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MEAN SEA LEVEL: THE OCEANOGRAPHER's POINT OF VIEW

ABSTRACT

Oceanographers obtain a fair approximation to mean sea level in the open ocean, using water density measurements. The assumptions underlying this method will be discussed, together with the observational basis for these assumptions

Further assumptions and measurements are needed to extrapolate from mean sea levels in deep water to the shore, where results can be compared with geodetic levelling. HAMON & GREIG (1972) have used these techniques in the east Australian region, to obtain results very different to those from geodetic levelling. Independent oceanographic considerations support the Hamon-Greig picture over the geodetic results. These considerations are outlined.

1. Introduction

This paper is concerned with a discrepancy between the distribution of mean sea level along the east Australian coast, as found on one hand by geodetic levelling on land, and on the other by steric levelling at sea. The aim is to discuss and to some extent amplify the results of HAMON & GREIG (1972); it will be assumed that the reader is unfamiliar with oceanographic methods and assumptions.

The Division of National Mapping of the Australian Department of National Development recently reported the results of an extensive levelling survey of the Australian continent (ROELSE et al 1971). Among other studies, this report dealt with levelling from one tide gauge to the next, around the coast. A particularly striking feature to emerge from this work was that mean sea level appeared to rise quite strongly and steadily from Coffs Harbour, near the Queensland-New South Wales border, to Bamaga at the tip of Cape York. The total rise was 1.75 ± 0.5 metres; the error estimate makes generous allowance for random errors, due to permitted misclosures in third-order levelling. From Coffs Harbour south to Eden (near the New South Wales-Victorian border), the mean sea level was found to be relatively flat.

We concern ourselves here only with the mean sea level in this region, from Bamaga to Eden.

2. The Discrepancy

HAMON & GREIG (1972) collected all available measurements of steric sea level, relative to an assumed "level of no motion" at a depth of about 1300 metres below the sea surface, between the edge of the continental shelf and about 700 km offshore (the terms will be explained shortly). The pattern they obtained was quite different from the geodetic result: they found steric sea level to be essentially flat from 10^{o}S to 30^{o}S - i.e. from about the latitude of Bamaga to that of Coffs Harbour - and to drop sharply by about 50 cm from the latitude of Coffs Harbour to that of Eden (38^{o}S).

Within the geographic "bins" used by HAMON & GREIG for averaging purposes, the individual observations differed from one another by amounts of order 10 cm; thus the observations are definitely not consistent with a rise of anything like 175 cm from Coffs Harbour to Bamaga.

3. The Steric Levelling Technique

The ocean is typically 4000 metres deep; the steric levelling technique is based on the assumption that at any depth below about 1000 metres, there is very little *steady* motion. More specifically, it is assumed that the pressure, averaged over tides and transient meteorological phenomena, is constant along any geopotential surface deep down in the ocean - i.e. the isobar map at such depths is flat (since flow should follow the isobars in the ocean, the two statements are equivalent). We shall discuss this important assumption in the next section; if it is temporarily accepted, the steric levelling method may be simply explained.

The method consists of measuring temperature and salinity at various depths above the assumed "level of no motion". From these, the water density may be accurately inferred from the equation of state for seawater. Furthermore, order-of-magnitude estimates of terms in the equations of motion for seawater show that the hydrostatic relation should hold with great accuracy - i.e. the rate of increase of pressure with depth $\partial p/\partial z$ must equal water density ρ times the acceleration due to gravity g:

$$\frac{\partial p}{\partial z} = -g\,\rho \qquad (1)$$

Therefore the density data may be used to integrate upwards from the "level of no motion", to find what depth of water is needed to produce the constant pressure at that level. If the water is warm and light at all depths, a slightly greater height of seawater is needed to produce the constant pressure than is required in regions where the water is cold and dense. For example, water in offshore parts of the East Australian Current is typically warmer by several degrees than water on the inshore side; so the steric levelling technique indicates that the sea surface should stand higher offshore than inshore, by an amount that is sometimes as high as 50 cm.

4. "The Level of No Motion" Assumption

This is the crucial assumption underlying the steric levelling method, and we must discuss why oceanographers believe it to be valid. Three arguments will be advanced in favour of it. First, the ocean above about 1000 metres is quite inhomogeneous - the temperature of the surface water varies from typical values of 1°C near the poles to 25°C or more near the equator - whereas from 1500 metres to the bottom, the water is homogeneous within about 1°C, over the whole world ocean. This suggests that the water has moved to level out major horizontal density differences; there is thus little potential energy to drive flow at these great depths.

Secondly, five direct observations of currents at depths below 2000 metres have been made in the east Australian region, by tracking floats that are carefully weighted to drift at the appropriate depth. The highest speed found was 10 cm/sec, whereas surface currents directly above were generally of order 100 cm/sec (BOLAND & HAMON 1970).

Finally, if the contour plots of steric sea level truly reflect the mean sea level, then ocean currents should run parallel to steric level contours, just as winds in the atmosphere follow the isobars. The reason is that on a geopotential surface just below the sea surface, high mean sea

sea level implies high pressure; thus contour maps of steric sea level should also serve as isobar maps. Direct observation of currents, in the East Australian Current and elsewhere in the ocean, indicate that this is consistently the case: errors (which are due to the current measurement as much as to the steric levelling technique) are of order 20-30%.

5. Extrapolation to the Coast

Steric levelling can only be performed in deep water: some other method must be found to extrapolate from beyond the edge of the continental shelf, when steric levelling is possible, in to the coast. To do this, we can in principle use the statement that pressure gradients associated with slope in the sea surface should be balanced (to within terms of order 10%) by Coriolis forces; in particular, a net southward surface current should imply (in the southern hemisphere) a rise in mean sea level from the coast, eastward towards the continental slope. Rather little quantitative information is available about average currents over the Australian continental shelf: but it is known that really strong currents, potentially capable of causing a 20 cm rise in sea level from the coast to the edge of the continental shelf, are present only along the coasts of south Queensland and northern and central New South Wales. Currents from Sydney southwards are rather weak, and directed (on average) towards the south, whereas nearshore currents in the Coral Sea are weak and generally directed towards the north.

Consequently, HAMON & GREIG concluded that extrapolation of steric levels to the coast should not qualitatively change the result quoted earlier: mean sea level should be roughly flat along the north Queensland coast, and should fall about 50 cm (perhaps a few centimetres more) from southern Queensland down to Eden.

6. Other Oceanographic Evidence

Fairly elaborate numerical models of the ocean have been run recently; one in particular, due to BRYAN & COX (1968) was found to give a surprisingly good qualitative representation of the East Australian Current (GODFREY, in press). In this model, the crucial element determining the dynamics of the current system was, precisely, the slope of mean sea level along the shoreline: this slope could *not* be balanced by Coriolis forces (water cannot flow out of the coast). Instead the pressure gradient accelerated the current, and there was a close correlation between the slope of mean sea level at the shoreline, and the strength of the longshore current.

Applied to the East Australian Current, the model suggests that the strong southward current region - south Queensland and north to central New South Wales - should also be a region of strong fall in mean sea level towards the south: whereas the weak northward current region (northern Queensland) should be a region of slow fall in mean sea level towards the north.

These deductions are compatible with HAMON & GREIG's results, but not with geodetic results.

7. Conclusion

Over recent decades, oceanographers have made a number of theoretical studies of ocean circulation, based on the Navier-Stokes equations (essentially, Newton's laws of motion, applied to fluids). The result is a body of theory which, though by no means rigorous, nevertheless provides a coherent qualitative explanation of a number of very different oceanographic phenomena. In the light of this theory, it is difficult to see how the slope of mean sea level along the east Australian coast found by geodetic levelling could be consistent with observed oceanographic phenomena in the region.

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9. Discussion

MUELLER: Could you tell us how close the isobaric surface at 1300 m depth is to an equipotemtial surface? In the case of a rotating homogeneous ellipsoid, there is no difference between an equipotential and an isobaric surface. It could be that the discrepancies observed here in Australia and elsewhere are due to the assumption that the isobaric surface is not really an equipotential surface.

GODFREY: If you don't have pressure surfaces coincident with potential surfaces in the oceans or in the atmosphere, you must have flow - at least in the open ocean. This comes out in order of magnitude estimates of the Coriolis force which turns out to be much larger than other forces. The only thing that balances a pressure gradient is flow along isobars.

MUELLER: It could be possible to assume that the departure of about ½ m or so over such long distances can be due to differences between those two reference surfaces?

GODFREY: Off-shore that could happen. You could have a smooth rise ....

MUELLER: This is of course the only place where you can make your comparisons.

GODFREY: If you have such a slope at the edge of the continental shelf, then that pressure gradient will have to drive a current. You can't have Coriolis forces there.

TAPLEY: What is this magic number 1300 m?

GODFREY: It does not matter very much what depth you take. In the next paper (by STURGES), 2000 m is used in the Atlantic, while a lot of people use 1000 m depth in the Pacific. It is largely a matter of history. However, whatever depth you use, provided the depth is low enough, you do not get very large errors. STURGES gives a figure for it.

ANGUS-LEPPAN: What happens from 75 km off-shore to the shore. Do you measure the profile over the last 75 km?

GODFREY: You reverse the principles and work backwards. If you know the currents parallel to the shore, you can work out the slope of the sea surface. The sea surface heights are measured directly off-shore and the currents worked out from it. Close in, we can measure the currents and then work out the slope of the sea surface. The flow is generally southward, and you work inwards from there to get the sea surface height at the coast.

ANGUS-LEPPAN: The slope is deduced from the current?

GODFREY: Yes, but we do not have good observations for currents. You can only get a qualitative figure; it is of the order of 5-10 cm.

GRAFAREND: What is the definition of the "heights" you refer to ? Are they dynamical heights or normal heights?

GODFREY: They are what's called "dynamic" heights.

LAMBERT: How do you know that the isobaric surface 1300 m down is parallel to the isobaric surface at the top?

GODFREY: At 1300 m, the isobaric surface follows the equipotential surface. At the sea surface it does not, as the latter is a free surface.

LAMBERT: Do you believe that the equipotential surface at the surface is parallel to the equipotential surface 1300 m down?

GODFREY: I believe if you took two equipotential surfaces, one at 1300 m and the other at the sea surface, the upper one will intersect the real ocean surface.

LAMBERT: The point is that you are taking as a datum the equipotential surface 1300 m down, and assuming that there is a surface parallel to it that is going to coincide with zero elevation.

HOLDAHL: They (the oceanographers) are assuming it is parallel in the dynamic sense but not in the geometric sense.

WERNER: Your term "bar" - does it refer to a normal atmosphere?

GODFREY: Yes. When we refer to 1300 dbar, we refer to a location where a pressure gauge would measure a pressure of 1300 dbar. We don't really mean 1300 m.

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DISCREPANCY BETWEEN GEODETIC AND OCEANOGRAPHIC LEVELLING ALONG CONTINENTAL BOUNDARIES

ABSTRACT

On the basis of levelling results on land, sea level along US coasts is found to rise to the north, with a total change in level of about 1 m. For comparison, the variation of sea level along continental boundaries is studied from a variety of oceanographic data. Along the western boundaries of the Atlantic and Pacific oceans, sea level *falls* approximately 80 cm between the equator and 40° N or S. Along the eastern boundaries, the change in level is about one-third this amount. This meridional slope is shown to be the correct amount required by the change, with latitude, of surface slope across the boundary currents. The meridional slope along the inshore edge of the Gulf Stream has the correct sign and magnitude to be balanced by the gradient of the Reynolds stresses. In the east-west direction, sea level along the Pacific coast of the United States stands about 70 cm higher than along the Atlantic coast, in agreement with levelling. It is argued that if the cause of the discrepancy in the north-south direction is an extremely small systematic error in levelling, it would have a much smaller impact on the parent field than if the error lies in some fundamental concept in physical oceanography.

1. Introduction

Sea level on the Pacific coast of the United States stands higher than on the Atlantic coast. This result has been apparent in the United States levelling network for nearly fifty years. The evidence from oceanographic data is about as old, but the agreement between these observations has been recognised by few oceanographers. A renewed interest in information about sea level near the coasts has been stirred by recent work in geodesy, especially the promise of very high accuracy altimeters in satellites and also by new work in numerical simulation models of ocean circulation in which slopes of sea level appear along coastal boundaries.

The most recent published analysis of first-order levelling in North America has shown (BRAATEN & McCOMBS 1963), in the 1963 Special Adjustment, that sea level along the Pacific coast stands 62 cm higher than along the Atlantic coast, in agreement with earlier levelling work. The uncertainty in the levelling was estimated to be about 6 cm. Over the latitudes of comparison, 32°N to 42°N, the difference between the two coasts is independent of latitude (see STURGES 1967, figure 1). On the basis of oceanic (or steric) levelling, MONTGOMERY (1969) has estimated that the sea-level difference between the two coasts should be 70 cm, in close agreement with the results of geodetic levelling.

2. Sea-level Differences from Hydrographic Data

One of the major reasons for the difference in sea level across the American continent has long been recognised as the difference in density of the waters of the Atlantic and Pacific oceans. REID (1961a) has shown that the surface of the Pacific Ocean should stand higher than the Atlantic by 40 cm on the basis of geopotential relative to the 1000 db surface. This difference increases with deeper reference surfaces; MONTGOMERY (1969) showed that the deepest significant reference surface is probably near 2000 db.

These differences discussed by REID, however, pertain to the zonally *averaged* level of the oceans. The largest slopes in the ocean are found in boundary currents, however, so it is clear that sea level *at the coasts* will depart considerably from the ocean-wide averages. In order to examine this effect, figure 1 shows, for the Atlantic and Pacific oceans, the height of steric sea level near the coasts. The quantity plotted is the geopotential anomaly of the sea surface relative to a deep pressure surface. (The anomaly is divided by gravity, to convert to length units.) To use this technique to infer sea-surface slope is usually called *steric levelling*. The data from the Pacific Ocean, relative to 1000 db, are from REID (1961b). In the Atlantic, relative to 2000 db, the data are from ANATI (1973), based on IGY data, and from DEFANT (1941). A single point in the Caribbean, from GORDON (1967), was adjusted from his map (1200 db) to 2000 db. Aside from the few points that are annual averages -- MONTGOMERY (1969), STURGES (1968) -- the data contain scatter caused by seasonal variations. PATTULLO et al (1955) have shown that these variations are typically 5 to 10 cm.

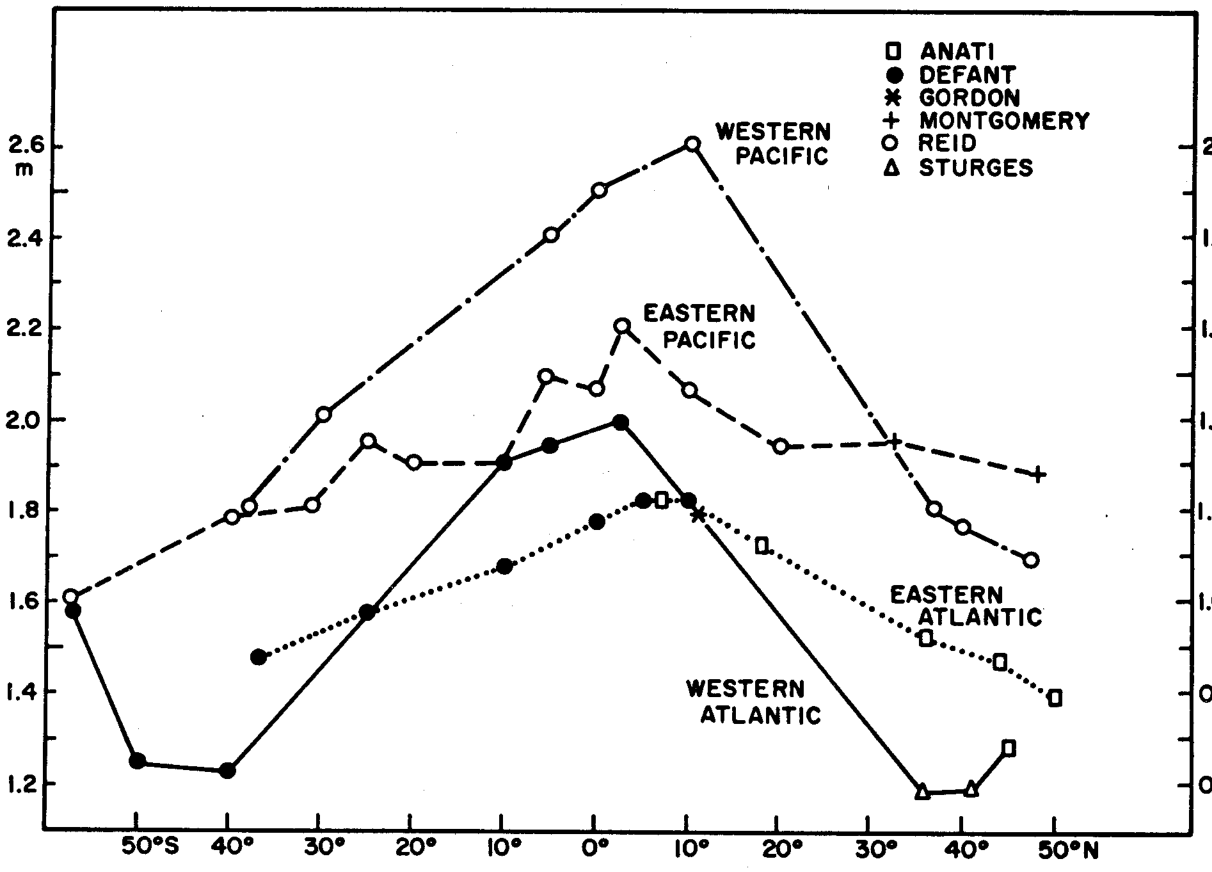

Figure 1. Dynamic height of the sea surface near continental boundaries versus latitude: Left ordinate, for Atlantic data, is relative to 2000 db; right ordinate, for Pacific data, is relative to 1000 db. Equivalence between the two scales is based on Montgomery's values and holds exactly only in the north-east Pacific.

The two scales in figure 1 (relative to 1000 db in the Pacific, relative to 2000 db in the Atlantic) are offset by 61 cm, the geopotential anomaly of the 1000-2000 db layer reported by MONTGOMERY (1969), for Ocean Weather Station Papa in the North Pacific.

The data in figure 1 are intended to apply as nearly as possible to *coastal* values, even though the data are obtained in water 1000 m to 2000 m deep. In regions with a narrow continental shelf, inside a weak boundary current, the change in level between deep water and the coast is a few centimeters at most; the change is also negligibly small at low latitudes. Inside a western boundary current, however, extrapolation to the coast would introduce large errors (by this technique), so no data points are included in figure 1 for these regions. As a result, we lose some detail in between; nevertheless, the average slope is correct.

A number of features in figure 1 deserve discussion. In both oceans, at low latitudes, the western side stands higher than the eastern side, as has long been recognised. The slope is reversed at higher latitudes. As REID (1961b) showed, for the Pacific, this difference closely parallels the wind stress at the sea surface. The cross-over for the Atlantic data in figure 1 is at a latitude which seems too low, but this problem is probably caused by the scatter of observations and relatively small difference in slope of the two lines.

There is a systematic difference in level between the Pacific and the Atlantic, caused by the density differences between the two oceans, and this difference is evident in figure 1. This difference is in agreement with the results of geodetic levelling across the United States, as mentioned above. It is also evident, however, that the coastal height depends on latitude, although the difference across the United States is a very weak function of latitude.

The difference across the American continent is a minimum at low southern latitudes; the apparent zero difference at 10^{o}S is not reliable, in light of seasonal variability.

