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Proc.Symposium on Earth's Gravitational Field & Secular Variations in Position (1973),205-212.

SECULAR VARIATIONS IN GRAVITY

1. Text

The problem of secular variations of gravity is one of the acute problems of gravimetry and has fundamental interest for such sciences as metrology, physics, astronomy, geodesy, geophysics and geology. This is explained by the fact that the complex set of data on recent crustal movements, gravity variations, irregularity of the Earth's rotation and oscillations of the world ocean level allows the evaluation of the displacement of masses within the Earth, thus providing new data about the dynamics and, consequently, the history of development of our planet.

This is a complex task which shall demand long-standing efforts of an extensive group of specialists. Its integral part is the study of gravity changes in time in a wide spectrum of frequencies caused both by endogenous and exogenous processes. In this study, the determination of the fact of gravity changes in time is decisive.

The initial stage of researches in this field started as far back as the twenties when such great discrepancies in the g values were obtained at repeated gravity determinations, that it was difficult to explain them only in terms of the errors of measurement. This gave grounds for some of the authors (ABAKELIA 1936a;ABEKELIA 1936b) to draw a conclusion about the possibility of considerable gravity variations in time, which reach several microgal per year, associating them with tectonic processes occurring in the Earth's interior.

Further works proved these conceptions to be erroneous. Thus PARIISKY (1937), analysing the processes which can cause the irregularity of the Earth's rotation, has shown for the first time that such processes can be either the vertical movements of large sections of the Earth's crust or the transformation of the matter within the Earth with a noticeable change of density. Both these reasons can cause the change of gravity on the Earth's surface, but they must be small and not exceed tenths of microgal.

Much later BARTA (1957; 1962; 1963; 1965; 1967) and then VOGEL (1968) have suggested a hypothesis about the possible gravity changes in time, caused by the displacement of the Earth's core in respect of its mantle. According to their calculations, the gravity variations can reach tenths of microgal per year and can have quasi-periodic character with the period of about 500 - 1000 years.

As a result of researches of vertical crustal movements carried out by KALASHNIKOVA (1970), it was established that a large part of depressions and uplifts develops independently of the basic gravitational fields. This provides grounds for supposing that vertical movements are caused by compression or extension of deep material without change of the general mass, i.e., in this case, transportation of the matter within the Earth does not cause gravity changes on its surface.

Comparison of these two concepts indicates our extremely limited knowledge of processes that cause the dynamics of the globe. At the same time, success in experimental gravimetry, in measurements of variations in the rotation rate of the Earth, in geodetic measurements provide opportunities to approach the solution of this problem not only from the point of view of theoretical studies, but also by special experiments. We already have the capability to accumulate experimental data, without which the solution of this most important problem is impossible.

What are the data available to us at the present moment?

Of outstanding importance in the field of experimental gravimetry for the last decade are the works of Sakuma who elaborated the instrument for absolute gravity determinations with the accuracy of ±1 to ±2 parts in 10^9. By means of this instrument he has established the gravity acceleration in Sèvres, France, with the rate of about 20 microgal per year. This result is of extreme importance and opens new ways to further studies in this field.

At the same time, since it is as yet the only result, it is rather difficult to have a clear conception even whether the phenomenon is of global, regional or local character. Only the establishment of similar stations in other regions of the globe or the undertaking of special additional measurements with highly sensitive gravimeters can provide grounds to obtain the answer to this problem in the future.

I am not aware of any attempts to find explanations of causes of this change. A certain exception is the paper by BURSA (1972). He showed the influence of the movement of the pole which can reach 5 microgal per year, and is in agreement with the results obtained by Sakuma.

During the last 15 - 20 years, several attempts were made to discover the secular or spontaneous changes of gravity of both regional and local character. Of greatest interest as regards the results obtained are the works of Torge (SCHLEUSENEER & TORGE 1971) and of the Japanese colleagues Satomura and Nakagawa (SATOMURA & NAKAGAWA 1972; NAKAGAWA 1972).

The changes of gravity obtained in Iceland by Torge are not large and on the verge of measurement accuracy. Though their analysis and processing are carefully done, actual comparisons can be carried out only between two epochs of measurements conducted in 1965 and 1970. Thus, for this region, only one result was obtained with relative reliability. And in the case where the measured values are on the accuracy limit, it is difficult to consider such conclusions as reliable. It is widely acknowledged that in similar cases it is sufficient to have an insignificant systematic effect on measurements in one of the epochs and the result, based even on very well statistically processed data, can be erroneous.

At the same time, the works of Torge deserve serious attention, since for the first time they experimentally establish the fact of gravity change caused, perhaps, by tectonic processes.

Of no lesser interest are the works on the study of gravity changes in the region of Lake Biwo-Ko. As a result of these measurements, a decrease of gravity was established in the southern part of the lake for three years on the average 50 - 100 microgal, i.e., with a rate of the order of 15-30 microgal per year. These results were compared with the variations of the level of the lake and changes of heights caused by recent crustal movements. However, agreement is obtained only in tendency. The changes in heights of water level can explain only a small part of the variations. The authors therefore tend to ascribe these changes to the variations in the crustal density caused by tectonic processes.

In the Soviet Union, the works on the study of secular gravity variations were started in 1935. But only in 1954-55 were the first measurements accomplished for that purpose along the line Potsdam - Petropavlovsk-on-Kamchatka, and on several points in the Caucasus and in middle Asia. The points were chosen in such a way as to cover with measurements, regions tectonically quiet and seismically active (BOULANGER & SCHEGLOV 1971). Repeated observations were carried out ten years later in 1964. Notable changes of gravity were not detected. All the measurements were much less than the errors of their determination. As the result of their careful processing, a conclusion was made that if gravity changes occurred at the indicated points, they did not exceed 10 microgal per year.

Similar observations were conducted at a number of points in eastern Europe in 1958 and 1968. The same result was obtained here. The changes of gravity were less than the errors in their determination and the annual rates could not be more than 10 microgal.

Commencing in 1970, systematic repeated observations of gravity were carried out at points along a closed polygon Moscow - Sverdlovsk - Perm - Petrozavodsk - Moscow. These measurements have also not detected changes in gravity. All deviations were less than their errors.

In recent years in the scientific publications of the Soviet Union, a number of works (SOBOKAR 1968; FAITELSON 1969a; FAITELSON 1969b; FAITELSON & AZARKINA 1970; FAITELSON & AZARKINA 1972; SHRAIBMAN 1971; SIROTOV & PARFENOV 1970) were published, which present data on gravity variations with time, obtained as the result of comparison of gravimetric maps compiled in different years. Comparisons were made of the maps of the Baikal region, the Ukraine, the Volga region and the northern Caucasus. According to the opinion of the authors of these works, they have discovered gravity variations which have a good correlation with the tectonics of these regions. Moreover, the variations reached several tenths of microgal per year.

The experience of highly accurate repeated relative determinations of gravity testifies to considerable difficulties of comparison of data of different epochs. It is done with reliability only for separate points with their precise identification and the presence of complete information on the metrology of measurements. At the slightest inaccuracy in this information, as a rule, the measurements show systematic errors which considerably distort the conclusions. It is therefore rather difficult to agree with the reality of the data obtained on the basis of comparison of repeated area surveys.

Thus, careful analysis of the world literature shows that at the present moment, extremely scanty data are available for characterising gravity changes in time. The problem arises, what is to be done to accumulate them at a greater rate, so that we can have as soon as possible, the data characterising the dynamics of the globe by gravimetric data.

In this connection, I would like to make a few comments on the possible organization of international works for the study of gravity changes in time.

In the first place, it would be necessary to find out the representativeness of secular variations of gravity obtained in Sèvres. This can be done by comparatively simple means using relative methods of measurements. In order to reveal the local component, it is necessary to establish a reliable control of the stability of the altitude of the station and organize a network of gravimetrical points located at distances ranging from 1-2 km to 25-30 km from Sèvres, and to tie them directly to the point at Sèvres. At the rate of changes around 20 microgal per year, using modern gravimeters, in 2-3 years we can obtain a reliable answer to the problem whether the observed phenomenon is local or regional.

In the case when the local character of the phenomenon is not evident, then gravimetrical measurements should be conducted along the line of latitudinal direction several hundreds of km long, and passing through Sèvres. Repeated observations on that line already in three years should be able to considerably elucidate this problem, and in five years produce an unequivocal answer about the character of the observed phenomena in Sèvres.

Considering the importance of measurement of secular variations of gravity in the equatorial zone, since actually in this zone the displacement of mass has its greatest influence on the Earth's rotation, it seems apparent to ask the countries having portable equipment for the absolute determination of gravity, to undertake absolute determinations annually at a number of points. In the first place, according the calculations of Barta, these observations should be organized in the region of Rangoon (Burma), Teheran (Iran), Tripoli (Libya), Accra (Ghana) and in the central part of Brazil. With an accuracy of measurements even of the order of ±1-2 parts in 10^8, we can expect in 3 - 5 years to obtain a positive answer about the possibility of displacement of the centre of the Earth's mass.

Since the measurements with instruments for absolute gravity determinations shall be conducted on a limited scale in the next few years, it would be useful to establish a network of international gravimetric points of high precision on the basis of the stations indicated above. These points of high precision should be located in regions with supposed maximum and minimum gravity changes. The connections of these points with points of absolute determinations by modern means can be carried out with an accuracy of the order ± 15 microgal. During repeated determinations once every 2 - 3 years, we can expect to obtain already in 5 - 6 years the first conception of the regional gravity changes. Using the network of points, the interested countries could conduct the study of local gravity changes.

Naturally, the suggested program can be realised only on the basis of international co-operation and in close collaboration of scientists, whose efforts provide the possibility of obtaining the necessary financial support and sufficient number of gravimeters well qualified for highly precise measurements.

In the Soviet Union, within the framework of this project, a program has been elaborated on for the study of secular variations of gravity for 10 years. This program envisages the establishment of a network of stations on the territory of eastern Europe and in the west of the Asiatic part of the USSR. This network will be based on Sèvres. The stations shall be located both in aseismic regions and in regions seismically and tectonically active. Simultaneously, studies of local gravity measurements shall be continued in middle Asia, the Baikal region, in Crimea, on the Caucasus, in the Ukraine amd Karelia.

The Soviet Geophysical Committee is prepared to participate in the establishment of the international network of gravimetrical points with the purpose of the study of secular variations of gravity.

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Table 1

Secular Change in Gravity (units milligal)

	$m_g(1955)$	$m_g(1967)$	$\delta = g_{1967} - g_{1955}$		$\delta' = \delta - \Delta$	$m_{\delta'}$	$\delta'/m_{\delta'}$
Potsdam	±0.00	±0.00	0.00	±0.00	-0.07	±0.04	-
Moscow	0.16	0.05	-0.10	0.17	-0.17	0.17	1.0
Riga	0.26	0.09	+0.31	0.28	+0.24	0.28	0.9
Kazan	0.25	0.07	-0.01	0.26	-0.08	0.26	0.3
Sverdlovsk	0.30	0.07	+0.02	0.31	-0.05	0.31	0.2
Tschita	0.51	0.12	+0.20	0.52	+0.13	0.52	0.2
Tahtamigda	0.58	0.11	+0.12	0.59	+0.05	0.59	0.1
Petropavlovsk	0.72	0.14	+0.16	0.73	+0.09	0.73	0.1
		Mean: Δ' =	+0.10	± 0.05			
Tbilisi	±0.32	±0.10	-0.03	±0.34	-0.10	±0.34	0.3
Ashkhabad	0.46	0.10	+0.03	0.47	-0.04	0.47	0.1
Dushanbe	0.51	0.12	-0.13	0.52	-0.20	0.52	0.4
Alma-Ata	0.49	0.11	+0.18	0.50	+0.11	0.50	0.2
Balhash	0.50	0.12	+0.18	0.51	+0.11	0.51	0.2
		Mean: Δ'' =	+0.05	± 0.06			
		Mean: Δ =	+0.07	± 0.04			

Table 2

The Tie "Potsdam" - "Moscow (Ledovo)"

Year	Δg	M(Δg)
1958	290.63	± 0.16 mgal
1968	.55	0.05
1970	.31	0.04
1971	.31	0.03

Table 3

Changes of Gravity on the European Part of the Soviet Union

Name of Point	Changes (mgal)	Error (mgal)
	1968 - 1973	
Moscow (Ledovo)	0.00	± 0.000
Kazan	-0.01	0.045
Sverdlovsk	+0.02	0.059
	1970/1971 - 1972/1973	
Moscow (Ledovo)	0.000	± 0.000
Kazan	-0.005	0.022
Sverdlovsk	+0.030	0.030
Perm	-0.005	0.038
Kirovsk	-0.005	0.038
Petrozavodsk	+0.005	0.030

Table 4

Results of Repeated Measurements in Countries of Eastern Europe
Units - mgal

Names of Points	1958 M(g)	1968 M(g)	Differences δg	$M(\delta g)$
Potsdam	±0.00	±0.00	0.00	± 0.00
Berlin	0.01	0.01	-0.04	0.01
Warsaw	0.09	0.02	-0.13	0.09
Prague	0.09	0.02	+0.10	0.09
Budapest	0.13	0.04	-0.04	0.14
Bucarest	0.16	0.04	0.00	0.16
Sofia	0.16	0.07	+0.07	0.17

4. Discussion

HOPKINS: Talking about 2/100 th mgal; are these instrumental errors?

