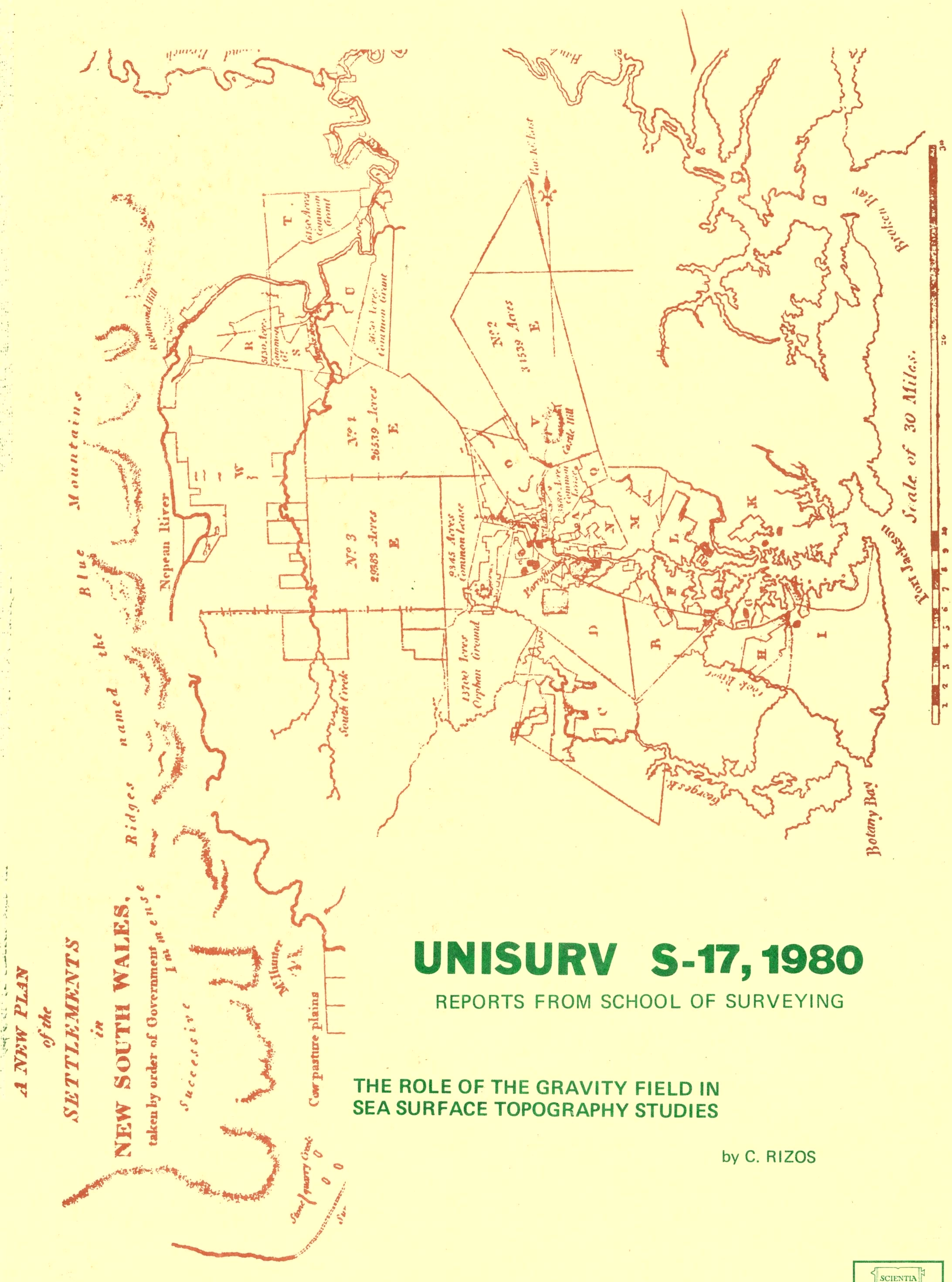

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THE ROLE OF THE GRAVITY FIELD IN SEA SURFACE TOPOGRAPHY STUDIES

by C. RIZOS

UNIVERSITY OF NEW SOUTH WALES
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CORRECTIONS

P.11, Paragraph 2, line 3:

Laboratory for Applications of Remote Sensing

P.52, Paragraph 1, line 1

proportional

P.52, Equation 2.19 should read

$$C(x, y) = \frac{E_G T_{\theta_V}}{K\pi} [\cdot \cdot \cdot \cdot \cdot \cdot] + \frac{L_P}{K}$$

P.173, Paragraph 1, Line 1

leave out 8.4

P.189, Paragraph 2, line 7

5 x 5 array

P.202, line 2

Deconvolution

ABSTRACT

GEOS-3 altimeter data provides estimates of the height of the instantaneous sea surface above a reference surface with geodetic accuracy. This has offered a stimulus to activity in physical geodesy to provide an oceanic geoid with sufficient precision to permit the mapping of deviations of the sea surface from the geoid - the Sea Surface Topography (SST). A primary objective of SST studies is, in the first instance, the establishment of long wavelength features of the stationary SST. The determination of short wavelength components of the stationary SST and variations in SST with time are performed at a subsequent stage. A secondary and no less important goal is the unification of regional levelling datums in relation to a datum level surface such as the geoid. Procedures and a set of relations between satellite-determined gravity field models, satellite altimetry and surface gravity measurements for achieving these objectives are given.

A prerequisite for remote sensing SST from satellites is a geoid model with at least ±10 cm resolution through wavelengths of interest. Gravity anomaly data banks for Australia and central North America, suitable for SST studies, have been prepared. Conceptual definitions of the geoid adequate for SST studies are presented. The validity of such definitions over geodetic time scales ($<10^2$ years) is studied in the context of possible earth expansion, secular variation in mean sea level (MSL) and mass redistributions in the earth's crust and mantle.

Methods for the determination of the geoid have been found lacking in certain respects under the more stringent accuracy requirements imposed. The effect of the atmosphere on solutions of Molodenskii's problem for the geoid height with a precision better than ±10 cm has been investigated. However, such geoid determinations have to circumvent a major problem: none of the data collected at the earth's surface - gravity and satellite altimetry - are related to the geoid to better than ±1-2 m.

The only source of geoid information which is potentially uncontaminated by data referenced to the sea surface is a satellite-determined gravity field model. The GEOS-3 altimeter data bank and the best such model available at present (GEM9) have been used to obtain preliminary estimates of some of the dominant parameters of the global stationary SST and for the dimensions of the ellipsoid of best fit to global MSL. The Australian and central North American gravity anomaly data banks have been analysed for the height of MSL above a unique geoid (defined by satellite altimetry) at the datum tide gauges.

Magnitudes of coastal sea surface slopes inferred from a comparison of the results of geodetic levelling with MSL defined by tide gauge readings are in significant disagreement with oceanographic estimates. A critical analysis of the possible sources for such discrepancies was carried out and it appears that some small systematic error may be present in the geodetic levelling data. The role which geodetic levelling can play in monitoring vertical crustal motion is discussed.

The most important requirement for significant advances in remote sensing SST using satellite altimetry data, and ultimately ocean surface circulation, is the refinement of the satellite-determined gravity field models to 2 parts in 10^9 *for wavelengths of interest. Consequently, given the choice, it is preferable to improve the density and distribution of high precision laser tracking systems for orbit determination and gravity field model improvements, rather than to carry out extensive surface gravity surveys.*

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1

INTRODUCTION

1.1 THE ROLE OF THE GEOID IN OCEAN DYNAMICS

Until recently, the principal role played by the oceans in classical geodesy has been a *passive* one as a datum for levelling. The most obvious level surface of the earth's gravity field to which all elevations could be referred is that corresponding to Mean Sea Level (MSL). It has therefore been conventional to define the geoid, the "mathematical figure of the earth", as that level surface corresponding to MSL (BOMFORD 1962, p.202).

The practical realisation of this global elevation datum at a coastal location is achieved by assuming that local MSL, as established from a minimum of one year's tide gauge observations, is a good estimate of the geoid. By virtue of the operations of geodetic levelling the spatial characteristics of this datum level surface are implicitly defined on land by the concept of "height above sea level", or more correctly by the orthometric height.

It has been a main task of scientific geodesy to determine the size and shape of the geoid from solutions of the Geodetic Boundary Value Problem (GBVP). The role of such solutions is for the definition of ellipsoidal elevations, and hence geocentric position. While such procedures are adequate for geodetic considerations to ±2m, significant inconsistencies arise when they are extended to instances where the required precision is ±10cm. The first uncertainty is whether every regional estimate of MSL coincides with the geoid locally.

It was recognised over 50 years ago (BOWIE 1929) that discrepancies existed between the datum level surface established from levelling operations and that indicated by tide gauges, on the assumption that the latter defined a unique level surface. These discrepancies are not unexpected. On the basis of both temperature and salinity measurements as well as current estimations in the oceans, oceanographers have maintained for some time that the free ocean surface, apart from periodic tidal and wind effects, does not lie along an equipotential surface of the earth's gravity field. Direct analogy with the solid Earth has resulted in departures of sea level from an equipotential surface being called Sea Surface Topography (SST). The total range of non-tidal SST is 3-4m with a time-varying component one quarter of the magnitude of the stationary part. Evidence available at the present time for the existence of SST therefore comes from two independent sources:

(1) The comparison of free net adjustments of geodetic levelling with MSL estimated at tide gauges along coastlines (section 3.4).

(2) The estimation of SST in ocean oceans from oceanographic considerations (such procedures being referred to as oceanic levelling - see section 3.3).

The results obtained from the oceanic levelling should be in good agreement with the coastal stationary SST determined from the geodetic levelling/MSL comparisons. The term "stationary" is used to describe those characteristics of the phenomenon which appear to be independent of time over the epoch of measurment. While this is so in some instances, there are many other cases where such estimates of stationary SST indicate discrepancies significantly larger than expected from the internal precisions of either procedure (see MATHER 1974a, Appendices). Although the ease with which "negligible" amounts of systematic error can contaminate levelling data has been adduced as a possible source of such discrepancies (STURGES 1974), the possibility of errors being introduced in extrapolating the results of oceanic levelling from the open ocean to the coast cannot be ruled out (section 3.4).

The Geodynamics Experimental Ocean Satellite (GEOS-3) was launched in April 1975 as part of the activities of NASA's Earth and Ocean Dynamics Applications Program (NASA 1972a). In addition to the usual instrumentation for accurate position fixing, GEOS-3 carried a 13.9 gigahertz radar altimeter for measuring the vertical distance to the sea surface. This provides a basis for an independent method of verifying the slopes of the sea surface.

Prior to the advent of satellite altimetry data no serious attention was paid to the choice of which level surface of the earth's gravity field was adopted as the geoid. However, an adequate conceptual definition of the geoid (sub-section 2.2.5) is a necessary prerequisite for SST determinations. In the final analysis, the choice of a particular level surface as the geoid will depend on,

- The basis on which MSL is sampled;
- The precision sought in the definition; and
- The potential users of the geoid so defined.

The data for sampling estimates of MSL used in the definition of the geoid can come from coastal tide gauges as well as from satellite altimetry. However, tide gauge estimates of MSL have to be treated with caution when used as indicators of the global characteristics of MSL due to the likely distorting influences of coastlines, the stability of tide gauge sites and other local factors. Altimetry-derived estimates of MSL are an acceptable alternative if the radial component of the orbital positions of altimeter-satellites are known to at least ± 50cm.

Oceanographers prefer a definition of the geoid based on non-coastal sampling of MSL. A geoid definition which satisfies such a requirement is the following:

> "The geoid for a selected epoch of measurement, is that level surface of the Earth's gravity field in relation to which the average non-tidal sea surface topography is zero as sampled globally in <u>ocean regions</u>".

The only means of obtaining a realistic global sampling of MSL is from satellite altimetry. Such a definition is acceptable to geodesists even though it can result in the mean of MSL as sampled at all geodetic levelling datums being non-zero.

The principal reasons for high precision determinations of the shape of the geoid are:

- the determination of sea surface topography for applications in oceanography; and
- the unification of levelling datums with a resolution equivalent to that of first order geodetic levelling by the determination of sea surface topography at regional levelling datums.

These tasks can be conveniently referred to as *Sea Surface Topography Studies*.

As a corollary, there is no reason for carrying out the burdensome task implicit in very accurate geoid determinations on the basis of geodetic considerations on continents alone.

Geoids which aspire to a role in SST determinations should achieve a precision of ±10cm as the magnitude of the phenomenon is not greater than ±2m. The requirement of a ±10cm marine geoid implies that this level of precision is sought for some range of wavelengths, e.g. $5^{o} \times 5^{o}$ area means, $1^{o} \times 1^{o}$ area means or point values; the latter being the most stringent requirement to satisfy. In addition, the geocentric position of the instantaneous sea surface as defined by satellite altimetry data should be known at the 10cm level.

The ability to remote sense SST from altimeter-equipped spacecraft opens up the possibility of using geodetic techniques for studying ocean dynamics, as ocean surface circulation is the result of the interplay between:

(1) the earth's rotation;

(2) the slopes of the sea surface in relation to the geoid;

(3) the horizontal atmospheric pressure gradients at the air/sea interface; and

(4) the frictional forces acting on the surface layer.

Table 1.1 (from MATHER ET AL 1978b) summarises the factors necessary to maintain a current of 10 cms^{-1} at latitude 45^{o}. Fast flowing quasi-stationary currents like the Gulf Stream in the western North Atlantic and the Kuroshio in the western North Pacific have velocities in excess of 10^{2} cms^{-1} and are maintained by steep SST gradients in excess of 1m per 10^{2}km, but with relatively short wavelengths. The wind velocities and/or atmospheric pressure gradients needed to maintain such currents are at least an order of magnitude larger than the strongest measured under extreme conditions at the surface of the Earth.

TABLE 1.1

Source: MATHER ET AL (1978b)

Magnitudes (Rounded Off) of Factors Which Can Maintain a Surface Layer Current Velocity of 10cm s^{-1} at Latitude 45°

Factor	Required Magnitude
Sea Surface Topography Gradient $\partial\zeta_s/\partial x_\alpha$	0.2 arcsec (10cm per 10^2km)
Atmospheric Pressure Gradient $\partial p_a/\partial x_\alpha$	30mb per 10^2km
Wind Stress*	40m s^{-1} (83 mph)

*F_α computed from the empirical relation:

$$F_\alpha = 2 \times 10^{-6} \frac{(W_1^2+W_2^2)^{\frac{1}{2}}}{\rho_w H} W_\alpha$$

H = Depth of mixed layer (10^2m)

ζ_s = Sea Surface Topography
p_a = Atmospheric Pressure
F^α = Wind Stress
W^α = Wind Speed
ρ^α = Water Density
α=1^w(E direct.) or α=2 (N direct.)

The stationary component of SST in regions of fast flowing steady state currents is 4-5 times larger than the time-varying constituents. Other more moderate current systems are maintained by the complex interplay of wind, SST and frictional forces on the surface layer. These circulation features exhibit time variations in the associated SST patterns from ±10cm to over ±50cm. Included in this category are eddies, which are of particular interest for navigation, climatology, oceanic engineering and the fishing industry, quite apart from purely scientific interest.

The use of remote sensors deployed on near-Earth satellites, in particular the satellite altimeter, provides the only realistic means of acquiring sufficient data for the quantification of ocean circulation in space and time.

The role of gravity field information in SST studies is twofold:

(a) To define the datum level surface (the geoid) in ocean areas to a precision of ± 10cm or better for wavelengths of interest; and

(b) To ensure that radial errors in the orbit determination of the altimeter-equipped spacecraft are held below the 10cm level in regions where direct satellite tracking coverage is inadequate.

The types of geodetic data presently available for satisfying these objectives are:

(1) <u>Gravity measured at the earth's surface</u>. These are usually expressed in the form of gravity anomalies (sub-section 4.1.4) using information on the "height" of the gravity station above the regional MSL levelling datum (assumed zero in ocean areas). The quality of the oceanic gravity anomaly data is generally regarded to be at least an order of magnitude inferior to land

based data.

(2) A global gravity field model defining the disturbing potential of the earth's gravity field. This is obtained from an analysis of orbital perturbations of near-Earth artificial satellites.

(3) Heights of the instantaneous sea surface above the reference ellipsoid ζ. In addition to containing the signature of the stationary SST, this data is subject to variations in time with periods of less than a day (due to tides, winds) to over a year, and with amplitudes from a few centimetres to half metre or more.

The need for the unambiguous combination of satellite altimetry, the satellite-determined gravity field model and surface gravity measurements to obtain sea surface topography has provided a considerable stimulus to physical geodesy in three main fields:

(i) Development of a solution to Molodenskii's GBVP with ±5cm resolution (see e.g. MATHER 1973a, MATHER 1975a)

(ii) Formulation of a procedure for the combination of gravity anomaly data (mostly restricted to land and continental shelf areas) and satellite altimetry (MATHER 1974d, MATHER ET AL 1976a).

(iii) Improvement in the precision of orbit determination through:

- gravity field model improvement (SMITH ET AL 1976, LERCH ET AL 1978a).
- deployment of high precision laser tracking systems.

In solutions of the GBVP for the geoid height, error accumulation in quadrature evaluations is largely a function of the magnitude and wavelength of errors in the gravity anomaly data. Such solutions need to take into account factors such as the ellipticity of meridians and the gravitational potential of the topography and atmosphere (section 4.2). Satisfactory results are obtained in practice only when the errors in such a data set decrease rapidly as a function of wavelength. For example, in the case of the gravity anomaly data set for Australia the long wavelength error sources, after correcting for all systematic errors that could be reliably modelled, are assessed as the following: (section 6.2)

(a) Errors with amplitude 0.15mgal and wavelength 7×10^3km due to residual errors in the Australian levelling survey.

(b) A constant error of ±0.06mgal due to the gravity value adopted for the National Base Station at Sydney being uncertain at this level.

(c) Errors with amplitude 0.2mgal and wavelength 7×10^3km due to residual errors in the Australian National Gravity Network.

A fourth significant source of error present in all gravity anomaly data sets is as a result of the adopted datum level surface to which the gravity anomalies are referred not necessarily coinciding with the geoid to better than ± 1m, causing systematic effects of ±0.3mgal in the entire anomaly data bank computed on this datum.

The lack of a global coverage of surface gravity data and the questionable quality of oceanic coverage has led to the use of combination techniques for geoid determinations. The input data for such solutions is the satellite-determined gravity field model and several regional gravity field determinations with representation on, say, a 10km grid providing surface gravity data within 20^{o} of the point of computation.

However, such solutions are of limited value in SST studies as they are restricted to those regions possessing adequate surface gravity field coverage - Europe, North America and Australia.

It is now recognised however (MATHER ET AL 1976a), that any determination of the shape of the geoid to ± 10cm precision from surface gravity information, satellite altimetry data or a combination of both is not possible as all observations are *related to the sea surface and not the geoid*. Consequently, the shape of the latter cannot be determined to better than the magnitude of the SST (i.e. ±1-2m). The role of the gravity field in SST studies is therefore somewhat different from that in the solution of the GBVP (MATHER 1978e). In the latter case, the objective was the geometrical mapping of a surface at which the measurements were made. In the SST application, the shape of the bounding surface is known from satellite altimetry. It is required to geometrically map (in concept) a datum level surface in ocean areas under circumstances where no measurements have been directly made in relation to it.

Consequently, attempts to find a means for determining SST from a solution of the GBVP, without making assumptions about the nature of the SST, have not been successful (MATHER ET AL 1976a). The only potential source of data on the earth's gravity field, and hence on the oceanic geoid, which are independent of any relationship to the geometry of the sea surface is a satellite-determined gravity field model. Such a model is capable, in theory, of resolving those features of the geoid with wavelengths greater than 10^3km. The possibility exists that the minimum resolvable wavelength could be decreased to about 500km if data from satellite-to-satellite tracking were included in the orbital analysis for the model of the disturbing potential.

1.2 SEA SURFACE TOPOGRAPHY STUDIES WITH GEOS-3 ALTIMETRY

The SST spectrum lends itself to convenient subdivision into two components, in the context of satellite remote sensing. The first is the stationary constituent, while all features which vary with time during the period of data acquisition comprises the second. The recovery of SST from satellite altimetry is one of the most exacting tasks facing not only specialists in defining oceanic geoids but also those engaged in satellite orbit determination. It was mentioned above that the only source of data on the geoid which is potentially unaffected by the existence of SST is a satellite-determined gravity field model. Therefore, assuming that the radial component of the orbital position of the GEOS-3 altimeter can be established to ± 10cm, the determination of the SST can be tackled in three distinct phases (MATHER 1978c):

(1) Stationary SST can be defined, in the first instance, through those wavelengths equivalent to those of the gravity field model (> 10^3km) (sub-section 5.2.2).

(2) Using the determination at (1), contributions to the SST with shorter wavelengths could be defined using quadrature evaluations of surface integrals if the gravity anomaly and satellite altimetry data were free from error (±0.03mgal or ± 10cm) through wavelengths of interest (sub-section 5.2.3).

(3) Variations in SST with time due to tides and mesoscale circulatory phenomena may be obtained from those in the sea surface height ζ, provided the geoid can be assumed not to change

shape in the same time interval by amounts in excess of ± 1-2cm.

The determination at (1) involves the definition of parameters of the global stationary SST corresponding to those wavelengths for which the earth's gravity field model is known with an acceptable level of accuracy. Upon adopting a low degree surface spherical harmonic representation for the stationary SST ζ_s, a study of the power spectrum of the SST as estimated by oceanographers in relation to that of the estimated errors in the best available gravity field model shows that it is only possible to recover seven harmonic coefficients of ζ_s (representing 80% of the total power), if the satellite altimetry is free of error. However, the GEOS-3 altimeter data are not error free, consequently any solution procedure that fails to take this factor into account will be unsuccessful.

Therefore the difficulties encountered in SST determinations ((1), (2) and (3) above) from GEOS-3 altimeter data can be summarised as:

(a) A gravity field model for the disturbing potential is still unknown at the 1% level.

(b) The geocentric position of the spacecraft cannot be defined at the decimetre level, at the present time.

Despite these adverse factors, a basis exists for the recovery of limited SST information from GEOS-3 altimetry on the following parts of the spectrum, in both space and time:

- Stationary components of very long wavelength.
- Time-varying components in regions where the magnitude of the variation exceeds the noise level of the altimeter, such as in the neighbourhood of the Gulf Stream.

In addition, GEOS-3 altimetry, in the short term, can play a significant role in gravity field model improvement.

A secondary and no less important goal of SST determinations is the unification of regional levelling datums in the context of a unique datum level surface - the geoid. Arguments for the unification of all geodetic levelling datums can be summarised as follows (MATHER 1978d):

(1) At present, elevation datums on different land masses cannot be related to the same datum level surface.

(2) Widespread occurrences of discrepancies between the geoid as defined by levelling and local estimates of MSL require that additional constraints be provided for large level networks.

(3) Present day global gravity anomaly data banks lack the resolution needed for high precision geodetic applications, such as ocean dynamic modelling and gravity field model improvement.

The simultaneous determination of SST at all ancillary tide gauges could play a further role in the adjustment of networks of geodetic levelling. It could be argued that such a definition could be provided in purely geometrical terms by relating tide gauges to three dimensional positional systems. However, the operations of geodetic levelling in addition to exhibiting the highest relative precision of any geodetic process at the moment, have practical utility by virtue of their direct relation to level surfaces of the earth's gravity field. For this reason, first order geodetic levelling results can still play a significant role in vertical crustal motion studies (section 8.3).

The principal problems that arise from the *direct* use of satellite altimetry for the determination of coastal SST are (MATHER ET AL 1978c):

(a) The geoid has to be defined through short wavelengths (< 10^2km) in continental shelf areas.

(b) As a result of the finite altimeter "footprint", the sea surface heights ζ have to be extrapolated from the open oceans to the coastal site over a distance not less than 20 km.

The first problem calls for the determination of the short wavelength SST. The resolution of the second problem calls for an oceanographic survey in order to extrapolate the sea surface slope by the principles of geostrophic levelling (§ 3.3.3.3), requiring that frictional forces at the ocean bottom and at the sea surface be adequately modelled.

In view of these difficulties, the most promising alternative procedure for the determination of the "height" of MSL at the regional elevation datum is based on an analysis of the continental gravity anomaly data bank related to this datum (section 5.3). The only role played by the GEOS-3 altimetry data is in defining a unique geoid (sub-section 5.1.3). Such an analysis can only be attempted at the present time, for Australia and central North America, as these are the only large continental areas for which a more or less homogeneous geodetic levelling network (and resultant gravity anomaly data set) exists.

It is the aim of this thesis to investigate the role that geodetic data such as satellite altimetry, surface gravity and models of the earth's disturbing potential can play in SST studies. Attention has been focussed on the following problems:

(1) The definition of a unique geoid adequate for geodetic and oceanographic applications at the ± 10cm level.

(2) Development of procedures for the determination of the shape of the marine geoid, and hence the resolution of the SST, both the stationary and time-varying constituents. It must be borne in mind however, that any procedure which utilises GEOS-3 altimeter data should have the capacity to achieve an unambiguous result in the presence of adverse signal-to-noise levels resulting from:

- an inability to define to oceanic geoid to a ± 10cm precision, except for very long wavelengths.
- the variable precision of the orbit determination of the GEOS-3 spacecraft.

(3) The feasibility of monitoring the behaviour of geodetic reference systems, particularly the geoid, as a function of time by campaigns of high precision geodetic observations of absolute gravity. This is essential for long term SST and vertical crustal motion studies.

(4) The role of geodetic levelling in satisfying geodetic objectives at the level of 1 part in 10^8.

(5) The determination of SST at regional levelling datums in relation to the geoid defined at (1).

1.3 SYNOPSIS OF CONTENTS

The concept of the geoid, together with gravity and gravitational potential, are introduced in section 2.1. The role of the geoid in "classical" geodesy is reviewed in section 2.2 and conceptual definitions adequate for high precision geodetic studies are presented. Methods for the determination of geoid undulations are discussed in section 2.3. Particular emphasis being given on their practicability for marine geoid determinations to ±10cm precision in support of sea surface topography studies. The continuity of a high precision geoid over geodetic time scales (< 10^2 years) is studied in section 2.4. Variations in geoid definition due to net volume changes or earth expansion as well as variations in shape as a consequence of tectonic plate motion are investigated.

Section 3.1 describes the characteristics of reference systems used in geodesy and oceanography, and the role played by Mean Sea Level. The vertical reference system in geodesy and the relationship between the results of geodetic levelling and the datum level surface for elevations - the geoid - is discussed in section 3.2. The variability of the ocean surface is studied and the determination of SST by techniques of oceanic levelling is investigated in section 3.3. A comparison of the results of geodetic and oceanic levelling at coastlines is presented in section 3.4. Possible reasons for the reported discrepancies in estimates of coastal SST as provided by these two independent techniques are adduced.

A formulation of the Geodetic Boundary Value Problem according to Molodenskii with a resolution of ±5cm is developed in section 4.1. A set of relations defining the disturbing potential and the gravity anomaly are presented. A solution of the GBVP based on Green's Third Identity is described in section 4.2. The role that satellite altimetry can play, in combination with surface gravity data, in solutions of the GBVP is discussed in section 4.3 and a mathematical relationship presented. Section 4.4 describes the "higher" reference model afforded by a model of the earth's disturbing potential and the basic mathematical relations are revised. Atmospheric effects in physical geodesy are investigated in section 4.5.

Section 5.1 investigates the possibility of determining a high precision marine geoid from combinations of surface gravity data, satellite altimetry and models of the disturbing potential. The practical realisation of a unique geoid definition from GEOS-3 altimetry data is attempted. The determination of sea surface topography using geodetic techniques (in contrast to determinations by the methods of oceanic levelling) is described in section 5.2. Procedures for the recovery of information on different parts of the space and time spectrum of SST are studied and the feasibility of using GEOS-3 altimetry is investigated. The secondary geodetic goal of determining SST at coastal sites is attempted in section 5.3 and a set of relations are developed.

Gravity anomaly data banks are needed in support of SST studies. The definition of the gravity anomaly to be used in solutions of the GBVP and SST studies is given in section 6.1. The preparation of the Australian gravity anomaly data bank (AUSGAD 77) is described in section 6.2. Section 6.3 describes the preparation of the central North American gravity anomaly data bank (CNAGAD 77).

Procedures for the determination of long wavelength stationary SST from GEOS-3 altimeter data and the best available gravity field model (GEM9) are described in section 7.1. Results for some of the dominant parameters in this part of the SST spectrum are presented in section 7.2. The best fitting ellipsoid to the global mean sea level surface is determined in section 7.3.

Geodetic levelling datum definition for sea surface topography and vertical crustal motion (VCM) studies is discussed in section 8.1. The determination of the "height" of MSL above a unique geoid defined by GEOS-3 altimetry at the regional levelling datum for Australia and central North America is attempted in Section 8.2. Such results are obtained from an analysis of AUSGAD 77 and CNAGAD 77. The stability of the geodetic levelling datum for vertical crustal motion studies is discussed in section 8.3 and a relation is developed that allows VCM to be defined from levelling, gravity and MSL data even if the spatial location of the elevation datum and the shape of the level surfaces of the earth's gravity field vary with time.

Conclusions and recommendations are presented in sections 9.1 to 9.6.

2

THE GEOID IN HIGH PRECISION GEODESY

2.1 INTRODUCTION

2.1.1 GRAVITY AND GEOPOTENTIAL

The purpose of this section is to briefly describe the fundamental quantities in physical geodesy and establish the terminology that will be used in this thesis. In particular, gravity and potential for both rigid and non-rigid models of the earth will be discussed.

2.1.1.1 *Gravitation*

According to Newtonian gravitational theory, an attracting force acts between two bodies, with magnitude proportional to the product of their masses and inversely proportional to the square of the separating distance. Such a gravitational force is exerted on a body at rest on or near the surface of the earth. As a consequence of the uneven mass distribution of the earth and atmosphere, the gravitational force F_G varies with position and is given by the relation,

$$F_G = G \iiint_V \frac{\rho}{r^2} dv \qquad (2.1)$$

where the volume integral illustrates the continuous distribution of density ρ within the earth and r is the distance from the body attracted by an element of volume dv. G is the gravitational constant whose value is (MORITZ 1979):

$$G = 6.672 \times 10^{-11}\ m^3 s^{-2} kg^{-1} \qquad (2.2)$$

The force F_G is directed towards the centre of mass of the solid earth and atmosphere : the geocentre.

The distinction between "gravitational force" and "gravitational acceleration" is of no consequence in this discussion.

2.1.1.2 *Deformation*

A second set of gravitation-like forces are exerted by extraterrestrial masses - the most dominant being those due to the Sun and Moon - and by mobile masses within the earth and atmosphere, e.g. groundwater movement, atmospheric and oceanic circulation. Such deformation forces F_D vary in magnitude and direction as a function of position and time in response to associated variations in the luni-solar ephemeris, the climatological cycle, etc.

These forces are a consequence of the earth being deformable and being located in a space containing other gravitating masses. As a corollary, if these deformation forces are ignored, this is tantamount to the earth being non-deformable and located in a space free of external forces. The treatment of deformation forces is discussed further in § 2.1.1.8.

2.1.1.3 *Rotation*

A force acting on a body due to its rotation with the earth is the centrifugal force F_R. The magnitude of this force is a function of the distance from the rotation axis, and is a maximum at the equator. The centrifugal force resolved in the direction of the geocentre at latitude ϕ is

$$F_R = \omega^2 \ p \ \sec\phi \tag{2.3}$$

where p is the distance from the rotation axis and ω is the angular velocity of the earth's rotation, whose value is (IBID)

$$\omega = 7.2921151467 \times 10^{-5} \ \text{rad s}^{-1} \tag{2.4}$$

The centrifugal force vector is directed away from the geocentre and increases in magnitude with increase in p. F_R is independent of the mass of the earth.

2.1.1.4 *Gravity*

The total force g, called gravity, is the resultant of the gravitational, deformation and centrifugal forces, and is the force acting on a body located in or travelling through the earth's gravity field. The deformation and centrifugal forces are less than 1% of the magnitude of the gravitational force, hence the "gravity" force is almost entirely comprised by the "gravitational" force.

The gravity vector has the physical dimensions of acceleration and is measured in "gals" (1 gal = 1 cm s^{-2}). The direction of the gravity vector is the direction of the plumbline. The components of this vector are:

$$g = F_G - F_R \pm F_D \tag{2.5}$$

	F_G	F_R	F_D
Order of Magnitude (mgal)	10^3	0-3	$<10^{-3}$

the positive direction being towards the geocentre.

2.1.1.5 *Geopotential*

For a body located in a gravity field, the capacity of an attracting force (gravity) acting upon it to do work - the potential energy - provides a measure of intensity at any point. This is usually expressed in terms of a scalar quantity called potential, which is the amount of work associated with the introduction of a "test" particle of unit mass, brought from an infinite distance to the required point (RESNICK & HALLIDAY 1966). The relationship between the attractive force and its potential can be illustrated in the following manner.

Denoting the attracting mass by M and assuming that the attracted mass is equal to unity, then the gravitational force F_G is given by:

$$F_G = \frac{GM}{r^2} \tag{2.6}$$

The potential of gravitation W_G is the scalar function defined by:

$$W_G = \frac{GM}{r} \tag{2.7}$$

For a continuous distribution of density, the earth's gravitational potential is given by:

$$W_G\ (x_1, x_2, x_3) = G \iiint_V \frac{\rho}{r}\, dv \tag{2.8}$$

where the quantities G, ρ, r and dv are the same as in equation 2.1.

Introducing a rectangular Cartesian coordinate system X_i , the components of the force F_G are given by:

$$F_{Gi} = \frac{\partial W_G}{\partial X_i} \tag{2.9}$$

or in vector symbolism,

$$F_G = - \vec{\nabla} W_G \tag{2.10}$$

the negative sign indicating that W_G increases in the direction away from the geocentre. The operator $\vec{\nabla}$ for such a coordinate system is

$$\vec{\nabla} = \frac{\partial}{\partial x_1}\vec{1} + \frac{\partial}{\partial x_2}\vec{2} + \frac{\partial}{\partial x_3}\vec{3} \tag{2.11}$$

$\vec{1}$, $\vec{2}$ & $\vec{3}$ being the unit vectors along the X_i axes. In other words, the force vector F_G is the gradient vector of the scalar W_G.

The potential of gravity W is termed the geopotential and is the sum of the potentials of gravitation W_G, deformation W_D and rotation W_R:

$$W = W_G + W_D + W_R \tag{2.12}$$

where the rotational potential is given by,

$$W_R = \tfrac{1}{2}\,\omega^2\,p^2 \tag{2.13}$$

the quantities ω and p being defined at equations 2.3 and 2.4.

In vector symbolism, the rotational (or centrifugal) force is related to W_R by

$$F_R = \vec{\nabla}\,W_R \tag{2.14}$$

The deformation force and its potential is discussed in § 2.1.1.8.

The combination of equations 2.5, 2.10 and 2.14 gives gravity, the gradient vector of geopotential:

$$g = -\vec{\nabla}\,W \tag{2.15}$$

2.1.1.6 *Laplace's Equation*

The geopotential W and its first derivative, the attractive force g, are continuous everywhere. However, the second derivative of the gravitational potential W_G is only continuous outside the attracting body - in the space exterior to the earth and its atmosphere. In this region it satisfies Laplace's Equation:

$$\vec{\nabla}^2 W_G = 0 \tag{2.16}$$

$\vec{\nabla}^2$ being the Laplacian operator defined by,

$$\vec{\nabla}^2 = \frac{\partial}{\partial x_1^2}\vec{1} + \frac{\partial}{\partial x_2^2}\vec{2} + \frac{\partial}{\partial x_3^2}\vec{3} \tag{2.17}$$

W_G is said to be harmonic in this space (e.g. GROTEN 1979, Section 5.12). The rotational potential W_R is everywhere non-harmonic ($\vec{\nabla}^2 W_R = 2\omega^2 \neq 0$), consequently the gravity potential W is non-harmonic in the space exterior to matter. Nevertheless, W and all its derivatives are continuous in this space.

2.1.1.7 *Equipotential or Level Surfaces*

The surfaces of constant geopotential,

$$W\,(x_1, x_2, x_3) = \text{constant} \tag{2.18}$$

are the equipotential or level surfaces of the earth's gravity field. They are also known as geops.

The gravity vector or the direction of the vertical at a point is perpendicular to the equipotential surface passing through that point. The plumbline intersects all equipotential surfaces normally and is therefore slightly curved.

Geodetic measurements are almost exclusively referred to the system of level surfaces and plumblines that constitute the earth's gravity field. The principal aim of geodesy is the determination of the position of points at or near the earth's surface from measurements made within the earth's gravity field. This is possible only if the locations of the equipotential surfaces are known. Therefore, in the first instance, it is necessary to determine the geopotential function

2.1.1.8 *Deformation of the Earth*

The concept of a non-rigid or deformable earth implies a redistribution of mass, with attendant variations in the earth's gravity field on a variety of time scales (see e.g. MELCHOIR 1966 for a discussion on luni-solar tidal variations and MUNK & MACDONALD 1960 for other geodynamical phenomena). All geodetic measurements are related, in some way, to the instantaneous gravity field.

The magnitude of gravitational deviations from a solid (non-deformable) earth model are smaller than 10^{-6}g (MATHER 1973c, p.118). The largest effect is the diurnal earth tide variation with magnitude o{10^{2}μgal}, while local atmospheric circulation, oceanic effects and changes in local groundwater content are of the order of 10μgal. The notation o{x} means "of the order of x". A far less significant effect is due to variations in the rotation rate ω, estimated at less than 1μgal (10^{-9}g) for a 10 msec variation in the length of day (IBID, p.121).

A rigid body model for the earth poses no difficulties for geodetic time scales (periods < 10^2 years) and in the context of observational accuracies which are no better than 1 part per million (1ppm). However, with significant improvements in metrology, the time variations in geodetic measurements need to be taken into account to ensure that the results are not degraded to below the level of precision sought. "Four dimensional geodesy" is a convenient term for high precision techniques which in addition to satisfying the traditional goal of geodesy, that of determining position and gravity, are concerned with the time variations of these quantities (ANGUS-LEPPAN 1973).

In modern high precision geodesy there are two options available for the treatment of deformation effects:

(1) The effects may be incorporated into the description of the model by adopting a non-rigid earth model and working with the raw observational data. Such data can only characterise an instant of time and represents a kinematic description of the earth's gravity field.

(2) The effects may be removed from the observations by modelling prior to any computations. The resultant information affords a dynamic representation of the gravity field.

In most cases it is preferable to adopt the approach at (2), the consequence of which is that no geodynamical information can be drawn from subsequent computations.

In the case of gravity, most of the deformation effects are 6-8 orders of magnitude smaller than g. The adoption of a standard model for the largest of these effects, i.e. the earth tides, allows observed gravity to be corrected for this departure of the earth from a rigid body model. However the accuracy of the majority of present day gravity observations is inadequate to warrant the modelling of such deformation effects. Such corrections are only necessary at fundamental gravity stations where determinations of g are made with the highest possible resolution, for the definition of global gravity standardisation networks or when attempting to study geodynamical phenomena of a smaller magnitude than the earth tide effect, e.g. locating changes in the position of the geocentre with time (MATHER ET AL 1977b).

In this thesis, unless otherwise stated, it will be assumed that the earth is non-deformable and located in a space free of external masses. Those deformation effects smaller than the phenomenon under study will be ignored, while those effects having a magnitude greater than the precision of the observation system will be removed from the data prior to analysis.

2.1.2 REFERENCE GRAVITY AND SPHEROPOTENTIAL

As a result of mass-density variations within the earth, geopotential surfaces have irregular shapes. Although analytical, they have no simple analytical expression, making them unsuitable as a basis for either a coordinate system (e.g. based on latitude and longitude) or for gravity computations. A simple analytic reference figure is chosen to best fit the most physically realisable equipotential surface. This geop closely *approximates* mean sea level (MSL) and is called the geoid. An unambiguous definition of the geoid is presented in section 2.2.

2.1.2.1 *Reference Ellipsoid*

The reference figure that is universally adopted in geodesy is a biaxial symmetrical ellipsoid centred at the geocentre, with its semi-major axis collinear with the earth's axis of rotation. The choice of geocentric ellipsoid is quite arbitrary. The shape of the reference ellipsoid is defined by the equatorial radius a and flattening f. In addition, as a best-fitting model for the earth, it has rotational characteristics and total mass that closely approximate those of the earth.

2.1.2.2 *Spheropotential*

The reference ellipsoid has both gravitational and rotational potential characteristics, and hence possesses a normal gravity field. The total potential U of the normal gravity field is known as the spheropotential,

$$U = U_G + U_R \tag{2.19}$$

U_G and U_R are the gravitational and rotational potentials respectively.

The angular velocity of the earth ω is well known (equation 2.14), hence the "real" rotational potential W_R and the "normal" rotational potential U_R are, for all intents and purposes, equal. Variations in ω due to the earth being non-rigid have already been established as being insignificant.

The reference ellipsoid has a homogeneous mass-density distribution, hence U_G varies with latitude but not with longitude and is easily computed from the parameters of the reference ellipsoid by formulae such as equation 5.22. The family of confocal ellipsoids are all equipotential surfaces of the normal gravity field. These surfaces, also known as spherops, are defined as:

$$U\ (x_1, x_2, x_3) = 0 \tag{2.20}$$

The normal plumbline intersects all spherops normally and is curved in the meridional plane only (HEISKANEN & MORITZ 1967, p.196).

The reference ellipsoid, a level surface of the normal gravity field (potential $U = U_0$), is the "normal" equivalent of the geoid, a level surface of the "real" (though non-deformable) gravity field (geopotential $W = W_0$).

Recommended parameters of a rotating gravitational equipotential ellipsoid known as the Geodetic Reference System 1967 (IAG 1971) are:

Geometric constants { $a = 6378160$ m (exact), $f^{-1} = 298.247167427$

Physical constants { $GM = 398603\ km^3 s^{-2}$, $\omega = 7.2921151467 \times 10^{-5}$ rad s^{-1} (exact)

Derived values of normal gravity and spheropotential for this ellipsoid are:

$\gamma_e = 978.03184558$ gal - normal gravity at the equator

$U_o = 6263703.052$ kgalm - normal potential of the ellipsoid

This reference system is the basis of most of the world's gravity anomaly data banks (see Chapter 6). The reference model used in the elevation datum study is one based on the parameters of the Geodetic Reference System 1980 (MORITZ 1979), with the exception of the value of a (= 6378137m for GRS 80), and is described in § 8.2.3.3.

2.1.2.3 *Gravity Anomaly and Normal Gravity*

The real gravity field can thus be split into a normal field and a remaining small "disturbing" part. The gravity anomaly Δg is the difference between g and the quantity γ, the normal or reference gravity of the model:

$$\Delta g = g - \gamma \tag{2.21}$$

The gravity anomaly is discussed in greater detail in sub-sections 2.3.5 and 4.1.4.

Normal gravity is the gradient vector of the spheropotential,

$$\gamma = -\vec{\nabla} U \tag{2.22}$$

and may be computed by formulae found in IBID (Section 6.2, 6.3). The direction of the normal gravity vector γ defines the normal to the spherop surfaces.

2.1.2.4 *Disturbing Potential*

The difference between the spheropotential and the geopotential is the anomalous or disturbing potential T:

$$T = W - U \tag{2.23}$$

On combining equations 2.12 and 2.19, neglecting the deformation component of the geopotential and noting that $U_R = W_R$, gives:

$$\vec{\nabla}^2 T = 0 \tag{2.24}$$

for the space exterior to the earth and its atmosphere. The disturbing potential is discussed in greater detail in sub-sections 2.3.3 and 4.1.3.

2.1.2.5 *Geoid Undulations*

The separation between the geoid and the reference ellipsoid is the geoid height or geoid undulation. The magnitude of the geoid undulations with respect to a "best-fitting" reference ellipsoid is of the order of ± 100m. A knowledge of the geoid undulations as a function of position would allow the geoid to be geometrically mapping in relation to the (known) reference ellipsoid. Methods of "determining" the geoid are described in sections 2.3 and 5.1.

Just as the geoid does not coincide everywhere with the reference ellipsoid, the whole family of geopotential surfaces depart from their numerically equivalent spheropotential surfaces. Once the geoid has been determined, the other geopotential surfaces can be located by the method of geodetic levelling, described in sub-section 3.2.1.

2.1.2.6 *Deflection of the Vertical*

Alternatively, the departures of the "real" gravity field from the "normal" gravity field may be described in terms of the difference in direction between the actual gravity vector (the plumbline) and the normal gravity vector. This quantity is the deflection of the vertical and forms the basis of a method of geoid determination described in sub-section 2.3.2.

2.2 THE GEOID : ITS DEFINITION

2.2.1 PREAMBLE

It was recognised last century that the mean surface of the oceans was part of a level surface of the earth's gravity field. This particular geop was first proposed by Gauss as the "mathematical figure of the earth" (HEISKANEN & MORITZ 1967, Section 2.2) and was subsequently termed the "geoid" by the German geodesist Listing (LELGEMANN 1976).

This definition was very useful for the practical determination of orthometric heights (see § 3.2.1.1) on the assumption that mean sea level, as defined by tide gauge readings, was coincident with the datum level surface to which all elevations were referred (see e.g. MATHER 1978d). As a corollary to this it was held that different height systems based on local MSL were related to this unique geop - the geoid.

2.2.2 COMMENTS ON THE GEOID TO A PRECISION OF ONE PART PER MILLION

As already mentioned, the geoid was assumed to be synonymous with the directly observable physical manifestation of the elevation datum afforded by the mean level of the oceans, albeit a coastal estimate. The geoid/MSL surface satisfied two main roles in classical geodesy (RAPP 1974a):

(i) The geoid represented the "figure of the earth" and it therefore was the ultimate aim of "scientific" geodesy to determine its shape and size.

(ii) At a practical level, the principal role of the geoid was a passive one as a datum for geodetic levelling.

The reference ellipsoid is the most suitable mathematical figure for the computation of horizontal position using traditional ground level geodetic observations. Such computations are performed with distance and angle measurements reduced to the ellipsoid with the aid of information on the elevations of survey stations above the reference ellipsoid (see e.g. BOMFORD 1962, Chapter 3).

In the context of such 1 part in 10^6 geodesy - an absolute accuracy of a few metres in position - the prime task was the determination of the shape (a,f) of the ellipsoid which "best fits" the local geoid from the results of regional surveys (e.g. CLARK 1968, p.408-409 gives some results for the determination of a and f). Such an ellipsoid was not necessarily centred at the geocentre. Nevertheless, it was possible to use the height above MSL (an observable quantity) to define position, in the vertical sense, for reduction purposes instead of the preferred height above ellipsoid (unobtainable from traditional survey techniques).

Implicit in such a procedure is the fact that the statements "height above MSL" and "height above geoid" are equivalent, and that the reference ellipsoid is one of best fit (in the geometric sense) to the geoid. Such conditions are easily satisfied in the context of geodesy to 1 part per million, where the geoid is considered to be a physical reality. As a consequence, the geoid can be located and monitored by a network of tide gauges. In other words, MSL is considered dependent only on the gravitational attraction of the earth and atmosphere, and the rotational potential. Averaging tide gauge readings over a period of time (1-19 years) was supposed to give the MSL surface free of wind, tides and other periodic effects.

However it has long been held by physical oceanographers that on the basis of air pressure, ocean temperature and salinity data, as well as from surface current observations, the free ocean surface (apart from periodic tidal and wind effects) does not lie along any equipotential surface of the earth's gravity field (MONTGOMERY 1969, p.147, DEFANT 1961, Chapter XI). The deviation of the ocean surface from a level surface is termed the *Sea Surface Topography* (SST) using the analogy that exists on land. In particular, departure of the ocean surface from the geoid is referred to as the *dynamic* sea surface topography. The distinction in labels will not be made as the datum for the SST, if critical to the discussion, will be clear from the context.

The stationary SST is the deviation of the MSL from the geoid and has an amplitude estimated at ± 1-2m (see Figure 3.3), while the magnitude of the time-varying component (deviation of the instantaneous sea surface from the geoid) is at the sub-metre level (see sub-section 3.2.2). Consequently the definition of the geoid as that geop corresponding to MSL poses no problems for 1 part in 10^6 geodesy, despite the acknowledged existence of SST.

2.2.3 COMMENTS ON A HIGH PRECISION GEOID

The aforementioned classical definition of the geoid is adequate for geodetic considerations to the ± 2m level but significant inconsistencies would arise if it were extended to instances where sub-metre precision was required. The primary uncertainty is whether each local estimate of MSL, based on the mean of at least one year's tide gauge data, coincides with the geoid locally, to the precision sought. The more stringent requirement for a geoid accurate to ±10cm (a resolution to 1 part in 10^8) for SST studies renders the traditional criterion for geoid identification so vague as to be useless under such accuracy requirements (see e.g. FISCHER 1977, MATHER 1974c for background).

Although recognising that mean sea level is a surface which is not level (in the geodetic sense), geodesists have refused to relinquish the central role that MSL plays in the definition of the geoid. Consequently, in order to satisfy the requirements of high precision geodesy, the physical manifestation of the geoid was modified to become the surface of a homogeneous ocean (free of density variations) under the influence of the earth's gravity field but excluding all other external forces such as luni-solar and planetary gravitation, ocean currents and circulation.

The literature is sprinkled with such "refined" definitions : for example, the geoid is that particular geop which coincides with the "idealised sea surface" (e.g. RAPP 1977, p.123, LELGEMANN 1976), the "undisturbed MSL surface" (e.g. FISCHER 1977, p.38, MOURAD ET AL 1975, p.1887, FUBARA & MOURAD 1971, p.5-2) or the surface of a "uniform and static ocean" (VONBUN ET AL 1975, p.1), to name a few. The result of such tautology is that the imaginary geoid surface is now represented by an ocean surface which itself has no reality. In addition, such definitions cannot form the basis for numerical evaluations at the $\pm$10cm level.

It is preferable that a unique definition for the geoid to $\pm$10cm should utilise, in some manner, the following observables:

- MSL at tide gauges and in the open oceans.
- non-tidal SST data in oceanic areas.
- the heights of continental levelling datums above a level surface, i.e. SST at coastal tide gauges.

Sub-section 2.2.4 discusses the reasons for the need of a geoid of $\pm$10cm accuracy, while sub-section 2.2.5 outlines possible conceptual definitions of the geoid to this level of accuracy.

2.2.4 THE NEED FOR A GEOID OF $\pm$10cm ACCURACY

Until recently it was inconsequential to carry out geoid determinations to a high precision as it was highly unlikely that such determinations could ever be put to any practical use. Further, the geoid could no longer lay claim to being the "mathematical figure of the earth" in the scientific sense, as that label could be applied to any of a number of surfaces including the physical surface of the earth (§ 2.3.5.3). In addition, the development of geodetic techniques for geometric position fixing from observations to extraterrestrial objects with $\pm$10cm precision appeared to obviate the need for a very accurate geoid on a global basis. In the short term however, geodetic levelling in combination with geoid height determinations will continue to define the ellipsoidal elevation, and hence geocentric position, to a few centimetres on a regional basis (see sub-section 3.2.2).

Interest in a high precision geoid has been generated within the last 10 years by the need to support certain oceanographic studies (see sub-section 1.1). A high precision geoid is needed for the following purposes (MATHER ET AL 1976a):

(1) The *determination* of sea surface topography.

(2) The *unification* of geodetic levelling datums.

(3) The *definition* of relations in physical geodesy.

As the magnitude of the SST is only of the order of $\pm$1-2m, a precision of at least an order of magnitude better is required in geodetic operations that support oceanographic studies described in Chapter 5. A corollary to this is the irrelevance of defining the geoid in continental areas to this same order of precision. It is the determination of the marine geoid to the ± 10cm level that is therefore the real challenge in physical geodesy (MATHER 1974c, Section 2).

Methods of determining the shape of the geoid are described in section 2.3, while practical possibilities for the evaluation of the marine geoid to ±10cm accuracy are discussed in section 5.1. However, an adequate *conceptual definition* must necessarily precede any determination. Such definitions are attempted in sub-section 2.2.5.

2.2.5 A CONCEPTUAL DEFINITION OF THE GEOID ADEQUATE FOR HIGH PRECISION GEODESY

2.2.5.1 *Preliminary Remarks*

A conceptual definition of the geoid for use in 1 part in 10^8 geodesy should reflect the real relationship between the observable MSL and the level surface, of the earth's gravity field. Obviously an a priori definition is not possible, rather data to be used in implementing such a definition is collected over a finite time period and the resultant geoid only characterises this *epoch*. The term "epoch" is used to refer to a time span of the order of 1-2 years, during which a system of observations is established for the purpose of determining position, ideally, on a global basis. In addition, such a geoid definition should have continuity over long time periods if it is to have relevance in the context of four dimensional geodesy, particularly for vertical crustal motion studies - discussed in section 8.3. This requires that the geoid be capable of being unambiguously identified from one epoch to another. Section 2.4 describes variations in the shape and definition of the geoid with time.

Obviously, in order that the geoid satisfy ± 10cm accuracy requirements in *four dimensions*, its spatial position must be capable of being monitored in four dimensions to this accuracy. The choice of a special equipotential surface as the geoid seems therefore, in a certain way, *arbitrary*. However, the geop selected as the geoid must still be capable of satisfying its traditional role under the less stringent requirments of 1 part in 10^6 geodesy. For example, adopting an arbitrary geopotential value for the geoid, say 6 x 10^6 kgalm, although providing a rigorous definition, lacks that special utility that the less precise geoid/MSL concept had for routine mapping and survey operations (sub-section 2.2.2). Ideally, the geoid defined in a global context should (IBID):

(i) Characterise a particular epoch.

(ii) Allow for the existence of stationary sea surface topography.

The first is necessary to ensure secular variations in the geoid are adequately handled (see section 2.4). The second would call for a global averaging procedure in the definition.

Data used in the definition of the geoid has traditionally come from tide gauge stations, the vast majority of which are located on coastlines. The averaging of tide gauge readings over a sufficiently long period of time enables *local* MSL to be determined to an accuracy better than ± 10cm.

The Geodynamics Experimental Ocean Satellite (GEOS-3) was launched in April 1975 and in addition to the usual instrumentation for accurate position fixing it carried a 13.9 gigahertz radar altimeter for measuring the vertical distance to the sea surface. Such altimetry data provides estimates of ζ, the height of the instantaneous sea surface above the reference ellipsoid (see sub-section 2.3.4) and is therefore another source of information for the definition/determination of the geoid.

Satellite altimetry data can provide estimates of the mean value of ζ for some $n^o \times m^o$ area, for some epoch of observation. Such sampling can be carried out on an equi-angular or equi-area basis for almost all oceanic regions. Both systems of areal subdivisions are based on surface areas defined by latitude and longitude increments n^o and m^o respectively. In the equi-area scheme, as the name suggests, each elemental subdivision represents (approximately) a constant surface area on the earth. Such sampling is preferable to that of an equi-angular system based on a geographic grid of latitude and longitude. Bands of latitude of constant width (n^o) gird a smaller surface area with increasing latitude ϕ. The number of elemental areas ($n^o \times m^o$) in such a band for the equi-angular system is constant at $I\{360/m\}$, while for the equi-area scheme it is $I\{360\cos\phi/m\}$. $I\{x\}$ represents the integer part of x. An equi-angular sampling would result in unacceptable aliasing due to the greater influence of samples from the polar regions on the outcome of any global averaging.

Note that ambiguity can occur in the selection of a level surface as the geoid depending on what data is used and how it is sampled. A variety of definitions have been proposed (e.g. MATHER 1975c, LELGEMANN 1976).

Different conceptual definitions of the geoid are discussed below (see MATHER 1978d, p.218 et seq.). Each one identifies a different level surface as the geoid, but the practical utility of one definition may be higher than the others.

2.2.5.2 *The "Geodetic" Definition*

> "The geoid, for a selected epoch of measurement, is that level surface of the earth's gravity field in relation to which the average non-tidal SST has zero mean at all the world's levelling datums."

The sampling is on an equi-area basis and restricted to land areas serviced by a geodetic levelling network tied to regional MSL. Landlocked countries or continental areas smaller than the sampling area ($n^o \times m^o$), such as islands, would require special treatment.

The input for this definition for each region would be the SST at the datum tide gauge, and would be constant for all land areas serviced by this levelling datum. The number of different values of SST is equal to the number of different elevation datums. See section 5.3 and 8.2 for a discussion on methods of estimating SST at the tide gauges defining regional elevation datums.

This scheme may be modified to allow sampling of *all* the world's tide gauges. However such a definition would be sensitive to any alteration in the original ensemble of tide gauges, either by inclusion of new or removal of old gauges (to a lesser extent this is also true for the original definition based on *datum* tide gauges).

It is likely that the geoid defined by tide gauge data alone will be unrepresentative of MSL in the open oceans as distorted patterns of sea level are expected in coastal areas (see section 3.4). Further, this definition does not incorporate samples from the 70% of the earth's surface that is oceanic.

In a modification proposed by LELGEMANN (1976) the existing levelling datums are corrected for local SST as deduced by oceanographers (Figure 3.3) and the geoid then chosen in such a manner that the sum of the squares of the departures of the "corrected" datum gauges from the geoid is a minimum.

2.2.5.3 *The "Oceanic" Definition*

> "The geoid, for a selected epoch of measurement, is that level surface of the earth's gravity field in relation to which the average non-tidal SST is zero as sampled globally on an equi-area basis in oceanic regions."

This definition is particularly favoured by oceanographers (e.g. MONTGOMERY 1969), the marine geoid being obtained from SST data provided by oceanic levelling techniques (see sub-section 3.3.3). In fact, locating the level surface to be the geoid has been mentioned as the "contribution" that oceanography can make to geodesy (IBID, p.150) - see sub-section 2.3.7.

The "oceanic" definition, if based on SST determined using satellite altimetry data, is acceptable to geodesists.

2.2.5.4 *The "Oceanic/Geodetic" Definition*

> "The geoid, for a selected epoch of measurement, is that level surface of the earth's gravity field in relation to which the average non-tidal SST is zero as sampled globally on an equi-area basis in both oceanic regions and at continental tide gauges."

In addition to incorporating satellite altimetry data (as for § 2.2.5.3), the heights of tide gauges above a datum level surface are included, as in the modification to the "geodetic" definition. The datum level surface represents a "preliminary" geoid that facilitates the evaluation of the SST for subsequent "refinement" of the geoid definition. For example, the definition at § 2.2.5.2 could provide the "preliminary" geoid.

Only those areal subdivisions which contain a tide gauge are included in the averaging process and, although their contribution is likely to be small in comparison to oceanic regions, aliasing may result due to the distorting effect of coastal phenomena on sea level. In addition, this definition suffers from the drawbacks of the "geodetic" definition, namely difficulty in determining SST at individual tide gauges and the questionable stability of such a geoid definition over long time periods.

2.2.5.5 *The "Geodetic Boundary Value Problem (GBVP)" Definition*

> "The geoid, for a selected epoch of measurement, is that level surface of the earth's gravity field in relation to which the average non-tidal SST has no zero degree harmonic in solutions of the Geodetic Boundary Value Problem."

The Geodetic Boundary Value Problem (GBVP) is posed in § 2.3.5.1 and the influence of the geoid definition on such solutions is discussed in sub-sections 4.2.2 and 5.1.3.

The GBVP definition of the geoid requires a complete global sampling for its realisation. Oceanic areas are sampled by satellite altimetry, as in the "oceanic" definition (§ 2.2.5.3), while land areas are represented by values of SST at the regional datum for elevations used in the compilation of gravity anomalies (see sub-section 3.2.2). The difference between this definition and that afforded by the "oceanic/geodetic hybrid" (§ 2.2.5.4) is that in the latter definition as many samples of coastal SST are needed as there are tide gauges, whereas in the "GBVP" definition a whole land mass serviced by one elevation datum (not necessarily a tide gauge) is represented by the single value of SST at the datum. For example, the Australian land area would be represented in such a averaging procedure by the value of SST at the Jervis Bay Datum (see § 6.2.2.3).

2.2.5.6 *Summary Remarks*

The definitions described in § 2.2.5.3, 2.2.5.4 and 2.2.5.5 provide a conceptual definition of the geoid that is more representative of MSL globally than that afforded by the "geodetic" definition (§ 2.2.5.2). However, any lack of a global coverage of data, especially satellite altimetry, could result in a realisation which is approximate. The altimetry data provided by the GEOS-3 spacecraft is restricted to regions between the parallels 65°N and 65°S. A geoid defined from such a limited coverage may be adequate for stationary SST studies, but may not have relevance in the context of high precision four dimensional geodesy.

In deciding upon a particular conceptual definition of the geoid to be adopted, thought should be given to the following points:

- No assumption should be made concerning the pattern of SST.
- Satellite altimetry data should play a direct role in the definition of the geoid as it is the only type of geodetic data that (potentially) covers the whole oceanic region.
- The purposes for which the geoid is to be used should be taken into account, e.g. continental geodesy or marine geodesy.

Neither the existence of sea surface topography or the possibility that geodetic levelling datums do not lie on a unique geop need inhibit the realisation of a definition for the geoid at the ±10cm level. In fact, upon adopting a unique definition, effects of zero degree can be isolated and further analysed for dynamic, gravitational and other scale effects (see § 5.1.3.1, 7.3 and 8.2.6).

The preferred definition for the geoid is the "oceanic" one (§ 2.2.5.3), and its implementation is attempted in § 5.1.3.2.

2.3 THE GEOID : ITS DETERMINATION

2.3.1 PREAMBLE

The object of a geoid "determination" is to define the shape of the geoid in relation to a reference figure. As mentioned in sub-section 2.1.2, the geoid surface is irregular, not conforming to any simple geometric figure, and is spatially related to the reference ellipsoid by the geoid undulation. The "determination" of the geoid is therefore primarily the task of computing the geoid-ellipsoid separation by some technique which utilises one or more of the following types of data available at present and for the foreseeable future,

(1) Gravity measured at the surface of the earth. Such data is mostly confined to land and continental shelf areas.

(2) Tracking data to near-earth artificial satellites. The orbital perturbations can be analysed for a model of the geopotential.

(3) The location of the instantaneous sea surface on a geocentric Cartesian coordinate system X_i as derived from satellite altimetry data.

Information from other sources such as geodetic levelling, horizontal geodetic surveys, oceanographic surveys, astronomical observations and three dimensional position fixes can also play a role.

In order to improve the areal distribution some types of data can be predicted from real data, for example gravity from satellite altimetry data (see e.g. KOCH 1970, RAPP 1974b, RAPP 1076a). Techniques of statistical geodesy, principally those making use of auto- and cross-covariance functions of and between different geodetic data types will not be discussed here. The interested reader is referred to such articles as MORITZ (1972), MORITZ (1973), NASH & JORDAN (1978) for an introduction to the role that statistical methods can play in satisfying traditional geodetic goals. The "classical" methods of utilising geodetic data for the determination of the earth's gravity field will only be considered.

There are a number of categories of techniques for the computation of geoid undulations. Each technique is based on a different principle, highlighting the relationship between a certain observable in the real gravity field and its corresponding "anomalous" quantity in the gravity field afforded by an ellipsoidal model. The three major data types expressed in terms of their anomalous components are:

(1) Gravity data is usually expressed in the form of gravity anomalies (equation 2.21).

(2) A geopotential model is ordinarily expressed in terms of the disturbing potential of the earth's gravity field (equation 2.23).

(3) The geometry of the sea surface is more conveniently represented in terms of the heights of the sea surface ζ above the reference ellipsoid.

For some techniques the geoid undulations are computed in an indirect fashion as, for example, from gravity anomalies. In other cases, the geometry of the undulations is established directly from linear or angular measurements. Others are based on the knowledge of the disturbing potential or combinations of some or all of the above.

It is not the intention here to give a thorough description of all the methods of geoid determination. The remainder of this section will be devoted to a brief outline of the various methods, examining the underlying theory, its deficiencies and the available quality and global distribution of data required for such calculations in order to establish the practicability for their use in marine geoid determinations to $\pm$ 10cm precision.

The geoid undulations may be determined in the form of point values, profiles or as a continuous surface on a variety of wavelengths.

The spectrum of geoid undulation information can be considered to be composed of short wavelength features (< 10^2km), medium wavelength features (10^2 - 10^3km) and long wavelength features (> 10^3km).

The techniques of geoid determination have been grouped into the following categories:

- Astrogeodetic methods.
- Passive satellite methods.
- Active satellite methods.
- Gravimetric methods.

- Combination methods.
- Non-geodetic methods.

These are described in following sections.

