GEODETIC SURVEYING

A.H.W. KEARSLEY

MONOGRAPH NO. 8
SCHOOL OF SURVEYING
THE UNIVERSITY OF NEW SOUTH WALES
KENSINGTON, N.S.W. 2033, AUSTRALIA

PREFACE

This monograph has been written to give students of surveying in Australia, and specifically at the University of New South Wales, an insight into the reasons for, and problems associated with, geodetic surveying . In this work I have tried to provide an overall perspective on this very practical topic. Many subjects deal with specific elements of the task. Network Solution by Least Squares, Positional Astronomy, Ellipsoidal and Projection Geometry - even practical field techniques - are treated in detail in various parts of an undergraduate course. But often the important relationship between these parts is not properly understood; the connecting links between the elements are missing because the subjects are taught by different lecturers, at different stages of the course, and of course, with differing emphases.

I have tried, therefore, to give theory only where it has seemed necessary in order to provide basic principles or to help illustrate a point. I have tried to use the Australian experience to amplify the text in the hope that this will, by its relevance, assist the student's understanding. Also much of the reference material will be more readily accessible - the reader will notice a fairly heavy usage of The Australian Surveyor in the reference list.

The monograph has as its beginnings the class notes in Geodesy I as they have evolved over a few decades. It is difficult, therefore, to know who was responsible for what in these original class notes so acknowledgement is given to all those others who have taught this part of the course in the past, including R. S. Mather, F. L. Clarke, S. M. Ganeshan and Jean Rüeger.

Acknowledgement is also made to the Institution of Surveyors, Australia, for many of the diagrams and also the Division of National Mapping, for the maps used. Reference is made in individual figures to these bodies where appropriate.

GEODETIC SURVEYING

with special reference to
Geodetic Surveying in Australia

A. H. W. KEARSLEY

Received August, 1983

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Published by
The School of Surveying
University of New South Wales
P.O. Box 1,
Kensington,
N. S. W., 2033
Australia

National Library of Australia
Card Number & I.S.B.N.
0 85839 0361

CONTENTS

GEODETIC SURVEYING

1. INTRODUCTION

1.1 The Objects of Geodetic Surveys (See TORGE, p.210)

Two objects of a geodetic control survey are:-

(a) to establish a system of points fixed in horizontal position, height and gravity which can form the basis for less precise surveys. These latter may take the form of topographical, cadastral and engineering surveys and maps or plans.

(b) to assist, in combination with gravity and with satellite and other extra-terrestrial observations, in determining the size, shape and density distribution of the earth, and in the investigation of geophysical phenomona of the Earth such as tectonic plate movement, polar motion, etc.

1.1.1 The Problems of Geodetic Surveying (see also TORGE, Section 1.2)

Geodetic Surveying differs from other surveys in at least one important way - it has no superior survey control network to tie into, it being itself the primary control on which other surveys are based. Coupled with this is the fact that it covers large areas of the Earth's surface. For example, the Australian Geodetic Survey extends about 30° in latitude and 35° in longitude (about 3,300 km by 3,900 km).

These factors mean that extreme care must be taken in both the measurement and the calculation of the geodetic survey. Because such large distances are covered, care must be taken to reduce to a minimum the sources of systematic errors in the observations. Errors which will be insignificant in local surveys quickly become a problem when extrapolated into a survey the size and extent of a geodetic survey. The fact that such large areas are encompassed means that it is no longer adequate to assume a plane as the model of the surface on which measurements are taken. This assumption very quickly breaks down. For example, the sum of the three perfectly observed angles of a small equilateral triangle of sides 20 km on a sphere should equal 180° 00′ 01". Assuming a plane model we would expect the sum to be 180° exactly, and the excess of 1" obtained from measurement would be interpreted as an error. In fact, of course, the model adopted is in error and when applied the correct model is used, no error exists.

The simplest analytical figure to closely approximate the "surface of the Earth" is an ellipsoid - an ellipse rotated about its minor axis.

The basic parameters used to define the ellipse so rotated are (see Figure 1.5):-

a = the semi-major axis

b = the semi-minor axis, and

$f = (a - b)/a$.

For small areas a sphere may suffice, but for distances greater than about 100 km it is found that the spherical model must be abandoned in favour of the ellipsoidal model, in order to maintain the accuracy in computation demanded by the precision of the observations. For this reason the ellipsoid is used universally as the model on which geodetic computations are based.

We have used the term "surface of the Earth" above, and the exact meaning of this should be explained. Intuitively, we think of the Earth's surface as its topographic surface, i.e. that represented by the interface between solid or liquid matter (ground level, ocean) and the atmosphere. This obviously departs a good deal (up to 8.5 km) from an ellipsoidal model. However if we use an imaginary surface called the geoid to represent the Earth's surface, the ellipsoidal fit to this surface is much closer. The geoid is defined (see Section 1.2) as the equipotential surface of the Earth (one which would be assumed by a fluid in a state of equilibrium) at mean sea level. For much (70%) of the Earth's surface this is fairly well approximated by the oceans, once tidal effects, wind stress effects and oceanographic disturbances are removed. For the land masses, it is possible to establish mean sea level at tide gauges, and to "carry" this level inland by means of spirit levelling. And herein lies the second advantage of using an equipotential to represent the "Earth's" surface. All terrestrial surveying done with respect to a level spirit bubble are referred to the local equipotential surface passing through the instrument axis. A small correction enables observations to be related to the geoid (the equipotential at mean sea level); thus all terrestrial surveying takes as its physical reference surface the geoid.

Summarising then, we use the geoid to represent the physical Earth's surface for two reasons.

1. It is much smoother than the topographic surface, thus the ellipsoidal model will approximate it more closely. (The maximum departure of the geoid from recent Earth-fitted ellipsoids is about 100 m).

2. It is a surface to which all survey measurements relate - hence it is simple to measure departures of the topographic surface from it. Hence topographic maps show heights of the topography above mean sea level.

1.1.2 The Operations involved in Geodetic Surveying

An overview of the steps involved in a geodetic survey is given in Figure 1.1. As can be seen, there are three main phases in the work - the planning, field and computational phases. In these notes we will follow the flow of the job as the logical development of the subject although not all aspects will be treated. For example, for students at UNSW the techniques and reductions of observations has already been covered in 29.005 Surveying V; the ellipsoidal geometry used to compute line elements on the ellipsoid is treated in Section A of Geodesy I; the theory and techniques adopted for network adjustment are given generally in Survey Computations II, and specifically in Geodesy II, Part A.

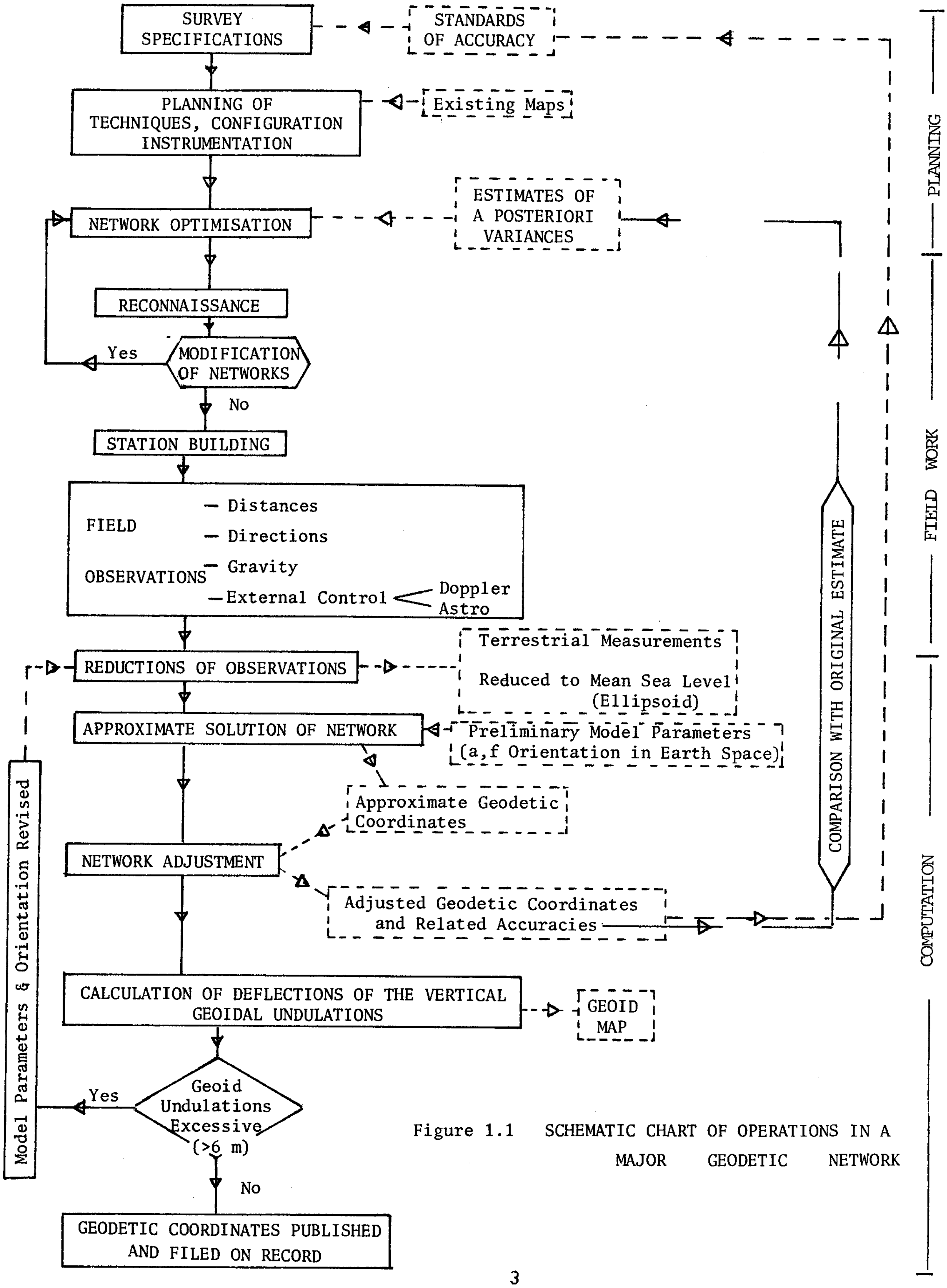

Figure 1.1 SCHEMATIC CHART OF OPERATIONS IN A MAJOR GEODETIC NETWORK

1.2 Introductory Definitions and Concepts

1.2.1 The Geoid, the Ellipsoid and their Relationships

The **Geoid** is the equipotential surface of the Earth at mean sea level (level which oceans find at state of equilibrium). It is not perfectly regular due to local anomalies in the gravitational field caused by the non-homogeneous nature of the surface, sub strata, etc.

The horizontal axis of the bubble tube of a levelled theodolite at sea level will be parallel to the Geoid and hence the vertical rotation axis of the theodolite will be normal to the geoid; this direction is called the **"vertical"** or **"plumbline"**.

The **Ellipsoid** is the mathematical surface which most closely approximates the geoid. The surface adopted for this purpose is an "oblate spheroid" obtained by rotating an ellipse about its minor axis (sometimes known as ellipsoid of revolution). A line perpendicular to the surface of the ellipsoid at a point on it is called the **"normal"** at that point.

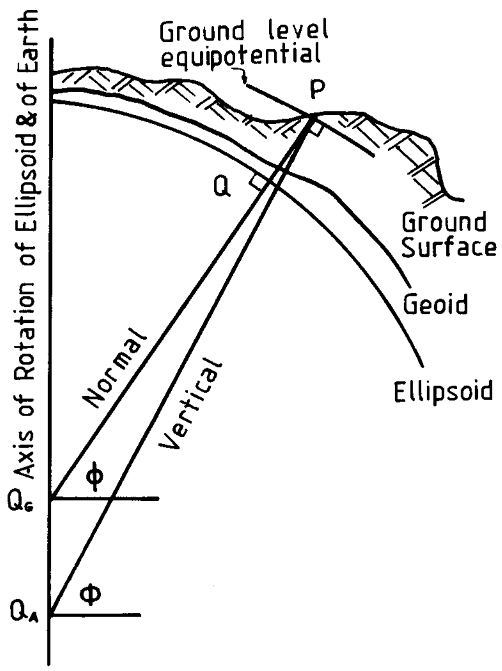

FIGURE 1.2: Meridianal Section through Ellipsoid

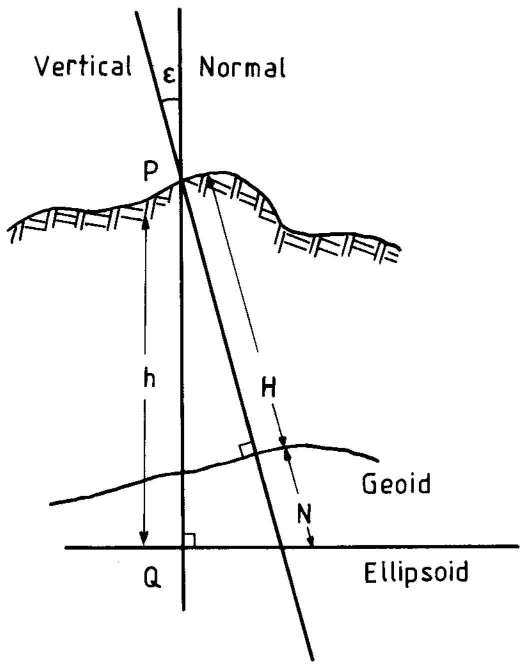

FIGURE 1.3: Local Relationship between Geoid and Ellipsoid

Geodetic Latitude (ϕ) of a point is the angle between the normal to the ellipsoid through the point and the equatorial plane of the ellipsoid.

Geodetic Longitude (λ) of a point is the angle between the meridian plane through the point and an arbitrarily defined zero meridian plane. This zero meridian plane is usually chosen as the meridian plane passing through Greenwich.

Geodetic Azimuth (α) of a line on the ellipsoid is the angle from the local geodetic meridian to the line, measured clocwisefrom $0^o$to 360^o.

Since Astronomical observations depend on the direction of gravity at the point, the Astronomical latitude and longitude are defined as follows:-

Astronomical Latitude (Φ) of a point is the angle between the meridianal component of the direction of gravity (or **Vertical**) at that point and the equitorial plane of the Earth.

Astronomical Longitude (Λ). The plane containing the vertical at a point P and parallel to the rotation axis of the earth is the **astronomic meridian** plane of P. The angle between some designated zero astronomic meridian plane (approx. Greenwich) and that of the point P is the astronomic longitude of P.

Astronomical Azimuth (A) of a line is the angle from the local astronomical meridian to the line, measured clockwise from 0 to 360.

The Deflection of the Vertical (ε) at a point is the angle at the Earth's surface between the normal and vertical through P (see Figure 1.2 & TORGE, 135-137). This solid angle can be split into two components, (i) the meridianal component, ξ, (+ve if the vertical is North of the normal) and (ii) the prime vertical component η, (+ve if the vertical is east of the normal). As is shown in Section 3.2.3,

$$\xi = \Phi - \phi \tag{1.1}$$

$$\eta = (\Lambda - \lambda) \cos \phi \tag{1.2}$$

The component of deflection in any azimuth α is

$$\varepsilon_\alpha = \xi \cos \alpha + \eta \sin \alpha \tag{1.3}$$

As can be seen from the above definitions, the components of the deflection reflect the tilt of the geoid with respect to the ellipsoid. The third element which relates the geoid to the ellipsoid is the "geoid - **ellipsoid separation**", N, (also known as the geoid undulation).

From inspection of Figure 1.3, for small ε,

$$h = H + N \tag{1.4}$$

where h is the ellipsoidal height of a point, and

H is the orthometric height or height above geoid.

"H" is the value obtained by spirit levelling from "mean sea level", defined by a tide gauge.

1.2.2 Geodetic Datum Definition (TORGE, 212-215)

One function of a geodetic survey is to establish a mathematical figure which is suitable for the geodetic computations for the region involved. In classical terms, this involved the selection of an ellipsoid (i.e. its semi-major axis, a; flattening f; the geodetic coordinates of the "origin of survey" or "fundamental station"; and the geoid-ellipsoid separation (N) at this station) such that the ellipsoid gave the "best possible" fit to the geoid over the extent of the geodetic survey. The "best possible" fit could be defined in terms of minimising the ΣN_i^2, or the $\Sigma (\xi_i^2 + \eta_i^2)$; determined throughout the region. This is done in the hope that the ellipsoid and geoid can be assumed to coincide, and that all distances, reduced for H to mean sea level can be assumed equal to ellipsoidal distance within allowable accuracy limits.

It is apparent that the size of the deflections, and of the geoid undulations, are directly dependent upon the particular ellipsoid chosen for the computation of the geodetic coordinates. The geoid is constant (within the limits of accuracy in which we are computing, viz $1:10^6$), hence the direction of the vertical in space is fixed at any point in the network. The size and orientation of the ellipsoid will determine the geodetic coordinates computed upon it, and hence the direction of the normal will depend upon the choice of these parameters. Hence, as stated above, the deflections are a function of the ellipsoid adopted for the geodetic datum. A simple example will help to illustrate this point.

Consider the meridianal section in Figure 1.4. The problem is how to obtain and locate the ellipse which is the best general fit to this section of the geoid. We will consider the second aspect first, i.e. the location of the ellipse when its dimensions have been decided on.

Figure 1.5 shows a meridian section of the ellipsoid of the selected dimensions. On any such ellipsoid, we may define the geodetic latitude of a point as the angle between the equator and the normal to the meridianal section of the ellipsoid; thus the geodetic latitude of X is ϕ . We might now decide that at X we are going to make the geodetic latitude equal to the astronomical latitude, already observed to be 30° 00′; if ϕ_X is to be 30° 00′, the position of X on the ellipse is defined exactly, and we may superimpose Figure 1.5 over Figure 1.4, with the normals to the geoid of Figure 1.4 and ellipsoid of Figure 1.5 coincident at X, and their surfaces coincident (or parallel; the possibility of a height difference (N) need not be considered here). If X and Y are now connected by survey, we could compute the distance XY along the meridian, and this would suffice to determine the geodetic latitude of Y. This might be computed to be 60° 0′. Figure 1.6a shows the result of the superimposition; the normals to geoid and ellipsoid at X coincide, but at Y the geoidal normal makes 60° 01′ with the equator, and the ellipsoidal normal, 60° 00′; the value of ξ at Y is therefore 1′00".

In Figure 1.6a, we have 00′ deflection along the meridian at X, and 01′ at Y; but we might equally well have started by making ϕ equal to Φ at Y; ϕ for Y is then 60° 1′, and the normals coincide at Y (Figure 1.6b). The geodetic latitude of X may now be computed from the known distance XY, and would be found to be very nearly 30° 1′; but its astronomical latitude is 30° 0′, and we now have a deflection of 1′ at X, and zero deflection at Y.

All deflections are thus relative, and depend on the point at which a value of the deflection is assumed; this is generally the origin from which an independent geodetic survey starts. They also depend on the ellipsoid selected; if zero deflection was assumed at X, it would be possible to arrange for zero deflection at Y also, by selecting that ellipse for which the fixed distance XY was equal to the arc length between 30° 0′ and 60° 01′; but this ellipse might be a bad fit at other points.

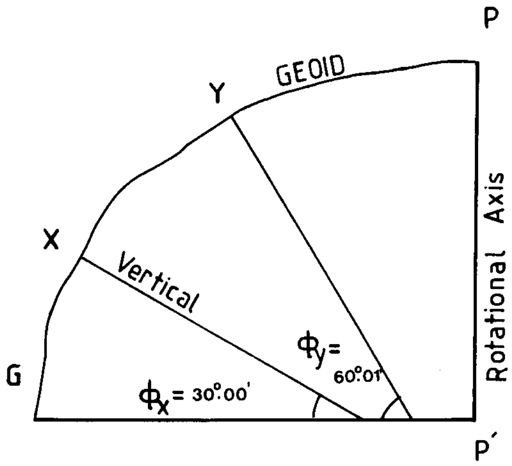

FIGURE 1.4: Geoidal Profile

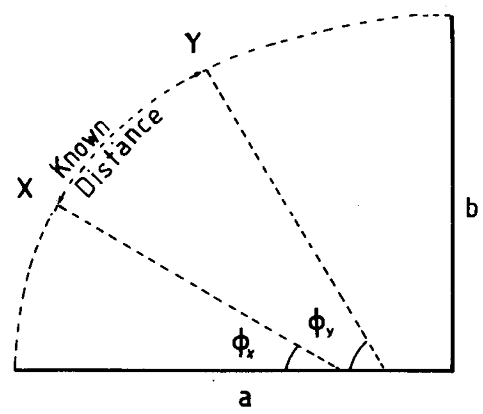

FIGURE 1.5: Ellipsoidal Profile

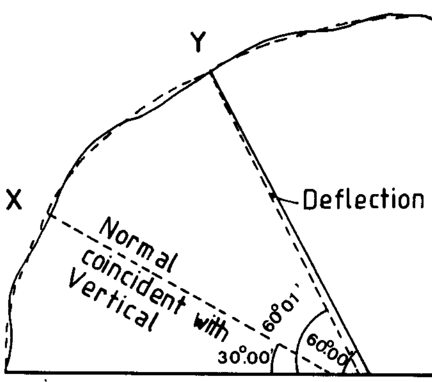

FIGURE 1.6a: Geoid and Ellipsoid: Normals coincident at X. Deflection 1′ at Y.

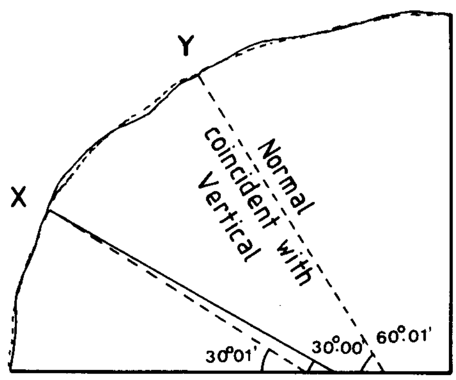

FIGURE 1.6b: Geoid and Ellipsoid: Normals coincident at Y. Deflection 1′ at X.

1.2.3 The "Development" Method of Geodetic Computation

The above simple illustration helps to emphasise two aspects of geodetic surveying, i.e.

(i) that the sizes and signs of the deflections of the vertical throughout the region are simply a function of the size and orientation of the ellipsoid chosen for the region; and

(ii) that one has to make certain assumptions in the first instance about the relationship of the ellipsoid to the geoid. In the example above, X was stated as known. In practice, distances are measured at ground level, and reduced (for the mean height of the line above sea level) only to the geoid, not the ellipsoid. This distance is then assumed to equal the distance on the ellipsoid* between the normals through the end points. This approximation was made because, at this stage of the computation, we have no knowledge of the geoid-ellipsoid separation. In fact, until we have a geodetic position (to produce a deflection) we cannot, using classical astro-geodetic methods, calculate the separation between these two surfaces. Approximate geodetic coordinates are calculated through the network using ellipsoidal geometry. These are based on some assumed value for ϕ, λ (usually Φ, Λ) at the origin of survey and on distances reduced to the geoid. One can now evaluate the deflections of the vertical at stations in the network where Φ, Λ are observed. Then, using a technique known as "astro-geodetic levelling" (see Section 4.3.3) a geoid map is produced and if necessary, corrections applied to the distances to reduce them to the ellipsoid. Also a new geodetic coordinate may be adopted at the origin of survey, in order to minimise the magnitude of N (or ξ, η) through the survey.

This technique is known as "the development method" (refer to the COMPUTATION PHASE of Figure 1.1).