BOULANGER: Yes. ±0.02 mgal is the error of the measurements. But each apparatus gives us more error than this. That is the error of the mean. We work with a minimum of 7-8 instruments and we measure each Δg 3 to 4 times. That is the error of the mean of all measurements.

HOLDAHL: You compared changes in gravity with changes in elevation. How did you measure the changes in elevation?

BOULANGER: In the fundamental sense we cannot control elevations, but it could be done in the following manner. We combine measurements of gravity with very comprehensive measurements of elevation in eastern Europe where we have many loops along which measurements are taken. These results give us vertical crustal movement. The movement is of the order of 1-3 mm per year.

HOLDAHL: Specifically what measuring method was used? Was it precise levelling?

BOULANGER: Yes. The points in the net were tied to check points at each station in the system of points. Having measured a change in gravity, you take the vertical component and relate it to the nearby points. The effects of microseisms, man made vibrations, temperature, etc. can be eliminated from the observations.

HOLDAHL: What is the distance between the two standard points?

BOULANGER: Quite different. Between satellite points situated around the principal points from 2 to 12 km.

HOLDAHL: Overall?

BOULANGER: Between principal points and standard points the mean is 300 - 400 km.

SAKUMA: Within two years, a portable gravity meter (for measuring absolute gravity) of 10 μgal accuracy will become available commerically.

MUELLER: When we get down to these sort of numbers, what do you do when the ground water level changes? How can you separate these changes and changes in climate?

BOULANGER: Yes; there is probably a change in level that can be attributed to water level changes to 0.02 mgal.

ECKHARDT: A base gradiometer at M I T (Mass. Inst. Technology) was sensitive to changes in water on the roof, sewage level, etc.

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Proc.Symposium on Earth's Gravitational Field & Secular Variations in Position (1973),213-217.

DISTORTION OF THE GRAVITY FIELD AND ITS CONSEQUENCES

1. Text

It is known that the geoid can be represented as a sum of two rotation-symmetrical forms, the axes of which are found in the plane of the equator. One of these axes originating from the centre of the Earth is directed towards India, while the other is directed towards Australia. The former has no ellipticity in contrast to the latter which has it (BARTA 1973a). (These two figures must not be confused with polar oblateness which also possesses a rotational symmetry.)

The combined form represents the true observed geoid with such an accuracy that it is worthwhile to publish the well known diagram again and to subject it to further discussion (figure 1). The proving force of similarity is a strong one because the computed figure is based only on the equatorial data of the geoid; thus the similarity is not a result of a simple approximation, but it is an evidence of principle of the suppositions used in the computations. Thus the equatorial data system of the geoid implicitly contains the four anomalies of the temperate zones too, together with sign, form and dimension.

Deviations between the two diagrams present themselves only in the orientation of the positive and negative anomalies of the temperate zones. On the observed figure (A), these directions show a certain oblique angle with the equator, while on the computed one (B) they are perpendicular to it. As it is easy to see, this is only as a consequence of our computations having been based only on equatorial data. Removing this constraint enables us to obtain also the oblique anomaly-directions seen on the measured diagram (A) and we can assure a similarity even better than seen in the figure.

This interpretation of the figure of the geoid has far reaching consequences within the domains of many branches of science. For example, the polar oblateness belonging to hydrostatic equilibrium increases owing to the reported ellipticity of the Australian form and if the axes of the two rotation-symmetrical forms do not fall exactly into the equatorial plane, then there must exist a deviation between the northern and southern hemispheres (the pear shape of the globe, etc.).

But let us disregard the geodetic aspects of the phenomenon and let us consider first of all, the consequences in connection with the internal structure of the Earth. In that respect, the most important consequence of the statement is that the geoid figure is determined only by a small number of relevant factors, i.e., the six big anomalies of the geoid are not formed by near-surface gravitational phenomena, but by two large scale effects only, against which the globe behaves as a fluid, and the influence of which makes itself felt on the whole surface of the Earth, even at opposite sides. These effects, owing just to their small number, do not intermingle with one another on the other side within the framework of measurement-noise, as it could be supposed in the case of a great number of acting sources. Thus they can be separated and discussed individually, so that we can draw exact

A

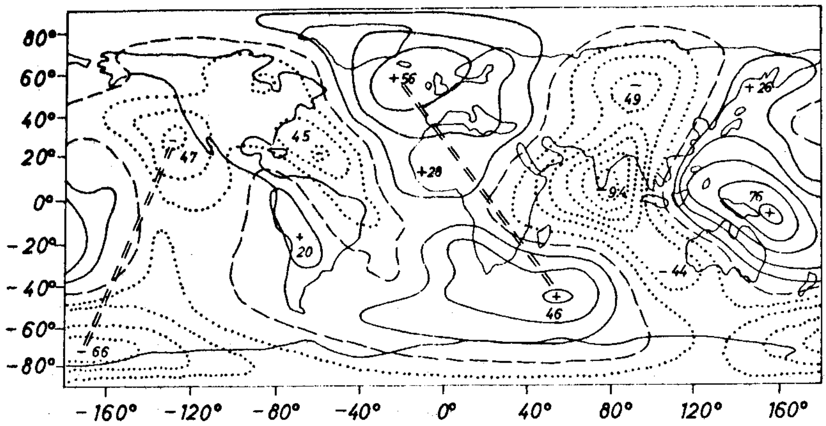

Figure 1 (A): The "Smithsonian 1966" geoid

B

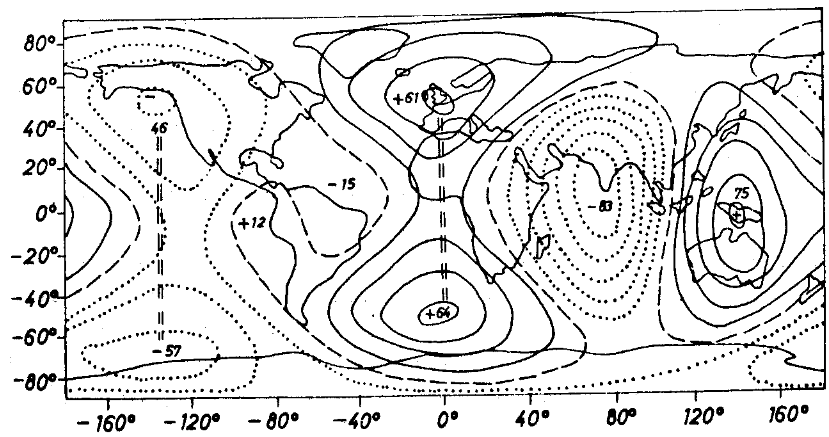

Figure 1(B): Geoid Figure Computed from Two Rotation-Symmetrical Anomaly Systems

consequences as regard their cause and different characteristics.

A mass anomaly of such a great dimension cannot exist in the material of the crust and mantle owing to the tendency towards isostasy. Under the influence of the gravity field, the anomalous mass would have been shifted up or down depending on its density during geologic ages. Thus, an anomaly of such a character can exist only either on the surface or at great depth. On the surface of the Earth, no sign of such a density anomaly can be found in the areas involved. Thus we have to direct our attention to the domains at great depth.

The mobile, plasmatic material of the outer core, however, is even less suitable for maintaining a significant inhomogeneity. It is strange enough that there is only one domain of the globe capable of holding the source of such a large-scale anomaly system, i.e., the inner core of the Earth. The inner core or its centre, respectively, represents namely a singular point of the gravity potential field. For this material domain the notion of "up" and "down" ceases to exist, because it does not exert any influence on itself as a unit, by its gravity field. The masses of the crust-mantle and the outer core (taken in first approximation as spherical shells), do not exert an attracting force on it, their gravitational interaction manifests itself in a pressure only. That is the reason why we need not - or even must not - suppose a central position of the inner core.

The eccentricity of our magnetic dipole - long known - indicates that the inner core does not show in fact a central symmetrical position. This is supported also by the fact that the component anomaly of the geoidal figure, which has a rotational symmetry from the direction of Australia, is showing an ellipticity in the direction of the eccentricity of the magnetic dipole. Thus both force fields indicate a general asymmetry of the internal structure of our globe. According to the indications of the two fields, the inner core of the Earth possesses an eccentric position in the direction towards Australia.

If the material of the crust and mantle shows a central symmetry in the form of spherical shells, then on the Gutenberg-Wiechert surface we have a constant pressure; the shells do not exert any internal effect. Thus they need not be taken into account in what follows. If however the material of the outer core can be compressed, then for the inner core of an arbitrary position, there exists a density distribution producing a constant pressure on the Lehman surface, this being the condition for a hydrostatic equilibrium of the inner core at the given location.

Thus assuming a suitable density distribution, one can arrive naturally at any desired value of the eccentricity. Therefore some force or effect must exist maintaining the eccentricity on the one hand and the inhomogeneity of the material of the external core on the other. One of such effects is the centrifugal force acting on the inner core, directed outwards and increasing linearly with the distance from the axis of rotation.

To define another such force we have to take recourse to the following considerations. When computing pressure we have taken into account the gravity interaction of the materials of the inner and outer core, but we did not consider the gravity effect of the material parts of the outer core on one another, though these are counteracting the inhomogeneity of the outer core in the direction of promoting the development of homogeneous conditions. This force is directed towards the centre and increases - as an approximation - quadratically with the distance from the centre. A rough estimation shows that the two forces equal one another at an approximate distance of 100 km from the centre. Thus the present eccentricity of the magnetic dipole amounting to 430 km can be related to the eccentricity of the inner core of the Earth, and what is more, it is an inevitable consequence of the hydrostatic equilibrium of the inner core when interpreted correctly (BARTA 1973b).

Thus the mechanism producing the eccentricity of the inner core of the globe is similar to the working method of speed regulators found on the old steam machines. The eccentricity does exist as long as the Earth rotates. In case of a stopping of rotation, central symmetry will be restored if the physical characteristics of the materials of the outer core remained unchanged.

The direction of the eccentricity however, would not be determined by the physical causes mentioned above. According to the reasoning, the rotation axis of the Earth is surrounded by a potential-trench where the inner core is lying, taking some direction. Owing to its eccentric position, it is not in an equilibrium state in the system Sun-Moon-Earth, so that it is shifting within this trench towards west, which may be the cause of the westward drift of the magnetic field.

Because of the westward drift of the the inner core, the material of the outer core must flow off the direction of movement and flow behind the inner core. The plasmatic material around the spot of divergence (under India) begins whirling, producing by its flow the magnetic secular variation; while by its dynamic effect, it produces the other rotation-symmetrical anomaly-configuration in agreement with the determination of the geoid (this has no ellipticity; the zonal spherical harmonics used for its representation have significant members only of odd orders (BARTA 1973a)).

The eccentricity and movement of the inner core furnishes - of course - much information about the material characteristics (compressibility, viscosity, etc.) of the outer core. Setting aside these opportunities for now, let us consider the possibilities for studying the variation phenomena represented by this movement.

The factual shifting of the core may cause a significant change in time of the internal mass distribution of our globe and of its gravity field as well. The interconnections are however very complex ones. The crust - and mantle - domain, not having the form of a spherical shell and in reality not being homogeneous, exerts an influence inside, while the asymmetrically located inner core is also influencing these masses above, and what is more, it keeps them moving too. Thus we cannot know what amplifying or compensating effects will result on the surface as a result of these complex interactions. It is clear that the anomalies determining the shape of the Earth must shift towards the west with the speed of the westward drift. We do not know any details as yet and many contradicting observations may support the development of deviating views. With the interpretation of measurements lacking sufficient accuracy, observational errors may be taken as signs of a secular change, while - owing to a great scattering of measurements - even the existence of such change can be contested.

Thus from the point of view of internal structure and processes of the Earth, it is of decisive importance to subject to an investigation - besides measurements regarding the eccentricity of the magnetic dipole and the magnetic secular variation - similar parameters of the gravity field too, and by increasing - as far as possible - the observational accuracy to the order of magnitude of the variations. Measurements should be arranged - of course - first of all on spots where variations of a maximum intensity may be expected. In case of an eccentricity and shifting of the inner core, it seems probable that the greatest changes will present themselves along the equator, and even here, at points where the sources of the two anomalies are near one another, i.e., between India and Australia.

But irrespective of any theory, a simple logical reasoning supports the inference that the greatest change in time is to be expected there, where the spatial gradient of the fields is the biggest one. The biggest positive anomaly of the geoidal figure can be found near New Guinea - northern Australia, while the greatest negative anomaly is near Ceylon.

It is not possible to establish a continuous measuring profile between these points owing to the presence of the ocean. Besides the theoretical considerations (the magnetic dipole being eccentric at present north of Australia towards the Marshall Islands, while the magnetic secular variation, also having a significant symmetry-point in the northern hemisphere near Pakistan) practical possibilities also indicate that the measuring profile should be laid along the line: north Australia - Indonesia - south east India - east India - Iran.

The surveying of this line as well as its repeated re-surveying would be needed for the accurate recognition of the inner structure of the Earth and if its internal processes. In the event of successful observations, we would obtain a way to understand the causes of tectonic movements, too. That is why Working Group 6 of the Inter-Union Commission on Geodynamics has included among its resolutions, the setting up of the above mentioned measuring line and has addressed the conference of the International Association of Geodesy at Sydney, as well as the teritorially competent national commissions to support the plan and to make arrangements for its implementation.

