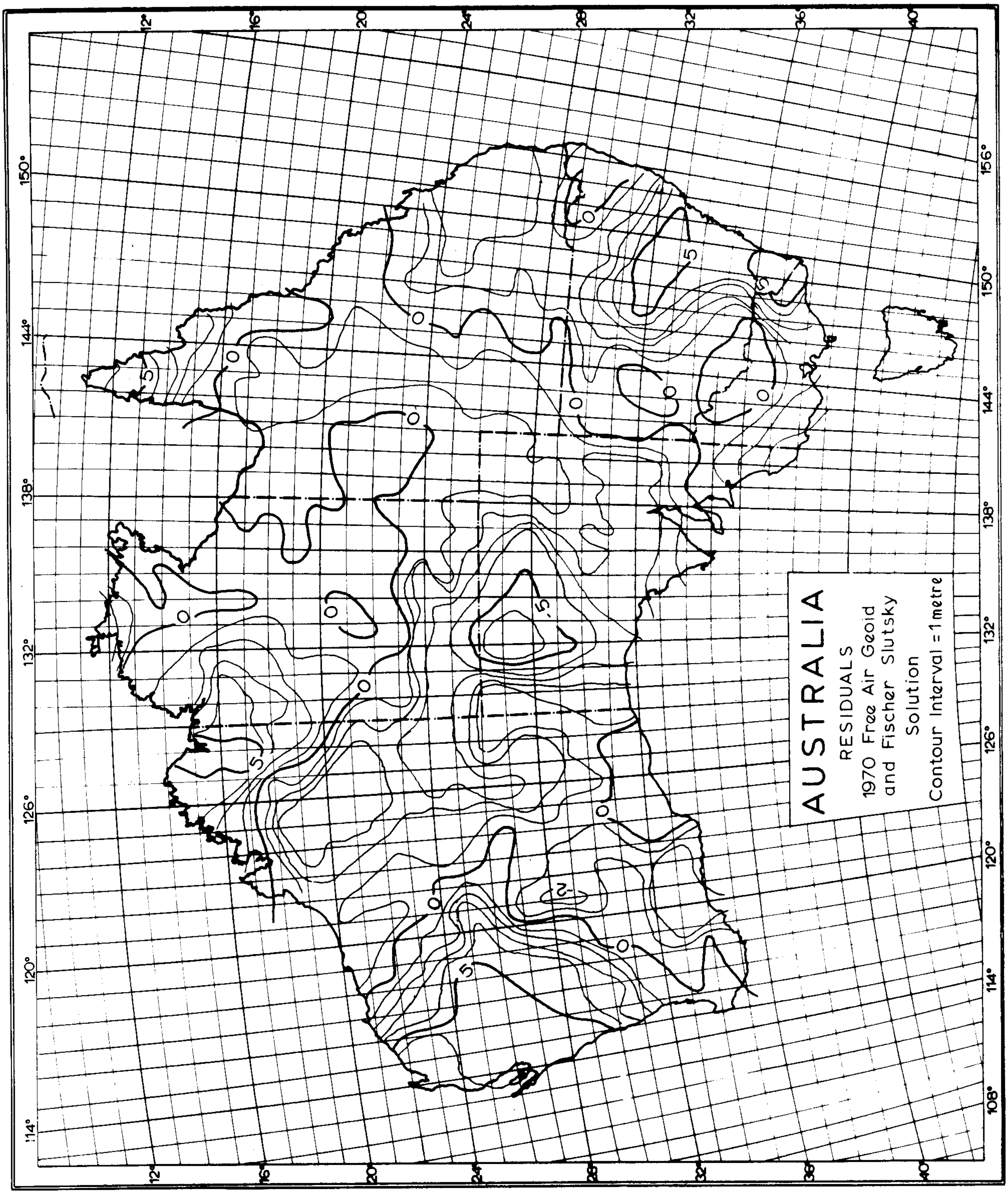

AUSTRALIA
RESIDUALS
1970 Free Air Geoid
and Fischer Slutsky
Solution
Contour Interval = 1 metre