The single data point inside the Caribbean Sea is from GORDON (1967, figure 8). Near the Panama Canal, sea level (on Gordon's map) stands 5 cm higher; the resulting difference, near 8^{o}N, between the Western Atlantic and Eastern Pacific is an estimate of the difference in sea level across the Panama Canal. This difference, near 8^{o} to 10^{o}N, is about 20 cm, but is subject to error from seasonal effects. The levelling across the Panama Canal, as cited by REID (1961a) gives a difference of 3/4 ft (23 cm) with the same sign as given here; the Pacific stands higher. Again, the levelling results agree with the findings of oceanic levelling.

The oceanic levelling is affected by the deep pressure surface used as a basis for the calculations. It has been shown (STURGES 1974) that the uncertainty introduced by this choice is only a few centimeters.

3. North-south Slope of Sea Level along North America

A feature that is apparent in Figure 1 is the meridional slope along each coast. The observation that sea level stands high at low latitudes is consistent with the primary thermal effect. In the Atlantic Ocean, the northeast trade winds, on the average, blow nearly perpendicularly into the coast of the American continent. The result is a piling-up of water there, balanced directly by the wind stress. One is struck by the observation (e.g. STOMMEL's 1965, figure 96), that the direction and magnitude of the annual mean wind stress and the orientation of the coastline seem to be optimised with regard to this piling-up of water at low latitudes. ANATI's figure 3 (sea surface topography relative to 200 db) suggests that the sea surface along the northeast coast of Brazil, near 0° to 5°N, stands about 30 cm higher than in the open ocean at about 30°N.

The slope of sea level along a coastal boundary will be an important effect in connection with the boundary currents. This effect is examined by consideration of the momentum balance in the cross-stream and down-stream directions.

4. Momentum Balance in the Cross-stream Direction

It is generally accepted that the flow in a western boundary current is in geostrophic balance in the cross-stream direction. That is, the momentum balance is given, to a very good approximation, by

$$fv = i_x g \tag{1}$$

where f is the Coriolis parameter, v is the down-stream velocity (here to the North, taken to be in the y-direction), i_x is the slope of the sea surface in the x-direction, and g is gravity. The total change in level across the flow, Δh, is given by the cross-stream integral of equation 1 after rearranging,

$$\Delta h = \frac{f\bar{v}X}{g} \tag{2}$$

where $\bar{v}$ is the average surface speed over the width X. To see the effect of a current flowing to a different latitude, we differentiate equation 2 with respect to y:

$$\frac{\delta \Delta h}{\delta y} = \frac{f}{g}\frac{\delta \bar{v}X}{\delta y} + \frac{\bar{v}X}{g}\frac{\delta f}{\delta y} \tag{3}$$

the left-hand side is the meridional slope of sea level along the coast, if the offshore edge of the current is assumed to remain level. The coastal slope can be corrected for slope of the offshore edge, as discussed below.

The first term on the right in equation 3 is the meridional change in the surface transport per unit depth. The second term is the result of variation of the Coriolis parameter with latitude. If these terms can be evaluated, they will provide an independent test of the meridional slope shown in figure 1. I have computed the necessary values from available observations in a single boundary current, the Gulf Stream. The values are given in table 1. The quantity $\bar{v}X$ was computed from the surface velocity at each section, and the total change in level, Δh, was calculated from equation 2. There seemed to be a small variation in $\bar{v}X$ over the latitude range of the available data, although there is a trend; the standard deviation is only 8% of the average value. Some of this scatter is surely observational error, but it is about the same magnitude as the variations in total transport as reported by SCHMITZ & RICHARDSON (1968). Presumably at lower latitudes, where the major flow is toward the west, the $\bar{v}X$ term may vary substantially. The total meridional slope of the sea surface required by the data in table 1 (through equation 3) is 2.8 ± .3 cm/deg. It should be emphasised that the data in table 1 (with two exceptions) are from direct velocity observations, and are independent of the density observations used for figure 1. The lowest latitude point, from the Caribbean Sea, is from geostrophic calculations; this point is included for completeness, but is not included in any calculations that follow. WARREN & VOLKMANN's (1968) velocity field was computed from hydrographic data plus Swallow floats. In all other cases, velocity was observed directly;

for some observations the surface velocity was determined by GEK, which in turn was calibrated with LORAN.

TABLE 1

Average surface transport in the Gulf Stream from direct observations.
The change in level across the stream, Δh, is computed from the geostrophic relation.

Latitude	$\bar{v}X$	Δh	Source
13° 0'N	$88\times10^{7}cm^{2}/s$	30 cm	GORDON 1967
$24^{\circ}20'$	110	68	RICHARDSON, SCHMITZ & NIILER 1969
$24^{\circ}45'$	102	64	RICHARDSON, SCHMITZ & NIILER 1969
$25^{\circ}30'$	111	71	RICHARDSON, SCHMITZ & NIILER 1969
$25^{\circ}40'$	100	65	WEBSTER 1961
$27^{\circ}26'$	108	75	RICHARDSON, SCHMITZ & NIILER 1969
$28^{\circ}21'$	106	75	RICHARDSON, SCHMITZ & NIILER 1969
30° 0'	111	83	WEBSTER 1965
$30^{\circ}20'$	114	86	RICHARDSON, SCHMITZ & NIILER 1969
$32^{\circ}24'$	117	94	RICHARDSON, SCHMITZ & NIILER 1969
35° 0'	100	86	WEBSTER 1965
$35^{\circ}45'$	117	103	WORTHINGTON & KAWAI 1970
38° 0'	123	103	VON ARX 1953
38° 0'	121	112	WARREN & VOLKMANN 1968
$38^{\circ}06'$	109	101	WORTHINGTON 1954
Mean	110 ± 8		

The question of the slope of the offshore edge was investigated by forming averages of hydrographic data in areas beyond the edge of the Gulf Stream, where a substantial accumulation of hydrographic data was available. The results of that study suggest that sea level in mid-ocean, i.e. beyond the edge of the Gulf Stream, rises slightly to the North with an average slope of 0.8 ± .2 cm/deg latitude; this slope, of course, is relative to the slope of the 3000 db surface, which may be assumed negligible for the present purpose. The meridional change in total elevation across the Gulf Stream, as required by the results in Table 1 (from 25° to 38°N), is 2.8 ± .3 cm/deg. The difference between these slopes, 2.0 ± .4 cm/deg. must appear as a north-south slope of sea level along the inshore edge of the current. Sea level should fall to the North by this amount.

5. Momentum Balance in the Downstream Direction

It is appropriate to determine whether the meridional slope found, in the preceding section, on the inshore edge of the Gulf Stream can be reconciled with the momentum equation in the downstream direction. We can omit the details (see STURGES 1974) and merely give the result: the balance is between the longshore pressure gradient, caused by the sea-surface slope, and a term involving the so-called Reynolds stresses.

The Reynolds stress term may be evaluated from the results of SCHMITZ & NIILER (1969) in the Florida Current. Their figure 1g gives the distribution of Reynolds stress across the stream near $30^{O}N$; the gradient at the inshore edge is calculated to be $+2.5 \times 10^{-4}$ cm/sec^2. The magnitude of the pressure gradient caused by the slope of the sea surface is $\sim 2 \times 10^{-4}$ cm/sec^2. The conclusion, therefore, is that the Reynolds stress term has the correct sign and approximate magnitude to balance the pressure gradient associated with the surface slope determined from equation 3.

In summary: the slope along the coast of the Western Atlantic indicated in figure 1 from $12^{O}N$ to $36^{O}N$ is -2.5 cm/deg. The uncertainties in the data are approximately ± 2 cm in the annual average at 36^{O}, ± 5 cm at 12^{O}, and ± 5 cm from the slope of the 2000 db surface (on the inshore edge of the stream). The two *independent* estimates of slope are thus -2.5 ± .5 cm/deg from steric levelling (figure 1), compared with -2.0 ± .4 cm/deg from geostrophic levelling. The two methods are independent, and the two results agree quite well.

6. Comparison with Land Levelling

The results of geodetic levelling (BRAATEN & McCOMBS 1963) are that sea level should rise strongly from South to North along US coasts, with a slope of 2.8×10^{-7} on both coasts. This discrepancy has been discussed elsewhere (STURGES 1967; STURGES 1968).

The slope along the Atlantic coast, as found by levelling, is in sharp contrast with the value of -2×10^{-7} (-2.2 cm/deg) found in the present paper. More recent work, done since the special levelling adjustment of 1963 (S. HOLDAHL, private communication) suggests that the slope indicated by levelling along the US Pacific coast is even larger than was found by BRAATEN & McCOMBS. A similar disagreement with levelling results in Australia has been discussed by HAMON & GRIEG (1972). In a previous study of sea level along the Pacific coast, it was shown (STURGES, 1967) that the geodetic levelling results are at variance with the oceanic results.

The "confrontation" may be described as follows. The oceanic results involve the density field, the observed currents, and the well-known equations of motion. Two independent methods give, quantitatively, the same result (in this paper). A third method, independent of the first two, gives qualitatively the same result. To put it colloquially, "it all hangs together." To change the oceanographic results will require a first-order (i.e. large) change in our understanding of at least one of the quantities involved.

By contrast, the presence of an extremely small systematic error in the geodetic levelling - a few microns per sight - would account for the discrepancy between the two sets of results, and would involve no difficulties with the understanding of any fundamental processes. My own personal inclination is to take this second choice. One suspects strongly, on the basis of the oceanographic results put forward here, that the geodetic levelling contains systematic errors.

By contrast, the levelling results in Brazil (RODRIGUEZ 1970) indicate that sea level should rise 29 cm from Imbituba ($28^{O}S$) to Fortaleza ($4^{O}S$). This slope agrees refreshingly well with that shown in figure 1.

It is obviously necessary that the present discrepancy between land levelling and oceanic levelling be resolved, in order that we may use their combined results to provide urgently-needed information about circulation, geodesy, and other areas of importance near our coasts.

7. Acknowledgment

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9. Discussion *

MARKOWITZ: Are the geodesists wrong? Is this discrepancy due to an error in the levelling?

HOLDAHL: It is very unlikely that the total discrepancy could be due to the accumulation of random levelling errors. The results of a recent adjustment showed the discrepancy between San Fransisco and Los Angeles to be 491 mm, whereas the standard error of the adjusted elevation difference between these points was only 40 mm.

MARKOWITZ: Then there is a genuine discrepancy?

HOLDAHL: Yes.

MUELLER: STURGES believes there is some systematic error in the levelling which no one knows anything about. This matter was discussed at the Fourth GEOP conference in Boulder last August, but I still dont understand after that session and the two presentations so far made here, what the basic reference surface is to which steric heights are referred. This is still a mystery to me. How do you define this surface which is 700 - 2000 m down?

GODFREY: The forces of pressure and gravity are essentially the same. The pressure is there because of gravity.

SYDENHAM: This can be resolved by the use of the (Division of) National Mapping laser ranging profiler which has actually picked up effects similar to this. This is an aircraft flown laser terrain profile recorder which has already, in tests, picked up about ½ m difference in sea surface in South Australian waters. It has a similar problem because it uses an isobaric surface for positioning reference.

BUCHWALD (Chairman): I think we all agree that this is a problem which needs resolving ....

WERNER: I notice from your comparison between geodetic levelling and (steric levelling), there is agreement with geodetic results when comparisons are made along a latitude circle. Along a meridian there is disagreement. I don't expect an answer but just introduce the point for discussion.

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SEA-SURFACE TOPOGRAPHY IN GEODESY WITH PARTICULAR REFERENCE TO AUSTRALIA

ABSTRACT

The relationship between the physical ocean surface and the more elusive geoid provides a problem worthy of attention in geodesy, as well as in oceanography. Co-operation between these two disciplines can feasibly lead to results which would have application to geodetic techniques. These points are discussed in the light of current methods and accuracies.

Present-day knowledge of the existence, and causes, of separation between the two surfaces is summarized, but progress in the study of the phenomenon in Australia is considered in detail. Areas of concern and possible techniques for study in this vicinity point to possible future activity in the field of sea-surface topography.

1. Definition of Sea-Surface Topography

The sea-surface topography is represented by the separations, ζ, along the vertical, between the ocean surface and the geoid. The values, which are of positive sign when the sea-surface is above the geoid, vary both with position on the geoid and with time. The heights are also dependent on the particular equipotential surface which is accepted as being the geoid. However, provided that the reference surface is used consistently and is chosen at approximately the mean position of the sea-surface, this effect, in practice, is not significant.

The sea-surface topography is often represented by the *mean* rather than instantaneous, value of ζ, over a period of time, which will be signified here by $\bar{\zeta}$.

It is useful for the following discussion to recognise ζ as being composed of two parts:

(i) a known portion, for which corrections can be estimated, and

(ii) a remaining, unexplained portion.

2. Interest in Sea-Surface Topography

The determination of $\bar{\zeta}$ has significance in practical geodesy, as it enables the definition of the position of the geoid at those places where, and at the times when the ocean surface is available. This may enable objects on and above the physical surface to be related to the geoid more easily than by conventional methods, or even where existing methods are impossible. Applications, related to height determinations, could include:

(i) height datum comparisons, both inter-continental and intra-continental. The former has application to satellite tracking, for example, by permitting observation station heights to be tied to one geoid. An accuracy of datum extension of 10 cm corresponds to one part in 10^8 of the earth's radius.

(ii) determination of geoid heights at sea. Elevations to 1 m are necessary for free-air corrections to 0.3 mgal for gravity reductions.

(iii) determination of altitudes of satellites relative to the geoid, which may be provided over the oceans by altimeter measurements of the distance between the satellite and sea-surface. This technique will be discussed further below.

In oceanography, interest lies in the value of the separation ζ, rather than in the spatial position of either the ocean surface or the geoid. Areas of concern include tides, ocean currents and water density, air-pressure and wind effects, tsunamis and surface waves. Although it may seem, therefore, that such interests are not relevant to geodesy, it must be stressed that the converse is true. Geodetic determinations of the sea-surface topography would assist oceanographic study of the ζ function which may be used, in turn, in the geodetic applications mentioned above.

The problem of sea-surface topography in geodesy is to determine and explain the unknown portion of the sea-surface topography in order to be able to specify ζ to an accuracy which is suitable for present purposes. Oceanographic knowledge presently provides estimates of ζ. If sample values of the true separation can be collected, it seems feasible that the value of sea-surface topography could be provided to an accuracy of 10 cm. The required level of accuracy of ζ depends on its intended use, but 10 cm seems adequate for geodetic purposes, as mentioned above. This approach to the problem assumes that the separation between the geoid and sea-surface cannot generally be determined whenever required and would, instead, be calculated if necessary using such data as ocean conditions. The following sections discuss the present state of knowledge of the expected and apparent sea-surface topographies, particularly around Australia.

3. Present Knowledge of the Existence of ζ

Observations of ζ are presently supplied by connections between geodetic levelling networks and tide-gauges. Levelling provides heights of the physical surface of the earth above an equipotential surface. The tide-gauges can be used to relate the sea-surface, or usually the mean sea-surface over a time period, to the physical surface. By combining levelling and tide-gauge observation, therefore, the sea-surface can be related to an equipotential surface.

Levelling in the United European Levelling Network, as reported by LEVALLOIS (1960) showed some variations between sea-level and levelled heights. If the levelling results were assumed to be correct, then sea-level had an apparent elevation of +28 cm at Kemi, Finland, an apparent elevation of +14 cm at Cascais, Portugal, and of -34 cm at Genoa on the Mediterranean Sea. Generally sea-levels had apparent elevations of the order of $\pm$10 cm in this network.

The first order levelling net of France of 1969 indicated that differences in sea-level around the coastline were up to half a metre (LEVALLOIS & MAILLARD 1970).

U.S. experience, as reported by STURGES (1967) and BRAATEN & McCOMBS (1963), based on the levelling used in the 1963 adjustment by the U.S. Coast and Geodetic Survey indicated that the sea-surface slopes upward from North to South by about 50 cm along both the Pacific and Atlantic coasts, and that at a given latitude, the Pacific Ocean is higher than the Atlantic Ocean.

The 1970 re-adjustment of the geodetic levellings of Great Britain, connected to about six gauges, showed maximum and minimum sea-levels to differ by 29 cm, between Aberdeen in Scotland and Newlyn in the south (KELSEY 1970).

RODRIGUEZ (1970) reported that, according to the Brazilian levelling, the sea-level variation is 41 cm between Rio de Janeiro and Fortaleza, which are separated by over 1500 km of coastline.

In Australia, since 1945, 160 000 km of levelling, mainly third order, has been undertaken under the direction of the Division of National Mapping of the Department of Minerals and Energy for the National Mapping Council of Australia. The network is described by LEPPERT (1970) and by ROELSE ET AL (1971). During the survey, connections were made to about 30 tide-gauges around the mainland coastline, with the intention of connecting the various local datums based on Mean Sea Level (Ibid). The results also provide an estimate of the sea-surface topography around this country. The levelling net, after adjustment, indicated that, if ζ at Port Lincoln in the Great Australian Bight was adopted as zero then ζ at Cape York would be of the order of 2 m. Further details of this result are given by ROELSE ET AL (1971).

To account for any effect of gravity variation on the results, an orthometric correction has been applied to the levelled height differences in the Australian net. The gravity values used in this correction were based on *theoretical* (normal) gravity values referred to the Reference System 1967 (ROELSE ET AL 1971, p. 65).

The levelling has also been converted into *geopotential* differences using *observed* gravity, (MITCHELL 1973). Potential differences, ΔW, between bench-marks in the levelling, were formed from the observed height differences Δh, using the relation:

$$\Delta W = \tfrac{1}{2}(g_1 + g_2)\Delta h \qquad (1)$$

g_1 and g_2, the gravity values at the two bench-marks, were interpolated from surrounding values available at 0.1 degree intervals. Bench-marks were at a spacing of the order of 3 km. However, the values of ζ in units of geopotential differences when compared with those in terms of height, were found to have been altered by amounts which are not significant in this discussion (Ibid).

The apparent sea-surface topography must be viewed in the light of the accuracy of the levelling results.

PETROV's analysis of reports about the United European Levelling Network showed that errors in the levelling do not permit any significance to be attached to sea-surface topography given by that net, as mentioned above (PETROV 1965, p. 247).

The accuracy of the levelling in the Australian network has been studied by ROELSE ET AL (1971) who, in their report on the adjustment, note that "...most of the levelling in the network was observed to third order standard" (Ibid, p. 75). This standard requires that the two levellings of any section between bench-marks must not differ by more than $12\sqrt{K}$ mm where K is the distance, in kilometres, between bench-marks, (NATIONAL MAPPING COUNCIL OF AUSTRALIA 1970). ROELSE ET AL (p. 76) have adopted a value of about $8.1\sqrt{K}$ mm for the precision of the *adjusted* orthometric levelling. It is estimated (Ibid, Annexure F) that the standard deviation of a height at the coastline when referred to a height at an origin towards the centre of the continent, via the adjusted orthometric levelling, is of the order of 35 cm. This is significantly less than the 200 cm variation in sea-level which has been shown by the levelling results. Thus, the existence of the sea-surface topography is not explained by the errors expected from type of levelling survey carried out in Australia.

Systematic errors in the levelling could exist undetected, to produce the apparent ζ values, but discussion on this would, at the moment, seem to be largely speculative.

The change in the shape of the geoid during tides in the equipotential surfaces (MELCHIOR 1966; JENSEN 1950) is not sufficient to contribute significantly to the apparent sea-surface topography.