When investigating the asymmetry of the globe, we inevitably risk the loss of the elegant and well applicable symmetry characteristics of the mathematical techniques applied as yet for the description of the fields of forces. Nevertheless, the accuracy of measurements has been increased recently to such an extent that the study could be extended to the details too. But clinging to the results of these more accurate measurements we have to re-examine these symmetry-characteristics and to apply an approximation procedure of higher order which adapts itself better to reality for the interpretation of the fields of forces.

In exchange for the loss of symmetry we get a compensation however, through the appearance of the possibility and idea of a variation itself and through the greater flexibility obtained by the abandonment of the rigorously symmetrical systems. It is possible that just on the basis of these advantages, some basic geophysical-geodetical problems will near their solution.

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Proc. Symposium on Earth's Gravitational Field & Secular Variations in Position (1973), 218-221.

GRAVITY CHANGE IN JAPAN

ABSTRACT

Gravity changes accompanying the Niigata earthquake, the Matsushiro swarm earthquakes, Mihara volcanic activity in Ooshima island, and crustal activities in Hokkaido, South Kanto and Shikoku districts have been found in Japan.

The correlation between gravity change and height change in Matsushiro is discussed under consideration of the mechanism of the Matsushiro swarm earthquakes. There are examples in which the gravity changes are often larger than expected. Some of them might be related with dilatancy of the crust. It is considered that the gravity survey is useful for earthquake prediction.

1. Text

Recent developments in gravimeter instrumentation and measuring techniques enable us to detect significant gravity change that may accompany various kinds of geophysical phenomena such as earthquakes, volcanic activity, tectonic movements and so on. Recently, many examples of gravity changes have been reported in Japan, as shown in table 1. These studies were recently reviewed by HAGIWARA & TAJIMA (1973a).

Table 1
Examples of Gravity Change in Japan

Phenomena	Location	Reference
Earthquake	Niigata	FUJII 1966
	Matsushiro	HARADA 1968
	Gifu	TANAKA & TSUKAHARA 1969
	Nemuro	OKAWA, YAMASHITA & YOKOYAMA 1973
Volcanic eruption	Oshima island	INOUCHI, KANO & FUJII 1972
Ground subsidence	Niigata	FUJITA 1971
	Tokyo	FUJITA 1971
	Mobara	HAGIWARA & TAJIMA 1973b
Tectonic movement	Kii & Muroto	FUJII & NAKANE 1972
Others	South Kanto	TAJIMA 1970
	Lake Biwa	SATOMURA & NAKAGAWA 1972

A typical example of gravity changes accompanying earthquake activity is the case experienced during the Matsushiro swarm earthquakes. This example seems to show that the observed gravity change cannot be interpreted by the deformation of continuous medium (TAZIMA 1970), but by the dilatancy model of earthquake occurrences (KASAHARA 1970; SCHOLZ, SYKES & AGGARWAL 1973).

Observed gravity changes associated with the Matsushiro swarm earhquakes are shown in figure 1. In stage I, the land around Mt. Minakami began to upheave by Δh and the gravity decreased by Δg. The observed $\Delta g/\Delta h$ is approximately equal to the free air gradient (-0.3086 mgal m^{-1}). In stage II, the most active period of earthquakes, the large amount of underground water started to flow out. The land upheaval and gravity decrease still continued. The observed gradient is quite different from the free air gradient and the Bouguer gradient (-0.1967 mgal m^{-1}). In stage III, with decrease of seismic activity, the land began to subside, the ground water flowed out of the crust and the gravity began to increase. The observed gradient returned to the free air gradient again. The results of seismological observation (pull and push) and geomagnetic survey showed the existence of east-west compression around Matsushiro.

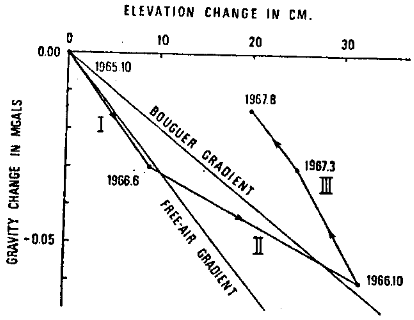

Figure 1 . Relation between Changes in Gravity and Elevation Observed at the Matsushiro First-order Gravity Station

When the thickness of the Bouguer plate varies from h to $h+\Delta h$, and the density from ρ to $\rho+\Delta\rho$, the gravity on the plate varies from g to $g+\Delta g$. Then the observed gradient is expressed as

$$\Delta g/\Delta h = -0.3086 + 2\pi k\rho + 2\pi k \Delta\rho h/\Delta h.$$

In stage I, the development of the open cracks in the porous medium might have caused the land upheaval. The gravity decreases along the free air gradient because of $\rho = -\Delta\rho h/\Delta h$. In stage II, the observed gradient of - 0.12 mgal m^{-1} results in $\Delta\rho \doteqdot +1 \times 10^{-4}$ g cm^{-3}. The porosity of the crust might have decreased together with the mass intrusion. The ground water might have also penetrated from neighbouring regions into the porous medium. A part of the ground water flowed out of the crust. In stage III, the closing of these cracks might have occurred. The ground water remaining in the crust might have been forced to come out of the crust when the cracks closed. Then the observed gradient is nearly equal to the free air gradient. The free air gradient in stage III may be explained mainly by the closing of the cracks. The explanation mentioned above is schematically shown in figure 2.

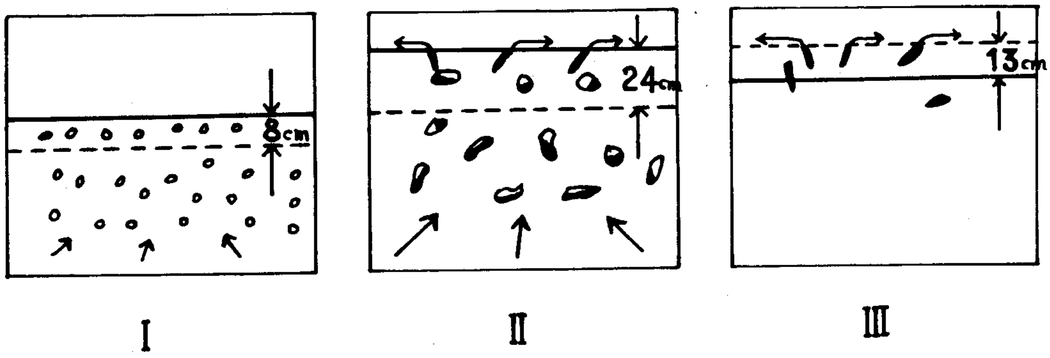

Figure 2 . A Dilatancy Model for Gravity Change at Matsushiro
I : Stage 1. Dilatancy II : Stage 2. Mass Intrusion & Density Increase III : Stage 3. Shrinkage

The writers proposed the pattern of the seismic crustal deformation (FUJITA & FUJII 1973). In figure 3, the β_1 phase corresponds to the dilatancy process and the γ_3 phase to the shrinkage process. It is expected that the free air gradient may appear in the β_1 and γ_3 phases. And the gravity change can be one of the precursory phenomena to earthquakes.

The gravity changes have been discussed around the accuracy of the gravity survey. At first, the accuracy of the gravity survey should be improved. In 1968, the Gravity Meters Comparison Group was organized. In 1969-1971, three symposia were held on Gravity and High Accuracy Levelling under the auspices of the Geodetic Society of Japan. In accordance with the resolution adopted at the symposia, the gravity survey of the south Kanto district has been carried out several times by using several La-Coste gravity meters since 1969 (NAKAGAWA 1971). It has been clarified that the gravity increase prevails in the south Kanto district. It is considered that the gravity value at the datum in the University of Tokyo may be decreasing by 0.01 mgal yr^{-1} (HAGIWARA & TAJIMA 1973a). It seems that the ground water level around the datum is dropping faster than the mean level of ground water in the other areas being surveyed.

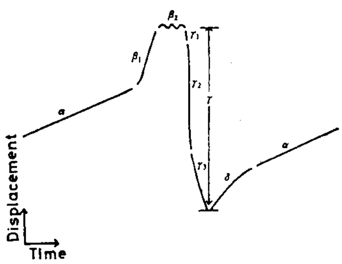

Figure 3. A Typical Pattern of Seismic Crustal Movement

α : Steady State
γ_1 : Pre-Seismic Slip
γ_2 : Co-Seismic Slip
γ_3 : Post-Seismic Slip
(γ_1, γ_2, γ_3) γ : Geodetic Slip
β_1 : Dilatant State
β_2 : Unstable State
δ : Transient State

When discussing the gravity change, the local conditions, such as open cracks and ground water level in the upper crust, cannot be neglected.

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3. Discussion

WALCOTT: The gravity change at Matsushiro is of great interest and, if correct, of considerable importance. You have small changes of 50 microgal but show your data as points. I presume there are error bounds on your gravity data. What are they?

FUJITA: ±0.02 - ±0.03 mgal.

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Proc. Symposium on Earth's Gravitational Field & Secular Variations in Position (1973), 222-226.

CONTRIBUTION TO THE VARIATIONAL FORMULATION OF THE BOUNDARY VALUE PROBLEM OF THE GRAVITY FIELD AND THE DISCRETE GRAVITY FIELD

ABSTRACT

The possibility of the variational formulation of the boundary value problem of the gravity potential is discussed. The method under consideration will be derived quite generally without any hypothesis about the mass distribution between the Earth's surface and the geoid. We shall start from the gravity data given on the Earth's surface. The relation between the variational formulation and the discrete gravity field is shown.

1. Introduction

In this contribution, a possibility of the solution of the variational formulation of the boundary value problem of the gravity field will be discussed. We start from the gravity data given on the Earth's surface and we do not make any hypothesis about the mass distribution between the Earth's surface and the geoid. The Earth's surface is assumed to be sufficiently smooth with a finite number of singular points and with the demand that the fundamental boundary value condition of gravity is fulfilled with the predetermined accuracy. Let us assume also that the total mass of the normal body is equal to the mass of the Earth and, further, both bodies turn around the same axis going through the identical centre of gravity with the same angular velocity ω.

2. Theory

We shall consider two cases of the formulation of studying the outer gravity field:

Case 1

1. The disturbing potential T fulfils the Laplace equation

$$\Delta T = 0$$

outside the Earth (i.e., in Ω).

2. It fulfils the condition

$$\frac{\partial T}{\partial n} + \frac{2T}{R} = -\Delta g$$

on the Earth's surface S, where n is the direction of the normal to the Earth, R is the distance from the point P on the Earth's surface to the centre of the Earth, Δg is the free air anomaly at the same point P.

3. It fulfils the condition of regularity

$$\lim_{r\to\infty} rT = 0$$

at infinity.

This problem is equivalent to another boundary value problem with a homogeneous boundary condition. By putting

$$t = T - w,$$

where T is the disturbing potential of the original problem and w is a suitable sufficiently smooth function fulfilling the boundary value conditions given in (2) and (3). We then obtain

$$\Delta t = 0 \qquad \text{outside the Earth,}$$

where

$$\theta = -\Delta w \qquad (1),$$

$$\frac{\partial t}{\partial n} + \frac{2t}{R} = 0 \qquad \text{on the Earth's surface S} \qquad (2),$$

and

$$\lim_{r\to\infty} rt = 0 \qquad \text{at infinity} \qquad (3),$$

or, in the operator's form

$$L\,t = -\theta.$$

Let us define the domain $D/\Omega/$ of the operator L as a set of all finite functions* twice continuously differentiable on the closure $\bar{\Omega}$ $(C^2/\Omega/)$, which satisfy the boundary conditions 2 and 3. We shall consider the completion of the $D/\Omega/$ with respect to the energetic norm. We call this space *energetical space* and denote it by $V/\Omega/$. In fact, we have

$$/Lt, t/ = /-\Delta t, t/ = \int_\Omega /\text{grad } t/^2\, d\Omega + \int_S \frac{2}{R} t^2\, dS \geq 0.$$

If $/Lt, t/ = 0$, then $t = C = \text{const.}$ As t is a finite function, we have $C = 0$ and hence $t = 0$. Thus $/Lt, t/ > 0$, $t \neq 0$ and the operator L is positive.

As the operator L is a positive operator, the problem leads to the minimization of the following functional

$$F/t/ = \int_\Omega /\text{grad } t/^2\, d\Omega + \int_S \frac{2}{R} t^2\, dS + \int_\Omega 2\theta\, t\, d\Omega \qquad (4).$$

This means that in order to find the disturbing potential T outside the Earth, we have to minimize the functional $F/t/$ from equation 4 over a special Hilbert space - the so-called energetical space $V/\Omega/$ (see NEDOMA 1973b) and using the expression

$$T = t + w.$$

As the author shows in the reference quoted, there exists exactly one disturbing potential T. This

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* A real function f belongs to the class of finite functions if there is a $r > 0$ and a sphere

$$K_r = \{/x,y,z/ \in R^3, x^2+y^2+z^2 \leq r^2,\ r > 0\}$$

such that $f/x/ = 0$ for $x \notin K_r$

is a consequence of the following facts:

a. That

$$\theta = -\Delta w = - \text{div grad } w = \text{div } F/x/,$$

where

$$|F/x/| = |-\text{grad } w| \in L_2/\Omega/,$$

and

$$\int_\Omega t_n \, \theta \, d\Omega = - \int_\Omega \text{grad } t_n \text{ grad } w \, d\Omega + \int_S t_n \frac{\partial w}{\partial n} \, dS,$$

where

$t_n \in V/\Omega/$ is the finite function, $\frac{\partial w}{\partial n} \in L_2/S/$ (see IBID);

and then the assumption of theorem 42.2 of MIKHLIN (1966) are satisfied.

b. That the disturbing potential T does not depend on the choice of the function w.