Fig. 16

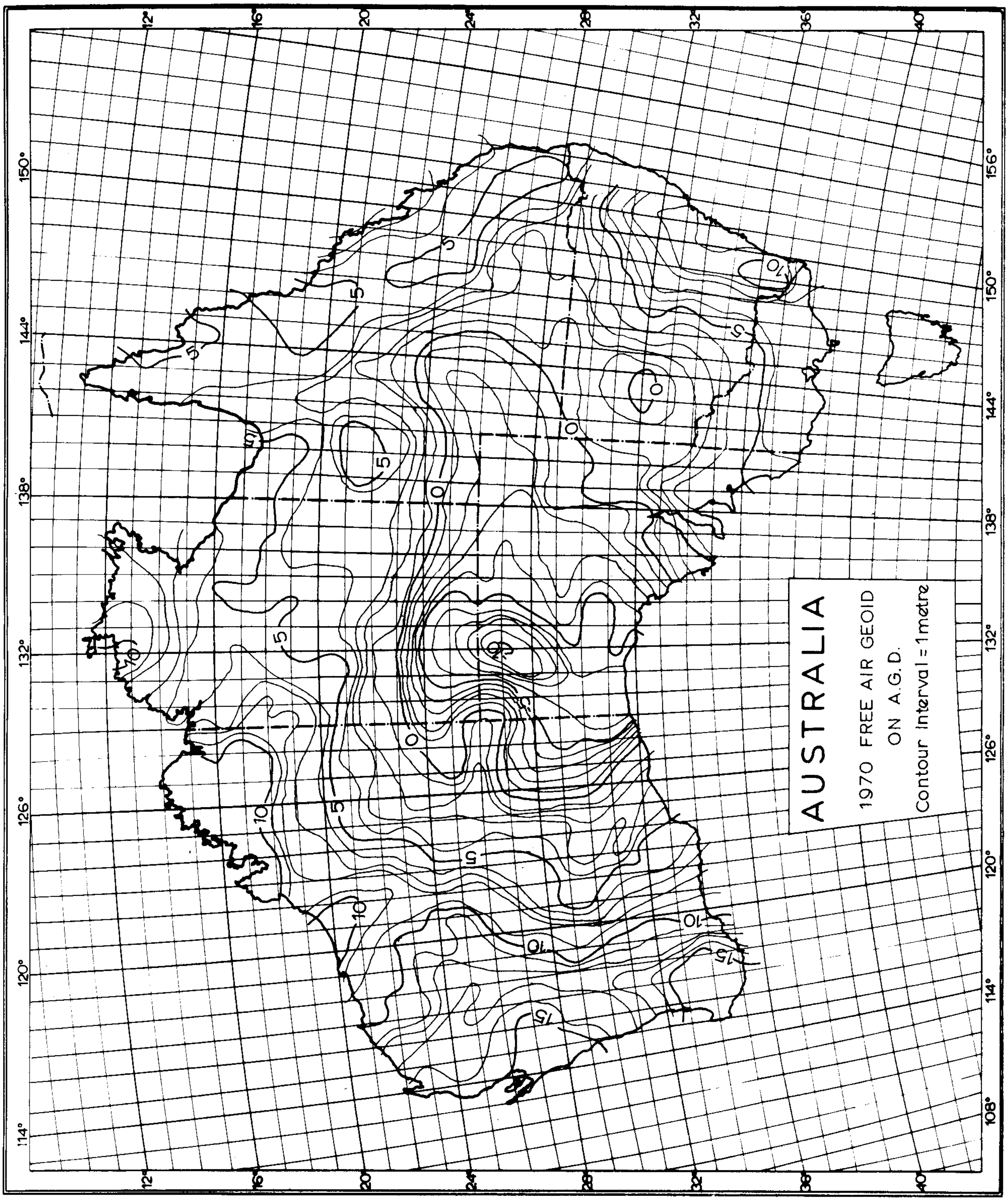

AUSTRALIA
1970 FREE AIR GEOID
ON A.G.D.
Contour Interval = 1 metre

Fig. 17

It is apparent that the solution of Fischer and Slutsky is of commendable accuracy when assessed in terms of the astro-geodetic station density of one point per 10,000 km^2 and the uneven distribution. There is also little doubt that the accuracy of the gravimetric determination is sufficiently adequate to make the drawing of any conclusions regarding the source of the discrepancies pure conjecture. This matter will be investigated further when the new astro-geodetic solution for the geoid, based on approximately 1500 astro-geodetic stations, is completed by the Division of National Mapping in 1971.

Comparisons between deflections of the vertical at individual stations on the A.G.D. is shown in table 4. The assessment of the accuracy of gravimetric computations of deflections of the vertical in circumstances where partially predicted fields are used, is a complex task. Correlation of predicted values can never be avoided, whatever statistical technique is used, the effect being directly proportional to the area over which uninterrupted prediction is performed. It therefore becomes mandatory to use analytical techniques and adequately filtered data to correctly assess these correlation terms. The results of individual comparisons in table 4 when viewed in this light, show that most residuals are of the order of precision of the astronomical observations. For example, at the Johnston origin, the astronomical latitude was based on circum-meridian observations to 16 pairs of stars over two nights with a standard deviation of ±0.80 sec, while the longitude was determined by 35° almucantar impersonal observations over two nights using 16 pairs with a standard deviation of ±1.16 sec, the instrument used being a DKM 3A theodolite.

