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ASTRONOMICAL PROGRAMS FOR THE STUDY OF SECULAR VARIATIONS IN POSITION †

ABSTRACT

Secular changes in relative position might be detected by stations which observe common stars. Five chains of stations on nearly the same latitude are in operation: the International Latitude Service (ILS), three PZT chains, and one astrolabe chain.

The latitude and longitude of a station change secularly in consequence of the secular motion of the mean pole of rotation. Corrections for such changes can be obtained from the secular motion determined from ILS observations.

At initial determination is made of the secular motion from BIH observations.

1. Introduction

Early theories of continental drift envisioned large drift rates. In a few cases large differences in position were obtained for two epochs of observation, e.g. 3" in latitude and 5 s in longitude for a field station in Greenland, for epochs 1870 and 1932. These large changes, however, could be accounted for by errors in the methods of observation. Continued observations at permanent observatories indicated that secular changes in position are very small, if not null.

Modern theories of continental drift, based on plate motions, lead to much smaller estimates of relative continental drift rates, of the order of 3 cm/yr. This corresponds to 0\".001/yr in arc and, for middle latitudes, about 0.1 ms/yr in longitude. The detection of such small drift rates through optical astronomy is possible but difficult. Not only is the rate extremely small, but new, precise definitions of reference axes and co-ordinates are required. The basic problem in formulating precise definitions is that no set of axes "fixed in the Earth", with respect to which positions may be referred, exists.

The detection of secular changes in position is simplified if the participating stations observe stars in common and use the same reduction constants. This eliminates errors in star positions, proper motions, and the observational ephemeris. The stations must therefore be on nearly the same latitudde. Five such stations, listed in table 1, are now in operation.

The International Latitude Service (ILS), which began observing in September 1899, was the first astronomical program formed in which all stations observe the same stars. Observations made during the past 74 years provide information on the periodic motions of the instantaneous axis of rotation about the axis of figure and the secular motion of the latter. The secular motion of the axis of figure, whose pole is identified with the mean pole of rotation, causes a drift in latitude and longitude which is not related to crustal displacements. Hence, this secular motion must be taken into account in determining relative drift in position.

† *Revised February 1974*

TABLE 1. Astronomical Chains

	Longitude	Latitude
1. ILS		
Mizusawa (Japan)	141° E	+39° 8' 3".602
Kitab (USSR)	67° E	1.850
Carloforte (Italy)	8° E	8.941
Gaithersburg (USA)	77° W	13.202
Ukiah (USA)	123° W	12.096
2. PZT		
Washington (USA)	77° W	+38° 55'
Mizusawa (Japan)	141° E	+39° 8'
Mt. Stromlo (Australia)	149° E	-35° 19'
Punta Indio (Argentina)	57° W	-35° 21'
Herstmonceux (UK)	0°	+50° 52'
Calgary (Canada)	114° W	+50° 52'
3. Astrolabe		
Paris (France)	2° E	+48° 50'
Dolbeau (Canada)	72° W	+48° 50'

2. The Gravity Vector; Astronomical Observations

Astronomical determinations of latitude and longitude are based on two axes, the *vertical* and the *instantaneous axis of rotation.*

At any point, P, on the Earth there exists a gravity vector $\underset{\sim}{G}$, whose direction is that of the vertical (plumb line) and whose magnitude is $\underline{g}$, the acceleration of gravity at P. $\underset{\sim}{G}$ is the resultant of the attractions of the Earth, Moon, and Sun and the centrifugal force of rotation. The magnitude undergoes variations, periodic certainly and secular probably.

When we say that $\underset{\sim}{G}$ is variable we must state with respect to what the variation occurs. The answer is simple as regards the magnitude, $\underline{g}$, which is measured in units of length and time. These units are defined by two electromagnetic radiations produced by quantum transitions: 1 metre = 1 650 763.73 wavelengths of a specific Kr-86 transition, and 1 second = 9 192 631 770 periods of a specific Cs-133 transition. Consideration is being given to using one transition to define both the units of length and time, and to using the speed of light as a base SI unit. However, no matter what changes are made two adopted constants will be involved in the measurement of $\underline{g}$.

Specifying the direction of $\underset{\sim}{G}$, rigorously, presents difficulties. At any instant $\underset{\sim}{G}$ has a definite orientation with respect to a set of axes fixed with respect to external galaxies. The orientation changes rapidly, however, and we wish to refer $\underset{\sim}{G}$ to axes fixed in the Earth. The Earth, however, is not a rigid body, and such axes cannot be defined rigorously.

Let E_r denote a fictitious rigid body which approximates the Earth. Associated with E_r is an axis of figure, A_F, an instantaneous axis of rotation, A_I, and a momentum axis, A_M. These are co-planar with A_M in between and close to A_I (about 3 cm at the Earth's surface, at maximum, for the motion observed). The 3 axes describe motions about each other in accordance with the representation of L. Poinsot, which involves rolling cones. A_M undergoes precession and nutation with respect to extra-galactic axes. A_F is fixed with respect to E_r, and A_I undergoes periodic motion about A_F (Euler nutation).

The actual Earth is not rigid and A_F does not necessarily retain a fixed position relative to the crust. However, there is an instantaneous axis for all practical observational purposes, A_I, which is used as a primary reference. It is the angles between the vertical at each station and their relation to A_I, which define the astronomical longitude and latitude of each station.

The newer techniques such as Doppler, laser ranging of the Moon and artificial satellites, and VLBI, do not employ the vertical. Hence, they cannot be used to determine astronomical longitude and latitude; they are used to determine other co-ordinates.

The co-latitude is the angle between the vertical and A_I. Latitude and time are determined in practice with specialised instruments which determine the angle between the zenith and stars whose *relative positions* are known with very high accuracy. The zenith is determined with a mercury reflecting basin or with a level.

The ILS chain includes 5 stations. A solution is made each month for 3 unknowns, x and y, the coordinates of the pole, and z, a constant term. All errors which affect the 5 stations alike, such as errors in star positions, proper motions, and constants of reduction, are absorbed by z. The observational data are the differences in the 5 mean monthly latitudes from the 5 initial latitudes adopted for each station.

3. Recently Started Programs

Beginning about 1957, and partly in connection with the International Geophysical year (IGY), a number of photographic zenith tubes (PZT) and astrolabes came into use. These instruments determine both time and latitude, and with high precision.

In consequence of the interest in verifying the hypothesis of continental drift, steps were taken to utilise these newer instruments in programs for the detection of secular variations in position. Definite secular changes in latitude had not been detected by the ILS observations. However, modern theories indicate that relative drifts are occurring chiefly in longitude; hence, the importance of time determinations with common stars.

A symposium on Continental Drift, Secular Motion of the Pole, and Rotation of the Earth, organised by the International Astronomical Union (IAU) in cooperation with the International Union of Geodesy and Geophysics (IUGG), was held in Stresa, Italy, in March 1967 (MARKOWITZ & GUINOT 1968). The symposium recommended that chains of two or more PZT's or astrolabes be established on nearly the same parallel of latitude, to observe the same stars. Also, the symposium adopted initial latitudes for the ILS stations, given in Table 1. These define an origin for the position of the pole, now called the *Conventional International Origin*, or CIO. These recommendations were adopted by both the IAU and the IUGG in 1967.

Chains in operation are listed in table 1. The Washington and Mizusawa PZT's have only about one-half of their lists in common. This, however, is sufficient to link the star lists. S. Takagi, at Mizusawa, has begun analyzing the observations.

Steps are being taken to install a PZT at each ILS station in addition to the visual latitude instrument. Only Mizusawa has a PZT now. The completion of a 5-station ILS-PZT chain would be of very high value in studying the secular motion of the pole and secular changes in position of stations.

4. The CIO

The reson for adopting the CIO was to end the confusion which had arisen because a multitude of origins had come into use, often not specifically defined. The CIO is specifically defined by the adopted initial latitudes of the 5 ILS stations, which may be regarded as five arbitrary constants. It does not matter what constants were selectdd since we are interested chiefly in the motion of the pole, and not in its absolute position. Nor does it matter that the 5 ILS stations may be in motion. The important thing is that the CIO provides a standard origin for specifying the latitude and longitude of a station and the co-ordinates of the pole.

5. Secular Motion of the Pole

A displacement of the pole of rotation from the origin to a point at x, y (with the conventional notation) causes changes in latitude, ϕ , and longitude, λ , as follows:

$$\phi = \phi_o + x \cos \lambda + y \sin \lambda \quad , \qquad (1)$$

$$\lambda = \lambda_o + (x \sin \lambda - y \cos \lambda)(\tan \phi)/15 \; , \qquad (2)$$

where ϕ_o and λ_o are the latitude and longitude when the pole is at the origin. (Note: West longitude is reckoned positive in astronomy; λ is in time in equation (2))

Secular changes in x and y would cause secular changes in latitudes and longitudes not caused by crustal displacements.

Figure 1 shows the recent motion of the pole of rotation and the motion of the mean pole since 1903.0, as derived from ILS observations.

6. Characteristics of the Secular Motion

The nature of the secular motion and indeed, its reality, have long been the subject of discussion.

The ILS secular motion was first determined, in 1916, by B. Wanach from a short interval, 1900-1915.

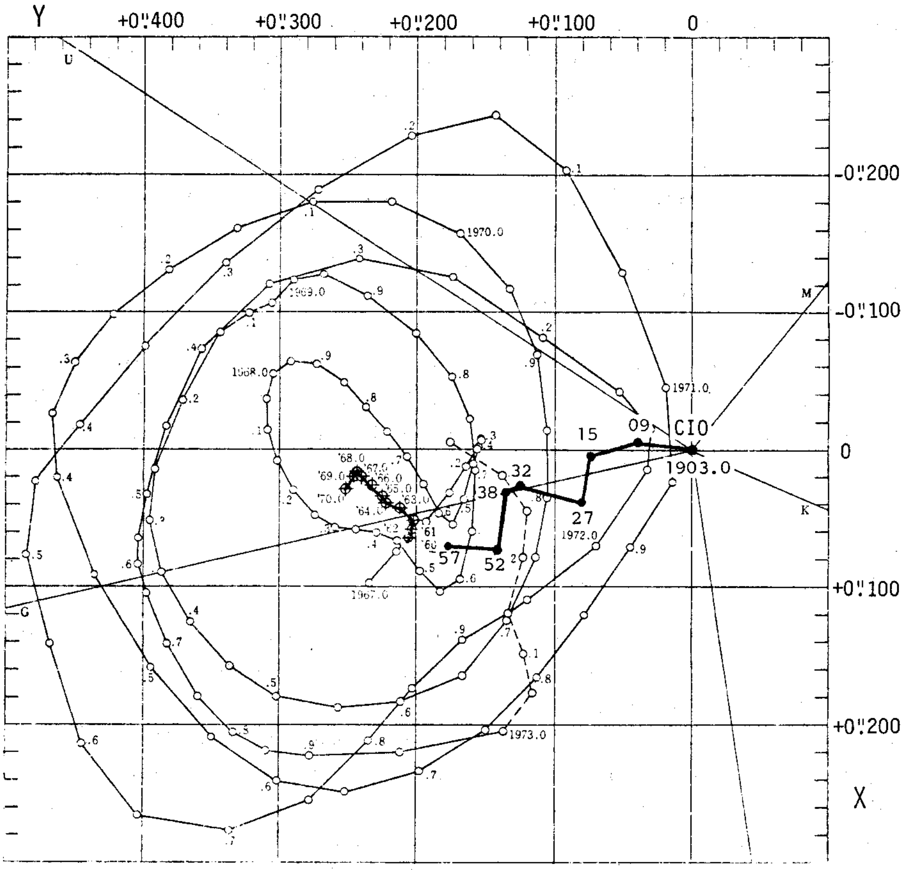

Figure 1. Secular Motion of the Pole

The ILS polar motion for 1968.00 to 1973.35; from YUMI (1973b). Crosses mark the position of the mean pole from 1960.0 to 1968.0, obtained by Yumi, based on 6-year running means of **x** and **y**. Solid circles are 6-year means for epochs from 1903.0 to 1957.0; from (MARKOWITZ 1960).

He obtained 0".003/yr along 55°W. N. Sekiguchi found in 1954 that a random motion was superimposed on a progressive motion. MARKOWITZ (1960) found that the secular motion was well represented by a progressive component of 0".0032/yr along 60°W plus a librational component of period 24 years. He noted that the motion is empirical and it was not known whether it would continue. Figure 1 shows that the progressive and librational motions have continued, although the librational motion is not strictly of period 24 yr.

The ILS observations thus indicate a secular motion of the mean pole relative to the stations. However, suggestions have been made that this motion is not a real motion of the pole. Alternate explanations which have been given are that the apparent motion is due to

(a) errors in proper motions, or

(b) non-polar changes in latitude of some stations, due either to crustal drift or to observational effects.

A study of the way in which the *definitive* ILS polar motion co-ordinates, x and y, are derived shows that they are independent of any external catalogues, e.g., GC or FK4, and therefore, of their errors in position or proper motion. The computation of control latitudes (MARKOWITZ 1961) confirms this independence. Hence explanation (a) is excluded, although (b) remains.