Case II

The same problem may be studied from the point of view of the other formulation in which any satellite's measurement can be used. We shall construct a certain surface of known form, which will be assumed to lie at a height H above the Earth's surface and that the angle between the direction of the normal n to it at the point Q and the direction of the normal n_1 to the level surface, passing through the same point Q, is small. We assume that it turns around the same axis passing through the same centre of gravity with the same angular velocity ω. We may then consider the fundamental boundary condition of gravity on the discussed surface in the form

$$\frac{\partial T}{\partial n} + \frac{2T}{R_1} = - \Delta g_1,$$

where n_1 is the normal to the mentioned surface Σ, R_1 is the distance of the point Q from the centre of the Earth and Δg_1 is the free-air anomaly obtained from the satellite measurement or from measurements at the Earth which were recalculated on the measured surface Σ. As in the case of the formulation discussed earlier, the disturbing potential T fulfils Laplace's equation outside the Earth and the fundamental boundary value condition of gravity on the Earth's surface. Using ideas analogous to the above, we obtain the equivalent problem for the disturbing potential t:

$\Delta t = \hat{\theta}$ in the space outside the Earth's surface, which we denote as $\hat{\Omega}$, and where

$$\hat{\theta} = -\Delta w_1 \quad (1'),$$

$$\frac{\partial t}{\partial n} + \frac{2t}{R} = 0 \quad \text{on the Earth's surface S} \quad (2'),$$

$$\frac{\partial t}{\partial n_1} + \frac{2t}{R_1} = 0 \quad \text{on the surface } \Sigma \quad (3'),$$

i.e.,

$$At = - \hat{\theta},$$

where A is a positive definite operator (see NEDOMA 1973b), as

$$(At, t) \geq \gamma^2 ||t||^2_{W_2^1},$$

$$\gamma = \min/\frac{2}{R}, \frac{2}{R_1}, 1/ \quad /\min \pi^{-2}/ /a-a_1/^{-2} + /b-b_1/^{-2} + /c-c_1/^{-2} /^{-1}, c_1 \pi^{-2}//a-a_1/^{-2} + /b-b_1/^{-2} + /c-c_1/^{-2}/^{-1},$$

$$+ c_2\pi^{-2}//a-a_1/^{-2} + /b-b_1/^{-2} + /c-c_1/^{-2}/^{-1}/,$$

$$c_1 \geq \frac{1}{f}\left|\frac{\partial f}{\partial n}\right| \Big|_S \quad , \qquad c_2 \geq \frac{1}{f}\left|\frac{\partial f}{\partial n_1}\right| \Big|_\Sigma \quad ,$$

where

$$f = \sin\frac{\pi x}{a-a_1}\sin\frac{\pi y}{b-b_1}\sin\frac{\pi z}{c-c_1} \quad , \quad f \neq 0 \quad \text{in } \hat{\Omega},$$

and where $a, a_1, \ldots, c_1$ are the parameters of the cube K (see figure 1) which contain the region $\hat{\Omega}$. The study of this problem leads to the minimization of the following functional

$$F/t/ = \int_{\hat{\Omega}} /\text{grad } t/^2 \, d\hat{\Omega} + \int_S \frac{2}{R} t^2 \, dS + \int_\Sigma \frac{2}{R_1} t^2 \, d\Sigma + \int_{\hat{\Omega}} 2\,\theta\, t \, d\hat{\Omega} \qquad (5).$$

Minimization of this functional takes in the special concrete Hilbert space $V/\hat{\Omega}/$, where $V/\hat{\Omega}/$ is the closure of $D/\hat{\Omega}/$ in the sense of the metric of the Sobolev space $W_2^1/\hat{\Omega}/$ (see IOSIDA 1965), and where $D/\hat{\Omega}/$ is the set of all functions $v \in C_2/\hat{\Omega}/$ which satisfy boundary conditions (2') and (3'). As shown in (NEDOMA 1973b), there exists exactly one disturbing potential T. This follows from the fact that the corresponding bilinear form $B/t,v/$, where

$$B/t,v/ = \int_{\hat{\Omega}} \text{grad } t \;\; \text{grad } v \, d\hat{\Omega} + \int_S \frac{2}{R} t\, v\, dS + \int_\Sigma \frac{2}{R_1} t\, v\, d\Sigma,$$

for all $v \in V/\hat{\Omega}/$ is V-elliptic *, and from the fact that T does not depend on the choise of the function w_1.

The space outside the Earth Ω and $\hat{\Omega}$ in the case where the gravity data from satellite observations are used, respectively, will be covered by a finite complex M_h of simple bodies, e.g., simple prisms, etc., in such a way that each surface has the following property:

Two elements of M_h have in common either a face or an edge or a vertex or are disjoint. The index h characterizes the maximum diameter of the elements of M_h. Let V_h be a finite dimensional subspace of V consisting of all functions whose restrictions to each element of M_h are polynomials of a prescribed degree. It is known that the disturbing potential t is the unique function in V which minimizes over V the functional at 4 and 5 respectively. The discrete disturbing potential t_h is a function from V_h which minimizes equation 4 (or equation 5) over V_h, i.e.,

$$F/t_h/ = \min_{v \in V_h} F/v/.$$

Let $\{\phi_k\}_{k=1}^n$ be a basis for V_h. Then $t_h/P/$ can be uniquely expressed in the form

$$t_h/P/ = \sum_k a_k \, \phi_k/P/ \qquad (6)$$

with some coefficients a, ..., a_n. Substituting successively $t_h/P/$ and ϕ_k $k = 1, \ldots, n$ into bilinear form corresponding to equations 4 and 5 respectively, we obtain the following system of linear algebraic equations for determining the unknown coefficients a, ..., a_n, i.e.,

$$\sum_{k=1}^{n} a_k (L\phi_k, \phi_m) = (-\theta, \phi_m), \qquad m=1, \ldots, n.$$

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* The bilinear form $B/u,v/$ is V-elliptic if

$$B/u,v/ \geq c \, \|v\|^2_{W_2^1/\hat{\Omega}/} \quad , \qquad v \in V, \quad c = \text{const} > 0.$$

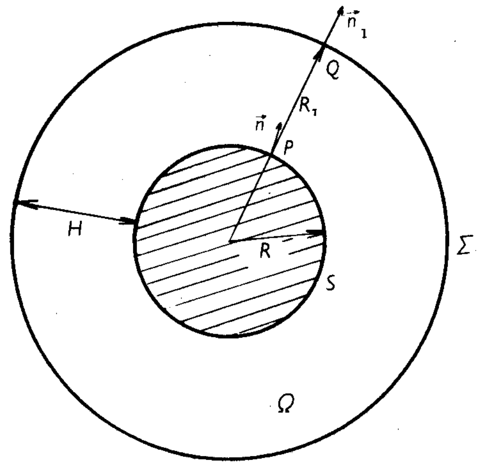

Figure 1.

Then the problem for studying the outer gravity field is transformed to the problem of studying the discrete outer gravity field. For a suitable choice of the basic functions, it may be shown that the discrete outer gravity field has similar properties to the outer gravity field (NEDOMA 1973a) concerning the maximum principle.

3. Conclusion

In the contribution presented, the possibility of studying the outer gravity field from the point of view of its discretization is discussed. This discretization of the outer gravity field was established from the variational formulation. An idea of a possible utilization of the satellite measurements performed on the Earth's surface was also given. However, there remains the question of transferring the gravity data received from the satellites to the above discussed surface Σ, as it turns with a different velocity around its axis than that of the satellite moving in its orbit. It appears that the discretization of the outer gravity field gives possibilities for using both types of measurements but the final answer cannot be given immediately.

4. Acknowledgments

I would like to thank Professor E. Grafarend who kindly delivered my lecture.

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Proc. Symposium on Earth's Gravitational Field & Secular Variations in Position (1973), 227-239.

GEOID, ISOSTATIC GEOID, ISOSTATIC CO-GEOID AND INDIRECT EFFECT OF GRAVITY IN INDIA

ABSTRACT

The terms relating to the geoid, isostatic geoid, isostatic co-geoid and indirect effect on gravity, are briefly explained in this paper with their implications in physical geodesy. After defining the geoid, the indirect effects of gravity are computed first with the help of Heiskanen's tables and then the isostatic geoidal heights derived for as many as 456 points uniformly distributed throughout the entire country after taking into account in the case of the latter, the shifting of the Earth's centre of gravity as is essentially needed in problems connected with the figure of the Earth in order to make the discussion complete. Finally the isostatic co-geoid (i.e., the actual geoid *minus* the computed isostatic geoid) is computed by making use of the actual geoidal data available in India till 1972. It has also been made clear that the procedure of deriving isostatic co-geoid direct from the geoid - isostatic differences is much more precise, economical and less time-consuming than what is normally obtained by first computing the isostatic deflections and then integrating the corresponding isostatic deflection anomalies circuit by circuit.

Relevant charts have been prepared and included to depict the geoid, isostatic geoid, isostatic co-geoid and indirect effect on gravity in contoured forms for extensive use in physical geodesy and various geophysical investigations in India.

1. Geoid

The figure of the Earth is a level surface whose potential is principally due to the gravitational attraction of the Earth's mass and the centrifugal force of the axial rotation. By convention, this level surface coincides with mean sea level which gives very nearly a physical representation of an equipotential surface, and is defined as the geoid (LAMBERT 1930,p.112). The elevations and depressions of this geoid with respect to the spheroidal surface of reference in use, i.e., the International spheroidal surface (see figure 1), and known as geoidal undulations, are depicted in maps in the form of geoidal contours. These undulations are normally determined either gravimetrically by the reputed Stokes' formula on utilizing the free air gravity anomalies all around the Earth or astro-geodetically as is usually done, by direct integration of plumb-line deflections observed at appropriate intervals or by satellite geodesy.

The Indian geoid fixed with respect to the adopted International spheroid of reference having the initial values at Kalianpur H.S. as

$$\xi_o = +2''.42 \quad ; \quad \eta_o = +3''.17 \quad ; \quad N_o = 9.5 \text{ m},$$

is obtained by the astro-geodetic method on utilizing about 1600 plumb-line deflections observed along geoidal circuits covering the entire country (GULATEE 1955,p.5).

Chart 1 depicts the positions of the geoidal circuits in India with the individual closure errors given

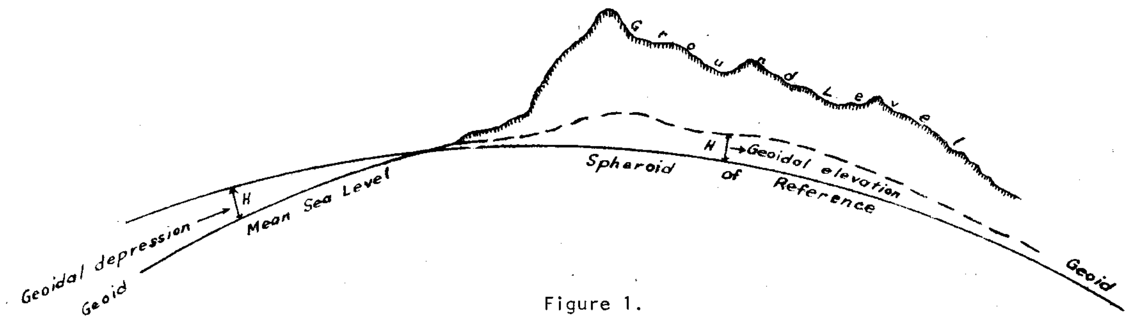

Ground Level
Geoidal elevation
Spheroid of Reference
Geoid
Geoidal depression
Mean Sea Level
Geoid
H
H

Figure 1.