2.3.2 ASTROGEODETIC METHODS OF GEOID DETERMINATION

The normal to the spherop passing through a point A on the surface of the earth intersects the local vertical (the normal to the geop through A) at some angle ε, called the deflection of the vertical. The angle ε is measured in the plane containing both normals and is at some arbitrary azimuth α,

$$\varepsilon = \xi \cos\alpha + \eta \sin\alpha \qquad (2.25)$$

where ξ and η are the components of the deflection of the vertical in the meridional and prime vertical directions respectively.

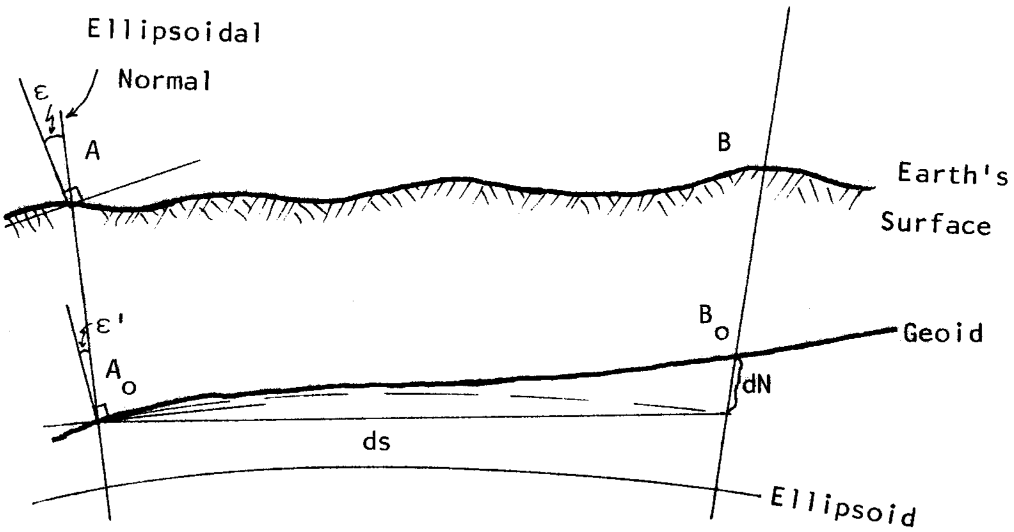
The increment dN in geoid height N for a distance ds (see Figure 2.1) gives the slope of the geoid with respect to the ellipsoid,

$$dN = -\varepsilon' \, ds \qquad (2.26)$$

where ε' is the deflection of the vertical referred to the point A_o on the geoid, related to the ground level deflection ε by formulae given in, e.g. HEISKANEN & MORITZ (1967, p.196).

FIGURE 2.1

Astrogeodetic Levelling



On integrating equation 2.26, the shape of the geoid can be determined if the deflections of the vertical along the profile AB are known. The basic relation is:

$$N_B = N_A - \int_A^B \varepsilon' \, ds \qquad (2.27)$$

The selection of suitable profiles and adoption of a value for the geoid height at the origin of profiles allows a geoidal map to be prepared. The method of determining grades of the geoid between stations by the use of equation 2.27 is known as astrogeodetic levelling and a detailed description of the method can be found IBID (Chapter 5).

The ground level deflections of the vertical can be obtained in a number of ways. Traditionally, the astrogeodetic geoid has been computed by comparing the geodetic and astronomic coordinates at the same point. The relationship between geodetic latitude ϕ, geodetic longitude λ on the one hand and astronomic latitude Φ, astronomic longitude Λ on the other hand is:

$$\xi = \Phi - \phi$$
$$\eta = (\Lambda - \lambda) \cos\phi \qquad (2.27a)$$

The geodetic coordinates are established for the purpose of providing horizontal control for mapping operations by methods of terrestrial triangulation, trilateration, etc. (see e.g. BOMFORD 1962, Chapters 1,2,3). The astronomic coordinates are determined directly from observations to stars (see e.g. CLARK 1968, Chapters 1,2) and are related to the direction of the gravity vector at the surface by means of the fluid in the vial of the observing instrument. Within the limits of observational accuracy, errors in star catalogues and evaluation of a series of corrections necessary for the reduction of astronomical measurements, the astronomic coordinates (Φ, Λ) are unique. This, however, is not the case with geodetic coordinates (ϕ, λ), which are related to an arbitrary "astrogeodetic" datum defined by the parameters a and f of a reference ellipsoid and adopted geodetic coordinates at the "origin of surveys" (IBID, Section 3.06). Consequently, the centre of the reference ellipsoid to which measured distances and angles are reduced in order to determine geodetic position is not necessarily coincident with the geocentre. Hence astrogeodetic geoids computed on different geodetic datums, with reference ellipsoids of different sizes and orientations, are incompatible without transformations of the type described in, e.g. HEISKANEN & MORITZ (1967, Section 5.9), MATHER (1970b).

Nowadays, geodetic coordinates can also be obtained by the techniques of satellite geodesy (e.g. MUELLER 1964, Section 2.51). Unlike conventional geodetic coordinates, satellite derived positions are obtained in relation to a geocentric reference ellipsoid. The resultant astrogeodetic geoid is earth-centred and of correct scale only if the absolute geoid undulation is known at one point in the geodetic network. Unfortunately, at present the density of stations with satellite-determined coordinates is too low to provide a geoid of adequate quality.

In moderately level areas a station spacing of, say, 25km may be necessary for the following approximation to equation 2.27 to be valid,

$$N_C - N_D = -\frac{\varepsilon'_D + \varepsilon'_C}{2} \cdot s \qquad (2.27b)$$

where C and D are two neighbouring astrogeodetic stations and s is the distance between them. In mountainous areas a spacing of 10km or less may be necessary to ensure a sub-metre precision. The average station spacing for the Australian astrogeodetic geoid was less than 35km (FRYER 1971). The smallest features of the geoid that can be resolved by the astrogeodetic technique are those with half wavelengths equal to the average station spacing.

The astrogeodetic method has been applied for the determination of geoidal sections on land by e.g. LEVALLOIS (1978) in Europe, FISCHER ET AL (1967) in Central and North America, FISCHER & SLUTSKY (1967) and FRYER (1971) in Australia.

A discussion of the method's practical aspects and accuracy can be found in BOMFORD (1962, Section 5.5). The deficiencies of astrogeodetic methods for the determination of the *marine geoid* to ± 10cm precision are:

(1) It is not practicable for geoid determinations at sea due to difficulties in obtaining deflections of the vertical of adequate precision.

(2) The limited accuracy of astronomic observations and atmospheric refraction corrections. For example, one metre accuracy in geoidal sections of 10^3km length and station spacings 20 - 100km in geodetically featureless areas would require that random errors in the astronomic latitude and longitude be held to less than 1 arcsec and systematic errors below 0.2 arcsec (BOMFORD 1962, Sections 5.42, 5.45).

Long profiles of astrogeodetic levelling suffer from rapid error accumulation away from the starting point as a result of:

(a) systematic errors in astronomic observations.

(b) errors in the geodetic network, particularly if it has been established by conventional ground surveys. In such a case the network is weakest on the peripheries and strongest in the vicinity of the "origin of surveys".

(c) errors in evaluating the curvature of the plumbline correction to ε for the computation of ε' (HEISKANEN & MORITZ 1967, p.196).

In practice, suitable profiles may be selected to form closed loops from which redundancies can be obtained and adjusted in some manner (see e.g. FRYER 1971).

(3) The astrogeodetic method is highly susceptible to local fluctuations in the grade of the level surfaces, and consequently the accuracy of the geoid determination falls off as a function of increasing astrogeodetic station spacing. Changes of up to 25 arcseconds have been known to occur in Australia over distances of 60km (see MATHER ET AL 1971, Figure 3.4), with more rapid changes in more mountainous areas.

(4) In the most commonly used technique of astrogeodetic levelling, the resultant geoid undulations are established in relation to an ellipsoid whose centre is not coincident with the geocentre. It is doubtful whether the transformation parameters are sufficiently well known at present for geoid determinations adequate for global studies in earth and ocean dynamics. See MATHER (1970b) for an attempt at determining the geocentric orientation vector for the Australian Geodetic Datum.

In conclusion, the astrogeodetic method of geoid determination is useful in the context of establishing geoid undulations in land areas, on a relative basis, to a precision of the order of a few metres. It is however unsuitable for the determination of a high precision marine geoid.

2.3.3 PASSIVE SATELLITE METHODS OF GEOID DETERMINATION

2.3.3.1 *Dynamic Methods*

The gravitational potential of the earth W_G is harmonic in the space exterior to the earth's atmosphere (i.e. satisfies Laplace's equation, equation 2.16). A solution to Laplace's differential equation at some point P with spherical coordinates (ϕ,λ,R) is expressed as an infinite series of spherical harmonics (HEISKANEN & MORITZ 1967, Section 1.9, 1.10):

$$W_{Gp} = \frac{GM}{R_p} \sum_{n=0}^{\infty} \left(\frac{a}{R_p}\right)^n \sum_{m=0}^{n} \sum_{\alpha=1}^{2} C_{\alpha nm} S_{\alpha nm} \qquad (2.28)$$

where a is the radius of some arbitrary sphere, R_p is the geocentric distance to P and GM is the product of the gravitational constant and the mass of the earth and atmosphere. $S_{\alpha nm}$ are the harmonic functions defined by:

$$S_{1nm} = P_{nm}(\sin\phi)\cos m\lambda \quad ; \quad S_{2nm} = P_{nm}(\sin\phi)\sin m\lambda \qquad (2.29)$$

where $P_{nm}(\sin\phi)$ is the Legendre function of degree n and order m (IBID, Section 1.11).

The spherical harmonic coefficients $C_{\alpha nm}$ can be determined from the analysis of orbital perturbations of artificial satellites by methods described in e.g. MUELLER (1964), KAULA (1966a). Such estimates of $C_{\alpha nm}$ are denoted by $C'_{\alpha nm}$. The coefficients $C'_{\alpha nm}$ together with constants a, GM and ω make up the geopotential or gravity field model of the earth. The Goddard Earth Model (GEM) series are examples of such models (SMITH ET AL 1976).

The geopotential W of a non-deformable earth is given by (equation 2.12):

$$W = W_G + W_R \qquad (2.30)$$

or

$$W = (W' + W_A) + W_R \qquad (2.31)$$

where W' is the gravitational potential of the solid earth and oceans, W_A is the gravitational potential of the atmosphere and W_R is the rotational potential (given by equation 2.13). The atmospheric potential W_A is discussed in section 4.5.

The gravitational potential W' can be represented by:

$$W'_p = \frac{GM}{R_p} \sum_{n=0}^{n'} \left(\frac{a}{R_p}\right)^n \sum_{m=0}^{n} \sum_{\alpha=1}^{2} C''_{\alpha nm} S_{\alpha nm} \qquad (2.32)$$

where $C''_{\alpha nm}$ are the coefficients $C'_{\alpha nm}$ adjusted for the atmospheric effect (see § 5.1.3.2). n' is the highest degree to which the coefficients $C'_{\alpha nm}$ are known.

The disturbing potential of the solid earth and oceans T' is defined by:

$$T' = W' - U_G \tag{2.33}$$

where U_G is the normal gravitational potential of the reference ellipsoid. T' is harmonic in the space exterior and down to the surface of the earth and is related to the disturbing potential T defined in equation 2.23 by:

$$T' = T - W_A \tag{2.34}$$

Or expressed in terms of a spherical harmonic representation:

$$T'_p = \frac{GM}{R_p} \sum_{n=0}^{n'} \left(\frac{a}{R_p}\right)^n \sum_{m=0}^{n} \sum_{\alpha=1}^{2} \bar{C}_{\alpha nm} S_{\alpha nm} \quad , \; n \neq 1 \tag{2.35}$$

where $\bar{C}_{\alpha nm}$ are the residual coefficients obtained after correcting $C''_{\alpha nm}$ for the harmonic representation of the normal gravitational potential U_G (see § 5.1.2.3). The zero degree coefficient $\bar{C}_{100}$ in such a representation of the earth's disturbing potential, excluding the atmosphere, has a non-zero value. Exclusion of the first degree harmonic in equation 2.35 has the effect of making the centre of the reference ellipsoid coincident with the geocentre.

T' can be downward continued to the surface of the ocean (downward continuation, through matter, to the continental geoid, although theoretically incorrect - RAPP (1971) - appears to present no problems in practice).

The geoid height N is related to the disturbing potential T by Brun's equation (HEISKANEN & MORITZ 1967, p.85):

$$N = T/\gamma \tag{2.36}$$

where γ is normal gravity.

The geoid undulations with respect to a geocentric reference ellipsoid can therefore be obtained from a gravity field model by:

$$N_p = \frac{GM}{R_p \gamma_p} \sum_{n=0}^{n'} \left(\frac{a}{R_p}\right)^n \sum_{m=0}^{n} \sum_{\alpha=1}^{2} \bar{C}_{\alpha nm} S_{\alpha nm} + \frac{W_{Ap}}{\gamma_p} \quad , \; n \neq 1 \tag{2.37}$$

Equation 2.37 requires a knowledge of the geocentric distance R_p, hence an estimate of the geoid height N and orthometric height H is needed, in addition to the geocentric distance to the reference ellipsoid R_{ep} (equation 5.43). For geoid determinations requiring a sub-metre precision, an approximate value of N can be used (obtained using R_{ep} in place of R_p in equation 2.37). The value of N obtained is used to update the estimate of R_p and, in turn, to recompute N. A value of N to a precision of a few centimetres can be obtained after approximately three iterations. Alternate methods for faster convergence to the desired result are given in RAPP (1971) and BLAHA (1978).

Geoid determinations based on equation 2.37 are discussed in greater detail in § 5.1.2.3.

From an inspection of equation 2.28 it can be seen that the effect of the higher degree harmonics is rapidly damped out as the term $(a/R)^n << 1$ for large n. As a result, only harmonics up to some limiting degree n' can be adequately determined from the analysis of tracking data. The value of n' at present is around 25 (approximately 800 coefficients $C'_{\alpha nm}$).

The harmonic coefficients of high degree cannot necessarily be considered applicable at the surface of the earth as their effect, dampened at the relatively high altitudes relevant to satellite orbits, is reasserted at lower altitudes. Therefore any evaluation of N based on geopotential models from satellite orbital analysis provides an *oversmoothened* representation at the earth's surface.

The magnitude of the coefficients decrease with increasingly higher degree according to the approximate relation (KAULA 1966b, p.4379):

$$C_{\alpha nm} \simeq 10^{-5}/\, n^2 \qquad (2.38)$$

Low degree harmonics, on the other hand, are less likely to be affected by the departure of (a/R) from unity and hence are more reliably determined than those of high degree. The upper limit to the degree of the harmonic coefficients which can be reliably determined is dependent primarily on the lowest possible altitude at which a satellite can orbit without suffering significant drag due to friction between the atmosphere and the spacecraft (about 500km).

The evaluation of $C_{\alpha nm}$ by the analysis of orbital perturbations is also affected by <u>resonance</u> between the values of the set of coefficients {n,m} and the satellite's orbital period (MATHER 1973a, p.72). This causes certain coefficients, which by themselves make no significant contribution towards the representation of the earth's gravity field, to have marked effects on the perturbations of those orbits with sympathetic parameters (see MATHER 1971, p.150, KAULA 1966a, Section 3.6). As it is likely that only a limited number of satellites will be available for determining the features of the disturbing potential then only a limited number of high degree "resonant" harmonic coefficients will be capable of evaluation. In such a case it is preferable to treat higher degree resonant effects as sources of orbital perturbations rather than signals from the gravity field which could be meaningfully translated into the geoid representation given by equation 2.37.

Gravity field models produced solely from satellite orbit analysis include the odd numbered GEM1, 3, 5, 7 and 9 (SMITH ET AL 1976, WAGNER ET AL 1976, LERCH ET AL 1977). In the case of GEM9 the harmonic coefficients have been determined to degree and order 20. This is equivalent to a smoothed representation of features of the geopotential, and hence the geoid, with wavelengths greater than 2 x 10^3 km (Figure 2.2 illustrates the implied geoid heights from GEM9). The geoid determined from equation 2.37 is unaffected by the existence of sea surface topography.

The geoid undulation information, in metres, above degree ℓ which is lost can be estimated from (CHOVITZ 1972):

$$N_{High} = 64/\ell \quad \text{(m)} \qquad (2.39)$$

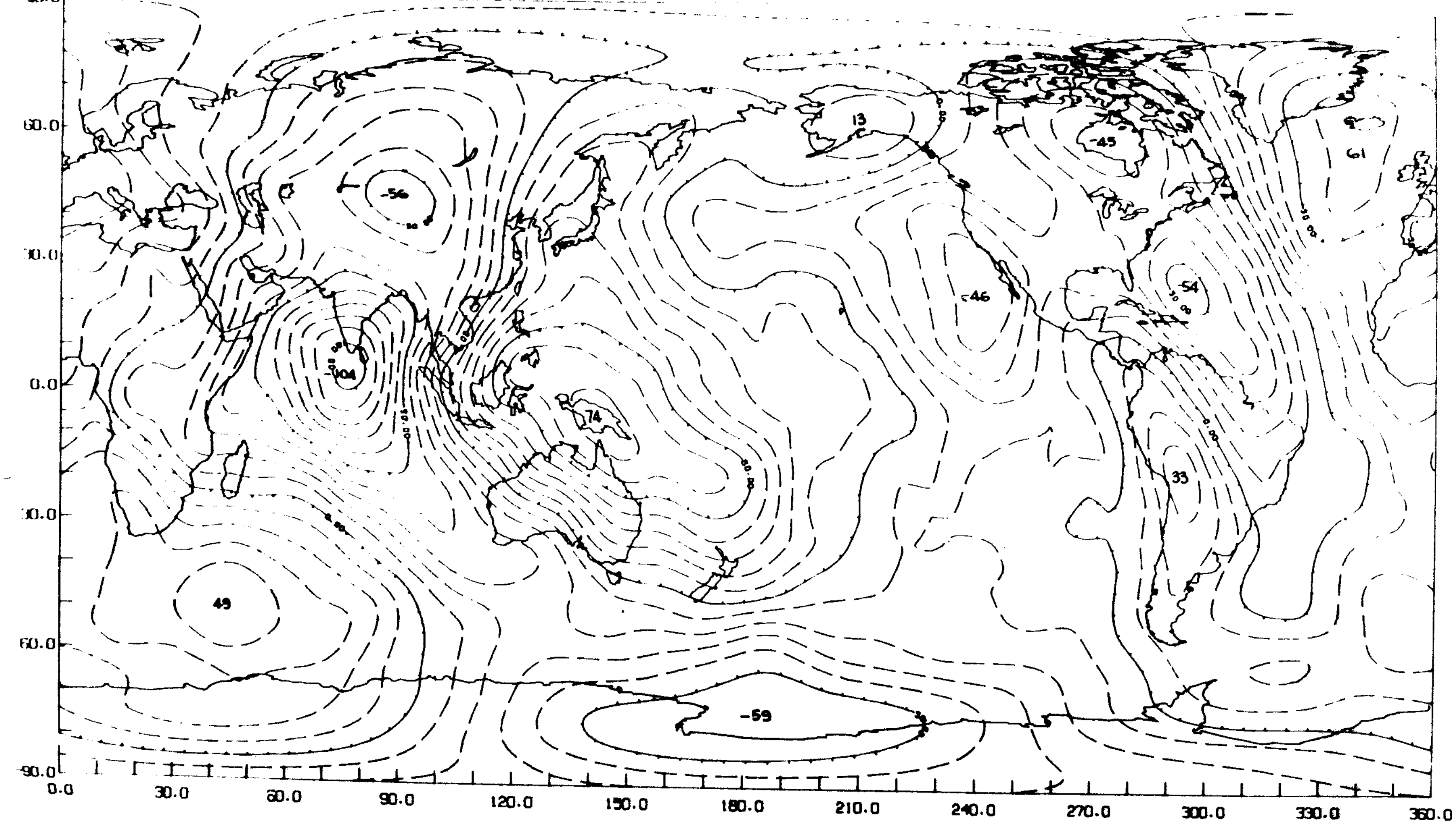


FIGURE 2.2

Satellite Geoid Computed from the GEM9 Model

(Contour Interval - 10 m)

For GEM9, approximately 3% of the geoid undulation signal is lost due to the truncation of ℓ to 20 (total signal ±100m). The possibility exists that the minimum wavelength resolved could be lowered to about 500km if satellite-to-satellite tracking data were analysed (e.g. MARSH ET AL 1977) and a low flying satellite mission of the GRAVSAT type were flown (NASA 1972b).

The accuracy of presently available gravity field models is of the order of ±1-2 kgalm (±1-2m in geoid height) in areas of high quality tracking for the wavelengths resolvable, i.e. does not include the short wavelength features. Gravity field model improvement is possible with the incorporation of additional data such as surface gravity measurements and satellite altimetry. The even numbered models GEM2, 4, 6, 8 and 10 (SMITH ET AL 1976, WAGNER ET AL 1976, LERCH ET AL 1977) are produced using surface gravity data as a supplement to satellite tracking data. GEM10B (LERCH ET AL 1978a) is a preliminary model which, in addition to surface gravimetry, incorporates satellite altimetry data from GEOS-3. A gravity field model complete to degree 180 has been recently developed by RAPP (1978b). Such a model is based on the analysis of a global $1^{\circ} \times 1^{\circ}$ gravity anomaly field (a portion of which has been predicted from satellite altimetry data - see RAPP (1978a)).

However, tracking data from third generation laser tracking systems, which have sub-decimetre precision, promise a significant improvement in the accuracy of geopotential models, if the data is collected from stations sited globally. It has been estimated (MATHER 1974b, p.87) that a minimum of 25 well distributed ±10cm tracking systems, operating without weather limitations, would provide data suitable for developing gravity field models with an accuracy of ±0.1kgalm in the geopotential for wavelengths greater than 10^3km.

The determination of a marine geoid as a continuous field with an accuracy of ±10cm (for geoidal features with wavelengths greater than 10^3km) from gravity field models is a strong possibility in the foreseeable future. The role such a geoid can play in SST studies is described in sub-section 5.2.2.

2.3.3.2 *Geometric Methods*

In principle the geoid can be mapped geometrically by using three dimensional position systems in combination with geodetic levelling results. The geoid undulation N, the orthometric height H (§ 3.2.1.1) and the ellipsoidal height h are related by the equation (HEISKANEN & MORITZ 1967, p.179):

$$h = N + H \qquad (2.40)$$

The ellipsoidal height at an observing site (e.g. satellite tracking station, VLBI station, lunar laser ranging station, etc.) can be obtained from geocentric Cartesian coordinates X_i derived from three dimensional position fixes (equation 3.3). The orthometric height H is deduced from geodetic levelling (sub-section 3.2.1). Therefore the geoid undulation N can be obtained directly from equation 2.40. However, this method only provides estimates at discrete points and is not a viable procedure for marine geoid determinations in support of SST studies.

2.3.4 AN ACTIVE SATELLITE METHOD OF GEOID DETERMINATION

The launching of the GEOS-3 satellite and the proposal to establish a series of SEASAT oceanographic satellites in the near future (NAGLER & McCANDLESS 1975) has opened up the possibility of remote sensing the *geometry* of the ocean surface. GEOS-3 was the first satellite equipped with a radar altimeter able to measure the distance from the spacecraft to the sea surface with geodetic accuracy. The altimeter was designed to operate in two modes: the intensive or short-pulse mode, having a nominal precision at the half metre level, and the less precise long-pulse mode (NASA 1972a).

Figure 2.3 illustrates the relationship between the altimeter range, the heights of the geoid N and the sea surface ζ above the reference ellipsoid and the sea surface topography ζ_S. The sea surface heights ζ are determined from a knowledge of the altimeter-equipped satellite's ephemeris and the altimeter ranges as described in § 5.1.2.2. The heights are given in relation to a reference ellipsoid centred at the geocentre. As the free ocean surface is almost coincident with the geoid, altimetry data can play a role in determining the marine geoid if the following requirements are satisfied:

(1) The position of the satellite is known with sufficient accuracy, particularly the radial component.

(2) The uncertainty in the altimeter measurement is less than the required accuracy in the geoid.

(3) The altimeter measures the distance between the satellite and the geoid.

Each of these is discussed further.

Values of sea surface height are directly influenced by radial errors in orbit determination. Consequently global fields of ζ deduced from satellite altimetry are prone to errors of long wavelength, with significant amplitudes, in regions where the tracking support for the altimeter-equipped spacecraft is inadequate, and the gravity field model lacks accuracy to compensate for the poor tracking coverage.

It appears that the reliability of gravity field models decrease with increasing distance from stations in the tracking network (see e.g. MATHER ET AL 1976a, Section 3, MATHER ET AL 1977a, Section 6). The uncertainty in such models at present appears to be anything between the 1-10% level in representing the disturbing potential in regions of low tracking station density; particularly in the southern ocean areas where different gravity field models exhibit significant discrepancies (see RAPP 1975). In short, it appears that errors in orbits determined by dynamical means, are functions of position.

On the other hand, it is possible to obtain orbits where the radial component of the satellite's position is known to a precision equivalent to that of the tracking system, if the spacecraft is continuously tracked. In such a scenario the position of the satellite is fixed geometrically and consequently no role is played by gravity field models. For a satellite with altitude 10^3km, this would require, in theory, a global network of approximately 125 tracking stations (MATHER 1974b, Section 5). The deployment of such a network of ±10cm laser tracking systems would define the geocentric position of the satellite to ±10cm. As it is unlikely that such a complement of systems will be operating in the foreseeable future (20 being the more likely number), an improvement in gravity field models to a resolution equivalent to the noise level of the tracking data (i.e. ±0.1kgalm) must be attempted in order to maintain orbital position to this accuracy during period of no tracking support.

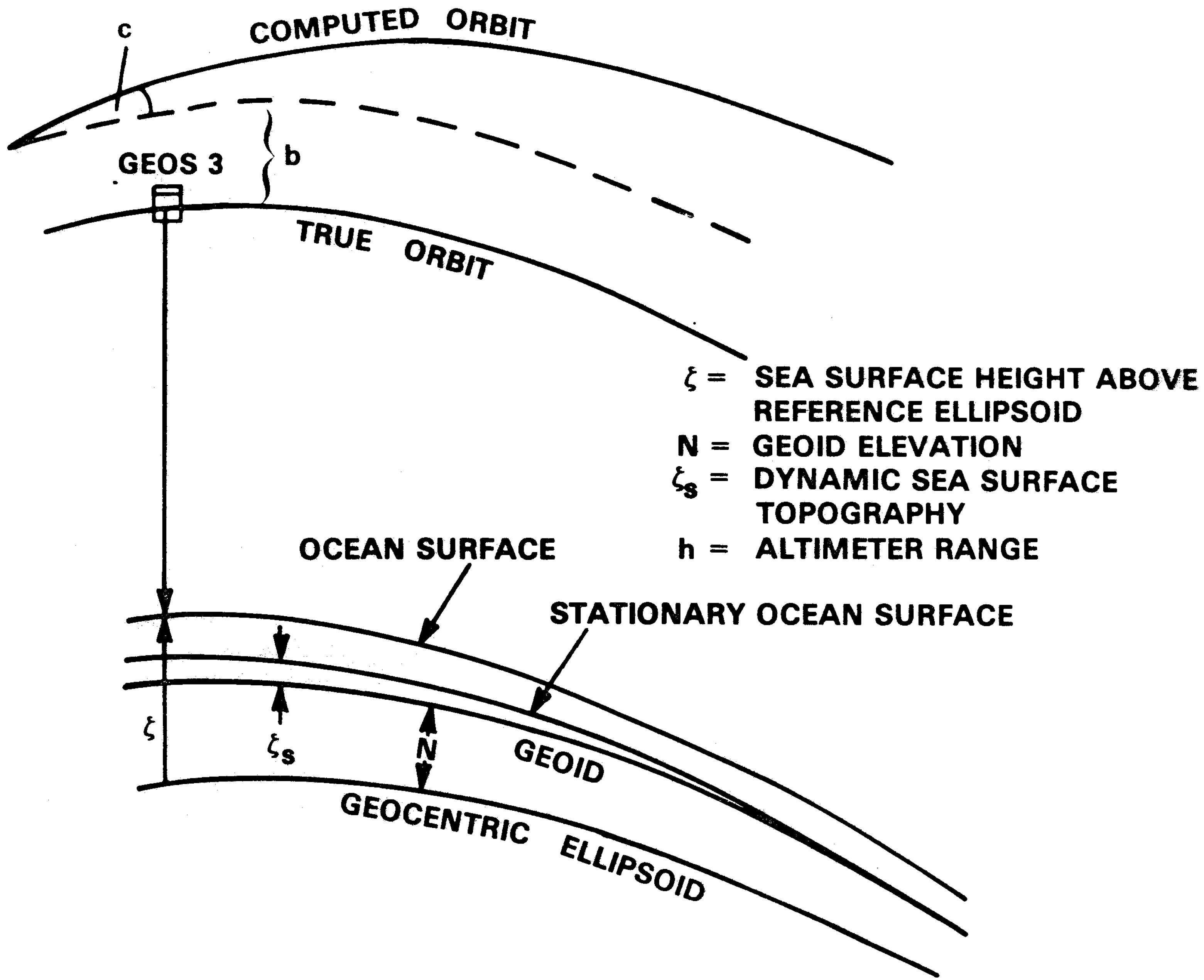


FIGURE 2.3 Sea Surface Topography in the Orbital Plane

Early studies of intensive mode GEOS-3 altimetry data collected in the Tasman and Coral Seas off eastern Australia indicated that passes of altimetry were subject to radial orbital errors ranging from 2 to in excess of 10 metres (MATHER ET AL 1977a).

As far as the altimeter hardware is concerned, the design requirement in the intensive mode was for a precision of ±50cm from averages over 0.1 seconds, such that the correlation of random error in the averaging procedure was held to below 0.33 (NASA 1972a). A description of altimeter hardware errors is given in ARGENTIERO ET AL (1974, p.9-12). From a comparison of overlapping passes the noise level of the GEOS-3 altimeter is assessed at ±20-30cm on a relative basis (MATHER & COLEMAN 1977, Section 3), assuming that the discrepancies were not of oceanographic origin. The root mean square of the discrepancies between sea surface heights on two overlapping passes can be reduced from over ±2m (due to radial orbital errors) to around ±30cm (the noise level of the altimeter) by modelling the orbital error by a bias b and tilt c (see Figure 2.3).

Although the precision of the altimeter range appears to be approximately ±30cm, the uncertainty in the accuracy is possibly greater. MARTIN & BUTLER (1977) have performed a pseudo-geometric calibration of the altimeter and claim the value of the altimeter bias obtained is accurate to ±20cm.

However, the existence of SST means that the altimeter does not measure to the geoid surface and the value of ζ deduced from such data is therefore not equivalent to the geoid height N. Nevertheless, detailed information on the geoid shape can be obtained from values of ζ to a resolution equivalent to the magnitude of the SST (±1-2m). The sea surface height can be treated as the "geoid height" in the context of 0.3 parts in 10^6 geodesy, and a map of sea surface heights is essentially an oceanic geoid map under such accuracy requirements. GEOS-3 altimetry data has provided estimates of the marine geoid to a precision of ±2m (e.g. BRACE 1977 globally and SMITH & CHAPPELL 1977 in the Indian Ocean) where the geoidal information from other sources was inadequate.

The shortest wavelength in the sea surface shape that can be resolved by GEOS-3 altimetry data based on 1 second averages is 15km, for two reasons:

- As the spacecraft is moving at approximately 7 km s^{-1}, data "points" are 7km apart.
- Each data "point" is in reality a "footprint" on the ocean surface, illuminated by a divergent radar beam of approximately 15km diameter.

A geoid of ±10cm precision can only be obtained from a sea surface model, which has been constructed from intersecting altimeter passes (see MATHER ET AL 1977a for details on the construction), if:

(i) the sea surface model has a precision of ± 10cm. Sea surface models constructed using GEOS-3 data appear to have an internal precision of ± 40cm on a regional basis (MATHER 1977).

(ii) a model for the sea surface topography with an accuracy of ±10cm were available. The ±2m uncertainty could be reduced to 20% of this figure if it could be assumed that the dominant second degree zonal harmonic in the SST (see Table 5.2) could be modelled from oceanographic charts (Figure 3.3). This, however, would hardly be of assistance for geodetic studies that aim to determine the SST.

Consequently, altimetry data alone cannot provide information on the shape of the geoid to better than the magnitude of the (unknown) SST.

2.3.5 GRAVIMETRIC METHODS FOR THE DETERMINATION OF THE GEOID

2.3.5.1 *Basic Relations*

Gravimetric methods for the determination of the geoid are based upon the principles of potential theory (e.g. MACMILLAN 1930). In 1849 George G. Stokes presented a practical method for the computation of geoid undulations using surface gravity measurements (STOKES 1849). An outline of Stokes' method is given in most geodesy textbooks (see e.g. HEISKANEN & MORITZ 1967, Chapters 1 & 2). Essentially the technique of solution is based on two mathematical properties.

(1) *Stokes' theorem* - stating that there is only one harmonic function V in a space free of matter exterior to a bounding surface S that assumes given boundary values on S (a function being harmonic if it satisfies Laplace's equation - equation 2.16).

(2) *Dirichlet's principle* - asserting that such a function exists.

The problem of computing the harmonic function (inside or outside of the surface S) from boundary values on S is an example of a boundary value problem (see IBID, Sections 1.16 & 1.17). Stokes' method specifically confronts the third boundary value problem of potential theory where the boundary values in the solution of Laplace's equation are a linear combination of the harmonic function and its normal derivative.

The determination of the disturbing potential of the earth's gravity field T (equation 2.23) is an example of a linearisation of the third BVP of potential theory. Such a free boundary value problem is known as the Geodetic Boundary Value Problem (GBVP). The surface S is the geoid and the boundary condition is provided by the gravity anomaly Δg (§ 2.1.2.3) defined by:

$$\Delta g = -\frac{\partial T}{\partial h} + \frac{1}{\gamma}\frac{\partial \gamma}{\partial h} T \qquad (2.41)$$

where h is the direction of the spherop normal, γ is normal gravity and $\partial\gamma/\partial h$ is the vertical gradient of normal gravity.

Equation 2.41 is also known as the fundamental equation of physical geodesy, which relates gravity anomalies on the geoid to the disturbing potential in the space exterior to the geoid. The gravimetric solution of Laplace's equation for the disturbing potential, and hence from Brun's equation (equation 2.36) the geoid height N, is an integral formula known as Stokes' integral (see e.g. IBID, p.94),

$$N = \frac{R}{4\pi\gamma} \iint f(\psi)\ \Delta g\ d\sigma \qquad (2.42)$$

where ψ is the angle between geocentric radii to the element of surface area $d\sigma$ and the point of computation P, R is the geocentric radius to P and

Stoke's function $f(\psi)$ is given by:

$$f(\psi) = \sin^{-1}(\psi/2) - 6\sin(\psi/2) + 1 - 5\cos\psi - 3\cos\psi \ell n\left(\sin(\psi/2) + \sin^2(\psi/2)\right) \quad (2.43)$$

The integration in equation 2.42 is over the whole globe, requiring values of Δg to be known at every point on the geoid.

The gravimetrically-determined geoid heights N refer to the same ellipsoid as do the gravity anomalies. Stokes' integral removes any first degree harmonic in the gravity anomalies and consequently the geoid undulations refer to a geocentric reference ellipsoid. Stokes' integral is, in addition, insensitive to any zero degree or scale effects in the gravity anomaly data. If the geoid heights are to have correct scale, the reference ellipsoid should satisfy the following conditions:

(i) Have the same potential as the geoid.

(ii) Enclose the same mass as the earth.

(This is discussed further in sub-sections 4.2.2 & 5.1.3).

Generalisations of equation 2.42 for some arbitrary reference ellipsoid not satisfying the above conditions are available (see IBID, Section 2.18, and section 4.2 of this thesis).

The accuracy of gravimetric methods of geoid determination are dependent on two main factors:

(a) The validity of several assumptions implicit in the derivation of Stokes' integral; and

(b) The techniques for the practical implementation of Stokes' method, the data quality and its distribution.

These are discussed further in § 2.3.5.2 & 2.3.5.4.

2.3.5.2 *Practical Considerations in the Application of Stokes' Integral*

A closer inspection of Stokes' method for the solution of the GBVP reveals some problems, namely:

(1) The bounding surface (the geoid) is assumed a sphere of radius R.

(2) The quantity Δg represents values on the geoid.

(3) The disturbing potential must be harmonic in the space exterior to the geoid (i.e. there must be no masses outside the geoid surface).

An oblate ellipsoid approximates the geoid to ±100m (see Figure 2.2), and deviates from a sphere by quantities of the order of the flattening f ($\simeq 3 \times 10^{-3}$). Treating the geoid as a sphere in expressions involving the disturbing potential therefore introduces relative errors of the order of Nf in the geoid height (i.e. ±30cm). A consequence of the spherical approximation made by Stokes is that the fundamental boundary condition (equation 2.41) becomes (IBID, Section 2.14):

$$\Delta g = -\frac{\partial T}{\partial r} - \frac{2T}{R} \quad (2.44)$$

where r is in the direction of the spherical radius.

Formulae analogous to equation 2.42 have been derived for an ellipsoidal model of the geoid (see RAPP 1974a, RAPP 1976b), and are treated as correction terms to the basic Stokes' integral (see sub-section 4.2.1 for details).

The problems associated with having gravity measured on the geoid with no exterior masses ((2) & (3)) may be treated together. Firstly, the atmosphere can be "removed" from solutions of the GBVP by methods described in section 4.5. The conditions required for Stokes' solution may then be created artificially by some mathematical procedure designed to *regularise* the earth. This is accomplished by the various gravity reductions (described in HEISKANEN & MORITZ 1967, Chapter 3), which attempt to:

(i) remove any offending topographic masses exterior to the geoid.

(ii) "lower" the gravity station to the geoid by the application of a "free air" reduction to gravity.

A theoretically exact reduction of gravity to the geoid requires a knowledge of the vertical gradient of gravity $\partial g/\partial h$, given by:

$$\frac{\partial g}{\partial h} = \frac{\partial \gamma}{\partial h} + \frac{\partial \Delta g}{\partial h} \tag{2.45}$$

In most cases the "anomalous" part of the vertical gradient is neglected and the free air correction to observed gravity F is assumed to be adequately modelled by the normal gradient (MATHER 1971, Section 6.7):

$$F = -\frac{\partial \gamma}{\partial h} = \frac{2\gamma}{a}(1 + f + m - 2f\sin^2\phi + o\{f^2\}) \tag{2.46}$$

where a and f are the equatorial radius and flattening of the reference ellipsoid, γ is the normal gravity (given by expressions found in § 6.1.2.1), ϕ is the geocentric latitude and the parameter m is defined by:

$$m = \frac{a\omega^2}{\gamma_e} \tag{2.47}$$

where ω is the angular rate of rotation of the reference ellipsoid and γ_e is the equatorial normal gravity on the ellipsoid.

For most practical purposes the free air reduction is approximated by:

$$F = 0.30855\ H^{(m)}\ \text{mgal} \tag{2.48}$$

where H is the height of the observing station above the geoid.

The removal or shifting of masses during commonly employed gravity reductions changes the geopotential and hence the shape of the geoid. The change in the geoid is known as the "indirect effect" of gravity reductions (HEISKANEN & MORITZ 1967, Section 3.6).

The use of a "regularised" gravity anomaly data set in Stokes' integral recovers not the "true" geoid but a different surface - the co-geoid. In addition, following the first regularisation (the reduction of gravity to the "geoid"), the gravity anomaly Δg_r needs to be reduced once more from the geoid to the co-geoid. This is the indirect effect on gravity. The co-geoid undulations N_{cg} are therefore given by:

$$N_{cg} = \frac{R}{4\pi\gamma} \iint f(\psi) \left(\Delta g_r + 0.30855\ \delta N\right) d\sigma \tag{2.49}$$

where δN is the geoid/co-geoid separation (in metres). The geoid-ellipsoid separation can then be determined thus,

$$N = N_o + N_{cg} + \delta N \tag{2.50}$$

where N_o is the zero degree term (see sub-section 4.2.2).

There are many methods of regularisation, each with its associated gravity reduction procedure, indirect effect and co-geoid. For example, the Bouguer reduction removes the topography above the geoid. However, such a reduction results in an indirect effect which is much larger than N because it does not take into account the fact that the earth is generally in isostatic equilibrium. An Isostatic reduction, on the other hand, strives to shift the offending topographic masses into the interior of the geoid according to some theory of isostasy (IBID) and consequently the indirect effect is an order of magnitude less than N.

The most pertinent reduction, in terms of its practical importance, is the Free Air Reduction. Although it has the effect of "lowering" gravity through "free-air", it is also a realistic form of regularisation. The free air reduction is in fact a special case of Helmert's second condensation reduction which condenses the topography, columnwise, to form a surface layer on the geoid. The effect on gravity of the regularisation due to Helmert's condensation method is zero for the free air reduction as the attraction of the topography (now a surface layer on the geoid) is assumed balanced by the attraction of its isostatic compensation below the geoid. The free air reduction increases the gravity observed at a point P on the earth's surface to give a value that would have been observed at the geoid had the topography above the geoid been balanced exactly by its compensation. The free air gravity anomaly is

$$\Delta g_f = g_p + H.F - \gamma_o \tag{2.51}$$

where F is defined in equations 2.46 and 2.48, H is the height of P above the geoid and γ_o is normal gravity on the reference ellipsoid.

The indirect effect due to regularisation by the free air reduction is a consequence of departures from isostatic equilibrium and is therefore small and generally of the order of a few metres, although it may be much larger in mountainous areas. The use of free air gravity anomalies Δg_f in Stokes' integral results in a co-geoid which, in the context of 1 part in 10^6 geodesy, is considered a good approximation to the real or non-regularised geoid. Such a geoid is sometimes termed the free air geoid (MATHER 1968). Regional free air geoids have been computed for Australia by MATHER (1970a), for Germany by BRENNECKE ET AL (1975), for Africa by OBENSON (1973), for Canada by NAGY & PAUL (1973) and for the U.S.A. by MATHER (1975b).

For geoid determinations of ±10cm accuracy however, the evaluation of the free air indirect effect is critical. ANDERSON (1976) made a detailed study of the effect of the topography on Stokes' solution of the GBVP. The result for δN, from the evaluation of the gravitational effect of the earth's topography and its compensation (using the Airy-Heiskanen isostatic model), is presented in IBID, Figure 8.5 (p.157). The indirect effect for the marine geoid is effectively zero, while on continents it is of the order of 2-4 metres except in such extensive mountainous areas as the Himalayas and the Andes where the effect is as large as 20 and 12 metres respectively.

The indirect effect for the Australian free air geoid was investigated by FRYER (1970). It was found that the horizontal gradient of the indirect effect is very low (there are no large mountainous areas), with variations in δN estimated as < 60cm with a very slow rate of change. Consequently, the regional gravimetric solution for the Australian free air geoid by MATHER (1970a) deviates from the non-regularised geoid by less than one metre (if errors arising from considerations of data quality are not included - see § 2.3.5.4).

However, the accuracy of the free air geoid, as with any other regularised geoid, is implicitly dependent on a knowledge of the distribution of masses between the geoid and the earth's surface. For example, the free air correction defined by equation 2.47 assumes that the "real" vertical gravity gradient is equal to the "normal" vertical gravity gradient. Furthermore, the evaluation of the indirect effect requires a knowledge of the density of the topography and the nature of its compensation. As this cannot be considered known to sufficient accuracy, geoid determinations at 1 part in 10^8 are subject to unacceptable levels of uncertainty.

2.3.5.3 *Molodenskii's Problem*

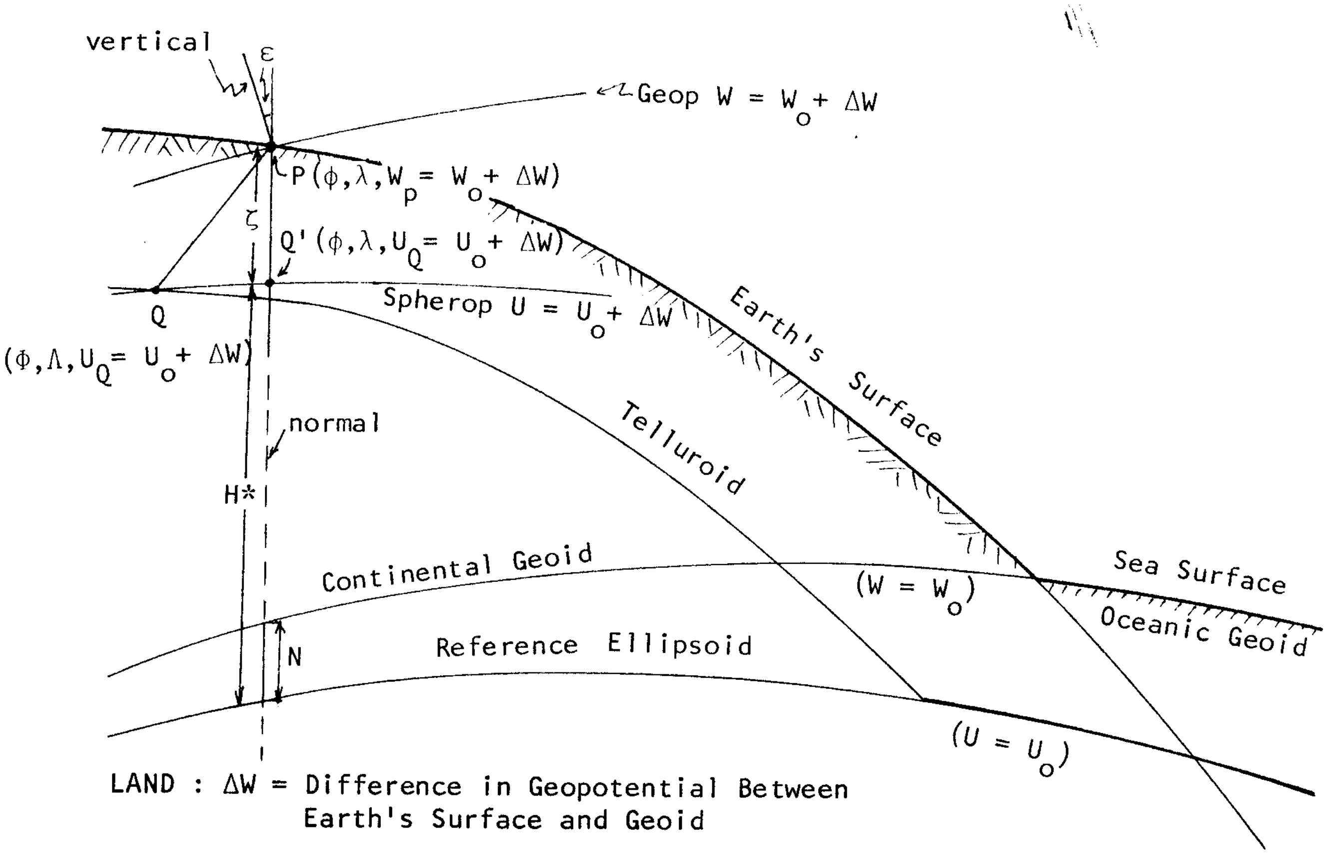
In order to overcome the drawback of not knowing the density of the masses exterior to the geoid (and having to make assumptions concerning it) in the conventional Stokesian solution to the GBVP, Molodenskii in 1945 proposed a different approach (MOLODENSKII ET AL 1962).

According to the reformulation of the GBVP, known as Molodenskii's problem (see HEISKANEN & MORITZ 1967, Section 8.3), the bounding surface is no longer the geoid but the earth's physical surface. A solution to Molodenskii's problem requires ground level gravity anomalies, which are free of the effect of gravity reductions that embody various assumptions concerning the density of the crust. As a corollary to this, the concept of the geoid is displaced from its central role as the basis of establishing the position of points, in a vertical sense, on the earth's surface (sub-section 2.2.2). Nevertheless, the gravimetric methods for the solution of Molodenskii's problem provide a means of determining the geoid in an <u>indirect manner</u>.

Traditionally, geoid heights, together with the deflections of the vertical, have provided a means of determining the geodetic latitude ϕ, geodetic longitude λ and ellipsoidal height h of a point on the earth's surface from the "natural" coordinates Φ, Λ and H (see equations 2.28 & 2.40). Restricting our attention to the radial component of position, the geometry of Molodenskii's solution to the GBVP is illustrated in Figure 2.4. (A complete description of the solution to the problem of determining geodetic position (ϕ,λ,h) by gravimetric techniques based on Molodenskii's formulation of the GBVP is given in MATHER (1973c)).

FIGURE 2.4

Geoid, the Earth's Surface and the Telluroid



LAND : ΔW = Difference in Geopotential Between Earth's Surface and Geoid

OCEAN : ΔW = 0

The ellipsoidal height h is now represented by:

$$h = H^* + \zeta \tag{2.52}$$

compared with equation 2.40 for the conventional geoid-ellipsoid system. The normal height H* (see § 3.2.1.2) replacing the orthometric height H and the height anomaly ζ replacing the geoid height N.

The normal height is the height of the spherop ($U = U_o + \Delta W$) above the ellipsoid, and can be determined from geodetic levelling data combined with gravity measurements by an analytical expression that is free of any terms involving crustal density (equation 3.17). The "telluroid" was originally defined as the locus of points Q whose spheropotential U equalled that of the geopotential W at the "equivalent" point P (Figure 2.4) on the earth's surface (see e.g. HEISKANEN & MORITZ 1967, p.292).

A more practical definition of the telluroid, allowing for the possibility that the potential of the geoid and reference ellipsoid are not equal (i.e. $W_o \neq U_o$), is the locus of those points Q which have the same difference in potential ΔW in relation to the reference ellipsoid as the difference in geopotential between the surface point P and the geoid.

The telluroid is not a level surface of the earth's gravity field. It closely mirrors the physical surface of the earth and would coincide with it at the point P if the geopotential W_p and spheropotential U_p were equal at that point. In such a case the normal height would coincide with the ellipsoidal height. In general however $W_P \neq U_P$, and hence the quantity

$$\zeta = h - H^* \tag{2.53}$$

is not zero. This is the reason it is termed the "height anomaly".

The gravity anomaly used in solutions of Molodenskii's problem is:

$$\Delta g = g_P - \gamma_{Q'} \tag{2.54}$$

It represents the difference between observed gravity at P and normal gravity at the point Q′, located on the same ellipsoidal normal as P. The normal gravity at Q′ is computed from the value at the ellipsoid γ_o by the application of the "normal" free air reduction (equation 2.46) *upwards*

$$\gamma_{Q'} = \gamma_o - H^*.F \tag{2.55}$$

The quantity defined by equations 2.54 & 2.55 is the ground level free air anomaly and is exact. In contrast, the conventional free air gravity anomaly defined by equation 2.51 involves the application of the normal free air reduction to the "real" gravity field in order to *lower* observed gravity to the geoid.

The solution to Molodenskii's problem for the height anomaly is outlined in section 4.2. Such a solution takes into account the following factors:

(1) The undulations of the topography (contributing 5-10% to ζ)

(2) The gravitational effect of the earth's atmosphere (contributing 0.5% to ζ).

(3) An ellipsoidal reference figure, rather than a sphere.

(4) Other Stokesian approximations such as the use of orthogonal properties of surface spherical harmonics on spheres that intersect the earth's surface and the neglect of the zero degree term N_o.

The contribution of (3) and (4) to ζ is approximately 0.3%.

The major term in the solution of Molodenskii's problem is Stokes' integral (equation 2.42) but based on ground level free air anomalies. This first order term contributes approximately 90% of the power to the height anomaly. The solution to Molodenskii's problem can be represented as:

$$\zeta = \zeta_N + \zeta_c \tag{2.56}$$

where ζ_N is the Stokesian contribution and ζ_c the remaining non-Stokesian contribution.

The geoid height is related to the height anomaly by (IBID, Section 8.12):

$$N = \zeta - \frac{(\bar{g} - \bar{\gamma})}{\bar{\gamma}} H \tag{2.57}$$

where $\bar{g}$ is the mean gravity along the vertical between the geoid and the earth's surface and $\bar{\gamma}$ is the mean normal gravity along the normal between the ellipsoid and the telluroid. It is possible to plot the height anomalies above the ellipsoid and obtain a surface called the quasi-geoid (IBID, Section 8.13). This surface is not a level surface and deviates from the true geoid by the quantity $(\bar{g}-\bar{\gamma})H/\bar{\gamma}$. In the absence of sea surface topography the quasi-geoid is, for all intents and purposes, coincident with the oceanic geoid and within a few decimetres of it elsewhere. The influence of the sea surface topography on solutions of the GBVP is discussed in § 2.3.5.5.

2.3.5.4 *The Computation of the Geoid Height from Gravity Anomalies*

The dominant component of the geoid height is that due to the Stokesian term N_S. The evaluation of the indirect effect δN (in the case of the Stokesian solution) or the non-Stokesian term ζ_c (in the case of the Molodenskii solution) is not discussed here.

The procedure used in evaluating Stokes' integral is based on a system of quadratures. The value at the ith computation point is given by:

$$N_{si}\ (\mathrm{cm}) = \frac{R^{(\mathrm{cm})} \times \pi^2}{4\pi\gamma\ 3.24 \times 10^4} \sum_{j=1}^{M} (n \times m)_j \mu_{ij} f(\psi_{ij}) \Delta g_j^{(\mathrm{mgal})} \quad (2.58)$$

where M is the total number of gravity anomalies, Δg_j is the representative gravity anomaly for an $(n^o \times m^o)_j$ area and μ_{ij} is $\cos\phi_j$ or $\sin\psi_{ij}$, depending upon the system of subdivision adopted for Δg. The first is for area means $(n^o \times m^o)$ defined by lines of latitude and longitude (i.e., equi-angular). The second system is based on templates (IBID, p.117) where the subdivision is defined by concentric circles about the ith computation point and ψ_{ij} is the angular distance from the computation point to the elemental area $(n^o \times m^o)_j$.

It is required that errors due to the adoption of the quadrature technique be kept to below ±10cm. Errors in each individual term within the summation can contribute to the overall error in two ways:

- Random or accidental fashion.
- Systematic fashion.

Obviously the magnitude of systematic errors should be significantly smaller than the accidental errors as the former hold the same sign over a considerable number of terms and can thus accumulate. A comprehensive study of the propagation of errors in evaluations of Stokes' integral by quadrature methods was made by MATHER (1973a, Section 4.3). The major conclusions were:

(1) Departures from linearity of Stokes' function $f(\psi)$ calls for a fine subdivision of the gravity anomaly field, particularly near the computation point.

(2) A procedure similar to Rice's circular ring method (RICE 1952) based on variable annuli thicknesses should be used for inner zone computations (within 2^o from computation point). The function $F(\psi) = f(\psi)\sin\psi$ should be used instead of $f(\psi)$ to circumvent the instability of the latter for small ψ.

(3) Interpolated values for the gravity anomalies are not necessarily inferior to observed ones, although care has to be taken in using prediction techniques in gravitationally disturbed areas or where observed data is insufficient.

(4) The most critical factor in quadrature evaluations is the extent

of correlation of error in the gravity anomaly values. This is discussed below.

An investigation of the propagation of systematic and random error characteristics in the gravity anomaly values through equation 2.58 indicates that an adequate sampling of the surface gravity field for ±10cm precision would be one which had:

(a) An error of representation of ±3mgal for the mean gravity anomaly for ($n^o \times m^o$) areas. $E\{\Delta g\}_{nm}$ is defined by (MATHER 1973c, Section 4.2):

$$\left(E\{\Delta g\}_{nm}\right)^2 = M\{(\Delta g_o - \Delta g)^2\} \qquad (2.59)$$

where Δg is the mean value, Δg_o are individual point gravity anomalies within the ($n^o \times m^o$) area and $M\{x\}$ is the mean of the quantity x. Such a representation is afforded by a 10km (0.1^o) grid in non-mountainous areas and is already available over large continental areas like North America, Australia and Europe.

The error of representation is the purely local, uncorrelated or random error in the gravity anomaly field and has nothing to do with the precision to which gravity can be established. It is merely a measure of how <u>representative</u> a value Δg is for an ($n^o \times m^o$) area as a whole.

For an equivalent precision, $1^o \times 1^o$ anomaly means would require that the error of representation be o{±0.3mgal}.

(b) No long wavelength systematic error $e_{\Delta g}$. The effect of an error $e_{\Delta g}$ in the gravity anomaly data set which is systematic over an $r^o \times r^o$ area (i.e. has a half wavelength of r^o) but behaves as an accidental error over the rest of the globe on the quadrature evaluation of the geoid height can be determined from the approximate relation (MATHER 1973a, p.65):

$$e_N{}^{(cm)} \nleq 10\, e_{\Delta g}{}^{(mgal)} \qquad (2.60)$$

For ±10cm precision and $r \simeq 20^o$ (the extent of major national levelling and gravity networks), the systematic error $e_{\Delta g}$ should not be in excess of ±50μgal for such wavelengths. The longer the wavelength of the systematic error, the lower the magnitude of $e_{\Delta g}$ that can be tolerated for ±10cm determinations.

The accuracy requirements for gravity anomaly data for the computation of local gravimetric geoids can be relaxed as there is no *differential* effect on the geoid shape by errors that hold their magnitude over the entire region for which the geoid is sought. However, such geoid determinations only satisfy the goal of ±10cm resolution on a <u>relative basis</u>.

Errors of considerable magnitude can be tolerated in gravity station elevations, provided they are purely local in character. A 10 metre error in elevation is equivalent to approximately 3mgal error in the gravity anomaly - the allowable error of representation of individual gravity values if sampled on a 0.1^o grid.

The sources of systematic error in gravity anomalies can be divided into two groups:

(i) Errors affecting the accuracy of the gravity measurement itself.

(ii) Errors introduced when gravity anomaly values are deduced from the gravity observations.

The majority of gravity determinations are made by differential means using gravimeters that measure the change in gravity force between a bench mark of the gravity control network and the gravity station being established. Errors in gravimeter ties seldom exceed ±0.2mgal, and are generally random if all necessary reductions have been carried out (e.g. for temperature effects, etc.). Ideally, for the needs of high precision geodesy, the spacing of stations comprising the global gravity control network should be approximately 10^3km in continental areas and the absolute gravity values established with a resolution of ±0.1mgal, such that errors in values at adjacent stations are uncorrelated. A wider station spacing would require a proportionate increase in the accuracy of gravity determinations as the wavelength of possible systematic errors is proportionately greater (see discussion on equation 2.60). A sparser spacing in the global gravity control network would in addition place a greater strain on regional networks like the ISOGAL system in Australia (see § 6.2.1.2) which provide regional control for local gravimeter surveys.

At present most gravity data banks are controlled by the International Gravity Standardisation Network 1971 (IGSN 71) (MORELLI ET AL 1971). The accuracy of this network is estimated to be ±0.1-0.2mgal, although the pattern of systematic error is not clear. This network could be improved to the desired accuracy and station density within the next decade or so if transportable absolute gravity measuring systems modelled on the Bureau International des Poids et Mesures apparatus (SAKUMA 1973) with accuracies of ±20μgal were deployed globally. These instruments would also strengthen the regional networks.

The sources of significant systematic error introduced in the process of evaluating gravity anomalies are:

(1) Errors in the computation of normal gravity. These may result from the adoption of an imprecise formula for γ or using gravity station latitudes based on regional geodetic datums rather than a geocentric system (see § 6.1.2.1 & 6.2.2.2).

(2) Errors in gravity station elevation. The effect of such errors (for continental data banks only) is a function of the wavelength and amplitude of the constituent errors in both the geodetic levelling network as well as the nature of the connections between the network and the individual gravity stations. While individual station heights could have errors of up to ±10m, any tendency towards correlation of errors between neighbouring gravity stations should be minimised. The effect of gravity station elevation errors in the Australian gravity anomaly data bank is discussed in § 6.2.3.5.

There is a tendency for gravity anomaly means to be lower than the true representative value. This is because gravity stations are normally established near roads and, as a result, tend to be located in valleys. Consequently the mean gravity station elevation for an ($n^o \times m^o$) area may be lower than the mean topographic elevation. This is particularly a problem in areas of rugged terrain.

In the case of oceanic gravity anomalies, the error in the height of the shipbourne gravimeter relative to MSL is expected to exhibit random characteristics.

(3) The most significant source of error for global gravimetric determinations of the geoid is due to the existence of sea surface topography. This factor limits the role of the gravimetrically determined geoid for SST studies and is discussed in the following sub-section.

2.3.5.5 *The Influence of SST on Solutions of the GBVP*

The existence of stationary SST affects the gravimetric determination of the height anomaly in two distinct ways:

(i) By causing systematic errors in the gravity anomaly data set due to the uncertainty in the relation between regional elevation datums and a universal datum level surface such as the geoid.

(ii) By altering the geometry of Molodenskii's problem, and consequently requiring the relationship between the geoid height N and the height anomaly ζ (equation 2.57) to be revised.

The height of the gravity station above the geoid is required in solutions of Molodenskii's problem for the upward continuation of normal gravity from the ellipsoid (equation 2.55), whereas in the case of Stokes' problem, the orthometric height is needed to downward continue observed gravity to the geoid (equation 2.51). Regional elevation datums are usually based on local MSL at one or more tide gauges and are not necessarily related to a unique geoid due to the existence of coastal stationary SST. Continental gravity anomalies are related to a level surface of the earth's gravity field that passes through the vertical datum of the region for which an elevation network has been established by geodetic levelling procedures (sub-section 3.2.1). Such a datum level surface, defined by a free net adjustment of geodetic levelling, is different for each regional elevation network (in some cases the gravity station elevations are based on an elevation datum that is not a level surface, e.g. the Australian Height Datum, however such elevations should be established on a free net adjustment if they are to be used for scientific purposes - see § 6.2.2.3).

Each oceanic gravity anomaly refers to that particular level surface that was tangential to the instantaneous sea surface at the time of observation and is therefore different from one gravity station to the next.

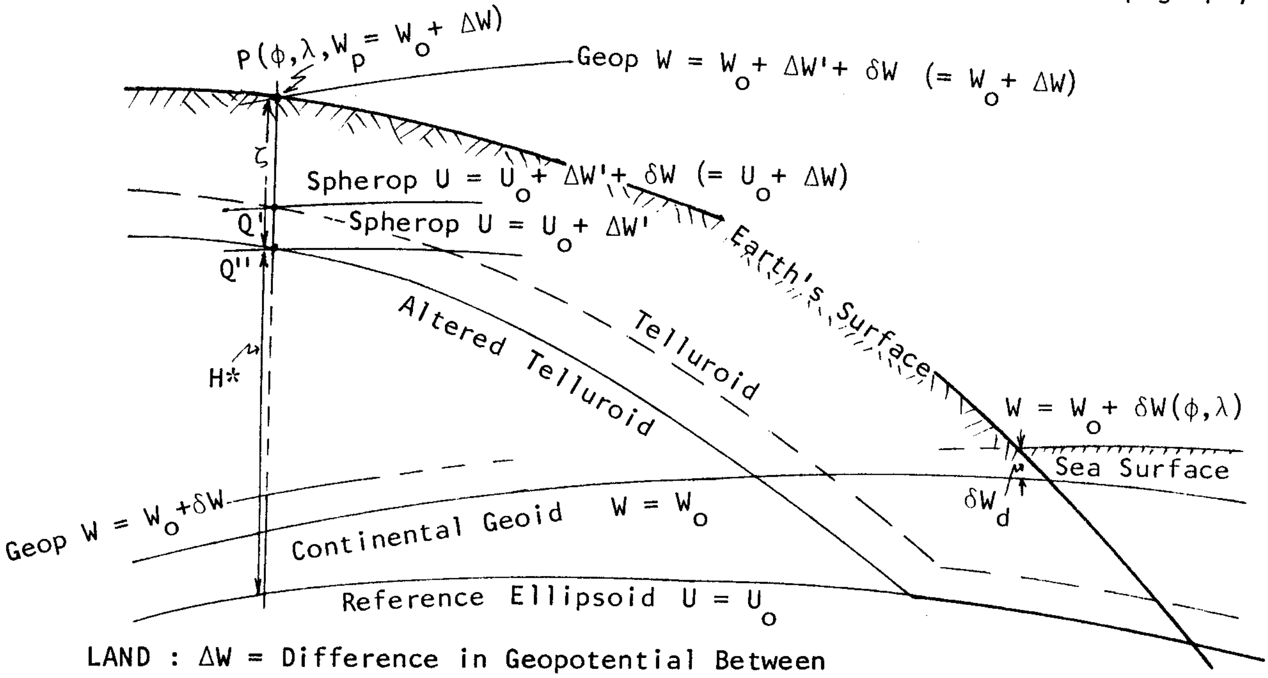
The separation between the geoid, (as defined in sub-section 2.2.5) and the elevation datum can reach 1-2m. This represents a systematic error of 0.3-0.6mgal in the gravity anomaly values, with half wavelengths of the order of 500 - 1000km (the average extent of continental levelling networks). The effect of inconsistent gravity station elevations on quadrature evaluations can be estimated from equation 2.58, using the above representative values. Such an effect on geoid height/height anomaly determinations results in an uncertainty of the order of ±15-50cm.