Plots of the variations in latitude of the 5 ILS stations showed progressive and oscillatory changes which agree in magnitude, phase, and sign with those expected from the derived progressive and librational motions of the mean pole (MARKOWITZ 1968; MARKOWITZ 1970). It is difficult to see how the phases of the variations at the five stations could maintain the proper relation if the variations were due to independent effects at the stations.

Nevertheless, an independent determination of the secular motion was greatly desired, and it was therefore decided to begin a study of the secular motion provided by the observations of the Bureau International de l'Heure (BIH). The importance of the BIH secular motion lies in the large number of stations which form the basic BIH system.

7. 1968 BIH System

In 1967 the BIH introduced a new method of deriving the polar motion, from both latitude and time observations, on a rigorously homogeneous basis (GUINOT & FEISSEL 1968). Details are given by FEISSEL (1972). The BIH reference system, called the *1968 BIH System*, is defined by a weighted set of instruments which were in operation in 1966. This includes 34 for latitude and 17 for time, with appreciable weight, at 29 observatories. The 5 ILS instruments are included, but their weight is only 0.09 of the total. Corrections are applied each year so that the reference system remains that of the 1968 BIH System. The system is unaffected by the addition of new participating stations, changes in catalogues used, or changes in the adopted co-ordinates of any station.

Corrections were applied back to 1962.0. However, as FEISSEL (1972) noted, the accuracy of the early years, particularly 1962 and 1963 is reduced; fewer stations were in operation than in 1966, and some corrections were uncertain. The annual differences in both x and y between the BIH and ILS show discontinuities at 1964/65, which indicates that the results from 1965 on are on the 1968 BIH System, but those for 1962, 1963, and 1964 may not be.

7.1. Proper Motions

The equations of condition contain x and y with coefficients sin λ, and cos λ, where λ is the station's longitude. In consequence, a fictitious, *constant* drift of the mean pole could occur if the average error in proper motion (in either co-ordinate) is correlated with longitude. There is no drift if the same error affects all stations.

The BIH stations, with few exceptions, are independent. Proper motions are tied nominally to the same system, FK4, but residual differences between stations may exist. GUINOT & FEISSEL (1968) estimated that the probable error of the fictitious drift is 0".000 2/yr. My estimate is of the same order. The fictitious drift due to proper motion errors is negligible for present purposes.

7.2 Systematic Errors

Because of the large number of instruments which form the 1968 BIH System, the BIH secular motion is much less affected than the ILS by systematic errors which may affect one or two stations, sometimes for years.

7.3 BIH Secular Motion

It follows that the BIH secular motion is determined with high accuracy, is negligibly affected by errors in proper motion, and is little affected by systematic errors at individual stations. We may use the results from 1965 with high confidence to check the ILS secular motion.

8. Comparison of Secular Motions

The co-ordinates of the barycentre are the 6-yr running means, which include nearly 5 Chandler periods, of **x** and **y**. The ILS values for the mean epochs 1965.0 to 1970.0 were given by YUMI (1973a; 1973b), and the BIH values were kindly furnished by Dr. B. GUINOT. The results are shown in figure 2. The BIH 6-yr means for epochs 1965.0 to 1967.0 include observations made through 1964, and the corresponding motion is shown dotted.

The ILS and BIH secular motions show certain similarities. A marked change in direction is indicated for the ILS about 1968 and for the BIH about 1967. More significant, however, is the similarity of motions during the last 8 years, which is given by mean epochs 1968.0 to 1970.0. The ILS motion is 0".007/yr along 37° W longitude and the BIH is 0".005/yr along 24° W, which is fairly good agreement. This 8-yr interval is too small, however, to permit drawing conclusions regarding the secular motion. We may be able to do so by 1980.

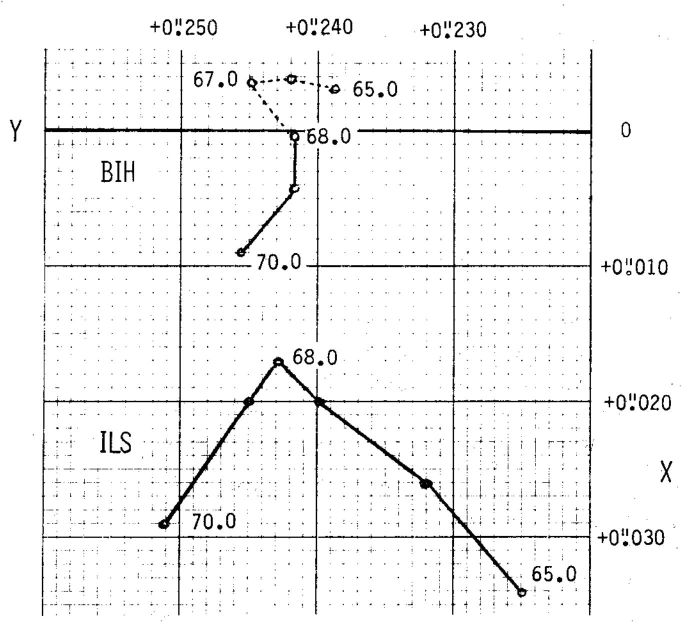

Figure 2. Mean Pole
ILS and BIH positions from 6-yr running means of **x** and **y**.

9. Discussion and Summary

The Earth is not a rigid body, and no axes fixed in the Earth can be defined to serve as a reference system for measuring secular changes in position. Use is made of the instantaneous axis of rotation and the vertical at a station to define astronomical latitude and longitude. These are determined with classical optical instruments, e.g., zenith telescope, PZT, and astrolabe.

The CIO, based on ILS observations, serves as a standard origin for latitude and longitude and for the polar motion.

The newer techniques, such as Doppler, laser ranging, and VLBI, do not employ the vertical; these would detect continental drift by techniques other than changes in astronomical latitude and longitude.

The axis of figure of the Earth, identified with the mean pole of rotation, has a secular motion, as indicated by the ILS observations. The BIH observations during the last 8 years indicate a similar secular motion, but the interval is too short to permit drawing conclusions. In a few more years the combination of ILS and BIH results should provide a reliable determination of the secular motion.

Several chains of stations on nearly the same parallel of latitude have been established to observe stars in common. These might detect secular changes in astronomical latitude and longitude.

The probable error of a yearly mean difference in longitude between two PZT's or astrolabes, as indicated by the BIH results, is about 3 ms. The probable error in drift rate would be about 0.1 ms/yr for 20 years of continuous observation and about 0.03 ms/yr for 40 years. If relative secular drifts in longitude of 0.1 ms/yr (about 3 cm/yr at latitude 45°) are occurring then we may obtain some evidence through classical optical astronomy towards the end of this century.

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11. Discussion

FREISLICH: Can you comment on the relative accuracy of the astrolabe as compared with the PZT?

MARKOWITZ: I cannot give a (simple) answer to this question.

BOMFORD: These days we do not think of our latitude observatories as fixed points. They are on tectonic plates. There may be no real secular motion of the pole at all. It may just be that the five International Latitude Stations and the thirty BIH stations are moving about with their tectonic plates, and what appears at the moment to be secular motion of the pole may just be random noise as the stations move around. Has this argument been put to you before and is there any good compelling argument in the other direction?

MARKOWITZ: What is obtained is the relative motion between the mean pole and the observing stations. Since the Earth is not rigid, one cannot say whether the stations are moving or the pole is moving. The suggestion that there is plate motion has been made before. However the problem which has not yet definitely been solved is whether the apparent motion observed is real or arises from local observational effects. If it turns out that the motion is real then we can attack the problem of trying to determine what is moving.

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AN ASTRONOMICAL MEASUREMENT OF THE PRESENT DAY DRIFT OF THE EURASIAN AND AMERICAN PLATES

ABSTRACT

A measurement of the displacement of the Eurasian and American plates on the Earth's spherical surface is made by using the 1962-1972 local drifts in latitude and longitude of 45 astronomical observatories with respect to the coordinates reference system of the Bureau International de l'Heure (BIH).

These displacements are described by three parameters: geographical coordinates of the rotation pole and angular velocity. Opposite to the geophysical computation of these parameters, which give the relative motions of the plates over several million years, the astronomical data used make it possible to measure the relative as well as the absolute (relative to an external reference system) motions over a short time period.

However, the noise level of one annual mean position derived from the observations is larger than the drift, the method used by the BIH in order to preserve a fixed reference system in the computation of the pole motion and the rotation of the earth gives local drifts which are homogeneous between 1962 and 1972 and independent of the Earth's rotation pole drift. Among the 80 observatories which measured time and/or latitude during this period, it was possible to compute one or both components of the local drifts of 35 stations belonging to the Eurasian plate and 10 belonging to the American plate.

For each of those two plates, the rotation parameters are obtained by a least-square fitting of the individual drifts of the stations it bears. The coordinates found for the Eulerian pole of their relative motion is in accordance with the geophysical results, while the angular velocity is found larger than the one admitted for the past 100 million years. The parameters of the absolute motion of the two plates are also computed, the BIH coordinates system being taken as an external reference system.

1. Introduction

The parameters of the relative motion of Eurasia and America have been determined on the basis of geophysical data. They are deduced from the relative displacement along the common boundary of the two plates and they describe the movement in a geological time scale. The geographical coordinates (Φ, Λ) of the eulerian pole of the rotation and the amplitude (ω) of the rotation vector according to different authors are given in Table 1.

Table 1

Rotation parameters of the motion of America relative to Eurasia

Reference	Φ	Λ	ω (10^{-7}deg/yr)	Name in Figure 1
BULLARD et al (1965)	73°N	97°E	--	BU 65
LE PICHON (1968)	78°N	102°E	- 2.8	LP 68
LE PICHON (1971)	79°N	16°W	--	LP 71
CHASE (1972)	48°N	155°E	- 2.36	CH 72

2. The Astronomical Measurements

A completely different kind of measurement of the motions can be made by using astronomical data : time and latitude observations give the position of a station referred to a system linked to a given star catalogue. In order to study the Earth's rotation, such observations are regularly made in a large number of stations. Once the effect of the variations in the Earth's rotation is removed from the universal time and latitude measurements made in a station, the residual values show secular variations due to errors in the star catalogue, possible local drifts or drift of the mean pole, but also to the motion of the plate to which the station belongs.

Precise latitude observations have been made since 1899. The time observations are actually a comparison of the universal time with a laboratory time-scale; the time scale used nowadays is the atomic time scale, TAI, given by atomic clocks and time signals, so that one can be sure that the local time scales actually used in the observatories are consistent within about 1 microsecond; but this accuracy was achieved only recently, and it can be concluded that the series of time observations made prior to 1955 may contain large systematic errors. Furthermore, the two modern instruments measuring both time and latitude, i.e. the PZT and the Danjon astrolabe which increase the accuracy of the observations themselves, were not in use before 1954-55.

The possibility of measuring the present day continental drift (which amplitude is of the order of 0".001 per year) by use of the residuals of some well chosen astronomical observatories has been examined. The difficulties encountered can be described as being due to the lack of homogeneous reference available during a period long enough to get rid of the observation errors.

A way to ensure the homogeneity of the reference is to consider concurrent observations of two or more instruments located on different plates and observing common stars. A pair of such stations makes it possible to measure the drift after about 30 years (MARKOWITZ 1968). A study of the latitude observations made since 70 years at the International Latitude Stations yields an evaluation of the position of the pole of the motion Eurasia-America : $\Phi = 82°$ N, $\Lambda = 57°$ W (PROVERBIO & QUESADA 1972).