1.2.4 The Australian Geodetic Datum

A review of the evolution of the Australian Geodetic Datum gives a good insight into the practical application of the above technique (see MATHER and FRYER, 1970; and LAMBERT, 1977). Maurice Trigonometrical Station in South Australia was used as the preliminary origin to obtain a set of homogeneous geodetic coordinates for the network (using a = 6378 165 m, f = 1/298.3; $\phi = \Phi$ and $\lambda = \Lambda$ at Maurice). Comparisons were made between Φ, Λ and ϕ, λ at 150 stations spread through the continent and deflections derived. The mean value of this set of deflections was then applied as corrections to the ϕ, λ at Maurice T.S. and new geodetic values calculated throughout the network based on these revised coordinates. This was the status in 1963.

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* To achieve an accuracy of $1:10^6$, this assumption must be correct to ±6m. That is, if the geoid is no greater than 6m from the ellipsoid, distances on both the geoid and ellipsoid are equal to within 1 part per million.

Because of the adoption of a new reference ellipsoid by the International Astronomical Union in 1965, (viz. a = 6378160 m; f = 1/298.25), this was adopted as the Australian National Spheroid (A.N.S.) in 1967, and a determination of the "best" orientation of this ellipsoid carried out. A new origin was chosen (Johnston Geodetic Station), the number of stations at which Φ, Λ observed increased to 533, and the geodetic datum defined such that it was approximately parallel with the geoid across Australia (see Section 4.3.3). The figure adopted for the Australian National Spheroid oriented with respect to the geoid in this way is known as the Australian Geodetic Datum (A.G.D.).

The geodetic coordinates of Johnston Origin, and its other details can be seen in Figure 1.7. The relationship of the geoid to the Australian Geodetic Datum is illustrated in Figure 1.8

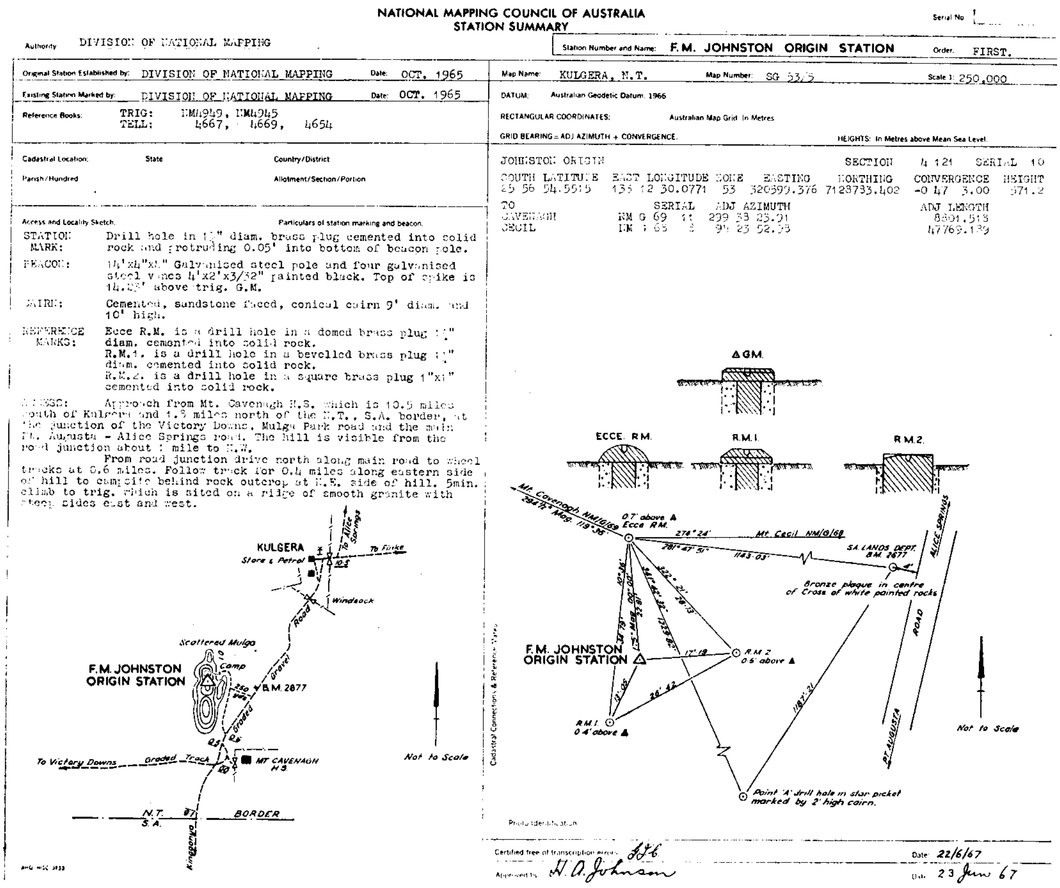


FIGURE 1.7: Details of Johnston Origin from LAMBERT, (1968), p.96.

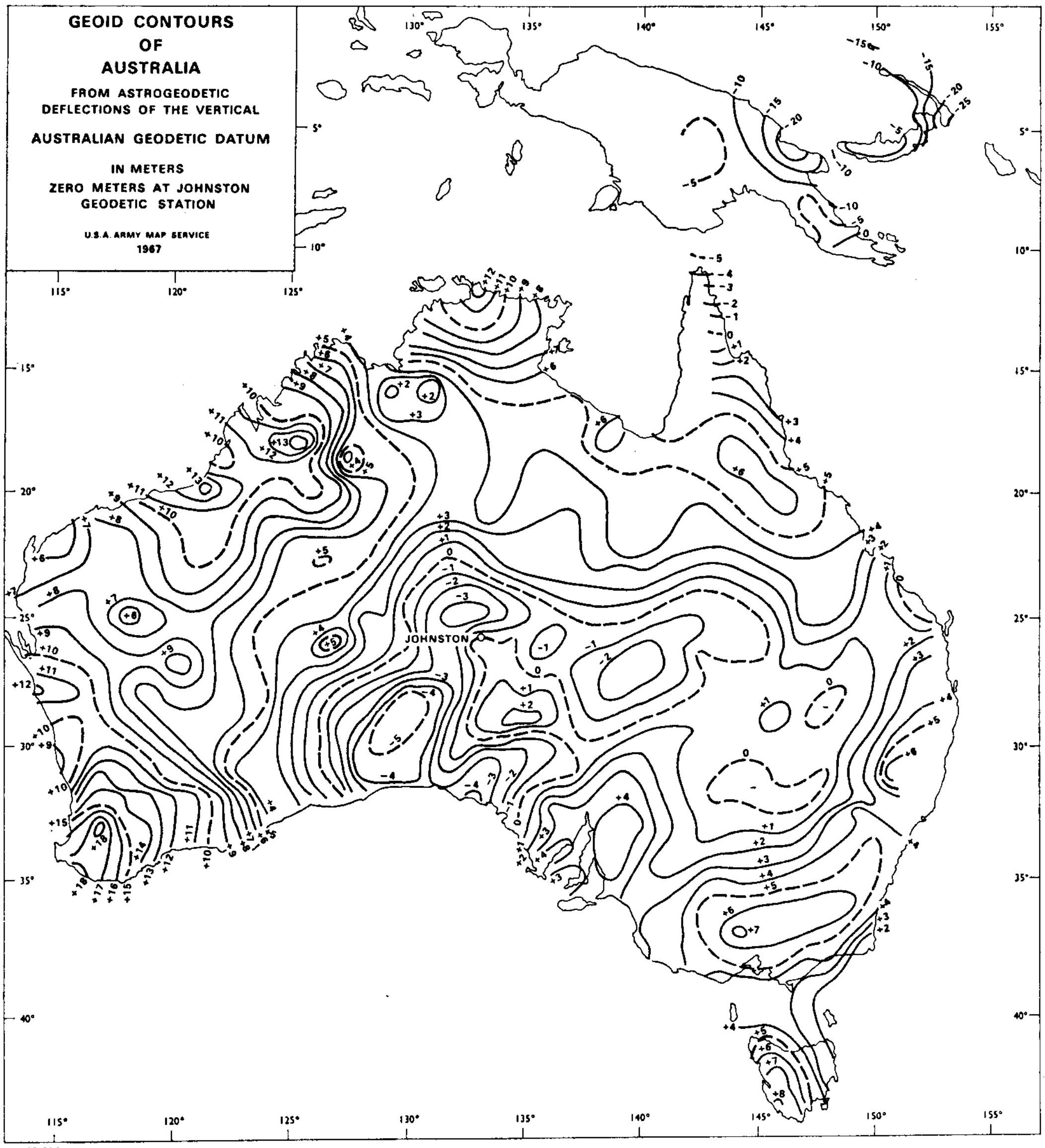


FIGURE 1.8: Geoid Contours on the Australian Geodetic Datum, in meters from FISCHER and SLUTZKY, (1967) p. 328.

1.2.5 Local and Global Datums

The constraints which were imposed in the selection of the A.G.D. means that it is essentially a **local datum**; it is useful for surveying and mapping in this small portion (2%) of the Earth's surface as the geoid-ellipsoid separation does not, on average, exceed 6m. The same technique has been used by many countries, as is illustrated in Table 1.1 and Figure 1.9. In each country, a particular ellipsoid has been oriented in space by choosing a geodetic latitude and longitude at a chosen origin, usually with the intention of fitting the model to the geoid of the region. This in turn (hopefully) eliminates the need to introduce a correction to observations reduced to the geoid to bring them to the ellipsoid.

This means that all parameters based upon this local ellipsoid (geodetic coordinates, deflections of the vertical, geoid undulations) are also essentially local and have no real relevance beyond the limits of the datum definition. If, however, it were possible to find an ellipsoid which was found to "best-fit" the geoid over the whole surface area of the Earth, this would produce an "absolute" definition of the datum. Such a datum is referred to as a **global datum**. The usefulness of such a datum is increased by the advent of position fixing methods based upon extra-terrestrial observations, e.g. position fixes by measurements from navigation satellites (see Section 2.1.2). Such positions are given with respect to a globally-based reference system, and it is simpler to relate these to existing geodetic networks if the latter, too, are referred to this reference system.

The most recent definition of a global ellipsoid was agreed upon at the meeting of the International Association of Geodesy in Canberra, December 1979. At this meeting it was decided to adopt a new set of parameters for Reference System, 1980 and the ellipsoidal elements of this were

$$a = 6\ 378137 \pm 1\ \text{m}, \quad f = 1/298.247 \text{ (MORITZ, 1979).}$$

An indication of the difference which the adoption of a global ellipsoid makes upon geoidal undulations, when compared to those based upon a local ellipsoid, may be seen by comparing Figure 1.10 with Figure 1.8. It can be seen that the geoid in the Australian region slopes evenly with respect to a global ellipsoid from about +70 m in the north east to about -25 m in the south west. This means that an assumption of coincidence (to ±6m) between this ellipsoid and the geoid is clearly wrong, and it is necessary to correct all distances onto the ellipsoid before computation upon this surface can be done.

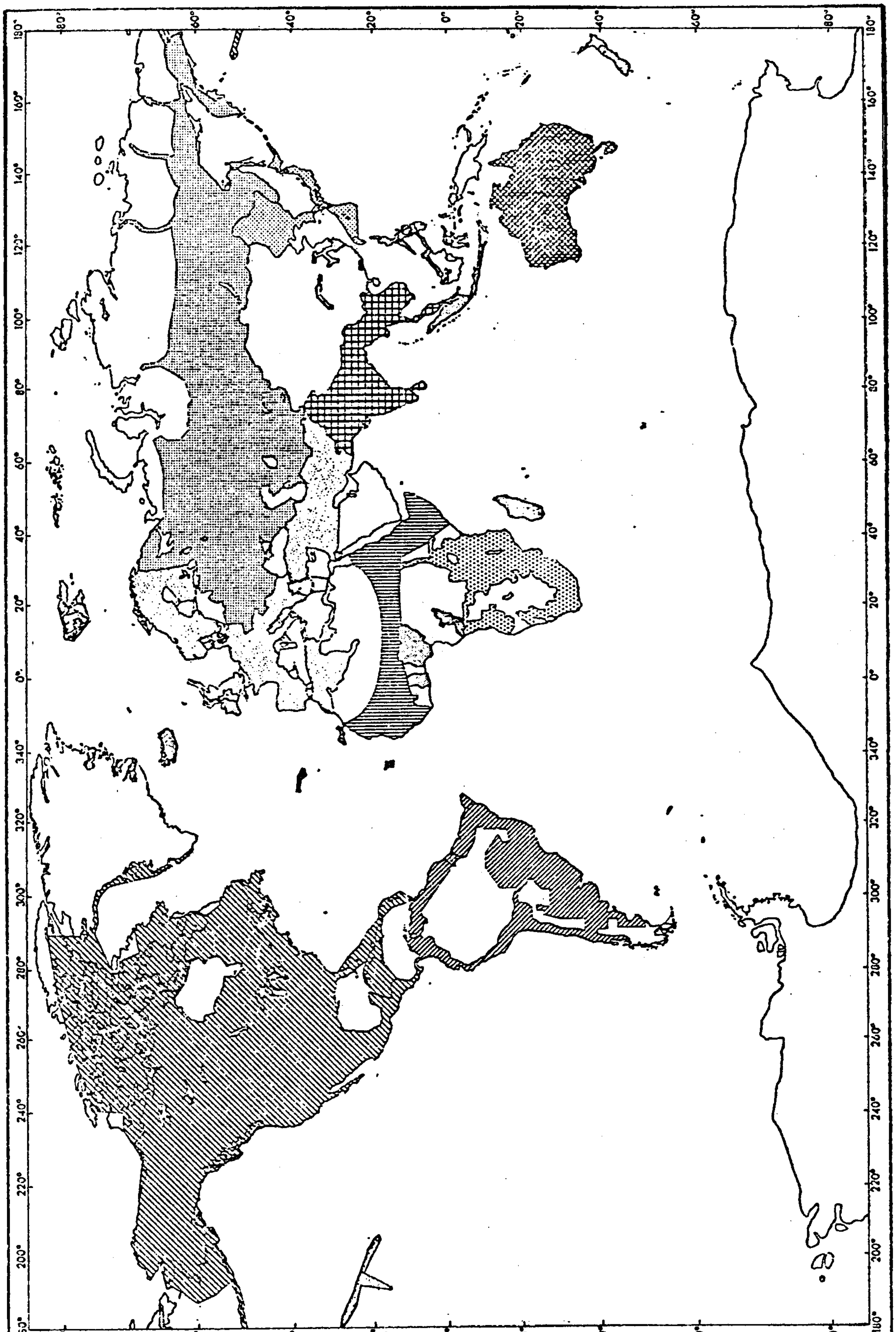


FIGURE 1.9: MAJOR GEODETIC DATUM BLOCKS from RAPP (1975).

TABLE 1.1: Reference Datums from RAPP (1975).

DATUM	SPHEROID	ORIGIN	LATITUDE	LONGITUDE (E)
Adindân	Clarke 1880	STATION Z_x	22°10'07."110	31°29'21."608
American Samoa 1962	Clarke 1866	BETTY 13 ECC	-14 20 08.341	189 17 07.750
Arc-Cape (South Africa)	Clarke 1880	Buffelsfontein	-33 59 32.000	25 30 44.622
Argentine	International	Campo Inchauspe	-35 58 17	297 49 48
Ascension Island 1958	International	Mean of three stations	-07 57	345 37
Australian Geodetic	Australian National	Johnston Geodetic Station	-25 56 54.55	133 12 30.08
Bermuda 1957	Clarke 1866	FT. GEORGE B 1937	32 22 44.360	295 19 01.890
Berne 1898	Bessel	Berne Observatory	46 57 08.660	07 26 22.335
Betio Island, 1966	International	1966 SECOR ASTRO	01 21 42.03	172 55 47.90
Camp Area Astro 1961-62 USGS	International	CAMP AREA ASTRO	-77 50 52.521	166 40 13.753
Canton Astro 1966	International	1966 CANTON SECOR ASTRO	-02 46 28.99	188 16 43.47
Cape Canaveral*	Clarke 1866	CENTRAL	28 29 32.364	279 25 21.230
Christmas Island Astro 1967	International	SAT.TRI.STA. 059 RM3	02 00 35.91	202 35 21.82
Chua Astro (Brazil-Geodetic)	International	CHUA	-19 45 41.16	311 53 52.44
Corrego Alegre (Brazil-Mapping)	International	CORREGO ALEGRE	-19 50 15.140	311 02 17.250
Easter Island 1967 Astro	International	SATRIG RM No. 1	-27 10 39.95	250 34 16.81
Efate (New Hebrides)	International	BELLE VUE IGN	-17 44 17.400	168 20 33.250
European (Europe 50)	International	Helmertturm	52 22 51.45	13 03 58.74
Graciosa Island (Azores)	International	SW BASE	39 03 54.934	331 57 36.118
Gizo, Provisional DOS	International	GUX 1	-09 27 05.272	159 58 31.752
Guam	Clarke 1866	TOGCHA LEE NO. 7	13 22 38.49	144 45 51.56
Heard Astro 1969	International	INTSATRIG 0044 ASTRO	-53 01 11.68	73 23 22.64
Iben Astro, Navy 1947 (Truk)	Clarke 1866	IBEN ASTRO	07 29 13.05	151 49 44.42
Indian	Everest	Kalianpur	24 07 11.26	77 39 17.57
Isla Socorro Astro	Clarke 1866	Station 038	18 43 44.93	249 02 39.28
Johnston Island 1961	International	JOHNSTON ISLAND 1961	16 44 49.729	190 29 04.781
Kourou (French Guiana)	International	POINT FONDAMENTAL	-05 15 53.699	-52 48 09.149
Kusaie, Astro 1962, 1965	International	ALLEN SODANO LIGHT	05 21 48.80	162 58 03.28
Luzon 1911 (Philippines)	Clarke 1866	BALANCAN	13 33 41.000	121 52 03.000
Midway Astro 1961	International	MIDWAY ASTRO 1961	28 11 34.50	182 36 24.28
New Zealand 1949	International	PAPATAHI	-41 19 08.900	175 02 51.000
North American 1927	Clarke 1866	MEADES RANCH	39 13 26.686	261 27 29.494
Old Bavarian	Bessel	Munich	48 08 20.000	11 34 26.483
Old Hawaiian	Clarke 1866	OAHU WEST BASE	21 18 13.89	202 09 04.21
Ordnance Survey G.B. 1936	Airy	Herstmonceux	50 51 55.271	00 20 45.882
OSGB 1970 (SN)	Airy	Herstmonceux	50 51 55.271	00 20 45.882
Palmer Astro 1969 (Antarctica)	International	ISTS 050	-64 46 35.71	295 56 39.53
Pico de las Nieves (Canaries)	International	PICO DE LAS NIEVES	27 57 41.273	344 25 49.476
Pitcairn Island Astro	International	PITCAIRN ASTRO 1967	-25 04 06.97	229 53 12.17
Potsdam	Bessel	Helmertturm	52 22 53.954	13 04 01.153
Provisional S. American 1956	International	LA CANOA	08 34 17.17	296 08 25.12
Provisional S. Chile 1963	International	HITO XVIII	-53 57 07.76	291 23 28.76
Pulkovo 1942	Krassovski	Pulkovo Observatory	59 46 18.55	30 19 42.09
Qornoq (Greenland)	International	No. 7008		
South American 1969	South American 1969	CHUA	-19 45 41.653	311 53 55.936
Southeast Island (Mahe)	Clarke 1880		-04 40 39.460	55 32 00.166
South Georgia Astro	International	ISTS 061 ASTRO POINT 1968	-54 16 38.93	323 30 43.97
Swallow Islands (Solomons)	International	1966 SECOR ASTRO	-10 18 21.42	166 17 56.79
Tananarive	International	Tananarive Observatory	-18 55 02.10	47 33 06.75
Tokyo	Bessel	Tokyo Observatory (old)	35 39 17.51	139 44 40.50
Tristan Astro 1968	International	INTSATRIG 069 RM No. 2	-37 03 26.79	347 40 53.21
Viti Levu 1916 (Fiji)	Clarke 1880	MONAVATU (latitude only)	-17 53 28.285	
		SUVA (longitude only)		178 25 35.835
Wake Island, Astronomic 1952	International	ASTRO 1952	19 17 19.991	166 38 46.294
White Sands*	Clarke 1866	KENT 1909	32 30 27.079	253 31 01.306
Yof Astro 1967 (Dakar)	Clarke 1880	YOF ASTRO 1967	14 44 41.62	342 30 52.98

* Local datums of special purpose, based on NAD 1927 values for the origin stations.

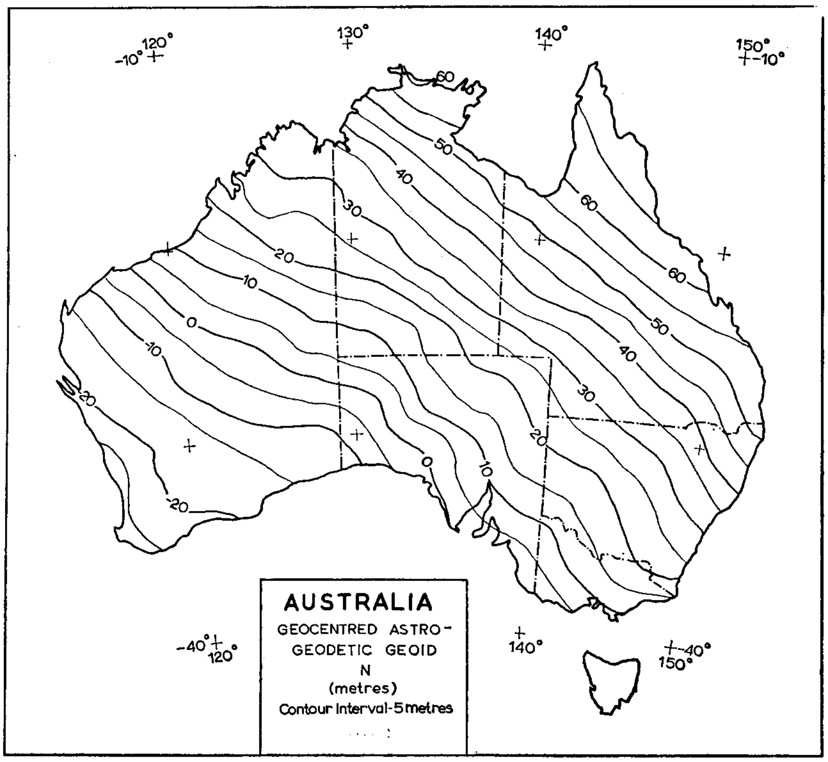


FIGURE 1.10: Australian Geoid Mapped with respect to a geocentric Ellipsoid, (MATHER and FRYER, 1970).

2. PLANNING THE GEODETIC SURVEY

2.1 Types of Survey used for Horizontal Control

2.1.1 Terrestrial Surveys

(a) Triangulation

Before the early 1950's the main survey measurement capable of geodetic accuracy over long distances were angular measurements. This meant that the main framework of a geodetic survey was nearly always a pure "triangulation" network (i.e. only angles were observed). Traverses (using flexible invar wires) were only considered where the topography rendered triangulation unsuitable.

(b) Traverse

The traverse became a feasible proposition with the advent in the 1950s of Electronic Distance Measuring devices (EDM), such as the Geodimeter and the Tellurometer. This coincided with the increased involvement by Australia in geodetic surveying and much of inland Australia has been surveyed by this method.

(c) Trilateration

The development of EDM also made possible the use of a "trilateration" technique, where lengths of sides of the geodetic figures were measured, but not angles. This proved especially useful in areas of low population, where control stations were widely separated and not intervisible (say 150 km).

(d) Combination of Triangulation and Trilateration

This is often called triangulation now. The combination of the two methods combines the advantages of both - the accumulation of scale (distance) error is kept under control by distance measurement and the "strength" of the figure is maintained by the angular measurement.

