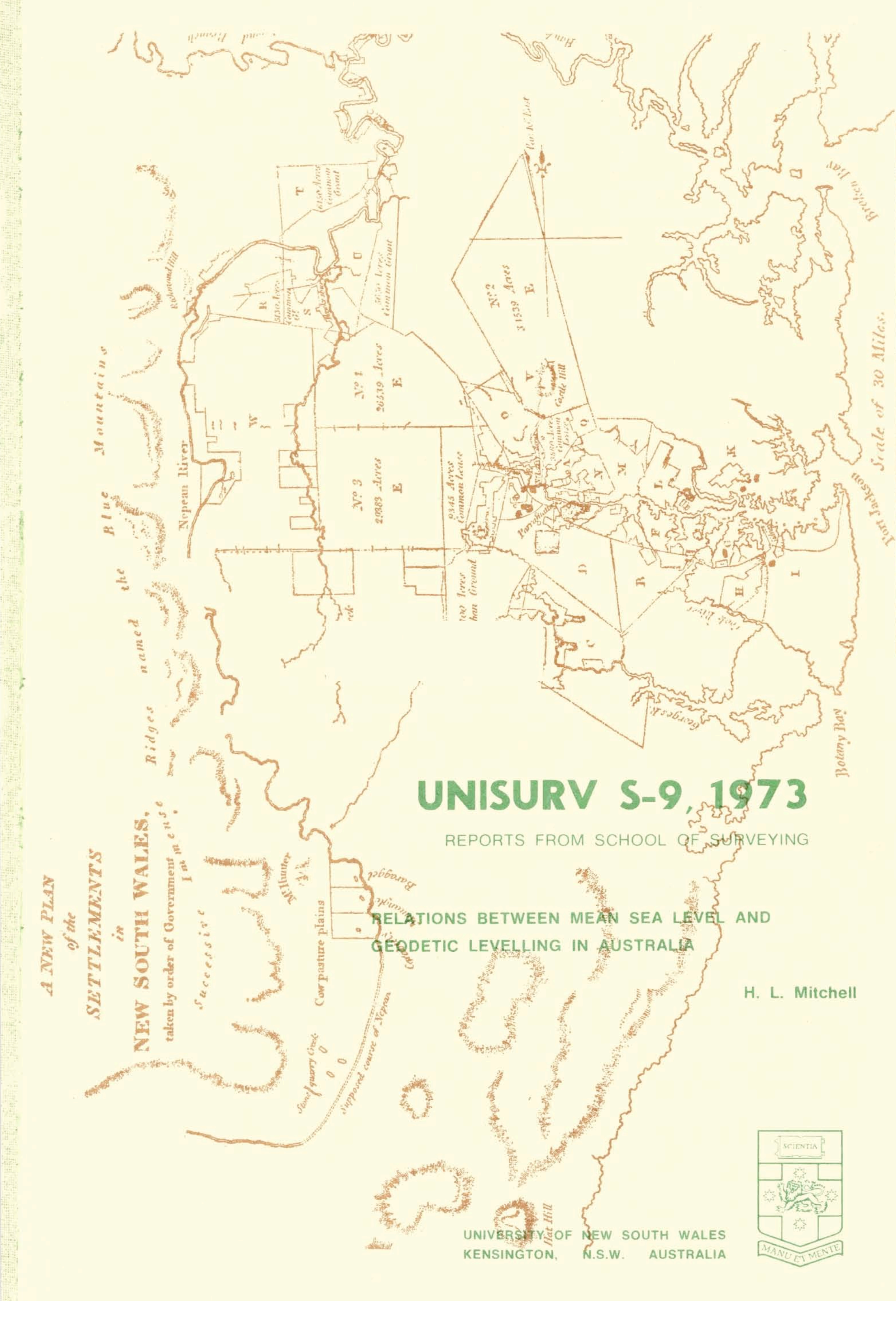

UNISURV S-9, 1973

REPORTS FROM SCHOOL OF SURVEYING

RELATIONS BETWEEN MEAN SEA LEVEL AND GEODETIC LEVELLING IN AUSTRALIA

H. L. Mitchell

UNIVERSITY OF NEW SOUTH WALES
KENSINGTON, N.S.W. AUSTRALIA

Reference to Districts.

A Northern Boundaries

B Liberty Plains

C Banks Town

D Parramatta

EEEE Ground reserved
for Govt. purposes

F Concord

G Petersham

H Bulanaming

I Sydney

K Hunters Hills

L Eastern Farms

M Field of Mars

N Ponds

O Toongabbey

P Prospect

Q

R Richmond Hill

S Green Hills

T Phillip

U Nelson

V Castle Hill

W Evan

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London: Published by John Booth, Duke Street, Portland Place, July 20th, 1810

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UNISURV REPORT No. S9

RELATIONS BETWEEN MEAN SEA LEVEL AND GEODETIC LEVELLING IN AUSTRALIA

H.L. Mitchell

Received June, 1973

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CORRIGENDA.

p.17 Equation 2.1 *For:* W, *read:* ΔW.

p.17 Line 14 *For:* n, *read:* h.

p.21 Line 16 *For:* station, *read:* section.

p.28 Line 1 *For:* 2.5^{-7}, *read:* 2.5×10^{-8}.

p.29 Line 10 *For:* e, *read:* e^2.

p.34 Line 1 *For:* M{e}, *read:* M{e^2}.

p.34 Line 16 *For:* divided, *read:* derived.

p.68 Line 1 *For:* $\ldots\sigma^2_{\Delta W_n}$, *read:* $\ldots+\sigma^2_{\Delta W_n}$

p.79 Line 20 *For:* 1.3 + 2.5 + 3.3 + 4.0 + 3(-3), *read:* 1 x 3 + 2 x 5 + 3 x 3 + 4 x 0 + 3(-3).

p.79 Last line *For:* 1.3 + 2.0 + 3(-3) + 4(-5) + 3(-3). *Read:* 1 x 3 + 2 x 0 + 3(-3) + 4(-5) + 3(-3).

p.97 Line 2 *Delete:* 'about friction'.

p.104 *Ordinate scale should read:* 1.5, 2.0, 2.5.

p.110 *Ordinate scale should read:* 45, 40, 35.

p.117 *For first paragraph, read:*

"gauge records after known variations have been eliminated, with air-pressure records over the same epoch for the same place. This has been undertaken in Australia by *Easton and Radok* (1970a) and by *Hamon* (1966). Their results (Tables 6.1 and 6.2) do not indicate that α is always equal to -1.01 cm $mbar^{-1}$. In Table 6.1, particularly, the co-efficients which range from -8.4 to +1.8 and which vary with time as well as with position, do not suggest any consistent value of α. The apparent error in the co-efficient may be attributable to sea-level fluctuations which are in phase with air-pressure variations which have not been eliminated from the original sea-level records. *Hamon* (1962; 1966) has suggested that the deduced values of α are influenced by shelf-waves, which travel along the continental shelf away from air-pressure disturbances. However, it is worth noting that the mean of all figures tested in the final column of Table 6.1 is -1.20 cm $mbar^{-1}$, which approaches the theoretical value, -1.01 cm $mbar^{-1}$.

p.126 Line 2 *For:* diurnal, *read:* semi-diurnal.

p.146 *For second last paragraph, read:*

"Rises or falls in the land level may be due to earthquakes in the short term, or post-glacial uplift, for example, in the long term. Sea-level is also considered to be affected over a long period, by the build-up of sediment on the ocean floor. A detailed coverage of causes of secular variation of sea-level is given by *Fairbridge* (1960).

p.169 Sixth last line *For:* density, *read:* directly.

p.171 Line 7 *For:* levelling tide-gauge, *Read:* levelling/tide-gauge.

p.173 Line 20 *For:* a, *read:* a^1.

p.176 *For sentence beginning on line 3, read:*

"Whether their findings are applicable at operational satellite altitudes and over different pulse widths is also unchecked.

p.187 Line 9 *For:* 200 m, *read:* 200 km.

p.187 Line 27 *For:* imperitive, *read:* imperative.

p.214 Line 20 *For:* observations, *read:* derivations.

SUMMARY

The Australian third-order geodetic levelling network which was connected to a number of tide-gauges around the coastline disclosed a two metre difference in sea level between the northern tip of Queensland and Spencer Gulf in South Australia. This study was initiated to examine the use of tide-gauges in conjunction with levelling in Australia, as the two metre discrepancy was not explained by the expected levelling survey errors.

The effect of non-parallelism of equipotential surfaces on the levelling net was investigated by converting all observed levelling into geopotential differences, using observed gravity values. Comparison with the original levelling showed that there were negligible differences between the two systems. The network of potentials was adjusted, by the method of conditions, and the accuracy of potential differences in the net was estimated. Not only did the adjustment maintain the aforementioned two metre deviation of sea level, but the error estimate also verified that the discrepancy could not be explained by the normal propagation of random levelling errors.

Attention was then turned to the known causes of the deviation of the sea-surface from a level surface. The Mean Sea Level over five years was accepted as a satisfactory mean of all tidal fluctuations. Existing data illustrated that the ocean density/current effect on sea level was a significant cause of the deviation of the sea-surface from a level surface. Using this data, corrections were applied to the Mean Sea Levels, thereby partially reducing the apparent variation in sea level. Corrections for air-pressure were also applied. A survey of

other recognised causes of deviations between the sea-surface and a level surface was undertaken, to no avail.

As the origin of the variation in sea level was still not apparent, the application of recent developments in satellite altimetry to the ultimate solution of the problem was investigated. Particular attention was paid to the feasibility of using the GEOS-C satellite which is due for launching in 1974.

TABLE OF CONTENTS

1. TIDE-GAUGES AND GEODETIC LEVELLING IN AUSTRALIA

1.1 Introduction

Between 1945 and 1970, approximately 160 000 km of levelling was carried out over the continent of Australia, under the direction of the Division of National Mapping of Australia's Department of Minerals and Energy, for the National Mapping Council of Australia. This levelling was designed to provide, as quickly as possible, suitable vertical control over Australia. During the survey, levelling connections were made between the network and a number of tide-gauges around the coastline of mainland Australia. It was intended that the tide-gauge survey would enable connections between the various datums which were based on local Mean Sea Level, (*Roelse et al*, 1971).

Geodetic levelling and tide-gauge records are often compared, on the assumptions that firstly, the mean position of the ocean surface at all points on the coastline closely approximates a level surface, and that secondly, the levelling network is capable of defining such a level surface. Thus, the mean sea level at all gauges would be expected to have the same elevation according to the levelling results, with some variation due to errors in the levelling and the failure of the mean position of the ocean surface to correspond exactly to a level surface. These assumptions are discussed more closely in Section 1.4.

During the collation of the Australian levelling results, the elevations provided by the levelling were found to differ markedly from those given by the tide-gauge records. The differences were of a magnitude which was larger than expected from the errors of the levelling, and which would not be immediately explained by departures of Mean Sea Level from a level surface. There were no reports of comparable experience overseas, where discrepancies were not of the magnitude of those obtained in Australia. The networks often covered smaller areas.

An explanation of the discrepancies seemed desirable, particularly if the levelling network was faulty.

The research programme described in this report was commenced in 1970 to study firstly, the Australian third-order levelling network and, secondly, the use of tide-gauge records in comparison with levelling results, in the hope of gaining some insight into the following points:

(i) whether the differences indicated in the Australian levelling network by the connections to tide-gauges are due to erroneous levelling network results;

(ii) whether the discrepancies are due to faulty tide-gauge data or to the reduction or application of the data;

(iii) whether levelled heights and sea levels at tide-gauges can be profitably compared, and, if so, what is the likely magnitude of agreement between the level surfaces? This point could be very relevant to any future levelling surveys carried out on this continent.

1.2 Levelling and Sea Level in Australia

Although the levelling which was used in the Australian network is of varying standards of accuracy, the net may be described as a *third-order* network. Most of the levelling is of third-order standard, for which differences between forward and reverse levelling are required to be less than $12\sqrt{K}$ mm, where K is the length of levelling in kilometres. A network of greater accuracy is planned by the Division of National Mapping, but the third-order network provided, in as short a time as was possible, a system of vertical control which was suitable for the pressing requirements of 1: 100 000 mapping and of geological and geophysical surveys. The Division of National Mapping has corrected the levelling observations for non-parallelism of the potential surfaces by an *orthometric correction* based on the theoretical value of gravity, (see Chapter 2). The network has also been adjusted, using approximately 760 sections of levelling formed into about 260 closed, interlocking loops, with circumferences between 200 and 2 000 km in length. Details of the network, including the levelling routes and the Division of National Mapping's adjustment, are given by *Leppert* (1970) and by *Roelse et al* (1971).

The distribution of the tide-gauges involved in this project is shown in Figure 1.1. The tide-gauge numbering system which is followed in this report, is also shown. Tide-gauges at Melville Bay (16) and Eden (30) were not included in the determination of the relationship between sea level and levelled heights. Although all gauges did not function continuously from January 1, 1966 to December 31, 1970, data was collected for this period by the Horace Lamb Centre for Oceanographical Research, of the Flinders University of South Australia. Complete details of the gauges and their records are given by *Easton* (1967a; 1967b; 1968; 1970) and by *Easton and Radok* (1968; 1970a; 1970b).

After the network adjustment in 1971, the heights of Mean Sea Level at 30 tide-gauges were calculated for the network. Mean Sea Levels were calculated from all available hourly data for the period January 1966 to December 1968 for each tide-gauge. As stated in Section 1.1, Mean Sea Levels at all gauges in the network are usually expected to be at a similar level, with some small variation due to the random errors of the levelling. The resultant heights of Mean Sea Level for the Australian network are shown in Figure 1.2, which was adapted from *Roelse et al* (1971, Annexure C). This figure features an apparent 2 m difference between sea levels at Port Lincoln (3) and Bamaga (20), with

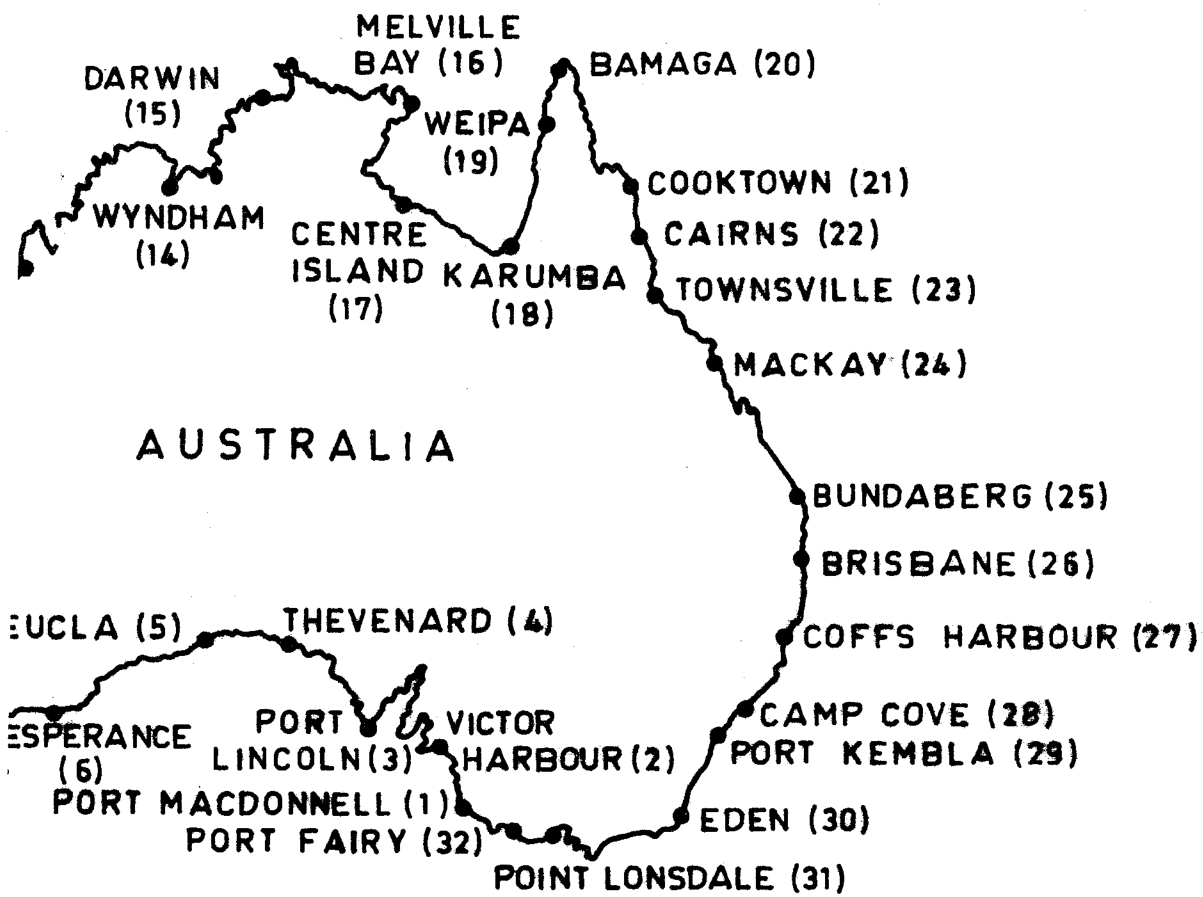


FIG. 1·1.
AROUND MAINLAND AUSTRALIA OF
.VED IN THE GEODETIC LEVELLING SURVEY.

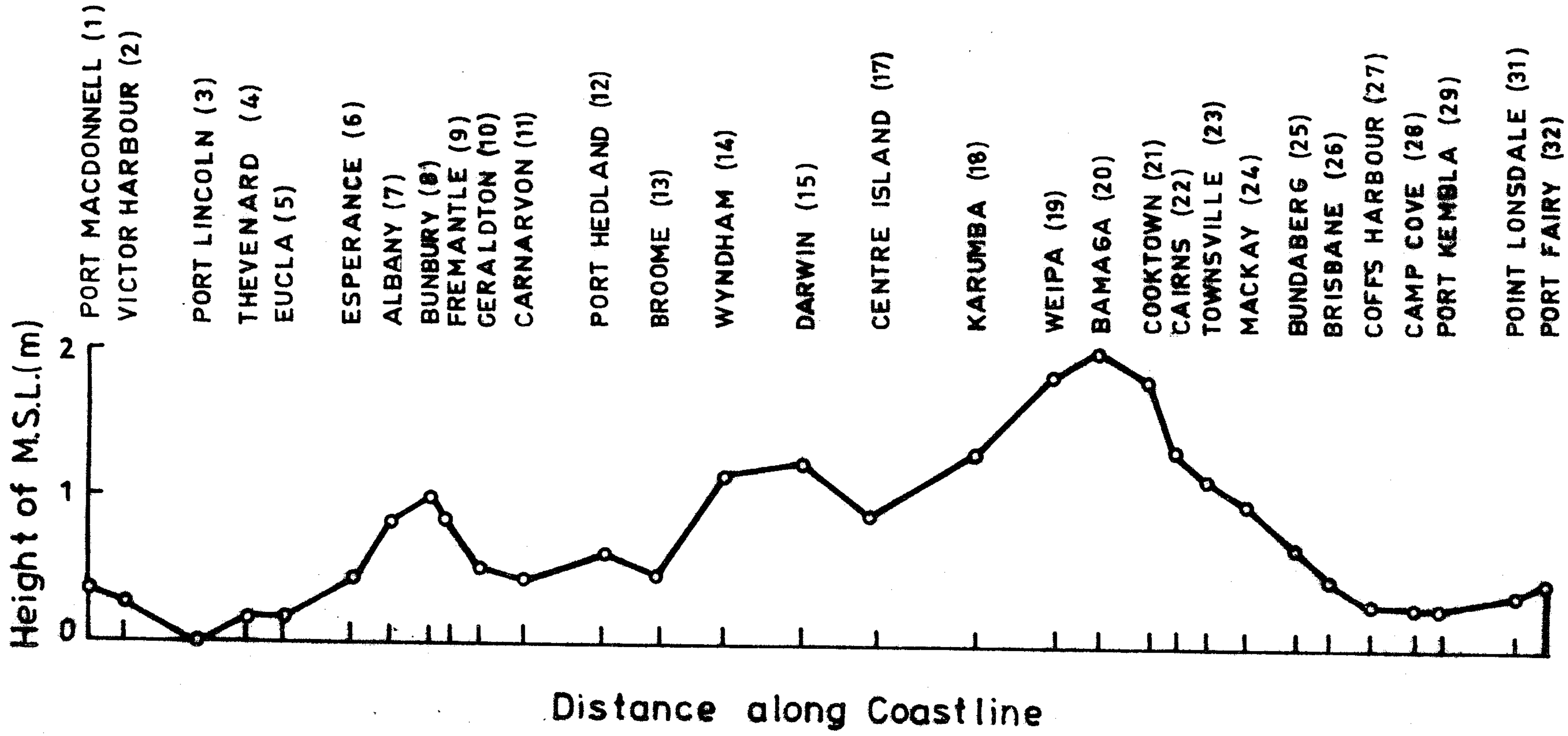


FIG. 1·2

HEIGHTS OF MEAN-SEA-LEVEL AT TIDE-GAUGES ACCORDING TO THE FREE ADJUSTMENT OF THE AUSTRALIAN LEVELLING ADAPTED FROM ROELSE et al (1971). THE HEIGHT OF M S L AT PORT LINCOLN HAS BEEN ADOPTED AS ZERO.

NUMBERING OF TIDE GAUGES AS IN FIGURE 1.1.

the steepest slope occurring between Coffs Harbour (27) and Bamaga (20). A rise between Port Lincoln (3) and Bunbury (8) is also prominent. Figure 1.3 shows the apparent variation in sea level as a function of latitude.

The accuracy of the levelling in this network has been discussed by *Roelse et al* (1971). After adjustment, the estimated standard deviation of heights in relation to the network origin, which is situated towards the centre of the continent, had values of 30 to 40 cm on the coastline (*ibid*, Annexure F). That is, the standard deviation of the height of any Mean Sea Level shown in figure 1.2, when referred to a height in the centre of the continent is of the order of 35 cm. This is significantly smaller than the 204 cm variation in sea level which has been shown by the levelling results. The existence of the discrepancies in sea level elevations is not explained, therefore, by the errors expected from the type of levelling survey carried out in Australia.

1.3 Overseas Experience

A number of overseas geodetic levelling networks have been connected to tide-gauges for a comparison of the sea-surface with levelled heights, but the discrepancies which were found are not as large as those in the Australian network.

Levelling in the United European Levelling Network, as reported by *Levallois* (1960) showed some variations between sea level and levelled heights. If the levelled heights were assumed to be correct, then sea level had an apparent elevation of +28 cm at Kemi, Finland, an apparent elevation of +14 cm at Cascais, Portugal, and of -34 cm at Genoa on the Mediterranean Sea. Generally, sea levels had apparent elevations of the order of ±10 cm in this network. However, Petrov's analysis of reports about the levelling shows that the errors in the levelling do not permit any significance to be attached to these results, (*Petrov*, 1965, p.247).

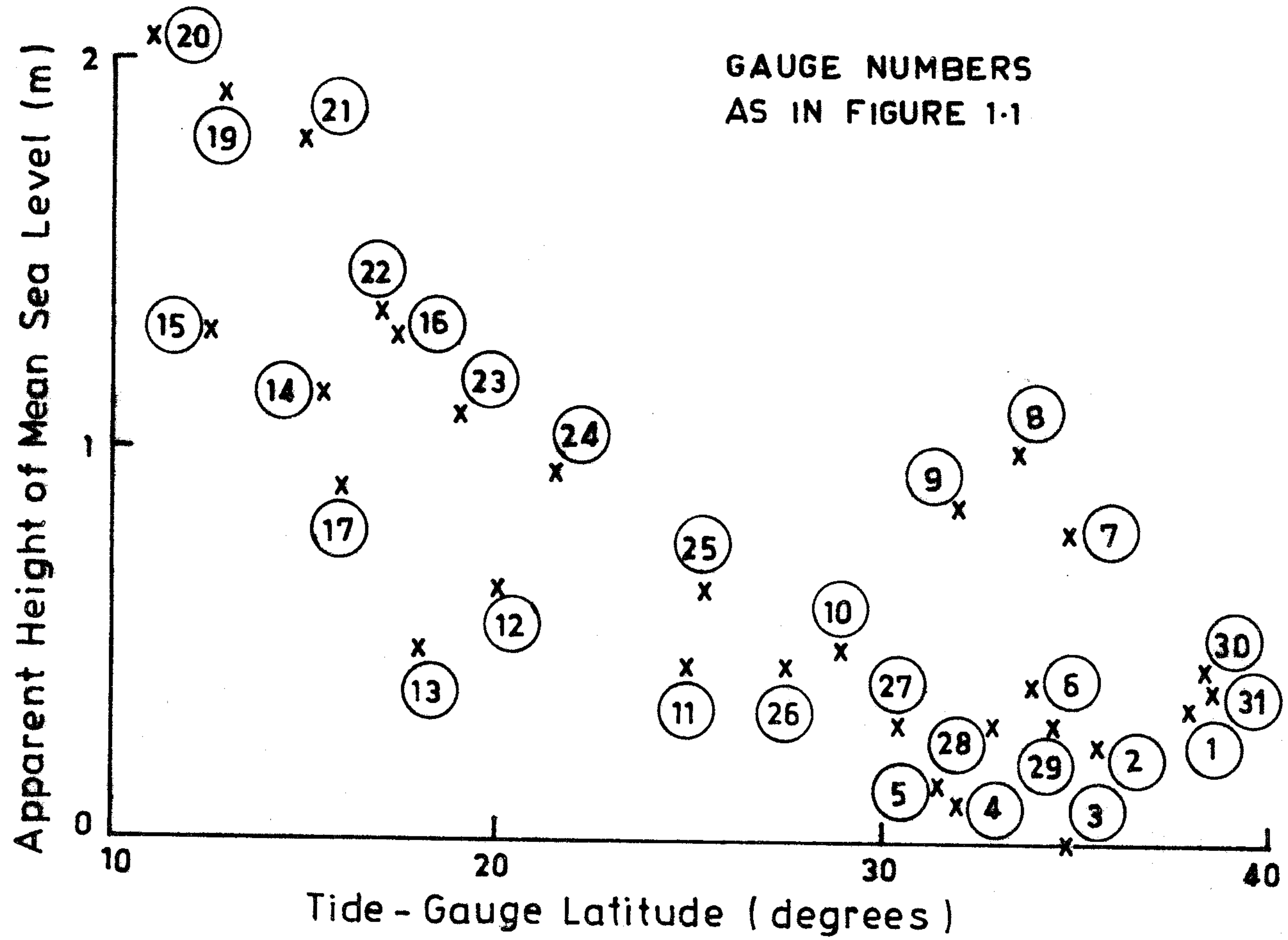


FIG. 1·3

RELATIONSHIP BETWEEN TIDE - GAUGE LATITUDES AND THE APPARENT HEIGHT OF SEA - LEVEL, AUSTRALIAN LEVELLING NETWORK, M.S.L. AT PORT LINCOLN (3) ASSUMED TO BE ZERO.

The most recent first-order levelling net of France indicated that between the Atlantic Ocean and Mediterranean Sea, sea-level fell by about 30 cm, the difference apparently being concentrated around the Straits of Gibralter, (*Levallois and Maillard*, 1970).

U.S. experience, as reported by *Sturges* (1967) and *Braaten and McCombs* (1963), based on the levelling used in the 1963 adjustment by the U.S. Coast and Geodetic Survey, indicated that the sea-surface slopes upward from south to north by about 50 cm along both the Pacific and Atlantic coasts, and that at a given latitude, the Pacific Ocean is higher than the Atlantic Ocean.

Petrov (1965, p. 246) reported that no conclusions could be drawn from levelling in the U.S.S.R. of about 1934, which showed the Black Sea to be 4 cm below the Baltic Sea.

The 1970 re-adjustment of the geodetic levellings of Great Britain, connected to about six gauges, showed maximum and minimum sea levels to differ by 29 cm, between Aberdeen in Scotland and Newlyn in the south, (*Kelsey*, 1970).

Rodriguez (1970) reported that, according to the Brazilian first order levelling, the sea level variation is 41 cm between Rio de Janeiro and Fortaleza, which are separated by over 1 500 km of coastline.

Although the overseas networks are of first-order standard, and the Australian network covers a large area, the apparent deviations between sea level and levelled heights in Australia are significantly larger than any others that have been reported.

1.4 Research Programme

The study which is described in this report was aimed at explaining the anomalous relationship between sea level and geodetic levelling in Australia, as illustrated in Figure 1.2. A research programme was designed to investigate the assumption which was stated in Section 1.1, that corrected sea levels at all coastal points should have the same heights according to a levelling network. This is the basic assumption which is necessary if a levelling network is connected to tide-gauges. More specifically, the comparison between sea level and levelling requires that both the sea level as given by the tide-gauges, and the levelling network, will define a level surface. The possibility of failure of firstly, the geodetic levelling network, and secondly, the sea-surface, to define a level surface around Australia, to the expected accuracy, has been considered in this study.

If some account is taken of the variation of gravity over the area of the survey, a levelling network should define a level surface, that is, an equipotential surface of the earth's gravitational field. The accuracy of the definition will depend on the standard of levelling. Generally, it is assumed that the levelling will define the particular level surface known as the *geoid*. This may be defined as a reference equipotential surface of the earth's gravitational field, and is chosen to closely approximate mean sea level. All points which lie on the geoid will have the same elevation in the network. This study of the Australian levelling firstly investigates methods of accounting for the gravity variation, (Chapter 2). Thereafter, in Chapter 3, an estimate of the accuracy of height differences in the net is made.

The assumption that the surface of the oceans coincide with an equipotential surface, or the geoid, is studied in Chapters 4, 5, 6 and 7. The surface of a volume of homogeneous liquid in hydrostatic equilibrium under the influence of the earth's gravitational field coincides with a surface of constant geopotential. If the *oceans* were homogeneous and in hydrostatic equilibrium, then at all points on the coastline, the position of the geoid would be defined by the ocean surface, as indicated by a tide-gauge. The oceans do not fulfill the requirements of homogeneity and hydrostatic equilibrium, and the sea surface deviates, with time and position, from an equipotential. Time variations are often assumed, in practice, to be accounted for by taking a mean of sea level observations, as obtained at tide-gauges, over a period of time. The position variations cannot be neglected if tide-gauge results are to be used to check the geoidal determination by geodetic levelling. Both position and time variations are important in this study. While the former term refers to the subject deviations of sea level from an equipotential surface, variations at any point, with time, are important as the Mean Sea Levels were calculated from hourly observations of sea level over a specific period of time, in this case, five years. Thus a station-dependent bias could be produced in the means of sea level. Further, time variations are valuable in tracing the causes of position variations.

The shortest-period effects on sea level are due to ocean swell, surface waves and chop, which have periods of the order of one minute and less. Their effects are periodic and may be regarded as time variations only. However, as tide-gauge recording mechanisms are insensitive to sea level variations of such short period, waves with a period less than a few minutes are inconsequential and need not be considered further.

One of the most widely known causes of variations of the level of the sea-surface is the tides, which will be studied in Chapter 4. Tides result from the variable attraction of the sun and the moon on the waters of the oceans. The resultant fluctuations of sea level have various periods, from twice daily, through daily and monthly to a tide which has a 1 600 year period. Amplitudes of tides vary from place to place. For the semi-diurnal tide, the amplitude may be of the order of 50 cm (e.g. Port Macdonnell, South Australia) or as large as 6 m (Spring tide at Secure Bay, Western Australia).

According to the isostatic theory of sea level, the surface of a column of ocean water which is of low density will be elevated relative to the surface of higher density water. Evaluation of the resulting sea level differences may be made either from the water densities or from the flow of ocean currents, and will be discussed more closely in Chapter 5. The flow of ocean currents is related to the pressure differentials which result from differences in water density. This, in turn, is a function of the salinity and temperature of the water.

Inter-action between the atmosphere and the oceans produces a number of time and position variations in sea level which are discussed in Chapter 6. Probably the most significant effect is that of air-pressure acting vertically on the sea-surface. Variations in atmospheric pressure produce corresponding changes in the level of the surface of the sea. Wind effects are also significant in the production of short-period effects, such as storm surges, as well as longer period phenomena. Short-period waves, including shelf waves and seiches result from inter-action between the atmosphere and the oceans.

Less significant influences on sea level are studied in Chapter 7. Erroneous gauge records may result from such gauge faults as clock or pen failure or from damage to gauge installations. River flow can also detrimentally affect gauge records of sea level. Mention is also made of tsunamis, slow aperiodic changes in sea level and variations in the volume of water in the oceans.

Despite the study of levelling network (Chapters 2 and 3) and of the variations of sea level discussed in Chapters 4, 5, 6 and 7, the apparent variation of Mean Sea Level around Australia cannot be completely accounted for. An independent study of this phenomenon by *Hamon and Greig* (1972) has also lead to the conclusion that the problem is presently insoluble. The application of the technique of satellite altimetry, which is presently under development, to provide information on the relationship between sea level and the geoid around Australia, is discussed in Chapter 8. The ultimate solution of the problem in Australia may require results from this source.

2. THE CONVERSION OF THE OBSERVED LEVELLING TO GEOPOTENTIAL

2.1 The Significance of Levelling Operations

The process of defining elevations, or heights, on the physical surface of the earth by the conventional method of spirit levelling has an inherent problem which necessitates the correction of results prior to the computation of elevations. Although it must be conceded that the levelling operation has many applications in practice, its use along conventional lines requires the validity of the assumption that any two geopotential surfaces are separated by a constant linear distance.

The process of levelling is outlined in any treatise on the subject of surveying, including *Bomford* (1952). Only a brief description is given here. A *level*, or levelling instrument, is set-up so that its line of sight is tangential to the equipotential or level surface at the instrument. Observations are then made onto linearly graduated staves, which stand vertically at points for which the elevation difference is required, see Figure 2.1. The difference in height between A and B may be calculated from the readings made on the staves. The process may be extended over an area, and, if the height of any one point in the area is given, the height differences thereby obtained may be used to form a system of heights.

A close study of this operation reveals that it contains assumptions which are not completely accurate, especially when applied over a large area. The line of sight of the level instrument, which is assumed to be in correct adjustment, defines a plane which is tangential to the equipotential surface which passes through the instrument. A staff defines a length, in space, in a direction perpendicular to the local equipotential surface. Thus, the spirit levelling operation combines a method of defining linear displacements along the vertical with a method of defining an equipotential surface. Consequently, certain relationships between these two physical quantities must be assumed to exist when levelling. Specifically, it must be assumed that a

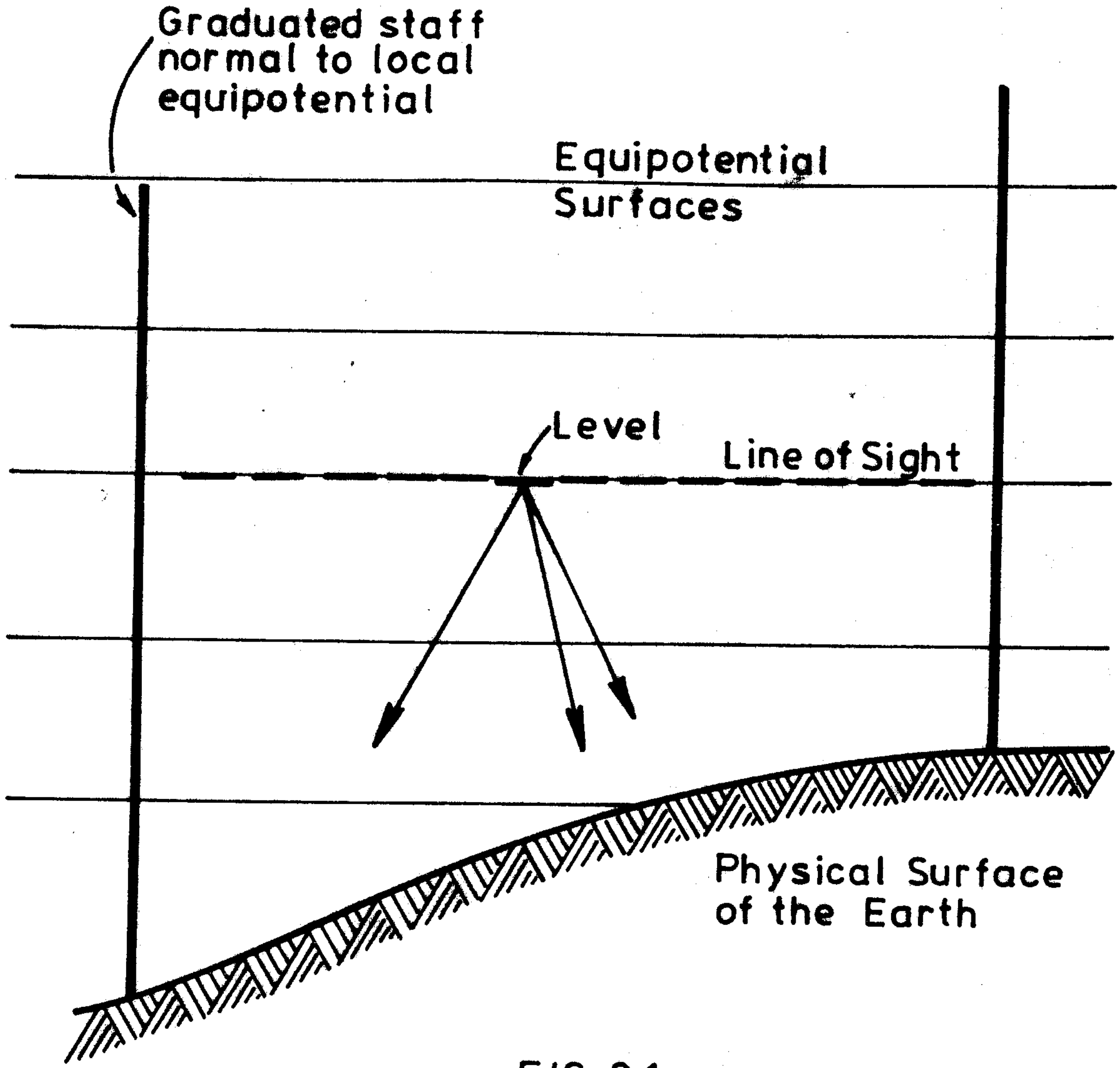


FIG. 2·1

DIGRAMMATIC REPRESENTATION
OF LEVELLING SET-UP

function exists for the *linear separation* between *equipotential surfaces*. That is, the equipotentials must be separated by an amount which is directly proportional to their potential differences. The surfaces need to be parallel. This is only approximately true. For a small or low order survey, this may not be significant. As the survey area is enlarged, the convergence of the equipotential surfaces towards the poles and the non-parallelism of equipotential surfaces around gravity anomalies are more effective.

The non-parallelism of equipotential surfaces ultimately makes the results of levelling route-dependent. This may be explained with the aid of Figure 2.2. Suppose that a levelling run followed approximately the route ABC in a determination of the levelled-height difference between A and C. If a level defines increments of an equipotential surface, then, according to this instrument, B and C have the same height, and the difference between A and C is x. Along the route ADC, the observed height difference would be x'. If the equipotential surfaces are not parallel, then x and x' are generally not equal. If the results of the levelling are route-dependent, then miscloses may result, from this cause, in closed levelling loops. The significance of this effect depends on the degree of convergence of the equipotential surfaces and on the accuracy which is required for the results. The solution to the problem may be found in one of the following three methods:

(i) The results may be used with an understanding of their restricted significance;

(ii) some account may be taken of the fact that the lines of sight of the level are not all parallel to each other, by a correction term for example. This correction would be equivalent to tilting the level so that its lines of sight are always parallel.

(iii) The length of the staves may be varied to suit the varying distance between the equipotential surfaces. That is

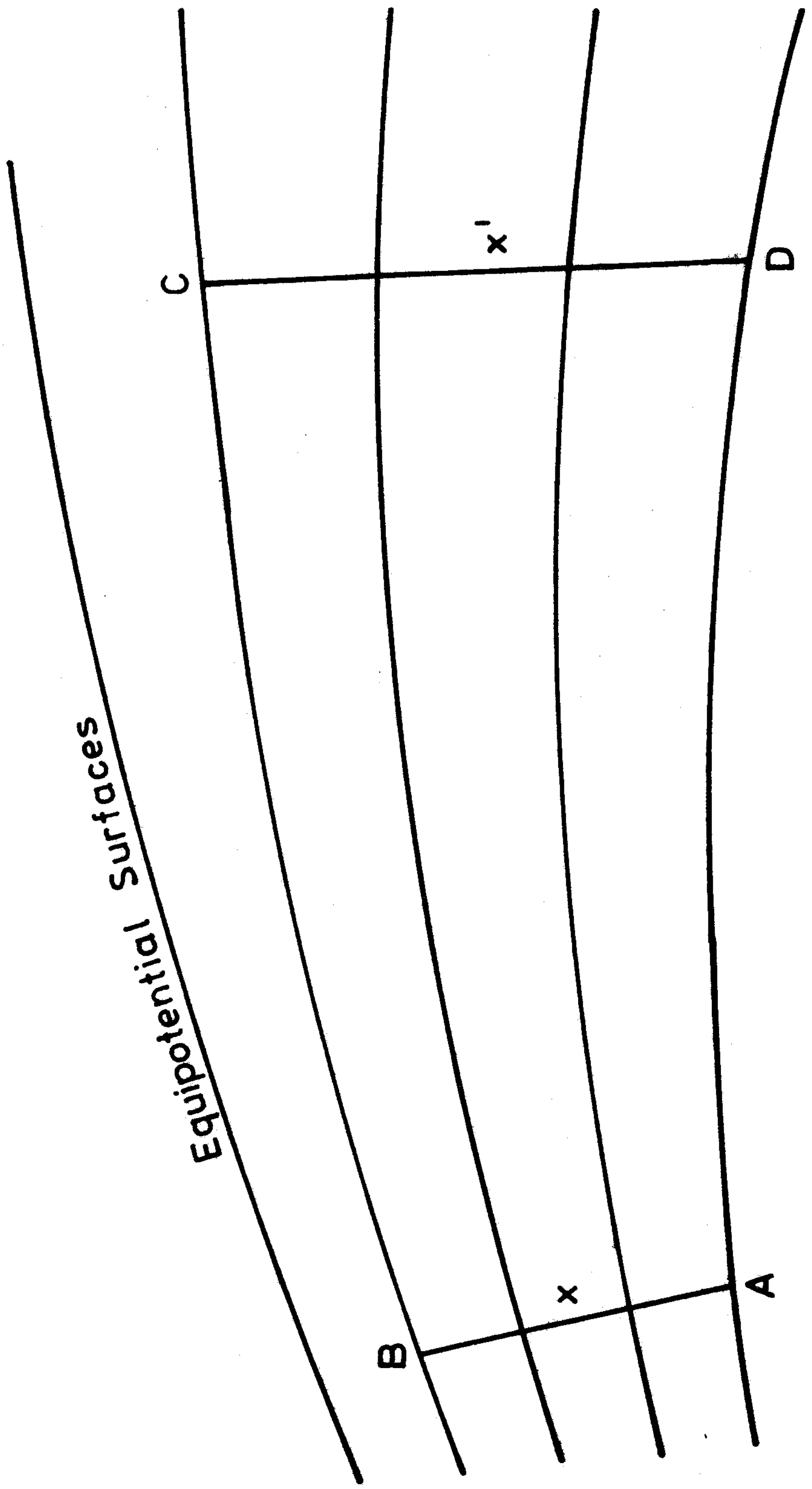


FIG. 2.2.

similar to (ii), except that now the staff readings are modified, instead of the level instrument being adjusted.

The most satisfactory solution is obtained by the third method. The readings on each staff would then be converted to a potential difference using the relationship,

$$W = - \int_{P_1}^{P_2} g.\ dh \qquad \text{.....(2.1)}$$

where: P_1 and P_2 are the terminal points of the linear difference observed on the staff. In practice, P_1 is 0 and P_2 is the recorded staff observation;

ΔW is the potential difference equivalent to the linear difference, $P_2 - P_1$;

g is the gravity value along the staff between P_1 and P_2;

n is the linear measurement between P_1 and P_2.

Thus, a set of results free from the inaccuracies caused by erroneous assumptions can be provided from levelling observations by multiplying the readings by the gravity value at the staff position, to convert all staff readings into potentials.

The significance of the study of the levelling process may now be apparent. Any levelling network over a large area may be affected by the non-parallelism of equipotential surfaces, although a solution to the problem has been presented. This Chapter is devoted to a study of this effect as possibly contributing to the apparent deviation between the geoid and sea level around Australia.

2.2 Systems of Vertical Control

In practice, levelled heights are not always converted into potentials by the method described above. The different systems in use for vertical control are discussed in this section.

The aim of a levelling network is to provide a system of vertical control, that is, a set of coordinates by which the position of a point in a vertical sense, may be defined. Various systems are used to fulfill this requirement, (see *Heiskanen and Moritz*, 1967, p.160 *et seq*).

Most commonly used of course is the system of *heights* or *elevations*. For the height of any point or the heights of a number of points to have significance, they must be referred to a known datum surface. The surface from which the heights are measured must be defined. It is not as obvious that the line along which the heights from the datum surface are measured must also be defined, as this line is not necessarily a straight line normal to the datum surface. It must be noted that heights are scalars with the physical dimension of length. *Orthometric heights* may be defined as the distance along the vertical between the point and the *geoid*. The vertical or plumb-line may possibly be curved. *Normal heights* refer to the distance to the *spheroid* along the normal which is straight. Over small areas, an arbitrary datum *plane* may be adopted. However, consideration is being given here to datums over larger areas where such assumptions are not satisfactory.

Some alternative systems of providing vertical control, which may be shown to have practical value, involve potential values:

(i) Geopotential is the gravitational potential at a point, usually related to the geoid. Dimensions are obviously the same as potential: $\text{length}^2\ \text{time}^{-2}$. The line along which the potential is measured is not significant. The surfaces of constant potential, or equipotential surfaces, are also known as *level* surfaces, the geoid being the equipotential surface of reference. A potential difference ΔW between points P_1 and P_2 is related to heights h by the relationship (2.1), quoted earlier,

$$\Delta W = -\int_{P_1}^{P_2} g \cdot dh$$

(ii) **Geopotential Numbers** are the same as geopotentials except that more suitable units are used: geopotential units, g.p.u., where

1 g.p.u. = 1 kgal m

As

$$g \doteqdot 0.98 \text{ kgals},$$

$$1\text{m} \doteqdot 0.98 \text{ g.p.u.}$$

Thus, in practice, one g.p.u. is about 2% longer than a metre.

(iii) **Dynamic Heights** are similar to geopotential numbers except that provision is made for the 2% difference between a metre and a g.p.u. by dividing the geopotential number by a factor close to 0.98. This factor is often a suitable gravity value for the area, such as the normal gravity value at the mid-latitude of the area.

The outlines of systems of vertical control and of the significance of levelling, indicate that account must be taken of the non-parallelism of equipotential surfaces in a large levelling network. More specifically, some account had to be taken in this survey. The Division of National Mapping, as reported by *Roelse et al* (1971), has produced a network of *orthometric heights* using a correction term to account for the non-parallelism, whilst retaining a *height* system of control, which is the most satisfactory system for practical purposes. For scientific application, the use of geopotentials seems preferable.

The remainder of this chapter consists of an outline of the conversion of the observed levelling in Australia to geopotential. The resultant potential differences were compared with the orthometric height differences produced by the Division of National Mapping, to determine the effects of using potentials rather than the orthometric heights. The effect of these systems on the geoid determination could then be estimated.

2.3 Geopotential Conversion Theory

The conversion of levelled height differences to geopotentials is based on equation 2.1,

$$\Delta W = - \int_{P_1}^{P_2} g . dh$$

Strictly, conversion of levelling results to geopotential requires that staff readings at all points along the levelling route should be converted to potential by combination with gravity. However, the variation of gravity along a levelling route is not so rapid as to require such frequent determinations of the gravity value. In practice, conversion to potential may be effected every few kilometres, usually at consecutive bench-marks, according to the formula

$$\Delta W = - g . \Delta h \qquad \text{.....(2.2)}$$

where Δh is the height difference obtained from all staff readings over a length of levelling. Obviously, if gravity does not vary over this run of levels, equation 2.2 is equivalent to repeated applications of equation 2.1. A more detailed study of the frequency of the application of this relationship, and the error which may result, is made in Section 2.7.

2.4 The Australian Levelling Data

The levelling network which covers Australia is described by *Roelse et al* (1971), who also give the orthometric correction, and by *Leppert* (1970). Some aspects of the levelling are repeated here.

The network comprises approximately 160 000 km of levelling which covers the entire Australian mainland with a density of levelling which is highest in the south-east and lowest in the central-west of the continent. The major portion of the network is of third-order standard, for which the two levellings of any section between bench-marks must not differ by more than 12 $\sqrt{K}$ mm, where K is the distance in kilometres between the bench-marks (*National Mapping Council of Australia*, 1970). Second-order levelling, for which an accuracy requirement of 8.4 $\sqrt{K}$ mm is specified, is also included in the survey, whilst in the south-east and south-west of the continent, some first-order levelling (4 $\sqrt{K}$ mm) has been observed.

Roelse et al (1971, p.76) adopt a value of about 8.1 $\sqrt{K}$ mm for the precision of the *adjusted* orthometric levelling. They estimate that the standard deviation of a height at the coast-line when referred to a height in the centre of the continent, via the adjusted orthometric levelling, is of the order of 35 cm, (*ibid*, Annexure F).

After application of the orthometric correction, which was based on normal gravity calculated on Reference System 1967, all heights and height differences in the network were related to the geoid. The corrections were applied to each station of levelling between bench-marks, the spacing of which is of the order of 3 km.

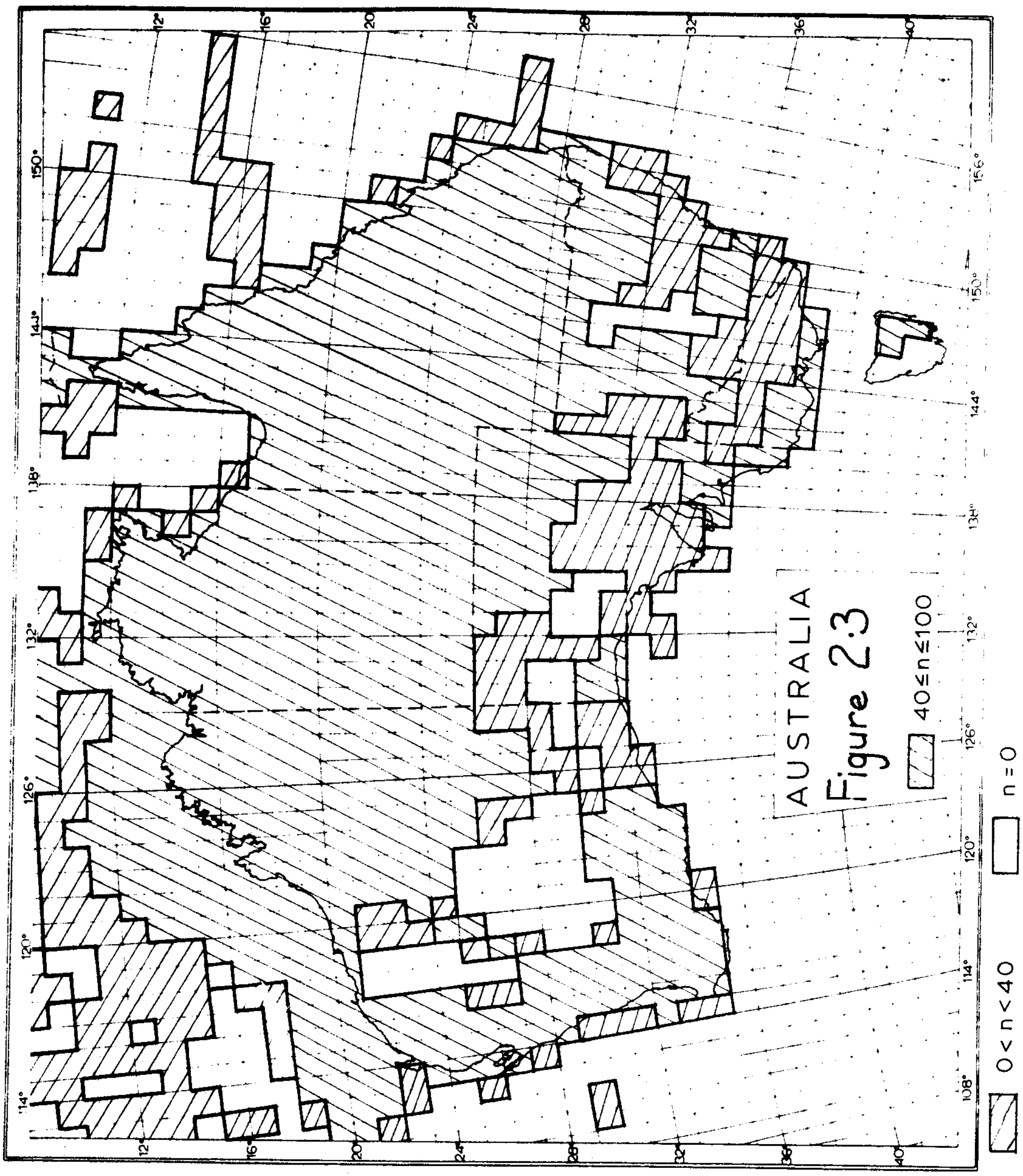
Almost 100 000 km of the original observed levelling which formed about 260 closed loops of levelling and which were used in the adjustment of the network, have been used in this study. This includes about 760 sections between loop junction-points and 36 000 sub-sections of levelling between bench-marks. The information which was made available included the mean of the observed height differences between all bench marks, the same value orthometrically corrected, the latitudes and longitudes to a minute of arc, and the measured distance between the bench-marks. This data is not identical to that used by the Division of National Mapping study as reported by *Roelse et al* (1971). The data set used here contains some additional levelling results obtained since the Division of National Mapping computations. However, the differences should not be significant to the fundamental sea-level result.

2.5 Gravity Data Available in Australia

The conversion to potential requires gravity values along the levelling route. Either *observed* or *normal* gravity values may be used. The disadvantage of the former is that they are not necessarily available where required, whilst the latter may be conveniently calculated whenever necessary. However, theoretical gravity cannot be as accurate as observed values. In the conversion to potential described in this chapter, observed gravity values were used. The Division of National Mapping's results were based on theoretical gravity, but it was considered preferable to use observed gravity here to ensure, particularly, that no systematic errors were produced in the Australian network by the use of normal gravity. The feasibility of the use of observed gravity was nevertheless dependent on the distribution of the available observed gravity. The source of data was the *U.N.S.W. Data Set* (*Mather*, 1970a, pp.72-74). The gravity values were not available in their observed form. For the potential conversion, the values of gravity were recalculated from the *free-air gravity anomalies* at intervals of 0.1 degrees of arc, which were part of the Data Set. The set of anomalies at 0.1 degree spacing included both observed anomalies and values which had been interpolated. Observed anomalies were given to 0.1 mgal and were based on observations which had an error of representation of ± 3 mgal, (*Mather*, 1970b). Prediction processes which were used to form the interpolated value are described by *Mather* (1970a, p.75; 1967). The estimated standard deviation of each interpolated anomaly was also available. The distribution of the *observed* values, as indicated in Figure 2.3, was considered satisfactory for the use of observed gravity in the geopotential calculation.

2.6 The Covariance Function over Australia

A free-air gravity-anomaly covariance function over Australia


AUSTRALIA
Figure 2.3
0 < n < 40
n = 0
40 ≤ n ≤ 100

was calculated, for use in later parts of this chapter: see sections 2.8 and 2.9. This function was calculated using a computer programme which is illustrated by the flow diagram, Figure 2.4. A listing is given in Appendix 1.1. The programme uses a number of areas of gravity anomaly values. In every area block, each anomaly at 0.1 degree spacing was multiplied by the anomaly at the centre of the area. For all anomaly separation distances, the *mean* product of anomalies was calculated, thereby forming the covariance function. Interpolated values of gravity, mentioned in section 2.5, were not used in the calculation of the function. The height correlation was neglected as it was not expected that the complexity of the programme which would be required to utilize height data, would be justified by the intended use of the results. The conversion to geopotential ultimately verified this assumption, (see section 2.10).

The results of the programme computation are shown in Figure 2.5. The reliability of the function was tested by forming the function twice using different parts of the data set, as well as using the complete data set. The comparison indicated that this function could be considered to have a reliability of ± 50 mgal^2. The covariance values are lower than those exhibited by a function calculated from data distributed over more diverse areas (*Kaula*, 1963, p. 524, Figure 2), and may be explained by the generally moderate anomaly values over the Australian region.

2.7 The Error Arising from the Spacing of Gravity Observations

The spacing of gravity observations in the conversion of levelling to geopotential has been considered by a number of authors, but notably by *Ramsayer* (1965) and *Levallois* (1964). The spacing of the levelling conversions to geopotential is restricted, in Australia, by the availability of gravity data. Consequently, it is more appropriate to calculate the error which results from the application of available gravity values, rather than to specify the desirable gravity spacing.