The above method is a strict one, but its use needs a coordination in the measurement process (location of the instruments, common program, equivalent reduction procedures) which is not easily

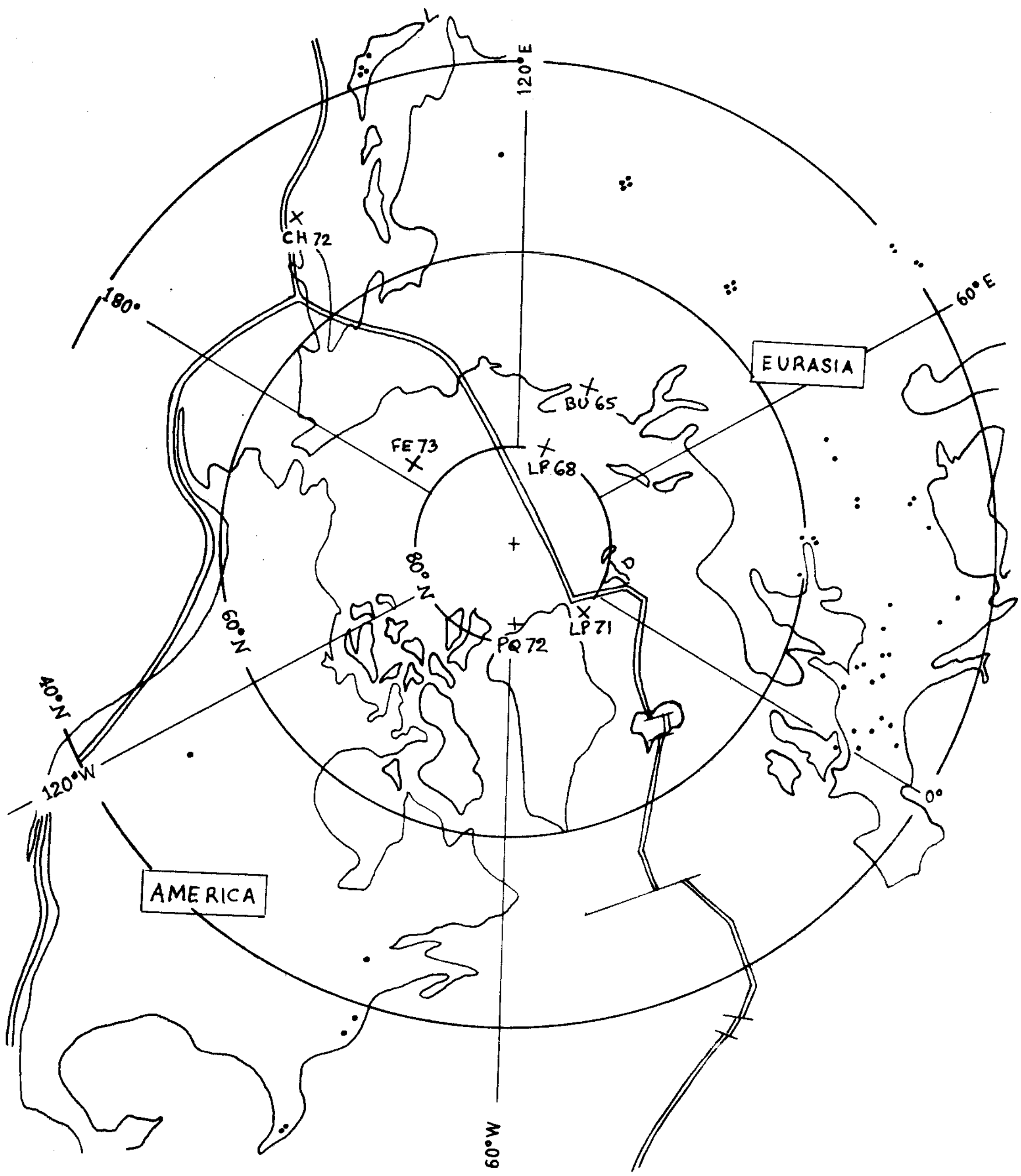

Figure 1 - America and Eurasia

● Time and latitude instruments (those of South America have the latitudes -0°, -23°, -23°, -33°, -34°.)

\+ proposed eulerian poles of the relative motion realised. Moreover, it does not allow the use of the numerous instruments which do not fulfil these conditions. The Bureau International de l'Heure computes time series of UT1-UTC and of the pole coordinates deduced from all the existing series of measured quantities directly related to the Earth Earth's rotation vector, i.e. the present 80 astronomical time and/or latitude series as well as the series obtained, sometimes as byproducts, by the modern methods (satellite observations, VLBI, etc). These series may show individual deviations related not only to continental drift. However, some plates bear a number of observations large enough to reduce the accidental errors and to make it possible to compute their motion relative to the BIH reference system. This provides the new way of measuring plate motions from astronomical observations which is used in this work. The parameters of the motion of a plate relative to the BIH system are obtained by a least square fitting in the individual drifts of the stations it bears, referred to this system; the errors caused by accidental deviations in the stars proper motions are averaged due to the number of stations.

The motion of a plate relative to another one is then computed by combining the two rotation vectors and thus eliminating the intermediate reference system; as the different star catalogues used are not independent, the effect of common systematic deviations in the proper motions is well eliminated if the stations of both plates are located in the same latitude zone.

3. The Local Drifts in the BIH Reference System and the Motion of the Plates

a) The data

In the computation method used by the BIH (GUINOT & FEISSEL 1968) predicted corrections are applied to the different series which contribute to the global computation (FEISSEL 1972). The correction used is the sum of a constant term a, and two periodical terms (annual and semi annual), its prediction being renewed every year according to its actual value for the preceding years. This method is applied in the current computation since 1968. It has been applied to older data, so that homogeneous time series of the pole coordinates and UT1-UTC are available starting from 1962.0. It gives also the individual annual residuals, a, of all the participating observatories, which allow the computation of the drifts 1962-1972 of the stations relative to the system BIH 1968. We thus obtained the individual drifts of $\frac{da}{dt}$ for about 40 instruments in latitude and 50 in longitude. These drifts are referred to the BIH system and they are not affected by the real drift of the mean Earth rotation pole relative to the plates.

The spacing of the stations on the different plates is very unequal. Only two of the six main plates bear enough observatories to make possible the computation of their motion. The American plate has 10 stations which give 8 values in latitude and 9 in time; the Eurasian plate has 35 observatories, 25 in latitude and 27 in time. Five of the ten American stations are in the same latitude zone as the Eurasian observatories (25° N to 60° N).

b) The equations of the Motion

Figure 2 shows P the Earth's pole, Π the Eulerian pole of a plate, A a point of this plate. The movement of A is a rotation with the angular velocity ω around Π. The velocity of the movement along a parallel of the pole Π is $V = \omega \sin \alpha$. The positive rotation is anticlockwise. For this study, we can assume that the Earth is a sphere centred on the centre of mass. If we call ϕ_A the annual residual in latitude and θ_A the annual residual in time observations, the drift in latitude

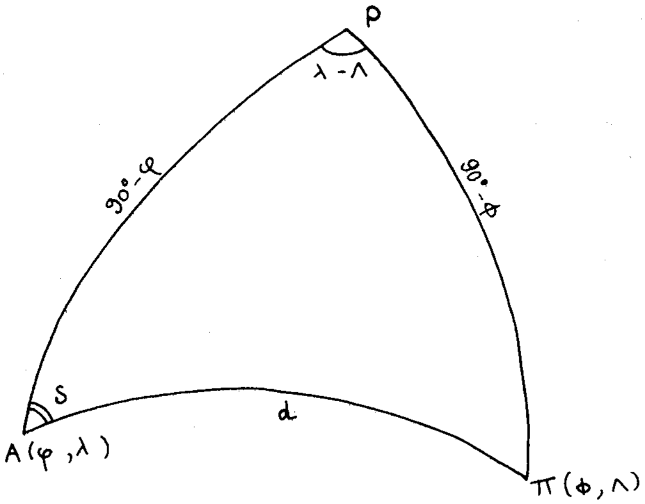

Figure 2

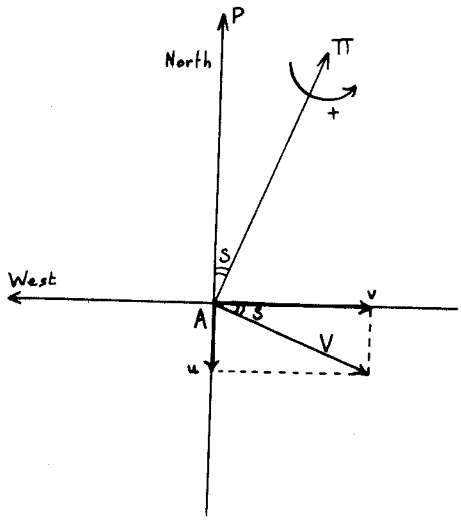

Figure 3

$u = \frac{d\phi_A}{dt}$ is the projection of V on the meridian of A ; the positive motion is northward. The drift along the great circle perpendicular to the meridian of A is

$$v = - \frac{d\theta_A}{dt} \cos \phi$$

the positive motion is westward, as for longitudes; if the local observed UT-UTC decreases, it means that the observatory has a westward drift. Taking into account the different positive directions of V, u, v, the velocity of A is related to the astronomical data by (Figure 3) :

$$\begin{cases} u = -\omega \sin \alpha \sin S & \text{(latitude)} \\ v = -\omega \sin \alpha \cos S & \text{(time)} \end{cases}$$

The trigonometric relations in the spherical triangle PAΠ yield:

$$u = -\omega \cos \Phi \sin (\lambda \ -\Lambda)$$

$$u = \omega \cos \Phi \ [\sin \Lambda \cos \lambda \ - \ \cos \Lambda \sin \lambda \ ]$$

One has too:

$$v = -\omega(\sin \Phi \cos \phi \ - \cos \Phi \sin \phi \ \cos (\lambda \ -\Lambda))$$

$$v = -\omega\sin \Phi \cos \phi \ + \omega \sin \phi \ \cos \Phi \cos \Lambda \cos \lambda \ + \ \omega \sin \phi \ \cos \Phi \sin \Lambda \sin \lambda$$

The change of coordinates:

$$\left| \begin{array}{l} x = \omega \cos \Phi \cos \Lambda \\ y = \omega \cos \Phi \sin \Lambda \\ z = \omega \sin \Phi \end{array} \right. , \text{ i.e. } \left| \begin{array}{l} \omega = (x^2 + y^2 + z^2)^{\frac{1}{2}} \\ \Phi = \text{arc cos} \left(\frac{x^2 + y^2}{\omega^2}\right)^{\frac{1}{2}} \\ \Lambda = \text{arc tg } \frac{y}{x} \end{array} \right.$$

yields

$$u = - x \sin \lambda \ + y \cos \lambda$$

$$v = + x \sin \phi \ \cos \lambda \ + y \sin \phi \ \sin \lambda \ - z \cos \phi$$

Or, when replacing u and v by their values,

$$\frac{d\phi_A}{dt} = - x \sin \lambda \ + y \cos \lambda \qquad (1)$$

$$- \frac{d\theta_A}{dt} \cos \phi \ = x \sin \phi \ \cos \lambda \ + y \sin \phi \ \sin \lambda \ - z \cos \phi \qquad (2)$$

The order of magnitude of the errors are 25° in Φ, 30° in Λ and 6×10^{-7}deg/yr on ω.

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6. Discussion

WHITTEN: Rather than ask a question, I would like to make a comment. Your paper is extremely interesting. Let us compare it with data presented by Markowitz. Looking at figure 2 of his paper, we see an illustration representing the change in latitude of the five ILS stations. About five years ago, after Markowitz had prepared his paper for the meeting in Stresa, I was interested in the effect of possible rotation of continents. I used the data Markowitz had shown, that over the seventy year period Gaithersburg had moved north with respect to Ukiah in California. Taking these points as the ends of a baseline, I made a rough calculation that there would be about 5° minus rotation (anticlockwise) in ten million years. This was in agreement with what geophysicists had said was happening in North America - twenty five degrees in the past fifty million years. I took the three Eurasian stations and found about a four degree clockwise rotation over the ten million year period. These numbers are in general agreement with the results you have found. The observations for the ILS stations do not include longitude, as Markowitz points out, so I could not get a true rotation, but I can get an element of rotation. I can't get the translation, but by using the latitude data, I think it is interesting to see that there is agreement with your work.

FEISSEL: For my computation I considered that Ukiah did not belong to the main American plate and I did not use Carloforte, which is too close to the Africa-Eurasia boundary. As the pole of the motion of America relative to Eurasia lies near the Earth's rotation pole, latitude variations give little information on the displacement.

WHITTEN: That is right; using the latitude only, you cannot determine the pole of rotation, but I could determine an element of plate rotation.

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THE EARTH'S SECULAR LIBRATIONS

ABSTRACT

Three components of the secular polar motion are described:

a) a minor libration which varies in frequency with the amplitude of the Chandler wobble;

b) the major libration which appears to be increasing in period and amplitude;

c) the progressive motion which is fairly steady in rate and direction.

Librations appear to affect the phase of the Chandler wobble. It is suggested that libration characteristics are due to time-dependent responses.

1. The Data

Figure 1 shows the positions of the barycentre according to YUMI & WAKO (1966) and YUMI (1966-1973a; 1973b). Those based on the data from Mizusawa (M), Carloforte (C) and Ukiah (U) extended from 1903 to 1963. Those that also include Kitab (K) and Gaithersburg (G) extend from 1936 to 1970. It is clear that different stations report very different progressive motions.

Figure 2 shows the result of simply applying -0.''004/yr to the y values as a rough correction for the progressive motion. The main feature is then the librations. Its departures from a smooth oscillation appear to vary in amplitude and frequency with R, the amplitude of the Chandler wobble. (R was greatest around 1910 and 1953). Arrows indicate the approximate direction of displacements by the minor libration.

Figure 3 indicates that the frequency response has not remained constant. The period of the strongest spectral line has increased, and the relative response at the shorter periods has changed. It was found that each of these corresponds to a particular value of R. Consequently, the strength of the line depends upon the number of times that a particular value of R occurs in the sample, e.g. small values of R were frequent before 1936, but not afterwards; this caused the 16-year period to be strong in the data for 1904-66, but not in the data for 1936-1967.