2.1.2 Accuracies of Geodetic Surveys

(a) Triangulation

As triangulation was originally the technique used to extend control, it is natural that it should have influenced the accuracy standards of the later techniques. Indeed, since it was necessary to match the accuracy of distance measurement with that of triangulation, it was natural that the standards of accuracy of geodetic triangulation provide the specifications for the early EDM equipment in its developmental stage.

Table 2.1 gives a table of specifications for classical primary and lower order triangulation.

TABLE 2.1: Specifications for Geodetic Surveys

Order	Triangle Sides	Triangle Miscloses	Relative Accurate	Base Length	Base Frequency	Astro Azimuth Frequency
Primary	15-150km	Av. 1" Max. 3"	1/40,000 to 1/150,000	5-30km	≅400 km apart	≅ 250 km apart
Secondary	10-40km	Max. 5"	1/15,000 to 1/35,000	Based upon Primary Network		
Tertiary	1-10km	Max. 15"	1/3,000 to 1/15,000	" " " "		

Normally, Secondary and Tertiary triangulations are based on sides in the Primary net, so that no additional bases or azimuths are required. However in a small country the most accurate survey required may be only of Secondary standard. In such a case a proportionally shorter baseline, measured to less rigorous standards than first order, could be used.

N.B.: Of these classifications only Primary is properly considered a Geodetic Survey. Also, with the development of position fixing from satellites, the role of astronomy in providing external control on the geodetic survey has been somewhat diminished.

It should be noted too that the statement of distance accuracy infers a relative accuracy between the two terminal stations of between $1:10^5$ to $2:10^6$.

(b) Traverse

A traverse is only considered to be of Primary standard if the traverse distances can all be measured with standard errors of between 1/40,000 and 1/150,000 and if the azimuths of the traverse lines are controlled so as to have standard errors no larger than those found in a properly adjusted Primary triangulation. This requires very frequent Laplace azimuth stations. If lengths are being measured by EDM (Tellurometer) and are of about 30-40 km, then Laplace azimuths at alternate stations are desirable (see Section 3.3.2).

(c) Trilateration

Conventional triangles, such as those mentioned in the above table, can be measured to the required precision with the EDM equipment currently available. However, because of the large uncertainty inherent in radar measurements, radar trilateration is of geodetic accuracy only if lines are long enough i.e. 250-700 km. With shorter lines the uncertainties of measurement due to large wave lengths used are greater than the permissible relative errors in primary work.

2.1.3 Extraterrestrial Surveys

(a) Satellite Techniques

With equipment which measures the Doppler shift of a radio signal of known frequency, it is possible to fix a 3-dimensional position to about ±1 m or better in a geocentric reference framework. This can be transformed into geodetic ϕ, λ and h referred to a geocentric ellipsoid by simple mathematical expressions (TORGE, 1980, p. 52). Geodetic coordinates of stations in the network will be calculated on a local ellipsoid. Comparisons of the geodetic position values from these two sources at a number of common stations throughout the network will provide the data needed to find the relation of the local reference system to the geocentric reference system. Once these transformation parameters have been unambiguously determined, positions from Doppler receivers can be converted to the local system and then, used in extension of control in the local system.

Obviously over short (20 km) lines this technique will not provide an adequate accuracy (in 20 km a positional uncertainty in both terminals of 1m results in a proportional accuracy of about 1:10 000). Over distances of the order of 200 km, proportional accuracy of geodetic standards are achievable and this technique is now being used widely to provide external control for geodetic surveys.

(b) Astronomical Techniques

Using instruments such as the Wild T3 or the DKM 3A, and taking observations over several nights, it is possible to establish the astronomical latitude and longitude (Φ, Λ) to about ±0.3", and ±0.6" respectively. This is equivalent to about ±10 m in Φ, and ±20 m in Λ; but cannot be used for external control on geodetic networks because the position relates to the local vertical, and not the normal to which the geodetic positions will refer.

However it is possible to establish the astronomical azimuth (A) of a line in the network to about ±0.6". If the deflection of the vertical in the prime vertical is also established at the observation point (Equation 1.2), the discrepancy between the local astronomical meridian and the local geodetic meridian can be found, and the azimuth derived after correction for this discrepancy used as control on the "skew" or "swing" of the network.

Stations where such observations are made are known as Laplace Stations. See Section 3.2.3 for a detailed discussion.

2.2 Designing the Network

There are two factors which will influence the design of the network. These are mainly

(i) Physical - relating to the nature of the terrain over which the control network is to be placed, and

(ii) Theoretical - whereby the shapes of the figures chosen for the network are analysed for their strength.

2.2.1 Physical Considerations

Obviously the type of survey adopted, and the location of the geodetic stations is influenced by the presence or otherwise of mountains which will allow lines of sufficient length to be observed. In small countries such as Great Britain or Denmark, which have hills spread through the region it is possible to construct a continuous triangulation network covering the entire country.

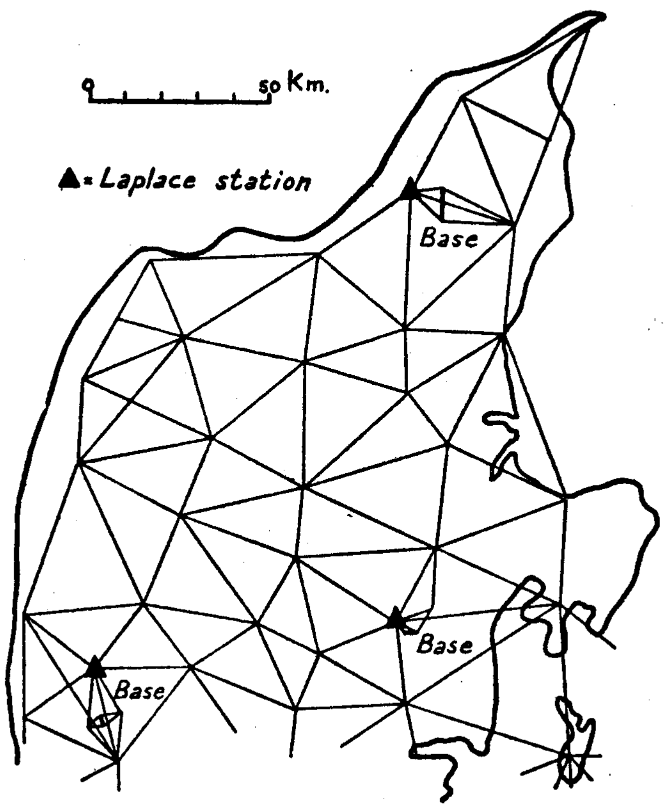


FIGURE 2.1: Triangulation Network of Denmark

In larger countries (e.g. India), with mountain ranges running the length and breadth of the country, a series of interlocking triangulation chains may be more suitable (see Figure 2.2).

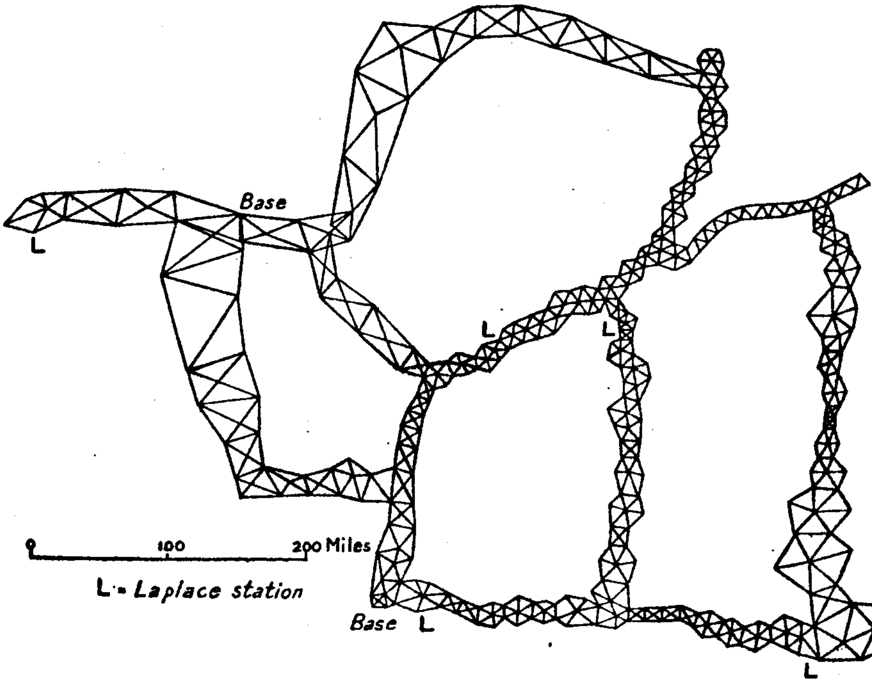


FIGURE 2.2: Part of the Triangulation of India

For countries with vast areas with little or no terrain, (e.g. Australia, Canada, U.S.A.) it may only be feasible to run traverses along the main roads, to form an interlocking series of loops of traverses (see Figure 2.3).

Note:- It will not be possible to start locating the geodetic stations without reliable maps, or a thorough reconnaissance survey. This consideration may override the two steps of the next phase of the work (Design and Optimisation, and Station Building) which will usually have to await the results of the reconnaissance survey.

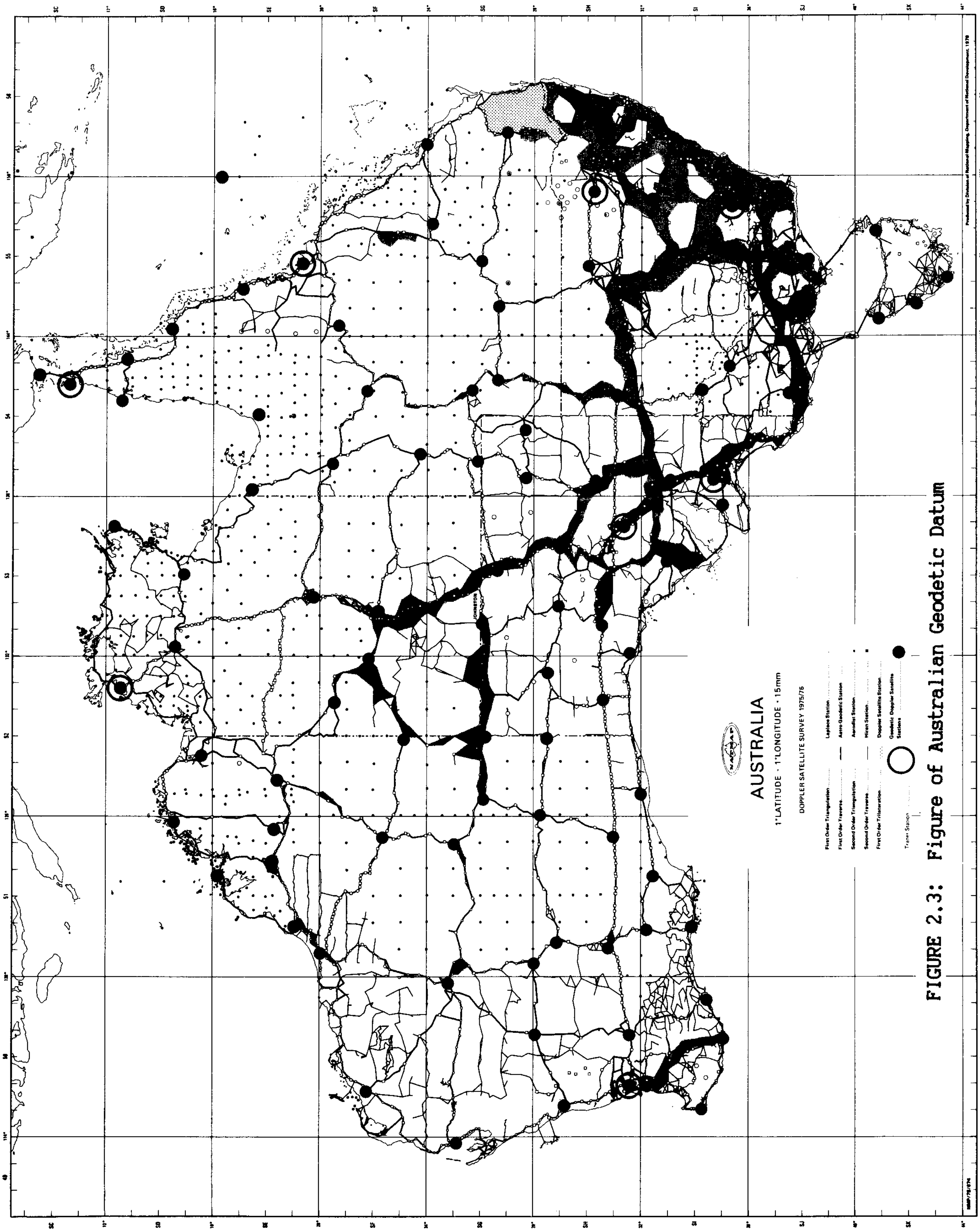


FIGURE 2.3: Figure of Australian Geodetic Datum

2.2.2 Structural Considerations

The aim of the survey is to provide a network of "well-conditioned" figures, i.e. figures, which, by dint of their shape and orientation with respect to the network, minimise the propagation of errors through the network. The network can, indeed, be thought of as an engineering structure, and the analogy between surveying adjustment problems and elastic structural analysis is a very close one (see WHITE and BETTS, 1968).

Over a period of time, three basic figures have evolved whose characteristics are well understood. They are the

(i) simple triangle (preferably equilateral)

(ii) braced quadrilateral (preferably square), and

(iii) centred polygons (see Figure 2.4).

They are often used as the basic figures, in combination with themselves or each other, in setting up a network or chain. Obviously it is necessary, for the network/chain to be continuous, for adjoining figures should have at least one common side.

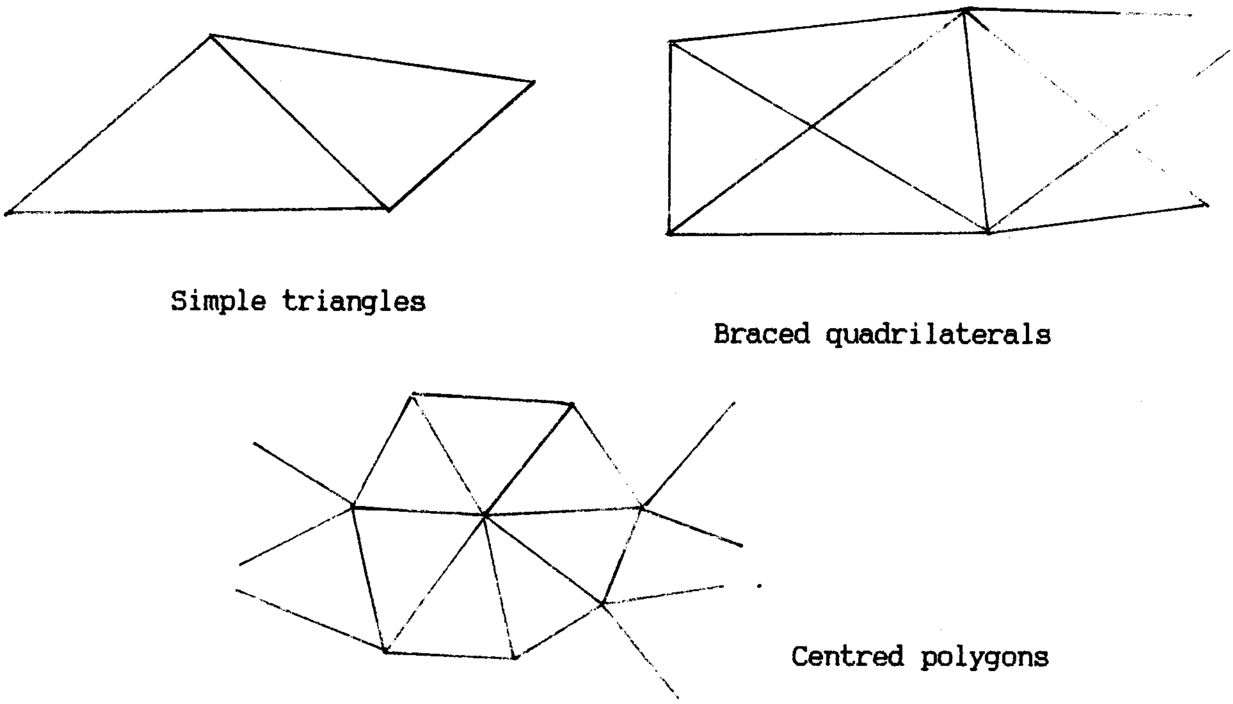


FIGURE 2.4: Common Figures of a Geodetic Network

We shall now consider the relative merits of these three alternatives, in terms of speed, accuracy, and area covered.

A fair estimate of speed will be the number of stations required for a certain length of chain. The controlling factor is likely to be the length of line that can be observed, and this will be assumed a constant length L. The accuracy of a layout will depend on the shape of the figures, and the number of conditions that must be satisfied.

Figure 2.5(a) shows a system of simple equilateral triangles with side length L. Every new station after the first adds a length L/2 to the chain; if there are S stations, the length of chain is the (S-1)L/2. There are (S-2) triangles. The number of conditions to be satisfied is one angle condition per triangle, so there are then (S-2) conditions. The area of each triangle is $\sqrt{3}/4\ L^2$; total area covered is then $0.43L^2(S-2)$.

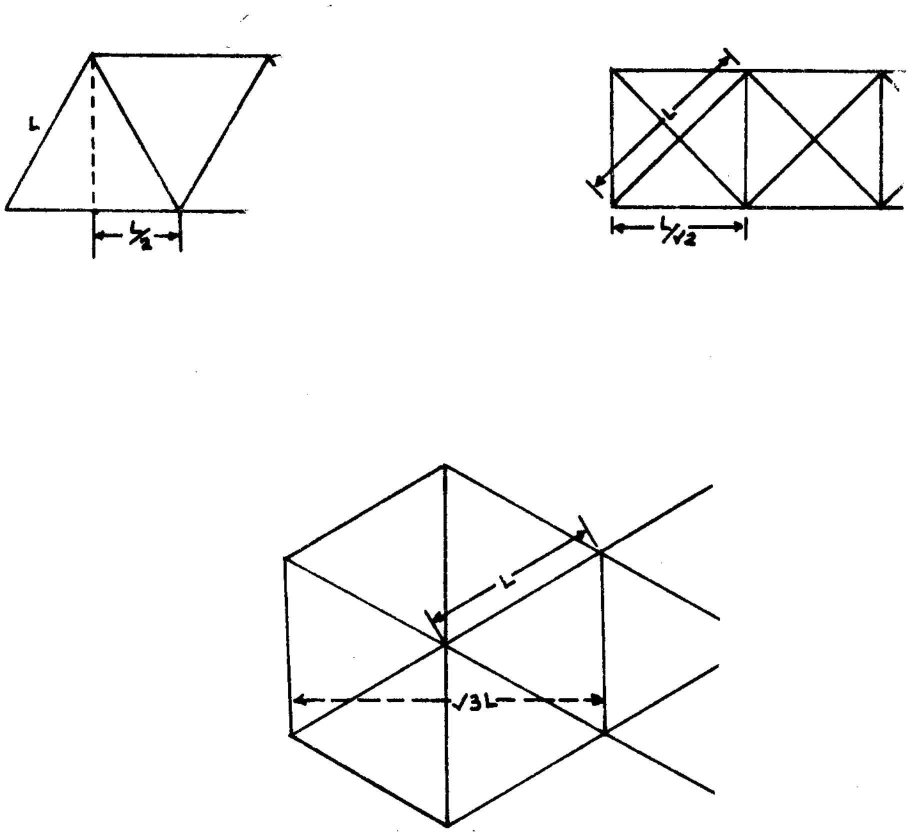


FIGURE 2.5: Basic Units with Longest Line of Sight 'L'.

Figure 2.5(b) shows a regular system of squares, with diagonals observed. The limiting factor is the length of the diagonals; if the diagonal is L, the side length is $L/\sqrt{2}$. Every pair of stations after the first 2 gives a new square, of which there will be (S-2)/2, of total area $(S-2)/2 \times L^2/2$. The number of conditions will be one side condition and three angle conditions per square; (three, not four, angle conditions, for only three are independent; if three are correct, the fourth follows automatically). The total number of conditions will be 4 x (S-2)/2, or 2(S-2).

Figure 2.5(c) shows regular hexagons; every five stations after the first pair give a new figure, of which there will be (S-2)/5; the length of each figure in the direction of the chain is $\sqrt{3}$ L, so the distance covered is $\sqrt{3}\ L \times (S-2)/5$. The number of conditions per figure is six independent angle conditions, and one side condition; the condition that the angles at the centre of the polygon add to 360° is not strictly an angle condition, but a "local condition"; it must be satisfied if the round of angles taken at the control point closes, and need not be considered. The area covered by each figure is six times the area of one triangle, or $6 \times \sqrt{3}/4\ L^2$; the total area is then $3\sqrt{3}/10\ L^2\ (S-2)$.

Tabulating these results, we have

	Distance	Conditions	Area
Triangles	(S-1)L/2	(S-2)	$.43\ L^2(S-2)$
Squares	.35 (S-2)L	2(S-2)	$.25\ L^2(S-2)$
Hexagons	.35 (S-2)L	1.4(S-2)	$.52\ L^2(S-2)$

For a chain of 22 stations, this given

	Distance	Conditions	Area
Triangles	10.5 L	20	$8.6\ L^2$
Squares	7 L	40	$5.0\ L^2$
Hexagons	7 L	28	$10.4\ L^2$

The deduction therefore is that triangles give most rapid progress; that squares with diagonals give best accuracy, and that hexagons cover the greatest area. It is generally preferred in a geodetic chain to use a system of squares or hexagons, so that computation is possible by two independent routes. In hilly country, the visibility will not restrict the observation of the diagonals of a square, and this layout may then be preferred; in flat country, hexagons would be better.

The choice of a figure depends not only on the type of figure but also on the shapes of the triangles composing it. We refer to triangles being "well-conditioned" if they are of a shape which gives only a small increase in error of a computed side. In general this requires the angles used in side computation across the figure to be no smaller than 30° and no larger than 150°, and the closer they can be made to 90° the better.

For the given direction of progress of the chain, the error propagation across 2.6(b) will be slower than across 2.6(a) because of the better sized angles used in the length transfer across the figure. Even though the triangles used in both are the same shape, in (a) they are not as well-conditioned as they are in (b), because of their orientation with respect to the chain.

Direction of progress of Chain

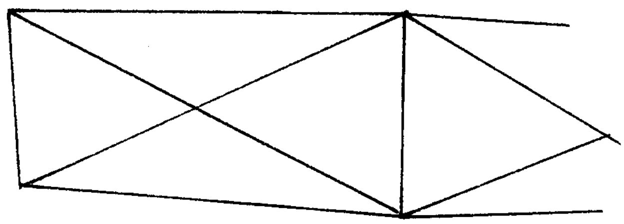


FIGURE 2.6(a)

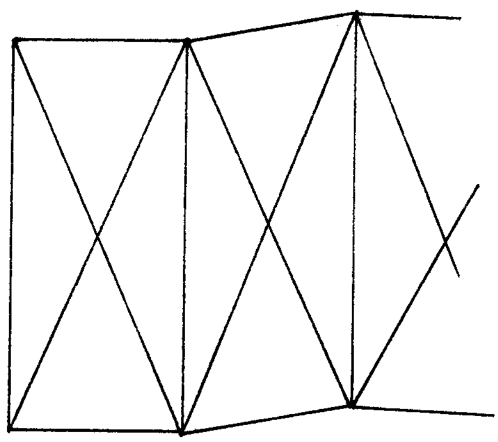


FIGURE 2.6(b)

2.2.3 Optimising the Network

After the network has been designed it is possible to estimate its error characteristics, based upon assumptions of the errors in the observed elements. This will indicate what weaknesses are present in the network, and enable the network to be "strengthened" (by using more accurate measurements, or incorporating, say, a distance where it was not previously thought necessary). At this stage, therefore, the surveying authority should be able to guarantee that the proposed network will fulfill the desired accuracy criteria (from Section 2.1.2).