Significant departures occur at some points in Western Australia which are surrounded by regions where significant field extensions were necessary in representing the near zones and also in the vicinity of the Snowy Mountains in the south east where the results appear to have been affected not only by the lower density gravity field in a region of rugged topography (see figure 10) but also by the extensive prediction necessary in the Tasman Sea. Computation of the topographical effect on deflections of the vertical through equation 71 were found to reach the magnitude of the estimated error in the astronomic latitude and longitude at only one station, viz., Wambrook (27 in table 4). The correction computed in this case significantly reduced the residual in ξ_2. These terms were not considered further in the present study as their effect on the geocentric orientation parameters is negligible.

4.4 THE BEST SET OF GEOCENTRIC ORIENTATION PARAMETERS

A study of table 3 shows that, on exclusion of type 3 solutions which are disproportionately representative of those stations with large residuals with significant effects on the determination due to these points being peripheral to the region covered by the datum, values can be assigned for the geocentric orientation parameters with confidence so that they lie within a range of 20 cm in ξ_3(i.e.,N) and 0.2 sec in ξ_1 and ξ_2. The exact values to be adopted still require clarification. An earlier study (Mather & Fryer 1970a, section 4) recommended that the set

$$O = \{ \Delta\xi_{1o} = -4.7 \text{ sec} ; \Delta\xi_{2o} = -4.4 \text{ sec} ; \Delta\xi_{3o} = 14.0 \text{ metres} \}$$

based on the 1968 free air geoid solution for Australia (Mather 1969b). The 1970 solution is based on a much improved data set as discussed earlier, although the land areas of Australia are still not completely represented at the present time. In addition, the earlier solution was seriously affected by systematic error due to lack of compatibility between data sets. This defect has been remedied in the current solution and the residuals on comparison of the two solutions are greatly reduced as discussed in the last section.