THE MODEL

In the words of JEFFREYS (1952):

> The behaviour can be represented better if we regard the solid as composed of pieces, all perfectly elastic when the stress is not too great, but flowing when it is great enough, the transitions being at different stresses for different portions. ... Thus the rate of deformation will increase more rapidly than in proportion to the

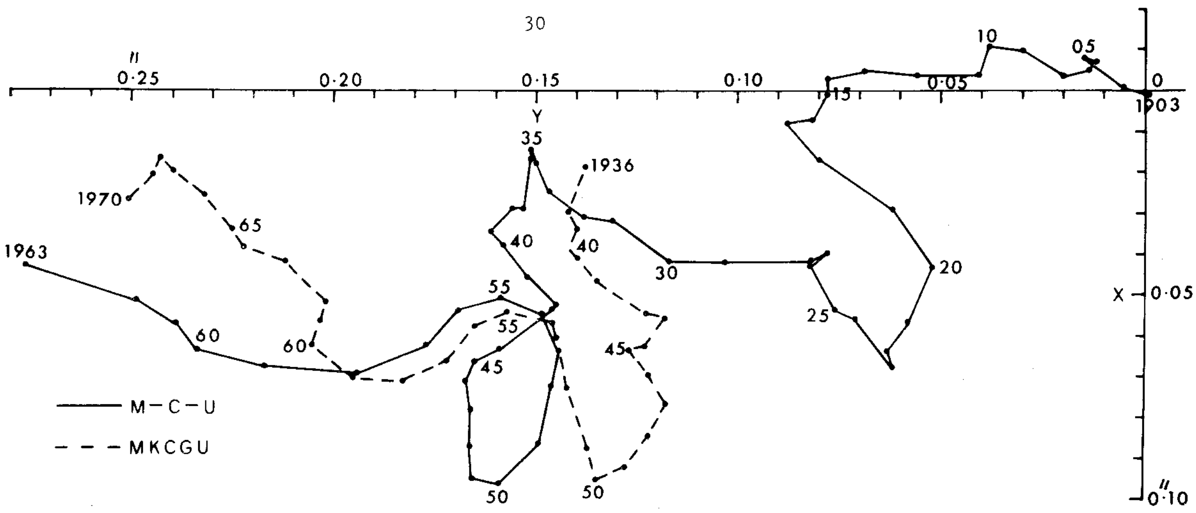

Figure 1. Positions of the barycentre

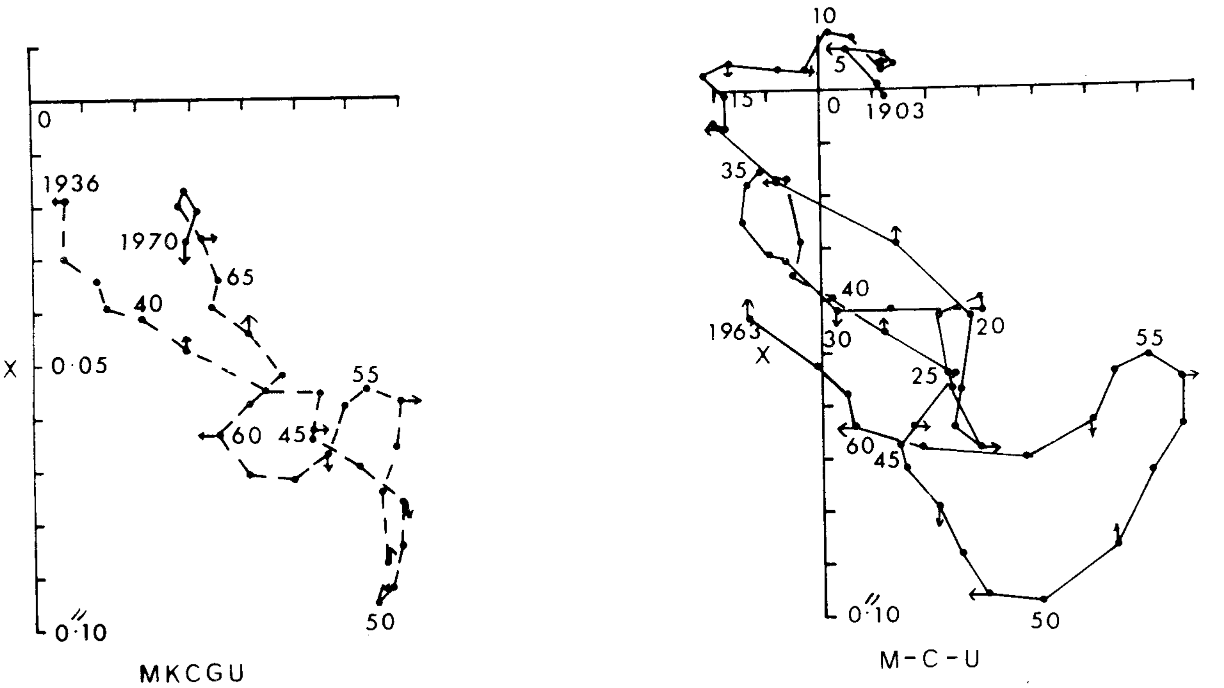

Figure 2. Positions of the barycentre partly
corrected for progressive motion

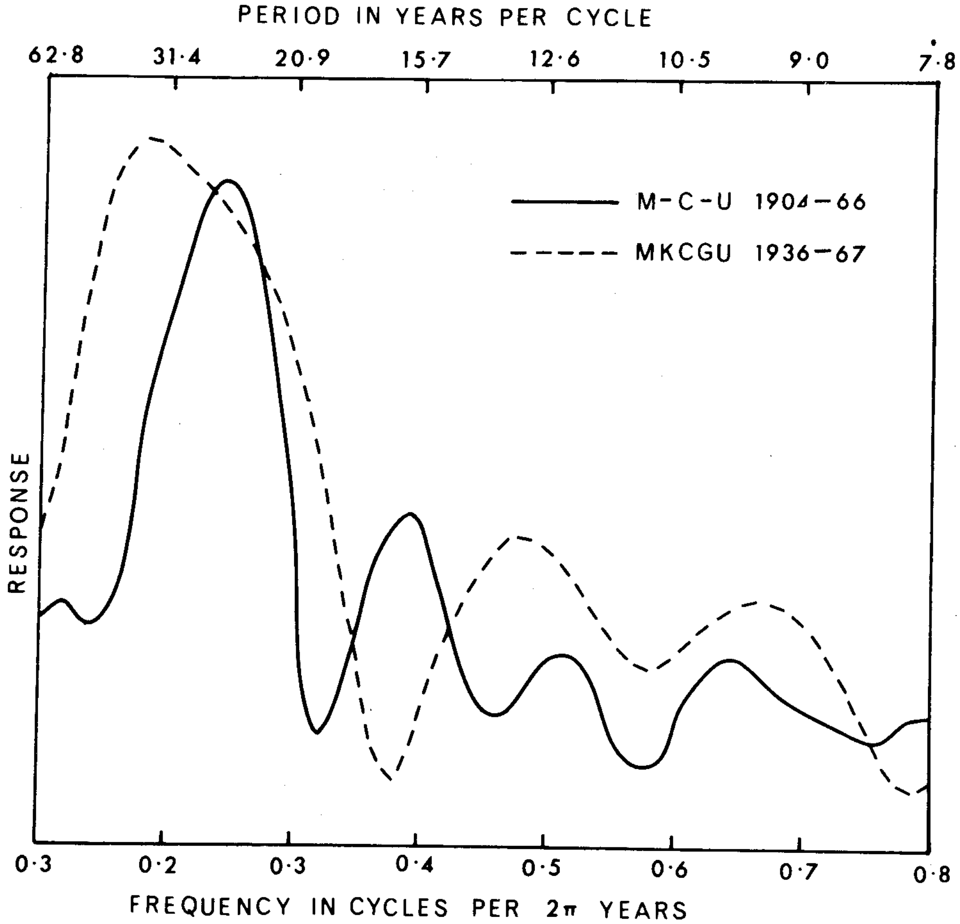

Figure 3. Frequency response of positions of the barycentre

> stress. ... If the stress is in an intermediate range the weaker parts may adjust themselves to a hydrostatic state, leaving the whole deforming stress to be borne by the stronger parts. If this happens the displacement will tend to a finite limit, the rate of approach being limited by the viscosity of the weak parts. If the stress is removed the strong parts will tend to spring back and, in doing so, apply deforming stresses to the weak ones, which will again flow, but in reverse direction.

This is the model which has been used here to interpret the strains that are associated with the polar motions. The presence of periodic strains is indicated by the deformations that accompany the wobbles. In the notation of MUNK & MACDONALD (1960), when the rotation pole P is displaced from its current position of rest 0 to some point $\underline{\underline{m}}$ then $\underline{\underline{m}}$ can serve as a measure of the stress $\underline{\underline{S}}$. Let the rate of relaxation of an excitation ψ be

$$\dot{\psi} = - \psi/\tau \qquad (1).$$

The strain in an actual body is indicated by the deformation excitation

$$\underline{\underline{\psi}}_D = (\underline{\underline{k}}/k_f)\underline{\underline{m}} \equiv (k/k_f)\underline{\underline{m}} - i(k/k_f)\ (\tfrac{1}{2}\mu/Q)\underline{\underline{m}} \qquad (2)$$

where k is the Love number; k_f is the value that k would have if the Earth had the shape of a rotating fluid and its density distribution were equal to that of the actual Earth; μ is the dimensionless rigidity; and $1/Q$ is the 'specific dissipation function' which indicates sharpness of resonance.

The existence of systematic stress is shown by the progressive motion. Along the meridian of this motion weak material is assumed to yield or fracture until the stress is borne by stronger material. Then the stress can rise until the yield strain is reached, and so the mean rigidity of the material that still resists will also rise. In the meridian normal to the progressive motion the rigidity would not become so great. Whenever the stress is reduced the material would tend to recover.

The strain ε depends not only on the stress S and the rigidity; as shown in Jeffreys' approximation (GUTENBERG 1959) there is also the time-dependent increase which depends on $\int S dt$ and the viscosity; and there is the time-dependent decrease (due to relief of stress) which depends on $\dot{\varepsilon}$ and the time of retardation. It is evident that terms in $\int S dt$ may be unimportant if the period is short but they can become important or even dominant if the period is long. The retarded elastic response is important in the period of the Chandler wobble, and the rate of response is important in the periods of librations.

2. The Minor Libration

An empirical formula is used provisionally for the frequency of this small libration. It was derived as follows:

Let R be the radius of the Chandler wobble
b be the semi-minor axis of the annual wobble
σ and σ_0 be the angular speed of the annual and Chandler wobble respectively

and let $\psi = R - (k/k_f)R.$ (3)

When the annual excitations with a steady excitation give rise to the annual and Chandler wobbles then the departure of the rotation pole P from its initial position O can be given in the form

$$\underline{\underline{m}} = \underline{\underline{R}}(1 - e^{i\sigma_0 t}) + \underline{\underline{m}}^+(1 - e^{i\sigma t}) - \underline{\underline{m}}^-(1 - e^{-i\sigma t}) \qquad (4).$$

During every period $2\pi(\sigma - \sigma_0)^{-1}$ the mean position of P is

$$\underline{\underline{R}} + \underline{\underline{m}}^+ - \underline{\underline{m}}^- = \underline{\underline{R}} + \underline{\underline{b}} \qquad (5).$$

Thus in each period $2\pi(\sigma - \sigma_o)^{-1}$ the rotation pole moves with apparent annual speed $R(\sigma_o - \sigma)$ about a mean position $\underline{\underline{R}} + \underline{\underline{b}}$. Consequently the apparent angular speed of the annual mean positions of the pole is

$$\frac{-R(\sigma - \sigma_0)}{R + b} = -\upsilon_1 \qquad (6).$$

3. The Major Libration

It is suggested that this libration arises from the Earth's response to stress in the presence of anelastic material.