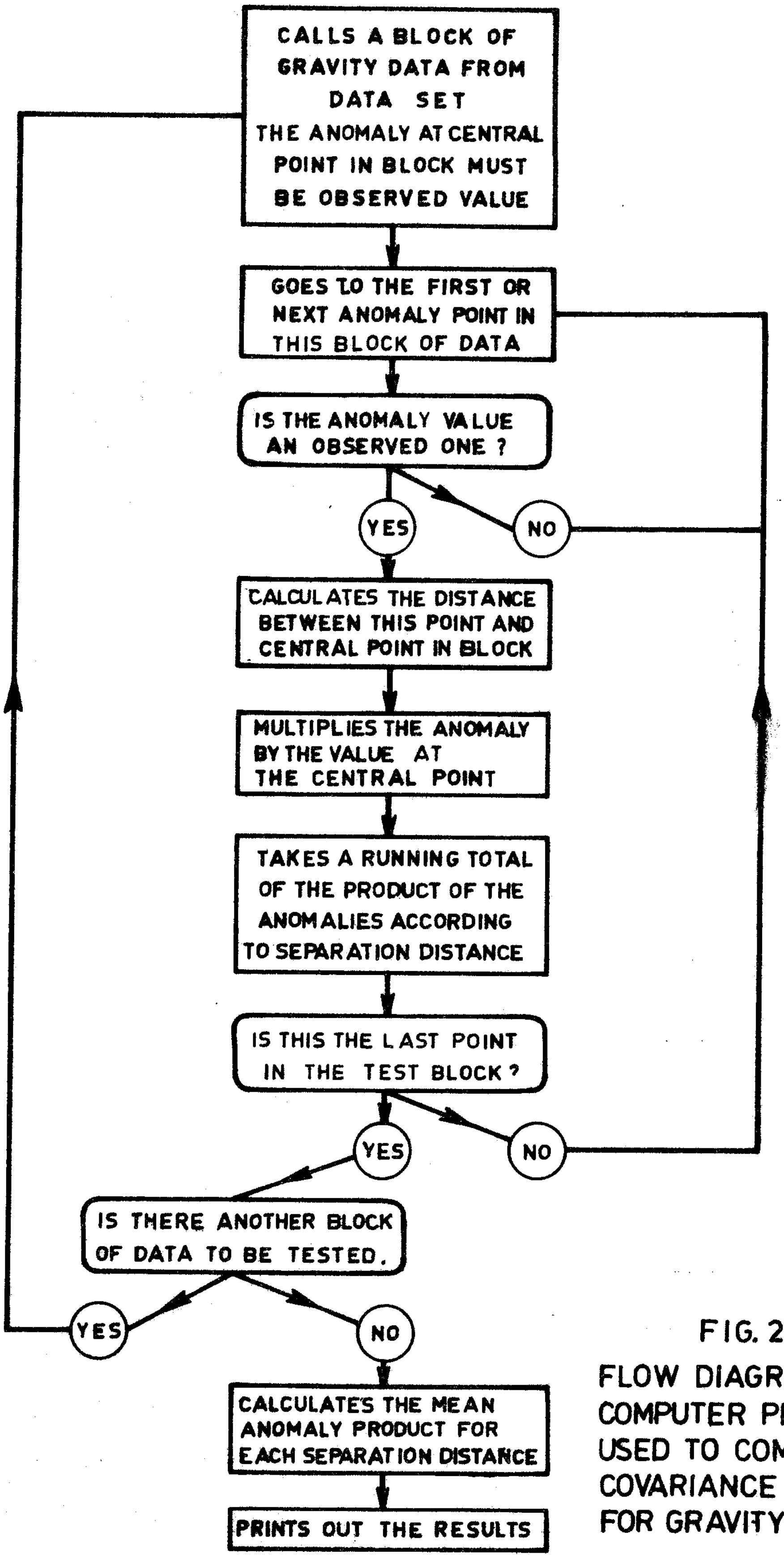
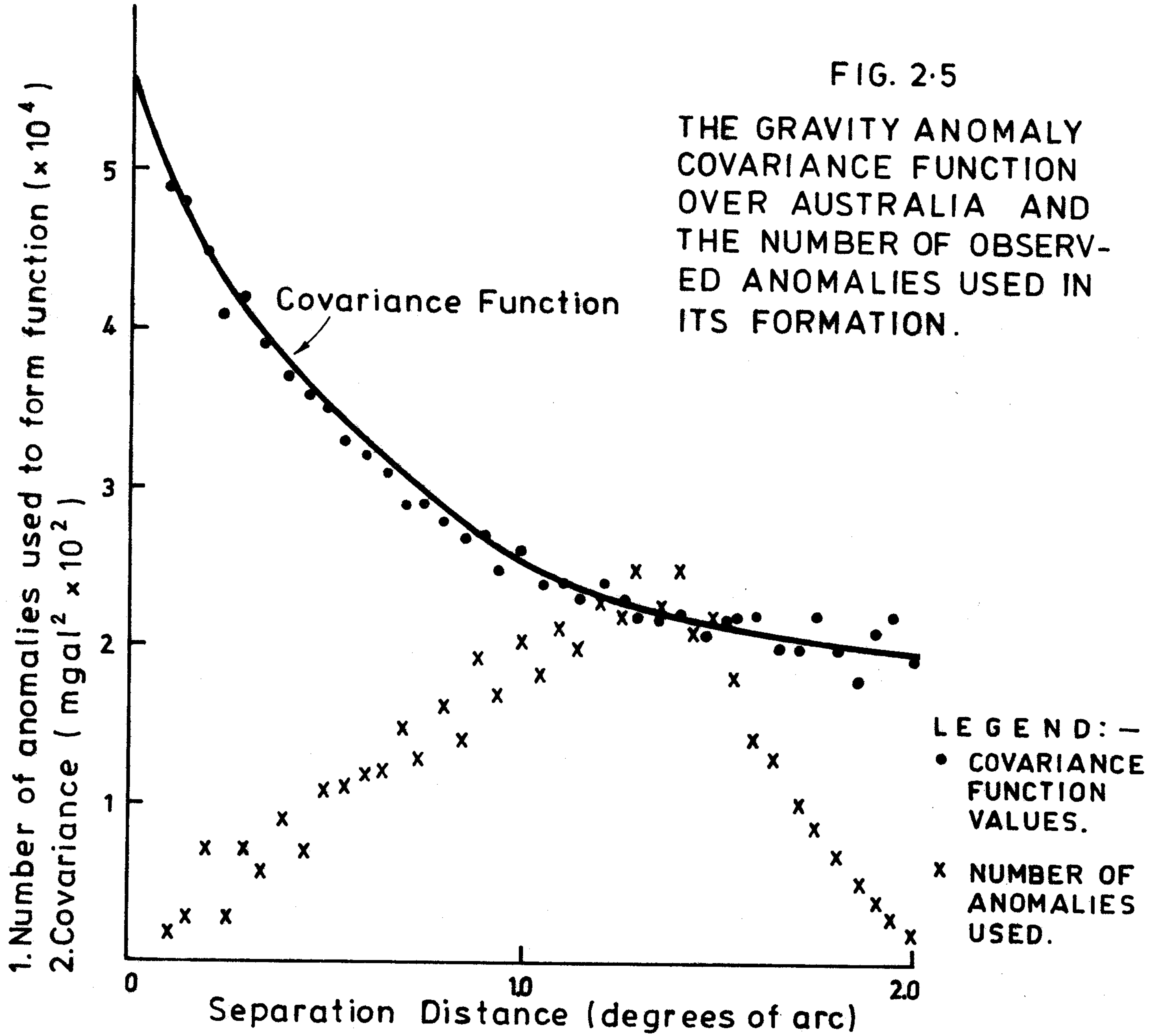


FIG. 2·4
FLOW DIAGRAM FOR COMPUTER PROGRAMME USED TO COMPUTE COVARIANCE FUNCTION FOR GRAVITY ANOMALIES

FIG. 2·5

THE GRAVITY ANOMALY COVARIANCE FUNCTION OVER AUSTRALIA AND THE NUMBER OF OBSERVED ANOMALIES USED IN ITS FORMATION.



Levallois (1964) considers the problem on a mathematical basis and has developed a formula by which the error due to the discontinuous application of gravity to levelling may be estimated. The formula is derived from an estimation, by Euler-MacLaurin theory, of the difference between the summation Σ g.dh and the integration $\int$g.dh,

$$e = \frac{\Delta h^2}{12} \left(\frac{\delta g}{\delta h}\right)_A^B + \frac{\Delta h . \Delta s}{12} \left(\frac{\delta g}{\delta s}\right)_A^B \qquad \text{.....(2.3)}$$

where e is the error in the geopotential produced by the application of gravity values at A and B instead of at all points continuously along the line of levelling from A to B, in metres; Δh is the elevation difference between A and B, in metres; Δs is the horizontal distance between A and B, in metres;

$\left(\frac{\delta g}{\delta h}\right)_A^B$ is the difference between the two *vertical* gradients of gravity at A and B;

$\left(\frac{\delta g}{\delta s}\right)_A^B$ is the difference between the two *horizontal* gradients of gravity at A and B.

The formula assumes continuity of gravity with height over the range A to B. *Levallois* (*ibid*) estimates that the values of

$\left(\frac{\delta g}{\delta h}\right)_A^B$ are not likely to exceed

0.3×10^{-6} gal cm^{-1} in flat country,
0.6×10^{-6} gal cm^{-1} in hilly country,
1.0×10^{-6} gal cm^{-1} in rugged country,

and that $\left(\frac{\delta g}{\delta s}\right)_A^B$ which may be estimated from changes in the deflection of the vertical, would be less than

2.5×10^{-7} gal cm^{-1} in flat country,
1.0×10^{-7} gal cm^{-1} in hilly country,
2.5×10^{-7} gal cm^{-1} in rugged country.

Maximum values between two bench-marks in Australia would be of the order of

200 m for Δh,
10 km for Δs.

Adopting maximum values of

$$10^{-6} \text{ gal cm}^{-1} \text{ for } \left(\frac{\delta g}{\delta h}\right)_A^B$$

and

$$2.5 \times 10^{-7} \text{ gal cm}^{-1} \text{ for } \left(\frac{\delta g}{\delta s}\right)_A^B$$

then

$$e = \frac{4 \times 10^8}{12} \times 10^{-6} + \frac{2 \times 10^2 \times 10^5}{12} (2.5 \times 10^{-7}) \text{ gal cm}$$

$$\doteqdot 3 \times 10^{-2} \text{ kgal cm.}$$

This is equivalent to only 0.03 cm in elevation. The application of gravity at each bench-mark will therefore be assumed to be satisfactory.

2.8 Gravity Prediction Processes

If the gravity anomaly is not available for any point, an estimated value may be obtained by interpolation of surrounding known values of gravity. The use of some linear combinations of surrounding gravity values were considered for this computation, based on an outline given by *Heiskanen and Moritz* (1967, pp.264 *et seq*).

The linear interpolation methods may be generalized by the formula

$$\overline{\Delta g} = \sum_{i=1}^{n} \alpha_i \, \Delta g_i \qquad .....(2.4)$$

where, $\overline{\Delta g}$ is the interpolated or predicted value of the gravity anomaly at the point P, under consideration;

Δg_i is the anomaly at the ith surrounding point being used in the interpolation;

α_i is the coefficient adopted for Δg_i. Preferably,

$$\sum_{i=1}^{n} \alpha_i = 1$$

If the true value of the anomaly at P is Δg, then the prediction error, e, is

$$e = \Delta g - \overline{\Delta g}$$

The mean value of e over an area is

$$M\{e^2\} = C_o - 2 \sum_{i=1}^{n} \alpha_i C_{ip} + \sum_{i=1}^{n} \sum_{k=1}^{n} \alpha_c \alpha_k C_k C_{ik}$$

where C_x is the gravity anomaly covariance for the separation distance x, or C_{yz} is the covariance for the distance between points y and z, (*ibid*, pp.266-267). Using the covariance function described in section 2.8, tests were made of the mean interpolation error over Australia for a number of prediction methods. For each prediction method, $M\{e^2\}$ was calculated according to two test cases:

Case 1. The point P is situated at the centre of a 0.1 degree square, gravity anomalies being available at one, two, three or four corners of the square.

Case 2. As for case 1, but using a 1.0 degree square. See Figure 2.6.

a. Using no nearby anomalies.

$$\overline{\Delta g} = 0$$

i.e. $n = 0$

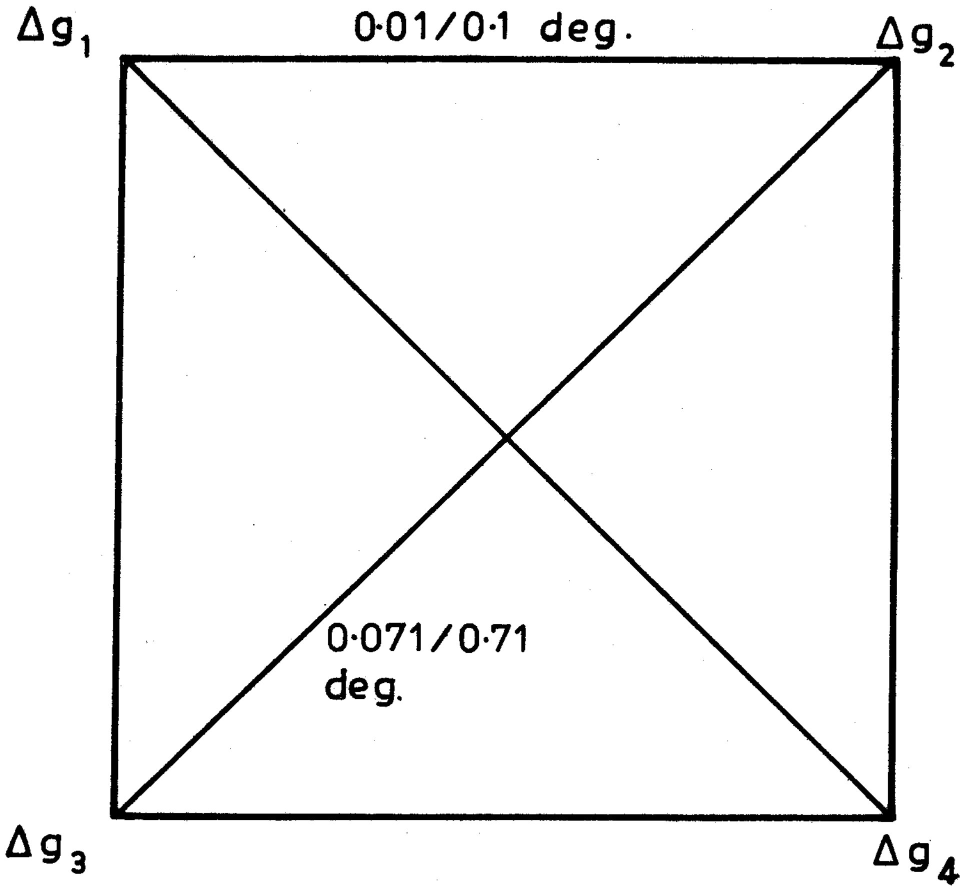


FIG 2·6

For both cases 1 and 2

$$M\{e^2\} = C_0 + 0$$
$$= 570 \text{ mgal}^2$$

b. Representation by the nearest anomaly.

$$n = 1$$
$$\alpha_1 = 1$$
$$\overline{\Delta g} = \Delta g_1$$
$$M\{e^2\} = C_0 - 2\,\alpha_1\,C_1 + \alpha_1\alpha_1\,C_{11}$$
$$= C_0 - 2C_1 + C_0$$
$$= 2C_0 - 2C_1$$

Case 1.

$$C_0 = 570$$
$$C_1 = C_{0.071}$$
$$= 525$$
$$M\{e^2\} = 2(570-525)$$
$$= 90 \text{ mgal}^2$$

Case 2.

$$C_0 = 570$$
$$C_1 = C_{0.71}$$
$$= 300$$
$$M\{e^2\} = 540 \text{ mgal}^2$$

c. A weighted mean of two anomalies.

Suppose the anomalies are weighted according to the inverse of their distance from P. Say,

$$\alpha_i = \frac{S_{i+1}}{S_1 + S_2} \quad (\text{if } i=2,\ i+1 = 1)$$

so that

$$\sum_{i=1}^{2} \alpha_i = 1$$

and where S_i is the distance from P to i.

$$\therefore \quad \overline{\Delta g} = \frac{S_2}{S_1 + S_2} \Delta g_1 + \frac{S_1}{S_1 + S_2} \Delta g_2$$

$$\therefore \quad M\{e^2\} = C_0 - 2 \sum_{i=1}^{2} \alpha_i C_{Pi} + \sum_{i=1}^{2} \sum_{k=1}^{2} \alpha_i \alpha_k C_{ik}$$

For both cases 1 and 2,

$$\alpha_i = {}^1/2$$
$$C_{P1} = C_{P2}$$

Case 1.

$$M\{e^2\} = C_0 - 2(\tfrac{1}{2} C_{0.071}) 2 + \tfrac{1}{4}(C_0 + C_{0.10}) \times 2$$

assuming anomalies are at adjacent corners, and not on diagonally opposite corners.

$$= {}^3/2\, C_0 - 2\, C_{0.071} + {}^1/2\, C_{0.10}$$
$$= {}^3/2(570) - 2(525) + {}^1/2(500)$$
$$= 855 - 1050 + 250$$
$$= 55 \text{ mgal}^2$$

Case 2.

$$C_0 = 570$$
$$C_{0.71} = 300$$
$$C_{1.00} = 255$$
$$M\{e^2\} = 855 - 600 + 128$$
$$= 383 \text{ mgal}^2$$

d. A geometric interpolation of three anomalies.

This may be done by passing a plane through three points as if the gravity anomaly value correspond to a third coordinate. *Heiskanen and Mortiz* (1967) show that

$$\alpha_i = \frac{(\lambda_{i+1} - \lambda)(\phi_{i-1} - \phi_{i+1}) - (\phi_{i+1} - \phi)(\lambda_{i-1} - \lambda_{i+1})}{(\lambda_{i+1} - \lambda_i)(\phi_{i-1} - \phi_{i+1}) - (\phi_{i+1} - \phi_i)(\lambda_{i-1} - \lambda_{i+1})}$$

where λ_j refers to longitude of j^{th} point,

ϕ_j refers to latitude of j^{th} point,

and $j + 1 = 1$ if $j = 3$

$j - 1 = 3$ if $j = 1$

Applied to Case 1 and Case 2, this formula for α_i will give values of 0.5, 0.0, 0.5. However, assume that each

$$\alpha_i = {}^1/3$$

Case 1,

$$M\{e^2\} = C_0 - 2 \sum_{i=1}^{4} \alpha_i C_{Pi} + \sum_{i=1}^{4} \sum_{k=1}^{4} \alpha_i \alpha_k C_{ik}$$

$$= C_o - 2.{}^1/3(C_1 + C_2 + C_3) + {}^1/9(C_{11} + C_{22} + \ldots\ldots C_{33})$$

$C_0 = 570$

$C_1 = C_2$

$= C_3$

$= C_{0.071}$

$= 525$

$C_{11} = C_{22}$

$= C_{33}$

$= 570$

$C_{12} = C_{13}$

$= C_{21}$

$= C_{31}$

$= C_{0.10}$

$= 500$

$C_{23} = C_{32}$

$= C_{0.141}$

$= 475$

$$M\{e\} = 570 - {}^{2}/3(525) \times 3 + {}^{1}/9(3 \times 570 + 4 \times 500$$
$$+ 2 \times 475)$$
$$= 570 - 1050 + 518$$
$$= 38 \text{ mgal}$$

Case 2.

$$C_0 = 570$$
$$C_1 = 300$$
$$C_{11} = 570$$
$$C_{12} = 255$$
$$C_{23} = 218$$
$$M\{e^2\} = 570 - {}^{2}/3(300) \times 3 + {}^{1}/9(3 \times 570 + 4 \times 255$$
$$+ 2 \times 218)$$
$$= 570 - 600 + 352$$
$$= 322 \text{ mgal}^2$$

e. Least-squares interpolation using the four nearest anomalies.

The α_i are divided by minimising $M\{e^2\}$. *Heiskanen and Moritz* (1967, p.268), show that

$$\overline{\Delta g} = \sum_{i=1}^{4} \left(\sum_{k=1}^{4} C_{ik}^{(-1)} \; C_k\right) \Delta g$$

where $C_{ik}^{(-1)}$ is the ik element in the inverse matrix of C_{ik}

$$\alpha_i = {}^{4}_{i=1} \; C_{ik}^{(-1)} \; C_k$$

The mean square error can be shown to be, (*ibid*, pp.268-269),

$$M\{e^2\} = C_0 - \sum_{i=1}^{4} \sum_{k=1}^{4} C_{ik}^{(-1)} \; C_i \; C_k$$

Case 1.

$$C_i = C_{0.071}$$
$$= 525$$
$$C_{ii} = C_{0.0}$$
$$= 570$$

The matrix of distances, ik, is

$$\begin{vmatrix} 0.000 & 0.100 & 0.100 & 0.141 \\ 0.100 & 0.000 & 0.141 & 0.100 \\ 0.100 & 0.141 & 0.000 & 0.100 \\ 0.141 & 0.100 & 0.100 & 0.000 \end{vmatrix}$$

$$C_{0.00} = 570$$

$$C_{0.10} = 500$$

$$C_{0.141} = 475$$

Then the matrix C_{ik} is

$$\begin{vmatrix} 570 & 500 & 500 & 475 \\ 500 & 570 & 475 & 500 \\ 500 & 475 & 570 & 500 \\ 475 & 500 & 500 & 570 \end{vmatrix}$$

for which the inverse is

$$\begin{vmatrix} 109.4 & -54.3 & -54.3 & 4.1 \\ -54.3 & 109.4 & 4.1 & -54.3 \\ -54.3 & 4.1 & 109.4 & -54.3 \\ 4.1 & -54.3 & -54.3 & 109.4 \end{vmatrix} \times 10^{-4}$$

$$\begin{aligned} M\{e^2\} &= 570 - 525 \times 525 \times (109.4 - 54.3 - \\ &\quad 54.3 + 4.1) \times 4 \times 10^{-4} \\ &= 570 - 540.2 \\ &= 30 \text{ mgal}^2 \end{aligned}$$

Case 2.

The matrix of distances, ik, is

$$\begin{vmatrix} 0.00 & 1.00 & 1.00 & 1.41 \\ 1.00 & 0.00 & 1.41 & 1.00 \\ 1.00 & 1.41 & 0.00 & 1.00 \\ 1.41 & 1.00 & 1.00 & 0.00 \end{vmatrix}$$

$C_{0.00} = 570$

$C_{1.00} = 255$

$C_{1.41} = 218$

So the matrix C_{ik} is

$$\begin{vmatrix} 570 & 255 & 255 & 218 \\ 255 & 570 & 218 & 255 \\ 255 & 218 & 570 & 255 \\ 218 & 255 & 255 & 570 \end{vmatrix}$$

$$C_i = C_{0.71} = 300$$

The inverse is

$$\begin{vmatrix} 25.1 & -7.1 & -7.1 & -3.3 \\ -7.1 & 25.1 & -3.3 & -7.1 \\ -7.1 & -3.3 & 25.1 & -7.1 \\ -3.3 & -7.1 & -7.1 & 25.1 \end{vmatrix} \times 10^{-4}$$

$$\begin{aligned} M\{e^2\} &= 570 - 300 \times 300 \times (25.1 -7.1 -7.1 -3.3) \\ &\quad \times 4 \times 10^{-4} \\ &= 570 - 273.6 \\ &= 296 \text{ mgal}^2 \end{aligned}$$

f. Simple interpolation process using the four nearest anomalies.

If the gravity anomalies are available at all four corners of a square surrounding the bench-mark, the simple prediction method described below may be used. If P_1, P_2, P_3 and P_4 in Figure 2.7 are the four points surrounding a bench-mark P_0, an anomaly at P_5 may be interpolated from Δg_1 and Δg_2 at P_1 and P_2; Δg_6 can be obtained from Δg_3 and Δg_4. The anomaly $\overline{\Delta g}$ at P_0 is then obtained by interpolation between P_5 and P_6.

Let the latitude and longitude coordinates of and the gravity anomaly at point P_i (i = 0,6) be ϕ_i, λ_i and g_i respectively. Then

$$g_5 = \frac{\lambda_0-\lambda_1}{\lambda_2-\lambda_1}(g_2-g_1) + g_1$$

$$g_6 = \frac{\lambda_0-\lambda_3}{\lambda_4-\lambda_3}(g_4-g_3) + g_3$$

$$g_0 = \frac{\lambda_0-\lambda_5}{\lambda_6-\lambda_5}(g_6-g_5) + g_5$$

It must be remembered that in the general case, the surrounding anomalies do not necessarily fill the corners of the surrounding square. The relationships for g_5 and g_6 would become complex.

Quite obviously, for Case 1 and Case 2, α_i will reduce to

$$\alpha_i = 0.25$$

Case 1.

$$C_i = C_{0.071}$$
$$= 525$$
$$C_{ii} = 570$$
$$C_{12} = C_{24}$$
$$= C_{43}$$
$$= C_{31}$$

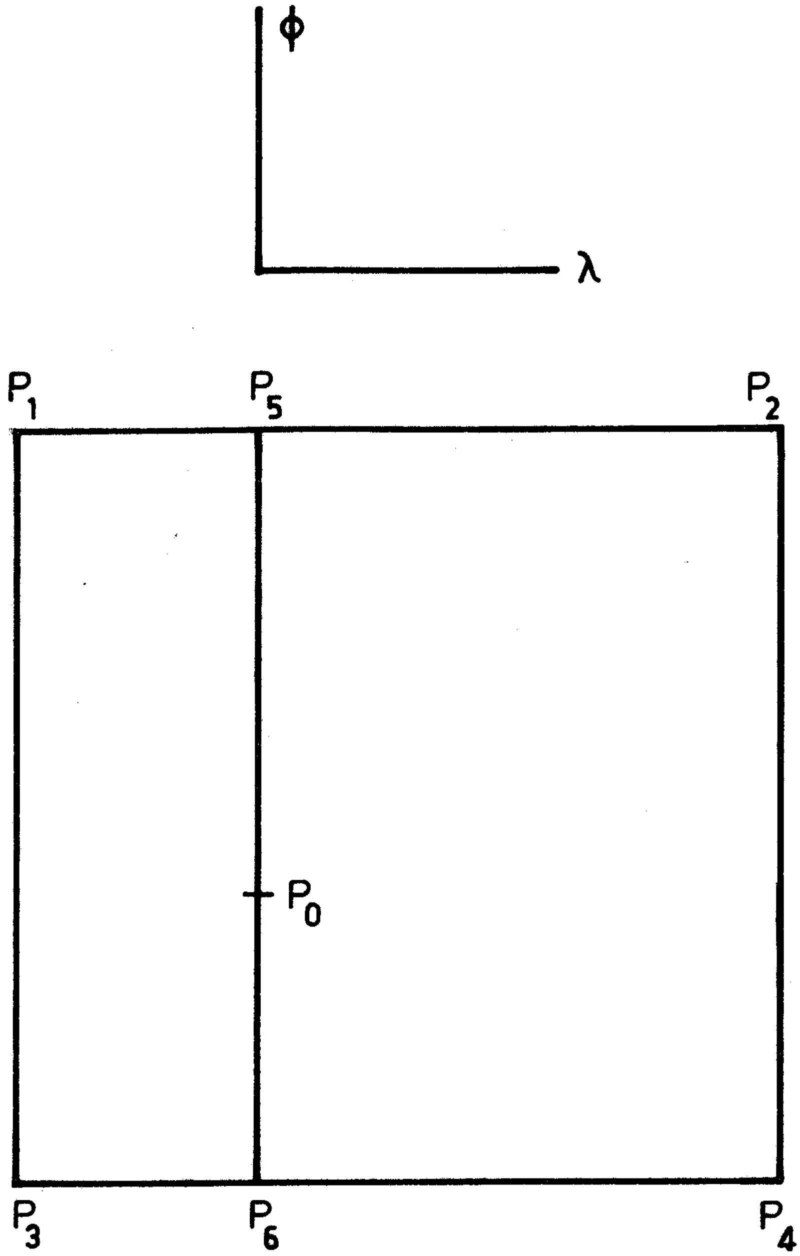


FIG. 2·7
SIMPLE GRAVITY ANOMALY INTERPOLATION PROCESS USING FOUR SURROUNDING VALUES.

$$= C_{0.10}$$
$$= 500$$
$$C_{14} = C_{23}$$
$$= C_{0.14}$$
$$= 475$$
$$\therefore \quad M\{e^2\} = 570 - 2(0.25 \times 4 \times 525) + 0.25 \times 0.25(570 \times 4 + 500 \times 8 + 475 \times 4)$$
$$\doteq 31 \text{ mgal}^2$$

Case 2.

$$C_i = C_{0.71}$$
$$= 300$$
$$C_{ii} = 570$$
$$C_{12} = 255$$
$$C_{14} = 218$$
$$M\{e^2\} = 570 - 2(0.25 \times 4 \times 300) + 0.25 \times 0.25(570 \times 4 + 255 \times 8 + 218 \times 4)$$
$$= 570 - 600 + 488$$
$$= 458$$

The results for the six anomaly prediction methods are shown in Table 2.1.

The estimates of the error due to interpolation are likely to be optimistic as the anomaly values which are used in the interpolation must be expected to exhibit correlation with elevation. Thus, the interpolated value will be affected by the average elevation of the corners of the surrounding 0.1 degree square. Unfortunately, heights at these corners were not available, although heights at the centres of the 0.1 degree squares were available. *Uotila* (1960) estimated the correlation of free-air anomalies with elevation to be about 0.11 mgal m^{-1} over limited regions so that if the average height of surrounding anomalies differs by 100 m from that of the bench-mark, the resultant error from this cause would be 11 mgal.

Table 2.1

Comparison of Anomaly Prediction Process

INTERPOLATION METHOD	MEAN ERROR			
	CASE 1		CASE 2	
	$M\{e^2\}$	$\sqrt{M\{e^2\}}$	$M\{e^2\}$	$\sqrt{M\{e^2\}}$
a. No anomalies	570	23.9	570	23.9
b. Single nearest	90	9.5	540	23.2
c. Mean of two	55	7.4	383	19.6
d. Three anomalies	38	6.2	322	17.9
e. Four: Least-Squares	30	5.5	296	17.2
f. Four: Geometrical	31	5.6	458	21.4

It was ultimately decided to use the simple geometrical interpolation method when all four anomalies were available at the corners of the surrounding 0.1 degree square. Otherwise, the nearest four anomalies would be sought and the least-square interpolation method would be applied. If the surrounding data at 0.1 degree spacing was available, the geometrical interpolation method was considered to be as accurate and as easy to use as any other method. The process does not require the determination of the nearest one, two or three anomalies. If anomalies were not available on the surrounding 0.1 degree square, geometrical interpolation becomes unwieldy, and the least-squares method would be preferable. Further, the least-squares method seemed no more complex than geometrical interpolation of three anomalies, but should be a slightly more accurate than the use of two values. The accuracy of the predicted anomaly has been estimated, but the subsequent effect on the levelling or geopotential network has not been considered. Levelling specifications require (see section 2.4) that $d(\Delta H) < 12\sqrt{K}$ mm, for third order levelling. The error in ΔW can be considered, from equation (2.2) to be

$$d(\Delta W) = -dg \cdot \Delta h \qquad \ldots\ldots(2.5)$$

where dg is the error in g. The error in ΔH is not relevant. Suppose that the error in ΔW due to interpolation is to be no greater than half the error in the levelling,

i.e.
$$d(\Delta W) = 0.5 \times d(\Delta H)$$
$$< 0.5 \times 12 \sqrt{K} \text{ mm}$$
$$-dg.\Delta H = 0.5 \times 12 \sqrt{K}$$

or,

$$dg = \frac{6\sqrt{K}}{\Delta H}$$

where dg is in kgals, ΔH in millimetres. The most difficult case to fulfill occurs when K is small and ΔH is large. If two bench-marks are separated by only 4 km in terrain which has a constant slope of 25 degrees, producing a large ΔH value,

i.e. $\Delta H \doteqdot 2$ km,

then dg has a maximum allowable limit given by

$$dg = \frac{6 \times \sqrt{4}}{2} \text{ mgal}$$
$$= 6 \text{ mgal}$$

As gravity anomalies over Australia are rarely less than this figure, the use of normal gravity could produce errors larger than the random error in the levelling. The use of observed rather than normal gravity to account for non-parallelism of the equipotential surfaces would seem advisable. The figure of 6 mgal also demonstrated that the most accurate interpolation processes should be applied.

2.9 The Computer Programme - The Main Routine

The conversion of the levelled height differences to geopotential was undertaken using a computer programme which utilizes four sub-routines.

The relationship between the sub-routines and the main programme is shown in Figure 2.8. The sub-routines are described separately from the main routine. The programme converts the observed height differences between successive bench-marks to geopotential differences by a process which is illustrated in the flow diagram, Figure 2.9. A listing of the programme is given in Appendix 1.2.

The value of gravity at the bench-marks must be calculated by, (*Heiskanen and Moritz*, 1967, p.146),

$$g = \gamma - f + \Delta g \qquad \ldots\ldots(2.6)$$

where γ is the value of normal gravity,

f is the free-air correction, and is positive when the bench-mark has a positive height,

Δg is the free-air anomaly of gravity.

Normal gravity at the bench-mark was calculated on the Reference System 1967 using the International Gravity Formula, by which,

$$\gamma = 978\ 049.0(1 + 5.288\ 4 \times 10^{-3} \sin^2\phi - 5.9 \times 10^{-6} \sin^2 2\phi)$$

where ϕ is the latitude of the bench-mark and γ is in milligals. This formula is correct to 0.03 mgals, as may be shown by its derivation. The latitude - induced errors are only one or two milligals, as the latitudes of all bench-marks in the network are known to a minute of arc.

As the anomalies in this Data Set are free-air anomalies, that is, they have been reduced to spheroid level, the elevation correction must be recalculated to produce the gravity value at bench-mark height. The *free-air correction* is given by

$$f = 0.3086\ H$$

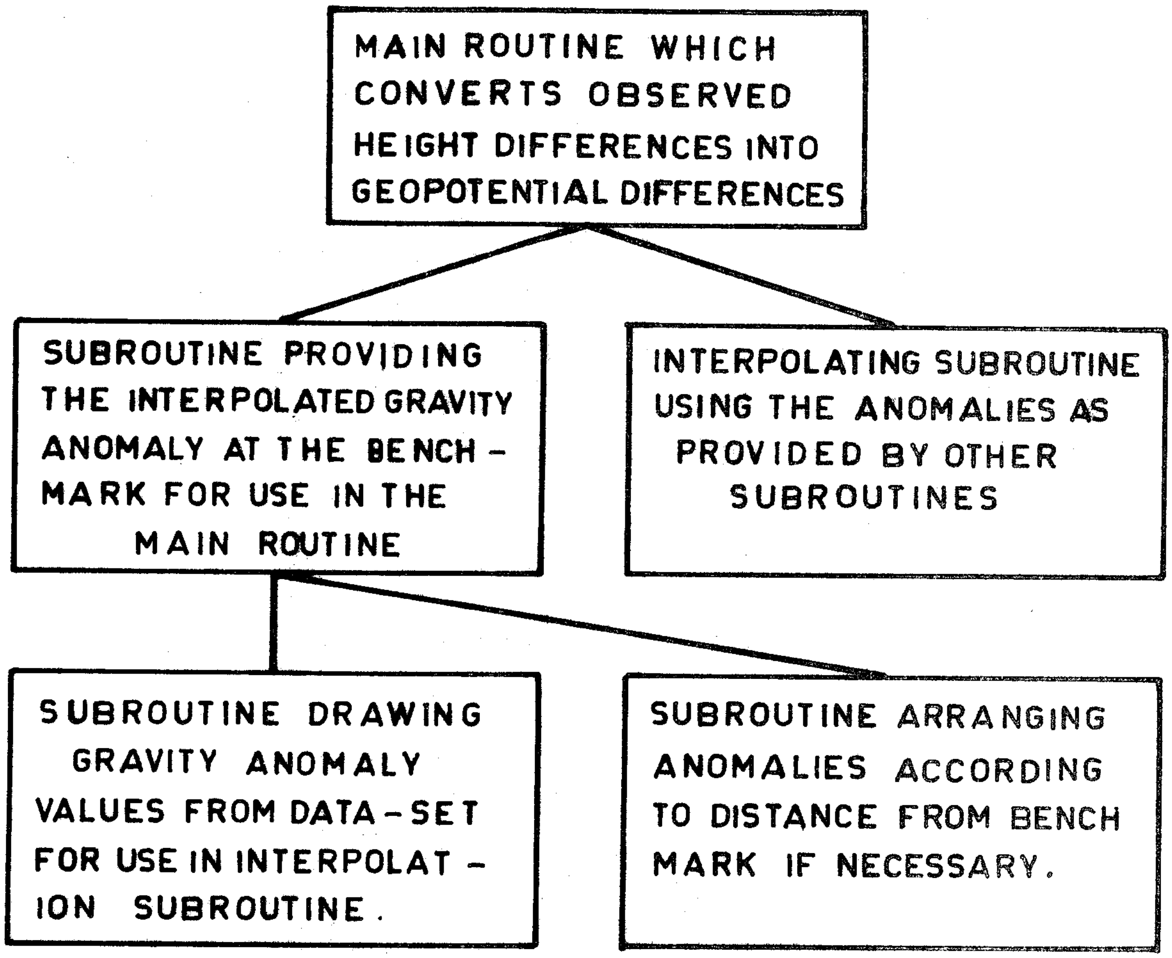


FIG. 2·8

RELATIONSHIP BETWEEN COMPUTING ROUTINES IN CONVERSION TO GEOPOTETIAL.

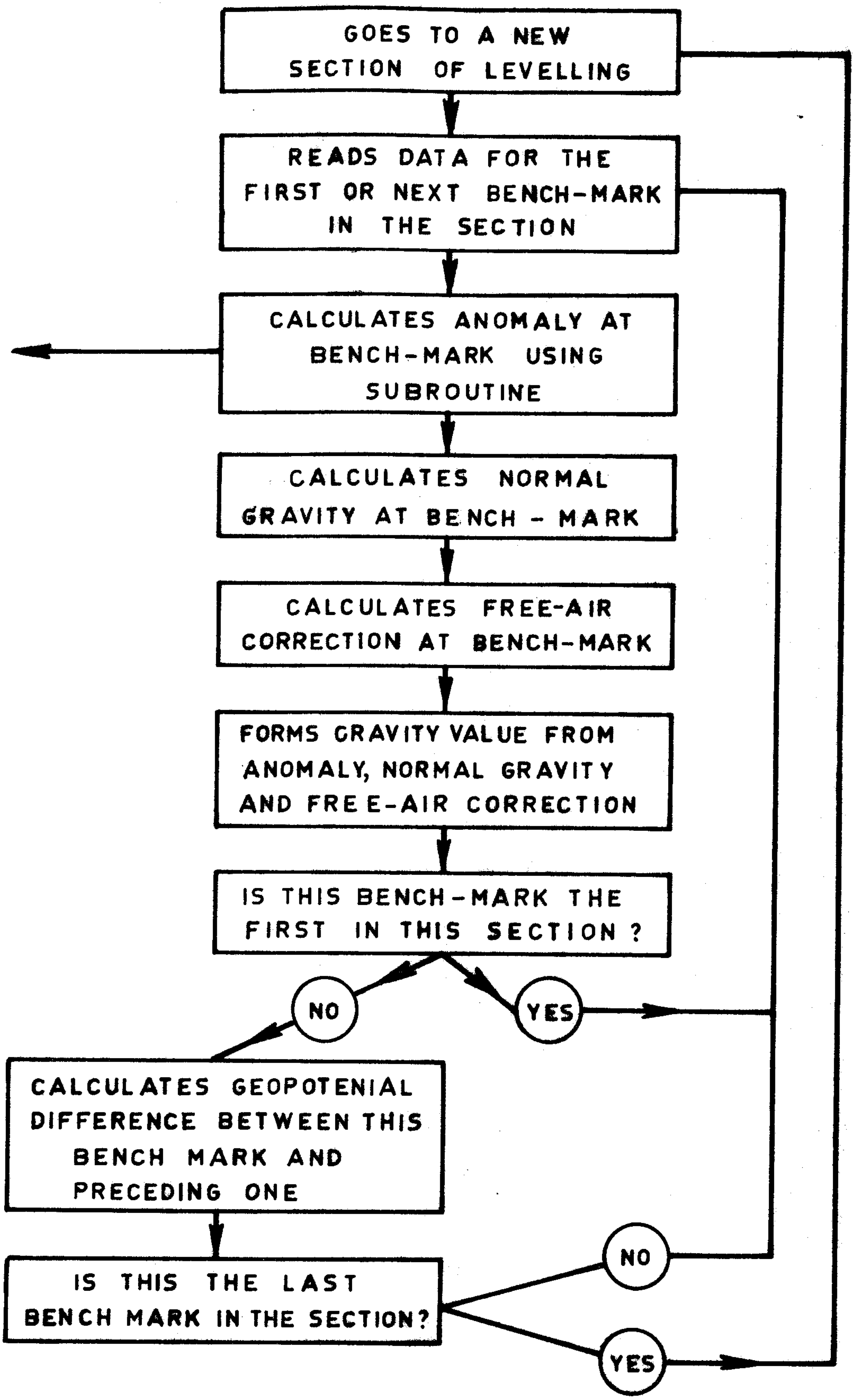


FIG. 2·9

FLOW DIAGRAM FOR MAIN ROUTINE OF PROGRAMME WHICH CONVERTS LEVELLING TO GEOPOTENTIAL.

where f is the correction in milligals,

H is the orthometric height in metres.

The neglected term in the formula is $3 H \gamma R^{-2}$, where R is the radius of the earth. The resultant error df is

$$df = 3 H^2 \gamma R^{-2}$$

For the highest mountains in Australia, for which,

$$H \doteq 2000 \text{ m},$$

then

$$df = 0.3 \text{ mgals}.$$

The conversion of the height difference ΔH, between two bench-marks is then calculated using formula (2.2),

$$\Delta W = -g.\Delta H$$

The value of g is taken as

$$g = \tfrac{1}{2}(g_1 + g_2) \qquad \text{.....(2.7)}$$

where g_1 and g_2 are the values of gravity at the first and second bench-marks.

The Anomaly Selection Sub-routine.

This was designed to find the four gravity anomalies nearest to each bench-mark being used in the main-routine calculations. The sub-routine is explained by means of the flow diagram, Figure 2.10 and the programme is listed in Appendix 1.3. When tested, this routine successfully found the nearest anomalies to the point under consideration.

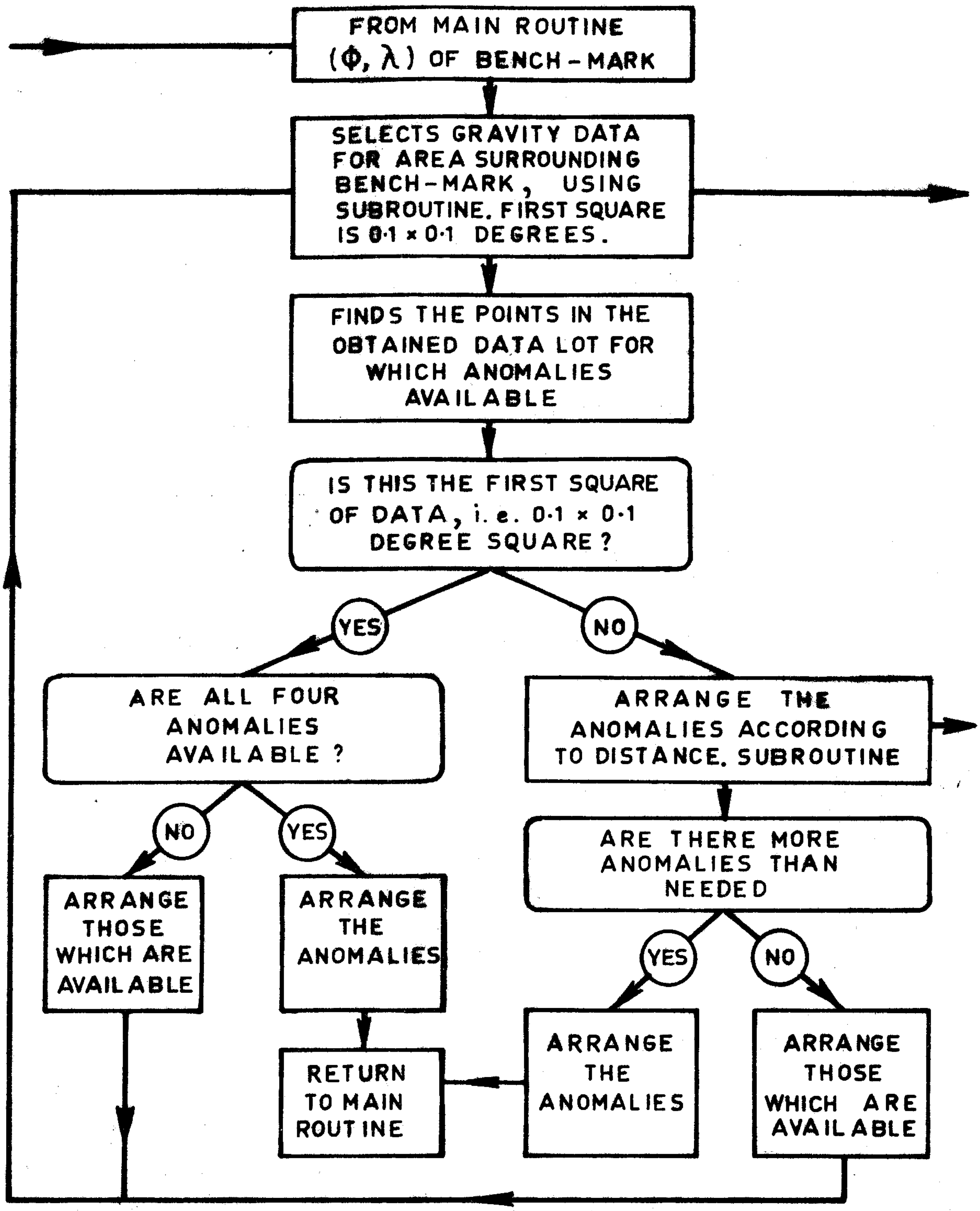


FIG. 2·10

FLOW DIAGRAM FOR SUB-ROUTINE WHICH SELECTS THE GRAVITY ANOMALY FOR THE MAIN ROUTINE .

Data Selection Sub-routine.

A block of free-air gravity anomalies covering a specified area in Australia were extracted from the disk-stored gravity Data Set, using a sub-routine made available by E.G. Anderson of the University of New South Wales. In the form used for this computation, anomalies at 0.1 degree intervals were provided.

Ordering Sub-routine.

The sub-routine which arranges the four anomalies in the order of their distance from the bench-mark is only required for the least-squares interpolation process, and is shown in full in Appendix 1.4.

The Interpolation Sub-routine.

This interpolates the available anomalies according to either the least-squares or the simple geometrical interpolation process with four anomalies. The programme is shown in Appendix 1.5.

The whole geopotential conversion routine was tested by comparing the results of the first section conversion by computer programme with results calculated by hand. The programme was then applied to each of the 760 sections of levelling involving 3600 bench-marks. An example of the print-out from the computer is shown in Table 2.2.

2.10 Analysis of the Results of the Conversion to Geopotential

Results showed that interpolated gravity anomalies were only occasionally greater than 50 mgals, and were generally of the order of 20 to 30 mgals. Free-air corrections ranged over 50 to 100 mgals, although occasional values reached 300 mgals.

A study of the results also showed that the least squares interpolation process was never used. Thus, for all bench-marks in this

Table 2.2

Example of Results of Conversion to Geopotential
Section Number 207, from Junction Point 135 to Junction Point 128

Bench Mark (from)	Bench Mark (to)	ϕ	λ	Dist. Between Bench Marks (miles)	Approx. Ht. (feet)	γ (mgal)	f (mgal)	Δg (mgal)	g (mgal)	g Of Previous BM (mgal)	ΔW (kgal m)	Observed ΔH (feet)	Ortho-metric Height Diff. (feet)
	ZM70	25°49'	117°18'	0.0	1273.927	979026.4	119.8	36.0	978942.6			0.0	0.0
ZM70	ZM71	25 47	117 18	1.9	1287.504	979024.1	121.1	36.0	978938.9	978942.6	4.0511	13.577	13.580
ZM71	ZM72	25 46	117 17	2.0	1314.773	979022.9	123.7	36.0	978935.2	978938.9	8.1368	27.270	27.272
ZM72	ZM73	25 44	117 16	2.0	1338.388	979020.5	125.9	36.0	978930.6	978935.2	7.0462	23.615	23.618
ZM73	ZM74	25 42	117 16	2.1	1363.036	979018.2	128.2	36.0	978925.9	978930.6	7.3544	24.648	24.651
ZM74	ZM75	25 40	117 15	2.3	1402.913	979015.8	132.0	36.0	978919.9	978925.9	11.8983	39.877	39.880
ZM75	ZM76	25 39	117 15	2.0	1426.376	979014.7	134.2	36.0	978916.4	978919.9	7.0007	23.463	23.465
ZM76	ZM77	25 37	117 15	2.0	1437.235	979012.3	135.2	36.0	978913.1	978916.4	3.2400	10.859	10.862
ZM77	ZM78	25 35	117 15	2.0	1480.926	979010.0	139.3	36.0	978906.7	978913.1	13.0361	43.691	43.694
ZM78	ZM79	25 34	117 15	2.0	1516.507	979008.8	142.6	36.0	978902.1	978906.7	10.6163	35.581	35.583
ZM79	ZM80	25 32	117 14	2.2	1475.702	979006.5	138.8	36.0	978903.6	978902.1	-12.1749	-40.805	-40.801
ZM80	ZM81	25 31	117 12	2.0	1457.450	979005.3	137.1	36.0	978904.2	978903.6	- 5.4455	-18.251	-18.249
ZM81	ZM82	25 29	117 10	2.0	1425.196	979003.0	134.1	36.0	978904.9	978904.2	- 9.6236	-32.254	-32.251
ZM82	ZM83	25 28	117 9	2.0	1403.280	979001.8	132.0	36.0	978905.8	978904.9	- 6.5391	-21.916	-21.914
ZM83	ZM84	25 27	117 8	2.5	1386.198	979000.7	130.4	36.0	978906.3	978905.8	- 5.0968	-17.082	-17.080
ZM84	ZM85	25 25	117 7	2.0	1360.698	978998.3	128.0	12.0	978882.3	978906.3	- 7.6083	-25.500	-25.497
ZM85	ZM86	25 24	117 6	1.9	1338.312	978997.2	125.9	- 0.0	978871.2	978882.3	- 6.6791	-22.386	-22.384
ZM86	ZM87	25 22	117 6	2.0	1324.692	978994.8	124.6	-24.0	978846.2	978871.2	- 4.0636	-13.620	-13.617
ZM87	ZM88	25 21	117 5	2.0	1307.465	978993.7	123.0	-36.0	978834.7	978846.2	- 5.1397	-17.227	-17.225
ZM88	ZM89	25 19	117 4	2.1	1291.518	978991.4	121.5	-36.0	978833.9	978834.7	- 4.7578	-15.947	-15.944
ZM89	ZM90	25 18	117 3	2.0	1277.591	978990.2	120.2	-36.0	978834.0	978833.9	- 4.1551	-13.927	-13.925
ZM90	ZM91	25 16	117 1	2.1	1261.532	978987.9	118.7	-36.0	978833.2	978834.0	- 4.7912	-16.059	-16.056
ZM91	ZM92	25 15	117 0	2.1	1250.427	978986.7	117.6	-36.0	978833.1	978833.2	- 3.3132	-11.105	-11.104
ZM92	ZM93	25 13	116 59	2.0	1234.428	978984.4	116.1	-36.0	978832.3	978833.1	- 4.7733	-15.999	-15.996
ZM93	ZM94	25 12	116 57	2.3	1215.604	978983.3	114.3	-36.0	978832.9	978832.3	- 5.6161	-18.824	-18.823
ZM94	ZL63	25 10	116 56	2.5	1215.506	978980.9	114.3	-36.0	978830.6	978832.9	- 0.0289	- 0.097	- 0.094
										TOTAL	-17.4262	-58.418	-58.355

network, anomalies were available at the corners of the 0.1 degree squares surrounding the bench-mark. The covariance function and the ordering sub-routine were not required, therefore, in the final computation.

Results obtained from the conversion of the observed levelling to geopotential differences have been analysed to determine whether any significant effect on the levelling had resulted from the use of observed gravity as opposed to normal gravity. Basically, the analysis was a simple comparison of geopotential values and the orthometric heights formed by the Division of National Mapping.

The orthometric heights and geopotential differences cannot, of course, be compared directly because of their dimensional dissimilarity. Orthometric heights have the dimension of length whilst geopotentials have the dimension of length^2 time^{-2}. Although these two scalars are *theoretically* impossible to compare, both are used for vertical control, and in *practice* some similarity does exist. Although the two methods of comparison which are described below, are open to criticism, their usage is perhaps justifiable if the limitations are realized. Orthometric heights and geopotential differences for a number of sections of levelling were compared by this method.

A Method for Comparing Geopotential and Orthometric Height Differences

In this test, orthometric heights are converted to potentials which are then compared with potentials formed from the levelling observations. The process for conversion of the orthometric heights is developed using Figure 2.11 in which P_1' and P_2' are two bench-marks. An orthometric height difference ΔH_0 has been observed between P_1' and P_2'. P_1 and P_2 are the points where the verticals through P_1' and P_2', respectively, intersect the geoid. Let the gravity values at P_1, P_1', P_2, P_2' be g_1, g_1', g_2 and g_2' respectively. If the orthometric heights of P_1' and P_2' are H_{0_1} and H_{0_2}, then theoretically,

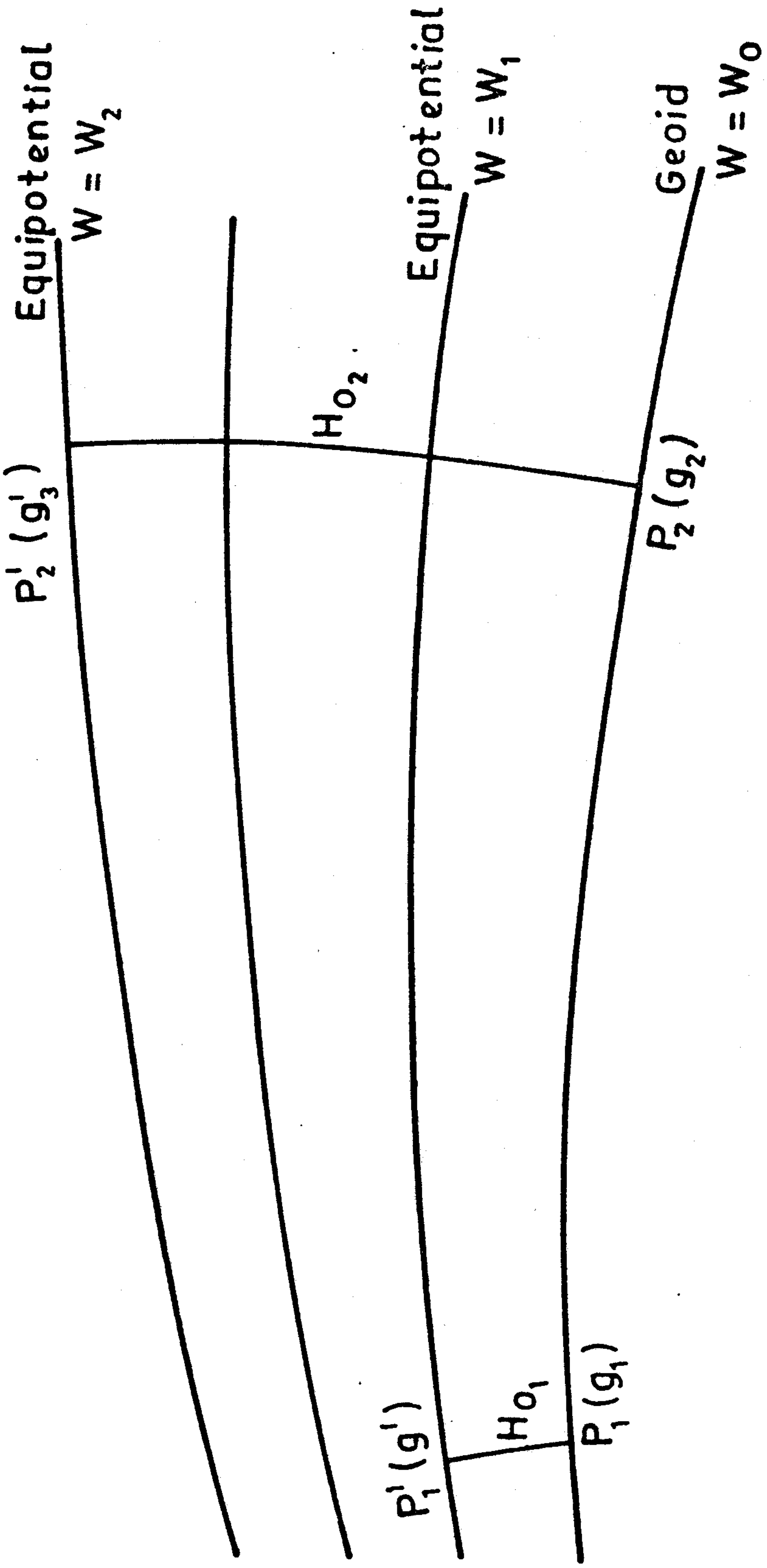

Equipotential
W = W$_2$
Equipotential
W = W$_1$
Geoid
W = W$_0$
H$_{O_2}$
P$_2'$ (g$_3'$)
P$_2$ (g$_2$)
H$_{O_1}$
P$_1'$ (g$'$)
P$_1$ (g$_1$)

FIG. 2·11

$$\Delta H_0 = H_{0_2} - H_{0_1}$$

Further, the geopotential difference between P_1' and P_2', ΔW, is

$$\Delta W = W_2 - W_1$$

where W_1 and W_2 are the potentials of the equipotential surfaces through P_1' and P_2', respectively.

$$W_1 = \int_0^{H_{0_1}} g \, dH_0$$

$$= \tfrac{1}{2} H_{0_1} (g_1 + g_1')$$

if g varies linearly with height.

$$W_2 = \tfrac{1}{2} H_{0_2} (g_2 + g_2')$$

$$\therefore \quad \Delta W = \tfrac{1}{2} H_{0_2} (g_2 + g_2') - \tfrac{1}{2} H_{0_1} (g_1 + g_1')$$

However, this relationship is difficult to use as ΔH_0 is known to a higher accuracy than H_{0_1} and H_{0_2} separately, and g_1 and g_2 cannot be measured.

Define

$$G_1 = \tfrac{1}{2}(g_1 + g_1')$$

$$G_2 = \tfrac{1}{2}(g_2 + g_2')$$

i.e.

$$\Delta W = H_{0_2} G_2 - H_{0_1} G_1$$

$$= (H_{0_2} - H_{0_1}) \frac{G_2 + G_1}{2} - (H_{0_2} - H_{0_1}) \frac{(G_2 + G_1)}{2} + H_{0_2} G_2 - H_{0_1} G_1$$

Let

$$T_1 = (H_{0_2} - H_{0_1}) \frac{G_2 + G_1}{2}$$

$$= \tfrac{1}{2} \Delta H_0 (G_2 + G_1)$$

and

$$T_2 = -(H_{0_2} - H_{0_1}) \frac{G_2 + G_1}{2} + H_{0_2} G_2 - H_{0_1} G_1$$

$$= \tfrac{1}{2}(H_{0_2}G_2 + H_{0_1}G_2 - H_{0_2}G_1 - H_{0_1}G_1)$$
$$= \tfrac{1}{2}(H_{0_2} + H_{0_1})(G_2 - G_1)$$

Let the height correction be $C.H_0$ where C is a constant variation of gravity with elevation. Then,

$$G_i = \tfrac{1}{2}(g_i + g_i') \qquad i = 1,2$$
$$= \tfrac{1}{2}(g_i' + C.H_{0_i} + g_i')$$
$$= g_i' + \tfrac{1}{2}.C.H_{0_i}$$

$$\therefore \quad G_2 + G_1 = g_1' + g_2' + \tfrac{1}{2}C(H_{0_1} + H_{0_2})$$
$$G_2 - G_1 = g_2' - g_1' + \tfrac{1}{2}C(H_{0_2} - H_{0_1})$$
$$\therefore \quad T_1 = \tfrac{1}{2}.\ H_0[g_1' + g_2' + \tfrac{1}{2}C(H_{0_1} + H_{0_2})]$$

and

$$T_2 = \tfrac{1}{2}(H_{0_2} + H_{0_1})\ [g_2' - g_1' + \tfrac{1}{2}.C(H_{0_1} - H_{0_2})]$$
$$= \tfrac{1}{2}(H_{0_2} + H_{0_1})\ [g_2' - g_1' + \tfrac{1}{2}.C.\Delta H_0]$$

$$\therefore \quad \Delta W = \tfrac{1}{2}\Delta H_0[g_1' + g_2' + \tfrac{1}{2}.C.(H_{0_1} + H_{0_2})] + \tfrac{1}{2}(H_{0_2} + H_{0_1})$$
$$[g_2' - g_1' + \tfrac{1}{2}.C.\Delta H_0] \qquad \ldots\ldots(2.8)$$

The value of C was adopted as 0.1967 mgal metre^{-1}, this being the free-air reduction minus the Bouguer plate correction (*Heiskanen and Moritz*, 1967, p.131). Further corrections, such as the terrain correction, were neglected, because of the expected insignificance of such corrections in Australia, (*ibid*, p.132).

As the magnitude of term T_1 is the same order as ΔW, the variables H_0, g_1', g_2', H_{0_1}, H_{0_2} are required to the same accuracy as used in the conversion to geopotential differences.

From the previous equations:

$$T_2 = \tfrac{1}{2}(H_{0_2} + H_{0_1})(g'_2 - g'_1 + \tfrac{1}{2}.C.\Delta H_0)$$

For levelling sections in this network, estimates of average to large values are:

500 metres for H_{0_i} $(i = 1,2)$
200 mgal for $(g'_1 - g'_2)$
500 metres for ΔH_0

$\therefore$ $T_2 \simeq 0.5 \times 100(200 - 0.5 \times 0.2 \times 500)$ mgal m
$\doteqdot 8$ kgal cm.

Hence it is seen that, if the values of H_{0_i}, $(g'_2 - g'_1)$ and H_0 are given to a metre, 10 mgals and a centimetre respectively, T_2 can be calculated with sufficient accuracy.