BRENNECKE & GROTEN (1977) investigated the effect of the assumption that oceanic gravity stations had zero elevation with respect to the geoid on global gravimetric determinations of geoid height. An analysis of the pattern of oceanographically-determined stationary SST (see Figure 3.3) showed that the errors in geoid height were functions of position (IBID, Figure 2) with amplitudes as large as ±60cm. This study only considered the effect of the SST on oceanic gravity anomalies and did not take into account the relative offsets of national levelling networks referred to earlier. However, as the wavelengths and amplitudes of these elevation datum offsets are similar to those of the dominant features of the stationary SST, it is not unreasonable to conclude that errors of the order of several decimetres can be expected to occur in gravimetric determinations of the geoid from a mixed set of oceanic and continental gravity anomalies. Such errors could be reduced by determining the geoid from combination solution techniques described in sub-section 2.3.6 or by modelling the effect of the SST (e.g. using Figure 3.3) on gravity anomaly values via a correction: $\zeta_s \times 0.3086$ (mgal), where ζ_s is the SST in metres.

The effect of SST on the geometry of Molodenskii's problem may be illustrated in the following manner. The use of geodetic levelling related to an offset datum for the computation of normal height results in values for H* which are no longer the height of the telluroid above the reference ellipsoid as defined in §2.3.5.3 (for simplicity it is assumed that Q and Q′, in Figure 2.4, coincide). Rather H* defines the height of the point Q″ above the ellipsoid - see Figure 2.5.

FIGURE 2.5

Geoid, the Earth's Surface, the Telluroid and Sea Surface Topography



LAND : ΔW = Difference in Geopotential Between Earth's Surface and Geoid
ΔW′= Difference in Geopotential Between Earth's Surface and MSL Datum obtained from Levelling

OCEAN : ΔW = δW(φ,λ) - the Potential of the SST
ΔW′= 0

The MSL elevation datum is offset by the quantity ζ_{sd} (or the SST δW in potential units). Consequently, in order to preserve the geometry implied in equation 2.52 and Figure 2.4, the height anomaly ζ (originally the separation between the surface point P and the "equivalent" point Q′) needs to be redefined to correspond to the distance between the "altered" telluroid (the locus of points Q″) and the earth′s surface. In the oceanic areas, the "altered" telluroid coincides with the reference ellipsoid and the oceanic height anomaly is identical to the sea surface height derived from satellite altimetry (see Figure 2.3).

However, gravimetric solutions of the GBVP for the height anomaly (as defined above) require a knowledge of the SST at the computation point. This is discussed further in sub-section 4.2.3.

The relationship between the geoid height N and the "altered" height anomaly is:

$$N = \zeta - \frac{(\bar{g} - \bar{\gamma})}{\bar{\gamma}} H + \frac{\delta W}{\gamma} \tag{2.61}$$

instead of equation 2.57. In oceanic areas the relationship is:

$$N = \zeta + \frac{\delta W}{\gamma} \tag{2.62}$$

For the remainder of this thesis the geometry of Molodenskii's problem is assumed to be as illustrated in Figure 2.5 and consequently, the *implicit* determination of the geoid height from gravimetric solutions of the height anomaly does not require a knowledge of the SST at the computation point (see equations 2.61 & 2.62).

2.3.6 THE GEOID FROM COMBINATION SOLUTIONS

The use of gravimetric methods for the determination of the geoid requires that the earth's gravity field be completely defined by gravity measurements. Owing to economic considerations and world politics this is not the case at the present time. The major concentration of unclassified gravity anomaly data is found in land and continental shelf areas, the open oceans being inadequately sampled (see Figure 5.1).

Accurate gravimetric determinations of the geoid are not possible from an incomplete gravity anomaly data coverage. If the evaluation of Stokes' integral where only to be carried out for spherical caps centred on the computation point, with angular radii ψ ranging from 30^o to 80^o, the resultant error in the geoid height can be as large as 10 metres (e.g. WONG & GORE 1969, Figure 3). Some method must therefore be developed for representing the unsurveyed areas beyond ψ.

2.3.6.1 *Using Satellite Determined Gravity Anomalies*

This technique makes use of the low degree spherical harmonic model of the earth's gravity field obtained from an analysis of the orbital perturbations of near-earth artificial satellites (see § 2.3.3.1). The gravity anomaly value at a point P with spherical coordinates (ϕ,λ,R) can be obtained from spherical harmonic coefficients $\bar{C}_{\alpha nm}$ (see discussion on equation 2.35), by applying the fundamental boundary condition for a spherical earth (equation 2.44) to the disturbing potential of the solid earth and oceans T' (defined by equation 2.35). The satellite-determined gravity anomaly Δg_s is given by:

$$\Delta g_{sp} = \frac{GM}{R_p^2} \sum_{n=0}^{n'} (n-1) \left(\frac{a}{R_p}\right)^n \sum_{m=0}^{n} \sum_{\alpha=1}^{2} \bar{C}_{\alpha nm} S_{\alpha nm} \ , \ n \neq 1 \tag{2.63}$$

where $S_{\alpha nm}$ is defined by equation 2.29 and all other quantities are as defined in sub-section 2.3.3. The gravity anomaly Δg_s is free of the effect of the atmosphere and therefore does not need to be treated in the same manner as anomalies based on observed surface gravity (section 4.5).

Stokes' integral may then be evaluated using a global gravity anomaly field comprising data provided by surface gravimetry supplemented with satellite determined values.

However, the most commonly used procedure for combining the representation afforded by equation 2.63 and the available surface gravimetry is based on the subdivision of the free air geoid undulation N into three components (RAPP 1973):

(1) N_1 - the contribution from Stokes' integral using "residual" gravity anomalies and an integration over a spherical cap out to some radius ψ_o. "Residual" gravity anomalies are formed by differencing the anomalies implied by the potential coefficients (equation 2.63) and the terrestrial gravity anomalies, i.e. $(\Delta g - \Delta g_s)$.

(2) N_2 - the geoid undulation deduced from the low degree spherical harmonic model of the geopotential as defined by equation 2.37.

(3) N_3 - the distant zone undulation contribution of the "residual" gravity anomalies through Stokes' integral from the region beyond ψ_o.

The N_1 component provides the short wavelength information on the geoid undulation that is missing from the spherical harmonic representation of the geoid N_2, while N_3 is assumed negligible for practical evaluations.

This combination technique has been applied extensively to determine geoid undulations with respect to a geocentric reference ellipsoid (e.g. VINCENT & MARSH 1973, MARSH & CHANG 1979).

The accuracy of the geoid computed using this procedure is dependent on the following factors:

- The accuracy of the gravity field model coefficients $\bar{C}_{\alpha nm}$ and the value of n' (required to be as high as possible).
- The value of ψ_o (required to be as large as possible).
- The factors affecting gravimetric determinations - described in § 2.3.5.4 and 2.3.5.5 (principally the density of data in the integration cap and the level of random and systematic error).

RAPP (1973) investigated the effect of these factors on geoid determinations as a function of ψ_o. It was estimated that for ψ_o equal to 10^o - 20^o, the total error is of the order of ±3m, of which approximately ±1.5m was attributed to potential coefficient errors in the then available gravity field models such as the Smithsonian Standard Earth II (GAPOSCHKIN & LAMBECK 1970). Present day gravity field models such as GEM9 (LERCH ET AL 1977) are a significant improvement. The effect of the SST on the computation of N_1 was not considered in this investigation.

BRENNECKE & GROTEN (1977) showed however, that the error in the gravimetrically-determined geoid height due the influence of the SST (§ 2.3.5.5) could be reduced if a low degree spherical harmonic model of the geopotential were incorporated in the geoid determination, as described above.

A possible source of error peculiar to combination solutions is the different manner in which the atmosphere influences the terrestrial (see sub-section 4.5.1) and satellite gravity anomalies (see RUMMEL & RAPP

1976).

2.3.6.2 *Using Truncation Functions*

An elegant technique for combining potential coefficient information and surface gravimetry was proposed by MOLODENSKII ET AL (1962). This technique combines a limited cap integration of terrestrial gravity anomalies with truncation functions $Q_n(\psi_o)$ to give the Stokesian part of the geoid height,

$$N = \frac{R}{4\pi\gamma}\int_0^{2\pi}\int_0^{\psi_o} f(\psi)\Delta g \sin\psi d\psi d\alpha + \frac{GM}{R\gamma}\sum_{n=2}^{n'} Q_n(\psi_o)(n-1)\left(\frac{a}{R}\right)^n \sum_{m=0}^{n}\sum_{\alpha=1}^{2} \bar{C}_{\alpha nm} S_{\alpha nm} \qquad (2.64)$$

The Molodenskii truncation functions are defined by (IBID, p.147):

$$Q_n(\psi_o) = \int_{-1}^{\cos\psi_o} f(\psi) P_{n0}(\cos\psi)\, d(\cos\psi) \qquad (2.65)$$

where $P_{n0}(\cos\psi)$ is the Legendre polynomial of degree n.

Values of $Q_n(\psi_o)$ for ψ_o in the range 0^o to 180^o, at 10^o increments, and for n up to 12 are given in OJENGBEDE (1973, Table 3.7). Note that $Q_n(180^o) = 0$.

GROTEN & RUMMEL (1974) have used this technique to determine the geoid for Europe. The accuracy of this method is dependent on the same factors that influence the other combination technique, but in addition is dependent on the accuracy with which equation 2.65 can be evaluated.

Procedures to improve the accuracy of combining a Stokesian integration in a limited cap area with long wavelength information from potential coefficients are described in e.g. COLOMBO (1979), RAPP (1979). In the approach described in RAPP (1979), equation 2.64 is modified to incorporate a modified Stokesian kernel and a set of modified Molodenskii truncation functions. Such an approach appears to be considerably superior for computational purposes than that presented in equation 2.65 (for more details see IBID).

Combination methods for the determination of a high precision detailed geoid appear to be the most promising techniques, although the accuracy requirements for both gravity anomaly data and potential coefficients are perhaps unattainable at the present time for oceanic geoid determinations to ±10cm (CHRISTODOULIDIS 1979).

2.3.7 NON-GEODETIC TECHNIQUES FOR THE DETERMINATION OF THE GEOID

The marine geoid can be determined indirectly from estimates of the stationary SST provided by the results of steric levelling. The method of steric levelling is described in § 3.3.3.2.

Figure 3.3 illustrates an oceanographically-determined SST model, the contours representing the deviation of the MSL surface from the surface of some adopted *standard ocean*. The standard ocean is a model of a deep homogeneous ocean with a constant temperature and salinity, whose surface is assumed to coincide with a level surface of the earth's gravity field. The average elevation of the sea surface above the surface of the standard ocean in Figure 3.3 is approximately +1.10m. If the "oceanic" definition of the geoid (§ 2.2.5.3) were adopted, the quantity +1.10m is equivalent to the height of the geoid above the surface of the standard ocean and the +1.10m contour would represent the line of intersection between the MSL surface and the geoid. In the Pacific Ocean this contour intersects Asia, North and South America at high latitudes, and in the Atlantic Ocean it intersects the U.S.A., western Africa and South Africa.

Given a reliable map of the topography of the MSL surface over the global oceans, the geoid can be determined geometrically in a straightforward manner. If the height of the sea surface ζ above the reference ellipsoid were known (e.g. from satellite altimetry - see sub-section 2.3.4), the geoid undulation may be determined easily from (see Figure 2.3):

$$N = \zeta - \zeta_s \qquad (2.66)$$

where ζ_s in the stationary SST with respect to the adopted definition of the geoid, obtained for example by subtracting 1.10m from the global field of oceanographically-determined SST depicted in Figure 3.3.

The shortest wavelength information in the resultant geoid is equivalent to the greater of the two limiting wavelengths contained in:

(i) the ocean surface model defined by the global values of (for GEOS-3 data this is less than 100km).

(ii) the stationary SST model (for Figure 3.3 this is 10^3km).

Similarly, the accuracy of the geoid is dependent on the accuracy with which both ζ and the SST ζ_s can be obtained.

Such a marine geoid however, cannot play a role in SST studies as an a priori model of the SST has been used in its determination.

2.3.8 SUMMARY

Each method of geoid determination described in the previous sections provides an estimate of the geoid that may differ in scale, orientation and shape at some level of resolution (see FUBARA & MOURAD 1971 for discussion).

These differences are due to a number of factors:

(1) The parameters of the reference ellipsoid (a,f,GM,ω) and its location with respect to the geocentre.

(2) The type of data together with its reduction to the "anomalous" quantity (gravity anomaly, deflection of the vertical, etc.), both in theory and in practice.

(3) The density, quality and distribution of the data in space and time.

The degradation in the accuracy of a geoid determination due to poor data quality or irregular distribution is quite a separate problem to that arising from a fundamental inadequacy in the technique. Nevertheless, to some level of resolution (e.g. 1 part in 10^6), the "geoid" as determined by a number of techniques can be made equivalent by:

(i) some form of transformation of the data or of the resultant geoid heights to ensure that the results relate to a common ellipsoid.

(ii) suitable treatment of any effects arising from dynamic, gravitational or scale inconsistencies in the various reference systems used.

(iii) suitable corrections for any effects in order to adjust the "approximate" geoid to one implied by the adopted definition.

It must be recognised though that in some cases the "approximate" geoid may not be a level surface of the earth's gravity field (e.g. the altimetric "geoid" in sub-section 2.3.4 is more correctly referred to as a "sea surface model").

For geoid determinations aiming for an accuracy of ±10cm, the basic precepts and assumptions inherent in all the methods need to be closely examined and, if found wanting, they may need to be refined or reformulated. For each technique it should be established whether any of the following shortcomings are present, and whether they may be easily remedied:

(a) Inadequate input data requiring a better quality, greater quantity or more complete coverage of such data. This may or may not be practicable.

(b) Whether the shortcoming in (a) can be overcome by the introduction of a different data source.

(c) A lack of conceptual sophistication in the technique which would require a certain amount of reformulation.

Some methods are more amenable to reformulation than others. Other methods become impracticable under the stringent precision requirements imposed due to an inability to overcome data inadequacies.

The most promising techniques for the determination of a high precision marine geoid are those based on satellite altimetry and surface gravity data in combination with satellite-determined geopotential models of the earth's gravity field. Each of these is discussed in greater detail in section 5.1.

2.4 THE GEOID IN FOUR DIMENSIONS

2.4.1 PREAMBLE

It was established in § 2.2.5.6 that an adequate conceptual definition for the geoid, for some specified epoch of time, is that level surface of the earth's gravity field which best fits global MSL as sampled on an equi-area basis in the world's oceans. The condition of best fit is defined as that of equal volume. In the case of the real (deformable) earth, the location of the level surfaces undergo changes on a variety of time scales. Consequently the geoid undulations are a function of time as well as position.

The dominant periodic variation is that due to the tide generating potential of the Sun and Moon. The radial disturbance in the level surfaces, which can reach a magnitude of 50cm, can be modelled to an accuracy of ±5cm (e.g. MELCHOIR 1973), although the additional effect of ocean loading will affect the accuracy (BRETREGER 1978). The tidal potential is also responsible for a permanent deformation of the earth's gravity field (see sub-section 7.1.5). In addition, short period mass changes will affect the spatial location of the level surfaces of the earth's gravity field, and hence the geoid. Included in this category is the effect of atmospheric circulation (see STOLZ & LARDEN 1979), variations in the water table and similar phenomena discussed in MUNK & MACDONALD (1960, Chapter 9).

Mass redistribution within the earth also causes the geocentre to move. The change in position of the geocentre in response to seasonal variations in air mass, ground water and sea level was investigated by STOLZ (1976a), STOLZ (1976b) but was found to be insignificant (of the order of 5mm).

As mentioned in § 2.1.1.8, interest is focussed primarily on the gravity field of a static, non-deformable earth located in a space free of external forces and therefore departures from rigidity which have periods less than one epoch are treated either as corrections to the observations, additional model parameters or measurement noise.

Secular variations in the geoid height however, influences the *continuity* of the geoid over geodetic time scales (up to 10^2 years). Such geoid variations are essentially of two types:

- Variations in the *definition* (e.g. net volume changes or changes in scale).
- Variations in the *shape* (changes in the geoid height as a function of position).

These are discussed in the following sections.

2.4.2 SECULAR VARIATION IN THE GEOID DEFINITION

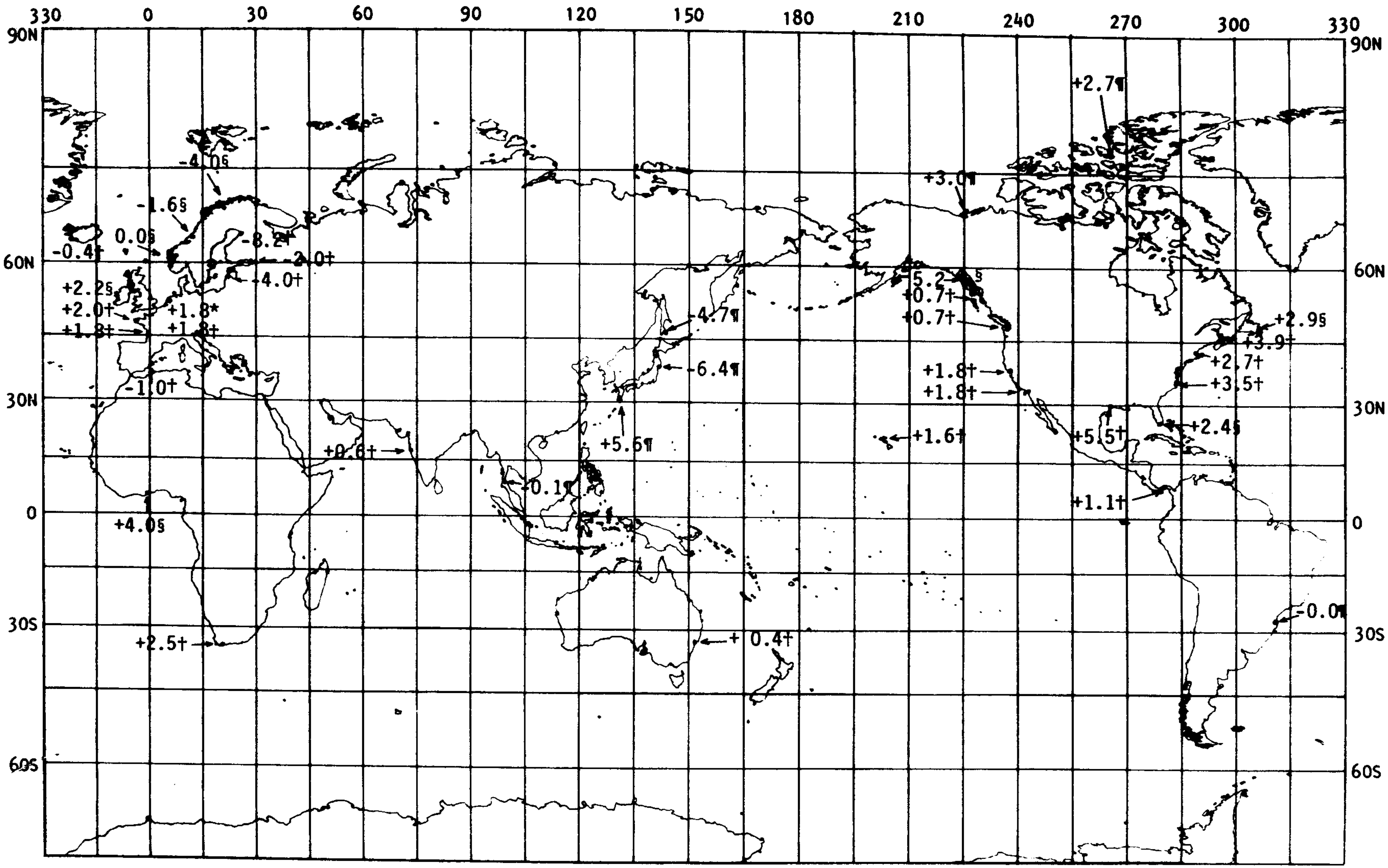
Such a variation could arise from an average global change in MSL as a result of changes in the net volume of sea water. A summary of systematic trends in the heights of sea level as recorded at coastal tide gauges to the present time is shown in Figure 2.6 (from MATHER 1978d). A positive secular change in MSL can be attributed to either a systematic sinking of the tide gauge support or a rise in sea level. While relatively large local changes (e.g. the Gulf of Bothnia) could be interpreted as being due to local uplift of an eustatic nature (rebound of the crust following the melting of paleo-icecaps), there is an apparent slow average rate of increase in global MSL of approximately 0.3mm/yr. Such an increase could be accounted for by a reduction in the ice cover over Antarctica at a rate of 0.5cm/yr (IBID). Therefore, on present evidence, secular changes in MSL do not affect the validity of the definition of the geoid to ±5cm, if such definitions are restricted to geodetic time scales. Nevertheless, the shape and size of the present geoid is very much different from that of the *paleogeoid* (in a time period > 10^4 years B.P.) (PARDI ET AL 1979).

A second type of geoid definition variation is a consequence of a scale change resulting from earth expansion or effects of a similar nature. There are two possible manifestations of such a scale change:

(a) The altered geoid may be a level surface with a different geopotential W_0 but with the same spatial location - a consequence of a change in the value of the universal gravitational constant G.

(b) The altered geoid may have the same geopotential W_0 but with a different spatial location in relation to the reference ellipsoid - a consequence of earth expansion.

The concept of an expanding earth has been the subject of some study (e.g. CAREY 1976). Earth expansion involves an overall decrease in the density



Key to Symbols Used for Length of Recording Period: * = > 100 yr; † = 50-100 yr; § = 20-50 yr; ¶ = < 20 yr.

FIGURE 2.6 Secular Changes in Mean Sea Level (mm yr^{-1})

Source: MATHER (1978d)

of the earth, without internal mass redistribution. However, the consequence of such an occurrence on the gravity field cannot be separated from that due to a reduction in the gravitational constant G, as predicted by the tensor-scalar theory of gravitation (DICKE 1967)(although it is more than likely that such effects are not mutually exclusive). This can be illustrated in the following manner. Consider the motion of a satellite during two epochs. An expanding earth would cause the geocentric coordinates of satellite tracking stations to change and therefore the ephemeris determined from tracking data would appear to indicate that the satellite orbit has been displaced away from the geocentre because,

(i) the measurement to the satellite remains unchanged between epochs.

(ii) the orbital behaviour of the satellite is unaffected because there is no change in the earth's total mass or its distribution.

However, a displacement of the satellite orbit between epochs would also be observed if the gravitational constant G were to decrease, although the station coordinates would remain unchanged. The relationship between the orbital radius and the value of G is discussed in section 7.3. An expansion of the orbit by 10cm, due to an expansion of the earth by this amount, is equivalent to a change of $-0.017\ km^3s^{-2}$ in GM (5 parts in 10^8). The detection of secular changes of the order of 1 part in 10^8/century is within the scope of present day metrology.

MATHER ET AL (1977b) have investigated the feasibility of using a global network of absolute gravity observing stations to monitor secular geodynamics. It was found that a rate of earth expansion/reduction in GM was a well defined quantity if the rate of expansion was in excess of 0.1cm/yr (or $0.0002\ km^3s^{-2}$/yr in GM). However, as with the satellite tracking technique this method is unable to resolve whether the change in the observed absolute gravity is due to a raising of the gravity station or a decrease in the value of G. Evidence presented by CAREY (1976) for an expansion in excess of 1cm/yr is given in support of his controversial repudiation of the Plate Tectonics hypothesis.

Current estimates of the rate of reduction in the value of G vary from 1 part in 10^{11}/year (DICKE 1967, Table 3) to 1 part in 10^{10}/year (NEWTON 1968, p.3765).

On the basis of these estimates for earth expansion/change in G /MSL increase, which are at the noise level of high precision geodetic measurement, <u>the definition of the geoid would be expected to remain valid for geodetic time spans</u>.

2.4.3 SECULAR VARIATION IN GEOID SHAPE

Another concern which has a consequence on the long-term stability of the geoid determination is geoid height changes as a function of position as well as time. The problem of whether the geoid could be considered invariant for geodetic time scales was investigated by MATHER ET AL (1979a).

Phenomena responsible for changes in the shape of the geoid can be separated into two groups:

- Those that can be characterised by a change in the shape of the ellipsoid that best fits the geoid.

- Those resulting from mass transfers during tectonic plate motion.

An example of the former would be the secular increase in the length of the day which could presage a change in the earth's meridional flattening after an appropriate delay (related to the rigidity of the earth). However, as such an increase is estimated to be only 0.1 msec/yr (BIH 1975, p.B23), the resultant change in the shape of the level surfaces is negligible. Similarly, plate motion could affect the meridional flattening if there was a net movement of mass with symmetry about the equator, giving rise to changes in the geopotential surfaces which would have the characteristics of a second degree zonal harmonic. It is also possible to envisage secular changes to the equatorial flattening of the best fitting ellipsoid due to plate motion. Such geometric changes in the figure of the earth, together with geocentre motion, could be monitored by a global absolute gravity network (MATHER ET AL 1977b). Figure 1 of MATHER ET AL (1979a, p.29) illustrates the variation in the shape of the geoid over a century for some modelled rates of earth expansion (sub-section 2.4.2) and changes in meridional and equatorial flattening. For more details see IBID (1979a).

The second type of geoid height change is the product of mass transfer mechanisms within the earth. These mechanisms operate over a variety of spatial and time scales (MATHER & LARDEN 1978):

(1) GLOBAL - where mass transfer is associated with the postulated convective processes within the earth's lithosphere necessary to "drive" tectonic plate motion.

(2) REGIONAL - where mass transfer is a product of frictional processes at plate boundaries.

(3) LOCAL - mass transfer due to fault displacement associated with earthquake activity.

Repeated gravimeter measurements can be used to study short wavelength gravity changes associated with regional and local geophysical phenomena (see e.g. BOULANGER 1973, FUJITA & FUJITA 1973, HAGIWARA 1977, GROTEN 1979). For a discussion of the role of gravity measurements in vertical crustal motion studies see sub-section 8.3.2. The effect of gravity changes δg on the shape of the regional geoid may be estimated from an evaluation of Stokes' integral (equation 2.42). Although a detailed study of local and regional mass transfer has not been undertaken, the relatively short wavelength effects are not expected to influence geoid definitions over geodetic time scales.

However, mass transfer on a global scale could present problems for the maintenance of a high precision geoid over time spans of the order of 10^2 years.

2.4.4 GEOID SHAPE CHANGES AS A RESULT OF TECTONIC PLATE MOTION

The following development is from an investigation by MATHER ET AL (1979a) of the time variations in geoid height for long to medium wavelengths (9×10^3 - 1×10^3 km) due to global tectonic plate motion.

The change in the geopotential at the earth's surface δW_m due to mass change δm at an element of surface area $d\sigma$ as a result of plate motion can be computed using the relation:

$$\delta W_m = \frac{G}{2R} \iint \operatorname{cosec}(\psi/2)\, \delta m \, d\sigma \qquad (2.67)$$

where all other quantities have been defined earlier.

There is at present no model for the global distribution of δm on a geodetic time scale. There is however, a tendency for subduction zones (areas where plate material is destroyed, such as at oceanic trenches and sites of continent-continent collisions) to be correlated with geoidal highs. Compare Figure 2.2 (GEM9 geoid) with the distribution of plate boundaries shown in Figure 2.7. In addition, gravity anomalies at mid ocean ridges (spreading zones where plate material is infused from the mantle) tend to the correlated with anomalies in the elevation of the sea floor (SCLATER ET AL 1975) and positive gravity anomalies to be correlated with subduction zones. Furthermore, it is plausible that mass transfer occurs in some way from spreading zones (equivalent to zones of mass "loss") to subduction zones (equivalent to zones of mass "accumulation").

Present estimates of plate velocities (see Table 2.1 - from SOLOMON & SLEEP 1974) over the past 5^6-10^6yr indicate that the rate of growth of different plates is highly variable and, as a result, mass transfer of some form must occur between colliding plates. For example, the Antarctica, Africa and American plates are growing whereas the Arabian, Australian, Pacific and Nazca plates experience significant shrinkage (GARFUNKEL 1975, p.4425). In view of the absence of local balance between spreading zones and subduction zones, and assuming that they *are* the major "sources" and "sinks" of lithospheric material, there must be a net influx of mass from shrinking plates to expanding ones through some form of sub-lithospheric flow. In the present situation, this appears to occur from the northern hemisphere to the southern hemisphere and should result in overall odd degree zonal harmonic characteristics in the global set of δm. The modelling of such sub-lithospheric flow has not been attempted here, instead two models for mass transfer based on widely differing concepts were used to estimate δW_m globally for a geodetic time span (10^2 years).

The plate motion model used in the present series of calculations is that proposed by SOLOMON & SLEEP (1974, p.2558). This model was augmented from KAULA (1975, Figure 1) to obtain 12 tectonic plates (see Figure 2.7). The plate boundaries were digitised in the form of straight line segments ℓ and the "type" of boundary (i.e. subduction, spreading, etc.) is as described in the above two references. The relative rotation rate vectors are given in Table 2.1 and are based on holding the African plate stationary.

TABLE 2.1

Adopted Values for Relative Plate Motions

Plate	Rotation Pole ϕ (deg)	λ (deg)	Relative Rotation Vector* 10^{-7} deg/yr	ω_1	ω_2	ω_3
ARB-Arabian	24.0	21.6	4.16	3.53	1.40	1.69
AFR-African	0.0	90.0	0.0			
ANT-Antarctic	3.4	308.8	2.58	-1.61	2.01	-0.16
AUS-Australian	15.0	47.9	6.33	4.10	4.54	1.64
CAR-Caribbean	39.0	309.5	3.63	-1.80	2.18	-2.29
COC-Cocos	-16.9	55.8	16.18	-8.70	-12.81	4.70
EUA-Eurasian	25.1	333.0	3.28	-2.64	1.35	-1.39
NAZ-Nazca	-37.4	31.2	5.23	-3.55	-2.16	3.18
NAM-Nth American	66.0	330.8	3.44	-1.22	0.68	-3.14
PAC-Pacific	57.6	296.4	11.06	-2.64	5.31	-9.34
PHL-Philippine	-65.5	62.1	9.72	1.88	3.56	-8.85
SAM-Sth American	66.0	330.8	3.44	-1.22	0.68	-3.14

* The listed rotation vector ω for a plate is appropriate for a coordinate frame in which AFR is stationary.
The rotation components ω_i are related to the geocentric Cartesian coordinate system X_i.
Source: SOLOMON & SLEEP (1974).

The two mass transfer models are:

MODEL 1

This model for δm assumes that mass is conserved within a plate over geodetic time spans, as a more sophisticated modelling of mass transfer of the type referred to earlier was not possible for such short time spans.

The mass δm accumulated or lost along a straight boundary segment of length ℓ can be represented by:

$$\delta m = \rho \, \ell \, d\ell \, T \tag{2.68}$$

where $d\ell$ is the distance the boundary segment moves in time dt (10^2 years), ρ is the density of matter accumulated or lost and T is the thickness of the tectonic plate.

The motion of the plate boundary segment ℓ is established in the following manner. The change in position ΔX_{ij} at the mid-point $P_j(\phi_j, \lambda_j)$ of a boundary segment of the kth plate which is rotating counterclockwise with components ω_{ik} about the geocentric Cartesian axes X_i (Table 2.1) after a time interval dt is given by:

$$\Delta X_{ij} = \varepsilon_{inm} \omega_{nk} X_{mj} dt + o\{\omega^2_{nk} X_{ij}\} \tag{2.69}$$

where subscript convention applies and,

$$\varepsilon_{inm} = \begin{cases} 0 & \text{if } i=n,\ n=m \text{ or } i=m \\ 1 & \text{if subscripts are ordered } 12312\ldots \\ -1 & \text{if subscripts are ordered } 13213\ldots \end{cases} \qquad (2.70)$$

The components $(\delta N_j, \delta E_j)$ of this motion in the north and east directions in the local horizon at P_j are given by:

$$\delta N_j = -\Delta X_{1j}\sin\phi_j\cos\lambda_j - \Delta X_{2j}\sin\phi_j\sin\lambda_j + \Delta X_{3j}\cos\phi_j \qquad (2.71)$$

and

$$\delta E_j = -\Delta X_{1j}\sin\lambda_j + \Delta X_{2j}\cos\lambda_j \qquad (2.72)$$

The component $d\ell_j$ of the motion of the boundary segment orthogonal to the plate boundary is obtained from:

$$d\ell_j = \delta N_j\sin\alpha_j - \delta E_j\cos\alpha_j \qquad (2.73)$$

where α_j is the azimuth of the plate boundary at P_j.

The density ρ_s of matter accumulated at subduction zones over such short time periods is assumed to be the representative value 2.67 gcm^{-3}. Values of T = 5km and T = 35km were adopted for the thicknesses of the oceanic and continental plates respectively. By enforcing the conservation of mass within a plate, the average density of matter being "lost" at spreading zones ρ_{sp} can be estimated from the relation:

$$\rho_{sp} = \frac{-\rho_s \sum\limits_{\substack{\text{subduction}\\ \text{zones}}} \ell\, d\ell\, T}{\sum\limits_{\substack{\text{spreading}\\ \text{zones}}} \ell\, d\ell\, T} \qquad (2.74)$$

TABLE 2.2

Density of mass "lost" at Spreading Centres using Model 1 for the transfer of mass due to Plate Motion (mass conservation per plate). Density of mass accumulating at Subduction Zones = 2.67 g cm^{-3}.

Plate Identification	ρ_{sp} for Model 1 (equation 2.74) (g cm^{-3})
ARB - Arabian	13.9
AFR - African	-
ANT - Antarctic	0.1
AUS - Australian	6.6
CAR - Caribbean	0.0
COC - Cocos	2.7
EUA - Eurasian	33.5
NAZ - Nazca	2.5
NAM - Nth American	14.2
PAC - Pacific	3.4
PHL - Philippine	0.0
SAM - Sth American	13.9

Source: MATHER ET AL (1979a)

Table 2.2 gives the values of ρ_{sp} for the 12 tectonic plates. The Arabian plate possesses no spreading zones on its boundaries, therefore in order to enforce mass conservation, the density of the subduction zones (ρ_s) was set to zero.

Figure 2.7 illustrates the values of δW_m (from equation 2.67) (expressed as geoid height changes using Brun's equation - equation 2.36) on adopting MODEL 1 for the mass transfer.

MODEL 2

For this model, mass conservation is enforced by introducing a negative mass Δm at the point whose geocentric Cartesian coordinates are X_{mi}. These quantities are defined as follows:

$$\Delta m = -\rho_s \sum_{\substack{\text{subduction}\\ \text{zones}}} \ell \, d\ell \, T \qquad (2.75)$$

and

$$X_{mi} = \frac{-\rho_s \sum_{\substack{\text{subduction}\\ \text{zones}}} \ell \, d\ell \, T \, X_{\ell i}}{\Delta m} \qquad (2.76)$$

where $X_{\ell i}$ are the distances of the mid-point of ℓ from the X_i axes centred at the geocentre. MODEL 2 acknowledges the temporary mass accumulation that occurs at subduction zones but replaces the large number of sources from which mass is "lost" in MODEL 1 (the spreading zones) by a single source for want of more information. MODEL 2 is equivalent to MODEL 1 if the mass loss in the latter were represented by a first degree harmonic model.

In both models it is assumed that all plate boundaries other than subduction zones and spreading zones are not subject to any mass change. The values of Δm and X_{mi} for this plate motion model, operating over 10^2 years, are:

$$\Delta m = -0.139 \times 10^{20} \text{ grams} \tag{2.77}$$

and

$$X_{m1} = -508.4 \text{ km} \; ; \quad X_{m2} = 718.9 \text{ km} \; ; \quad X_{m3} = 523.0 \text{ km} \tag{2.78}$$

The significance of a positive value for X_{m3} is that there are more subduction zones in the northern hemisphere than in the southern hemisphere, confirming suggestions that the expanding plates tend to be clustered in the southern hemisphere (GARFUNKEL 1975).

Figure 2.8 illustrates the change in the shape of the geoid for δm implied by MODEL 2. The pattern of contours in Figures 2.7 and 2.8 is a function of the distribution of plate boundaries and the similarity in the two is a reflection of the mass accumulation at subduction zones and the apparent dominant first degree harmonic characteristic of the mass "loss" which is common to both models. The most consistent features are high rates of change over the Himalayas and the Philippine plate.

The mass transfers due to plate motion do not appear to make significant contributions to the change in the shape of the geoid over time spans of 10^2 years. It must be emphasised however, that these transfer mechanisms (MODEL 1 & 2) are implausible for a geological time scale (> 10^5 years) as convective processes within the mantle transfer mass accumulating at subduction zones, from shrinking plates to expanding plates.

Recently MATHER & LARDEN (1978) conducted a more comprehensive study of mass transfer models using the same computer software as was used in the previous study. Their investigation considered the following factors:

(1) The effect of varying the plate parameters ρ_s, ρ_{sp} and T.

(2) The effect of more detailed plate boundary classification, e.g. differentiating between subduction zones of continental crust and continental lithosphere, oceanic crust and oceanic lithosphere from continent-continent collisions such as that occurring at the Himalayas.

(3) The effect of varying the conditions introduced to enforce mass conservation.

The changes in the geoid shape deduced from a wide variety of mass transfer models are shown in Figures 3 to 10 (IBID, p.18 et. seq.) and confirm that the geoid height changes over geodetic time scales are too small to be of concern. This contention would require revision if it were established that a steady rate of plate motion substantially greater than that modelled in these studies were taking place, and that the mass transfer had the characteristics of low degree harmonics as would be the case if transport of mass were occurring from the shrinking plates to the expanding plates, particularly if the latter were concentrated in a particular part of the earth (e.g. in the southern hemisphere).

A global network of high precision absolute gravity stations could continuously monitor gravity, and would enable variations in the shape of the geoid due to secular geodynamics to be defined through Stokes' integral.

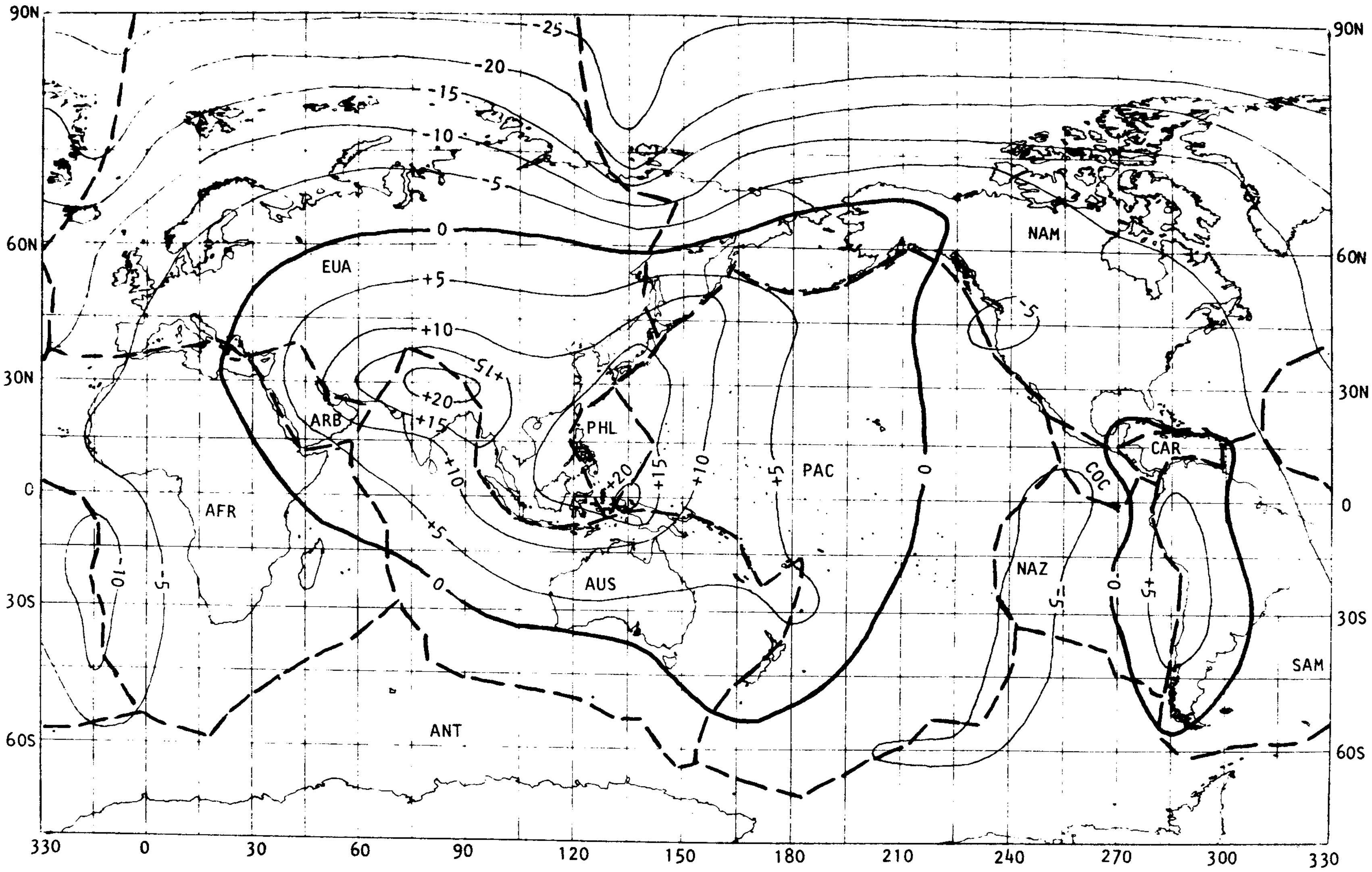
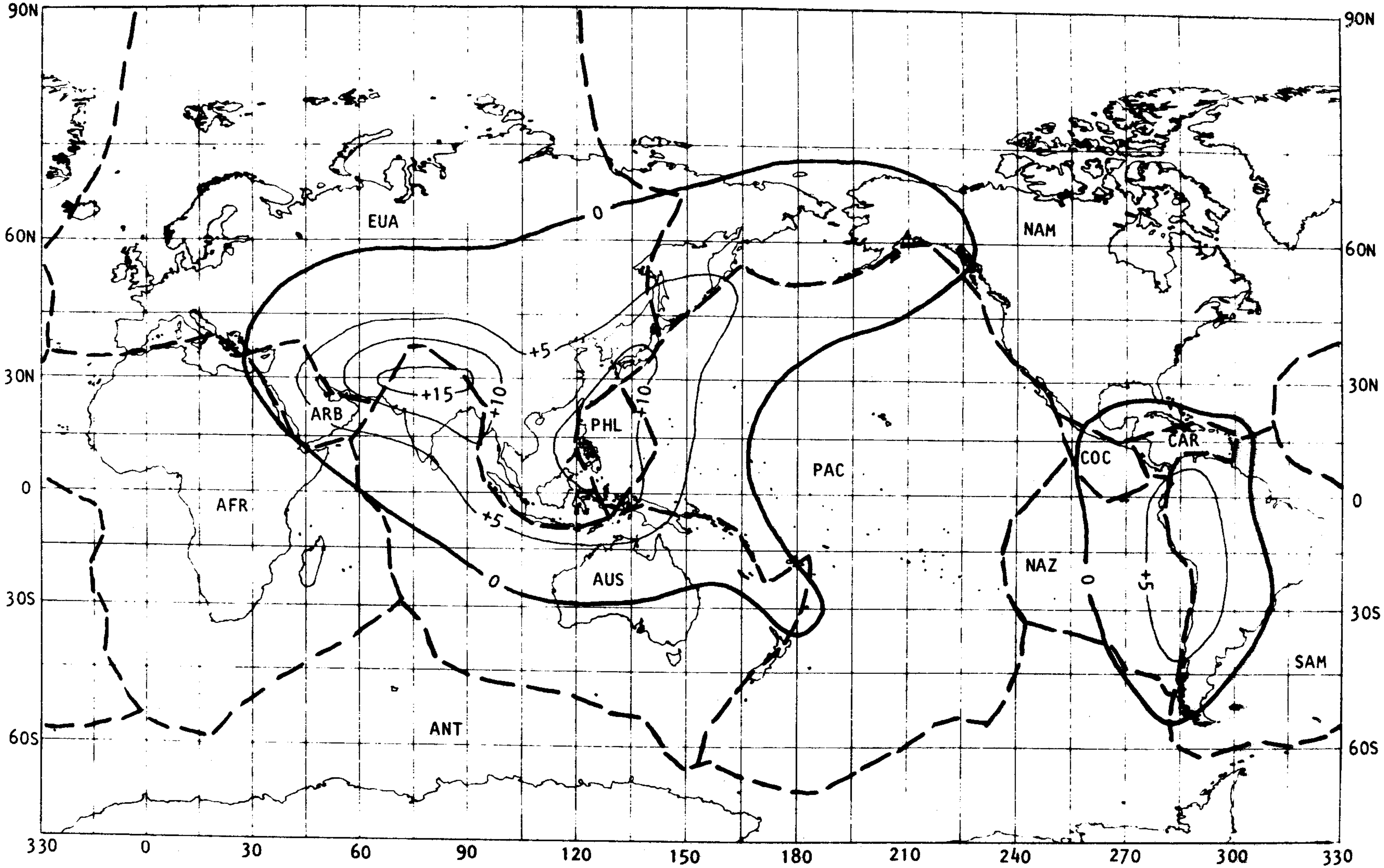


FIGURE 2.7 Variation in Geoid Undulations Due to Mass Transfers Associated With Plate Motion Using Model 1 for Mass "Loss"

Source: *MATHER ET AL (1979a)* (Contour Interval - 5 mm per century)



F I G U R E 2.8 Variation in Geoid Undulations Due to Mass Transfers Associated With Plate Motion Using Model 2 for Mass "Loss"

Source: *MATHER ET AL (1979a)* (Contour Interval - 5 mm per century)

3

VERTICAL REFERENCE SYSTEMS AND THEIR RELATION TO MEAN SEA LEVEL

3.1 INTRODUCTION

3.1.1 REFERENCE SYSTEMS IN GEODESY

Geodesy is the field of applied science that deals with the size and shape of the earth and the location of points on or near its surface. It is therefore a task of geodesy to define and maintain a reference system which enables position to be defined. The relevance of such a reference system is governed by the ease with which geodetic observations can be referred to it.

There are a variety of possible coordinate reference systems (see e.g. CAPPELLARI ET AL 1976, Chapter 3), each defined by an origin for coordinates, a reference plane and a principal direction in the reference plane. Geodetic coordinate systems can be divided into two categories:

- Body-centred inertial coordinate systems.
- Body-centred rotating coordinate systems.

The manipulation of "traditional" geodetic measurements such as astronomic observations, horizontal and vertical observations, slope distances and geodetic levelling is most easily carried out on a geodetic reference system that rotates with the earth. In such a case there is a one-to-one correspondence between the physical points on the earth's surface and a set of mathematical coordinates that do not contain the effect of the earth's diurnal rotation. An example of such a reference system is the right-handed Cartesian coordinate system, the location and orientation of the X_i axes being dictated by circumstances. The "natural" reference system has the origin at the geocentre, the X_3 axis coinciding with the axis of rotation and the X_1, X_2 axes, which complete the reference frame, lying respectively in the plane of the reference meridian (conventionally the Greenwich meridian) and at right angles to it.

Another commonly used rotating reference system is that afforded by the various curvilinear coordinate systems where the position of a point is expressed in relation to an arbitrarily defined geometric figure. This figure is commonly an oblate ellipsoid of revolution which closely approximates the geoid. Such a geodetic coordinate system is defined in terms of five parameters a, f, X_{1o}, X_{2o} and X_{3o}, where (a,f) denote the semi-major axis and flattening of the reference ellipsoid and X_{io} are the cartesian coordinates of the centre of the reference ellipsoid with respect to the geocentre. The semi-minor axis of the ellipsoid is assumed parallel to the earth's axis of rotation.

The geodetic latitude ϕ of a point P on the earth's surface is defined to be the angle between the normal at P and the plane containing the ellipsoidal equator. The geodetic longitude λ of P is the angle between two meridian planes, both containing the semi-minor axis, one containing P and the other some arbitrary reference meridian such as the mean Greenwich meridian. The geodetic coordinate triad is completed by including the quantity h, the height of P above the reference ellipsoid as measured along the spherop normal.

The geodetic coordinates (ϕ,λ,h) of a point P are related to those in the geocentric Cartesian coordinate system by the well known formulae (e.g. HEISKANEN & MORITZ 1967, p.205):

$$\begin{aligned} X_1 &= (\nu + h)\cos\phi\cos\lambda + X_{1o} \\ X_2 &= (\nu + h)\cos\phi\sin\lambda + X_{2o} \\ X_3 &= \left(\nu(1-f)^2 + h\right)\sin\phi + X_{3o} \end{aligned} \tag{3.1}$$

where the radius of curvature in the meridional plane is:

$$\nu = a\left(1-(2f-f^2)\sin^2\phi\right)^{-\frac{1}{2}} \tag{3.2}$$

The reverse relations are (CAPPELLARI ET AL 1976, p.3-44 et seq.):

$$\lambda = \tan^{-1}(X_2'/X_1')$$

$$\text{iterate}\left[\begin{aligned} \phi &= \tan^{-1}\left(\frac{X_3'(\nu+h)\sin\lambda}{X_2'(\nu(1-f)^2+h)}\right) = \tan^{-1}\left(\frac{X_3'(\nu+h)\cos\lambda}{X_1'(\nu(1-f)^2+h)}\right) \\ h &= \frac{X_1'}{\cos\phi\cos\lambda} - \nu = \frac{X_2'}{\cos\phi\sin\lambda} - \nu = \frac{X_3'}{\sin\phi} - \nu(1-f)^2 \end{aligned}\right. \tag{3.3}$$

where

$$X_i' = X_i - X_{io} \tag{3.4}$$

Space techniques currently claim to define position in the X_i system with sub-metre precision in each coordinate (although there is some doubt about possible systematic effects) at satellite tracking stations within a global network responsible for maintaining satellite ephemerides (e.g. ANDERLE 1974, SMITH ET AL 1976) or at fixed "observatory" type installations such as those that range to the Moon (see e.g. BENDER ET AL 1973). There is some degradation in accuracy for "stand alone" systems such as the commercial Doppler equipment. The accuracy with which position can be determined at sea is far lower. However, the accuracy promised by space techniques is not translated into the regional geodetic coordinate system (equation 3.3) due to the uncertainty in the values of the geocentric translation parameters X_{io}.

Unlike the geodetic coordinates (ϕ,λ,h), the natural coordinates Φ (astronomic latitude), Λ (astronomic longitude) and H (height above mean sea level) are measurable. The earth's gravity field is inextricably involved in all observations related to this coordinate system. The angles Φ and Λ define the direction of the plumbline or local vertical at a point on the earth's surface and are deduced from astronomical observations. The orthometric height H is obtained from the results of geodetic levelling combined with gravity measurements as described in § 3.2.1.1.

Because of irregularities in the earth's gravity field, there is no simple mathematical relation comparable to equation 3.1 or 3.3 between (Φ,Λ,H) and (ϕ,λ,h). It is possible to relate the natural coordinates to geodetic coordinates by (equations 2.27a, 2.40 and 2.52):

$$\begin{aligned} \phi &= \Phi - \xi \\ \lambda &= \Lambda - \eta\sec\phi \\ h &= H + N = H^* + \zeta \end{aligned} \tag{3.5}$$

where ξ and η are components of the deflection of the vertical (sub-section 2.3.2), N is the height of the geoid above the reference ellipsoid, H^* and ζ are the normal height and height anomaly in the Earth-Telluroid system (§ 2.3.5.3).

It is the vertical component of the geodetic reference system that is of most concern in sea surface topography studies. This is discussed in section 3.2. The determination of the quantities N and ζ was described in section 2.3. Evaluation of H and H^* is described in sub-section 3.2.1. The relevance of geodetic levelling in high precision geodesy is discussed in sub-section 3.2.2.

3.1.2 THE GEODETIC SYSTEM OF REFERENCE IN FOUR DIMENSIONS

The physical concepts upon which geocentric reference systems (geodetic or Cartesian) are based - the geocentre and the axis of rotation - can only be considered *instantaneous*, as the real earth is non-rigid. Deformation of the earth, and the subsequent mass redistributions, result in movement of the geocentre and the pole of rotation with respect to a fixed point on the earth (tectonic plate motion, earth tides, etc., further complicate this picture). Such considerations are of no consequence in studies of geodetic position to precisions of 1 part in 10^6.

Departures of the rotational characteristics of the earth from those of a rigid body model are essentially of two types:

(i) Those which are a consequence of variations in the rate of rotation (important in satellite geodesy where transformations between inertial reference systems and earth-fixed systems are necessary).

(ii) Those due to changes of the position of the poles of the rotation axis with respect to the earth's crust, i.e. polar motion.

A detailed description of the techniques that can be used to monitor the rotational characteristics of the earth and the motion of the geocentre is beyond the scope of this thesis. For a useful summary see KOLACZEK & WEIFFENBACH (1975).

Such manifestations of the departure of the real earth from a rigid body model have periods ranging from less than one day to many years. Effects due to variations in position of the instantaneous geocentric reference system which have periods shorter than one epoch can be suitably eliminated by modelling (e.g. earth tides and short period polar motion - see § 2.1.1.8). In such a case, the reference system can be considered "fixed" for some epoch of measurement. However, the location and orientation of the geocentric reference system may vary from epoch to epoch.

It is possible to relate a coordinate system X_i at epoch $(\tau = t)$ to the χ_i system at some <u>selected epoch of reference</u> $(\tau = t_o)$. The coordinates of a point P at the earth's surface on the two reference systems are related by equations of the form (MATHER 1973b, p.107):

$$\chi_i = X_i + \chi_{io} - \varepsilon_{ijk} X_j \omega_k \tag{3.6}$$

where ε_{ijk} is defined in equation 2.70.

χ_{io} are the average Cartesian coordinates of the geocentre for epoch $(\tau = t)$ with respect to the origin of the χ_i coordinate system and ω_i are the counterclockwise rotations of the X_i axes about the X_i axes.

An example of a system of coordinates χ_i which allows position to be unambiguously referred irrespective of epoch is the CIO - Greenwich system In the CIO - Greenwich system the X_3 axis is made coincident with the average pole position (the Conventional International Origin) between 1900 and 1905, as defined by the five astronomic observatories of the International Latitude Service. At the present time, the position of the pole with respect to CIO is monitored by the 1968 BIH system (GUINOT & FEISSEL 1968). The $\chi_1\chi_3$ plane is defined by the CIO pole and the point of intersection of the plane of zero longitude and the equator, as maintained in an average sense by the network of astronomic observatories that make up the BIH system.

In such a system, the counterclockwise rotations ω_i are related to the conventional components x and y of polar motion (IBID, p.9) between epoch $(\tau = t)$ and the reference epoch $(\tau = t_o)$ by:

$$\omega_1 = y \quad ; \quad \omega_2 = x \quad ; \quad \omega_3 = 0 \tag{3.7}$$

Equations similar to 3.6 and 3.7 can be used to relate coordinates on any regional Cartesian system to CIO - Greenwich if:

(1) The <u>displacement of the regional origin</u> from the geocentre is known; and

(2) the <u>appropriate rotations</u> ω_i are given.

Furthermore, any global three dimensional position determination can be referred to the χ_i axes irrespective of epoch of observation provided (MATHER ET AL 1979a, p.17):

- the relation between the instantaneous system of reference and the representative reference system for that epoch $(\tau = t)$ were known; and
- polar motion and geocentre motion were defined between the epoch of reference $(\tau = t_o)$ and the representative epoch of observation $(\tau = t)$.

The relationships between geodetic coordinate systems at different epochs are much clumsier mathematical expressions (see e.g. HEISKANEN & MORITZ 1967, Section 5-4, 5-9). There are essentially three transformations involved:

(i) A correction for the change in dimensions (a,f) of the adopted reference ellipsoid.

(ii) A translation due to non-coincidence of the centres of the ellipsoid at the epoch of observation and the ellipsoid at the reference epoch.

(iii) A transformation due to non-coincidence of the semi-minor axes of the ellipsoid at epoch ($\tau = t$) and the ellipsoid at the reference epoch ($\tau = t_o$).

In practice, it is much easier to convert geodetic coordinates at some epoch into equivalent Cartesian coordinates using equation 3.1, transform these (equation 3.6) into some other Cartesian reference system and then, if required, to determine the transformed geodetic coordinates using equation 3.3.

3.1.3 REFERENCE SYSTEMS IN OCEANOGRAPHY

The role of a reference system in oceanography is to facilitate the quantitative description of the character of the ocean waters and their movements. However, the horizontal precision with which a system of reference need be defined is not critical. Precisions of the order of 1 in 10^4 (approximately 0.5-1 km) are adequate for mapping the spatial characteristics of sea water and its movements. Consequently a system of horizontal reference based on latitude and longitude, without the sophistication demanded by high precision geodesy (sub-section 3.1.2), is normally adopted.

The vertical component is given by the height of the sea surface above a reference ellipsoid. However, the vertical reference system in oceanography is subject to a unique problem: the spatial location of the sea surface undergoes variations with time (see sub-section 3.3.2).

For 1 part in 10^6 geodesy, the MSL surface can be assumed to define the geoid and consequently the height of MSL can be considered the height of the geoid above the reference ellipsoid. In reality, MSL departs from the geoid by amounts of the order of $\pm$ 1-2m and, in addition, the open ocean surface deviates in space and time from MSL due to the action of winds, waves, tides and variable currents. Nevertheless, the motion of the physical ocean surface does not seriously interfere with geodetic concepts at the level of accuracy of a few metres.

In physical oceanography however, the study of the vertical motion of the sea level is often as important as the mean position of the sea surface. Further, the stricter requirements of $\pm$ 10cm geodesy have led to a re-examination of SST, its variation with time, and its role in the definition of the geoid (see sub-sections 2.2.3 and 2.2.5). Therefore, the establishment of a vertical system of reference for the unambiguous determination of the stationary and time-varying constituents of SST is an essential prerequisite for satisfying geodetic and oceanographic goals.

The systems of reference used in oceanic levelling are described in sub-section 3.3.3. SST in the open oceans can, in principle, be determined by geodetic techniques as discussed in section 5.2, and are based on the reference system described earlier in sub-section 3.1.1.

3.2 VERTICAL REFERENCE SYSTEMS FOR GEODESY

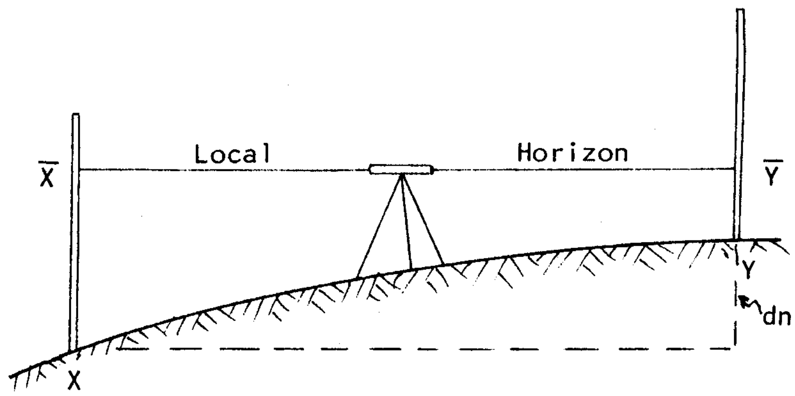
3.2.1 GEODETIC LEVELLING

The vertical position of a point P on the earth's surface is most commonly expressed in a system of heights above the reference ellipsoid. As previously mentioned, the ellipsoidal height consists of two components (equation 3.5). In the conventional geoid/earth's surface system, one component is the height of the reference geop (the geoid) above the reference ellipsoid. The other component is the orthometric height or the height of the surface point above the geoid as measured along the vertical through P, and is ordinarily obtained from geodetic levelling operations. The geodetic levelling procedures include spirit levelling and the less precise trigonometrical levelling technique. These procedures are described in detail in textbooks of surveying (see e.g. BOMFORD 1962, Chapter 4). Attention will be restricted to the method of spirit levelling.

The principle of spirit levelling is very simple although laborious to execute. To measure the *difference* in height between two point X and Y, linearly graduated staves are set up vertically at these two points and a levelling instrument between them (see Figure 3.1). The instrument's line of sight $\overline{X}\overline{Y}$ is tangential to that equipotential surface of the earth's gravity field passing through the instrument's horizontal axis. Since the line $\overline{X}\overline{Y}$ is horizontal, the difference in height between X and Y is determined from the readings made on the staves by an observer at the levelling instrument. This process is extended over an entire area so that if the height of one point is given, the height differences thereby obtained are used to form a network of heights.

FIGURE 3.1

Spirit Levelling



A closer study of this operation reveals an assumption that is not strictly correct. If the points X and Y are so far apart that the procedure shown in Figure 3.1 has to be applied repeatedly, then the sum of the levelled height differences between X and Y will not necessarily be equal to the difference in the orthometric heights at X and Y. This is due to the non-parallelism of the level surfaces of the earth's gravity field. The levelling increment dn given by the staff readings measures linear separation and not the separation between level surfaces. A consequence of this is that the results of spirit levelling are route dependent.

The solution to this "holonomity" problem for a rigid earth model (earth tide effects on observations to X and Y, although small, can be taken into account, see e.g. ANGUS-LEPPAN 1975, Section 5.6) is to convert the linear staff increments to potential differences dW;

$$dW = -g \, dn \tag{3.8}$$

(If observed gravity g is not available it is possible to use normal gravity γ - obtaining "normal" potential differences). Thus, geodetic levelling combined with a knowledge of gravity can provide geopotential differences which are not route dependent. In the case referred to above, the geopotential difference between X and Y is obtained from the levelling operation by:

$$W_Y - W_X = -\int_X^Y g \, dn \tag{3.9}$$

where W_X and W_Y is the geopotential at X and Y respectively.

If the levelling line returns to X, the total integral is zero. (This is not strictly true for the integration of normal potential differences) because of the effect of gravity anomalies on the levelling instrument).

A system of vertical reference consists of geopotential differences ΔW of points with respect to the geoid. For a point P this is defined by (HEISKANEN & MORITZ 1967, Section 4.1):

$$\Delta W = W_P - W_o = -\int_{geoid}^P g \, dn \tag{3.10}$$

the path of integration being the levelling route from the geoid to P.

Alternative systems of vertical reference utilising the results of geodetic levelling and taking into account the non-parallelism of level surfaces include geopotential numbers and dynamic heights. These systems, described in IBID (Section 4.2), represent efforts to reconcile the practical utility of a height system of control for mapping operations with the scientific rigour of the geopotential system.

3.2.1.1 *Othometric Height*

For most mapping applications the height system of vertical control is preferred and the results of spirit levelling, computed in the geopotential system, need to be converted into orthometric height. The relationship between orthometric height H, and geopotential difference ΔW (obtained from levelling increments using equation 3.10) is given by (IBID):

$$H_P = -\frac{\Delta W_P}{\bar{g}} \tag{3.11}$$

where the subscript p refers to the point of evaluation and $\bar{g}$ is the mean value of gravity along the plumbline between P and the geoid. The value of $\bar{g}$, and hence the orthometric height, can never be known precisely as the density of matter between the earth's surface and the geoid is unknown.

An approximation for $\bar{g}$ is provided by the Prey reduction:

$$\bar{g} = g - \left(\tfrac{1}{2}\frac{\partial\gamma}{\partial h} + 2\pi G\rho\right) H \tag{3.12}$$

where $\partial\gamma/\partial h$ is the normal free air gravity gradient (equation 2.46), G the universal gravitation constant and ρ is the average density. For $\partial\gamma/\partial h^{(mgal)} = 0.3086\ H^{(m)}$, $\rho = 2.67\ gcm^{-3}$ and $G = 6.67 \times 10^{-11} m^3 s^{-2} kg^{-1}$, equation (3.12) becomes:

$$\bar{g}^{(mgal)} = g^{(mgal)} + 0.0424\ H^{(km)} \tag{3.13}$$

This is the basis of the Helmert method for determining the orthometric height,

$$H_P = \frac{-\Delta W_P}{g + 0.0424\ H_P} \tag{3.14}$$

An approximate value for H_P can be used on the right hand side of equation 3.14 as g does not strongly depend on it. This procedure corresponds to replacing the topography above the geoid with an infinite Bouguer plate of constant density ρ and thickness H and is one of a number of methods of estimating g described in IBID (1967). The magnitude of errors in orthometric height due to crustal density errors are given in MATHER (1974c, Table 1). For example, a 20% error in ρ (equation 3.12) and an orthometric height of 10^3m will be in error by 50 ppm, i.e. 5cm. Therefore, for high precision scientific purposes it is more satisfactory to work in the system of geopotential differences.