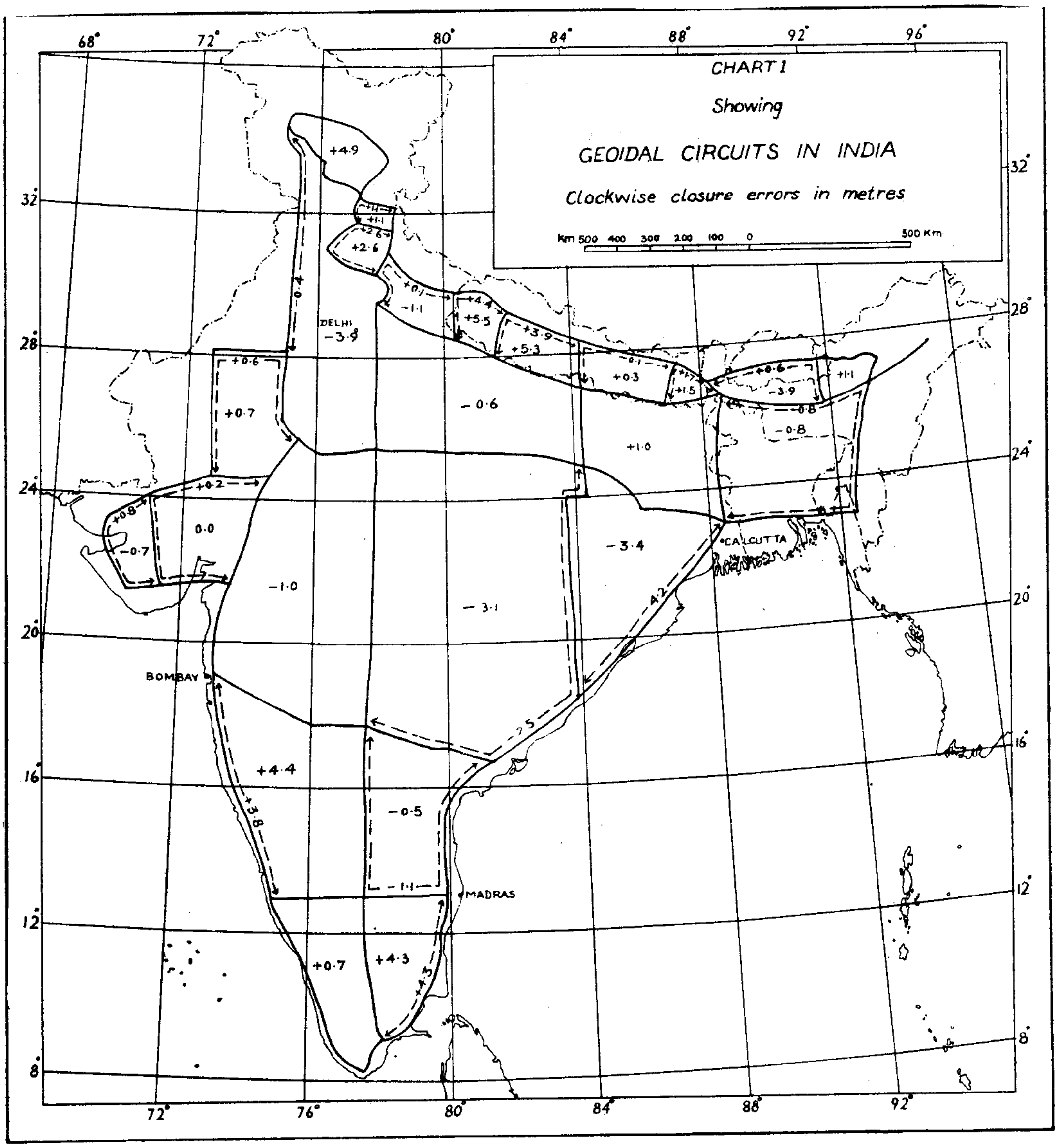


CHART 1
Showing
GEOIDAL CIRCUITS IN INDIA
Clockwise closure errors in metres
DELHI
CALCUTTA
BOMBAY
MADRAS

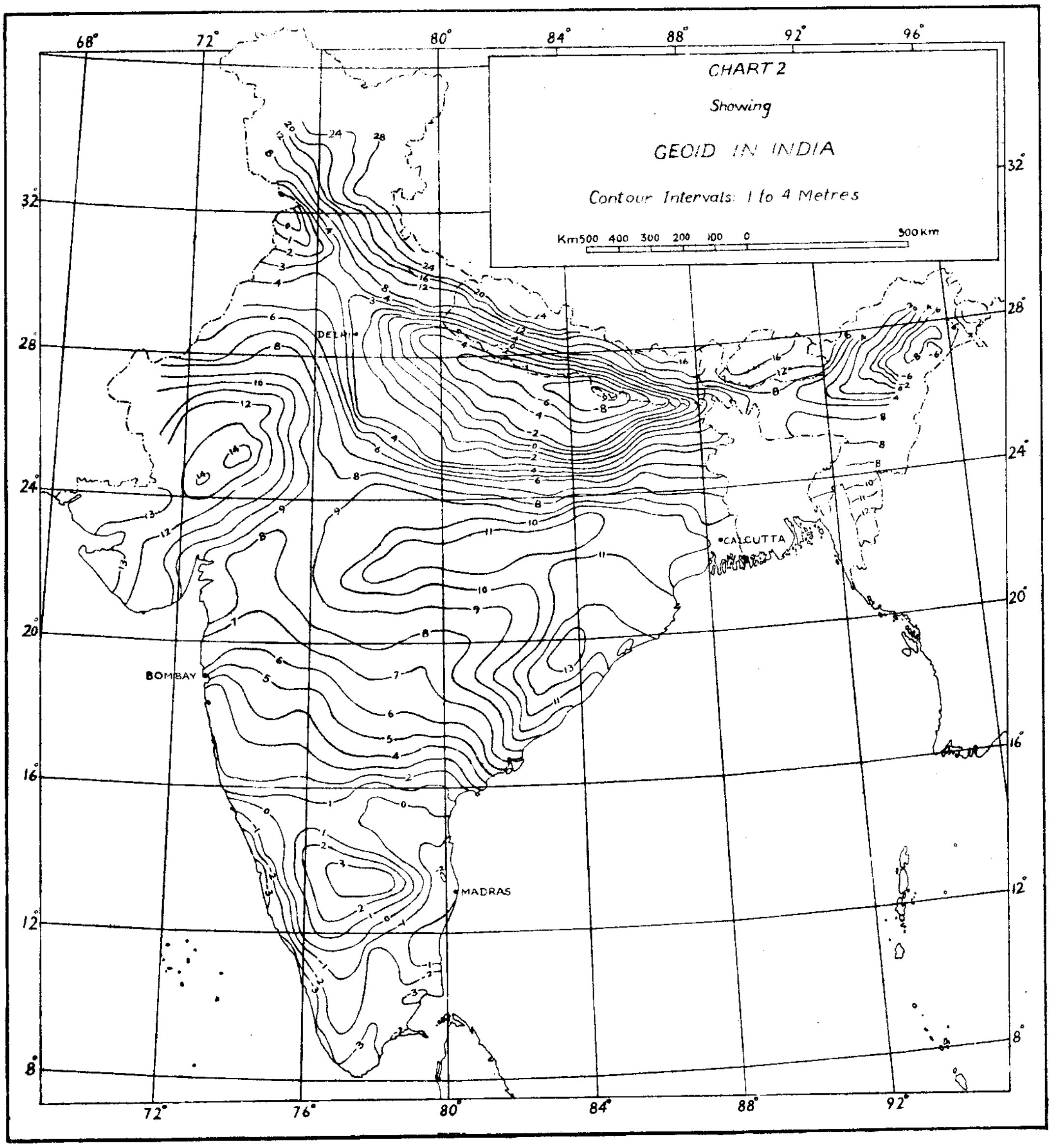
within brackets, along which plumb-line deflection stations were established at 20 - 30 km intervals covering the entire country. The general procedure followed in carrying out the circuit to circuit figural adjustment of the closure errors is also indicated in the same chart.

Chart 2 depicts the geoid in India, based on the arbitrarily assigned value of +9.5 m for the geoid-spheroid separation at the initial point at Kalianpur H.S., showing the geoidal contours at intervals of 3 m in the high hills of the Himalayas and 1 m in the rest of the country. The chart comprises the synthesized results of about 1600 plumb-line deflections observed throughout the country, providing thereby a more realistic picture regarding not only the relative shape of the Indian part of the geoid, but also the underground crustal variations inside the Earth, than that provided by the isolated plumb-line deflections alone. But on referring to chart 1, it will appear necessary to have a much closer network of geoidal circuits in India in order to ensure a more precise and dependable geoidal map for the country. However, while delineating the geoidal lines inside the wider gaps between the existing deflection lines, attempts have been made to utilize the general trends of the theoretical geoid contours based on the isostatic considerations also (see chart 6) in order to make them more representative and precise. In fact, the geoidal undulations as given in chart 2, constitute a complete geoidal cover for India, and display a number of interesting features in the form of geoidal abnormalities. The chief among these is indicated by a broad band of average breadth 500 km with an average elevation of over 10 m running across the whole breadth of peninsular India, which has been given the appellation of the Hidden Range of Burrard, lying practically parallel to the Himalayan Range. This if followed by a region of marked depression of over 8 m along the Himalayan foot-hills, characterized by the alluvial plains of the Ganges, and then again by a region of elevated geoid of the order of 15 to 20 m or more towards the high Himalayas. On the whole, India in fact is a region of excessive geoid which again appears to rise as as we go east, west and north of the Indian frontiers. But unfortunately, the Indian part of the geoidal framework being yet based on a more or less arbitrary orientation, no definite conclusions can be drawn on them without further considerations.

2. Indirect Effect on Gravity

According to Clairaut's theorem, if the value of gravity is given at a single point on or outside a level surface whose form is known and coincides with the physical surface of a uniformly rotating body, then gravity due to the body at all points outside the level surface is uniquely determined. But the converse is not necessarily always true. In case, however, the level surface is not very different from an ellipsoid of revolution, the values of gravity on the level surface pre-determine the form of the surface itself. But in applying this theorem to the Earth, we face an obvious difficulty because of the implied condition for the level surface to coincide with the physical surface of the body, being not fulfilled by the Earth's geoidal surface (LAMBERT 1930,p.113). "The expedient adopted in the circumstances, is to substitute in imagination for the actual Earth, an idealized Earth in which the masses lying outside the geoid are replaced by masses within it. This process of substituting internal masses for external ones, changes, in general, both the values of gravity and the form of the geoid" (IBID).

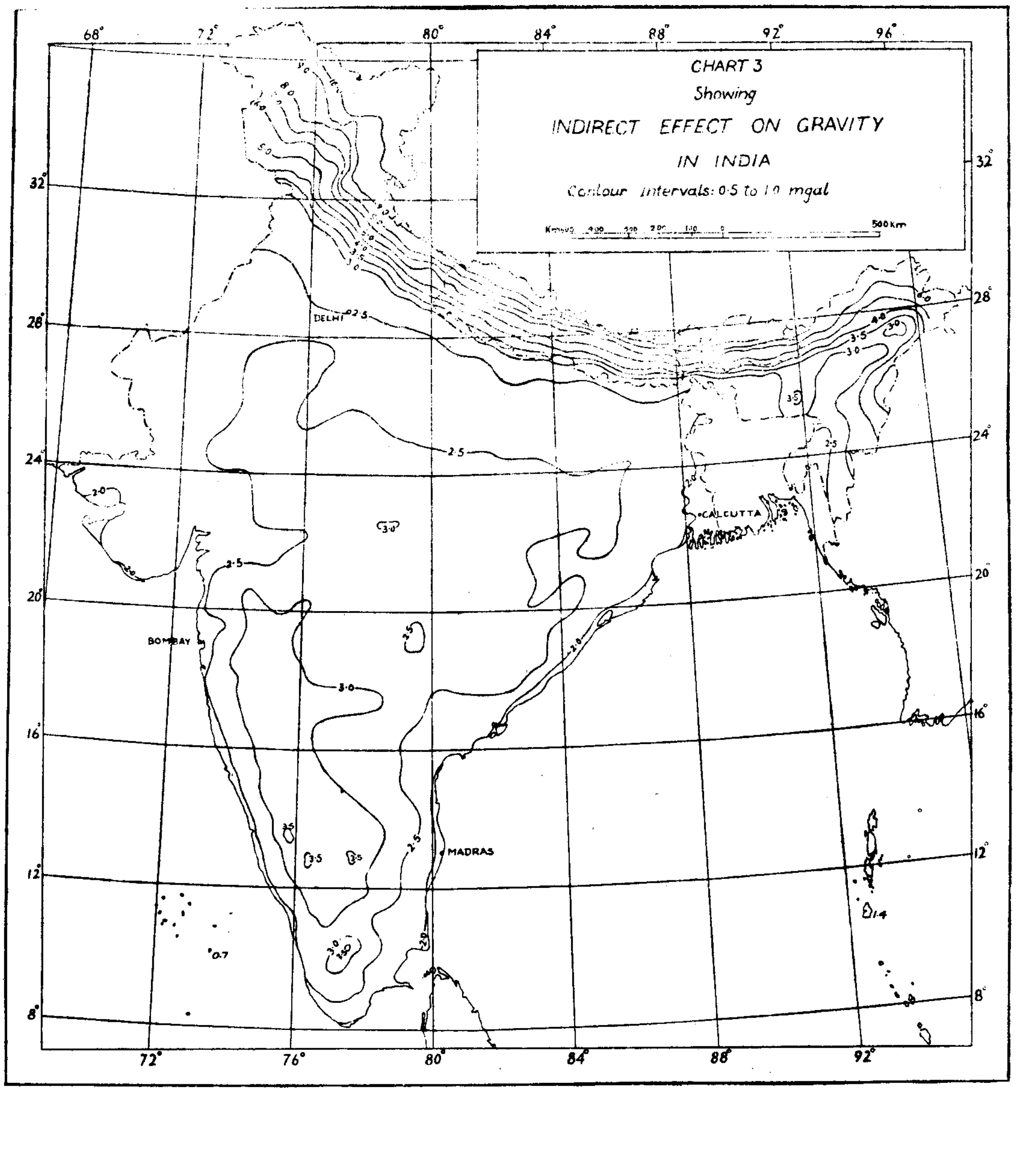
When the masses involved and their old and new positions are known, those changes become calculable. Similarly, the transfer of matter implied in the usual isostatic reduction of observed values of gravity to the sea level, also means a warping of the geoid giving rise to a new surface which may be called an isostatically reduced warped geoid, with its centre of gravity having been shifted from that of the Earth during the process of computation. Hence, in applying the isostatic method to the



problem of the figure of the Earth, it must be remembered that the geoid differs from the warped geoid and an allowance has to be made accordingly by reducing observations to the latter instead of the former, in the form of an additional correction which is termed the *indirect effect* $\Delta_2 g$. Why the warped geoid referred to, is not required to be corrected at this stage, for the shifting of its centre of gravity will be explained later.