Let $\overline{\underline{\underline{R}}}$ be the mean position of the modified excitation pole of the free wobble with respect to 0 during each period $2\pi(\sigma - \sigma_0)^{-1}$. The corresponding anelastic deformation excitation due to strong material is $\overline{\underline{\underline{R}}}\ \underline{k}/k_f$ so the unrelieved mean excitation is $\overline{\underline{\underline{R}}}(1 - \underline{k}/k_f)$ $\overline{\underline{\underline{\psi}}}$. This excitation urges the weak material to yield or fracture and thus relieve the remaining stress. The consequent motion of $\overline{\underline{\underline{\psi}}}$ with respect to 0 would be

$$\frac{d\overline{\underline{\underline{\psi}}}}{dt} = -(1 - \frac{\underline{k}}{k_f} + \frac{i}{2}\frac{k}{k_f}\frac{\mu}{Q})\frac{\overline{\underline{\underline{R}}}}{\tau} \qquad (7).$$

The real term shows the rate at which $\overline{\underline{\underline{\psi}}}$ would attempt to move to 0. However, it is well known that R has a continuing mean value of about 0".15. Therefore there is evidently an energy source as well, and this compensates for the damping and maintains the value of $\overline{\underline{\underline{\psi}}}$, so the real term can be disregarded. Then

$$\frac{d\overline{\underline{\underline{\psi}}}}{dt} = \frac{k_f - \underline{k}}{k_f}\frac{d\overline{\underline{\underline{R}}}}{dt} = -\frac{i}{2}\frac{k}{k_f}\frac{\mu}{Q}\frac{\overline{\underline{\underline{R}}}}{\tau}\ .$$

Therefore $\overline{\underline{\underline{R}}}$ revolves with radius $\overline{R}$ and frequency

$$-\upsilon_2 = -\frac{k\mu}{2Q(k_f - \underline{k})\tau} \qquad (8).$$

A simple example is that in which the strong material on its own would respond to the Chandler wobble as a Kelvin-Voigt body, i.e. like a spring with rigidity $\tilde{\mu}_s$ in parallel with a dashpot with viscosity $\tilde{\eta}_s$. There would be no permanent strain; and the relaxation time $\tau_s = \tilde{\eta}_s/\tilde{\mu}_s$. It can be shown (MUNK & MACDONALD 1960) that $1/(2Q) = \sigma_0\tau_s/(1 + \mu)$. The relaxation time for the actual Earth is τ. $(\tau > \tau_s)$. Then equation 8 becomes

$$-\upsilon_2 = -\frac{k}{k_f - \underline{k}}\frac{\mu}{1 + \mu}\frac{\tau_s}{\tau}\sigma_0 \simeq -\frac{k}{k_f}\frac{\tau_s}{\tau}\sigma_0 \qquad (9).$$

Let us now consider some of the effects of a stress $S \sin \upsilon_2 t$ in a material that gives a retarded elastic response. The stress equation is of the type $\ddot{y} + 2p\dot{y} + q^2y = q^2 s \sin \upsilon_2 t$ in which $\upsilon^2 < q^2$. Then the steady state solution is $y \simeq SA \sin(\upsilon_2 t - \tan^{-1}\upsilon_2\tau)$ where $A = (1 + \upsilon_2^2\tau^2)^{-\frac{1}{2}}$ (SOKOLNIKOFF 1941). Similarly in the case of $\overline{R}\exp(-i\upsilon_2 t)$; let $t = 0$ be an instant when this excitation is acting in the meridian of reference, and let $\tau = \tilde{\eta}/\tilde{\mu}$ be the relaxation time along that meridian. Let τ_i be the relaxation time along the meridian which is normal to this. Then the deformation excitation is

$$\overline{R}\,A\cos(\upsilon_2 t - \tan^{-1}\upsilon_2\tau) - i\,\overline{R}\,A_i\sin(\upsilon_2 t - \tan^{-1}\upsilon_2\tau_i)$$

$$\simeq \overline{R}\,A\{\sin\upsilon_2 t + i(A_i/A)\cos(\upsilon_2 t + \lambda)\} \qquad (10),$$

where $$\lambda = \tan^{-1}\upsilon_2\tau - \tan^{-1}\upsilon_2\tau_i \simeq (\upsilon_2\tau_i)^{-1} \qquad (11).$$

According to these expressions the oblique axis should be A_i/A times greater than that in the meridian of reference, and it should lie $\pi/2 - \lambda$ radians to the west of that meridian, approximately. It seems likely that τ would be greatest in the meridian of the progressive motion.

4. Variations

The model helps to explain several phenomena. The observed period of the libration has been increasing, as can be seen in Figure 1 and in the line shift in Figure 3. The rate of increase is about 0.4 year/year. This can be expected if the stress is continuing to make the strong material give way. Then the mean value of $\tilde{\mu}_s$ would rise, τ_s would fall, and υ_2 would decrease according to equation (9). υ_2 would also decrease if the more viscous conditions increase or last longer and thus cause τ to rise.

The observed amplitude of the librations varies with the Chandler amplitude (e.g. R was relatively large about 1910 and 1953 and small from about 1922 to 1942), and it appears to have been increasing as well. According to equation (10) there should be an increase in amplitude on account of the decrease in υ_2 in A.

A small eastwards rotation of this libration axis would also occur, according to equation (11) but further treatment will be needed to verify this.

The progressive motion appears to be far more stable, as shown by the compact libration pattern when a constant rate correction is applied.

Finally, the model is relevant to the problem of why the Chandlerian wobble *was not* disturbed in phase when it *was* disturbed in amplitude (around 1910 and 1953), and *was* disturbed in phase when it *was not* disturbed in amplitude (1925.5 to 1928.5) (GUINOT 1972). The explanation appears to be that the period of the wobble is strongly related to the librations as well as to the amplitude of the wobble (ABRAHAM & BOOTS 1972). This is because the period of the wobble increases with the integrated deformation, and this is affected by librations.

Least squares solutions are now being fitted to obtain greater precision.

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DEFLECTIONS AT SEA

ABSTRACT

As the frontiers of geodesy are pushed forward into the ocean areas, the determination of ocean geoids is being discussed in anticipation of the forthcoming GEOS-C satellite altimeter experiment. Independent methods will be needed for checks and verification, but also for interpolating the finer details of the geoid and/or deflections of the vertical in areas of interest. This paper explores the possibility of utilising bathymetric data to determine deflections of the vertical at sea, if gravimetric anomalies are not or not sufficiently available. A Pacific atoll where a number of astrogeodetic deflections of the vertical have been observed, is studied as a test case to determine the accuracy with which the given deflection values can be reproduced.

1. Introduction

The current extension of geodesy from detailed and highly accurate land data into the vast ocean regions takes several different forms. Foremost is satellite geodesy which establishes positions of islands and ships with increasing accuracy. The feasibility of placing geodetic markers on the ocean floor provides the prospect of a geodetic control network with calibration ranges in the oceans, to aid all other endeavours with recoverable checkpoints. Satellite altimetry is expected to yield geoidal profiles and slopes of the ocean surface. Methods of determining deflections of the vertical directly by a ship's inertial navigation system are tried and the possibility of observing astros from stabilised platforms is explored. The standard procedure of deriving geoid undulations and deflections of the vertical from gravity anomalies by STOKES' and VENING MEINESZ' formulas requires an extensive collection of gravimetric measurements at sea, with great density around the points of interest. The ocean bottom topography is being mapped with increasing completeness as well as detail, but so far this data has not been used for direct computation of deflections. Suppose good bathymetric maps of an area are available, then it should be possible to compute from them deflections directly as the horizontal components of mass attractions. This paper explores the feasibility of this approach from a practical viewpoint; that is, real data from existing maps will be used and the results will be tested against observed deflection values. The exasperating difficulties and pitfalls of numerical data will thus show up, while they do not appear in theoretical treatises or smooth textbook examples. A realistic feel for achievable accuracies can be gained from the comparison of computed with observed deflection values. As a significance gauge for deflections at sea, we quote from the literature (MOURAD 1971), that VON ARX achieved an accuracy of ± 12 seconds of arc for astrogeodetic deflections across the Puerto Rican Trench; and that BUTERA et al determined deflections across the same Trench from the difference between the positions from a ship's inertial navigation system (SINS) and from LORAC or LORAN, which differed by up to 70" from those by VON ARX.

2. The Test Area

An atoll in the Marshall Islands in the Pacific Ocean was selected as a test case to see whether and how accurately deflection values could be produced from bathymetric maps, as an alternative procedure to computing deflections from an extensive collection of gravity anomalies. An evaluation of the accuracy achieved was provided by the comparison with a set of sixteen astrogeodetic deflections, observed on the islands surrounding the lagoon, and referred to a local geodetic datum. The deflection values range from -24.49" to +28.89" in the meridian, and from -19.12" to +21.83" in the prime vertical, rather large variations across an area of less than $3/4^{\circ}$ in diameter.

Considering the dependence of astrogeodetic deflection values on the arbitrary choice of the geodetic datum, one might wonder whether or not a change to a world datum or a best fitting regional datum might be appropriate before a comparison is made with values computed from ocean bottom topography. As it turns out, a change to any other datum for such a small area amounts practically to a blanket correction in either component, and would thus be easily recognisable, if needed in the comparison.

The bathymetric maps available for this study were Hydrographic Office charts at about 1:1 000 000, at 1:100 000, 12 000, 10 000 and Army Map Service maps at 1:25 000. Here we ran into our first exasperating difficulties with real numbers: plotting the sixteen stations on these maps according to the given coordinates made them fall into the water. Then a reference statement for an overall blanket correction was discovered which should align the geodetic coordinates with the map grid. This correction brought the stations nearer to the islands but not yet onto them. A search of the field books and station descriptions helped to locate the stations on the large scale maps in proper relation to buildings or roads. The correct relationship was thus established, station by station, between the geodetic coordinates on the local datum and the map coordinates.

Next came the reading of the bathymetric information, requiring a decision about the degree of detail and the extent of coverage to be used, versus the effort to be expended. It was decided to use the "rectangular" method throughout (FISCHER 1966), with area units of $\frac{1}{2}' \times \frac{1}{2}'$, 2.5' x 2.5', and 5' x 5', and the delineation between them to be assigned by trial and error, and by the detail, or rather the lack of detail shown on the maps. The trial was compounded by the poor quality of the maps, with little or different information given on different- scale sheets in some regions. Eventually, the overall extent of coverage adopted for this pilot study was an area of 5° in latitude and 4° in longitude, so that each station was at least 1.5° from the boundary. A refinement in the vicinity of each station was added, however: the 0.5' x 0.5' square containing the station had been subdivided into four parts by the meridian and prime vertical through the station; this arrangement was now replaced by 5" x 5" readings, and by 10" x 10" readings in the eight 0.5' x 0.5' squares around it. A further refinement of 2" x 2" instead of the 5" x 5" was tried for one island. The gain seemed too insignificant to justify the expense for the others. It is assumed that the adopted coverage provides 80-90% of even the large deflection values.

A verification of this assumption may be seen in Figure 1, where the accrual with distance is plotted for each deflection component at a specific station, showing a gradual levelling off. The vertical line marks the end of the centrally symmetric coverage for this station. The additional accrual stems from the lopsidedness of a fixed data area for all stations of the region. This brings up the old question of a necessarily limited data extent: should one use the same fixed area for all sixteen stations of the region, or should one use centrally symmetric areas for the individual stations, which then would differ from station to station. For purposes of computing interpolation values between given deflections, the first approach works on the assumption that distant neglected masses will affect all stations in the same or slowly varying amount, which can be taken care of within the interpolation process; this approach is less expensive, since the area can be kept relatively small. To reproduce one individual deflection value independently, one would have to go out far enough so that the accrual from the next distance-belt would be insignificantly small. Figure 1 suggests a distance of about 350 km.

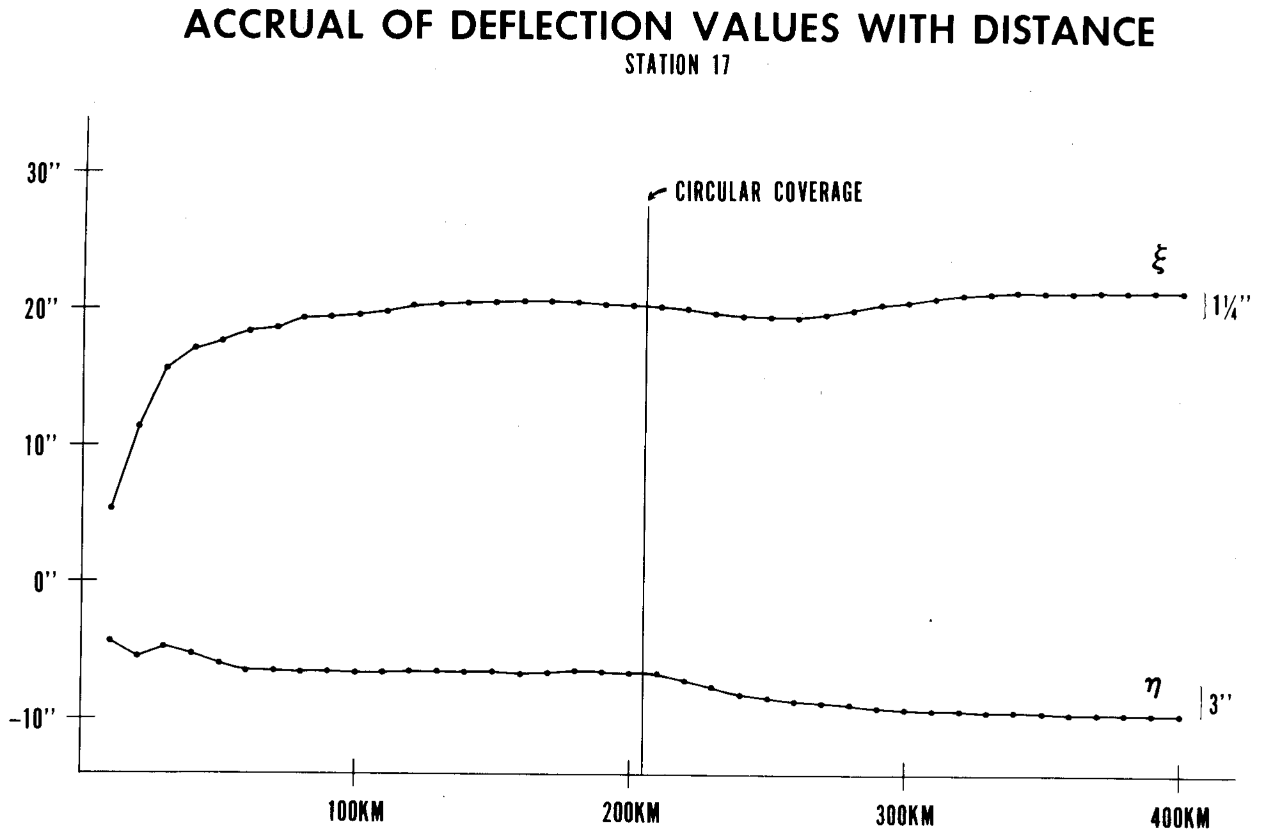

Figure 1

3. Isostatic Models

The basic expression for the horizontal attraction of a mass m on a station A, formulated as meridional and prime vertical deflection components in a rectangular coordinate system, is given by

$$\left.\begin{matrix}\xi \\ \eta\end{matrix}\right\} = - \frac{k\ m}{g\ s^3} \left\{\begin{matrix}x_m - x_A \\ y_m - x_A\end{matrix}\right. ,$$

where k = gravitational constant
g = mean value of gravity
s = distance Am

For a numerical integration of the effect of all given masses around A, the data area is subdivided into a system of rectangular area units with pre-assigned dimensions $(a \times b)_i$. The mass m_i of a rectangular column with the given cross section $(a \times b)_i$ is described by the height H_i and the density σ_i. For the small region of this study the assumption of a flat earth is adequate. As a further simplification, the mass m_i is treated as a point mass at the center of the column.