The job has now reached the stage where the design can be put into effect: the geodetic stations can be placed and the observing phase commenced.

3. NETWORK OBSERVATION

3.1 Creating the Network

3.1.1 Reconnaissance Surveys

BOMFORD (1971) mentions three forms of reconnaissance, which he implies are alternatives. These consist of:-

(a) Examination of existing maps (if any)

This may enable a "paper" layout of a possible chain, or a number of alternative chains. In suitable country (i.e. very hilly or moderately mountainous) he claims a scheme may be set out with 95% certainty. Personal experience, admittedly in relatively poor triangulation country, indicates that the paper layout is only a first try, and requires careful examination of the ground. It should be remembered, that, if a geodetic survey is not in the area, maps may not be of sufficient accuracy for this task.

A useful formula in determining intervisibility is

$$h = h_A + (h_B - h_A)\frac{d_A}{d_A + d_B} - 0.066\, d_A d_B \text{ metres} \qquad (3.1)$$

where h_A, h_B are the heights of the two stations, A & B, in metres

h is the height of the ray of light between them distance d_A km from A, d_B km from B.

Example:

Reduced Level of Inst. Axis at A is 500 metres

" " of Beacon at B is 400 metres

AB is 30 kilometres

C is 10 kilometres from A, and its R.L. is 460 metres

(i) Can the line AB be observed directly?

$$h = 500 - 100 \times \frac{10}{30} - .066 \times 10 \times 20$$

$$= 500 - 33.3 - 13.2$$

$$= 453.5$$

Answer: NO!

(ii) To what R.L. must the beacon at B be lifted so that the line of sight from A will clear C by 3 metres.

This is done by direct proportion.
i.e. 10 km from A, line of sight is to be lifted 9.5 metres.
30 km " A, " " " " " " " 28.5 metres.
.'. Required R.L. of Beacon at B will be 428.5 metres.

(This can be checked by substituting back into Equation 3.1 since that the new value of h is in fact 463 m).

(b) **Aerial Reconnaissance.** This has been practised in Canada. Helicopters were used for similar purposes in Australia in the middle 1950's and proved extremely useful. A special plane table is mounted in the aircraft which is flown on a straight line marked on the table. The position of the aircraft is estimated from the map and rays drawn by alidade to hills thought to be possible stations. Intervisibility is checked by low level runs.

(c) **Ground Reconnaissance.** This is the most tedious method, but probably the one with the highest chance of being correct. All prospective stations are visited, and intervisibility physically proven. It has the added advantage that feasibility of routes to prospective stations, location of water, fuel and sources of supply of food and other requisites are known before the actual measuring phase of the survey commences.

The objectives of the reconnaissance are to choose suitable stations forming a well conditioned triangulation chain, such that it is physically possible to make the required measurements (i.e. that stations are intervisible either from ground level or by using suitable observing towers). It includes the choice of bases and extension nets (although these will almost certainly not be required in modern triangulation).

3.1.2 Station Building

This phase includes the marking and permanent beaconing of the geodetic stations.

The essentials are

(a) A distinctive, and as far as possible, indestructible, mark at a below ground level. It may be a stainless steel or brass mark driven into rock, or set in concrete poured "in situ". A mark cut on rock is sometimes a suitable substitute.

(b) Provision of two or three reference, or witness marks. These should be of a suitably durable nature. The bearings and distances of each from the station mark, and from each other, and their relative heights, should be carefully measured (see Figure 1.7).

(c) If required, a beacon should be erected. This may be a metal, wooden or PVC pole carrying vanes. For stations which will be infrequently occupied, the pole may be supported by a cairn of rocks. In such cases, the observations are often made from a satellite station, usually one of the witness marks.

For stations which will be frequently occupied (those in areas of future closer development) special beacons consisting of a short pole and vanes supported by a concrete pillar (see Figure 3.1) or a tripod or quadripod structure may be used. A theodolite may be screwed onto the pillar in the former case, thus gaining the advantages of a constrained centring system, but unfortunately removing the ability to sight to the mast and vanes. The latter system allows the retention of the geodetic beacon whilst observing at the station, but of course does not have the advantage of constrained centring.

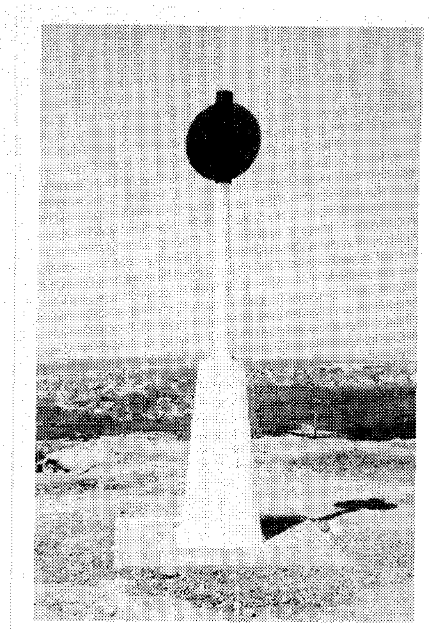


FIGURE 3.1: Observing Pillar and Beacon

The station marking phase of the operation is normally performed by parties of field hands under the control of an experienced piling overseer. These parties would usually also have to clear "lanes" of any timber which may obstruct lines of sight to adjacent geodetic stations.

A very good insight into the logistics and manpower involved in setting up and observing the geodetic network for Australia can be gained by reading JOHNSTON (1964) and FORD (1979).

3.2 Triangulation

3.2.1 Baselines and Base Extension Net

Before the advent of EDM, baselines were measured to an accuracy of 1:5 x 10^5 to 1:1 x 10^6 by means of bars, tapes or wires. This was an extremely tedious, painstaking and highly expensive undertaking. For this reason, baselines measured prior to about 1950 were generally considerably shorter than the primary triangulation side. A very careful triangulation network was employed which worked from the baseline through a series of triangles graded in size until a length approaching that of a primary side was reached. This side was then used in the first figure of the triangulation chain proper. This is called a Base Extension Net.

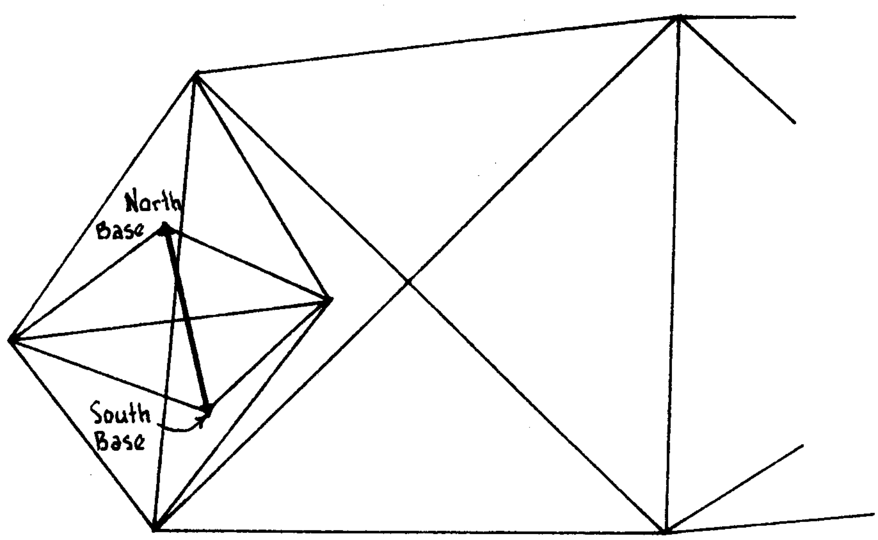


FIGURE 3.2: Baseline Extension Network

An interesting insight into the use of baselines to introduce and control scale in early triangulation can be found in FURBER (1898).

The first baseline to be measured was at Lake George (near Canberra). Measurement started in 1868, but because of an abnormal rise in the lake this work was abandoned and a new site located in 1870. The forward and backward measurement of this new baseline was completed in 1872, the difference between the two measurements being 0.542 inches (1cm). The total length of the baseline was 5.5 miles (nearly 9km), giving a proportional error of 1:644 000. This stands as a tribute to the care and professionalism of those early surveyors, especially when one recognises that this distance was not measured by EDM or flexible invar bands,but by laying end-to-end three standardised wooden rods, supported by trestles and insulated from variations in temperature. Each pine rod was only 10ft (3m) long.

The second baseline was located at Richmond, 50 km west of Sydney. It was measured with similar apparatus, using the pine rods for the forward measurement and three standardised steel rods for the check measurement.

The comparison between the two measurements showed a difference of 0.662 inches (1.7 cm), which over the baseline length of 7 miles (> 11km) gives a proportional error of about 1:670 000.

The comparison of the distance carried through the triangulation from Lake George to Richmond is impressive. Let the report speak for itself (IBID, p.181).

> "The combined errors of the two bases and of the intervening triangulation produced an aparrent discrepancy of only **one and two third inches** (4 cm) in the length of the Lake George base. The bases were assumed to be correct and an adjustment of the triangles was made made in order to eliminate this small, apparent difference."

3.2.2 The Observing Routine

Direction observations may be subject to systematic errors as a result of

(i) drag - a change in the absolute orientation of the theodolite circle in the direction of the rotation of the instrument about its vertical axis;

(ii) twist - a change in the absolute orientation of the circle which is independent of the amount or direction of rotation of the theodolite;

(iii) circle graduation errors - a cyclic error in the graduation of the theodolite circle graduations;

(iv) micrometer graduation errors - errors in the graduation of the micrometer scale;

(v) residual instrumental errors, such as non-verticality of the vertical axis.

To minimise the effects of these systematic errors a special observing routine is constructed. The details of this routine vary, and it will be necessary to refer to the instruction manual for each particular authority for details of their observing procedure. An example of such a routine is given below.

We must first of all decide on the number of arcs (n) to be measured (see below). This we can do by studying the random errors of pointing, reading and circle graduation (as distinct from cyclic errors of graduation).

(a) Point to Target 1 in F.L. Set micrometer reading to about zero and set circle reading to about 0°00′.

(b) Unclamp and rotate telescope through 360° in clockwise direction.Sight to target 1, make final pointing with tangent screw approaching from left, read and record.

(c) Swing clockwise to target 2, intersect approaching always from left, read and record.

Continue swinging clockwise through full set of targets until last target has been sighted, read and recorded.

(d) Change face, turn theodolite anticlockwise through about 540°, point to last target approaching from right, read and record.

(e) Swing anticlockwise to second last target, approaching from right, read and record. Continue anticlockwise through targets till first target has been sighted, read and recorded.

(f) Change face, with telescope pointed to first target, increase micrometer reading by about $\frac{r}{n}$, and increase circle reading by $\frac{180°}{n}$, where r is the range (run) of the micrometer and n the number of arcs.

(g) Repeat steps (b) to (f) until the n arcs have been read. If any steep sights are included, the striding level may be used. Observations will include striding level readings.

(a) Choice of "n" for 1st Order Surveys: a simplified analysis.

One of the criteria for 1st Order Triangulation is that errors in triangle angle sums should average < 1", with a maximum of 3" (see Section 2.1.2). RAINSFORD (1959) has shown that the errors in triangle angle sums, for a large number of triangles, approximates fairly well the normal distribution. If we accept this, it seems reasonable, since errors in triangle closures of greater than 3" are rarely encountered, to assume 3" to represent the 99% Confidence Interval, i.e. that 3" = $3\sigma_T$ where σ_T is the Standard Error of a triangle closure.

i.e. $\sigma_T = 1''$

This means that, of a large number of triangles, 68% would have errors of 1" or less, which would seem to be reasonable if the mean misclose is not to exceed 1" (it can be shown that this should give a mean misclose of about 0.8").

Each triangle contains 3 angles, each of which is defined by the difference between mean of two observed directions.

$$\therefore \quad \sigma_T^2 = 6\,\sigma_m^2$$

where σ_m is the standard error of the mean of 2n observations of a direction (n arcs, 2 semi-arcs in each).

$$\text{i.e.} \quad \sigma^2_m = \frac{\sigma_T^2}{6} = \frac{1}{6}$$

Let σ_D be the Standard Error of a single observation of a direction (estimated by RICHARDUS, (1965) as 1.5" for a Wild T3 or similar instrument).

Then

$$\sigma_D^2 = 2n. \sigma^2_m$$

$$2.25 = 2n. \frac{1}{6}$$

$$\therefore n = 6.75$$

The most convenient value for n is probably 8. This is a widely used value. Many survey departments, at least in Australia, have for many years made a practice of observing 32-36 arcs, usually spread over two observing sessions. This seems unreasonably high, particularly when we consider that our estimate of σ_T = 1" is probably low by about 25%, and Richardus' estimate of σ_D is considered by many authorities to be very conservative.

(For σ_T = 1.25", σ_D = 1", n = 2!!! However, we need n > 4 for reduction of cyclic graduation error).

(b) Recording, Reducing and Testing of Internal Precision

Methods for recording, reducing and testing the observations will vary depending largely upon the observing technique and will differ from department to department. One particular method which has been followed fairly widely in practice, is described by RICHARDUS (1965, p.167) and the reader is referred to this. The outcome of the reductions will be a set of directions referred to zero on a reference object, along with a statement of their estimated precision.

3.2.3 Laplace Observations for Azimuth Control

(a) The Need for Azimuth Control

In a triangulation network (or any system which depends heavily upon terrestrial observations for the evaluation of the network, e.g. traverse), it is necessary to introduce some control on the azimuth component of the network. Systematic errors can creep into the most tightly observed network, especially if there is any likelihood of horizontal refraction. The most spectacular example of this phenomenom has been documented by JOHNSTON (1962) where the mean directions of sets observed in the region of the Great Australian Bight differed by up to 8", with the variation in one set increasing from 42.82" to 55.53" over one hour's observations.

Such large variations rendered the observations useless for geodetic purposes, and special observing routines had to be devised to counter the effect of refraction. However, this example shows that horizontal refraction can certainly occur, and if present even in a minute way can cause the triangle chain to "swing" or "skew" upon accumulation.

To keep this effect within acceptable limits it is traditional to observe an astronomical azimuth at about 250 km intervals in the network (see Section 2.1.3). Stations where this is done are called **Laplace Stations.**

One must immediately recognise that it is not possible to directly compare the azimuth evaluated by astronomy with that being carried through the network. The former refer to a meridian defined by the local vertical (the vertical axis to the theodolite); the latter to the geodetic meridian which would be derived from the geodetic longitude computed through the network.

Before a comparison can be made, therefore, it is necessary to establish the difference between the local astronomical meridian and the geodetic meridian. This is done by observing the astronomical **longitude** of the Laplace station. Comparing this with the geodetic longitude computed through the network yields a deflection of the vertical in the prime vertical (η - see Figure 3.3). This in turn gives the discrepancy between the meridians at the horizon ($M_A M_G$). The astronomical azimuth can now be corrected for this discrepancy, and a proper comparison of the geodetic and astronomic (corrected) azimuth given (see Equation 3.8).

(b) Simplified Derivation of the Laplace Equation

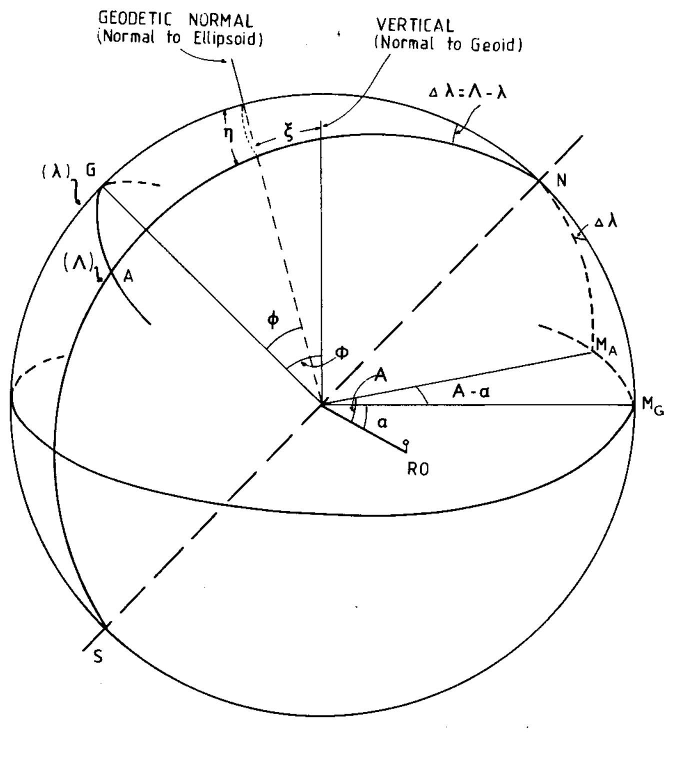


FIGURE 3.3 Local Astronomical and Geodetic Meridians Projected onto the Celestial Sphere.

Small values of ξ, η (< 20") are implied throughout this derivation.

On the celestial sphere,

$\xi'' = (\Phi - \phi)''$
= deflection of the Vertical in the meridian from (1.1).

$\eta'' = (\Lambda - \lambda)'' \cos\phi$
= deflection of the vertical in prime vertical from (1.2).

or $\Lambda - \lambda = \eta \sec\phi$

Now, the geodetic azimuth of R.O. $\alpha = M_G \rightarrow$ R.O. (Clockwise)
and the astronomic azimuth of R.O. $A = M_A \rightarrow$ R.O. (Clockwise)

$$\therefore M_A M_G = A - \alpha$$

In $\Delta\ NM_AM_G$ by Napiers Rules of Circular Parts

$$\sin\phi = \tan(A - \alpha)\cot(\Lambda - \lambda)$$

$$(A - \alpha)'' = (\Lambda - \lambda)'' \sin\phi$$

or $\alpha = A - (\Lambda - \lambda)\sin\phi$ ...Laplace's Equation ..(3.8).

Substituting for $\Lambda - \lambda$

$$\alpha = A - \eta\tan\phi$$

This derived geodetic azimuth is known as **The Laplace Azimuth.**

(c) Effect of Errors in λ on Azimuth Correction

The geodetic and astronomic longitudes in Australia are subject to an observational σ of less than 1.0". Astronomic azimuths are determined with about the same precision. Geodetic azimuths, when are carried through the triangulation, are subject to an error about 10 times greater. The network may therefore be greatly strengthened by correcting the geodetic azimuths at Laplace stations by means of the Laplace Equation.

In the early days of geodetic surveying, the Laplace Azimuth was held fixed in an adjustment. Modern techniques allow one to supply a variance factor to this value so it can "float" in the adjustment.

(d) Example on the Use of Laplace Stations in Controlling Azimuth

Calculated geodetic longitude for (say) Woodford, λ, = +151° 01′ 49.00",
and the observed astro longitude for Woodford, Λ, = +151° 01′ 53.85".

$$\therefore \Lambda - \lambda = +4.85$$

$$\text{and } (\Lambda - \lambda)\sin\phi = -2.64 \quad (\phi = -33°)$$

Observed astro azimuth, Woodford to Razorback A = 143° 16′ 13.55"

∴ Laplace azimuth $\alpha = A - (\Lambda - \lambda) \sin \phi$ = 143° 16′ 16.19"

Calculated geodetic azimuth (obtained through network) = 143° 16′ 15.64"

So misclose in calculated geodetic azimuth -0.55"

∴ Correction to be distributed through scheme +0.55"

NOTE: If the "uncorrected" astronomical azimuth has been used the misclose would have been stated as +2.09. This would be incorrect as the observed astronomical azimuth refers to a different meridian to that inferred by the geodetic parameters (longitude and azimuth).

3.3 Primary Traverse

The principles, instruments, reductions and computations associated with Electronic Distance Measurement are treated comprehensively in RUEGER (1980) and will not be covered here.

3.3.1 Traverse vs Triangulation

Traverse was for long regarded as an inferior substitute for triangulation since

(i) It was generally held to be of lower accuracy

(ii) It provides fewer control stations than triangulation.

However the accuracy of the traverse can be increased to that of Primary triangulation or better by

(1) Measuring each traverse line to Primary triangulation baseline accuracy.

(2) Measuring traverse angles by procedures similar to that of Primary triangulation angles.

(3) Having Laplace azimuths at each station.

Primary triangulation usually has closing errors on baselines of $< 1:10^5$ and scale error remaining after adjustment between bases should be less than $1:2 \times 10^5$. Overall accuracy between points separated by continental distances should be $\cong 1:5 \times 10^5$.

Primary traverses must be of similar accuracy in order to supersede triangulation as the main control framework.

In suitable country primary traverse can be measured using invar tapes in catenary, but the method in general use in Australia employs the Tellurometer and Geodimeter for distance measurement.

The advantages of EDM traverse compared to triangulation may be summarised as follows:

(a) the ease with which a single line can be selected compared to a chain of triangles (particularly in flat country),

(b) the ease of organising observations to only 2 stations at a time compared to several scattered stations,

(c) the presence of the radio - telephone is part of the equipment, hence communication between field parties is easy,

(d) a system of traverse is easier to adjust than a system of triangulation, and, importantly,

(e) there is no accumulation of scale error.

3.3.2 Azimuth Control in Traverses

It is necessary to determine the frequency of Laplace Azimuths needed to match directional accuracy of any single traverse line to that of its distance accuracy.

Procedure:

The accuracy of tellurometer measurement of length over geodetic lines

$$\frac{\sigma_D}{D} \geq \pm 1:200,000$$

This is roughly comparable to 1" of arc in direction. To increase the possibility of getting this accuracy in direction take, say, the 87% confidence interval i.e. 87% of all directions are within ±1" from mean.

$$\therefore \quad \sigma_\theta = \frac{1.0''}{1.5} \text{ (since 87\% confidence} = 1.5\,\sigma)$$

Thus, a σ_θ of 0.67" is desired.

1) Now, say $\sigma_{\text{single obs}}$ for the Wild T3 is ≅ ±0.6" (RICHARDUS, 1965, gives a value of 1.5")

σ_{angle} " " " " " ≅ $\pm\sqrt{(.36 + .36)} = \pm.85''$

and so σ of mean of 16 measures of an angle (16 semi-arcs or 8 arcs)

$$\cong \pm 0.21'' = \left(\frac{.85}{\sqrt{16}}\right)$$

2) The σ_α for a Laplace Determination ≅ ±0.6"

3) Therefore, the σ of the derived azimuth through a traverse after n angles (each measured by 8 arcs) would be

$$\sigma_\alpha \cong \pm\sqrt{[0.6^2 + n(.21)^2]} = \pm\sqrt{[0.36 + 0.044\ n]}$$

So equating this to our desired accuracy for $\sigma_\alpha = \pm 0.67$

$$= \pm\sqrt{[0.36 + 0.044\ n]}$$

$$\therefore\ n < 2$$

i.e. Laplace azimuths are needed at about every second station to meet the stated criteria.

3.3.3 Traverses in the Australian Geodetic Network

As can be seen by referring to a map of the geodetic network of Australia (Figure 2.3) the bulk of the national control network was done by traversing. Indeed, without the traverse (and EDM) it is unlikely that the geodetic control network would yet have reached the coverage which already existed by the mid 1960's. By this time the whole of inland Australia had been covered by interconnecting traverses to form loops, connecting onto existing triangulation where possible. In 1966 the whole Australian geodetic survey was adjusted in phases, the first to fix the main loops and subsequent phases the portions between the fixed intersections of these loops. These resulting coordinate values form the basis for most control surveying and mapping at present.