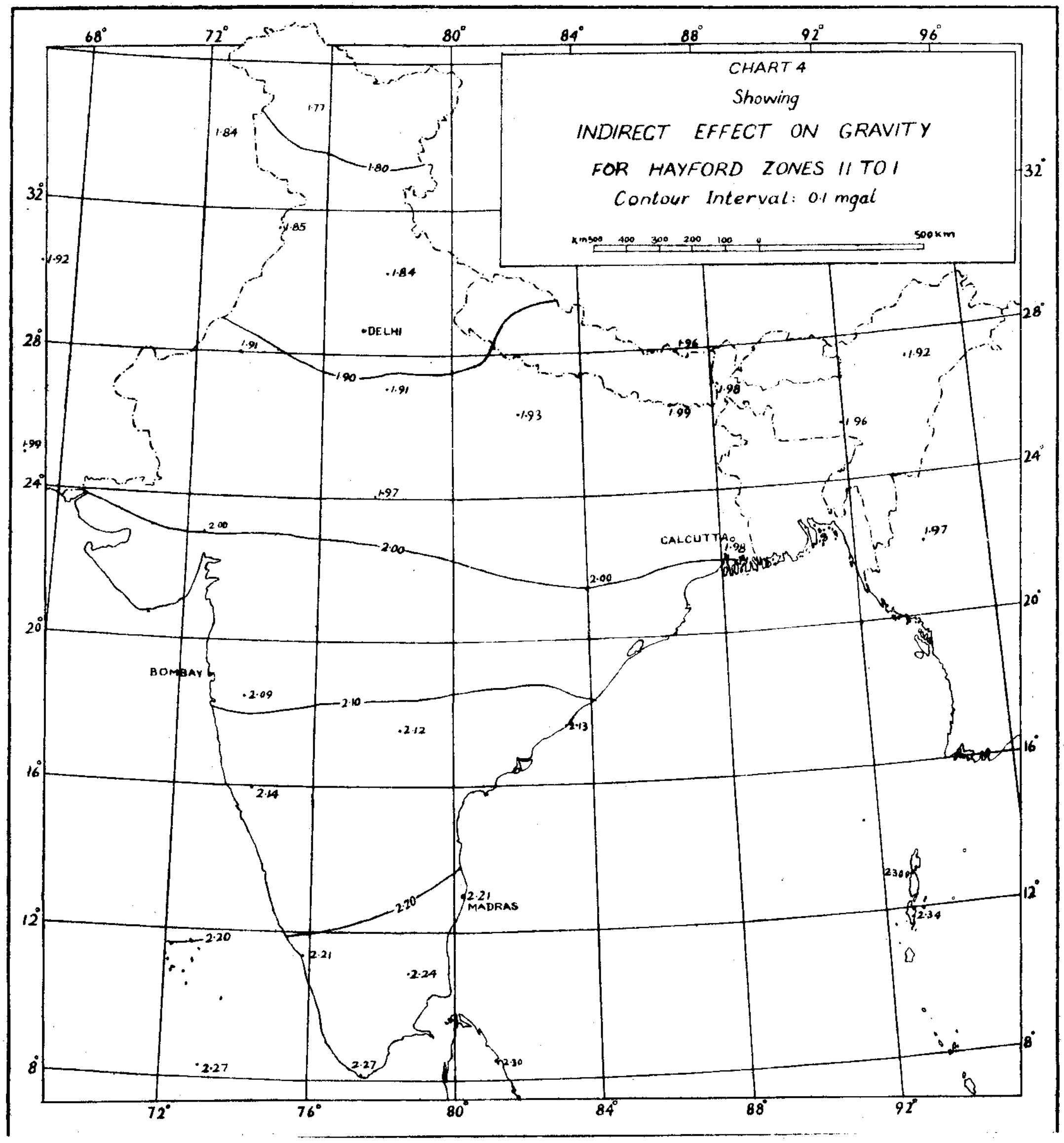
Now, the deviation of the warped geoid varying quite slowly, the topography and the compensation corresponding to the matter between the geoid and the warped geoid are practically equal, so that the resultant isostatic correction becomes very negligible reducing the indirect effect $\Delta_2 g$ ultimately to a simple free air correction only. In the free air method of reduction also, the intervening topography being condensed immediately below the level of the warped geoid, the indirect effect $\Delta_2 g$ is reduced to the same simple free air correction as before (IBID,p.143). The necessary tables have been provided by HEISKANEN & NISKANEN (1941) for direct evaluation of the indirect effect $\Delta_2 g$, i.e., the correction required to reduce the gravity values from the geoid to the warped geoid, obtained on account of the isostatically reduced visible topography considered on the entire surface of the Earth, so that the reduced isostatic anomalies can be rightly utilized in Stokes' formula. But the question of displacement of the Earth's centre of gravity due to the mass transfer implied in the isostatic reduction, is necessarily irrelevant at this stage as far as the reduced isostatic gravity anomalies are concerned; for, if the shifting occurs, the direct and indirect effects, $\Delta_1 g$ and $\Delta_2 g$ are both affected, and we do not ordinarily treat $\Delta_1 g$ and $\Delta_2 g$ separately, but only their sum which is never affected (IBID,p.162).

Chart 3 depicts the indirect effect on gravity $\Delta_2 g$ for India on Hayford's hypothesis of d = 113.7 km,



showing the $\Delta_2 g$ contours at intervals of 1 mgal in the high hills of the Himalayan region, and at 0.5 mgal in the rest of the country. The chart has been prepared for the first time for the Indian peninsula to the present high degree of precision, by making use of as many as 456 computed points throughout the country, in accordance with the recommendations of the International Union of Geodesy & Geophysics. The computations are based on the tables of HEISKANEN & NISKANEN (1941) for the effects up to Hayford zone 12, and a specially designed chart 4 for the effects of zones 11 to 1 is included in this paper.

The indirect effect has practically no use in geophysics by way of delineations of local structures, but plays an important role in physical geodesy in correcting relevant isostatic gravity anomalies for



use in the Stokes and Vening Meinesz formulae for the determination of absolute undulations of the geoid and plumb-line deflections.

3. Isostatic Geoid and Isostatic co-geoid

As already mentioned, the overall transfer of the Earth's external topography above the geoid implied in the isostatic method of reduction, means not only the warping of the geoid but would also obviously displace the centre of gravity of the Earth. Moreover, since Stokes' formula furnishes the undulations of the same warped geoid having its centre of gravity coincident with that of the Earth and in no other position, the warped geoid has to be corrected for its deformation due to the displacement of its centre of gravity, while deducing the undulations of the geoid from the corresponding undulations derived from the Stokes formula. The warped geoid in its true position, duly corrected for the displacement of its centre of gravity, is termed as the *co-geoid* or more appropriately the *isostatic co-geoid* and is known in the Survey of India as the *compensated geoid*, as shown in figure 2. In fact there are many co-geoids of which the considered isostatic co-geoid on Hayford's hypothesis for $d = 113.7$ km, is one, as there are systems of substituting internal masses for external ones lying outside the geoid.

The necessary formulae are in existence for computing the required corrections for the displacement of the centre of gravity of the Earth in both magnitude and direction implied by the isostatic method of reduction on Hayford's hypothesis. The resultant displacement $\bar{X}_R$ of the centre of gravity of the Earth in passing from the geoid to the isostatic co-geoid, due to the addition of the isostatically compensated topography in the form of a cap of angular radius θ, height h, density ρ, Earth's mean density ρ_m and mean radius a, and depth of compensation d, is denoted by (LAMBERT 1930,p.162)

$$\bar{X}_R = \frac{3}{4} \frac{\rho}{\rho_m} \sin^2\theta \frac{h\,d}{a} ,$$

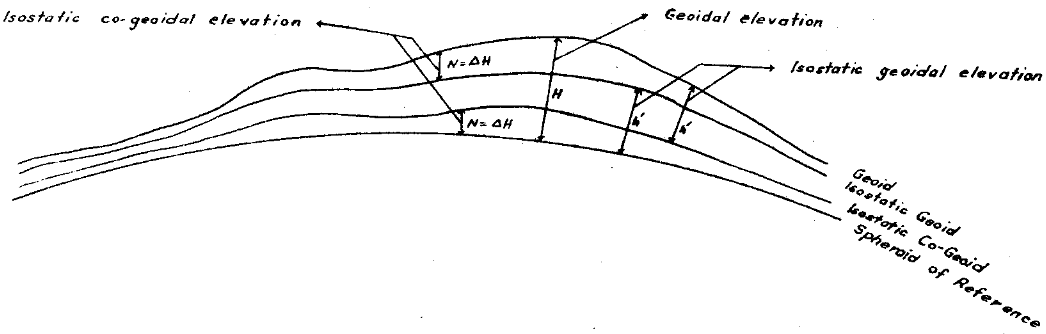


Figure 2.

and the summation for the separate zones furnishes the total resultant displacement. In practical applications, if the continents are considered as extra loads and the oceans as deficits of matter on the Earth's surface, the addition of these surpluses and deficits is to displace the centre of gravity. Let the latitude $\overline{\Phi}$ and longitude $\overline{\lambda}$ denote the direction and $\overline{r}$ denote the amount of the displacement when the excesses and deficits are isostatically compensated on Hayford's hypothesis to a depth d (IBID, p.168). The results as obtained by Lambert and Prey for the case (d = 100 km) are as follows

$$\overline{r} = 4.9 \text{ m} \quad ; \quad \overline{\Phi} = 43^{\circ}\ 57'\ \text{N} \quad ; \quad \overline{\lambda} = 31^{\circ}\ 01'\ \text{E}.$$

For any other depth d', the values of $\overline{\Phi}$ and $\overline{\lambda}$ remain unchanged while the value of $\overline{r}$ gets changed approximately in the ratio d'/d (IBID,pp.168-169), the corresponding values in the case of Hayford's compensation for the depth of 113.7 km reduce to

$$\overline{r} = 5.6 \text{ m} \quad ; \quad \overline{\Phi} = 43^{\circ}\ 57'\ \text{N} \quad ; \quad \overline{\lambda} = 31^{\circ}\ 01'\ \text{E}.$$

Now, since the displacement for any other point on the Earth's surface varies as the cosine of its angular distance from $(\overline{\Phi},\overline{\lambda})$, the corrections can be worked out as above for only a limited number of uniformly distributed points and the values of the isostatic geoidal undulations as deduced from the corresponding values of the indirect effect on gravity $(\Delta_2 g)$, with the help of the usual free air corrections only, can be adequately corrected for the displacement of the Earth's centre of gravity without any difficulty.