A computer programme, shown in Appendix 1.6, was written, to calculate ΔW values according to formula (2.8) and to compare them with the values obtained in the conversion to geopotential. Every tenth of the 757 levelling sections was compared. The resultant differences are illustrated in Figure 2.12, which shows that 60% are less than 3 mm whilst 8% are 2 cm or more. The mean difference without regard to sign was 0.57 cm, whilst the mean when the sign of the differences was considered was -0.12 cm. Approximately 30 sections of levelling are sufficient to cross the continent from one side to the other in any direction, so that the total effect on a continent-wide levelling run should only be about 4 cm. This is smaller than the estimated accuracy of the levelling. The majority of differences are then regarded as insignificant. In an attempt to see whether there are any obvious causes of the differences greater than 1 cm, these have been examined and the following points were found:

FIG. 2.12

HISTOGRAM

of

DIFFERENCES

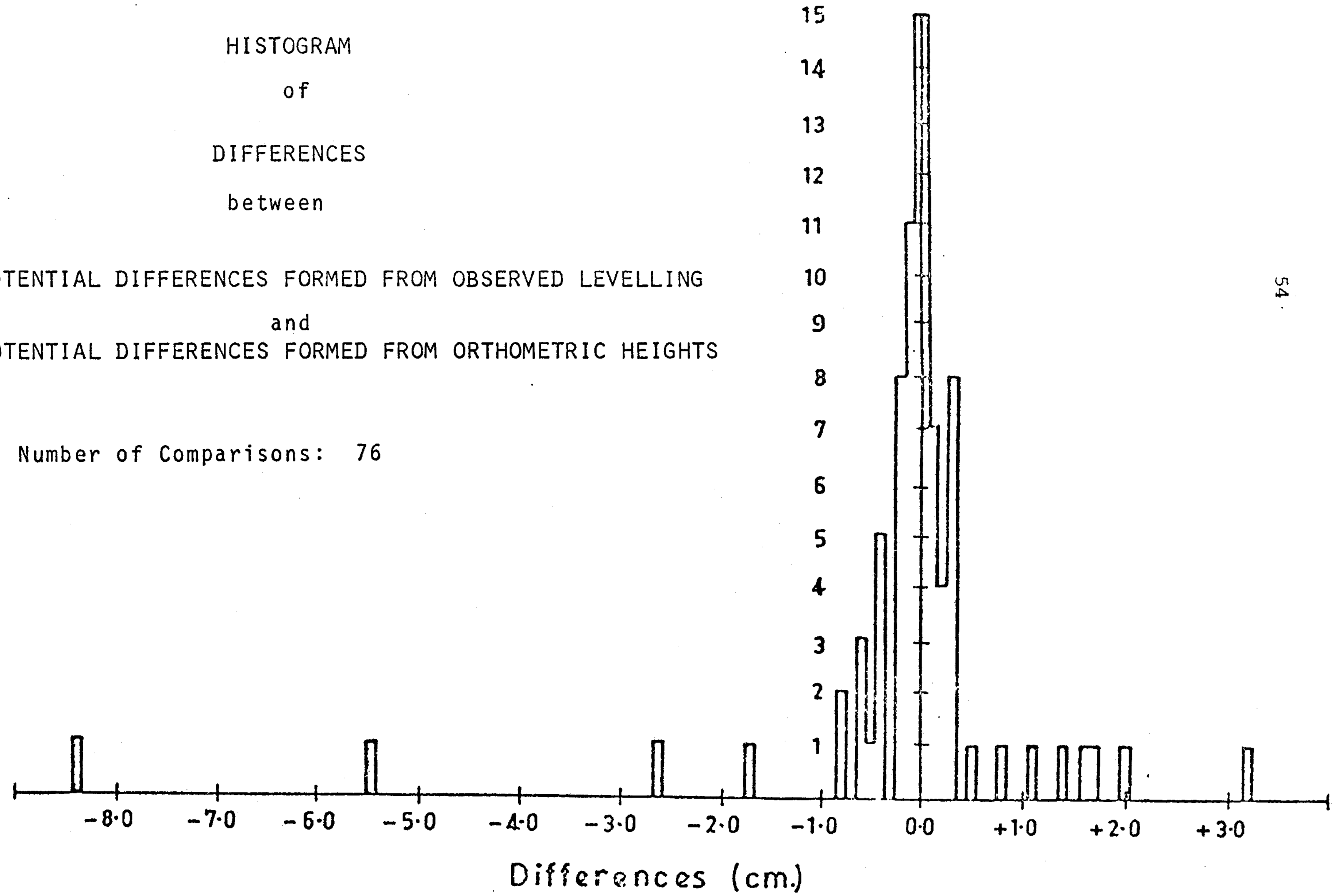
between

GEOPOTENTIAL DIFFERENCES FORMED FROM OBSERVED LEVELLING

and

GEOPOTENTIAL DIFFERENCES FORMED FROM ORTHOMETRIC HEIGHTS

Total Number of Comparisons: 76



(i) there is definitely no correlation between the large differences and the horizontal length of the section of levelling;

(ii) there is an apparent correlation between the differences and ΔW;

(iii) a degree of correlation exists between the differences and the mean elevation of the levelling section.

As the differences are not significant to levelling of this accuracy, further explanation has not been sought.

Comparison of Trans-Continential Runs of Levelling

To test for the existence of any cumulative influence in the conversion to geopotential differences, two levelling routes traversing Australia, one in a north-south and the other in an east-west direction were compiled in the newly formed geopotential differences and in the orthometric heights. The differences which are shown in Table 2.3, are so small that the effect of the dimensional difference can be considered to be insignificant, as it must be remembered that the standard deviation of a height at the coastline with respect to a height at the *centre* of the continent was estimated as about 35 cm using *adjusted* orthometric heights.

2.11 Conclusions

It does not seem difficult to extract the relevant conclusions from this chapter. The effects of using observed gravity and of using geopotentials differences on the existing system of vertical control have been so small as to show that these modifications contribute negligibly to the cause of the MSL/geoid deviation. Although this type of conversion may offer scope for detailed analysis, this is not warranted in a study of a 200 cm separation between the geoid and MSL.

Table 2.3

Comparison of Orthometric Height Differences
and Geopotential Differences in Trans-Continental Levelling

	NORTH-SOUTH LEVELLING RUN	EAST-WEST LEVELLING RUN
Total difference in orthometric height	4.119 m	7.892 m
Approximate conversion to potential by multiplying by γ where γ = 0.979000 kgal	4.033 kgal m	8.714 kgal m
Total difference in geopotential	4.044 kgal m	8.695 kgal m
Difference	1.1 kgal cm	1.9 kgal cm

Conclusions could however be drawn for levelling networks in general, although the results obtained in Australia do not necessarily apply to other areas where gravity values may have different characteristics. Nevertheless, if gravity observations were made along the levelling route of surveys, the only significant sources of error are eliminated. Furthermore, the conversion of the levelling loses much of the complexity which was necessary in this study. The geopotential calculation could very easily form part of the levelling reductions.

3. ADJUSTMENT OF THE NETWORK OF GEOPOTENTIAL DIFFERENCES

3.1 Methods of Adjustment

The network of newly formed geopotential differences contains inconsistencies, or misclosures, as does the original network of observed levelling. The effects of these levelling observation inaccuracies must be removed before a homogeneous system of vertical control can be produced from the geopotentials. Thus, until an adjustment is undertaken, height differences in the network are dependent on the route of the levelling along which the calculations are made. The Division of National Mapping has undertaken an adjustment of their network of orthometric height observations, (*Roelse et al*, 1971). In this adjustment, the continent was divided into four regions, each of the smaller networks being adjusted by the *parametric* method, and then the regions were recombined using a *conditions* method adjustment. The adjustment described here was undertaken *independently* of the Division of National Mapping calculations, in a single phase. The adjustment was also undertaken as a step towards a statistical analysis, by which the accuracy of the heights in the adjusted network could be estimated. The apparent deviation between MSL and the geoid could then be compared with the evaluated levelling accuracy. Division of National Mapping results indicate that the levelling should easily distinguish MSL variations of 2 m; see Chapter 1. The statistical analysis is also independent of the results described by *Roelse et al* (*ibid*).

The aim of the process described in this chapter was, then, to adjust the network of geopotential differences corresponding to the observed levelling, and thereafter, to analyse the adjustment to obtain statistics for the accuracy of the adjusted geopotential network. Both the adjustment and the analysis were to be undertaken by completely standard processes. The techniques of adjustment were accepted as valid.

The two processes by which a levelling, or geopotential, network may be adjusted, the *parametric* and the *condition* methods, were both considered as possible alternatives for the adjustment. The theory of adjustment is not discussed in this Chapter, but processes used here are explained by *Allman* (1967), although the theory may actually be found in any number of articles on the subject. The symbols and terminology used here follow *Allman* (*ibid*). However, the infrequently-used statistical analysis is based on the application of the general law of propagation of variances applied to the weight coefficients of the variates, and will be discussed further in Section 3.4.

Complete network data was available for the adjustment. The geopotential differences for all sections were available (Chapter 2) along with the distances between Junction Points in the net. The loop and junction point numbering systems adopted were the same as those used by the Division of National Mapping.

In a network as large as this, the advantages and disadvantages of each method of adjustment must be weighed carefully. The adjustment programme was thought likely to approach computer storage limits and was expected to be demanding on computer time. Moreover, the suitability of the available data for easy application to each method had to be considered. The ease of programming and the use of results from each of the methods also had to be investigated. The methods are compared below on the basis that the Australian network contains 760 sections of levelling forming 260 loops:

(1) In the condition method, the number of normal equations to be solved is equal to the number of loops, 260. The number of normal equations in the parametric method is equal to the difference between the numbers of sections and loops, about 500 in this case.

(2) The condition method requires the formation of 260 condition equations and 260 correlate equations for the Australian network; the parametric method requires 760 observation equations and 500 parameters. This aspect is not as significant as (1) because the calculations can be "streamlined" so that not all equations need to be completely formed in the final computation.

(3) Each condition must be derived manually for the condition method. That is, the identifying numbers of each section in every loop of the network must be compiled from a plan of the network. This is important when 260 loops are to be formed from 760 levelling sections. Although the parametric method requires approximate geopotentials of all network junction points, these are available in the Australian network.

(4) The parametric method requires iteration, whereas final solutions are obtained from the first calculation by conditions.

(5) The parametric method produces the adjusted potentials of the junction points, but the condition methods adjusts the geopotential differences. Consequently, the latter requires that the potentials of the junction points be calculated from the adjusted geopotential differences. This disadvantage is not severe in this study, as only the potentials of the *coastal* junction points are ultimately required.

(6) The calculation of the weight coefficients of the estimates of the variates is calculated by the multiplication of three matrices in the parametric method; five matrices are combined with conditions. However, the inverses of the normals are used in both cases, and the use of the large normal equations is a prohibitive factor, see (1).

As the only serious disadvantage of the condition method is given in (3) above, and as it seemed preferable to use manual labour time rather than risk approaching computer time and storage limits, the condition method was used for the programme described below.

3.2 Computer Programme

The flow diagram for the computer programme which was designed to adjust the network of geopotential differences is shown in figure 3.1. The programme itself is listed in Appendix 2.1. The detailed statistical analysis of the adjusted network was undertaken by *another* programme and is discussed in Section 3.4.

Two lots of data are required: firstly, the geopotential differences for each section as described previously with an estimate of the variance for each observation; and secondly, the data showing the composition of the 260 loops of levelling. The latter was prepared manually from numbered diagrams of the levelling network.

On the basis that the random error in levelling is proportional to the square root of the length of levelling, the variance of each section of geopotential differences was adopted as the distance between the junction points of the section, in miles, as the data was given in miles. This immediately assumes that the *variance factor* is one kgal m per mile of levelling; or 0.63 kgal m per kilometre. Although obviously erroneous, the effect may be rectified later.

The programme, if written to follow exactly the traditional matrix algebra forms of the calculation, would be unnecessarily unwieldy for this specialized computation. Some measures have been taken to streamline calculations. The matrix of conditions is not written in full. Instead, the equivalent matrix consists of one row for each equation, whilst the columns show the sections involved in the relevant condition equation, with the appropriate signs. Thus, they take the form of the condition equations of the conventional, non-matrix manipulation. As a result of this abbreviation, however, the resultant programme is more complex than would be needed for simple matrix manipulation, although the actual processes are abbreviated. The matrix of normal equations is not formed completely as only the upper triangular matrix is required. Moreover, this matrix is not actually formed at all, as the equivalent

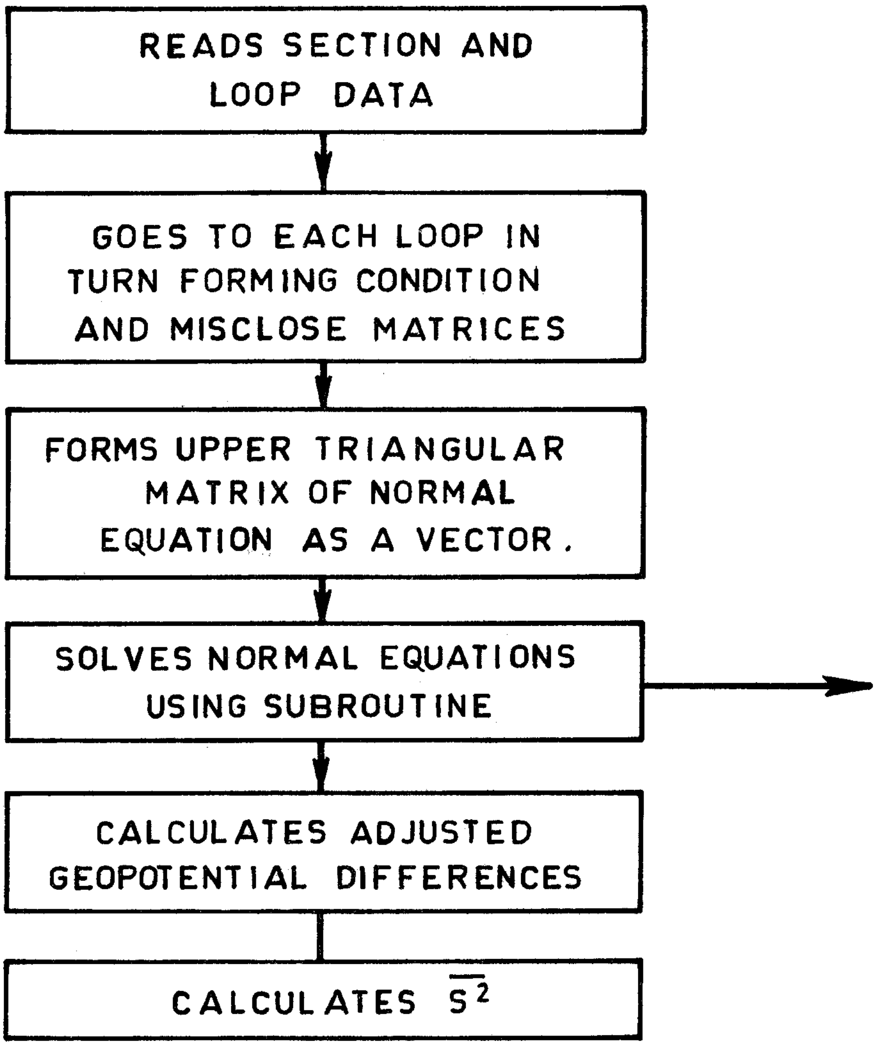


FIG. 3·1

FLOW DIAGRAM FOR ADJUSTMENT PROGRAMME

vector is formed immediately for application to the solving subroutine. This routine uses a Gaussian elimination with back-substitution, and was made available by Messrs. D.C. Black and S. French, School of Civil Engineering, University of N.S.W. A listing appears in Appendix 2.2.

The only attempt at statistical analysis in this programme is the calculation of the estimate of the variance factor. More correctly, the programme calculates a value which must be divided by the number of redundancies, which is equal to the number of loops minus one.

The adjustment programme was tested on two levelling nets for which the adjusted values were known. The first net had only four loops and nine sections, the second had ten loops and fifty-seven sections. The adjusted values and variance factor estimate agreed with the known values.

3.3 Results

The calculations using the described programme were undertaken to adjust the 757 sections of geopotential differences formed into 260 loops. A geopotential value for a suitable junction point occurring on the coast, was *adopted* and, using the adjusted geopotential differences, the potential values of junction points along a levelling route around the coastline, were calculated. The misclose of the geopotentials remaining after adjustment, in this loop around the continent, was only 0.0004 kgal m, equivalent to 0.4 mm. The geopotential values of the MSL's at the thirty tide-gauges around Australia were calculated from the geopotential differences between the relevant coastal junction point and Mean Sea Level. Potential values at inland junction points were not calculated. Figure 3.2 show the resultant geopotential values of MSL compared with the orthometric heights of MSL provided by the Division of National Mapping. It must be realized that the Division of National

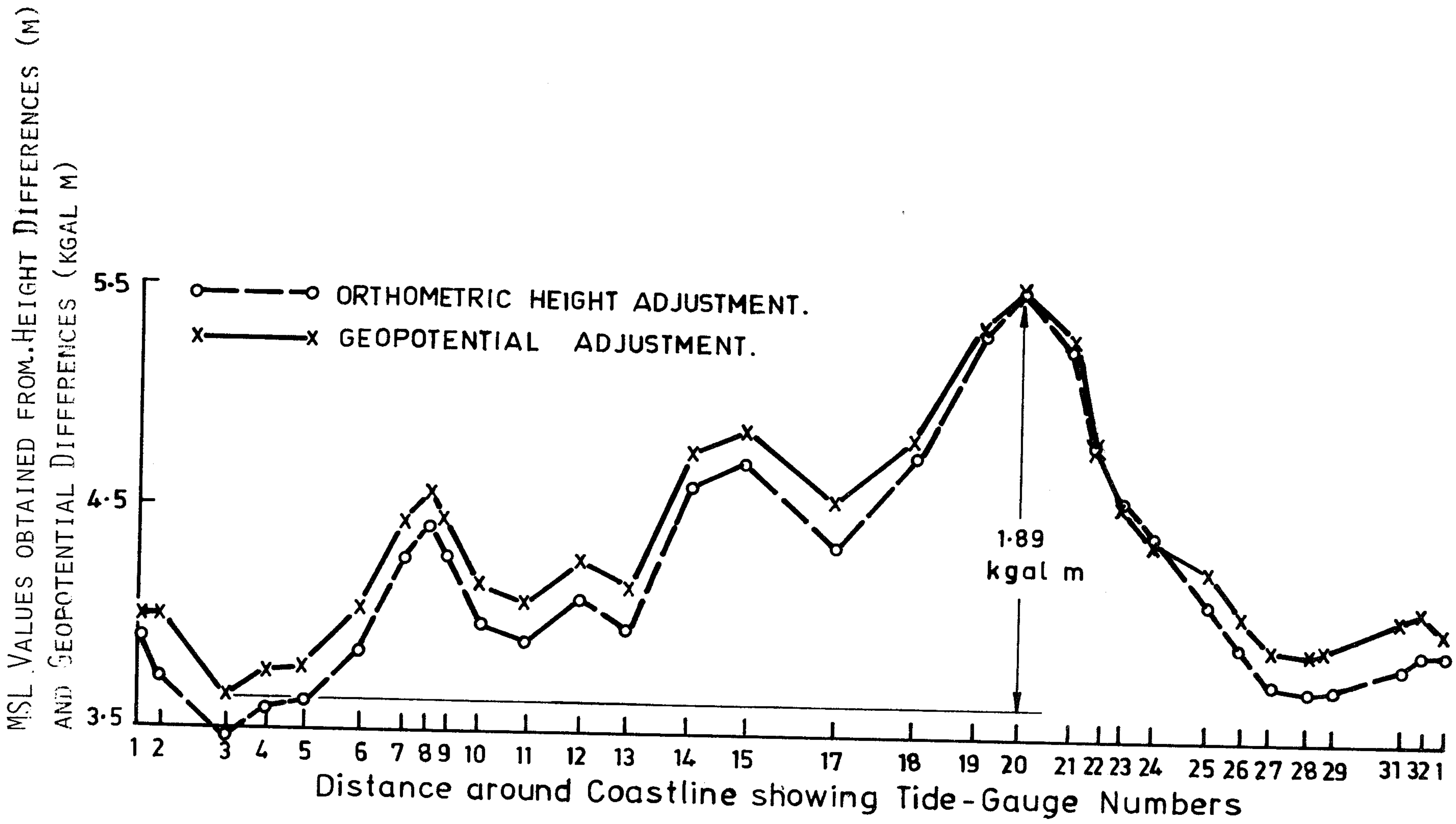


FIG. 3·2
VALUES OF MEAN-SEA-LEVEL
ACCORDING TO THE ADJUSTED NETWORK OF GEOPOTENTIAL
DIFFERENCES COMPARED WITH THE ADJUSTED HEIGHT VALUES
PRODUCED BY THE DIVISION OF NATIONAL MAPPING.

Mapping results are in different units than the potentials. Approximately a two percent difference must therefore be expected, the orthometric height differences appearing slightly larger in this figure. Differences between the two graphs may be explained by the data which were used in the adjustments. After the orthometric-height network was adjusted, further levelling information became available. This may account for the apparent modifications in the potentials adjustment, as this used the more recent data.

The value of the *minimum* was calculated in the programme as 0.2143 kgal m, which, when divided by the number of redundancies, 259, gives an estimate of the variance factor of 0.0008274 (kgal m)^2. As the weight coefficients of the observations was 0.1 times the distance in miles, or 0.06215 times the distance in km, the estimate of the variance of the levelling is given by

$$\sigma^2 = 0.0008274 \times 0.06215 \times \sqrt{K} \text{ kgal m}^2$$

where K is the distance in km. The standard deviation,

$$\sigma = 0.007171 \times \sqrt{K} \text{ kgal m}$$
$$\doteqdot 7.2 \sqrt{K} \text{ kgal mm.}$$

The Division of National Mapping adopted a standard deviation for the levelling of 0.0338 $\sqrt{M}$ feet, where M is the distance along the levelling route in miles. This is equivalent to:

$$0.0338 \times 0.3048 \times 0.979 \times \sqrt{0.622} \times \sqrt{K} \text{ kgal m}$$
$$= 0.007952 \sqrt{K} \text{ kgal m}$$
$$\doteqdot 8.0 \sqrt{K} \text{ kgal mm.}$$

To test the validity of the σ value derived for the potential adjustment, the programme was re-run using weight coefficients for the observations of 51.4 kgal mm per km, equal to σ^2. The resultant estimate of the variance factor was unity, verifying the value of σ above. Theory relevant to the processes described above is given in *Richardus* (1966, pp.74-76).

A slightly modified version of the adjustment routine was used to calculate:

(i) the corrections to each section of geopotential in the adjustment.

(ii) the algebraic mean of all corrections in (i). This was found to be -0.0014 kgal m, equivalent to about 1.4 mm in height.

(iii) the mean of the absolute values of all corrections was 0.0375 kgal m, or about 3.75 cm.

(iv) a value of the variance factor, calculated by a different method to that discussed above. This was formed, as a check, according to, (*Richardus*, 1966, p.74),

$$\bar{S}^2 = \sum_{i=1}^{n} (w_i e_i e_i) / r$$

where w is the weight coefficient for each observation and was adopted as in the original adjustment,
e is the correction found in (i),
r is the number of redundancies, and in this case is equal to 756. The value of the variance factor was found to be the same as that calculated in the original programme.

More intricate statistical analysis is undertaken in Section 3.4.

3.4 Statistical Analysis

As the statistical analysis undertaken in this study is not amongst the commonly used adjustment processes, the theory of the analysis will be outlined. Moreover, an outline of the theory will simplify the explanation of the computer programme and its results. The *aim* of the analysis is to enable the estimation of the standard deviation of a difference between the geopotential values of two points in the network, after adjustment. Ultimately, this result can be applied to estimate the standard deviation of the difference in potential of two MSL's, thereby indicating whether the apparent MSL/geoid deviations are explicable by levelling inaccuracy. The calculation is based on the *general law of propagation of variances*, by which, if

$$y = y(x_1, x_2 \ldots\ldots x_n)$$

then the variance of y is given by

$$\sigma y^2 = \left(\frac{\partial f}{\partial x_1}\right)^2 \sigma^2_{x_1} + \left(\frac{\partial f}{\partial x_2}\right)^2 \sigma^2_{x_2} + \ldots\ldots \left(\frac{\partial f}{\partial x_n}\right)^2 \sigma^2_{x_n}$$
$$+2\,\frac{\partial f}{\partial x_1}\frac{\partial f}{\partial x_2}\,\sigma_{x_1 x_2} + 2.\frac{\partial f}{\partial x_1}\frac{\partial f}{\partial x_3}\,\sigma_{x_1 x_3} \ldots\ldots$$
$$+2\,\frac{\partial f}{\partial x_{n-1}}\frac{\partial f}{\partial x_n}\,\sigma_{x_{n-1} x_n}$$

where $\sigma^2_{x_i}$ is the variance of x_i, and

$\sigma_{x_i x_j}$ is the covariance of x_i and x_j.

In a network of geopotentials, let the potentials of two junction points, A and B, be W_A and W_B respectively. These potentials may be related according to

$$W_B - W_A = \Delta W_{AB}$$
$$= \sum_{i=1}^{n} \Delta W_i,$$

where the ΔW_i, (i=1, n) are the potential differences of the n sections of geopotential between junction points A and B. The sign of W_i is dependent on whether the geopotential difference for the ith section is given in the direction A to B or in the direction B to A. According to the general law of propagation of variances, the variance of ΔW_{AB}, using the aforementioned notations, is

$$\sigma^2_{\Delta W_{AB}} = \sigma^2_{\Delta W_1} + \sigma^2_{\Delta W_2} + \ldots \sigma^2_{\Delta W_n}$$
$$+ 2\sigma_{\Delta W_1 \, \Delta W_n} + \ldots\ldots + 2\sigma_{\Delta W_{n-1} \, W_n}$$

The values of $\sigma^2_{\Delta W_i}$ and $\sigma_{\Delta W_i \, \Delta W_j}$, the variances and covariances of the adjusted coefficients are now required. *Allman* (1967, pp.222-223), describes the derivation of the weight coefficients of the estimates of the variates. Multiplied by the variance factor, these produce the estimates of the variances of the adjusted values (*ibid*, p.216). In matrix notation, the weight coefficients of the variates are given by the matrix Q_{PP},

$$Q_{PP} = G - Q_{VV}$$

where Q_{VV} is the matrix of the weight coefficients of the corrections.

$$Q_{VV} = G \, B^T \, Q_{KK} \, B \, G^T$$

where G is the matrix of weight coefficients of the observations,

B is the matrix of coefficients in the condition equations,

Q_{KK} is the matrix of weight coefficients for the correlates and is equal to the inverse of the matrix of coefficients of the normal equations (*ibid*, pp.222-223).

A computer programme was written to form the variances according to the theory just described and to apply the general law of propagation of variances. The computation was actually undertaken in two parts, the first being the formation of the inverse of the normals, and the second being the calculation of the variances and covariances, and of the variance of the potential difference between two junction points. Measures were taken to streamline the programme, especially as the analysis programme was considered likely to be more time and computer-space consuming than the original adjustment routine. As only a limited number of junction points are involved in the calculation of the standard deviations, not all the covariances and variances need to be calculated. Before the routine was used, the required variances and covariances were selected and found to lie in two groups of numbers. Thus, not all the matrix Q_{PP} was found necessary, but the full matrix B was used. Only the upper triangular matrix of Q_{PP} was formed. Although these streamline procedures made programming difficult, they enabled the computation to be undertaken on the available computer.

The first part of the programme which forms the matrix Q_{KK} is listed in Appendix 2.3. Appendix 2.4 shows the subroutine which inverts the matrix of normal equations, and which was made available by Messrs. D.C. Black and S. French, School of Civil Engineering, University of N.S.W. The second part of the programme continues the formation of the variances and covariances and then calculates the variance of the potential differences. It is listed in Appendix 2.5, whilst Appendices 2.6 and 2.7 show the two subroutines which were used. The programme requires the use of suitable weight coefficients of the observations; that is, in this network, 7.2 $\sqrt{K}$ kgal mm was used. When tested on a levelling network of three loops and six sections, the programme correctly calculated the standard deviation of a geopotential difference between two junction points separated by three sections of levelling.

Data preparation for the programme involved the compilation of the numbers of the sections of geopotential differences between the two junction points for which the standard deviation was required. The first part of the programme, the formation of the inverse of the normals, was undertaken only once. The second part of the programme was used seven times. Thus, in all, $\sigma^2_{\Delta W_{AB}}$ was calculated from the single normal inverse for *seven* pairs of junction points A and B. The results of these tests are shown in Figure 3.3, which gives the standard deviations of the differences in adjusted potential between the pairs of points. The significance is discussed below.

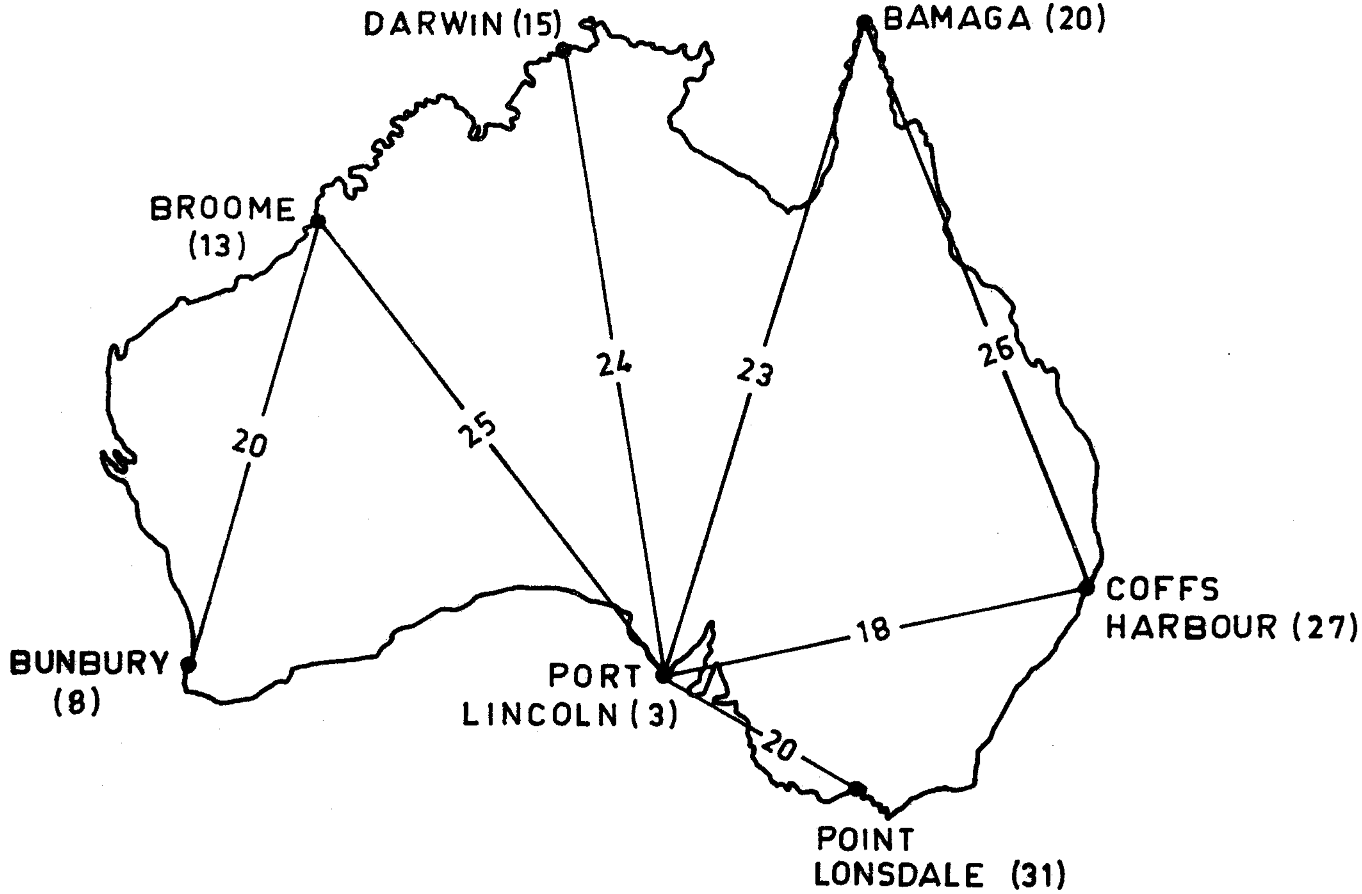


FIG. 3·3
STANDARD DEVIATIONS OF DIFFERENCES IN POTENTIALS AT POINTS SHOWN IN ADJUSTED NET WORK.
STANDARD DEVIATIONS IN kgal cm

3.5 Conclusions

The aims of the adjustment and the statistical analysis were to, firstly, produce a network of adjusted geopotential values which could be compared with the orthometric height network produced by the Division of National Mapping, and, secondly, to study the accuracy of the geopotentials in the network. As the latter point has greater significance in the search for the causes of the apparent geoid-MSL deviations, it must be considered to be more important.

The adjustment itself (Sections 3.1 to 3.3) produced a network in which the geopotential values of MSL agreed closely with the orthometric heights in the Division of National Mapping net. Slight differences are possibly due to the more recent levelling observations in some sections of the potential conversion and adjustment in this study, as mentioned in Section 2.4.

For the statistical analysis, the most significant tide gauges are at Bamaga and Port Lincoln, between which MSL is shown to have a potential difference of 1.89 kgal m, (Figure 3.2). According to the statistical analysis, this potential difference has a standard deviation of 0.28 kgal m. In other words, the *corrected* MSL's at Bamaga and Port Lincoln have potentials differing by about 1.9 kgal m ± 0.3, with 67% confidence, or ±0.6 with 95% confidence. The error estimate falls well below the geopotential difference itself. It is worth noting that the estimate of the variance of the levelling according to this adjustment is similar to that produced by the Division of National Mapping, (*Roelse et al*, 1971, p.76).

Further analysis of the random error effects in the levelling are possible, but, as in the geopotential conversion described in Chapter 2, closer study does not seem warranted.

In terms of the random errors, the MSL/geoid deviations must be considered to be almost unaffected. Systematic errors however remain uninvestigated. Aspects of the levelling network itself are considered in the discussion in Chapter 9.

4. TIDES

4.1 Introduction

The combination of gravity and levelling, the subsequent adjustment of the network of geopotentials and the estimate of the random error propagation, produced no clues to the origin of the apparent deviation between Mean Sea Level and the geoid. Consequently, attention should be turned to the definition of the position of a level surface by the sea-surface. Virtually, a survey was taken of the present state of knowledge, in the field of oceanography, of deviations between sea level and the geoid. A number of causes of time and position deviations between the two surfaces exist, as initially mentioned in Chapter 1. A summary of the present state of knowledge of each phenomenon is presented in the Chapters 4 to 7. The impact on the deviation between MSL and the geoid is estimated or evaluated wherever possible.

The variations of the level of the sea-surface are dominated by tides, because of the comparatively large magnitude and the comparatively high frequency of occurrence of tides. Smaller and less frequent fluctuations in sea level are masked by tidal variations. Consequently, the influence of tides was examined before any other oceanographic phenomenon.

The *ultimate aim* of the study of tides was to ensure that Mean Sea Level derived from gauge readings over five years was free from the influence of the rise and fall of tides. It was also proposed to eliminate time variations of tides from the records of sea level, whilst preserving other fluctuations, thereby enabling later examination of the remaining time-variations.

Although the most obvious way to account for the tidal effect is to subtract the theoretical height of the tide at any time from the recorded gauge height of sea level, this is not possible. Predictions of tides based on the physical laws of gravitational attraction, applied to the known positions and masses of the sun and the moon, do not agree with observed times and heights of existent tides. "Equilibrium theories" of tides have been rigorously developed by various mathematicians since about 1900 (see for example, *Darwin*, 1907; *Doodson*, 1922; 1928; 1958) but the discrepancies between the simplified ocean models and the real oceans are so great as to prevent successful theoretical predictions of tidal amplitudes and phases. Fortunately, the *periods* of the tides derived theoretically from the movements of the sun and the moon occur in reality. Using these periods, it is possible to devise methods which help eliminate significant tidal effects from gauge records.

4.2 The Movements of the Sun and the Moon

A brief outline of the movements of the sun and the moon are given as background to the tidal periods. Although the movements of the sun and moon are periodic, they are nevertheless complex, and they are difficult to express mathematically. As the *details* of the solar and lunar movements are irrelevant, however, a description of the more important movements of these two bodies is given. Some indications of the tidal periods associated with these movements are also included.

The general configuration of the sun-moon-earth system is well known: the moon orbits the earth, with a period between successive perigees of 27.55455 days, while this orbit rotates in its own plane in 8.847 years. The earth, rotating on its own axis to face the sun every 24 hours, completes an orbit around the sun in a year of 365.242199 days. Immediately, a number of periodic gravitational influences are apparent. A 24 hour cyclic attraction occurs because any point on earth faces the sun once every day. Actually, because of the mechanics of the system, a 12-hour-period tide is produced. Similarly, there is an equivalent lunar gravitational variation. Because the moon rotates about the earth in the direction of the earth's rotation, this period is not 24 hours, but 24 hours plus 1/27.55455 days, i.e. 1.035050 days (24 hours 50.47 minutes). Coincidence of the sun and the moon in the meridian occurs every 29.53059 days, at new moon.

Because of the eccentricity of the earth's orbit, the distance between the sun and the earth, and consequently the gravitational attraction between the sun and the earth, has a 365.26 day period. In a year the sun's declination varies through a cycle as a result of the 23½ degree dihedral angle between the ecliptic and the earth's equatorial plane. Owing to the angle of about $5^{\circ}09'$ between the moon's orbit and the ecliptic, the moons declination varies with every lunar orbit about the earth. The dihedral angle itself varies over 173 days.

Gravitational attraction between the sun and the moon causes disturbances of the moon's orbit, known as *evection* and *variation*, which have 31.812 and 14.765 day cycles respectively.

Figure 4.1 illustrates some of the aforementioned relationships.

4.3 The Theoretical Development of the Tidal Periods

For any point on the earth's surface, the variation, with time, of the gravitational attraction due to the sun and the moon may be calculated by applying the laws of Newtonian gravitational attraction to the masses and relative positions of the sun, moon and earth. One approach to producing an expression of the *tide generating potential* in mathematical form and the subsequent development into a harmonic series has been outlined by *Melchior* (1958; 1966). The different tidal periods may be extracted from the harmonic expression of the tide of an equipotential surface at a given latitude. The expression can be developed for both the sun and the moon.

A rigorous expansion was undertaken by *Doodson* (1922). Table 4.1 shows the various harmonic terms, given by *Doodson* (1928). Each term is given a symbol which is indicative of the period and/or origin. These symbols are also applied to the *tides* of corresponding period.

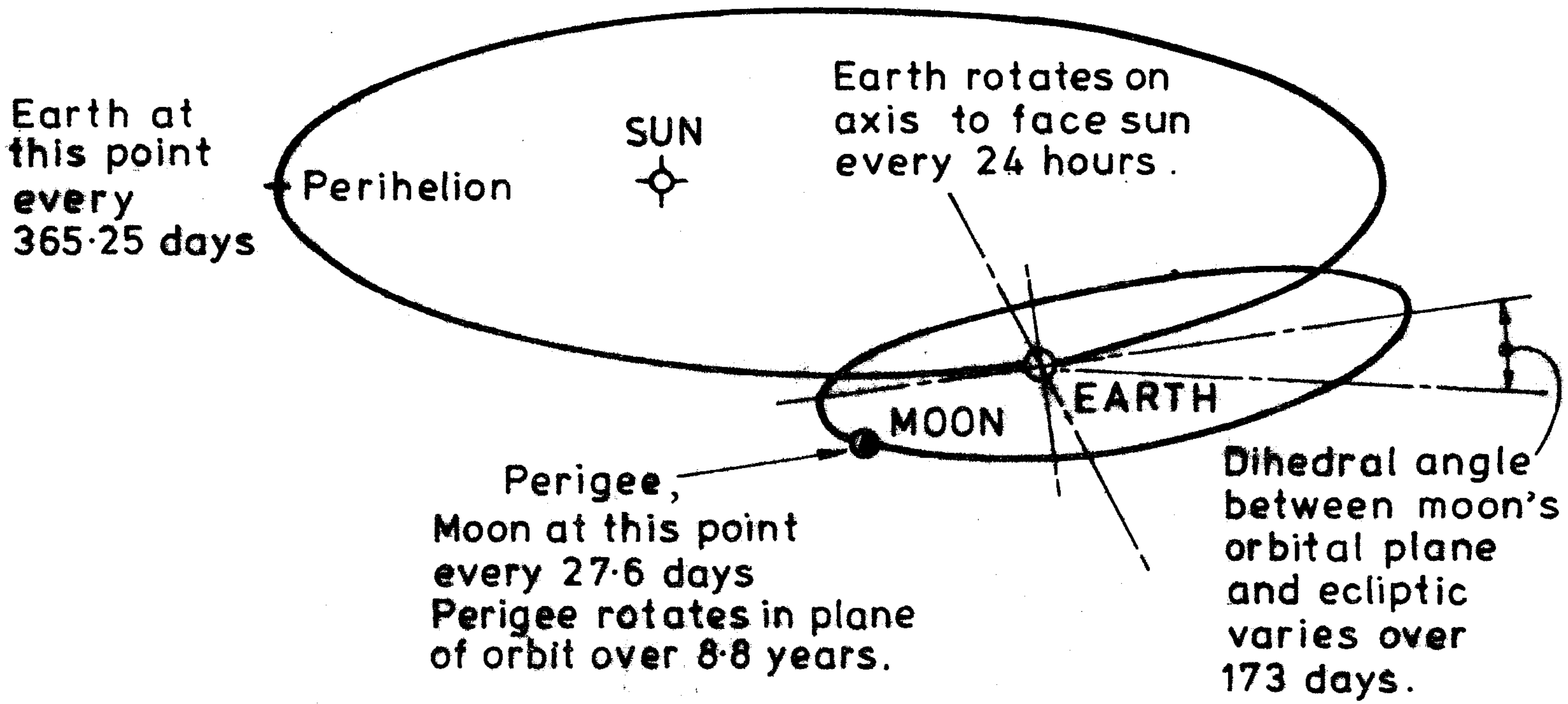


FIG. 4·1

BASIC RELATIONSHIPS BETWEEN EARTH SUN AND MOON

Table 4.1

Tidal components according to the harmonic analyses by *Doodson* (1922), taken from *Doodson* (1928).

SYMBOL	SPEED (deg./solar hr.)	REMARKS	SYMBOL	SPEED (deg./solar hr.)	REMARKS
S_o	0.000 0000	Constant	M_2	28.984 1042	
Sa	0.041 0686	Annual	MKS_2	29.066 2415	
Ssa	0.082 1373	Semi-annual	λ_2	29.455 6253	
Mm	0.544 3747	Monthly	$_2$	29.528 4789	
Msf	1.015 8958	Approx.	T_2	29.958 9333	
Mf	1.098 0331	Fortnightly	S_2	30.000 0000	Semi-diurnal
$2Q_1$	12.854 2862		R_2	30.041 0667	
γ_1	12.927 1398		K_2	30.082 1373	
Q_1	13.398 6609		MSN_2	30.544 3747	
ρ_1	13.471 5145		KJ_2	30.626 5120	
O_1	13.943 0356		$2SM_2$	31.015 8958	
MP_1	14.025 1729		MO_3	42.927 1398	
M_1	14.492 0521		M_3	43.476 1563	
χ_1	14.569 5476		SO_3	43.943 0356	Ter-diurnal
π_1	14.917 8647		MK_3	44.025 1729	
P_1	14.988 9314	Diurnal	SK_3	45.041 0686	
S_1	15.000 0000		MN_4	57.423 8337	
K_1	15.041 0686		M_4	57.968 2084	
ψ_1	15.082 1353		SN_4	58.439 7295	
ϕ	15.123 2059		MS_4	58.984 1042	Quarter diurnal
θ^1_1	15.512 5897		MK_4	59.066 2415	
J_1	15.585 4433		S_4	60.000 0000	
SO_1	16.056 9644		SK_4	60.082 1373	
OO_1	16.139 1017		$2MN_6$	86.407 9380	
OO_2	27.341 6964		M_6	86.952 3127	
MNS_2	27.423 8337		MSN_6	87.423 8337	
$2N_2$	27.895 3548		$2MS_6$	87.968 2084	Sixth-diurnal
μ_2	27.968 2084	Semi-diurnal	$2MK_6$	88.050 3457	
N_2	28.439 7295		$2SM_6$	88.984 1042	
ν_2	28.512 5831		MSK_6	89.066 2415	
OP_2	28.901 9669				

Other harmonic component lists are given, for example by *Doodson and Warburg* (1941), *Groves* (1955) and *Melchior* (1958; 1966). Some frequencies may be slightly different, e.g. *Groves* shows the speed of M_1 as 14.4966939 degrees/solar hour, against *Doodson's* 14.4920521, whilst other lists include higher degree components (e.g. eigth-diurnal) which are thought to be due to shallow water effects. Some components shown in Table 4.1 have both lunar and solar origins. Thus K_1 could be shown as ${}^{m}K_1$ and ${}^{s}K_1$, both of speed 15.0410686 as done by *Melchior*, (1966).

4.4 Theory of Filters

Using the tidal periods obtained by harmonic development, the method of weighted combinations of ordinates is often used to eliminate tidal effects from gauge records of sea level. Combination of ordinates was investigated as having possible application to this study. The theory is given by *Holloway* (1958) although *Melchior* (1966) discusses ordinate combination in greater detail. Utilization of this theory for the problem at hand is discussed in Section 4.5.

Consider a time series, $y(t)$, composed of n different sinusoidal series, which have various periods, amplitudes and phases. The ordinate at time t may be written

$$y(t) = \sum_{k=1}^{n} y_k(t),$$

where $y_k(t)$ is a component series. Ordinates of $y_k(t)$ at time t may be written

$$y_k(t) = A_k \sin(2\pi f_k t + \phi_k)$$

where A_k is the amplitude, f_k the frequency, and ϕ_k the phase at time $t=0$, for the series $y_k(t)$.

$$\therefore \quad y(t) = \sum_{k=1}^{n} A_k \sin(2\pi f_k t + \phi_k)$$

The $y(t)$ series may be considered to be either a continuous or discrete function in the range

$$-\infty < t < \infty$$

Ordinates in the discrete form of the function may be assumed to be separated by times Δt.

Another series, $Y(t)$, which may similarly be discrete or continuous, will now be defined. In discrete form,

$$Y(t) = \sum_{i=a}^{a+m} w_i \, y\,(t + i\,\Delta t) \qquad .....(4.1)$$

where w_i are numerical constants, known as *weights* or *multipliers*, which may be positive, negative or zero.

The ordinate of $Y(t)$ at time t is a combination of the ordinates of $y(t + i\,\Delta t)$.

As an example, consider the time series for which $y(t)$ is given in the domain t=10, 11....20 in Table 4.2. Given that

$$Y(t) = \sum_{i=-2}^{+2} w_i \, y\,(t + i\,\Delta t)$$

where w_i are shown in Table 4.3, values for $Y(13)$, $Y(14)$....$Y(17)$ can be calculated (Table 4.4) using $\Delta t = 1$.

$$\begin{aligned}
Y(13) &= w_{-2}\, y(11) + w_{-1}\, y(12) + w_0\, y(13) \\
&\quad + w_1\, y(14) + w_2\, y(15) \\
&= 1.3 + 2.5 + 3.3 + 4.0 + 3(-3) \\
&= 13 \\
Y(14) &= 1.y(12) + 2.y(13) + 3.y(14) \\
&= + 4.y(15) + 3.y(16) \\
&= -16 \\
Y(15) &= 1.3 + 2.0 + 3(-3) + 4(-5) + 3(-3)
\end{aligned}$$

Table 4.2

The function y(t)

t	y(t)	t	y(t)
10	0	16	-5
11	3	17	-3
12	5	18	-6
13	3	19	-8
14	0	20	-6
15	-3		

Table 4.3

The Weights or Multipliers w_i

i	w_i
-2	1
-1	2
0	3
1	4
2	3

Table 4.4

The function $Y(t)$

t	y(t)
13	+13
14	-16
15	-35
16	-51
17	-70

$= -35$

$Y(16)$ and $Y(17)$ are calculated similarly.

A continuous series $Y(t)$ may be similarly derived from $y(t)$ using a *weight function*, $w(t)$, instead of the individual weights w_i. Only discrete series, as would be practically applied to tide-gauge records, will be considered here.

From equation (4.1), the discrete function

$$Y(t) = \sum_{i=a}^{a+m} w_i \, y(t + i \, \Delta t)$$

$$= \sum_{i=a}^{a+m} w_i \{\sum_{k=1}^{n} y_k(t + i \, \Delta t)\}$$

$$= \sum_{i=a}^{a+m} w_i \{y_1(t + i \, \Delta t) + y_2(t + i \, \Delta t) + \dots + y_n(t + i \, \Delta t)\}$$

$$= \sum_{i=a}^{a+m} w_i y_1(t + i \, \Delta t) + \sum_{i=a}^{a+m} w_i y_2(t + i \, \Delta t) + \dots \sum_{i=a}^{a+m} w_i y_n(t + i \, \Delta t),$$

so that $Y(t)$ may be considered to be a combination of n series, $Y_k(t)$, where

$$Y_k(t) = \sum_{i=a}^{a+m} w_i y_k(t + i \, \Delta t)$$

and

$$Y(t) = \sum_{k=1}^{n} Y_k(t)$$

Notice that

$$Y_k(t) = \sum_{i=a}^{a+m} w_i \, A_k \sin(2\pi \, f_k \, t + \phi_k + i \, \Delta t)$$

The significant point is that the transformation of the series $y(t)$ to $Y(t)$ using the weights may also be considered as n separate transformations of $y_k(t)$ to $Y_k(t)$. For each $y_k(t)$ series, there will exist a corresponding $Y_k(t)$ series. Furthermore, $Y_k(t)$ are sinusoidal functions with the frequency f_k, but not necessarily with the phase ϕ_k.

Furthermore, there will be no phase difference between $y_k(t)$ and $Y_k(t)$ if the w_i values are symmetrical about w_0, that is, if the limits of i are given as

$$-a < i < +a$$

and if

$$w_{-i} = w_i$$

If there is no phase difference between the $y_k(t)$ and the $Y_k(t)$ series, then the effect of the application of the weight series is to change $y_k(t)$ in amplitude only. The weight series is simply an attenuator or magnifier of the original series.

The transformed series $Y(t)$ is then the original $y(t)$ series with the amplitudes of the component series *modified*. Therefore, a set of multipliers may be used to "filter" a series by reducing the effect on certain component series whilst either leaving unaffected or magnifying other components.

By adopting certain $R(f)$ functions, a filter may be designed to suit frequency response requirements. Restrictions, such as making the set of weights symmetrical, may be necessary.

It is useful to note that the frequency response $R(f)$ resulting from the application of M filters of response $R_k(f)$ is given by,

$$R(f) = \prod_{k=1}^{M} R_k(f)$$

(*Holloway*, 1958, p.370).

4.5 The Application of Ordinate Combination to Tide-Gauge Records

Filtering by ordinate combination is often applied to data collected in time series, such as in meteorology, oceanography, and geophysics, to smooth the results by reducing high frequency noise. Alternatively, short period fluctuations may be studied by eliminating longer period variations through filtering processes. The variation in the level of the sea-surface, with time, at any point as recorded by a tide-gauge is a time-series composed of a large number of variations of different magnitudes. Some variations are sinusoidally periodic, others are not. It may be possible to design a filter which suitably attenuates or magnifies components of sea level variation. Specifically, a filter which reduces to zero, or to almost zero, the magnitudes of tidal periods, whilst leaving other variations unaffected, has application to the suppression of tides from the continuous records of sea level at any gauge. The possible relevance of filter theory and the availability of alternatives to the problem of tidal effects on gauge records will be discussed.

For reasons which will be apparent later, it is convenient to define the term *Mean Sea Level*. MSL is the mean height of the sea-surface *for a given epoch*. The MSL must be defined for a given period: a simple but often neglected fact. Moreover, the MSL calculated over a given time interval of length T will not necessarily be the same as another MSL over the same period length T, but which is taken at a different epoch. For any MSL, not only the period length but also the times or dates of the observations, must be specified.

A number of methods which are used in practice to reduce the effect of tides on sea level are discussed below in an evaluation of their application to the present problem. Although not all the methods are classed as *filters*, most are in fact special cases of the general filtering principles described above.

(i) Planimetric integration of the area under the curve obtained from tide gauge records: a method of determining mean sea level if a continuous tide gauge record is available. The area under the curve over a given period is measured by a planimeter. See (iii).

(ii) Mean Tide Level: the mean height of a number of sea levels at successive low and high tides. The result obtained is, however, unsuitable for the elimination of tidal effects. Mean tide level is not as accurate an estimator of the mean level of the sea-surface as a conventional MSL, although *Hamon and Stacey* (1960, p.270) say that the differences over a month are of the order of less than a centimetre.

(iii) Mean Sea Level: probably the most frequently used estimator of the height of the sea-surface without tides. Some of the commonly used MSL's are -

a) daily MSL calculated from 24 hourly gauge heights.

b) daily MSL calculated from three gauge readings at eight-hourly intervals.

c) monthly MSL calculated from all hourly gauge readings over a month.

d) annual MSL from all hourly readings in the given year.

e) annual MSL from 365 daily readings of sea-level.

MSL's are re-considered in greater detail in Section 2.6.

(iv) Numerical Filters: among the more commonly used filters of the form described previously is one developed by *Doodson*, (see *Doodson and Warburg*, 1941). Thirty-nine weights are applied to observations at hourly intervals to eliminate the M_2, S_2, M_4, S_6, K_1, O_1 and MO_3 tides. Often known as *(Doodson's)* X_0 *filter*, this filter is a combination of three filters, one of which eliminates the S_1, S_2, S_4 and S_8 tides, another eliminates S_6 whilst the third eliminates M_1, M_2, M_3, M_5, M_6 and O_1. Table 4.5 shows the 39 weights.

Rossiter (1958; 1960) describes a Z_0 filter which uses eleven weights applied to readings at three-hour intervals, to reduce the major tidal components with a frequency of once or more per day.

Nineteen separate filters, with between 15 and 51 weights were developed by *Groves*, (1955) on the basis of the correlation between the tidal constituents in the semi-harmonic development by *Darwin* (see *Schureman*, 1941).

Easton (1970) has used a filter, described as X_{41}, using 41 hourly heights with the weights shown in Table 4.6.

4.6 The Filtering Characteristics of Mean Sea Level

The application of a numerical filter, of the form described previously, to a time series, produces for each $y(t)$ a corresponding $Y(t)$ in the filtered series. When a MSL is calculated, a single figure is used to cover the full period of the MSL calculation. Some principles of filters may nevertheless be applied to MSL. The value of the MSL may be considered to be the $Y(t)$ corresponding to any $y(t)$ in the time interval over which the mean was calculated. The set of weights is

$$w_i = 1/n$$

where n is the number of readings used in the mean. The limits of i will depend on the time of $y(t)$. Thus, for the first $y(t)$ in the gauge record interval

Table 4.5

Doodson's X_0 Filter

i	$w_i \times 30$	i	$w_i \times 30$	i	$w_i \times 30$
-19	1	-6	1	7	1
-18	0	-5	0	8	0
-17	1	-4	2	9	2
-16	0	-3	1	10	0
-15	0	-2	1	11	1
-14	1	-1	2	12	1
-13	0	0	0	13	0
-12	1	1	2	14	1
-11	1	2	1	15	0
-10	0	3	1	16	0
- 9	2	4	2	17	1
- 8	0	5	0	18	0
- 7	1	6	1	19	1

Table 4.6

Easton's X_{41} Filter

i	$w_i \times 20$	i	$w_i \times 20$
0	4	±11	3
± 1	5	±12	3
± 2	5	±13	2
± 3	5	±14	2
± 4	5	±15	1
± 5	3	±16	1
± 6	3	±17	2
± 7	3	±18	2
± 8	3	±19	2
± 9	4	±20	1
±10	3		

$$i = 0,\ N-1$$

For the central point,

$$i = {}^{-N}/2,\ {}^{N}/2$$

For the last point,

$$i = 1-N,\ 0$$

As an example, a six-hour mean sea level calculated from the readings

$$0,\ 3,\ 4,\ 2,\ -2,\ -3,$$

produces a filtered $Y(t)$ series,

$$^{2}/3,\ ^{2}/3,\ ^{2}/3,\ ^{2}/3,\ ^{2}/3,\ ^{2}/3$$

$$Y(1) = \sum_{i=0}^{5} w_i\ Y(t + i\ \Delta t)$$

$$Y(2) = \sum_{i=-1}^{4} w_i\ y(t + i\ \Delta t)$$

$$Y(3) = \sum_{i=-2}^{3} w_i\ y(t + i\ \Delta t)$$

$$Y(6) = \sum_{i=-5}^{0} w_i\ y(t + i\ \Delta t)$$

and

$$w_1 = w_2 = w_3\ \ldots\ldots = w_6 = {}^{1}/6$$

The use of different limits of i produces a phase difference between the original $y(t_i)$ and the filtered $Y(t_i)$ at time t_i. Quite obviously, if the phase difference between $Y_k(t)$ and $y_k(t)$ at time t, is x, then the phase difference at time $(t + \Delta t)$ is $x + \Delta t$. Thus the $Y_k(t_i)$ is not dependent on t, but is the same for all $y(t)$ over the period of the MSL calculation.

The use of monthly MSL in this study has a number of advantages which are outlined below.

A monthly MSL or MMSL, is an adequate diminisher of all tides with periods up to a month. Table 4.7 shows frequency response factors for a number of tides for MMSL's from *Pattullo et al* (1955). The contributions to MMSL by diurnal and semi-diurnal species were estimated to be of the order of a millimetre, with a maximum of a centimetre if all components happended to be in phase. Fortnightly tides were considered as possibly contributing more than a centimetre. Annual effects can be seen from Table 4.7 to retain at least 95% of their amplitude. For reasons which will be clarified in later chapters, it is not expedient to filter the annual and semi-annual tides.

A simple arithmetic mean also has the advantage of having only a limited effect on non-sinusoidal variations. At this point, only tides can be considered to be truly sinusoidal variations of sea level. A straight arithmetic mean is therefore safer, with respect to non-tidal influences, than a weighted mean or filter. *Hamon and De Castillejo* (1964) are critical of the use of monthly means to reduce meteorological effects. However, their aim is to obtain an MSL which is free from meteorological influences. The aim in this study is to reduce tidal effects only.

When compared with any form of filter, MMSL is easily calculated. This is significant when about 30 gauges' results over five years are to be reduced.

4.7 Tide Gauge Data Reductions in Australia

Easton (1967a; 1967b; 1970), *Easton and Radok* (1968; 1970a; 1970b) and *Radok* (1971) have undertaken calculations with the records from the gauges used in this study, as well as records from other gauges. Where data was available, the following values were produced for each gauge.

Table 4.7

Frequency Responses of MMSL on main tides

TIDAL COMPONENT	PERIOD (days)	FREQUENCY RESPONSES			
		28 day	29 day	30 day	31 day
M_2	0.5175	0.0019	0.0006	-0.0006	-0.0016
S_2	0.5000	0.0000	0.0000	0.0000	0.0000
N_2	0.5274	-0.0016	0.0003	0.0021	0.0035
K_2	0.4986	0.0027	0.0027	0.0027	0.0026
K_1	0.997	0.0027	-0.0027	0.0027	-0.0027
O_1	1.076	0.0010	0.0016	-0.0040	0.0060
P_1	1.003	-0.0027	0.0027	-0.0027	0.0027
Mf	13.66	0.0242	0.0564	0.0837	0.1050
Msf	14.77	-0.0537	-0.0183	0.0156	0.0466
Ssa	182.72	0.9618	0.9591	0.9562	0.9533
Sa	365.24	0.9903	0.9897	0.9890	0.9882

(i) Daily Mean Sea Levels;

which were calculated using the X_{41} filter described in Section 4.5, and are then not strictly MSL's.