To gain insight into the work and the problems associated with these traverses it is worth reading the various papers which have been published in The Australian Surveyor, notably JOHNSTON (1962), JOHNSTON, (1965) and FORD (1979).

3.4 Trilateration

3.4.1 Long Sides using Aircraft and Shoran Type Equipment

(a) Uses and Applications:- (see BOMFORD, 1971, pp. 94-102)

This provides a net of lines 300-700 km in length, used mainly for either

(i) Connection between two continental or islands triangulation systems e.g. Europe - Iceland - Greenland - Canada ; Crete - North Africa ; Florida - Trinidad in West Indies

(ii) Cover of large uninhabited areas, e.g. Australia, North Canada, with the basic framework. This will need to be broken down by triangulation or traverse, but makes it possible to start breaking down anywhere in the network as and when required for topographic mapping and puts all surveys on same origin. Both the above applications have been made obsolete by the advent of satellite position fixing methods. However, this technique was used widely to provide control in much of outback Australia, and a short description is given below.

(b) Method of Measurement

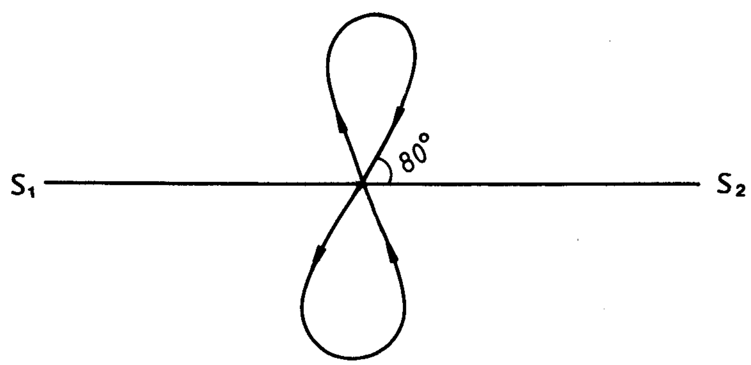
The measurement technique may be as follows:-

(i) Fly figure-of-eight pattern across line, constantly monitoring sum of distances $AS_1 + AS_2$. Three "8's" giving six crossings per day for two days is regarded in Canadian practice as a minimum requirement. (If measured lengths differ by > $1:10^5$, a third set is observed).

(ii) The sum of $AS_1 + AS_2$ (or return transmission times) is recorded at intervals of 150m along-line of flight (≅ perpendicular to measured line) and for 1.5 - 3 km either side of S_1S_2. Plotting $AS_1 + AS_2$ vs time gives a parabola. The minimum of the parabola gives the required distance, $AS_1 + AS_2$ when the aircraft was over the line S_1S_2.

Given heights above ellipsoid of S_1 and S_1 and the aircraft, calibration constants of the equipment, meteorological data (or a good estimate) all along lines AS_1, AS_1, and the velocity of transmission, the ellipsoidal geodetic distance P_1P_2 may be computed.

Care had to be taken in the siting of S_1, S_2 so that rays do not interfere with nearby structures or reflect from nearby bodies of water, causing erroneous signals.



Plan View

FIGURE 3.4a

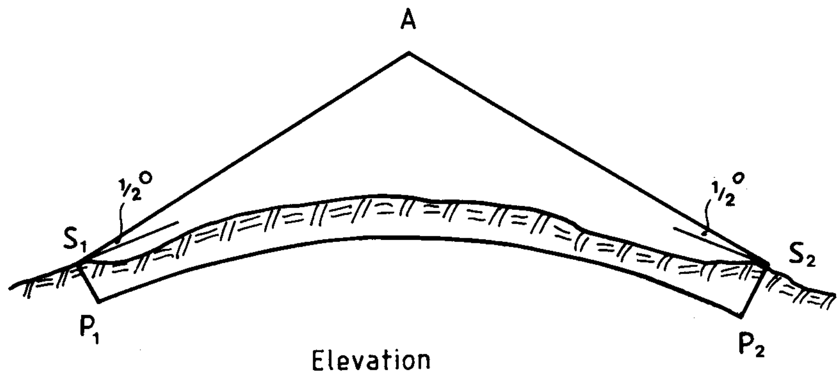


FIGURE 3.4b

3.4.2 Main Systems

	SHORAN	HIRAN	AERODIST
Tech. Details	Uses metre waves Signals in pulses of 0.8s every 930s Returned by Transponders at ground station. One operator at aircraft and two at each ground stations.	Same as for Shoran	Similar to Tellurometer system. Continuous signal 1200-1470 MHz (20cm) modulated to 1.5 MHz. Ranges simultaneously recorded by pen recorder in aircraft. One operator in aircraft and one at each ground station.
Weight of Equipment	Airborne 340 kg Transponders 680 kg each	Same as for Shoran	Master 15 kg
Atmospheric Condition	Works in cloud or mist. Lines must be clear of surface trees and buildings.	Same as for Shoran	
Range	Up to 700 km	Up to 700 km	Up to 400 km
Sensitivity	8 m range, error varies with variation of signal strength.	3.5 m due to modification to keep signal strength continuously controlled to a constant.	$\pm$ 1m + $\frac{1}{100,000}$

3.5 Satellite Surveying

As mentioned in Section 2.1.3, satellite techniques for position fixing are a viable alternative or an important addition to terrestrial techniques. The position which results from such a fix will necessarily be with respect to the reference system used for the satellite, i.e. an X-Y-Z system with its origin at the centre of the Earth's mass (the geocentre) and the axes aligned in accordance with the Conventional International Origin (CIO - see TORGE, p.39). These coordinates (X,Y,Z) can be transformed directly to geodetic coordinate (ϕ, λ and h) by adopting an ellipsoid of suitable a and f (TORGE, p.52).

The significance of this to geodetic surveying is that these coordinates refer to a **global** ellipsoid, not to a **local** ellipsoid as is usually the case with coordinates derived for terrestrial survey (see Section 1.2.2). This means that, before satellite-derived coordinates can be used in geodetic networks transformation must be applied to them to find their values with respect to the local reference system.

3.5.1 Basic Principles

The Doppler effect most familiar to us is in the field of sound wave propagation, where it is sensed as the change in the pitch of a note (e.g. train whistle, siren) emitted by a vehicle as it passes a listener. As the vehicle approaches the pitch of the note increases: as the vehicle passes and departs, the pitch appears to drop suddenly and then continues to drop until it takes up a constant tone.

The change in the pitch of the tone i.e. the change in apparent frequency, is therefore a function of the velocity of the emitter with respect to the receiver. If the relative velocity of the transmitter is known, its position fixed with time and the **change in frequency** (or **Doppler shift**) of the received frequency measured, it should be possible to calculate the position of the receiver with respect to the emitter.

Take for example, this simple case below (see HARG et al, 1980).

Example 1

In this example the sound source is stationary and its position is taken as the origin of a two-dimensional Cartesian coordinate system. The signal emitted has a frequency (f_E) of 300 Hz and a wavelength (λ) of 1.104 m based on the velocity of sound in air of 331.3 m/s.

The propagations of the sound waves outward from the source is indicated in Figure 3.5 by the concentric rings centred on S. Points A, B, C & D show the locations of observers travelling with different relative velocities to S. The observer at A is stationary, while those at B, C & D are moving at 20 m/s in a direction parallel to the x-axis.

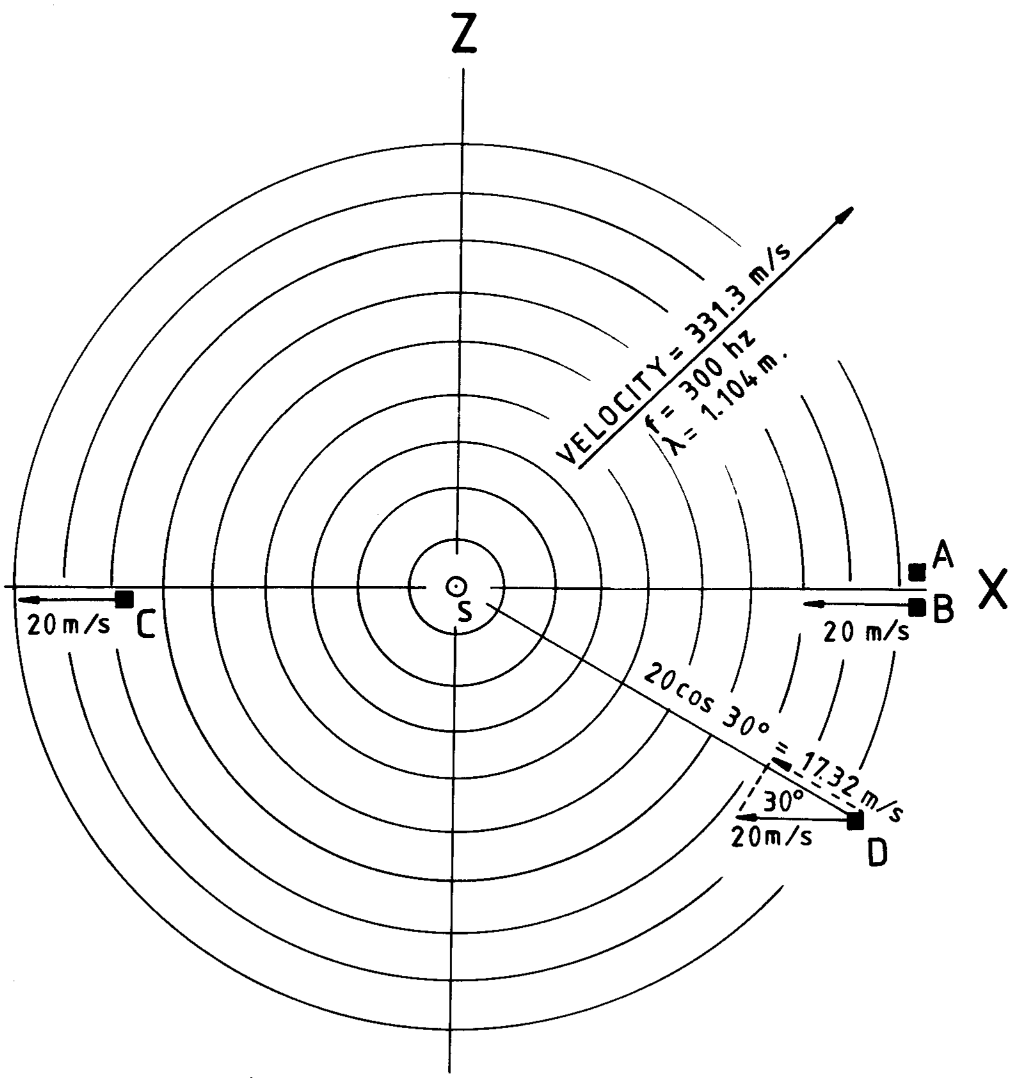


FIGURE 3.5: Relationship between relative velocity and frequency for cases A, B, C and D.

The observer at point A will receive 300 cycles in 1 second, i.e. he will observe a frequency of 300 Hz (f_R or received frequency). The Doppler shift, Δf, will therefore be zero.

The observer at B is moving directly towards S at 20 m/s. The number of cycles received at B in 1 second will equal 300 (the number received at A) plus the number crossed in travelling forward for 1 second. B moves ahead by 20m in 1 second. Since the length of each wave is 1.104m, B crosses 20m/1.104m or 18.12 cycles in the 1 second of moving toward the sound source. Hence the frequency observed at B (f_R) will be (300 + 18.12)Hz, and the Doppler shift Δf

$$\Delta f = f_R - f_E = 318.12 - 300$$
$$= 18.12 \text{ Hz.}$$

Using similar reasoning we can compute the Doppler Shift at all 4 stations, as shown in the table below.

At	Relative Velocity V_C (m/s)	f_R	$\Delta f = f_R - f_E$
A	0	300	0
B	+20	300+20/1.104	18.12
C	-20	300-20/1.104	-18.12
D	20 cos 30° = 17.32	300+17.32/1.104	+15.69

f_E = 300 Hz

$$\text{Clearly } \Delta f = \frac{V \cos \theta}{\lambda_m} = \frac{\text{(closing or relative) velocity}}{\text{wavelength}}, \quad (3.9)$$

where θ is the closing angle or angle of approach of the observer,
$V_C = V \cos \theta$,
and V_C is the relative or closing velocity
where V is the absolute velocity.

By reversing the above equation,

$$\cos \theta = \frac{\Delta f \ \lambda}{V}$$

$$\text{or } \theta = \cos^{-1} \left(\frac{\text{closing velocity}}{\text{absolute velocity}}\right)$$

$$\text{or } \theta = \cos^{-1} \left(\frac{\text{observed Doppler shift}}{\text{maximum Doppler shift}}\right) = \cos^{-1} \frac{\Delta f}{V/\lambda} \quad (3.10)$$

Thus, if f_E and f_R are known or measured, and if the absolute velocity of the observer is known at at least two points whose locations are given, it is possible to fix the position of the source of the signal by intersection.

Alternatively, if we have the known frequency source on the moving platform and if the position of this source is known with time, it is possible to measure the Doppler shift of the signal received at discrete instants of time. The closing angles can again be determined and the location of the stationary receiver fixed by intersection.

Example 2

A second simple example will help to illustrate this procedure. In this example, the vehicle emitting the sound wave is travelling at 20 m/s in a straight line on the X-Z plane (see Figure 3.6). An observer at (X_0, Z_0) receives the sound at times t_1, t_2, .. at which times the coordinates of the vehicle are known (see Figure 3.6).

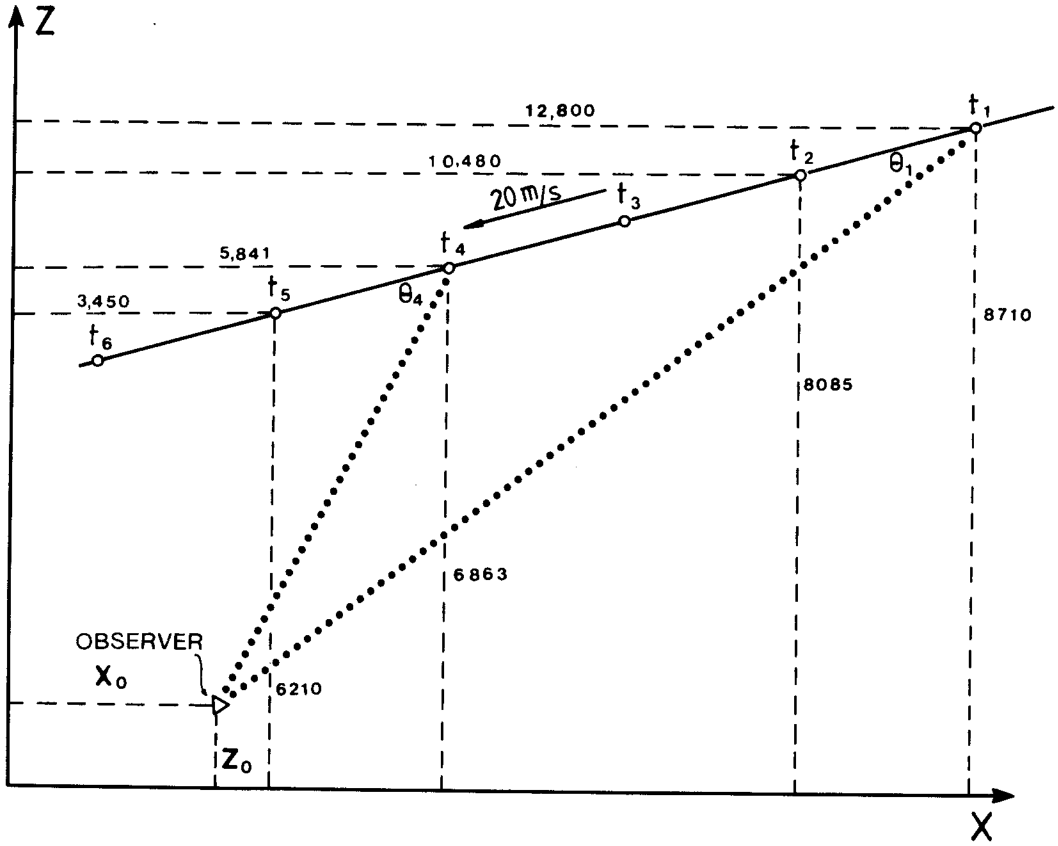


FIGURE 3.6: The coordinates of a point can be determined by observing the frequency of a passing sound source.

If, at time t_1, the observed frequency is 316.6 Hz. Remembering, from Example 1, that f_E = 300 Hz and, $\lambda = 1.104$, the maximum Doppler shift is therefore $V/\lambda = 20/1.104 = 18.12$ Hz.

The closing angle at t_1, $\theta_1 = \cos^{-1}(16.6/18.12)$ (Equation 3.10). If at time t_4, the observed frequency is 312.26, the closing angle at this time will be

$$\theta_4 = \cos^{-1}(12.26/18.12)$$

$$= 47° \ 25'$$

It remains a straightforward intersection problem to show that

$$X_0 = 2513 \text{ m}$$

$$Z_0 = 527 \text{ m}.$$

NOTE that if the observed frequency changed by 0.1 Hz at both t_1 and t_4, the position of the observer changes to X_0 = 2411.5 m (-102 m), Z_0 = 213.6 m (-313 m). Obviously, the precision of the position is critically dependent upon the precision of the recorded frequency.

By observing frequencies at t_1, t_2 etc. a series of closing angles, and hence positions, can be determined from the 1 vehicle "pass", and a final estimate for X_0, Z_0 obtained by taking a mean of all values so obtained, suitably weighted.

Position from Frequency Differences

The difference in the distances or ranges between the observer and point 1, and the observer and point 2 (see Figure 3.7)

$$\Delta r = r_{0-1} - r_{0-2}$$

$$= \sqrt{[(X_1 - X_0)^2 + (Z_1 - Z_0)^2]} - \sqrt{[(X_2 - X_0)^2 + (Z_2 - Z_0)^2]}$$

$$= \sqrt{[\Delta X_{1,0}^2 + \Delta Z_{1,0}^2]} - \sqrt{[\Delta X_{2,0}^2 + \Delta Z_{2,0}^2]} \qquad (3.10)$$

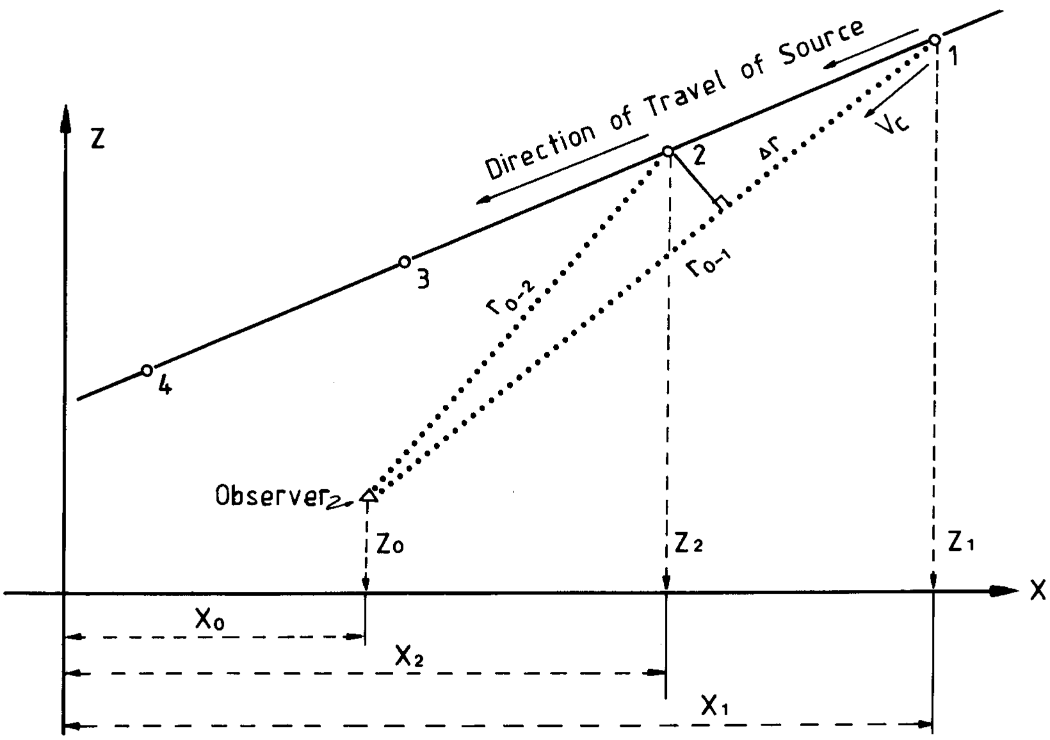


FIGURE 3.7: Relationship between change in frequency and change in range.

From (3.9) $$\Delta f = V_C/\lambda$$

$$= \frac{V_C \ \Delta t/\lambda}{\Delta t} \qquad (3.11)$$

Also we can say

$$\Delta r = V_C \ \Delta t \quad \text{for small } \Delta r.$$

Thus $$V_C = \Delta r/\Delta t \qquad (3.12)$$

From the basic wave equation

$$\frac{1}{\lambda} = \frac{f_E}{V} \qquad (3.13)$$

Substituting (3.12) and (3.13) into (3.11) we get

$$\Delta f \Delta t = V \ \Delta t/\lambda_C = \frac{\Delta r}{\Delta t} \ \Delta t \ \frac{f_E}{V}$$

or $$\Delta r = \frac{V}{f_E} \ \Delta f \ \Delta t \qquad (3.14)$$

In other words, we now have an expression for the change in range in terms of the velocity and frequency of the emitted wave, the Doppler shift and the time between successive ranges.

Equating (3.14) and (3.10) gives

$$\Delta t \ \Delta f = \frac{f_E}{V} \{\sqrt{[\Delta X^2_{1,0} + \Delta Z^2_{1,0}]} - \sqrt{[\Delta X^2_{2,0} + \Delta Z^2_{2,0}]}\} \qquad (3.15)$$

in which only two parameters (X_0, Z_0) are unknown. The others are either known (V, f_E, X_i, Z_i) or measured (Δt, Δf).

A number of observation equations in the form of (3.15) can be set up for successive observations, and the solution for the observer's position obtained by a suitable adjustment.

3.5.2 The Transit System

The development of the Doppler-Measuring-System started about 1938, when this technique was used by Germans for rocket orbit tracking. Further developments were executed in the U.S.A., finally by the Applied Physics Laboratory (APL) of the John Hopkins University in Washington D.C. The latter used the principle to determine the orbit data of the first (Russian) Satellite in 1957 from the radio signals transmitted by the satellite. APL is the author of the U.S. Navy Navigation Satellite System (N.N.S.S.), used at present by the U.S. Navy. In this system six satellites (as of September 1981) orbit the Earth in polar orbits (see Figure 3.8), transmitting at two frequencies - 400 MHz and 150 MHz. (The altitude of the orbit about 1100 km above the Earth which means they circle the Earth about once every 110 minutes).

These satellites are tracked by a basic network of 13 tracking stations (TRANET = TRAcking NETwork System) which are distributed around the world in order to determine the precise satellite ephemeris (±3 m) and to enable prediction of the future orbits. These predictions form the broadcast satellite ephemeris or the raw ephemeris (±25m). The latter is transmitted to the satellites for storage and retransmitted, along with the broadcast frequencies, to the receivers either on the land (for geodetic positioning) or on-board ships (for navigation). Using the principles outlined in Section 3.5.1 (adapted for 3-dimensions), it is possible to determine the locations of the receiver. In the case of ships, the accuracy of the position is about ±50m to ±100m. Using special techniques described below it is possible to get geodetic position in X, Y and Z to about ±0.5m.