As the results obtained from table 3, code 19, indicated that inner zone contributions had marginal effects on the orientation parameters, it was decided to include the values determined from comparisons between ξ_3 values on a one degree grid, as shown at code 12, in the analysis, being listed as type 4 in table 3. It can be seen that the values of $\Delta\xi_{1o}$ and $\Delta\xi_{2o}$ obtained from this solution are not significantly different from those obtained from the type 2 determination at code 6 in table 3. Due to the similarity in the results obtained from solution types 1 and 3, it was decided that the values deduced for the astro-geodetic geoid at the 38 selected stations were not representative of the datum as a whole and a number of composite solutions were investigated with *minimal concern for marginal increases in the root mean square residual in* ξ_3 *at the 38 stations.*

Two composite solutions were considered. The first was defined by the orientation correction parameters

$$O = \{ \Delta\xi_{1o} = -4.22 \text{ sec} ; \Delta\xi_{2o} = -4.54 \text{ sec} ; \Delta\xi_{3o} = 9.82 \text{ metres}\},$$

being a combination of codes 3 and 6 in table 3. The resulting root mean square residuals at the 38 stations are shown in code 14. These results are only marginally different from those at code 13 where the orientation parameters were adopted from code 12, which was based on comparisons on the continent wide one degree grid. The forced mean values of $\sigma_{\xi 1}$ and $\sigma_{\xi 2}$ are a measure of the non-representative nature of the residuals in ξ_3, there being larger increases in $\sigma_{\xi 3}$ than in either $\sigma_{\xi 1}$ and $\sigma_{\xi 2}$, both of which, as expected, are almost unchanged. The incorporation of these forced means into the orientation parameters themselves results in comparisons given at table 3, code 15, which, as envisaged, produces changes in the root mean square residual in ξ_3 but not in those of ξ_1 and ξ_2.

It can therefore be concluded that the following set of geocentric orientation parameters best fit the Australian Geodetic Datum

$$O = \{\Delta\xi_{1o} = -4.2\pm0.2 \text{ sec} : \Delta\xi_{2o} = -4.5\pm0.2 \text{ sec} : \Delta\xi_{3o} = 10.0\pm0.2\text{metres}\} \quad ....(148).$$

The above set is significantly different from the one obtained by the use of the 1968 free air geoid and quoted earlier. The latter should be disregarded as based on an inferior data set.

The error estimates given for the above set of orientation parameters are based on detectable errors and do not reflect the magnitude of any systematic effects in those low degree harmonics of the earth's gravitational field which have near planar variations over the Australian region as discussed in section 3.3. Even degree zonal harmonics, being determined from secular variations must be considered the most reliable of the suspect harmonics. Accuracy estimates between Rapp's solution for these coefficients from gravity data alone and the Smithsonian Astrophysical Institution Standard Earth solution (Rapp 1969,p231) indicate that the maximum values possible in the error estimates of $\Delta\xi_{3o}$ as ± 5 metres and ± 0.4 sec in $\Delta\xi_{1o}$ and $\Delta\xi_{20}$. These limiting values would be smaller if the estimates of the errors in the coefficients were closer to values given by Cook for those in C_{20} and C_{40} (Cook 1965,p.181).

It is therefore a distinct possibility that $\Delta\xi_{3o}$ may be subject to changes of the order of 1 or 2 metres but it is unlikely that $\Delta\xi_{1o}$ and $\Delta\xi_{2o}$ will change by more than the limits specified in equation 148. The final set of geocentric orientation parameters defining the geocentric orientation vector O at the Johnston origin of the Australian Geodetic Datum, on including the zero degree term of -2.8 metres from table 1 are

$$O = \begin{Bmatrix} \Delta\xi_{1o} = \Delta\xi_o = 4.2 \pm 0.2 \text{ sec} \\ \Delta\xi_{2o} = \Delta\eta_o = -4.5 \pm 0.2 \text{ sec} \\ \Delta\xi_{3o} = \Delta N_o = 7.2 \pm 0.2 \text{ metres} \end{Bmatrix} \quad \ldots.(149)$$

5. CONCLUSIONS

5.1 THE 1970 FREE AIR GEOID FOR AUSTRALIA

The geoid across Australia can be related to a geocentric reference spheroid if it is computed by the use of a set of free air gravity anomalies, which, to the order of the flattening, approximate to the gravity anomaly, provided the anomaly set, on global analysis using a surface harmonic series, has no harmonics of first degree and first order of second degree. The height anomaly, the geoid/spheroid separation and the results of astro-geodetic levelling can be assumed to be equal for the Australian region, provided they are related to the same common datum, as the systematic errors in the representation of the incompletely surveyed earth's gravity field at the present time provides sufficient "noise" to drown out the "signal" due to such indirect effects. The resulting free air geoid, called the 1970 free air geoid for Australia, is based on a totally compatible composite data set where the gravity field within 20 degrees of the Australian coastline has been defined either by observations or predictions in accordance with criteria and procedures specified in sections 3.2 to 3.4. Such a solution provides a good approximation to both the height anomaly and the geoid/spheroid separation.