(ii) Monthly Mean Sea Levels;

The arithmetic means of all available hourly readings, for the appropriate calendar months.

(iii) Annual Mean Sea Levels;

meaned from all available hourly readings of any year.

(iv) Amplitudes and phases of tidal constituents;

records from 49 mainland Australian gauges for the period 1966 to 1968 were analysed by *Easton* (1970) for the amplitude and phases of a number of tidal constituents. Analysis for 64 components was undertaken for gauges for which over 370 days of records were available. Otherwise, 25 components were sought.

4.8 Conclusions

Despite a study of the origins of tides and the theoretical predictions of tides, no suitable way of completely removing their effect was found.

Monthly Mean Sea Levels were found to be virtually free from the influence of the fluctuation of tides which had a period less than a month. The MMSL's are therefore suitable for studies of the time variation of non-tidal effects on sea-level. More importantly, the Five-Year Mean Sea Level which is used to represent the long-term ocean surface is also unaffected by the periodic nature of these tides.

Although a true mean of the *time* variations is obtained, the mean itself may deviate from the geoid. That is, the five-year MSL may still exhibit a position-dependent effect due to tidal influence. However, negligible mention of the position effect of tides has been found in the oceanographic literature, and, in this study, this aspect has been regarded as unknown.

Although the theory and practice of filters were considered, their use was finally rejected. The five year Mean Sea Level was accepted as an alternative estimate of sea level, affected, to less than a centimetre, by the astronomically induced tides with a period shorter than a month. The annual and semi-annual tides remained unstudied. Alternatives to Mean Sea Levels would be unnecessarily complex to calculate. This was significant as five years of hourly readings were available for each gauge. Nevertheless, the decision to use MSL's over a month and over five years, to estimate the tideless sea-level was aided by the background given in Sections 4.2 to 4.5.

5. OCEAN WATER DENSITY AND CURRENTS

5.1 Introduction

In the preceding Chapter, it was concluded that the five-year Mean Sea Level should be free from the influence of tides. Position effects on sea level resulting from tidal action were not resolved. The most significant, *recognised* cause of position variations in sea level is the distribution of the density of ocean water. Being the most important effect, it is discussed before the secondary effects in Chapters 6 and 7.

The theoretical background is included, in Section 5.2, because the relationship between Mean Sea Level and the geoid has been studied as a levelling problem. Thus, the theory outlined here is not familiar in the field of geodetic levelling.

Ocean current flow is closely related to ocean water densities, and the two phenomena have been considered simultaneously.

The surface of low density ocean water is elevated with respect to the surface of high density water. The resulting sea-surface topography may be calculated from a knowledge of densities if certain assumptions are made. Pressure differences, resulting from density distribution, create current flow in the oceans.

5.2 Theoretical Relationships

The basic equation for development of the relevant theory is the hydrostatic equation. The pressure p at a point P at a depth h, is

$$p = \bar{\rho}\,\bar{g}\,h \qquad \text{.....(5.1)}$$

where $\bar{\rho}$ is the mean water density along the vertical above the point P;
$\bar{g}$ is the mean value of gravity above P.

Equation 5.1 may be more generally written

$$dp = \rho \; g \; dh$$

or

$$p = \int \rho \; g \; dh \qquad \text{.....(5.2)}$$

It should be noted that if the hydrostatic equation is applied to ocean water, then, in the form of equation 5.2, the pressure p results from the overlying atmosphere as well as from the overlying ocean water.

Thus, so far, the position of pressure surfaces referred to the depth of the water have been associated with the density of the fluid.

Equipotential surfaces may be introduced to replace heights as a reference system.

According to equation (2.2)

$$dW = - g \; dh$$

where dW is the potential increment along the vertical on which g and h are measured. Dynamic heights are more convenient than the potentials, and in accepted oceanographic symbolism

$$dD = g \; dh \qquad \text{.....(5.3)}$$

where D is the dynamic height measured along the vertical.

Pressures and dynamic heights, or potentials, can now be related by combining equations 5.2 and 5.3,

$$dp = \rho \; dD$$

$$\therefore \qquad dD = \frac{1}{\rho} \cdot dp \qquad \text{.....(5.4)}$$

The dynamic height difference between pressure levels is therefore related to the densities between the points. If the points are P_1 and P_2,

$$D_{P_1-P_2} = \int_{P_1}^{P_2} \frac{1}{\rho} \cdot dp \qquad \ldots\ldots(5.5)$$

If, at some point, the pressure and the potential are known, the potential of another point on the same vertical with known pressure can be calculated using the density distribution between the points. Furthermore, if the point for which the dynamic height is required is on the sea-surface, the pressure is equal to that due to the atmosphere only, and the sea level may be related to the point of known pressure and potential. According to the isostatic theory of ocean pressure surfaces, there is a depth at which an equipotential and a pressure surface do coincide, although there is doubt as to the level at which this occurs. Thus, the above calculations may be applied to an area of ocean to calculate the dynamic height of the sea-surface, referred to a known equipotential surface, if the density distribution can be ascertained. The necessity to recognise the coincident pressure and potential surfaces is essential.

Practical application of the theory is simplified by calculating the dynamic height *anomaly* of the sea-surface. This is the difference between the measured dynamic height of the sea-surface and its dynamic height under conditions of standard density. The dynamic depth is given by

$$D = D_{35,O,P} + \Delta D$$

where $D_{35,O,P}$ is the dynamic depth of the pressure surface under standard conditions, and ΔD is the dynamic depth anomaly.

In the oceans, density is a function of temperature, salinity and pressure. Increases in temperature cause decreases in water density. An *increase* in density is produced by increases in salinity and pressure. Calculations of the dynamic depth of pressure surfaces thus require the density variations of the water to be known. It is more usual to calculate specific gravity, or its inverse, specific volume, rather than density. At a given salinity, temperature and pressure, the specific volume α_{STP}

is calculated as,

$$\alpha_{STP} = \alpha_{35,O,P} + \delta \qquad (5.6)$$

where $\alpha_{35,O,P}$ is the specific volume at a standard solinity of 35%, at a standard temperature of 0^0C and at a given pressure P. The anomaly of specific volume, δ, is a correction term accounting for the deviations from standard salinity and temperature. The dynamic depth anomaly of specific volume,

$$\Delta D = \int_{P_1}^{P_2} \delta \, dp.$$

For oceanographic purposes, pressure is measured in decibars where

$$1 \text{ dbar} = 10^5 \text{ dyne cm}^{-2}$$

Specific gravity is the ratio of the density of the given water to the density of pure water at 4°C. Usually, it is of the order of 1.025 for ocean water. As it is generally required to six significant figures for oceanographic purposes, it is easier to use the expression, σ, where

$$\sigma = (\rho - 1) \times 10^3$$

If, for example

$$\rho = 1.02572 \text{ g cm}^{-3}$$

then

$$\sigma = 25.75$$

Because

$$P \doteqdot \rho \, g \, h$$

$$\doteqdot \rho \, D,$$

it can be seen that, using

$$\rho \doteqdot 1.02, \text{ and } g \doteqdot 0.98,$$

and if h is in metres, p in decibars will be almost equal to the depth.

If $\rho = 1.025 \text{ g cm}^{-3}$

and $g = 0.980$ gals

Then

$$P = 1.025 \times 0.980 \times h$$
$$= 1.005 \times 10^5 \times h \text{ dynes cm}^{-2}$$
$$= 1.005 \times h \text{ dbar}$$

In practice, when D is calculated from p and ρ, it is usual to adopt the pressure in decibars as being equal to the depth in metres, thereby producing an error of only 0.5%. The pressure at any point will be due to air-pressure as well as to the water pressure. This is taken into account in the calculations by reducing all sea levels to a constant air-pressure and then continuing on the assumption that the surface of the sea is an isobaric surface.

Currents flow according to the pressure differences which arise as a consequence of the intersection of barometric (constant pressure) and equipotential surfaces. A component of gravity must then exist along an isobaric surface. As the force cannot be balanced by a pressure force along an isobaric surface, movement of the water must result. However, the current movement is modified by Coriolis effects, so that, in the Southern Hemisphere, currents are forced to turn to the left. The flow is clockwise around a high density area and anticlockwise around a low density area in the southern hemisphere. As low density water corresponds to a high point with respect to an equipotential surface, currents flow anticlockwise around areas of large dynamic depth anomaly. Perpendicular to the direction of flow of the current, there will therefore be an apparent slope of the sea-surface. This slope is upward from right to left when looking in the direction of a current movement in the southern hemisphere. Just as it is possible to calculate the dynamic height anomalies from density anomalies, it is also possible to calculate the velocity of currents and the slope across the currents, from densities, if certain assumptions are made about friction. The slope across a current is a function of the speed of the earth's rotation, gravity, latitude and current velocity.

Wind-stress causes a bank-up of water in the direction of the wind, with a deflection to the left in the Southern Hemisphere, according to the Coriolis effect. Winds consistently blowing in any direction will cause a redistribution of water-mass as the wind moves the less-dense upper layers. Currents arise, according to the processes described above.

It must be stressed that the above outline of density and current effects is greatly oversimplified. The hydrostatic equation requires equilibrium, a condition which may be said never to exist in the oceans. The effects of friction and of the ocean bed topography have been completely neglected in the outline above. Further details of the complex theory of currents and density may be obtained from *Dietrich* (1957, pp.291 *et seq*), *Fomin* (1964), *Neumann* (1968), *Sverdrup et al* (1942, pp.389 *et seq*).

5.3 Application of Density Data Around Australia

Various estimates of the effect of density of ocean water around Australia indicate that the application of corrections for this study is undoubtedly worthwhile. *Lisitzin* (1965, p.14) has produced a world map of density effects on sea level, showing a change in sea level up the East Australian coast of the order of 80 cm. Other publications indicating dynamic anomalies around Australia have been produced by *Hamon* (1961; 1965b); *Hamon and Tranter* (1971); *Wyrtki* (1962a; 1962b).

As much data as possible has been collected for the Australian region in a study of the density effects. It was found most suitable to divide the Australian coastline into four regions, as shown in figure 5.1. The regional break-up is based on the type and amount of data available. The sections may be classed as:

1. Eastern Australia: Coral Sea, Tasman Sea and Pacific Ocean.

2. Western Australia.

3. Southern Australia: between about Albany, W.A., and Bass Strait.

4. Northern Australia.

Density data around Australia has been collected principally by the Division of Fisheries and Oceanography, CSIRO, Australia. Information was made available directly from this source, but some data has come from papers published by CSIRO officers. There are a number of problems involved in the application of density data to Australian MSL's. Difficulties arise mainly from the scarcity of data around such a vast continent, as follows:

1. Time variations of currents and densities are not recorded in detail. This information would require monthly readings at points in the ocean, preferably over a number of years. Yet only occasionally is there more than one dynamic anomaly for any one degree square. Consequently, individual monthly MSL's cannot be corrected.

2. There are large areas where no data has been collected, especially off Northern Australia.

3. The dynamic anomalies for any one degree square have an apparent accuracy of the order of 10 cm. This estimate is based on the results in those squares of ocean where more than one observation of density was made. It is confirmed in those areas where observations are closely spaced, such as off the Eastern Australian coast.

4. Many of the anomalies are distant from the coast. The distribution of observation points is not necessarily suited to a study of coastal M.S.L.

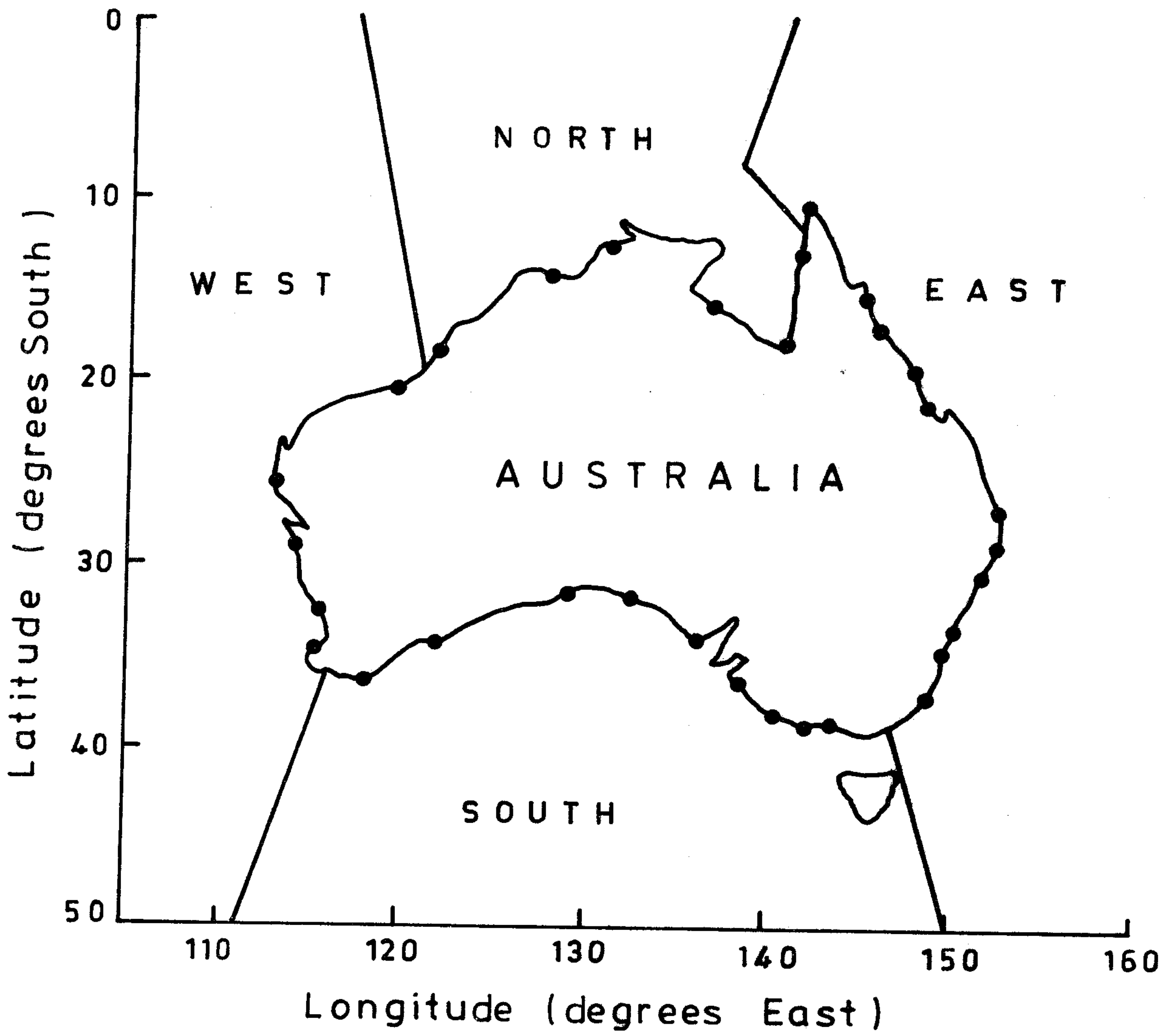


FIG. 5·1
SUBDIVISION OF ASTRALIAN OCEAN AREAS FOR DENSITY STUDY.

Each sector of coastline is considered individually:

1. Eastern Australia

Because of the existence of the East Australian Current, which ranks amongst the world's significant current flows, current and density studies for Australian waters have been concentrated in this region. The area's density and currents have been studied by *Hamon* (1961; 1965b; 1968), *Hamon and Kerr* (1968), *Hamon and Tranter* (1971), *Newell* (1961), *Rochford* (1959; 1968), *Woodhead* (1970), and *Wyrtki* (1960; 1962b). Figure 5.2 shows the geopotential topography of this region related to a 1750 dbar reference surface, according to *Wyrtki* (1962b).

Eastern Australia has also been selected as a separate region for study because of the data available. All CSIRO cruise-collected data for this region has been compiled into a single data set, which was made available by that authority for this project. The data consists of all one degree square means of dynamic depth anomalies related to both a 900 dbar and to a 1300 dbar surface, and of the original observed salinities and temperatures. The distribution of observations related to the 1300 dbar surface is shown in figure 5.3. In about 70% of one degree squares shown, only one temperature and one salinity observation has been made. About 20% of squares contain two to five observations, the rest contain more.

Details of the data summarized in figure 5.3 is exemplified in figure 5.4. A study of the values indicates that their variation is too marked to allow simple contour interpolation. Instead the values have been grouped with 10 cm ranges. For example, all points for which

$$1.90 < \Delta D < 1.99 \text{ kgal m}$$

have been grouped to indicate approximately the probable position of the 1.95 m contour. The information has also been plotted as a function of latitude, figure 5.5. The corrections finally applied to the five-year MSL's are shown in Table 5.1.

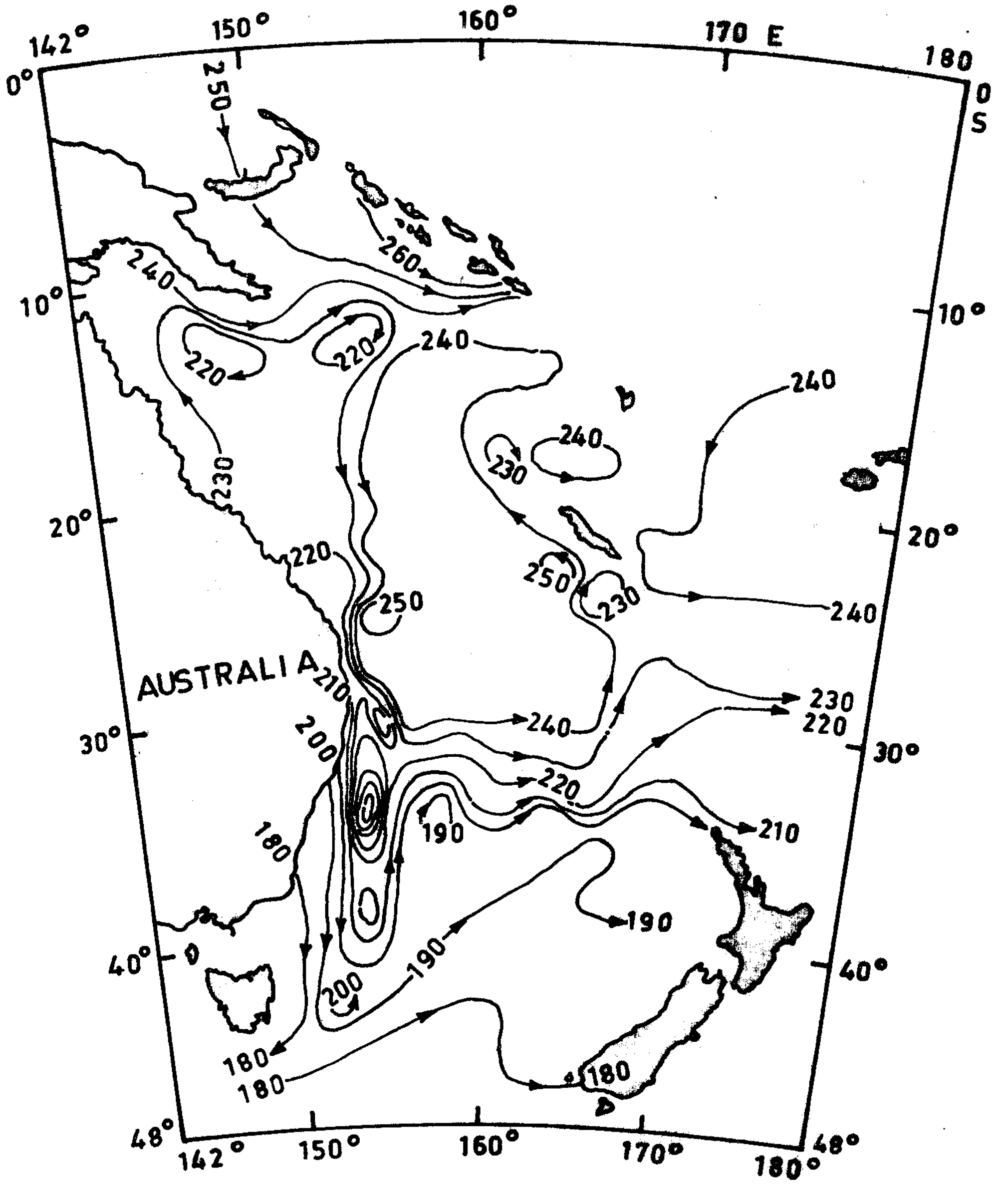


FIG. 5·2

GEOPOTENTIAL TOPOGRAPHY OF THE SEA SURFACE RELATIVE TO 1750 DECIBARS IN DYNAMIC cm., EASTERN AUSTRALIA, ACCORDING TO WYRTKI (1962b, p. 90)

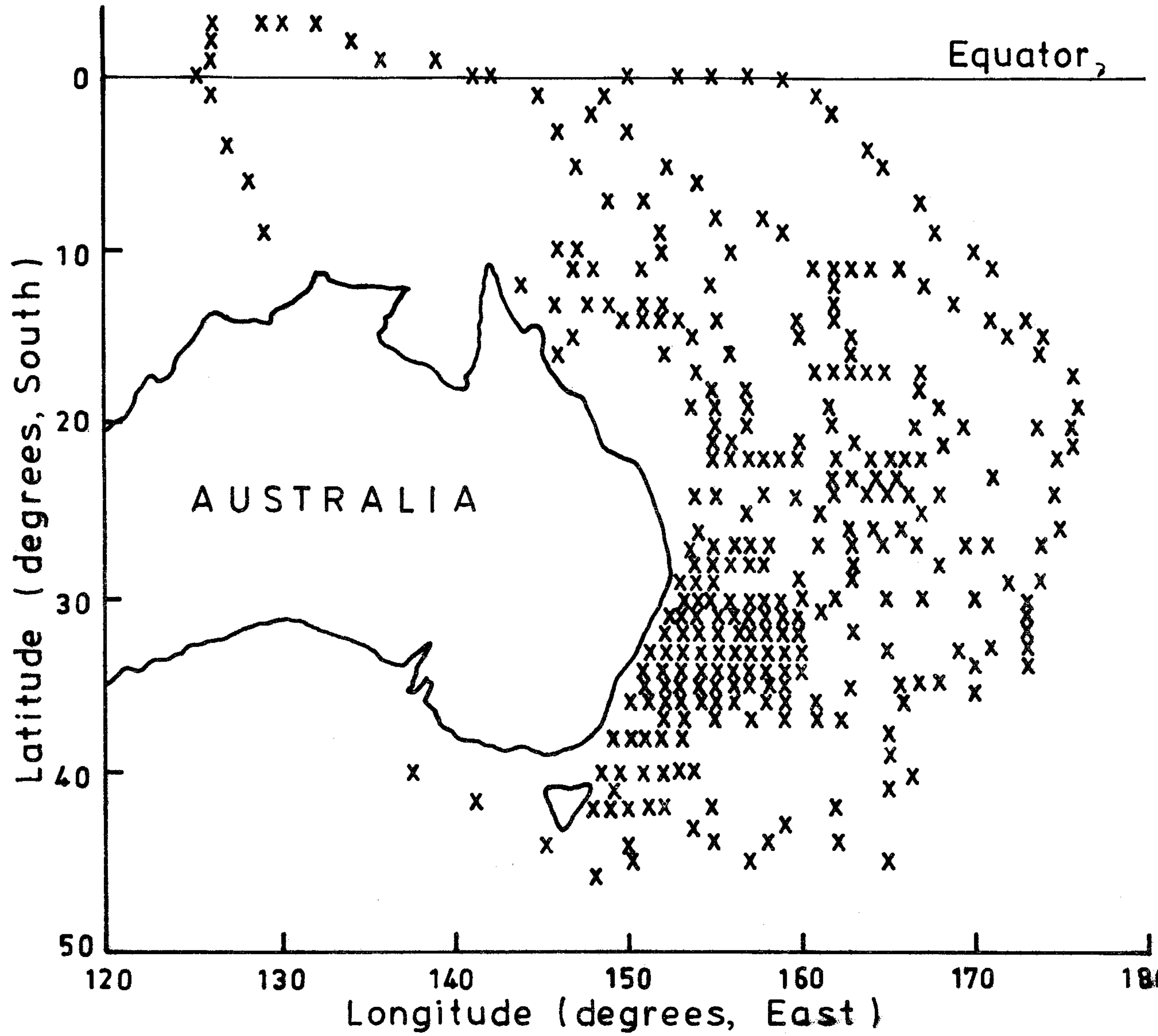


FIG. 5·3
DISTRIBUTION OF OCEAN DENSITY DATA, EASTERN AUSTRALIA.

Each 'x' represents a 1° x 1° square of ocean for which a dynamic depth anomaly has been calculated from density data.

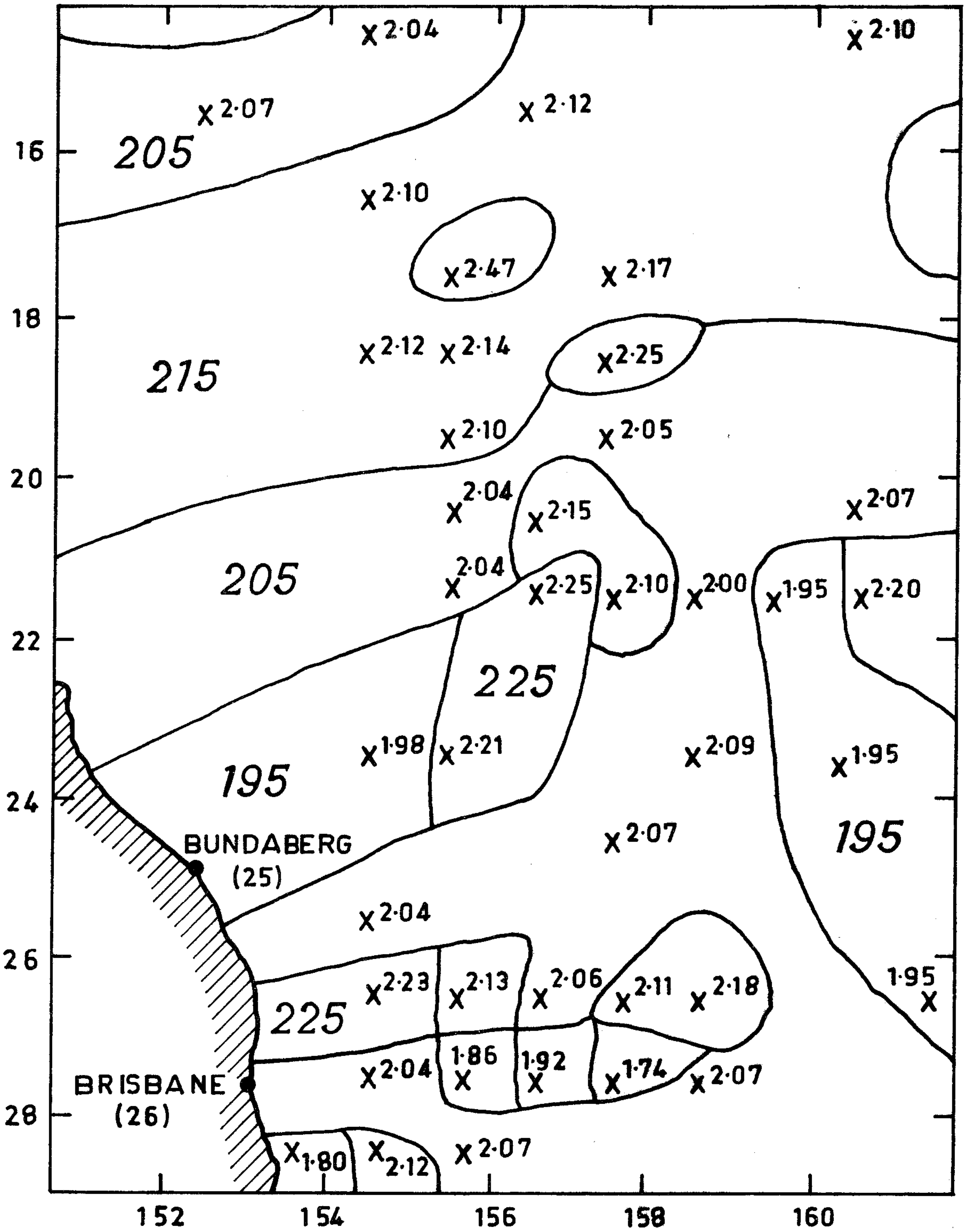


FIG. 5·4
EXCERPT FROM MAP OF ALL AVAILABLE SEA-SURFACE DYNAMIC HEIGHT ANOMALIES FOR EASTERN AUSTRALIA.

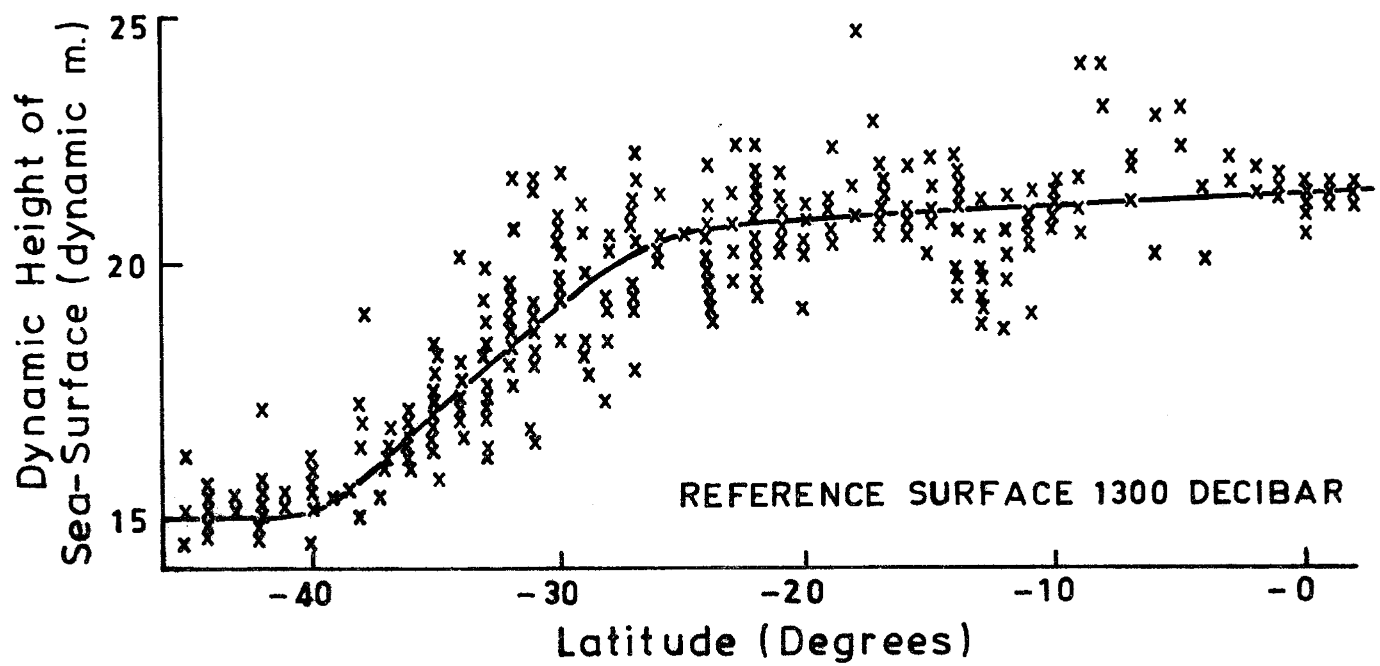


FIG. 5·5

RELATIONSHIP BETWEEN DYNAMIC HEIGHT OF SEA-SURFACE AND LATITUDE OFF THE EAST COAST OF AUSTRALIA. COMPILED FROM ALL AVAILABLE C.S.I.R.O. DATA, FOR LONGITUDES BETWEEN 145 AND 175 DEGREES EAST.

Table 5.1

Density Corrections to Mean Sea Levels

GAUGE NAME	REGIONAL GROUPING	LATITUDE*	DENSITY CORRECTION (kgal cm)	POTENTIAL OF MSL (kgal cm)	CORRECTED MSL (kgal cm)
31 Point Lonsdale	South		150	406	256
32 Port Fairy			150	409	259
1 Port MacDonnell			160	400	240
2 Victor Harbour			160	402	242
3 Port Lincoln			160	363	203
4 Thevenard			160	375	215
5 Eucla			160	378	218
6 Esperance			160	403	243
7 Albany			160	445	285
8 Bunbury	West	35.0	153	457	304
9 Fremantle		32.0	154	445	291
10 Geraldton		28.8	162	414	252
11 Carnarvon		24.8	177	407	230
12 Port Hedland		20.4	192	426	234
13 Broome	North		210	413	203
14 Wyndham			210	476	266
15 Darwin			210	486	276
17 Centre Island			210	456	246
18 Karumba			210	483	273
19 Weipa			210	537	327
20 Bamaga	East		215	552	337
21 Cooktown			205	531	326
22 Cairns			205	480	275
23 Townsville			215	456	241
24 Mackay			215	441	226
25 Bundaberg			195	424	229
26 Brisbane			205	406	201
27 Coffs Harbour			195	390	195
28 Camp Cove			165	391	226
29 Port Kembla			165	391	226
30 Eden			155	381	226

* Used to interpolate figure 5.7, where relevant.

\+ After adjustment. See figure 3.2.

2. Western Australia

Figures for the western area are given by *Hamon* (1965a) and *Wyrtki* (1962a). Figure 5.6 taken from the former report illustrates dynamic depth anomalies. Final figures were taken from figure 5.7, which was given by *Hamon* (*ibid*). This was considered to be the most suitable form of data covering the region. Again, results are shown in Table 5.1.

The data for this region shows a constant difference between the density anomalies given for the west coast and those elsewhere. This evidently results from the use of a 1750 dbar reference surface on the west coast, whilst a 1300 dbar reference was used elsewhere. The approximately constant dynamic depth anomaly of the 1300 dbar surface, referred to the 1750 dbar surface, is a constant additive factor on the western results. This effect may be recognised from the geopotential topography of the 1100 dbar surface, relative to 1750, given by *Wyrtki* (1962b, fig. 6, p.95). Table 5.2 shows the effect of different reference levels for the same cruise observations. A -30 cm correction has been applied to values abstracted from fig. 5.7, to make them compatible with other regions.

3. Southern Australia

is also considered by *Hamon* (1965a) and *Wyrtki* (1962a), although the data shown in these reports is scant. More extensive data, from various cruises, is shown in Figure 5.8. (Data from the cruise information shown in Figure 5.9 was combined into a single value in Figure 5.8). The values were combined as for the Eastern region, using, instead, a 5 cm range, and results are listed in Table 5.1.

4. North Australia

Data distribution is indicated by *Wyrtki* (1962a), *Hamon* (1965a) and in Figure 5.3. The sparseness of data has led to its separation as a region in this study. Final values were adopted as 210 cm, on the basis of Figures 5.5, 5.7 and 5.10.

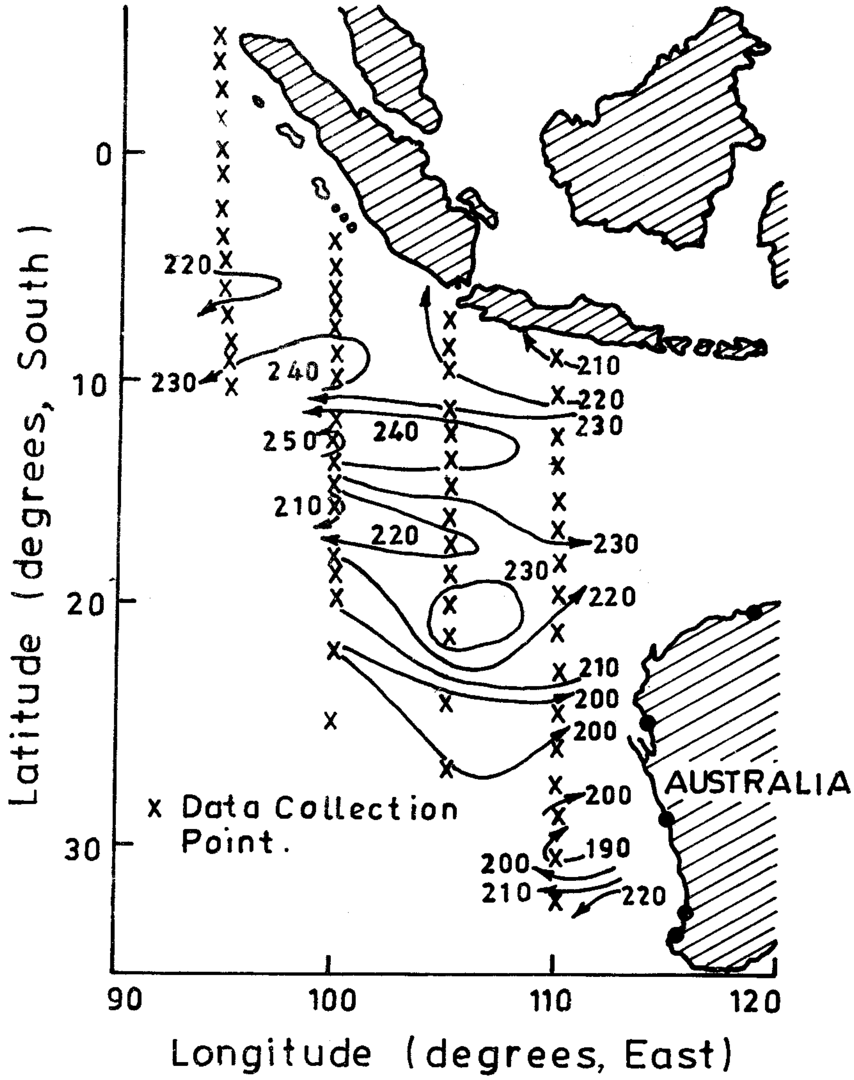


FIG. 5·6.

DYNAMIC TOPOGRAPHY OF THE SEA SURFACE RELATIVE TO 1750 DECIBAR IN kgal cm. FROM HAMON (1965a, p. 260) DATA COLLECTED IN TWO 1962 C.S.I.R.O. CRUISES.

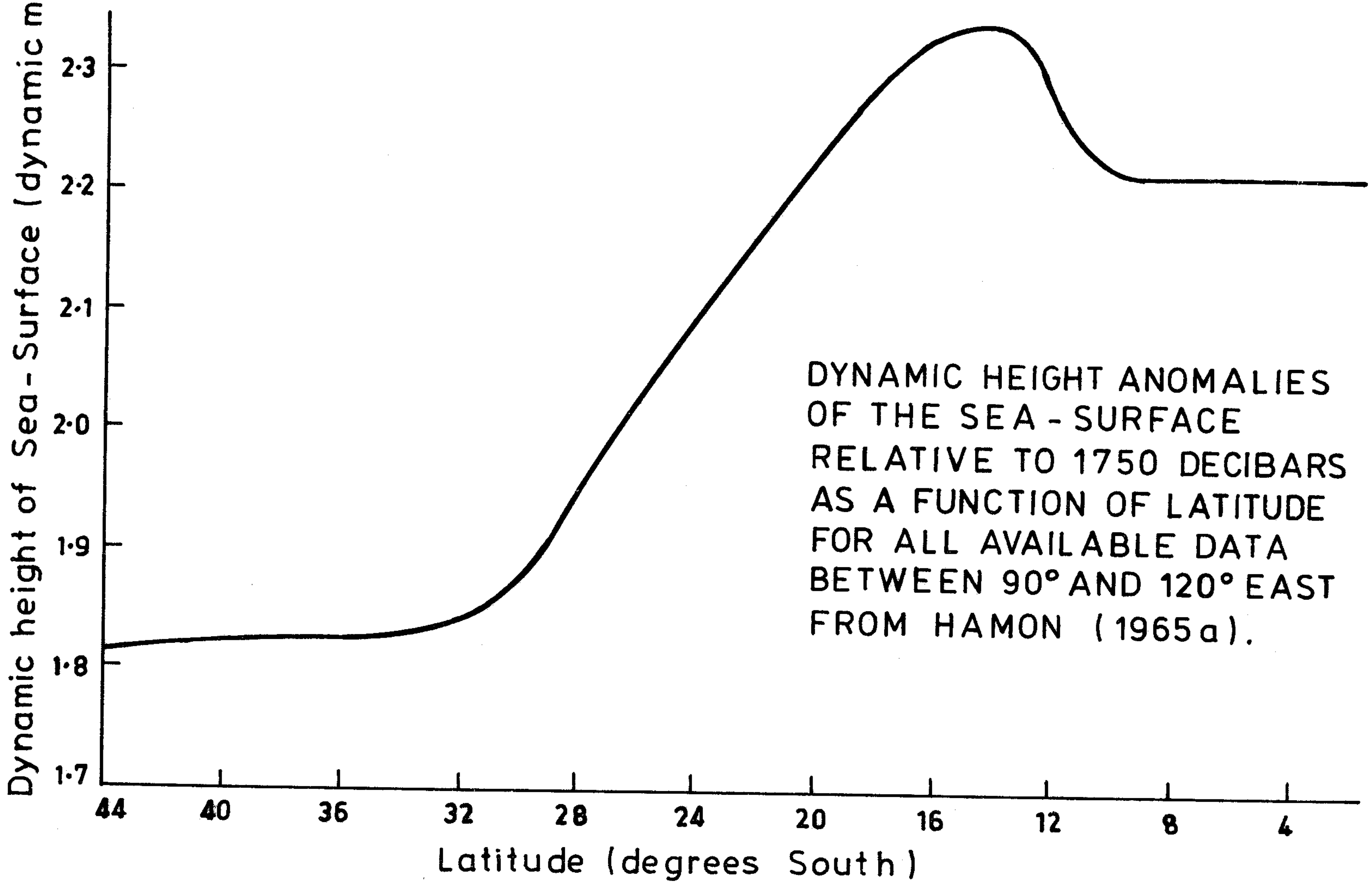

Dynamic height of Sea - Surface (dynamic m.)
2·3
2·2
2·1
2·0
1·9
1·8
1·7
44
40
36
32
28
24
20
16
12
8
4
Latitude (degrees South)
DYNAMIC HEIGHT ANOMALIES
OF THE SEA - SURFACE
RELATIVE TO 1750 DECIBARS
AS A FUNCTION OF LATITUDE
FOR ALL AVAILABLE DATA
BETWEEN 90° AND 120° EAST
FROM HAMON (1965a).

FIG. 5·7

Table 5.2

Comparison of Dynamic Heights of Sea-Surface Calculated with respect to the 1300 and 1750 Decibar Reference Levels, (from the first cruise of *HMAS Diamantina* in 1961).

CRUISE STATION NUMBER	DYNAMIC HT. RELATIVE TO 1300 dbar	DYNAMIC HT. RELATIVE TO 1750 dbar	DIFFERENCE (cm)
13	1.6581	1.96	-31
15	1.5332	1.82	-29
17	1.5629	1.87	-31
19	1.5307	1.84	-31
23	1.5408	1.84	-30
27	1.5824	1.91	-33
31	1.5369	1.87	-33
32	1.4995	1.79	-29
38	1.6158	1.89	-27
41	1.5746	1.85	-28
		Mean	-30

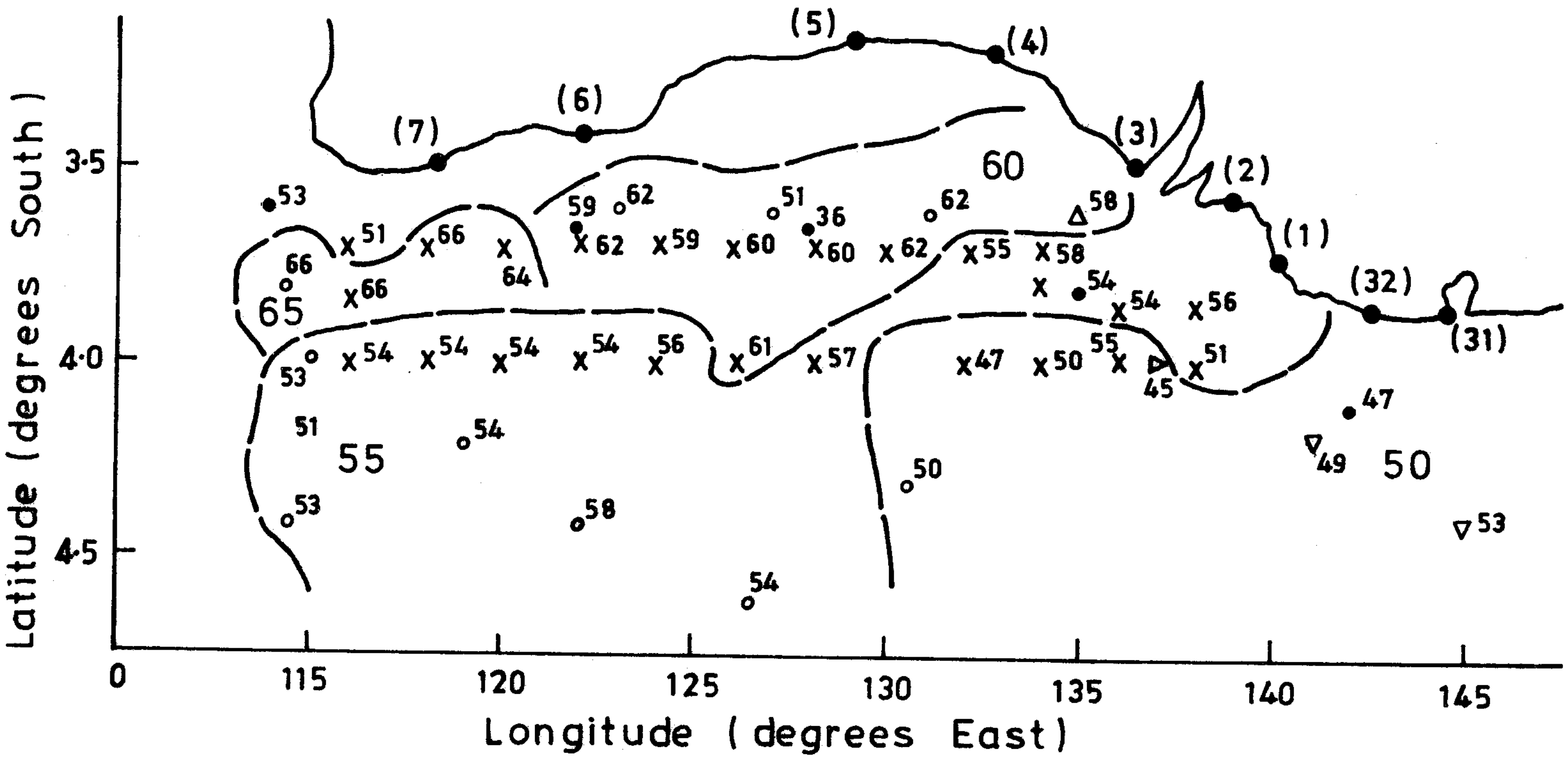


FIG. 5·8

DENSITY DATA OFF SOUTHERN AUSTRALIA

NUMBERING OF TIDE GAUGES AS IN FIGURE 1.1.

CRUISE LEGEND

- x 1/60
- ● 7/69
- Δ 2/66 MEAN (See Fig. 5·9)
- o 1/61
- ∇ EAST COAST DATA.

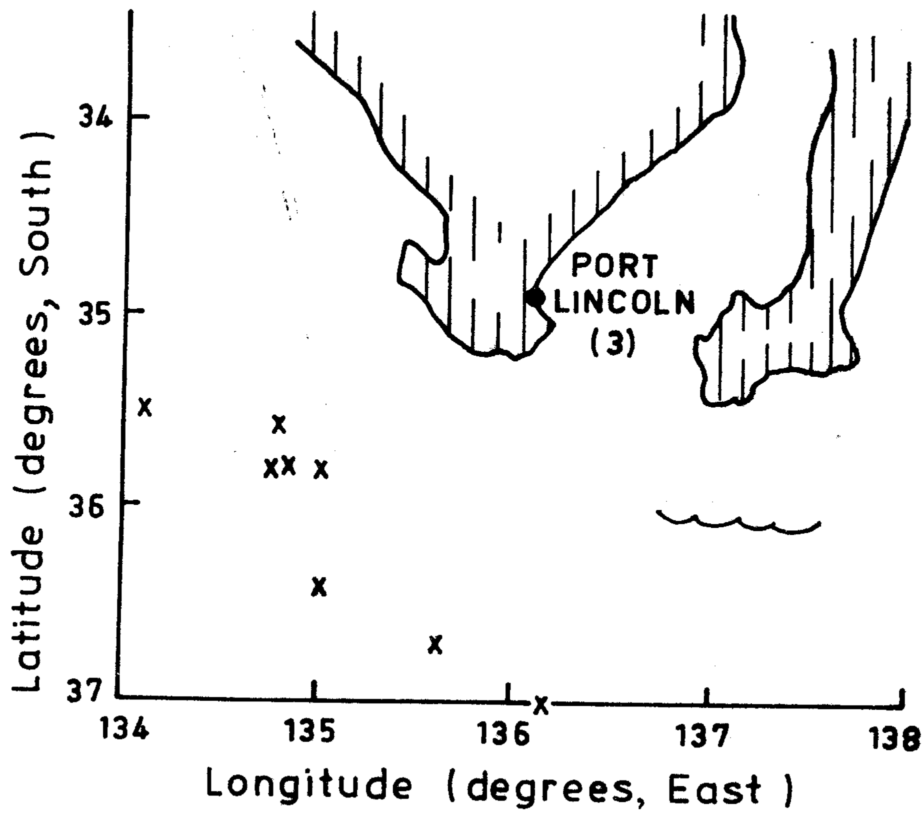
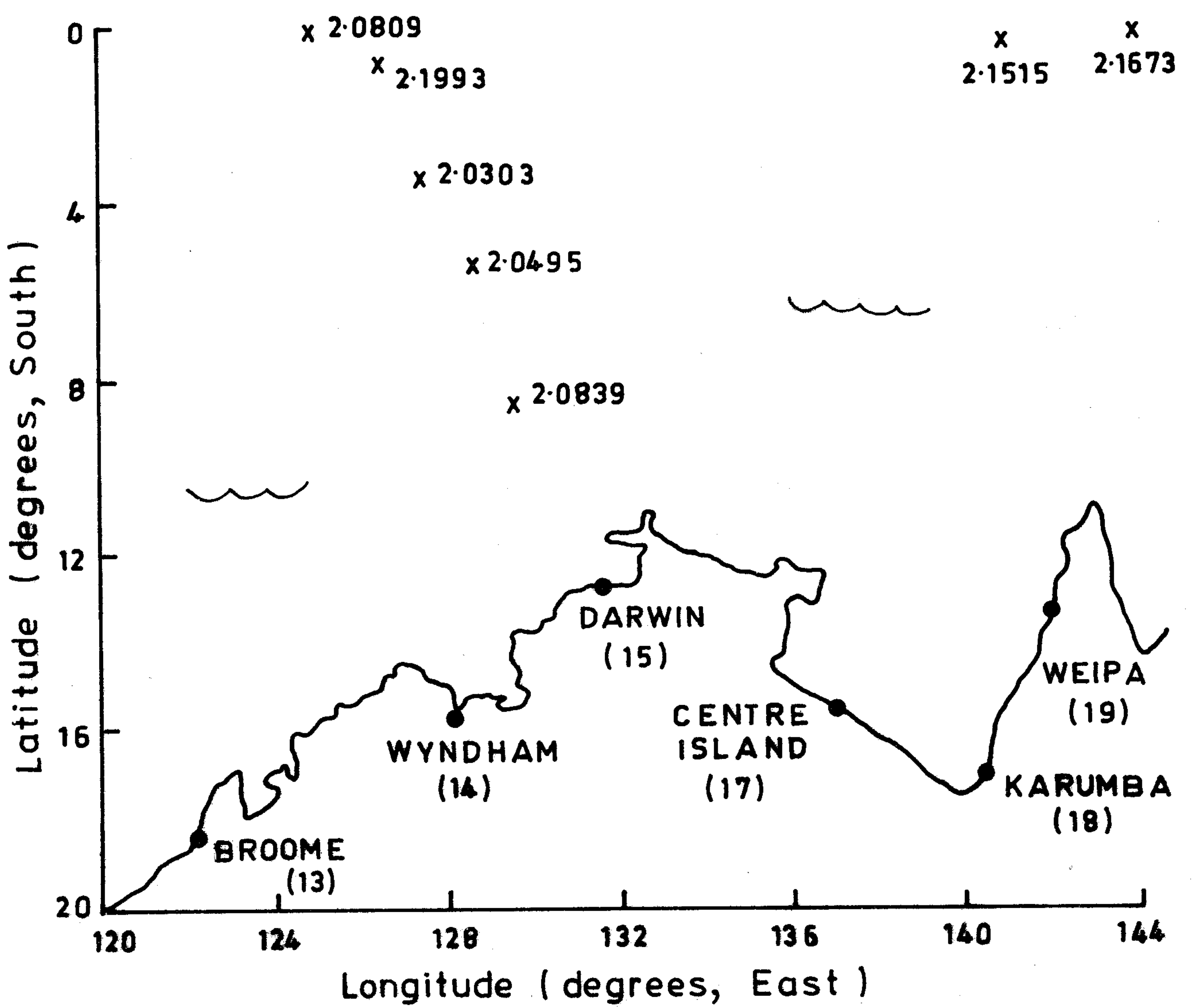


FIG. 5·9.
DATA POINTS FOR C.S.I.R.O.
CRUISE No. Dm 2/66.

FIG. 5·10



2·0809
2·1993
2·0303
2·0495
2·0839
2·1515
2·1673
BROOME
(13)
WYNDHAM
(14)
DARWIN
(15)
CENTRE
ISLAND
(17)
KARUMBA
(18)
WEIPA
(19)
Latitude (degrees, South)
0
4
8
12
16
20
120
124
128
132
136
140
144
Longitude (degrees, East)

Almost all studies of ocean current flows around Australia illustrate the significant time variations of the ocean currents and eddies. The accurate prediction of current and eddy positions is virtually impossible although there is a discernable annual cycle. Evidence of the time and position variations is given by *Wyrtki* (1960; 1961), *Hamon and Kerr* (1968) and to a lesser extent by *Hamon* (1961; 1965a; 1965b), *Hamon and Tranter* (1971) and *Wyrtki* (1962b). Figure 5.11 from *Hamon* (1965, p.918) illustrates these time changes. Their significance will again be discussed in Chapter 9.

5.4 Discussion

Figure 5.12 shows the density corrections around the coastline as a function of the geoid-MSL deviations. Although there is an apparent correlation, this may be partially explained as a correlation with latitude which is not *necessarily* due to density.

Undoubtedly, the density or current effect on sea level is one of the most significant aspects of variations of sea level with position as well as with time. It is then one of the most important effects for this study. Corrections are only of about 10 cm accuracy, however. The lack of accurate time variation figures makes it virtually impossible to study the effect of currents and density more closely. This situation arises from practical difficulties of measurement over the times and areas involved. It is also feasible that the theory of dynamic topography corrections is imperfect. Points mentioned in this discussion will be reviewed in Chapter 9.

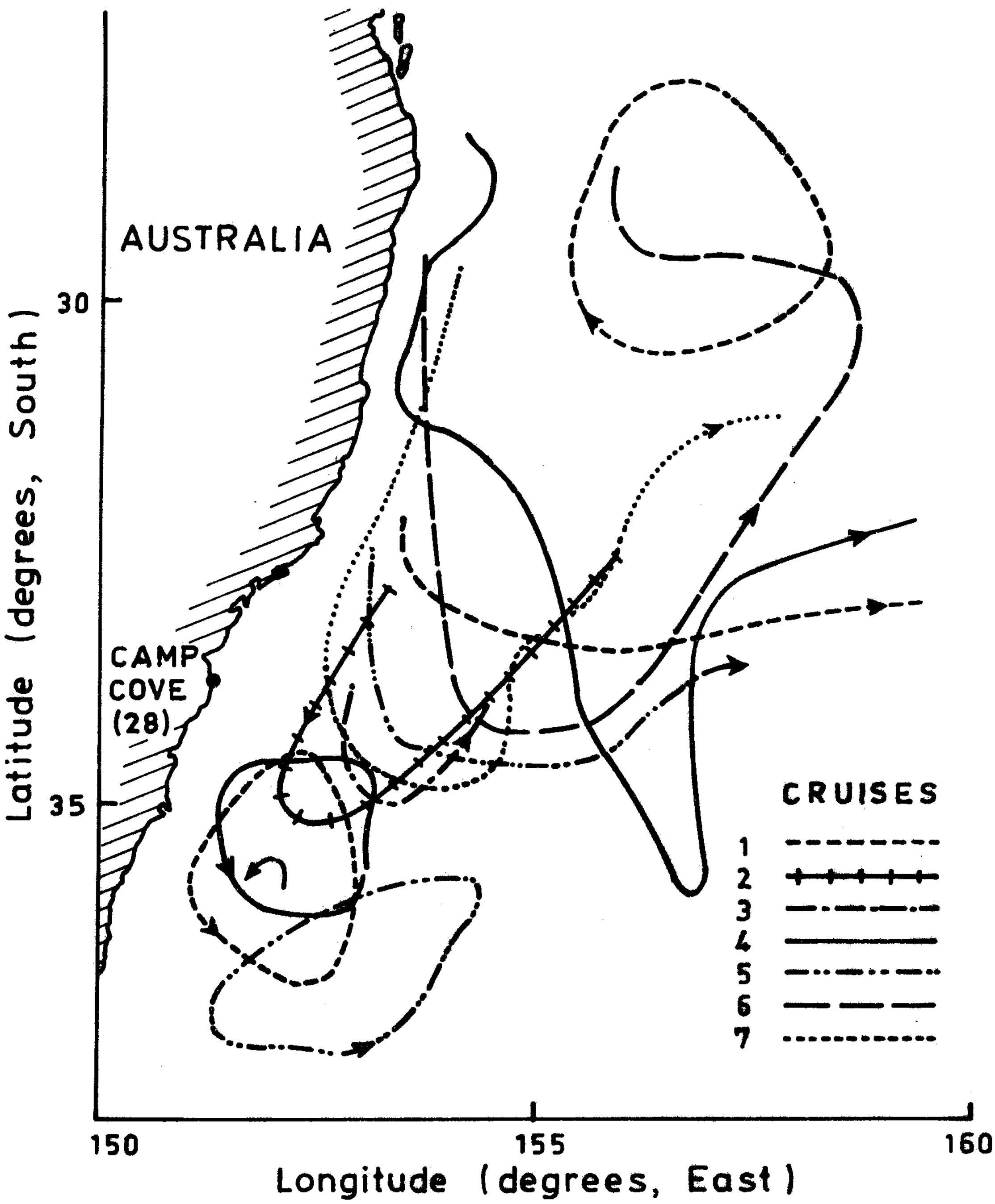


FIG. 5·11
VARYING POSITION OF 190 kgal cm CONTOUR.
SEVEN C.S.I.R.O. DATA COLLECTION CRUISES,
1960-1964, AFTER HAMON (1965b, p. 918).