4. Known Sources of Sea-Surface Topography

Although the surface of an undisturbed homogeneous liquid should coincide with an equipotential surface, the sea-surface, in practice, is not expected to fulfill such a condition exactly. Known sources of separations between the surfaces are summarized below. Where possible, they will be related to the Australian sea-surface topography result.

Under generation in a wind field, waves and swell may have heights up to the order of 10 m and periods up to 15 sec. (e.g. SCOTT 1969). However, because of their periodicity and comparatively high frequency, these waves are generally considered to be filtered, by the mechanism of the tide-gauges, from their records. Thus, the state of the sea and swell should not have contributed to the sea-surface topography indicated by the Australian survey.

Large values of ζ are also caused by the tidal fluctuations. Their effect on sea-surface topography is assumed to vary with time according to

$$\zeta_{tides} = \sum_{i=1}^{n} a_i \cos (2\pi f_i t + \phi_i) \tag{2}$$

where $a_i \cos (2\pi f_i t + \phi_i)$ represents one of n tidal constituents. A mean of observations over a suitable period of time should therefore eliminate tidal influence. The effectiveness of the mean depends on the period over which it is taken, and on the amplitudes, a_i, and frequencies, f_i, of the tides. Means of sea-level produced from tide-gauge records over a number of years as in the case of the Australian survey, may be considered to be free from the variable effects of any tides which have a magnitude of more than a few centimetres.

Subtraction of the tide-height from an observed level of the sea-surface is generally only an accurate process if records of the tides over a number of years are available for that particular point, as evidenced by co-range charts given by EASTON (1970).

UNOKI & ISOZAKI (1965) have discussed the influence of tides on ζ within enclosed bays and harbours, although the order of magnitude of the effect was only centimetres. Little other mention has been found in the literature on the position dependent effect of the tides.

Two oceanographic phenomena which significantly affect sea-surface topography are the distribution of water density and the existence of ocean currents. They are closely related and may be discussed simultaneously.

Theoretically, the value of ζ at a given time and place is assumed to depend on the average density in a column of water between the surface and a specified equipotential surface deep in the ocean. Density is a function of salinity and temperature, which are measurable, and of pressure, which depends on the dynamic depth. Variations of ζ with time and position may therefore be calculated from differences in density. The surface of less dense water is higher, with respect to the geoid, than that of more dense water.

Ocean current positions are also indicative of the density-induced portion of sea-surface topography. The currents arise as a result of the pressure differences, which result from the density distribution. Their flow is modified by the Coriolis effect, so that, for example, currents in the southern hemisphere circulate in an anticlockwise direction around areas of high ζ value. The situation is analogous to the relationship between atmospheric winds and barometric pressure differences. The complete theory of ocean water densities and currents, and their relationships to sea-surface topography is complex (DIETRICH 1957, pp. 291 et seq; FOMIN 1964; NEUMANN 1968; SVERDRUP ET AL 1942, pp. 389 et seq).

The effects of ocean currents and density in practical cases have been considered by BOWDEN (1960), DONN ET AL (1964, pp. 247 et seq), LISITZIN (1965), STURGES (1967; 1968). See also a review by HAMON (1970). LISITZIN (1965, p. 16) has produced a world map of density effects on the sea-surface, showing the resultant variation in ζ around the Australian coast to be of the order of 80 cm. Other papers indicating ζ values due to density in waters around Australia have been published by HAMON (1961; 1965 A,), HAMON & TRANTER (1971), WYRTKI (1962 A; 1962B) and HAMON & GREIG (1972). The density data, which has been collected principally by the Division of Fisheries and Oceanography, CSIRO, Australia, shows a contribution of about 60 cm to the apparent variation of sea-surface topography between Port Lincoln and Cape York (MITCHELL 1973). The existence of time variations of ζ due to density is illustrated by the changes in ocean current positions near Australia (WYRTKI 1960; 1961; HAMON & KERR 1968), but lack of complete data makes it difficult to estimate accurately the change of ζ with time.

The possibility of correcting for density variations depends directly on the available salinity and temperature data. It must be noted, however, that the density/current influence is one of the major contributors to variable sea-surface topography.

Interaction between the atmosphere and the oceans produces a number of variations of ζ with both position and time. The variation of observed sea-level due to barometric pressure is generally given by the linear relationship

$$\zeta_{barom} = \alpha \, \Delta p \tag{3}$$

where Δp is the deviation of the atmospheric pressure from a standard value which is independent of time and position, and α is a constant. Theoretically, α has a value of -1.01 cm mbar^{-1} (HAMON 1966, p. 2883). However analysis of sea-level records in conjunction with air-pressure records has not always produced a value for this factor of -1.01 cm mbar^{-1} (DONN ET AL 1964, pp. 247 et seq; EASTON & RADOK 1970 A, p. 6; HAMON 1958, pp. 189 et seq; 1962; 1966; LISITZIN & PATTULLO 1961, p. 845; ISOZAKI 1969).

However, using this value of α and air-pressure observations, obtained from *Bureau of Meteorology (Monthly)* the corrections needed to reduce all mean levels of the sea-surface in the Australian survey to a standard atmospheric pressure, have been calculated (MITCHELL 1973). The corrections reduced the apparent variation in ζ around Australia by the order of 10 cm.

Another atmospheric action on sea-level, often operating simultaneously with barometric pressure effects, is wind. The magnitude of the influence is dependent on the depth and width of the continental shelf and on the velocity of the wind acting across the shelf towards the shore. Various formulae can be used to estimate this effect (e.g. STURGES 1967, p. 3630; HAMON 1958, p. 191; CREPON 1970; SILVESTER 1970). Theoretically, the influence of winds on a long term mean sea level is expected to be negligible (STURGES 1967; HAMON 1958). Studies of the tide-gauges used in the Australian survey (EASTON 1968) and of their records (EASTON 1967 A, 1967 B, 1970; EASTON & RADOK 1968, 1970 A, 1970 B) have produced some information on wind effects at these sites. Most notable is the evidence of a persistent 15 cm effect at Mackay (EASTON 1970, pp. 47 and 252).

Some minor effects on the value of ζ as determined by the conventional process with tide-gauges, can be considered to have negligible influence on the Australian results. These include:

(i) tsunamis,

(ii) the secular change in ζ which is only of the order of 0.5 mm per year around Australia (IAPO 1955),

(iii) erroneous gauge recording. The faults described in the reports on the gauges and their recording, should not be significant in this discussion (EASTON 1967 A, 1967 B, 1968, 1970; EASTON & RADOK 1968, 1970 A),

(iv) river flow past the tide gauges, a number of which are situated in rivers or near their mouths (EASTON 1968). The river flow at individual gauges is not considered likely to produce ζ effects as a regular function of position on the coastline, and should not, therefore, affect the overall picture of ζ variation around Australia.

5. Investigation of Anomalous Sea-Surface Topography

Around Australia, the apparent sea-surface topography is presently only partially accounted for by known oceanographic phenomena.

It must therefore be considered as a possibility that the estimate of $\overline{\zeta}$ by the networks of height or geopotential differences over this continent is erroneous. A latitude dependent systematic error of 0.3 mm per km would explain a significant portion of the apparent sea surface topography. It is, therefore, important to recognise that any future determination of the sea-surface relative to the geoid should, if possible, be independent of the existing levelling.

The most significant known cause of a deviation between the *mean* sea-surface and the geoid is undoubtedly that of ocean water density. It is credible that its contribution may be larger than indicated in the above discussion. It is plausible that the relationship between the measured densities at sea and the sea-surface topography is imperfect, being affected, for example, by sea bed topography, ocean current eddies or friction, to an extent which is presently underestimated.

Attention could be paid to wind effects on the sea-surface. Although not expected to produce a full metre variation in sea-level, their contribution could nevertheless be significant. Extensive study should at least prove conclusively either that the wind effects are not significant or that they are worthy of further study. The vast areas of shallow water less than 200 m deep in the Gulf of Carpentaria, Arafura Sea and Torres Strait regions arouse interest.

The possibility of a position dependent tidal influence could be examined.

Estimates of the effect of river flow past tide-gauges in Australia would reduce the uncertainty in present values of the sea-surface topography.

6. Sea Surface Topography from Satellite Altimetry

As it is possible that erroneous levelling or tide-gauge results contribute to the apparent sea-surface topography around Australia, a determination of ζ independent of the levelling or potential networks would assist further investigation of the problem. Satellite altimetry has been considered as one suitable method for determining sea-surface topography (STURGES 1971). The principles may be explained with the aid of Figure 1, which shows the development of a portion of the surface which contains the verticals through successive satellite positions. Along any vertical, the following scalars, which are shown positive in the diagram, are involved:

(i) a, the distance, measured by an altimeter, between the satellite and the instantaneous sea-surface,

(ii) h_{si}, the height of the satellite above the ith spheroid. Two reference surfaces are included to indicate that there are many spheroids, with various dimensions and positions, to which reference may be made,

(iii) N_i, the separation between the geoid and the ith spheroid.

The required value of ζ is then given by

$$\zeta = h_{si} - a - N_i \qquad (4)$$

Such an altimetering system has been planned for the GEOS-C satellite, and in the following summary of the application of satellite altimetry, GEOS-C is emphasized.

Errors in the value of a, which will be measured from GEOS-C by an altimeter reflecting a *radar* beam from the sea-surface, will result from refraction through the atmosphere. Corrections must be made to minimise the errors (WEIFFENBACH 1971, p. 1-3; SIRY 1971, p. 7-30). The state of the sea at the time of the satellite's passage, which will also affect the measured value of a, may have to be determined to apply necessary corrections, (WEIFFENBACH 1971, pp. 1-6). Possibly the altimeter readings themselves will provide an estimate of the sea-state (SHAPIRO ET AL 1971).

The spheroid height is obtained by either tracking the satellite from stations whose co-ordinates are known, or by predicting the satellite height using knowledge of the gravity field combined with corrections obtained from tracking. The former method, which relates the satellite height to a local spheroid, cannot be applied to the Australian problem as heights of observation stations cannot be determined independent of the levelling processes. The latter method, however, relates the satellite height to a geocentric spheroid. The determination of h_{si} is possibly the major cause of errors propagated into ζ.

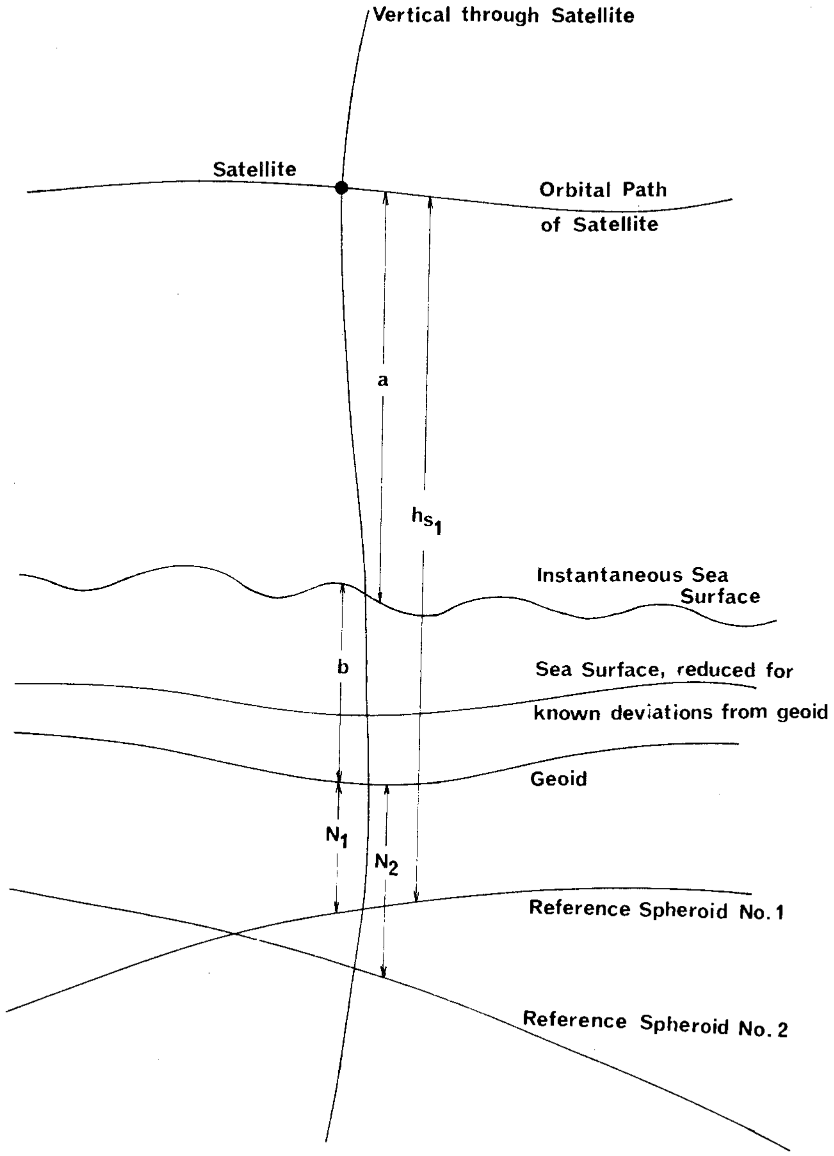

Figure 1

Of the two main methods of determining the geoid-spheroid separation, i.e. the astro-geodetic levelling and gravimetric method, the former cannot be applied to the Australian situation. It relates the geoid to a *local* spheroid. The gravimetrically determined geoid is related to the geocentric spheroid and also has the advantage, for satellite altimetry, of being determinable on the continental shelf and in the open ocean. Although the value of N on continental shelves around Australia must presently be extrapolated from land areas, the *possibility* of development to 10 cm accuracy will make possible the future use of satellite altimetry to determine the sea-surface topography around this continent.

Because the altimeter will operate for many hours over lifetime of a satellite, 1.5 to 2 years, a large number of values of a and h_{si}, obtained on many different orbital crossings, will be involved in the determination of the sea-surface topography. For GEOS-C, typical estimates are of the order of 20 observations per one degree square (WEIFFENBACH 1971, pp. 1 - 9; CHOVITZ 1971). Thus, unlike tide-gauge records only a limited number of observations of the ocean surface are obtained by satellite altimetry. The corrections for known ζ influences will be more difficult to calculate, unless they are suitably meaned by as few as 20 measurements over 10^4 km^2 of ocean. Hopefully, the determination of sea and swell conditions by the altimeter itself will enable satisfactory corrections to be computed for the influence of surface waves as mentioned previously. Although a large number of observations will be taken over a square kilometre of ocean, the tides, whose spring range may be as large as 10 m around the mainland Australian coastline (EASTON 1970, p. 159) are likely to influence results. Tide-gauge recording during the altimeter's operation would be imperative, therefore, but difficulty will arise as the times and ranges of these tides on the continental shelf and, more particularly, at sea, cannot be extrapolated from coastal values. Corrections for barometric pressure, which may be of the order of 15 cm under typical conditions, can easily be applied.

The mean of the number of observations taken should be effectively free from the time influences of surges and ocean-density variations.

Australia, being a large area surrounded by ocean and covered by homogeneous levelling and gravity networks, seems suited to an application of satellite altimetry to check the unexpectedly large values of ζ. However, the region lacks sufficient tracking facilities to ensure that errors in h_{si} are kept to acceptable levels. The feasibility of satellite altimetry's application to this continent also depends on the accuracy of the determination of the geoid-spheroid separation, and of the tidal conditions, on the continental shelf areas.

Attention is drawn to the possibility that ζ varies across the continental shelf. This would require careful consideration in the analysis of altimetry results, as well as in any eventual application of sea-surface topography to geodetic techniques.

7. Summary

It is suggested that, through the use of satellite altimetry, the presently uncertain portion of the sea surface topography may be deduced. Such a determination would be particularly valuable in Australia where the position variation of the mean ζ value is exceptionally large. Oceanographic research or studies of the levelling survey, based on the results from satellite altimetry may provide a more complete knowledge of the sea-surface topography which could, in turn, be usefully applied to geodetic methods.

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THE INFLUENCE OF STATIONARY SEA SURFACE TOPOGRAPHY ON GEODETIC CONSIDERATIONS

ABSTRACT

The comparison of the results of geodetic levelling with the mean level of the sea as defined by tide gauge readings, has indicated the apparent existence of widespread departures of sea level from an equipotential surface of the Earth's gravitational field.

While the evidence available at present for the existence of this quasi-stationary sea surface topography is confined to coastal regions, there should be repercussions of fundamental significance in high precision geodesy if this phenomenon were to prevail over the global oceans with amplitudes and wavelengths which are not negligible. This arises primarliy through the widespread adoption of "mean sea level" as a datum for elevations, and affects global determinations of the highest resolution in both physical geodesy and vertical crustal motion studies.

These problems are summarized and desirable goals indicated for satellite altimetry and laser ranging techniques which would not only eliminate the errors arising from the assumption outlined above, but also confirm the global characteristics of stationary sea surface topography in non-coastal areas.

1. Introduction

1.1 Preamble

It has been widely reported in the literature (see Appendix 1) that Mean Sea Level as defined by the mean of tide gauge readings over some limited epoch, does not correspond to a single equipotential surface of the Earth's gravitational field insofar as the latter is implied from the results of a continental geodetic levelling operation. This phenomenon has been called stationary *sea surface topography* (APEL 1972) and its possible existence on a world wide scale poses problems in geodetic terms due to the reliance on Mean Sea Level as a means for providing the datum of reference for first order geodetic levelling on a global scale. The evidence available at present for providing some estimate of the possible magnitude of this phenomenon is summarized in Appendix 1.

On the basis of this evidence, sea surface topography which is quasi-stationary, could be expected to have amplitudes as great as 2 m and wavelengths as great as 8000 km if the coastal samples were representative of the Earth's oceans as a whole. Without making a case one way or the other for the existence of such a possibility, it is of interest to assess the effect it has on relevant geodetic considerations. In the first instance, it can be stated that Mean Sea Level does indeed provide an adequate global datum for first order geodetic levelling *insofar as* the results of levelling can be combined with those obtained from horizontal survey methods, to give position in three dimensions with a precision of 1 part in 10^6 in each co-ordinate.

First order geodetic levelling retains the capacity to provide a higher relative resolution than any of the other traditional geodetic processes at the present time. It is relatively simple in both principle and procedure, though somewhat laborious to execute. The resolution obtained from first order levelling procedures is approximately 30 times superior to the requirements of geodesy at 1 part in 10^6. This resolution is admittedly lost on transformation of the results of levelling to notions of position in three dimensions, where the best results obtained to date are generally conceded as having a precision approaching ±6 m in each co-ordinate.

The superior resolution of first order geodetic levelling is used at present primarily for either sophisticated engineering projects which are of local relevance, or for vertical crustal motion studies where global considerations, which should be of consequence, are not resolved with an equivalent degree of certainty due to the levelling being referred in all cases to the regional definition of Mean Sea Level. On the basis of the extrapolation of coastal estimates of quasi-stationary sea surface topography to the global oceans, the possibility of an uncertainty of up to ±2 m should be anticipated in the inter-connection between two such studies due to a lack of continuity in the definition of the datum for levelling. This figure is an order of magnitude greater than the internal precision expected from a continental network of first order geodetic levelling.

Present practice in the measurement of vertical crustal motion, relies on the resolution of the phenomenon afforded in most instances by orthometric elevations. The latter are referred to a datum usually provided by a convenient tide gauge. The Mean Sea Level datums so defined are assumed to be situated on a unique equipotential surface of the Earth's gravitational field called the *geoid*. It cannot be considered desirable to base extended studies over long time spans with reference to a surface whose shape and Earth space position in relation to the Earth's surface, change as a function of time, even on excluding tidal phenomena. This is a characteristic which can be expected from the geoid if constant mass re-distributions were to take place in the Earth's interior.