The vertical separation between the geoid and the isostatic co-geoid in its proper position, when reckoned positively downwards and superimposed on the surface of the spheroid of reference in the opposite sense, is termed the *isostatic geoid* on Hayford's hypothesis for d = 113.7 km (see figure 2). Indeed, if the particular hypothesis of isostasy used in the computation were exactly fulfilled in nature, and if the computations were rigorously made, the *isostatic geoid* and the *geoid* would have been exactly identical. But since the Earth is not constituted in that way, this can never be the case in actual practice. The undulations of the isostatic co-geoid or the Survey of India compensated geoid are thus equivalent to the *differences* (actual geoid *minus* isostatic geoid), i.e.,

$$N = H - h' = \Delta H \quad \text{(see figure 2)},$$

which can also rightly be designated as *isostatic geoidal anomalies* on the relevant Hayford's hypothesis for d = 113.7 km, providing an indirect measure of the regional departures of isostatic equilibrium inside the Earth. Similarly the undulations of the geoid are obtainable from the *sums* (isostatic co-geoid - derived gravimetrically from Stokes' formula - *plus* isostatic geoid), i.e.,

$$H = N + h' = \Delta H + h' \quad \text{(see figure 2)}.$$

The undulations of the isostatic co-geoid or the compensated geoid are determined either gravimetrically from Stokes' formula on utilizing the isostatic gravity anomalies all around the Earth or astro-geodetically, as is usually done, by direct integration of Hayford anomalies or more precisely isostatic deflection anomalies, i.e., the observed deflection *minus* the isostatic deflection computed on the Hayford hypothesis on considering compensated topography right up to the antipodes. The Indian isostatic co-geoid or compensated geoid has been obtained by the astro-geodetic method on utilizing about 1031 isostatic deflection anomalies computed till 1956 and adopting the initial value of the elevation of the isostatic co-geoid above the International spheroid of reference, at Kalianpur H.S. as +6.7 m which was originally derived by subtracting the approximate value of the isostatic geoidal elevation computed on considering compensated topography upto a distance of 600 km only instead of the whole Earth, from that of the natural geoid more or less arbitrarily fixed with respect to the International spheroid of reference. As mentioned earlier, since the necessary tables are available (HEISKANEN & NISKANEN 1941) for the precise computations of the indirect effect $\Delta_2 g$ on considering the compensated topography all around the Earth, which can be straight away converted into the corresponding values of the undulations of the isostatic co-geoid by making use of the usual free air reduction factor and the corrections for the displacement of the Earth's centre of gravity from the relevant chart, it has been easily possible to derive for the first time the isostatic geoidal values with all the needed corrections applied, for as many as 456 uniformly distributed points in India, including the one at the initial point, which will provide not only a reliable isostatic geoidal picture for the whole country, but also a firm value for the isostatic co-geoidal elevation of +4.8 m instead of the old approximate figure of +6.7 m at Kalianpur H.S., relative to the more or less arbitrarily defined geoidal undulations in India. Since the undulations of the isostatic co-geoid as determined gravimetrically are considered absolute, these would, however, obviously differ from those obtained astro-geodetically, because of the arbitrariness in the orientation of the Indian geoid with respect to the International spheroid of reference. In fact, the astro-geodetic method of computing the undulations of the isostatic co-geoid by making use of the isostatic geoid as described above, is obviously much simpler and less time-consuming and therefore much more economical than that of direct integration of the Hayford or isostatic deflection anomalies on Hayford's hypothesis for d = 113.7 km, adopted later by the Survey of India as a routine procedure considering the latter presumably more precise (GULATEE 1955,p.7). But it can be seen from the comparative statement given below that if the considered corrections are duly applied, the procedure now adopted in this articale, apart from its remarkable simplicity in application, is much more accurate also than the existing one, because of not only the possible accumulation of errors in the individual isostatic co-geoidal circuits deduced from the large number of point to point isostatic deflection anomaly values, but also partly due to the computed isostatic deflection in use being not duly corrected for the displacement of the Earth's centre of gravity in the isostatic method of reduction.

Chart 5 depicts the corrections for the displacement of the Earth's centre of gravity on Hayford's hypothesis for (d = 113.7 km) in contoured forms at an interval of 0.1 m, for the entire country, to be applied to the undulations of the isostatic co-geoid in passing from the geoid to the isostatic co-geoid or straight away to those of the isostatic geoid with respect to the International spheroid of reference. The correction contours are based on only seven uniformly distributed points (see chart 5) computed by making use of the figures derived by Lambert and Prey.

Chart 6 depicts the isostatic geoid in India on Hayford's hypothesis for (d = 113.7 km) , prepared for the first time in its proper form, considering the compensated topography all round the Earth and also duly corrected for the shifting in the Earth's centre of gravity by making use of chart 5, and shows the isostatic geoidal contours at intervals of 3 m in the high hills of the Himalayas and 1 m in the rest of the country. The chart has been drawn by utilizing a dense network of computed isostatic geoidal elevations comprising the same 456 uniformly distributed points as for the indirect effects $\Delta_2 g$, thus making both charts 3 and 6 extremely precise and dependable. But critical examination of charts 2 and 6 will obviously indicate a lack of resemblance between the two, which will only show that Bowie's generalized statement

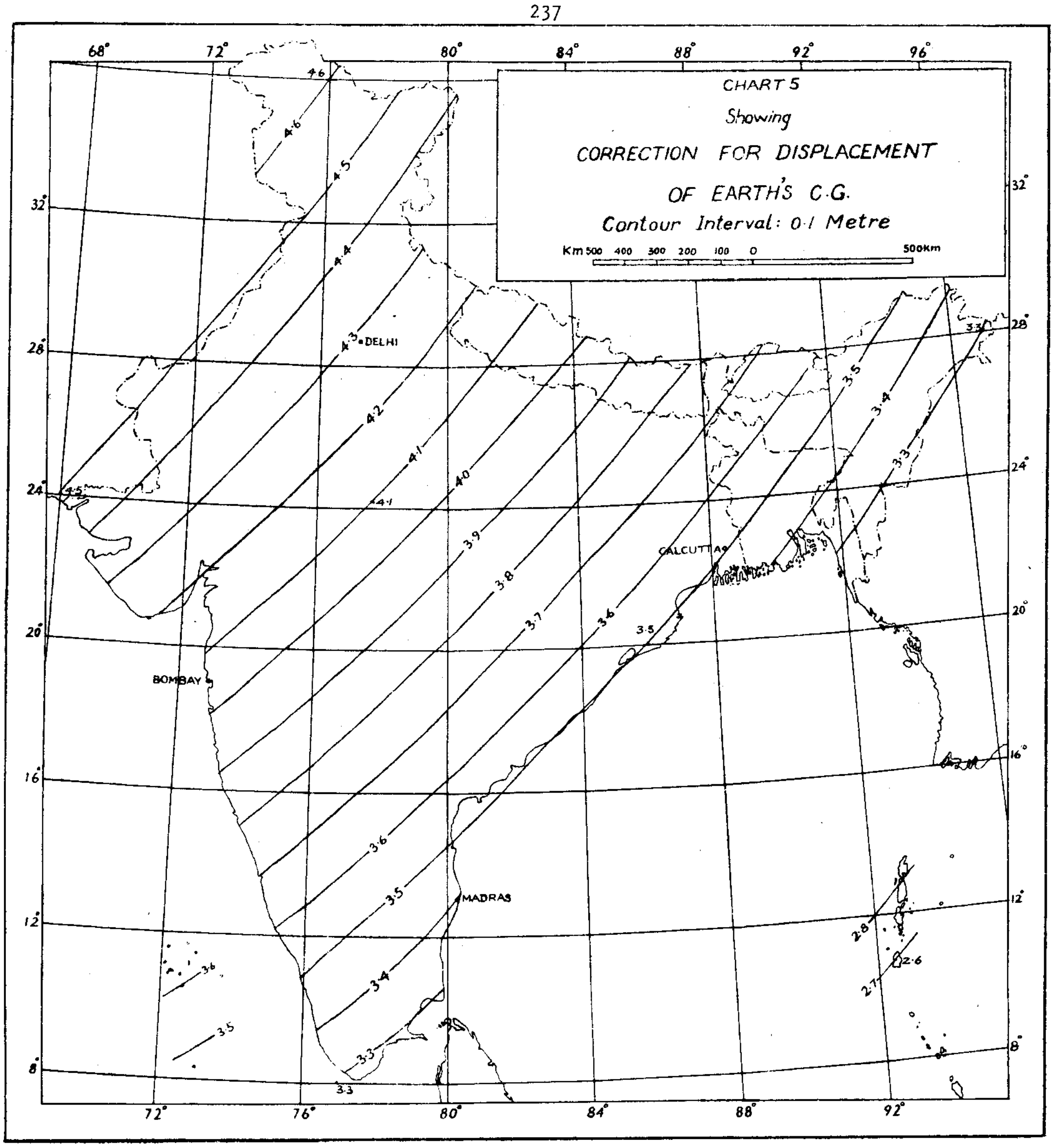
"The proof of isostasy has made it possible to compute an approximate geoid without all these observational data"

(BOWIE 1927,p.227) is not correct at least so far as India is concerned. In fact, the above

COMPARATIVE STATEMENT OF ISOSTATIC CO-GEOIDAL UNDULATIONS IN A CIRCUIT IN SOUTH INDIA DERIVED BY BOTH GEOID-ISOSTATIC GEOID AND ISOSTATIC DEFLECTION ANOMALY METHODS