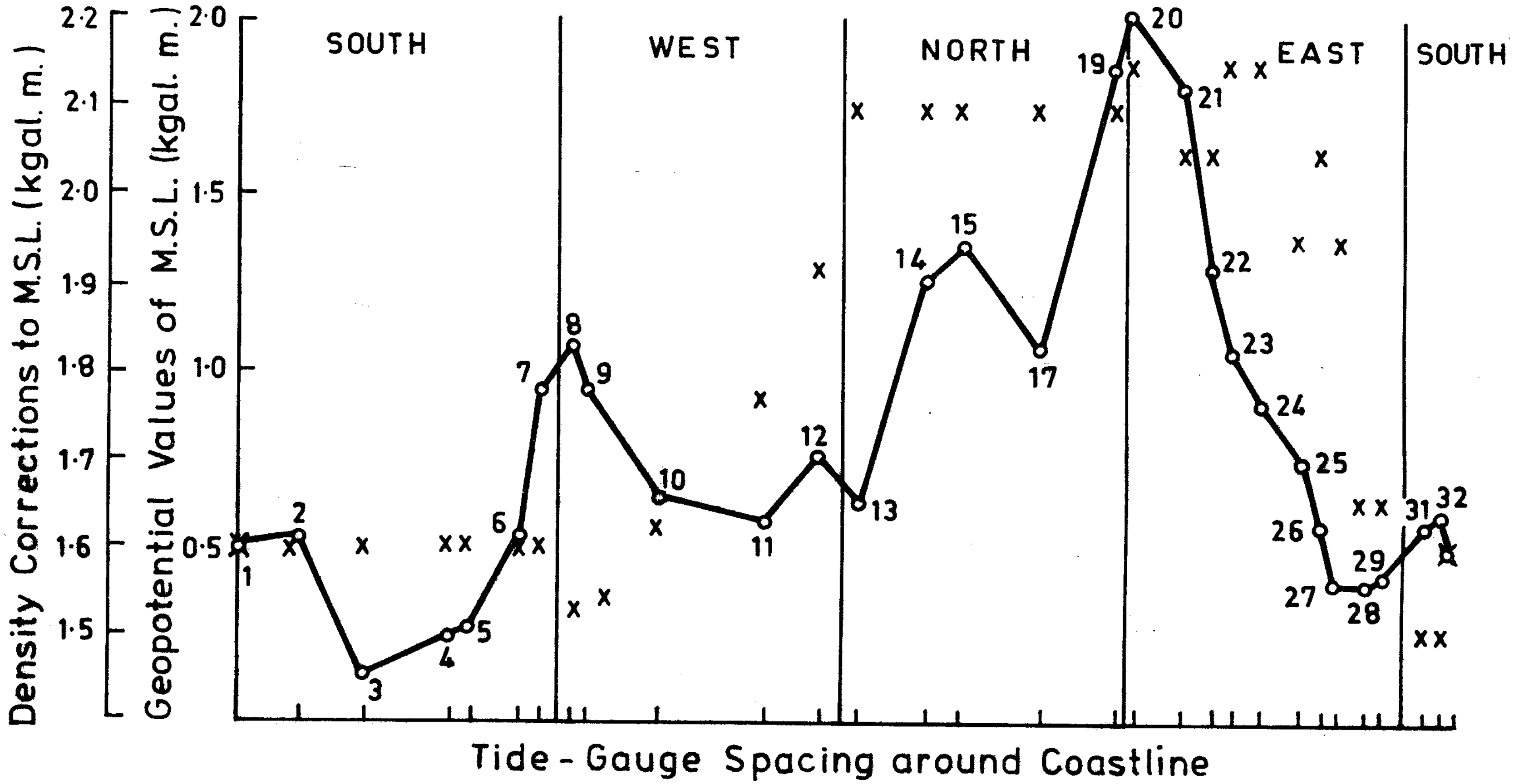


FIG. 5·12

COMPARISON OF DENSITY CORRECTIONS WITH APPARENT GEOPOTENTIAL VALUE OF M.S.L., AS IN FIGURE 3·2.

LEGEND:-

x DENSITY CORRECTIONS

o—o GEOPOTENTIALS OF M.S.L.

6. ATMOSPHERIC EFFECTS ON SEA LEVEL

6.1 Introduction

There are many interactions between the earth's atmosphere and the oceans, which result, directly or indirectly, in deviations with time or position, of the sea-surface from an equipotential surface. The most significant effects, air-pressure and wind, will be considered in Sections 6.2 and 6.3; other effects will be discussed in Section 6.4.

6.2 Air-Pressure Variations

In simple terms, the sea-surface should act as an "inverted barometer" to variations in atmospheric pressure. That is, an increase in air-pressure should result in a depression of the sea-surface, and *vice-versa*, according to a *barometric factor*, α, which is the ratio of the increase in the sea level resulting from a decrease in the air-pressure. According to the isostatic theory of ocean pressures, (see Chapter 5, Section 5.2) there is at some depth below the ocean surface, an equipotential surface which coincides with a surface of constant pressure. If the air-pressure at a point on the surface increases, then there must be a decrease in the pressure exerted by the water above the isostatic-equipotential surface, to maintain the constant pressure. Consequently, at that point in the ocean, the total mass of water above the isostatic surface must decrease. Thus, the surface level drops, unless the density of the water changes.

Theoretically, α has a value of -1.01 cm mbar^{-1}, (*Hamon*, 1966, p.2883) because the pressure of 1.01 cm of water is equal to 1 mbar of atmospheric pressure. The negative sign arises because an *increase* in sea level should result from a *decrease* in air-pressure. However, there is debate as to whether this factor is always -1.01 cm mbar^{-1} (see *Hamon*, 1962; 1966; *Lisitzin and Pattullo*, 1961; and others). Normally, α is calculated by correlating gauge record residuals; i.e.

gauge records after known variations have been eliminated with air-pressure records over the same epoch for the same place. This has been undertaken in Australia by *Easton and Radok* (1970a) and by *Hamon* (1966). Their results (Tables 6.1 and 6.2) indicate that α is not equal to -1.01 cm mbar^{-1}. It has been suggested (*Hamon*, 1962; 1966) that this deviation of α from -1.01 is due to the occurrence of shelf-waves which travel along the continental shelf away from the air-pressure disturbances.

In consideration of whether it is important to correct MMSL's for air-pressure variations, the following calculations have been made, on the assumption that α is equal to -1.01 cm mbar^{-1}.

(A) Time Variations of Air-Pressure

Variations of mean-monthly-air-pressure around Australia have been obtained from the *Bureau of Meteorology* (1970) which shows mean barometer readings calculated for each month from many years of records. Table 6.3 lists the maximum and minimum monthly means for five coastal cities as an indication of variations with *time* of sea level, due to air-pressure variations with time.

(B) Position Variations of Air-Pressure

Mean air-pressures for the month of October 1970 for six barometers situated on the coast, with an elevation close to zero, were obtained from the *Bureau of Meteorology* (Monthly). Results shown in Table 6.4 are indicative of position variations of sea level due to air-pressure differences.

Corrections were made to the MMSL's for air-pressure variations using a barometric factor of -1.01. From Tables 6.3 and 6.4 it appeared that time and position variations are of a magnitude which would support further investigation. It was hoped that insight into the

Table 6.1

Values of the barometric coefficient for Australian ports. Extracted from *Easton & Radok* (1970a, Table 4.1)

	PORT	BAROMETER	1966	1967	1968	TOTAL
1	Port MacDonnell	Mt. Gambier	1.26	0.60	-0.90	-0.12
2	Victor Hbr.	Adelaide	1.54	1.41	0.03	0.29
3	Pt. Lincoln	Pt. Lincoln	1.42	1.81	-0.22	0.40
4	Thevenard	Ceduna	1.56	1.80	0.22	0.59
6	Esperance	Esperance	0.78	1.15	-1.00	0.22
7	Albany	Albany	0.05	-0.21	-0.76	-0.28
8	Bunbury	Bunbury	1.44	0.42	-0.02	0.41
10	Geraldton	Geraldton	0.61	-0.31	-0.01	0.03
11	Carnarvon	Carnarvon	0.59	-1.32	-0.32	-0.55
12	Pt. Hedland	Pt. Hedland	-1.11	-1.32	-0.74	-1.06
13	Broome	Broome	-3.39	-1.32	0.29	-1.21
14	Wyndham	Darwin	-2.57	-5.37	-1.93	-3.32
15	Darwin	Darwin	-3.10	-3.42	-5.01	-2.66
16	Melville Bay	Darwin	-5.51	-6.47	-5.01	-5.38
17	Centre Is.	Vanderlin Is.	-5.93	-6.14	-5.91	-5.68
20	Bamaga	Thursday Is.	-4.69	-8.40	-3.54	-5.36
21	Cooktown	Cooktown	-0.71	-0.78	-0.97	-0.71
22	Cairns	Cairns	-1.12	-1.51	-1.27	-1.16
23	Townsville	Townsville	-0.66	-1.55	-1.18	-0.96
24	Mackay	Mackay	-0.58	-1.25	-1.69	-1.18
26	Brisbane	Brisbane	-0.31	-0.25	-0.39	-0.31
27	Coffs Hbr.	Coffs Hbr.	-0.38	0.36	-0.20	-0.03
28	Camp Cove	Sydney	-0.38	-0.21	-0.84	-0.46
29	Pt. Kembla	Sydney	-0.36	-0.26	-0.45	-0.54
30	Eden	Eden	-0.94	-0.85	-1.77	-0.93

Table 6.2

Barometric Coefficients Calculated by *Hamon* (1966)

STATION	NUMBER OF DAYS USED	REGRESSION COEFFICIENT cm $mbar^{-1}$		AIR PRESSURE NOTE
22 Cairns	58	-0.14	±0.28	2
23 Townsville	58	+0.41	±0.54	2
24 Mackay	58	+0.29	±0.62	2
Urangan	105	-0.84	±0.27	2
27 Coffs Harbour	105	-0.11	±0.23	3
Newcastle	61	-0.35	±0.19	2
Sydney	168	-0.52	±0.12	3
29 Port Kembla	105	-0.36	±0.20	2
30 Eden	105	-0.71	±0.23	2
Hobart	61	-1.07	±0.11	2
1 Port Macdonnell	56	-1.26	±0.17	2
7 Albany	58	-1.24	±0.14	2
8 Bunbury	104	-1.03	±0.40	1
9 Fremantle	104	-1.79	±0.31	1
10 Geraldton	104	-2.39	±0.55	1
12 Port Hedland	58	-1.70	±0.98	2

Air Pressure Note - Daily mean atmospheric pressure computed from

1. 9 a.m. and 3 p.m. pressures
2. four 6-hourly readings
3. eight 3-hourly readings

Table 6.3

Time variations of sea-level due to air-pressure

CITY	MONTHLY AIR-PRESSURE (mbar)			YEARS USED TO CALCULATE AIR PRESSURE	SEA LEVEL VARIATION DUE TO PRESS. DIFFS. (cm)
	MAXIMUM	MINIMUM	DIFFERENCE		
Perth, W.A.	1018.8	1012.6	6.2	84	6.3
Darwin, N.T.	1012.7	1006.6	6.1	85	6.2
Adelaide, S.A.	1020.1	1013.2	6.9	112	7.0
Brisbane, Qld.	1018.9	1011.7	7.2	82	7.3
Sydney, N.S.W.	1018.8	1012.0	6.8	59	6.9

Table 6.4

Position variations of air-pressure

BAROMETER AND HEIGHT (METRES)	AIR-PRESSURE, OCTOBER 1970		
	9 A.M. (mbar)	3 P.M. (mbar)	MEAN (mbar)
Carnarvon Airport, 4	1014.0	1011.5	1012.8
Bunbury PO, 5	1016.6	1014.8	1015.7
Port Lincoln, 5	1016.5	1014.3	1015.4
Coffs Hbr, A MO, 5	1016.4	1013.1	1014.8
Townsvill, A MO, 6	1014.3	1011.1	1012.7
Cooktown, 5	1012.4	1009.7	1011.0
Esperance PO, 4	1014.4	1012.5	1013.5

deviation of α from -1.01 cm mbar^{-1} would be gained.

The results of *Hamon* (1966) and *Easton* (1970) were not used because of the dissimilarity in the analysed α, which is evidenced by the differences between the two sets of results, and the differences in *Easton's* own results for separate years. An analysis of α in this study was not likely to produce results any different from those of *Easton* (*ibid*).

Ideally, the air-pressures used in the correction of MMSL's at any tide-gauge would be procured by a barometer operating at the site of the tide-gauge. As barometers were not situated next to each gauge used in the Australian survey, the air-pressures had to be obtained by interpolation or adoption from distant barometers. Consideration was given to the variation of air-pressures with *horizontal* and *vertical* position changes.

The relationship between air-pressure and elevation along a column of air may be expressed by the general formula,

$$H_2 - H_1 = K(\log P_1 - \log P_2)\ A.B.C. \qquad .....(6.1)$$

where H_1 and H_2 are the elevations at the top and bottom of the air column, P_1 and P_2 are the corresponding air-pressures, and K is a constant. A, B and C are terms which account for the effect of temperature and moisture, the effect of gravity variation with elevation, and the effect of gravity variation with latitude, respectively.

> The value of K is, according to *Clark* (1963, p.555), "variously estimated as 60,159 to 60,384 for foot units", i.e. between 18,336 and 18,405 for H in metres.

As A, B and C in equation (6.1) are approximately unity, the equation is of the order of

$$H_2 - H_1 = \Delta H$$
$$= K(\log P_1 - \log P_2)$$
$$\doteq 20{,}000 \log (P_1/P_2)$$

$$\therefore \qquad \log P_1/P_2 \doteq \Delta H/20{,}000$$
$$P_1 \doteq P_2 \text{ antilog } \frac{\Delta H}{2 \times 10^4}$$
$$= P_2 \; 10^{\Delta H/2 \times 10^4}$$

If a barometer is situated at an elevation of 1000 m, the pressure difference between the barometer and sea level will be

$$\Delta P = P_1 - P_2$$
$$= P_2 \text{ antilog } \frac{\Delta H}{2 \times 10^4} - P_2$$
$$= P_2 \left(\text{antilog } \frac{\Delta H}{2 \times 10^4} - 1\right)$$
$$= 1000 \left(\text{antilog } \frac{10^3}{2 \times 10^4} - 1\right)$$
$$= 1000 (\text{antilog } 0.05 - 1)$$
$$= 1000 (1.123 - 1)$$
$$\doteq 120 \text{ mbar}$$

To obtain corrections to MSL to a centimetre, the air-pressure is required to a millibar. Then the value of K correct to 1% will be sufficient for barometer elevations to 1000 m. As the values given above differ by only 69 in 18,000, or approximately one part in 250, the value of 18,400 was used in the calculations of the air-pressure corrections.

The factor A which accounts for temperature and an average amount of moisture was taken from *Clark* (*ibid*) to be

$$A = 1 + \frac{t_1 + t_2 - 64}{900}$$

where t_1 and t_2 are the temperatures in degrees Fahrenheit at elevations H_1 and H_2 along the air-column. Fahrenheit degrees will be maintained because the data for the years 1966-1970 is in these units. If $t_1 = t_2 = 80^{\circ}F$,

then

$$A = 1 + \frac{80 + 80 - 64}{900}$$

$$= 1 + 96/900$$

$$\doteq 1.1$$

i.e. $$A \simeq 1 + \frac{2t - 64}{100}$$

$$= \frac{t + 418}{450}$$

$$\therefore \quad dA = dt/450$$

$$\therefore \quad dA/A = \frac{dt}{450} \Big| \frac{t + 418}{450}$$

$$= {}^{dt}/t + 418$$

If dA/A is required to be 1% or less to ensure that the air-pressure correction is correct to 1%,

$$\frac{dt}{t + 418} = 0.01$$

$$dt = 0.01(t + 418)$$

If $$t = 80$$

$$dt = 0.01 \ (498)$$

$$= 5$$

Thus, an error in the temperature of up to about $5^{\circ}F$ was tolerable.

The term *B* which accounts for the variation of gravity with latitude is taken from the International Gravity Formula, and is

$$B = 1 + 0.0026 \cos 2\phi$$
$$\simeq 1.003$$

This term will be neglected.

The variation of gravity with height at sea level is

$$\frac{dg}{dH} = 0.3086 \text{ mgal m}^{-1}$$

As the value of gravity is of the order of 10^6 mgal, a 1% variation in gravity would require a height variation of the magnitude of 30 km, and the term C is clearly negligible.

The relationship between elevation and air-pressure used in this study was then

$$\Delta H = K(\log P_1 - \log P_2)\ (1 + \frac{t_1 + t_2 - 64}{900}) \qquad .....(6.2)$$

where ΔH, K, P_1, P_2, t_1 and t_2 take their previously described meanings. The required accuracy of H may be examined from this formula

$$\Delta H \doteq K \log P_1/P_2$$

For a pressure variation of 1 mbar,

$$P_1/P_2 \doteq 1001/1000$$
$$= 1.001$$
$$\Delta H \doteq 20,000 \times \log 1.001$$
$$= 2 \times 10^4 \times 0.00043$$
$$\doteq 9$$

A height difference of approximately 10 m will produce a pressure change of 1 mbar, so that barometer elevations to about 3 m should be satisfactory.

The equation (6.2) was transformed to allow the calculations of a pressure correction from given barometer elevations

$$H_2 - H_1 = K(\log P_1 - \log P_2)\ (1 + \frac{t_1 + t_2 - 64}{900})$$

If $\quad t_1 = t_2$

$\quad\quad = t$

and $\quad H = 0,$

as the corrections are being made to sea level, P_2 is known and P_1 is unknown,

$$H_2 = K \log\ (P_1/P_2)(1 + \frac{2t - 64}{900})$$

$$\therefore \quad \log P_1/P_2 = \frac{H_2}{K(1 + \frac{t-32}{450})}$$

$$\therefore \quad P_1/P_2 = \text{Antilog} \left\{\frac{H_2}{K(1 + \frac{t-32}{450})}\right\}$$

$$P_1 = P_2 \text{ Antilog } \frac{H_2 \cdot 450}{K.(450 + t-32)}$$

$$= P_2 \text{ Antilog } \frac{H_2 \cdot 450}{18,400\ (t + 418)}$$

$$= P_2 \text{ Antilog } (H \times 0.0245)/(t + 418)$$

$$\Delta P = P_1 - P_2$$

$$= P_2\ (\text{Antilog } \frac{H.(0.0245)}{t + 918} - 1) \qquad \text{.....(6.3)}$$

Air-pressure and temperature data was taken from the *Bureau of Meteorology* (Monthly) publication, which lists for stations where barometers are available, the monthly mean 9 a.m. air-pressure at station level, the monthly mean 3 p.m. air-pressure, both to 0.1 mbar. Further, the monthly mean 9 a.m. and 3 p.m. temperatures to 0.1° Fahrenheit were also available from this source. Barometer elevations were made available by the *Bureau of Meteorology* directly from their records. Elevations are known to less than half a metre.

A problem to be faced when using 9 a.m. and 3 p.m. monthly means is the diurnal cycle of air pressure, the cause of which is uncertain, but which produces maximum pressures at approximately 10 a.m. and 10 p.m. and minimum at 4 a.m. and 4 p.m., the variation being larger the nearer the equator. The effect of this phenomenon at 9 a.m. and 3 p.m. is demonstrated in Table 6.5. Its effect on the mean of two pressures was assumed to be negligible as the 9 a.m. and 3 p.m. observations occur an hour before the 10 a.m. maximum and the 4 p.m. minimum.

As a study of the variation of air-pressure with position, a number of pressures around Victoria for the month of November, 1970, were extracted from the data source, *Bureau of Meteorology* (Monthly), and reduced to sea level using the equation 6.3. The resulting distribution of pressures is shown in Figure 6.1.

The MMSL's for the years 1966 to 1970 were corrected for air pressure effects according to methods described above. Of the 31 gauges connected to the levelling network, only for Karumba (18) were MMSL's not available for this project. All Monthly Mean Sea Levels were reduced to a standard pressure of 1000 mbar, chosen for convenience. Table 6.6 shows an example of the application of corrections, which are summarized for all gauges in Table 6.7. For most gauges barometers were found to be available only a few kilometres from the tide-gauge. However, for five gauges, the nearest barometers were 20, 30, 50, 60 and 70 km distant. Barometer site elevations were abstracted from *Bureau of Meteorology* records, where the heights were given to better than half a metre.

The number of months for which both air-pressure and Mean Sea Level data was available is included in Table 6.7. Corrections were not undertaken for sea levels at Eucla (5), where only 29 months of MMSL's were available for the period 1966 to 1970. Furthermore, for only twelve of these months were air-pressures available from Eucla. The next closest barometer is at Forrest, over 100 km distant. Similar difficulties were encountered for Eden (30). For the years 1966 to 1970,

FIG. 6·1

AIR-PRESSURES REDUCED TO SEA LEVEL, VICTORIA
January, 1970

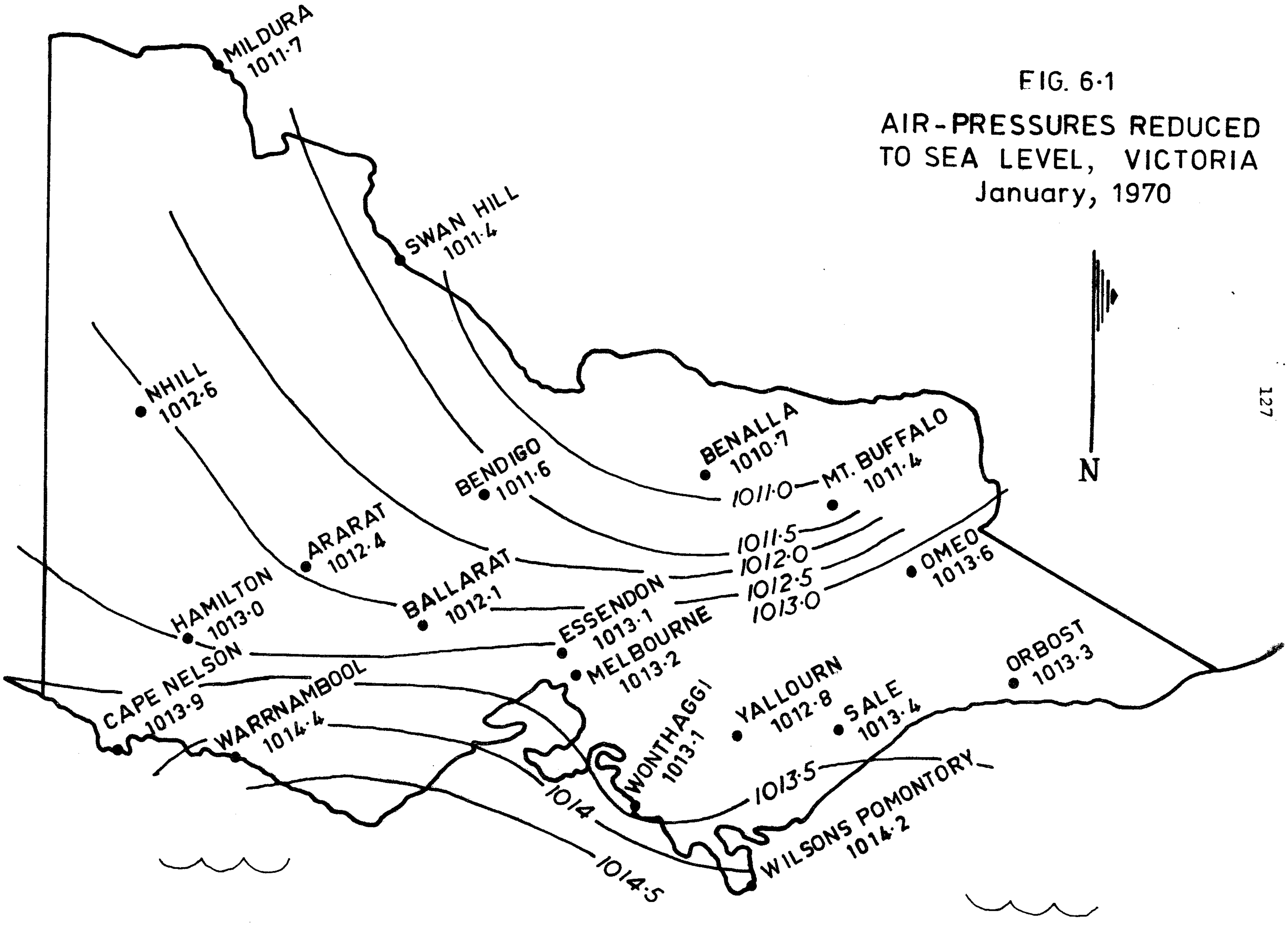


Table 6.5

Monthly Mean air-pressures at 9 a.m. and 3 p.m.
for some Australian gauges,
indicating the diurnal variation.

STATION	STATION LEVEL AIR-PRESSURE (mbar)			APPROXIMATE LATITUDE (Degrees)
	9 A.M.	3 P.M.	DIFFERENCE	
Cape Don, N.T.	1007.7	1005.2	2.5	12
Alice Springs, N.T.	953.5	949.7	3.8	24
Charleville, Qld.	981.4	977.3	4.1	26
Brisbane, Qld.	1013.5	1009.9	3.6	27
Forrest, W.A.	998.3	994.8	3.5	31
Perth, W.A.	1014.5	1012.3	2.2	32
Sydney, N.S.W.	1015.9	1013.1	2.8	34
Canberra, A.C.T.	949.1	946.3	2.8	35
Melbourne, Vic.	1005.6	1004.0	1.6	38

Table 6.6

Air Pressure correction calculations
Townsville (23), 1966-1968

YEAR & MONTH		MONTHLY MEAN SEA LEVEL		AIR-PRESSURE (mbar)		TEMPERATURE (° FAHRENHEIT)		AIR-PRESSURE CORRECTION TO 1000 mbar	ADJUSTED MMSL (metres)
		FEET	METRES	9 A.M.	3 P.M.	9 A.M.	3 P.M.		
1966	J	5.56	1.695	1011.4	1009.2	80.1	83.6	10.96	1.806
	F	5.74	1.750	1008.9	1006.2	82.5	86.5	8.20	1.833
	M	5.84	1.780	1012.0	1009.3	82.0	85.5	11.31	1.894
	A	5.87	1.789	1014.6	1011.5	79.2	83.8	13.71	1.927
	M	5.69	1.734	1017.7	1014.5	72.8	78.1	16.77	1.903
	J	5.51	1.679	1019.5	1016.4	68.5	75.4	18.63	1.867
	J	5.44	1.658	1017.8	1014.4	66.3	74.8	16.78	1.827
	A	5.36	1.634	1018.5	1015.5	70.5	75.5	17.68	1.843
	S	5.09	1.551	1016.9	1013.3	75.6	79.5	15.77	1.710
	O	5.20	1.585	1016.6	1013.2	77.6	80.5	15.57	1.742
	N	5.20	1.585	1013.1	1010.1	80.5	82.5	12.26	1.709
	D	5.26	1.603	1011.8	1009.3	81.2	84.2	11.21	1.719
1967	J	5.47	1.667	1007.0	1004.3	84.2	87.5	6.30	1.731
	F	5.68	1.731	1009.9	1007.6	81.5	84.9	9.41	1.826
	M	5.85	1.783	1009.4	1006.7	79.1	83.1	8.71	1.871
	A	5.61	1.710	1015.4	1012.3	76.8	81.9	14.51	1.857
	M	5.29	1.612	1016.5	1013.4	73.1	77.9	15.62	1.770
	J	5.22	1.591	1016.0	1012.9	66.9	73.9	15.13	1.744
	J	5.12	1.561	1018.0	1014.8	65.2	73.7	17.08	1.734
	A	5.13	1.564	1016.7	1013.3	70.0	76.0	15.67	1.722
	S	4.85	1.478	1017.3	1013.9	73.8	78.3	16.27	1.642
	O	5.06	1.542	1015.8	1012.6	78.2	80.9	14.86	1.692
	N	5.07	1.545	1012.5	1009.3	81.7	84.0	11.56	1.662
	D	5.26	1.603	1010.0	1007.0	82.3	85.2	9.16	1.695
1968	J	5.51	1.679	1006.8	1004.2	81.7	85.9	6.15	1.741
	F	5.57	1.698	1008.9	1006.2	79.3	83.5	8.26	1.781
	M	5.66	1.725	1011.4	1008.7	80.7	85.0	10.71	1.833
	A	5.38	1.640	1014.0	1010.8	79.2	83.8	13.06	1.772
	M	5.32	1.622	1015.4	1012.0	71.4	77.7	14.37	1.767
	J	5.33	1.625	1018.3	1014.9	66.8	75.4	17.28	1.800
	J	5.13	1.564	1018.9	1015.8	65.7	72.9	18.03	1.746
	A	5.06	1.542	1019.9	1013.3	69.6	79.1	15.77	1.701
	S	5.02	1.530	1018.0	1014.7	74.6	77.6	17.02	1.702
	O	5.09	1.551	1016.6	1013.3	78.7	80.9	15.62	1.709
	N	5.09	1.551	1013.5	1010.1	81.7	84.8	12.46	1.677
	D	5.19	1.582	1011.2	1008.5	82.1	85.5	10.51	1.688

Table 6.7

Summary of Air-Pressure Corrections to Sea-Level

TIDE-GAUGE IN LEVELLING SURVEY	NUMBER OF MONTHS OF DATA USED	MEAN AIR-PRESSURE CORRECTION (cm)	MSL AFTER DENSITY CORRECTIONS, TABLE 5.1	MSL AFTER AIR-PRESSUR CORRECTION (kgal cm)
1 Port MacDonnell	55	16.87	240	257
2 Victor Harbour	60	18.52	242	261
3 Port Lincoln	57	17.48	203	220
4 Thevenard	60	18.05	215	233
5 Eucla	29	-	218	-
6 Esperance	54	16.32	243	259
7 Albany	59	16.77	285	302
8 Bunbury	60	16.98	304	321
9 Fremantle	40	16.86	291	308
10 Geraldton	55	15.44	252	267
11 Carnarvon	45	14.78	230	245
12 Port Hedland	55	11.63	234	246
13 Broome	48	10.41	203	213
14 Wyndham	20	-	266	-
15 Darwin	58	9.89	276	286
17 Centre Island	45	11.78	246	258
18 Karumba	Uncorrected	-	273	-
19 Weipa	36	9.39	327	336
20 Bamaga	36	10.04	337	347
21 Cooktown	36	11.67	326	338
22 Cairns	60	12.84	275	288
23 Townsville	60	13.34	241	254
24 Mackay	59	16.93	226	243
25 Bundaberg	60	15.27	229	244
26 Brisbane	58	16.28	201	217
27 Coffs Harbour	58	16.22	195	211
28 Camp Cove	60	16.44	226	242
29 Port Kembla	60	16.39	226	242
30 Eden	Uncorrected	-	226	-
31 Point Lonsdale	58	16.76	256	273
32 Port Fairy	38	16.47	259	275

complete or partially complete sea level data was available for only 21 months and the nearest barometer is at Montague Island, approximately 100 km from Eden.

Figure 6.2 was formed by combining the geopotential values of Mean Sea Level after the correction for the density effect (see Table 5.1), with the air-pressure corrections from Table 6.8. No relationship between the pressure corrections and the apparent sea levels is noticeable. The air-pressure corrections do have a correlation with latitude, which is illustrated in Figure 6.3.

To test whether the relationship between air-pressure and sea level would be modified by using barometric factors different from 1.01 cm $mbar^{-1}$, the phases of sea level and air-pressure have been studied. The mean sea levels for each month over the period 1966 to 1970 were combined for each month to indicate the annual cycle of sea level. Figure 6.4 shows the results for Townsville. The cycle of sea level using MMSL's which were *corrected* for air-pressure, at Townsville, are also shown in Figure 6.4. A cycle of air-pressure corrections was calculated similarly; results for Townsville are shown in Figure 6.5. These calculations were also undertaken for four other gauges for which almost complete data was available. Sixty months of data was available for Cairns (22), Camp Cove (28) and Thevenard (4), as well as for Townsville (13), whilst 58 months were used for Darwin (15). From these results it could be concluded that;

(i) the air-pressure corrections have not made significant changes to the time-variations of MMSL's, except at Camp Cove (28).

(ii) the air-pressures have consistent lows in the second or third month, and a high six months later. The MMSL's do not appear to have a consistent low or high time at all gauges, although the gauges seem to be reasonably consistent within themselves.

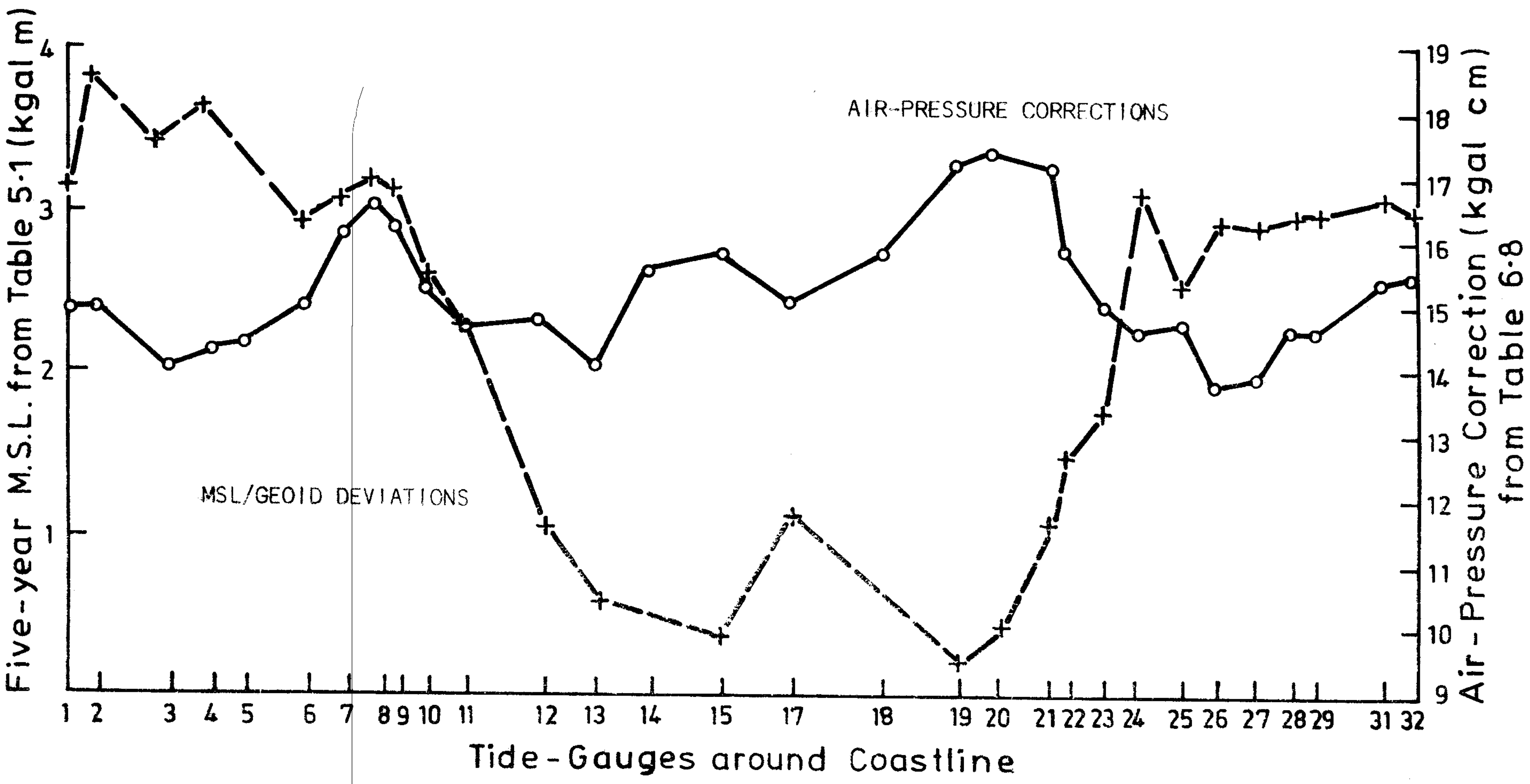


FIG. 6·2

RELATIONSHIP BETWEEN AIR-PRESSURE CORRECTIONS AND APPARENT DEVIATIONS BETWEEN M.S.L. AND GEOID.

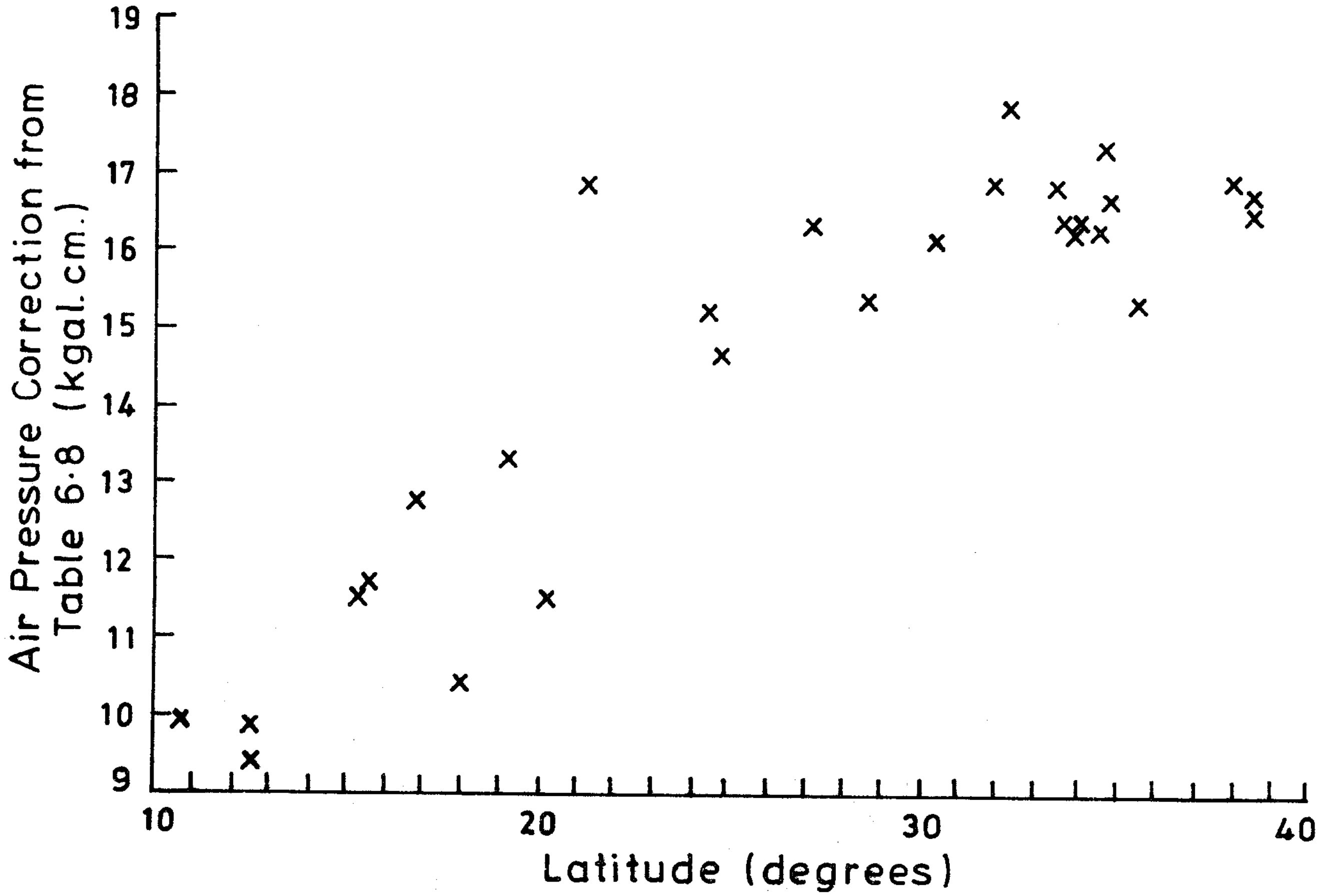


FIG. 6·3
COMPARISON OF AIR-PRESSURE CORRECTIONS TO S.L. WITH LATITUDE

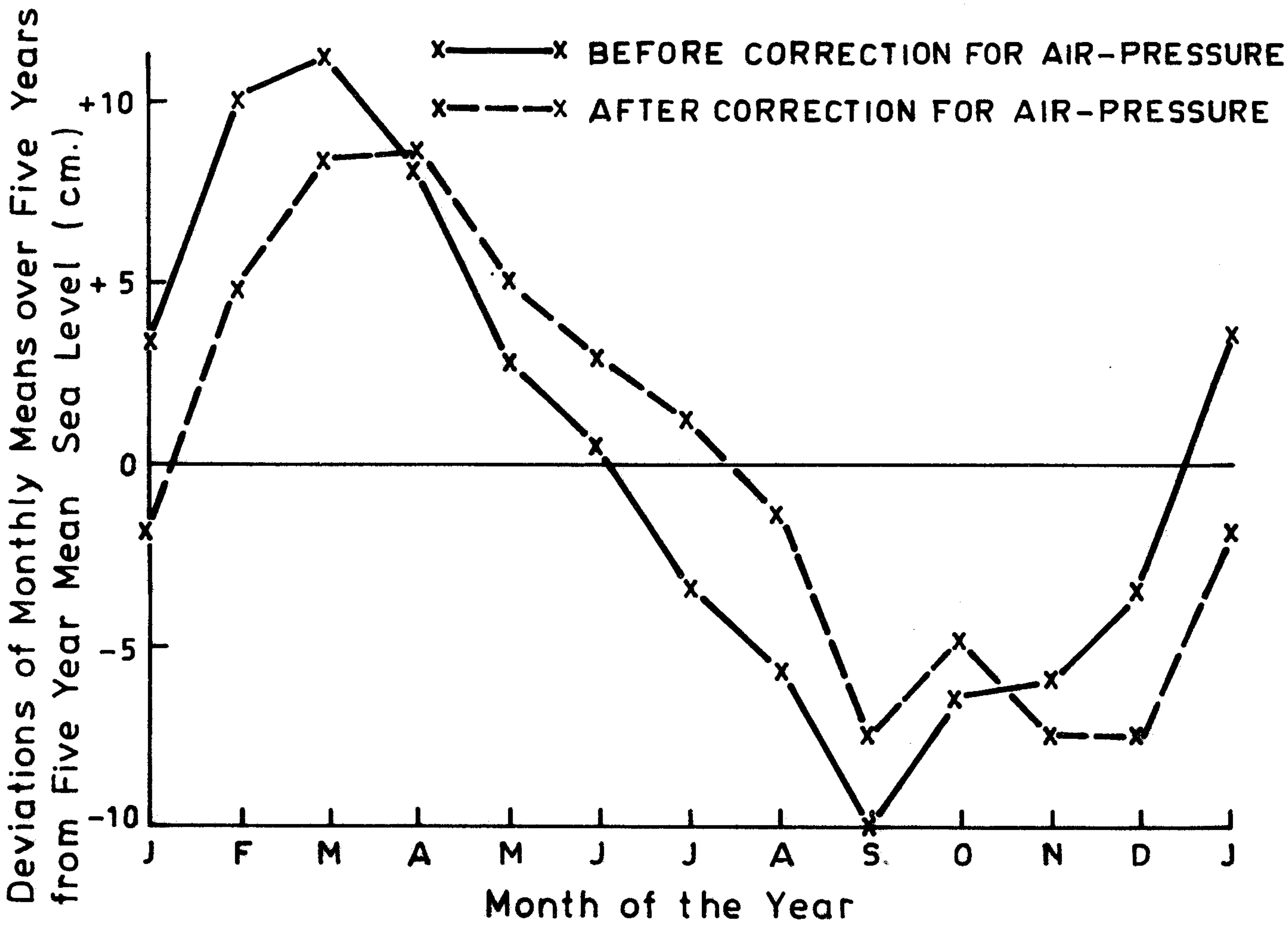


FIG. 6·4

ANNUAL CYCLE OF SEA LEVEL AT TOWNSVILLE (23)

DATA: 1966-1970

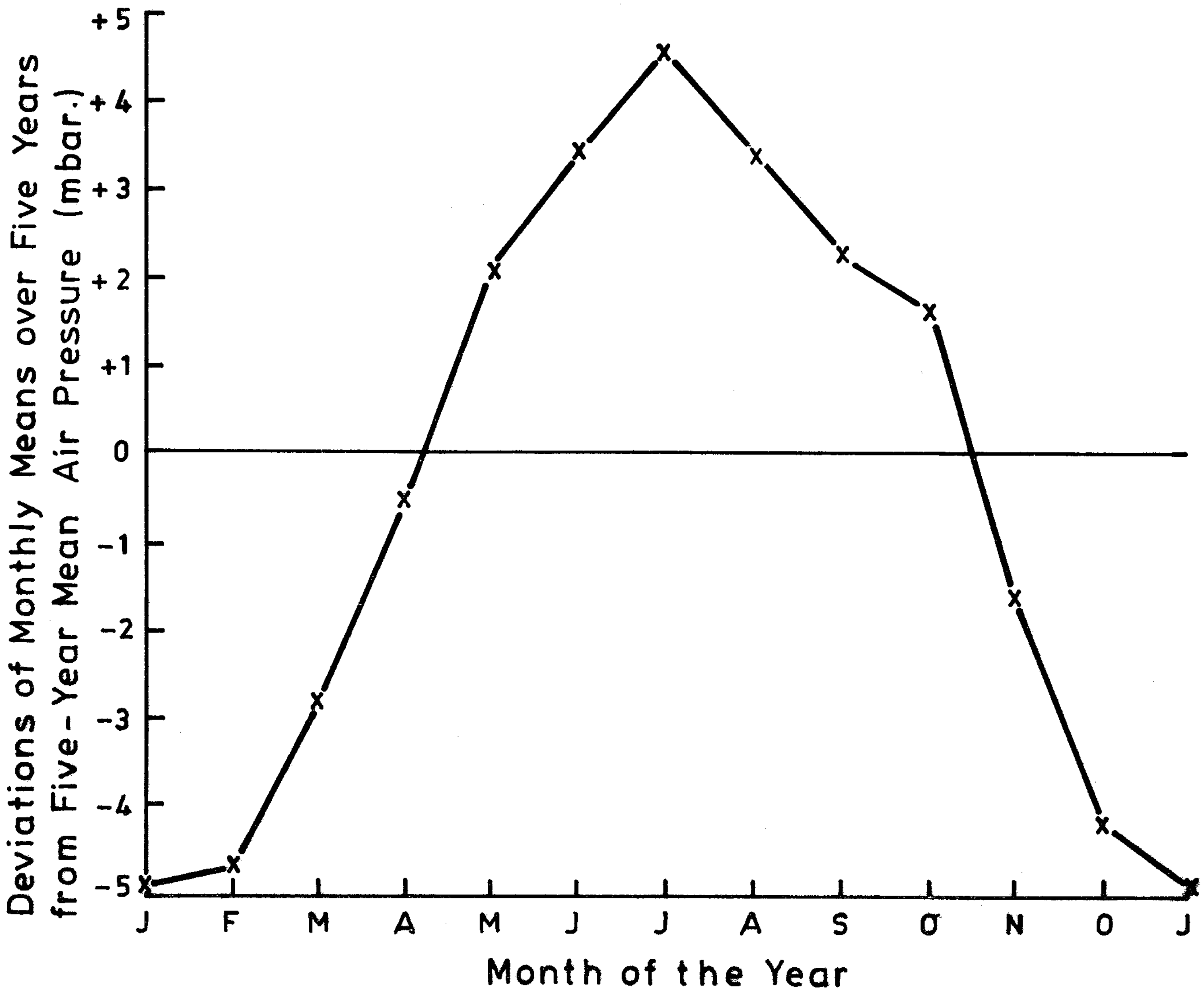


FIG. 6·5
ANNUAL CYCLE OF AIR PRESSURE TOWNSVILLE (23)
DATA: 1966 - 1970

That is, there does not appear to be any phase agreement between pressures and MMSL's.

(iii) small variations in the pressure curves rarely have corresponding fluctuations in the MMSL's.

Therefore, it would not be expected that the use of any value of α would necessarily improve the MMSL graphs. The effect of air-pressure can be discounted as a cause of the geoid-MSL discrepancies. Although the position effects of air-pressure as applied were comparatively small, their calculation seems to be justified by allowing the determination of the above conclusion.

6.3 Wind Effects

The wind can be considered to have three effects on sea level. Firstly, *wind stress*, or the friction between the moving air and the water surface, results in transportation of the upper layers of the oceanic water, which in turn produces a redistribution of the water density. This effect results in current flow, which was considered in detail in Chapter 5. Secondly, *wind pile-up* may occur: the water which is moved by the wind is piled against an obstruction, such as a coastline, with a consequent rise in sea level if the resultant water pile-up cannot dissipate. The third wind effect is that of *storm surges*.

Because of the discussion of density in the previous chapter, wind stress will not be considered further in this chapter. Wind pile-up and storm surges will be considered simultaneously, the former being assumed to be a longer period version of the latter.

Formulae with varying degrees of complexity are used to calculate the effects of wind on Mean Sea Level.

Hamon (1958, p.191) has used

$$3 \times 10^{-6} < \frac{g.h \sin \theta}{U_a^2} < 4.5 \times 10^{-6}$$

where U_a is the wind velocity, g is the value of gravity, h is the water depth and θ is the resulting slope of the sea-surface. For a 50 km per hour wind on a continental shelf of depth 150 m, θ will be such as to cause an uplift of 3 cm over a shelf width of 40 km.

In his study of sea level on the U.S. Pacific coast, *Sturges* (1967, p.3630) calculated the effect of wind-stress by the formula

$$S = \tau\alpha/gh$$

where S is the slope of the sea-surface, α is the specific volume, τ is the magnitude of the wind-stress and g and h are as before. Using a value for τ of 0.2 dynes cm^{-2} the effect on sea level was calculated by *Sturges* to be of the order of 0.15 cm, for the U.S. Pacific coast.

Crepon (1970) has developed formulae, which are too complex to be reproduced here. Instantaneous (daily) sea level deviations of up to 30 cm were calculated for Sète on the French Mediterranean coast where the shelf is 100 km long and 90 m deep.

For analysis of sea level *Doodson* (1960, p.70) used a formula which gave the deviation of sea level due to winds from one direction as a constant, to be determined by analysis, multiplied by the difference in air-pressure between the point under consideration and a point 500 km in the direction being considered.

Although the long-term wind pile-up may be theoretically quite small (e.g. *Hamon*, 1958, and *Sturges*, 1967), significant Australian variations in sea level are reported by *Easton* (1968; 1970) and *Easton and Radok* (1970a; 1970b). *Easton* (1968) describes the results of his own visits to 22 of the 30 tide-gauges used in the Australian levelling survey. Where relevant, the effects of winds on the gauge records, if known, are described. As an example, on the subject of the Thevenard (4) tide-gauge, *Easton* notes that a wind East of South lowers the tide while a wind West of North raises the tide.

Two quotations taken from *Easton* (1970) which show the effect of wind are given below.

> "..... a caution in the South Australian Tide Tables states that for Thevenard 'Easterly winds lower sea level by 1 foot to 2 feet, and low waters at spring tides often fall below datum in such conditions'". (p.33). One foot is approximately 30 cm.

Qutoing from the Australian National Tide Tables, in reference to Fremantle (9):

> "Before and after westerly gales, a high level of about 4 feet is maintained for possibly 6 days. In the ımmer months, especially during Easterly weather, a very low level is experienced for the same period". (p.35).

Observations by *Easton and Radok* (1970b) include

> "..... winds are so important in determining levels in the South Australian Gulfs that extreme levels may occur during any month"

and,

> "..... a study of the winds at Mackay over several years has shown that normal sea level heights are attained under the influence of 10 knot winds, e.g. in March 1967, winds reaching 30 knots were reflected in positive anomalies, winds below 10 knots in negative anomalies".

Easton and Radok (1970a) also suggest that, after a comment similar to that above,

> "It may be expected that similar conclusions can be drawn for other ports, and that this mechanism is responsible for the variations in the monthly mean sea levels along the South coast and elsewhere".

An intended quantitative study of wind effects proved prohibitive owing mainly to the shortage of wind data. To study wind effects, both the *velocity* and *direction* of winds are required at each point under consideration. A general description of winds may be obtained from the *Bureau of Meteorology* (1970). The *Bureau of Meteorology* (Monthly) wind data includes the *miles per day* figure at those stations for which such a figure was observed. Directions are not available from this source.

Wind roses compiled by the Bureau of Meteorology (see *Bureau of Meteorology*, 1970, pp.44-45) show frequencies of winds from each direction for each month of the year. Velocities are not available. Judging by the marked differences between the 9 a.m. and 3 p.m. wind roses, it would be inaccurate to use only two observations per day to estimate daily winds. Wind roses are given for 16 coastal places.

The Bureau of Meteorology does have available tables of velocity against direction, shown for each three hours of the day, tabulated for each month of the year. The records are obtained from averages over as many years as possible. That is, monthly means, divided into each three hours of the day, are available for velocity and direction for a number of places. A vast amount of data would be involved in an analysis with this data. Eight directions for twelve months in about ten velocities every three hours produces 7680 pieces of information for each station. The distribution of wind data points shown in Figure 6.6 is not suitable for a complete Australia-wide study.

A test for correlation between wind and the geoid-MSL deviations by relating the continental shelf-width and apparent deviations of sea level, Figure 6.7, is inconclusive.

Conclusions

Over a short period, surges may alter sea level by up to a couple of metres, but, on a five year mean, the effect of wind is reduced to the

FIG. 6·6

ANEMOMETER DISTRIBUTION
ON MAINLAND AUSTRALIA FROM
THE BUREAU OF METEOROLOGY (MONTHLY)

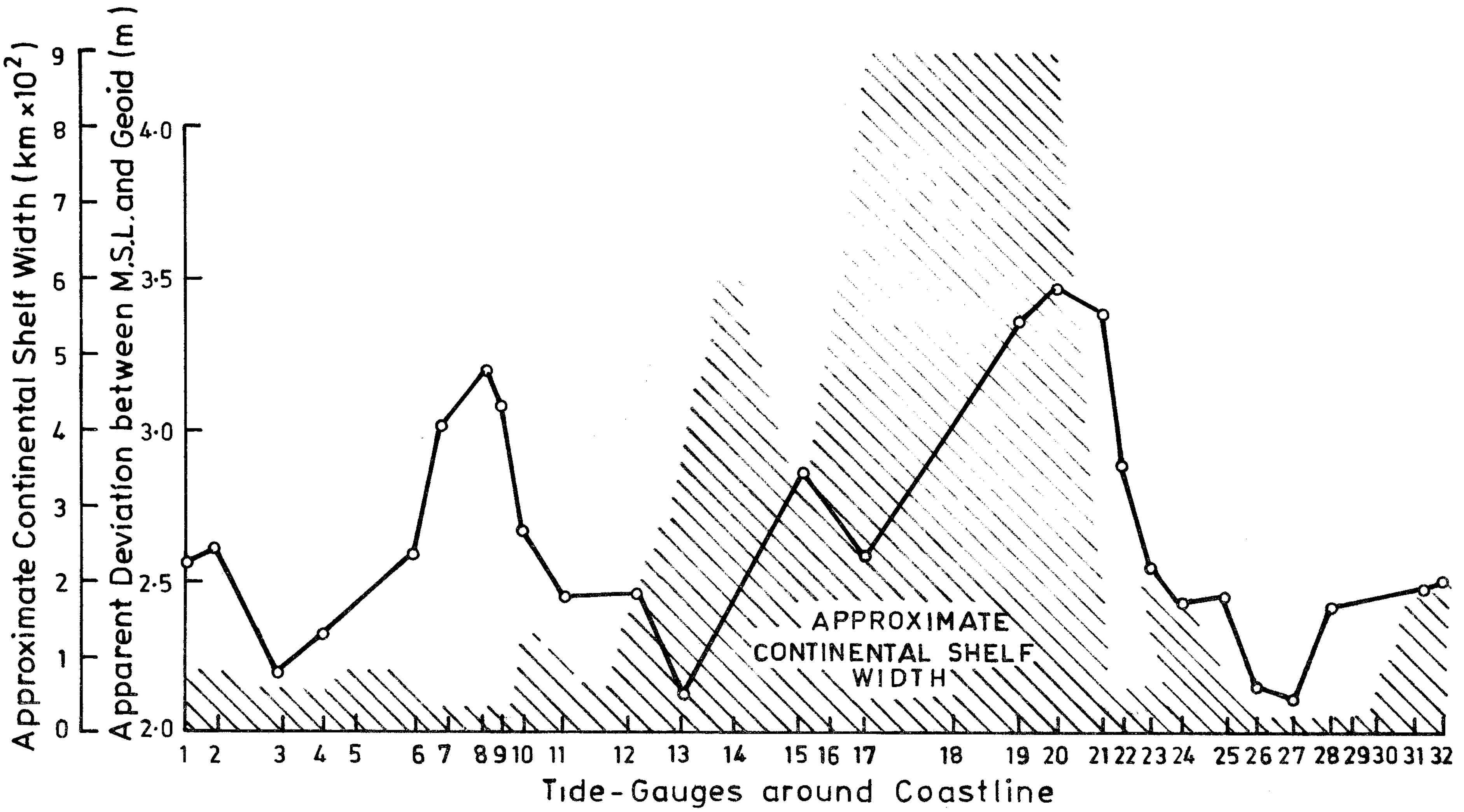


FIG. 6·7

RELATIONSHIP BETWEEN DEVIATIONS OF M. S. L. FROM GEOID AND CONTINENTAL SHELF WIDTH M.S.L./GEOID RELATIONSHIP FROM TABLE 6·8; SHELF-WIDTH TAKEN TO BE DISTANCE OF 200 m. CONTOUR FROM COAST.

order of centimetres. Thus, wind pile-up is not likely to account for a position-dependent variation in sea level of over 100 cm. However, there are definite effects of wind on sea level, illustrated by the abovementioned reports by *Easton* and *Easton and Radok*. Such variations are worthy of study. Corrections to sea level for wind could be a contribution to an accurate sea level determination. Lack of wind data at present would seem to be the biggest hindrance to a study of wind effects. Attention could be focussed on one gauge for which extensive wind and tide-gauge data are available. Such an involved study could not be embarked upon for this project, especially in light of the expected influence over five years.

6.4 Other Atmospheric Influences on Sea Level

Most influences of atmospheric occurrences on the ocean can be classed as *long waves*. This term refers to waves with a period between gravity waves or swell and the semi-diurnal tide, although the upper limit is sometimes extended. The causes and nature of the long waves often overlap, as they all result from atmospheric disturbances.

Seiches, the large scale oscillations of water masses, occur in bays, harbours and lakes, and occasionally on the continental shelf, as a result of resonance of a wave in the mass of water at the natural frequency of oscillation of the water. The causes of the originating wave are considered to be very numerous. Most commonly, they are thought to be due to air-pressure fluctuations or wind. Seiche periods are usually between five minutes and an hour, commonly about 15 minutes. Magnitudes are of the order of 30 cm, and up to about a metre. Although their occurrence is commonly evidenced on gauge records, they have little significance to MMSL's because of their periodicity. Moreover, for short period seiches, interpolation of hourly readings is not affected by high frequency fluctuations of the tide gauge recorder trace.

Surf beats have a period of about one to four minutes and an amplitude which is dependent on the magnitude of the surf swell. Although

Munk (1949) originally attributed the phenomenon to variations in the volume of water transported to shore by surf waves, he later discounted this (*Munk*, 1962). *Donn et al* (1964) remarked that

> "the mechanism of surf-beat generation is still not fully understood"

Storm surges often have periods which classify them as *long waves* but their effects have been discussed in Section 6.3.

Shelf Waves apparently originate at air-pressure disturbances and travel with a velocity of about 400 cm sec^{-1} along the continental shelf. Although their amplitude is only of the order of 4 cm, it is believed by *Hamon* (1962; 1966) that they could cause the barometric factor produced by analysis of sea level records to differ from the theoretical value of -1.01. Theoretical waves have been mathematically demonstrated by *Adams and Buchwald* (1968; 1969). Their effect on MMSL's will be assumed to be insignificant on the basis of their small amplitude, which is apparently dependent on the width and depth of the continental shelf and on the atmospheric pressure fluctuations which caused the waves.

Donn et al (1964) discuss the various interactions between the atmosphere and the sea-surface, including the above-mentioned surf-beat and surges.