The primary input of the ocean depths read from the maps and associated with the adopted density value $\sigma = 1.027$ for sea water must be supplemented by reasonable assumptions about the rock density in the area, variations from an average density value, the existence of isostatic compensation, depth of compensation, different types of isostatic theories, or other density models. One may hope that, in turn, a distinction between some of these assumptions can be made from comparing the computation results with the observed deflection values.

As a first model, we tried the textbook approach of PRATT-HAYFORD's isostatic theory with density $\sigma = 2.67$ and depth of compensation at 113.7 km. A land column with zero elevation above mean sea level is taken as the standard, and a water column as deficient in mass by comparison. Traditionally, the water column is thought to be compressed to rock density, leaving an "empty" space as a clear deficiency, which is compensated by an excess mass distributed from the ocean bottom down to 113.7 km below mean sea level. For computational simplification, the deficiency and the excess are thought to be concentrated at the midpoints of their respective columns.

The sixteen deflections computed from this model were compared with the sixteen given astrogeodetic deflection values. The residuals (observed minus computed values) are listed in Table 1. Variations of this model with rock density values from 2.3 to 2.8 were computed to see the sensitivity of this parameter. With a criterion of the average error in absolute value and of the number of "hits" within 1.5" the density value 2.5 seemed best, with 2.4 a close second. To pinpoint a bias which could be interpreted as a datum shift, seemed to be premature and insignificant at this stage. Figure 2 is a graphic representation of these residuals which permits a visual interpolation for other density values and the following conclusions:

a. The "best" σ would be indicated by the shortest vector, that is the perpendicular to a line connecting the other solutions. Apparently, no "best" value would lead to zero residuals.

b. The "best" σ would be different for different stations or different regions. The east and west regions seem to behave differently, with 2.4 better in the east, and 2.5 - 2.6 better in the west.

c. An overall "good" value would be between 2.4 and 2.5, but it would not be 2.67.

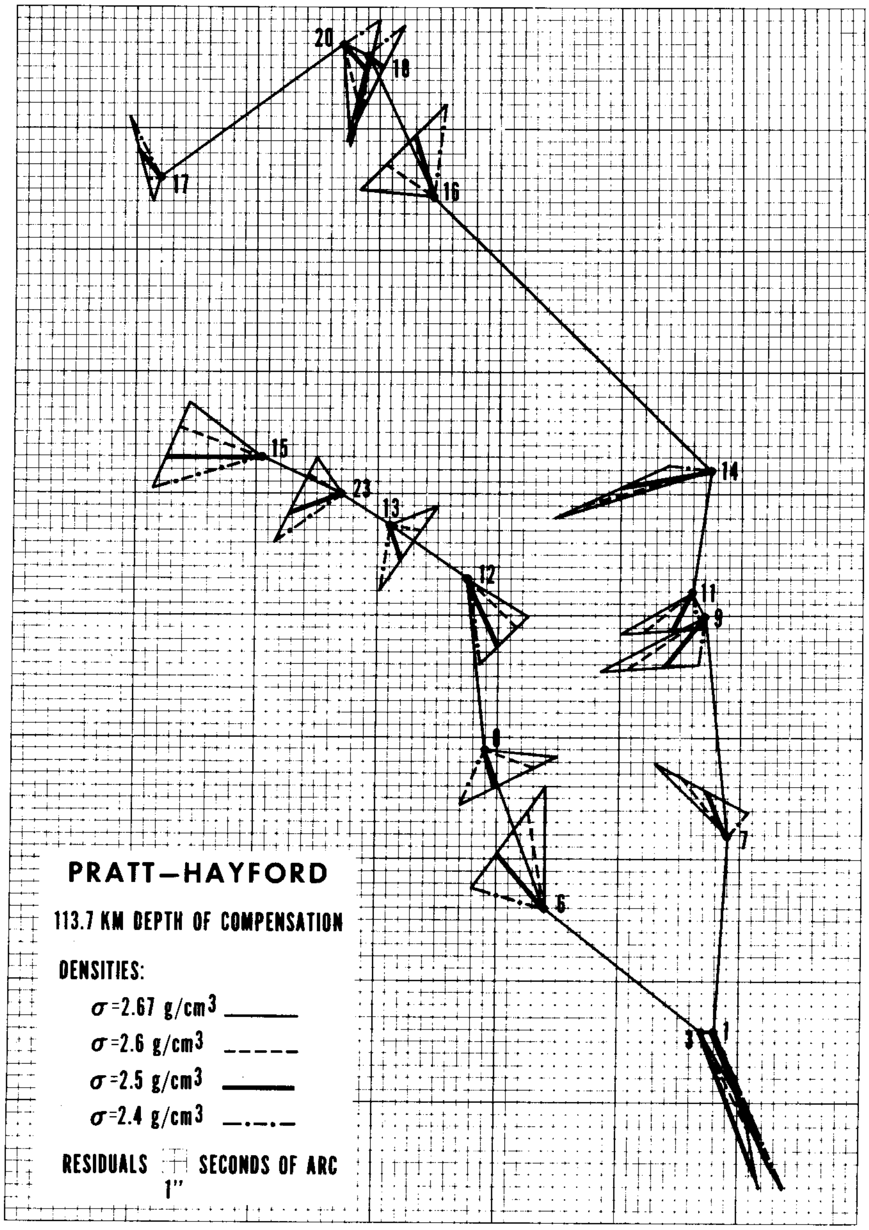

Figure 2. Pratt-Hayford Residuals for Various Rock Densities

TABLE 1. Pratt-Hayford Isostatic Models
Residuals for Various Densities and Depths of Compensation

Sta	Depth of Compensation 113.7 km, σ =2.67 v_ξ''	v_η''	σ =2.5 v_ξ''	v_η''	σ =2.4 v_ξ''	v_η''	57 km, σ =2.4 v_ξ''	v_η''	227 km, σ =2.4 v_ξ''	v_η''	σ =2.5 v_ξ''	v_η''
1	-3.26	+1.39	-5.26	+2.36	-6.44	+2.94	-8.70	+3.61	-5.51	+3.43	-4.26	+2.90
3	-2.95	+1.75	-5.16	+2.04	-6.46	+2.21	-8.79	+2.69	-5.48	+2.75	-4.11	+2.62
6	+5.05	+0.01	+2.35	-1.95	+0.76	-3.10	-2.09	-4.01	+2.00	-2.42	+3.69	-1.22
7	+2.92	-3.12	+1.74	-0.72	+1.05	+0.69	-0.64	+2.52	+1.71	+0.66	+2.45	-0.76
8	-0.29	+3.00	-1.47	+0.54	-2.17	-0.90	-4.68	-1.84	-1.10	-0.42	-0.33	+1.06
9	-2.24	-4.21	-2.07	-1.67	-1.97	-0.17	-2.23	+2.41	-1.81	-0.63	-1.90	-2.15
11	-1.74	-2.97	-1.59	-0.89	-1.50	+0.34	-1.60	+2.87	-1.40	-0.12	-1.48	-1.39
12	-1.47	+2.49	-2.70	+1.28	-3.42	+0.57	-4.69	+0.58	-2.92	+0.69	-2.16	+1.41
13	+0.72	+1.92	-1.40	+0.53	-2.66	-0.29	-3.99	-0.77	-2.19	-0.04	-0.90	+0.79
14	-2.07	-6.51	-0.66	-3.60	+0.17	-1.88	+1.27	+1.26	-0.19	-2.59	-1.05	-4.36
15	+2.30	-3.09	0.00	-4.01	-1.35	-4.55	-2.58	-5.63	-0.95	-4.10	+0.43	-3.53
16	+0.29	-2.92	+2.46	-0.73	+3.73	+0.57	+6.64	+2.19	+2.63	+0.16	+1.28	-1.16
17	-1.03	-0.27	+1.26	-0.96	+2.61	-1.36	+5.13	-2.06	+1.78	-1.13	+0.37	-0.71
18	-3.45	-0.82	-0.48	+0.62	+1.28	+1.46	+5.22	+2.47	-0.24	+1.12	-2.11	+0.25
20	-4.19	+0.27	-0.96	+1.03	+0.94	+1.47	+4.97	+2.32	-0.61	+1.16	-2.63	+0.69
23	+1.46	-0.91	-0.72	-2.04	-2.00	-2.71	-3.27	-3.39	-1.59	-2.40	-0.28	-1.71
a	2.2	2.2	1.9	1.6	2.4	1.6	4.2	2.5	2.0	1.5	1.8	1.7
b	6/16	6/16	8/16	9/16	7/16	10/16	2/16	3/16	6/16	10/16	8/16	10/16

a = $\Sigma |v| / 16$
b = number of $|v| \leq 1.5''$ out of 16 cases

A variation of the traditional depth of compensation at 113.7 km to half and twice its value, holding the density value of 2.4, leads to other sets of residuals which are listed also in Table 1, and graphed in Figure 3. It appears that 57 km is no improvement, while 227 km is a strong competitor, although the gain is small and not evident at all stations. It suggests, however, that isostatic compensation takes place further down rather than nearer to the ocean bottom.

Although the PRATT-HAYFORD theory may be too simple for geophysical insight, it is a very convenient tool as an intermediary step for interpolating deflections. In fact, the thirty-two deflection components were modelled with an average error of 1.8" and 1.7" respectively and 18 "hits" within 1.5", without even applying any further possible sophistication in the model or the procedure. That is not too bad for a first try, considering the 12" accuracy quoted in MOURAD's Report.

The AIRY-HEISKANEN theory of isostasy takes more cognisance of geophysical factors and some of its findings have been corroborated by seismic evidence. We know now that the crust under the oceans is thinner than in the continental blocks, and we know some density magnitudes involved. Figure 4 is a simplified density model for our area adapted from the literature (HEISKANEN & VENING MEINESZ 1958; STRAHLER 1971). It takes the 5 km ocean depth shown on the maps, as the standard, and assumes underneath a 1 km sediment layer, a 6 km basaltic crust, and heavier ultrabasic rock beyond the Mohorovičić discontinuity at 12 km depth. The thin crust under the oceans can be interpreted as the result of a compensating antiroot, coming up from a theoretical depth of compensation at around