Tests of the 1970 free air geoid with the astro-geodetic solution of Fischer and Slutsky indicated that the former was a marked improvement over the 1968 solution, the root mean square residual for the Australian region reducing from ±5.3 metres to ± 2.5 metres. A more comprehensive analysis of this solution will have to await the completion of the detailed astro-geodetic determination of the geoid being prepared by the Division of National Mapping. The current indications are that its accuracy is at least on par with an astro-geodetic solution prepared from an average station spacing of 100 km.

5.2 THE SET OF GEOCENTRIC ORIENTATION PARAMETERS FOR THE AUSTRALIAN GEODETIC DATUM

The composite data set described in sections 3.2 to 3.4 was used in two ways to obtain the required orientation parameters. In the first case, it was used to compare gravimetric values of the deflections of the vertical ξ_1 and ξ_2 as well as the separation ξ_3 at the corners of a one degree grid which covered the entire datum and the results are set out in table 3 as the type 4 solution. It was also used to compute these quantities at 38 astro-geodetic stations evenly spaced over the datum and shown in figure 12.
The solution on the grid was compared with the Fischer and Slutsky astro-geodetic geoid on the A.G.D. from which orientation parameters were defined for transforming the earth space location of the Australian National Spheroid, as specified by the geodetic co-ordinates adopted at the Johnston origin, to a geocentric one.
The geocentric orientation parameters obtained by the first technique were in good agreement with a composite set established

by the second, this procedure being resorted to as the values of the astro-geodetic geoid deduced at the 38 stations were not representative of the entire datum.

The following conclusions were drawn after testing various possible sets of orientation parameters.

(i) The inner zone field has a marginal effect on the determination of sets of geocentric orientation parameters for a region the size of Australia.

(ii) The non-Stokesian topography dependent terms have purely local effects to within the limits of precision sought in this investigation and have not been considered.

(iii) The use of the free air geoid alone instead of the height anomaly and surface deflections of the vertical which comprise the complete gravimetric solution, should give orientation parameters with errors less than ±0.1 sec in each of $\Delta\xi_{1o}$ and $\Delta\xi_{2o}$ and ± 0.1 metres in $\Delta\xi_{3o}$, provided the gravity field is adequately defined and no serious systematic effects exist due to errors in the values adopted for the coefficients of low degree harmonics of the earth's gravity field. The current data set does not quite meet these requirements as can be seen from equation 149.

iv) It is possible to estimate the precision of deflections of the vertical obtained from gravimetry using the data set currently available for the Australian region. The use of free air anomalies in the Vening Meinesz integrals does not give the total surface deflections of the vertical, but nevertheless matches the astro-geodetic values with a root mean square residual of ±1.0 sec in the meridian component and ± 1.8 sec in the prime vertical component. These results should be interpreted in the context of the accuracy of the astro-geodetic determinations which are estimated as being ±0.5 sec in ξ_1 and ± 1.0 sec in ξ_2

for determinations in the Australian network. These figures quoted for the residuals are considerably improved if only those stations where the inner zone gravity field within 3^{o} of the computation point has been adequately defined, are considered, when the root mean square residual approaches that of the estimated precision of the astronomical values. These conditions are satisfied only in western Queensland,the Northern Territory and adjacent areas. The contributions of the topography dependent terms to the surface deflections of the vertical are small, being of significant magnitude only at station 27 in table 4, when computations using equation 71 significantly reduce the magnitude of the residual in ξ_2.