It is evident that if the definition of the geoidal datum for elevations on a world-wide basis is to match the resolution of first order geodetic levelling,

(a) the definition should characterize a particular epoch; and

(b) the existence of stationary sea surface topography, which must be construed as a non-gravitational phenomenon, will have to be allowed for,

The first is necessary due to reported secular variations in sea level (e.g., DONN ET AL 1964). The second would call for an averaging procedure in the definition of the geoid. It is therefore of interest to re-examine the geoidal concept in the context of determinations of secular variations in position with an ultimate goal of 1 part in 10^8.

1.2 A Guide to Notation

1.2.1 Recurring Symbols

A_n = surface harmonic of degree n having the form

$$A_n = \sum_{m=0}^{n} p_{nm}(\sin \phi) \left(A_{1nm}\cos m\lambda + A_{2nm}\sin m\lambda\right)$$

a = equatorial radius of the ellipsoid of reference

dS = element of area on the surface of integration

dt = interval of time between two contiguous epochs

$d\sigma$ = element of surface area on unit sphere

e = eccentricity of meridian ellipse

f = flattening of meridian ellipse

$f(\psi)$ = Stokes' function $\equiv$ $\operatorname{cosec} \tfrac{1}{2}\psi + 1 - 5\cos\psi - 6\sin\tfrac{1}{2}\psi - 3\cos\psi\,\log\left(\sin\tfrac{1}{2}\psi(1+\sin\tfrac{1}{2}\psi)\right)$

G = gravitational constant

G_n = surface harmonic of degree n in the expansion of

$$\Delta g' = \sum_{n=0}^{\infty} G_n, \quad n \neq 1$$

g = observed gravity at the surface of the Earth

$\bar{g}$ = mean value of gravity along local vertical between the Earth's surface and the geoid.

h = elevation above ellipsoid

h' = orthometric elevation

h_d = height anomaly

$\vec{i}$ = set of unit vectors along the axes of the X_i Cartesian co-ordinate system

$m = \dfrac{a\omega^2}{\gamma_e}$

$\vec{N}$ = unit vector normal to dS ($d\sigma$)

R = mean radius of the Earth

r = distance between the element of surface area dS and the point of computation P

T = time

U = potential of the reference system

U_o = potential of the equipotential reference ellipsoid

V_d = disturbing potential

W = geopotential

W_o = potential of the geoid

X_i = geocentric rectangular Cartesian co-ordinate system $X_1X_2X_3$, with the X_3 axis coincident with the axis of rotation, and the X_1X_3 plane defining the meridian of reference

α = azimuth

γ = normal gravity due to the reference system; subscript $_o$ refers to values on the reference ellipsoid; subscript $_e$ refers to equatorial value

Δg = gravity anomaly at the surface of the Earth

$\Delta g' = \Delta g - 2\dfrac{\delta W}{R}$

ΔW = geopotential difference with respect to the geoid

δW = geopotential difference between elevation datum and the geoid

λ = longitude, positive east

τ = epoch of observation

ρ = density of crustal material

ϕ = latitude, positive north

ψ = angular distance at geocentre between the point of computation P and the element of surface area dS ($d\sigma$)

ν = radius of curvature of the reference ellipsoid in the prime vertical normal section

ω = angular velocity of rotation of the Earth

$$\vec{\nabla} = \sum_{i=1}^{3} \frac{\partial}{\partial X_i} \vec{i}$$

1.2.2 Convention

$a = b + o\{b^2\}$ $\equiv$ terms whose order of magnitude is equivalent to or less than b^2 have been neglected

2. The Sea Surface and the Geoid

As pointed out in the introduction, the adoption of the definition of the geoid as *the equipotential surface of the Earth's gravitational field corresponding to Mean Sea Level*, poses no problems in the context of determinations of position in Earth space to 1 part in 10^6; co-ordinates of geodetic stations being computed on a three dimensional geocentric or quasi-geocentric Cartesian co-ordinate system X_i, on combining the observational data from both horizontal surveys and levelling operations. The required relations are

$$X_1 = (\nu + h)\cos\phi\cos\lambda \ ; \quad X_2 = (\nu + h)\cos\phi\sin\lambda \ ; \quad X_3 = \left(\nu(1 - e^2) + h\right)\sin\phi \qquad (1),$$

where all the symbols are described in section 1.2. The ellipsoidal elevation h needs to be known to o{6 m} to ensure that the precision of the X_i's are equivalent to those of the horizontal co-ordinates on a global basis. Regional determinations call for differences in ellipsoidal elevations between adjacent geodetic stations to be evaluated to o{±30 cm}. Conversely global solutions for geocentric position obtained from geometrical satellite geodesy (e.g, MARSH ET AL 1973) could be used to deduce ellipsoidal elevations to this same precision using procedures similar to that summarized in Appendix 2.

The results obtained by the use of the latter procedure obviate the necessity for the definition of orthometric elevations and the geoid vis-a-vis the reference ellipsoid, whose Earth space position is implicit in the numerical magnitudes of the ellipsoidal elevations, and the Earth's surface. It is not uncommon at the present time, to deduce geoid heights (e.g., IBID) with a resolution at the 5% level, from orthometric elevations supplied in the case of most tracking stations by the local geodetic authority (e.g., NASA 1971).

If it were pessimistically assumed that sea surface topography were to exist in a stationary state with amplitudes equivalent to the maximum reported in Appendix 1, and with long wavelengths in the non-coastal oceanic regions, a further 2% uncertainty would be introduced into the computed values of the geoid heights at the tracking stations. This would be difficult to detect if the resolution were only at the level of 1 part in 10^6. This uncertainty is of course introduced on account of the probability that each regional height datum would not lie on a unique equipotential surface of the Earth's gravitational field.

It is also necessary to take into account the nature of orthometric elevations when attempting to deduce geoid heights from three dimensional solutions. It is conventional to define orthometric elevations h' in terms of the observed difference of geopotential ΔW with respect to mean sea level which is assumed to coincide with the geoid, by the equation

$$h' = \frac{\Delta W}{\bar{g}} \qquad (2),$$

where $\bar{g}$ is the mean value of gravity as sampled along the local vertical between the Earth's surface and the geoid. If the Earth's surface were assumed to be planar, any value g_p at P on the vertical is related to observed gravity g at the surface of the Earth by a relation of the form

$$g_p = g + \frac{(\Delta h)^i}{i!}\frac{d^i g}{dh^i} - 4\pi G\int_0^{\Delta h}\rho\, dh \qquad (3),$$

where Δh is the depth of P below the Earth's surface and the differential coefficients $d^i g/dh^i$ are free air effects. Errors occur in the value of $\bar{g}$ due to errors in the values assumed for ρ as the stratification of matter between the Earth's surface and the geoid are not known. As the error $e_{h'}$ in h' due to an error $e_{\bar{g}}$ in $\bar{g}$ is obtained from equation 2 as

$$\frac{e_{h'}}{h'} = \frac{e_{\bar{g}}}{\bar{g}} \qquad (4),$$

it follows that errors in $\bar{g}$ due to incorrect modeling of the Earth's crust will have to be held to below 1 part in 10^5 in mountainous areas if orthometric heights are to be unaffected by crustal model errors in excess of the estimated precision of first order geodetic levelling over continental extents. Table 1 gives estimates of the magnitudes of errors in h' as functions of both h' and crustal modeling errors expressed as percentages of the value ($\rho = 2.67$ g cm^{-3}) most commonly adopted as a mean density for the upper crust.

Table 1

Errors in Orthometric Heights Due to Crustal Density Errors (100% = 2.67)

Errors expressed as parts per million in orthometric elevations

h (m)	Percentage density error				
	10	20	30	40	50
500	11	23	35	46	57
1 000	23	46	77	91	124
3 000	68	137	206	274	342
5 000	114	228	343	457	571
7 000	160	320	480	640	800
10 000	228	457	685	914	1142

As it is extremely unlikely on present trends that the stratification of crustal matter between the Earth's surface and the geoid will be known well enough such that $e_{\bar{g}}$ could in fact be kept to $o\{\pm 10^{-5}\bar{g}\}$, it could be argued that a plausible but nevertheless arbitrary model could be adopted for the Earth's crust. This would define not only both h' and N without ambiguity, but also the equipotential surface for this prescribed mass distribution which coincides with average mean sea level over the oceans, assuming that no stationary sea surface topography exists.

While orthometric height differences will continue to be a basic high precision geodetic measurement, it is difficult to put forward a forceful case for the retention of the concept of orthometric elevations in the context of geodesy to 1 part in 10^8. A corollary to this contention is the irrelevance of defining the geoid *in continental areas* to this same order of precision. This conclusion is by no means new, having been the basis for the decision taken in the nineteen-fifties to use geopotential differences as the means for representing the results of first order geodetic levelling in lieu of orthometric elevations.

Two points have to be taken into consideration when re-assessing the role of the geoid vis-a-vis geodetic levelling. The *first* is the consequence of the various Mean Sea Level datums not lying on the same equipotential surface of the Earth's gravitational field. While this will not cause concern when attempting to use the results of levelling in conjunction with three-dimensional co-ordinates obtained from satellite solutions to 1 part in 10^6, it has effects of significance on determinations in physical geodesy where formulations at the present time are based on the ability to refer all differences of geopotential ΔW to a common surface of equal potential $W = W_o$.

The *second* problem arises in the use of geodetic levelling techniques for the evaluation of vertical crustal movement, which at the time of writing, is largely based on measurements of differential changes obtained on re-levelling the same network of benchmarks

(a) after some finite but limited time lapse; and

(b) in relation to a datum surface which is assumed to be invariant in Earth space between successive levellings.

The definition of vertical crustal motion in the short term is unlikely to be improved by the use of geopotential differences ΔW in lieu of orthometric elevations, to any significant extent. The fundamental technique itself founders on the assumptions which are implicit in the adopted surface of reference and which could cause concern in long period studies when attempting to relate results from different areas into a single cohesive dynamic Earth model. It is of relevance to note that the practice of defining vertical crustal motion instead of a three dimensional crustal motion vector, is testimony to the superiority of first order geodetic levelling over all three dimensional position determinations which have been made to date.

These two questions are dealt with in the following sections.

3. Stationary Sea Surface Topography and Solutions in Physical Geodesy

The following is a revised presentation in a convenient summary form, of an aspect of an earlier development (MATHER 1973a). All basic definitions in physical geodesy are framed in the context of the system afforded by the physical surface of the Earth and the telluroid which is the locus of the points Q $(\phi_a, \lambda_a, U{=}U_o{+}\Delta W)$ illustrated in figure 1, in relation to points P $(\phi_g, \lambda_g, W{=}W_o{+}\Delta W)$ defining the Earth's surface. The disturbing potential V_{dp} at P is given by the generalized Bruns' equation

$$V_{dp} = W_o - U_o + \gamma h_{dp} + o\{f^2 V_d\} \tag{5}$$

The validity of equation 5 is not questionable so long as no interpretation is forced on the value W_o which is the geopotential of the datum of reference for the geodetic levelling from which the geopotential differences ΔW are measured. U_o is conventionally assumed to be the potential of the reference system on the surface of the equipotential ellipsoid of reference $U = U_o$. Such a concept, while questionable in geophysical terms, is totally acceptable in the geodetic sense.

The gravity anomaly Δg is defined as (e.g., MATHER 1971, p.101)

$$\Delta g = g_p - \gamma_{p'} = g_p - \left\{\gamma_o - \frac{2\Delta W}{a}\left(1 + f + m - \tfrac{1}{2}\frac{\Delta W}{a\gamma} - 2f\sin^2\phi + o\{f^2\}\right)\right\} \tag{6}$$

this being an instance of a free boundary value problem where it is assumed that the co-ordinates (ϕ_g, λ_g) are sufficiently well known to eliminate second order effects on the results (MATHER 1973a, p.16).

At the time of writing, Δg is considered to be the basic observed quantity in solutions of the geodetic

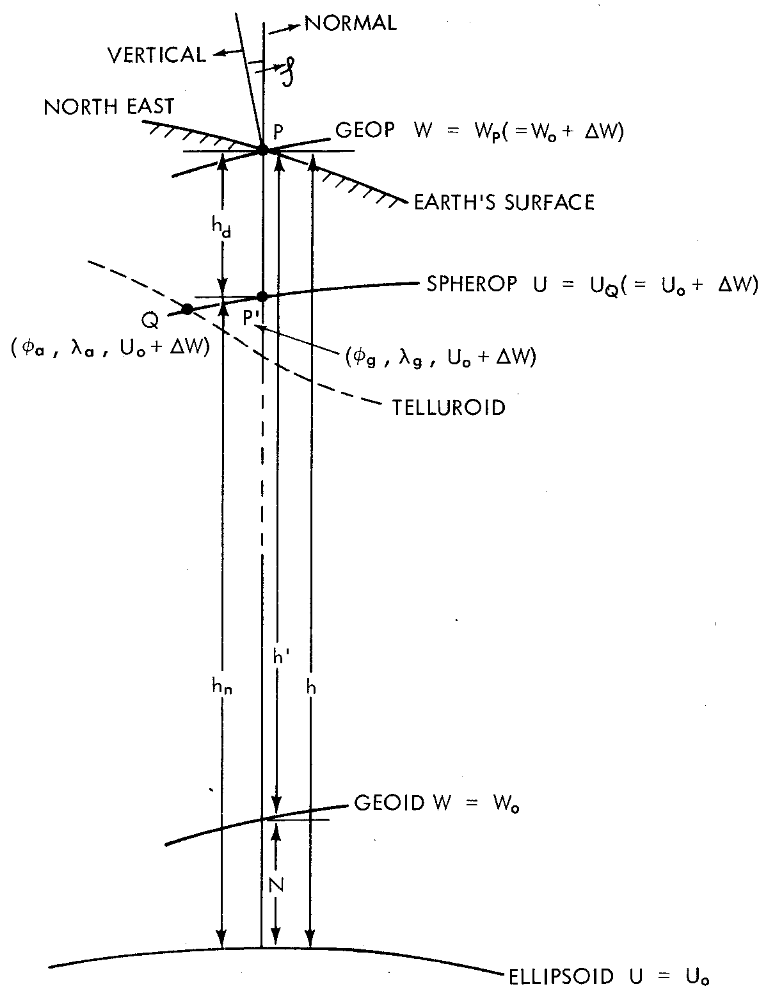

Figure 1. Systems of Elevations

boundary value problem, being used to solve integrals which are of the type

$$V_{dp} = \frac{1}{2\pi}\iint \left(V_d\, \vec{\nabla}\cdot\vec{N}\, \frac{1}{r} - \frac{1}{r}\, \vec{\nabla}\cdot\vec{N}\, V_d \right) dS \qquad (7).$$

The gravity anomaly Δg is introduced into equation 7 by the use of the inter-relation between equation 5 and the expression (IBID,p.18)

$$\Delta g = -\frac{\partial V_d}{\partial h} + h_d \frac{\partial \gamma}{\partial h} + o\{f\ \Delta g\} \qquad (8).$$

Problems arise when the geopotential differences ΔW are referred to the equipotential surface ($W = W_o + \delta W$) of the Earth's gravitational field which does not coincide with the geoid ($W = W_o$). These are, of course, the various local estimates of the geoid , which is defined as the equipotential surface which best fits Mean Sea Level as sampled at a limited selection of tide gauge stations distributed globally. Individual tide gauges situated on the equipotential surface ($W = W_o + \delta W$) will not, in the general case, coincide with the geoid ($W = W_o$). The disturbing potential V_{dp} at a point

P whose geopotential difference has been established in such a manner, can be more accurately described by the equation

$$V_{dp} = W_p - U_p = (W_o + \delta W - U_o) + \gamma h_d + o\{f^2 V_d\} \qquad (9).$$

The use of equation 9 along with the simplified relation

$$\frac{\partial\gamma}{\partial h} = -\frac{2\gamma}{R} + o\{f\ \frac{\partial\gamma}{\partial h}\} \qquad (10)$$

and equation 8, gives the gravity anomaly in such a case as

$$\Delta g = -\frac{\partial V_d}{\partial h} - \frac{2}{R}\{V_d - (W_o + \delta W - U_o)\} + o\{f\ \Delta g\} \qquad (11).$$

W_o is at best known to o{±10 kgal m} at the present time, while the most pessimistic estimate of δW on the basis of the evidence currently available can be put at o{±2 kgal m}, with equivalent uncertainties of o{±3 mgal} and o{±½ mgal} respectively in Δg. The first effect is of zero degree while the second is likely to have wavelengths as great as 10^3 to 10^4 km. The effect of tide gauges not lying on the same equipotential surface of the Earth's gravitational field, which can be interpreted as being due to the existence of stationary sea surface topography, influences the solution of the geodetic boundary value problem. This effect can be illustrated with the least complexity and adequate accuracy, by formulating its consequences in the Stokesian case where the Earth is represented without any matter exterior to the bounding equipotential which constitutes the Earth's surface.

Equation 7 is well known to transform according to the relation

$$V_{dp} = \frac{1}{2\pi}\iint \frac{1}{r}\left\{\Delta g + \frac{3V_d}{2R} - \frac{2}{R}(W_o + \delta W - U_o)\right\} dS \qquad (12),$$

on using equation 11 and the relation

$$\vec{\nabla}\cdot\vec{N}\ \frac{1}{r} = -\frac{1}{2Rr}$$

which is valid in the case of a spherical equipotential surface. It is standard practice to adopt a spherical harmonic representation

$$V_d = \sum_{n=0}^{\infty} \frac{A_n}{R^{n+1}}, \quad n \neq 1 \qquad (13)$$

for the disturbing potential as a means of combining the effects of the gravity anomaly and the disturbing potential, when the use of the conventional procedure using equation 11 gives

$$\Delta g = -\frac{\partial V_d}{\partial h} - \frac{2}{R}\{V_d - (W_o + \delta W - U_o)\} + o\{f\ \Delta g\} = \sum_{n=0}^{\infty}(n-1)\frac{A_n}{R^{n+2}} + \frac{2}{R}(W_o + \delta W - U_o) +$$

$$o\{f\ \Delta g\},\ n \neq 1 \qquad (14).$$

Equation 12 can therefore be written as

$$V_{dp} = \frac{1}{4\pi}\iint \frac{1}{r}\sum_{n=0}^{\infty}(2n+1)\frac{A_n}{R^{n+2}}\,dS, \quad n \neq 1 \qquad (15).$$

Stokesian practice calls for the replacement

$$A_n = \frac{R^{n+1}}{n-1} G_n$$

which is valid on the surface of a sphere of radius R. G_n is the n-th degree harmonic in the representation of the amended gravity anomaly $\Delta g'$ which is given by the equations

$$\Delta g' = \sum_{n=0}^{\infty} G_n = \sum_{n=0}^{\infty}(n-1)\frac{A_n}{R^{n+2}}, \; n \neq 1 = \Delta g - 2\frac{W_o - U_o}{R} - 2\frac{\delta W}{R} \qquad (16),$$

the second equality following from equation 11.