In mapping operations, an orthometric correction is evaluated and added to observed levelling increments dn to obtain an orthometric height difference. The orthometric height difference between X and Y is given by (HEISKANEN & MORITZ 1967, equation A-33):

$$O_{XY} = \int_X^Y \frac{g - \gamma_o}{\gamma_o}\,dn + \frac{(\bar{g}_X - \gamma_o)}{\gamma_o} H_X - \frac{(\bar{g}_Y - \gamma_o)}{\gamma_o} H_Y \tag{3.15}$$

where $\bar{g}_X$ and $\bar{g}_Y$ are the mean values of gravity along the plumblines through X and Y, H_X and H_Y are approximate orthometric heights.

Normal orthometric height is defined by equation 3.11 replacing $\bar{g}$ by the mean value of normal gravity along the ellipsoidal normal.

3.2.1.2 *Normal Height*

In the conventional approach to determining the orthometric height (equation 3.11) and the geoid height by the gravimetric technique (subsection 2.3.5), the density of matter exterior to the geoid needs to be known to sufficient accuracy or failing this, some assumptions made concerning it. As such a practice is unsatisfactory for high precision geodesy, Molodenskii proposed a different approach. The geometric principles are described in § 2.3.5.3 (Figure 2.4), the ellipsoidal height h is now defined by,

$$h = H^* + \zeta \tag{3.16}$$

The normal height H^* replacing the orthometric height and the height anomaly ζ replacing the geoid height. The problem is now the determination of the separation vector between "equivalent" points P ($\phi,\lambda,W_P = W_O + \Delta W$) on the earth's surface and Q ($\Phi,\Lambda,U_Q = U_O + \Delta W$) on the associated spherop $U = U_O$. The locus of points Q is called the telluroid. The normal height H^* is the height of the telluroid above the reference ellipsoid as measured along the ellipsoidal normal through Q and can be determined from levelling data combined with gravity measurements. No assumptions are made concerning the density of the crust as such a height system is based on the external normal gravity field of a reference ellipsoid defined by geometric parameters (a,f) and gravity parameters (GM,ω). The normal height is related to geopotential differences by an analytical expression (IBID, Section 8.3):

$$H^* = \frac{-\Delta W}{\gamma_o}\left(1 - \frac{\Delta W}{a\gamma_o}(1 + m + f - 2f\sin^2\phi_c) + \left(\frac{\Delta W}{a\gamma_o}\right)^2 + o\{f^3\}\right) \tag{3.17}$$

where γ_o is normal gravity on the reference ellipsoid, m is defined by equation 2.47 and ϕ_c is the geocentric latitude. All quantities in equation 3.17 can be defined without approximation.

The "vignal" height is numerically equal to the normal height but is conceptually different in that it is regarded as an approximation to the orthometric height, and is thus the height of the quasi-geoid above the reference ellipsoid (see § 2.3.5.3). HOLDAHL (1979a) reports a test level run across the USA using vignal heights, normal orthometric heights and Helmert derived orthometric heights. These heights failed to agree by up to 1 metre in the Rocky Mountains, the Helmert heights being considered the true orthometric heights.

3.2.2 THE ROLE OF GEODETIC LEVELLING NETWORKS IN HIGH PRECISION GEODESY

The ellipsoidal height h can be computed directly from three dimensional position determinations using equations 3.3. Such determinations make use of space techniques such as ranging to artificial satellites and the Moon, range-rate tracking of satellite and Very Long Baseline Interferometry (see e.g. MATHER & ANGUS-LEPPAN 1973 for a description of modern high precision position fixing techniques). It could be argued that such techniques therefore obviate the need for either a system of orthometric/geoid heights or normal heights/height anomalies for high precision geodesy. In this context ellipsoidal height derived from geometric information provided by modern global position fixing techniques could be judged as the best definition of earth space position. Although accuracies of ± 10cm in each coordinate have not been reliably achieved to date, technological advances should result in this happening in the near future.

The advantages of high precision three dimensional position determinations over conventional geodetic techniques include:

(1) <u>Total definition of spatial information</u> X_i as opposed to determining the vertical component of position separate from the horizontal component (ϕ,λ).

(2) Global relevance of results, as opposed to a regional one.

(3) Independent of the direct influence of the earth's gravity field. This is of particular concern in crustal motion studies as secular variations in the position of level surfaces affects the relevance of the results of relevelling (see sub-section 8.3.2).

(4) It is possible to deduce geoid heights/height anomalies at locations whose X_i coordinates are known from the orthometric height information supplied by a local geodetic authority (equations 3.3 & 3.5).

As a corollary to this is that the task of defining the geoid to the sub-metre level, particularly in continental areas, is no longer necessary for satisfying traditional geodetic roles.

Nevertheless, first order geodetic levelling retains the capacity to provide a higher relative resolution than any of the other geodetic processes at the present time. The propagation of random error in first order levelling is not expected to exceed $\pm 2\sqrt{D}$ (mm), where D is the length of the line of levelling in kilometres (i.e. the difference in backward and forward levelling between bench marks should not exceed $4\sqrt{D}$ (mm)). The adjustment of continental networks of first order levelling provides height data at bench marks which are internally consistent at the ± 20cm level or approximately 30 times superior to the requirements of geodesy at 1 part per million (however there are some doubts concerning the validity of such claims in light of recent investigations of long range geodetic levelling - see sub-section 3.4.3). This superior resolution is admittedly lost on transformation of the levelling results to notions of position in three dimensions using equation 3.5, due to:

(i) Lack of equivalent accuracy in geoid height determinations (see section 2.3 for discussion).

(ii) Departure of the levelling datum from the geoid (see § 2.3.5.5).

Neither of these is a problem if levelling information is only used on a differential basis over medium to short distances.

Geodetic levelling information is essential for precise engineering operations that are related to the dynamics of flow. Grades of ellipsoidal height dh obtained from three dimensional position fixes differ from grades of orthometric height dH obtained from levelling because of tilts of the level surfaces with respect to the ellipsoid. The relation is given by (MATHER 1978d, Section 4):

$$dh = dH - \varepsilon_d ds \qquad (3.18)$$

where ε_d is the component of the deflection in the direction of the grade over the distance ds. For example, in Australia it is not uncommon to have grades of the level surfaces in relation to the ellipsoid which are as large as 10m in 10^2km, and for such values to change by equivalent amounts over distances of the order of 10^2km (MATHER ET AL 1971, Figure 3.4). Consequently, the differences (dh - dH) can be as large as 10cm per km in relatively disturbed areas. This limits the role geometric determinations of height can play in replacing levelling operations for sophisticated engineering works - unless the shape of the level surfaces for short wavelengths is known to a high precision.

Geodetic levelling also has a role to play in vertical crustal motion studies. However, such information is not exactly equivalent to the desired geometric displacements unless appropriately combined with gravity and MSL data as a function of time (see sub-section 8.3.2). The recovery of equivalent information form satellite techniques using ±10cm systems is not expected to be competitive in the short term as levelling produces data with a more favourable signal-to-noise ratio. In addition, when levelling data is combined with accurate position determinations and gravity data, as functions of time, it is a valuable source of information for studying precursory phenomena associated with step-wise mass transfer due to earthquakes (see e.g. WHITCOMB 1976).

A third important need for continental levelling networks is in the establishment of networks of gravity and height anomalies on land for supporting ocean dynamic studies (see sub-section 6.1.1).

The fact that MSL at the levelling datum is not coincident with the geoid has to be taken into account in vertical crustal motion and SST studies (Figure 5.2 illustrates the situation). The geopotential difference ΔW at a point P as deduced from geodetic levelling data is related to the regional MSL datum, not the geoid (see § 2.3.5.5),

$$\Delta W_P = - \int_{\text{MSL datum}}^{P} g \, dn \tag{3.19}$$

This expression supersedes equation 3.10.

3.3 VERTICAL REFERENCE SYSTEMS IN OCEANOGRAPHY

3.3.1 PREAMBLE

The same geodetic principles of vertical reference may be applied in oceanic regions. The two components of ellipsoidal height are:

(i) the height of the marine geoid above the reference ellipsoid.

(ii) The deviation of the instantaneous or mean sea level surface from the geoid - the SST.

These two quantities, which make up the sea surface height ζ (Figure 2.3), are incorporated into the definition of the height anomaly as described in § 2.3.5.5 (Figure 2.5). To understand the relationship of sea level, the geoid and a reference system in four dimensions, it is necessary to investigate the time variations in ζ. This is attempted in sub-section 3.3.2. The quantity equivalent to "orthometric height" in oceanic areas - the sea surface topography - is obtained from the results of oceanic levelling. The techniques of oceanic levelling are described in sub-section 3.3.3.

3.3.2 VARIABILITY OF SEA LEVEL

The sea surface height undergoes time variations encompassing four broad spectral ranges (MATHER 1978c):

(a) Short period variations - those with periods less than a day.

(b) Intermediate period variations - those having periods between one and 10^2 days.

(c) Long period variations - periods from 10^2 days to 2 years.

(d) Quasi-stationary - constant over an epoch for which data is collected by surface and space techniques. Such a quasi-stationary surface is defined by MSL, although it is subject to periodic, random or secular time variations that are not detectable during the epoch of study (e.g. secular change in sea level due to the melting of icecaps - see sub-section 2.4.2).

The oceanographic phenomena affecting the sea surface height in these four time scales are described in the following sections.

3.3.2.1 *Short Period Phenomena*

Because of their comparatively large magnitude and pronounced periodicity the most dominant effect is that due to the tides. Although very marked tidal ranges of over 10m have been observed at localities such as the Bay of Fundy (near Newfoundland) and the Gulf of St. Malo (France), such occurrences are rare. The open ocean tidal range, although not well known, is not expected to exceed 1 metre and in many cases is much lower (see co-range tidal charts in, e.g. LISITZIN 1974). Fortunately, the periods of the tides derived theoretically from the motions of the Sun and Moon do occur in reality. The most pronounced harmonic constituents of the tide are the semi-diurnal (twice daily) constituents M_2, S_2 and the diurnal (daily) constituents K_1, O_1 and P_1 (IBID, p.12). Monthly mean levels of the sea computed from hourly tide gauge observations are virtually free from the dominant influence of the tides, almost all of which have a period less than a month. Further, sea level deduced from a year's such observations will adequately represent the long term non-tidal ocean surface. The largest tidal terms neglected being the long period nodal tide (period of 18.61 years and an amplitude of several centimetres), the semi-annual and annual tide (amplitude of the order of 1-2cm) and the "pole tide" (with a period of 14 months and a maximum amplitude of a couple of centimetres).

It has been estimated that the tidal potential, in addition to being responsible for ocean tides, produces geopotential variations due to self-potential of the order of 10-15cm (HENDERSHOTT 1973). This causes SST variations with the same periods as the tidal potential as a result of both a change in sea surface height and geoid height as a function of time (see § 5.2.2.3).

Tsunamis are short duration waves which radiate from coastal or submarine tectonic activity. These waves may travel with velocities of between 150 and 800 km/hr and with wavelengths of up to 500 km (MITCHELL 1973). They are of little consequence in the open oceans, having amplitudes of the order of 1-2 decimetres. Although the increase in amplitude when approaching coastlines has resulted in waves of great destructive power, their effects on MSL determinations is negligible.

Other short period oceanographic phenomena include those due to the direct action of wind, such as storm surges (water piling up against a coastline during periods of high wind), waves and swells. These effects may alter sea level by up to a couple of metres (IBID, p.136-142) although the effect of such intermittent occurrences on long period MSL is probably less than 10cm.

Variations in atmospheric pressure cause the sea level to react, at least theoretically, as an inverted barometer, depressing sea level 1 cm for every millibar increase in air pressure. MITCHELL (1973) has studied this effect for the oceanic areas surrounding Australia and has concluded that air pressure variations do not make significant changes to the monthly sea level tide gauge records as such barometric variations are usually associated with changes in direction and force of wind and can therefore not be distinguished from wind effects per se.

3.3.2.2 *Intermediate Period Variations*

Eddies are "mesoscale" circulatory patterns which frequently are spawned when western boundary currents transporting warm water from the equator to higher latitudes, form meanders which finally break off and become separated from the main stream. Western boundary currents are the most energetic circulatory features in the oceans and are found in the western margins of ocean basins, such as in the North Pacific and North Atlantic. The Kuroshio off Japan, the Gulf Stream off the east coast of North America and the East Australia current form a number of eddies each year (see Figure 3.6). Eastern boundary currents are usually slow and broad, being fed by the wind drifts of large scale ocean circulation and do not normally partition into eddies, although the West Australian current appears to be an exception (ANDREWS 1977).

Although eddies were first observed in the Gulf Stream about 40 years ago (ISELIN 1940), there is much that is unknown about their movements and life history. In the Atlantic Ocean, cyclonic eddies consisting of pockets of cold continental shelf water which have been pinched off by a Gulf Steam meander, rotate in a counterclockwise direction as they travel SW to decay in the vicinity of the Sargasso Sea. Anticyclonic eddies form on the western margins of the Gulf Stream where warm mid-Atlantic water is pinched off, imparting a clockwise rotation and proceed down the Florida coast to decay. The motion of major eddies and the location of both the edge of the continental shelf water and the Gulf Stream are monitored and a monthly record is published by the US National Weather Service in, for example, NOAA (1978). The rotations of cyclonic and anticyclonic eddies are reversed in the southern hemisphere.

The mechanism of eddy formation in the East Australia current appears to be somewhat different (NILSSON ET AL 1977). The eddies are triggered by or otherwise associated with the bottom topography of the area and appear to represent the decay of a portion of the East Australia current and not merely an insignificant offshoot of the main stream, as is the case with the more powerful Gulf Stream and Kuroshio current.

Typical eddies are of the order of 100-300 km in diameter with a mean life of 2-3 years (see e.g. RICHARDSON ET AL 1973, CHENEY & RICHARDSON 1976, RICHARDSON ET AL 1977). The average speed of eddies is about 2km/day, although they can reach speeds of 10km/day in the Gulf Stream or even "stall" for several weeks in the same location as is sometimes the case with East Australian eddies (NILSSON ET AL 1977). The density contrasts they exhibit with respect to the surrounding water (see § 3.3.3.2) result in eddies being manifest as either "mounds" (warm water or anticyclonic eddies) or "depressions" (cold water or cyclonic eddies) in the sea surface with amplitudes up to 1 metre. Consequently they may be detected by the use of satellite altimetry, as described in § 5.2.3.4.

Other intermediate period phenomena include variable, ill-defined surface currents such as the East Australia current. Surface currents can be identified by faster than normal surface velocities and distortions in the thermocline - the depth at which the temperature gradient is a maximum (in low and middle latitudes this is located between 200 and 1000m depending on the time of the year - PICKARD 1968). In addition, the physical manifestations of the main stream change rapidly in space and time.

Although the SST associated with migrating eddies and short period currents is of a significant amplitude, they tend to affect MSL on a regional basis and in a random fashion.

3.3.2.3 *Long Period Phenomena*

The dividing line between effects included in this category and those of the previous group is indistinct, and long period phenomena include extreme cases of effects already mentioned.

For example, one of the most dramatic occurrences in the Pacific Ocean is the El Nino (WYRTKI 1979), which is caused by a prolonged period of strong SE trade winds lasting 18 months or longer, during which sea level is increased by about 10cm in the western Pacific and lowered by about 5cm in its eastern part. When the wind field decreases in strength the potential energy stored in this east-west slope of sea level is released and water flows back across the Pacific leaving a pronounced signature in sea level records.

However, the most prevalent effects are seasonal in nature. Climate fluctuations cause seasonal variations in atmospheric pressure, wind stress and heating of the oceans, giving rise to corresponding changes in sea level. A theoretical investigation of the seasonal variability in the oceans by GILL & NIILER (1973) (although restricted to the North Pacific and North Atlantic) has shown that the most important response is that produced by expansion and contraction of the water column above the seasonal thermocline due to varying fluxes of heat. These steric changes in sea level can be in excess of 10cm. On the other hand, changes in atmospheric pressure are only of importance at high latitudes, causing a barometric effect on sea level of the order of 10cm. LISITZIN (1974), quoting an older study, obtains a similar pattern for this effect, although numerically the values differ considerably. Analysis of observational data by WYRTKI (1975) indicates that in fact the seasonal variation can be much larger in restricted areas such as the Kuroshio. Nevertheless, the average long period variability in sea surface height is estimated to be only of the order of ±10cm.

3.3.2.4 *Quasi-Stationary Phenomena*

These phenomena are the main cause for MSL deviating from a level surface of the earth's gravity field as a function of position. A phenomenon can only be considered "stationary" for the period of time that it is under observation. Two stationary oceanographic phenomena are:

- The distribution of water density.
- The pattern of major surface currents.

The courses of ocean currents are closely related to the spatial variation of water density as the dominant driving force of surface circulation is the horizontal gradient of SST (see section 1.1).

The application of geostrophic principles (§ 3.3.3.3) to surface velocity measurements can supplement the steric levelling results based on water density measurements (§ 3.3.3.2) and provide additional information on the stationary SST.

3.3.3 OCEANIC LEVELLING

In addition to the stationary component, ellipsoidal height (and hence SST) may vary with periods of less than one day to more than a year, with amplitudes ranging from a few centimetres to over 50cm. However, oceanic levelling relies on data collected during the course of surface ship surveys and because of the relatively slow data acquisition rate it is only possible to estimate the stationary component of the sea surface height.

Information about the shape of MSL relative to a level surface has come from two main sources (see discussion in sub-section 3.4.1):

(1) The comparison of free net adjustments of geodetic levelling data with MSL estimated at coastal tide gauges.

(2) The determination of SST in open oceans using the techniques of oceanic levelling. At the present time only the stationary component can be determined on a global basis by these techniques.

There are three main methods of estimating the spatial variation of the stationary SST in the open ocean (MONTGOMERY 1969):

(a) Pipeline Levelling (also known as Hydrostatic Levelling by geodesists).

(b) Steric levelling.

(c) Geostrophic Levelling.

These are discussed in greater detail in the following sections.

3.3.3.1 *Pipeline Levelling*

The implementation of this technique involves observing the free surface of a fluid in a long tube. The fluid in the pipeline responds to gravity and the free surfaces at each end define a profile of an equipotential surface of the earth's gravity field. When compared with sea level, the *grade* of the SST can be determined. Limitations due to practical considerations restrict this method to bridging narrow channels (IBID 1969) and it therefore plays no significant role in defining the global SST.

3.3.3.2 *Steric Levelling*

A description of the theoretical basis of this method and its execution in practice can be found in most textbooks on physical oceanography (e.g. FOMIN 1964, LISITZIN 1974, p.86 et seq.). In principle, temperature and salinity of sea water are measured and variations in water density as a function of position are determined. From this information it is inferred that "columns" of sea water with different vertical density distributions will stand at a varying "height" above some deep reference surface if they are to exert a constant pressure at this surface.

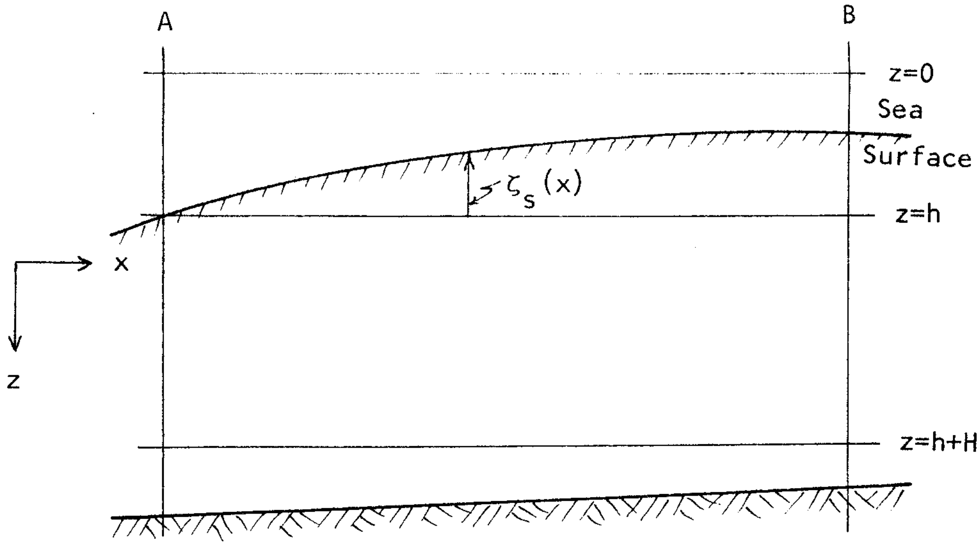
The basis of the method of steric levelling is the hydrostatic equation (e.g. DEFANT 1961, p.337):

$$g\,\rho(x,z) = \frac{\partial p}{\partial z} \tag{3.20}$$

relating the density of the fluid ρ and gravity g to the vertical gradient of pressure p acting upon an element of ocean in static equilibrium (large expanses of ocean can be considered to be in static equilibrium). The positive z direction is oriented along the downward vertical and x is some arbitrary direction normal to z, as indicated in Figure 3.2.

FIGURE 3.2

Reference System for Steric Levelling



$\zeta_s(x)$ is the SST referred to some level surface of the earth's gravity field, z = h. The relationship between the hydrostatic principles embodied in equation 3.20 and the SST can be established in the following manner.

Let z = 0 be a level surface at considerable elevation above the ocean surface, at which the atmospheric pressure can be assumed zero. Integrating equation 3.20 along the local vertical to some depth H (Figure 3.2) gives:

$$p = \int_0^h g\,\rho_a\,dz + \int_h^{h+H} g\,\rho_w\,dz \qquad (3.21)$$

where p is the pressure at depth H due to the overlying atmosphere (density ρ_a) and ocean (density ρ_w). Differentiation of equation 3.21 with respect to variable x results in,

$$\frac{\partial p}{\partial x} = \int_h^{h+H} \frac{\partial}{\partial x}(g\rho_w)\,dz + \frac{\partial p_a}{\partial x} + \frac{\partial h}{\partial x} g_s(\rho_{ws} - \rho_{as}) \qquad (3.22)$$

where the subscript s refers to values at the ocean surface. Note that the horizontal pressure gradient is comprised of two parts:

(1) The first term is dependent on the depth variable z and is the effect due to the horizontal inhomogeneity of the density field.

(2) The remaining terms do not change along the vertical (i.e. they are independent of z) and are due to the slope of the free sea surface.

In order to transform the integral in equation 3.22 into a form ordinarily used by oceanographers, the density ρ_w is replaced by the specific volume $\alpha_w = \rho_w^{-1}$ and the integration is performed with respect to the pressure p, to give:

$$\int_{h}^{h+H} \frac{\partial}{\partial x}(g\rho_w)\,dz = \int_{p_a}^{p_H} \frac{\partial}{\partial x}\left(\frac{g}{\alpha_w}\right)\frac{dz}{dp}\cdot dp \tag{3.23}$$

p_a and p_H are the pressure at the ocean surface (i.e. atmospheric pressure) and at $z = h + H$ respectively. According to the hydrostatic equation the vertical gradient of water pressure is related to specific volume and gravity by:

$$\frac{\partial z}{\partial p} = \frac{\alpha_w}{g} \tag{3.24}$$

Combination of equations 3.23 and 3.24 gives, to sufficient accuracy:

$$\int_{h}^{h+H} \frac{\partial}{\partial x}(g\rho_w)\,dz = \frac{1}{\bar{\alpha}_w}\,\frac{\partial}{\partial x}\int_{p_H}^{p_a} \alpha_w\,dp \tag{3.25}$$

where $\bar{\alpha}_w$ is the mean specific volume. The geopotential separation w' of the sea surface and the surface $z = h + H$ is defined by (MITCHELL 1978, Section 2):

$$w' = \int_{p_a}^{p_H} \alpha_w\,dp \tag{3.26}$$

In the oceanographic literature (e.g. FOMIN 1964, p.6) w' is termed the <u>dynamic height</u>, but is expressed in units of potential (kgalm).

If it is assumed that the ocean layer is non-accelerated and that frictional forces are negligible, i.e. steady state conditions apply, the Coriolis force balances the horizontal pressure gradient force (FOMIN 1964, p.4):

$$f\,v\,\rho_w = \frac{\partial p}{\partial x} \tag{3.27}$$

the velocity v is in the direction normal to the plane containing the x and z axes, and the Coriolis parameter f is defined by:

$$f = 2\,\omega\,\sin\phi \tag{3.28}$$

ω being the angular rotation of the earth.

Further, let it be assumed that at some depth D the horizontal pressure gradient is zero and therefore, according to equation 3.27, the current velocity v at this isobaric surface is zero. Such a surface, known as the "level of no motion", is in addition an <u>equipotential surface of the earth's gravity field</u>, for if the gravitational force were not balanced by pressure forces at this level there would be a resultant force acting on water particles causing motion. This is discussed further in sub-section 3.4.2.

At the level of no motion (i.e. v = 0), $z = h + D$, the combination of equations 3.22, 3.24, 3.26 and 3.27 gives:

$$\frac{\partial h}{\partial x} = \frac{-1}{\bar{g}(\bar{\rho}_w - \bar{\rho}_a)} \left\{ \bar{\rho}_w \frac{\partial w}{\partial x} + \frac{\partial p_a}{\partial x} \right\} \quad (3.29)$$

where replacing gravity, atmospheric and sea water density by representative mean values introduces errors of less than one centimetre to SST determinations (MITCHELL 1978, p.13) from equation 3.31. The quantity w, the geopotential separation between the level of no motion and the ocean surface is defined by (equation 3.26):

$$w = \int_{p_a}^{p_D} \alpha_w \, dp \quad (3.30)$$

The potential difference between two points on the same vertical, with pressures p_D and p_a respectively in equation 3.30, has the same characteristics as the geopotential difference ΔW (equation 3.10), i.e. the quantity has a positive magnitude when integration is carried out in the direction of the inward vertical (the positive z direction).

Integrating equation 3.29 with respect to x and noting that $\bar{\rho}_a \ll \bar{\rho}_w$ gives:

$$\zeta_s = -\frac{1}{\bar{g}} \left\{ w + \frac{p_a}{\bar{\rho}_w} + w_c \right\} \quad (3.31)$$

where w_c is the integration constant which is evaluated from the boundary conditions: $w = w_o$, $p_a = p_o$ when $\zeta_s = 0$. An expression for w_c is:

$$w_c = -\frac{p_o}{\bar{\rho}_w} - w_o \quad (3.32)$$

where w_o is the geopotential thickness (or "dynamic height") of the layer between the level of no motion and the zero SST surface. In linear units, the separation between the surfaces z = h and z = h + D is:

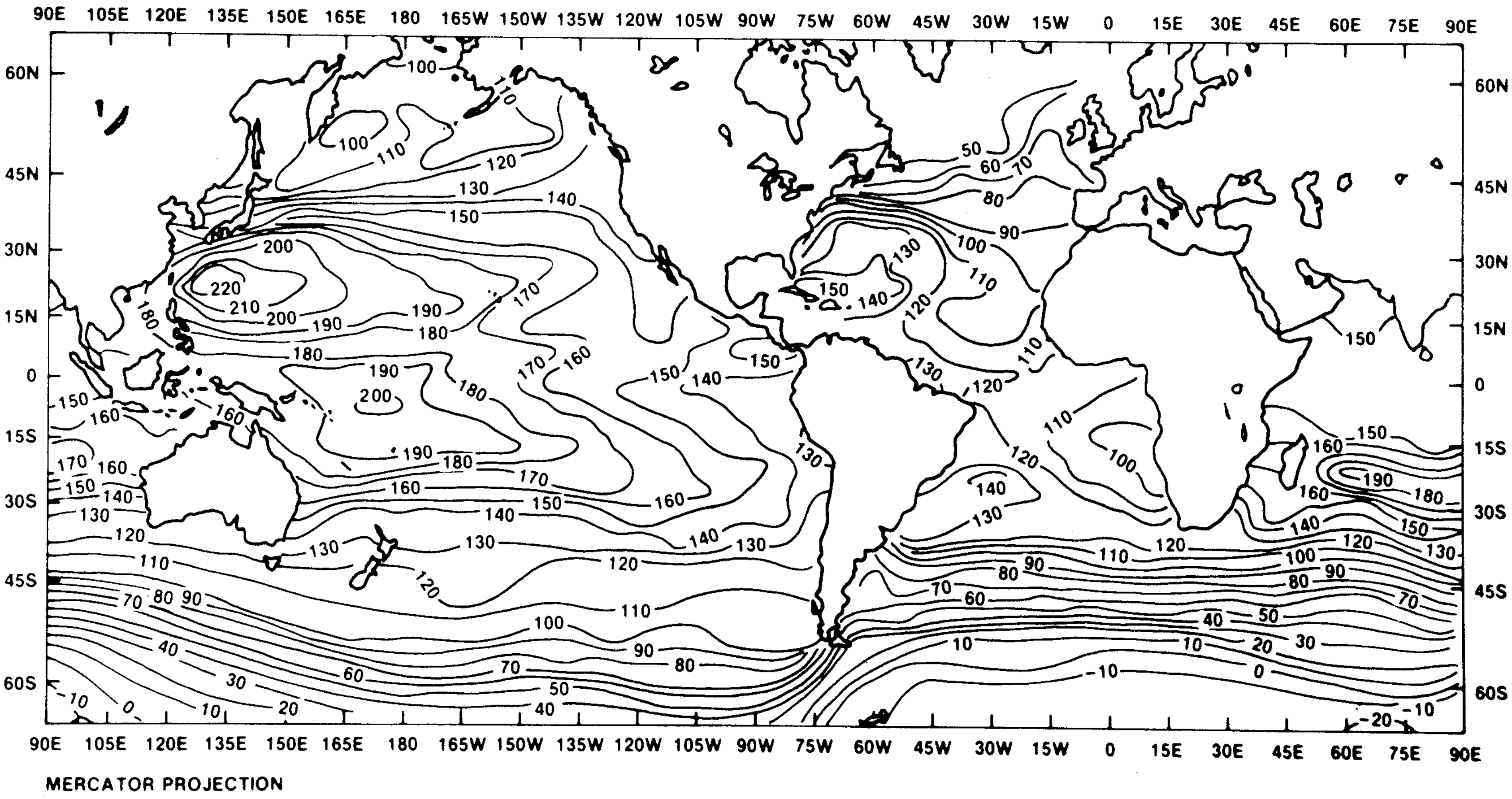
$$d = w_o/g \quad (3.33)$$

where d varies with latitude and can be computed using formulae given in FISCHER (1975, p.19).

Equation 3.31 can be written in terms of the <u>steric</u> or <u>dynamic height anomaly</u> $(w - w_o)$:

$$\zeta_s = -\frac{1}{\bar{g}} \left\{ (w - w_o) + \frac{dp_a}{\bar{\rho}_w} \right\} \quad (3.34)$$

where $dp_a = p_a - p_o$, is the deviation of the atmospheric pressure from some "standard" value p_o (say 1 atmosphere). Setting a value for w_o is equivalent to adopting a "standard" or "normal" value for the specific volume of the world's oceans α_o (for example, 273°K and a salinity of 35 parts per thousand) which is independent of location. The quantities w_o and p_o allow the departures of the real sea surface (ζ_s) from that of a "standard ocean" under the influence of a "standard atmosphere" to be



F I G U R E 3.3

Stationary Sea Surface Topography Relative to 1000 db Surface

Wavelengths > 10^3 km

Source: MATHER ET AL (1978b) (Contour Interval - 10 cm) computed from sea water temperature, salinity and atmospheric pressure data. Further, by defining the surface of the "standard ocean" as the geoid (through the use of appropriate values for w_o and p_o), the oceanographically-determined quantity ζ_s (equation 3.34) is equivalent to the SST defined in the geodetic context (sub-section 2.2.2).

Equation 3.34 may be expressed in terms of the observable quantities using equation 3.30:

$$\zeta_s = \frac{1}{\bar{g}} \left(\int_{p_a}^{p_o} (\alpha_w - \alpha_o)\, dp - \frac{dp_a}{\rho_w} \right) \qquad (3.35)$$

The atmospheric correction $dp_a/\bar{g}\rho_w$ in centimetres is numerically equivalent to the stationary atmospheric pressure anomaly dp_a in millibars. This correction has been mapped by STOMMEL (1964) and LISITZIN (1974, p.61). The range is of the order of 30cm and for the most part reduces the contrasts in the relief of the steric anomalies obtained from sea water density considerations alone.

The technique of steric levelling has been applied widely by oceanographers to determine the SST both on a regional and global basis. For example, it has been applied in the Atlantic Ocean by REID ET AL (1977), in the Pacific Ocean by WYRTIKI (1975) and REID (1961) and in the Antarctic by GORDON & BYE (1972). A global map of SST based on a reference pressure p_D of 1000 decibars (approximately 1000m) was prepared by STOMMEL (1964, chart 1), although no allowance was made for the atmospheric pressure anomalies (last term in equation 3.35). A world map by LISITZIN (1965, Fig. 2) illustrated the SST in relation to a reference isobaric surface of 4000 db and, in addition, had been adjusted for the stationary atmospheric effect. A more recent result has been obtained by LEVITAS & DORT (1977), although it is not clear whether the atmospheric effect was considered or what the reference level was.

Figure 3.3 (from MATHER ET AL 1978b, Fig. 1) is a representation of the long wavelength SST (> 10^3 km) as deduced from the results of steric levelling adjusted for the atmospheric effect. It has been compiled from the results of LISITZIN (1974), WYRTIKI (1975) and REID ET AL (1977) and is based on a 1000 db reference level. It must be emphasised however, that the contours represent the stationary SST and the real sea surface may depart from such a model by several decimetres due to temporal variations in sea level (see sub-section 3.3.2). In addition, no attempt has been made to represent details near coasts or in enclosed seas.

The mean value of SST depicted in Figure 3.3 is +1.10m and is due to the oceanographic datum for the dynamic height anomalies (i.e. the quantity α_o in equation 3.35) not being coincident with the "oceanic" definition of the geoid (§ 2.2.5.3). The mean SST for the northern hemisphere oceans is +1.28m, while the value for the southern hemisphere oceans is +1.00m. The Pacific Ocean mean SST is +1.47m, standing on average higher than any other ocean because of its low sea water density. The Indian Ocean value is +1.17m, while the mean SST in the Atlantic Ocean is only +0.90m, as it is composed of water with the highest average density. The difference in average levels of the Pacific and Atlantic Oceans is of the order of 60cm, and is in close agreement with the value obtained from transcontinental geodetic levelling (STURGES 1974).

3.3.3.3 *Geostrophic Levelling*

This levelling technique is based on the hydrodynamic concepts embodied in equation 3.27. At the surface of the ocean, noting that $\rho_w >> \rho_a$, equation 3.22 in combination with equation 3.27 becomes:

$$f\,v = g\,\frac{\partial \zeta_s}{\partial x} + \frac{\partial p_a}{\rho_w \partial x} \tag{3.36}$$

where f is the Coriolis parameter defined by equation 3.28 and v is the velocity of the geostrophic component of surface flow at right angles to the x direction. In the northern hemisphere the geostrophic current runs parallel to the SST contours such that the "high" SST is to the right of the current direction, and visa versa for the southern hemisphere.

Steady (geostrophic) flow is maintained by horizontal pressure forces produced by SST gradients and, to a lesser degree, by horizontal atmospheric pressure gradients. The equation of motion at 3.36 neglects all frictional forces, non-linear terms and is correct to the order of the flattening. In addition, a non-accelerated system is assumed and the pressure field is considered to be in hydrostatic balance (equation 8.2 is a generalisation of equation 3.36).

The Dynamic Method of physical oceanography (see e.g. FOMIN 1964) allows the ocean current vector to be determined from the horizontal gradients of the SST deduced from the results of steric levelling. By a reversal of this procedure the grades of SST across surface currents can be obtained from equation 3.36 if observations of the velocity vector have been made. This is the principle of geostrophic levelling. This technique has been applied on a regional basis to determine sea level slope over limited distances, e.g. across the Gulf Stream by MONTGOMERY (1969) and STURGES (1974).

The technique has also been used to investigate the relationship between the geostrophic mass transport of the North Equatorial Countercurrent and differences in sea level observed at a series of islands on either side of it (WYRTIKI 1973). In most circumstances however, geostrophic levelling principles are used to extrapolate the results of steric levelling from the open oceans into the coast (GODFREY 1973).

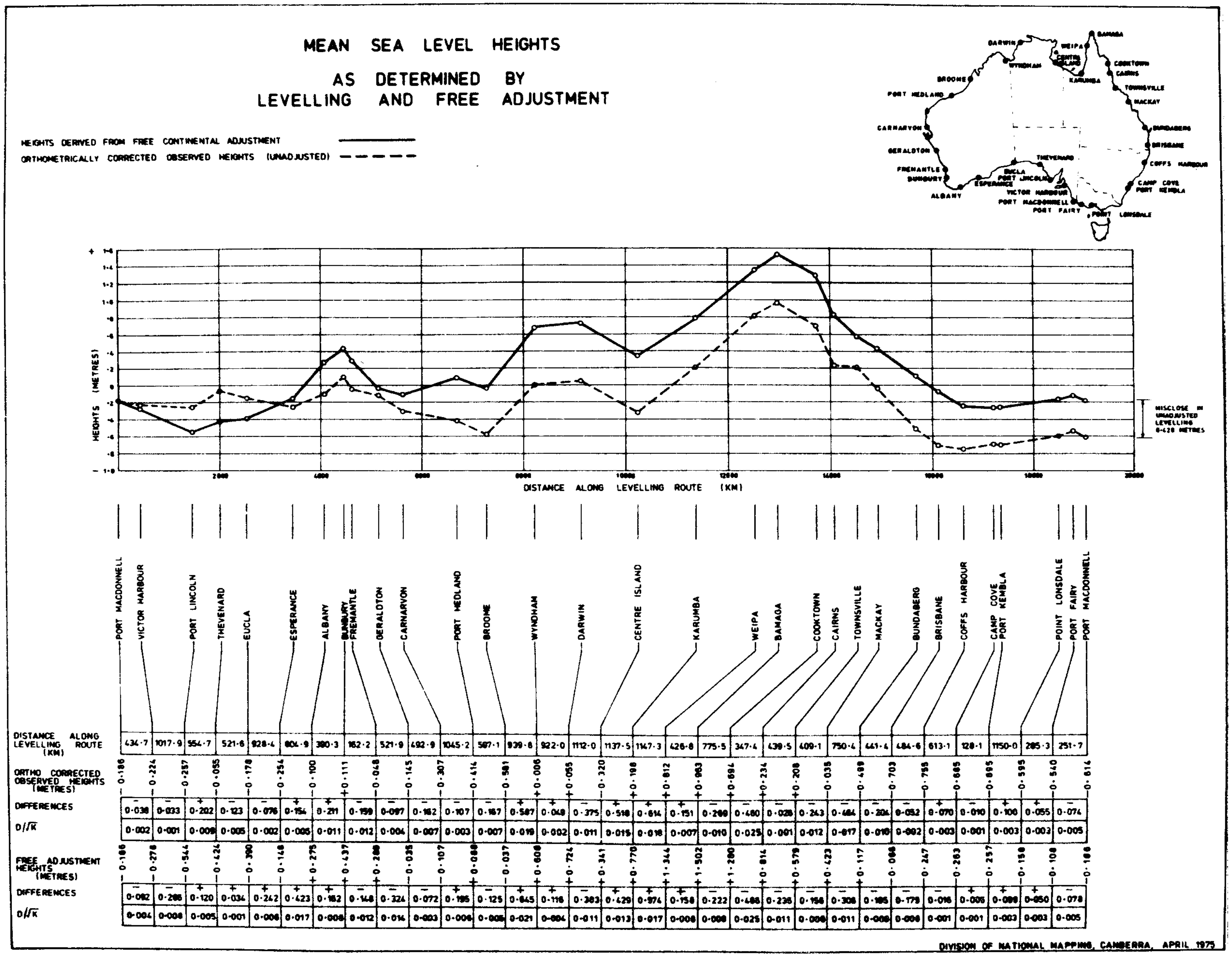
3.4 A COMPARISON OF THE RESULTS OF GEODETIC AND OCEANIC LEVELLING

3.4.1 BACKGROUND

A minimum of one year's tide gauge data should provide an estimate of local MSL with an uncertainty less than ±10cm. If there were no stationary SST the MSL surface would define the geoid with an equivalent precision. In such a case, the MSL at all tide gauges connected to a geodetic levelling network would be expected to have zero orthometric height, to within the internal precision of such a network.

It was recognised over 50 years ago (BOWIE 1929) that discrepancies existed between the elevations of coastal tide gauges established by geodetic levelling. It was found that the MSL surface defined by more than one tide gauge does not lie along a unique level surface of the earth's gravity field due to the existence of stationary coastal SST. In the case of the geodetic levelling network of Australia, the datum level surface for elevations can be considered to be that level surface passing through the local mean water mark at the Coffs Harbour tide gauge (see Figure 3.4). The 1971 free net adjustment of 160000km of third order spirit levelling

FIGURE 3.4



Source: *LEPPART ET AL (1975)*

covering the whole Australian continent (see ROELSE ET AL 1971) allowed this datum level surface to be referenced by all points in the levelling network, including 30 tide gauges scattered around the coastline. Figure 3.4 shows the "heights" of MSL at these tide gauges as determined by levelling and the subsequent free adjustment (from LEPPERT ET AL 1975). According to these results there exists an apparent systematic rise of MSL with respect to the datum level surface of 1.72m (observed) or 1.75m (adjusted) in the direction from Coffs Harbour in the south to Bamaga at the northern tip of Australia. This difference is equivalent to a slope of 0.1 arcsec over 3700km or a grade of 0.5mm per km.

The MSL/level surface discrepancies exhibit a systematic trend observed mostly along coastlines significantly correlated with changes in latitude. A summary of reported discrepancies from other countries can be found in MATHER (1974a, Appendix 1), of which Table 3.1 is an abstract of typical values.

TABLE 3.1

Reported Stationary Coastal Sea Surface Topography

NATION	FROM LATITUDE (°N)	TO LATITUDE (°N)	OBSERVED SLOPE ANGULAR (ARCSEC)	OBSERVED SLOPE LINEAR (cm)	DIRECTION	COAST
EUROPE	36	69	0.01	13	NORTH	WEST
U.K.	50	57	0.09	30	NORTH	EAST
BRAZIL	23	4	0.04	40	NORTH	EAST
U.S.A.	23	44	0.06	59	NORTH	EAST
U.S.A.	32	49	0.06	50	NORTH	WEST
AUSTRALIA	30	11	0.18	170	NORTH	EAST

Due to oceanographic considerations, the free ocean surface (apart from periodic tidal and wind effects) does not lie along any particular level surface of the earth's gravity field. However, in many cases SST determined by the methods of oceanic levelling (sub-section 3.3.3) does not completely account for the reported discrepancies.

In the case of Australia, oceanic levelling detected an apparent rise in the MSL surface with respect to the datum level surface of only 30cm between Coffs Harbour and Bamaga (HAMON & GRIEG 1972), leaving approximately 1.4m of the reported 1.7m discrepancy unexplained (Figure 3.4). The oceanic levelling results claim to be accurate to about ± 10cm and although the time variations in the sea level are not well known, their effect is estimated to be unlikely to exceed 20cm along the east coast of Australia (IBID). MITCHELL (1973) has investigated the relationship between the third order geodetic levelling data, level surfaces of the earth's gravity field and oceanic levelling data, but no satisfactory explanation was found for the conflicting estimates of coastal SST along the Australian coast.

In the USA, slopes of the sea surface deduced from tide gauge connections are in the opposite direction to those deduced from oceanic levelling. The geodetic levelling/MSL connections indicate that the sea surface slopes downward in the direction of the equator, in contrast to an expected poleward drop in sea level with respect to the earth's level surfaces (see e.g. STURGES 1974).

The original 1929 USA levelling adjustment indicated an approximate overall downward slope towards the equator of 4cm/degree latitude on the Atlantic coast and a slope of 1.8cm/degree latitude, in the same direction, on the Pacific coast (BALAZS 1973). The 1963 free net adjustment gave results which were similar to the 1929 adjustment, a systematic slope of 3cm/degree latitude on the Atlantic coast and 2.6cm/degree latitude on the Pacific coast. However, the 1973 east coast adjustment (based on post-1963 data) indicated that the marked slope north of Charleston, South Carolina was absent though all the other characteristics of the slopes reported earlier were confirmed (see MATHER 1974a, Appendix 2). On the west coast, post-1963 data increased the slope to 6cm/degree latitude.

Repeated first order levelling surveys along the Californian coast for the periods 1968-69, 1968-71, 1971-72, 1973-75 and 1977-78 indicate a steady trend from a positive slope (south to north) to a negative one (BALAZS 1979, Figure 6). The inference that southern California is rising with respect to northern California at an average rate of 7cm/year is not supported by the tide gauge records.

Rather, it appears that geodetic levelling is unable to provide a reliable result. Interestingly, the latest relevelling results on the west coast of the United States between San Francisco and San Diego give a similar result to observed steric results.

However, both geodetic and oceanic levelling indicate higher sea level on the Pacific coast than on the Atlantic coast. It is estimated that for the latitude of Norfolk (37^{o}N), steric sea level is 70cm higher on the Pacific coast (MONTGOMERY 1969, STURGES 1974), in good agreement with geodetic levelling (BALAZS 1979).

The cases referred to above are just some examples of continental geodetic levelling networks indicating stationary coastal SST that are in disagreement with oceanographic estimates by up to 5 times the reported internal precisions of such networks (see MATHER 1974a, Appendices), even after considering the possible biasing due to time variations in SST. Almost without exception the discrepancies are in the meridional direction where the slope of the sea surface is a maximum (see Figure 3.3).

In order to resolve the conflict arising from such disparate estimates of stationary SST, it is necessary to study the factors influencing the determinations of coastal SST for possible ambiguities (see e.g. COLEMAN ET AL 1979 for an investigation of the reported slopes along the eastern coast of Australia).

Factors which are responsible for oceanographic estimates of SST deviating from those inferred by geodetic levelling connections to tide gauges are:

(1) The methods of estimating sea surface slopes by oceanic levelling (sub-section 3.3.3) may have flaws in theory, in practice or both, and the level surface to which they are referred is tilted in relation to the geoid.

(2) The geodetic levelling procedures (described in e.g. BOMFORD 1962, Chapter 4) are subject to some small systematic error, which appears to be latitude and time dependent.

(3) The MSL surface deduced from coastal tide gauge readings may be biased at individual gauges due to faulty meter operation or

inappropriate siting.

These points are discussed further in the following sections.

3.4.2 COMMENTS ON OCEANIC LEVELLING

Steric levelling is the most important of the oceanic levelling techniques, and is capable of determining the topography of the sea surface only in relation to a more or less arbitrarily selected isobaric surface. Geostrophic levelling provides estimates of the slope of the sea surface in relation to level surfaces of the earth's gravity field.

Steric levelling however, has the ability to determine SST with respect to a uniquely defined geoid only if the condition that there is no horizontal geostrophic flow at the reference isobaric surface (assumed coincident with a level surface) is satisfied over the entire deep ocean area. Attempts to identify such a reference surface have been based upon certain telltale characteristics which include:

- identifying the dissolved oxygen minimum.
- distortion in the thickness of isopyncal layers.
- salinity distribution.
- a deep isobaric surface.

A summary of these techniques together with arguments pro and contra can be found in FOMIN (1964, p.117 et seq.).

There is as yet no universally accepted method for the detection of the level of no motion. However, a technique commonly used is one based on the observation that the velocity of currents decrease with depth, and therefore a reference isobaric surface at great depth is a good approximation to a zero velocity surface. Despite shortcomings, this approach is a makeshift in the absence of anything better and forms the basis for the theoretical development in § 3.3.3.2. Most charts of stationary SST are based on a reference surface located at between 1000 and 4000 decibars (1000 to 4000 metres depth).

If the determination of SST were insensitive to the choice of reference pressure, the geopotential thickness of the water layer between two possible deep isobaric surfaces would be constant. STURGES (1974) reports an experiment where the reference surface in the oceans off the Pacific and Atlantic coasts of the USA was varied by 500db on either side of the 2000db level. Although the absolute values of the SST changed by up to 40cm, the difference between the steric anomalies of the sea surface on the Pacific and Atlantic coasts remained relatively constant. The uncertainty was estimated at ±10cm. However, a more disturbing result is given by REID (1961, Section 3). In the Pacific Ocean, in addition to a chart of the SST with respect to the 1000db surface, he prepared a map of the geopotential differences between the 1000db and 2000db surfaces. Such a map exhibited systematic variation "somewhat similar" to the SST but with a smaller range, implying that a change in the reference pressure level does in fact affect the accuracy of the slopes of the sea surface deduced from such charts. The total range of geopotential difference between the 1000db and 2000db surfaces over the whole Pacific Ocean was about 50cm as against a range of over 2m in the SST. A conservative estimate of the uncertainty in the stationary SST due to a lack of knowledge of where the "true" reference level is located is therefore of the order of ±20-30cm, such an uncertainty having a significant amplitude for long wavelengths.

What is the effect on the resulting SST if the deep isobaric reference surface is tilted in relation to the level surfaces of the earth's gravity field? In such a case the geopotential difference between the surface of the "normal" ocean and the reference surface would vary with position and horizontal geostrophic flow at the reference surface would occur. This is analogous to the problem of sensitivity of steric levelling to the choice of reference pressure referred to above. It is difficult to estimate the amount of dislevelment of the reference isobaric surface that may exist in SST results. If the tilts exhibit systematic variation similar to the SST pattern but with smaller magnitudes than the sea surface slopes, then the degradation in accuracy is of the same order as adopting an "incorrect" reference pressure, i.e. o{±20cm}. If the sea surface slopes, with respect to the oceanographic reference surface, are analysed for the implied geostrophic surface current using equation 3.36 (after accounting for the horizontal atmospheric pressure gradient), then these currents could be compared with actual current observations and major discrepancies identified. However such discrepancies should not be confused with those arising from the limitations inherent in both the steric and geostrophic levelling procedures, and departures of the observed circulation from a steady-state model.

A computation of the global geostrophic circulation (neglecting the atmospheric contribution) implied by the oceanographically-determined SST (as depicted in Figure 3.3) was carried out and is illustrated in Figure 3.5. Only the direction of the surface current vector is shown. A comparison with a chart of the major ocean circulation features (Figure 3.6) indicates good agreement with the major circulation gyres in the northern hemisphere. However, the patterns obtained in the southern oceans are not entirely consistent with those illustrated in Figure 3.6. The conclusion that can be made from such a study is that the detection of broad tilts of the deep reference surface with respect to a level surface using the Dynamic Method is inconclusive because of possible errors in the assumption that the surface circulation is predominantly geostrophic in nature (i.e. strongly influenced by the pattern of stationary SST shown in Figure 3.3).

The extent to which ocean surface circulation is not geostrophic therefore influences the accuracy of the results of geostrophic levelling. In the open oceans purely geostrophic flow is rarely observed for the following reasons:

- the wind produced component of the current.
- the bottom topography effect.

Both of these may be neglected (as is usually done) or allowed for in some arbitrary manner.

According to FOMIN (1964, p.43), the frictional effect of the wind at the air/sea interface has three manifestations:

(1) Pure drift of the surface water resulting directly from the action of the wind, and consequently distorting the geostrophic nature of surface currents.

(2) The wind, by setting up a sea surface slope in addition to that caused by hydrostatic considerations (i.e. steric levelling), induces a gradient current of a geostrophic nature.

(3) A convective current resulting from mass transport in the wind driven current at (1). This is irrelevant for this discussion as its velocity is zero at the surface and increases with depth to

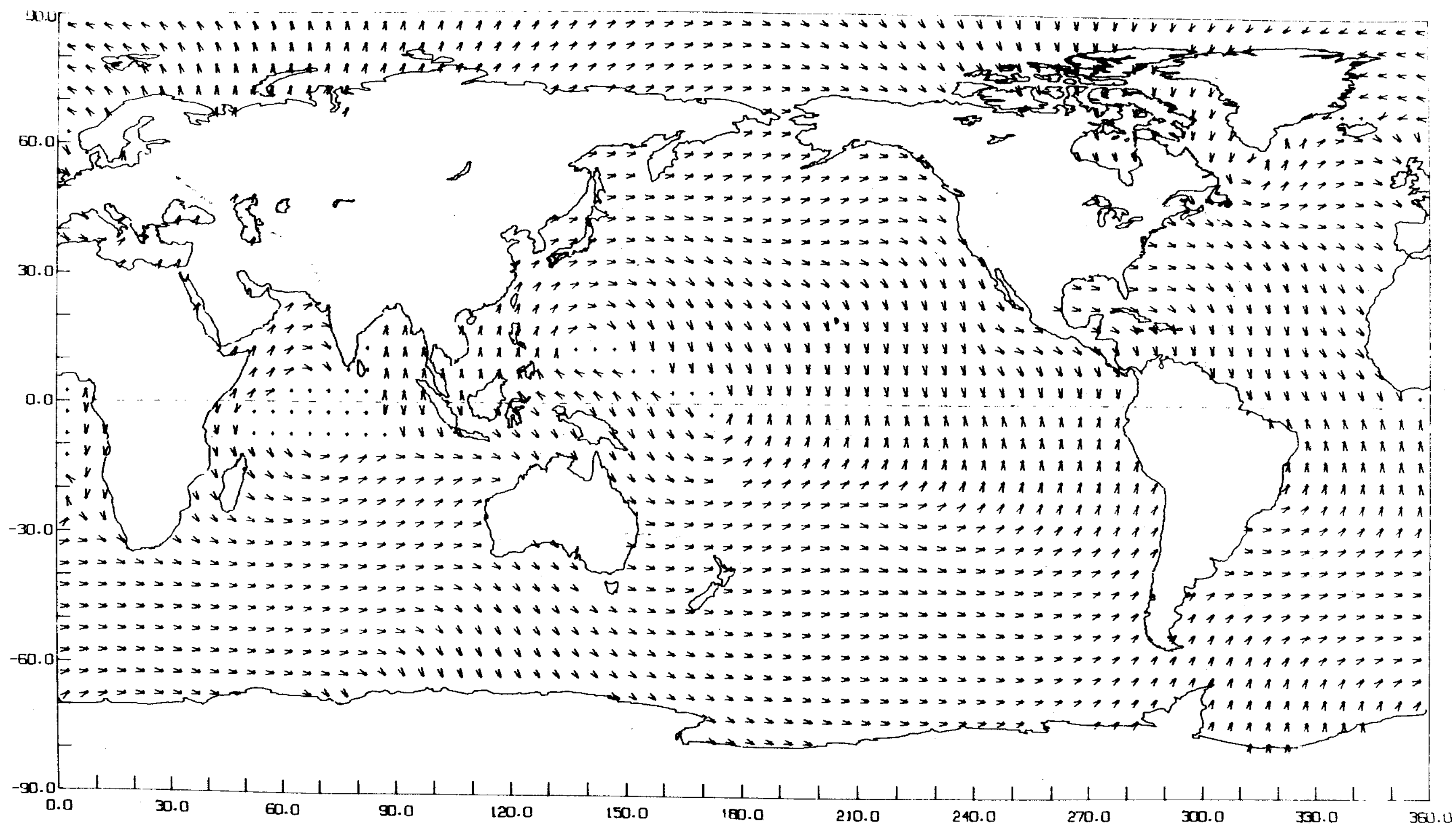
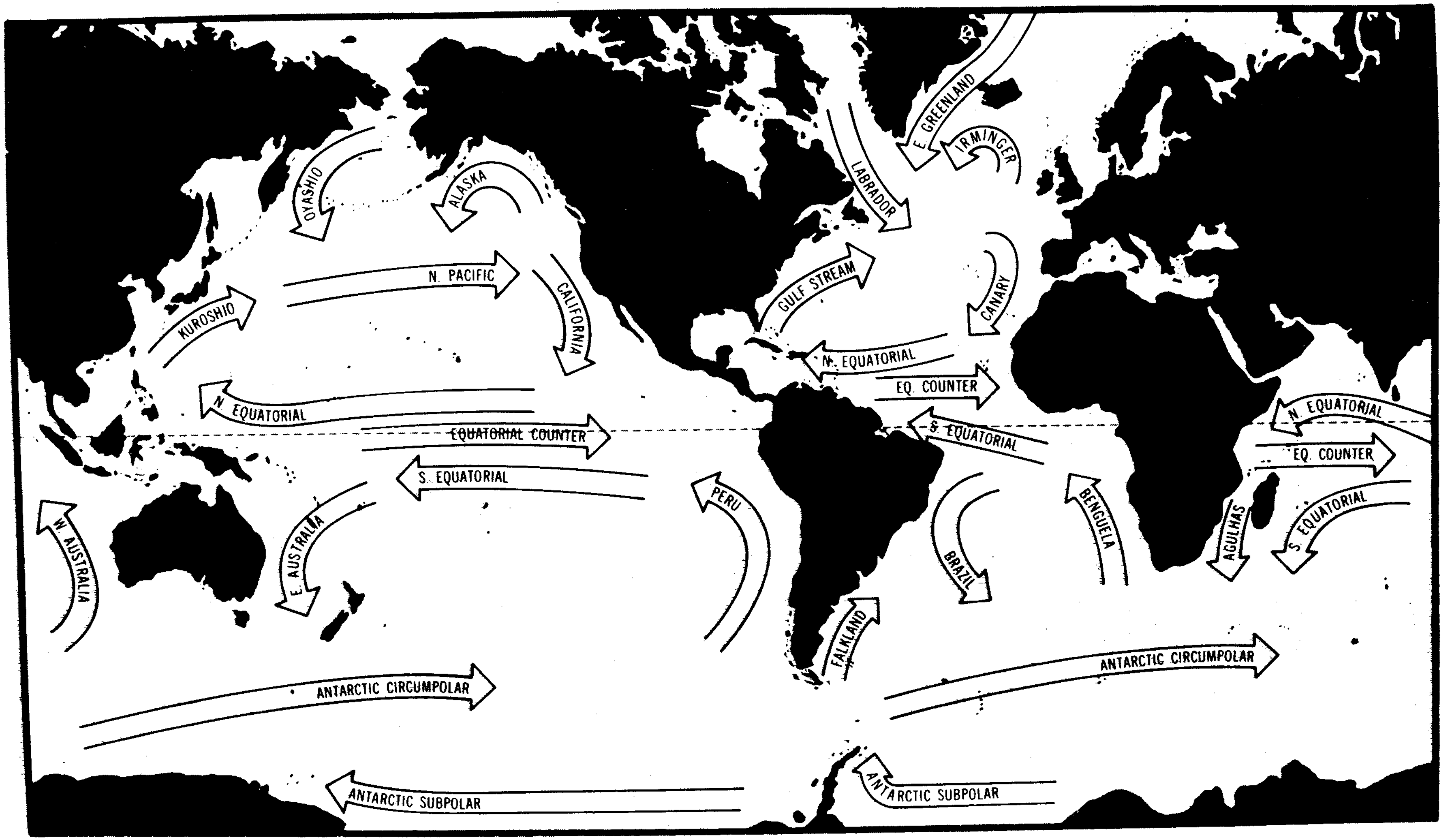


FIGURE 3.5

Global Geostrophic Circulation Implied from Oceanographically Determined SST Based on a Surface Harmonic Analysis to (8,8) of Figure 3.3

(Current Directions Only)



F I G U R E 3.6

Major Ocean Surface Circulation

Source: *"Sea and Air - The Marine Environment" by Williams, J., Higginson, J.J. & Rohrbough, J.D. Naval Institute Press 1973* some limiting value.

As with the wind produced drift current at (1), the effect of uneven bottom topography is to further distort the geostrophic characteristics of the surface current. For example, steady flow over an oceanic ridge can cause significant deflections of the velocity vector, especially in high latitudes where currents "tend to follow the isobars of the sea bottom" (DEFANT 1961, Chapter 13.2). Distortions due to a combination of wind and bottom topography effects are such that the comparison of average SST deduced by geostrophic levelling with that obtained from steric levelling in the well-surveyed North Atlantic Ocean indicate discrepancies at the 25% level (see e.g. STURGES 1974, p.828, GODFREY 1973).

Physical oceanographers maintain however, that extrapolated SST over distances of several hundred kilometres from deep ocean areas to coastal sites are unlikely to introduce errors in excess of ±20cm in the results (HAMON & GRIEG 1972, p.7160). On the eastern coast of Australia, where the bottom topography of the narrow uniform continental shelf is regular, such an error estimate may be valid. But rarely have steric sea levels been extrapolated across shelf areas for *direct* comparison with geodetic levelling results at tide gauge sites. As pointed out in COLEMAN ET AL (1979, Section 3.1), the procedures to be followed for achieving these comparisons seem to be under conjecture amongst oceanographers (e.g. CSANADY 1979), for along a coastal boundary the mean longshore pressure gradient may no longer be in geostrophic balance and so must be balanced by other forces (e.g. the longshore wind stress), as pointed out by CHASE (1979). Thus an error estimate of ± 20cm for the results of extrapolated stationary SST may well be optimistic.

SST determined by the application of hydrostatic and geostrophic principles (Figures 3.3 and 3.7) lacks continuity in space and time due to the data having been collected during the course of sporadic surface ship surveys. At best they can be considered to provide an estimate of the long wavelength features of the stationary SST and to be within ±20cm of the instantaneous SST. This however, is not considered a problem for MSL/geodetic levelling comparisons.

3.4.3 COMMENTS ON GEODETIC LEVELLING

In order to explain the residual discrepancy between geodetic levelling connections and MSL at tide gauges after correction for the oceanographically-determined SST, physical oceanographers allege that "negligible" amounts of systematic error accumulate in large levelling networks (e.g. STURGES 1974, p.830).

Most of the Australian levelling is of third order accuracy for which differences between forward and backward levelling between bench marks are required to be less than $12\sqrt{D}$ (mm), where D is the length of the line of levelling in kilometres (ROELSE ET AL 1971, App.H). The systematic error accumulation is expected to be about an order of magnitude smaller (BOMFORD 1962, p.238). The 1971 adjustment of the Australian levelling data was carried out using orthometric heights based on normal gravity rather than the more acceptable observed gravity. In a subsequent investigation into the unexplained discrepancy in the geodetic levelling/MSL comparison by MITCHELL (1973), the levelled heights were converted into orthometric heights using observed gravity. However, when the Australian levelling network was readjusted it was found that the bench mark heights were not altered by significant amounts. The estimated standard deviation of heights of MSL at the 30 coastal tide gauges in relation to the network origin in the centre of the continent is ± 30-40cm (ROELSE ET AL 1971, App.F). Admittedly this accuracy estimate is based on the internal statistics of the adjustment.

Sources of systematic errors in the levelling procedure have been identified (see e.g. BOMFORD 1962, Section 4.19). Some errors do not necessarily accumulate with an increase in the length of the levelled line, which is the simplistic assumption made in estimating the a priori uncertainty of lines of levelling for use in the subsequent adjustment. Other sources of error can be eliminated by the use of certain levelling and computation procedures as described in e.g. RAPPLEYE (1948), while others are assumed negligible or ignored due to lack of information concerning their behaviour. The standard deviation of ± 30-40cm for geodetically-determined MSL heights in Australia must therefore be considered optimistic. A review of possible systematic errors in levelling by ANGUS-LEPPAN (1975) was unable to identify an error source that could explain the sea level discrepancies shown in Figure 3.4.

More disturbing are the results of numerous relevellings along the Californian coast (BALAZS 1979). The apparent change in MSL "heights" is not corroborated by tide gauge data, and is an order of magnitude greater than could be reasonably expected from known levelling errors based on analyses of loop misclosures.

Alarming discrepancies have also been noted in the relevelling and adjustment of the first order trans-Canadian level line running from Halifax on the Atlantic coast to Vancouver on the Pacific coast (LACHAPELLE ET AL 1977, LACHAPELLE 1978a). Comparison of two level runs (one observed in the 1910's and the other in the 1960's) reveals a sustained linear trend in the differences at common bench marks along the line. The total discrepancy at the end points is 2.2m.