See STANSELL (1978) for a more detailed description of the TRANSIT Satellite System.

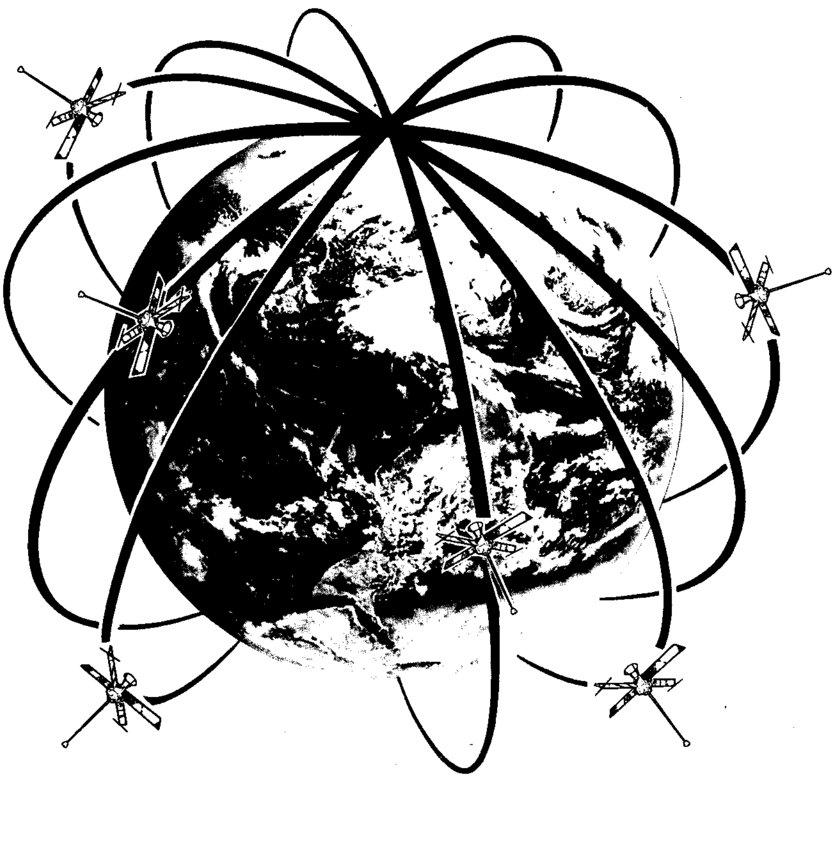


FIGURE 3.8: Worldwide NAVSAT Cover.

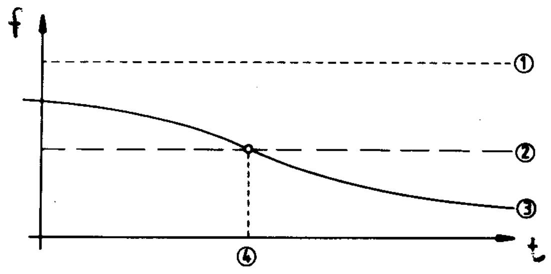


FIGURE 3.9: Change of Received Frequency with Time.

f = frequency, t = time.
1 = Constant Doppler Receiver reference frequency.
2 = Constant frequency emitted by the satellite.
3 = Doppler shifted satellite frequency received by Doppler receiver.
4 = Time of closest approach of satellite with respect to Doppler receiver.

3.5.3 Observational Equipment and Techniques

Portable Doppler Receivers consist of an antenna, which can be set up over a survey mark on its tripod, the receiver with recording facilities (e.g. cassette tape) and a 12 volt power supply by its side. The antenna is connected to the receiver by a cable during operation. The instrument has only to be set up and switched on. It automatically searches for the satellite signals, locks onto them, takes the measurements and records them on magnetic tape (cassettes).

Instruments of the first generation are:

Geoceiver (GEOdetic Doppler ReCEIVER) by Magnavox (U.S.A.)
ITT 5500 by International Telegraph and Telephone (U.S.A.)

Instruments of the second generation:

Magnavox 702 by Magnavox (U.S.A.)
JMR-1 by JMR - Instruments Inc. (U.S.A.)
Marconi CMA-722 by Marconi (Canada)

A few instruments which could be described as 3rd generation are now available, notably the Magnavox MX 1502. The main difference between 2nd and 3rd generation equipment is that the latter has a powerful micro-computing facility built in which enables it to perform many computational tasks (e.g. translocation, described below, or navigation). (See HOAR, 1982, p.4-5 for more detail).

The navigation satellites (NAVSAT) transmit information and 2-minute time pulses on two phase locked frequencies (400 MHz and 150 MHz), to provide a means of eliminating ionospheric refraction from the data. Because of the relative movement of the satellite with respect to the receiver, the emitted constant frequency is Doppler shifted upon arrival at the receiver to a continuously variable frequency, (see Figure 3.9). The latter is mixed with a constant reference frequency in the receiver and the number of cycles of the beat frequency (Doppler counts) are counted in the receiver for every 4.6 second time interval. The integrated Doppler counts are stored for the high and the low frequency, together with data, time, satellite number, number of passes, ambient temperature and pressure on the cassette, for later processing.

Observational Techniques

There are two main techniques classified according to whether the coordinates of the position of the receiver, (actually a point marked on the antenna) is "absolute" or "relative". The technique giving an "absolute" position is called "point positioning", whilst that giving a relative position is known generally as "translocation".

(i) **Point Positioning**

In point positioning the equipment is placed at the station to be fixed and many satellite passes, requiring up to 5 days occupation, are observed. For geodetic purposes it is necessary to use the "precise ephemeris" of the satellites observed and this is (obviously) only available after the observational period, and is the product of the tracking of the satellites by the TRANET (see Section 3.5.2). This "precise ephemeris" gives the "exact" coordinates (±3 m) of the satellites with time and is only available to government agencies friendly with the U.S. Government.

The position obtained is "absolute" only in the sense that it relates to the geocentric reference system in which the satellite positions are calculated. The coordinates are geodetic (i.e. relate to the mathematical reference system rather than the physical reference system of the geoid). Estimates of the precision obtainable from this technique are that the 1σ value of position obtained from 40 satellite passes is about 0.7m in each of the X, Y and Z coordinates.

(ii) **Translocation**

Translocation refers to a technique employing more than 1 receiver operating simultaneously at different stations. The computational procedure tries to eliminate any uncertain elements (e.g. satellite orbits) which are common to the observations at the receivers employed, and produces a **difference** in position between the stations occupied. Thus if one of the stations is a geodetic station the coordinate differences can be applied to the known values to obtain the geodetic positions at the unknown stations.

Before this technique can be successful, approximate coordinates must be known. These are based on the **broadcast** ephemerides of the satellites, e.g. the coordinates of the satellites predicted at certain times from the tracking of the satellites in their orbits.

The terminology used often in connection with translocation is as follows:-

(a) **Short Arc**, in which several known stations are occupied, as well as the unknown ones. Identical satellite passes are observed simultaneously, and the orbital parameters as well as the coordinates of the unknown stations, are solved for in the least squares solution.

(b) Semi-Short Arc, which is similar to Short-Arc except that some of the parameters used to describe the satellite orbits are held fixed.

(c) Long Arc, where again several known and unknown stations are occupied, but no restrictions are made upon the satellites observed. Orbit ephemerides must now be used in the solution as known values, and the only unknowns are the coordinates of the unfixed stations occupied by receivers.

Field tests of the translocation technique, used in optimum configuration, show that relative positions of points will have precisions of about ±30cm in horizontal position and ±60cm in elevation (HOAR, 1982, p.4-11).

3.5.4 Applications

Doppler Satellite Receivers open new possibilities in Geodetic Surveying:

(1) Relative point positioning on existing control networks for the purpose of deriving datum transformations between independent networks and/or readjusting and possibly improving the accuracy of existing control networks.

(2) Establishment of coordinates relative to an earth-centred coordinate system for providing worldwide ties between datums.

(3) Point positioning in remote areas lacking any control for future control extensions by conventional methods.

(4) Point positioning in support of mapping, particularly in remote areas.

(5) Point positioning on existing astronomic positions for obtaining deflections of the vertical.

(6) Point positioning on existing vertical control to determine the height of the geoid directly. Coordinates of points are computed first in a earth centred coordinate system, as described earlier. They can be transformed later into an existing datum based on a specified ellipsoid. Ellipsoidal heights are obtained and can be compared with orthometric heights from levelling or trigonometric levelling. The difference is the height of the geoid.

(7) Point positioning in support of geophysical surveys as well as oil drillings in remote areas and also off-shore oil wells.

(8) Determination of large soil and ice movements.

(The above list is according to RUTSCHEIDT, 1974).

In Australia, four JMR-1 receivers have been used by NATMAP since 1975 for point positioning in the primary control. Short arc methods are expected to give finally a relative accuracy (between points) of between ±0.2 and ±0.4m. Geoceivers are used by the Royal Australian Survey Corps in the point positioning mode using precise ephemerides.

Doppler-derived positions, establishment at the junction points of geodetic traverses are being used to augment the geodetic control network, and to establish the transformation parameters to relate the Australian Geodetic Datum to the global reference system used by the TRANSIT System (ALLMAN et al, 1979; LEPPERT, 1978).

In February and March, and June-July of 1982, the School's two Magnavox MX1502 satellite receivers were involved in the observation of multi-station networks throughout Australia. In all, 62 first order stations were observed in blocks of up to 13 stations. This data set formed an important part of the combined readjustment of the primary network for Australia. All Doppler observations were computed using the School's Doppler data processing software on the University's Cyber 171 mainframe computer.

A total of 13 organisations, both public and private, were involved in the project, namely:

Division of National Mapping (NATMAP), Australia

Queensland Dept. of Mapping and Surveying

Queensland Private Survey Sector

Mt. Isa Mines

NSW Division of Telecom

Esso Australia

B.H.P. Engineering

Dept. of Aviation, Victoria

Western Lands Commission, N.S.W.

South Australian Dept. of Lands

Northern Territory Dept. of Lands

Division of Surveys & Mapping,Victoria

School of Surveying, University of N.S.W.

The project resulted in a significant strengthening of the primary network.

4. COMPUTING THE NETWORK

4.1 Reduction of Observation

4.1.1 Corrections to Observed Directions

Much of the reduction to observed directions is carried out in the field by the recorder. This is done as observation proceeds as it forms a valuable check against observing errors. Indeed, a skilled recorder would be in a position to obtain grand means and estimates of precisions almost a soon as the observer completes his observations.

The grand means and their precisions should be checked and abstracted onto prepared forms ready for keying onto a computer readable medium (i.e. magnetic tape).

It may be necessary to correct the observations for systematic errors, such as eccentric standpoints/signals, or height above datum. These corrections are described below.

Corrections to Obtain Reduced Directions

(i) Correction for Phase of Signal

Most Primary observations are made at night to lamps mounted on the trig. beacons or over the eccentric station at the trig. station. Observing to cylindrical poles in daylight introduces an error known as phase error.

If the signal has a highly reflective surface, sunlight will be reflected towards the observer from a single vertical line, which will appear to be very bright. The observer will sight to this bright line. The correction to be applied is

$$c'' = \frac{r \cos \frac{1}{2} a}{D} \cdot \rho'' \qquad (4.1)$$

where r = radius of cylindrical signal,
D = distance from observer to signal,
a = angle measured clockwise from the sun to the signal, at the observer's position, and
ρ'' = 206265 (no. of seconds/radians).

If the cylinder has a matt finish, part of the signal will be illuminated and part in shadow. The observer will bisect the illuminated section. In this case, the correction will be

$$c'' = \frac{r \cos^2 \frac{1}{2} a}{D} \cdot \rho'' \qquad (4.2)$$

N.B. The derivation of (4.1) and (4.2) may be found in CLARK, 1957, p.227.

(ii) Eccentricity Correction

(a) Eccentricity of Instrument:

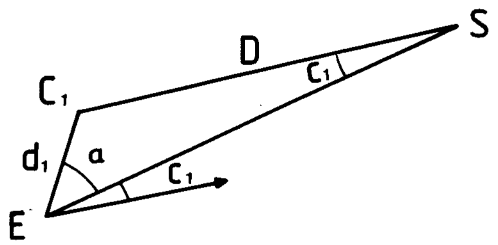


FIGURE 4.1: Correction for Eccentricity of Instrument

Let E be the eccentric station
C_1 be the trig. station
S be the distance signal

Let α be the angle measured clockwise from EC_1 to ES
D = distance C_1S
d_1 = distance EC_1 (<< CS)

c_1 = direction C_1S - direction ES.

By sine rule,
$$\frac{\sin c_1}{d_1} = \frac{\sin \alpha}{D}$$

$$\sin c_1 = \frac{d_1}{D} \sin \alpha$$

The maximum value of $c_1 \cong \frac{d_1}{D} \cong \frac{50 \text{ metres}}{50\text{km}} \cong .001$ radians.

The difference between c_1 and $\sin c_1$ is therefore $\cong 2 \times 10^{-10}$ radians $\cong 4 \times 10^{-5}$ secs.

Therefore, we can write

$$c_1 = \frac{d_1 \sin \alpha}{D}$$

or

$$c_1'' = \frac{d_1 \sin \alpha}{D} \rho'' \qquad (4.3)$$

(b) Eccentricity of Signal:

By similar reasoning, if β is the angle measured clockwise from SC_2 to SC_1

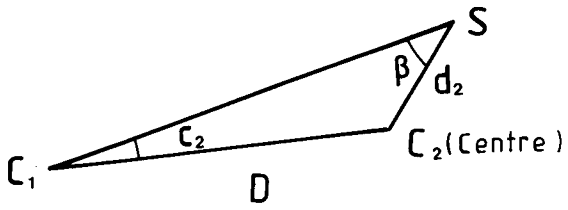


FIGURE 4.2: Correction for Eccentricity of Signal

$$c_2 = \frac{d_2 \sin \beta}{D} .$$

$$c_2'' = \frac{d_2 \sin \beta}{D} . \rho'' \qquad (4.4)$$

If both signal and instrument are eccentric, the the total correction is

$$c'' = \frac{\rho''}{D}(d_1 \sin \alpha + d_2 \sin \beta) \qquad (4.5)$$

using the notation defined above.

(iii) Reduction of Directions to the Ellipsoid

This is required only in the most refined Primary Triangulation at considerable elevations above the ellipsoid (h), especially in low latitudes.

In general, the normals to the ellipsoid passing through points A and B on the topographical surface are not coplanar (due to reference surface not being spherical). The projections of A and B along the normals onto the ellipsoid are A_1 and B_1. The plane AA_1B, which contains the observed direction AB, and the plane $AA_1 B_1$, which contains its projection on the ellipsoid, are not the same. The observed direction AB should be corrected to yield the ellipsoidal direction A_1B_1.

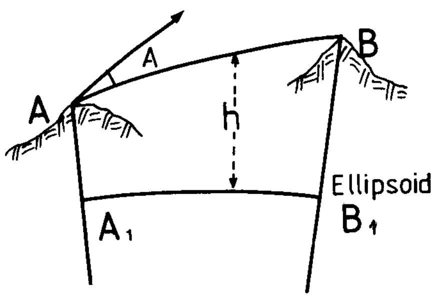


FIGURE 4.3: Reduction of Directions to the Ellipsoid

The correction is given by

$$c'' = \frac{e\ h}{2a} \sin 2\alpha \cos^2\phi\ \rho'' \qquad (4.6)$$

where $c'' = (A_1B_1 - AB)''$,
e = eccentricity of ellipsoid,
h = mean ellipsoidal height of A and B,
a = semi-major axis of ellipsoid,
α = azimuth of AB, and
ϕ = the mid-latitude of line AB.

We can see that $\sin 2\alpha$ has max value = 1 when $\alpha = 45°$.

$\cos^2 \phi = 1$, when $\phi = 0$.

Consider a case where $h \cong 3\text{km}$

$$c'' \cong \frac{7 \times 10^{-3} \times 3 \times 10^3}{2 \times 6 \times 10^6} \times 2 \times 10^5$$

$$\cong 3.5 \times 10^{-1}$$

In Australia, maximum value of $\cos^2$ is about about 0.93,
maximum value of h is about is about 2km.

$\therefore\ c'' < 0.2''$ anywhere in Australia.

4.1.2 Corrections to Distances

The observing routine, and hence the method of recording and reducing distances, will depend largely on the EDM instrument being used. As for directions, a skilled recorder should be able to produce grand means and estimates of precision shortly after observations have ceased. These values will then be checked and abstracted in readiness for the network computation.

Observed distances will need various reductions to bring them to the standardised mean sea level value to be used in the computation. These may be summarised as:

(i) Corrections for the refractive index of the atmosphere (RÜEGER, 1980, 35 - 41).

(ii) Reductions of the measured distance to the mathematical model (IBID, 41-56).

(iii) Corrections for the eccentricity of the measuring equipment to the geodetic station(s).

4.2 Calculations for Station Positions

4.2.1 Approximate Coordinates

As a first step to a rigorous adjustment, it is often necessary to obtain approximate coordinates for the stations in the network.

(a) **Directions**

Approximate azimuths can be carried through the network from one Laplace azimuth to the next by the application of the relevant direction observations. Any resultant misclose is adjusted so that a good approximation for the azimuths of all lines in the network can be obtained.

The following example will help to illustrate the method.

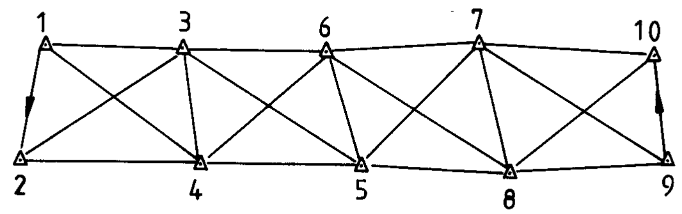


FIGURE 4.4: Example of Triangulation

Assume all directions in Fig. 4.4 have been observed and that Laplace azimuths on lines 1, 2 and 9, 10 determined.

The azimuth of line 1, 3 ($\alpha_{1,3}$) can be determined by the application of the difference between the observed direction on line 1-2 ($D_{1,2}$) and Laplace azimuth on this line ($\alpha_{1,2}$); in other words, by applying the orientation correction at station 1 (O_1) to the observed direction, i.e.

$$O_1 = \alpha_{1,2} - D_{1,2} \quad (4.7)$$

$$\alpha_{1,3} = D_{1,3} + O_1 \quad (4.8)$$

The reverse azimuth of line 1,3 ($\alpha_{3,1}$) can be determined by finding the convergence of the meridians - the change in the direction of the true norths - between stations 3 and 1 (see BENNETT & FREISLICH, 1980, pp.3-6). If this convergence is called $\gamma_{3,1}$, then

$$\alpha_{3,1} = \alpha_{1,3} \pm 180 + \gamma_{3,1} \quad (4.9)$$

In a similar way, $\alpha_{3,6}$, $\alpha_{6,7}$, $\alpha_{7,10}$ and hence $\alpha_{10,9}$ and $\alpha_{9,10}$ can be calculated.

The value of the azimuth of line 9,10 carried through the triangulation can be compared with the Laplace azimuth observed at station 9 ($\alpha'_{9,10}$). A misclose which results can now be distributed back through the chain to establish good first-approximations for the azimuths of the lines along the northern border of the chain. The azimuths of all lines can now easily be found.

Please note that the Laplace azimuths $\alpha_{1,2}$ and $\alpha_{9,10}$ are evaluated with respect to the geodetic meridian (see Eqn. 3.8) so that all values $\alpha_{i,i+1}$ are geodetic azimuths.

(b) **Distances**

Triangulation. Observed ellipsoidal angles in the triangle can be reduced to the equivalent plane angles by applying Legendre's Theorem. Ellipsoidal distances for all lines in the network can now be computed using plane trigonometry and the equivalent plane angles.

For example (referring again to Fig. 4.4), say the ellipsoidal distance on line 1,2 has already been evaluated and that ellipsoidal angles at β_1, β_2, and β_3 have been calculated (e.g. $\beta_1 = D_{2,1} - D_{3,1}$).

The spherical excess (e) for triangle 1,2,3 can be calculated (see MAUGHAN, 1975A,pp. 81-83)) and the equivalent plane angles then found

$$\beta'_i = \beta_i - e/3 \tag{4.10}$$

Ellipsoidal distances $s_{1,3}$, $s_{2,3}$ can now be found from the plane trigonometry sin rule, using the equivalent plane angles, viz

$$\frac{s_{1,3}}{\sin \beta'_2} = \frac{s_{2,3}}{\sin \beta'_1} = \frac{s_{1,2}}{\sin \beta'_3}$$

Traverses and Combined Triangulation/Trilateration. The reduced distances from Section 4.1.3 can be considered to be ellipsoidal distances and as such are suitable for computation on the ellipsoid.

(c) **Approximate Coordinates**

Having now obtained ellipsoidal bearings and distances for all lines in the network, it is now possible to compute approximate geodetic coordinates (ϕ, λ) for all stations, providing geodetic coordinates are given for at least 1 station in the network (e.g. Station 1, Fig. 4.4). Such coordinates can be computed using expressions for the "direct problem" as derived in, for example, the Puissant, Clarke's medium Line or Robbins Formulae.

4.2.2 Adjustment of the Network

Network adjustment is based upon the principles of least squares (e.g. MAUGHAN, 1975B). Most adjustments by least squares are carried out on the plane. It is therefore necessary, before this step in the computation can proceed, to project the line elements s, α and the geodetic coordinates ϕ, λ onto a projection surface to obtain their respective plane counterparts S, θ, N and E.

The use of projections in geodetic computations is covered fully in MATHER (1972) and will not be treated here.

As can be seen, this section 4.2 treats the computation in a broad, descriptive fashion only. Its aim is to provide a perspective on how various aspects of geodetic computation link together to form the chain of computation for the geodetic coordinates in the network.

4.3 Mapping the Geoid

4.3.1 Evaluating the Deflection of the Vertical

As a result of the geodetic computations from Section 4.2, we now have the geodetic (or ellipsoidal) coordinates of all points in the network. (Remember, these values will be dependent upon the size and orientation of the ellipsoid chosen for the geodetic datum - See Section 1.2.2). If astronomic position (Φ, Λ) has also been determined at a geodetic station, the geodetic position (ϕ, λ) can be compared with the astronomic position, yielding the components of the "deflection of the vertical" from the normal to the ellipsoid, viz:-

$$\xi = \Phi - \phi \quad \text{(from 1.1)} \qquad (4.12)$$

$$\eta = (\Lambda - \lambda)\cos\phi \quad \text{(from 1.2)} \qquad (4.13)$$

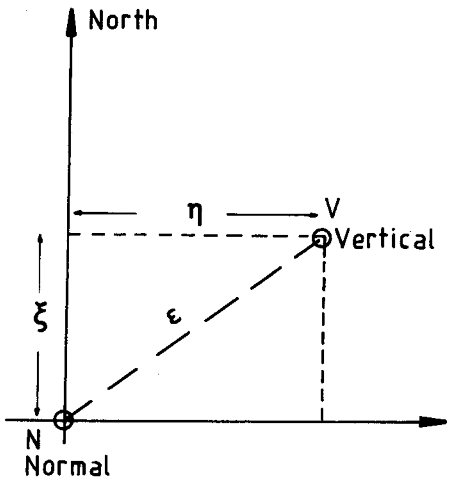
where ξ is the component in the meridian,

η is the component in the prime vertical.

Consider Figure 4.5a. It is easy to see that the actual deflection angle (ε), i.e. the solid angle between the vertical and the normal, is

$$\varepsilon = \sqrt{(\xi^2 + \eta^2)} \qquad (4.14)$$

It is now necessary to compute the component of the deflection (ε_α) along any specified azimuth (α). In Figure 4.5b, ε_α is the distance from N to the projection of V onto the line of azimuth α, i.e. NG.