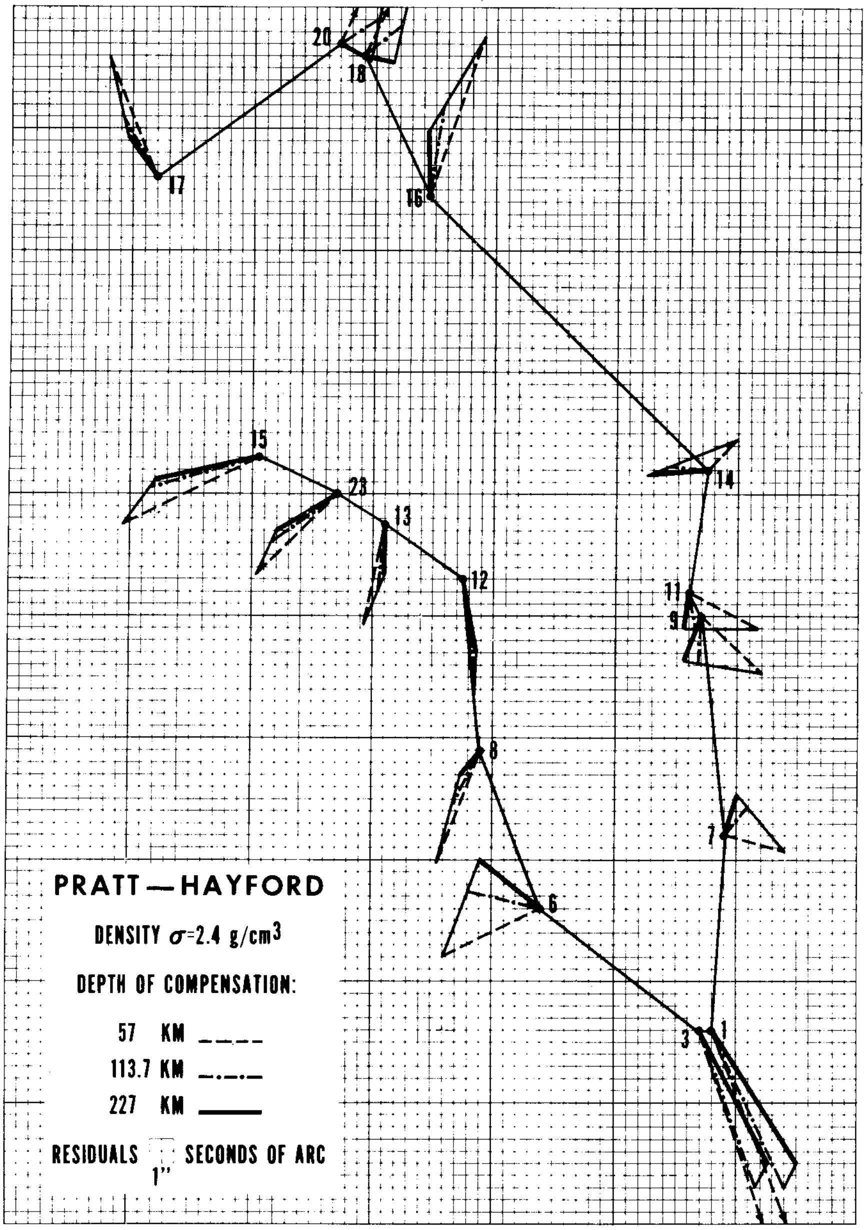

Figure 3. Pratt-Hayford Residuals for Various Depth of Compensation

30 km, which balances the mass deficiency of the ocean waters. So the overall fact of isostasy is already included in our standard model, which will now be modified by the specific depth readings D, in order to compute the deflections at each station.

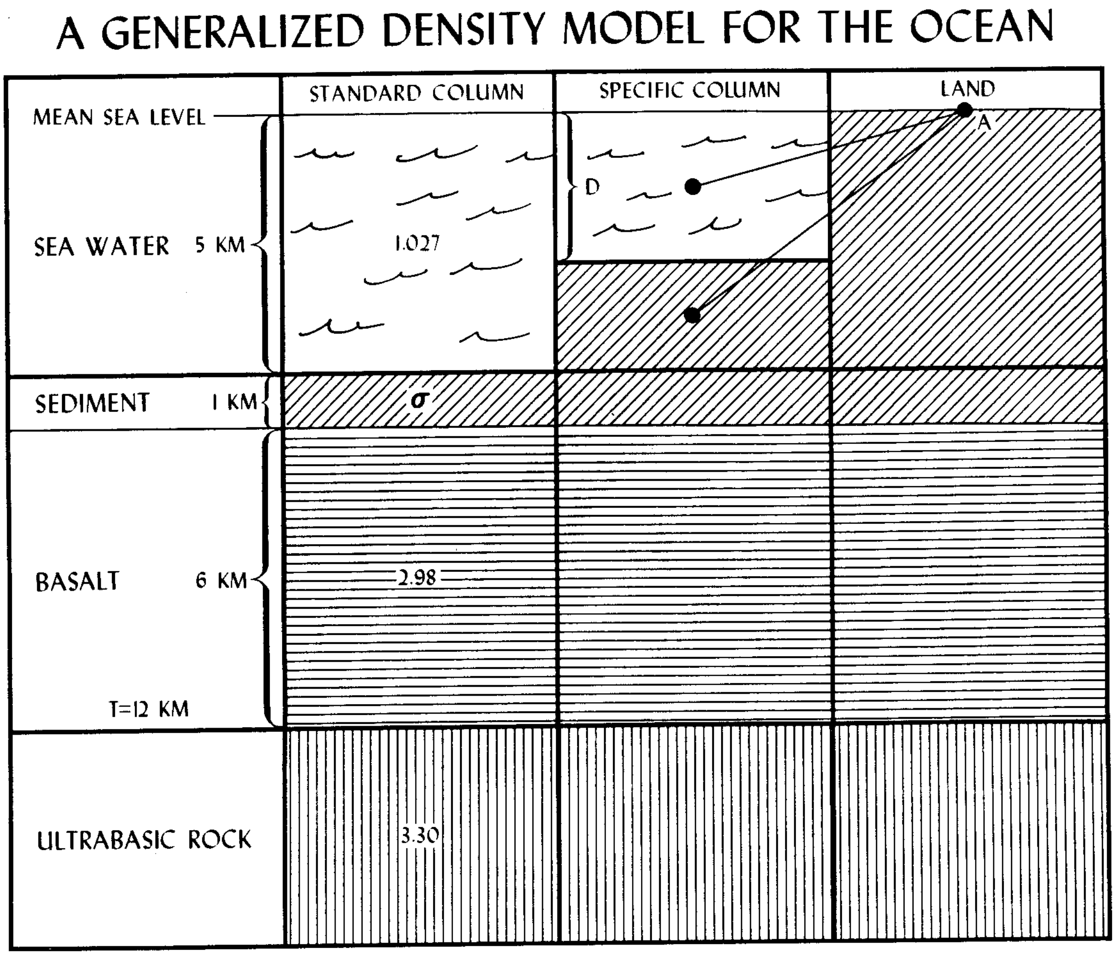

Figure 4

To allow for a similar variation of parameters (density and depth of compensation) as for the PRATT-HAYFORD theory above, the computations were carried out in two steps: first, the effect of the "visible" topography was computed, that is the land and water distribution down to 5 km depth as read from the maps. The effect of assumed compensating masses, modifying the standard model of Figure 4, is computed separately and added.

The effect of the "visible" topography was computed for various rock densities and 2.4 was found to give the smallest residuals. The depth of compensation was varied from T = 12 km to T = 30 km and T = 55 km, at an assumed boundary of rigidity within the ultrabasic layer. The residuals are listed in Table 2 and graphed in Figure 5. Interpolating by sight for the "best" T, one can see again that east and west behave differently, but that an overall "best" depth of compensation is further down

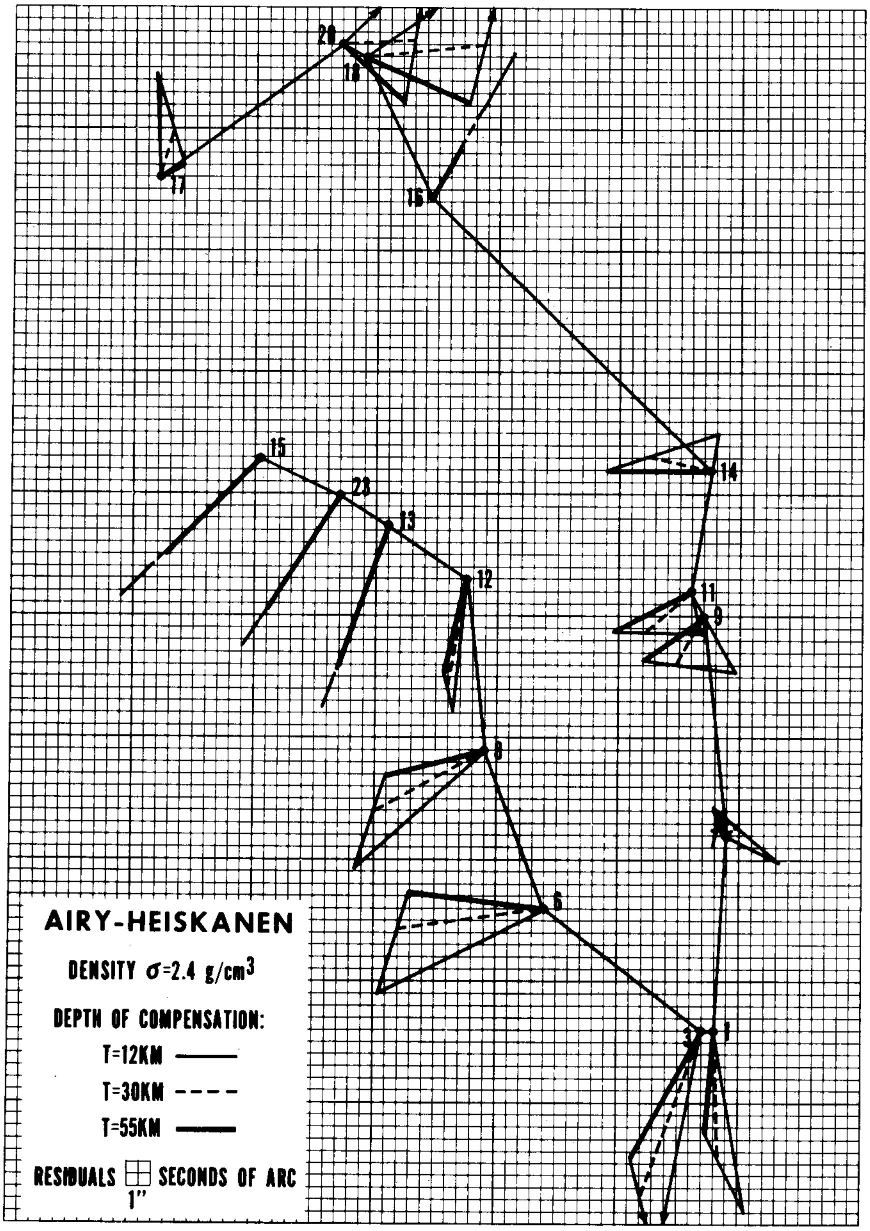

Figure 5. Airy-Heiskanen Residuals for Various Depths of Compensation

rather than up. Residuals in Table 2 are less attractive than those in Table 1.

It appears that further improvement would come from considering the different behaviour of the different regions.

TABLE 2. Airy-Heiskanen Isostatic Models
Residuals for Various Depths of Compensation

σ =2.4 Sta	T=12 km v_ξ"	v_η"	T=30 km v_ξ"	v_η"	T=55 km v_ξ"	v_η"
1	-7.51	+1.13	-5.50	+0.13	-4.29	-0.26
3	-9.15	-1.66	-7.09	-2.43	-5.83	-2.71
6	-3.49	-6.78	-0.89	-5.91	+0.69	-5.52
7	-1.31	+2.29	+0.29	+0.39	+1.22	-0.62
8	-4.81	-5.32	-2.41	-4.52	-1.03	-4.19
9	-2.30	+1.29	-2.06	-1.15	-1.87	-2.55
11	-1.81	+0.37	-1.73	-2.01	-1.62	-3.39
12	-5.40	-0.59	-4.30	-0.72	-3.68	-0.85
13	-7.50	-2.67	-6.39	-2.29	-5.81	-2.10
14	+1.51	+0.37	+0.47	-2.62	-0.09	-4.33
15	-5.70	-5.74	-4.54	-4.69	-4.06	-4.04
16	+6.06	+3.35	+3.55	+1.90	+2.00	+1.15
17	+4.24	-0.13	+1.96	+0.50	+0.54	+1.07
18	+3.93	+5.52	+0.44	+4.73	-1.86	+4.29
20	+3.74	+3.47	+0.13	+2.83	-2.21	+2.48
23	-6.24	-4.00	-5.18	-3.39	-4.69	-3.04
a	4.6	2.8	2.9	2.5	2.6	2.7
b	1/16	6/16	5/16	5/16	5/16	5/16

a = Σ |v| / 16
b = number of |v| $\leq$ 1.5" out of 16 cases

4. Other Density Models

Since the practical goal of this study and the degree of its success lies in small residuals (what is small?), the residuals themselves may help to define a density model that will decrease them further. This approach permits to accommodate the apparent differences of specific regions.

Figure 6 shows the residuals due to the "visible" topography down to a depth of 5 km, assuming 2.4 as the "best" overall rock density. The direction and magnitude of the residual vectors point to or from additional mass deficiencies or excesses that would counteract these vectors. A deepening of the sediment layer into the basalt below 6 km depth (see Fig.4) would constitute a mass deficit against the model, with a density difference $\Delta\sigma$ = 2.4 - 2.98 = -.58. Similarly, a deepening of the basalt beyond the 12 km depth would act as a deficiency of $\Delta\sigma$ = 2.98 - 3.3 = -.32. A mass excess could be produced by letting the ultrabasic rock come up above the 12 km line with a density difference of +.32 or by letting the basalt come up above the 6 km line with a density difference of +.58. In the latter case one must take care not to contradict the map readings of the water depths. A little inspection with some simplified computational experimentation placed some hypothetical yet reasonable "holes" H and "loads" L where the residual vectors seemed to indicate (Fig.6).

Obviously, the choices are not unique; while some are clearly better than others, some are also equivalent. A large mass at the 12 km density discontinuity may have the same deflection effect as a smaller mass at the 6 km level, and so on. Table 3 gives the details of one choice of a fairly obvious and simple combination of such holes and loads, all on the 6 km level. For simplicity, they were treated as point masses at the center of the layer.