Similar trends have been observed in the "rapid first order" relevelling along the north-east coast of Australia, carried out in 1975-1976 to investigate the anomalous sea surface slope detected in the 1971 adjustment. The 1971 results gave a positive slope of MSL of 1m from Coffs Harbour (30°S) to Cairns (17°S) - see Figure 3.4. The new levelling indicates an apparent slope of the sea surface of -50cm (GRANGER 1977) (See Figure 3.7). Furthermore, the direction of the slope is now reversed and although it now matches the average grade reported for the Pacific and Atlantic coasts of the USA, it also contradicts the oceanographic evidence for a positive slope of MSL towards the equator. As with the USA experiences, the discrepancies appear to be far greater than the generally accepted internal precision of geodetic levelling, even after taking into account the fact that the original Australian levelling was of a lower standard to the subsequent one.

Such results all suggest that some unaccounted systematic error exists in the levelling process, and significantly, it is <u>not necessarily restricted to level lines that run in a north-south direction</u> as was previously thought. Even more disconcerting is the <u>apparent time dependent nature of these levelling errors</u>. Is the geodetic levelling procedure capable of supporting high precision geodetic studies? The test of whether geodetic levelling can be considered a "scientific experiment" can be stated as (from ANDERSON 1979a):

> "Before primary level surveys can validly assume the role of scientific experiments, they must be shown to possess the fundamental qualities which have been traditionally considered essential to the scientific experiment method. Predominant among

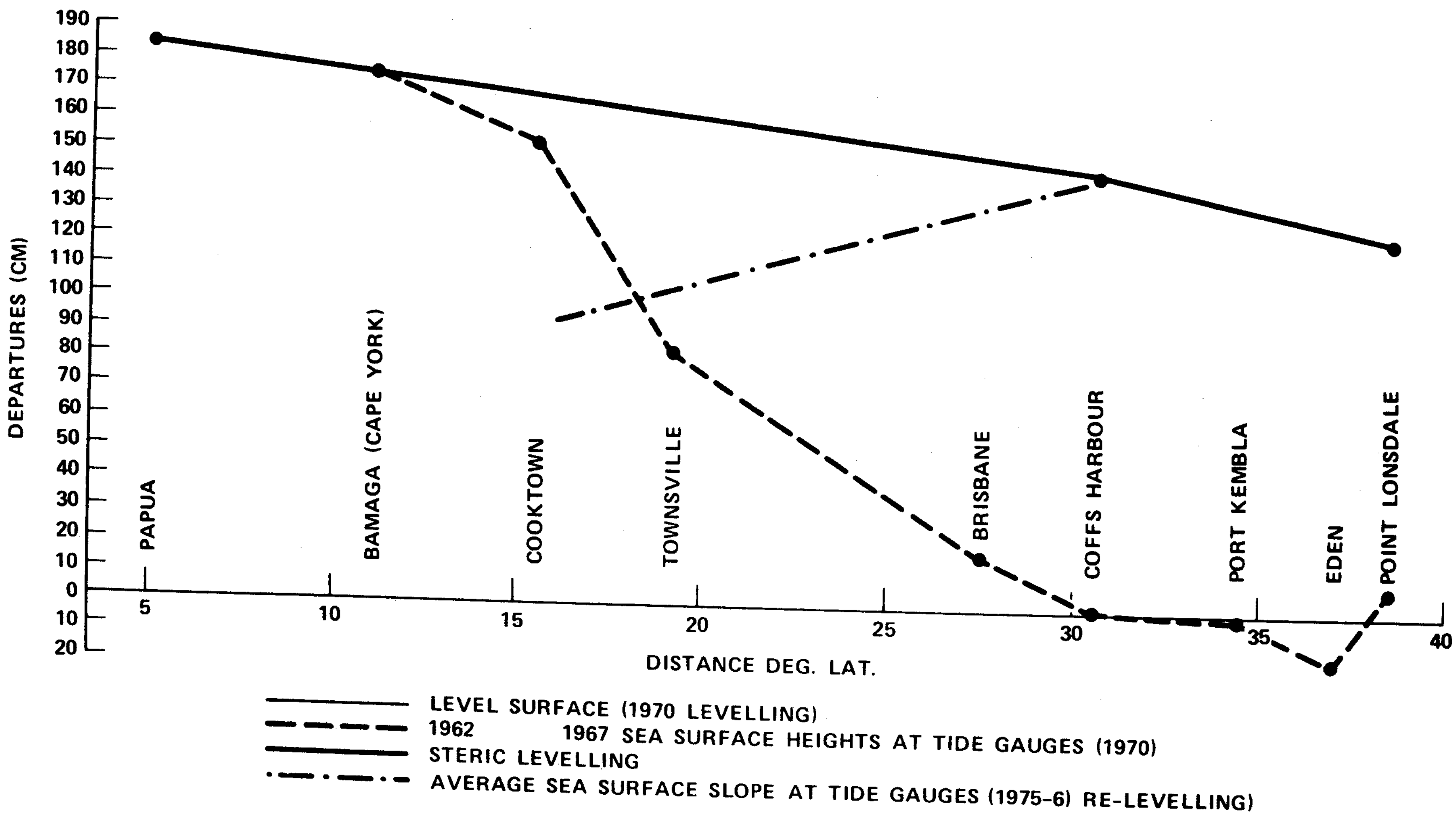


FIGURE 3.7

Heights of Mean Sea Level Along the East Coast of Australia

Source: GRANGER (1977)

> these qualities is that of repeatability. Specifically, the result must be demonstratably capable of duplication within the a priori tolerances associated with the experiment procedure".

The mounting evidence has raised doubts as to the validity of using the results of geodetic levelling for such scientific purposes as SST studies.

What are the unaccounted systematic effects that give rise to an accumulation of errors in lines of geodetic levelling? A great deal of activity has been directed in this regard in response to the task of redefining the North American Vertical Datum (NAVD). Some of the most recent results are reported by HOLDAHL (1979a), HOLDAHL (1979b). The object has been firstly to identify the systematic error(s) and then develop methods of eliminating or minimising its/their effect. Two likely candidates have been identified. The first is the astronomic error resulting from the tilting of the level surfaces due to the tidal potential exerted by the Sun and Moon. Although the correction can be easily computed it has hitherto not been applied to levelling data. It is very small, at most 0.1mm/km, but tends to accumulate in the north-south direction. The accumulated correction along the Pacific coast of the USA is estimated to be only 7cm (HOLDAHL 1979b). ANGUS-LEPPAN (1975) estimates the error in Australia to be of smaller magnitude, of the order of 0.03mm/km.

However, it is the error due to atmospheric refraction that is regarded as the most serious problem with geodetic levelling. It has long been suspected that accumulation of refraction error depends on the slope of the terrain being levelled, length of sight and the vertical air temperature gradient, but only recently have experiments indicated that the error may be much greater than was previously assumed (HOLDAHL 1979a). Refraction error can be minimised by adopting suitable levelling procedures (e.g. by limiting and balancing sight lengths), and the remaining error removed by the application of a suitable correction to the data, a process to be attempted in the forthcoming redefinition of the NAVD. In order to evaluate this correction additional field measurements of meteorological parameters are required. The refinement of historical levelling data would require some form of modelling of the temperature gradients, for example by techniques suggested in ANGUS-LEPPAN & WEBB (1971).

What has prompted the investigation of this effect is the realisation that greater levelling refraction will occur on southern terrain slopes than on northern ones, the error therefore accumulating in the north-south direction. It has been suggested (HOLDAHL 1979b) that a north-south accumulation of 0.3mm may occur per km of levelling and that its full effect is normally not seen in miscloses between forward and backward levelling. An investigation by ANGUS-LEPPAN (1979) showed that the refraction error is the resultant of complex interrelations between the sun's radiation, the height of the sightline above the surface, length of sight, slope and direction of slope. This study confirmed that refraction is an error of significant magnitude, estimated to be anything from a few centimetres/100km to over 0.5m/100km, depending upon the values of the abovementioned parameters.

However, that it is not clear how these suspected systematic errors can exhibit a time-varying influence on levelling results (as seen in the repeated levellings along the Californian coast) or influence levelling surveys carried out in an east-west direction (as with the trans-Canadian runs). A question mark therefore hangs over the accuracy of geodetic levelling networks and they should only be used with caution for scientific investigations.

3.4.4 ESTIMATION OF MSL FROM TIDE GAUGE OBSERVATIONS

The comparison of the spatial location of the level surfaces of the earth's gravity field in relation to MSL defined by the operations of geodetic levelling on the one hand and oceanic levelling on the other, takes place on the coastline. The connection is achieved through the use of one or more tide gauge instrument(s) that record the local sea level variations from some "mean" mark M, and whose "elevation" has been established by geodetic levelling as described earlier.

How well does this level M, as determined from tide gauge observations over a long period of time, relate to the physical mean level of the oceans N, a hypothetical state of rest defined by oceanic levelling techniques? The time history of each tide gauge contains the time (though not the spatial) signature of influences acting on the physical sea level. The problem related to SST studies is the estimation of the level M from the trends, cycles and noise present in the tide gauges records.

A comprehensive analysis of the relationship between M and the various global, regional and local influences on sea level that operate over a variety of time scales was made by WEMELSFELDER (1972). For example, the averaging of tide gauge records establishes the level M which is free of the major periodic effects and short term trends in sea level due to:

(1) Short period phenomena such as tides and effects of meteorological origin.

(2) The seasonal effect of temperature, winds, air pressure, river discharge, etc., that are a consequence of the earth's orbital motion.

The effect of secular trends in the apparent heights of MSL which contain, amongst other things, the signature of vertical crustal motion, should be minimised by ensuring that the collection of levelling data (geodetic and oceanic) and the determination of M are carried out in the same epoch. In general, the global trends in MSL are much smaller than the SST or the discrepancies between geodetic and oceanic levelling (see Figure 2.6).

Instrumental and gauge site peculiarities influence the determination of M to a far greater extent and can bias the local estimate of N (IBID 1972). MITCHELL (1973, Section 7.4) made a study of 30 tide gauges around Australia and found that many suffered from faults such as time and height errors as well as failure of the individual tide gauges to estimate the level N of the open ocean due to the silting of the float and/or the effect of nearby riverflow. Although it is impossible to accurately quantify the effect of such faults, it may run to 10-15cm (IBID 1972). However such biases should not produce spurious deviations of MSL (or more correctly, M) from the geoid as a function of position on the coastline, as seen along the east coast of Australia (Figure 3.4). Furthermore, they cannot be responsible for the observed discrepancies in relevelling.

3.4.5 CONCLUSION

It must be concluded therefore that the magnitude of systematic error in long lines of levelling is greater than previously estimated. However, spirit levelling will continue to play an important role in regional geodetic studies, for example in the determination of vertical crustal motion (sub-section 8.3.1). In addition, it seems that sea surface slopes obtained from oceanographic techniques have uncertainties that are perhaps double those generally quoted.

The serious doubts which currently exist concerning the reliability of precise geodetic levelling networks prompts the development of new techniques for independently estimating the height of MSL at tide gauge sites from satellite altimetry data. This is attempted in section 8.2. The determination of the SST in the open oceans is described in section 5.2 and preliminary results based on GEOS-3 altimeter data are given in Chapter 7.

4

COMPARISON OF RELATIONS BETWEEN THE GRAVITY ANOMALY, THE HEIGHT ANOMALY AND THE SEA SURFACE TOPOGRAPHY

Section 4.1 defines the basic geodetic quantities: gravity anomalies, height anomalies and disturbing potential and their relationship to one another. All basic definitions in modern physical geodesy are framed in the context of the Geodetic Boundary Value Problem (GBVP) according to Molodenskii (see § 2.3.5.3). Section 4.2 describes a solution to the GBVP by which the geoid height can be determined with a resolution of ±10cm from gravity anomaly data. The role that satellite altimetry data can play in the solution of the GBVP is discussed in Section 4.3. A "higher" reference model for sea surface topography studies is described in section 4.4 and the basic definitions in physical geodesy are revised for such a system. As the precision sought in solutions of the GBVP is at the ±10cm level, the effect of the atmosphere on geodetic quantities must be taken into account. Section 4.5 investigates the gravitational effect of the atmosphere on high precision geoid determinations.

4.1 BASIC RELATIONS

4.1.1 INTRODUCTION

The contribution geodesy can make to sea surface topography studies is twofold:

(1) The determination of the stationary and time-varying components of SST in the open oceans.

(2) Relating geodetic levelling results to a unique geoid in order to facilitate the unification of all continental levelling datums with a precision comparable to that of first order levelling, and investigating the time dependent nature of these relationships due to vertical crustal motion, secular variation in global mean sea level and changes in the shape of the geoid (these last two factors having been discussed in section 2.4).

A necessary prerequisite for these studies is the development of procedures for the determination of a high precision marine geoid from all data available at present and for the foreseeable future. The role that the high precision geoid can play in determinations of SST in the open oceans and at tide gauge levelling datums is discussed in chapter 5. Preliminary results obtained from GEOS-3 satellite altimetry data will be presented in chapters 7 and 8.

The mathematical relationships between data collected in the course of gravity, levelling and oceanographic surveys will be established in this chapter.

The types of data presently available for satisfying the above mentioned geodetic objectives are the following:

(i) Gravity measured at the earth's surface. This data is usually expressed in the form of gravity anomalies Δg using information on the "height" of the gravity station above the regional levelling datum (assumed zero in oceanic areas). For SST studies the precision sought in Δg is around $\pm 30\mu gal$ (see § 2.3.5.4). Expressions for the gravity anomaly are derived in sub-section 4.1.4.

(ii) The disturbing potential T of the earth's gravity field, as obtained from an analysis of orbital perturbations of near-earth artificial satellites. Assuming that the tracking data from ground stations and high altitude synchronous satellites can resolve orbital position to ± 10cm, the satellite ephemeris and the resultant global gravity field model that defines T (see § 2.3.3.1) is unaffected by the existence of SST and is therefore a valuable input for SST studies. Expressions for the disturbing potential are derived in sub-section 4.1.3.

(iii) Heights of the instantaneous sea surface above the reference ellipsoid ζ as obtained from satellite altimetry (see sub-section 2.3.4). The sea surface height ζ (equivalent to the height anomaly in formulations of the GBVP - see § 2.3.5.5), in addition to containing the signature of the stationary SST, is subject to short period variations of up to 1 metre due to tides and mesoscale variations in SST (see sub-section 3.3.2). Values of ζ determined from satellite altimetry are directly influenced by errors in orbit determination and consequently, global sets of ζ are prone to errors of long wavelength with significant amplitudes in regions where tracking data is inadequate and where gravity field models lack the precision to compensate for inadequacies in the tracking coverage.

(iv) Regional levelling surveys used for the establishment of gravity anomaly data banks. Elevation errors up to ± 10m (equivalent to an error of ± 3mgal in the gravity anomaly) can be tolerated for quadrature solutions of the GBVP if systematic errors of long wavelength in the levelling data are kept below ± 0.15m (equivalent to $\pm 50\mu gal$) (see § 2.3.5.4).

(v) Estimates of SST obtained by oceanographic techniques described in sub-section 3.3.3. These can be used as corrections to Δg and ζ in order to relate them to the geoid, provide constraints in the adjustment of large levelling networks or furnish an a priori model against which stationary SST determined by geodetic techniques can be compared (see sub-section 7.1.4).

The following section describes the formulation of the GBVP according to Molodenskii, providing a framework in which the relationship between geoid height N and gravity anomalies Δg can be established at the ±10cm level.

4.1.2 MATHEMATICAL FORMULATION OF THE GEODETIC BOUNDARY VALUE PROBLEM ACCORDING TO MOLODENSKII

It was noted in § 2.3.5.2 that, for accuracies of ±10cm in the final result, it is unsatisfactory to make assumptions concerning the crustal density exterior to the geoid in solutions of the GBVP according to Stokes. This problem was circumvented by adopting an approach suggested by Molodenskii (see § 2.3.5.3) and for which several solutions correct to the order of the flattening (± 30cm) have been proposed. MATHER (1973c) classifies such solutions into the following categories:

(1) Solutions based on surface layer techniques - proposed by Molodenskii.

(2) Solutions requiring gravity data to be sampled at discrete points on the earth's surface - as suggested by Bjerhammer.

(3) Solutions based on Green's third identity (HEISKANEN & MORITZ 1967, p.11).

A solution with ±5cm resolution was presented by MATHER (1973a), and took into account the effect of the topography, the interactions between the topographic slopes and the level surfaces of the earth's gravity field and the atmosphere, as well as identifying the gravity anomaly to be used. However, such a solution did not take into account the SST. It was based on the assumption that all gravity data could be related to the geoid with ±5cm precision.

A revised solution which considered the effect of the stationary SST was proposed by MATHER (1975a). The following formulation has been developed from IBID (1975a).

The basic formulation of the GBVP for the earth's surface is given in MATHER (1973a, Section 2.4) and stems from the application of Green's theorem (MATHER 1971, p.23) to the two scalars r^{-1} and θ. θ is a potential that is harmonic in the volume exterior to a bounding surface S. According to Green's third identity, the expression obtained for the scalar θ at a point P on the surface S (assuming that all matter is contained within S) is:

$$\iiint_{V_i} \frac{1}{r} \vec{\nabla}^2\theta \, dv_i = -2\pi\theta_P + \iint_S \left(\frac{1}{r} \vec{\nabla}.\vec{N}\theta - \theta\vec{\nabla}.\vec{N} \frac{1}{r}\right) ds \qquad (4.1)$$

where r is the distance of the relevant element of surface area ds or volume dv_i (interior to the bounding surface S) from the point P. $\vec{N}$ is the unit vector defining the outward normal to S and $\vec{\nabla}$ is the operator defined in equation 2.11.

The formulation of the boundary value condition for the physical surface of the earth is obtained as follows.

The geopotential W of a non-deformable earth is given by (equation 2.31):

$$W = W' + W_A + W_R \tag{4.2}$$

where W' is the gravitational potential due to the solid earth and oceans, W_A is the gravitational potential of the atmosphere and W_R is the rotational potential.

W' satisfies Laplace's equation (equation 2.16) and is therefore harmonic at all points exterior to S. Application of equation 4.1 to W' exterior to S gives:

$$2\pi W'_p = - \iint_S \left(\frac{1}{r} \vec{\nabla}.\vec{N}\, W' - W'\, \vec{\nabla}.\vec{N}\, \frac{1}{r}\right) ds \tag{4.3}$$

Similarly, application of equation 4.1 to the rotational potential W_R for v_i interior to S gives:

$$2\omega^2 \iiint_{v_i} \frac{1}{r}\, dv_i + 2\pi W_{Rp} = \iint_S \left(\frac{1}{r} \vec{\nabla}.\vec{N}\, W_R - W_R\, \vec{\nabla}.\vec{N}\right) ds \tag{4.4}$$

Differencing equations 4.3 and 4.4,

$$2\pi (W'_p - W_{Rp}) = - \iint_S \left(\frac{1}{r} \vec{\nabla}.\vec{N}(W'+W_R) - (W'+W_R)\vec{\nabla}.\vec{N}\, \frac{1}{r}\right) ds + 2\omega^2 \iiint_{v_i} \frac{1}{r}\, dv_i \tag{4.5}$$

Green's third identity can also be applied to the spheropotential U. The scalar U is the gravity potential due to an ellipsoid of revolution having the same gravitational and rotational characteristics as the earth and defined by (equation 2.19):

$$U = U_G + W_R \tag{4.6}$$

where U_G is the gravitational potential of the ellipsoid. Following the same procedure that led to equation 4.5,

$$2\pi (U_{Gp} - W_{Rp}) = - \iint_S \left(\frac{1}{r} \vec{\nabla}.\vec{N}(U_G+W_R) - (U_G+W_R)\vec{\nabla}.\vec{N}\, \frac{1}{r}\right) ds + 2\omega^2 \iiint_{v_i} \frac{1}{r}\, dv_i \tag{4.7}$$

As the integrations in equations 4.5 and 4.7 are taken over the same surface and volume, it follows from appropriate differencing and rearrangement that:

$$T'_p = \frac{1}{2\pi} \iint_S \left(T' \vec{\nabla} . \vec{N} \frac{1}{r} - \frac{1}{r} \vec{\nabla} . \vec{N}\, T' \right) ds \qquad (4.8)$$

where (equation 2.33):

$$T' = W' - U_G \qquad (4.9)$$

Equation 4.8 relates the earth's surface S to the disturbing potential T' and its normal derivative $\vec{\nabla}.\vec{N}T'$ and is the most direct linearisation of the BVP of physical geodesy according to Molodenskii.

A solution for the disturbing potential of the solid earth and ocean can be obtained from the integral equation 4.8, in addition to one based on the differential equation 2.24 (a solution for which is given in MATHER 1971, Section 3 & 4, in terms of spherical harmonic functions - see § 2.3.3.1). A solution for equation 4.8 is given in section 4.2.

4.1.3 THE DISTURBING POTENTIAL

The relationship between T', the earth's geopotential W, the atmospheric potential W_A and the normal potential U at the point P on the earth's surface is (equations 4.2, 4.6 & 4.9):

$$T'_p = W_p - U_p - W_{Ap} \qquad (4.10)$$

and (equation 2.34):

$$T_p = T'_p - W_{Ap} \qquad (4.11)$$

where T is the disturbing potential of the earth <u>and</u> atmosphere.

From the conventional geometry of Molodenskii's problem (see Figure 2.4), it can be seen that:

$$T' = (W_o + \Delta W) - \left(U_o + \Delta W + \zeta\frac{\partial U}{\partial h}\right) - W_A \qquad (4.12)$$

where ζ is the height anomaly, W_o is the potential of the geoid, U_o is the spheropotential of the reference ellipsoid, ΔW is the geopotential difference obtained from geodetic levelling (see equation 3.10) and $\partial U/\partial h$ is the radial derivative of the spheropotential along the spherop normal defined by (equation 2.22):

$$-\frac{\partial U}{\partial h} = \gamma \qquad (4.13)$$

Equation 4.12 can be rewritten in the form (MATHER 1973a, p.25):

$$T' = (W_o - U_o) - W_A + \gamma\zeta \qquad (4.14)$$

This is a generalisation of <u>Brun's formula</u> (HEISKANEN & MORITZ 1967, p.85) for the basic harmonic quantity at the surface of measurement.

Equation 4.14 is valid as long as the geopotential of a point can be established from the geopotential difference ΔW with respect to W_o:

$$W_p = W_o + \Delta W \tag{4.15}$$

Problems arise however when ΔW is referred to the equipotential surface $(W = W_o + \delta W)$ passing through the local MSL defining the levelling datum, which does not coincide, in general, with the geoid $(W = W_o)$. The geopotential difference ΔW is therefore, in reality, related to the levelling increments by (equation 3.19):

$$\Delta W = - \int_{\text{MSL Datum}}^{P} g \; dn \tag{4.16}$$

Individual tide gauges situated on the equipotential surface $(W = W_o + \delta W)$ will therefore not coincide with the geoid due to the existence of SST. This problem was discussed in § 2.3.5.5, where the definition of the height anomaly ζ has been modified to take into account the SST. The expression for T' given in equation 4.14 can be more accurately expressed in the context of 1 part in 10^8 geodesy by:

$$T' = (W_o + \delta W - U_o) - W_A + \gamma\zeta \tag{4.17}$$

or

$$\zeta = \frac{1}{\gamma}\left(T' - (W_o + \delta W - U_o) + W_A\right) \tag{4.18}$$

where ζ is now the geometric quantity depicted in Figure 2.5 (rather than in Figure 2.4).

The quantity δW is defined by:

$$\delta W = \begin{cases} -g\,\zeta_{sd} & \text{on land} \\ -g\,\zeta_s & \text{at sea} \end{cases} \tag{4.19}$$

where ζ_{sd} is the height of the MSL datum above the geoid, ζ_s is the SST in the open oceans and g is observed gravity.

As T' satisfies Laplace's equation exterior to the surface of the earth it may be defined in terms of spherical harmonic functions over some sphere of radius a (see e.g. HEISKANEN & MORITZ 1967, equation 1-87b):

$$T' = \frac{GM}{a}\sum_{n=0}^{\infty}\left(\frac{a}{R}\right)^{n+1} T'_n \quad , \quad n \neq 1$$

or (4.20)

$$T' = \frac{GM}{R}\sum_{n=0}^{\infty}\left(\frac{a}{R}\right)^{n} T'_n \quad , \quad n \neq 1$$

where the surface spherical harmonic T'_n is given by (equation 2.35):

$$T'_n = \sum_{m=0}^{n} \sum_{\alpha=1}^{2} \bar{C}_{\alpha nm} S_{\alpha nm} \tag{4.21}$$

$\bar{C}_{\alpha nm}$ are spherical harmonic coefficients of degree n and order m in the spherical harmonic representation of T'. The zero degree coefficient $\bar{C}_{100}$ in such a representation of the disturbing potential of the earth excluding atmosphere has a value which is non-zero. The zero degree term of the disturbing potential T' is $G\Delta M/R$ where ΔM is the difference in mass between the solid earth and oceans, and the reference ellipsoid. The spherical harmonic functions $S_{\alpha nm}$ are defined by equation 2.29.

The exclusion of the first degree harmonic in equation 4.20 has the effect of placing the centre of the reference ellipsoid at the centre of mass of the solid earth and oceans.

Spherical harmonic expressions relate quantities on the surface of a sphere, while geodetic observations are made at or near the irregular surface of the earth. For solutions seeking a resolution of ±5cm this necessitates that all relations derived using the properties of spherical harmonics are obtained between quantities which refer to a sphere. The minimum sphere which best approximates the earth and satisfies the condition that harmonic quantities, in the space exterior to the surface of measurement, exist at all points on its surface, is the Brillouin sphere. The Brillouin sphere has its centre at the centre of mass of the solid earth and oceans, and includes all the topography.

Quantities on the Brillouin sphere are denoted by an overbar. If the radius of the Brillouin sphere is $\bar{R}$, the following quantity may be defined:

$$\bar{T}' = \frac{GM}{\bar{R}} \sum_{n=0}^{\infty} \left(\frac{a}{\bar{R}}\right)^n T'_n \quad , \quad n \neq 1 \tag{4.22}$$

4.1.4 THE GRAVITY ANOMALY

Differentiation of equation 4.10 along the spherop normal through P gives:

$$\left(\frac{\partial T'}{\partial h}\right)_P = \left(\frac{\partial W}{\partial h}\right)_P - \left(\frac{\partial U}{\partial h}\right)_P - \left(\frac{\partial W_A}{\partial h}\right)_P \tag{4.23}$$

The radial derivative of the spheropotential along the spherop normal is the normal gravity at P (equation 4.13). This is often an unknown quantity (except when the ellipsoidal height of P is known, e.g. from satellite altimetry). However, $\gamma_{Q''}$ on the telluroid (Figure 2.5) is known and can be related to γ_P by a Taylor's series expansion (neglecting all but the first term):

$$\gamma_P = \gamma_{Q''} + \zeta \frac{\partial \gamma}{\partial h} \tag{4.24}$$

where $\partial\gamma/\partial h$ is the gradient of normal gravity along the spherop normal, defined by (HEISKANEN & MORITZ 1967, p.293 - see equation 2.46)

$$\frac{\partial\gamma}{\partial h} = \frac{-2\gamma}{R}(1 + f + m - 3f\sin^2\phi + o\{f^2\}) \tag{4.25}$$

All quantities have been defined in § 2.3.5.2. Observed gravity g is the radial derivative of the geopotential along the vertical through P (equation 2.15):

$$\left(\frac{\partial W}{\partial h_v}\right)_P = -g_P \tag{4.26}$$

Any periodic variations of the geopotential due to earth tides, etc., can be removed from gravity observations by applying suitable corrections.

Equation 4.26 can be transformed to a derivative along the spherop normal to give:

$$\begin{aligned}\left(\frac{\partial W}{\partial h}\right)_P &\simeq -g_P \cos\varepsilon \\ &\simeq -g_P(1 - \tfrac{1}{2}\varepsilon^2)\end{aligned} \tag{4.27}$$

where ε is the deflection of the vertical.

The gravity anomaly Δg, in the context of the GBVP according to Molodenskii, is defined in reality by (see Figure 2.5 and note the difference from equation 2.54):

$$\Delta g_P = g_P - \gamma_{Q''} \tag{4.28}$$

A combination of equations 4.23, 4.24, 4.26, 4.27 & 4.28 gives (MATHER 1973a, p.26):

$$\frac{\partial T'}{\partial h} = -\Delta g + \zeta\frac{\partial\gamma}{\partial h} + \tfrac{1}{2}g\varepsilon^2 - \frac{\partial W_A}{\partial h} + o\{1\mu gal\} \tag{4.29}$$

The resulting expression for the gravity anomaly Δg is therefore (from equations 4.18, 4.23 and 4.29):

$$\Delta g = \left(-\frac{\partial T'}{\partial h} - \frac{2T'}{R}\right) - \frac{2T'}{R}c_\phi + \frac{2}{R}(W_o - U_o) - \delta g_a + \tfrac{1}{2}g\varepsilon^2 + \frac{2\delta W}{R} + o\{f^2\Delta g\} \tag{4.30}$$

where

$$c_\phi = m + f - 3f\sin^2\phi \tag{4.31}$$

and the atmospheric effect on the gravity anomaly is given by (sub-section

4.5.1):

$$\delta g_a = \frac{\partial W_A}{\partial h} + \frac{2W_A}{R} \qquad (4.32)$$

The gravity anomaly Δg can be computed from gravity and levelling data from equations 2.54, 2.55 & 3.17.

The "pseudo" gravity anomaly $\Delta g'$ (MATHER ET AL 1976a, p.23) can be defined by:

$$\Delta g' = \frac{-\partial T'}{\partial h} - \frac{2T'}{R} \qquad (4.33)$$

The term "pseudo" is used in order to differentiate this quantity from the ground level gravity anomaly which is deduced from surface gravity observations. The "pseudo" gravity anomaly can only be evaluated from satellite-determined gravity field models (equation 4.35). The quantity $\Delta g'$ is harmonic in the space exterior to and on the surface of measurement and takes the values $\overline{\Delta g}'$ on the Brillouin sphere,

$$\overline{\Delta g}' = \frac{-\partial \overline{T}'}{\partial h} - \frac{2\overline{T}'}{\overline{R}} \qquad (4.34)$$

$\Delta g'$ can be represented by a spherical harmonic expansion of the form (from equations 4.20 and 4.33):

$$\Delta g' = \frac{GM}{R^2} \sum_{n=0}^{\infty} (n-1) \left(\frac{a}{R}\right)^n T'_n \quad , \quad n \neq 1 \qquad (4.35)$$

or, in the case of $\overline{\Delta g}'$ (equations 4.22 and 4.34):

$$\overline{\Delta g}' = \frac{GM}{\overline{R}^2} \sum_{n=0}^{\infty} (n-1) \left(\frac{a}{\overline{R}}\right)^n T'_n \quad , \quad n \neq 1 \qquad (4.36)$$

$\overline{\Delta g}'$ may also be obtained by upward continuation:

$$\overline{\Delta g}' = \Delta g' + \delta\Delta g' \qquad (4.37)$$

where $\delta\Delta g'$ is the change in the gravity anomaly from the earth's surface to the Brillouin sphere and, to the first order, is given by (MATHER 1973a,p.39)

$$\delta\Delta g' = \frac{(\overline{R}-R)^i}{i!} \frac{\partial^i \Delta g}{\partial h^i} = -\gamma\left(\frac{\partial \xi}{\partial x_1} + \frac{\partial \eta}{\partial x_2} - \frac{\xi \tan\phi}{R} - \frac{2\zeta}{R^2}\right)(\overline{R}-R) + o\left\{dR\frac{\partial^2 \Delta g}{\partial h^2}\right\} \qquad (4.38)$$

where $\partial\xi/\partial x_1$ and $\partial\eta/\partial x_2$ are the horizontal gradients of the components of the deflection of the vertical in the meridional and prime vertical directions. The magnitude of $\delta\Delta g'$ varies from 1mgal to 10mgal as the quantities $\partial\xi/\partial x_1$, $\partial\eta/\partial x_2$ vary from 1 arcsec per 10^2km to 10 arcsec per 10^2km (MATHER 1974b, p.99). dR is $(\bar{R} - R)$.

On the surface of measurement, $\Delta g'$ is given by (equations 4.30 and 4.33):

$$\Delta g' = \Delta g + \frac{2T'}{R} c_\phi - \frac{2}{R}(W_o - U_o) - \frac{2}{R}\,\delta W - \tfrac{1}{2}g\varepsilon^2 + \delta g_a \qquad (4.39)$$

However, this equation contains unknown terms due to the SST (δW) and the potential of the geoid (W_o). That part of the pseudo gravity anomaly that can be computed Δg_c is given by:

$$\Delta g_c = \Delta g + \frac{2T'}{R} c_\phi - \tfrac{1}{2}g\varepsilon^2 + \delta g_a \qquad (4.40)$$

or, in terms of the unknown quantities:

$$\Delta g_c = \Delta g' + \frac{2}{R}(W_o - U_o) + \frac{2}{R}\,\delta W \qquad (4.41)$$

Similarly, the computable part of $\overline{\Delta g}'$ is denoted $\overline{\Delta g}_c$ and is the gravity anomaly used in practical solutions of the GBVP described in the following section.

4.2 SOLUTION OF THE GEODETIC BOUNDARY VALUE PROBLEM

4.2.1 DEVELOPMENT OF THE SOLUTION

The following is an outline of the development given in MATHER (1973a), MATHER (1975a).

Adopt a local X_i Cartesian coordinate system centred on element ds with the X_3 axis oriented along the spherop normal and the X_1 and X_2 axes oriented north and east respectively. It can be shown that (MATHER 1970a, p.14):

$$\vec{N} = \cos\beta\left(-\tan\beta_1\vec{1} - \tan\beta_2\vec{2} + \vec{3}\right) \qquad (4.42)$$

where β is the slope of the topography at ds, and β_1, β_2 the components of ground slope in the north and east directions respectively.

Therefore,

$$\vec{\nabla}.\vec{N}\,\frac{1}{r} = \frac{\cos\beta}{r^3}\left(x_1\tan\beta_1 + x_2\tan\beta_2 - x_3\right) \qquad (4.43)$$

and

$$\vec{\nabla}.\vec{N}\,T' = \cos\beta\left(-\frac{\partial T'}{\partial x_1}\tan\beta_1 - \frac{\partial T'}{\partial x_2}\tan\beta_2 + \frac{\partial T'}{\partial x_3}\right) \qquad (4.44)$$

The boundary condition (equation 4.8) can now be rewritten as:

$$T' = I_1 + I_2 \tag{4.45}$$

where the integral I_1 contains the terms relating to the spherop normal:

$$I_1 = \frac{1}{2\pi} \iint \frac{R^2}{r} \left(-\frac{\partial T'}{\partial x_3} - T'\frac{x_3}{r^2}\right) d\sigma \tag{4.46}$$

and the integral I_2 contains the remaining terms relating to the horizontal disturbing potential gradient and the slope of the topography:

$$I_2 = \frac{1}{2\pi} \iint \frac{R^2}{r} \left(\frac{x_1 \tan\beta_1}{r^2} T' + \frac{x_2 \tan\beta_2}{r^2} T' + \frac{\partial T'}{\partial x_1} \tan\beta_1 + \frac{\partial T'}{\partial x_2} \tan\beta_2\right) d\sigma \tag{4.47}$$

where, in these two integral equations, $R^2 d\sigma$ is the element of surface area ds (equation 4.8) projected onto the associated spherop surface through P:

$$\cos\beta \; ds = R^2 d\sigma \tag{4.48}$$

R is the distance from the geocentre to the point P.

Integral I_1 would be equivalent to Stokes' integral (equation 2.42) if R were constant over the surface of integration. This is not the case for solutions striving for ±10cm precision. I_1 can be further partitioned:

$$I_1 = I_{11} + I_{12} \tag{4.49}$$

where the term,

$$I_{11} = \frac{1}{2\pi} \iint \frac{R^2}{r} \left(-\frac{\partial T'}{\partial h} - \frac{T'}{2R}\right) d\sigma \tag{4.50}$$

contains the major Stokesian term, and

$$I_{12} = \frac{1}{2\pi} \iint \frac{R^2}{r} T' \left(\frac{x_3}{r^2} - \frac{1}{2R}\right) d\sigma \tag{4.51}$$

deals with quantities relating to the topography and ellipticity of meridians. The I_{12} term is f times smaller than I_{11} except in regions of rugged topography situated very close to the point of computation, when the contribution of I_{12} is still at least an order of magnitude smaller than that of I_{11} (see discussion in MATHER 1973a, p.A-3).

The standard Stokesian manipulation then requires that $\partial T'/\partial h$ and T' in equation 4.50 be replaced by solid spherical harmonic representations (equation 4.20) to give an expression involving the pseudo gravity anomaly $\Delta g'$ (equation 4.35). However, such replacement is only valid if the harmonic expansion of $\Delta g'$ refers to a sphere of radius R and does not exist if $R < R_p$ (R_p is the geocentric radius to the computation point), the quantity $\Delta g'$ not being harmonic inside matter.

Conditions for the recovery of Stokes' integral can be obtained in the following manner (IBID, p.38).

$\Delta g'$ and T' take values $\overline{\Delta g}'$ and $\overline{T}'$ on the Brillouin sphere:

$$\overline{T}' = T' + \delta T' \qquad (4.52)$$

where

$$\delta T' = \frac{(\overline{R}-R)^i}{i!} \frac{\partial^i T'}{\partial h^i} \qquad (4.53)$$

$\overline{\Delta g}'$ is defined by equations 4.37 and 4.38.

Integral I_{11} can now be separated into Stokesian and non-Stokesian parts:

$$I_{11} = I_{111} + I_{112} \qquad (4.54)$$

where

$$I_{111} = \frac{1}{2\pi} \iint \frac{\overline{R}^2}{\overline{r}} \left(-\frac{\partial \overline{T}'}{\partial h} - \frac{\overline{T}'}{2\overline{R}}\right) d\sigma \qquad (4.55)$$

the integration now being performed on the Brillouin sphere. The distance $\overline{r}$ is the chord distance between $\overline{P}$ (the projection of the computation point P onto the Brillouin sphere) and an elemental surface $d\sigma$. Using the definition of the pseudo gravity anomaly in equation 4.34, I_{111} becomes:

$$I_{111} = \frac{1}{2\pi} \iint \frac{\overline{R}^2}{\overline{r}} \left(\overline{\Delta g}' + \frac{3\overline{T}'}{2\overline{R}}\right) d\sigma \qquad (4.56)$$

On following the standard Stokesian procedure (IBID, p.40 et seq.), the solution can be written as the generalised Stokes′ integral:

$$I_{111} = -\overline{R}M\{\overline{\Delta g}'\} + \frac{\overline{R}}{4\pi} \iint f(\psi)\, \overline{\Delta g}'\, d\sigma \qquad (4.57)$$

where Stokes′ function $f(\psi)$ is defined in equation 2.43.

$M\{x\}$ refers to the global mean value of x and ψ is the angle between geocentric radii to the element of surface area $d\sigma$ and the point of computation P.

The term I_{112} contains the upward continuation terms and can be written in the form (IBID, p.40):

$$I_{112} = \frac{1}{2\pi} \iint \frac{R^2}{r} \left\{\Delta g'\left(c_\Delta + \frac{3}{2}\frac{dR}{R}\right) + \frac{3T'}{2R}\left(c_\Delta + \frac{3dR}{R}\right) - \delta\Delta g' + o\{f^2 \Delta g\}\right\} d\sigma \qquad (4.58)$$

where

$$c_\Delta = \frac{R^2}{r}\frac{\bar{r}}{\bar{R}^2} - 1 \tag{4.59}$$

and

$$\bar{r} = 2\bar{R}\sin(\psi/2) \tag{4.60}$$

The integral I_{111} completely defines the Stokesian contribution to the disturbing potential T', while the sum of the effects in equations 4.47 , 4.51 and 4.58 define the indirect contribution T_c,

$$T' = -\bar{R}M\{\overline{\Delta g}'\} + \frac{\bar{R}}{4\pi}\iint f(\psi)\ \overline{\Delta g}'\ d\sigma + T_c \tag{4.61}$$

The solution for the height anomaly ζ can be found from equation 4.18:

$$\zeta = \zeta_N + \zeta_c \tag{4.62}$$

where the Stokesian term ζ_N is

$$\zeta_N = \frac{1}{\gamma}\left(-(W_o - U_o) - \bar{R}M\{\overline{\Delta g}'\} - \delta W\right) + \frac{\bar{R}}{4\pi\gamma}\iint f(\psi)\ \overline{\Delta g}'\ d\sigma \tag{4.63}$$

In practice, the Stokesian term can only be evaluated at a point P by using the computable gravity anomaly on the Brillouin sphere $\overline{\Delta g}_c$. Equation 4.63 can be transformed using equation 4.41, to give:

$$\zeta_{Np} = \frac{1}{\gamma}\left((W_o - U_o) - \bar{R}M\{\overline{\Delta g}'\} + 2M\{\delta W\} - \delta W_p\right) \tag{4.64}$$

$$+ \frac{\bar{R}}{4\pi\gamma}\iint f(\psi)\ \overline{\Delta g}_c d\sigma - \frac{1}{2\pi\gamma}\iint f(\psi)\ \delta W\ d\sigma$$

The indirect effect for the height anomaly is

$$\zeta_c = \frac{1}{\gamma}\left(W_A + T_c\right)$$

$$= \frac{1}{\gamma}\sum_{\alpha=1}^{2}\left[W_A + \frac{1}{2\pi}\iint \frac{R^2}{r}\left[\frac{\partial T'}{\partial x_\alpha}\tan\beta_\alpha + T'\left(\frac{x_\alpha \tan\beta_\alpha}{r^2} + \frac{3}{2}(c_\Delta + \frac{3dR}{R})\right.\right.\right. \tag{4.65}$$

$$\left.\left.\left. - \left(\frac{x_3}{r^2} - \frac{1}{2R}\right)\right) - \delta\Delta g' + \Delta g'\left(c_\Delta + \frac{3dR}{2R}\right) + o\{f^2\Delta g\}\right] d\sigma\right]$$

Equation 4.65 can be evaluated using Δg_c in place of $\Delta g'$ without loss of precision.

4.2.2 THE ZERO DEGREE TERM

The terms of zero degree in equation 4.64, $(W_O - U_O)$, $\bar{R}M\{\overline{\Delta g_C}\}$ and $M\{\delta W\}$, contain some information on the scale of the system. The term $M\{\delta W\}$ is discussed in sub-section 4.2.3.

The effect of the term $(W_O - U_O)$ can only be reliably separated from that of $\bar{R}M\{\overline{\Delta g_C}\}$ if a global representation for the gravity anomaly as sampled at the surface of the earth is available (see § 5.1.3.1).

The zero degree term ζ_O in the height anomaly (neglecting the term relating to the SST for the moment) is:

$$\zeta_o = \frac{1}{\gamma}\left((W_o - U_o) - RM\{\overline{\Delta g_c}\}\right) + \zeta_{co} \tag{4.66}$$

where ζ_{CO} is the contribution of zero degree of the indirect effect, due mainly to the zero degree term in the atmospheric potential - see § 4.5.2.1).

The zero degree term ζ_o can be interpreted in the following manner.

Taking the global mean values, on the Brillouin sphere, of the quantities in equation 4.18, and noting that the zero degree term for the disturbing potential of the earth plus atmosphere is $G\Delta M/\bar{R}$, gives the following expression:

$$\zeta_o = \frac{1}{\gamma}\left(\frac{G\Delta M}{\bar{R}} - (W_o - U_o)\right) \tag{4.67}$$

Comparing equations 4.66 & 4.67 gives the following relation for the global mean value of gravity anomalies:

$$\bar{R}M\{\overline{\Delta g_c}\} = -\frac{G\Delta M}{\bar{R}} + 2(W_o - U_o) \tag{4.68}$$

where ΔM is the difference in mass between the solid earth *plus* atmosphere and the reference ellipsoid. The following observations can be made from equation 4.67:

(1) If the reference ellipsoid does not have the same potential as the geoid $(W_o \neq U_o)$, then ζ_o is non-zero.

(2) If the reference ellipsoid does not have the same mass as the earth with atmosphere $(\Delta M \neq 0)$, then ζ_o is non-zero.

(3) If, however, $M\{\overline{\Delta g_c}\}$ in equation 4.68 is non-zero, then either $\Delta M \neq 0$ or $W_o \neq U_o$ or both. If $M\{\overline{\Delta g_C}\} = 0$ and $\zeta_O = 0$ then the reference ellipsoid both encloses the same mass as the earth with atmosphere and has the same potential as the geoid.

The problems of scale in gravimetric solutions, the potential of the geoid and the choice of parameters of the reference ellipsoid are investigated further in sub-section 5.1.3.

4.2.3 THE EFFECT OF SEA SURFACE TOPOGRAPHY ON SOLUTIONS OF THE GBVP

The influence of the SST on gravimetric determinations of the height anomaly was described in § 2.3.5.5.

The net effect e_{SST} of the existence of SST on solutions of the GBVP (through the major Stokesian term ζ_N) is (see equation 4.64):

$$e_{SST} = -\frac{\delta W}{\gamma} + \frac{2}{\gamma} M\{\delta W\} - \frac{1}{2\pi\gamma} \iint f(\psi)\ \delta W\ d\sigma \qquad (4.69)$$

The input δW from land gravity represents the difference in potential between the geodetic levelling datum and the geoid, while the input from oceanic areas would be the difference in potential between the sea surface and the geoid (see equation 4.19).

The magnitude of the constant term $M\{\delta W\}$ is dependent on the definition of the geoid adopted (see sub-section 2.2.5). Setting $M\{\delta W\} = 0$ is tantamount to defining a unique geoid according to § 2.2.5.5 ("GBVP" definition).

The error in the height anomaly due to the datum for the gravity anomaly data being affected by the SST has been estimated (see § 2.3.5.5) as being of the order of 15-60cm. This is the indirect effect of SST on the height anomaly, representing the contribution through the quadratures evaluation of Stokes' integral (equation 4.64) and is the last term in equation 4.69.

Such an effect is a consequence of the gravity anomaly being defined by (equation 4.28):

$$\Delta g = g_P - \gamma_{Q''} \qquad (4.70)$$

where $\gamma_{Q''}$ is the normal gravity at a point displaced by the (unknown) quantity δW from the telluroid (see Figure 2.5) or, in other words, the error is due to the uncertainty in the definition of the disturbing potential T' (equation 4.17) used to construct the gravity anomaly in equation 4.30.

However, the most significant contribution to the error in the height anomaly e_{SST} is the magnitude of the stationary SST $(-\delta W/\gamma)$ at the computation point. This (unknown) term was incorporated into the definition of the height anomaly (see § 2.3.5.5) in order to allow the ellipsoidal height to be defined by (equation 2.52):

$$h = H^* + \zeta \qquad (4.71)$$

where the normal height H^* is the height of the "altered" telluroid above the reference ellipsoid and the height anomaly is the remaining separation between the surface point P and the "equivalent" point Q'' (Figure 2.5). The magnitude of e_{SST} is therefore o{1-2m} through the first term in equation 4.69, with a further 15-60cm due to the nature of the propagation of δW through quadrature evaluations of equation 4.63.

As a consequence of the relationship between geoid height N and the height anomaly ζ (equation 2.62), it follows directly that the effect of the SST on the geoid height e_N is:

$$e_N = \frac{2}{\gamma} M\{\delta W\} - \frac{1}{2\pi\gamma} \iint f(\psi)\ \delta W\ d\sigma \tag{4.72}$$

In other words, to determine the geoid height *implicitly* from the height anomaly there is no need to make any assumptions concerning the magnitude of the SST at the computation point itself. The determination of the geoid height N from solutions of the GBVP according to Molodenskii is therefore subject to a lower error due to the existence of SST than the height anomaly as defined in § 2.3.5.5. Procedures for the determination of a high precision marine geoid for SST studies from surface gravity data are discussed further in § 5.1.2.1.

4.3 SATELLITE ALTIMETRY AND THE GBVP

The main problem with gravimetric solutions of the GBVP is the lack of a global representation of gravity anomalies for the foreseeable future and the questionable quality of oceanic coverage, widely conceded as being at least an order of magnitude inferior to land based data.

An alternative to equation 4.64 is the use of a combination approach discussed in sub-section 2.3.6. For a spherical cap of angular distance ψ_o, a computational scheme could take the form of (equation 2.64):

$$\zeta_N = \zeta_{No} + e_{SST} + \frac{R}{4\pi\gamma_0}\int_0^{2\pi}\int_0^{\psi_o} f(\psi)\ \overline{\Delta g_c}\ \sin\psi d\psi d\alpha + \tag{4.73}$$

$$\frac{GM}{R\gamma}\sum_{n=2}^{n'} Q_n(\psi_o)(n-1)\left(\frac{a}{R}\right)^n \sum_{m=0}^{n}\sum_{\alpha=1}^{2} \overline{C}_{\alpha nm} S_{\alpha nm}$$

where ζ_{No} is the zero degree term (defined by $\zeta_o - \zeta_{co}$ in equation 4.66), e_{SST} is the effect of the SST (equation 4.69) and $Q_n(\psi_o)$ are the Molodenskii truncation functions (see § 2.3.6.2), all other quantities having been defined in § 2.3.3.1. Only a finite number of spherical harmonic coefficients $\overline{C}_{\alpha nm}$ are known (n' is of the order of 20). A further innovation is the use of a modified Stokes' kernel and modified Molodenskii functions as suggested by RAPP (1979), though the principles remain unchanged.

A rigorous inner cap calculation using available surface gravity anomaly data to, say, 20° ($=\psi_o$) from the computation point will provide information on the short wavelength features of ζ_N. The distant zone contribution from the harmonic representation will provide long wavelength information if the coefficients $\overline{C}_{\alpha nm}$ are known to sufficient accuracy (see discussion in § 2.3.3.1). If a "higher" reference model is adopted (see section 4.4), the long wavelength geoid height information will be incorporated into the reference system.

The height anomaly ζ may be deduced <u>directly</u> from the altimeter measurement (see sub-section 2.3.4). Satellite altimetry, in principle, can provide the most accurate estimates of ζ in the oceans, while gravimetric techniques have the potential to provide the most accurate values of geoid height but, unfortunately, restricted to regions of high quality surface gravimetry (usually land and continental shelf areas).

The quantity $\overline{N}'$ may be defined by:

$$\overline{N}' = \frac{\overline{T}'}{\gamma} \tag{4.74}$$

where $\overline{T}'$ is the disturbing potential of the solid earth and oceans on the Brillouin sphere. Combination of equations 4.17, 4.52 and 4.74 gives:

$$\overline{N}' = \frac{1}{\gamma}\left((W_o - U_o) + \delta W - W_A + \delta T'\right) + \zeta \tag{4.75}$$

Values of $\overline{N}'$ are deduced from:

(i) satellite altimetry in ocean areas.

(ii) estimates of ζ in continental areas from gravimetric solutions of the GBVP.

An alternate formulation for the fundemental relationship between the two types of measured quantities (gravity anomalies and height anomalies) was developed by MATHER ET AL (1976a). This relationship is similar to the inverse relation to Stokes' problem as propounded by MOLODENSKII ET AL (1962, p.50) where the gravity anomalies are obtained from geoid height data in the case of a spherical earth with no matter exterior to the bounding equipotential.

For some surface point P, the inverse relation is (MATHER ET AL 1976a, p.29):

$$\overline{\Delta g}'_p + \frac{\gamma}{R}\overline{N}'_p = \frac{\gamma}{4\pi R}\iint M_1(\psi)\left(\overline{N}' - \overline{N}'_p\right)d\sigma \tag{4.76}$$

where

$$\begin{aligned} M_1(\psi) &= \sum_{n=2}^{\infty} n(2n+1)P_{n0}\cos\psi \\ &= -\frac{1}{4}\operatorname{cosec}^3\frac{\psi}{2} - 3\cos\psi \end{aligned} \tag{4.77}$$

Computational procedures for the recovery of gravity anomalies using this inverse relation have been investigated by COLEMAN & MATHER (1976). Characteristics of the kernel function $M_1(\psi)$ were studied and an optimum annular subdivision for successful quadrature evaluation was proposed.

Equation 4.75 can be expressed in terms of the computable quantity $\overline{N}_c$:

$$\overline{N}_c = \zeta - \frac{1}{\gamma}(W_A - \delta T') \tag{4.78}$$

or, in terms of the unknown quantities:

$$\overline{N}' = \overline{N}_c + \frac{1}{\gamma}\left((W_o - U_o) + \delta W\right) \tag{4.79}$$

Expressing equation 4.76 in terms of the computable quantities $\overline{N}_c$ (equation 4.79) and $\overline{\Delta g}_c$ (equation 4.41), gives the actual relationship between satellite altimetry data and surface gravimetry:

$$\overline{\Delta g}_{cp} + \frac{1}{R}\left(\gamma\overline{N}_{cp} - (W_o - U_o) - \delta W_p\right) = \frac{\gamma}{4\pi R}\iint M_1(\psi)\left(\overline{N}_c - \overline{N}_{cp}\right)d\sigma$$

$$+ \frac{1}{4\pi R}\iint M_1(\psi)\left(\delta W - \delta W_p\right)d\sigma \qquad (4.80)$$

4.4 THE "HIGHER" REFERENCE SYSTEM

The system of reference based on an equipotential ellipsoid with the same rotational characteristics and mass as the earth, and the same volume as global MSL or the geoid, gives rise to a disturbing potential T at the surface of the earth whose magnitude is of the order of $\pm 10^2$kgalm. If the reference ellipsoid does not have the same mass as the earth, or is not a geometric best fit to the geoid a non-zero scale term will be present in T (notions of scale were discussed in sub-section 4.2.2).

The corresponding geoid heights on such a reference system are of the order of $\pm 10^2$m (see Figure 2.2). Moreover, detailed gravimetric geoid studies, such as for Australia (MATHER 1970a), indicate that the high frequency (short wavelength) contribution to the geoid height does not exceed 5% of the dominant contribution from a low degree gravity field model such as GEM9 (LERCH ET AL 1977).

It is therefore possible to formulate a "higher" system of geodetic reference than that afforded by a rotating gravitational equipotential ellipsoid. Such a system was proposed by MATHER (1974b), where the total reference potential is obtained by superimposing a gravitational potential model ΔU onto the spheropotential of the ellipsoid. ΔU is defined by a discrete set of spherical harmonics:

$$\Delta U = \frac{GM}{R}\sum_{n=2}^{n'}\left(\frac{a}{R}\right)^n \sum_{m=0}^{n}\sum_{\alpha=1}^{2} C'_{\alpha nm} S_{\alpha nm} \qquad (4.81)$$

where $C'_{\alpha nm}$ are estimates of the spherical harmonic coefficients of the geopotential $C_{\alpha nm}$ to some maximum degree n', as obtained from satellite orbit analysis (see § 2.3.3.1). The spherical harmonic functions $S_{\alpha nm}$ are defined by equation 2.29.

In such a scheme the reference ellipsoid is no longer an equipotential surface of the earth's normal gravity field, instead its spheropotential is given by:

$$U = U_o + \Delta U_s \qquad (4.82)$$

where

$$\Delta U_s = \frac{GM}{R_e}\sum_{n=2}^{n'}\left(\frac{a}{R_e}\right)^n \sum_{m=0}^{n}\sum_{\alpha=1}^{2} C^*_{\alpha nm} S_{\alpha nm} \qquad (4.83)$$

and R_e is the distance from the geocentre to the ellipsoid. The coefficients defining the "higher" reference model are:

$$C^*_{\alpha nm} = C'_{\alpha nm} \tag{4.84}$$

except when $\alpha = 1$, $m = 0$ and $n = 2$, 4 and 6.

In these three special cases,

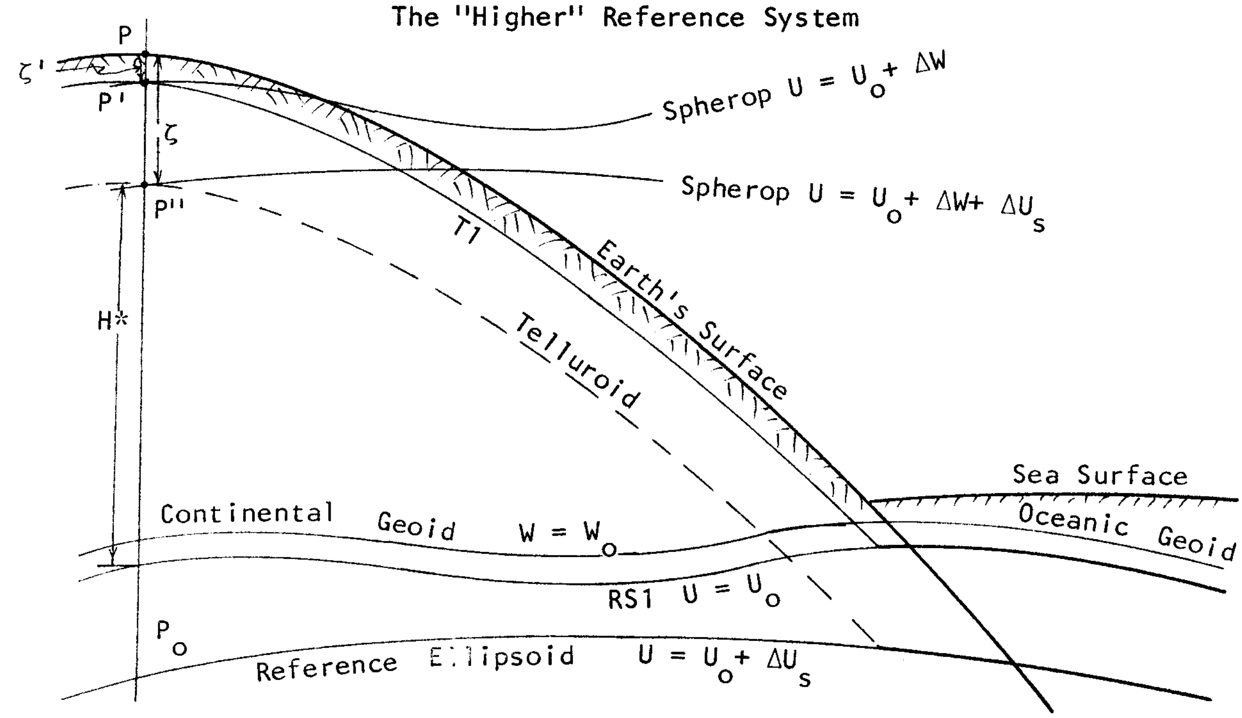
$$C^*_{120} = 0 \tag{4.85}$$

and C^*_{140}, C^*_{160} have been corrected for the effect of the ellipsoidal flattening implied in the value of C'_{120} by relations given in equation 5.23 (see § 5.1.2.3).

The equipotential reference surface ($U = U_o$) is irregular and approximates the geoid to within ±10m. The new telluroid, labelled T1 (see Figure 4.1), is the locus of points P'' ($\Phi, \Lambda, U_o + \Delta W$) where ΔW is obtained from geodetic levelling related to MSL as described in § 2.3.5.5 (Figure 2.5). Consequently the height anomaly ζ', and hence the geoid height, for the T1-Earth surface system can be expected to be about 5% of that for the Telluroid-Earth surface system described in § 2.3.5.3.

FIGURE 4.1

The "Higher" Reference System



LAND : W = Difference in Geopotential Between Earth's Surface and MSL Datum for Levelling (see Figure 2.5)

OCEAN : ΔW = 0

Subsequent mathematical formulations will be based on the "higher" reference model therefore the fundamental quantities and their relations, described in sections 4.1 and 4.2, need to be modified.

The disturbing potential of the solid earth and oceans for the "higher" reference system is (equation 4.17):

$$T'' = (W_o - U_o) + \delta W - W_A + \gamma\zeta' \tag{4.86}$$

where

$$T'' = T' - \Delta U_s \tag{4.87}$$

and, in terms of spherical harmonic representation (equation 4.20):

$$T'' = \frac{GM}{R}\sum_{n=0}^{\infty}\left(\frac{a}{R}\right)^n \sum_{m=0}^{n}\sum_{\alpha=1}^{2} dC_{\alpha nm} S_{\alpha nm} \tag{4.88}$$

The coefficients $dC_{\alpha nm}$ are defined by:

$$dC_{\alpha nm} = C_{\alpha nm} - C'_{\alpha nm} - \left(\frac{R_a}{a}\right)^n V_{s\alpha nm} \tag{4.89}$$

where $C_{\alpha nm}$ are the "true" spherical harmonic coefficients of the earth's geopotential (equation 2.28) and $V_{s\alpha nm}$ are the spherical harmonic coefficients of a representation of the atmospheric potential W_A for all points exterior to the geocentric sphere of radius R_a enclosing the earth's atmosphere.

If the coefficients $C'_{\alpha nm}$ and the atmospheric representation are known without error i.e. $dC_{\alpha nm}$ contains terms only related to the atmospheric potential, then T'' only contains the short wavelength components of the geopotential (those wavelengths corresponding to degrees higher than n' in the representation of ΔU_s in equation 4.83) and all wavelengths in the atmospheric potential.

In other words, although T'' is unknown, it is banded with harmonics of wavelength less than ℓ (= o{10^3km}) where ℓ is the shortest full wavelength (corresponding to degree n') in the gravity field which perturbs the orbits of near-earth satellites above the noise level of the tracking. In addition, if the "higher" reference model were indeed representative of the long wavelength features of the geopotential, and hence the geoid height, the contribution of the last term in equation 4.73 would be zero if expressed in terms of the redefined quantities for ζ and Δg_c (equation 4.94).

The gravity anomaly $\Delta g''$ on the "higher" reference model is defined by (from equation 4.33):

$$\Delta g'' = -\frac{\partial T''}{\partial h} - \frac{2T''}{R} \tag{4.90}$$

or, in terms of a spherical harmonic representation (equation 4.35):

$$\Delta g'' = \frac{GM}{R^2}\sum_{n=0}^{\infty}(n-1)\left(\frac{a}{R}\right)^n \sum_{m=0}^{n}\sum_{\alpha=1}^{2} dC_{\alpha nm} S_{\alpha nm}\ , \quad n\neq 1 \tag{4.91}$$

It may also be defined in terms of the "pseudo" gravity anomaly $\Delta g'$:

$$\Delta g'' = \Delta g' - \delta\gamma \tag{4.92}$$

where $\delta\gamma$ is the change in normal gravity from the telluroid to the T1 surface (see figure 4.1) and is given by (MATHER 1974b, p.95),

$$\delta\gamma = \frac{GM}{R^2}\sum_{n=2}^{n'}(n-1)\left(\frac{a}{R}\right)^n \sum_{m=0}^{n}\sum_{\alpha=1}^{2} C^*_{\alpha nm} S_{\alpha nm} + o\{f\delta\gamma\} \ , \quad n\neq 1 \tag{4.93}$$

While the adoption of a "higher" reference system reduces the magnitude of the disturbing potential (and height anomaly) to less than 10% of their values on the rotating equipotential ellipsoidal system, a similar reduction does not occur in the case of the gravity anomaly. That is, $\Delta g''$ is not less than 10% of $\Delta g'$, as gravity is more heavily influenced by local mass anomalies and therefore unrepresented in the "higher" reference model, unless n' is correspondingly large.

All other quantities defining the GBVP for the T1-Earth Surface system will be denoted by the superscript ", e.g. on the Brillouin sphere $\overline{\Delta g}'$ becomes $\overline{\Delta g}''$ after applying the normal gravity correction $\delta\gamma$. The only exception will be the computable quantities $\overline{\Delta g}_c$ and $\overline{N}_c$, the system of reference being clear from the context. For example (equations 4.40 and 4.41):

$$\begin{aligned}\overline{\Delta g}_c &= (\Delta g - \delta\gamma) + \frac{2T''}{R}c_\phi - \tfrac{1}{2}g\varepsilon^2 + \delta g_a + \delta\Delta g'' \\ &= \overline{\Delta g}'' + \frac{2}{R}(W_o - U_o) + \frac{2}{R}\,\delta W\end{aligned} \tag{4.94}$$

where U_o is the spheropotential of the "higher" reference surface, not the ellipsoid and $\delta\Delta g''$ is the correction necessary to upward continue $\Delta g''$ to the Brillouin sphere.

4.5 ATMOSPHERIC EFFECTS IN PHYSICAL GEODESY

4.5.1 THE INFLUENCE OF THE ATMOSPHERE ON SOLUTIONS OF THE GBVP

It is an axiom in potential theory that the attraction of a homogeneous ellipsoidal shell at any point within it should be zero (see e.g. MACMILLAN 1930, p.10). Therefore, the net gravitational attraction at a point on the physical surface of the earth of all the atmosphere which could be considered to be situated in homogeneous ellipsoidal shells lying exterior to the point is zero. The attraction at a point P would be due to a mass M', which is less than that of the solid earth, oceans and atmosphere (M), by amounts up to 1 part in 10^6 (the ratio of the atmospheric mass to the total mass of the earth).