6.5 Conclusions

The study of atmospheric effects on sea level was a fruitless search with respect to the 1 or 2 m deviation between sea level and the geoid. However, the study was not wasted. Although time consuming, the investigation described in Section 6.2 showed that air pressure is not a direct cause of significant variations with time. This is indicated by phase differences between air-pressure and sea-level fluctuations. If the time variations are uncorrelated, then presumably the position changes of air-pressure are not related to position variations of sea-level. The wind-effect on sea level could be of greater significance but it seems to be more difficult to study. MMSL's could well benefit from corrections for wind influences. Only a warning of such is possible now. The lesser atmospheric interferences seem to be comparatively insignificant. Section 6.4 has merely drawn attention to the existence of obscure atmospheric influences.

7. MINOR EFFECTS ON SEA LEVEL

7.1 Tsunamis

Tsunamis, often known by the deceptive term "tidal waves", are long waves which radiate from coastal or submarine tectonic activity, such as earthquakes and volcanoes, or even from coastal landslides and rockfalls. These waves may travel in the open ocean at velocities between about 150 and 800 km $hour^{-1}$, with periods from 15 to 60 minutes. Wavelengths may be 500 km.

Although tsunamis appear to be of little consequence in the open ocean, the increase in their amplitude when approaching coastlines has often resulted in waves of the order of 10 m high, and sometimes up to 30 m striking coastlines, with disastrous consequences.

In Australia, tsunamis have mainly been noticed on the Pacific coast, which faces a tectonically active area, but the effects on sea level are usually not more than a metre. Significant tsunami effects have been noted on about five occasions in the last century: in 1871 due to a Peruvian earthquake; in 1873; after the 1883 Krakatoan eruption; and after Chilean earthquakes in 1960 and 1964. On the East coast, abnormal sea level variations of up to a metre in 1960 and up to 30 cm due to the 1964 tsunami, were recorded.

It was concluded that tsunamis can be neglected as an effect on MMSL's in this period, for the following reasons:

(i) *United States Coast and Geodetic Survey* (Annual) reports show no significant tsunamis as having occurred during 1966 to 1968. At the time of writing, 1969 and 1970 data is unavailable from the source shown.

(ii) a tsunami is a periodic wave form so that its effect on a MMSL should be negligible.

(iii) tsunami effects would not pass unnoticed on continuous gauge records yet *Easton* (1970) has made no mention of their effects during 1966 to 1968, in his study of the gauge records;

(iv) tsunamis have too short a duration (less than a day) to significantly affect an MMSL. Even an *aperiodic* tsunami of 15 cm amplitude lasting for a full day, would affect an MMSL by only 5 mm.

7.2 Secular Changes of Sea Level

The term secular variations of sea level, refers to slow aperiodic or linear variations of the level of the sea surface at a tide-gauge. Such changes are due principally to constant increases in the mass of water in the oceans, to a rise or fall of the earth's crust in the locality of the tide-gauge, or to a change in the capacity of the oceans.

A change in the *amount of water* in the oceans results from evaporation, precipitation and river influx to oceans or from the melting of polar ice-caps. Evaporation, precipitation and river flow are short-term effects related to the cycle of water between the land, atmosphere and oceans and, being somewhat periodic, will be discussed in Section 7.3. Melting of the polar ice-caps can be attributed to an increase in the temperature of the atmosphere.

Rises or falls in the land level may be due to earthquakes in the short term or post-glacial uplift, for example, in the long term. Sedimentation on the ocean floor, causing a drop in the carrying capacity of the oceans is also considered to affect sea-level over a long period. *Fairbridge* (1960) gives a detailed coverage of causes of secular variations of sea level.

The magnitude of secular variations varies from place to place particularly because the various tectonic regions have different land uplift or sinking rates. In the Fenno-Scandian area, for example, changes of apparent sea level of approximately 9 cm per year, (Furuogrund, Sweden), (*IAPO*, 1955) have been measured between 1916 and 1951, as a result of the significant post-glacial uplift of the area. Other examples are shown in Table 7.1.

In Australia, the effects are comparatively small, owing to the tectonic inactivity of the area, and are only of the order of 0.5 mm per year (*IAPO*, *ibid*).

The most satisfactory method of accounting for secular variations is to analyse many years of results from a gauge to determine the change which is assumed to be linear with time. However, few of the 31 gauges being used in this study have been recording for many years. From the records of Fort Denison, N.S.W. for 1897 to 1951, and Williamstown, Victoria, for 1916 to 1930, *IAPO* (*ibid*) deduced falls of sea level of 0.4 and 0.7 mm per year respectively.

Secular effects on the 31 gauges were assumed to be negligible, for the following reasons:-

(i) it is impossible to obtain the secular variation by analysis, for the Australian gauges. Studies of annual means show that other variations outweigh the secular change, which cannot be observed in records over five years;

(ii) it is also impossible to analyse results from other gauges and to interpolate the required values. Only two gauges with long records are available in Australia;

(iii) both these gauges' records, analysed by *IAPO* (*ibid*) showed less than 1 mm per year secular change. For these gauges, this variation would only be of the order of 3 mm for the period 1966-70.

Table 7.1

Examples of secular variations of sea-level around the world, from *IAPO* (1955)

GAUGE SITE	YEARS	SECULAR VARIATION & STD. DEVIATION (mm/yr)
Stockholm, Sweden	1889-1951	-4.2 ±0.4
Wismar, Germany	1882-1943	1.4 ±0.3
Aarhus, Denmark	1889-1951	0.5 ±0.2
Marselle, France	1885-1951	1.4 ±0.2
Trieste, Italy	1905-1950	1.1 ±0.4
Forteau Bay, Canada	1899-1915	-0.8 ±1.3
New York, U.S.A.	1921-1951	4.2 ±0.5
San Francisco, U.S.A.	1898-1951	1.8 ±0.2
Hososima, Japan	1900-1951	0.8 ±0.4
Aberdeen, U.K.	1862-1913	-0.5 ±0.3

(iv) any secular effect will be reduced by observing all gauges for the same period if the changes are of the same order at each gauge.

7.3 Variations of the Mass of Water in the Oceans

The level of the ocean surface is, obviously, dependent on the volume of water contained in that ocean. Apart from long period or secular increases in the amount of water in the oceans, there is a shorter period variation due to the following, (see *Donn et. al*, 1964):

(i) the cycling of the water between the land areas, the oceans and the atmosphere. Varying amounts of water are held in the atmosphere, in the ground as groundwater, on the ground in creeks and rivers or as snow and ice, resulting in a fluctuation in the amount of water which must exist in the oceans.

(ii) a variation in the amount of water held in an ocean due to the transport of water between oceans in the Southern Hemisphere and those in the Northern Hemisphere, on a seasonal cycle.

(iii) a seasonal variation in the amount of ocean water which is stored as ice in the polar regions.

The magnitude of these changes is apparently of the order of a few centimetres. *Donn et al* (*ibid*, p.250) in an attempt to account for changes in sea level which could not be explained by other known effects, note that *Van Hylckama* (1956)

> "calculated that the spring minus (autumn) water storage on the continents is 0.75 x 10^{19} grams. This value corresponds to a sea level change of about 1-2 cm".

Using the residuals which they have calculated themselves, *Donn et al* (*ibid*) consider these residuals to be due to a change in mass of water in the oceans, as there is

> " ..... actually more water in the North in September than there is in March, and the reverse South of the Equator. Further, if one integrates over the entire ocean, there is more water in the Pacific Ocean from 60^{o}N to 50^{o}S in September than there is in March. This difference amounts to 1.8 x 10^{18} g. or approximately 1 cm of water if spread evenly out over the entire surface. We must add immediately that the uncertainty of the calculation is very large in view of possibly large experimental errors. The probability that this estimate is correct even to one significant figure is low ..... ".

The effects of water mass changes were accepted as having negligible significance to the time or position variations of sea level in this study. Although knowledge of the precise effects is scare, indications are that the magnitudes are very small. To undertake an involved study of the variations of sea level due to water mass changes is unwarranted in an explanation of sea level deviations from the geoid. Furthermore, as all gauges were observed simultaneously, the position effect on the three year mean is insignificant.

7.4 Gauging Faults

A tide gauge will not necessarily record accurately the times and heights of sea-level, with consequent effects on the monthly-mean sea level which is calculated from the records. To facilitate understanding of gauge faults which are given below, a description of an automatically recording tide-gauge which is representative of the gauges used in the Australian survey, is given in the diagram, Figure 7.1. The causes of faulty readings may be divided into three groups which are detailed below:

(i) Time errors of the gauge occur when the level of the sea at time t is shown on the output record as having occurred at time $t + \Delta t$. This results from an error along the time axis of the recorded graph. The main cause is, expectedly, faulty clock times from a clock which gains or loses time or which is incorrect by a constant time interval due to a lack of checking. It may also be caused by shrinkage of the chart paper along the time axis producing a change of scale, or by eccentricity of the chart drum which results in a variation of the time scale with each revolution of the drum.

(ii) Height errors of the gauge occur when the level of the water surface in the well is not correctly recorded on the chart and may arise if

- (a) the height recorder is incorrectly set when the chart is changed on the drum;
- (b) the chart paper distorts in the direction of the height axis;
- (c) there is friction or slippage in the height recording mechanism;
- (d) sudden changes in the level of the gauge occur. For example, gauge support structures are known to have been struck by ships; or
- (e) there are slow changes in the level of the gauge, due to the support structure sinking into the ocean bed.

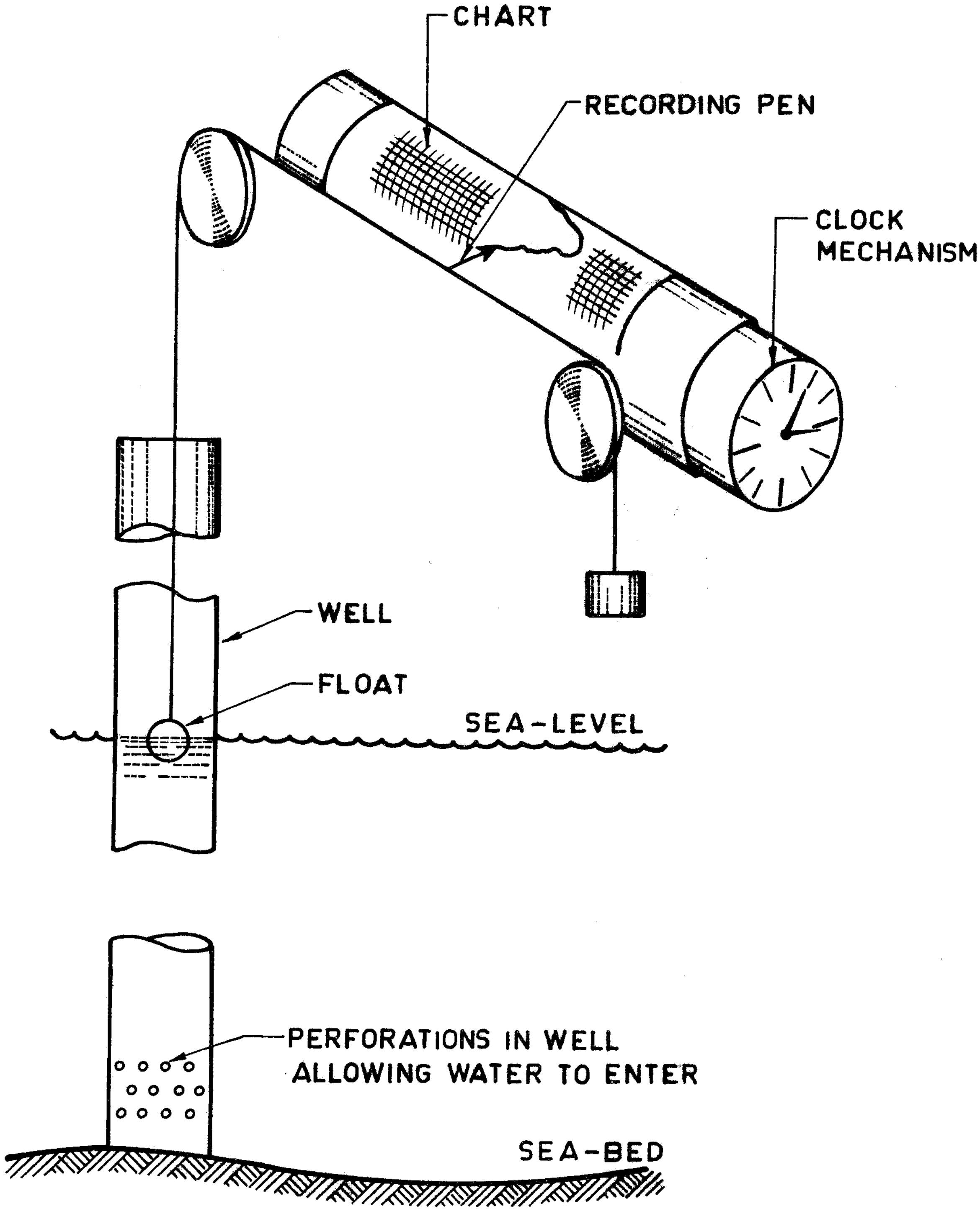


FIG. 7·1
DIAGRAMMATIC REPRESENTATION OF TYPICAL AUTOMATICALLY RECORDING TIDE-GAUGE.

(iii) Failure of the level of the float to correspond to the level of the open-sea-surface. This may occur if the float is holed or is overgrown with marine life so that its density changes and it floats at an incorrect level, or it may happen if the water level around the gauge is affected by the flow of river water.

Further details of errors from tide-gauges are given by *Gordon* (1960).

Most of the abovementioned faults are difficult to discover or quantify, even by inspection of each gauge site. Although such visits were not possible for this project, A.K. Easton (of the Horace Lamb Centre for Oceanographical Research, South Australia), has visited many of the 31 gauges in the Australian survey and has studied their operation and records, (*Easton*, 1967a; 1967b; 1968; 1970; and *Easton and Radok*, 1968; 1970a), thereby providing useful information on the possible existence of gauging faults.

Easton's surveys showed that the conditions and operating methods of many gauges left much to be desired, as is evidenced by the numerous gaps in records, resulting from gauge damage, loss of records from 33 Australian gauges for the two years 1966 and 1967, *Easton and Radok* (1968) note that,

> "Altogether 12 stations worked for more than 700 days, 25 for more than 600 days, 31 for more than 500 days ..... "

with another two gauges providing records for 326 and 280 days only. Time and height checks were classified as *good, satisfactory* or *unsatisfactory*. Some gauges are situated in rivers, others are at river mouths while some are a sufficient distance from rivers for river flow to have a negligible effect on recorded sea levels.

Although the height and time checks were unsatisfactory for some purposes, the effect on MMSL's should not be significant, as may be shown below:

(i) **Height errors.** Suppose that MMSL is calculated from a month of hourly readings, and that the gauge is checked n times per month. The error in the MMSL from height check errors will have a standard deviation of

$$\sigma_{MMSL} = \sigma / \sqrt{n}$$

where σ is the standard deviation of the height checking. If checks are made every 3 days say, so that, for a month,

$$n = 10$$

and if

$$\sigma = 10 \text{ cm},$$

then

$$\sigma_{MMSL} \doteq 10 / \sqrt{10}$$
$$\doteq 3 \text{ cm}$$

Thus, a height check error of about 10 cm standard deviation will produce an MMSL standard deviation of about 3 cm. It appears from Easton's reports that checks would rarely be worse than 10 cm, and it may be assumed that, provided checks are made every few days, MMSL's will be affected to only several centimetres.

(ii) **Time errors.** A *constant* time error will not affect an MMSL, which requires height readings at equally spaced time intervals, independent of the local time. Therefore, variable time errors only need to be considered. Such errors occur when the resetting of the chart is deficient.

> Suppose the tide is semi-diurnal with a range of 6 m, so that the tide height varies 6 m in approximately six hours, or 8 cm in five minutes. Using the theory of height errors, a clock error with a standard deviation of five minutes of time has a similar effect to a height check with 10 cm standard deviation, producing an MMSL with a standard deviation from this cause of a few centimetres.

Easton and Radok (1968) delineate the causes of gaps in records for 1966-1967 due, apart from clock and pen failure, to inlet blockages, slippage of float wires, sticking of tide rolls, holes in the floats and accidential damage to the gauge installations, as well as gaps due to records being lost. The gauges and the times for which gaps occured are listed, (*ibid*). Gauge errors due to these causes may therefore be assumed to have been studied by *Easton and Radok* for most gauges. A gap in the record is indicative of these effects.

Sinking of the tide-gauge support structure, resulting in a long-period change in the gauge-zero level, can only be determined by repeated levelling connections to the gauge from a reliable bench mark, unless the change is so marked as to be noticeable by eye. Gauges situated on wharves (such as Fremantle and Geraldton) could not be expected to be as susceptible to sinking as gauges on jetties, especially at jetty ends. The sinking of gauges which have their own piling structures, such as Centre Island and Port Fairy, would be very difficult to determine. No mention is made by *Easton* of this effect being noticed, and it must be assumed to be imperceptible. As there was no alternative, it could only be assumed that sinking of gauge supports was non-existent.

River water flow can be a serious problem in the calculation of MMSL. Quantification of the influence on gauge records is virtually impossible. Furthermore, the alteration of sea level is always of the same sign, and is not accounted for in a monthly mean. To provide a warning of river-water effects at certain tide-gauges, the siting of the tide-gauges relative to nearby rivers has been sought, particularly from descriptions and aerial photographs provided by *Easton* (1968). Mention is made of river positions, where relevant, in the lists below:

Thevenard (4). Sticking of the mechanism in December 1967 is displayed by gaps in the records.

Albany (7). Records distorted by partial blockage of well during 1967-1968.

Bunbury (8). The river mouth is approximately 2 km from the gauge site. More seriously, trains travelling along the pier on which the gauge is situated disturb the gauge, having caused occasional time errors of six hours. As *Easton* (1967a) infers, records from Bunbury may be unreliable.

Fremantle (9). Definitely affected by flow of the Swan River, in which the gauge is situated.

Carnarvon (11). The gauge is situated next to the river mouth. Good records have not been produced from this gauge because of blockage of inlets and the tide well's becoming detached. Instability of the structure is evidenced by results from levellings to the recorder platform, reported in *Easton* (1968, p.128).

Port Hedland (12). Gauge sited on the river, about a kilometre from the mouth.

Broome (13). Height errors up to about 30 cm have resulted from an improper method used to adjust the gauge for height.

Wyndham (14). The site was not visited by Easton.

Darwin (15). The gauge is situated on a river estuary, but *Easton* (*ibid*) remarks that

> "nevertheless, river discharges have little effect on tidal variations".

The effect of such discharges on sea level is not indicated.

Karumba (18). No information is available.

Weipa (19). Gauge is situated in the river, about 8 km upstream.

Bamaga (20). Gauge records are prone to shocks from work on the wharf. Loading and unloading of ships has caused jumps of the height trace of the order of 6 cm. However, this should not affect MSL interpretations of hourly readings whilst an occasional error would have little effect on an MMSL calculated from a month of hourly readings.

Cooktown (21). The gauge is situated on the mouth of the Endeavour River.

Cairns (22). Gauged sited at river mouth.

Townsville (23). *Easton* (*ibid*, p.213) stated that

> "gauge level is influenced by outflow from Ross River, part of which flows into Ross Creek".

Mackay (24) Bundaberg (25) Brisbane (26)	The gauges were not visited by Easton and their relationships to rivers is not known.

Coffs Harbour (27). Gauge sometimes set one unit (a foot) in error, so results are open to doubt. The gauge was not visited by Easton and river influences are unknown.

Camp Cove (28) Port Kembla (29) Eden (30)	Gauge site and river effects are unknown.

Port Fairy (32). The gauge is situated next to a river mouth and is possibly affected by freshwater flow.

Although the 31 gauges under study suffer from various faults and errors, there is little quantitative information on the effects on sea level records. With a visit to the gauges precluded, the only solution is to study the works of *Easton* and *Easton and Radok*, from which the following conclusions can be drawn:

(i) Records from the following gauges are less reliable than others for reasons apart from river-water effects.

Albany (7)
Bunbury (8)
Carnarvon (11)
Broome (13)
Coffs Harbour (27)

(ii) River water effects may be expected as follows -

Fremantle (9) Townsville (23)	Definitely affected	
Bunbury (8) Carnarvon (11) Port Hedland (12) Darwin (15)	Weipa (19) Cooktown (21) Cairns (22) Port Fairy (32)	Possibly affected

(iii) While no information is available for Karumba, the records only, and not the gauge itself, have been studied by Easton and Radok for

Wyndham (14)
Mackay (24)
Bundaberg (25)
Brisbane (26)
Coffs Harbour (27)
Camp Cove (28)
Port Kembla (29)
Eden (30)

7.5 Tides with a period longer than a month

In Chapter 4, the effect of tides with periods of a month or less on a monthly mean sea level, was shown to be negligible. Consideration must be given to the tides which are not reduced by this filtering process, that is, tides which have a period longer than a month. In this category, the only tides with an amplitude of more than a millimetre or two are the solar semi-annual (Ssa) tide, the solar annual (Sa), the nodal tide and the pole tide. Their periods are 182.6211 days, 365.2422 days, 18.6 years and 428 days respectively.

Strictly, the "pole tide" is not a tide, as it is not produced by the gravitational attraction of the Sun or Moon. As its *characteristics* are the same as a tide, it may be included in this discussion. The pole tide occurs as a result of the wobble or Chandler nutation of about 0.1 seconds of arc, of the earth's rotational axis. Because of the aspherical shape of the earth, the libration produces apparent variations in the depth of the oceans which tend to remain in a fixed position in space (*Haubrich and Munk*, 1959).

The theoretical magnitude of the pole tide is approximately 5 mm. An analysis for the power spectra of 11 European tide-gauges from 10,000 MMSL's by *Haubrich and Munk* (*ibid*) showed the existence of the pole tide at four of the stations only, where the amplitudes were 10.8, 8.7, 9.4 and 4.8 mm.

According to the equilibrium theory, the magnitude of the nodal tide is given by

$$H = 18.5\ (\sin^2 \phi - {}^1/3)\ \cos N \text{ mm} \qquad \text{.....(7.1)}$$

(*Rossiter*, 1962), where N is the mean longitude of the moon's ascending mode, which decreases from 0.3 degrees by 19.33 degrees per year since January 1, 1932. The ϕ is the latitude of the station at which H is determined. There has been argument as to whether the nodal tide should follow this equilibrium law, especially by *Proudman* (1960), *Doodson* (1960) and *Rossiter* (1960b; 1960c; 1962; 1967). However, *Rossiter* (1967) has analysed the records from a number of European tide-gauges for the phase and amplitude of the nodal tide, and obtained magnitudes between one and 32 mm, the average being about 4 mm. He concludes that although there is evidence that the nodal tide follows the equilibrium law, further study is required to ultimately answer the question.

According to *Rossiter* (1962), equilibrium theory gives the magnitudes of the annual and semi-annual solar tides as

$$4.8\ (^1/3 - \sin^2 \phi)\ \text{mm}$$

and

$$29.4\ (^1/3 - \sin^2 \phi)\ \text{mm} \qquad \ldots\ldots(7.2)$$

Respectively, where ϕ is the latitude. Analysis of records from 49 mainland Australian gauges for the years 1966 to 1968 has been undertaken by *Easton* (1970). The results are apparently affected by other sea level variations of similar period as, although the annual and semi-annual tides are

"generally agreed to have negligible amplitude"

(*Hamon*, 1958, p.188), *Easton's* (*ibid*) analyses show amplitudes of between 1.43 cm and 8.17 cm for the semi-annual tide, and 2.77 and *29.35* cm for the annual tide. That there are large variations in sea level due to causes other than tides has been shown in preceding Chapters. It is generally recognised that the main causes of variations in MMSL's are of meteorological and oceanographic origin (e.g. *Hamon*, 1958, or *Rossiter*, 1962) and it would seem unwise to accept Easton's analyses as being indicative of the Sa and Ssa components.

To account for these four tides by subtracting magnitudes derived from gauge record analysis was virtually impossible. It can be seen from the European analyses by *Rossiter* (1967) and *Haubrich and Munk* (1959) that an analysis from one gauge is not necessarily applicable to another gauge, for the nodal and pole tides. The analyses of *Easton* (1970) are apparently influenced by other variations so it was concluded that there were no existing analyses which could be applied to the Australian gauges. To undertake analysis for the pole and nodal tides at Australian gauges was also impossible as the gauges were mostly operative for only a few years, whilst proper analysis for annual tide components necessitates the removal of other annual sea level variations from the data.

Subtraction of the equilibrium form of the tide was possible, but, because of the small magnitudes of the theoretical tide, it was not necessarily expedient to do so. Tables 7.2 and 7.3 show the amplitudes of semi-annual, annual and nodal tides for some Australian latitudes. Formulae 7.1 and 7.2 were used.

Thus, if at any time, at a latitude of 10°, the Ssa tide has an amplitude of +0.88 cm and the annual +0.14 cm, the resulting slope of the sea-surface is 1.30 cm over 30 degrees of arc along a meridian. The maximum effect on any MMSL is of the order of 0.9 cm. As it is possible to account for the annual and semi-annual tides by taking means of an integral number of years of MMSL's, these tides would not affect the final determination of an equipotential surface.

As stated earlier, the magnitude of the pole tide is theoretically about 0.5 cm.

As the equilibrium forms of the tide are small and are not reliable, subtraction of these tides was not considered suitable for these purposes. Further, the phases of the equilibrium tides are possibly in error as much as the magnitudes.

In an attempt to see whether the analyses of Easton could be used to estimate the magnitude of the annual and semi-annual tides, (especially the latter, which is less likely than the former to be overshadowed by meteorological or oceanographical variations of the same period), the following comparisons were made.

(i) Comparison of the analysed magnitude of the semi-annual tide with the analysed magnitudes of the solar semi-diurnal and lunar fortnightly tides for a number of gauges to determine whether the amplitude of any gauge was dependent on the characteristics of the tide gauge site. If this

Table 7.2

Magnitudes of the Equilibrium Form of the Annual and Semi-annual Tides

LATITUDE (degrees)	Ssa (cm)	Sa (cm)
10	0.88	0.14
20	0.62	0.10
30	0.24	0.04
40	-0.24	-0.04

Table 7.3

Magnitudes of the Equilibrium Form of the Nodal Time (cm)

LATITUDE (degrees)	JAN. 1966	JAN. 1967	JAN. 1968	JAN. 1969
10	-0.26	-0.41	-0.50	-0.55
20	-0.18	-0.28	-0.35	-0.39
30	-0.07	-0.11	-0.13	-0.15
40	+0.07	+0.11	+0.13	+0.15

was so, a large S_2 or Mf amplitude may occur with a large Ssa amplitude. The results of the comparison from thirty-six Australian gauges are shown in figures 7.2a and 7.2b. The Sa tide was also included; figure 7.2c.

(ii) Comparison of the annual and semi-annual tides with station latitude, for 34 gauges, figure 7.3.

(iii) Relationship between the two tides and $(\sin^2 \phi - 1/3)$ where ϕ is the gauge latitude, in order to study correlation between the equilibrium form of the tide and the observed amplitude at 34 gauges. Figure 7.4.

No relationships between the amplitude of the semi-annual tide and the amplitudes of the solar diurnal, lunar fortnightly and annual tide are apparent. Although correlation between the annual tide and latitude is apparent, this may be due to the meteorological - oceanographic annual variations, as the relationship is much less pronounced with the semi-annual variation. There is no exact relationship between the semi-annual tide and the factor $(1/3 - \sin^2 \phi)$, whilst the annual tide shows a more distinct trend which may be due to the abovementioned latitude dependence of meteorological - oceanographic factors.

It seemed necessary to accept the effect of the long-period tides, especially the annual and semi-annual tides as being unknown. The pole and nodal tides do appear to have negligible amplitude, but consideration must be given to the other two tides in any future study. For example, any analysis of the MMSL's should include the Sa and Ssa tides as unknowns.

The *position* effects of the Sa and Ssa tides are eliminated by taking observations at gauges over the full five years 1966-1970.

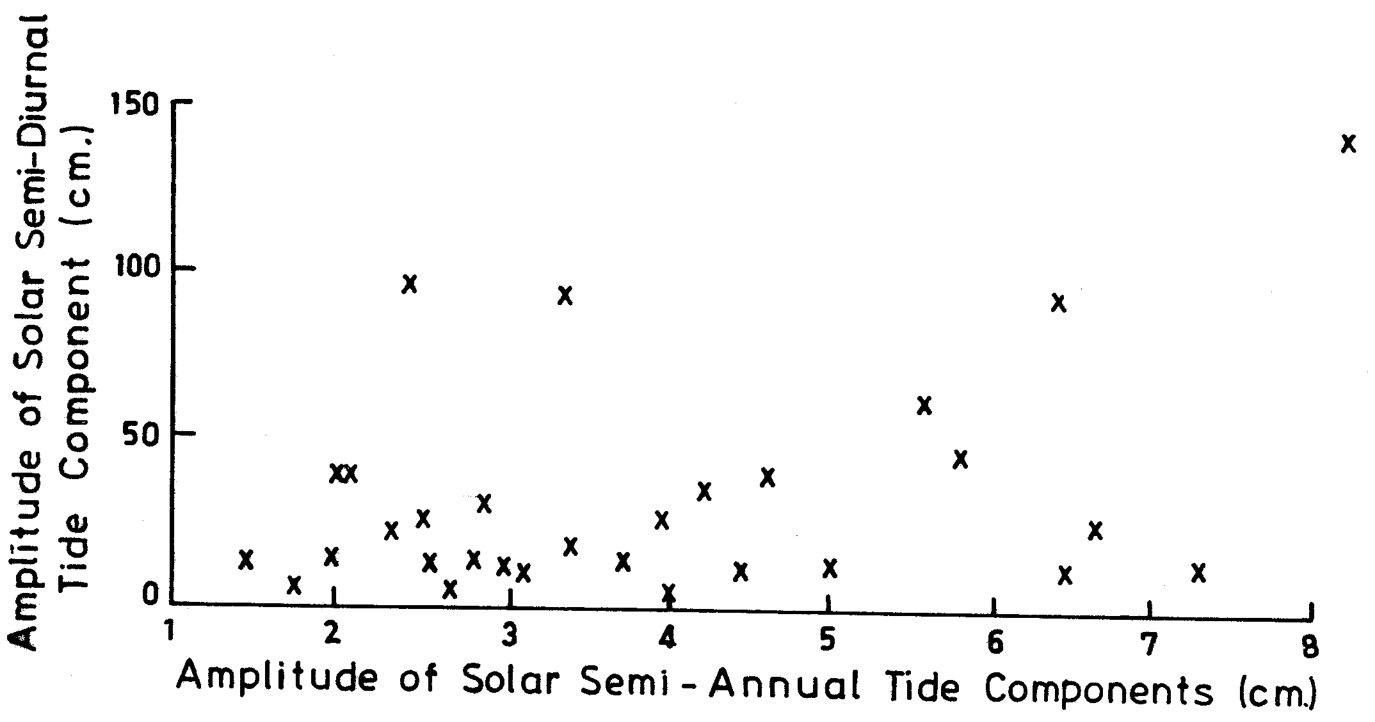


FIG. 7·2 a

RELATIONSHIP BETWEEN AMPLITUDES OF SEMI-DIURNAL AND SEMI-ANNUAL TIDES.

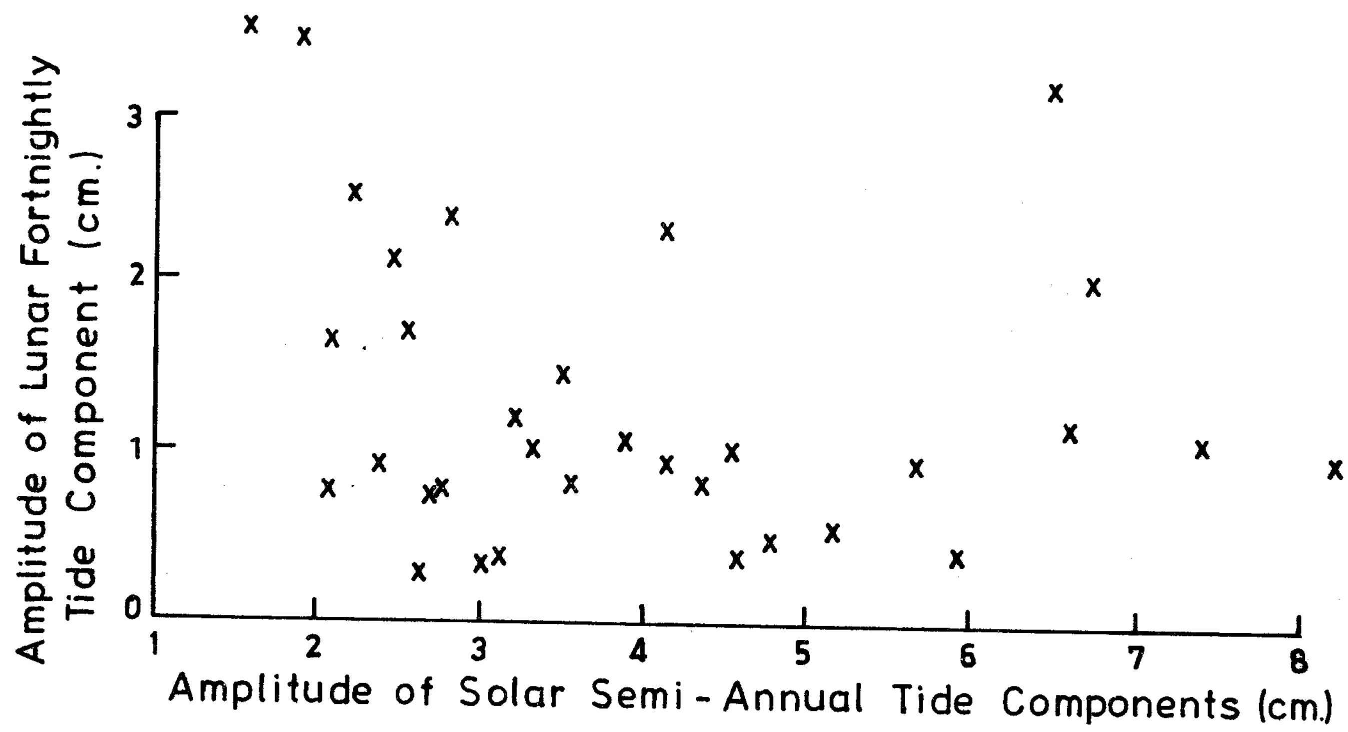


FIG. 7·2 b
RELATIONSHIP BETWEEN AMPLITUDES OF
FORTNIGHTLY AND SEMI-ANNUAL TIDES.

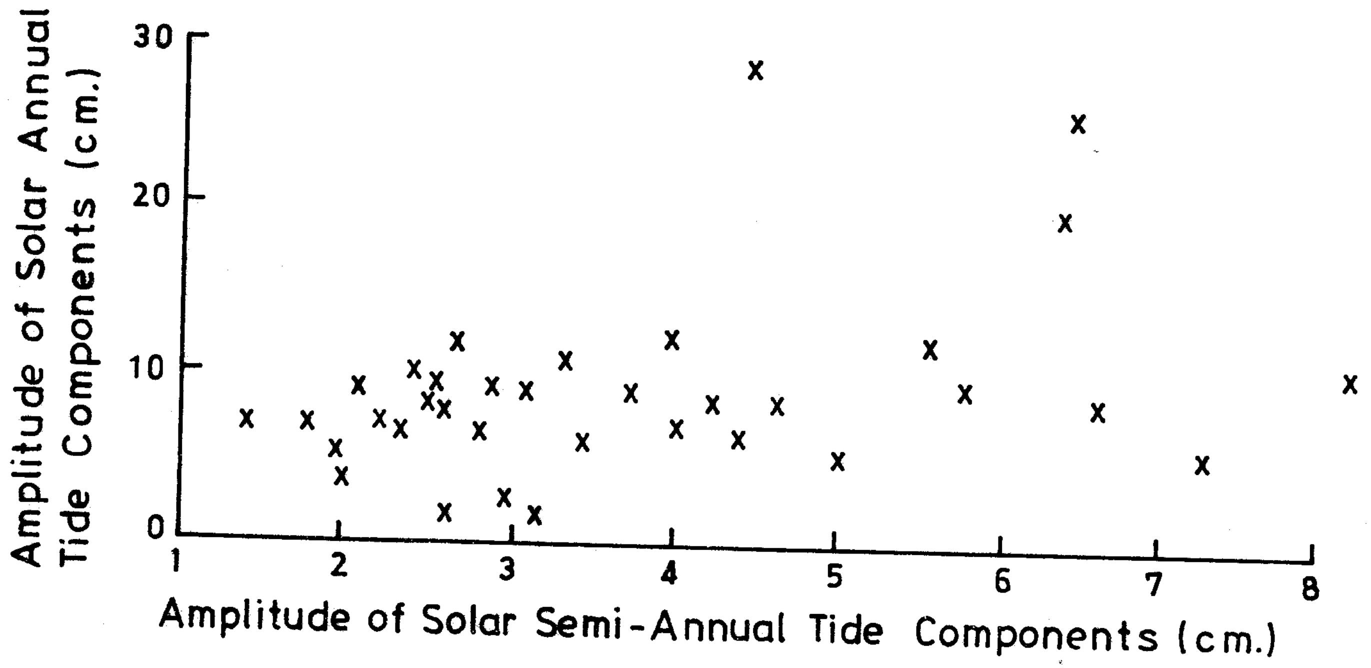


FIG. 7·2c
RELATIONSHIP BETWEEN AMPLITUDES OF ANNUAL AND SEMI-ANNUAL TIDES.

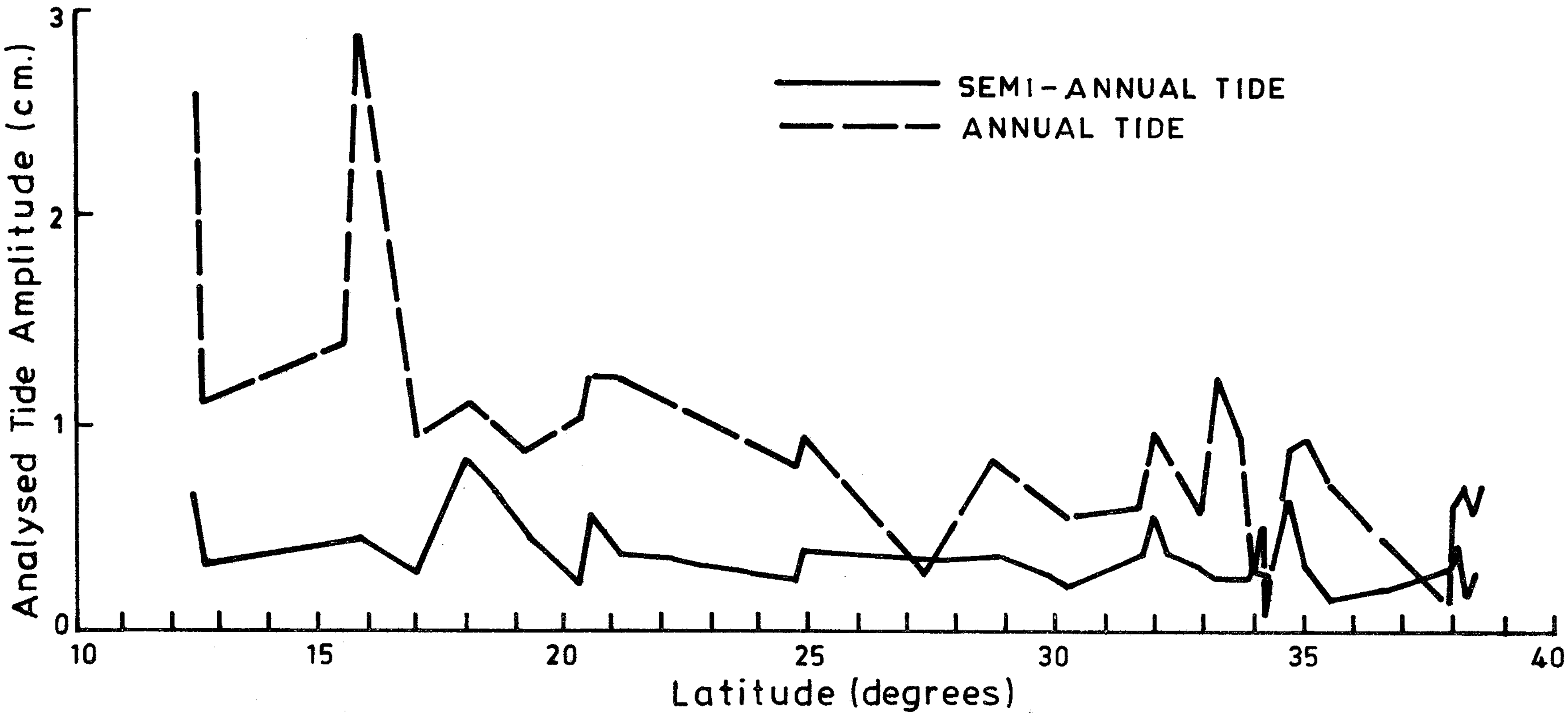


FIG. 7·3
RELATIONSHIP BETWEEN ANALYSED AMPLITUDE OF ANNUAL AND SEMI-ANNUAL TIDES AND THE LATITUDE OF TIDE-GAUGE SITE.

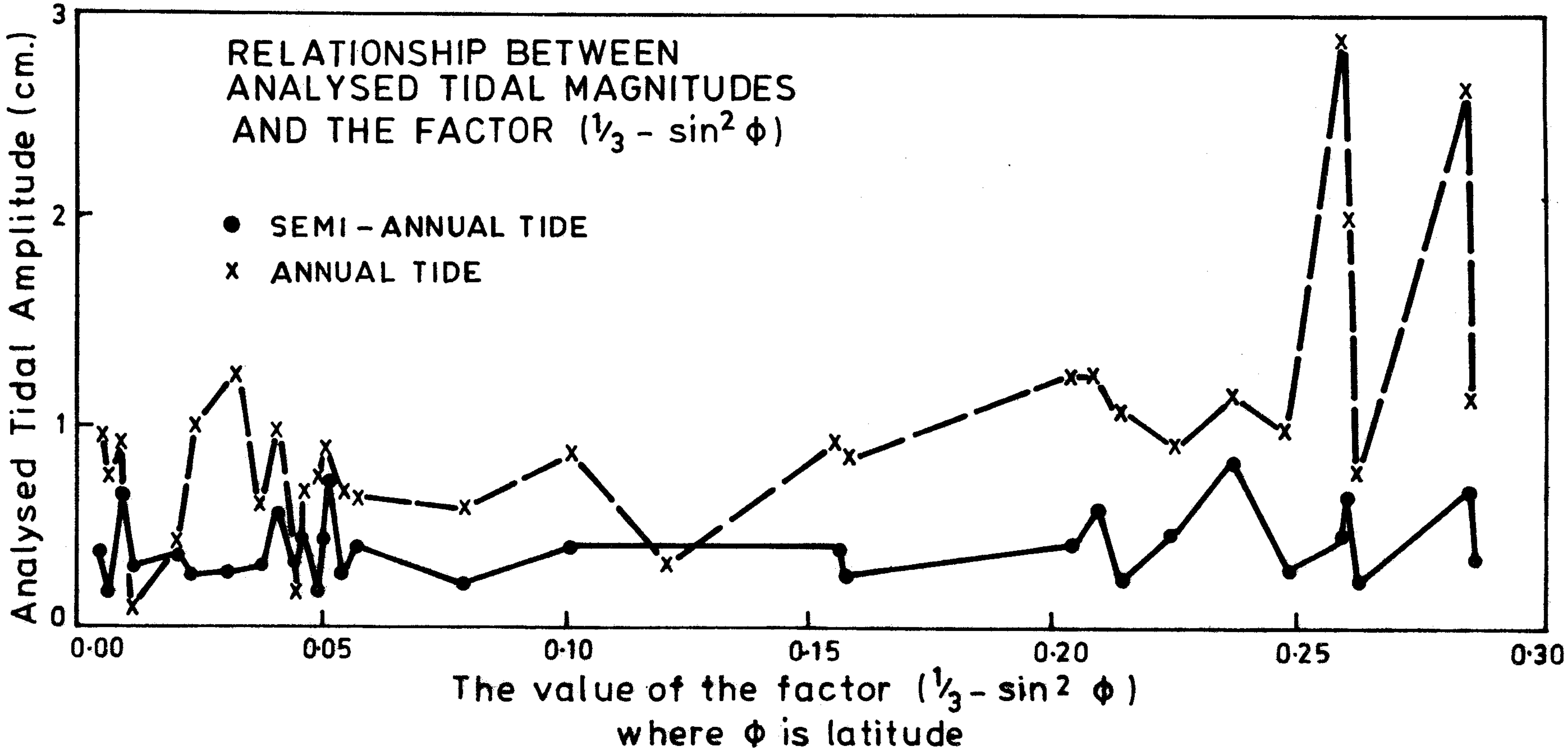

RELATIONSHIP BETWEEN
ANALYSED TIDAL MAGNITUDES
AND THE FACTOR (1/3 - sin² φ)
● SEMI - ANNUAL TIDE
x ANNUAL TIDE
Analysed Tidal Amplitude (cm.)
0
1
2
3
0·00
0·05
0·10
0·15
0·20
0·25
0·30
The value of the factor (1/3 - sin² φ)
where φ is latitude

FIG. 7·4

7.6 Conclusions

The influences of tsunamis, of the secular variation and of variations in the mass of water in the oceans can satisfactorily be assumed to be negligible for this particular study.

The existence of gauging faults must be noted, the effects of river water appearing to be the most serious gauge error. This freshwater flow may detrimentally effect monthly means, and be a hindrance to studies of sea level. However, river flow at individual gauges should not produce deviations of five-year MSL's from the geoid as a regular function of position on the coastline as shown in Figures 1.2, 3.2, 6.2 or 6.7.

Tides with a period longer than a month seem to be little more than a nuisance. Again, the ultimate effect on a MSL calculated over five years can be assumed to be negligible, so that they are considered to have no real significance to this study of MSL-geoid relationships.

8. THE USE OF SATELLITE ALTIMETRY TO STUDY MEAN SEA LEVEL AROUND AUSTRALIA

8.1 Principles of Satellite Altimetry

The deviation of the Mean Sea Level Surface from the geoid which was originally described in Chapter 1, still remains largely unexplained. However, recent developments in the use of a satellite altimeter may provide a method by which the relationship between MSL and the geoid may be determined independently of levelling observations and of tide-gauge records. The successful, accurate operation of the method would provide evidence on the existence of the MSL-geoid separations which have been studied in preceding Chapters.

The method is based on an artificial earth satellite carrying an altimeter which is capable of measuring the elevation of the satellite above the sea-surface. If the satellite height is related to the geoid, the sea-surface may, in turn, be related to the geoid. The principles and practice of the method are outlined in this Chapter, and the feasibility of its application to the Australian problem is developed. Emphasis is placed on the intended first satellite to use satellite altimetry, GEOS-C.

In mid-1974, the U.S. National Aeronautics and Space Administration expects to launch the GEOS-C satellite, the third in the GEOS (Geodetic Earth Orbiting Satellite) series; but the first to carry a specially designed radar altimeter, with which it is proposed to measure the distance between the satellite and an area of ocean density below the satellite. GEOS-C is expected to be only the first in a series of altimeter-bearing satellites to be used in earth and ocean studies: earth physics, oceanography and geodesy. Although the first altimetering satellite has yet to be launched, the use of satellite altimetry has received a lot of attention, particularly in the U.S.

Geoid studies utilizing altimetry are based on the assumption that, to a suitable accuracy, either sea level coincides with the geoid or corrections for deviations between the surfaces are available. If the satellite altitude is related to a reference ellipsoid, and if the altimeter provides the distance between the satellite and sea level, the geoid (assumed to be relatable to sea level) can be fixed with respect to the spheroid.

Oceanographic studies may be aimed at studying time variations of sea level (resulting from tides, or ocean current changes for example) or at relations between sea level and the geoid. *Sturges* (1971) describes a number of areas of oceanographic interest, on both large and small scales, which may have application for satellite altimetry. This includes the variable positions of ocean eddies and currents and the slope of sea level along the U.S. coasts. *Siry* (1971) suggests a study of Gulf Stream variations, long-period tides and geoid-MSL separations.

Satellite altimetry also has application in meteorological studies of atmospheric conditions and in oceanographic studies of sea-surface conditions.

An altimeter-satellite has, then, been already suggested as a means for studying MSL-geoid relationships similar to the problem in Australia. The method may have even greater application in Australia than elsewhere, because, as described in Chapter 1, the MSL-geoid deviations are suspected to be largest in Australia.

Because GEOS-C is expected to be the first in a series of altimetering satellites, testing of principles and equipment in altimetry will form a significant part of the GEOS-C experiment. GEOS-C will also be used for studies unrelated to its altimeter, such as dynamic satellite studies of the geopotential field.

The application of satellite altimetry to the problem of the relationship between MSL and levelling around Australia is based on the comparison of the MSL-geoid separation determined by satellite altimetry with the separation obtained by the levelling and tide-gauge study. It is important to recognise that any comparison will only be to the accuracies possible from the system. Thus, if the satellite altimetry and levelling tide-gauge results agree, it would seem that the levelling results are reliable, whereas, if the altimetry indicates a coincidence between sea level and the geoid then errors in the levelling would seem likely. It is also possible that neither of these results may be found.

The principles of determining the MSL-geoid separation by an altimetering satellite may now be examined in further detail. Figure 8.1 represents a number of surfaces in the plane of an orbit of the satellite at an instant of the satellite's altimetering over the sea.

The instantaneous sea-surface refers to the position of the sea-surface at the instant when the satellite is in the position S.

Two reference spheroids, surfaces 5 and 6, are included to indicate that there are many spheroids, of varying dimensions and positions, to which reference may be taken.

In the diagram,

a'	is the altimetered distance between the satellite and the instantaneous sea-surface
a	is the distance S to MSL
b	is the correction for sea-surface effects to form MSL from instantaneous MSL
c	is the *required* MSL-geoid separation
N_i	are the geoid-spheroid separation distances, and h_{S_i} the spheroid height of the satellite

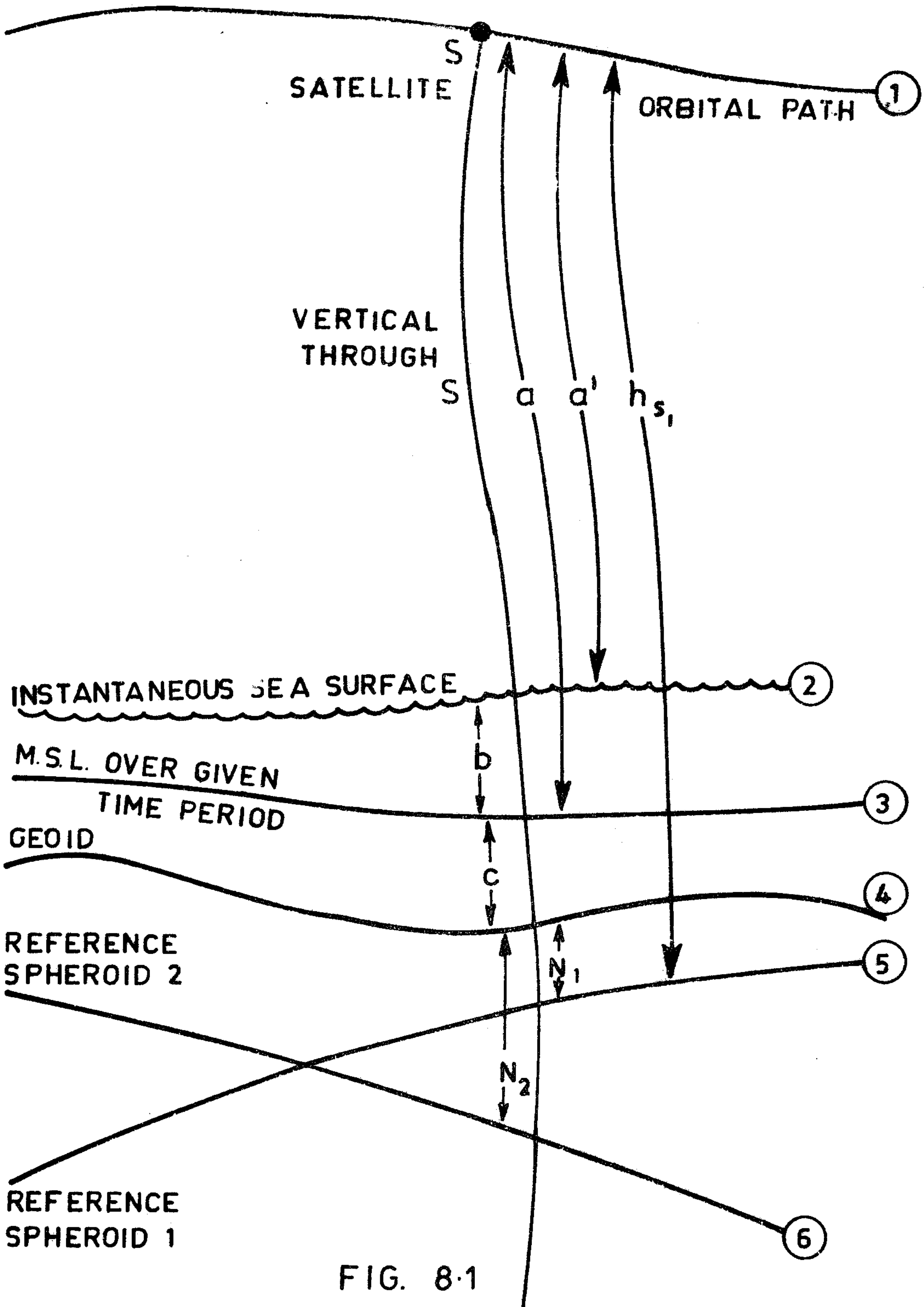


FIG. 8·1

PRINCIPLES OF SATELLITE ALTIMETRY :—
SEE TEXT FOR EXPLANATION.

To manipulate these figures, the following sign conventions will be adopted

(i) S is always above the sea-surface so a' and a are positive in the figure.

(ii) Assume b is positive as shown. Thus, if the instantaneous sea level is above MSL, a subtractive correction must be applied to the instantaneous value. a = a' + b.

(iii) Conventionally, N is positive when the geoid is above the spheroid as in the diagram.

(iv) Assume c is positive as shown, i.e. when MSL is above the geoid.

(v) h_s is positive as in the figure.

The required value of c is given by

$$c = h_{s_i} - a - N_i$$

$$= h_{s_i} - a' - b - N_i \qquad \text{.....(8.1)}$$

N_i and h_{s_i} must be referred to the same spheroid; see Section 8.5.

There will be a number of measurements of a and h_s and one of the problems will be the combination of these observations. Mean heights over an area of ocean should be produced. Although limitless varieties of areas of ocean could be adopted, a two degree square was initially selected as a seemingly suitable size and shape. A *square* of ocean seemed convenient for data reduction purposes, if coordinates are to be given in latitude and longitude. An area smaller than a two degree square was considered likely to produce an excessive number of distances for the separation between the geoid and sea level. The Australian coastline is almost 20,000 km long, equivalent to about 170 degrees of arc. Larger squares were less likely to fit the irregularities of the coastline. Ultimately, the two degree square did seem to be a satisfactory choice.

8.2 Altimeter Details

The measurements of the distance between the satellite and the instantaneous sea-surface, a' in Figure 8.1, are considered to be related to the altimeter. Details, especially probable sources of error, are best studied under three headings.

A. Instrumentation: the details of the altimetering device. The instrument which has been designed for use in GEOS-C will be a *radar* device, which has the advantage of all-weather operation. It will operate at 13.6 GHz frequency, and will be capable of being used in two modes. In the *global*, or *synoptic* mode, the radar pulse is intended to be 300 ns. The more accurate *localized*, or *intensive*, mode will use a 25 ns pulse. Because of battery recharging requirements, the operation of the altimeter is restricted to approximately two hours out of eight in the global mode and one hour in eight in the localized mode, that is, about six and three hours per day respectively. In the global mode, altimeter measurements will be taken twice per second and, in the intensive mode, six times per second. The "design altitude measurement errors" are ±2 and ±0.5 m for the global and localized modes respectively. Details of the design and operation of the altimeter are given by *Oakes* (1971).

The satellite will be kept oriented to the local *vertical* to within a degree of arc. Because of the radial propogation of the radar beam and the spherical earth surface, this tilt will not affect measurements. The shortest distance between altimeter and sea-surface will still be measured (*Hudson*, 1971, p.3). *Siry* (1971 , p.7-30) provides an error breakdown for the altimetry, allowing a 2 m error due to spacecraft attitude. As a safe estimate, a 2 m allowance will be made here.

Owing to the divergence of the radar beam from the altimeter, the beam strikes a large area on the surface of the ocean. The radar beamwidth will be 2.7 degrees, and as the altitude of the spacecraft will be about 900 km, the resultant reflecting area on the sea-surface should be of about 40 km diameter, i.e. about 0.4 degrees of arc in diameter. However, *Weiffenbach* (1971) says that the "footprint" will have a diameter in the range 1 to 10 km. Owing to the movement of the satellite, there is also an *along-track* distribution of the sea-surface reflection area, which *Weiffenbach* (*ibid*) says is 10 to 20 km.

B. **Refraction:** the effects of the atmosphere through which the radar beam passes. *Siry* (1971, p.7-30) mentions in the error budget a 20 cm allowance for refraction effects. *Weiffenbach* (1971, p.1-3) discusses the refraction effects on a radar altimeter and notes that the "uncorrected ionospheric range error" would "have a maximum of about 15 cm for daytime observations and about 3 cm at night" and "even a rather crude correction can reduce ionospheric altitude errors to acceptable levels".

C. **Reflection:** of the beam from the sea-surface. A number of papers have been written on the theoretical and practical effects of sea-surface reflection on the measured distance. Conclusions from the test by *Shapiro et al* (1971), who placed a one nanosecond pulse radar altimeter 21 m above the sea-surface, indicated that "reflecting properties of the ocean biases the mean sea level by about 5% of the significant wave height", (*ibid*, p.11-1). Longer pulses as expected for GEOS-C, are apparently less affected by the sea-state. Furthermore, the tests showed that the wave height can be extracted from the altimeter data with a reduction of 6%. Even for a 5 m significant wave height, only a 30 cm error in SWH would result - even if the bias was unknown. The reflecting effect correction should then be possible to

5% of 30 cm, or 1.5 cm. Further testing is considered by the experimenters to be necessary before the results can be applied to SWH greater than 2 m. The effect of increased altitude to satellite levels and of different pulse widths is also unchecked. However, it will be assumed that the reflection error can be reduced to negligible levels, especially if the wave height is determinable from the altimeter data, as this eliminates the necessity for constant sea-state recording.

The measurement of a' is therefore assumed to be affected by a 2 m instrument tilt error, a 20 cm refraction error and a negligible reflection effect.

The standard deviation of a' is assumed then -

$$\sigma^2_{a'} = (200^2 + 20^2)\ \text{cm}^2$$

8.3 GEOS-C Orbit Details

A study of the orbit details is necessitated by the evaluation of much of the statistical theory associated with this study of satellite altimetry. GEOS-C has necessarily been adopted as the subject satellite.

The *inclination* of the orbit of GEOS-C has been constantly revised but the latest estimate (*NASA*, 1972) is 115 degrees, that is, -65 degrees, a retrograde orbit of inclination 65 degrees. In that case, latitudes of points on the earth over which the satellite passes will vary from +65 degrees to -65 degrees. As the latitude extremes of mainland Australia are -11 and -39 degrees, any inclination greater than about 45 degrees is suitable for a sea level study around Australia, and 65 degrees is satisfactory.