(v) The root mean square residual of the comparisons between the free air geoid and the astro-geodetic solution after datum translation varies between ±2.2 metres at the 38 stations to ±2.5 metres over the entire continental extent. The significance of these figures must be evaluated in the light of the following observations.

(a) The astro-geodetic geoid matches a gravimetric solution without inner zone effects *with smaller residuals* than a complete solution, indicating that the former is over-smoothened, which is not surprising in view of the fact that the station density is one per 10,000 km^2.

(b) The residuals are position dependent, inferring the existence of systematic error in both the astro-geodetic and gravimetric solutions.

The errors in the former are essentially due to oversmoothing , thus underestimating geoidal highs in mountainous regions, as occurs in the Snowy Mountains region, and overestimating geoidal lows, an instance of which clearly occurs in the Officer Basin region near the Johnston origin. The errors in the gravimetric solution are due to correlated prediction errors which continue to be of significance in determinations at certain regions on the continental margin.

(vi) The best set of geocentric orientation parameters which can be deduced from the 1970 data set are given by

$$O = \left\{ \begin{array}{lllr} \Delta\xi_o & = \Delta\xi_{1o} & = & -4.2 \pm 0.2 \text{ sec} \\ \Delta\eta_o & = \Delta\xi_{2o} & = & -4.5 \pm 0.2 \text{ sec} \\ \Delta N & = \Delta\xi_{3o} & = & 7.2 \pm 0.2 \text{ metres} \end{array} \right\} ,$$

which also includes the effect of the zero degree term, but assumes that the potential of the geoid is equal to that of the spheroid, as any other zero degree consideration can only be interpreted on the basis of global analysis (e.g.,Mather 1968b,p.528).

5.3 CONCLUSION

The current investigation is of a preliminary nature in that results were of prime importance and speed of execution was the essence of the entire study. In many instances field readings were made available to the investigator before final plans had been issued. The bulk of the U.N.S.W. data set has been primarily compiled from Bouguer anomaly maps and the free air anomalies regenerated using the digital representation of topographical maps available in 1964. The investigator felt that such a procedure was warranted in view of the gaps in the anomaly field and the precision attainable from astronomical observations, and subsequently confirmed by the results of this investigation.

The quality of the gravity data set available at present makes it desirable that the entire U.N.S.W. data set be re-assembled from individual gravity readings as it presently contains errors due to both the interpolation of Bouguer anomaly maps as well as elevation errors consequent to the digitising of topographical maps which have largely been superseded. The re-assembly of the data will take approximately 24 man-months, exclusive of predictions.

This will be a pre-requisite to any investigations of the topography dependent terms in the expressions for the surface deflections of the vertical, which, in Australia, are capable of serious investigation only in the Snowy Mountains region. The establishment of the gravity field in the Tasman Sea to the east of this area is also a pre-requisite to such a study.

In a more general context, the only restriction on the gravimetric determination of the geocentric orientation vector for a geodetic datum is the precision of the definition of the gravity field and the extent to which prediction has to be resorted to. Results can be seriously jeopardised by the nature of the predictions performed as correlation errors significantly affect results which are based on extensive predictions of the near zone gravity field within 10 degrees of the computation point, the effect decreasing with increase of distance between the predicted values and the computation point. The chances of obtaining satsifactory predictions from purely local statistical processes alone are limited. The combination of statistical and analytical techniques is to be preferred, the latter assessing the regional trends while the former estimates the likely deviations of individual stations from these trends. Marine gravity surveys for geodesy should concentrate on establishing "noise" free estimates of area means at discrete intervals in order that analytical techniques could be successfully applied during that interim period preceding the complete global representation of the earth's gravitational field.

In the short term, gravimetric determinations of the geocentric orientation vector will provide a valuable independent check on any global satellite triangulation scheme. In addition, it will afford a definition of local geodetic datums with respect to fundamental invariants of earth space. The adoption of the technique is strongly recommended in view of its long term geophysical significance.

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