The continuation of the Stokesian manipulation, including the term of zero degree, gives

$$\begin{aligned} V_{dp} &= \frac{1}{4\pi}\iint \frac{1}{r}\sum_{n=0}^{\infty}\frac{2n+1}{n-1} G_n\,dS, \quad n \neq 1 \\ &= -\frac{R\,G_o}{4\pi}\int_0^{\pi}\int_0^{2\pi} \cos\tfrac{1}{2}\psi\,d\psi\,d\alpha + \frac{1}{4\pi R}\iint f(\psi)\,\Delta g'\,dS \\ &= 2(W_o - U_o) - R\,M\{\Delta g\} + 2M\{\delta W\} + \frac{1}{4\pi R}\iint f(\psi)\left(\Delta g - 2\frac{\delta W}{R}\right)dS \qquad (17), \end{aligned}$$

as the term $(W_o - U_o)$ has no effect through Stokes' integral being a term of zero degree. On using equation 9, and if M{ } refers to the global mean value, equation 17 becomes

$$h_d = \frac{W_o - U_o}{\gamma} + \frac{2\,M\{\delta W\} - \delta W_p}{\gamma} - \frac{R}{\gamma}M\{\Delta g\} + \frac{R}{4\pi\gamma}\iint f(\psi)\left(\Delta g - 2\frac{\delta W}{R}\right)d\sigma \qquad (18).$$

$M\{\delta W\} = 0$ if the geoid is defined as the mean equipotential surface through the global set of elevation datums. The net effect $e_{h_{dp}}$ of the existence of stationary sea surface topography on the solution h_{dp} of the boundary value problem at P is therefore

$$e_{h_{dp}} = -\frac{\delta W_p}{\gamma} - \frac{1}{2\pi\gamma}\iint f(\psi)\,\delta W\,d\sigma \qquad (19).$$

Notes

(i) Equation 19 defines effects of consequence in practical evaluations in physical geodesy as δW holds the same sign over large extents of surface area.

(ii) It was implied in an earlier study (MATHER 1973a,p.87) that an iterative procedure would be necessary if quasi-stationary sea surface topography were widespread phenomena with significant amplitudes and wavelengths. The use of equation 19 would however restrict the iterative procedure to merely the correction terms which would be more economic to compute. The expression at 19 can be considered to be of adequate accuracy for all foreseeable practical solutions of the geodetic boundary value problem.

(iii) The earlier study referred to above estimated that the existence of values of o{±2 kgal m} for δW with wavelengths of 4000 km would contribute errors of o{±30 cm} to determinations of geoid heights in ocean areas if not allowed for by procedures similar to those outlined above.

(iv) As a corollary, it would appear at first glance that no determination of stationary sea surface topography by a combination of satellite altimetry, physical geodesy and tidal analysis, as illustrated in figure 2, can be achieved without resorting to an iterative procedure. A first iteration excluding the effect embodied in equation 19, should define the stationary sea surface topography to o{±30 cm} as discussed at (iii). Consequently, δW in equation 19 can still be as large as o{±0.3 kgal m}, giving effects of o{±5 cm} in h_d. The apparently less plausible long wavelengths in the stationary sea surface topography cause greater systematic errors in the solution of the geodetic boundary value problem.

(v) An alternative technique with some appeal, especially when repeating the solution to get rid of small systematic effects, is the use of randomization techniques on the value of δW when using equation 19. The use of such techniques on the gravity data is not advisable as in the first instance, it precludes the use of equation 19.

(vi) It is important to note particulars of elevation datums when compiling gravity data banks so that δW can be correctly computed.

4. Geodetic Reference Systems for Determination of Vertical Crustal Motion

The techniques in use at present for the determination of vertical crustal motion can at best, be classified as regional in concept rather than global. The basic principles utilized, more from necessity rather than by design, can be summarized as follows. Points of reference (benchmarks) are selected to represent the terrain in the region considered. These benchmarks are linked during a selected epoch ($\tau = T$) by first order geodetic levelling procedures to some datum of reference, usually a convenient tide gauge which is assessed as lying "outside" the area subject to vertical crustal motion. The levelling is repeated at a subsequent epoch ($\tau = T + dT$) and the differences in geopotential between the levellings at each benchmark, as converted to their linear equivalents, give an estimate of vertical crustal motion.

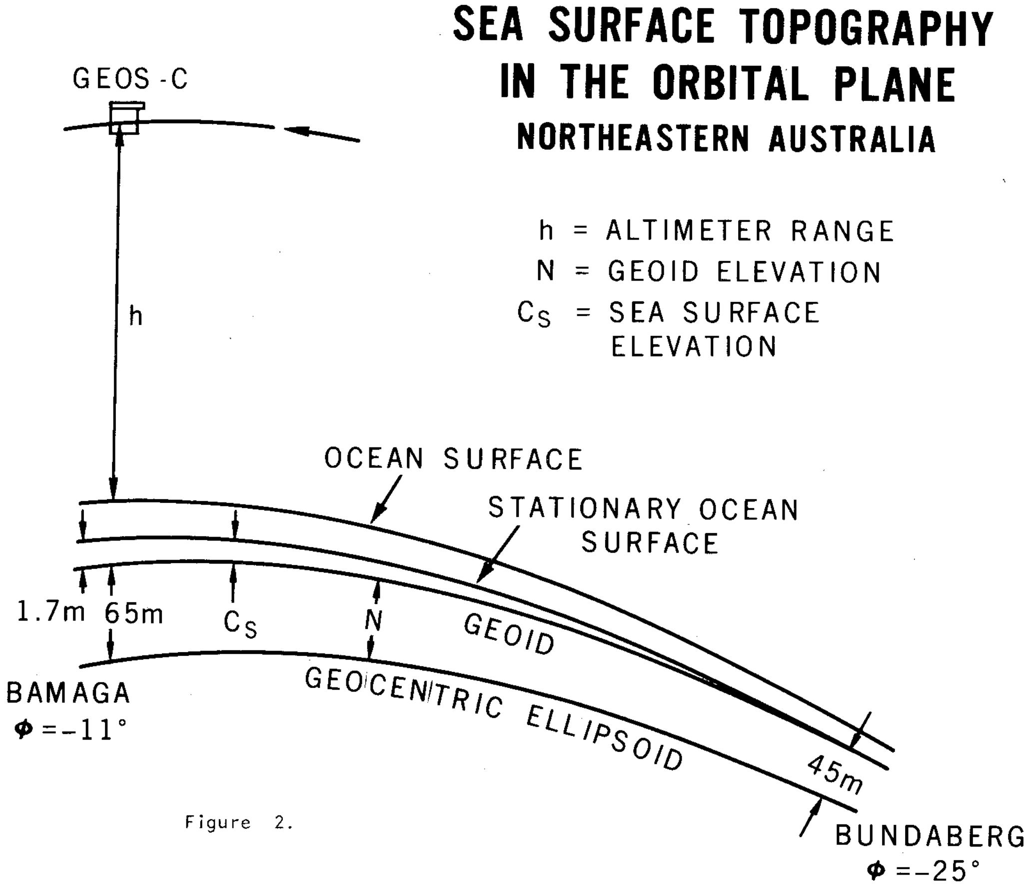

Figure 2.

The technique described above makes the following assumptions.

(i) The location of the datum for elevations is invariant in Earth space between epochs

(ii) The change in geopotential obtained between the epochs at any benchmark is a measure of the Earth space displacement along the local vertical.

The validity of the first assumption or any departures from its basic premise, is partly dependent on the criteria adopted for the definition of the observation space. It is possible to define a three dimensional Cartesian system of reference X_i for position in Earth space such that the differences between the co-ordinates $X_i(\tau = T + dT)$ and $X_i(\tau = T)$ give the displacement vector of the point considered in Earth space (MATHER 1973b). Elevation datums which were incorporated in such a world wide network of geodetic control would provide space characteristics of global relevance for determinations of vertical crustal motion using levelling techniques *only if* there were no change in the shape and spacing between adjacent geops (equipotential surfaces of the Earth's gravitational field) in Earth space. This follows from the nature of the levelling process where the level is always set tangential to the instantaneous surface geop prior to observing incremental differences in orthometric elevations.

The latter contention is therefore based on the same foundation as the assumption at (ii) above. It is however inconsistent with the concept of crustal motion at *some* order of magnitude, as the latter implies changes in the Earth space position of a proportion of the mass elements constituting the Earth. It is therefore more desirable in principle, that studies of vertical crustal motion be based on observations referred to a system of geodetic reference with an unambiguous location in Earth space, *provided no loss of resolution occurs in doing so.*

Assuming that it is considered desirable to evaluate vertical crustal motion as opposed to the crustal motion vector, i.e., it is necessary to determine incremental changes in position along the local vertical, the surface of reference adopted should be such that its normal approximates closely to the direction of the local vertical. It is common knowledge that the normals of an ellipsoid of revolution which has the same volume as the geoid, and is concentric with the geocentre, do not depart from surface verticals by amounts much in excess of $o\{\frac{1}{3}$ mrad$\}$. Three dimensional Cartesian co-ordinates X_i obtained in the manner referred to earlier for all benchmarks in the region of vertical crustal motion determination, can be converted to positional parameters (ϕ,λ,h) with respect to the ellipsoid, if desired, using procedures similar to that described in Appendix 2. The changes

$$dh = h(\tau = T + dt) - h(\tau = T)$$

obtained at any benchmark using such a system of reference, would be a measure of vertical crustal motion which is free from the errors of assumption given at (i) and (ii) above.

The adoption of this procedure would be of practical relevance only if the resulting estimate of vertical crustal motion had the same resolution as the results obtained from first order geodetic levelling. If this were possible, the three dimensional technique would, in addition, completely define the crustal motion vector in Earth space. The precision requirements for the geocentric co-ordinates in such a solution would be $o\{\pm 10$ cm$\}$ in each co-ordinate. While no such achievements have been realized to date, there is promise that recent technological developments could well result in this happening in the foreseeable future by the use of global networks of either laser ranging systems to satellites and the moon, or VLBI.

The advantages of doing so are

(a) the total definition of the crustal motion vector, as opposed to the vertical component only;

(b) the global relevance of the results; and

(c) an independence from temporal variations in the shape and Earth space position of the equipotential surfaces of the Earth's gravitational field.

The main obstacles to implementation are logistic. It is all important to have a truly global network of ground stations to comprise the fundamental geodetic net. It is equally important to fabricate transportable systems to complement the fixed stations. Observations to extra-terrestrial objects which are banded in declination may cause a weakness in determinations of the X_3 co-ordinate, as pointed out by Kaula for determinations from laser ranging to the moon (KAULA 1973).

In the final analysis, the goal of four dimensional position determinations for the crustal motion vector must be the resolution of vertical crustal motion with the same precision as that presently achieved by first order geodetic levelling.

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6. Appendix 1

Reported Stationary Topography of Mean Sea Level

REGION	OCEAN/SEA	APPROXIMATE POSITION				ESTIMATED STATIONARY COASTAL TOPOGRAPHY(+VE N,E)						
		Latitude		Longitude		From Geodetic Levelling				From Steric Levelling		
		deg N		deg E		Slope		Epoch	Source	Slope (m)	Epoch	Source
		From	To	From	To	(m)	(rad)					
AUSTRALIA												
North East	Pacific	-30	-11	153	142	+1.75	+0.74	1966-68	7	+0.2 ±0.1	1960-64	6
South East	Southern;Tasman	-35	-30	136	153	+0.30	+0.17	1966-68	7	+0.35±0.1	1960-64	6
South	Southern	-35	-35	118	136	-0.82	-0.52	1966-68	7	0.0 ±0.1	1960-64	6
South West	Indian	-35	-29	118	115	-0.31	-0.45	1966-68	7	+0.0 ±0.1	1960-64	6
North West	Indian;Timor	-29	-17	115	137	+0.38	+0.17	1966-68	7	+0.5 ±0.1	1960-64	6
North	Carpentaria	-17	-11	137	142	+1.16	+1.38	1966-68	7	+0.05±0.1	1960-64	6
BRAZIL	Atlantic	-24	-4	-42	-38	+0.41	+0.17	1949-57	2	+0.4 ±0.1	*	9
EUROPE (WESTERN)												
Malaga-Genoa	Mediterranean	37	44	-4	9	-0.04	-0.06	1950.0	5			
Cadiz-Dieppe	Atlantic	37	50	-6	2	0.00	0.00	1950.0	5			
Dieppe-Cuxhaven	North	50	53	2	9	+0.21	+0.34	1950.0	5			
DENMARK	North;Baltic	55½	55½	9½	12½	0.02	+0.06	1950.0	5	-0.09±0.1	1950.0	8
FINLAND	Baltic	60½	65½	27	24½	0.03	+0.06	1950.0	5	+0.03±0.1	1950.0	8
GREAT BRITAIN	Atlantic;North	50	57	-5½	-2	+0.29	+0.34	1950.0	4	+0.13±0.1	1950.0	8
JAPAN East	Pacific	32	35	132	140	-0.18	-0.23	1950.0	3			
West	Japan	35	40	132	140	+0.03	0.00	1950.0	3			
NORWAY South	North;Atlantic	59	63½	10	9	+0.01	+0.00	1916-53	10	+0.04±0.1	1950.0	8
North West	Atlantic	63½	70	9	19	-0.03	-0.06	1950.0	5	+0.03±0.1	1950.0	8
SWEDEN	Baltic	58½	65	11	21	+0.31	+0.34	1950.0	5	+0.09±0.1	1950.0	8
UNITED STATES												
East Coast (N)	Atlantic	33	45	-81	-67	+0.00	+0.00	1966.9	1	+0.0 ±0.1	*	9
East Coast (S)	Atlantic	24½	33	-80	-81	+0.33	+0.34	1966.9	1	-0.2 ±0.1	*	9
Gulf Coast	Gulf of Mexico	24½	29	-87	-95	+0.49	+0.52	1966.9	1			
West Coast (S)	Pacific	33	35	-117	-121	0.32	+0.17	1969.3	1	-0.1 ±0.1	*	9
West Coast (N)	Pacific	35	48½	-121	-125	0.63	+0.40	1969.3	1	-0.2 ±0.1	*	9

KEY TO SOURCE :- 1 = BALAZS 1973; 2 = DE ALENCAR 1968,p.11; 3 HAYASHI 1969,p.18; 4 = KELSEY 1970,p.366; 5 = LEVALLOIS & MAILLARD 1970, p.330; 6 = MITCHELL 1973,p.105; 7 = ROELSE ET AL 1971; 8 ROSSITER 1967,p.292; 9 = STURGES 1973,figure 1; 10 = TROVAAG & JELSTRUP 1956, p.167

* No epoch specified. See uncertainty estimates in previous column.

7. Appendix 2

RECOVERY OF VERTICAL COMPONENT

ADOPT A REFERENCE ELLIPSOID, HENCE (a,f)

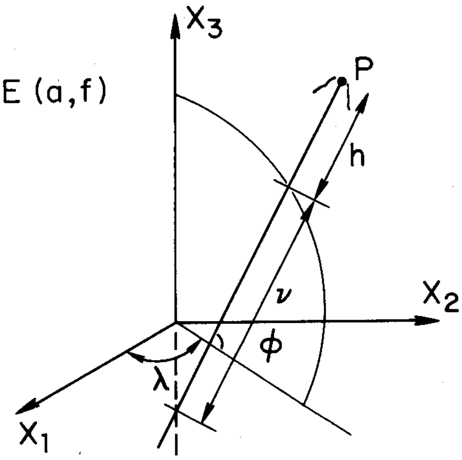
$$\nu = \frac{a}{\left[1-(2f-f^2)\sin^2\phi\right]^{1/2}}$$

$$X_1 = (\nu+h)\cos\phi \quad \cos\lambda$$

$$X_2 = (\nu+h)\cos\phi \quad \sin\lambda$$

$$X_3 = \left[\nu(1-f)^2 + h\right] \quad \sin\phi$$

$$\lambda = \tan^{-1}(X_2/X_1)$$



ITERATE BETWEEN

$$\phi = \tan^{-1}\left[\frac{X_3(\nu+h)\sin\lambda}{X_2\left[\nu(1-f)^2+h\right]}\right] = \tan^{-1}\left[\frac{X_3(\nu+h)\cos\lambda}{X_1\left[\nu(1-f)^2+h\right]}\right]$$

$$h = \frac{X_1}{\cos\phi\cos\lambda} - \nu = \frac{X_2}{\cos\phi\sin\lambda} - \nu = \frac{X_3}{\sin\phi} - \nu(1-f)^2$$

8. Discussion

WALCOTT: Are there differences between geodetic re-levelling and tidal changes at the same location? I ask this because the European map of recent crustal movements shows some peculiarities when you get near the coast. Contours tend to smooth out and not show the closures forming domes and basins which are mapped inland.

HOLLWEY: As far as the British Isles are concerned, the answer is "Yes".

MELCHIOR: On page 5 of your Compendium (TITLE: *Quasi-Stationary Sea Surface Topography and Variations of Mean Sea Level with Time - An Interim Compendium (1973)* - Final Version to appear shortly in *UNISURV Rep.* G-Series, University of New South Wales), you state that only a limited number of organizations were able to provide information. But you do not mention the Permanent Service for Mean Sea Level.

MATHER: I wrote to Dr. Lennon and he referred me to the *Report of the Symposium on Coastal Geodesy*

(SIGL, R. ed., Munich 1970) as a "state of the art" document and considerable material from that source has been incorporated.

MELCHIOR: It is a matter of great concern for IUGG how the Permanent Services are really working, and what kind of data they provide to geodesists and geophysicists. We are really in a very difficult situation in the Union because the ICSU is channelling money to the Permanent Services. At the last meeting of the Union Executive Committee, we discussed the situation and in a few weeks, I shall send documents to all Associations and to many interested geodesists concerning data provided by the Permanent Services. In this case, the Permanent Service on Mean Sea Level should provide a lot of data, not only to oceanographers but also to geodesists. It should be a first task for IAG to say what they expect to receive from this service and if they are satisfied; and if something has to be improved. Such a meeting as this could say something about it.

If the work of this service is satisfactory (I hope so), or if you need more help from such a service - for I expect you will in preparing such a compendium - I should expect that the Permanent Service of Mean Sea Level should help.

MATHER: All I can say is that when I wrote to national government agencies in various parts of the world, those who replied wanted to have a copy of the compendium. Presumably the data is not readily available.

RAPP: On the equation that you showed for the effect of MSL, you had the integration of Stokes' function times δW. Implicitly you said that δW changed from place to place, e.g., you had (one value for a) datum like Australia and another for NAD, etc. A more difficult problem occurs in the case of the 70% of the Earth's surface which is oceanic. How accurately do people who measure gravity at sea measure their heights? This is going to change for every one of your points. For example, for Australia, you have one correction for *all* points, but at sea, unless you can determine very uniquely the height of the gravity meter with respect to sea level, you have a problem.

BARLOW: I think the gravity meters themselves have limited resolution and no one worries at all about this effect.

MATHER: We are really concerned about effects which retain the same sign over large areas. If it changes sign often, then the effect is minimized.

MUELLER: I did not catch one of your comments about disagreement in the meridian and agreement in the prime vertical. What did you mean by that?

MATHER: On levelling across a continent generally along a parallel, and on comparing the steric levelling with the geodetic levelling, good agreement is obtained from the data available at present.

RAPP: I might remark that STURGES' answer to that is the way the sun shines on the levelling staffs.

MATHER: I have read that comment.

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E N G L A N D

Proc. Symposium on Earth's Gravitational Field & Secular Variations in Position (1973), 600-621.

SEA LEVEL VARIATIONS IN TIME AND SPACE ALONG THE EAST COAST OF ENGLAND

1. Introduction

In many respects, the problems arising from variations in the level of the sea from place to place become more complex when we introduce a fourth dimension of time into the situation. It follows that with the introduction of this extra dimension, there will be a repetition of connections by geodetic levelling to tidal observatories. Furthermore, with an improvement in the quality of our measuring systems both at the Observatory and in the geodetic levelling, the opportunity is lost to sweep into a category of observational error any results which do not fit a very simple model of the natural phenomena involved. This leaves open the question of a more complex model to fit the results or the possibility of a source of error in the observations not hitherto considered.