TABLE 3. A Tentative Density Model

Feature	Position of Center horizontal	vertical	Thickness	Horizontal Extent	Density Difference
Hole 1	see Fig.6	- 8 km	4 km	12' x 12'	-.58
Hole 2	"	- 8	4	8' x 8'	-.58
Load 1	"	- 5	2	8' x 8'	+.58
Load 2	"	- 5	2	10' x 10'	+.58
Load 3	"	- 5	2	6' x 6'	+.58

The cumulative effect on the residuals of adding one such point mass after another is given in Table 4 and in Figure 6, showing a dramatic improvement. In 23 of the 32 cases the error is within 1.5", with none larger than 2.9". No optimization was applied, although it is clear that further refinements could be made.

Seven additional observed deflection stations were used as test points for this preliminary model. Table 5 shows an r.m.s. error of 1.17" in each component at these test points.

5. Conclusions

This study showed that bathymetric data can be utilised to compute deflections of the vertical with an r.m.s. accuracy of about 1½" or better in each component, as tested against observed deflection values. This result compares favourably with the current attempts to observe deflections at sea, with an accuracy of ± 12" so far.

Considering modern techniques of ocean bottom mapping, one can expect even better results from these than from the maps which were available for this pilot study. Although the numerical results must be considered preliminary, the feasibility of the bathymetric method has been demonstrated. Geophysical interpresentation and a distinction between several possible density models will have to

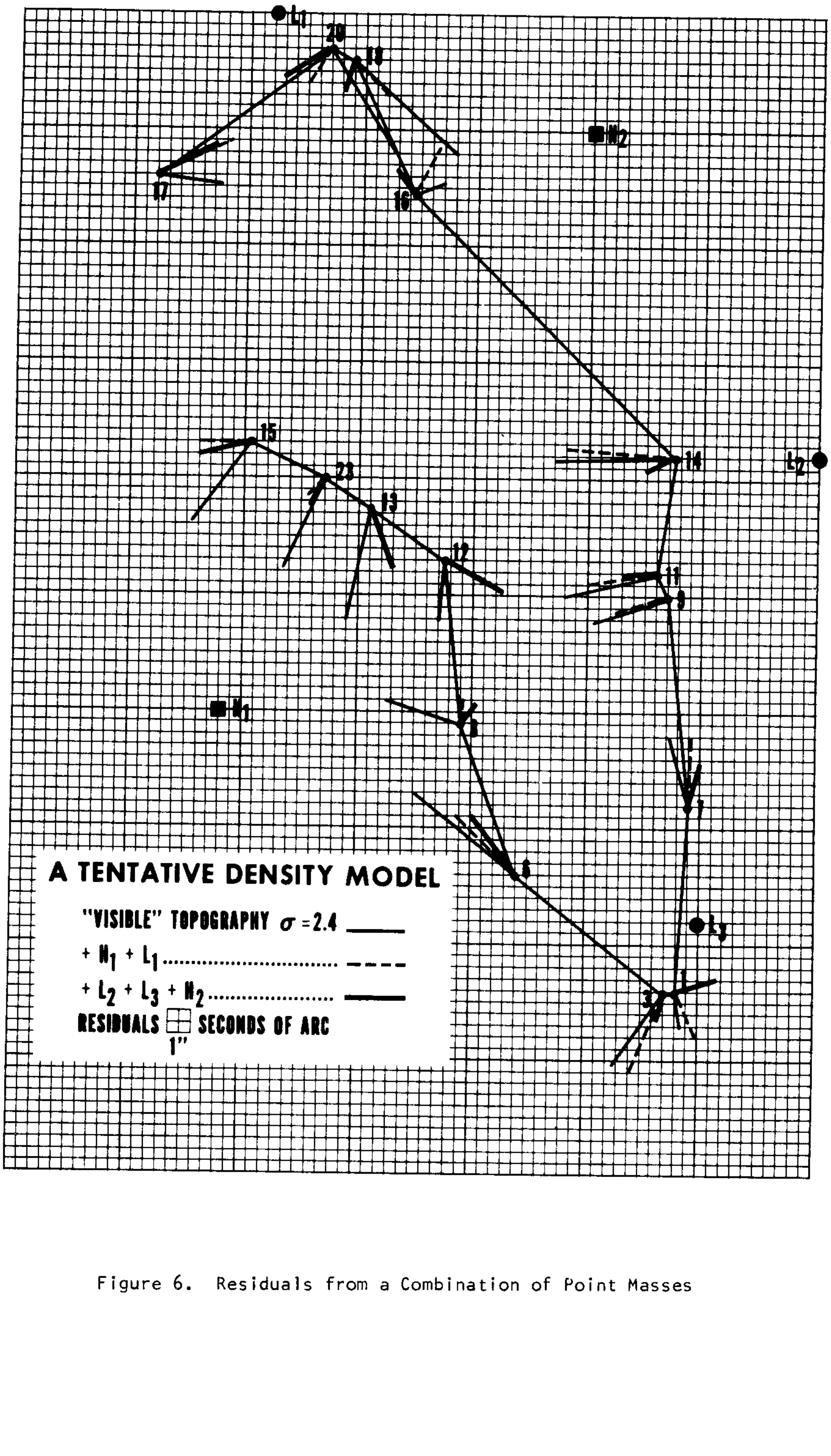

Figure 6. Residuals from a Combination of Point Masses

TABLE 4. Effect of a Tentative Density Model
Residuals from a Combination of Point Masses

	"Visible" Topography				+ H_1		+ L_1		+ L_2		+ L_3		+ H_2	
	σ =2.5		σ =2.4											
Sta	v_ξ''	v_η''	v_ξ''	v_η''	v_ξ''	v_η''	v_ξ''	v_η''	v_ξ''	v_η''	v_ξ''	v_η''	v_ξ''	v_η''
1	-0.23	-0.40	-1.62	+0.18	-2.04	+0.88	-1.99	+0.86	-1.72	+0.93	+0.54	+1.72	+0.49	+1.73
3	-1.65	-2.40	-3.10	-2.16	-3.54	-1.44	-3.49	-1.46	-3.22	-1.38	-1.09	-0.32	-1.24	-0.31
6	+5.19	-3.21	+3.44	-4.39	+2.54	-2.73	+2.61	-2.75	+2.87	-2.56	+2.69	-1.85	+2.49	-1.87
7	+3.86	-2.28	+3.06	-0.89	+2.87	+0.10	+2.93	+0.07	+3.50	+0.28	+1.95	+0.43	+1.72	+0.47
8	+1.99	-1.74	+1.11	-3.15	+0.95	+0.17	+1.05	+0.14	+1.30	+0.48	+1.11	+0.71	+0.82	+0.65
9	-1.12	-4.82	-1.08	-3.25	-0.81	-2.21	-0.72	-2.27	+0.51	-0.94	+0.23	-0.91	-0.24	-0.83
11	-0.98	-5.41	-0.94	-4.06	-0.61	-3.01	-0.51	-3.08	+0.59	-1.54	+0.35	-1.51	-0.16	-1.43
12	-1.73	+0.52	-2.54	-0.21	-0.99	+2.12	-0.83	+2.07	-0.68	+2.60	-0.81	+2.69	-1.29	+2.53
13	-3.57	-0.29	-4.79	-1.11	-2.27	+0.76	-2.06	+0.72	-2.01	+1.13	-2.09	+1.20	-2.56	+0.92
14	-0.99	-7.19	-0.17	-5.38	+0.25	-4.62	+0.36	-4.72	+0.36	-1.54	+0.22	-1.53	-0.62	-1.31
15	-1.87	-1.83	-3.21	-2.47	-0.37	-2.16	-0.08	-2.15	-0.09	-1.89	-0.14	-1.84	-0.47	-2.20
16	-0.78	-0.06	+0.60	+1.20	+1.34	+1.48	+2.07	+0.89	+1.88	+1.19	+1.83	+1.21	+1.13	-0.77
17	-1.82	+3.19	-0.39	+2.64	+0.43	+2.53	+1.42	+3.24	+1.36	+3.39	+1.33	+3.41	+1.27	+2.89
18	-5.86	+3.83	-3.92	+4.55	-3.37	+4.66	-1.59	+1.55	-1.73	+1.72	-1.76	+1.73	-1.38	+0.39
20	-6.45	+2.41	-4.36	+2.80	-3.82	+2.88	-1.42	-1.11	-1.55	-0.95	-1.58	-0.94	-1.23	-2.08
23	-2.52	-1.13	-3.78	-1.85	-0.95	-0.57	-0.70	-0.60	-0.68	-0.26	-0.75	-0.20	-1.18	-0.53
a	2.5	2.5	2.4	2.5	1.7	2.0	1.5	1.7	1.5	1.4	1.2	1.4	1.1	1.3
b	5/16	5/16	6/16	5/16	9/16	7/16	9/16	8/16	8/16	9/16	10/16	8/16	13/16	10/16

a = $\Sigma |v| / 16$
b = number of $|v| \leq 1.5''$ out of 16 cases

TABLE 5. Seven Test Points
Model: Density 2.4, Two "Holes" and Three "Loads"

Station	Observed Deflection		Residuals	
	ξ''	η''	v_ξ''	v_η''
2	- 23.59	+ 11.69	- 0.80	+ 1.86
4	- 23.79	+ 6.95	- 0.45	+ 0.72
5	- 21.82	+ 11.73	+ 1.41	+ 1.91
10	+ 0.05	+ 21.11	+ 1.81	- 0.37
19	+ 28.89	+ 10.49	+ 0.21	+ 0.09
21	+ 26.27	+ 10.14	- 1.87	- 1.28
22	+ 28.75	+ 11.16	+ 0.03	+ 0.50
$\Sigma \lvert v \rvert /7$			0.94	0.96
r.m.s.			1.17	1.17

wait, however, until the truncation error due to the limits of the data area has been sufficiently reduced.

6. Acknowledgment

Mr Philip Y. WYATT III, Defense Mapping Agency Topographic Center, assembled the data tapes from 14 000 readings and carried out the computations on the UNIVAC 1108.

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8. Discussion

KAULA: On the question of determining gravity anomalies from the topography at sea with a wavelength of a couple of hundred kilometres. I was wondering if you had considered using a regional type of isostatic compensation?

FISCHER: Yes, I did. I had started with the usual approach given in the textbooks: local compensation and regional. The Vening Meinesz regional compensation was considered at length but was put aside for the moment because the differences in results as compared to the other hypotheses were too small to explain the large residuals. I had bigger problems at that stage as the effect of various density models on the residuals is larger than the fine distinction between local compensation and regional, with the data used so far. When I started, it was thought that computing the deflections from mass attractions should tell us what type of compensation applied here. Maybe it will later on, after we have studied further the effect of truncating the data area, and of various density distributions.

BOMFORD: I do not quite understand how you locate your masses.

FISCHER: This is obtained from the residual vectors (Observed minus Computed): if that vector is large, and since the observed values cannot be changed, you have to make another assumption about the computed value. A mass excess or mass deficiency is added to the model to make the residual smaller. The vector points towards the area where a deficiency is needed to make that vector smaller. A point mass is placed there, which could be spread out later to make it more realistic, if one wants to. The point mass is assumed to be of a certain size, say a 10' × 10' square and a specified density difference, located at the centre of the mass column; and the resulting effect studied. It is a little trial and error, but using simple computation which does not require a big computer.

BOMFORD: You did a number of computations, iterating to make your residuals as small as possible?

FISCHER: Theoretically you could do so by making a lot of such assumptions. That is why I didn't pursue it any further beyond using just big ones - five of them - that have major effects. Hundreds could be used if desired, but of very local effect which would not help you predict effects elsewhere. Some conclusions can be drawn from the major effects: there is either some deficiency here or excess there; and there is some isostatic compensation. One could combine such findings, for example with the Vening Meinesz regional model or some other hypothesis so that the residuals would be small. Then one could try some geophysical interpretation. While that is premature, I was careful not to put any deficiency or excess arbitrarily which might contradict other evidence such as the depth readings from the maps.

GRAFAREND: How did you define your datum?

FISCHER: The datum of the astrogeodetic deflections is a given local datum. This is not the same datum as the one corresponding to the computations of mass attractions. I considered changing to a world datum or a regionally well-fitting datum. Since the Earth is practically flat for such a small region, such a datum transformation amounts to a constant correction of, say, about 3" in latitude and longitude to the given deflection values. One can apply a tentative correction on this account to begin with, or one can leave it for the inspection of the residuals. If the residuals were constant (rather than near zero), they could be interpreted as a datum shift. If the purpose is the interpolation of astrogeodetic deflections, such constant correction is taken care of within the interpolation process.

DOOLEY: Is there any gravity information in the area in which you are working?

FISCHER: Yes, but not enough for a computation of gravimetric deflections.

KEARSLEY: I would be interested to know how the station geodetic position was found.

FISCHER: There is a first order survey on a local datum. That survey connects all these islands and the positions are internally consistent. There are also some geoceiver stations. We checked the distances between these with the survey and they agreed. So we believe the claim that the survey is first-order.