The *altitude* planned for this satellite has been estimated as about 800 to 900 km. The "nominal height" should finally be 927 km (*NASA*, 1972). The period of revolution for a satellite at this height is about 105 minutes, giving a velocity across the face of the earth of:

360 degrees per 105 minutes
i.e. 3.4 degrees per minute
or 200 degrees per hour

Weiffenbach (1971) assumes a figure of 240 degrees per hour.

This satellite is expected or intended to orbit for 18 months to two years; calculations in this study will be based on an assumption of a 1.5 year lifetime.

As the earth rotates 360 degrees in 24 hours, the orbits of GEOS-C will be longitudinally separated by about 26 degrees, nodal regression being comparatively insignificant.

If the satellite orbits once every 105 minutes of time over a 1.5 year period, there will be:

1.5 x 365 x 24 x 60/105 orbits
i.e. about 7500 orbits in 1.5 years

The average spacing of equatorial crossings for each orbit will be three minutes of arc, or 1.5 minutes of arc for both north and south bound satellite passages. However, the altimeter does not operate continuously, and this must be taken into account. If the altimeter observes twice per second and if it operates for 25% of the time over 1.5 years, there will be 2.5×10^7 data points, assuming, that is, that the satellite operates for 3300 hours. *Oakes* (1971, p.18-3) says that the altimeter is designed for 1500 hours of operation over 18 months. *Weiffenbach* (1971, p.1-7) says that the altimeter would run for 2100 hours over 18 months if it ran for 3.84 hours per day, that is, for 16% of the day. *Chovitz* (1971) mentions 10^7 data points in a two year lifetime, assuming that the altimeter operates for 20 minutes per revolution, i.e. 20% of the time. *Hudson's* (1971) figure is 5.5×10^5 points in 1500 hours.

These data points will be spread over an area of the earth given by

$$A = 4 \pi R^2 \sin i$$

where R is the radius of the earth and i is the satellite inclination, see Figure 8.2.

On a hemisphere,

$$\Delta A = \text{Circumference} \times R \, \Delta\phi$$
$$= 2 \pi R \cos \phi \, . \, R \, \Delta\phi$$
$$A = \int 2 \pi R^2 \cos \phi \, d\phi$$
$$= \left[2 \pi R^2 \sin \phi \right]_{\phi_1}^{\phi_2}$$

Area from equator to latitude ϕ,

$$A = 4 \pi R^2 \sin \phi$$

For whole sphere,

$$A = 4 \pi R^2$$

For a radius of 6400 km and a satellite inclination of 65°,

$$A = 4 \pi . 6400^2 \sin 65^\circ \text{ km}^2$$
$$= 4.7 \times 10^8 \text{ km}^2$$

Assuming that there are 10^6 observation points, there would be 2×10^{-3} data points per square kilometre, or 25 per square degree of arc. That is, each data point would represent, on the average 500 km^2 or

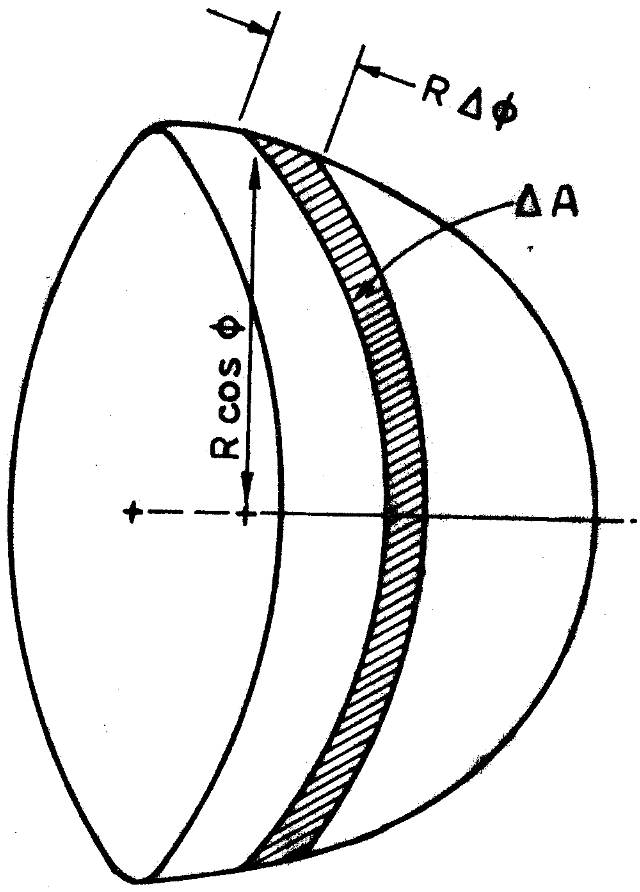


FIG. 8·2

0.04 degree squares. *Weiffenbach's* (1971, pp. 1-7 and 1-9) figures are 19 samples per degree square for a 65 degree inclination. *Chovitz* (1971) assumes 15 points per degree square over two years.

The problem of correlation between observations in the same orbit will be discussed in Section 8.7.

All figures in this section are of order-of-magnitude only, as the data collection will depend on the controlled operation of the altimeter from the ground. It is most likely that the *global* mode of operation will be used over Australia. Although the intensive mode may increase accuracy it cannot be used to produce as many data points.

In summary, a satellite passage separation of 1.5 degrees at the equator will be assumed, producing about 10^6 observations, each degree square being represented be about 20 observations. The global mode will be assumed to be used.

8.4 Determination of Spheroid Height of Satellite

The height of the satellite above the spheriod, h_s, is determined by a combination of orbit prediction and observations from earth onto the satellite. In this section, the determination of h_s, with emphasis on possible accuracies, will be discussed. Firstly, the tracking of the satellite, that is, the observations from earth, will be outlined, followed by a description of the methods of using these observations. Their suitability for application to the Australian problem will also be considered.

Most of the existing ways of tracking a satellite are to be used on GEOS-C. *Weiffenbach* (1971, p. 1-4) says,

> "If all ground stations...are employed, GEOS-C will be the most intensively tracked satellite ever".

Six observation methods will be considered individually.

a) Ground Laser Tracking
Lasers are used to measure the *distance* to the satellite. *Weiffenbach* (*ibid*, p.1-4) says that GEOS-C will be tracked by twelve or more lasers of 0.3 to 1 m accuracies. Laser accuracies are, however, expected to improve to 0.1 to 0.3 m over the next year or two (i.e. 1972-1973) and possibly even to 5 to 10 cm, according to *Smith* (1971, p.10-2). Laser tracking instruments are usually capable of angular measurements of azimuth and elevation to about a minute of arc. Laser trackers can be classed as transportable, although their siting is dependent on weather conditions (laser operation is restricted to clear skies) and the occurrence of air traffic. The sites of a number of laser trackers around the world are given by *Martin* (1971, p.9-7). GEOS-C will be suited for laser observations by carrying the appropriate retroreflector.

b) Radar Observations
GEOS-C will be fitted with two C-band transponders for radar ranging.

c) Doppler Tracking
In this method, range *rate* is observed. GEOS-C will be fitted with two transmitters for Doppler tracking.

d) Satellite to Satellite Tracking (S.S.T.)
The principle of S.S.T. is to use two satellites, the altitude of the first being generally quite high. A higher altitude satellite is purturbed to a lesser extent by the earth's gravity field, than are low altitude satellites. Its orbit can be accurately determined. Further, as low degree harmonics of the gravitational field are well known, it is easier to accurately predict a high orbit than a low orbit. The second satellite is in a comparatively low orbit where its track is disturbed by the gravity field. At a low altitude, it may then be possible to use an altimeter or to study the satellite's perturbations due to the gravity field. The position of the high orbit satellite would be refined by tracking, whilst by measurement between the satellites, the position of the lower may be fixed.

It is planned to place, on GEOS-C, an S-band transponder to enable *range* and *range-rate* measurements in conjunction with the Applications Technology Satellite, ATS-F, which will be in a synchronous orbit and which will be tracked from a number of stations (see *Siry*, 1971, p.7-27) including one in Australia.

Vonbun (1971) discusses the errors arising through S.S.T., and notes (p.6-2) that

"S.S.T., as presently configured should be able to 'detect' satellite height variations in the submeter level",

although this refers to relative height variations.

Siry (1971) also discusses the ATS-F satellite, but his diagram showing GEOS-C height errors against time, when GEOS-C is tracked by S.S.T. and other methods, shows the errors as being never less than 2 m, and averaging 3 to 4 m.

Another satellite planned for S.S.T. usage is the GEOPAUSE to be placed at an altitude of 4.6 times the earth's radius, that is; at about the limit of the earth's gravitational field, the geopause. In its orbit,

"at this height uncertainties in only a few gravitational harmonic terms correspond to orbit perturbation amplitudes above the decimeter level",

(*Siry*, 1971, p.7-6).

In the opinion of *Smith* (1971), S.S.T. is the only way to determine sea level topography to 10 cm, and he expected that range-rate accuracies of 0.3 mm sec^{-1} which were possible in 1971 should be improved to 0.05 mm sec^{-1} by 1973.

e) **Angle Measurements**
with cameras capable of measuring azimuth and altitude angles of the satellite to about a second of arc can also be valuable, (see *Berbert and Loveless*, 1971) although restricted to night-time recording.

f) **Very Long Base Interferometry**
will probably not be applied to GEOS-C, and its accuracy will probably be restricted to at least 10 cm by refraction considerations (*Smith* 1971, p.10-5)

The tracking information can be used to determine the satellite's position in two separate ways, which will be referred to here as the *long-arc* and *short-arc* methods.

The term *long-arc* will be applied to the method of predicting the satellite's position, over the whole orbit or any part of it, according to the known gravitational field of the earth. Tracking data is used, as available, to correct the predicted position in the orbit. It must be noted that, because the satellite position is predicted from knowledge of the whole of the earth's gravity field, its position will be related to a geocentric spheroid. If the gravity field of the earth and all other forces on the satellite, such as atmospheric drag and solar radiation pressure were perfectly known, it would be possible to predict the position of the satellite without error. However, owing to inaccuracies in these factors, the satellite's position cannot be predicted faultlessly, and a watch on the satellite, by tracking from earth, to apply corrections, is necessary.

Errors will arise in this system then, from

(a) inaccurate knowledge of the earth's gravity field in making the predictions.

(b) inaccurate corrections, because of imperfect tracking, due in turn to

 (i) timing errors

 (ii) satellite position measurement errors

 (iii) tracking station coordinate errors which must be related to the *geocentric spheroid.*

(c) errors in atmospheric drag, and solar radiation calculations, etcetera.

An estimate of errors in a satellite's altitude calculated by the long-arc method may be obtained from a study by *Martin* (1971). He simulated the errors in a satellite position, in three directions, which would result from this method of position determination. Martin propogated sources of error, including tracking station coordinate errors, tracking observation errors and geopotential field errors, into all coordinates. The radial coordinate, i.e. altitude, errors were then studied. A number of simulations were undertaken by *Martin* who varied two aspects of the tracking. In one set of studies, the tracking methods were varied, over

(i) eight range measurement devices (i.e. lasers and radars) with world-wide distribution assuming a range bias of 2m and station coordinate uncertainties of 5 m in all three directions.

(ii) twelve Doppler trackers.

(iii) and the combination of the twenty trackers.

Martin's second variation was the number of hours of the satellite's orbit which were tracked to produce the positions of the satellite in one orbit. Times of two, four and six hours were used. Although Martin suggests that his result be considered tentative, the following points from the study can be noted -

(i) the biggest source of error was considered to be gravitational field model errors, so that the errors were not likely to improve with increased tracking.

(ii) the position errors in any orbit were found to be as good when the tracking of one orbit was used as when tracking over more than one orbit was used in the determination.

(iii) errors were least when the satellite was actually being tracked from earth and with good tracking geometry, radial errors of the order of 2 m or less appear feasible, (*ibid*, p.9-16).

There are two tracking stations in Australia used in this study: a range measurer at Carnarvon, and a Doppler tracker at Smithfield.

It should be noted that the ATS-F satellite to be used for satellite to satellite tracking of GEOS-C will have its position on a *geocentric* system, so that the data could be combined with data for the GEOS-C altitude obtained by the ordinary long-arc methods.

It may be concluded that the accuracy of the satellite altitude determination by the long-arc method is very dependent on the knowledge of the gravity field, that at present, accuracies of 2 m seem likely. *Mather* (1973) suggests that the tracking would be enhanced by placing a laser-tracker near Townsville (23). Such a device should ensure that the error in the satellite altitude would be less than the resolution of the altimeter, particularly in the area of the greatest apparent sea level deviation from the geoid, that is, along the Queensland coast, between Bamaga (20) and Brisbane (26). Further conclusions will be drawn in Section 8.7.

In the *short-arc* method, the satellite's position is determined by observation only from stations on earth. Its position is thus usually only determined over the short-arc in which it is continually tracked, say 20° of arc on the earth's surface. As the tracking station coordinates are usually related to a local datum, the satellite's position is generally given on a local datum system from short-arc calculations. Errors in the satellite's position calculation result from errors in the positions of the tracking stations and from observation errors.

The method however has serious drawbacks in the problem of the Australian study. As stated, errors in the satellite position will result from errors in the tracking station coordinates in all three directions. But the height of the tracking station above either the local datum or the geocentric spheroid will involve the height of the physical surface of the earth above some datum as determined by levelling. Any error in the levelling will then be propagated through the position calculations and will affect the calculated satellite height. However, the satellite is being used to check the very levelling network which would be used to determine the tracking station elevation. The tracking station heights should be, therefore, independent of the levelling. In studies suggested by *Siry* (1971) for the Caribbean, the tracking stations are located close to the oceans so that the heights can be obtained by levelling between MSL and the tracking stations. Again, the assumption that MSL coincides with the geoid is to be *avoided*. The only solution to the problem is to use a single tracking station with an *adopted* height above the geoid. The resultant error in height could be assumed to be systematic across the observation area. The orientation of the coordinate system for a single station will then depend heavily on the latitude and longitude values assumed for the station. For a local datum system, the geodetic latitude and longitude are obtained by geodetic connection from the origin station; and will necessarily contain error. Adoption of coordinates at the tracking station will define the orientation of the spheroid to which the coordinates refer. This is an unacceptable procedure as the spheroid must be the same as that to which the geoid-spheroid separation refers. It is not even possible to relate the station to the geocentric spheroid as this would require deflections of the vertical, which, although available, will be subject to error. A one second error in latitude or longitude at the tracking station tilts the local spheroid by a second also. This is equivalent to a tilt of one part in 200,000 or a full metre in 200 m. If it is argued that the heights of a number of tracking stations could be related by high-precision levelling, independent of the levelling network under discussion, then the reply is that if this were possible it would have *been* used to check the levelling. Such a check may have the same systematic errors in the levelling or potential conversion as the original network and it is therefore to be avoided.

If a single tracking station is adopted, note that the calculation of the satellite position from the tracking station position is a *spheroidal* calculation. The satellite's position from the tracking station may be measured by various observing methods, although *range and angle* data is most suitable, i.e. preferably laser ranger and co-located one second camera. Anything less could not be expected to be sufficiently accurate. Even a camera is restricted to observing GEOS-C in the earth's shadow only. The area of coverage by the single-station short-arc method is restricted by the minimum elevation angle of the tracking device, and the decreasing accuracy as the elevation angle is reduced. *Siry* (1971) has reproduced figures on the accuracy of the satellite altitude based on camera/range measurements. The figures indicate that one-second accuracy cameras are virtually imperitive for the study discussed here. His accuracy values are based on 2m for range measurers, 1 m for station heights and a range bias of 2 m. In this study, the range bias and height error will not be effective whilst a range error of much less than 2 m should be possible.

If the tracking device can track to five degree elevation angles, the ground coverage is about 60 degrees and for 20 degrees it is a 16 degree diameter circle.

8.5 Geoid-Spheroid Separation

A review of the various methods which may be used to determine the separation between the geoid and spheroid is given by *Fubara and Mourad* (1971). The only methods to have been applied in Australia are astro-geodetic levelling and the gravimetric (Stokes' integral) method; and only these methods should be assumed to have any application to satellite altimetry over Australia. The methods themselves will not be described here. The significant aspect of the methods to this study is their accuracy and in particular, the accuracy of differences of the geoid-spheroid separation at one point relative to another point.

It is important to notice the different spheroids to which the separations relate. Astrogeodetic levelling relates the geoid to a local datum whilst the gravimetrically determined geoid is related to a geocentric spheroid.

Astrogeoditic levelling in Australia is described by *Mather et al* (1971, p. 1 *et. seq.* and p. 10 *et. seq.*) who include a determination of the geoid over Australia (*ibid*, fig. 3.1). Details of the gravimetric determinations of the geoid over Australia are also described by *Mather et al* (*ibid*, p.1 *et. seq.* and p. 16 *et. seq.*). After the spheroids for both geoids were reduced to a common datum, the subsequent rms residual difference between the two geoids was $\pm$ 1.6 m, (*ibid.*, pp. 23, 24, 33). Such an accuracy needs to be improved for satellite-altimetry studies, which require accuracies of differences to be of the order of 10 cm.

However, it is now possible that a 10 cm solution may be obtained through present improvements in gravimetric solution processes. Astrogeodetic levelling cannot be considered likely to achieve such an accuracy in Australia in the forseeable future.

It is also important to notice that the astrogeodetic geoid solution cannot easily be extended out to sea, thereby limiting again, its application to satellite altimetry. The gravimetric geoid may be determined at sea although dense gravity anomaly ocean coverage is required, and is not necessarily available. A study of either geoids mentioned previously (i.e. *Mather et al*, 1971, fig. 3.1 and *Mather*, 1970, p. 111) indicates that the geoid undulations cannot be extrapolated about two degrees from the coast without errors of at least 10 cm being introduced. It can then be expected that geoid determination one or two degrees from the coast is required for satellite altimetry.

Another problem of the geoid-spheroid separation is the determination of a mean separation over the area of a 2 degree x 2 degree square on the earth's surface. Again, either figure for the geoid indicates that care must be taken when determining this mean. A 10 cm error will possibly be difficult to avoid.

8.6 Sea Level Effects on Altimeter Readings

If the Mean Sea Level derived from the tide-gauge readings over a number of years is to be compared with Mean Sea Level derived from satellite altimetry, it is necessary to ensure that both the gauges' and the altimeter's data provide the same MSL, at least to an acceptable accuracy. Even if the gauge MSL is not used, the accuracy of the altimetry MSL must be estimated. The tide-gauge readings may be taken every hour over one or more years, but the altimeter observations are only obtained when the satellite passes overhead. The subsequent determination of MSL from a limited number of observations of the instantaneous sea-level apparently has received only limited consideration in the GEOS-C project to this point.

Firstly, consider the probable pattern of satellite orbit crossings over any two degree square on the earth, for which Figure 8.3 has been constructed. The inclination angles of these orbit crossings are drawn at 63 degrees to the prime vertical. Not only is this convenient, as

$$\tan 63^{o} = 2$$

but the inclination across Australia will be about 63 degrees as explained using Figure 8.4. On the earth are shown

(i) the equator,

(ii) the projection of the satellite orbit,

(iii) the parallel of latitude, ϕ,

(iv) the meridian passing through the point of intersection of the orbit and the parallel.

The inclination of the orbit at latitude ϕ, shown as x in the diagram, is required. The spheroid triangle is reproduced in the figure. By Napier's Algorithm of Spherical Trigonometry,

$$\cos i = \cos \phi \sin (90-x)$$

$$\cos x = \frac{\cos i}{\cos \phi}$$

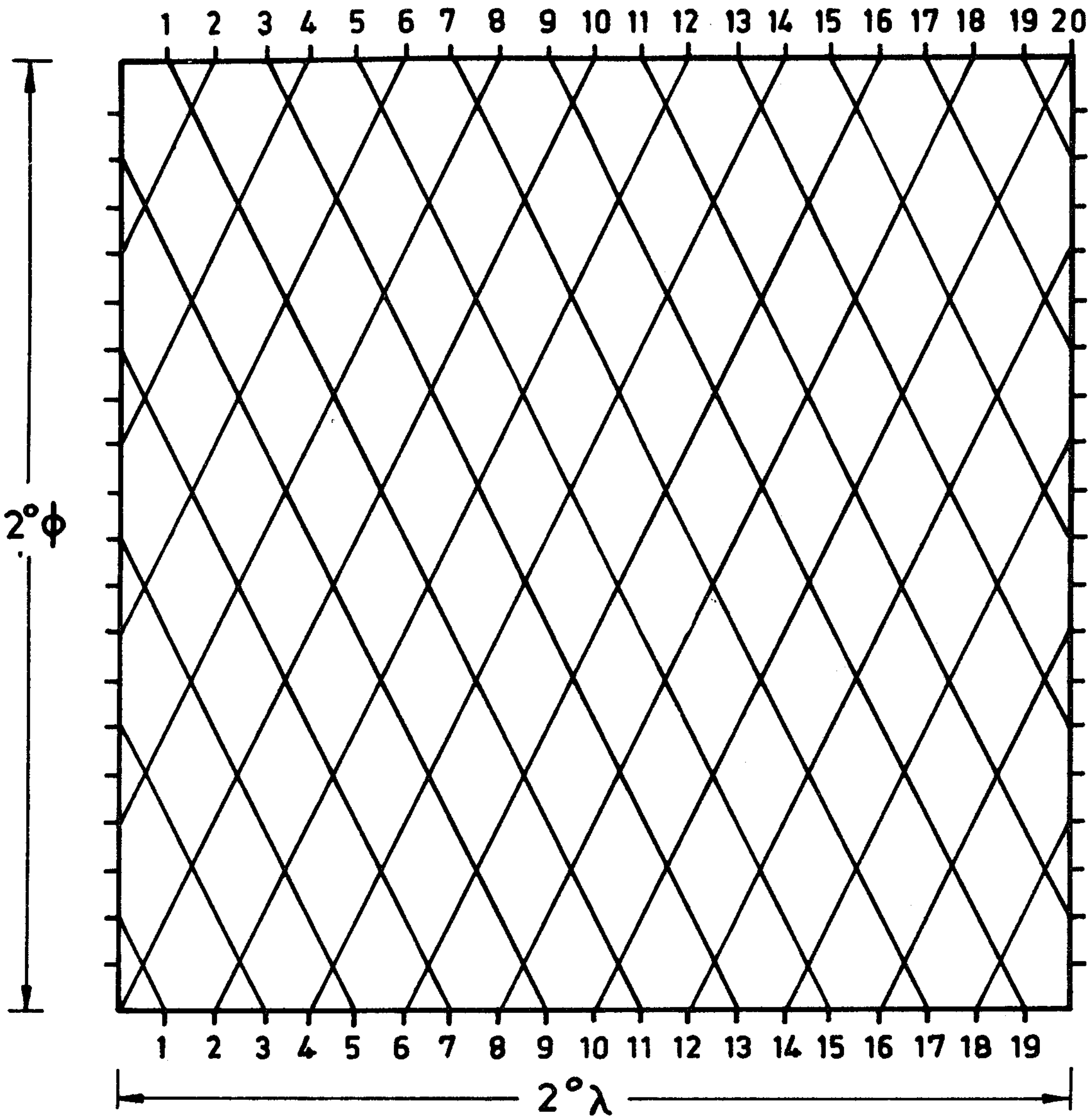
Over Australia, ϕ varies between 10 and 40 degrees; Table 8.1 has been constructed assuming that $i = 65$ degrees. The orbital crossings are spaced at six minutes of arc, according to the results deduced in Section 8.3.

A computer programme was developed to find the error resulting from determining Mean Sea Level from only a limited number of observations.

Table 8.1

Satellite Orbit Inclinations

ϕ (deg.)	cos ϕ	cos x	x (deg.)
10	0.98	0.43	65
20	0.94	0.45	63
30	0.87	0.48	61
40	0.77	0.55	57



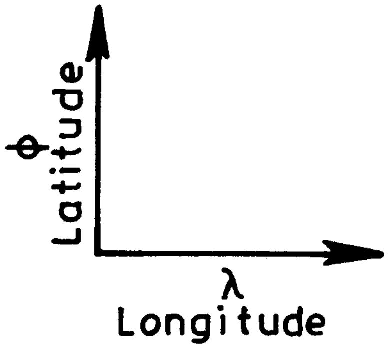


FIG. 8·3

ORBITAL CROSSING OVER
A 2° SQUARE OF OCEAN.

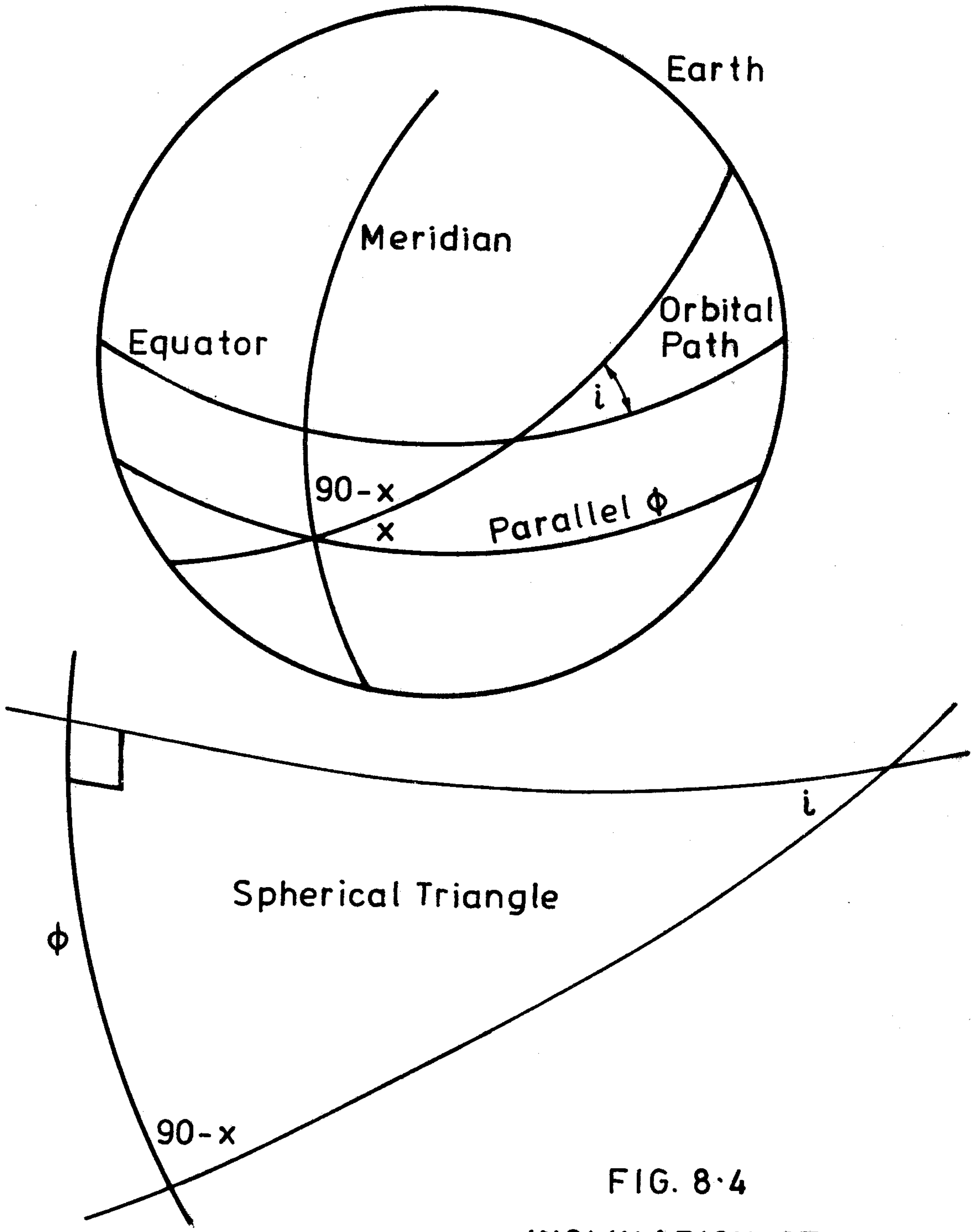


FIG. 8·4

INCLINATION OF SATELLITE ORBITAL PATH

That is, the effect of calculating a mean of a sine curve from only a limited number of samplings of the sin curve was to be estimated. The computer programme is shown in Appendix 3. The calculations involved sampling the curve

$$y = \sin 2 \pi f t$$

where f is the frequency, at *random* points, corresponding to the orbital crossings. The mean, $\bar{y}$, of the y values was calculated a number of times so that the distribution of $\bar{y}$ could be estimated. After testing of the programme to ensure satisfactory working, the calculation of the distribution of the mean was undertaken using various frequencies and multipliers. The results of the calculations are summarized in Table 8.2. The figures show the number of times, out of the number of tests made, that $\bar{y}$ had the value shown. The value of $\bar{y}$ was never greater than 0.5 times the sine curve amplitude. Table 8.2 shows that test numbers 1 to 4, which used the same number of weights, although the weight distribution and the frequencies were different, produced basically the same distribution. When the number of orbits was doubled, test 5, the distribution function gathered closer about the real mean, as could be expected. The results for tests 1 to 4 were combined to produce Figure 8.5, showing the distribution function for $\bar{y}$. The weights shown in Table 8.3 were based on the length of the orbital passages shown in Figure 8.3, on the basis that the longer the travel time, the greater the number of altimeter observations. Figure 8.5 shows that 67% of the $\bar{y}$ lie between 0.00 and 0.13 times the sine curve amplitude. The standard deviation for a weight calculated from 29 orbital crossings was then adopted as being 0.13 times the sine curve amplitude.

The various known causes of time and position variations in sea level were discussed in Chapter 1, as well as in Chapters 4, 5, 6 and 7. Their significance to satellite altimetry and the error due to limited sampling are discussed below.

Table 8.2

Results of calculations by computer programme

Test Number		1	2	3	4	5
Weights used		29 weights, see Table 8.3		29 weights 1,2 29	29 equal weights	58 equal weights
Frequency used		2.0	6.4		9.8	2.4
Number of tests		300				243
	0.00	85	82	78	94	102
Results	0.05	78	76	66	74	79
	0.10	55	61	63	55	37
	0.15	36	36	39	41	21
	0.20	25	23	30	21	2
	0.30	5	7	5	2	0
	0.35	0	4	4	2	0
	0.40	0	0	1	0	0
	0.45	0	0	0	0	0
	0.50	1	0	0	0	0
	0.55	0	0	0	0	0

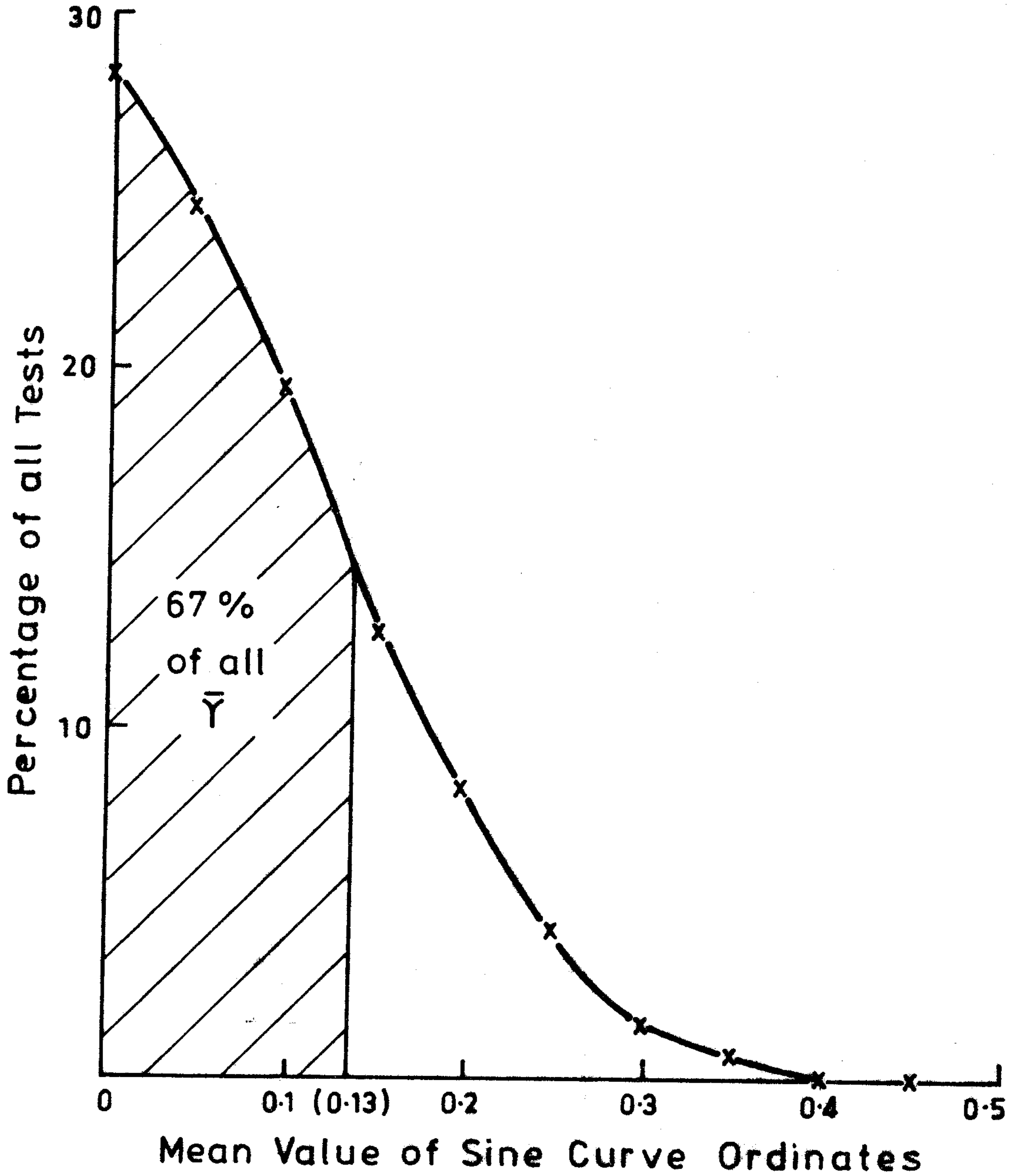


FIG. 8·5
DISTRIBUTION OF MEAN VALUE OF SINE CURVE FROM LIMITED SAMPLING.
AMPLITUDE OF SINE CURVE ASSUMED TO BE UNITY.

Table 8.3

Orbital weights derived from figure 8.3

Orbit Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Weight	56	56	56	56	56	56	45	45	34	34	22	22	11	11

Orbit Number	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
Weight	6	6	17	17	28	28	39	39	50	50	56	56	56	56	56

1. Gravity Waves

have been discussed in Section 8.2, where it was decided that the gravity waves would not be effective if, firstly, the significant wave height could be determined to within 6% from altimeter records and if, secondly, the resultant 5% altimeter error exists for altimeter heights of 900 km and for significant wave heights greater than 2 m. *Weiffenbach* (1971, p.1-6) does suggest, however, that significant wave heights of all oceans are about 1.5 m.

2. Tides with a period less than a month

Specifically, emphasis is placed on the diurnal and semi-diurnal tides. Around Australia, these may have amplitudes of up to about 3 m. According to the limited altimeter sampling effect described above, this would produce a standard deviation of MSL of about 40 cm. As this is excessive, it is essential that tide-gauges operate during altimeter operation times. Gauge records will provide tidal phases and magnitudes at the gauge site. However, the magnitudes and phases vary over the ocean. *Easton* (1970) provides co-range and co-tidal charts to cover the Australian coastline. They show, for example, (*ibid*, p.226, fig. 5.4.8) ranges varying about 60 cm over a two degree square of ocean. Phases are shown to vary generally over about half an hour, although variations up to a few hours are shown over two degrees. It may be advantageous or even imperative that such charts be used during GEOS-C operation. If by these charts, the height of the tide under the altimeter is estimable to 15 cm, the limited sampling effect will be reduced to about 2 cm. Neglecting these charts could produce errors as large as the effects of the tides themselves, causing ultimately the aforementioned 40 cm standard deviation from this cause.

3. Tsunamis

The conclusions reached in Chapter 7 on the subject of tsunamis should also be applicable here. The existence of tsunamis must be acknowledged and account should be taken if any are found to exist during altimeter data collection over Australia. It must be noted, though, that the magnitude of tsunamis is slight at sea, and is only significant when the wave strikes impeding terrain. The effect then is certain to be negligible. However, it is interesting to note the intended use of satellite altimetry to determine the existence of tsunamis for warning services.

4 & 5. Secular Effects on Sea Level and Variations in the Mass of Water in the Oceans

will be negligible and ineffectual.

6. Gauge Recording Errors

including river flow influences, are equivalent to errors in the levelling network and need not be considered as altimetry effects.

7. Tides with Periods Greater than a Month

are considered to have a negligible magnitude. The annual cycle may be included in the discussion below, on ocean currents and density.

8. Atmospheric Effects on Sea Level

(i) Air Pressure: if effects are of the magnitude indicated in Chapter 6, Section 6.2, their influence on MSL derived from satellite altimetry may be considered to be negligible. Furthermore, the effect can be reduced by the application of corrections based on observed atmospheric pressures. The problem then is that the frequency response of sea level to air-pressure must be known, so that the relevant air-pressure can be applied. That is, whether monthly means, daily means or hourly readings should be used to correct MSL will depend on the rate at which sea level responds to changes in atmospheric pressure. Error is only likely to arise if the assumed barometric factor deviates from the theoretical value. However, the effects of this may be discussed elsewhere in this thesis.

(ii) **Wind Effects:** On the assumption that they contribute to the annual cycle, the influences of winds can be considered with density and current effects.

(iii) **Storm Surges** - emphasize the necessity for tide-gauge recording. The magnitude of the surges over a two degree square area however, must be estimated. If a 1 m storm surge were to occur every 30 days, the influence on readings should only be about 3 cm. Until more details of surge effects are obtained, this figure will be used in the error budget.

(iv) Other atmospheric effects described in Chapter 6 are considered to have negligible magnitude.

9. Current and Density Variations

with time must be assumed to contribute to the observed annual cycle described in Chapter 5. Other phenomena have already been mentioned as contributors to this cycle. As magnitudes of the annual cycle are of the order of 20 cm, the standard deviation of MSL due to the limited sampling should be only about 3 cm.

8.7 Conclusions

It has been shown in section 8.1 that the deviation between MSL and the geoid obtained by satellite altimetry is given by equation

$$c = h_{s_i} - a' - b - N_i$$

This is applicable to either a single observation or to the mean value of a number of observations. If applied to two areas of ocean, each a two degree square as described previously, then

$$\Delta c = h_{s_2} - h_{s_1} - a_2' + a_1' - b_2 + b_1 - N_2 + N_1$$

$$= \Delta h_s + (a_1' - a_2') + (b_1 - b_2) - \Delta N$$

where Δh_s is the difference between the mean satellite altitudes over square 2 and over square number 1, whilst ΔN is the difference in geoid-spheroid separation distances of these areas. By the general law of propagation of variances, if σ_x^2 is the variance of x,

$$\sigma_{\Delta c}^2 = \sigma_{\Delta h_s}^2 + 2\sigma_{a'}^2 + 2\sigma_b^2 + \sigma_{\Delta N}^2$$

assuming that

(i) there is no correlation between any of the quantities h_s, a, b and N.

(ii) $\sigma_{a'}^2 = \sigma_{a'_2}^2 = \sigma_{a'}^2$

(iii) There is no covariance between a'_1 and a'_2

(iv) $\sigma_{b_1}^2 = \sigma_{b_2}^2 = \sigma_b^2$

(v) There is no covariance between b_1 and b_2.

The most difficult variances to estimate are $\sigma_{\Delta h_s}^2$ and $\sigma_{\Delta N}^2$; see Sections 8.4 and 8.5. The variance of individual altimeter measurements was given as

$$\sigma^2 = (200^2 + 20^2)\ \text{cm}^2$$

For readings meaned over 29 orbits, see Section 8.6, the variance will be reduced to

$$\sigma_{a'}^2 = \frac{1}{29}(200^2 + 20^2)\ \text{cm}^2 \doteqdot 7\ \text{cm}^2$$

assuming no correlation between altimeter measurements over any ocean space nor between refraction errors of various orbits.

The variance of b, on the basis of points described in Section 8.6, can be written,

$$\sigma_b^2 = \sigma^2_{TIDES} + \sigma^2_{SURGES} + \sigma^2_{ANNUAL\ CYCLE}$$
$$= 4 + 9 + 9 \text{ cm}^2$$
$$= 22 \text{ cm}^2$$

$$\therefore \quad \sigma^2_{\Delta c} = \sigma^2_{\Delta h} + 7 + 22 + \sigma^2_{\Delta N} \text{ cm}^2$$
$$= \sigma^2_{\Delta h} + \sigma^2_{\Delta N} + 29 \text{ cm}^2$$

As the values of $\sigma^2_{\Delta h}$ and $\sigma^2_{\Delta N}$ are not finalized, a nomogram, Figure 8.6, has been constructed showing the value of $\sigma_{\Delta c}$ as functions of $\sigma_{\Delta h}$ and $\sigma_{\Delta N}$. It is worth noting that if

$$\sigma_{\Delta h} = 20 \text{ cm} \quad \text{and}$$
$$\sigma_{\Delta N} = 20 \text{ cm}$$

then

$$\sigma_{\Delta c} = 29 \text{ cm}$$

Conclusions may be drawn directly from the nomogram. However, to search for a 1 m MSL-geoid deviation, $\sigma_{\Delta c}$ of the order of 25 cm is desirable. The area is shaded on the figure.

Other conclusions arising from this Chapter, to be read with points made already in Sections 8.4 and 8.6, are

1. More information about the variation of tidal phases and magnitudes may be valuable.

2. Tide-gauge recording is absolutely imperative.

3. *Long-arc* tracking should be used.

4. The use of the intensive mode over Australian coastal regions is desirable.

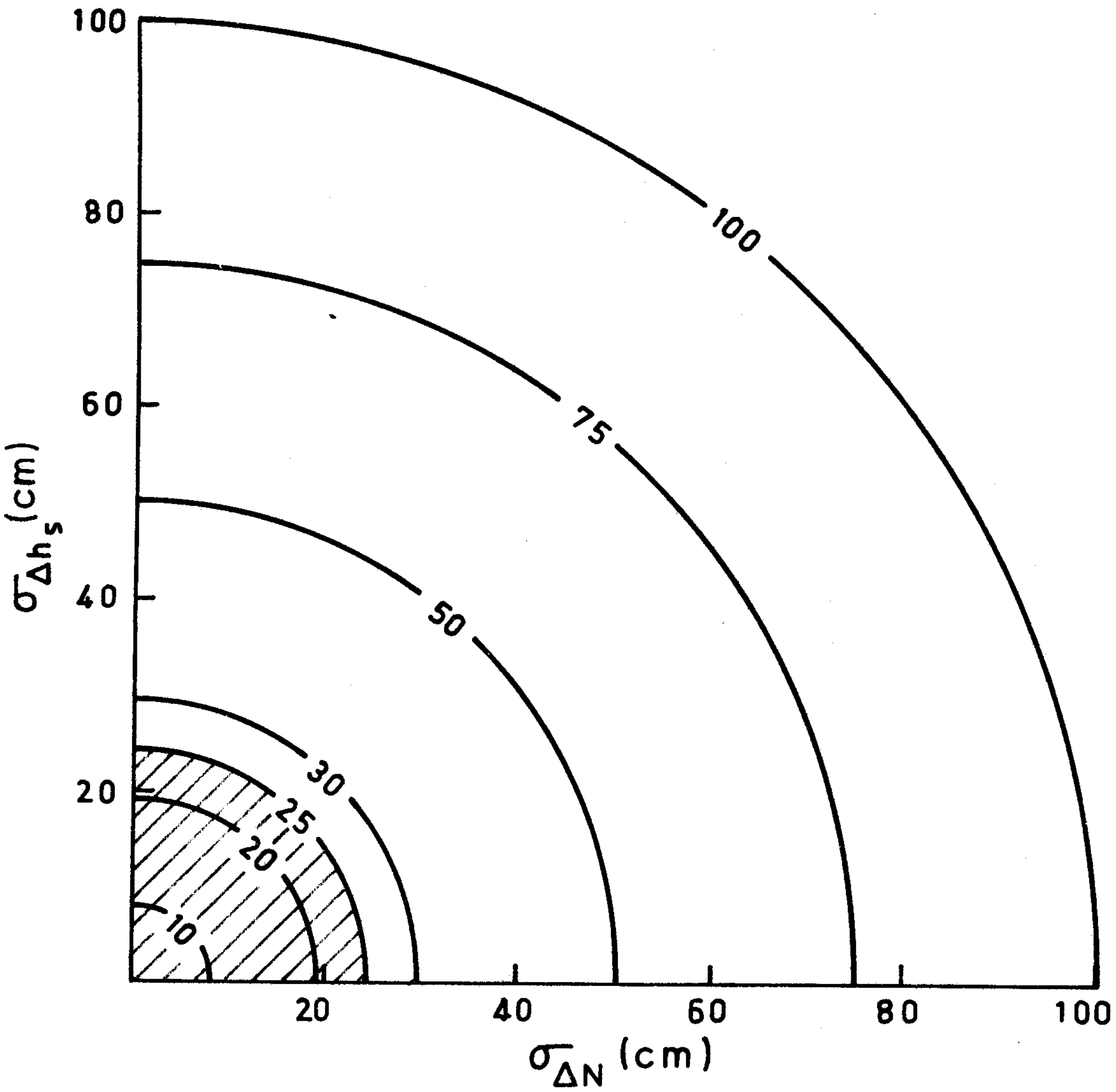


FIG. 8·6
VALUES OF $\sigma_{\Delta C}$ AS FUNCTIONS OF $\sigma_{\Delta N}$ AND $\sigma_{\Delta h_s}$ (cm)

5. The determination of c should be undertaken both on and off the continental shelf to see whether the geoid-MSL deviations are a continental-shelf phenomenon.

6. The largest sources of error are in the determination of h_s and N.

7. Following from 6, the method could be improved if the height of the satellite above the geoid could be calculated directly, for example, from the gravity field.

   Thus $h_o \approx h_s - N$.

   The computation of h_o directly could reduce errors.

8. Results from GEOS-C will at least indicate the future value of satellite altimetry for problems of sea-surface topography.

9. DISCUSSION AND CONCLUSIONS

9.1 Discussion of Results

Levelling Network

The transformation of the observed levelling in the Australian network into geopotential, minimally altered the apparent deviations between Mean Sea Level and the geoid. The subsequent adjustment of the geopotential network was also noteworthy for its failure to significantly alter these results. The estimates of standard deviations refer only to random errors. The figures tendered in Chapter 3 are of the same order as those given by the Division of National Mapping (*Roelse et al*, 1971) , and they do not account for a metre variation in sea level around Australia. Nevertheless, geopotential network error may contribute to the apparent sea-surface topography. Specifically, a systematic error should be suspected. A very small systematic error would create an apparent sea level variation of the magnitude found in Australia. The slope of the sea-surface before density and air-pressure corrections, regarded as a function of latitude, is equivalent to a divergence of approximately 2 m over 24 degrees of arc, or 2 m in 2700 km. This corresponds to a slope of 0.15 seconds of arc, which could result from a systematic error of 0.07 mm in observed height differences for any levelling set-up, assuming staffs to be spaced at 100 m. Between bench-marks at an assumed 3 km separation, the required systematic height error would be only 0.2 cm, or 7 cm between junction points 100 km apart. Such a small systematic error would remain undetected without a comparison with tide-gauges.

The existence of imperfections in the levelling is indicated by *Roelse et al* (1971, pp.54-55). The accumulated discrepancy between forward and reverse runs of levelling is shown as a function of the length of the levelling run. Over a distance of 50 km in the example shown (*ibid*, p.55), the accumulated difference is 15 cm, which is about twice the allowable error by the $12\sqrt{K}$ mm formula which was mentioned in Section 2.2. Thus, the difference is not explained by the expected or allowed random error. *Lister* (1972) has undertaken similar calculations for a line of 1200 km length and for a closed loop of 900 km. The accumulated differences were 90 and 150 cm, respectively, which were of the order of two and three times the admissable $12\sqrt{K}$ mm difference. A line of 1300 km which was calculated for this report accumulated a difference of 80 cm. The Division of National Mapping suspects that these results arise from improper automatic level usage, but that the mean values of the two-way levelling are unaffected by the phenomenon. A closer study could verify or disprove this presumption. However, the existence of the results suggests that other unusual results could be produced from detailed analysis of the levelling.

Roelse et al (*ibid*, p.59 and Annexure E) have also detected a relationship between the apparent height of M.S.L. at each tide gauge and its distance from the Johnston Geodetic Origin. No other pattern in the corrected heights of MSL, as given in Table 6.8, is apparent.

Tide-Gauge Survey

The most significant cause of a deviation between sea level and the geoid is undoubtedly ocean water density. It is credible that its contribution may be larger than indicated by the figures in Table 5.1. It is plausible that the measurement of densities at sea and the theory of relationships between density, currents and sea surface topography are not perfect. The differences between mathematical models of the oceans and actual conditions have made theoretical predictions of tidal magnitudes and phases virtually impossible. Ocean current theory incorporating restrictions due to friction, ocean-bed topography and so on, has been highly developed; see, for example, *Neumann* (1968). Nevertheless, predictions of the sea-surface topography from ocean density observations alone, as used in Chapter 5, may suffer from imperfect theory.

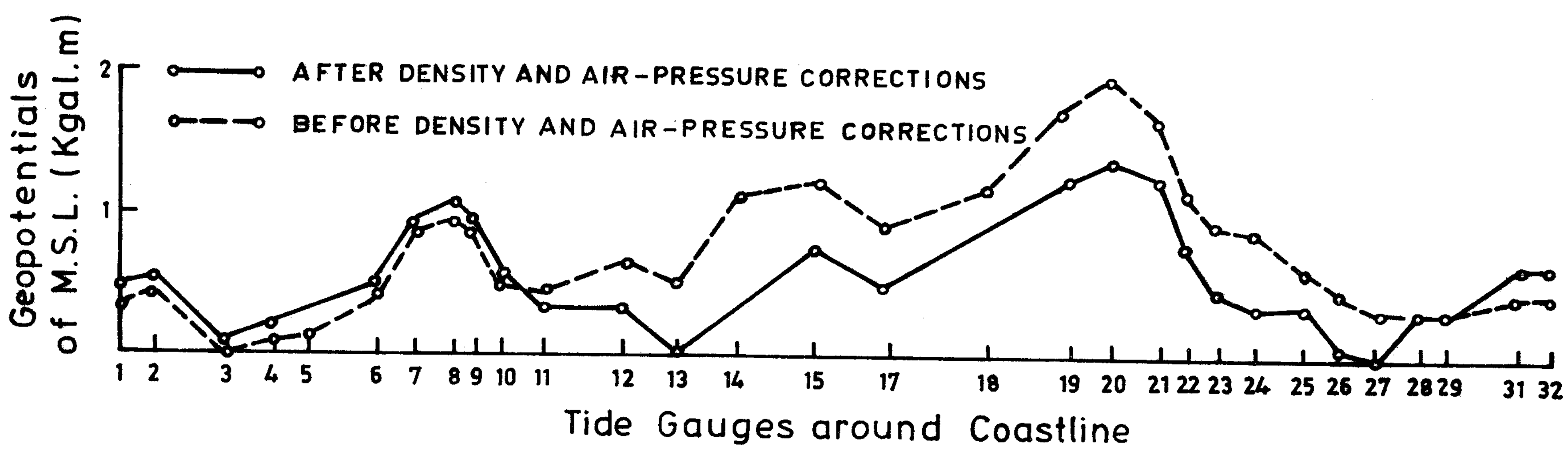


FIG. 9·1

FINAL GEOPOTENTIALS OF M.S.L. AT TIDE-GAUGES AFTER CORRECTION FOR DENSITY AND AIR-PRESSURE, M.S.L. AT BRISBANE ASSUMED TO BE ZERO. POTENTIALS OF M.S.L. BEFORE CORRECTION (FIGURE 3·2) ARE ALSO SHOWN, PORT LINCOLN M.S.L. AS ZERO. MEAN SEA LEVELS 1966-1970.

NUMBERING OF TIDE GAUGES AS IN FIGURE 1.1.

The study of position-dependent deviations of sea level is likely to be enhanced by extensive time-variation data of ocean phenomena. *Hamon and Stacey* (1960) have studied the relationship between time-variations of sea level and of density; see also *Hamon* (1958). Using the data from a number of gauges around Australia for the five years 1966 to 1970, the correlation of sea level with barometer readings has been studied: see Figures 6.4, 6.5 and 6.6. Other annual cycles are illustrated in Figure 9.2. A full study of the correlation between the annual cycle and wind or density has been virtually impossible. However, using the data given by *Wyrtki* (1961, Plates 1c to 6c) for variations of sea level at Darwin, Figure 9.3 was constructed. Agreement between the two variations is significant and is evidence that:

(i) the density variation may be a major contribution to the annual cycle;

(ii) the density effect on sea level according to conventional theory is correct;

(iii) an error up to the order of 15 cm could result from the calculation of sea-surface topography from a single density observation;

(iv) as a result of (ii), density would not contribute a further metre to the deviation in sea level.

Nevertheless, it is notable that the density corrections have been of the appropriate sign which reduces the apparent sea level slope between Port Lincoln and Bamaga. Furthermore, the air-pressure corrections also reduced this slope, although only slightly, using the given barometric factor.

Flexure of the Earth's Crust.

In a highly accurate survey, the effects on the levelling and on the gauge records of tides in the solid earth and of the loading tide due

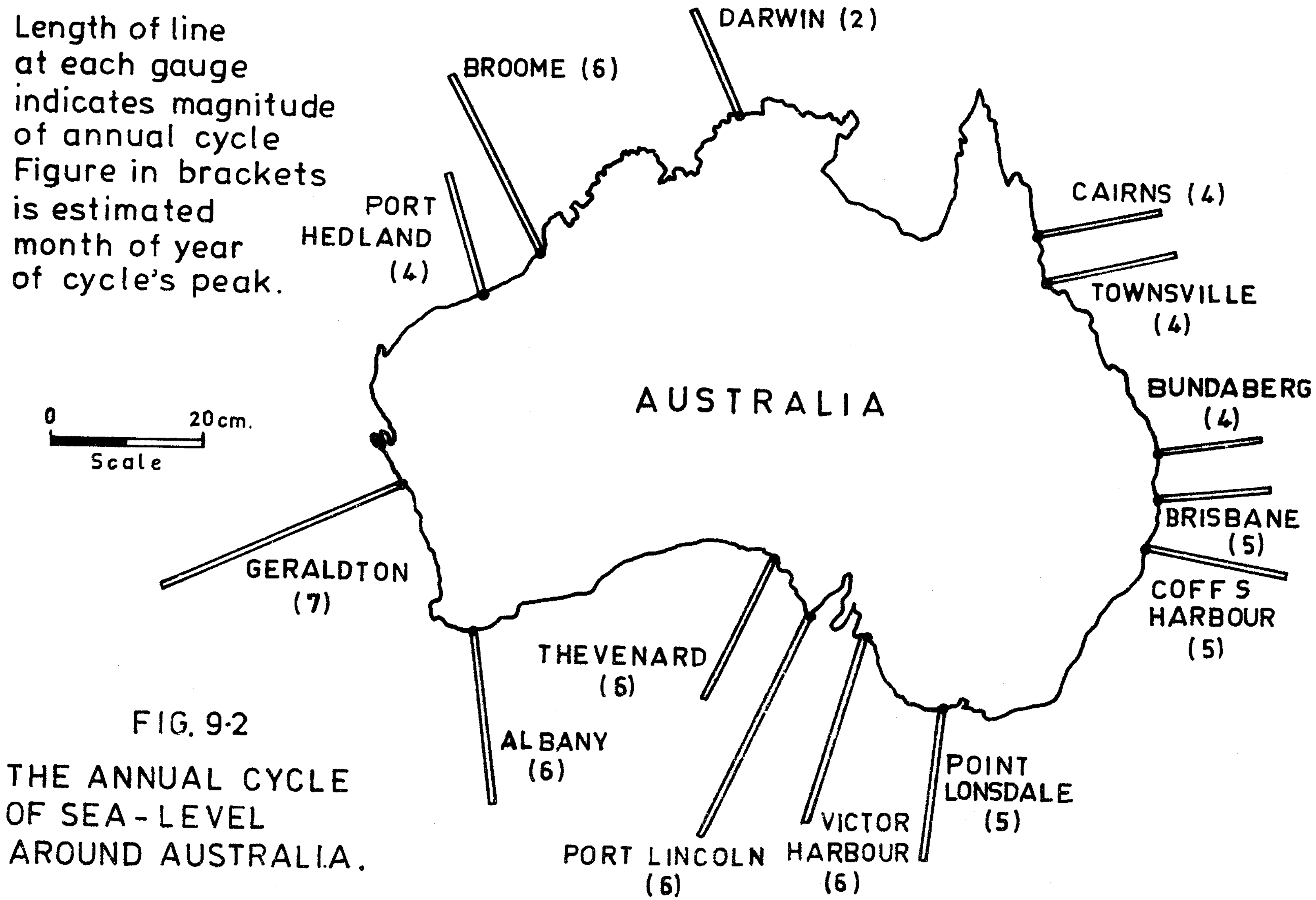


FIG. 9·2

THE ANNUAL CYCLE
OF SEA-LEVEL
AROUND AUSTRALIA.

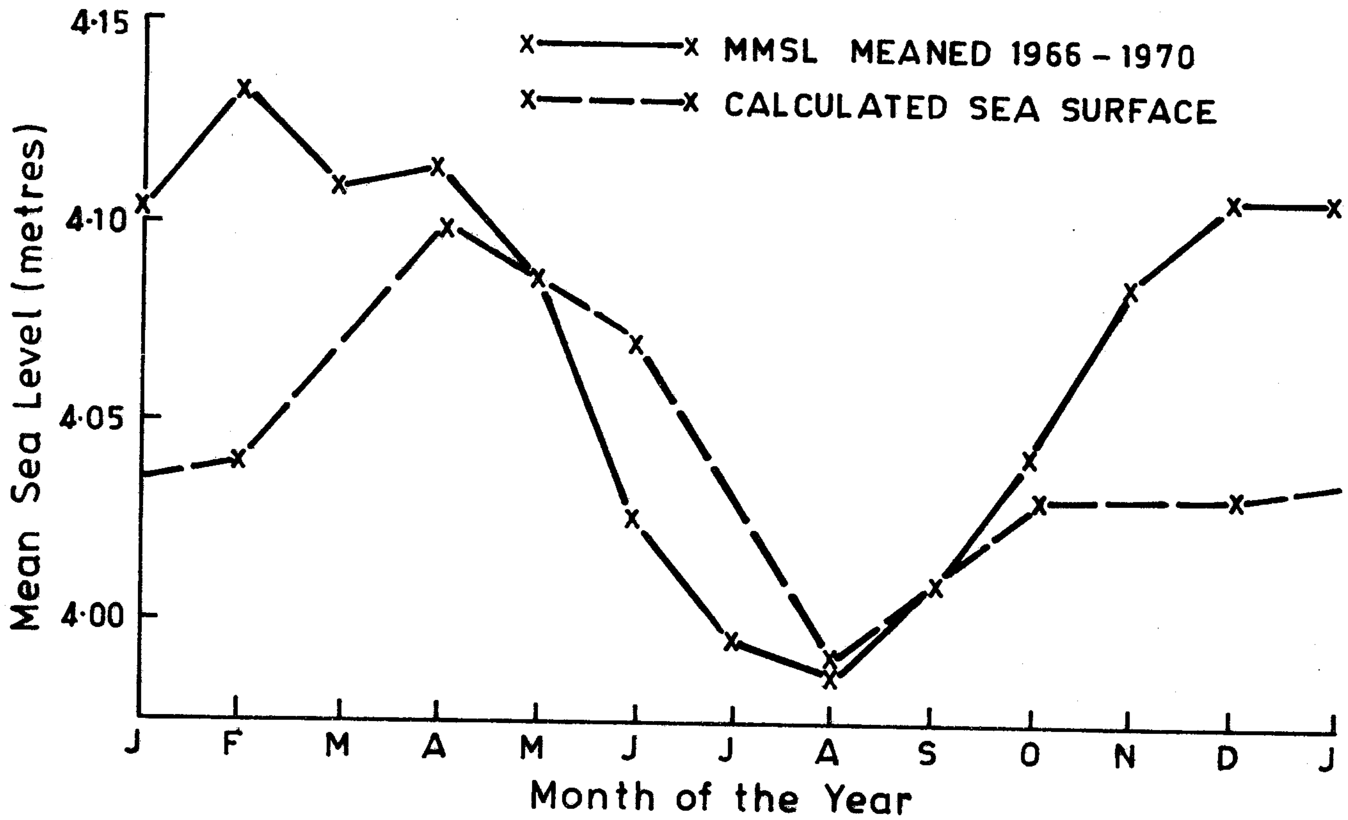


FIG. 9·3

OBSERVED MONTHLY MEAN SEA LEVELS AND CALCULATED SEA SURFACE TOPOGRAPHIES, DARWIN (15) to the mass of ocean tide water on the crust, could be worthy of consideration as a source of error. The magnitudes of their influences are expected to be too small to significantly contribute to a 1 or 2 m divergence of apparent sea level, (*Jensen*, 1950; *Melchior*, 1958; 1966; 1967).