The United Kingdom is fortunate in possessing a Second and Third Geodetic Levelling Network of a standard sufficiently reliable to permit comparison. As well, the main island is sufficiently small to permit detailed study of results from both geodetic levelling and tidal observatories. At the same time, it is sufficiently large for this result to be meaningful. The oceanographic and meteorological conditions provide a variation in data and geological information is both detailed and reliable.

Considering first the situation exposed by the completion of the Third Geodetic Levelling, this is illustrated by figure 1; a more detailed study of these results is given in CARMODY & BURNETT (1960).

The conclusions from the levelling alone points to rise of Dunbar relative to Newlyn of 0.58ft, but from the tidal observations alone, assuming Mean Sea Level has risen at Dunbar and Newlyn at the same rate, there seems to be evidence of a rise of Dunbar relative to Newlyn of 0.17ft. It will also be seen from figure 1 that Felixstowe and Newlyn seem to move in sympathy and that the critical area of uncertainty is the coastline from the Thames estuary to the Firth of Forth. Along this stretch of coast, there are now reliable tidal observatories at Southend, Lowestoft, Immingham and North Shields. Note: the Felixstowe Observatory is no longer in use. It is this stretch of coast which I am examining here in some detail.

2. The Determination of Mean Sea Level

No uniform method of reduction of sea level observations exists; ROSSITER (1958) lists the following ways in which the basic daily mean is normally computed -

A. Direct average of 1, 3, 6 or 8 heights per day at fixed hours.

B. Direct average of 24 or 25 hourly heights.

C. Numerical filters applied to hourly heights.

D. Direct average of high and low water heights.

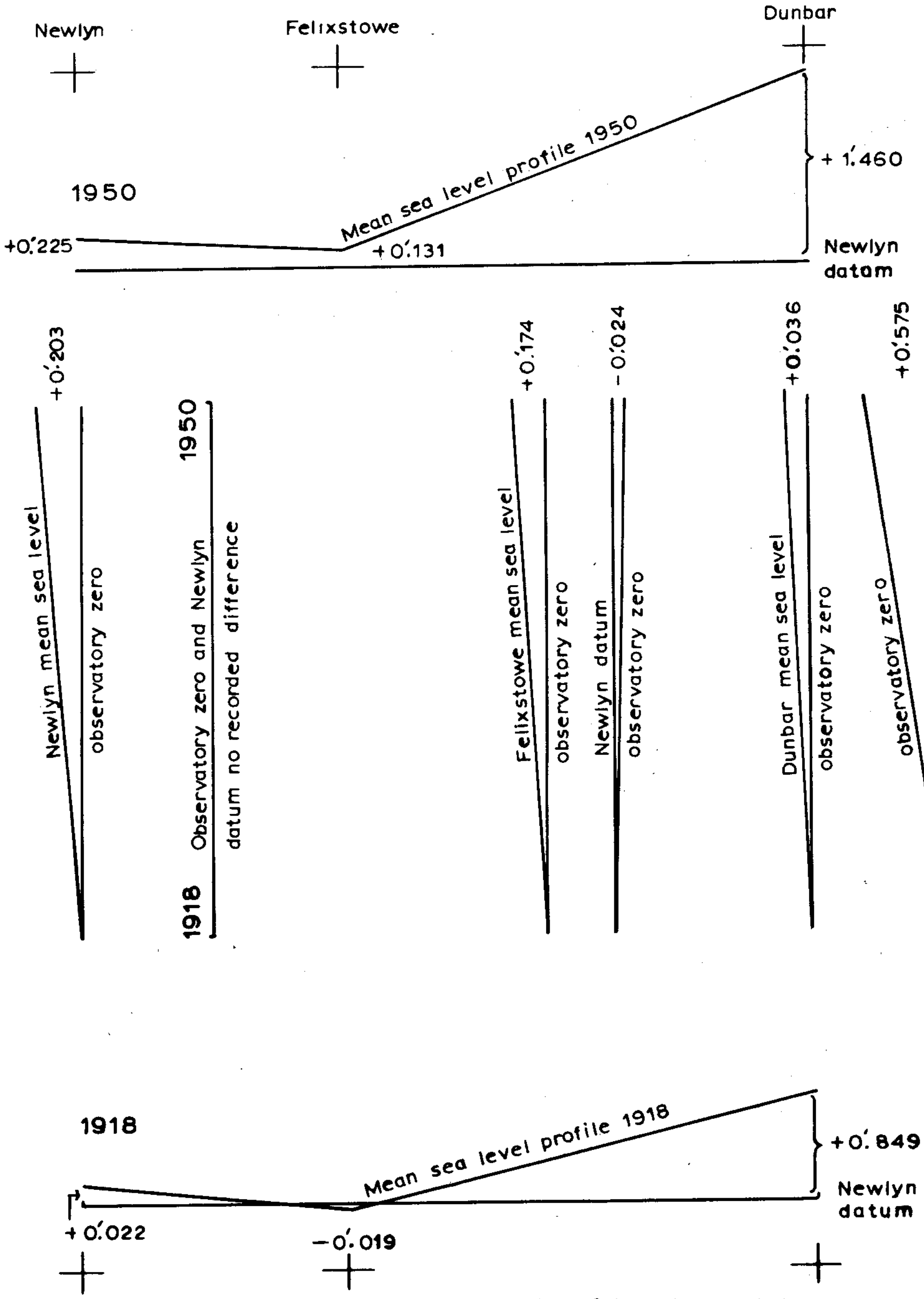

Figure 1. Diagram to Represent Results of Second and Third Geodetic Levelling of the United Kingdom

E. Integration of daily records of sea level changes by planimeter.

to which I would add -

F. The average of hourly heights taken during the hours of daylight. For example, the mean of 12 or 13 consecutive observations in every 24 hours.

It is obvious that A) can only be meaningfully used in regions where tidal ranges are small. Similarly, D) and F) must be treated with considerable caution, although they are commonly the resort of survey parties establishing a temporary datum. B), C) and E) give results which are commonly described as Mean Sea Level.

If, however, we adopt a definition which seems the most suitable for the problem of geodesy, namely:

"the level which results from processing a set of sea observations in such a way that the contributions made by the tidal oscillations are removed" (LENNON, 1965)

the following points emerge:

Let Mean 'Sea' Level so defined be Zo and let the contributions made by the tidal oscillations at time t be Y_t, then the level of the sea at time t is given by h_t, where

$$h_t = Zo + Y_t. \tag{1}$$

If a harmonic tidal constituent be represented by

$$\zeta_H = R \cos (\sigma H + pD - \varepsilon) \qquad \text{(after ROSSITER, 1958)}$$

with R the amplitude and ε the phase lag, both constant at any one place σ the speed in degrees per mean solar hour, p the speed in degrees per mean solar day, H the time in hours, and D the time in days, then

$$Y_t = \sum_{i=1}^{n} . R_i . \cos (\sigma_i H + p_i D - \varepsilon_i) \tag{2}$$

where R_i, σ_i, p_i and ε_i represent values unique for each one of n tidal constituents significant at the locality.

The problem is now reduced to one of combining values of ζ_H in such a way that in their summation

$$\Sigma Y_t \simeq 0 \quad \text{or} \int_0^{24} Y_t \simeq 0.$$

This is most nearly achieved by a numerical filter of which the X_o filter of Doodson (DOODSON & WARBURG 1941) is the most suitable. It can be shown that:

$$\sum_{i=0}^{m-1} R \cos (\sigma q t_j + pD - \varepsilon) = (\sin \frac{qm\sigma}{2} / \sin \frac{qm}{2}) R \cos (pD - \varepsilon + \frac{m-1}{2} q\sigma) \tag{3}$$

where terms are as above and in addition the summation sequence involves m values at intervals of q hours. Further, H in equation (1) is related to q and t_i by $t_i q = H$

From equation (3) it is possible to devise a numerical filter which enables a quantity X_o to be evaluated which approximates to the LENNON (1965) definition of Mean Sea Level. For each day,

$$X_o = \frac{1}{30}\,(h_{17} + h^1_{19} + h^1_{22} + h_0 + h_1 + 2h_3 + h_5 + h_6 + 2h_8 + h_9$$

$$+ h_{10} + 2h_{11} + 2h_{13} + h_{14} + h_{15} + 2h_{16} + h_{18} + h_{19}$$

$$+ 2h_{21} + h_{23} + h_{24} + h^{11}_2 + h^{11}_5 + h^{11}_7)$$

where h^1_n is the height at hour n on the previous day, h_n is the height at hour n on the current day, and $h^{11}n$ is the height at hour n on the following day.

The efficiency of the X_o filter and in particular the tidal coefficients it removes is illustrated by figures 2 and 3.

A comparison of the daily means computed by taking the mean of the 24 hour readings and by using the X_o filter was carried out for the years 1967, 1968 and 1969 for the four east coast observatories mentioned above. The result of this comparison is given by table 1 below.

TABLE 1

	Southend	Lowestoft	Immingham	N.Shields
Max. -ve diff. 1967	-0.73	-0.50	-0.58	-0.66
Max. +ve diff. 1967	+0.49	+0.43	+0.54	+0.40
Max. -ve diff. 1968	-0.63	-0.61	-0.89	-0.51
Max. +ve diff. 1968	+0.66	+0.55	+0.57	+0.47
Max. -ve diff. 1969	-0.66	-0.72	-0.74	-0.68
Max. +ve diff. 1969	+0.50	+0.70	+0.68	+0.38
Neap Tide Range	11.0	3.4	10.4	6.0
Spring Tide Range	17.1	6.4	20.9	14.1

Units Feet

Using the various daily values, it is found that in the two sets of 144 monthly means differences for the same month of 0.048 ft. occur in only one case; for the remainder, differences between 0.015 and 0.04 ft. occur in only seven cases, and in all other cases differences are less than 0.015 ft. It seems that for the conditions pertaining along the east coast it is unnecessary to use the filtered values. This conclusion agrees with theory as expressed by table 2, obtained by use of tables 1 and 3 of ROSSITER (1958).

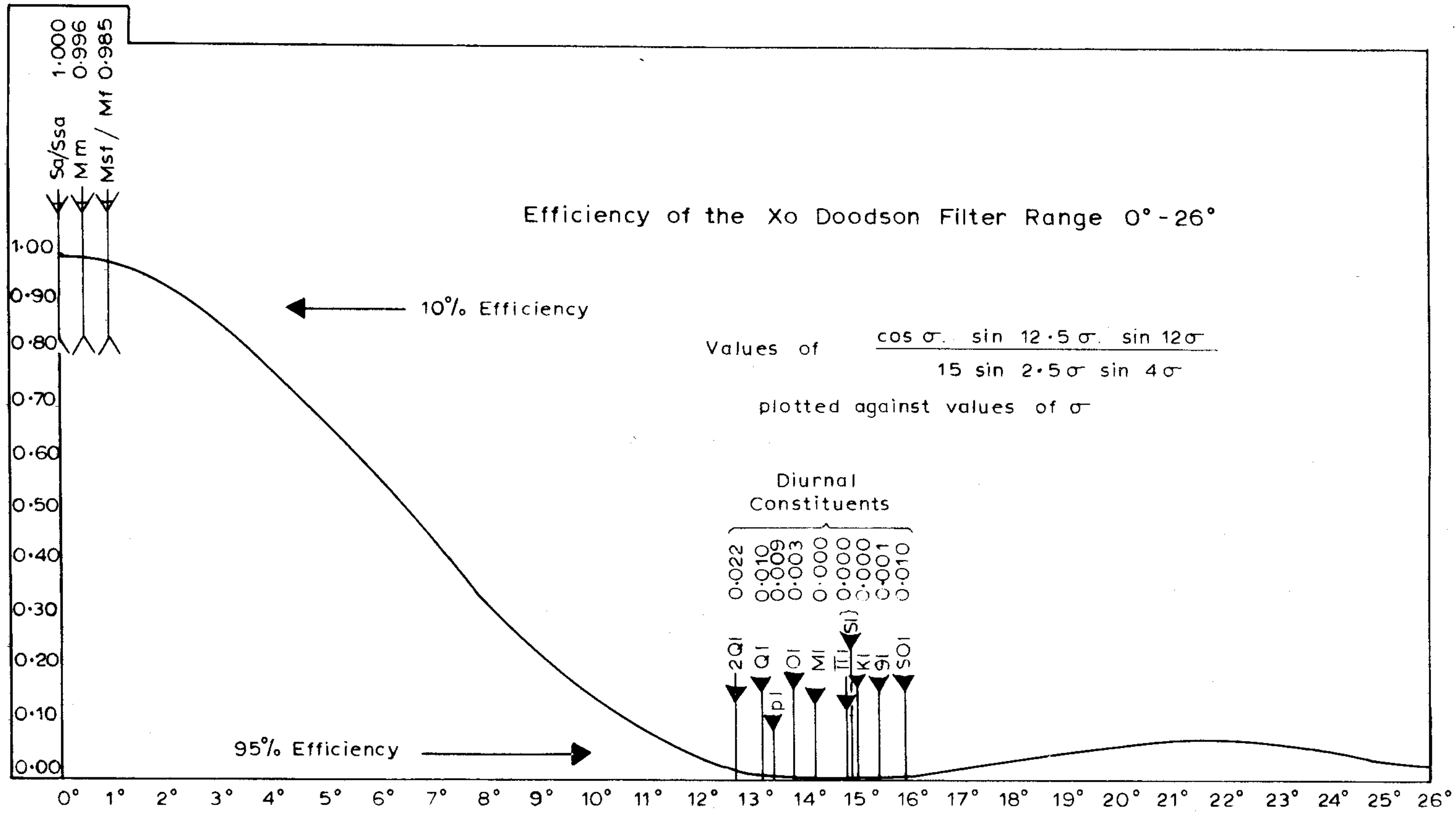
Efficiency of the Xo Doodson Filter Range 0° - 26°
Values of $\frac{\cos\sigma \cdot \sin 12{\cdot}5\sigma \cdot \sin 12\sigma}{15 \sin 2{\cdot}5\sigma \sin 4\sigma}$
plotted against values of σ
10% Efficiency
95% Efficiency
Sa/Ssa 1·000
Mm 0·996
Msf / Mf 0·985
Diurnal Constituents
2Q1 0·022
Q1 0·010
ρ1 0·009
O1 0·003
M1 0·000
π1 0·000
(S1) 0·000
K1 0·000
J1 0·001
SO1 0·010
1·00
0·90
0·80
0·70
0·60
0·50
0·40
0·30
0·20
0·10
0·00
0° 1° 2° 3° 4° 5° 6° 7° 8° 9° 10° 11° 12° 13° 14° 15° 16° 17° 18° 19° 20° 21° 22° 23° 24° 25° 26°

Figure 2

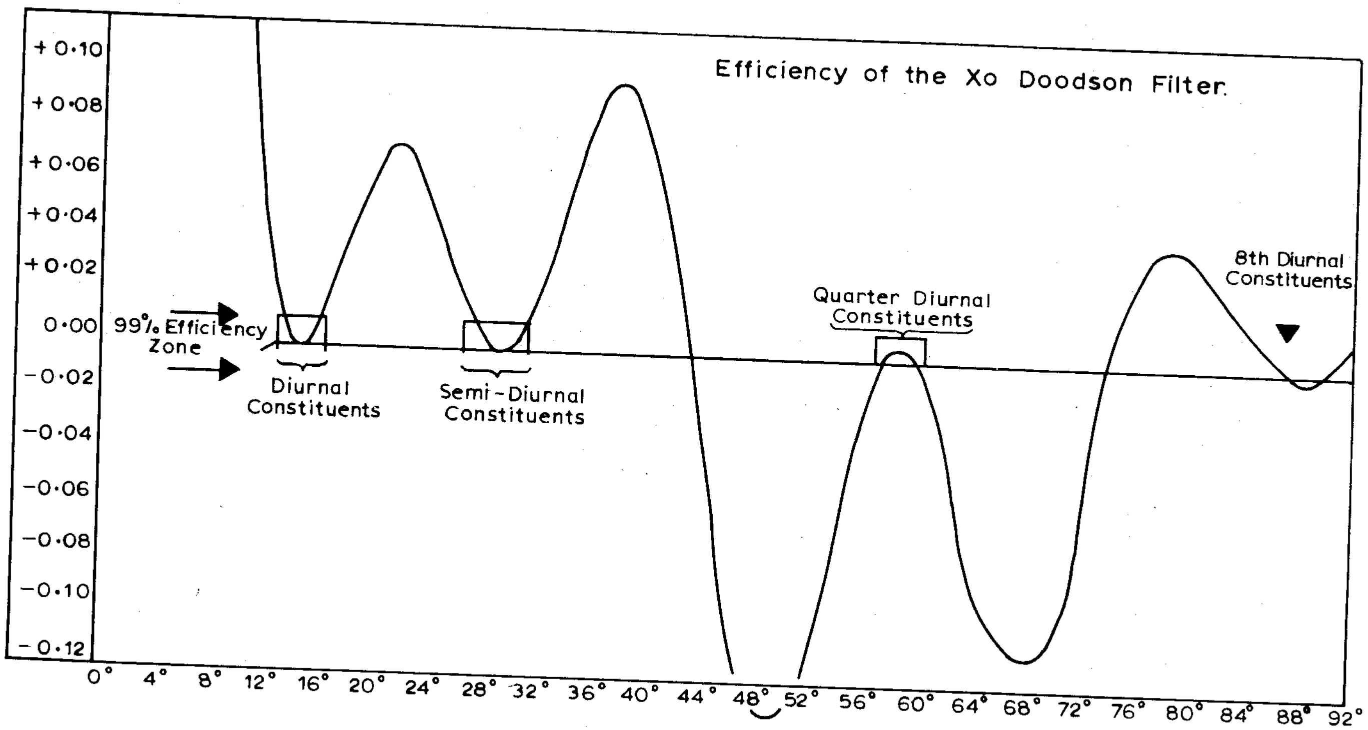
Efficiency of the Xo Doodson Filter.
+0·10
+0·08
+0·06
+0·04
+0·02
0·00
−0·02
−0·04
−0·06
−0·08
−0·10
−0·12
99% Efficiency Zone
Diurnal Constituents
Semi-Diurnal Constituents
Quarter Diurnal Constituents
8th Diurnal Constituents
0° 4° 8° 12° 16° 20° 24° 28° 32° 36° 40° 44° 48° 52° 56° 60° 64° 68° 72° 76° 80° 84° 88° 92°

Figure 3

Table 2

Coefficients of Contributions from Listed Tidal Constituents								
	30 day Means							
	K_1	O_1	M_2	N_2	M_3	M_4	M_6	MSf
To X_o Value	.0001	.0002	-.0000	-.0001	.0001	-.0000	-.0000	.0155
24h Value	-.0027	.0040	-.0005	.0021	-.0006	-.0006	-.0006	.0155
	365 day Means							
To X_o Value	.0000	-.0000	-.0000	-.0000	-.0000	-.0000	-.0000	.0127
24h Value	.0000	-.0007	-.0003	.0000	-.0001	.0002	-.0001	.0127

Note: The maximum contribution to the mean over the period specified is obtained by multiplying the amplitude of the constituent by the coefficient given above.

3. Variations of Monthly Mean Sea Level at the Four Observatories (1967-69)

Monthly Mean Sea Level at each of the four stations varies during the period considered within a range of approximately 1ft. (300mm) and with a standard deviation of 0.2 to 0.3ft (60 to 90mm). Regardless of methods used to process the hourly height these statistics remain unchanged. Against this background it was decided to establish how far the variation pattern might be related between stations and how far monthly values at one station might be predicted from one or more adjacent stations.