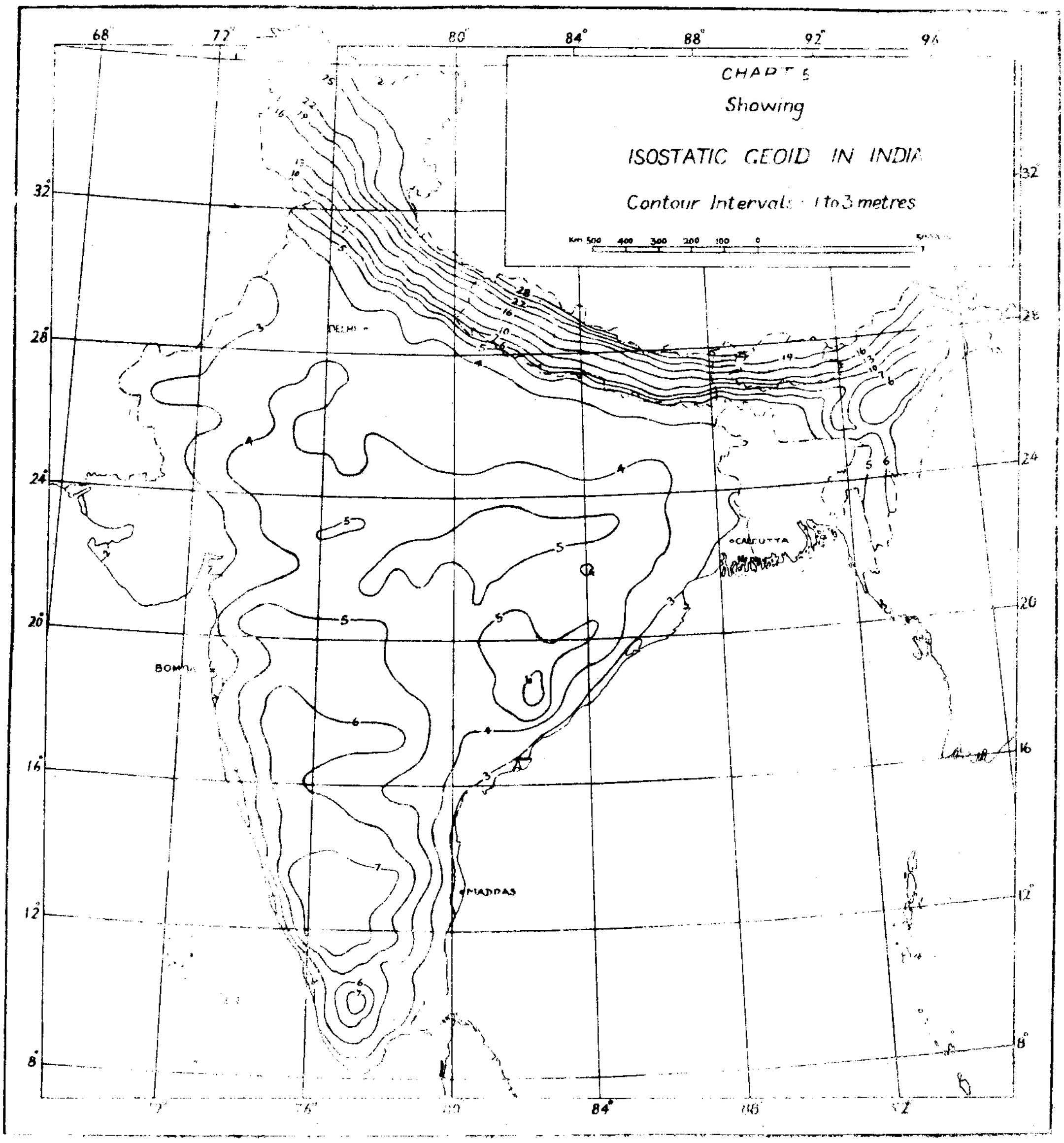
Sl. No.	Sheet No.	Name of Station	Geoid	Isostatic geoid (corrected for shifting of earth's C.G.)	Isostatic co-geoid by Geoid-Isostatic Geoid Method No. 1 (4)−(5)	Isostatic co-geoid by Isostatic deflection anomaly Method No. 2 after having corrected for initial error of 1·9 m (including shifting of earth's C.G.) in isostatic geoid at Kalianpur H.S.	Discrepancy Method No. 1 *minus* Method No. 2 (6)−(7)	Correction to Method No. 2 for variation of error due to shifting of earth's C.G.	Isostatic co-geoid by Method No. 2 after correction* (7) + (9)	Discrepancy Method No. 2 *minus* Method No. 2 after correction* (6)−(10)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
			metres	*metres*	*metres*	*metres*	*metres*	*metres*	*metres*	*metres*
1	56 G	Alipur ..	+4·0	+6·1	−2·1	+0·2	−2·3	−0·4	−0·2	−1·9
2	56 G	Sodaseopet ..	+4·2	+6·1	−1·9	+0·5	−2·4	−0·4	+0·1	−2·0
3	56 K	Muttanji ..	+4·2	+6·1	−1·9	+0·7	−2·6	−0·4	+0·3	−2·2
4	,,	Bolarum ..	+4·3	+6·0	−1·7	+0·9	−2·6	−0·5	+0·4	−2·1
5	56 K	Tuprampet ..	+3·6	+5·8	−2·2	+0·5	−2·7	−0·5	0·0	−2·2
6	56 O	Yalmanaid ..	+3·1	+5·4	−2·3	+0·5	−2·8	−0·5	0·0	−2·3
7	,,	Katangur ..	+2·6	+5·1	−2·5	+0·5	−3·0	−0·5	0·0	−2·5
8	,,	Suriapet ..	+2·5	+4·8	−2·3	+0·8	−3·1	−0·6	+0·2	−2·5
9	56 O	Munagala ..	+2·6	+4·5	−1·9	+0·9	−2·8	−0·6	+0·3	−2·2
10	65 D	Anumanchipalli ..	+2·8	+4·3	−1·5	+1·5	−3·0	−0·6	+0·9	−2·4
11	,,	Nandigama ..	+3·0	+4·0	−1·0	+1·9	−2·9	−0·6	+1·3	−2·3
12	,,	Ibrahimpatnam ..	+3·2	+3·6	−0·4	+2·5	−2·9	−0·6	+1·9	−2·3
13	,,	Anantavaram ..	+2·6	+3·6	−1·0	+1·9	−2·9	−0·6	+1·3	−2·3
14	,,	Dhulipalla S. ..	+1·0	+3·8	−2·8	+0·2	−3·0	−0·6	−0·4	−2·4
15	65 D	Kitapa S. ..	+0·6	+3·7	−3·1	−0·3	−2·8	−0·6	−0·9	−2·2
16	57 M	Danapa H.S. ..	0·0	+3·5	−3·5	−0·8	−2·7	−0·6	−1·4	−2·1
17	66 A	Medaimata ..	+0·1	+3·5	−3·4	−0·3	−3·1	−0·6	−0·9	−2·5
18	66 A	Ongole H.S. ..	−0·1	+3·3	−3·4	−0·4	−3·0	−0·7	−1·1	−2·3
19	57 M	Puripad ..	−0·5	+3·3	−3·8	−0·9	−2·9	−0·7	−1·6	−2·2
20	57 M	Darutippa S. ..	−1·1	+3·3	−4·4	−1·4	−3·0	−0·7	−2·1	−2·3
21	57 N	Nishanbadu ..	−1·9	+3·0	−4·9	−2·0	−2·9	−0·7	−2·7	−2·2
22	,,	Kistama H.S. ..	−2·2	+3·4	−5·6	−2·9	−2·7	−0·7	−3·6	−2·0
23	57 N	Bandaldura ..	−3·0	+3·0	−6·0	−3·0	−3·0	−0·7	−3·7	−2·3
24	66 B	Gudali H.S. ..	−2·9	+3·0	−5·9	−2·9	−3·0	−0·7	−3·6	−2·3
25	57 O	Anmapudi ..	−3·1	+3·0	−6·1	−3·2	−2·9	−0·7	−3·9	−2·2
26	,,	Satyanedu ..	−3·3	+3·4	−6·7	−3·6	−3·1	−0·8	−4·4	−2·3
27	57 O	Chambedu ..	−3·1	+3·1	−6·2	−3·1	−3·1	−0·8	−3·9	−2·3
28	57 P	Bimantangal ..	−2·6	+3·1	−5·7	−2·5	−3·2	−0·8	−3·3	−2·4
29	,,	Sembarambakkam ..	−2·6	+3·6	−6·2	−3·2	−3·0	−0·8	−4·0	−2·2
30	57 P	Chinnasamudram ..	−2·3	+4·1	−6·4	−3·5	−2·9	−0·8	−4·3	−2·1
31	57 O	Timmayyapalli ..	−1·2	+4·9	−6·1	−3·4	−2·7	−0·8	−4·2	−1·9
32	57 K	Venkatagiri ..	+0·1	+5·9	−5·8	−2·9	−2·9	−0·7	−3·6	−2·2
33	,,	Pattikonda ..	+1·2	+6·4	−5·2	−2·6	−2·6	−0·7	−3·3	−1·9
34	,,	Anantapura ..	+2·0	+5·9	−4·9	−2·4	−2·5	−0·7	−3·1	−1·8
35	57 K	Chunchadenahalli ..	+2·2	+7·2	−5·0	−2·4	−2·6	−0·7	−3·1	−1·9
36	57 G	Hoskote ..	+2·2	+7·6	−5·4	−2·7	−2·7	−0·7	−3·4	−2·0
37	,,	Bangalore ..	+1·9	+7·7	−5·8	−2·7	−3·1	−0·7	−3·4	−2·4
38	,,	Halasurbetta ..	+2·2	+7·6	−5·4	−2·5	−2·9	−0·7	−3·2	−2·2
39	,,	Kalkote h.s. ..	+2·2	+7·5	−5·3	−2·3	−3·0	−0·7	−3·0	−2·3
40	,,	Gudem hill staff ..	+1·7	+7·2	−5·5	−2·5	−3·0	−0·6	−3·1	−2·4
41	,,	Konduru white ridge ..	+1·1	+6·9	−5·8	−3·0	−2·8	−0·6	−3·6	−2·2
42	57 G	Bommasandra s. ..	+0·6	−6·8	−6·2	−3·3	−2·9	−0·6	−3·9	−2·3
43	57 F	Penukonda h.s. ..	+0·4	+6·7	−6·3	−3·4	−2·9	−0·6	−4·0	−2·3
44	,,	Urakonda H.S. ..	−0·2	+6·5	−6·7	−3·8	−2·9	−0·6	−4·4	−2·3
45	,,	Gollapalle h.s. ..	−0·9	+6·3	−7·2	−4·2	−3·0	−0·6	−4·8	−2·4
46	,,	Devarakonda H.S. ..	−1·2	+6·2	−7·4	−4·5	−2·9	−0·6	−5·1	−2·3
47	57 F	Kanampalli Rock peak	−1·7	+6·0	−7·7	−4·7	−3·0	−0·6	−5·3	−2·4
48	57 E	Namthabad S. ..	−1·8	+5·9	−7·7	−4·9	−2·8	−0·6	−5·5	−2·2
49	,,	Koitkonda h.s. ..	−1·6	+5·8	−7·4	−4·7	−2·7	−0·5	−5·2	−2·2
50	,,	Pullikonda H.S. ..	−1·5	+5·6	−7·1	−4·6	−2·5	−0·5	−5·1	−2·0
51	,,	Palakurti rock ..	−1·5	+5·4	−6·9	−4·5	−2·4	−0·5	−5·0	−1·9
52	57 E	Kere Belagal H.S. ..	−1·3	+5·4	−6·7	−4·2	−2·5	−0·5	−4·7	−2·0
53	56 H	Induvasi h.s. ..	−0·7	+5·4	−6·1	−3·6	−2·5	−0·5	−4·1	−2·0
54	,,	Tuagat h.s. ..	−0·4	+5·4	−5·8	−3·4	−2·4	−0·5	−3·9	−1·9
55	,,	Tonsalgutta s. ..	0·0	+5·4	−5·4	−3·0	−2·4	−0·5	−3·5	−1·9
56	,,	Jaganpalli rock ..	+0·5	+5·6	−5·1	−2·5	−2·6	−0·5	−3·0	−2·1
57	,,	Impagat H.S. ..	+1·5	+5·9	−4·4	−1·8	−2·6	−0·5	−2·3	−2·1
58	56 H	Dorapalli h.s. ..	+2·2	+6·3	−4·1	−1·4	−2·7	−0·5	−1·9	−2·2
59	56 G	Linganapalle h.s. ..	+2·7	+6·3	−3·6	−0·9	−2·7	−0·4	−1·3	−2·3
60	,,	Devanur s. ..	+2·9	+6·2	−3·3	−0·7	−2·6	−0·4	−1·1	−2·2
61	,,	Marepalli s. .	+3·2	+6·2	−3·0	−0·4	−2·6	−0·4	−0·8	−2·2
62	,,	Kandenmarai T.S. ..	+3·7	+6·2	−2·5	−0·1	−2·4	−0·4	−0·5	−2·0
63	56 G	Alipur ..	+4·0	+6·1	−2·1	+0·2	−2·3	−0·4	−0·2	−1·9

* This still includes the error of + 9 m in co-geoid accumulated from Kalianpur H.S. to Alipur.



discrepancies are in a way, indirect measures of the deviation of isostatic equilibrium prevalent in the respective regions inside the Earth, and more vividly and completely represented in chart 7 (see next paragraph). But the isostatic geoidal undulations as given in chart 6, being duly corrected for the shifting of the Earth's centre of gravity, have ample applications in physical geodesy in obtaining the actual geoidal undulations directly from the corresponding isostatic co-geoidal values computed gravimetrically by the Stokes formula.

Chart 7 depicts the isostatic co-geoid in India on Hayford's hypothesis for (d = 113.7 km), prepared with the help of the same charts 2 and 6 by making use of their differences only at a sufficiently large number of, or simply the same 456 common points (as for $\Delta_2 g$) and as such, is expected to be as accurate as chart 2, the precision of chart 6 being unquestionable as indicated before. The chart

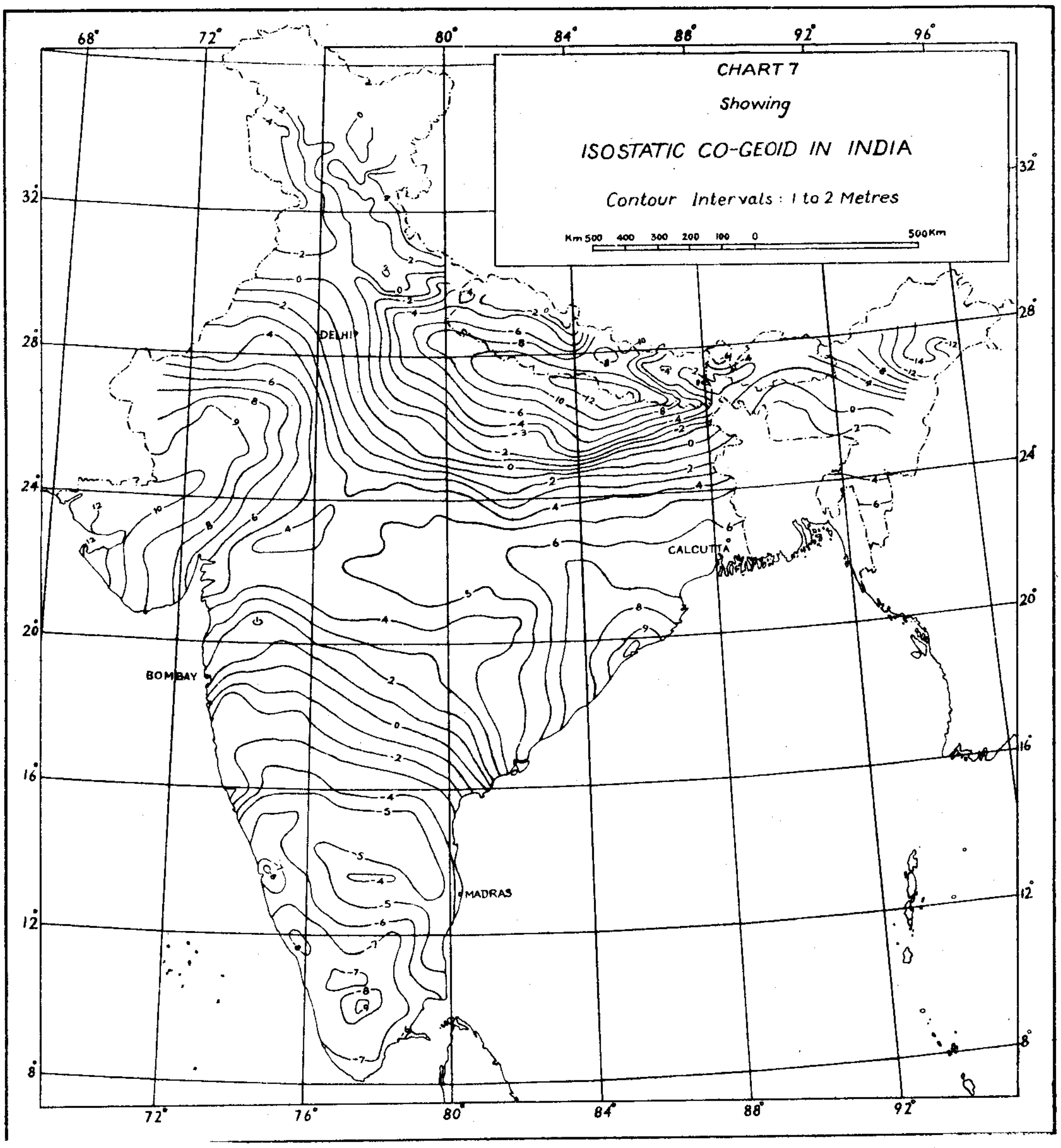


gives the isostatic co-geoidal contours relative to the assigned geoidal elevation of +9.5 m at Kalianpur H.S., at intervals of 3 m in the high hills of the Himalayan region and 1 m in the rest of the country.

The isostatic co-geoidal undulations as given in chart 7, indicate more or less, the same pattern of feature, slightly reduced, as the actual geoidal undulations in chart 2, showing thereby that the isostatic co-geoid has no significant correlation with the isostatic geoid derived according to isostatic theory from a purely theoretical consideration of the Earth's topographical features.

4. Acknowledgments

I am thankful to the Director, Geodetic and Research Branch, Survey of India, for his guidance, encouragement and offering of necessary facilities and data for writing this paper. My grateful



thanks are also due to the Surveyor General of India for his kindly granting me necessary permission to present the paper at the symposium.

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