This can be *interpreted* as gravity being too small by amounts which vary as a function of the elevation of individual points. The maximum effect is at sea level, where the total atmospheric mass does not contribute to the magnitude of the surface gravity (being influenced only by the mass of the solid earth and oceans M_e), while the minimum effect is at high altitudes. Such a treatment was developed by ECKER & MITTERMAYER (1969) and included in the definition of the Geodetic Reference System 1967 (GRS 67) (IAG 1971). This treatment implies that the effect on observed gravity of all the atmosphere at elevations above that of the station on the surface of the earth could be evaluated as being due to a series of homogeneous ellipsoidal shells. ECKER & MITTERMAYER (1969, p.79) present their results as a series of positive corrections $\delta g_a'$ to be added to observed gravity. The net result is a gravity anomaly which is *unaffected* by the fact that observed gravity has been measured at different altitudes with an exterior atmosphere.

Such a description would be strictly true for a static atmosphere only in the case of those shells lying wholly exterior to the Brillouin sphere. The use of such corrections in solutions of the GBVP (section 4.2) seeking precisions in excess of the order of the flattening (±30cm in ζ) can at best be considered to be intuitive rather than within the strict framework of potential theory. No consideration has been given to the fact that the density of some of those ellipsoidal shells which are not exterior to the Brillouin sphere are not homogeneous, being partly composed of topographic crust ($\rho \simeq 2.67$ g cm^{-3}) rather than atmosphere ($\rho \simeq 0.003$ g cm^{-3}).

The effect of the atmosphere on gravity and potential was studied by ANDERSON (1976, Section 9) while the validity of the assumption concerning homogeneous ellipsoidal shells inherent in the development by ECKER & MITTERMAYER (1969) was investigated by ANDERSON ET AL (1975). The following is a development from IBID (1975).

The effect of making such a correction to the gravity anomaly on the Stokesian term in the solution of the geodetic boundary value problem is given by (equation 4.40 and 4.64):

$$\delta\zeta_a' = - \frac{\bar{R}}{\gamma} M\{\delta g_a'\} + \frac{\bar{R}}{4\pi\gamma} \iint f(\psi)\ \delta g_a'\ d\sigma \qquad (4.95)$$

where $\bar{R}$ is the radius of the Brillouin sphere, $M\{\delta g_a'\}$ is the global mean value of $\delta g_a'$, $f(\psi)$ is Stokes' function, and ψ is the angular distance between the element of surface area $d\sigma$ on the unit sphere and the point of computation at which normal gravity is γ. $\delta g_a'$ is largely of the form:

$$\delta g_a' = M\{\delta g_a'\} + \Delta\delta g_a'(\phi,\lambda) \qquad (4.96)$$

where (ϕ,λ) are geocentric surface coordinates. $M\{\delta g_a'\}$ = o{0.8mgal} and has no effect through Stokes' integral. The variable part $\Delta\delta g_a'$ is much smaller (< o{0.3mgal}) and is highly correlated with topography. This particular approach will hereafter be referred to as the GRS 67 approach.

An alternate treatment of the atmospheric effects in physical geodesy is a consequence of the development of a solution of the geodetic boundary value problem to define the height anomaly ζ in oceanic regions to ±5cm (sub-section 4.1.2). In this approach, hereafter called the BVP approach, the potential due to the atmosphere W_A is separated from that of the solid Earth and oceans T'.

The principal effects produced by the atmosphere in the BVP approach are twofold:

(1) The gravity anomaly $\overline{\Delta g}_c$ on the "higher" reference system used in

the evaluation of the Stokesian term is given by (equation 4.94):

$$\overline{\Delta g}_c = \Delta g + \delta g_a + c_g \quad (4.97)$$

where

$$c_g = \frac{2T''}{R} c_\phi - \tfrac{1}{2} g \varepsilon^2 + \delta\Delta g'' \quad (4.98)$$

All quantities are as defined in sections 4.1.4 and 4.4. The correction to the conventional gravity anomaly Δg is given by (equation 4.32):

$$\delta g_a = \frac{\partial W_A}{\partial h} + \frac{2W_A}{R} \quad (4.99)$$

(2) The dominant contribution of the atmosphere to the height anomaly is one of zero degree, given by (equations 4.64, 4.65 & 4.99):

$$\delta\zeta_{ao} = -\frac{\bar{R}}{\gamma} M\{\delta g_a\} + \frac{W_A}{\gamma}$$
$$= \frac{1}{\gamma}\left(-2\bar{R}M\{\frac{W_A}{R}\} + W_A - \bar{R}M\{\frac{\partial W_A}{\partial h}\}\right) \quad (4.100)$$

where $M\{\ \}$ refers to the global mean value and R is the geocentric distance. Values not in terms defining global means refer to the point of evaluation.

The net effect $\delta\zeta_a$ on ζ in this case due to the contribution of the atmosphere, excluding secondary effects through the non-Stokesian terms (through the use of $\Delta g'$ in equation 4.65) is given by:

$$\delta\zeta_a = \frac{1}{\gamma}\left(-2\bar{R}M\{W_A\} + W_A - \bar{R}M\{\partial W_A/\partial h\}\right) + \frac{\bar{R}}{4\pi\gamma}\iint f(\psi)\delta g_a d\sigma \quad (4.101)$$

W_A is of the form:

$$W_A = M\{W_A\} + \delta W_A(\phi,\lambda) \quad (4.102)$$

where $\delta W_A(\phi,\lambda)$ is up to an order of magnitude smaller than $M\{W_A\}$.

4.5.2 AN EVALUATION OF THE EFFECT OF THE ATMOSPHERE ON GEOID DETERMINATIONS

The atmospheric potential W_A and the three components of the derivative $\partial W_A/\partial x_i$ were computed by ANDERSON (1976, Section 9) from the direct evaluation of the basic Newtonian integrals (equations 2.1 & 2.8):

$$W_A = G \iiint_{atmos} \frac{\rho}{r}\, dv \qquad (4.103)$$

and

$$\frac{\partial W_A}{\partial x_i} = - G \iiint_{atmos} \frac{x_i - \xi_i}{r^3}\, \rho\, dv \qquad (4.104)$$

where

- G is the universal gravitation constant.
- dv is the element of volume of the atmosphere.
- ρ is the atmospheric density.
- r is the distance of dv from the computation point.
- x_i are the three dimensional rectangular coordinates of the computation point.
- ξ_i are the coordinates of dv on this same Cartesian system.

The atmospheric density model used was the vertical density distribution tabulated for the NACA Standard Atmosphere (see ANDERSON ET AL 1975, Figure 1). It was assumed that this vertical density variation was globally applicable, i.e. changes in density due to variations in latitude or season were not considered.

The results of the evaluation of equations 4.103 and 4.104 are given in ANDERSON (1976, Figures 9.4 to 9.9) in the form of maps of W_A and its local vertical and horizontal derivatives at the earth's surface and at satellite orbit altitudes (10^3km). Coefficients of low degree surface spherical harmonic representations of W_A and $\partial W_A/\partial h$ are listed in IBID (Table 9.6) and maps of W_A and $\partial W_A/\partial h$ based on these harmonic representations are given in ANDERSON ET AL (1975, Figures 3 and 4). The mass of the atmosphere M_a has been estimated to be 5.242×10^{21}g.

A low degree surface spherical harmonic representation of δg_a can be constructed from these results (equation 4.99):

$$\delta g_a = \sum_{n=0}^{n''} \sum_{m=0}^{n} P_{nm}(\sin\phi)\left(\delta g_{1nm}\cos m\lambda + \delta g_{2nm}\sin m\lambda\right) \qquad (4.105)$$

In order that these results be comparable with evaluations obtained by the use of the GRS 67 approach, a global equi-angular 5^o elevation data bank was used to derive the corrections $\delta g_a'$ (ECKER & MITTERMAYER 1969, p.79) for this $5^o \times 5^o$ grid. The resulting global distribution of $\delta g_a'$ was analysed for the coefficients $\delta g_{\alpha nm}'$ of a low degree surface harmonic representation of the form in equation 4.105.

4.5.2.1 *The Contribution of Zero Degree*

The effect of zero degree in the GRS 67 approach ($\delta\zeta_{ao}'$) in the contribution to the height anomaly ζ is obtained from equation 4.95:

$$\delta\zeta'_{ao} = -\frac{R}{\gamma} M\{\delta g'_a\} \tag{4.106}$$

where

$$M\{\delta g'_a\} = \delta g'_{100} \tag{4.107}$$

The result is given in Table 4.1.

TABLE 4.1

The Low Degree Contributions of the Atmosphere to Geodetic Parameters

Degree	Effect	BVP Approach	GRS 67 Approach
0	Height Anomaly	$M\{W_A/R\}=0.856$ mgal	$M\{\delta g'_a\}=\delta g'_{100}$
		$M\{\partial W_A/\partial h\}=0.001$ mgal	$=0.847$ mgal
		$\delta\zeta_{ao}=-5.57$ m	$\delta\zeta'_{ao}=-5.51$ m
1	Geocentric Coordinates	$V_{110}=-0.003$ kgalm	$\delta g'_{110}=-0.0046$ mgal
	of Centre of	$V_{111}=-0.026$ kgalm	$\delta g'_{111}=-0.0123$ mgal
	Reference Ellipsoid	$V_{211}=-0.027$ kgalm	$\delta g'_{211}=-0.0117$ mgal
		$\overline{X}_{e_1} = 0.027$ m	$\Delta X_1= 0.040$ m
		$\overline{X}_{e_2} = 0.028$ m	$\Delta X_2= 0.038$ m
		$\overline{X}_{e_3} = 0.003$ m	$\Delta X_3= 0.015$ m

Source: ANDERSON ET AL (1975)

In the BVP approach, the effect of zero degree ($\delta\zeta_{ao}$) on the height anomaly is given by equation 4.100, which for all practical purposes is:

$$\delta\zeta_{ao} = \frac{1}{\gamma}\left(-RM\{W_A/R\} - RM\{\partial W_A/\partial h\}\right) \tag{4.108}$$

where $M\{W_A/R\}$ and $M\{\partial W_A/\partial h\}$ are obtained from the respective terms of zero degree obtained from the computations described in ANDERSON (1976, Chapter 9). The constituent values and their effects on ζ are also listed in Table 4.1.

The contribution of the atmosphere to the term of zero degree in the height anomaly as computed by these two totally different techniques agree to 6cm.

The global correction of -5.5m implied by the zero degree term $\delta\zeta_{ao}$ does not imply that all geoid heights as known at the present time are too large by this amount.

This figure is of significance only when analysing zero degree terms in the solution of the geodetic boundary value problem obtained by the use of a global distribution of surface gravity information. If the atmospheric correction δg_a were not allowed for, any deductions about scale from such a global coverage of surface gravity data would point toward a mean Earth ellipsoid which would be too small by 5.5m (assuming a perfect value for GM and W_o being equal to U_o - see sub-sections 4.2.2 & 5.1.3). This is not the case at the present time as all low degree information about the earth's gravity field comes from satellite orbit analysis which is unaffected by the atmosphere.

The influence of the atmosphere on the zero degree term of the height anomaly in combination solutions (sub-section 2.3.6) is a function of the size of the integration cap for surface gravity anomalies and the topography within the cap. RUMMEL & RAPP (1976) have shown that for cap sizes of $\psi = 10^o$ and $\psi = 20^o$ it reaches 1.2m and 2.3m respectively.

4.5.2.2 *The Significance of the First Degree Harmonic*

In the BVP approach, the atmospheric potential at the surface of the earth can be represented by a surface spherical harmonic series of the form:

$$W_A = \sum_{n=0}^{\infty} \sum_{m=0}^{n} P_{nm}(\sin\phi)\left(V_{g1nm}\cos m\lambda + V_{g2nm}\sin nm\lambda\right) \qquad (4.109)$$

On expressing the inverse of the distance (r) from the general mass element of the atmosphere to the computation point P (in equation 4.103) as a set of zonal harmonics (MATHER 1971, p.39), W_A is given by:

$$W_A = G \iiint \frac{1}{R} \sum_{n=0}^{\infty} \left(\frac{R_P}{R}\right)^n P_{n0}(\cos\psi)\, \rho\, dv \qquad (4.110)$$

where (ϕ,λ,R) define a set of geocentric spherical coordinates, the subscript p referring to values at the point of computation, and ψ is the angular distance at the geocentre corresponding to r.

On considering the terms of degree 1 in each of the equations 4.109 and 4.110, and noting that

$$R = R_p(1 + o\{10^{-3}\}) \qquad (4.111)$$

it can be shown that (e.g. IBID, p.107):

$$\frac{GM_e}{R^2} \bar{X}_{ei}(1 + o\{10^{-3}\}) = V_{ai} \qquad (4.112)$$

where

$$V_{a1} = V_{g111} \quad ; \quad V_{a2} = V_{g211} \quad ; \quad V_{a3} = V_{g110} \qquad (4.113)$$

$\bar{X}_{ei}$ are the coordinates of the centre of mass of the solid earth and oceans on a geocentric Cartesian coordinate system. Values of V_{ai} and $\bar{X}_{ei}$ are given in Table 4.1. The centre of mass of the solid earth and oceans can therefore be assumed to coincide with the geocentre to less than ±5cm.

In the GRS 67 approach, if the atmospheric corrections $\delta g_a'$ have a non-zero first degree harmonic $\delta g_{a1}'$, the use of such a correction to the gravity anomaly Δg_c (equation 4.97) for the evaluation of Stokes' integral would result in a determination with respect to an ellipsoid whose centre is not coincident with the geocentre. If the former can be represented as a set of coordinate difference ΔX_i with respect to the geocentre (the origin of the Cartesian coordinate system), ΔX_i produces a radial displacement dr at the surface location (ϕ,λ) given by:

$$dr = \Delta X_1 \cos\phi\cos\lambda + \Delta X_2 \cos\phi\sin\lambda + \Delta X_3 \sin\phi \tag{4.114}$$

The resulting change $\delta\gamma$ in normal gravity is given by:

$$\delta\gamma = c\ dr \tag{4.115}$$

where c is interpreted as the vertical gradient of normal gravity ($\simeq$ 0.31mgal/m). The resulting change $d\Delta g$ in the gravity anomaly is thus:

$$d\Delta g = -\ \delta\gamma = -\ c\ dr \tag{4.116}$$

The Earth space effect of such a change could be represented by the introduction of a term of first degree in the surface spherical harmonic representation of $\overline{\Delta g}_c$. In the case of the atmospheric effect of degree one $(\delta g'_{a1})$:

$$\delta g'_{a1} = P_{10}(\sin\phi)\delta g'_{110} + P_{11}(\sin\phi)\delta g'_{111}\cos\lambda + P_{11}(\sin\phi)\delta g'_{211}\sin\lambda \tag{4.117}$$

where

$$P_{10}(\sin\phi) = \sin\phi \qquad \text{and} \qquad P_{11}(\sin\phi) = \cos\phi \tag{4.118}$$

Values of ΔX_i are given in Table 4.1. The figures obtained in this interpretation of the GRS 67 approach for the geocentric location of the centre of the reference ellipsoid, differ by less than 3cm from the position defined for it by the BVP approach.

Consequently, for all practical purposes, the existence of the atmosphere does not move the centre of the reference ellipsoid from the geocentre in solutions of the GBVP using Stokes' integral.

4.5.2.3 *Higher Degree Effects of the Atmosphere*

Attention will be confined only to those effects of the atmosphere in the BVP approach which contribute through the dominant Stokesian term ζ_N.

The evaluation of the higher degree effects $\delta\zeta_a$ of the atmosphere on ζ_N can be estimated from the low degree surface spherical harmonic representation of δg_a (equation 4.105). Stokes' integral can be written as (HEISKANEN & MORITZ 1967, Section 2.17):

$$\zeta_N = \frac{\bar{R}}{4\pi\gamma} \iint \sum_{n=2}^{\infty} \frac{2n+1}{n-1} P_{n0}(\cos\psi)\, \Delta g_n\, d\sigma \tag{4.119}$$

where Δg_n is the surface spherical harmonic of degree n in the representation of $\overline{\Delta g}'$ (equation 4.36). The contribution $\delta\zeta_a$ to ζ_N can be estimated, to the order of the flattening, as:

$$\delta\zeta_a = \frac{\bar{R}}{\gamma}\sum_{n=2}^{\infty}\frac{1}{(n-1)}P_{nm}(\sin\phi)\left(\delta g_{1nm}\cos m\lambda + \delta g_{2nm}\sin nm\lambda\right) \qquad (4.120)$$

on using the orthogonal properties of surface spherical harmonics.

In the GRS 67 approach, the higher degree effects $\delta\zeta_a'$ of the atmosphere are estimated from the surface spherical harmonic series in equation 4.120 using the coefficients $\delta g'_{\alpha nm}$.

Figures 4.2 and 4.3 illustrate the results of the evaluation of equation 4.120 in the case of the BVP and GRS 67 approaches for a model of δg_a and $\delta g_a'$ respectively. The model coefficients $\delta g_{\alpha nm}$ and $\delta g'_{\alpha nm}$ are based on a surface harmonic representation for values of n in the range $2 \leq n \leq 6$ (i.e. $n'' = 6$).

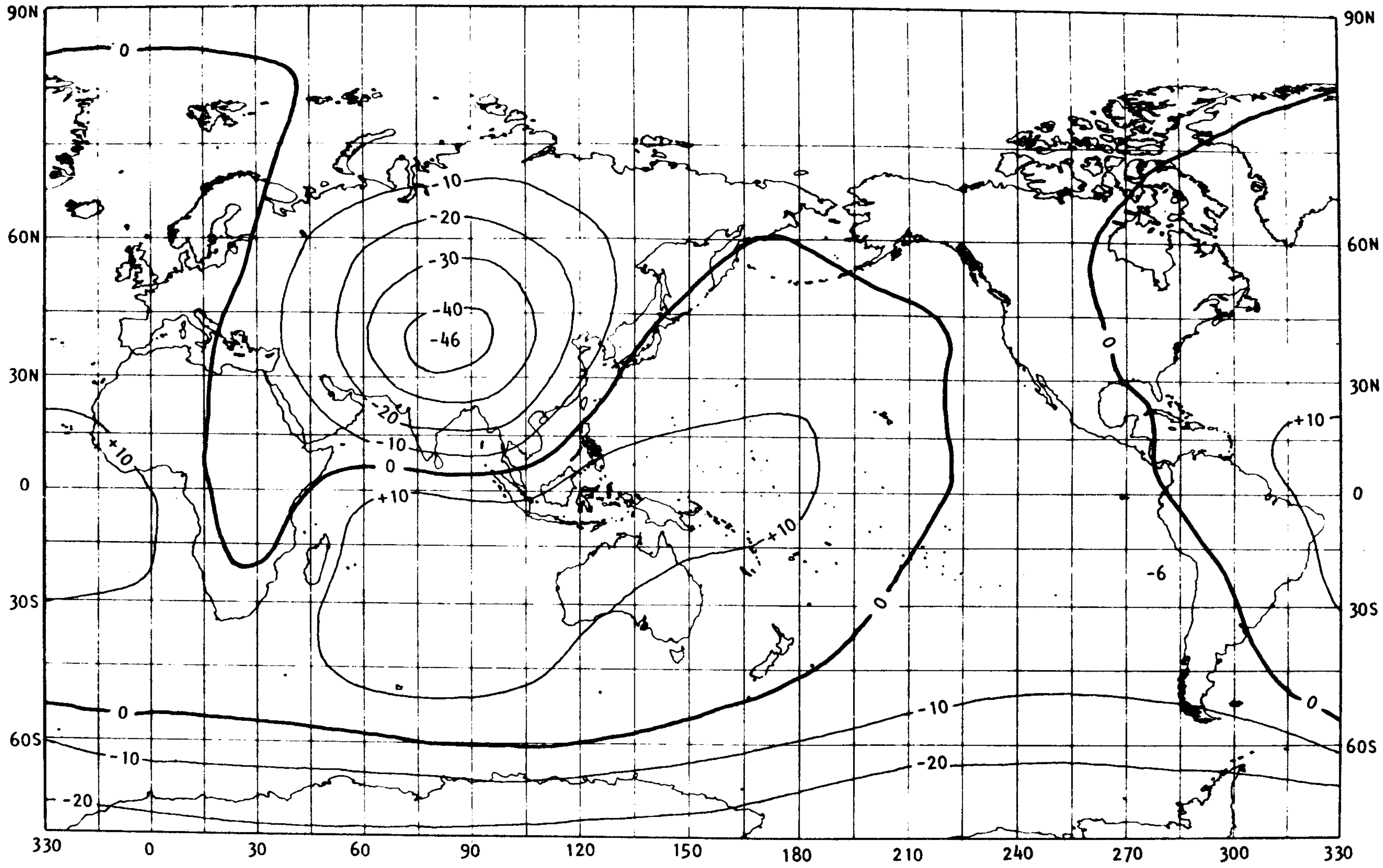
The contribution of the atmosphere to solutions for the height anomaly/geoid height is strongly correlated with the earth's topography. As the oceans predominate, the lows over the mountains are larger in numerical magnitude than the highs over oceans in approximately the same 2:1 ratio of oceanic areas to continental areas. The contribution varies from approximately -40cm in the Himalayas to +15cm over most of the oceans. Note that the lows in these figures have been flattened by the restricted range of harmonics used.

The effects of the atmosphere on the shape of the geops as computed by these two approaches have similar characteristics with discrepancies which are of the order of a few centimetres, with the exception of some discrepancies at the 10cm level in extensive mountain areas, e.g. the Himalayas. The results are in reasonable agreement with those obtained by RUMMEL & RAPP (1976, Figure 1) for a spherical harmonic development to degree and order 36 (in equation 4.120).

The spatial variations in the contributions of the atmosphere to the height anomaly are too large to ignore in any work requiring a precision in excess of the order of the flattening (±30cm). The treatment of the atmospheric correction to surface gravity as suggested in the GRS 67 approach appears to be adequate for SST studies.

The question of a time and spatially varying density distribution for the atmosphere was studied by CHRISTODOULIDIS (1976). It was found that the combined seasonal and latitude dependent atmospheric correction for gravity had a maximum magnitude of the order of 40μgal, and could therefore be neglected.

Atmospheric effects must be correctly allowed for prior to the combination of surface gravity with data deduced from satellite orbital analysis (spherical harmonic coefficients $C'_{\alpha nm}$ - § 2.3.3.1). Biases may be introduced into some of the coefficients $C'_{\alpha nm}$ in the representation of geopotential if the atmospheric corrections δg_a are not applied to the gravity anomalies (see equation 4.97) prior to their incorporation in solutions for gravity field models such as GEM10 (LERCH ET AL 1977).

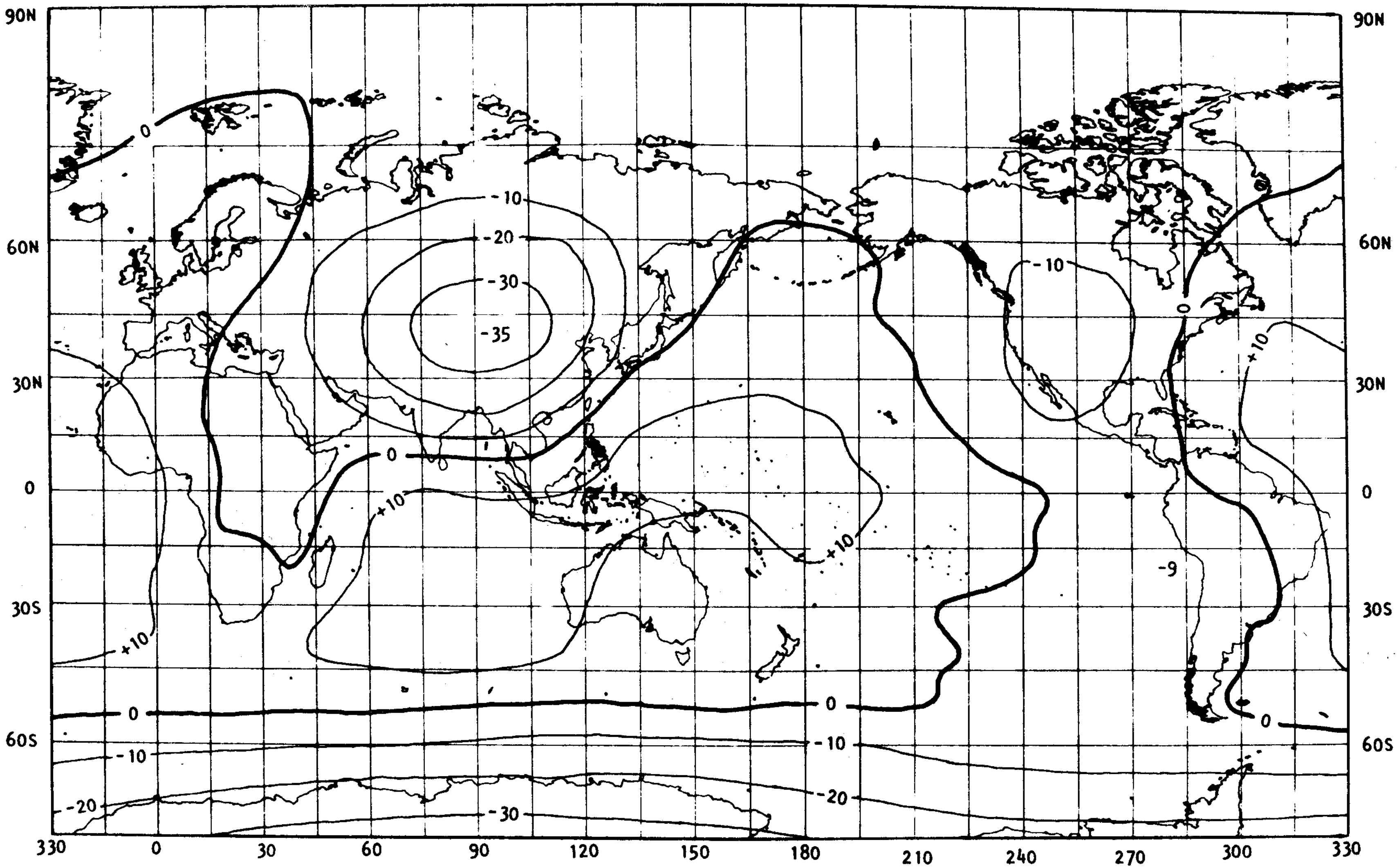


F I G U R E 4.2

Stokesian Contribution of the Atmosphere to the Height Anomaly
- Boundary Value Problem Approach

Contour Interval - 10 cm

Source: ANDERSON ET AL (1975) (Based on a Spherical Harmonic Analysis of Data on a 30° Grid to (6,6))



F I G U R E 4.3

Stokesian Contribution of the Atmosphere to the Height Anomaly
- Approach Implied by GRS67 Contour Interval - 10 cm

Source: ANDERSON ET AL (1975) (Based on a Spherical Harmonic Analysis of Data on a 5° Grid to (6,6))

5

THE ROLE OF THE GRAVITY FIELD IN SEA SURFACE TOPOGRAPHY STUDIES

It was noted in sub-section 2.2.4 that the definition of a marine geoid with ±10cm resolution is a necessary prerequisite for SST studies. Procedures and a set of relations for the determination of a high precision marine geoid are presented in section 5.1. The role that gravity field information such as satellite altimetry, surface gravity and satellite-determined geopotential models can play in the determination of SST is discussed in sections 5.2 and 5.3. Techniques for the evaluation of both the stationary and time-varying constituents of the SST in the open oceans are described in section 5.2. The determination of coastal SST at tide gauge sites (i.e. the height of the MSL surface above a unique geoid) connected to continental levelling networks is discussed in section 5.3.

5.1 ON DETERMINING THE MARINE GEOID TO ±10CM ACCURACY

5.1.1 PREAMBLE

Information on the shape of the boundary surface has conventionally come from measurements made on or in relation to it, and involve the combination of the results of geodetic levelling (to determine orthometric or normal height) and gravimetric computations (for the height anomaly or geoid height). However, it is no longer essential to follow such a procedure as the geometry of the earth's surface can be established to sub-metre precision by modern techniques such as:

(1) Three dimensional position determinations from observations to satellites and other extra-terrestrial objects.

(2) Satellite altimetry in the open oceans.

The only reason for carrying out the burdensome task implicit in very accurate determinations of the geoid appears to be in support of SST studies (see sub-section 2.2.4). Furthermore, such determinations are restricted to oceanic areas.

The marine geoid needs to be determined in four dimensions in the following context:

(i) A conceptual definition must precede determinations seeking a precision better than ±30cm (see sub-section 2.2.5).

(ii) The time dependent geometry of this datum level surface needs to be defined over geodetic time scales ($o\{10^2\}$ years) (see section 2.4).

(iii) Data is collected at or in relation to the instantaneous sea surface or MSL, which deviates by up to ±2m from the datum level surface.

(iv) Short period variations in the SST can be almost as large as the stationary component (see sub-section 3.3.2).

5.1.2 THE SHAPE OF THE MARINE GEOID

There are three basic equations relating geoid height N and geodetic quantities to the SST.

The first is an integral based on convolutions using an operator of Stokes' type. A combination of equations 2.62, 4.64 and 4.65 gives:

$$N = \frac{1}{\gamma}\left((W_o - U_o) - \bar{R}M\{\overline{\Delta g}_c\} + 2M\{\delta W\}\right) + \frac{\bar{R}}{4\pi\gamma}\iint f(\psi)\overline{\Delta g}_c d\sigma - \frac{1}{2\pi\gamma}\iint f(\psi)\delta W d\sigma + \zeta_c \qquad (5.1)$$

where ζ_c is the non-Stokesian contribution. All other quantities having been defined in sections 4.1 and 4.2.

The second is (equation 2.62):

$$N = \zeta + \frac{\delta W}{\gamma} \qquad (5.2)$$

where ζ is the oceanic height anomaly deduced from satellite altimetry (sub-section 2.3.4).

The third is based on Brun's equation (equations 2.62 and 4.18):

$$N = \frac{1}{\gamma}\left(T' - (W_o - U_o) + W_A\right) \qquad (5.3)$$

where T' is the disturbing potential of the solid earth and oceans (sub-section 4.1.3), which can be obtained from an analysis of satellite tracking data (§ 2.3.3.1). Note that this expression does not contain any SST term.

The global set of values of δW in equations 5.1 and 5.2 are defined as follows (equation 4.19):

At SEA - the difference in geopotential between the geoid and the sea surface to or at which the altimeter range or gravity measurement is made (possibly reduced for some adopted ocean tide model and, where relevant, any other periodic effect on the sea surface that can be reliably modelled).

On LAND - the difference in geopotential between the MSL datum, to which the geopotential differences ΔW in equation 4.16 are referred for the computation of the gravity anomaly Δg (equations 5.88 and 5.89), and the geoid. Unless the levelling network has been distorted during adjustment (as e.g. the Australian Height Datum - § 6.2.2.3), the value of δW would hold its magnitude and sign over large continental areas.

All three equations are fundamentally equivalent and the choice of one over another is merely a matter of convenience and a function of the nature of the input data. Each relation is examined in more detail in the following sections.

5.1.2.1 *Geoid Determinations Using Surface Gravity Data*

The criteria to be satisfied by the gravity anomaly Δg_c in equation 5.1 can be summarised as follows (see § 2.3.5.4):

(1) Global gravity standardisation networks should have a station spacing of 10^3km in continental areas. The determination of gravity at these stations should have a precision of $\pm$ 0.1mgal with no significant correlation of errors in adjacent values.

(2) Geopotential heights of land gravity stations should be based on levelling data with correlated errors held to below $\pm$0.15kgalm, although individual station elevations can have uncorrelated errors an order of magnitude greater.

(3) The contribution of long wavelength systematic errors in gravity anomalies due to (1) and (2) should be kept to below $\pm 50\mu$gal.

(4) Gravity station spacing is to be such that the error of representation is $\pm$3mgal (equivalent to 10km spacing in non-mountainous areas).

(5) Geocentric coordinates should be used instead of regional geodetic values for the computation of normal gravity.

It is possible to envisage all land gravity data achieving the quality control specified above, e.g. the Australian Gravity Anomaly Data Bank has been upgraded by the removal of all sources of systematic error that could be reliably modelled (see Chapter 6).

However, a major problem with determinations of geoid height using equation 5.1 is the lack of complete global coverage of gravity anomalies. Detailed coverage is, in the main, restricted to land and continental shelf areas, as shown in Figure 5.1 (from RAPP 1977). The use of a "higher" reference model (section 4.4) of adequate precision obviates the need for a global integration of gravity anomalies. The following relation can be used:

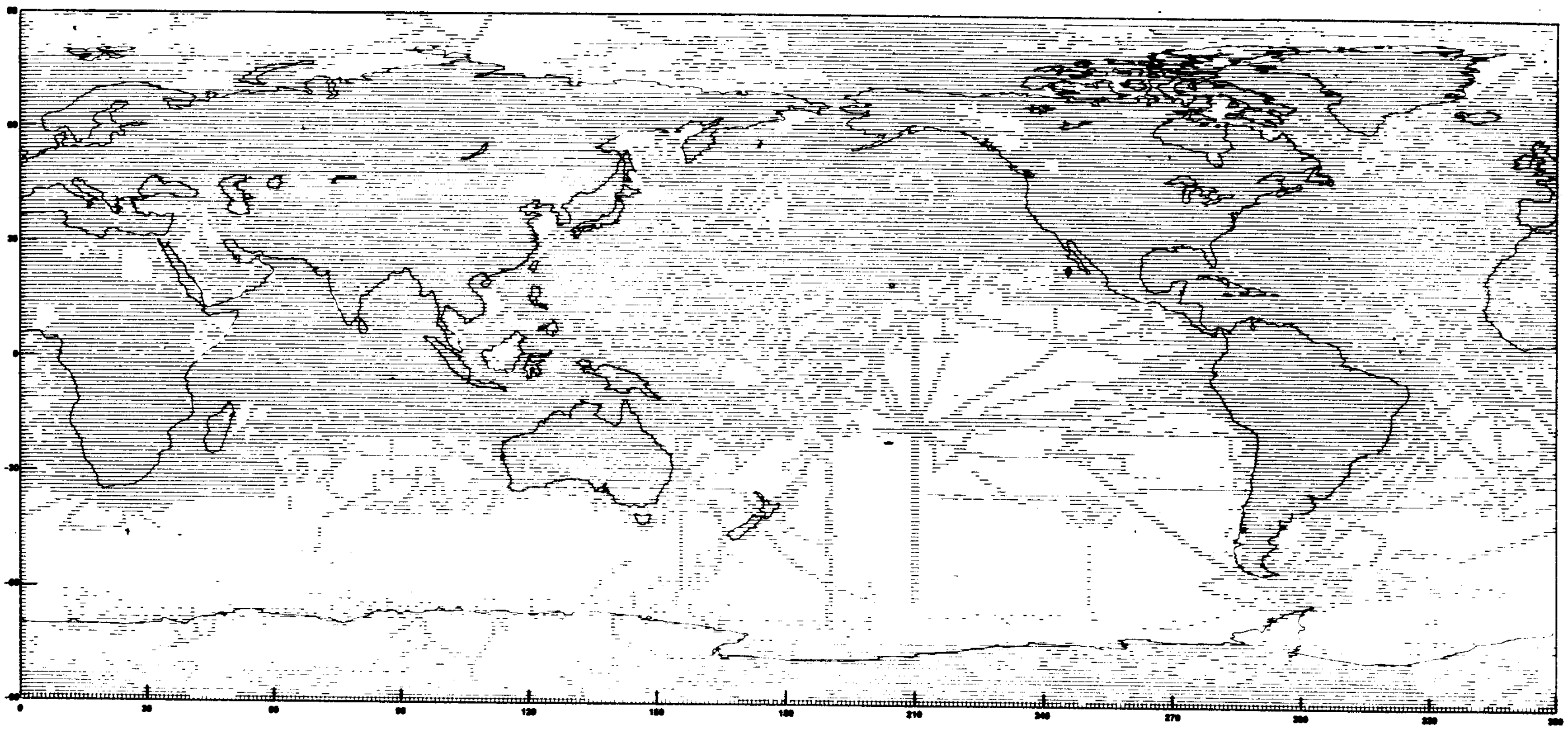
$$N = \frac{1}{\gamma}\left((W_o - U_o) - \bar{R}M\{\overline{\Delta g}_c\} + 2M\{\delta W\}\right) + \frac{\bar{R}}{4\pi\gamma}\int_0^{\psi_o}\int_0^{2\pi} f(\psi)\,\overline{\Delta g}_c \sin\psi \, d\alpha \, d\psi$$

$$- \frac{1}{2\pi\gamma}\iint f(\psi)\,\delta W d\sigma + \zeta_c \qquad (5.4)$$

Nevertheless, considerable degradation is to be expected due to the questionable quality of oceanic gravity anomalies. For example, it only requires, as a common occurrence, a linear drift of 0.1mgal to be

FIGURE 5.1

DISTRIBUTION OF 38,406 1° x 1° MEAN FREE AIR GRAVITY ANOMALIES



Source: RAPP (1977)

systematic over 10^3km to make the data inadequate for high precision geoid determinations (MATHER ET AL 1976a, p.42). In addition, a complete oceanic coverage of gravity measurements appears to be a long range goal.

As gravity anomaly data has to be defined at sea for equations 5.1 or 5.4 it is worthwhile investigating techniques for strengthening the gravity field representation in these areas. Such techniques should use the available gravity data, satellite altimetry and geopotential information, and recognise that systematic biases may exist in the various data types. A number of techniques using satellite altimetry data have been proposed, e.g. some are based on the principles of least squares collocation (e.g. RAPP 1974b, RAPP 1976a, RAPP 1978a, etc.) and claim an accuracy of the order of $\pm$6mgal for a $1^o \times 1^o$ mean gravity anomaly value, although the systematic errors in the prediction process are unclear.

Another method is based on the inverse of Stokes' integral (KOCH 1970, MATHER 1974d). The relation between the gravity anomaly $\overline{\Delta g}_c$ and the altimeter-derived height anomaly ζ is (equation 4.80):

$$\overline{\Delta g}_{cp} = \frac{\gamma}{4\pi R}\iint M_1(\psi)\left(\overline{N}_c - \overline{N}_{cp}\right)d\sigma + \frac{1}{4\pi R}\iint M_1(\psi)\left(\delta W - \delta W_p\right)d\sigma$$

$$- \left(\gamma\overline{N}_{cp} - (W_o - U_o) - \delta W_p\right)/R \qquad (5.5)$$

where the quantity $\overline{N}_c$ is defined by (equation 4.78):

$$\overline{N}_c = \zeta - \frac{1}{\gamma}(W_A - \delta T') \qquad (5.6)$$

All other quantities having been defined in section 4.3.

The $M_1(\psi)$ function rapidly decreases with increase in ψ (COLEMAN & MATHER 1976, p.24) and an integration out to a radius of 5^o (=500km) is adequate for most prediction purposes. However, the nature of the $M_1(\psi)$ function is unsuited to computing a high frequency phenomenon like the gravity anomaly, because the smoothened signal $(\overline{N}_c - \overline{N}_{cp})$ has lost an estimated 30% of its high frequency characteristics in the region closest to the point of evaluation (IBID, Table 3) due to the finite altimeter footprint - 10 to 15km in diameter.

The predicted quantity $\tilde{\Delta g}_p$ can be obtained from equation 5.5:

$$\tilde{\Delta g}_p = \frac{\gamma}{4\pi R}\iint M_1(\psi)\left(\overline{N}_c - \overline{N}_{cp}\right)d\sigma - \frac{\gamma}{R}\overline{N}_{cp} \qquad (5.7)$$

$\tilde{\Delta g}_p$ is related to the desired quantity $\overline{\Delta g}_{cp}$ by:

$$\tilde{\Delta g}_p = \overline{\Delta g}_{cp} - \frac{1}{4\pi R}\iint M_1(\psi)\left(\delta W - \delta W_p\right)d\sigma - \frac{1}{R}\left((W_o - U_o) - \delta W_p\right) \qquad (5.8)$$

the discrepancy being mainly of long wavelength.

The accuracy with which this smoothened gravity anomaly $\tilde{\Delta g}_p$ can be determined is dependent on the accuracy of the orbit determination from which values of ζ (and hence $\bar{N}_c$) are derived. Any long wavelength errors in $\Delta\gamma_p$ will produce biased results in evaluations of equation 5.4.

The effect on the quadrature evaluation of equation 5.7 of an error $e_\zeta^{(m)}$ in $(\bar{N}_c - \bar{N}_{cp})$ which have systematic error characteristics over an $n^o \times m^o$ area but behave as an accidental error over greater extents is given by (MATHER 1974d, p.103):

$$e_{\Delta g}^{(mgal)} \ngtr o\{0.2(n^o \times m^o)^{\frac{1}{2}} e_\zeta^{(m)}\} \qquad (5.9)$$

Consequently, data based on $1^o \times 1^o$ area means may have errors as large as ± 10m in $\bar{N}_c$ without introducing errors in excess of ± 2mgal, provided the errors e_ζ representing contiguous squares are not correlated. In addition, an error of ± 6m in $\bar{N}_{cp}$, through the last term in equation 5.7, results in an error of ± 1mgal in the predicted quantity. Such uncertainties can be tolerated if the systematic component of the error is less than $\pm 50\mu$gal, i.e. values of $\bar{N}_c$ at contiguous points of evaluation Pi and Pi+1 should have uncorrelated errors below the ± 30cm level.

This is unlikely to be true in the case of satellite altimetry, where sources of error with long wavelength include:

(i) errors in tracking station coordinates.

(ii) errors in the long wavelength components of the gravity field model (if continuous tracking of the altimeter-equipped spacecraft is not available).

The use of mean values for station coordinates as obtained from a number of satellite solutions using different configurations of both spacecraft and tracking stations should give, in principle, a set of coordinates of each station whose errors can be treated as uncorrelated. Reliance on the geopotential model can only be eliminated in the eventuality of continuous tracking using 24 hour all-weather 10cm systems. Systematic errors in satellite altimetry are discussed further in § 5.1.2.2.

It appears that GEOS-3 altimetry data contains errors of the order of ± 1-2m with wavelengths greater than 10^3km, which would result in unacceptably high levels of systematic error in the values of $\widetilde{\Delta g}$ obtained using equation 5.7. Further, the quantity $\widetilde{\Delta g}$ deviates from the required gravity anomaly $\bar{\Delta g}_c$ by up to ± 0.3mgal with long wavelength ($> 10^3$km) due to the existence of the SST term $\delta W/\bar{R}$ (equation 5.8). Therefore, the accuracy of gravimetric determinations of geoid height will degrade by an amount which is a function of the percentage of predicted gravity anomalies in the cap integration of equation 5.4.

The non-Stokesian term ζ_c is given by (equation 4.65):

$$\zeta_c = \frac{1}{\gamma}\sum_{\alpha=1}^{2}\left[W_A + \frac{1}{2\pi}\iint \frac{R^2}{r}\left[\frac{\partial T'}{\partial x_\alpha}\tan\beta_\alpha + T'\left(\frac{x_\alpha \tan\beta_\alpha}{r^2} + \frac{3}{2}\left(c_\Delta + \frac{3dR}{R}\right) - \right.\right.\right.$$

$$\left.\left.\left.(x_3/r^2 - 1/2R)\right) - \delta\Delta g' + \Delta g'\left(c_\Delta + \frac{3dR}{2R}\right)\right] d\sigma\right] \qquad (5.10)$$

all terms are defined in sub-section 4.2.1

For the computation of ζ_c, the disturbing potential T' can be estimated from satellite altimetry using equations 4.74 and 4.78. This estimate of T' ignores the unknown terms $(W_o - U_o)$ and δW, however estimates of $\partial T'/\partial x_\alpha$

will be largely unaffected as $(W_o - U_o)$ has no effect on the gradient, while δW is constant for considerable continental extents. In oceanic areas, where δW is more variable, the effect is scaled by the (unknown) $\tan\beta_\alpha$ term which is estimated to be of the order of 10^{-4} or less (MATHER 1975a, Section 2) and is thus negligible. The only effect of possible consequence is through the term $(x_3/r^2 - 1/2R)$ in the expression multiplying T' where some distorting effects at the ± 30cm level may occur. These will only be of consequence where the great mountain ranges occur (less than 3% of the earth's surface area). The effect on quadrature evaluations in oceanic regions is probably less than ± 5cm. It is reasonable to conclude therefore that the computation of ζ_c is unlikely to introduce errors in excess of ±5cm in the final result if the unknown terms $(W_o - U_o)$ and δW are neglected (IBID 1975a).

However the major drawback to using equation 5.4 for the computation of geoid height N to ±10cm accuracy is due to the terms containing the unknown SST (δW). Assuming that the dominant long wavelength stationary SST is known (e.g. see Figure 3.3) and capable of being modelled (see sub-section 7.1.4), then N can be determined by the use of an iterative procedure described in MATHER (1973a, Section 4.2).

It is far more realistic to admit that all gravity data is related to the ocean surface whose departures from the geoid are *not known*. Consequently it is not possible to solve the GBVP (section 4.2) to a precision of ±10cm due to the existence of the unknown quantities δW with global distribution in equation 5.1 or 5.4. The effect of the SST on solutions of the GBVP is discussed in § 2.3.5.5 and 4.2.3.

In summary, determinations of the marine geoid using gravimetric techniques are faced with the following difficulties:

(1) Lack of a global coverage of gravity anomaly data. Although a global integration of gravity anomalies is not necessary if the solution procedure incorporates information from a satellite-determined gravity field model, a limited coverage of gravity anomaly data implies an incomplete gravimetric marine geoid.

(2) Gravity anomalies predicted from satellite altimetry data will contain unacceptable levels of systematic error if radial orbital errors of long wavelength have magnitudes in excess of a few decimetres.

(3) Errors in quadrature evaluations of the geoid height due to the existence of the (unknown) SST are of the order of ±15-60cm (see § 2.3.5.5), although such errors are expected to decrease with the incorporation of information on the long wavelength features of the geoid through the "higher" reference model (a form of "combination" solution - see § 2.3.6.1).

The last factor is the most serious problem in determinations of the shape of the marine geoid to a precision adequate for SST studies.

5.1.2.2 *Geoid Determinations Using Satellite Altimetry Data*

While the desired output of altimetry data is the height anomaly ζ, the initial quantity obtained is the instantaneous position $X_{ij}(t)$ of the point P_j on the ocean surface at time t in relation to a three dimensional Cartesian coordinate system X_i centred at the geocentre (see section 3.1). The positions $X_{ij}(t)$ are obtained from:

(i) a set of observations made by a network of tracking stations, which enable the satellite position $X_{is}(t)$ to be determined; and

(ii) altimeter range measurements $h(t)$ along the local vertical from the satellite to the sea surface. For GEOS-3 this is of the order of 800km.

The relationship between the satellite position, the altimeter range and the three dimensional coordinates of the sea surface is (see Figure 2.3) (e.g. MATHER 1978c, p.291 et seq.):

$$x_{ij}(t) = x_{is}(t) - h(t)\ell_{ij} + o\{10^{-8}h\} \qquad (5.11)$$

where the direction cosines ℓ_{ij} of the vertical at P_j are modelled by those of the ellipsoidal normal to P_j, whose geodetic coordinates are (ϕ_{gj}, λ_j):

$$\ell_{1j} = \cos\phi_{gj}\cos\lambda_j \; ; \; \ell_{2j} = \cos\phi_{gj}\sin\lambda_j \; ; \; \ell_{3j} = \sin\phi_{gj} \qquad (5.12)$$

The height anomaly $\zeta_j(t)$ in relation to a reference ellipsoid (equatorial radius a, flattening f) is given by (equation 3.3):

$$\zeta_j(t) = \frac{x_{1j}(t)}{\ell_{1j}} - \nu = \frac{x_{2j}(t)}{\ell_{2j}} - \nu = \frac{x_{3j}(t)}{\ell_{3j}} - \nu(1-f)^2 \qquad (5.13)$$

where the radius of curvature in the meridional plane is (CAPPELLARI ET AL 1976, p.3-42):

$$\nu = a\left(1 - (2f-f^2)\sin^2\phi_{gj}\right)^{-\frac{1}{2}} \qquad (5.14)$$

A purely geometrical definition of the altimeter's position $x_{is}(t)$ can be obtained if there is continuous tracking coverage. In theory, this would require approximately 125 well distributed all-weather tracking systems if the minimum sighting elevation angle is 25^{o} (MATHER 1974b, p.103). Such a density of high precision laser tracking stations is highly improbable for the foreseeable future. Consequently, the maintenance of accurate orbital position will depend upon the quality of the gravity field model used in the orbit integration procedure. The model of the geopotential widely used at present is a spherical harmonic representation based on a set of coefficients $C_{\alpha nm}$ (see § 2.3.3.1).

The coefficients $C_{\alpha nm}$ are obtained from the analysis of satellite tracking data as described in e.g. MUELLER (1964). Only harmonics up to some limiting degree n' are adequately determined in this way, e.g. n' for GEM9 (LERCH ET AL 1977) is around 20, with some resonance coefficients obtained to degree 30. It is reasonable to expect that features in the geopotential at satellite altitudes that have extents greater than approximately $2 \times 10^4 (n')^{-1}$km, i.e. 10^3km for GEM9, and amplitudes in excess of the noise level of the tracking can be resolved in such gravity field models. However, at the present time the precision of gravity field models is about 1-2 orders of magnitude inferior to that required for determining $x_{is}(t)$ to ±10cm, i.e. there are uncertainties at the 1-10% level in the representation of the disturbing potential deduced from such a model (the higher uncertainty being associated with the southern oceans of the world).

Neglecting error sources within the radar altimeter hardware, the definition of the sea surface position from altimetry and present day gravity field models/tracking coverage is influenced by orbital errors which have the following characteristics (MATHER 1978c, p.293):

(1) Errors in orbit determination are reflected in the values of X_{ij}. Such uncertainties are due primarily to errors in the gravity field model used and to errors in the tracking station coordinates held fixed in the orbit integration procedure. The radial component of such orbital error is of particular importance as it is directly related to the derived height anomaly (equations 5.11 and 5.13). The horizontal component of position can be subject to errors of up to two orders of magnitude larger without affecting SST studies.

(2) The orbit determination errors are likely to be related to the distribution of tracking stations at the surface of the earth. The present generation of ±10cm laser tracking systems operate almost exclusively in the USA and its environs (see Figure 7.2). Consequently, gravity field models are weakest in the southern oceans where high precision tracking systems that can provide data for gravity field model improvement are almost nonexistent (see RAPP 1975, Figures 1 & 2 for a comparison of independently determined gravity field models in the southern oceans).

(3) The errors in the radial component of the sea surface position are expected to be a function of the distance from the nearest tracking station. Such a pattern has already been noticed during the analysis of GEOS-3 altimetry data off the east coast of Australia (MATHER ET AL 1977a, Figure 8). This is more likely to be the case for altimeter orbits integrated using a purely satellite-determined gravity field model (such as an odd numbered GEM model - see § 2.3.3.1), than a model incorporating surface gravity and perhaps satellite altimetry data (such as the even numbered GEM models - § 2.3.3.1).

(4) The error in the resulting height anomaly ζ is therefore most probably correlated with position. For a discussion see MATHER ET AL (1976a, p.36 et seq.).

The role that altimetry data on its own can play in the determination of the geoid was discussed in sub-section 2.3.4. If it is assumed that the above mentioned errors could be minimised, and hence the sea surface height ζ could be determined to a precision of ± 10cm, the use of equation 5.2 to obtain the geoid height N is only possible *if the SST is known*. The relation is given by (equation 4.19 and 5.2):

$$N = \zeta - \zeta_s \qquad (5.15)$$

where, in this context, ζ_s is the dynamic sea surface topography (the word "dynamic" is used to indicate that ζ_s is the departure of the sea surface from that particular level surface of the earth's gravity field that is labelled the geoid and not some arbitrary level surface).

Therefore, as is the case with surface gravimetry, the use of satellite altimetry data for the determination of a high precision marine geoid will not be successful unless assumptions are made concerning the nature of the SST. Note that the errors arising from the gravimetric method described in § 5.1.2.1 due to the unknown SST are approximately one quarter those implied from the method based on satellite altimetry. The role that satellite altimetry data can play in gravity field model improvement is discussed in sub-section 5.2.4.

As the main *justification* for the computation of a high precision marine geoid is for the determination of the SST it is not considered desirable to assume characteristics for the global distribution of ζ_s (e.g. as depicted in Figure 3.3) in processing the data (ζ) prior to obtaining a geoid solution. The problem can be most succinctly stated as : it is required, in concept, to geometrically map the geoid in oceanic areas under circumstances where no measurements (Δg or ζ) have been directly made in relation to it.

5.1.2.3 *Geoid Determinations Using Information on the Geopotential Obtained from the Analysis of Satellite Tracking Data*

The only source of data regarding the earth's gravity field (and hence the geoid) which is independent of any relationship to the geometry of the ocean surface is satellite tracking data from which gravity field models are computed. Assuming the logistic limitations described in § 2.3.3.1 were overcome, the appropriate analysis of tracking data should define a spherical harmonic representation of the geopotential to some degree n' with a precision equivalent to that of the tracking data (hopefully ±0.1kgalm) for all the wavelengths in the model.

Such a model, if appropriately downward continued through the earth's atmosphere to the ocean surface, can be expected to define those features of the geoid with wavelength greater than ℓ (=o{10^3km} for n' = 20) and amplitudes in excess of the noise level of the tracking, with an equivalent precision.

The satellite-determined model of the geopotential, in terms of a spherical harmonic series, is expressed in the form (equations 2.28 & 2.30):

$$W = \frac{GM}{R}\sum_{n=0}^{n'}\left(\frac{a}{R}\right)^n \sum_{m=0}^{n}\sum_{\alpha=1}^{2} C'_{\alpha nm} S_{\alpha nm} + W_R \qquad (5.16)$$

where $C'_{\alpha nm}$ are the unnormalised estimates of the "true" harmonic coefficients $C_{\alpha nm}$ and W_R is the rotational potential. The functions $S_{\alpha nm}$ are defined by (equation 2.29):

$$S_{1nm} = P_{nm}(\sin\phi)\cos m\lambda \quad ; \quad S_{2nm} = P_{nm}(\sin\phi)\sin m\lambda \qquad (5.17)$$

where $P_{nm}(\sin\phi)$ is the associated Legendre function of degree n and order m (HEISKANEN & MORITZ 1967, Section 1.11).

W cannot be downward continued to the ocean surface due to the presence of the atmosphere. The atmospheric potential V_s at satellite altitudes can be represented by a truncated spherical harmonic series to degree n'' by (MATHER ET AL 1978c, p.16):

$$V_s = \frac{GM}{R}\sum_{n=0}^{n''}\left(\frac{R_a}{R}\right)^n \sum_{m=0}^{n}\sum_{\alpha=1}^{2} V_{s\alpha nm} S_{\alpha nm} \qquad (5.18)$$

where $V_{s\alpha nm}$ are harmonic coefficients of degree n and order m, and R_a is the radius of the minimum geocentric sphere enclosing the earth's atmosphere (say, R_a = 6420km).

Downward continuation is only possible when considering the potential W' of the solid earth and oceans, defined by:

$$W' = W - V_s$$
$$= \frac{GM}{R} \sum_{n=0}^{n'} \left(\frac{a}{R}\right)^n \sum_{m=0}^{n} \sum_{\alpha=1}^{2} C''_{\alpha nm} S_{\alpha nm} + W_R \qquad (5.19)$$

where

$$C''_{\alpha nm} = C'_{\alpha nm} - \left(\frac{R_a}{R}\right)^n V_{s\alpha nm} \qquad (5.20)$$

The potential W' will contain non-zero first degree harmonics if the origin of coordinates (ϕ,λ,R) remains at the geocentre. However, it was shown in § 4.5.2.2 that enforcing the condition

$$C''_{110} = C''_{111} = C''_{211} = 0 \qquad (5.21)$$

will introduce errors less than ±5cm due to the non-coincidence of the geocentre and the centre of mass of the solid earth and oceans.

The zero degree coefficient V_{s100} is approximately +4.8kGalm and represents the effect of the difference in mass between the solid earth and oceans and the total earth mass at satellite altitudes (ANDERSON 1976, Chapter 9).

The spheropotential U of a gravitating equipotential ellipsoid in the space exterior to its surface is given by (MATHER 1971, p.85):

$$U = \frac{GM}{R}\left(1 + \left(\frac{a}{R}\right)^2 P_{20}(\sin\phi)\tilde{C}_{120} + \left(\frac{a}{R}\right)^4 P_{40}(\sin\phi)\tilde{C}_{140} + \left(\frac{a}{R}\right)^6 P_{60}(\sin\phi)\tilde{C}_{160}\right) + o\{0.01\text{kgalm}\} + U_R \qquad (5.22)$$

where the even degree zonal coefficients $\tilde{C}_{1(2p)0}$ are related to the parameters of the reference ellipsoid (a,f,GM,ω) by (MATHER 1978e, p.31):

$$\tilde{C}_{1(2p)0} = \frac{(-1)^p \sin^{2p}\alpha}{2p+1}\left(1 + \frac{m' \sin\alpha\, 2p}{3q_2(\alpha)(2p+3)}\right) \qquad (5.23)$$

where p is a positive integer and

$$m' = \frac{a^3\omega^2}{GM} \qquad (5.24)$$

In this context, the only significant even degree zonal harmonic coefficients are those of the second, fourth and sixth degree.

The α parameter is defined by:

$$\sin\alpha = (2f-f^2)^{\frac{1}{2}} \tag{5.25}$$

and

$$\cos\alpha = 1 - f \tag{5.26}$$

$q_2(\alpha)$ is defined by (MATHER 1971, p.83):

$$q_2(\alpha) = \frac{1}{2}\left(\alpha(3\cot^2\alpha + 1) - 3\cot\alpha\right) \tag{5.27}$$

In order to ensure that the flattening f of the reference ellipsoid is equivalent to that implied by the second degree zonal harmonic C'_{120} of the geopotential W, the following condition is enforced:

$$\tilde{C}_{120} = C'_{120} \tag{5.28}$$

In such a case, f is <u>derived</u> from the quantities GM, ω, a and C'_{120} (MATHER 1978e, equation A-28):

$$f = 1 - \left[1 - \left(\frac{2m\sin^3\alpha}{15q_2(\alpha)} - 3C'_{120}\right)\right]^{\frac{1}{2}} \tag{5.29}$$

This computation must be performed iteratively, with the procedure being rather unstable unless sufficient significant digits are carried in the computations (for details see IBID, Section 10.3).

This value of f, together with the other parameters of the reference ellipsoid, is used in equation 5.23 to evaluate the coefficients $\tilde{C}_{120}$ (= C'_{120}), $\tilde{C}_{140}$ and $\tilde{C}_{160}$.
The disturbing potential T' of the solid earth and oceans (under the assumption that $W_R = U_R$) is given by (equation 4.9):

$$\begin{aligned} T' &= W' - U \\ &= \frac{GM}{R}\sum_{n=0}^{n'}\left(\frac{a}{R}\right)^n \sum_{m=0}^{n}\sum_{\alpha=1}^{2}\overline{C}_{\alpha nm}S_{\alpha nm} \quad , \quad n\neq 1 \end{aligned} \tag{5.30}$$

where

$$\overline{C}_{\alpha nm} = C''_{\alpha nm} \tag{5.31}$$

except when $\alpha = 1$, m = 0 and n = 2, 4 and 6.

In these three special cases C''_{120}, C''_{140} and C''_{160} are corrected for the flattening of the reference ellipsoid:

$$\bar{C}_{120} = C''_{120} - \mathcal{C}_{120} = V_{s120}$$
$$\bar{C}_{140} = C''_{140} - \mathcal{C}_{140} \qquad (5.32)$$
$$\bar{C}_{160} = C''_{160} - \mathcal{C}_{160}$$

Equation 5.30 holds in the space exterior and down to the earth's surface. A second expression that is valid at the surface of measurement is obtained from a combination of equations 2.62 and 4.17:

$$T' = (W_o - U_o) + \gamma N - W_A \qquad (5.33)$$

where N is the geoid height, W_o the geopotential of the geoid, U_o the spheropotential of the reference ellipsoid and W_A is the potential of the atmosphere at the earth's surface, represented by a surface spherical harmonic series of the form (equation 4.109):

$$W_A = \sum_{n=0}^{n''} \sum_{m=0}^{n} \sum_{\alpha=1}^{2} V_{g\alpha nm} S_{\alpha nm} \qquad (5.34)$$

The zero degree coefficient V_{g100} is approximately +5.5kgalm (see § 4.5.2.1).

The atmospheric potential at satellite altitudes (V_s) and at the earth's surface (W_A) have been evaluated by ANDERSON (1976, Chapter 9). Values of the coefficients $V_{s\alpha nm}$ and $V_{g\alpha nm}$ to degree and order 5 are listed in IBID (Table 9.6).

The determination of a high precision geoid using a gravity field model defined by a finite set of coefficients $C'_{\alpha nm}$ was discussed in § 2.3.3.1. It was concluded that if the low degree harmonics of the gravity field are known to some degree n' with an error less than $10^{-8}W$ the disturbing potential T' at the ocean surface can be determined to a precision of ±0.1kgalm. Hence the geoid shape, for those wavelengths corresponding to such a low degree harmonic model $C'_{\alpha nm}$, can be established to this same precision by (equations 5.30, 5.33 and 5.34):

$$N = \frac{GM}{R_o \gamma} \sum_{\substack{n=0 \\ n\neq 1}}^{n'} \left(\frac{a}{R_o}\right)^n \sum_{m=0}^{n} \sum_{\alpha=1}^{2} \left(\bar{C}_{\alpha nm} + \frac{R_o}{GM} V_{g\alpha nm}\right) S_{\alpha nm} - (W_o - U_o) , \qquad (5.35)$$

where R_o is the geocentric distance to the ocean surface (see equations 5.42 & 5.43).

The term $(W_o - U_o)$ gives some information on the scale of the system (see § 5.1.3.1). The computation of W_o is described in § 5.1.3.2, while the quantity U_o is obtained from (MATHER 1971, p.83):

$$U_o = \frac{GM}{a} \frac{\alpha}{\sin\alpha} + \frac{1}{3} a^2 \omega^2 \qquad (5.36)$$

5.1.2.4 *Summary Remarks*

Neither surface gravity observations nor satellite altimetry data *on their* *own* can provide information on the geoid with a precision adequate for SST determinations, because all measurements are made either at or in relation to sea level and not the geoid. Satellite altimetry data cannot provide geoidal information (point values or area means) with a precision in excess of the magnitude of the SST, while techniques utilising gravity anomaly data promise a precision approximately a quarter of the magnitude of the SST (i.e. ± 15-60cm).

The incorporation of a gravity field model, determined under conditions of adequate tracking and with a precision of ±0.1kgalm for wavelengths greater than ℓ, in any solution procedure is vital for the definition of a high precision geoid.

It must be emphasised that the techniques discussed in this section refer to the geometric determination of a family of equipotential surfaces of the earth's gravity field, any of which may be selected as the geoid, in the manner described in sub-section 2.2.5. However, an adequate conceptual definition of the geoid at the ± 10cm level is essential for the unambiguous combination of gravity field models, satellite altimetry and surface gravity observations for the determination of SST.

The definition of a unique geoid is attempted in sub-section 5.1.3. The estimation of SST is discussed in section 5.2.

5.1.3 THE POTENTIAL OF THE GEOID

5.1.3.1 *Scale and the Potential of the Geoid*

The potential of the geoid W_o is one of the parameters defining the zero degree term in equations involving the disturbing potential, gravity anomalies and satellite altimetry (e.g. equations 5.1, 5.3 & 5.5). For example, the zero degree term for the gravimetric solution of the geoid is (equation 5.1):

$$N_o = \frac{1}{\gamma}\left((W_o - U_o) - \bar{R}M\{\overline{\Delta g_c}\} + 2M\{\delta W\}\right) + \zeta_{co} \qquad (5.37)$$

The interpretation of the zero degree term in solutions of the GBVP was given in sub-section 4.2.2.

The four components of N_o which contain information on the scale of the reference system are:

(1) ζ_{co} - the zero degree contribution from the non-Stokesian part of the gravimetric solution. This term can be assumed known from the global evaluation of equation 5.10 (the significance of the zero degree term in the atmospheric potential W_A has been discussed in § 4.5.2.1).

(2) $(W_o - U_o)$ - the difference in potential between the geoid and the reference ellipsoid. This quantity completely defines the zero degree term in geoid determinations from geopotential models (equation 5.35).

(3) $M\{\overline{\Delta g_c}\}$ - the global mean value of the "computable" gravity anomaly on the Brillouin sphere (equation 4.40).

(4) $M\{\delta W\}$ - the global mean value of the SST.