Satellite Altimetry.

A full examination of satellite altimetry was given in Chapter 8, but it is worth recognising that the sea-surface topography may be associated with the continental shelf, in which case the slope of the sea-surface may not exist in the deep-sea. The slope will only show in altimetry results when the continental shelf extends to sea.

Consequently, the proportion of shallow water in the two degree square in the altimetry reduction could be important.

9.2 Conclusions

Conclusions may be applied to either the Australian network or to any levelling being connected to tide-gauges.

In the Australian net, the 2 kgal m apparent separation between the two equipotential surfaces defined by Mean Sea Level and by the geopotential network is only partially explicable. The deviation is striking as reports of overseas comparisons between levelling and sea levels have not described differences of this magnitude. The figure of 2 kgal m, which is based on adjusted geopotentials and which has an estimated standard deviation of about 0.3 kgal m is virtually unchanged from the original observed levelling value. The figure is reduced by about a third by accounting for air-pressure and water density effects according to existing theoretical relationships.

Satellite altimetry over the oceans, as proposed for GEOS-C, could be a valuable contribution to the problem's solution. Successful operation should result if the satellite height and geoid-spheroid separation error figures can be reduced to the levels discussed in Chapter 8. Even if not completely successful, the GEOS-C project should nevertheless be a satisfactory test for future satellite altimetry projects. By indicating whether the apparent geoid - MSL deviations are of oceanographic origin or are due to a levelling phenomenon, satellite altimetry results can halve the future work involved in this project. Tide-gauge operation during GEOS-C flight is essential.

If the geoid-MSL separations are considered to be of oceanographic origin, wind effects would be worthy of study, mainly by the collection of suitable direction and velocity data. Density effects could also be the subject of further study, but the air pressure factor could be given third priority. Time variation data for wind and density should at least be extended.

Alternatively, if the geopotential network is regarded as faulty, a systematic error should be suspected. Differences between the forward and back runs of observed levelling, as mentioned in Section 9.1, could provide a fruitful study.

Conclusions applicable to levelling networks generally will be evident from a final solution to the Australian network problem. The following points can still be made:

1. The spacing of tide-gauges as existent around Australia should be satisfactory.

2. A five year Mean Sea Level produced from hourly gauge readings is also satisfactory. Tidal influences, which often have over-rated significance, are

suitably meaned over five-years. Tide-gauge observation over 18.6 years is needless, unless working to millimetre accuracy. Other variable influences, including currents, should be reduced by a five-year mean to a level which produces an error smaller than that expected from the levelling. The variability of annual means (e.g. *Easton and Radok*, 1970b) suggests that *five*-year MSL's should be repeatable to a few centimetres at most gauges, or to 10 cm at occasional tide-gauges.

3. Density, wind and air-pressure data should be collected during the period of the gauge survey.

4. Gravity observations at bench-marks make conversion to geopotentials an extremely simple process.

The apparent observations of mean sea level from the geoid have been approached as a problem in geodetic levelling. Nevertheless, both the definition, by levelling, of a vertical control system and the tide-gauge provision of sea level have been viewed as methods of fixing the position of a potential surface. Thus, the fields of levelling and oceanography have been combined to question the assumption that the accuracy of geodetic levelling could be checked by connections to tide-gauges. As the problem has been seen from a geodetic levelling point of view, the study of oceanic phenomena in Chapters 4 to 7 has been descriptive and necessarily covers material familiar in the field of oceanography. However, it is felt that this was the only way to approach the problem, and that the necessary groundwork for a further investigation of the subject has been covered.

Areas for continued study were apparent but investigation was restricted to the point at which a new study of one of the aspects mentioned in this report would be prefered.

The investigation has shown that large scale levelling surveys cannot be compared to simple, observed Mean Sea Levels.

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APPENDIX 1

Computer Programmes related to the Geopotential Calculation

1.1 Covariance function calculation.

1.2 Potential conversion: main routine.

1.3 Gravity-anomaly-selection subroutine.

1.4 Sorting subroutine.

1.5 Interpolation subroutine.

1.6 Potential conversion analysis.

APPENDIX 3.1

```
C       ************ CALCULATION OF COVARIANCE FUNCTION *****************
C
C       ****************************************************************
C       * TO CALCULATE THE GRAVITY AUTOCORRELATION FUNCTION FOR AUSTRALIA*
C       * NEGLECTING HEIGHT CORRELATION. USING FREE-AIR ANOMALIES AT ONE *
C       * TENTH OF A DEGREE ANOMALY SPACING.                             *
C       ****************************************************************
C
        DIMENSION IG(31,31),COVAR(31,31),IDIST(31,31),NUM(45),TOT(45),AVER
       #(45)
        REAL*8 SUMSQ,TOT
C
     20 DO 75 K=1,45
        NUM(K)=0
        TOT(K)=0.0
     75 CONTINUE
C
        NGRD=5
        LN=31
        LE=31
C                             ******************************************
C                             * DO LOOP TO SELECT DIFFERENT BLOCKS OF GRAV. DATA*
C                             ******************************************
        DO 460 LON=11455,15055,200
        DO 460 LAT1=1255,4255,300
        LAT=-LAT1
        NCHK=0
C                             ******************************************
C                             * GRAVITY DATA CALLED FROM DATA-SET ON DISK *
C                             ******************************************
        CALL GRAV(IG,LAT,LON,LN,LE,NGRD,NCHK)
        IF(NCHK)460,125,460
C
    125 JG=IG(16,16)
```

```
C                         ***********************************************
C                         * IF GRAVITY NOT OBS'D AT CENTRE, AVOID THIS ARRAY*
C                         ***********************************************
      IF(JG.LT.4900.AND.JG.GT.-4900) GO TO 140
      GO TO 460
  140 RADLAT=LATI*1.74533E-4
      WRITE(3,450) LATI,LON
  450 FORMAT(' ','TEST OF AREA WHICH IS CENTERED AT',2I6)
C                         ***********************************************
C                         * DO LOOP TO COMPARE ANOMALY AT EACH (I,J) WITH   *
C                         * THE ANOMALY AT POINT JG.                        *
C                         ***********************************************
      DO 400 I=1,31
      DO 400 J=1,31
C                         ***********************************************
C                         * IF GRAV. AT (I,J) NOT OBSERVED, GO TO NEW (I,J) *
C                         ***********************************************
      IF(IG(I,J).GT.4900.OR.IG(I,J).LT.-4900) GO TO 400
C                         ***********************************************
C                         * PRODUCT OF ANOMALIES AND SEPARATION DISTANCE    *
C                         ***********************************************
      COVAR(I,J)=IG(I,J)*JG
      DIST=2*SQRT(((J-16)*COS(RADLAT))**2+(I-16)**2)
      IF(DIST.GT.45) GO TO 400
C                         ***********************************************
C                         * ROUND OFF DIST. TO NEAREST 0.05 DEGREES.        *
C                         ***********************************************
      IDIST(I,J)=DIST
      DEC=DIST-IDIST(I,J)
      IF(DEC.LT.0.5) GO TO 200
  175 IDIST(I,J)=IDIST(I,J)+1
  200 L=IDIST(I,J)
      IF(L)375,400,375
  375 NUM(L)=NUM(L)+1
      TOT(L)=TOT(L)+COVAR(I,J)
  400 CONTINUE
C
  460 CONTINUE
```

```
C                           *****************************************************
C                           * PRINT OUT RESULTS FOR EACH DISTANCE               *
C                           *****************************************************
  465 WRITE(3,475)
  475 FORMAT('1','DISTANCE S (DEGS X 20)    NO. OF POINTS COMPARED    TO
     #TAL PRODUCT     COVARIANCE'//' ',2(22('_'),4X),13('_'),4X,10('_'),4
     #X,8('_'))
      DO 525 L=1,45
      IF(NUM(L).EQ.0) GO TO 525
  485 AVER(L)=TOT(L)/NUM(L)
      WRITE(3,500) L,NUM(L),TOT(L),AVER(L)
  500 FORMAT(' ',9X,I3,20X,I6,13X,F12.1,4X,F8.1)
  525 CONTINUE
  600 STOP
      END
```

APPENDIX 1.2

```
C      CONVERSION OF OBSERVED LEVELLING TO GEOPOTENTIALS...MAIN ROUTINE
C
C      *****************************************************************
C      * OBSERVED LEVELLING-HEIGHT DIIFERENCES BETWEEN EACH BENCH-MARK  *
C      * IS CONVERTED TO A GEOPOTENTIAL DIFFERENCE USING THE VALUE OF   *
C      * GRAVITY INTERPOLATED FROM SURROUNDING FOUR OBSERVED ANOMALIES. *
C      *****************************************************************
C
       DIMENSION MGAL(4,4),IG(32,32),MAXMIN(32,2),LIST(32,2),COVAR(200),C
      #IK(4,4),CPI(4)
       REAL*8 RADIAN,PHI,EQGRAV,GAMMA,DSIN,SECT
       IREAD=4
       IWRITE=2
C      *****************************************************************
       READ(1,100) (COVAR(I),I=1,200)
  100  FORMAT(20F3.0)
       WRITE(3,150)
  150  FORMAT('0 THE COVARIANCE FUNCTION IS'/)
       WRITE(3,160) (COVAR(I),I=1,200)
  160  FORMAT(' ',20X,10F8.7)
       DATA CHAR/'E'/
   24  READ(1,25) ISKIP,LAST1
   25  FORMAT(2I5)
       WRITE(3,26) ISKIP,LAST1
   26  FORMAT('      ISKIP AND LAST1    ',2I5)
C
       DO 90 IA=1,ISKIP
   30  READ(IREAD,40) E
   40  FORMAT(1X,A1)
       IF(E.NE.CHAR) GO TO 30
   90  CONTINUE
       IAD=0
C      *****************************************************************
       B=5.2884E-3
       C4=5.9E-6
       EQGRAV=9.780490E5
       FEET=3.048E-1
       RADIAN=1.74532925D-02
C      *****************************************************************
```

```
C               ********************************************************
C               * READS NUMBERS OF JUNCTION-POINTS AT ENDS OF SECTION     *
C               ********************************************************
      READ(IREAD,1503) JPA,AJPHT,JPA1,ANAME1,ANAME2,ANAME3,IBMA1,IBMA2,L
     #ATDA,LATMA,LONDA,LONMA
      GO TO 1402
 1400 READ(IREAD,1501) JPA,AJPHT,JPA1,ANAME1,ANAME2,ANAME3,IBMA1,IBMA2,L
     #ATDA,LATMA,LONDA,LONMA
 1550 WRITE(3,1551) JPA,AJPHT,JPA1,ANAME1,ANAME2,ANAME3,IBMA1,IBMA2,LATD
     #A,LATMA,LONDA,LONMA
 1551 FORMAT('0',I5,F12.3,I5,2X,4A4,A3,2I3,2I4)
 1402 READ(IREAD,1500) JPB,BJPHT,JPB1,BNAME1,BNAME2,BNAME3,IBMB1,IBMB2,L
     #ATDB,LATMB,LONDB,LONMB
 1500 FORMAT(   I4,F9.3/I4,4A4,A3,10X,2I3,I4,I3)
 1501 FORMAT(//I4,F9.3/I4,4A4,A3,10X,2I3,I4,I3)
 1503 FORMAT(/ I4,F9.3/I4,4A4,A3,10X,2I3,I4,I3)
 1560 WRITE(3,1551) JPB,BJPHT,JPB1,BNAME1,BNAME2,BNAME3,IBMB1,IBMB2,LATD
     #B,LATMB,LONDB,LONMB
      HT=AJPHT
      IAD=IAD+1
C               ********************************************************
C               * READS DETAILS OF NEXT SUB-SECTION BETWEEN BENCH-MARKS *
C               ********************************************************
 1959 READ(IREAD,2000) STATE, SECT,BM11,BM12,BM21,BM22,IBM,DST1,DELTAH,D
     #ELTAO,LATDEG,LATMIN,LODEG,LOMIN
 2000 FORMAT(A5,A8,A4,A3,4X,A4,A3,1X,I3,F6.2,2F9.3,I2,3I3)
      HT=HT+DELTAH
C               ********************************************************
C               * DETERMINATION OF GRAVITY ANOMALY AT BENCH-MARK BEGINS *
C               ********************************************************
      LEFT=4
C
      DO 20 KA=1,LEFT
      DO 15 KB=1,3
      MGAL(KA,KB)=0
   15 CONTINUE
      MGAL(KA,4)=6000
   20 CONTINUE
```

```
      KEY=0
      LN=1
  300 LN=LN*2
      LNSQD=LN*LN
C     ************************************************************************
      CALL SELECT(MGAL,LN,LEFT,LNSQD,KEY,IG,LIST,MAXMIN,LATDEG,LATMIN,LO
     #DEG,LOMIN,DECLAT,DECLON)
C     ************************************************************************
      IF(LEFT) 400,400,300
C     ************************************************************************
  400 CALL INTERP(MGAL,KEY,DECLAT,DECLON,COVAR,ANOM)
C     ************************************************************************
C
      WRITE(3,450) KEY,ANOM
  450 FORMAT('+',T42,I2,T69,F6.1)
C
      PHI=(LATDEG+LATMIN/60.0D0)*RADIAN
      GAMMA=EQGRAV+EQGRAV*B*(DSIN(PHI))**2-EQGRAV*C4*(DSIN(2.0E0*PHI))**
     #2
      FAA=HT*3.086E-1*FEET
      GRAV2=GAMMA+ANOM-FAA
C
C               ************************************************************
C               * TESTS WHETHER CALCULATING FIRST SUB-SECTION AFTER JP    *
C               ************************************************************
      IF(DST1.EQ.0.AND.DELTAH.EQ.0) GO TO 1799
      GO TO 2999
 1799 GRAV1=GRAV2
      WRITE(3,2899) BM11,BM12,BM21,BM22,LATDEG,LATMIN,LODEG,LOMIN,DST1,H
     #T,GAMMA,FAA,GRAV2,DELTAH,DELTAO
 2899 FORMAT('+',A4,A3,A4,A3,2I3,I4,I3,F5.1,F9.3,12X,F9.1,F6.1,6X,F9.1,2
     #7X,2F9.3)
C
      SUMDC=0.0
      SUMDH=0.0
      SUMDO=0.0
      SUMDD=0.0
```

```
C
C               ******************************************************
C               * DATA FOR SECOND SUB-SECTION READ                   *
C               ******************************************************
      GO TO 1950
 2900 DELTAC=0.5*(GRAV1+GRAV2)*DELTAH*1.0E-6*FEET
      DCFEET=DELTAC*100.0/DELTAO/FEET-100.0
      WRITE(3,2950) BM11,BM12,BM21,BM22,LATDEG,LATMIN,LODEG,LOMIN,DST1,H
     #T,GAMMA,FAA,GRAV2,GRAV1,DELTAC,DCFEET,DELTAH,DELTAO
 2950 FORMAT('+',A4,A3,A4,A3,2I3,I4,I3,F5.1,F9.3,12X,F9.1,F6.1,6X,2F9.1,
     #2F9.4,2F9.3)
      SUMDC=SUMDC+DELTAC
      SUMDH=SUMDH+DELTAH
      SUMDO=SUMDO+DELTAO
      SUMDD=SUMDD+DST1
      SUMDF=SUMDC*100.0/SUMDO/FEET-100.0
C               ******************************************************
C               * TESTS WHETHER CALCULATING LAST SUB-SECTION BEFORE JP *
C               ******************************************************
      IF(IBM) 3300,2050,3000
 2050 GRAV1=GRAV2
      DANOM1=DANOM2
      GO TO 1950
C               ******************************************************
C               **READS DATA FOR NEXT SUB-SECTION                    *
C               ******************************************************
 3000 WRITE(3,3100) SUMDC,SUMDF,SUMDH,SUMDO
 3100 FORMAT(' ',T94,2F9.4,2F9.3)
      WRITE(3,2800) SECT,STATE
 2800 FORMAT(' ',T45,'SECTION NUMBER',A8,' IN STATE OF',A3/10X,90('-'))
      SUMDD2=SUMDD**2
      WRITE(3,3050) STATE,SECT,JPA,JPB,SUMDC,SUMDD2
      WRITE(IWRITE,3050) STATE, SECT,JPA,JPB,SUMDC,SUMDD2
 3050 FORMAT(' ',A3,A8,2I5,F12.5,F9.2)
      SUMDC=0.0
      SUMDH=0.0
      SUMDO=0.0
      SUMDD=0.0
      IF(IAD.EQ.30) GO TO 3300
      IF(IAD.EQ.LAST1) GO TO 24
      GO TO 1401
 3300 STOP
      END
```

APPENDIX 3.5

```
      SUBROUTINE SELECT(MGAL,LN,LEFT,LNSQD,KEY,IG,LIST,MAXMIN,LATDEG,LATM
     *MIN,LODEG,LOMIN,DECLAT,DECLON)
C
C     *********************   SUBROUTINE SELECT   *********************
C
C     *****************************************************************
C     * TO SELECT THE FOUR GRAVITY ANOMALY VALUES WHICH ARE NEAREST TO *
C     * THE BENCH-MARK FOR WHICH THE GRAVITY ANOMALY IS DESIRED. THE   *
C     * ANOMALY AT THE B.M. IS INTERPOLATED FROM THESE FOUR ANOMALIES  *
C     * USING SUBROUTINE 'INTERP',WHICH,WITH 'SELECT',IS A PART OF THE *
C     * MAIN PROGRAMME 'GEOLEV'. INPUT REQUIRED FOR SELECT IS THE LAT. *
C     * AND LONG. (DEGREES + MINUTES) OF THE B.M.                      *
C     *****************************************************************
      DIMENSION MGAL(LEFT,4),IG(LN,LN),LIST(LNSQD,2),MAXMIN(LNSQD,2)
      WRITE(3,5)
    5 FORMAT(' ',T47,'S')
C           ***********************************************************
C           *    SOME CONSTANTS FIXED FOR THE WHOLE PROGRAMME         *
C           ***********************************************************
      KOUNT=0
      NGRD=5
      NCHK=0
      LF=LN
C           ***********************************************************
C           * CALCULATES VARIABLES REQUIRED IN SELECTION OF ANOMALY   *
C           * DATA FROM DATA BANK USING THE SUBROUTINE GRAV           *
C           ***********************************************************
      DECLAT=-(LATDEG+LATMIN/6.0E1)*1.0E1
      ILAT=DECLAT
      REMLAT=ILAT-DECLAT
      IF(REMLAT.GT.0.5) ILAT=ILAT-1
      LAT=ILAT*1.0E1+5-5*LN
      DECLON=(LODEG+LOMIN/6.0E1)*1.0E1
      ILON=DECLON
      REMLON=DECLON-ILON
      IF(REMLON.GT.0.5) ILON=ILON+1
      LON=ILON*1.0E1+5-5*LN
```

```
C              ****************************************************************
C              * ANOMALY DATA NOW CALLED FROM DATA BANK BY GRAV               *
C              ****************************************************************
      WRITE(3,6)
    6 FORMAT('+',T42,'G')
      CALL GRAV(IG,LAT,LON,LN,LE,NGRD,NCHK)
C              * CALCULATES COSINE OF LATITUDE, CONSTANT FOR DATA LOT         *
C              ****************************************************************
      COSPHI=COS(LAT*1.74533E-4)
C              ****************************************************************
C              * LISTS THE POINTS IN THE DATA LOT WHERE ANOMALIES ARE         *
C              * AVAILABLE UNLESS THEY WERE INCLUDED IN PRECEDING LOT         *
C              ****************************************************************
      KC=1
      DO 50  IH=1,LN
      DO 50  J=1,LN
 2200 LATIJ=LAT+10*IH-10
      LONIJ=LON+10*J-10
      IF(IG(IH,J).EQ.5200) GO TO 50
      IF(LN.EQ.2) GO TO 30
      LN1=LN/4
      LN2=3*LN1+1
      IF(IH.GT.LN1.AND.IH.LT.LN2.AND.J.GT.LN1.AND.J.LT.LN2) GO TO 50
   30 KOUNT=KOUNT+1
      LIST(KC,1)=100*IH+J
      LIST(KC,2)=SQRT((LATIJ-10*DECLAT)**2+((LONIJ-10*DECLON)*COSPHI)**2
     #)*500
      KC=KC+1
   50 CONTINUE
C              ****************************************************************
C              * TESTS WHETHER THE INNERMOST SQUARE IS BEING CONSIDERED       *
C              ****************************************************************
      IF(LN-2) 100,100,350
```

```
C          ***************************************************************
C          * IF AT INNERMOST SQUARE, TEST WHETHER 4 ANOMALIES THERE *
C          ***************************************************************
  100 IF(KOUNT-4) 250,150,150
C          ***************************************************************
C          * IF ALL FOUR AVAILABLE, AIM OF SUBROUTINE IS FULFILLED  *
C          ***************************************************************
  150 KEY=1
      DO 200 IA=1,2
      DO 200 JA=1,2
      KD=2*IA+JA-2
      MGAL(KD,1)=IG(IA,JA)
      LATIT=LAT+10*IA-10
      LONGIT=LON+10*JA-10
      MGAL(KD,2)=LATIT
      MGAL(KD,3)=LONGIT
      MGAL(KD,4)=LIST(KD,2)
  200 CONTINUE
      GO TO 550
C          ***************************************************************
C          * CONSIDERS CASE OF INNNERMOST SQUARE, LESS THAN 4 ANOMS.*
C          ***************************************************************
C          ***************************************************************
C          * USES ANOMALIES AVAILABLE TO FORM THE ARRAY MGAL          *
C          ***************************************************************
  250 IF(KOUNT) 550,550,290
  290 DO 300 KE=1,KOUNT
      IB=LIST(KE,1)/100
      JB=LIST(KE,1)-100*IB
      MGAL(KE,1)=IG(IB,JB)
      LATIT=LAT+10*IB-10
      LONGIT=LON+10*JB-10
      MGAL(KE,2)=LATIT
      MGAL(KE,3)=LONGIT
      MGAL(KE,4)=LIST(KE,2)
  300 CONTINUE
      GO TO 550
```

```
C                ***********************************************************
C                * IF MORE ANOMALIES ARE AVAILABLE THAN NEEDED, USES ORDER*
C                *TO FIND WHICH ONES ARE THE NEAREST                       *
C                ***********************************************************
  350 CALL ORDER(LIST,KOUNT,MAXMIN)
C                ***********************************************************
C                * USES THE RESULTS FROM ORDER TO ADD TO ANOMALY LIST      *
C                ***********************************************************
      IF(KOUNT-LEFT) 400,400,450
  400 LIMIT=KOUNT
      GO TO 500
  450 LIMIT=LEFT
  500 KH=4-LEFT
      DO 530 KG=1,LIMIT
      KH=KH+1
      IC=MAXMIN(KG,1)/100
      JC=MAXMIN(KG,1)-100*IC
      LATIT=LAT+10*IC-10
      LONGIT=LON+10*JC-10
      MGAL(KH,1)=IG(IC,JC)
      MGAL(KH,2)=LATIT
      MGAL(KH,3)=LONGIT
      MGAL(KH,4)=MAXMIN(KG,2)
  530 CONTINUE
C                ***********************************************************
C                * PRINTS OUT THE LIST OBTAINED SO FAR FOR THE ARRAY MGAL *
C                ***********************************************************
  550 NOWHAV=4-LEFT
      LEFT=4-KOUNT
      RETURN
      END
```

APPENDIX [illegible].4

```
      SUBROUTINE ORDER(LIST,KOUNT,MAXMIN)
C
C     ********************** SUBROUTINE  ORDER  ********************
C
C     ***************************************************************
C     * REARRANGES GIVEN LIST OF ANOMALIES AND THEIR DISTANCES FROM A  *
C     * BENCH-MARK, INTO AN ORDER DEPENDING ON THEIR DISTANCE FROM THE *
C     * BENCH-MARK. FIRST IN LIST IS NEAREST, ETC.                     *
C     ***************************************************************
C
      DIMENSION LIST(KOUNT,2),MAXMIN(KOUNT,2)
      WRITE(3,5)
    5 FORMAT('+',T51,'O')
C
      DO 10 KA=1,KOUNT
      MAXMIN(KA,1)=0
      MAXMIN(KA,2)=0
   10 CONTINUE
C
      DO 100 KB=1,KOUNT
      LESS=1
      IEQUAL=0
C
      NUMBER=LIST(KB,2)
      DO 50 KC=1,KOUNT
C
      IF(LIST(KC,2)-NUMBER) 30,40,50
   30 LESS=LESS+1
      GO TO 50
C
   40 IEQUAL=IEQUAL +1
   50 CONTINUE
C
      IF(IEQUAL-1) 70,70,60
   60 IF(MAXMIN(LESS,1)) 70,70,65
   65 LESS=LESS+1
      GO TO 60
C
   70 MAXMIN(LESS,1)=LIST(KB,1)
      MAXMIN(LESS,2)=LIST(KB,2)
  100 CONTINUE
      RETURN
      END
```

APPENDIX 1.3

```
      SUBROUTINE  INTERP(MGAL,KEY,DECLAT,DECLON,COVAR,ANOM)
C
C     ******************** SUBROUTINE  INTERP ***********************
C
C     ****************************************************************
C     * INTERPOLATES VALUE OF GRAVITY ONOMALY AT BENCH-MARK , USING   *
C     * ANOMALIES AND INTERPOLATION METHOD DIRECTED BY 'SELECT'       *
C     ****************************************************************
C
      DIMENSION MGAL(4,4),COVAR(200),CIK(4,4),CPI(4)
C
      ANOM=0.0
      COSPHI=COS(DECLAT*1.745335E-3)
C
C               **************************************************
C               * KEY TO  INTERPOLATION METHOD REQUIRED          *
C               **************************************************
      WRITE(3,100) KEY
  100 FORMAT('+',T44,I2,6X,' I')
      IF(KEY) 800,500,300
C               **************************************************
C               * SIMPLE GEOMETRICAL  INTERPOLATION METHOD       *
C               **************************************************
  300 ILAT=DECLAT
      REMLAT=ILAT-DECLAT
      IF(REMLAT.GT.0.5) ILAT=ILAT-1
      LATIT=ILAT*1.0E1-5
      ILON=DECLON
      REMLON=DECLON-ILON
      IF(REMLON.GT.0.5) ILON=ILON+1
      LONGIT=ILON*1.0E1-5
      DLAT2=(DECLAT*10.0-LATIT)/10.0
      DLAT1=1-DLAT2
      DLON1=(DECLON*10.0-LONGIT)/10.0
      DLON2=1-DLON1
```

```
      DO 400  IB=1,4
      IF(IB.GT.2) GO TO 310
  304 IF(IB.EQ.2) GO TO 308
  306 ALPHA=DLAT1*DLON2
      GO TO 316
  308 ALPHA=DLAT1*DLON1
      GO TO 316
  310 IF(IB.EQ.4) GO TO 314
  312 ALPHA=DLAT2*DLON2
      GO TO 316
  314 ALPHA=DLAT2*DLON1
  316 IF(IABS(MGAL(IB,1))-5000) 320,360,360
  320 ANOM=ANOM+MGAL(IB,1)*ALPHA*1.0E-1
      GO TO 400
  360 IANOM=MGAL(IB,1)/1.0E3
      ANOM=ANOM+IANOM*ALPHA*1.0E-1
  400 CONTINUE
C
      GO TO 800
```

```
C               ****************************************************************
C               * LEAST-SQUARES INTERPOLATION METHOD                           *
C               ****************************************************************
  500 DO 600 IA=1,4
      IF(MGAL(IA,4).GT.19950) MGAL(IA,4)=19950
      CPI(IA)=COVAR((MGAL(IA,4)+50)/100)
      DO 600 IB=1,4
      IDIST=SQRT((MGAL(IA,2)-MGAL(IB,2))**2+((MGAL(IA,3)-MGAL(IB,3))*COS
     #PHI)**2)+0.5
      IF(IDIST.GT.200) IDIST=200
      IF(IDIST.EQ.0) GO TO 510
      CIK(IA,IB)=COVAR(IDIST)
      GO TO 600
  510 CIK(IA,IB)=565
  600 CONTINUE
C     ******************************************************************
      CALL RINV(CIK)
C     ******************************************************************
      DO 630 IA=1,4
      DO 630 IB=1,4
  630 CONTINUE
      DO 604 IA=1,4
      IF(IABS(MGAL(IA,1))-5000) 520,560,560
  520 ANOMA=MGAL(IA,1)
      GO TO 580
  560 IANOM=MGAL(IA,1)/1.0E3
      ANOMA=IANOM
  580 ANOMB=0.0
      DO 605 IB=1,4
      ANOMB=ANOMB+CIK(IA,IB)*CPI(IB)
  605 CONTINUE
      ANOM=ANOM+ANOMB*ANOMA
  604 CONTINUE
C
  800 RETURN
      END
```

```
      SUBROUTINE RINV(CIK)
C     ********************** SUBROUTINE RINV **************************
C
C     ********************************************************************
C     * INVERTS MATRIX OF GRAVITY ANOMALY COVARIANCES FOR 'INTERP'       *
C     ********************************************************************
C
      DIMENSION CIK(4,4)
      DOUBLE PRECISION TEMP
      DO 60 I=1,4
      IP1=I+1
      TEMP=CIK(I,I)
      DO 70 J=1,4
   70 CIK(I,J)=CIK(I,J)/TEMP
      CIK(I,I)=1./TEMP
      DO 80 J=1,4
      IF(I.EQ.J) GO TO 80
      TEMP=CIK(J,I)
      CIK(J,I)=0.
      DO 90 K=1,4
      CIK(J,K)=CIK(J,K)-TEMP*CIK(I,K)
   90 CONTINUE
   80 CONTINUE
   60 CONTINUE
  800 RETURN
      END
```

APPENDIX [illegible].6

```
C     ******************** PROGRAMME ANALYSIS *************************
C
C     ****************************************************************
C     * TO COMPARE GEOPOTENTIAL DIFFERENCES WITH ORTHOMETRIC HEIGHT  *
C     * DIFFERENCES OBTAINED FROM THE SAME OBSERVED LEVELLING        *
C     ****************************************************************
C
C
      F=0.059954
C
      WRITE(3,100)
  100 FORMAT('1',' LOOP  LIST JP1  JP2  ORTHO-HT      G1          G2',6X
     #,'HT1      HT2     BY ME          BY NM',11X,'DIFF'/ )
C
      DO 600 I=1,25
      READ(1,200) LOOP,LIST,JP1,JP2,DELTAO,G1,G2,HT2,HT1,DELTAC
  200 FORMAT(I3,I2,2I3,F9.3,2F9.1,18X,2F5.0,F10.4)
C
      SUMHT=HT2+HT1
      SUMG=G1+G2
      TERM1=0.25*DELTAO*(2.0*SUMG+F*SUMHT)
      TERM2=0.25*SUMHT*(2*(G2-G1)+F*DELTAO)
      DELTAH=(TERM1+TERM2)*1.0E-6
      DHMET=DELTAH*0.3048
      DIFF=(DELTAC-DHMET)*1.0E2
      WRITE(3,300)LOOP,LIST,JP1,JP2,DELTAO,G1,G2,HT1,HT2,DELTAC,DHMET,DE
     #LTAH,DIFF
  300 FORMAT(' ',4I5,F10.3,2F11.1,2F8.1,3F11.4,F7.2)
  600 CONTINUE
C
      STOP
      END
```

APPENDIX 2

Computer programmes related to the geopotential network adjustment and the estimate of accuracy

APPENDIX 2.1

```
C       ************************** ADJUST  ******************************
C
C       *****************************************************************
C       * TO ADJUST AUSTRALIAN NETWORK OF GEOPOTENTIALS BY CONDITIONS   *
C       *****************************************************************
C
        DIMENSION ISECT(757,3),DIFFHT(757,3),JSECT(12,2),MATRXB(261,12),D(
       #261),D2(261),VECTOR(34191),ID(261),LPSECT(261)
        DOUBLE PRECISION VECTOR,D
C       DOUBLE PRECISION VECTOR,D
        READ(1,50) LOOPS,NSECT
   50   FORMAT(2I3)
        WRITE(3,75) LOOPS,NSECT
   75   FORMAT('  IN THE NETWORK, THERE ARE',I3,' LOOPS,',I3,'SECTIONS')
C
        SBARSQ=0.0
        DO 80 J=1,LOOPS
        D(J)=0.0
        DO 30 J1=1,12
        MATRXB(J,J1)=0
   80   CONTINUE
        DO 85 J=1,NSECT
        DIFFHT(J,3)=0.0
   85   CONTINUE
C
        NVECTR=LOOPS*(LOOPS+1)/2
        DO 100 J=1,NVECTR
        VECTOR(J)=0.0
  100   CONTINUE
C
        ID(1)=1
        DO 150 J=2,LOOPS
        ID(J)=ID(J-1)+LOOPS-J+2
  150   CONTINUE
        WRITE(3,160) (ID(K),K=1,LOOPS)
  160   FORMAT('    (ID(K),K=1,LOOPS)=',10I8)
C
        WRITE(3,290)
  290   FORMAT('1   LEVELLING SECTION DATA'/20X,'SECTION      JUNCTION POINT
       # NUMBER      LEVELLED HEIGHT      VARIANCE'/20X,'NUMBER          (FROM
       #)          (T?)         DIFFERENCE',10X,'OF LINE')
```

```
      DO 300 J=1,NSECT
      ISECT(J,1)=J
      READ(1,350) (ISECT(J,I),I=2,3),(DIFFHT(J,N),N=1,2)
  350 FORMAT(12X,2I5,F12.5,F9.2)
      DIFFHT(J,2)=SQRT(DIFFHT(J,2))*0.1*8.242E-4
      WRITE(3,575) (ISECT(J,M),M=1,3),(DIFFHT(J,N),N=1,2)
  575 FORMAT(' ',21X,I3,2(9X,I4),7X,F10.4,8X,F9.7)
  300 CONTINUE
C
      WRITE(3,410)
  410 FORMAT('1  NO.   LOOP   SECT    D(LOOP)     DIFFHT      D(LOOP)')
      DO 500 J=1,LOOPS
      READ(1,420) LOOP,LPSECT(J)
  420 FORMAT(2I3)
      J3=LPSECT(J)
      READ(1,430) (JSECT(J1,1),JSECT(J1,2),J1=1,J3)
  430 FORMAT(26I3)
      DO 450 J2=1,J3
      MATRXB(LOOP,J2)=JSECT(J2,1)*JSECT(J2,2)
      D(LOOP)=D(LOOP)-DIFFHT(JSECT(J2,1),1)*JSECT(J2,2)
  450 CONTINUE
      D2(LOOP)=D(LOOP)
      WRITE(3,490) LOOP,D(LOOP),(MATRXB(LOOP,J2),J2=1,J3)
  490 FORMAT(' ',T30,I5,F10.4,12I4)
  500 CONTINUE
C
      DO 600 L1=1,LOOPS
      DO 600 L2=1,LOOPS
      IF(L1.GT.L2) GO TO 600
      JVECTR=L1*(LOOPS-0.5*(L1-1))+(L2-LOOPS)
      J4=LPSECT(L1)
      DO 590 M1=1,J4
      MB1=MATRXB(L1,M1)
      J5=LPSECT(L2)
      DO 590 M2=1,J5
      MB2=MATRXB(L2,M2)
      IF(IABS(MB1).NE.IABS(MB2)) GO TO 590
      VECTOR(JVECTR)=VECTOR(JVECTR)+DIFFHT(IABS(MB1),2)*MB1/MB2
  590 CONTINUE
  600 CONTINUE
```

```
C
      WRITE(3,555) (L,VECTOR(L),L=1,400)
C
  555 FORMAT((' ',9(I4,F10.4)))
C     *********************************************************************
      CALL SOLN1(D,VECTOR,ID,LOOPS,3)
C     *********************************************************************
C
      WRITE(3,630) (VECTOR(JVECTR),JVECTR=1,6),(D(J),J=1,3)
  630 FORMAT(' ',9F10.4)
C
      DO 650 L=1,LOOPS
      J3=LPSECT(L)
      DO 650 N=1,J3
      MB1=MATRXB(L,N)
      MB2=IABS(MB1)
      IF(MB1) 660,650,660
  660 DIFFHT(MB2,1)=DIFFHT(MB2,1)+DIFFHT(MB2,2)*D(L)*MB1/MB2
  650 CONTINUE
C
      DO 800 L=1,LOOPS
      SBARSQ=SBARSQ+D(L)*D2(L)
      WRITE(3,550) L,SBARSQ
  550 FORMAT(' ',T20,I4,6X,F10.4)
  800 CONTINUE
C
      WRITE(3,850) (L,DIFFHT(L,1),                               L=1,NSECT)
  850 FORMAT((' ',5(I4, F10.4)))
C
C     WRITE(2,851) (L,DIFFHT(L,1),DIFFHT(L,2),L=1,NSECT)
C 851 FORMAT(3(I4,F10.4,F8.2)/)
C
      STOP
      END
```

APPENDIX 2.2

```
C       *****************SUBROUTINE SOLN1 ********************************
C
        SUBROUTINE  SOLN1(C,V,ID,LL,NW)                                  SOLN   1
C                                                                        SOLN   3
C       SUBROUTINE FOR THE SOLUTION OF BANDED SYMMETRIC FRAME STIFFNESS  SOLN   4
C       EQUATIONS BY GAUSSIAN ELIMINATION                                SOLN   5
C                                                                        SOLN   6
C       THE SIMPLE INTEGERS I,J,K REFER TO THE PIVOT ROW, THE ROW BEING  SOLN   7
C       ELIMINATED AND THE COLUMN OF THE ROW BEING ELIMINATED IF THE     SOLN   8
C       COEFFICIENTS WERE STORED IN THE FULL SQUARE MATRIX FORM. NIJ REFERSOLN   9
C       THE POSITION OF THE I,J ELEMENT IN THE VECTOR FORM. THE PROCEDURE SOLN  10
C       HAS THE SAME APPEARANCE AS STANDARD GAUSSIAN ELIMINATION.        SOLN  11
C       NOTE HOWEVER THE RANGE OF THE DO LOOPS ARE MODIFIED.             SOLN  12
C                                                                        SOLN  13
        DIMENSION  V(6),C(3),ID(3)
        DOUBLE  PRECISION  V,C,TEMP,TEMP2                                SOLN  18
        DOUBLE  PRECISION  DABS                                          SOLN  19
C                                                                        SOLN  20
C       ELIMINATION                                                      SOLN  21
C                                                                        SOLN  22
        DO 10 I=1,LL                                                     SOLN  23
        NII=ID(I)                                                        SOLN  24
        IF  (DABS(V(NII)) .GE. 1.E-20)  GO  TO  20                       SOLN  25
        WRITE(NW,30)I                                                    SOLN  26
   30   FORMAT(////,35H ***** ZERO ON DIAGONAL OF EQUATION,I3,23H SOLUTIONSOLN  27
     1  ABORTED *****)                                                   SOLN  28
        STOP                                                             SOLN  29
   20   TEMP=1./V(NII)                                                   SOLN  30
        JS=I+1                                                           SOLN  31
        IF  ( I .NE.LL)  GO  TO  4                                       SOLN  32
        NBAN =1                                                          SOLN  33
        GO  TO  6                                                        SOLN  34
    4   NBAN =ID(I+1)-ID(I)                                              SOLN  35
    6   JF=I+NBAN-1                                                      SOLN  36
C                                                                        SOLN  37
C       NORMALIZE THE I'TH ROW                                           SOLN  38
C                                                                        SOLN  39
        C(I) =C(I)*TEMP                                                  SOLN  43
```

```
      IF(JS.GT.JF) GO TO 10                                             SOLN  44
      NIJ=NII                                                           SOLN  45
      DO 40 J=JS,JF                                                     SOLN  46
      NIJ=NIJ+1                                                         SOLN  47
      V(NIJ)=V(NIJ)*TEMP                                                SOLN  48
   40 CONTINUE                                                          SOLN  49
C                                                                       SOLN  50
C     USE THIS ROW TO ELIMINATE THE TERMS IN THE I'TH COLUMN OF THE     SOLN  51
C     MATRIX OF COEFFICIENTS.   THESE TERMS ARE KNOWN FORM SYMMETRY BEFOSOLN  52
C     NORMALIZATION.                                                    SOLN  53
C                                                                       SOLN  54
      NIJ=NII                                                           SOLN  55
      DO 60 J=JS,JF                                                     SOLN  56
      NIJ=NIJ+1                                                         SOLN  57
      IF  (DABS(V(NIJ)) .LE. 1.E-20)  GO  TO  60                        SOLN  58
      TEMP2=V(NIJ)/TEMP                                                 SOLN  59
      NIK=NIJ-1                                                         SOLN  60
      NJK=ID(J)-1                                                       SOLN  61
      DO 70 K=J,JF                                                      SOLN  62
      NIK=NIK+1                                                         SOLN  63
      NJK=NJK+1                                                         SOLN  64
      V(NJK)=V(NJK)-TEMP2*V(NIK)                                        SOLN  65
   70 CONTINUE                                                          SOLN  66
      C(J) =C(J)-TEMP2*C(I)                                             SOLN  70
   60 CONTINUE                                                          SOLN  71
   10 CONTINUE                                                          SOLN  72
C                                                                       SOLN  73
C     BACK SUBSTITUTION                                                 SOLN  74
C                                                                       SOLN  75
      LL1=LL-1                                                          SOLN  76
      DO 90 IN=1,LL1                                                    SOLN  77
      I=LL-IN                                                           SOLN  78
      NIJ=ID(I)                                                         SOLN  79
      JS=I+1                                                            SOLN  80
      JF=I+ID(I+1)-NIJ-1                                                SOLN  81
      DO 91 J=JS,JF                                                     SOLN  82
      NIJ=NIJ+1                                                         SOLN  83
      C(I) =C(I)-C(J)*V(NIJ)                                            SOLN  87
   91 CONTINUE                                                          SOLN  88
   90 CONTINUE                                                          SOLN  89
      RETURN                                                            SOLN  90
      END                                                               SOLN  91
```

```
C         ******************** ADJANAL (PART 1) ****************************
C
C         ******************************************************************
C         * ESTIMATION OF STANDARD DEVIATION OF GEOPOTENTIAL DIFFERENCE IN *
C         * ADJUSTED NETWORK. PART 1: INVERSION OF MATRIX OF NORMALS       *
C         ******************************************************************
C
          READ( ,5 ) LOOPS,NSECT,LIMIT1,LIMIT2,LIMIT3,LIMIT4,PREC,MAXSEC
          FORMAT(6I4,F8.7,I4)
          WRITE(3,75) LOOPS,NSECT,LIMIT1,LIMIT2,LIMIT3,LIMIT4,PREC,MAXSEC
     75   FORMAT('   IN THE NETWORK, THERE ARE',I3,' LOOPS,',I3,'SECTIONS',4I
         *4,F9.7,I7)
C
          DO 6   J=1,LOOPS
          DO 6   J1=1,MAXSEC
          MATRXB(J,J1)=0
      6   CONTINUE
          DO 35  J=1,NSECT
          DIFFHT(J,3)=0.0
     35   CONTINUE
C
          NVECTR=LOOPS*LOOPS
          DO 6   J=1,NVECTR
          VECTR (J)=0.
          CONTINUE
C
C         ******************************************************************
          READ( ,65 ) (DIFFHT(L,1),DIFFHT(L,2),L=1,NSECT)
     65   FORMAT(5(4X,F1 .4,F8.2))
          WRITE(3,70 ) (DIFFHT(L,1),DIFFHT(L,2),L=1,NSECT)
     70   FORMAT(' ',(5(F 0.2,4X)))
C
          WRITE(3,400)
    400   FORMAT('      NO.   LOOP   SECT    D(LOOP)     DIFFHT      D(LOOP)')
          DO 50  J=1,LOOPS
          READ( ,420) LOOP,LPSECT(J)
    420   FORMAT(2I )
          J3=LPSECT(J)
          READ( ,430 ) (JSECT(J1,1),JSECT(J1,2),J1=1,J3)
    430   FORMAT( I )
          DO 45  J2=1,J3
          MATRXB(LOOP,J2)=JSECT(J2,1)*JSECT(J2,2)
     45   CONTINUE
```

```
      WRITE(3,490) LOOP,              (MATRXB(LOOP,J2),J2=1,J3)
  490 FORMAT(' ',T30,I5,            12I4)
  500 CONTINUE
C
      DO 600 L1=1,LOOPS
      DO 600 L2=1,LOOPS
C
      JVECT1=LOOPS*(L1-1)+L2
      IF(L1.GT.L2) GO TO 595
C
      J4=LPSECT(L1)
      DO 590 M1=1,J4
      MB1=MATRXB(L1,M1)
      J5=LPSECT(L2)
      DO 590 M2=1,J5
      MB2=MATRXB(L2,M2)
      IF(IABS(MB1).NE.IABS(MB2)) GO TO 590
      VECTR1(JVECT1)=VECTR1(JVECT1)+DIFFHT(IABS(MB1),2)*MB1/MB2
  590 CONTINUE
      GO TO 600
C
  595 JVECT2=LOOPS*(L2-1)+L1
      VECTR1(JVECT1)=VECTR1(JVECT2)
  600 CONTINUE
C     **********************************************************************
      CALL MATI3D(VECTR1,LOOPS,LOOPS,LIG,PREC)
C     **********************************************************************
      WRITE(7) LOOPS,LPSECT,MATRXB,NSECT,DIFFHT,LIMIT1,LIMIT2,LIMIT3,LIM
     #IT4,VECTR1,GBQBG
C
      WRITE(3,610) (VECTR1(L),L=1,9)
  610 FORMAT(9F10.4)
C     **********************************************************************
      REWIND 7
```

APPENDIX 2.4

```
      SUBROUTINE MAT13D(A,N,NDIM,LIG,PREC)                           M13     1
C
C     ***************** SUBROUTINE MAT13D ****************************
C
C     ***************************************************************
C    MATINV=INVERSION, PARTIAL PIVOT SELECTION ONLY
C    MATRIX INVERSION ROUTINE WITH PIVOT SELECTION OPTIMISATION      M13     2
C    THE ORIGINAL -A- MATRIX IS DESTROYED                            M13     3
C    IF A PIVOT IS LESS THAN -PREC- THE ROUTINE PRINT AN ERROR MESSAGE AM13  4
C    GOES IMMEDIATELY TO RETURN                                      M13     5
C     ***************************************************************M13     6
C
      DIMENSION A(68121),LIG(261)
      DO 5J=1,N                                                      M13     8
    5 LIG(J) = 0                                                     M13     8
      KN=N*NDIM                                                      M13    10
      DO4K=1,N                                                       M13    11
      IF(MOD(K,10).NE.0) GO TO 100
C    ORDERING THE PIVOTS
  100 KK= (K-1)*NDIM+K                                               M13    12
      PVT =   ABS(A(KK))                                             M13    13
      DO 15 L=KK,KN,NDIM
      P1 =   ABS(A(L))                                               M13    15
      IF(P1-PVT)15,15,16
   16 PVT=P1                                                         M13    17
      LIG(K)=(L-K)/NDIM+1                                            M13    18
   15 CONTINUE                                                       M13    19
      IF(PVT.LT.PREC)GO TO 21                                        M13    20
      IF(LIG(K).EQ.K)GO TO 6                                         M13    21
      IX=(LIG(K)-K)*NDIM                                             M13    22
      LM=(LIG(K)-1)*NDIM+1                                           M13    23
      LL=LM+N-1                                                      M13    24
      DO 17L=LM,LL                                                   M13    25
      AP=A(L)                                                        M13    26
      LX=L-IX                                                        M13    27
      A(L)=A(LX)                                                     M13    28
   17 A(LX)=AP                                                       M13    29
                                                                     M13    30
```

```
C     INVERSION
    6 COM=A(KK)
      A(KK)=1.
      DO 1 J=K,KN,NDIM
    1 A(J)=A(J)/COM
      DO 4 I=1,N
      IX=I-K
      IF(IX)2,4,2
    2 IK=(K-1)*NDIM+I
      COM=A(IK)
      A(IK)=0.
      DO 3 J=I,KN,NDIM
      KJ=J-IX
    3 A(J)=A(J)-COM*A(KJ)
    4 CONTINUE
C     REORDERING THE ROWS AND COLUMNS
      K=N
   18 IF(LIG(K).EQ.0)GO TO 20
      L=LIG(K)
      DO 19 M=L,KN,NDIM
      AP=A(M)
      KM=M-L+K
      A(M)=A(KM)
   19 A(KM)=AP
   20 K=K-1
      IF(K.NE.0) GO TO 18
      LIG(1)=
      RETURN
   22 LIG(1)=
      RETURN
      END
```

APPENDIX 1.5

```
C      ****************  ADJANAL (PART 3) ***********************************
C
C      ***********************************************************************
C      * ESTIMATION OF STANDARD DEVIATION OF GEOPOTENTIAL DIFFERENCES IN*
C      * ADJUSTED NETWORK. PART 2: APPLICATION OF GENERAL LAW OF          *
C      * PROPOGATION OF VARIANCES TO VARIANCES/COVARIANCES OF ADJUSTED    *
C      * POTENTIAL DIFFERENCES                                             *
C      ***********************************************************************
C
       DIMENSION DIFFHT(757,3),JSECT(12,2),MATRXB(261,12),LPSECT(261),GBQB
      #BG(128,128),VECTR1(6821),LIG(261)
C
       READ(7) LOOPS,LPSECT,MATRXB,NSECT,DIFFHT,LIMIT1,LIMIT2,LIMIT3,LIMI
      #T4,VECTR1,GBQBG
C
       READ(2,11) LIMIT1,LIMIT2,LIMIT3,LIMIT4
    11 FORMAT(4I3)
C      ***********************************************************************
       CALL Q(LOOPS,LPSECT,MATRXB,NSECT,DIFFHT,LIMIT1,LIMIT2,LIMIT3,LIMIT
      #4,VECTR1,GBQBG)
C      ***********************************************************************
       CALL GLPV(GBQBG)
C      ***********************************************************************
C
       STOP
       END
```

APPENDIX [illegible]

```
      SUBROUTINE Q(LOOPS,LPSECT,MATRXB,NSECT,DIFFHT,LIMIT1,LIMIT2,LIMIT3
     #,LIMIT4,VECTR1,GBQBG)
C     *********************** SUBROUTINE Q ****************************
C
C     ****************************************************************
C     * TO FORM THE MATRICES USED TO CALCULATE STANDARD DEVIATION OF  *
C     * DIFFERENCES IN GEOPOTENTIAL IN AUSTRALIAN NET.                *
C     ****************************************************************
C
      DIMENSION LPSECT(261),MATRXB(261,12),DIFFHT(757,3),GBQ(128,260),GBQ
     #QBG(128,128),VECTR1(68121)
C
      IRANGE=LIMIT4-LIMIT3+LIMIT2-LIMIT1+2
      DO 100 L=1,IRANGE
      DO 150 L1=1,LOOPS
      GBQ(L,L1)=0.0
  150 CONTINUE
      DO 100 L1=1,IRANGE
      GBQBG(L,L1)=0.0
  100 CONTINUE
C
C
      DO 750 L=1,LOOPS
      L6=LPSECT(L)
      DO 740 L1=1,L6
      L2=MATRXB(L,L1)
      L3=IABS(L2)
      IF(L3.LT.LIMIT1.OR.L3.GT.LIMIT2) GO TO 71
      L31=L3-LIMIT1+1
      GO TO 72
   71 IF(L3.LT.LIMIT3.OR.L3.GT.LIMIT4) GO TO 74
      L31=L3-LIMIT1+LIMIT2-LIMIT3+2
   72 DO 73 L5=1,LOOPS
      VEC=VECTR1((LOOPS*(L5-1)+L))
      GBQ(L31,L5)=GBQ(L31,L5)+L3/L2*VEC*DIFFHT(L3,2)
   73 CONTINUE
   74 CONTINUE
  750 CONTINUE
```

```
      WRITE(3,724) L,L1,L2,L3,L31,L5,L6,VEC,DIFFHT(L3,2),GBQ(L31,L5)
  724 FORMAT('          L,L1,L2,L3,L31,L5,L6,VEC,DIFFHT(L3,2),GBQ(L31,L5)',7I
     #4,3F10.4)
C
C                 ****************************************************
C                 * CALCULATE THE MATRIX Q(XX)                       *
C                 ****************************************************
C
      DO 751 L=1,LOOPS
      L6=LPSECT(L)
      DO 741 L1=1,L6
      L2=MATRXB(L,L1)
      L3=IABS(L2)
      IF(L3.LT.LIMIT1.OR.L3.GT.LIMIT2) GO TO 711
      L31=L3-LIMIT1+1
      GO TO 742
  711 IF(L3.LT.LIMIT3.OR.L3.GT.LIMIT4) GO TO 741
      L31=L3-LIMIT1+LIMIT2-LIMIT3+2
  742 DO 731 L5=1,NSECT
      IF(L5.LT.L3) GO TO 731
  721 IF(L5.LT.LIMIT1.OR.L5.GT.LIMIT2) GO TO 712
      L51=L5-LIMIT1+1
      GO TO 743
  712 IF(L5.LT.LIMIT3.OR.L5.GT.LIMIT4) GO TO 731
      L51=L5-LIMIT1+LIMIT2-LIMIT3+2
  743 GBQBG(L51,L31)=GBQBG(L51,L31)-L3/L2*GBQ(L51,L)*DIFFHT(L3, 2)
C
  731 CONTINUE
  741 CONTINUE
  751 CONTINUE
```

```
      WRITE(3,732) L,L1,L2,L3,L31,L5,L51,L6,   GBQ(L51,L),DIFFHT(L3, 2),G
     #BQBG(L51,L31)
  732 FORMAT('    8I4,3F10.4)
C     ****************************************************************
C
      IRANGE=LIMIT2-LIMIT1+1
      JRANGE=LIMIT4-LIMIT3+1
      KRANGE=IRANGE+JRANGE
      DO 850 L=1,KRANGE
      IF(L.GT.IRANGE) GO TO 810
      L1=LIMIT1+L-1
      GO TO 820
  810 L1=LIMIT3+L-1-IRANGE
  820 GBQBG(L,L)=GBQBG(L,L)+DIFFHT(L1,2)
  850 CONTINUE
      WRITE(3,840) (GBQBG(L,L),L=1,IRANGE)
  840 FORMAT(' ',10F10.4)
C
      RETURN
      END
```

APPENDIX .7

```
      SUBROUTINE GLPV(GBQBG)
C     ********************** SUBROUTINE GLPV **************************
C
C     *****************************************************************
C     * VARIANCE OF REDUCED GEOPOTENTIAL AT ONE TIDE-GAUGE CALCULATED *
C     * WITH RESPECT TO GEOPOTENTIAL AT ANOTHER GAUGE, USING GENERAL  *
C     * LAW OF PROPOGATION OF VARIANCES. REQUIRES A LEVELLING ROUTE   *
C     *****************************************************************
C
      DIMENSION IAR(128,2),GBQBG(128,128)
      SIGMA=0.
C
      READ(1,100) IRANGE
  100 FORMAT(I3)
      WRITE(3,150) IRANGE
  150 FORMAT(' IRANGE=',I3)
      READ(1,200) (IAR(L,1),IAR(L,2),L=1,IRANGE)
  200 FORMAT(2I4)
      WRITE(3,250) (IAR(L,1),IAR(L,2),L=1,IRANGE)
  250 FORMAT(' ',2I5)
      DO 400 L=1,IRANGE
      L1=IAR(L,2)
      L4=IABS(L1)
      DO 400 L2=1,IRANGE
      L3=IAR(L2,2)
      L5=IABS(L3)
      IF(L4.LT.L5) GO TO 350
  275 SIGMA=SIGMA+GBQBG(L4,L5)*L1*L3/L4/L5
      GO TO 400
  350 SIGMA=SIGMA+GBQBG(L5,L4)*L1*L3/L4/L5
  400 CONTINUE
C
      WRITE(3,300) L,L1,L2,L3,L4,L5,GBQBG(L4,L5),SIGMA,IRANGE
  300 FORMAT(' L,L1,L2,L3,L4,L5,GBQBG(L4,L5),SIGMA & IRANGE ARE',6I4,F10
     #.4,F10.5,I4)
C
      RETURN
      END
```

APPENDIX 3

Computer programme for estimating the mean value of a sine curve from limited sampling

APPENDIX I

```
C       ************************* SINE ************************************
C
        DIMENSION MULTS(29),LIST(21)
        PI=3.14159
        IX=1234567
        IY=1334567
C
        DO 100 I=1,21
        LIST(I)=0.0
  100   CONTINUE
C
        READ(1,200) IORBIT,FREQ,NTEST
  200   FORMAT(I3,F3.1,I3)
        WRITE(3,250) IORBIT,FREQ,NTEST
  250   FORMAT('    NUMBER OF ORBITS: ',I4/'    SINE FREQUENCY: ',F3.1/'    N
       #UMBER OF TEST RUNS: ',I4)
C
        TWOPIF=2.0*PI*FREQ
        READ(1,300)(MULTS(I),I=1,IORBIT),ISUM
  300   FORMAT(29I2,I4)
        WRITE(3,350) (MULTS(I),I=1,IORBIT),ISUM
  350   FORMAT('    LIST OF ORBITAL WEIGHTS: ',29I3/'  SUM OF WEIGHTS: ',I4
       #)
C
        DO [illegible] ITEST=1,NTEST
        YSUM=0.0
C
```

```
      DO 600 I=1,IORBIT
      IX=IY
C
C     ******************************************************************
      CALL RANDUM(IX,IY,YFL)
C     ******************************************************************
      Y=SIN(TWOPIF*YFL)*MULTS(I)
      YSUM=YSUM+Y
  600 CONTINUE
C
      YMEAN=(ABS(YSUM/ISUM))*20.0+1.0
      MEAN=(IABS(INT(YSUM/ISUM*20)))+1
      DIFF=YMEAN-MEAN
      IF(DIFF.LT.0.5) GO TO 650
      MEAN=MEAN+1
  650 LIST(MEAN)=LIST(MEAN)+1
  800 CONTINUE
C
      WRITE(3,900) (LIST(I),I=1,21)
  900 FORMAT('  DISTRIBUTION OF MEANS: ',21I3)
C
      STOP
      END
```

```
      SUBROUTINE RANDOM(IX,IY,YFL)
C
C     ********************** SUBROUTINE RANDOM ************************
C
C     *****************************************************************
C     * RANDOM NUMBER GENERATOR: SEE IBM SCIENTIFIC-SUBROUTINE-PACKAGE *
C     *****************************************************************
      IY=IX*65539
      IF(IY) 5,6,6
    5 IY=IY+2147483647+1
    6 YFL=IY
      YFL=YFL*.4656613E-9
C
      RETURN
      END
```

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