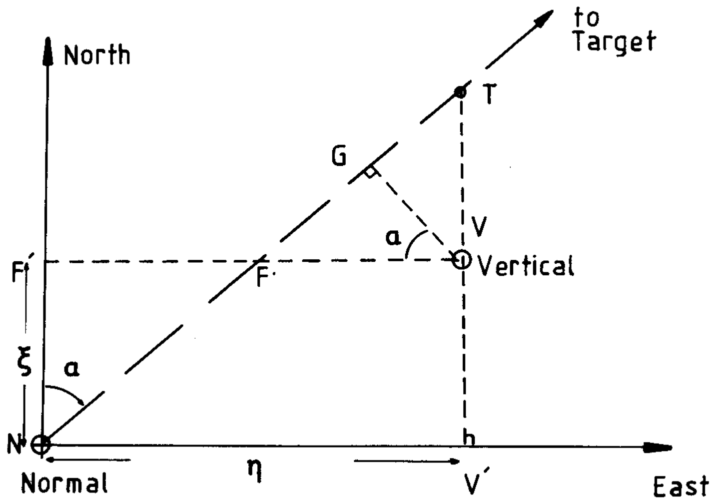


FIGURE 4.5: Deflections Along the Line of Azimuth α

From the geometry of the constructions,

$$NTV' = \alpha,$$

and $$GVF = \alpha.$$

Now, in triangle NF′F,

$$F'F = \xi \tan \alpha$$

$$NF = \xi / \cos \alpha.$$

Thus, since $$F'V = \eta$$

$$FV = F'V - F'F = \eta - \xi \tan \alpha$$

In triangle GVF

$$\sin \alpha = GF/(\eta - \xi \tan \alpha)$$

$$\therefore \quad GF = (\eta \sin \alpha - \xi \sin^2 \alpha / \cos \alpha)$$

Now

$$NG = \varepsilon_\alpha = NF + FG$$

$$= \xi/\cos\alpha + \eta\sin\alpha - \xi\sin^2\alpha/\cos\alpha$$

$$= \frac{\xi}{\cos\alpha}(1 - \sin^2\alpha) + \eta\sin\alpha$$

$$\therefore\ \xi_\alpha = \xi\cos\alpha + \eta\sin\alpha \qquad (4.15)$$

See also Equation (1.3).

Thus, if the components of the deflection of the vertical (ξ, η) can be evaluated (Equations 4.12, 4.13), it is possible to find the component of the deflection of the vertical along any line whose azimuth is given.

4.3.2 Evaluating Geoidal Undulations and Astro-Geodetic Levelling

To determine a 'geoid section', or the profile of the geoid with respect to the ellipsoid, astronomical Φ and Λ are observed in a section of the geodetic network, at geodetic stations separated by 20 to 30 km. Stations at which ϕ, λ and Φ, Λ are determined are called astro-geodetic stations. The increment in the geoid ellipsoid separation (ΔN) can be calculated from the component of the deflection of the vertical in the direction of the profile, as follows.

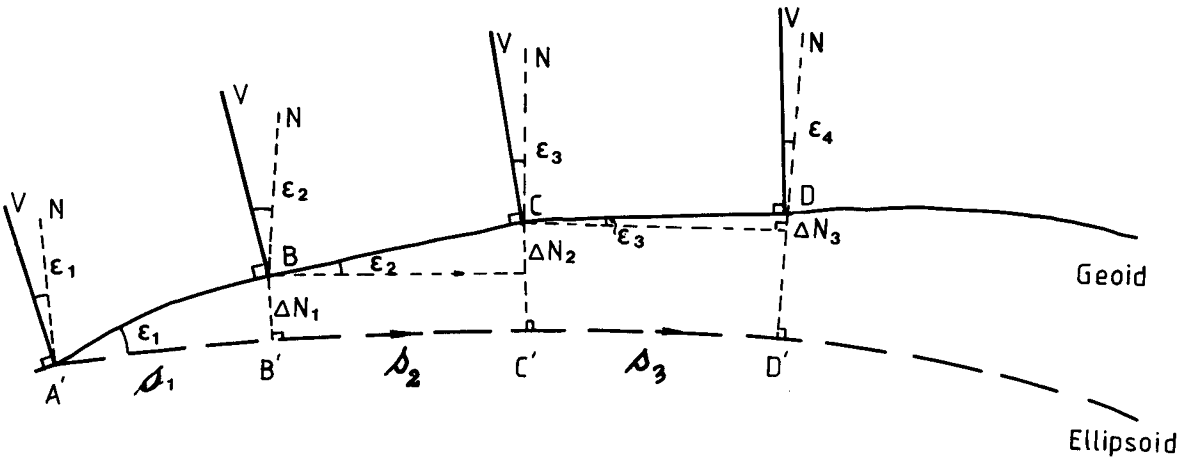


FIGURE 4.6: Geoid Profile

The deflection of the vertical can be thought of as the tilt of the geoid with respect to the ellipsoid. Thus for a smooth geoid, the tilt of the geoid in the plane of the section AB at A equals the deflection of the vertical in that same section, ξ_1.

Obviously,

$$\Delta N_1 = -AB'\tan\varepsilon_1$$

$$\therefore\ \Delta N_1 = -s_1\tan\varepsilon_1$$

Generally, $\Delta N_i = -s_i \tan \varepsilon_i$

(Note from Equation 1.1, 1.2: ε is +ve if the vertical is north & east of the normal. Thus a +ve deflection will result in a -ve increment in N.)

Thus, accumulating through the profile between A and D,

$$N_D - N_A = \sum_i \Delta N_i = -\sum_{i=1}^{3} s_i \tan\varepsilon_i \qquad (4.16)$$

Geoid profiles are joined to form loops, and ΔN carried around loop to obtain loop misclosures. Finally, the geoid-ellipsoid separation is defined at one station in the network (usually the origin of the survey), enabling N to be carried through the geoid profiles to give N at all astro-geodetic stations. From this, geoid maps, showing contours of the geoid with respect to the ellipsoid, can be drawn using standard draughting techniques.

4.3.3 Astro-Geodetic Levelling in Australia

Geodetic instrumentalities associated with the National Mapping Council of Australia have combined forces to establish over 600 astro-geodetic stations throughout the Australian region. The deflections of the vertical at these stations are shown in Figure 4.7, where the vectorial respresentation of the deflection is equivalent to the distance NT in Figure 4.5a.

The loop closures of the adjacent geoid profiles are shown in Figure 4.8, and the map of the geoid across Australia with respect to the Australian Geodetic Datum is illustrated in Figure 1.8.

The magnitude of the misclosures in Figure 4.8 deserves some comment.

The deflections ξ and η in Australia are usually determined to better than ±1". This implies an error in ε (σ_ε) of say 1.5". The standard error in a 30 km line, determined by EDM, may be (say) 0.3 m. By simple error analysis we can estimate the expected error in the change in N for 1 line ($\sigma_{\Delta N}$) to be

$$\sigma_{\Delta N} = (S^2\sigma^2_\varepsilon + \varepsilon^2\sigma^2_s)^{0.5}$$

$$= (0.05 + 2 \times 10^{-2})^{0.5}$$

$$= .05 \text{ m}$$

(The correlation between σ_ε and σ_s can be assumed to be negligible).

Taking this around a loop consisting of 100 such lines, the expected misclose (σ_M) might be

$$\sigma_M^2 = (100)\,(.05)^2$$

$$\therefore \; \sigma_M = 0.5 \text{ m}.$$

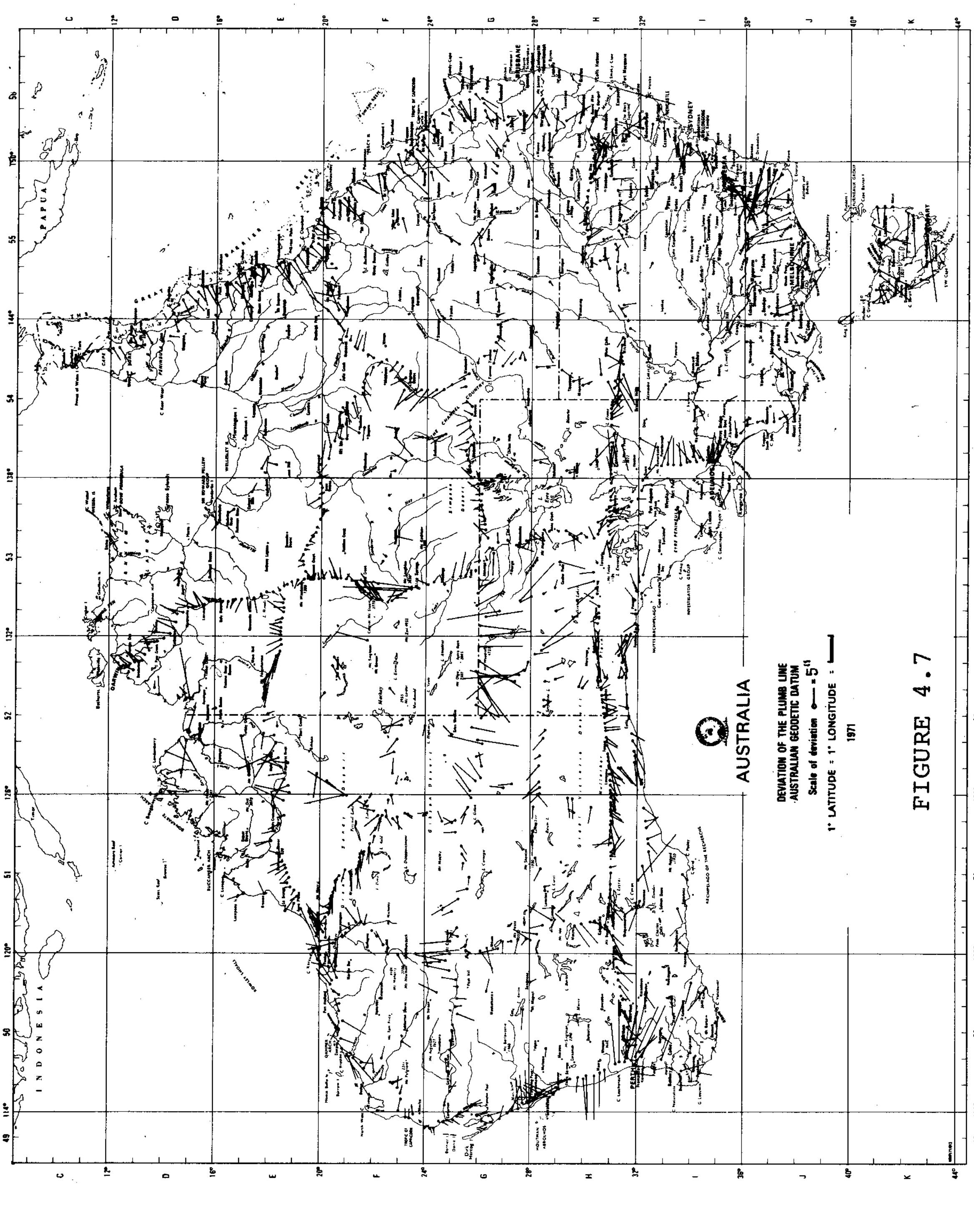

AUSTRALIA
DEVIATION OF THE PLUMB LINE
AUSTRALIAN GEODETIC DATUM
1971
INDONESIA
PAPUA

FIGURE 4.7

The actual miscloses shown in Figure 4.8 are significantly larger than this. The most probable reason is that the assumptions made in the derivation of Equation 4.16 are not accurate. These assumptions are that the geoid tilts evenly with respect to the ellipsoid for the extent of each line, and that the tilt is represented by the deflection determined at the initial station of each line.

In an attempt to satisfy the assumptions made

(i) the line is kept as short as feasible, especially in mountainous areas where it is expected the geoid will undulate more rapidly, and

(ii) the tilt is determined as the mean of the line component of the deflections at each terminal of the line.

Ideally, one would like to determine the deflection at each point midway between changes of grade of the geoid, in the same way spot heights are taken at changes of grade of the terrain for contouring. In this way the deflection will more truly represent the tilt, and the assumptions be better satisfied. Unfortunately, it is not really possible to do this as

(i) one does not really know the behaviour of the geoid until it has been mapped i.e. until after the astro-geodetic survey has been completed, and

(ii) even if it was known, it is not possible to locate stations at will. The requirements of the geodetic survey will restrict the geodetic stations to the tops of the mountains.

A worked example will help illustrate the method.

Name	Φ ° ' "	Λ ° ' "	ϕ "	λ "	$\alpha_{i,i+1}$ ° '	$s_{i,i+1}$ km
Kaputar	-30 16 25.58	150 09 40.05	30.12	48.03	80 00	32.0
Blue Nobby	-30 13 16.21	150 29 31.86	23.09	33.15	81 30	17.1
Gulf Creek	-30 11 53.73	150 40 03.46	51.42	05.99	184 15	32.1
Newry	-30 29 20.13	150 38 22.71	20.95	22.18	269 45	33.2
Byar	-30 29 19.75	150 17 20.37	16.21	25.16	332 00	26.6

The aim is to:

(1) Calculate ξ, η at each station.

(2) Assuming N at Kaputar (Kap) is 10 m on the Australian National Spheroid (Section 1.2.4), calculate N at other stations around the loop.

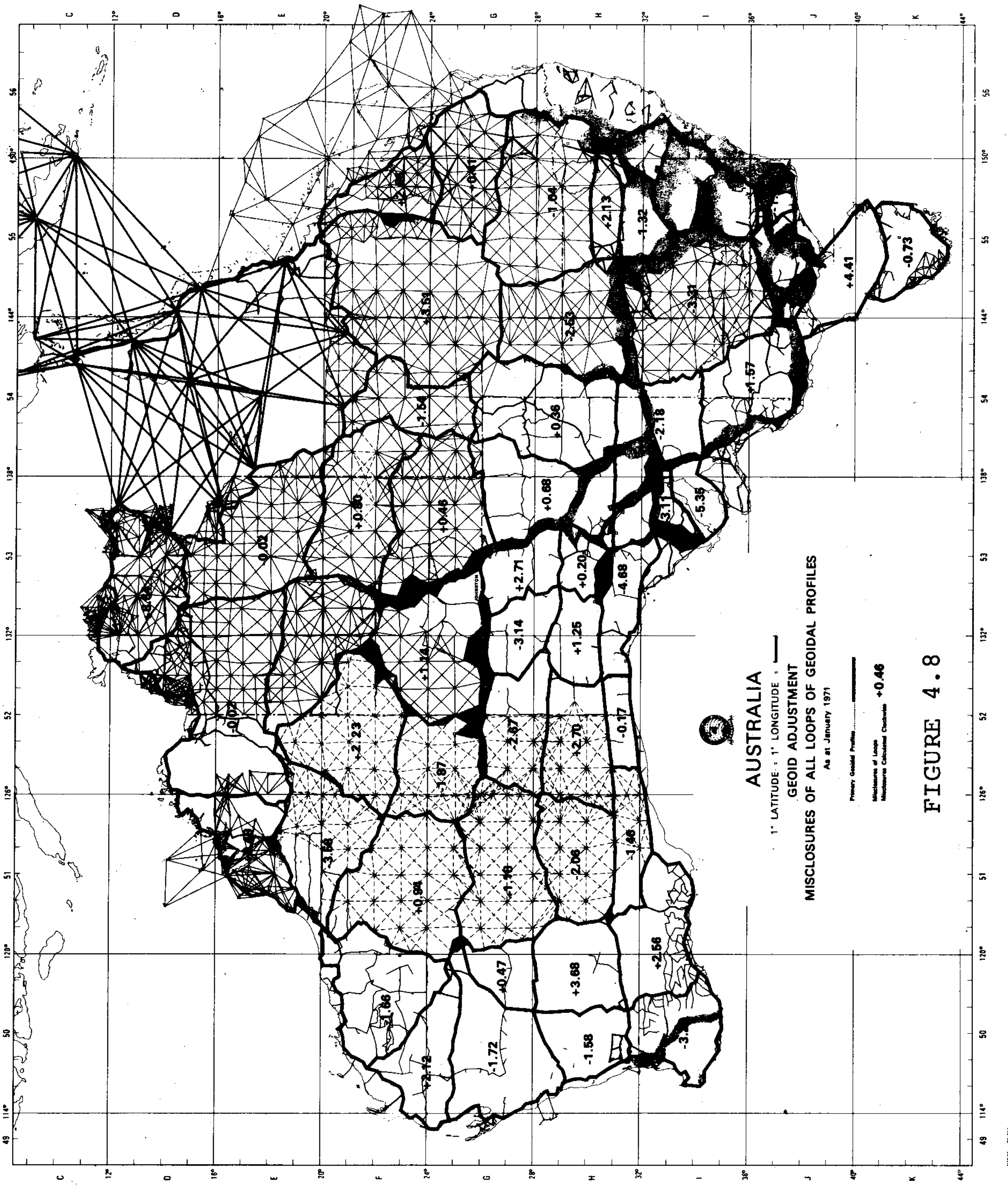

AUSTRALIA
1° LATITUDE : 1° LONGITUDE
GEOID ADJUSTMENT
MISCLOSURES OF ALL LOOPS OF GEOIDAL PROFILES
As at January 1971

FIGURE 4.8

Solutions

STN. (i)	$\Delta\phi=\xi$ "	$\Delta\lambda$ "	η "	$\varepsilon_{i,i+1}$	$-\varepsilon_{i+1,i}$	ε_{α}^{*} "	$\Delta N=-s.\varepsilon_{\alpha}$ m	N m
Kap (1)	4.54	-7.98	-6.89	-6.00	0.92	-2.95	+0.458	10.000
BN (2)	6.88	-1.29	-1.11	-0.08	-2.51	-1.30	+0.107	10.458
GC (3)	-2.31	-2.53	-2.19	+2.47	-0.85	0.81	-0.126	10.565
Newry (4)	0.82	0.53	0.46	-0.46	+4.15	1.84	-0.297	10.439
Byar (5)	-3.54	-4.79	-4.13	-1.19	+7.24	3.02	-0.390	10.142
Kap (6)	4.54	-7.98	-6.89	-6.00	0.92	-2.95	+0.458	9.752

							Σ -0.248	

$$^{*}\ \varepsilon_{\alpha} = \xi \cos\alpha + \eta \sin\alpha \qquad (4.15)$$

4.3.4 An Astro-Geodetic Geoid for Australia

The most comprehensive geoid determination from astro-geodesy for Australia was performed by FISCHER & SLUTSKY (1967). The description below outlines the approach taken by the authors of this paper.

The geodetic values for 600 stations, spaced uniformly through Australia, were computed on the Australian Geodetic Datum (AGD), i.e. the ellipsoid chosen for computation was the Australian National Spheroid (ANS) and the origin - Johnston Origin - defined in accordance with the values given in Section 1.2.3. It should be recalled that this value was deliberately defined by analyzing the deflections of the vertical at 275 stations in the network, and choosing a value to give a reasonably good approximation of the geoid by the ANS.

Using the 600 values for both ξ and η "contour" charts for these parameters were prepared for Australia with a contour interval of one second. Along each degree of latitude and longitude, and at intervals of half a degree, values of ξ and η were interpolated from the charts. The change in geoid-ellipsoid separation was then computed from Equation 4.16, suitably modified for the specific circumstance. The value of N was then accumulated through the grid based upon a value of N at Johnston origin of 0.00 metres.

An adjustment, analogous to an adjustment of interconnecting loops of levelling runs, was then performed to provide final values of N at the grid intersection points. From this the contours of the geoid were plotted, resulting in the map of the geoid shown in Figure 1.8.

4.3.5 Gravimetric Techniques for Determining Geoidal Undulation

The theory for this method, stated simply, is as follows. The solution requires a good global knowledge of gravity and a detailed knowledge of gravity in the region (100 km) of the point at which the geoidal undulation is being computed. By defining a computational model for the earth - taken to be an ellipsoid of stated size, mass and rotating at a known rate about its minor axis - it is possible to compute a theoretical value for gravity for any point whose position (latitude) is given. This theoretical value is known as normal gravity and is denoted by γ. The difference between this and observed terrestrial gravity reduced to the geoid (g) gives the gravity anomaly, where

$$\Delta g = g - \gamma \qquad (4.16)$$

An expression developed by Stokes in the middle of last century enables one to compute the separation between two equipotential surfaces if the forces on these two surfaces are known. Applied to the Earth, it enables the computation of the separation between the geoid (a physical equipotential surface) and the ellipsoid (a theoretical equipotential surface modelling the geoid). The form of the expression is

$$N_P = C \iint \Delta g \, S(\Psi) \, d\sigma \qquad (4.17)$$

where
- C is a constant
- N_P is the geoid-ellipsoid separation at point P
- Δg is defined in (4.16) and is determined over the surface
- $S(\Psi)$ is Stokes' Function, and is a function of
- Ψ the angular distance between P and $d\sigma$, and
- $d\sigma$ is the element of surface area used in the integration.

Although the theory for this was developed over 100 years ago it is only in the last few decades that any meaningful evaluation of the Stokes' integral can be made. This is due to two recent developments: the artificial near-earth satellite, the study of whose orbits have given insight into the large scale features of the Earth's gravity field; and the electronic computer, whose ability to handle large volumes of data and to perform mathematical operations at high speed has made possible the evaluation of the lengthy and tedious computations involved.

Data

In Australia a number of geoid determinations from gravimetry have been performed, thanks initially to the work of R. S. Mather at the School of Surveying, U.N.S.W. Global gravity has been obtained from international bodies, such as the International Gravity Bureau (IGB) in Paris or the Department of Geodetic Science, Ohio State University. The more detailed data for the Australian region has come from the Bureau of Mineral Resources (BMR) in Canberra. This authority completed a gravity survey for the country at a density of about 1 point every 10 km for geophysical exploration, but this data has proved invaluable to the geodesist, as it provides good detailed coverage in the region of the computation point. Individual researchers have undertaken their own gravity surveys to "fill in the gaps" in the BMR surveys where required.

However, there are still large gaps in the data, mainly in the ocean regions off the south-east, southern and south-western coasts of Australia. Research is at present in progress to see if gravity deduced from geoidal undulations derived from satellite altimetry over the oceans can be used to provide more information on the detailed gravity field in these regions.

Results

The geoidal undulations obtained by evaluating Stokes' Integral are often termed "absolute". The reason for this is that the ellipsoid to which the undulations refer is, by dint of the mathematical conditions imposed in the derivation of Stokes' integral, a geocentric ellipsoid. That is, the origin of the ellipsoid is constrained to the Earth's centre of mass or the geocentre. However, this term is misleading in the sense that, because of the deficiencies in the gravity information in some areas of the globe (notably the ocean regions, see above) the geoid calculated may contain quite significant biases and tilts with respect to the true geoid. So it should be recognised that the term "absolute", as used here, is used to distinguish the ellipsoid from a "relative" determination, as would result from the classical astro-geodetic solution (Section 4.3).

Precision

The absolute precision of N will be dependent upon the distance of the computation point to areas of poorly surveyed gravity. It appears that there may be systematic errors in the earlier geoids calculated for Australia of about 10 m. However, relative precision of N for points close to each other (say 1° or 110 km) will be much better, of the order of 10 cm. This is because the systematic errors are significantly the same over short distance.

4.3.6 Combination of Doppler and Levelling Surveys

The position which is obtained from a Doppler-satellite survey is a three-dimensional X,Y,Z coordinate with respect to an axis system centred at the geocentre and oriented according to an internationally defined convention. It is a straightforward matter to transform the X,Y,Z coordinates into a ϕ, λ, h system on an ellipsoid, once the semi-major axis (a) and flattening (f) of the ellipsoid is stated (TORGE, p.52). The height h is obviously a height above the ellipsoid, being the difference between the distance along the normal from the minor axis to the point on the ground occupied by the receiver, and the distance along the same normal to the surface of the ellipsoid (Figure 1.3) for small c.