If it were assumed that U_o is equal to W_o, this would be tantamount to imposing a second scale constraint in physical geodesy. The first is the velocity of light c, implicit in the definition of the length standard (and hence in a and GM). The dilemma of a dual scale can be illustrated in the following manner.

Changes d(GM) in GM and da in a for a gravitating reference ellipsoid will produce changes dU_o in U_o according to the relation (MATHER 1978d, p.246):

$$dU_o = \frac{d(GM)}{GM} - \frac{GM}{a^2}da + o\{f dU_o\} \tag{5.38}$$

The quantity dU_o can be interpreted as the change in U_o necessary to make it equal to the unknown potential of the geoid W_o, and is defined by changes in either or both of the fundamental constants of gravitation (GM) and length (a).

If GM is known free of error (i.e. d(GM) = 0), then the enforcing of the condition that $W_o = U_o$ implies that the adopted ellipsoid is one of best fit to the geoid, i.e. da is the necessary correction to the semi-major axis of the reference ellipsoid to ensure that it encloses the same volume as the geoid. If this were not true (i.e. d(GM) ≠ 0), enforcing the condition $W_o = U_o$ results in a second scale constraint through the adopted value of GM.

It was shown in sub-section 2.4.2 that a basis does not exist for determining GM and a independently, for a best fitting model of the earth, from the analysis of near-earth satellite orbits (see e.g. MATHER 1973b for further discussion).

A value of a for the best fitting ellipsoid to a geoid based on the "oceanic" definition (see § 2.2.5.3) can be determined from satellite altimetry data by a procedure in section 7.3. However, both a and the resultant W_o (obtained by methods described in § 5.1.3.2) are dependent on the values of GM, ω and f adopted for the reference ellipsoid, and ultimately on the velocity of light c. This is the preferred procedure in dealing with the scale effects. That is, the term $(W_o - U_o)$ is treated as an unknown and its evaluation ensures that the value of GM, upon which it is based, is consistent with the accepted constant for the velocity of light. The relationship between GM, a and c is discussed further in section 7.3.

Upon adopting the "GBVP" definition (§ 2.2.5.5) for the geoid, the term $M\{\delta W\}$ is set to zero. Any other conceptual definition of the geoid based on oceanic and/or tide gauge data will not necessarily result in $M\{\delta W\} = 0$ in solutions of the GBVP. However, other definitions may have more utility in certain circumstances. The magnitude of the zero degree term in the SST is discussed further in § 5.1.3.2 and 5.3.2.

As mentioned in sub-section 4.2.2, the zero degree effect $(W_o - U_o)$ can be separated reliably from that of $M\{\overline{\Delta g_c}\}$ in solutions of the GBVP only if a global representation is available for the gravity anomaly as sampled at the earth's surface. The term $(W_o - U_o)$ can be evaluated if a geometrical relation between the geoid and reference ellipsoid is established *independently* of any BVP formulation, e.g. by using satellite altimetry (see § 5.1.3.2). The quantity $M\{\overline{\Delta g_c}\}$ can also be estimated by the technique described in HEISKANEN & MORITZ (1967, Section 2-20). This technique requires an estimation of the zero degree component of the Stokesian solution for the height anomaly ζ_{No} from the measurement of at least one terrestrial baseline and the astronomic latitude and longitude of its end points. The unknown quantity may then be estimated from (equation

4.66):

$$\bar{R}\, M\{\overline{\Delta g_c}\} = (W_o - U_o) - \gamma\, \zeta_{No} \quad (5.39)$$

and is dependent on the parameters of the reference ellipsoid implicit in the value of $(W_o - U_o)$.

However, if $M\{\overline{\Delta g_c}\}$ is known from an analysis of global gravity anomalies and ζ_{No} is known, the potential of the geoid W_o can be estimated from equation 5.39. Such a procedure would ensure that the reference system is consistent with the velocity of light implied in the distance measurements and normal gravity and, in addition, the value of W_o would be consistent with the "GBVP" definition for the geoid. Furthermore, the mass of the earth can be deduced from equation 4.68. However this procedure is essentially land based and requires a knowledge of the geoid shape along the baseline (IBID 1967). In this sense it is a "classical" method of providing scale in gravimetric geodesy, but is unsuited for high precision marine geoid studies.

A geoid definition that is favoured by oceanographers and one which geodesists can live with (despite the fact that $M\{\delta W\} \neq 0$ for solutions of the GBVP) is the "oceanic" definition (§ 2.2.5.3):

> "The geoid is that level surface of the earth's gravity field in relation to which the average non-tidal SST is zero as sampled globally in oceanic areas".

Such a definition has the major advantage that satellite altimetry data can be used in its implementation and it enables the quantity $(W_o - U_o)$, which completely defines the zero degree term in geoid solutions based on gravity field models (equation 5.35), to be evaluated. The determination of W_o for such a geoid is described in the following section.

5.1.3.2 *Practical Realisation of a High Precision Geoid Definition*

It was established in § 5.1.2.3 that the geopotential which can be downward continued to the surface of the earth is that due to the solid earth and oceans. At the surface of the oceans W_e can be represented by the following truncated spherical harmonic series (equation 5.19):

$$(W')_{ss} = \frac{GM}{R_o} \sum_{n=0}^{n'} \left(\frac{a}{R_o}\right)^n \sum_{m=0}^{n} \sum_{\alpha=1}^{2} C''_{\alpha nm} S_{\alpha nm} + (W_R)_{ss} \quad , \; n \neq 1 \quad (5.40)$$

where the coefficients $C''_{\alpha nm}$ are defined by equation 5.20 and the rotational potential $(W_R)_{ss}$ is given by:

$$(W_R)_{ss} = \frac{1}{2} R_o^2 \omega^2 \cos^2\phi \quad (5.41)$$

R_o is the geocentric distance to the sea surface (ϕ,λ) and is defined by:

$$R_o = R_e + \zeta \quad (5.42)$$

where the height anomaly ζ is obtained from satellite altimetry (see § 5.1.2.2) and R_e is the geocentric radius to the reference ellipsoid (equatorial radius a, flattening f) given by:

$$R_e = \frac{a(1-f)}{\left(1 - (2f-f^2)\cos^2\phi\right)^{\frac{1}{2}}} \tag{5.43}$$

where ϕ is the geocentric latitude. All other quantities have been defined in § 5.1.2.3.

The geopotential of the earth with atmosphere, at the ocean surface, is given by:

$$W_{ss} = (W')_{ss} + W_A \tag{5.44}$$

where the gravitational potential of the atmosphere at the earth's surface can be represented by a surface spherical harmonic model with coefficients $V_{g\alpha nm}$ (equation 5.34).

The long wavelength features of the geopotential at the ocean surface can therefore be represented by:

$$W_{ss} = \frac{GM}{R_o} \sum_{n=0}^{n'} \left(\frac{a}{R_o}\right)^n \sum_{m=0}^{n} \sum_{\alpha=1}^{2} \left[C'_{\alpha nm} + \delta C_{\alpha nm} \right] S_{\alpha nm} + \tfrac{1}{2} R_o^2 \omega^2 \cos^2\phi \ , \ n \neq 1 \tag{5.45}$$

where the spherical harmonic coefficients $C'_{\alpha nm}$ of the best available gravity field model (e.g. GEM9 - LERCH ET AL 1977) have been corrected ($\delta C_{\alpha nm}$) for the downward continuation through the atmosphere of the satellite sensed geopotential. $\delta C_{\alpha nm}$ are defined by (equations 5.20 and 5.34):

$$\delta C_{\alpha nm} = \frac{R_o}{GM} V_{g\alpha nm} - \left(\frac{R_a}{a}\right)^n V_{s\alpha nm} + o\{f\delta C_{\alpha nm}\} \tag{5.46}$$

Table 5.1 lists the corrections $\delta C_{\alpha nm}$ as obtained from estimates of $V_{g\alpha nm}$ and $V_{s\alpha nm}$ given by ANDERSON (1976, Table 9.6).

TABLE 5.1

The Differential Effect of the Atmosphere in Geopotential Computations from Satellite-Determined Potential Coefficients ($\delta C_{\alpha nm}$ in Equation 5.46)

$-\frac{GM}{R}\delta C_{\alpha nm}$ (in kgalcm)

ORDER / DEGREE	0	1		2		3		4		5		6		DEGREE VARIANCE (kGal cm)^2
	$\alpha=1$	$\alpha=1$	$\alpha=2$	$\alpha=1$	$\alpha=2$	$\alpha=1$	$\alpha=2$	$\alpha=1$	$\alpha=2$	$\alpha=1$	$\alpha=2$	$\alpha=1$	$\alpha=2$	
0	-0.4													0.19
1	+0.1	0.0	-0.2											0.05
2	+0.2	-0.0	-0.2	0.0	-0.0									0.05
3	+0.2	+0.0	+0.1	0.1	-0.5	+0.0	+0.1							0.26
4	-0.4	0.0	-0.0	-0.0	+0.0	0.1	0.2	0.0	0.0					0.21
5	+0.2	0.0	0.1	-0.1	-0.0	0.0	0.1	0.1	0.0	0.0	-0.1			0.08
6	+1.1	-0.0	0.0	0.1	-0.0	-0.0	-0.0	-0.0	0.0	-0.1	-0.1	0.0	-0.0	1.13

Source: MATHER ET AL (1978c)

Note that the effect of the correction to W_{SS} per harmonic coefficient never exceeds ±0.5kgalcm for degrees less than 6. This is due to the fact that the gravitational effect of the long wavelength components (> 2 x 10^3km) contribute approximately 98% of the strength of signal which varies by less than ±5kgalcm over the surface of the oceans (IBID, p.209-210). Consequently, the *differential effect* of the atmosphere $\delta C_{\alpha nm}$ is insignificant through low degree terms.

If R_o and $C'_{\alpha nm}$ are "free from error", it follows that over the oceans,

$$W_{ss} = W_o + \delta W + v_w \tag{5.47}$$

where δW is potential due to the SST, W_o is the geopotential of the geoid and v_w is the high frequency contribution to W_{SS} that is not modelled in equation 5.45. This high frequency component is estimated as ±4kgalm (MATHER ET AL 1978c, p.18).

It therefore follows, upon adopting the "oceanic" definition for the geoid (§ 2.2.5.3), that the global mean value of W_{SS}, as sampled on an equi-area basis in the oceans, is an estimate of the quantity W_o. That is, enforcing the condition that $M\{\delta W\} = 0$ for the oceans and assuming that $M\{v_w\}$ is negligible, the geoid is defined as:

$$W_o = M\{W_{ss}\} \tag{5.48}$$

It must be emphasised however, that the numerical value of W_o is dependent on the value adopted for GM (in equation 5.45) and the equatorial radius a (in equations 5.43 and 5.45), which are implicit in estimates of the height anomaly ζ. For GM = 398600.47 km ^{3} s^{-2} and a = 6378140m the value obtained for W_o is:

$$W_o = 6263682.76 \pm 0.4 \text{ kgalm} \tag{5.49}$$

The value of GM used in this computation is consistent with the adopted velocity of light c = 2.99792458 x 10^8 m s^{-1} (MORITZ 1979). For more details on how the estimate at 5.49 was obtained see § 8.2.3.2.

If the geocentric distance R_o to the continental levelling datums were available (e.g. from three dimensional position determinations) a basis exists, in theory, for the realisation of the "Geodetic" (§ 2.2.5.2) and "GBVP" (§ 2.2.5.5) definitions for the geoid.

On setting $M\{\delta W\} = 0$ and evaluating equation 5.48 from estimates of W_{ss} at regional levelling datums only (equation 5.45), the value of W_o obtained would be consistent with the "geodetic" definition. The global mean value of W_{ss} from both land (represented by the value of W_{ss} at the regional levelling datums) and oceanic areas provides an estimate of W_o consistent with the "GBVP" definition. Neither of these is likely to give a stable value for W_o due to the poor precision of R_o. The global mean value of estimates of W_{ss} confined to oceanic areas, where errors in the altimeter-derived quantity R_o are likely to exhibit random characteristics over the global oceans, is considered a good estimate of W_o. Nevertheless, scale ambiguities may still be present due to the values used for GM and a in the computations. This is discussed further in sub-section 8.2.6.

5.2 THE DETERMINATION OF SEA SURFACE TOPOGRAPHY USING GEODETIC TECHNIQUES

5.2.1 PREAMBLE

The geodetic principles underlying the determination of SST have been investigated in e.g. MATHER (1974d), MATHER (1975a), MATHER ET AL (1976a), etc, and are summarised in MATHER (1978c).

In § 5.1.2.4 it was noted that all high frequency information on the earth's gravity field (surface gravity measurements and satellite altimetry data) suffered from the drawback that the data was collected in relation to the sea surface and not the geoid. In contrast, low frequency information on the shape of the earth's level surfaces from satellite-determined geopotential models is not influenced by the existence of the SST. Therefore, the most favourable procedure for determining the SST ζ_s appears to the the following (MATHER ET AL 1976a).

The spectrum of the stationary SST in the oceans can be constituted as follows:

$$\zeta_s = \zeta_{s\ell} + \zeta_{ss} \tag{5.50}$$

where $\zeta_{s\ell}$ are the components of the SST with wavelengths longer than those in the earth's gravity field which perturb the altimeter-equipped satellite by amounts greater than the noise level of the tracking. This limiting wavelength is ℓ. In the absence of observations to low-flying geodetic satellites such as the proposed GRAVSAT (NASA 1972b), it is estimated that $\ell = o\{10^3 \text{km}\}$ for satellites having altitudes of the order of 800km and tracked by a global network of tracking stations. ζ_{ss} refers to all contributions to the SST with shorter wavelengths.

In addition, the stationary and time-varying components of the SST can be treated separately.

5.2.2 DETERMINATION OF THE LONG WAVELENGTH COMPONENTS OF THE SST

5.2.2.1 *Introduction*

The first stage in the determination of the stationary SST is the evaluation of $\zeta_{s\ell}$ from the available gravity, satellite altimetry and disturbing potential data. The determination of the long wavelength time-varying component is discussed in § 5.2.2.3.

The following two relations can be used:

<u>RELATION 1</u> (equations 4.19, 4.86 & 4.88)

$$T'' = \frac{GM}{R}\sum_{n=0}^{\infty}\left(\frac{a}{R}\right)^n \sum_{m=0}^{n}\sum_{\alpha=1}^{2} dC_{\alpha nm} S_{\alpha nm} \quad , \quad n\neq 1 \tag{5.51}$$

and

$$T'' = (W_o - U_o) - W_A + \gamma\zeta' + \gamma\zeta_s \tag{5.52}$$

where $dC_{\alpha nm}$ are spherical harmonic coefficients of the disturbing potential T'', defined by equation 4.89, ζ' is the height anomaly on the "higher" reference model (section 4.4) and ζ_s is the SST.

The first equality applies in the space at and exterior to the surface of measurement, while the second is valid at the surface itself. All coordinates (ϕ,λ,R) are geocentric spherical coordinates.

<u>RELATION 2</u> (equations 4.19, 4.90, 4.91 & 4.94)

$$\Delta g'' = -\left(\frac{\partial T''}{\partial h} + \frac{2T''}{R}\right) = \frac{GM}{R^2}\sum_{n=0}^{\infty}(n-1)\left(\frac{a}{R}\right)^n \sum_{m=0}^{n}\sum_{\alpha=1}^{2} dC_{\alpha nm} S_{\alpha nm} \quad , \quad n\neq 1 \tag{5.53}$$

and

$$\Delta g'' = \Delta g_c - \frac{2}{R}\left((W_o - U_o) - \gamma\zeta_s\right) \tag{5.54}$$

The first equality of 5.53 provides the definition of $\Delta g''$ (section 4.4), the second equality applies in the space exterior and at the surface of measurement, while the equality at 5.54 is valid at the latter.

RELATIONS 1 and 2 can be used to determine $\zeta_{s\ell}$ if the coefficients $dC_{\alpha nm}$ are known to the equivalent of o{±0.1kgalm} i.e. approximately an order of magnitude smaller than ζ_s. If $dC_{\alpha nm}$ are not sufficiently well known, RELATIONS 1 and 2 may be used in gravity model improvement, as described in sub-section 5.2.4.

The disturbing potential of the solid earth and oceans on the "higher" reference model can be represented in the following form:

$$T'' = T''_\ell + T''_s \tag{5.55}$$

where T''_ℓ refers to those contributions to T'' with wavelength greater than ℓ, corresponding to the highest degree n' in the spherical harmonic representation of the geopotential in equation 5.16. T''_s includes all terms of shorter wavelength.

In practice, the satellite-determined gravity field model $C'_{\alpha nm}$ is already incorporated into the "higher" reference model and hence reflected in the values of Δg_c (equation 4.94) and ζ' (equation 7.2). As a result:

$$dC_{\alpha nm} = \left(\frac{R_a}{a}\right)^n V_{s\alpha nm} + o\{\pm 0.1\text{kgalm}\} \text{ for } n \leq n' \tag{5.56}$$

Therefore T'' at the earth's surface is unknown but bounded with harmonics of wavelength less than ℓ, such that

$$T'' = T_s^* - W_A \tag{5.57}$$

where T_s^* is the (unknown) short wavelength component of the disturbing potential of the earth and atmosphere on the "higher" reference model. In addition:

$$\Delta g'' = \Delta g_s^* + \delta g_a \tag{5.58}$$

where Δg_s^* is the (unknown) high frequency component corresponding to the first equality in equation 5.53 and δg_a is the effect of the atmosphere on gravity (equation 4.99).

The long wavelength component of the stationary SST can be modelled by a low degree surface spherical harmonic series of the form:

$$\zeta_{s\ell} = \sum_{n=0}^{n'} \sum_{m=0}^{n} \sum_{\alpha=1}^{2} \zeta_{s\alpha nm} S_{\alpha nm} \tag{5.59}$$

where the maximum degree n' corresponds to the limiting wavelength ℓ.

5.2.2.2 *Practical Possibilities for the Determination of the Long Wavelength Components of the Stationary SST*

RELATIONS 1 and 2 can be rewritten as the following observation equations:

<u>OBSERVATION EQUATION 1</u> - for values of ζ' deduced from altimetry

$$\left(\frac{1}{\gamma}T_s^* + \zeta_{ss}\right) = v_\zeta = \zeta' - \sum_{n=0}^{n'} \sum_{m=0}^{n} \sum_{\alpha=1}^{2} \zeta_{s\alpha nm} S_{\alpha nm} + \frac{1}{\gamma}(W_o - U_o)$$

	$\left(\frac{1}{\gamma}T_s^* + \zeta_{ss}\right)$	ζ'	$\sum\sum\sum \zeta_{s\alpha nm} S_{\alpha nm}$	$\frac{1}{\gamma}(W_o - U_o)$
Order of magnitude (m)	±4	±4 known	±2 local value	±1 zero degree
Range of wavelengths (ω)	$\omega < \ell$	$0 < \omega < \infty$	$\omega > \ell$	$\omega = \infty$

(5.60)

If the stationary component of the SST is to be studied, the tidal signal should be removed from estimates of ζ' prior to its use in this equation. This can be done by averaging the values of ζ' over a sufficiently long period of time.

<u>OBSERVATION EQUATION 2</u> - for values of Δg_c in oceanic areas

$$\left(\Delta g_s^* + \zeta_{ss}\frac{\gamma}{R}\right) = v_{\Delta g} = \Delta g_d + \frac{2\gamma}{R}\sum_{n=0}^{n'}\sum_{m=0}^{n}\sum_{\alpha=1}^{2}\zeta_{s\alpha nm}S_{\alpha nm} - \frac{2}{R}(W_o - U_o)$$

Order of magnitude (mgal)	±10	±10 known	±½ local value	±¼ zero degree
Range of wavelengths (ω)	$\omega<\ell$	$0<\omega<\infty$	$\omega>\ell$	$\omega=\infty$

(5.61)

where

$$\Delta g_d = \Delta g_c - \delta g_a \qquad (5.62)$$

The values of ζ' and Δg_d used in these observation equations are in the form of area means, usually $1^o \times 1^o$ equi-area block means.

The value of $(W_o - U_o)$ can be considered a known quantity, its magnitude being dependent on the definition adopted for the geoid (see discussion in § 5.1.3.2). For example, consider the case where the possible definitions of the geoid are based only on the pattern of long wavelength stationary SST. The adoption of the "oceanic" definition (§ 2.2.5.3) requires that the spherical harmonic series (equation 5.59) only represent $\zeta_{s\ell}$ in the oceanic areas. The zero degree harmonic ζ_{s100} in such a case is forced to take the value zero. As a consequence, land areas (not considered in equations 5.60 and 5.61) would have to be represented by discrete values at each regional levelling datum. The global mean value of these heights would not be necessarily zero for such a geoid definition. However, the "geodetic" definition (§ 2.2.5.2) can be enforced by making this global mean value zero and adjusting the value of W_o (§ 5.1.3.2) accordingly.

If the sum of the global mean value of $\zeta_{s\ell}$ at levelling datums and the zero degree coefficient ζ_{s100} is forced to take the value zero, this would be equivalent to adopting the "oceanic/geodetic hybrid" definition for the geoid (§ 2.2.5.4).

If the harmonic representation for $\zeta_{s\ell}$ is adopted without differentiating between land and oceanic data input and the quantity ζ_{s100} is zero, this is tantamount to adopting the "GBVP" definition for the geoid (§ 2.2.5.5). The value of $\zeta_{s\ell}$ generated on land from such a model (equation 5.59) should be constant for locations on the same regional elevation datum, and equal to the height of the datum above the geoid.

At first glance, it would appear that the signal-to-noise problems preclude the recovery of SST from equations 5.60 and 5.61. However, the conditions for a favourable solution are based on the band-limited nature of the signal being recovered, as can be seen from the wavelength ranges listed. Chapter 7 presents preliminary estimates of the long wavelength components of the stationary SST from GEOS-3 altimetry data, using OBSERVATION EQUATION 1.

The resolution in the results is expected to improve with a global distribution of observations at sea. The conditions for a favourable solution can be improved further by using values of ζ' and Δg_d in the form of appropriately large area means equivalent to the Nyquist frequency implicit in the harmonic representation of $\zeta_{s\ell}$ to degree n'.

In practice however, confidence in the successful recovery of information on the stationary SST will only increase if the radial errors in orbit determination, and hence ζ', can be lowered (ideally to ±10cm). The use of OBSERVATION EQUATION 2 is not recommended for SST determinations due to the questionable quality of oceanic gravity data.

5.2.2.3 *Time Variations in the Long Wavelength SST*

Equation 5.60 can be used to study the time variations of $\zeta_{s\ell}$. The quantity $\Delta\zeta_{s\ell}$ can be defined by (MATHER 1978d):

$$\Delta\zeta_{s\ell} = \zeta_t + \delta\zeta_s \qquad (5.63)$$

where $\Delta\zeta_{s\ell}$ is the time variation in $\zeta_{s\ell}$, ζ_t is the tidal variation and $\delta\zeta_s$ is the remaining non-tidal variation in the long wavelength SST for the epoch of observation. The quantity $\Delta\zeta_{s\ell}$ contains effects of both known frequencies (e.g. seasonal variations, tides) and unknown frequencies.

The modification of equation 5.60 to incorporate the term $\Delta\zeta_{s\ell}$ would enable, in principle, the evaluation of coefficients for the adopted functional representations of ζ_t and $\delta\zeta_s$ (see e.g. IBID, p.231 et seq.), in addition to those of the stationary component $\zeta_{s\ell}$. However, the magnitude of the signal-to-noise ratio is critical in such determinations.

The tidal component ζ_t can be analysed for the five major tidal constituents (M_2, S_2, K_1, O_1 and P_1) by techniques described in MASTERS ET AL (1979).

The non-tidal time-varying constituents of the SST are expected to have frequencies ranging from a few days for transient phenomena (with middle to short wavelengths) to a year for seasonal variations, with amplitudes varying from a few centimetres to at least half a metre.

The time variation in ζ', between epochs $\tau = t_1$ and $\tau = t_2$, after removal of the tidal signal, can be obtained from:

$$\delta\zeta_{sa} = \zeta'(t_2) - \zeta'(t_1) \qquad (5.64)$$

If the non-tidal time variations in the sea surface height $\delta\zeta_{sa}$ is produced by mass transport due to ocean currents, the "true" variation in the SST $\delta\zeta_s$ is given by (MATHER 1978c, Section 7):

$$\delta\zeta_s = \delta\zeta_{sa} + \delta N \qquad (5.65)$$

where δN is the change in the height of the geoid as a consequence of mass redistributions in the ocean. Equation 5.65 relates changes in the long wavelength SST ($\delta\zeta_s$) to changes in both the spatial location of the sea surface ($\delta\zeta_{sa}$) and the geoid (δN). δN can be computed from:

$$\delta N = \frac{G\rho_w R}{2\gamma} \iint \operatorname{cosec}(\psi/2)\,\delta\zeta_{sa}\,d\sigma \qquad (5.66)$$

where ρ_w is the density of sea water and ψ is the angle between the geocentric radii to the element of volume $\delta\zeta_{sa}d\sigma$ and the point of computation.

However, it must be emphasised that the quantity $\delta\zeta_{sa}$ in equation 5.64 contains, in addition to the non-tidal variations in ζ', the following effects:

(i) Any time dependent measuring errors in the radar altimeter.

(ii) Orbital errors due to an inadequate gravity field model, insufficient tracking coverage or both.

(iii) Short wavelength time variations in the sea surface height.

The effect at (ii) is the most critical as it causes errors in the sea surface heights along an altimeter profile that are characterised by a bias and tilt to passes having lengths in excess of about 10^3km. Consequently, orbital errors will mask any time variations in ζ' which can also be modelled by a bias and a tilt.

All information in the time-varying spectrum of the SST with wavelengths greater than twice the length of pass will be lost unless the geocentric position of the altimeter-equipped satellite can be determined with an accuracy of ±10cm from a combination of high precision tracking and orbit integration with a gravity field model of equivalent precision. The recovery of information on the short wavelength time-varying component of the SST is more promising and is discussed in § 5.2.3.4.

5.2.3 DETERMINATION OF THE SHORT WAVELENGTH COMPONENTS OF THE SEA SURFACE TOPOGRAPHY

The determination of components of the stationary SST with wavelengths less than ℓ can only be attempted as a *second stage* once $\zeta_{s\ell}$ has been recovered. Given estimates of $\zeta_{s\ell}$, the high frequency information on the earth's gravity field (surface gravimetry and satellite altimetry) can be related to the geoid with a precision an order of magnitude better than the SST, i.e. ±10-20cm.

There are two expressions which relate gravity anomalies and height anomalies to the SST. They are:

- An integral equation based on an operator of the Stokesian type.
- An integral equation based on an operator which is the inverse to a Stokesian type.

The role that these two equations can play in the determination of the short wavelength component of the stationary SST ζ_{ss} is discussed in § 5.2.3.1, 5.2.3.2 and 5.2.3.3. The evaluation of the time-varying component is the subject of § 5.2.3.4.

5.2.3.1 ζ_{ss} *Based on Solutions of the Stokesian Type BVP*

The solution to the Stokesian type GBVP at a point P is given by (equations 5.1, 5.2 and 5.6):

$$\overline{N}_{cp} = \frac{1}{\gamma}\left((W_o - U_o) - \overline{R}M\{\overline{\Delta g}_c\}\right) - 2M\{\zeta_s\} + \zeta_{sp} + \frac{1}{2\pi}\iint f(\psi)\,\zeta_s\,d\sigma + \frac{\overline{R}}{4\pi\gamma}\iint f(\psi)\,\overline{\Delta g}_c\,d\sigma + \left(\zeta_{cp} - \frac{W_A}{\gamma} + \delta T''\right) \qquad (5.67)$$

where $\overline{N}_c$ is the computable part of $\overline{T}''$ for the "higher" reference system on the Brillouin sphere, defined by (from equation 4.78):

$$\overline{N}_c = \zeta' - \frac{1}{\gamma}\left(W_A - \delta T''\right) \tag{5.68}$$

For most practical purposes $\delta T''$, the change in the disturbing potential T'' between the earth's surface and the Brillouin sphere, is given by:

$$\delta T'' = dR\frac{\partial T''}{\partial R} + o\left\{\frac{(dR)^2}{2}\frac{\partial^2 T''}{\partial h^2}\right\} \tag{5.69}$$

where dR is the distance between the earth's surface and the Brillouin sphere measured along the ellipsoidal normal through P. All other quantities having been defined in sub-section 5.1.2.

The zero degree component of the SST can be separated into long and short wavelength contributions:

$$M\{\zeta_s\} = \zeta_{s100} + M\{\zeta_{ss}\} \tag{5.70}$$

By restricting the harmonic representation of $\zeta_{s\ell}$ to oceanic areas (see discussion in § 5.2.2.2), ζ_{s100} is the zero degree harmonic coefficient corresponding to the global mean value of the long wavelength stationary SST - tantamount to adopting the "oceanic" definition for the geoid (§ 2.2.5.3).

In addition, the use of the orthogonal properties of surface spherical harmonics (HEISKANEN & MORITZ 1967, Section 1-13) allows the following separation:

$$\frac{1}{2\pi}\iint f(\psi)\,\zeta_s\,d\sigma = \sum_{n=2}^{n'}\frac{2}{n-1}\sum_{m=0}^{n}\sum_{\alpha=1}^{2}\zeta_{s\alpha nm}S_{\alpha nm} + \frac{1}{2\pi}\iint f(\psi)\,\zeta_{ss}\,d\sigma \tag{5.71}$$

The SST at the computation point P (ϕ,λ) can be written in the form:

$$\zeta_{sp} = \zeta_{s100} + \sum_{n=2}^{n'}\sum_{m=0}^{n}\sum_{\alpha=1}^{2}\zeta_{s\alpha nm}S_{\alpha nm}(\phi,\lambda) + \zeta_{ssp} \tag{5.72}$$

Note that the first degree harmonic in $\zeta_{s\ell}$ has been removed (the significance of this is described in § 7.2.3.2).

If the non-Stokesian term is neglected, as it is of minor importance in oceanic areas, equation 5.67 can be rewritten in terms of:

(a) the computable quantities $\overline{\Delta g}_c$ and $\overline{N}_c$.

(b) quantities derived from the analysis of satellite altimetry, such as $(W_o - U_Q)$ (see § 5.1.3.2) and the long wavelength stationary SST (see § 5.2.2.2).

(c) the unknown ζ_{ss}.

The result is (combination of equations 5.67, 5.70, 5.71 and 5.72):

$$-2M\{\zeta_{ss}\} + \zeta_{ssp} + \frac{1}{2\pi}\iint f(\psi)\zeta_{ss}d\sigma = \overline{N}_{cp} - \frac{1}{\gamma}((W_o - U_o) - \overline{R}M\{\overline{\Delta g}_c\})$$

$$+ \zeta_{s100} - \frac{\overline{R}}{4\pi\gamma}\iint f(\psi)\overline{\Delta g}_c d\sigma - \sum_{n=2}^{n'}\frac{n+1}{n-1}\sum_{m=0}^{n}\sum_{\alpha=1}^{2}\zeta_{s\alpha nm}S_{\alpha nm} \qquad (5.73)$$

where the subscript p refers to the point of evaluation. The left hand side of equation 5.73 contains all the unknowns in the stationary SST, while the right hand side includes all the known quantities.

It is possible, in principle, to recover all features in ζ_{ss} from equation 5.73 with wavelengths greater than 20km using 0.1^o area mean values of $\overline{\Delta g}_c$ and $\overline{N}_c$. A suitable model for ζ_{ss} could be a two dimensional Fourier series of the form (MATHER 1978c, Section 7):

$$\zeta_{ss} = \sum_{n=0}^{n_{max}}\sum_{m=0}^{n_{max}}\sum_{\alpha=1}^{2} K_{\alpha nm}F_{\alpha nm} \qquad (5.74)$$

where $K_{\alpha nm}$ are coefficients of the Fourier functions $F_{\alpha nm}$ defined by:

$$F_{1nm} = \cos(n\Delta\phi + m\Delta\lambda) \quad ; \quad F_{2nm} = \sin(n\Delta\phi + m\Delta\lambda) \qquad (5.75)$$

where $\Delta\phi$ and $\Delta\lambda$ are differences in (ϕ,λ) at the point P in relation to some convenient point of reference with relevance in an area extending ℓ^2 km^2.

Equation 5.73 may be written as an observation equation at the general point $P_i(\phi_i,\lambda_i)$:

$$\sum_n\sum_m\sum_\alpha K_{\alpha nm}\left(F_{\alpha nm}(\phi_i,\lambda_i) + \frac{1}{2\pi}\sum_j F_{\alpha nm}(\phi_j,\lambda_j)f(\psi_{ij})d\sigma_j\right) - B_i = v_i \qquad (5.76)$$

where the known quantity B_i is the right hand side of equation 5.73 and v_i are the residuals to be minimised in the solution procedure. The term $M\{\zeta_{ss}\}$ can be assumed zero to satisfy the condition for the "oceanic" definition of the geoid.

The coefficients $K_{\alpha nm}$ obtained from a solution of the set of observation equations at 5.76 for all P_i where there is data provide a continuous representation of ζ_{ss} in the ℓ^2 km^2 area considered. Such a representation, in combination with the solution for $\zeta_{s\ell}$, provides, in theory, an estimate of the stationary SST in oceanic areas for all wavelengths greater than (say) 20km.

5.2.3.2 ζ_{ss} *Based on the Inverse Solution to the Stokesian Type BVP*

Such a solution is expressed by (equation 4.80):

$$\overline{\Delta g}_{cp} + \frac{1}{R}\left(\gamma\overline{N}_{cp} - (W_o - U_o) + \gamma\zeta_{sp}\right) = \frac{\gamma}{4\pi R}\iint M_1(\psi)\left(\overline{N}_c - \overline{N}_{cp}\right)d\sigma - \frac{\gamma}{4\pi R}\iint M_1(\psi)\left(\zeta_s - \zeta_{sp}\right)d\sigma \tag{5.77}$$

$M_1(\psi)$ is the inverse of Stokes' operator and is defined by equation 4.77. All other quantities have been defined in § 5.2.3.1.

The use of the orthogonal properties of surface spherical harmonics leads to the following separation of long and short wavelength components:

$$\frac{\gamma}{4\pi R}\iint M_1(\psi)\left(\zeta_s - \zeta_{sp}\right)d\sigma = \frac{\gamma}{R}\sum_{n=2}^{n'} n \sum_{m=0}^{n}\sum_{\alpha=1}^{2}\zeta_{s\alpha nm}S_{\alpha nm}(\phi,\lambda) + \frac{\gamma}{4\pi R}\iint M_1(\psi)\left(\zeta_{ss} - \zeta_{ssp}\right)d\sigma \tag{5.78}$$

Equation 5.77 can be rearranged, in a similar manner to equation 5.67, into the (now) known long wavelength constituents of the SST and terms involving the unknown short wavelength component ζ_{ss}. Combination of equations 5.72, 5.77 and 5.78 gives:

$$-\frac{\gamma}{R}\left[\zeta_{ssp} + \frac{1}{4\pi}\iint M_1(\psi)\left(\zeta_{ss} - \zeta_{ssp}\right)d\sigma\right] = \overline{\Delta g}_{cp} + \frac{1}{R}\left(\gamma\overline{N}_{cp} - (W_o - U_o)\right) - \frac{\gamma}{4\pi R}\iint M_1(\psi)\left(\overline{N}_c - \overline{N}_{cp}\right)d\sigma + \frac{\gamma}{R}\zeta_{s100} + \frac{\gamma}{R}\sum_{n=2}^{n'}(n+1)\sum_{m=0}^{n}\sum_{\alpha=1}^{2}\zeta_{s\alpha nm}S_{\alpha nm} \tag{5.79}$$

The left hand side of equation 5.79 contains all the unknowns while the right hand side contains all quantities whose values can be determined.

Upon adopting a regional two dimensional Fourier representation for ζ_{ss} (equations 5.74 and 5.75), equation 5.79 can be written as an observation equation for the general point $P_i(\phi_i,\lambda_i)$:

$$\gamma\sum_n\sum_m\sum_\alpha K_{\alpha nm}\left[F_{\alpha nm}(\phi_i,\lambda_i) + \frac{1}{4\pi R}\sum_j\left(F_{\alpha nm}(\phi_j,\lambda_j) - F_{\alpha nm}(\phi_i,\lambda_i)\right)M_1(\psi)\,d\sigma_j\right] - B_i = v_i \tag{5.80}$$

where the known term B_i is the right hand side of equation 5.79 and v_i is the residual to be minimised in the solution procedure for the coefficients $K_{\alpha nm}$.

5.2.3.3 *Viable Possibilities for the Recovery of ζ_{ss} from Surface Gravity and GEOS-3 Altimetry Data*

The preferential choice of either of the solution procedures described in § 5.2.3.1 and § 5.2.3.2 is a matter of convenience.

The surface integrals on the right hand sides of equations 5.73 and 5.79 are adequately evaluated only if data of acceptable quality and global distribution are available. For the foreseeable future, the results obtained using either equation 5.73 or 5.79 are vulnerable to systematic errors in both:

(1) The gravity anomaly field $e_{\Delta g}$ (see discussion in § 2.3.5.4).

(2) The altimeter orbit determination e_{ζ} (see discussion in § 5.1.2.1).

The use of equation 5.73 as an observation equation in oceanic areas is subject to the serious limitation imposed by the lack of a reliable global marine gravity anomaly field. On the other hand, equation 5.79 based on the inverse of Stokes' operator suffers from the fact that the global field of ζ' is discontinuous over land. However, it is easier to compute ζ' at 30% of the earth's surface than gravity anomalies at sea for 70% of the globe. In addition, equation 5.73 is the least promising due to the slow convergence of Stokes' integral as a function of ψ. Therefore it would appear that the approach based on Stokes' operator (§ 5.2.3.1) is not the most viable for practical computations.

Equation 5.79 has the advantage that the function $M_1(\psi)$ rapidly fades with increasing ψ. Over 80% of the signal comes from contributions in the range $0^o < \psi < 5^o$ (COLEMAN & MATHER 1967). In fact regions beyond 500km from the computation point may be represented by truncation functions $Q_n(\psi_o)$ as discussed in IBID (Section 4). It appears that the approach described in § 5.2.3.2 is the most viable procedure for determining the stationary SST with wavelengths shorter than those in the satellite-determined gravity field model. Its implementation calls for the recovery of the coefficients $K_{\alpha nm}$ in equation 5.80.

A sound basis would exist for the recovery of ζ_{ss} if the input data ($\overline{\Delta g}_c$ and $\overline{N}_c$) were in the form of area means large enough to maximise the signal-to-noise. However, in order to avoid aliasing effects from frequencies greater than the Nyquist frequency, the area means should have extents not greater than $1/2\ell_r$. In addition, the input data should ideally have errors with equivalent wavelengths held to below ±10cm in $\overline{N}_c$ and ±30μgal in $\overline{\Delta g}_c$.

The finite nature of the GEOS-3 altimeter "footprint" (10-15km in diameter) means that such altimetry data cannot resolve the quantity $(\overline{N}_c - \overline{N}_{cp})$ for wavelengths shorter than 10km. The region within 0.1^o of the computation point, which is incapable of discrimination from GEOS-3 data, contributes about 40% of the signal to the surface integral containing $M_1(\psi)$ (IBID, Table 1). This lost contribution is expected to be of short wavelength. The use of a 100km lower limit for the shortest recoverable wavelength of the stationary SST (ℓ_r) will, to some extent, circumvent the problem of obtaining meaningful estimates of $(\overline{N}_c - \overline{N}_{cp})$.

The coefficients $K_{\alpha nm}$ obtained from the preferred solution of equation 5.80, in conjunction with the low degree spherical harmonic representation of $\zeta_{s\ell}$ can provide an estimate of ζ_s in a region for all wavelengths down to 10^2km.

However, the use of equation 5.79 in practice is hampered by the following factors:

(1) 80% of the signal comes from terms in the surface integral for areas within 5^o of the computation point when the "higher" reference model is used.

(2) Values of ζ' that are required on land within 500km of the coastline must be provided by three dimensional position determinations to ± 10cm precision. The use of equation 5.73 to determine $\bar{N}_c$ is not advised because of the need to neglect the short wavelength component of the SST ζ_{ss} for such evaluations. Furthermore, modern gravity anomaly data banks are inadequate for gravimetric determinations of $\bar{N}_c$ to ±10cm, e.g. the precision of the Australian Gravity Data Bank is estimated to be only adequate for geoid determinations to ±30cm (see Chapter 6).

(3) Oceanic gravity anomaly data will be unable to achieve the necessary precision in the foreseeable future.

Therefore, at the present time it is unlikely that GEOS-3 altimetry data can be used in combination with surface gravity data for the successful evaluation of the short wavelength stationary SST.

5.2.3.4 *Time Variations in the Short Wavelength SST*

Equation 5.64 can be used, in principle, for the study of time variations in the SST for all wavelengths down to the diameter of the altimeter "footprint". If the variations in sea surface height over short wavelengths were produced by mass transport (e.g. ocean tides), the time variation $\delta\zeta_{ss}$ in ζ_{ss} could be represented by equations 5.65 and 5.66. For the purposes of this discussion it will be assumed that δN in equation 5.65 is zero, and such effects will not be included in the following formulation.

The small scale time variations in the SST between times t and t+dt can be described by (equation 5.64):

$$\delta\zeta_{ss} = \zeta'(t+dt) - \zeta'(t) \tag{5.81}$$

If ζ' could be defined with an accuracy of ±10cm, the variation in ζ' could be analysed on a global or regional basis for:

(i) the time variations in $\zeta_{s\ell}$ (equations 5.64, 5.65 & 5.66).

(ii) the time variations in ζ_{ss} (equation 5.81).

However, the GEOS-3 altimetry data does not satisfy this accuracy requirement (see § 5.1.2.2 and 7.1.2) and special techniques must be developed for the study of temporal variations in sea surface height.

It was concluded in § 5.2.2.3 that because GEOS-3 altimetry data is influenced by orbital uncertainties with long wavelength (> ℓ), meaningful estimates of time variations in the SST with equivalent wavelengths is not possible.

The recovery of $\delta\zeta_{ss}$ however is more promising and has been investigated in a number of papers (e.g. MATHER & COLEMAN 1977, MATHER ET AL 1978a). All solution techniques are based on modelling the orbital error for each altimeter pass i with length in excess of 10^3km by a bias b_i and a tilt c_i.

Early studies of intensive mode GEOS-3 altimetry in the Tasman and Coral Seas off eastern Australia (MATHER ET AL 1977a, MATHER 1977) indicated that passes of altimetry data were subject to orbital errors varying from ±2m to in excess of ±10m, primarily as a result of inadequate tracking of the GEOS-3 satellite, poor gravity field definition in the southern oceanic areas and time tag errors. It is possible to model the sea surface height ζ at the point (ϕ,λ) in terms of observed sea surface height ζ_{ij} from the jth element of the ith pass of altimetry using the relation (MATHER ET AL 1977a, p.38):

$$\zeta = \zeta_{ij} + b_i + c_i(t_{ij} - t_{i1})/\Delta t_i + \zeta_t + v_r \qquad (5.82)$$

where b_i is the bias and c_i is the tilt of the ith pass due to orbital error; ζ_t is the height of the combined earth and ocean tide; t_{ij}, t_{i1} are the times of the jth and first elements in the ith pass; Δt_i is the duration of the pass and v_r is the result of unmodelled effects (including sea roughness) in the sea surface height.

The noise level of the GEOS-3 has been estimated as ±20cm (IBID 1977a), therefore a basis exists for studying time variations in ζ on a <u>regional basis</u> if:

- the corrections b_i and c_i can be determined; or
- the technique of analysis does not require that b_i and c_i be known.

There are two techniques that can be used (MATHER ET AL 1978a):

(1) the method of *regional models*.

(2) the analysis of *overlapping passes*.

The construction of regional models is based on the establishment of a framework of control at crossover points formed by the intersection of passes for the subsequent adjustment of the sea surface model. The estimates of ζ from values of ζ_{ij} and ζ_{kl} on the ith and kth pass, which intersect at a point P, can be obtained from equation 5.82. The combination of these two estimates gives an observation equation of the form (MATHER ET AL 1977a, p.38):

$$v = \zeta_{ij} - \zeta_{kl} + (b_i - b_k) + c_i(t_{ij} - t_{i1})/\Delta t_i - c_k(t_{kl} - t_{k1})/\Delta t_k \qquad (5.83)$$

on assuming that the tidal signal ζ_t can be treated as being included in either ζ or the residual v. For a discussion on methods of recovering the tidal signal from satellite altimetry data see MASTERS ET AL (1979).

The construction of a regional sea surface model is carried out by forming observation equations (5.83) at all crossover points in some $n^o \times m^o$ area, for some specified time period (say one month) and subsequently solving for all the biases and tilts that would minimise the discrepancies v at all the crossover points. For further details see MATHER ET AL 1977a (Section 4).

The technique of regional models improves the resolution of the altimetry data to around ±40cm on a regional basis. However, the resultant sea surface model is <u>insensitive to absolute datum</u> as the model is based on relations (equation 5.83) which are essentially differential in nature. Monthly models have been used by MATHER ET AL (1978a) in the western Sargasso Sea to study short wavelength variations in the SST with time spans greater than a month. This investigation showed that such regional models, if appropriately analysed, provide a means of monitoring the motion of eddies (§ 3.3.2.2) in the western North Atlantic Ocean (Gulf Stream eddies exhibit sea surface height variations in excess of ±50cm over distances larger than 10^2km).

The use of such sea surface models for the study of short wavelength stationary SST is not possible until an absolute datum is established and a geoidal model with ±10cm precision for wavelengths of interest is available.

GEOS-3 satellite groundtracks approximately repeat themselves every 526 revolutions or 37.18 days (MATHER 1977, Section 3). All passes with common groundtrack can be fitted to each other by applying a correction for bias and tilt (equation 5.82) and the residuals of these overlapping passes can be subjected to spectral analysis (as described in MATHER ET AL 1978a, Section 4). The corrections b_i and c_i provide no useful information for the study of temporal variations in the sea surface height. The residuals may, however, be analysed for the tidal signal (MASTERS ET AL 1979).

The average root mean square value of the residuals obtained from the analysis of overlapping passes in the Sargasso Sea is ±33cm (MATHER ET AL 1978a, Section 4). Such residuals contain information on the temporal variations of the sea surface height (and hence the SST) due to seasonal effects and ocean eddies. However, SST variations with lifetimes less than approximately one month cannot be detected by this technique.

IBID (1978a) reported a very high correlation (0.98) between the results of satellite infrared imagery (at present used to monitor eddies in the North Atlantic - see e.g. NOAA 1978) and equivalent sea surface topography features detected in the spectral analysis of overlapping pass residuals.

Regional models and overlapping pass analysis appear to have successfully recovered short wavelength temporal variations in sea surface shape in areas of fast moving surface currents like the Gulf Stream, where amplitudes in the time-varying component of SST are in excess of ±50cm. However, time variations with amplitudes as small as ±10cm can only be detected if radial orbital errors can be significantly reduced.

The accuracy of gravity field models need to be improved by at least an order of magnitude (ideally to 2 parts in 10^9 or 1kgalcm) before further progress can be made in studying short and long wavelength features of both the stationary and time-varying components of the SST. The role that satellite altimetry data can play, in the interim, towards the improvement of gravity field models is discussed in the next section.

5.2.4 GRAVITY MODEL IMPROVEMENT USING SATELLITE ALTIMETRY DATA

The greatest limiting factor to gravity field model improvement at the present time is the absence of a global network of high precision satellite tracking stations. Such a network would provide high quality data for satellite orbit determination and, after appropriate analysis, a gravity field model with a precision equivalent to that of the tracking data. However, in the short term, satellite altimetry data can play a role in gravity field model improvement.

The role of the gravity field model in determinations of the long wavelength components of the stationary SST has been described in § 5.2.2.1. An absence of high precision continuous tracking requires that the gravity field model be able to define the geocentric position of the altimeter-equipped spacecraft to ±10cm. For SST studies, the spherical harmonic coefficients $C'_{\alpha nm}$ of the satellite-determined gravity field model need to be known to at least a precision of 1 part in 10^8 in each harmonic (equivalent to ±6kgalcm in the geopotential), preferably better. Present gravity field models such as GEM9 (LERCH ET AL 1977) appear to satisfy this requirement to degree four (see Table 5.2). Holding these harmonics fixed, it is possible to attempt a gravity field model improvement along the lines

TABLE 5.2

Degree Variances of SST and GEM9 Errors

DEGREE (n)	SEA SURFACE TOPOGRAPHY + (cm^2)	GEM 9 ERROR ++ $(kGal\ cm)^2$
1	528.6	-
2	2170.3	15
3	107.0	138
4	109.7	83
5	23.7	391
6	31.1	266
7	3.9	865
8	4.5	619
9	2.7	1435
10	0.7	1193
11	0.4	2415
12	0.1	1815
13	0.1	2452
14	0.0(4)	2049
15	0.0(7)	2575
16	0.0(1)	2248

+BASED ON A CONSTRAINED SOLUTION TO (16, 16) FROM 5° x 5° AREA MEANS IN OCEAN AREAS ONLY FIT TO DATA = ± 9 cm

++BASED ON COMPARISONS WITH SURFACE GRAVITY (LERCH ET AL. 1977). COMPARISONS WITH ALTIMETRY ARE A FACTOR OF TWO BETTER ON THE AVERAGE

Source: MATHER (1978e)

proposed by MATHER (1978e, Section 6).

The observation equations 5.60 and 5.61 can be used to analyse the available ζ' and $\overline{\Delta g_c}$ data for a set of coefficients $C_{\alpha nm}$ for degree n>4 on the assumption that the contribution of the SST (both the stationary and time-varying constituents) is insignificant. As the SST is o{±2m}, the adoption of such a procedure is unlikely to define a model of the geopotential to better than the magnitude of the SST. Furthermore, the geoid model generated from the resulting coefficients (see § 5.1.2.3) will have errors with wavelengths equivalent to those of the SST. Additional distortions can be expected due to the inclusion of unreliable oceanic gravity anomaly data in the model improvement, via equation 5.61.

Table 5.2 lists the error degree variances of GEM9 and the degree variances of the estimated global SST (based on Figure 3.3 and analyses described in sub-section 7.1.4). Considerable scope exists for refining gravity field models using satellite altimetry data, despite the existence of (unmodelled) SST. A first attempt at such model improvement using GEOS-3 altimetry and surface gravity data is GEM10B (LERCH ET AL 1978a).

The GEM10B model is a considerable improvement over all previously published GEM gravity field models in almost every respect (LERCH ET AL 1978b). GEM10B is the product of the analysis 700 passes of GEOS-3 altimetry together with long wavelength information from GEM10 (obtained from the analysis of both tracking data and surface gravity anomaly data) and represents a model of the geopotential to degree and order 36 (equation 5.16). A preliminary gravity field model (GEM10C) has been developed from an analysis of the global GEOS-3 altimetry data bank and contains information on those wavelengths in the geopotential corresponding to degrees 37 to 180 (IBID 1978b).

The evaluation of some of the *dominant* features of the stationary SST (by techniques described in § 5.2.2.2 and attempted in section 7.2) makes it possible to remove their effect from the altimetry and surface gravity data sets. The time-varying component of SST due to tides and mesoscale variations in ζ can only be eliminated by using altimetry data well distributed in time as well as space (oceanic gravity data cannot easily be corrected for the temporal variations in SST - the location of the gravimeter is assumed to be at mean sea level).

An observation equation based on satellite altimetry can be constructed from equation 5.60:

$$v = \frac{GM}{R_o} \sum_{n=2}^{n'} \left(\frac{a}{R_o}\right)^n \sum_{m=0}^{n} \sum_{\alpha=1}^{2} \Delta C_{\alpha nm} S_{\alpha nm} - \gamma \zeta_{sr} \tag{5.84}$$

where R_o is the geocentric distance to the sea surface, v is the residual to be minimised, $\Delta C_{\alpha nm}$ are spherical harmonic coefficients representing corrections to $C^*_{\alpha nm}$ and

$$\zeta_{sr} = \zeta' - \sum_{n=0}^{n''} \sum_{m=0}^{n} \sum_{\alpha=1}^{2} \zeta_{s\alpha nm} S_{\alpha nm} \tag{5.85}$$

where n'' is the maximum degree to which the coefficients $\zeta_{s\alpha nm}$ have been determined. ζ' is the height anomaly on the "higher" reference model, defined by coefficients of the a priori model $C^*_{\alpha nm}$ (see equation 4.84 for the relationship between these coefficients and those of the satellite-determined geopotential model $C'_{\alpha nm}$).

It is not recommended that oceanic gravity anomaly data be used as the quality is inferior to that of the altimetry, and does not contribute to a complete areal coverage.

However, areal representation can be improved by forming observation equations on land areas. These are of the form (equation 5.61):

$$v = \frac{GM}{R^2} \sum_{n=2}^{n'} (n-1) \left(\frac{a}{R}\right)^n \sum_{m=0}^{n} \sum_{\alpha=1}^{2} \Delta C_{\alpha nm} S_{\alpha nm} - \Delta g_r \qquad (5.86)$$

where Δg_r is the gravity anomaly on the "higher" reference system, defined by:

$$\Delta g_r = \Delta g_c - \delta g_a + \frac{2\gamma}{R} \sum_{n=2}^{n''} \sum_{m=0}^{n} \sum_{\alpha=1}^{2} \zeta_{s\alpha nm} S_{\alpha nm} \qquad (5.87)$$

Equation 5.87 assumes that the dominant harmonic representation of the stationary SST adequately models the SST at the regional levelling datum used in the computation of Δg_c (this is discussed further in sub-section 5.3.2).

If the altimetry data were of adequate precision, the use of equation 5.84 (possibly with land gravity anomaly data through equation 5.86) in conjunction with a low degree surface spherical harmonic model of the stationary SST should improve the gravity field model to a resolution of 1 part in 10^8 of the geopotential. It is doubtful however, whether such models can play a role in determining short wavelength features of the global stationary SST.

If, on the other hand, the long wavelength features of the SST are not known, an iterative procedure would be needed to successfully refine the gravity field model. At each iteration the gravity field model would be used to reintegrate the orbits of the altimeter-equipped spacecraft and obtain sea surface heights ζ' with improved accuracy. These, in turn, could be used to improve the long wavelength stationary SST model $\zeta_{s\alpha nm}$ (§ 5.2.2.2) and, subsequently, be used in equation 5.85 to prepare the input data for another iteration.

If the determination of a long wavelength model of the stationary SST by geodetic techniques is not possible, an oceanographically-determined model for $\zeta_{s\ell}$ could be used instead (see Table 7.3).

An alternative to the above mentioned iterative procedure was used in the development of GEM10B and GEM10C (LERCH ET AL 1978b). Individual altimeter passes were biased and tilted to fit an a priori long wavelength geoidal model defined by GEM10 (LERCH ET AL 1977). The resulting gravity field information in these adjusted altimeter passes is of a higher frequency than that afforded by the GEM10 model and improvement of the low degree harmonics of the gravity field model ($n < n'$) is not possible. The incorporation of GEOS-3 altimetry in the process of gravity field model development has resulted in model errors dropping from previous levels of ±3-6m to ±1-2m for the implied geoid model in all but high latitudes (MATHER 1978b, p.241).

Ultimately, gravity field model improvement to 2 parts in 10^9 for SST studies can only be achieved in the presence of direct tracking, although gravity field models refined by the use of satellite altimetry data as described above can be used as input models. In theory, data from a global network of at least 25 laser tracking systems capable of ± 10cm precision are required for providing data for the necessary orbital analysis (MATHER 1974b, p.104). Consequently, given the choice, it is preferable to improve the density and distribution of high precision tracking systems for orbit determination and gravity field model improvement, rather than to carry out extensive ground gravity surveys.

The possibility exists that the minimum wavelength that can be resolved could be decreased to about 500km if satellite-to-satellite tracking to low-flying GRAVSAT type satellites (NASA 1972b) were included in gravity field model solutions. Research in this area is still at a very early stage and the feasibility of determining the short wavelength components of the earth's gravity field from satellite-to-satellite tracking has been studied by e.g. RUMMEL (1979). The desired resolution in radial acceleration is $o\{10^{-5}cms^{-2}\}$ (MATHER 1978e, p.22), though the present resolution is almost two orders of magnitude worse (MARSH ET AL 1977).

5.3 DETERMINATION OF HEIGHT OF MEAN SEA LEVEL IN RELATION TO THE GEOID AT CONTINENTAL LEVELLING DATUMS

5.3.1 RELATIONSHIP BETWEEN CONTINENTAL GRAVITY ANOMALY DATA AND SST AT THE LEVELLING DATUM

It was established in § 2.3.5.4 and 2.3.5.5, that the solution of the GBVP to ± 10cm by gravimetric techniques is hampered by the inadequate precision of modern continental gravity anomaly data banks. If the surface gravity values g have been compiled in relation to a global gravity standardisation network with an estimated precision of ± 0.2mgal, such as IGSN 71 (MORELLI ET AL 1971), the precision of the gravity anomaly given by (equation 6.1 and 6.3):

$$\Delta g = g - \gamma - \frac{2\Delta W}{a}\left(1 + f + m + \frac{\Delta W}{2a\gamma} - 2\sin^2\phi + o\{f^2\Delta g\}\right) \qquad (5.88)$$

is influenced primarily by the errors in ΔW, the difference in geopotential between a point P with geodetic coordinates (ϕ,λ), on the earth's surface and the regional datum level surface. The quantity ΔW is obtained from the results of geodetic levelling and gravity surveys as described in sub-section 3.2.1. It is defined by (equation 3.19):

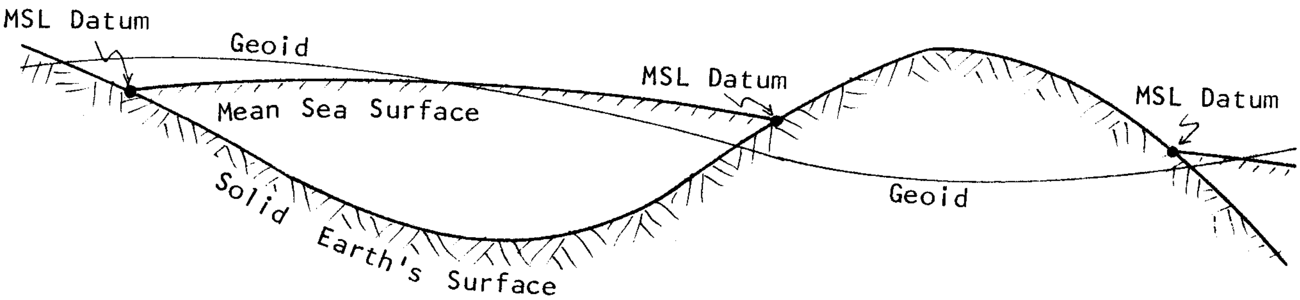
$$\Delta W = -\int_{\text{MSL datum}}^{P} g\, dn \qquad (5.89)$$

where dn are the observed level staff increments.

This is illustrated in Figure 5.2.

FIGURE 5.2

The Geoid, the Sea Surface and the Levelling Datums



In the context of surface integral evaluations (such as equation 5.4), the accuracy of the result is a function of the magnitude and wavelength of the errors in the global gravity anomaly field. This is dependent, amongst other things, on the errors in the networks of ΔW used to establish such gravity anomaly data banks.

In the case of the gravity anomaly data bank for Australia (AUSGAD 77) (see sub-section 6.2.3), the long wavelength error sources have been assessed as:

(i) Residual errors in the Australian levelling survey of amplitudes ±50cm.

(ii) A constant error of ±0.06mgal due to the uncertainty in the adopted gravity datum value.

(iii) Residual errors in the Australian National Gravity Network of amplitude ±0.2mgal.

The establishment of gravity datum and gravity networks with precisions an order of magnitude better than at present is within the capabilities of today's metrology.

However, aside from the limited precision of the Australian levelling survey, the most significant source of error in geoid determinations using such gravity anomaly data banks is the effect of the adopted local elevation datum not coinciding with the geoid to better than ±1-2m. This discrepancy, due to the stationary SST at the tide gauge, is denoted ζ_{sd} and causes a systematic error of ±0.3mgal per ±1m of SST in the entire gravity anomaly data bank based on this elevation datum. It is therefore desirable to determine the height of MSL ζ_{sd} at the datum tide gauge in relation to a uniquely defined geoid (Figure 5.2), to a resolution of ±10cm. In addition, the estimation of the "heights" of MSL at other coastal tide gauges will provide an extra set of constraints for controlling the propagation of systematic error in the adjustment of continental levelling networks (see e.g. ANDERSON 1979b).

The problem of determining the stationary SST at regional levelling datums and the subsequent unification of continental levelling networks is investigated in Chapter 8. The description of a procedure for defining vertical datums with respect to the geoid, using geodetic data which has been collected in relation to MSL is given in the following section.

5.3.2 PRACTICAL POSSIBILITIES FOR THE DETERMINATION OF ζ_{sd}

The task of vertical datum definition on land is *complementary* to that of determining SST in oceanic areas (section 5.2).

At first glance it would appear that both equations 5.52 and 5.54 can play a role in determining the height of MSL above the geoid at tide gauge sites. Satellite altimetry data has the potential for estimating ζ_{sd} at *any* coastal site, whereas continental gravity anomaly data can only resolve ζ_{sd} at the tide gauge defining the elevation datum.

The principal problems that arise from the direct use of satellite altimetry for the determination of ζ_{sd} (aside from those due to orbital errors discussed in § 5.1.2.2) are the following:

(1) The geoid has to be defined through short wavelengths (< 10^2km) in continental shelf areas.

(2) Due to the finite altimeter "footprint", the sea surface heights ζ have to be extrapolated from the open oceans to the coastal site over a distance not less than 20km.

The first problem calls for the determination of the short wavelength SST ζ_{sd} (§ 5.2.3.3), which is subsequently used to correct the gravity anomaly data for use in a gravimetric solution for a detailed regional geoid (§ 5.1.2.1). The resolution of the second problem requires an oceanographic survey in order to extrapolate the sea surface slope by the principles of geostrophic levelling (see § 3.3.3.3), ensuring that frictional forces at the ocean bottom and sea surface are adequately modelled.

In view of these difficulties, the most promising alternative procedure for the determination of the "height" of MSL is based on the regional gravity anomaly data which has been established using levelling data related to a local MSL datum (equations 5.88 and 5.89).

The following is a development from MATHER & RIZOS (1978) and MATHER ET AL (1978c).

The spectrum of the global stationary SST can be best represented by:

$$\zeta_s = k_o(\phi,\lambda)\left(\sum_{n=0}^{n''}\sum_{m=0}^{n}\sum_{\alpha=1}^{2}\zeta_{s\alpha nm}S_{\alpha nm} + \zeta_{ss}\right) + \left(1 - k_o(\phi,\lambda)\right)\zeta_{sd} \tag{5.90}$$

Such a representation does not affect the observation equations developed for the determination of the long wavelength components of ζ_s at sea in § 5.2.2.2. The first degree harmonic is excluded to ensure that the SST model is in relation to a geocentric reference system.

The oceanic function $k_o(\phi,\lambda)$ is defined by:

$$k_o(\phi,\lambda) = \begin{cases} 0 & \text{if } (\phi,\lambda) \text{ is land} \\ 1 & \text{if } (\phi,\lambda) \text{ is ocean} \end{cases} \tag{5.91}$$

According to such a representation, two possibilities exist for the definition of the geoid that allows satellite altimetry data to play a role.

(1) *An "Oceanic" Definition of the Geoid:*

The geoid in such a case is that level surface of the earth's gravity field which best fits MSL in oceanic areas. Such a definition can be realised by representing the oceanic SST by the relation (see § 5.2.2.2):

$$\zeta_s = k_o(\phi,\lambda)\left(\sum_{n=2}^{n''}\sum_{m=0}^{n}\sum_{\alpha=1}^{2}\zeta_{s\alpha nm}S_{\alpha nm} + \zeta_{ss}\right) \tag{5.92}$$

where the zero degree harmonic coefficient ζ_{s100} is excluded. In addition, it is assumed that the mean value of the short wavelength SST ζ_{ss} over the global oceans is zero.

The observation equation for gravity anomaly data, based on the representation at equation 5.90, takes the form (equation 5.61):

$$v_{\Delta g} = \Delta g_d + \frac{2\gamma}{R}k_o(\phi,\lambda)\sum_{n=2}^{n''}\sum_{m=0}^{n}\sum_{\alpha=1}^{2}\zeta_{s\alpha nm}S_{\alpha nm} + \frac{2\gamma}{R}\left(1 - k_o(\phi,\lambda)\right)\zeta_{sd} - \frac{2}{R}(W_o - U_o) \tag{5.93}$$

where the residual $v_{\Delta g}$ contains the effect of the neglected short wavelength SST. In such an observation equation, the surface spherical harmonic series only models ζ_s in oceanic areas, while land areas are modelled by the discrete value ζ_{sd} at the regional levelling datum.