The method of correlation and regression analysis used, the statistical processes involved, and the computer program are not relevant to this paper and are fairly standard; here I am only concerned with the results obtained.

Figure 4 illustrates the changes in monthly mean sea level over the period January 1967 to December 1969 at the four stations Southend, Lowestoft, Immingham and North Shields. It is evident from the graphs that there is a close relationship. In 26 cases sea level changes from month to month at all stations with the same sign, in 7 of the 9 remaining cases, 3 out of 4 of the changes are the same. In April/May 1967 the two northerly stations show an increase followed by a decrease in May/June 1967 whilst the southerly stations show successively a decrease followed by an increase.

Examining the statistics in table 3, the Immingham correlations are in all cases the strongest. Not surprisingly Immingham/North Shields (0.925) is the strongest, but this is closely followed by the Immingham/Southend figure (0.910). CARTWRIGHT (1967) draws artention to the way in which daily means at Southend and Immingham are closely related and this result seems to be repeated with monthly means. The weakest correlation is given by North Shields/Southend (0.773) which is still, however, very significant.

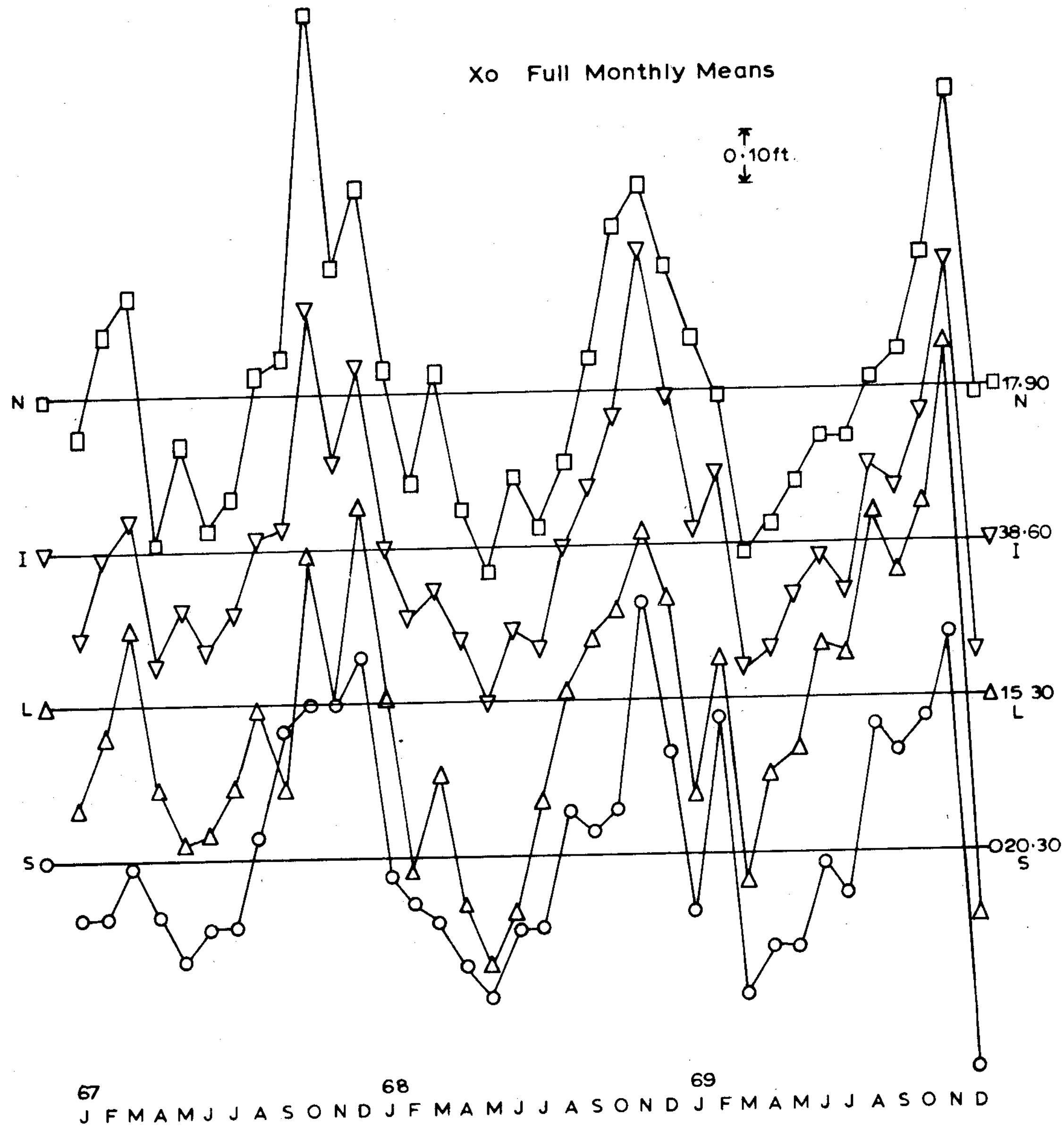

Figure 4

The results of a multiple (or where necessary linear) correlation of all combinations of 3 of the 4 stations are given in table 4. On the basis of the regression equation, it should be possible to predict monthly mean sea level at one station given monthly means at one or more of the other stations. In table 5 are given the results of such a prediction for Southend 1961 using 3 of the regression equations. The ability of this treatment to detect unreliable monthly means, as for instance Immingham May 1961, is more significant than its use in prediction. In addition, isolated gaps might be more successfully filled by this method than by prediction from harmonic constituents.

For example, the predicted values for North Shields December 1969 are all consistently lower than observed. In that month only 16 daily means were obtained and it might have been better to use a predicted monthly mean for purposes of an investigation into changes of Mean Sea Level.

TABLE 3

X_o Full Month Monthly Means Correlation Coefficients			
	Lowestoft	Immingham	N.Shields
Southend	0.851	0.910	0.773
Lowestoft	-	0.882	0.776
Immingham	-	-	0.925

TABLE 4

X_o Full Month Regression Analysis Between Monthly Means

Ref.	Std. Dev. of Variable	Intercept $x10^{-2}$	Regression Coeff. of Independent Variables			Multiple Correl. Coeff.	Std. Err. of Est.$x10^{-2}$	Degs. of F'dom	F Value
Southend			Low.	Imm.	N.Sh.				
020/1	0.223	-20.219	+0.110	+1.185	-0.387	0.930	0.085	3:32	68.813
020/5		- 9.792	+0.175	+0.710		0.916	0.092	2:33	85.915
020/6		+ 8.096	+0.510		+0.247	0.869	0.113	2:33	51.062
020/7		-23.862		+1.338	-0.420	0.928	0.085	2:33	102.963
020/8		+ 9.805	+0.687			0.851	0.119	1:34	89.120
020/9		-14.393		+0.899		0.910	0.094	1:34	164.247
20/10		+ 8.246			+0.673	0.773	0.143	1:34	50.424
Lowestoft			South.	Imm.	N.Sh.				
020/02	0.276	-26.945	+0.257	+1.041	-0.192	0.892	0.130	3:32	41.344
020/11		-21.322	+0.341	+0.765		0.890	0.130	2:33	62.535
020/12		- 6.066	+0.771		+0.317	0.871	0.139	2:33	51.856
020/13		-33.078		+1.392	-0.300	0.888	0.130	2:33	61.676
020/14		- 6.096	+1.052			0.851	0.147	1:34	89.120
020/15		-26.310		+1.077		0.882	0.132	1:34	118.989
020/16		+ 0.295			+0.826	0.776	0.176	1:34	51.416
Immingham			South.	Low.	N.Sh.				
020/03	0.226	+20.812	+0.380	+0.144	+0.441	0.979	0.048	3:32	243.647
020/17		+23.889		+0.338	+0.534	0.961	0.064	2:33	199.674
020/18		+19.941	+0.491		+0.486	0.975	0.052	2:33	317.721
020/19		+21.838	+0.586	+0.319		0.933	0.084	2:33	110.762
020/20		+19.894	+0.922			0.910	0.094	1:34	164.247
020/21		+27.589		+0.722		0.882	0.108	1:34	118.989
020/22		+23.989			+0.817	0.925	0.027	1:34	202.193
North Shields			South.	Low.	Imm.				
020/04	0.256	-31.202	-0.432	-0.092	+1.535	0.941	0.090	3:32	83.002
020/23		+23.292	+0.469	+0.398		0.805	0.156	2:33	30.372
020/24		-26.972		-0.167	+1.229	0.929	0.097	2:33	104.194
020/25		-29.242	-0.464		+1.465	0.940	0.090	2:33	125.831
020/26		- 0.095	+0.887			0.773	0.165	1:34	50.424
020/27		+ 6.923		+0.720		0.776	0.164	1:34	51.416
020/28		-22.565			+0.048	0.925	0.098	1:34	202.193

F. Distribution:

D. of F.	5%	$2\frac{1}{2}$%	1%
3 : 32	2.90	3.56	4.46
2 : 33	3.28	4.14	5.32
1 : 34	4.13	5.50	7.45

TABLE 5

Actual and Predicted Monthly Mean Sea Level: Southend 1961 (units feet)

Month	Actual Mean+	Predicted 020/5	Res.	Predicted 020/8	Res.	Predicted 020/9	Res.
Jan.	20.10	20.00	-.10	20.18	+.08	19.99	-.11
Feb.	20.28	20.13	-.15	20.22	-.06	20.13	-.15
Mar.	20.40	20.21	-.19	20.36	-.04	20.19	-.21
Apr.	20.08	20.08	0	20.18	+.10	20.09	+.01
May	20.45	(20.21)	-.24	20.38	-.07	(20.19)	-.26
June	20.21	20.18	-.03	20.28	-.07	20.19	-.02
July	20.38	20.18	-.20	20.30	-.08	20.18	-.20
Aug.	20.21*	20.20	-.01	20.25	+.04	20.21	0
Sept.	20.35	20.34	-.01	20.24	-.11	20.40	+.05
Oct.	20.38	20.48	+.10	20.34	-.04	20.55	+.17
Nov.	20.68	20.47	-.21	20.48	-.20	20.48	-.20
Dec.	20.57	20.44	-.13	20.54	-.03	20.43	-.14
		σ = .15		σ = .09		σ = .16	

Prediction 020/5 uses Immingham & Lowestoft

Prediction 020/8 uses Lowestoft only

Prediction 020/9 uses Immingham only

NOTES: + Source Monographie 21 (IUGG), Monthly and Annual Mean Heights of Sea Level 1959-61 except *where corrected value given in Monographie 30.

() Immingham Data indicated as being unreliable

4. Results and Differences in Height of Mean Sea Level

The relationship between tidal observatory datums will depend, inter alia, upon the adjustment of the connecting geodetic levelling. Three alternative relationships are given in table 6.

TABLE 6

Corrections to Reduce Monthly Mean Value (X_o Full Month) of Sea Level to A.O.D. Newlyn (ft)

	(1) Reduction based on pract. Adj.effectively 2nd Geodetic Levelling	(2)-(1)	(2) Reduction based on 2nd Scientific Adj. of 3rd Geod. Levelling	(3)-(2)	(3) Reduction based on limited Adj. of 3rd Geod. Levelling
Southend	-20.000	+0.200	-19.800	+0.030	-19.770
Lowestoft	-15.010	+0.094	-14.916	+0.054	-14.862
Immingham	-37.695	+0.063	-37.532	+0.101	-37.431
N.Shields	-16.952	+0.248	-16.704	+0.107	-16.597

For the whole period under review, the alternative height A.O.D. Newlyn of mean sea level is given in table 7 and the "slopes" in table 8.

TABLE 7

Mean Sea Level 1967-68-69, A.O.D. Newlyn Various Network Adjustments (ft.)

	Southend (20.318)	Lowestoft (15.282)	Immingham (38.627)	N.Shields (17.925)
M.S.L. based on Adj. (1)	0.318	0.272	0.932	0.973
M.S.L. based on Adj. (2)	0.518	0.366	1.095	1.221
M.S.L. based on Adj. (3)	0.548	0.420	1.196	1.328

TABLE 8

Slope Mean Sea Level 1967-68-69 Various Network Adjustments (ft.)

	Adj. (1)	Adj. (2)	Adj. (3)
Southend - Lowestoft	- 0.046	- 0.152	- 0.128
Southend - Immingham	+ 0.614	+ 0.577	+ 0.648
Southend - N. Shields	+ 0.655	+ 0.703	+ 0.780
Lowestoft - Immingham	+ 0.660	+ 0.729	+ 0,776
Lowestoft - N.Shields	+ 0.701	+ 0.855	+ 0.908
Immingham - N.Shields	+ 0.041	+ 0.126	+ 0.132

5. Magnitude of Possible Errors in the Results of Differences in Height of Mean Sea Level

There are two possible courses of error in the results given above. Firstly, the levelling is liable to error. Secondly, the observations of the sea level and the reduction to monthly and annual means are liable to error.

Considering the leveliing, we find that an estimate of the precision per mile of the Third Geodetic Levelling (computed using the IAG Oslo 1948 formulae) is 0.00469 ft. $\sqrt{M}$ (1.13mm $\sqrt{K}$). For a free network extending from Southend to North Shields, this gives a figure of 0.08 ft. The results of the adjustment can be illustrated by the differences in the difference in height between the

FBM's at Ipswich and Hexham given by table 9.

TABLE 9

(units ft.)	Ipswich to Hexham
(1) Second Geodetic Levelling	+ 512.086
(2) Second Scientific Adjustment of the Third Geodetic Levelling	+ 512.2097
(3) Adjustment of Limited Network of Third Geodetic Levelling	+ 512.2520

The magnitude of the total corrections applied from Ipswich to Hexham in the second scientific adjustment is -0.0442 being made up of -0.0481 from Hexham to Bulmer, +0.0889 from Bulmer to Caster and -0.0850 from Caster to Ipswich.

The difference between the Second and Third Geodetic Levelling seems to point to a rise of land in the north relative to the south east, but there is an uncertainty of around 0.1 ft.

There is a further uncertainty in the connection to the Southend Tidal Observatory of the order of 0.05 ft.

Considering the observations of sea level, table 10 gives an estimate of the standard errors in gauge reductions. This does not, however, express the full uncertainty in the determination of mean sea level. To these values must be added a more fundamental uncertainty of around 0.1 ft. arising from the unreliability of the stilling well type of tide gauge (see LENNON 1968).

TABLE 10

	Correction to reduce observations reduced to tide gauge zero to heights A.O.D. Newlyn second scientific adjustment (ft)		Estimated Standard errors (feet)	
			Gauge Reduction	Levelling Connection
Southend	Add	0.200	± 0.075	± 0.05
Tilbury	Add	0.198	± 0.060	Not Sig.
Lowestoft	Subtract	14.916	± 0.100	" "
Immingham	Subtract	37.532	± 0.050	" "
N. Shields	Subtract	6.704	± 0.100	" "

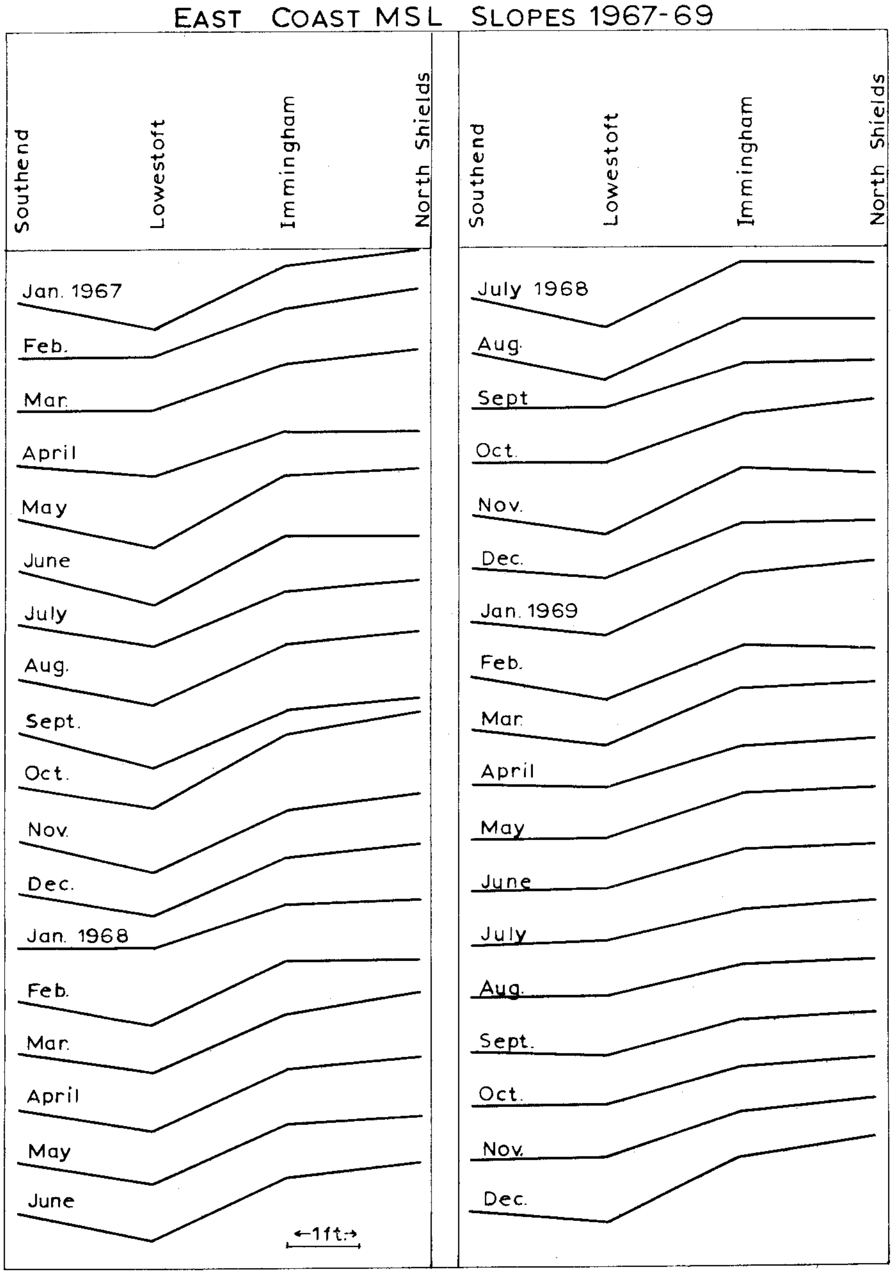
EAST COAST MSL SLOPES 1967-69
Southend
Lowestoft
Immingham
North Shields
Jan. 1967
Feb.
Mar.
April
May
June
July
Aug.
Sept.
Oct.
Nov.
Dec.
Jan. 1968
Feb.
Mar.
April
May
June
←1ft.→
July 1968
Aug.
Sept
Oct.
Nov.
Dec.
Jan. 1969
Feb.
Mar.
April
May
June
July
Aug.
Sept.
Oct.
Nov.
Dec.

Figure 5.