If the orthometric height of the ground point (H) is also determined (Section 5.1) then the geoidal undulation N is, for most practical purposes, the difference between the ellipsoidal height h and the orthometric height. Please note that the deflection of the vertical in Figure 1.2 is greatly exaggerated for the purpose of illustration. It will not usually exceed 20". Thus

$$N = h - H \tag{4.18}$$

Comments

(1) The value for N obtained this way will be "absolute" (as defined in Section 4.4.1) because the ellipsoid is geocentric.

(2) The precision of N will be a function of the precision two heights used to obtain its value.

(i) The precision of h will depend on which ephemeris, broadcast or precise, is used in obtaining its value, and in which mode (e.g. single-point or translocation) the observation is made.

(ii) The precision of H will depend on how the orthometric height has been determined. This precision could vary enormously, from the most accurate as obtained by precise levelling to the least accurate as determined by barometric levelling. These various methods and their precisions are considered in Section 5.

5. DETERMINATION OF HEIGHT

5.1 Datum of Australia

The first essential in a system for height determination is the datum or reference surface for the heights. It is usual, for obvious reasons, to adopt mean sea level (MSL) as this reference surface. This means, in practice, the establishment of a tide gauge and its observation over a long period to get a good estimate of mean sea level at the gauge. In fact, the full cycle of the moon is about 19 years, and since this is the main body influencing tide it is theoretically necessary to observe a tide gauge for this period of time. However it is possible to obtain an estimate of MSL to about ±20 cm from observations taken over 2 years.

Having established MSL it is then a straightforward surveying exercise to run levels from the tide gauge to the levelling network and thence to establish the heights of all points in the network above MSL. Height determined in this way is often referred to as the "orthometric height".

The datum for Australia, the Australian Height Datum or AHD, has been determined by adopting 30 tide gauges established at about 1,000 km intervals around the coast of mainland Australia. The tide gauges were connected to the geodetic levelling network and held to zero in the network adjustment which was performed subsequently (GRANGER, 1972). As a result, the AHD is not, strictly speaking, an equipotential surface. A "free-net" adjustment (with only 1 tide gauge - Jervis Bay - held fixed at zero) gives height values for some of the other tide gauges of up to 1 metre, an indication of the amount by which AHD departs from an equipotential surface (see Figure 5.1).

5.2 Measurement of Height

5.2.1 Spirit Levelling

(The principles of spirit levelling are familiar to all surveyors and will not be repeated here. Refer to TORGE (1980, 111-115) for a brief summary).

According to LAMBERT ET AL (1976) a levelling network of about 161,000km, mostly to third-order accuracy, has been adopted to provide a network of height stations for Australia (Figure 5.2). The object of the levelling is "to provide a framework of level traverses mainly along roads and tracks with permanent marks placed at intervals of between 1 and 5 miles" (LEPPERT, 1965, 593). This has been augmented by about 83,000 km of levelling by various agencies, connecting the levelling network into selected geodetic and Aerodist stations. This has provided control in the heighting of the geodetic network, established mainly by trigonometric levelling (see Section 5.2.2).

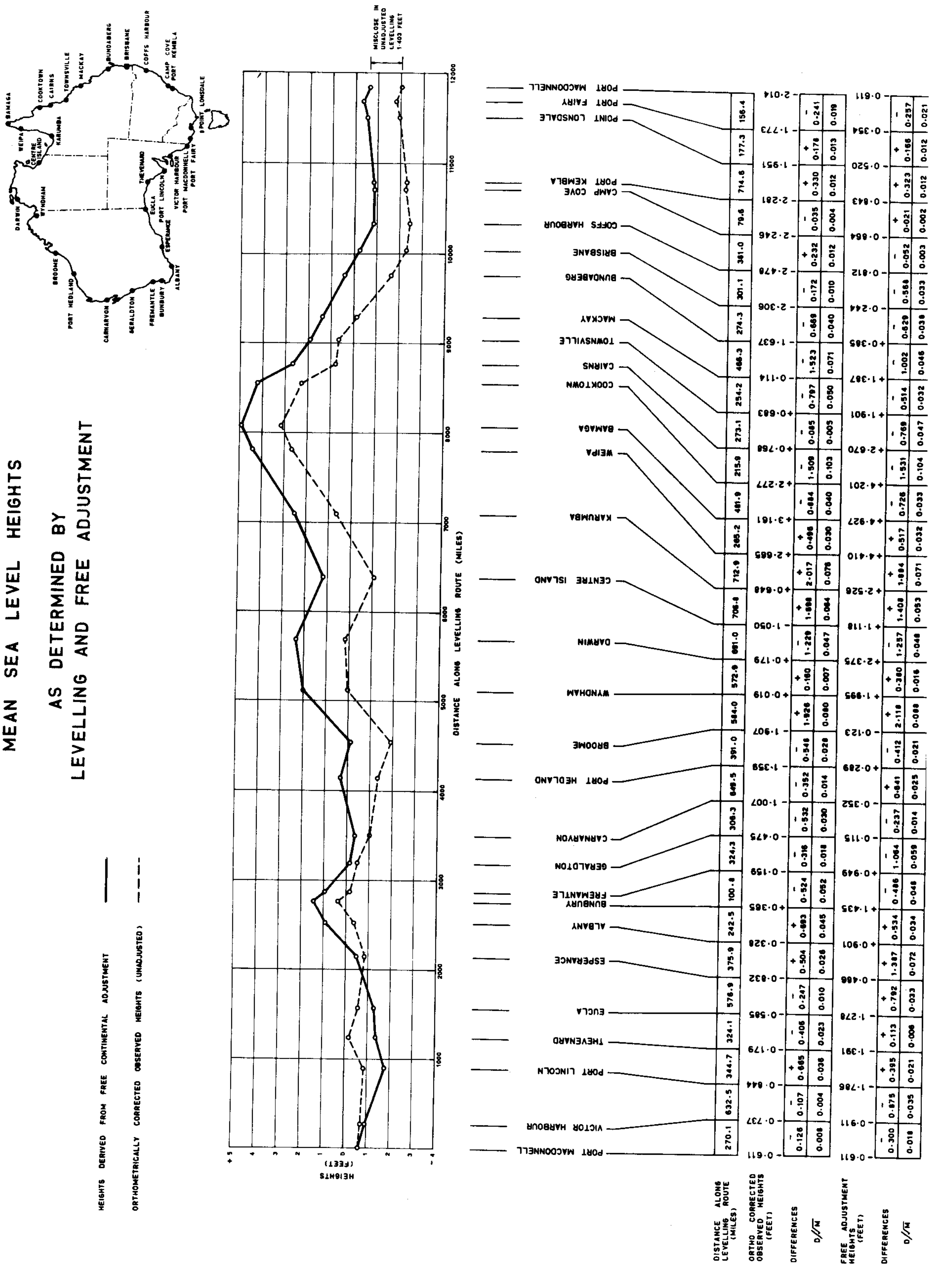


FIGURE 5.1: Mean Sea Level Heights from Geodetic Levelling

The procedure and instrumentation has been described thoroughly in IBID (594-605). The precision of the survey was assessed as follows:-

Average loop closure ±6 $mm/km^{0.5}$. A reasonable estimate of the unit weight standard deviation of the survey was ±7 $mm/km^{0.5}$ for all States except N.S.W., where some problems in the adjusted values were encountered, and either the mathematical model or the data suspected.

The rationale for adopting 3rd-order levelling (as opposed to 2nd or 1st order levelling normally required for geodetic surveys) was mainly one of expediency:-

(1) The cost of the actual survey was $7 million cf. a figure of 3.5 times that if 25% had been done to 1st-order standards, 25% to 2nd-order and the remainder to 3rd-order.

(2) It was considerably faster than a higher order survey (not only in that less care had to be taken, but also that more of the work could be contracted out to the private sector).

(3) It provided adequate precision for mapping, engineering and the coordination of levelling surveys already undertaken in the course of gravity surveys. Plans (as at 1976) are underway to upgrade selected sections of the network to 1st-order standards.

5.2.2 Trigonometric Levelling

The most common means of extending height through a network is by using the technique known as trigonometric levelling (TORGE, 1980, 109-111). Vertical angles between geodetic stations can be observed during the period in which the station is occupied for horizontal angle observation, although, of course, horizontal angles are often observed at night time, and vertical angles around midday. The reason for midday observations is that it is at this time that the atmosphere is likely to be most stable and hence the coefficient of refraction in the vertical plane most likely to assume its theoretical value.

This assumption is, however, only accidentally correct and to assist in controlling the errors in refraction, vertical angles are best observed by the "simultaneous, reciprocal" method. This entails occupation of the stations at both ends of the line and, possibly, radio contact between both observing parties to ensure simultaneity. The assumption made with these observations is that the coefficient of refraction is identical at both ends of the line, and can be solved for as one of the unknowns - the height difference being the other. This assumption is also open to question, as the coefficient is a function of the vertical temperature gradient of the atmosphere through which the light ray passes. This is in turn a function of the cloud cover, wind conditions, ground cover and profile of the terrain at the observing station, and it is coincidental if these are similar at both ends of the line.

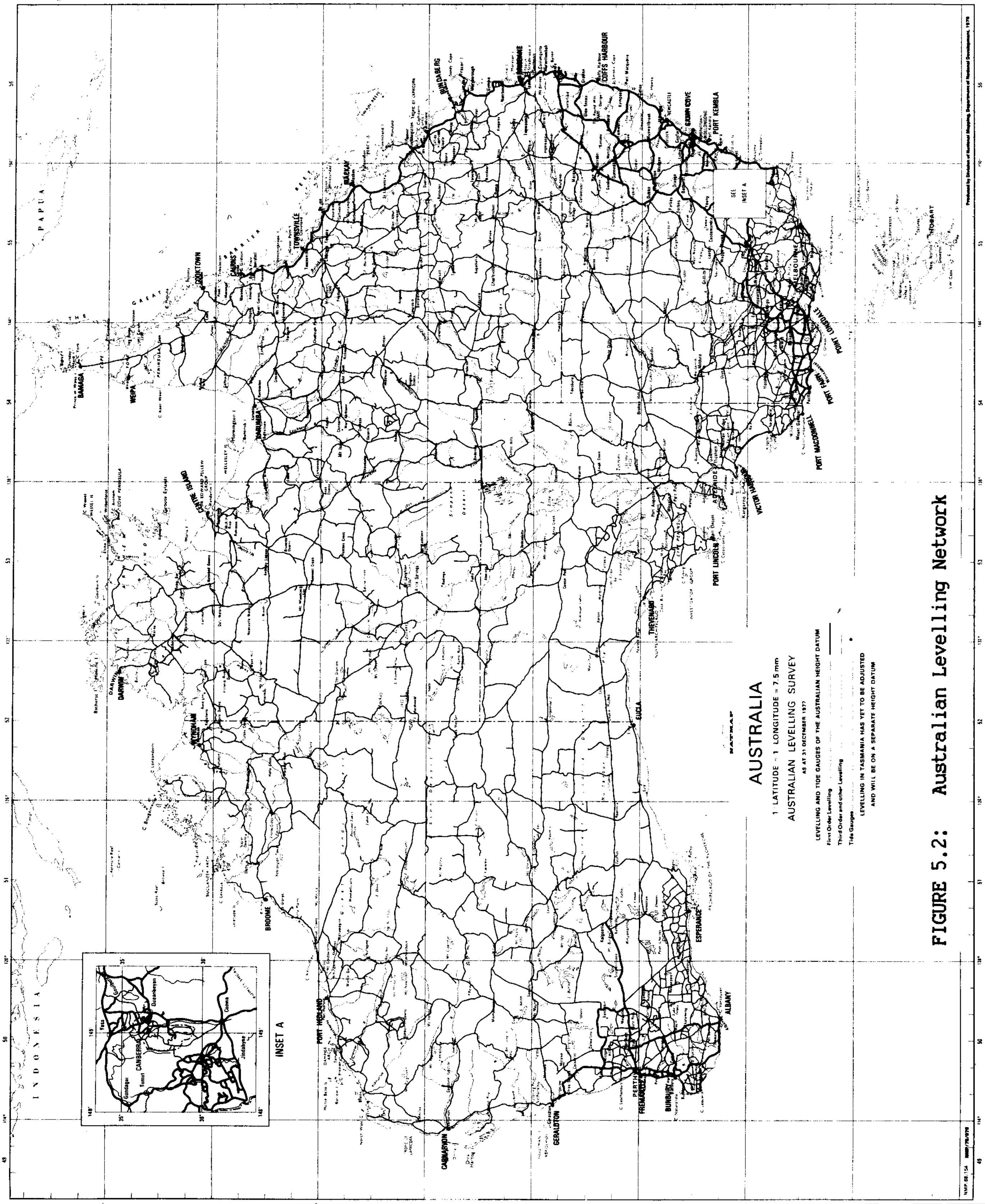


FIGURE 5.2: Australian Levelling Network

The net result is that trigonometric levelling is only capable of giving heights of a low order of accuracy for lines of geodetic length. This is usually quite adequate for distance reductions (Section 4.1.2) where to retain a precision of $1:10^6$ in the computation the height of the line should be known to ±6 m. However, this precision may not match that obtained for ellipsoidal heights from satellite techniques and may therefore produce a deterioration in the precision of the geoidal undulation which results from a comparison of these two (Section 4.3.6).

For an early reference on this topic, see FURBER, 1898. Results of a recent investigation into the use of trigonometric heighting in conjunction with EDM traversing can be found in (RUEGER and BRUNNER, 1981).

5.2.3 Heights from Satellite Surveying

This technique involves the principles outlined in Section 4.3.5, and Equation 4.16 is reorganized such that H (orthometric height) is found from h, the ellipsoidal height measured by the Doppler receiver, and N, the geoid-ellipsoid separation, viz

$$H = h - N.$$

The precision of H derived in this way will depend upon the technique used to determine h (for point positioning σ_h = ±1.5 m; for translocation σ_h = ±0.5 m), and upon the way in which N was determined. This will vary greatly. The most recent Doppler receivers have global geoidal models recorded in memory and these can be expected to be good to say 1-2 m in areas where the geoid is smooth, but an order of magnitude worse than this where the geoid is bumpy (in, for example, mountainous regions). N determined by relative means from gravimetry may be accurate to 0.3 m, but an absolute N from this approach may have systematic errors in it approaching 10 m.

At best, the precision of H determined by this method would be about ±0.6 m.

NOTE: Care must be taken to ensure that the value of N used in the above solution gives the separation between the geoid and the particular ellipsoid used in determining h. Most local geoids maps refer to a local ellipsoid (e.g. ANS) and these values would have to be transformed onto the reference ellipsoid used by the equipment used to transform the X,Y,Z coordinates to the ϕ, λ, h system.

6. GEODETIC DATA BASES

It seems almost an anti-climax that the final product from the enormous effort put into the planning, observation and computation of the geodetic network should be a list of coordinates. Yet without this list, the foregoing investment of the community's skill, time, and money is wasted and it is of utmost importance that the information is accurate and easily accessed by those members of the public who may have need of it. The data should consist of the basic information, such as identifying name, geodetic position and height. But some versions of the data base may consist of much more. For example, as well as geodetic position, the station's projection coordinates (in two zones if necessary) and the deflections of the vertical could be given. As well as orthometric height, geoid-ellipsoid data could also be given. The amount of data will be essentially determined by the purpose of the particular listing, i.e. by the use to which the list will be put.

A data base may appear in various forms.

6.1 Hard Copy Data Bases

(a) Computer listing of the results of the computations. One of the primary sources of the data base but hardly in a form suitable for general usage.

(b) A register of geodetic stations, published by the responsible authority (e.g. Central Mapping Authority, 1980). This simply lists the coordinates of the geodetic stations in a given region. It can be widely disseminated and easily read. However, the information may be inadequate for specialist users, and it is a difficult task to update for corrections.

(c) Microfiche. A microfilm version of (b). It is probably the most efficient way to produce and store hard copy, but it requires special equipment for reading, and suffers in the same way as (b). The Bureau of Mineral Resources publishes details of its gravity stations in this way.

(d) Card System. Each geodetic station has a card on which are listed all particulars relevant to the station (see Figures 6.1, 6.2). This would be the most comprehensive version offered by a hard copy; but would hardly be suitable for publication. It is a system housed and maintained by the geodetic authority but the public could have access to it if it is so desired.

6.2 Computer-Based Data Bases

It is possible to foresee that the public will have access to a computer-based data base in the near future. To the author's knowledge such systems do not exist yet for geodetic data, although some very sophisticated systems are already operating for Land Information System (notably LOTS - see SEDUNARY, 1981). Here, the user has access to the data base through an interactive terminal and can obtain hard copy of desired information through a line printer attached to the terminal.

It is possible that in future the general public could access this data via TELECOM lines through terminals in their own offices, or obtain copies of the data files in computer readable form e.g. magnetic tape or floppy disc.

For a thorough insight into the contemporary approach to geodetic data bases the reader is referred to (TSCHERNING, 1981).

Authority DIVISION OF NATIONAL MAPPING

Original Station Established by: Division of National Mapping Date: 1960

Existing Station Marked by: Division of National Mapping Date: 1960

Reference Books: NM1999,NM2217,NM2091,NM2504,NM2326,NM7406,NM10868,NM12809

Cadastral Location: State Queensland County/District

Parish/Hundred Allotment/Section/Portion

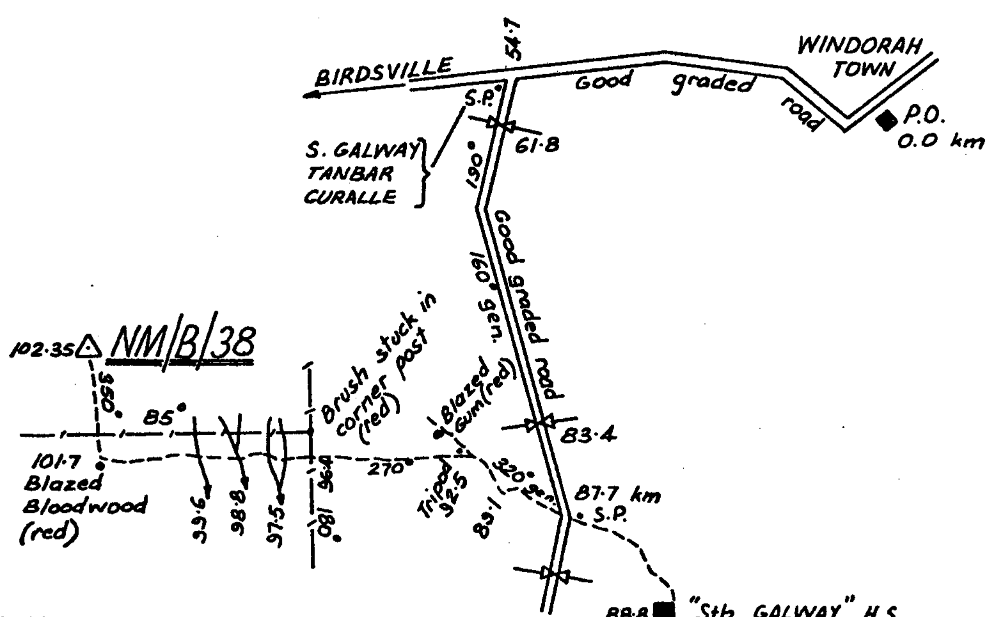
Access and Locality Sketch: Particulars of station marking and beacon:

Station Mark : 0.013 copper tube set in concrete.

Beacon : 3.04 x 0.10 x 0.10 oregon pole with four 0.91 x 0.61 bondwood vanes set 0.06 below top and strutted with four 3.0 lengths of unimetal A circle of piled rocks about 0.3 high and 4.5 in diameter surrounding the beacon.

Reference Marks: Three 0.013 copper tubes set in concrete. RM2 is surrounded by a 0.5 m diameter circle of small rocks.

Access : Approach from Windorah. Take Birdsville road and turn south at Galway S.P.. Turn off this good graded road at 87.7 km and follow station track on bearings shown to blazed Bloodwood at 101.7 km across country through scatted timbers to trig site which is on the ridge. It is necessary to undo the fence at 96.4 km and 101.7 km.



Station revisited by:
National Mapping 1974.
Marks in good condition.

FIGURE 6.1: Station Summary - access and station mark details (NATMAP, 1976).

Serial No

Station Number and Name: NM/B/38 Order: FIRST

Map Name: Canterbury Map Number: SG 54-7 Scale 1: 250 000

DATUM: Australian Geodetic Datum 1966 All dimensions in metres except where otherwise shown.

RECTANGULAR COORDINATES: Australian Map Grid: In Metres AHD HEIGHT 192.405

GRID BEARING = ADJ AZIMUTH + CONVERGENCE HEIGHTS: In Metres on the Australian Height Datum

NM B 38 SECTION 17 15 SERIAL 6

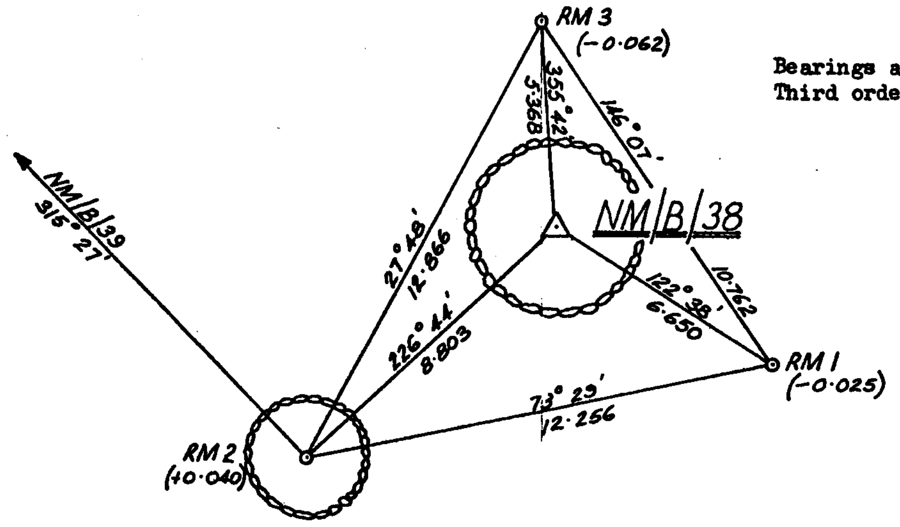
SOUTH LATITUDE	EAST LONGITUDE	ZONE	EASTING	NORTHING	CONVERGENCE
25. 36. 31.4978	141. 59. 21.2100	54.	599328.431	7167263.088	+0. 25. 39.36

TO		SERIAL		ADJ AZIMUTH	ADJ LENGTH
	NM B	37	5	131. 42. 2.34	54975.092
BUTLER	NM B	39	10	315. 25. 44.09	28434.995
	NM B	40	7	337. 56. 44.60	40433.524

SECTION NM74 QD1 SERIAL 3

	NM B	309	4	230 57 35.66	40088.290

Cadastral Connections & Reference Marks.



Bearings are true.
Third order level connection, 1967.

Photo Identification: Spot photo: Station mark CAB 3453 Exp 36 -40.
Mapping photo: Canterbury CAB 92 Run 8/5018.

Certified free of transcription errors: [signature] Date: 7-5-75

Approved by: K. [signature] Date: 21-8-75

FIGURE 6.2: Station Summary - Positional Information (NATMAP, 1976).

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46.* Aust.J.Geod.Photo.Surv. No. 31, (December 1979) 177 pp.
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