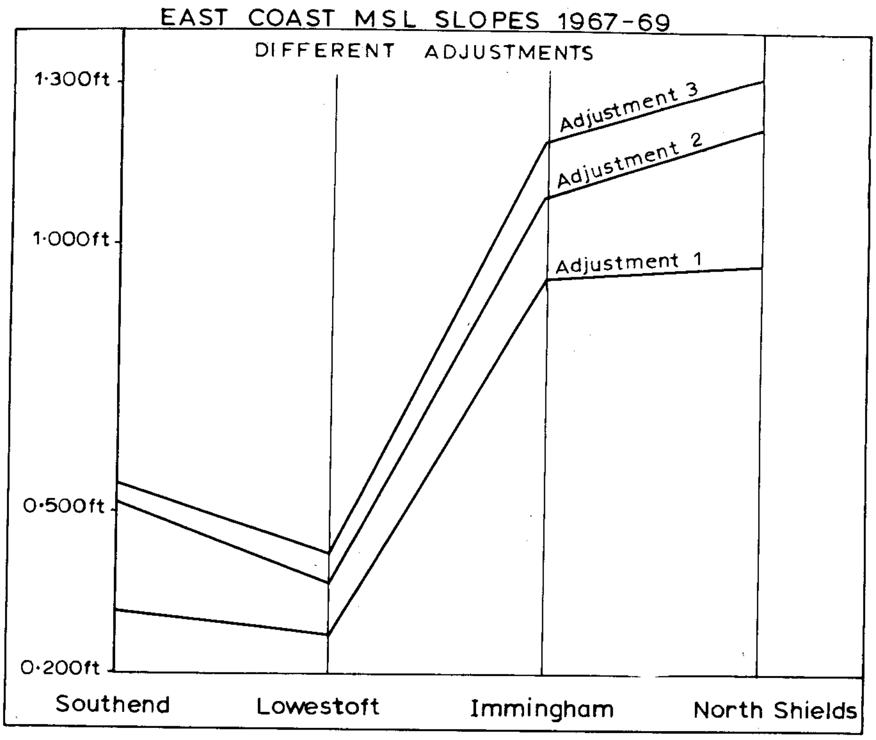
EAST COAST MSL SLOPES 1967-69
DIFFERENT ADJUSTMENTS
1·300ft
1·000ft
0·500ft
0·200ft
Adjustment 3
Adjustment 2
Adjustment 1
Southend
Lowestoft
Immingham
North Shields

Figure 6

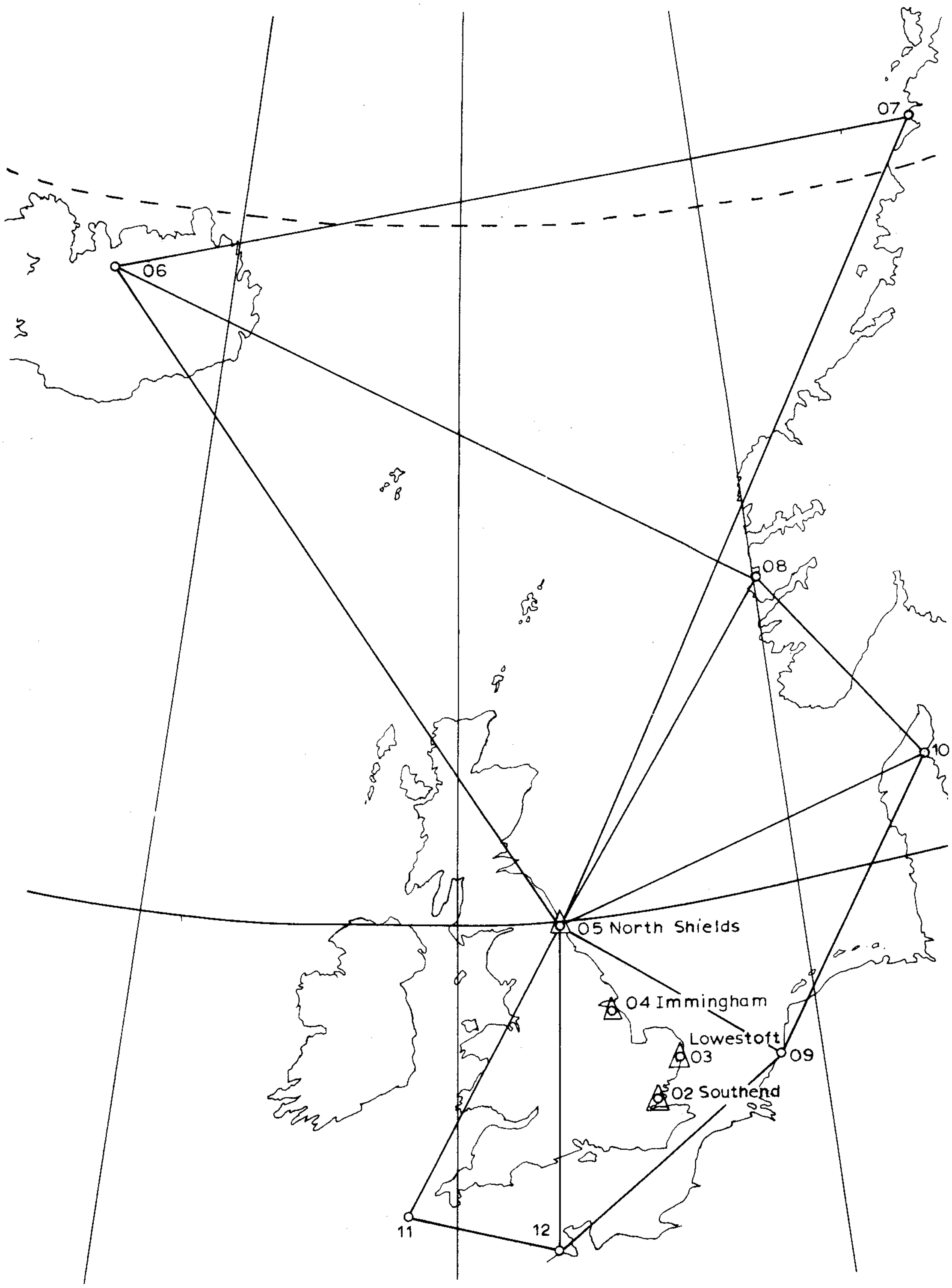

Figure 7. Location of Barometric Pressure Stations
North Shields Triads

6. Validity of Results

Bearing in mind the uncertainties expressed above, the validity of the results detailed in tables 7 and 8 may be examined. The standard deviations computed from the monthly variation of "slope" (see figure 5) are not in themselves a measure of reliability. Part of the differences from month to month may be due to the different magnitude of the seasonal influences and none of the difference can be attributed to the errors in the levelling or to the systematic errors in the sea level measuring and recording system. The standard deviations do, however, indicate a steadiness in the combined sea level measuring and recording system.

Considering each result (see figure 6)

Southend to Lowestoft

The negative "slope" with quite a large variation from month to month produces a standard deviation nearly as large as the "slope". This is hardly a significant result. The inherent errors of the tide gauge and the possible errors in the connections to the Southend Tidal Observatory reinforce this conclusion. Theindications are that from Southend to Lowestoft there is a slight negative "slope" for the period being considered in this study. This "slope" is about the same magnitude as the uncertainty in the result.

Southend to Immingham

The close correlation found elsewhere between the behaviour of these two tide gauges is reflected in the smaller standard deviation. Uncertainties in the levelling and in the tide gauge records make it unlikely that this result can be expressed to better than ±0.15 ft. and to regard the "slope" as 0.58 ft. with a standard error of 0.15 ft. seems reasonable.

Southend to North Shields

This result indicates a northward continuation of the "slope" detected from Southend to Immingham. The uncertainties in this result are, however, greater than in the previous case; the North Shields tide gauge is less reliable and the less steady determination of the "slope" indicated by the greater standard deviation points to an estimate of 0.70 ft. with a standard error of 0.2 ft.

Summarising, we have -

TABLE 11

	Slope*	Standard Error
(1) Southend - Immingham	+ 0.58	± 0.15
(2) Southend - N.Shields	+ 0.70	± 0.20
(3) Lowestoft - Immingham	+ 0.73	± 0.25
(4) Lowestoft - N.Shields	+ 0.85	± 0.30

* Based on second scientific adjustment of the 3rd Geodetic Levelling

The Immingham to North Shields "slope" is only slightly more significant than the Southend to Lowestoft result. It is, however, an extension of the Southend to Immingham "slope" and there is ample evidence to regard this result as indicative of a slight positive "slope" to the north.

In conclusion, the evidence supports a general thesis that there is a slope of Mean Sea Level for the period 1967-69 from the mouth of the Thames to the Tyne.

Possible errors in the levelling and sea level observations and reductions are not sufficient to cast serious doubt on this thesis. The magnitude of the slope is around 0.7 ft.

7. The Meteorological Contribution to Changes in Sea Level Along the East Coast of England.

The meteorological conditions over the North Sea and the eastern part of the North Atlantic are generally regarded as significantly contributing to changes in sea level along the east coast of England. The response of the southern North Sea to the meteorological changes over this larger area will take the form of water movements and surface gradients. In a theoretical sense, it is possible to deduce this response from a knowledge of the spatial and time distribution of the forcing function; this is $f(\tau, B)$ where τ is the mean wind stress and B is the air pressure. Such deductive studies would be extremely difficult and the practice has been to seek relationships by means of a regression analysis between the time variations of sea level at a given station and the forcing functions, see ROSSITER (1967).

Wind stress is a function of wind velocity and its influence on water movements and surface gradients will depend upon its velocity and direction. In the absence of systematically observed values of these quantities a further relationship must be assumed; that is that wind velocity W is a function of the air pressure gradient ΔB. In effect, the assumptions that are made are that $\tau \propto W$, $W \propto \Delta B$ and hence $\tau \propto \Delta B$. It is arguable that τ is more correctly proportional to W^2; for such a case, however, it is probable that W^2 is only proportional to ΔB^2 when ΔB^2 is a function of the mean of the squares of daily differences of barometric pressure. This information was not readily available from the sources used and this line of investigation was not pursued.

It follows that the forcing function was assumed to be represented by three variables; firstly, the mean monthly air pressure over the selected area, secondly the wind velocity and, thirdly, the wind direction. For purposes of the regression, the second and third of these variables were represented, the relationships above being accepted, by the mean monthly air pressure gradients along the three sides of a triangle.

Ideally this triangle should be as nearly equilateral in form as possible, as such a triad will most effectively represent all wind vectors.

On this basis, the eleven stations where monthly mean air pressure was known were grouped to form a variety of triads. The choice of the triad or triads which would be most significant was not easy and only samples of the full set of results are given.

In tabular form these are:

	Southend	Lowestoft	Immingham	North Shields
1. A three triad network which covers whole sea area north of the tidal station ("Theoretical best")	014/2 (02 06 08) (06 07 08) (02 08 09)	015/2 (03 06 08) (06 07 08) (03 09 10)	015/2 (04 06 08) (06 07 08) (04 09 10)	015/2 (05 06 08) (06 07 08) (05 09 10)

	Southend	Lowestoft	Immingham	North Shields
2. A three triad network which gives largest multiple correlation coefficient ("Actual best")	014/10 (04 06 08) (04 09 08) (08 09 10)	014/6 (02 06 08) (06 07 08) (02 09 08)	014/8 (05 06 08) (05 08 09) (05 11 12)	014/10 (04 06 08) (04 08 10) (04 09 10)
3. SaSsa + Density + triad which involves nearest air pressure station to tidal station and covers most of sea area ("Theoretical triad")	019/2 (02 06 08) level of Sig. 1% (F = 4.775)	019/2 (02 06 08) Level of Sig. 1% (F = 9.050)	019/2 (04 06 08) Level of Sig. 1% (F = 10.655)	019/2 (05 06 08) Level of Sig. 1% (F = 29.590)
4. SaSsu and Density + triad which gives largest MCC selected ("best response") from all reasonable combinations of the 11 stations	019/5 (04 06 08) (F = 4.944)	-19/6 (04 06 10) (F = 14.719)	019/6 (04 06 10) (F = 12.675)	019/4 (04 06 08) (F = 29.900)

8. Prediction of Mean Sea Level from Results of the Regression Analysis

Mention is made in ROSSITER (1967) of evaluation of the height of sea level under isobaric conditions from the results of a regression analysis of mean sea level observations at 97 stations. When a similar method was used here to estimate the difference of height of sea level under both isobaric conditions and conditions ofuniform density between the four stations, it was found that the isobaric contribution is largely cancelled out by the contribution from assumed density.

Table 12 compares the normal and isobaric (1015 millibars) heights AOD Newlyn 2nd Scientific Adjustment.

TABLE 12

1967-69 Units of ft.

	Southend	Lowestoft	Immingham	North Shields
h_n	+0.518	+0.366	+1.095	+1.221
h_i	+0.501	+0.278	+1.046	+1.109
$(h_n - h_i)$	+0.016	+0.088	+0.049	+0.112
$(h_n - h_i)$ cm	+0.5	+2.7	+1.5	+3.4

These figures substantially agree with table 9 and figure 9 of ROSSITER (1967); the -3cm anomaly at North Shields having been replaced by a +3.4 cm value.

If a larger barometric pressure had been selected for the isobaric surface, the apparent "slope" of 0.70 ft. from Southend to North Shields would nearly disappear. For example, 1050 millibars gives an isobaric height with uniform density as in table 13.

TABLE 13

	Southend	Lowestoft	Immingham	N.Shields
h_i	+0.293	-0.084	+0.556	+0.205
$(h_n - h_i)$	+0.225	+0.450	+0.539	+1.016

This poses the question as to the contribution of the longshore mean current and whether it reinforces the barometric contribution. The whole of the predictions made in this section are based upon the regression analyses. We find for the four formulae used the statistics given in table 14.

TABLE 14

	S_o	ϕ	R	F
Southend	22.279	16.323	0.765	4.775
Lowestoft	27.554	16.350	0.853	9.050
Immingham	22.567	12.602	0.871	10.655
N.Shields	25.567	9.314	0.947	25.590

Where S_o is the standard deviation of the dependent variable, i.e. the monthly mean sea level values,

ϕ is the standard error of the estimate,
R is the multiple correlation coefficient,
F is the F value.

From these figures, it is apparent that much of the season variation is unaccounted for at Southend compared with the results at Immingham and North Shields. The ratio of R^2 Southend to North Shields is 58.5 : 90 which indicates that over four times as much of the variation is unaccounted for at Southend as at North Shields and that factors which have not been taken into account and play only a minor role in effecting sea level at North Shields are significant at Southend. Furthermore, the significance of these unknown factors increases from north to south. (100 -90) : (100 -76) : (100 -73) : (100 -58)
10 : 24 : 27 : 42

9. Conclusions

The available evidence points clearly to an apparent upward slope of mean sea level from the Thames estuary northwards. In 1950 this apparent slope over some 360 miles (580 km) was of the order of 1.3 ft. (40 cm); this compares with an earlier determination in 1918 of 0.85 ft. (26 cm) based on a different geodetic levelling. However, these results are confused by seeming land movement which may be of the magnitude of 0.2 ft. (6 cm) or of 0.6 ft. (18 cm). In 1968 the mean sea level slope over 270 miles (440 km) of the same coastline was found to be 0.7 ft. (21 cm); unfortunately, a value over the whole length of the original coastline was not possible.

Investigations into the possible sources of error in the levelling and sea level measurement are sufficient to account for some of the variation in the slope with time, and residual uncertainties may be accounted for by crustal movements. Systematic errors such as suggested by EDGE (1959) and SIMONSEN (1965) are not of sufficient magnitude to account for the slope.

Accordingly, an oceanic or meteorological origin for the slope which is found in many north-south coastlines, must be sought. Techniques involving the statistical treatment of a fairly simple model proved fairly successful when considering annual values. ROSSITER (1967). When monthly values of mean sea level are used, even when the annual and semi-annual tide is allowed for, the results of a statistical treatment of the simple model are not particularly satisfactory. It would seem that a more complex model is needed.

This paper is somewhat negative in that it is establishing a physical or apparent physical feature of the sea level topography, but offers no explanation of its origin. It would seem that the following is needed:

a) A more detailed study of the environment of each tidal observatory. For example, the Southend station may not be stable.

b) A more detailed study of the process of levelling to a tidal observatory. This process is subject to errors arising from local deviations of the vertical due to the variable attraction of the vertical by the oceanic tide (LENNON 1961), and due to the variable crustal loading from the same source.

c) A more complete study of the changes, month by month, in the marine environment. There is a surprising lack of detailed information over periods of greater than a few days.

(d) A more complex model than that of ROSSITER (1967) of the interaction of the oceanic and meteorological effects on the sea level topography must be designed.

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11. Discussion

BUCHWALD (Chairman): I can now see why I was chosen as chairman - hoping that, as a mathematician, I'd stay neutral.

HOLDAHL: I have a question regarding changes in height due to loading by ocean tides. How far inland can this effect be detected? 10 km? 150 km? Or more? MELCHIOR may wish to comment.

MELCHIOR: When we had one station in Czechoslavokia which is at the centre of Europe, and at 1500 m depth, loading effects were still sensible; and if you look at the paper I have distributed you will see that in Europe we have an advance of phase of $2\frac{1}{2}°$ in gravity tides in east Switzerland. Not only the Altantic ocean influences events but also the Pacific.

HOLDAHL: In other words, you say that the whole of England would be affected?

MELCHIOR: Yes.

WALCOTT: It may be worth mentioning that Land's End (Newlyn) which is the fundamental bench mark in England, has the largest ocean tide displacement effect known in the world.

HOLLWEY: That is most interesting. It is just a datum and you could change the datum to a point in the Midlands and it would not really matter. It's only a reference point for heighting.

SYDENHAM: Is there any plan to modify the levelling using the Channel Tunnel as a reference line under a large stretch of sea surface?

HOLLWEY: Once the Channel Tunnel is built, we will certainly level through it.

GRAFAREND: What we are really measuring in levelling is the difference of potential. But I do not see from this point of view why we need to have any datum for heights. Why do we need to have a datum for heights?

HOLLWEY: We don't need MSL as a datum. But we do need some point as a reference.

GRAFAREND: What for?

BOMFORD: We need some numbers, and for numbers you must have a datum.

GRAFAREND: You can use any arbitrary number.

HOLLWEY: What you need to know is whether point A is higher than point B.

GRAFAREND: Right. Therefore we need only differences.

HOLLWEY: This is what we really have in the United Kingdom.

ANGUS-LEPPAN: There are practical reasons why we should have a physical datum. If you adjust your levelling net to Mean Sea Level instead of an equipotential surface, you distort the whole system. This is why we have to solve the problem of whether MSL is an equipotential, and if not, whether the difference is significant in practice.

HOLLWEY: I would not adjust to M S L or a particular equipotential surface. What we seek for a level net is consistency within itself.

GRAFAREND: The problem in geodesy is that every nation has its own height datum. It is absurd for every country to have this. We perhaps need a global datum. For example take Europe, where nearly every country has its own datum and there are datum problems in adjusting data across national boundaries.

HOLDAHL: Are you familiar with the reasons why corrections are not applied to levelling observations to compensate for loading by ocean tides? As I see it, ocean loading would produce an effect similar to the Earth tide which disturbs a large area and has a short period. The effect on height determination would be small and random if good levelling procedures were used. I was wondering whether this was the way English geodesists have reasoned?

HOLLWEY: The presence of the 12 cm Earth tide at Newlyn indicates that the magnitude is such that there there is an argument for examining possible corrections. The effect is, as you say, simultaneous and should in the normal case be cancelled out. The next stage in this argument is that there is no need to keep a record of time of observation which could be correlated with the tidal effect. This is a foolish extension of the argument because once you have no record you cannot prove that the effect is cancelled out. At the moment, as no record is kept, we cannot make such an analysis.

BUCHWALD: I use my prerogative as chairman to sum up. Regarding the two metre slope in the sea surface between the north and south of the east coast (of Australia), the work of GODFREY and his co-workers is an attempt in my opinion, to explain the phenomenon using the dynamics of the ocean and it does not really matter what datum we use and so on. Neither the dynamics nor the statics of the ocean as known at present, explains this two metre difference. Both oceanographers and geodesists should work at it to see if there is some systematic reason why there is this difference.