(2) *A "GBVP" Definition of the Geoid:*

In this case the global stationary SST is represented by:

$$\zeta_s = \sum_{n=2}^{n''}\sum_{m=0}^{n}\sum_{\alpha=1}^{2}\zeta_{s\alpha nm}S_{\alpha nm} + \zeta_{ss} \tag{5.94}$$

and the implementation of such a definition requires that the zero degree coefficient ζ_{s100} be excluded (and the global mean value of ζ_{ss} be assumed zero). However, no distinction is made between land and oceanic areas. The value of ζ_s generated on land from such a model should be constant for locations on the same regional levelling network (equal to ζ_{sd} at the datum tide gauge). It would be unrealistic not to expect aliasing effects in coastal areas due to the harmonic model for $\zeta_{s\ell}$ if observation equations like equation 5.61 were formed on land as well as at sea.

As was mentioned in § 5.2.2.2, the choice of geoid definition does not alter the form of the observation equations (5.60 and 5.61) used for the recovery of oceanic SST, merely the interpretation of the zero degree term in the surface spherical harmonic representation of $\zeta_{s\ell}$.

The equation relating values of $\overline{\Delta g_c}$ on land for the "higher" reference model to ζ_{sd} is given by (equation 4.94):

$$\Delta g_c = \Delta g'' + \frac{2}{R}(W_o - U_o) - \frac{2\gamma}{R}\zeta_{sd} \tag{5.95}$$

The numerical value for ζ_{sd} depends on the known quantity $(W_o - U_o)$ and the values of $\Delta g''$. The magnitude of the zero degree term $(W_o - U_o)$ is dependent on the specific definition for the geoid adopted and the values of the fundamental constants, as discussed in § 5.1.3.2. The preferred definition is the "oceanic" definition, and consequently the value of W_o is that defined in equation 5.49.

Equation 5.95 is equivalent to the following observation equation (see equation 5.61) (MATHER ET AL 1978c, p.9):

	$v_{\Delta g}$	=	$(\Delta g_c - \delta g_a)$	+	$\frac{2\gamma}{R}\zeta_{sd}$	-	$\frac{2}{R}(W_o - U_o)$
Order of magnitude (mgal)	± 10		± 10 known		$\pm\frac{1}{2}$ unknown constant		$\pm\frac{1}{4}$ zero degree
Range of wavelengths (ω)	$\omega < \ell$		$0 < \omega < \infty$		$\omega = \infty$		$\omega = \infty$

(5.96)

Observation equation 5.96 forms the basis for the evaluation of ζ_{sd} from the analysis of data collected in relation to the regional levelling datum if the area covered by this datum is larger in extent than the highest full harmonic in the "higher" reference model which is free from error (ℓ^2 km^2). This proviso means that there is no zero degree term in the residuals $v_{\Delta g}$. The only role played by the altimetry data is in defining $(W_o - U_o)$.

The estimation of ζ_{sd} for the Australian and central North American geodetic levelling networks (based on gravity anomaly data banks prepared in the manner described in Chapter 6) is attempted in section 8.2. These results are influenced by the extent to which the gravity field model is error free through the critical wavelengths. Therefore the desired goal for datum level studies is a gravity field model with a precision of 2 parts in 10^9 for all wavelengths longer than ℓ and where ℓ is made as short as possible.

6

GRAVITY ANOMALY DATA BANKS FOR SEA SURFACE TOPOGRAPHY STUDIES

6.1 INTRODUCTION

6.1.1 THE ROLE OF GRAVITY DATA IN SEA SURFACE TOPOGRAPHY DETERMINATIONS

Satellite altimetry data is collected as part of the National Aeronautics & Space Administration's Earth and Ocean Dynamics Applications Program (NASA 1972b). The continued evolution of such programs calls for a review of the adequacy of existing gravity data banks for use in combination solutions with a view to improving gravity field models (see sub-section 5.2.4), and ultimately in defining sea surface topography.

The goal of ±10cm resolution in sea surface topography studies using geodetic techniques implies a precision of ±30μgal in the gravity anomaly data. This precision is unattainable at present in field measurements where errors of 1-2 orders of magnitude greater than this occur in individual observations. It was shown in § 2.3.5.4 that meaningful results for the geoid height are obtained from solutions of the Geodetic Boundary Value Problem only if certain criteria are satisfied by gravity anomaly data banks. These have been summarised in § 5.1.2.1. The most important requirement is that long wavelength systematic errors in continental gravity anomaly data be kept below ±50μgal, though individual errors in anomalies may be as large as ± 3mgal.

The effect that the displacement of the regional levelling datum from the geoid (used to compute the gravity anomaly) has on solutions of the GBVP has been discussed at length in § 2.3.5.5 and 4.2.3. It was established that the solution of the GBVP from surface gravity data is not capable of defining the oceanic geoid with a precision sufficient for the determination of SST without making prior assumptions concerning the nature of the SST. Nevertheless, a gravity anomaly data bank that satisfies the precision requirements listed in § 5.1.2.1 and related to a geopotential network based on one Mean Sea Level datum, could still provide valuable local geoid information as the errors in continental geoid determinations due to the SST will be of long wavelength.

It was concluded in § 5.1.2.4 that a meaningful determination of SST would only be possible, in the first instance, through wavelengths equivalent to those in the gravity field which affect the orbit of an altimeter-equipped satellite above the noise level of the tracking. Therefore it could be argued that gravity anomalies are not required for the definition of SST. Such a proposition overlooks the contribution that gravity data banks can make towards the determination of the short wavelength component of SST (sub-section 5.2.3), improving the gravity field model (sub-section 5.2.4) and the unification of level datums (sub-section 5.3.2).

A long wavelength model for the global stationary SST (determined by methods described in § 5.2.2.2) would provide the basis for improving the gravity anomaly data banks in order that they may acquire a global relevance in high precision geodetic studies. For example, a gravity anomaly data bank free of the effect of the dominant wavelengths of SST, would allow high precision global geoids to be computed. In addition, such "corrected" data could play a vital role in improving gravity field models. These gravity field models, based on data related to the geoid instead of the sea surface, could be subsequently used to obtain refined satellite orbits and hence, more accurate sea surface heights. The successive refinement of the orbits at each iteration would make a valuable contribution to the study of the time-varying component of SST.

It would therefore be most desirable to carry out a unique determination of the sea surface topography, including values at those coastal tide gauges serving as datum level surfaces, using all the available surface gravity data and values of ζ derived from satellite altimetry for the epoch of observation. Furthermore, this would permit a unification of all the level datums on a global basis. It would also provide a consistent definition of the geoid as the level surface such that ζ_s has zero mean (for any of a number of possible definitions described in section 2.5) when averaged from an equal area global sample established in the following manner:

- ζ_s is estimated from satellite altimetry data in oceanic areas;
- ζ_s for any land area is the value at the level datum in relation to which the local elevation is established.

It is therefore essential that the gravity anomaly data banks be <u>free from errors with wavelengths in excess of those sought in the sea surface topography</u>, as their neglect would seriously bias such determinations.

6.1.2 BASIC RELATIONS

6.1.2.1 *The Gravity Anomaly* Δg

The gravity anomaly Δg, described in sub-section 4.1.4, has a precise definition on the Earth-Telluroid system as illustrated in Figure 2.5 for the case of the fixed boundary value problem. Δg is defined as (equation 4.28):

$$\Delta g = g_P - \gamma_{Q''} \qquad (6.1)$$

where g_P is observed gravity at P on the Earth's surface and $\gamma_{Q''}$ is the value of normal gravity at the point Q'', lying on the same ellipsoidal normal as P but with gravity potential $(U = U_o + \Delta W)$. U_o is the potential of the reference ellipsoid and ΔW is the measured difference of geopotential between the levelling datum and P, given by the relation (equation 3.19):

$$\Delta W = -\int_{\text{Level Datum}}^{P} g\, dn \qquad (6.2)$$

where dn is obtained from geodetic levelling as the orthometric height difference.

The levelling datum has an unknown vertical displacement ζ_{sd} above the geoid. $\gamma_{Q''}$ is related to normal gravity γ_o on an equipotential ellipsoid by the formula (e.g., MATHER 1971, p.101):

$$\gamma_{Q''} = \gamma_o + \frac{2\Delta W}{a}\left(1 + f + m + \frac{\Delta W}{2a\gamma} - 2f\sin^2\phi + o\{f^2\}\right) \qquad (6.3)$$

where ϕ is the geodetic latitude. The closed formula for γ_o is (e.g. IBID, p.88):

$$\gamma_o = \gamma_e(1-\sin^2\alpha_o\sin^2\phi)^{\frac{1}{2}}\left(1 + m\left(1 + \frac{3}{2}\Phi\right)\frac{\cos^2\alpha_o\sin^2\phi}{1-\sin^2\alpha_o\sin^2\phi}\right) \qquad (6.4)$$

where $m = a\omega^2/\gamma_e$ and,

$$\Phi = \sec^2\alpha_o\left(1 - \frac{4}{7}\tan^2\alpha_o + \frac{68}{147}\tan^4\alpha_o + o\{f^3\}\right) \qquad (6.5)$$

$$\sin^2\alpha_o = 2f - f^2 \quad ; \quad \cos\alpha_o = 1 - f \qquad (6.6)$$

and

$$\gamma_e = \left(\frac{GM}{a^2} - a\omega^2\cos\alpha_o\left(1 + \tfrac{1}{2}\Phi\right)\right)/\cos\alpha_o \qquad (6.7)$$

M is the mass of the Earth implicit in the reference model adopted.

It is conventional to compute normal gravity γ_o in practice by a formula of the type:

$$\gamma_o = \gamma_e\left(1 + \beta_1\sin^2\phi + \beta_2\sin^2 2\phi\right) \qquad (6.8)$$

where values of the constants γ_e, β_1 and β_2 for Geodetic Reference System 1967 (GRS 67) are (IAG 1971, p.60):

$$\gamma_e = 978031.85 \text{ mgal} \; ; \; \beta_1 = 5.3024 \times 10^{-3} \; ; \; \beta_2 = -5.9 \times 10^{-6} \qquad (6.9)$$

the last two constants having an accuracy of ± 0.1mgal.

An alternate form for computing γ_o is the Chebyshev formula (IBID):

$$\gamma_o = \gamma_e\left(1 + \beta_3 \sin^2\phi + \beta_4 \sin^4\phi\right) \qquad (6.10)$$

where values of β_3 and β_4 for GRS 67 are:

$$\beta_3 = 5.278895 \times 10^{-3} \; ; \; \beta_4 = 2.3462 \times 10^{-5} \qquad (6.11)$$

The use of the Chebyshev form in numerical computations gives values of γ_o which differ from those computed using equation 6.4, by amounts normally less than 4μgal. Table 6.1 illustrates the errors involved in using equations 6.8 and 6.9 in lieu of equation 6.4 for the computation of γ_o. From these results, it can be concluded that equation 6.8 must be avoided when computing normal gravity for data banks to be used in sea surface topography determinations as the resulting errors (Table 6.1, Column 7) have a second degree zonal structure similar to that of sea surface topography as deduced from oceanographic considerations (Figure 3.3).

TABLE 6.1

The Computation of Normal Gravity γ_o

1	2	3	4	5	6	7
Latitude *Deg N*	Values of Normal Gravity γ_o (mgal)			Differences (μgal)		
	Exact Form *Equation 6.4*	Chebyshev Form *Equation 6.10*	Conventional Form *Equation 6.8*	2 − 3	3 − 4	2 − 4
0	978 031.845 6	978 031.850 0	978 031.800 0	-4.4	50.0	45.6
10	978 187.549 7	978 187.552 2	978 187.499 5	-2.5	52.7	50.2
20	978 636.113 2	978 636.111 8	978 636.052 7	1.4	59.1	60.4
30	979 324.019 3	979 324.016 0	979 323.951 2	3.3	64.9	68.2
40	980 168.965 9	980 168.964 6	980 168.899 1	1.4	65.5	66.8
50	981 069.479 7	981 069.482 1	981 069.423 9	-2.4	58.2	55.8
60	981 916.948 8	981 916.953 0	981 916.909 1	-4.3	44.0	39.7
70	982 608.719 6	982 608.721 8	982 608.694 7	-2.1	27.0	24.9
80	983 060.681 6	983 060.679 7	983 060.666 3	1.9	13.4	15.3
90	983 217.727 9	983 217.724 0	983 217.715 8	3.9	8.2	12.1

Source: MATHER ET AL (1976b)

6.1.2.2 *Gravity Anomalies on the "Higher" Reference Model*

The concept of a "higher" reference model is introduced for two reasons (see section 4.4):

- It results in the surface of reference (T1 in Figure 4.1) modelling the Earth's surface more closely (±10m) than the telluroid ($\pm 10^2$m).
- It enables the gravity force field model used in the altimeter-satellite orbit analysis to be kept separate when handling data recorded at the surface of the Earth.

Figure 4.1 illustrates the basic relations in the case of the fixed boundary value problem. The reference ellipsoid potential is now $(U = U_o + \Delta U_s)$, where ΔU_s are the values on the reference ellipsoid of the potential given by equation 4.84.

The effect of "higher" reference model on normal gravity ($\delta\gamma$) is given by (equation 4.93):

$$\delta\gamma = \gamma \sum_{n=0}^{n'} (n-1) \sum_{m=0}^{n} \sum_{\alpha=1}^{2} C^{*}_{\alpha nm} S_{\alpha nm} \left(1 + o\{f\}\right) \tag{6.12}$$

$C^{*}_{\alpha nm}$ being the coefficients of the gravity field model containing terms to degree n'. A more exact expression for $\delta\gamma$ is given in MATHER (1974b). the quantity $\gamma_{Q''}+\delta\gamma$ is normal gravity at P'' on T1. The gravity anomaly on the "higher" reference model is thus $(\Delta g - \delta\gamma)$.

6.1.2.3 *The Computable Part of the Spherically Harmonic Pseudo Gravity Anomaly* $\Delta g''$

The quantity $\Delta g''$ is defined as (equation 4.90):

$$\Delta g'' = - \frac{\partial T''}{\partial h} - \frac{2T''}{R} \tag{6.13}$$

where T'' is the disturbing potential of the solid Earth and oceans for the "higher" reference model, given by (equation 4.86):

$$T'' = \gamma\zeta' + (W_o - U_o) + \gamma\zeta_s - W_A \tag{6.14}$$

where W_A is the potential of the atmosphere and ζ' is the height of the Earth's surface (sea surface in oceanic regions) above the reference surface. The latter coincides with the spherop $(U = U_o)$ at the "higher" reference system in oceanic areas.

The pseudo gravity anomaly on the "higher" reference system is (equation 4.91):

$$\Delta g'' = \frac{GM}{R^2} \sum_{n=0}^{\infty} (n-1) \left(\frac{a}{R}\right)^n \sum_{m=0}^{n} \sum_{\alpha=1}^{2} dC_{\alpha nm} S_{\alpha nm} \quad , \quad n \neq 1$$

and (according to equation 4.39): (6.15)

$$\Delta g'' = \Delta g - \delta\gamma + \delta g_a + \frac{2T''}{R} c_\phi - \tfrac{1}{2} g\varepsilon^2 + \frac{2\gamma\zeta_s}{R} - \frac{2}{R}(W_o - U_o)$$

All quantities in equation 6.15 are defined in sub-section 4.1.4 and section 4.4. The first equality holds in the space exterior to and on the surface of the earth. The second equality holds on the surface of measurement. Initially, both $(W_o - U_o)$ and ζ_s are unknown. It is therefore useful to define the quantity:

$$\Delta g_c = \Delta g - \delta\gamma + \delta g_a + \frac{2T''}{R} c_\phi - \tfrac{1}{2} g\varepsilon^2 \tag{6.16}$$

which can be numerically evaluated prior to sea surface topography determinations. For the "higher" reference model $T'' = o\{10\text{kgalm}\}$. Thus $2T''c_\phi/R = o\{10\mu\text{gal}\}$ as $c_\phi = o\{f\}$ with an effect of about ± 5 cm in computations of sea surface topography through wavelengths greater than or equivalent to those sought in such determinations. It can therefore be neglected at the present time in all computations. Effects of wavelengths in excess of 2×10^2km through the last term on the right in equation 6.16 are also not of significance as ε is of order ± 10 arcsec in its spectrum of variations, with $\frac{1}{2}g\varepsilon^2 = o\{\pm 1\mu\text{gal}\}$.

If consideration is restricted to data which will only be used for the determination of long wavelength features in the sea surface topography (i.e. wavelengths greater than 10^3km), it would therefore suffice if the relation:

$$\Delta g_c = \Delta g - \delta\gamma + \delta g_a \tag{6.17}$$

were adopted for the computable part of $\Delta g''$ in equation 6.15.

6.2 AN AUSTRALIAN GRAVITY ANOMALY DATA BANK FOR SEA SURFACE TOPOGRAPHY STUDIES (AUSGAD 77)

Gravity data banks used in geodesy are unsuitable for the determination of SST as they are usually in the form of free air anomaly means. Gravity data for the Australian region is based on the homogeneous Isogal control network with elevations referred to the Australian Height Datum. This section describes the construction of a new gravity anomaly data bank (AUSGAD 77) for Australia, corrected for all identifiable systematic errors.

The data is reduced to gravity anomalies for the Earth-Telluroid system using the free net adjustment of the Australian levelling survey provided by the Division of National Mapping, Canberra and the geopotential network for Australia produced by MITCHELL (1972). The datum level surface adopted for Australia is the tide gauge zero at Jervis Bay. All latitudes used in the computation of normal gravity were converted to a geocentric ellipsoid consistent with Geodetic Reference System 1967 (GRS 67). The observed gravity values in AUSGAD 77 are based on the datum and scale adopted for Australia by BOULANGER ET AL (1973).

The preparation of AUSGAD 77 is described below and effectively supersedes an earlier data bank - AUSGAD 76 (MATHER ET AL 1976b).

6.2.1 THE FREE AIR ANOMALY DATA BANK FOR AUSTRALIA

The preparation of the free air gravity anomaly data bank is developed from MATHER ET AL (1976b).

6.2.1.1 *The Grid System*

The Australian continent is covered by a grid of gravity measurements with a minimum average station density of one per 130km^2 over most of the region. The data set gives an almost continuous sampling of the gravity field on a tenth degree grid. Both an equi-angular and an equi-area data bank were prepared for the region, the latter being for internal use at the Department of Geodesy, University of New South Wales. Only the tenth degree equi-angular free air anomaly data bank and others derived from it will be dealt with in this chapter. The basic subdivisional areas coincide with integer values along meridians and parallels.

6.2.1.2 *Establishment of Gravity Values in Australia*

Nearly all surveys since 1950 are tied to control networks and are of sufficient accuracy for inclusion in a homogeneous data bank suitable for geodetic purposes. All of the surveys necessary to provide a complete regional coverage of onshore Australia have been re-computed to a common datum and scale, and, together with preliminary marine data, form the 1976 Bureau of Mineral Resource, Geology & Geophysics (BMR) computer compatible data bank. That data bank is the basis of the Australian Gravity Anomaly Data Bank for 1976 (AUSGAD 76), see IBID. The 1977 BMR data bank includes additional marine gravity data and the 315000 point gravity values form the basis for AUSGAD 77.

About 85% of the onshore area of Australia is covered by systematic reconnaissance gravity surveys carried out between 1959 and 1974 by BMR or by private companies under contract to BMR. These surveys used helicopters for transport, microbarometers to determine height differences, and Worden or La Coste & Romberg gravity meters to measure gravity intervals between stations whose positions are permanently recorded on aerial photographs. The average station spacing is 11km except in South Australia and Tasmania where it is 7km. A cell pattern of flights minimises systematic errors (HASTIE & WALKER 1962). In each 15 minute by 15 minute cell, a four-leaf clover pattern of flights is flown from the centre of the cell to establish the regular grid of singly-read gravity stations and provide drift-controlled gravity ties between the cell centre and the corner points, all of which are semi-permanently marked on the ground. Re-occupation of the cell corners during flights in neighbouring cells builds up a uniform network of gravity links in which propagation of error can be minimised by least squares adjustment with free nodes at the cell centres and cell corners. Fixed nodes are provided wherever the flights re-occupy gravity control points of the Australian National Gravity Network (ANGN) or elevation control points defining the Australian Height Datum (AHD) and subsidiary spirit levelling traverses. Individual stations have a precision of about ± 0.5mgal in observed gravity and about ± 10m in elevation. For further comment regarding the latter, see § 6.2.3.5.

Data for the remaining 15% of the onshore area have been obtained by re-computing semi-detailed or reconnaissance surveys by other organisations. The precision of the data from these surveys is at least equal to that of the helicopter-borne reconnaissance surveys.

For the marine surveys, precision of the data can be estimated from the root mean square (rms) deviation of the differences of the gravity mis-ties at line intersections. The rms values range from ± 2 to ± 6mgal, with a strong probability that the errors are systematic over considerable areas.

For geodetic purposes, random errors in individual station values are less important than systematic errors of long wavelength in the data bank, provided the coverage is sufficiently dense and uniform. The latter arise from errors in the datum and scale of the gravity control network and from a lack of precision in that network. The ANGN consists of a series of east-west traverses between airports of nearly equal gravity joined by three north-south traverses. These traverses were established during a series of surveys in 1964-1967 known as the Isogal Project (BARLOW 1970). The values of observed gravity determined for the base stations of the ANGN are known as the May 1965 Isogal values. Their datum is 979979.00mgal at the station Melbourne A, designated 45474 A by the International Gravity Bureau (MORELLI ET AL 1971), and is compatible with the Potsdam datum.

Their scale is defined by the mean Australia milligal determined from an adjustment of the ANGN in 1962 (NATREP 1975). The interval precision of the net was sufficient to maintain datum and scale to better than ±0.2mgal equivalent throughout Australia. The values of observed gravity in the 1977 BMR data bank are held in the computer to the datum and scale of the May 1965 Isogal values.

In 1973 a new datum and scale were chosen to put the gravity control network as nearly as possible on to an absolute basis (BOULANGER ET AL 1973).

Three nearly independent values for the station Sydney A, designated 45331A by the International Gravity Bureau (IGB) are given by:

- International Gravity Standardisation Network 1971 (IGSN 71) (MORELLI ET AL 1971) — 979 671.86 ± 0.021 mgal (1σ)
- an absolute determination of gravity at an adjacent site (BELL ET AL 1973) — 979 672.00 ± 0.20 mgal (3σ)
- Soviet tie to Moscow and Potsdam — 979 671.84 ± 0.06 mgal (1σ) (GUSEV 1973)

The values agreeing within their stated accuracy, the IGSN 71 value was adopted. The change from the May 1965 Isogal value at Sydney A is -13.88mgal. σ is the standard deviation.

The milligal scale for the network has been determined accurately along only the most easterly of the north-south traverses, known as the Australian Calibration Line (ACL). The present scale was adopted in 1973 (BOULANGER ET AL 1973) and is based on Australian-Soviet gravity surveys along the ACL (WELLMAN ET AL 1974). Scale along this line, which had a gravity range of 3000mgal, was defined to an accuracy (σ) of 2.5 parts in 10^5 by a group of eight Soviet GAG-2 gravity meters. Other work and calculations (WELLMAN ET AL 1974) have shown that this scale is compatible, to within experimental error, with absolute measurements and Gulf, Cambridge and OVM pendulum measurements in other countries. This scale does differ from that which can be calculated from the IGSN 71 values in Australia by 15 parts in 10^5, equivalent to 0.34mgal in the gravity interval Hobart-Port Moresby, a half wavelength distance of 3.5×10^3 km. The adopted scale is considered to be more accurate. The mean Australian milligal used as scale for the May 1965 Isogal values was shown to be in error by 5 parts in 10^4.

The internal precision of the Isogal network has recently been investigated by McCRACKEN (1977). Gravity intervals between base stations observed during the 1964-1967 Isogal surveys were corrected for Earth tides and meter drift and converted to intervals consistent with the 1973 gravity scale. The resulting intervals were adjusted to remove loop misclosures and a new set of gravity values (Isogal-74 values) were determined for the base stations with an estimated uncertainty of the order of ±0.1mgal. The May 1965 Isogal values were estimated to have uncertainties of the order of ±0.2mgal.

The May 1965 Isogal values can be adjusted for the 1973 change of datum and scale by a linear transformation:

$$g_{1973}^{(mgal)} = 979\ 671.86 + 1.000\ 511\ 8\left(g_{1965}^{(mgal)} - 979\ 685.74\right)$$

with an estimated σ of less than ±0.2mgal throughout Australia (DOOLEY & BARLOW 1976).

McCRACKEN (1977) compared the Isogal-74 values with those computed using the above linear transformation. The largest differences are found near Darwin (-0.21mgal), in Tasmania (-0.23mgal) and near Perth (+0.26mgal). The great majority of values agree to ±0.1mgal or better.

The values of observed gravity for AUSGAD 77 were obtained from the 1977 BMR data bank by applying the above linear transformation. They are considered to be on an absolute scale and datum to within ±0.3mgal (3σ) and to be compatible with IGSN 71 to the same accuracy. For further discussions on gravity datum and scale in Australia, see § 6.2.3.2 & § 6.2.3.3.

6.2.1.3 *Unsurveyed Areas*

Tenth degree equi-angular area means were computed for all subdivisions which were represented with gravity data. Most squares were represented by a single value. Free air anomalies were predicted at approximately 5% of the squares in the data bank containing over 76000 tenth degree area means. Prediction did not therefore constitute a significant operation in the preparation of AUSGAD 77.

MATHER (1975c) reports a lack of correlation between free air anomalies and height in North America for distances in excess of 50km. As the prediction interval in the preparation of the Australian data bank was always less than 50km, it was decided to convert all free air anomalies to Bouguer anomalies prior to predicting values for the unsurveyed areas. Elevations from the Australian tenth degree elevation data bank were used for this purpose. The prediction in each case was performed from a set of neighbouring values. The number in the set varied from three in the case of an optimum geometric disposition, to a maximum of eight - one for each octant. A linear predictor of the form:

$$E\{\Delta g_b\} = \sum_{i=1}^{n} (w_i \, \Delta g_{bi}) \Big/ \sum_{i=1}^{n} w_i$$

where w_i is a weight coefficient ranging from 5 to 1 as the distance from the point of prediction varied from 10km to 50km. The predicted free air anomaly is obtained from $E\{\Delta g_b\}$ by removing the Bouguer reduction.

6.2.1.4 *Final Free Air Anomaly Data Bank* - Δg_f

The final tenth degree equi-angular data bank was used to compute one degree equi-angular area means. One hundred tenth degree values were used to compute each one degree area mean. The resulting one degree equi-angular mean free air anomalies Δg_f are given in Tables 6.2A, B & C and illustrated in Figure 6.1. The free air anomalies were computed according to the formula (equation 2.51):

$$\Delta g_f = g - \gamma_o + 0.3086^{(mgal)} \, H^{(m)} \tag{6.18}$$

where H is the orthometric elevation on AHD. γ_o was computed using equation 6.10 in relation to GRS 67, defined by the parameters (IAG 1971, p.58):

$$a = 6\ 378\ 160 \text{ m} \quad ; \qquad f^{-1} = 298.247\ 167\ 427 \tag{6.19}$$

$$GM = 398\ 603.0 \text{ km}^3\text{s}^{-2} \quad ; \qquad \omega = 7.292\ 115\ 147 \times 10^{-5} \text{rad s}^{-1}$$

Longitude - Deg E

Latitude Deg S

Latitude	110.5			113.5			116.5			119.5	120.5			123.5			126.5			129.5
10.5														-117.3	-59.5	0.5	40.2	60.6	61.1	41.5
										26.6	-17.8	-10.3	40.7	24.1	28.0	29.5	44.5	35.0	24.8	11.8
									20.5	23.5	20.2	10.0	32.8	38.8	34.0	15.3	16.0	24.6	20.6	0.3
13.5									0.5	21.8	17.5	4.7	27.1	18.5	28.2	13.0	3.1	6.1	24.0	2.7
									-19.0	-7.3	4.0	23.6	23.5	23.8	24.3	16.7	22.1	34.1	8.0	5.0
					-1.1	-13.3	-8.6	-21.7	-37.0	-10.7	18.6	40.2	24.2	24.5	12.1	23.0	31.8	16.5	0.8	-8.8
16.5		-17.5	-4.8	9.1	2.0	-22.2	-43.0	-58.2	-22.7	12.5	23.8	29.1	24.0	27.8	3.6	27.6	30.8	28.5	-6.5	-16.6
	-10.3	-13.1	-11.7	11.2	-14.3	-20.2	-23.6	-43.0	-4.2	12.7	-3.8	5.0	8.8	18.8	18.3	5.3	15.1	47.1	-7.6	15.5
	-7.3	-20.5	-22.6	-8.0	-35.3	-37.5	-15.5	-5.2	10.6	8.0	9.5	0.5	6.8	7.5	1.0	4.0	10.5	16.3	-1.1	22.0
19.5	-7.6	-28.8	-10.7	-2.7	-47.5	16.8	11.5	-2.1	10.5	14.6	26.3	24.5	1.3	-9.7	5.7	0.8	-5.1	-2.3	19.0	26.6
20.5	-31.5	-16.1	-24.3	-35.3	-12.5	4.2	-9.5	-21.3	-20.5	2.0	8.0	27.3	5.5	-4.0	-6.0	0.8	0.8	4.5	5.8	2.7
	-29.1	-41.7	-38.5	-14.7	6.1	28.8	0.5	-10.0	-34.0	-9.5	2.0	0.7	20.7	-6.1	-13.2	-13.2	-7.0	7.8	7.7	11.6
	-35.5	-50.3	-19.3	14.3	19.0	17.3	22.0	1.2	-4.8	-17.2	-9.0	-5.1	22.0	10.1	-8.2	-16.5	-21.1	12.5	-1.0	-3.5
23.5	-54.0	-38.6	5.5	3.8	4.5	13.0	19.5	0.5	14.3	4.0	2.2	3.1	-7.0	-18.6	-8.8	0.5	-14.7	-2.5	-16.1	-17.3
	-37.7	-6.0	24.2	-10.0	-6.1	-1.2	-6.5	-21.2	-10.0	17.8	17.3	7.7	-4.2	-24.0	-19.0	4.3	-35.0	-22.5	-15.7	-24.2
	-19.5	-3.6	19.3	-6.6	-8.1	3.5	-3.0	-27.6	-22.5	-9.1	-5.7	0.5	-14.0	-19.0	-14.2	-15.6	0.8	8.6	-4.2	19.3
26.5	-23.5	-17.2	-2.1	-1.0	2.3	1.7	13.0	-7.0	-23.0	-1.6	3.7	-7.5	-16.3	-9.5	-4.3	-17.2	-12.3	9.0	21.3	56.1
		-33.6	-12.1	4.5	34.1	11.0	14.5	4.0	-18.7	-10.5	-7.1	-3.2	-3.2	-9.0	-8.0	-23.0	-31.0	-22.1	-49.0	-51.5
		-42.7	-32.3	-9.8	33.6	4.6	-4.0	-6.8	-3.8	-9.0	-18.1	-7.0	-2.8	-2.1	-21.0	-24.1	-20.5	-33.6	-46.8	-44.6
29.5			-44.2	-44.3	9.1	-26.8	-2.2	-3.8	-3.7	-5.8	-19.8	-29.1	-11.5	-23.0	-10.0	-13.1	-24.5	-35.0	-44.5	-34.1
30.5				-54.5	-14.7	-55.5	8.1	-1.0	-9.6	-9.5	-14.8	-10.3	-17.0	-42.3	5.5	-25.2	-15.3	-30.1	-47.3	-34.2
				-38.2	-22.2	-77.3	7.0	-10.1	-6.1	-2.1	-3.7	-7.6	-33.0	-14.6	-21.8	-16.0	-5.0	-21.2	-33.3	-20.7
	-18.3	-37.1	-36.5	-40.7	-6.6	-60.1	24.2	5.8	-2.1	-5.1	-14.3	-25.5	-15.2	-5.3	-16.8	8.8	10.6	1.5	-11.5	-16.7
33.5	17.8	-18.5	-17.5	-28.6	7.6	-23.0	31.7	21.1	4.7	-15.3	-20.0	-15.0	-20.0	-20.6	0.8	15.1	4.7	-10.0	-32.8	-6.8
	15.5	7.1	-8.1	-31.8	13.5	-12.0	13.6	8.3	-1.3	-9.6	-21.0	-34.6	-26.2	-25.8	-27.5	-39.1	-60.0	-61.8	-63.7	-64.1
	-36.5	-33.5	-45.2	-56.8	-39.0	-28.0	-1.5	-6.0	-35.8	-55.3	-48.2	-51.1	-44.5	-41.5	-47.0		-55.3	-55.0		-78.5
36.5						-49.0	-42.2	-45.2	-48.5								-51.0	-58.5		
						-24.7														
						-15.3														
39.5																				
40.5																				
43.5																				
46.5																				
49.5																				

TABLE 6.2A

Free Air Anomalies for Australia

- Western Section (mgal)

$1^{o} \times 1^{o}$ Equi-Angular Area Means on GRS 67

Longitude - Deg E

Latitude Deg S	130.5			133.5			136.5			139.5	140.5			143.5			146.5			149.5
10.5	47.2	42.7	47.6	30.5					23.0		-22.5	20.1	35.0	53.7	22.6	22.7	7.5	-25.0	7.3	57.7
	46.1	38.5	36.8	34.1	16.5	46.0	21.5	24.3	26.5	0.3	6.1	1.2	32.3	41.3	7.8	27.5	29.0	2.6	-4.8	-12.5
	36.5	31.6	30.6	37.2	15.8	41.2	17.8	15.2	19.0	0.1	2.5	6.8	28.3	38.1	-17.0	9.1	16.5	12.1	6.2	7.5
13.5	19.7	29.0	31.0	40.5	16.6	17.2	17.3	19.1	6.7	2.8	0.5	15.5	15.0	36.8	-13.5	0.6	12.3	1.1	-6.5	8.0
	11.7	5.5	14.5	20.5	11.7	11.3	27.0	17.2	0.5	0.5	-5.5	12.3	16.5	27.0	47.2	27.8	-1.0	24.0	18.2	-8.0
	-3.7	-2.0	-5.7	4.6	10.6	8.0	23.0	14.6	-1.2	-5.7	-7.8	-3.6	15.0	27.5	36.7	35.0	-14.5	24.0	30.7	32.0
16.5	2.5	0.5	10.5	-6.7	2.0	13.0	9.5	3.2	15.1	15.1	6.5	3.8	16.1	15.7	26.6	27.5	1.2	3.2	18.0	23.7
	16.1	5.7	13.8	-3.8	13.6	17.5	13.2	3.5	12.8	25.6	8.5	2.6	12.3	13.2	25.2	33.7	20.2	-16.1	9.5	23.2
	17.0	-4.0	0.1	8.0	6.8	3.5	9.1	17.0	1.3	19.5	17.0	-4.2	14.2	24.3	39.3	38.5	12.7	-2.0	-5.2	0.8
19.5	7.6	-10.1	-7.3	6.5	6.7	0.1	-3.1	9.5	1.5	27.1	25.7	-8.3	7.0	35.5	45.7	33.2	25.8	27.0	19.7	10.5
20.5	12.7	-4.0	-17.0	-10.5	14.2	0.2	3.0	1.5	11.5	37.2	31.3	2.0	-6.0	0.3	23.3	32.1	31.1	26.2	18.5	27.5
	2.2	5.5	17.7	22.6	12.7	11.7	13.0	0.3	14.5	36.0	29.8	15.3	0.5	6.0	0.3	14.5	27.3	25.2	24.3	25.1
	-6.1	4.7	-6.2	7.5	22.8	12.5	14.0	11.5	13.5	22.3	35.2	20.6	3.3	0.2	8.5	16.6	27.1	31.8	17.5	28.3
23.5	-21.1	-44.3	-18.5	0.6	-6.0	8.0	14.5	7.3	2.0	18.6	30.0	11.6	-5.1	-13.6	-1.7	10.0	23.5	33.8	15.1	17.3
	-44.0	-31.2	-36.5	-37.0	-34.1	-3.8	10.2	5.0	4.3	13.6	11.6	-8.3	-12.7	-18.0	-2.3	1.1	16.2	31.6	20.8	16.5
	-19.2	-45.8	-45.5	-33.5	-7.7	-13.5	-14.2	-11.1	-11.5	-7.6	-12.8	-8.6	-15.8	-17.1	-12.3	0.8	9.6	25.6	20.0	10.0
26.5	32.0	19.1	7.0	-1.5	-13.3	-23.5	-9.6	-15.8	-5.6	-14.7	-11.3	-16.5	-10.0	-13.7	-5.6	-3.1	9.1	8.7	0.6	-6.5
	-30.5	-50.7	-74.0	-15.0	-3.8	-20.1	-10.5	-17.0	-18.7	-14.7	-16.5	-7.5	-5.7	-2.5	4.2	9.7	13.6	-2.8	2.8	0.6
	-45.8	-39.0	6.6	3.0	1.5	-5.6	-3.7	-13.2	-11.5	-17.7	-16.7	-5.6	-6.6	-5.7	2.8	3.1	-5.3	5.3	2.1	-2.5
29.5	-32.2	2.0	7.8	4.1	6.6	6.5	-10.0	-8.5	0.7	-6.7	0.0	13.5	1.7	8.0	5.0	4.0	7.2	12.0	2.1	-1.7
30.5	-31.5	1.0	7.2	-5.5	-21.0	-12.7	-8.1	-4.0	19.5	7.5	-5.7	8.8	12.7	3.6	-3.0	6.5	13.0	11.3	-2.7	3.5
	-37.5	3.7	-22.8	-29.0	-16.0	1.3	0.2	-10.5	14.1	-7.7	-1.3	28.0	7.0	-21.8	-17.6	1.0	14.5	12.0	6.6	23.2
	-15.5	19.6	-48.2	-46.5	-10.0	4.3	9.8	-6.0	19.8	20.2	15.6	17.3	-14.2	-29.6	-12.0	1.3	-2.2	16.2	18.8	26.2
33.5	-26.0	5.8	-14.3	-15.6	-19.5	-1.5	14.5	-1.6	23.0	15.5	-1.5	-14.5	-16.3	-9.3	-10.5	-1.0	0.6	14.1	17.8	51.6
	-31.6	-41.7	-24.0	14.3	15.7	19.5	10.2	19.7	8.5	13.2	-16.7	-15.3	-6.6	-3.1	-4.0	0.2	-3.8	8.2	17.7	42.1
	-67.0	-41.1	-33.7	-37.3	-37.2	1.8	23.3	18.6	9.5	0.6	0.0	2.5	-3.5	0.7	0.7	-3.6	1.2	0.8	39.1	46.8
36.5		-62.5	-72.1	-75.0	-73.0	-67.5	-1.8	21.2	11.6	0.5	9.2	9.8	7.1	1.8	2.7	-1.1	0.2	20.2	55.7	49.5
							-60.0	-42.1	-21.1	-1.0	7.0	21.5	32.5	19.7	19.2	12.5	57.0	31.0	20.0	32.5
									-47.2	-43.5	-18.0	7.8	11.5	17.1	0.0	7.5	6.5	-19.0	-17.2	-25.0
39.5										-17.5	-42.0	-43.6	-21.5	26.7	40.6	-14.0	0.3	-6.8	21.0	-39.0
40.5												-39.8	-53.6	23.5	20.6	17.0		10.5	30.1	-30.0
													-42.0	-49.3	28.3	39.8	29.6	23.0	24.6	-37.6
														-43.5	-24.3	30.5	43.0	31.3	13.7	-44.2
43.5															-38.6	9.0	31.2	24.0	-25.2	-27.6
														-20.2	-37.6	-27.0	-2.0	-17.5	-33.3	-22.7
														-14.6	-32.1	-12.3	-10.6	-27.8	-27.0	
46.5																		-32.0	-28.5	
49.5																				

TABLE 6.2B

Free Air Anomalies for Australia

\- Central Section (mgal)

$1^{o} \times 1^{o}$ Equi-Angular Area Means on GRS 67

Longitude - Deg E

Latitude Deg S	150.5			153.5			156.5			159.5	160.5			163.5	166.5	169.5
10.5	96.5	63.5	59.6	55.0	54.6	49.7	45.0						62.8	-2.1		
	-2.6	44.6	71.8	68.6	54.8	22.3										
	-5.6															
13.5	10.6															
	-12.6	-15.5	-4.5													
	-30.0	13.6	0.0	-12.6	-2.7	10.5										
16.5	28.5	30.5	13.6	9.2	8.0	6.0	4.7									
	20.5	21.0	24.0	15.5	10.6	27.3	8.0									
	-2.0	-6.3	3.0	-2.5	18.0	21.1	12.8									
19.5	13.7	37.0	28.0	4.3	0.0	-3.5	19.7									
20.5		14.0	21.5	-4.1	-7.0	5.1	1.3									
	19.8	23.1	34.5	12.5	-23.5	-2.2	13.1	7.5	-17.6	-18.0						
	12.3	20.0	15.3	15.2	5.6	16.5	-4.5	-1.5	-15.8	33.6						
23.5	27.5	16.3	15.8	2.2	-1.8	0.5	11.0	0.8	-4.2	25.0						
	35.5	14.7	21.0	12.6	-32.3	-9.7	-1.2	3.7	-4.7	10.2						
	36.7	29.0	21.0	42.5	-32.8	-9.1	-13.5	3.5	-8.1	36.6						
26.5	24.5	32.1	30.3	41.5	-48.6	-7.8	-11.7	2.5	-12.2	1.5						
	5.7	36.5	28.5	40.0	-33.5	17.7	-20.8	1.2	0.2	-1.0						
	0.0	22.3	26.1	40.5	-45.3	1.6	-26.6	-5.8	-2.5	-3.1						
29.5	13.0	33.6	19.0	41.0	-33.3	-11.1	-17.0	3.2	13.5	8.0						
30.5	30.5	49.6	26.0	16.0	-22.1	-21.3	18.6	2.3	-18.3	0.3						
	17.5	45.5	33.0	-2.5	-17.6	-15.8	-20.7	2.5	-9.1	39.5						
	9.0	20.6	39.8	-59.7	-15.5	-20.0	5.2	-5.0	-27.6	-18.3						
33.5	28.3	32.2	9.0	-55.2	-30.8	-1.3	35.5	-4.6	-10.6	-17.7						
	42.7	11.1					13.5		-13.2	-9.5						
	34.5	-37.2				-11.1	9.2	-1.2								
36.5	0.8			-28.3	-20.8	-18.7	30.8									
	0.3	-36.5	-26.2	-27.0			11.7									
	-49.7	-54.8					9.2									
39.5	-32.3	-17.8	-12.8	0.5	-9.6	-10.0	-6.3									
40.5																
		-19.1														
43.5	3.7	-16.7														
	-12.2	-18.8														
46.5																
49.5																

TABLE 6.2C

Free Air Anomalies for Australia

- Eastern Section (mgal)

$1^{\circ} \times 1^{\circ}$ Equi-Angular Area Means on GRS 67

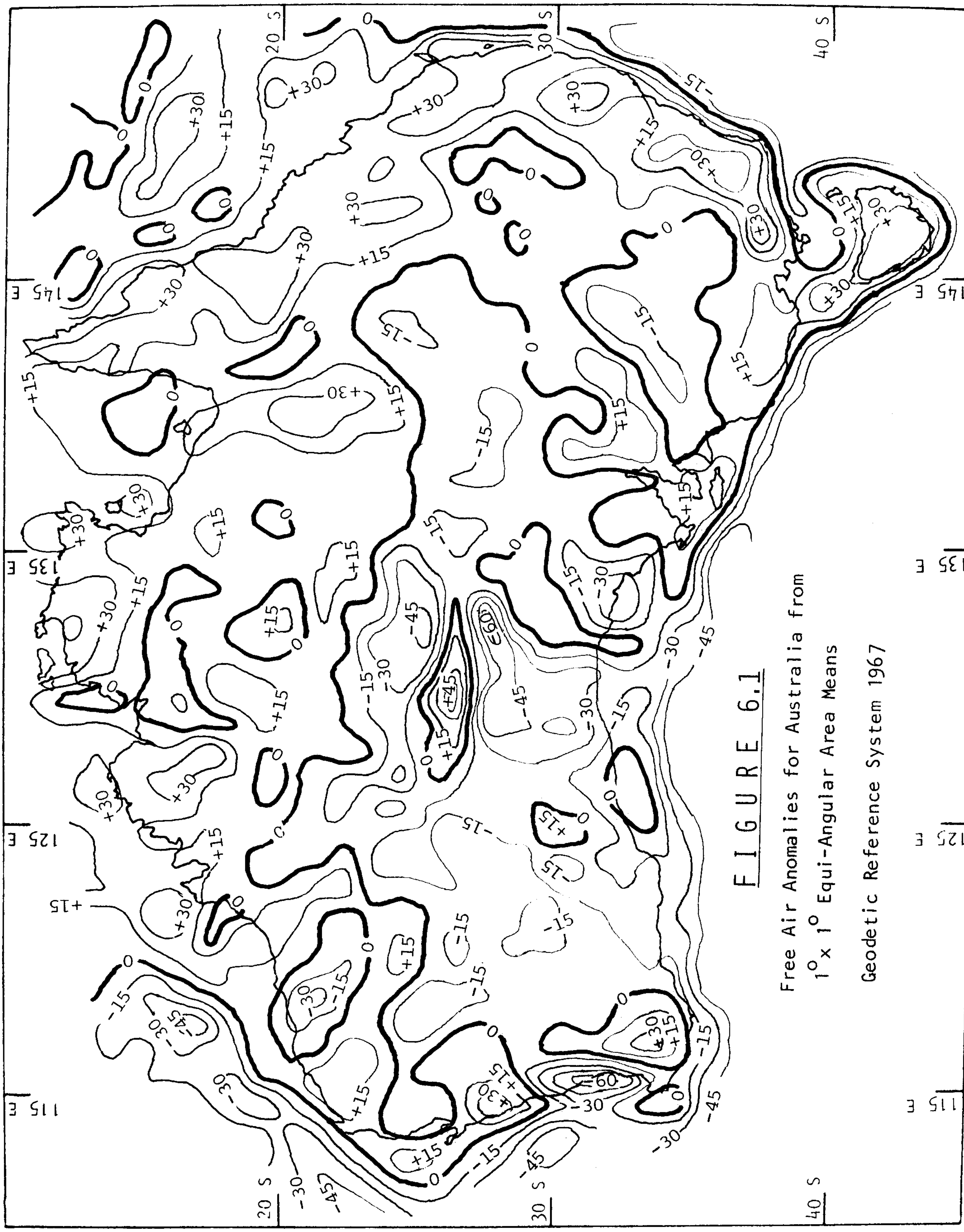


FIGURE 6.1

Free Air Anomalies for Australia from

$1^{\circ} \times 1^{\circ}$ Equi-Angular Area Means

Geodetic Reference System 1967

Contour Interval - 15 mgal

The entire gravity data bank is controlled by the Isogal gravity control network described in § 6.2.1.2 and the AHD (GRANGER 1972) obtained from the Australian levelling survey (ROELSE ET AL 1971). However, the AHD is not a level surface of the earth's gravity field. It is defined as follows (GRANGER 1972, p.228):

> "On 5th May 1971, the Division of National Mapping Council of Australia, carried out a simultaneous adjustment of 97320 kilometres of the two-way levelling in Australia holding mean sea level fixed at thirty tide gauges around the mainland coast. The resulting datum surface, with minor modifications in two metropolitan areas, has been called the Australian Height Datum (AHD) 1971."

Tables 6.3A & B give the error of representation $E\{\Delta g\}_{10}$ of values in the $1^{o} \times 1^{o}$ gravity anomaly data bank, computed from:

$$E\{\Delta g\}_{10} = \pm\left(10^{-2} \sum_{i=1}^{100} \left(\Delta g_i - M\{\Delta g\}_{10}\right)^2\right)^{\frac{1}{2}} \qquad (6.20)$$

where $M\{\Delta g\}_{10}$ is the mean free air anomaly for the $1^{o} \times 1^{o}$ square and Δg_i is the tenth degree equi-angular area mean at the ith subdivision.

No attempt has been made to adjust free air anomalies to the mean elevation of the surface subdivision in instances where gravity station elevations differ from the former, as the reliability of the mean elevation data bank for Australia was not considered to be of sufficient precision to warrant making such a correction. This should not pose a serious drawback in sea surface topography studies as the Australian land mass does not contain much topographic variation except in the east and south-east. These are regions where the free air anomaly means may tend to underestimate the true area mean. However, the possibility exists that all free air anomaly values for Australia may require a positive correction of around 0.2mgal on the average to account for this effect.

6.2.2 CORRECTIONS NEEDED TO PRODUCE A GRAVITY ANOMALY DATA BANK FOR AUSTRALIA FOR SEA SURFACE TOPOGRAPHY DETERMINATIONS

6.2.2.1 *Preamble*

The free air anomaly data bank represented in Figure 6.1 is inadequate for sea surface topography determinations for the following reasons:

(i) Latitudes are based on Australian Geodetic Datum (AGD) coordinates and not those on an equivalent geocentric ellipsoid.

(ii) Elevations are based on AHD and not on a free net adjustment of the Australian levelling survey.

(iii) The quantity required in the first instance is the gravity anomaly Δg computed using equations 6.1 and 6.3, and not the free air anomaly Δg_f obtained from equation 6.18.

(iv) The quantity required for sea surface topography determinations is the computable part Δg_c of the spherically harmonic pseudo gravity anomaly $\Delta g''$, given by equation 6.17 for the "higher" reference model.

§ 6.2.2.2 to 6.2.2.5 deal in depth with the corrections to be made on these counts.

Longitude - Deg E

Latitude - Deg S

Lat	110.5	111.5	112.5	113.5	114.5	115.5	116.5	117.5	118.5	119.5
10.5										
										0
									4	6
13.5									15	27
									18	17
					5	6	4	6	22	12
16.5		4	21	21	38	59	26	30	27	13
	2	2	14	18	16	19	9	27	21	11
	10	21	9	7	16	20	19	22	8	10
19.5	9	13	9	14	12	37	27	15	12	8
20.5	17	15	16	14	33	26	20	11	12	19
	13	10	17	26	20	12	24	20	12	22
	7	11	20	24	9	8	14	20	26	16
23.5	8	17	12	6	10	8	8	11	14	20
	11	15	7	9	9	15	15	17	18	18
	7	16	7	11	9	14	18	21	23	21
26.5	16	16	13	6	12	16	10	13	17	20
		10	14	8	19	19	6	12	9	12
		4	16	19	16	15	10	8	14	10
29.5			19	31	30	35	10	11	9	12
30.5				14	46	39	11	7	7	18
				6	19	27	20	8	7	9
	20	3	5	13	30	19	12	10	8	10
33.5	16	16	12	16	24	39	9	10	8	11
	15	16	10	15	15	31	23	19	16	19
	16	17	14	10	33	39	37	38	42	14
36.5						18	3	0	2	
						27				
						10				
39.5										
40.5										
43.5										
46.5										
49.5										

Lat	120.5	121.5	122.5	123.5	124.5	125.5	126.5	127.5	128.5	129.5
10.5				11	53	41	18	11	15	8
	20	10	27	44	15	14	8	12	14	14
	15	10	20	14	8	17	15	7	7	9
13.5	19	25	14	9	11	8	10	13	28	28
	12	20	10	10	8	10	10	9	18	18
	19	6	11	7	8	15	7	13	10	10
16.5	7	10	12	15	11	15	10	10	20	15
	14	12	9	8	15	16	14	19	32	18
	19	10	8	7	13	13	12	22	14	12
19.5	16	13	7	7	18	13	8	10	13	12
20.5	23	16	15	5	8	9	6	6	13	16
	16	13	9	6	4	6	13	11	13	18
	11	11	13	13	8	7	9	12	11	10
23.5	19	11	11	15	12	10	10	8	13	32
	14	12	12	7	10	13	17	16	10	20
	12	13	16	6	8	14	26	41	20	24
26.5	14	11	18	14	6	7	20	41	41	28
	9	10	11	9	9	7	6	9	15	22
	11	16	11	7	8	9	18	9	7	10
29.5	11	12	10	14	21	16	15	15	7	7
30.5	8	14	13	14	27	8	10	20	9	7
	6	8	21	34	16	10	13	23	8	7
	7	15	31	20	19	8	8	11	15	7
33.5	14	12	8	7	12	11	11	20	38	15
	31	23	26	41	37	27	20	15	12	16
	7	13	5	4	6		13	14		5
36.5							0	0		
39.5										

Lat	130.5	131.5	132.5	133.5	134.5	135.5	136.5	137.5	138.5	139.5
10.5	6	7	7	0					4	
	11	14	12	10	6	11	6	0	8	0
	13	10	10	12	7	16	11	6	12	5
13.5	11	11	14	11	8	9	10	3	11	5
	14	7	10	9	5	4	5	17	4	6
	13	8	7	7	8	8	12	13	7	6
16.5	9	9	6	7	10	8	9	6	10	13
	9	6	5	11	11	10	8	8	9	5
	9	10	9	8	5	6	15	8	10	12
19.5	12	11	11	9	7	8	9	9	14	10
20.5	10	25	20	15	12	16	12	6	11	15
	25	25	27	13	9	7	12	9	10	16
	32	30	37	23	13	11	9	7	9	11
23.5	47	34	52	48	33	17	12	8	11	8
	18	15	14	10	11	18	9	10	10	14
	19	20	16	14	9	9	8	6	10	6
26.5	30	33	33	28	13	13	4	5	6	8
	33	42	25	43	12	12	8	8	10	6
	11	28	23	14	10	10	11	9	5	5
29.5	8	11	5	12	16	15	6	10	8	15
30.5	7	15	8	17	9	10	7	6	28	27
	7	18	22	14	12	12	7	8	29	15
	8	14	9	7	13	10	6	10	18	12
33.5	17	27	17	19	15	12	14	12	25	13
	7	7	17	14	12	15	11	12	16	12
	17	18	7	8	28	37	12	14	20	10
36.5		9	14	15	14	42	48	18	16	13
							11	25	31	15
									15	11
39.5										13

TABLE 6.3A

Error of Representation of Free Air Anomalies

- Western Section (± mgal)

Free Air Anomalies - $1^{o} \times 1^{o}$ Squares

Longitude - Deg E

Latitude - Deg S	140.5			143.5			146.5			149.5	150.5			153.5			156.5			159.5
10.5	13	10	12	10	13	17	24	19	43	56	32	41	44	40	30	39	25			
	7	10	11	8	17	9	14	9	12	9	19	61	54	31	31	80				
	6	15	16	13	15	13	16	15	9	8	10									
13.5	6	12	10	15	16	11	16	11	8	13	3									
	14	16	15	14	20	32	19	8	21	15	9	5	1							
	6	12	11	18	11	18	14	14	8	8	11	22	16	6	9	13				
16.5	12	10	9	12	16	16	26	18	12	14	6	10	13	13	13	17	19			
	9	5	8	11	12	26	13	17	18	16	12	11	12	14	16	25	20			
	10	5	9	14	15	24	18	19	14	14	8	9	18	8	23	25	19			
19.5	17	9	13	16	12	14	15	13	7	6	8	8	14	8	14	13	29			
20.5	17	9	7	9	17	13	9	13	17	7		13	10	13	17	26	16			
	9	8	8	7	4	11	5	8	16	14	8	7	7	16	18	27	26	22	25	3
	12	9	6	8	7	8	11	10	10	16	14	5	7	11	22	63	16	10	5	54
23.5	7	11	10	8	8	11	7	9	8	16	18	7	13	19	32	32	21	13	10	62
	12	7	7	7	9	6	10	16	8	5	15	13	6	49	19	34	15	13	16	46
	9	10	7	7	9	9	9	16	13	11	22	11	9	22	36	37	13	13	16	51
26.5	7	7	10	8	10	10	14	13	8	6	21	12	13	23	27	26	9	18	12	29
	7	6	8	4	8	7	9	6	7	7	13	11	18	18	32	63	14	11	15	17
	7	9	10	8	9	5	9	9	9	7	9	16	20	22	21	65	17	11	20	15
29.5	6	11	8	9	8	10	10	11	9	6	12	24	21	23	20	22	21	26	18	27
30.5	11	14	12	12	18	9	10	7	14	12	22	21	42	47	19	19	81	20	18	19
	9	14	19	8	11	12	9	11	10	12	13	31	28	60	11	13	13	18	19	49
	11	18	14	11	12	8	10	11	9	15	15	18	18	32	14	12	37	21	21	28
33.5	11	7	11	8	7	5	7	8	21	22	27	13	46	14	12	32	71	18	20	21
	9	6	4	6	5	7	8	13	11	15	21	39					4		11	22
	7	8	5	8	9	8	10	16	36	21	30	40				4	27	5		
36.5	6	7	8	7	7	9	23	49	52	25	48			6	15	15	11			
	8	11	12	12	18	20	48	41	33	19	56	8	5	4			7			
	27	16	11	17	13	11	26	12	36	42	27	0					11			
39.5	7	3	23	16	12	2	6	13	22	19	1	4	5	28	7	7	15			
40.5		7	12	34	17	5		7	14	14										
			5	15	24	21	35	26	17	10										
				10	29	23	23	19	15	10		2								
43.5					22	32	27	13	35	12	24	13								
				5	48	25	20	25	13	15	11	9								
				2	25	24	22	10	7											
46.5								4	4											
49.5																				

Latitude - Deg S	160.5			163.5			166.5			169.5
			44	13						

T A B L E 6.3B

Error of Representation of Free Air Anomalies

- Eastern Section (± mgal)

Free Air Anomalies - $1^{\circ} \times 1^{\circ}$ Squares

6.2.2.2 *Latitude Corrections for the Geocentric Orientation Vector*

The centre of the ellipsoid defining a regional geodetic datum is, in general, displaced from the geocentre. Estimates of the magnitudes of these displacements are generally available (see e.g. LERCH ET AL 1974). The vector separation of centres is called the geocentric orientation vector and can be represented by three geocentric orientation parameters:

(1) $\Delta\xi_o$ in the meridian

(2) $\Delta\eta_o$ in the prime vertical

(3) ΔN_o along the normal

in the local Laplacian triad at the origin of the regional datum. The change $\Delta\xi_o$ obtained on shifting the centre of the regional ellipsoid to coincide with the geocentre is related to the resultant change in latitude $\delta\phi_o$ according to:

$$\Delta\xi_o = - \delta\phi_o \tag{6.21}$$

Gravity anomalies computed using latitudes referred to the regional ellipsoid can be corrected so that they refer to an equivalent geocentric ellipsoid by applying the correction δg_ϕ at the point whose latitude is ϕ and longitude λ, defined by (MATHER 1973a, p.16):

$$\delta g_\phi = -\gamma_e\beta_1\sin2\phi\left[\Delta\xi_o(\cos\phi_o\cos\phi + \sin\phi_o\sin\phi\cos\delta\lambda) + \Delta\eta_o\sin\phi\sin\delta\lambda - \frac{\Delta N_o}{R}(\sin\phi_o\cos\phi - \cos\phi_o\sin\phi\cos\delta\lambda)\right] + o\{f\delta g_\phi\} \tag{6.22}$$

where

$$\delta\lambda = \lambda - \lambda_o \tag{6.23}$$

(ϕ_o,λ_o) being the latitude and longitude of the regional geodetic origin, R the mean radius of the Earth, γ_e and β_1 having the same definition as in equation 6.8. Several estimates are available for $\Delta\xi_o$, $\Delta\eta_o$ and ΔN_o for the AGD. Figure 6.2 illustrates the corrections using values obtained by MATHER ET AL (1971, pp.15 & 24):

Australian Geodetic Datum Geocentric Orientation Vector:

$$\Delta\xi_o = -4.0 \text{ arcsec} \; ; \quad \Delta\eta_o = -4.1 \text{ arcsec} \; ; \quad \Delta N_o = 14.3 \text{ m} \tag{6.24}$$

As the latitudes of all gravity stations in Australia are based on data related to the AGD, the corrections shown in Figure 6.2 should be applied prior to use in high precision gravimetric computations. The precision of the corrections defined by equation 6.24 is currently estimated at $\pm$0.2 arcsec or its equivalent. Thus the errors in the corrections illustrated in Figure 6.2 are unlikely to be greater than $\pm 5\mu$gal. This is adequate for sea surface topography determinations at the $\pm$10cm level.

6.2.2.3 *Corrections δg_h for Conversion of Elevations from AHD to a Free Net Adjustment Based on the Jervis Bay Datum Level Surface*

These corrections are necessary if the Australian gravity data bank is to play a meaningful role in the definition of sea surface topography for

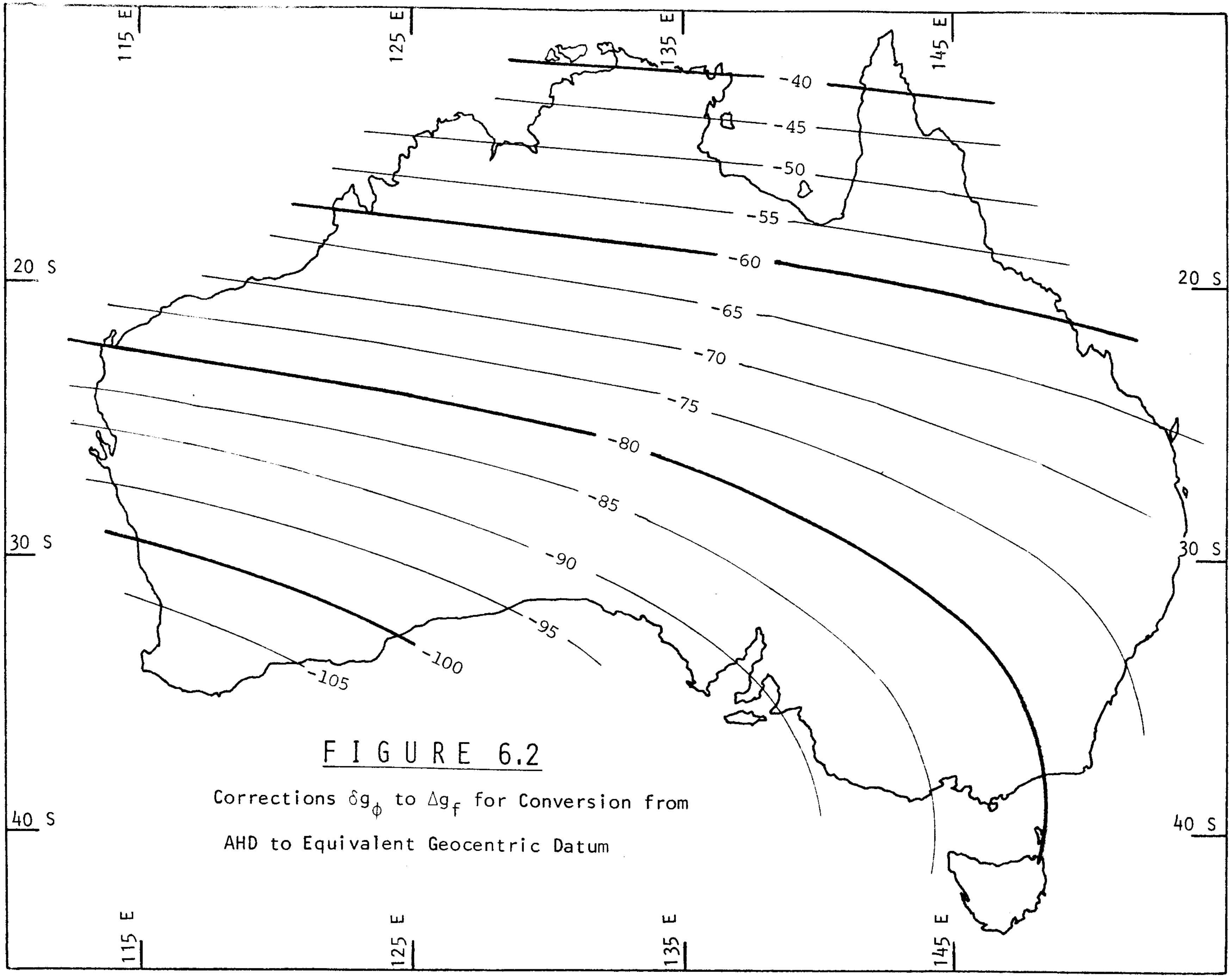


FIGURE 6.2

Corrections δg_ϕ to Δg_f for Conversion from AHD to Equivalent Geocentric Datum

Source: MATHER ET AL (1976b)

reasons given in sub-section 6.1.1. Uncertainty surrounds the relation of the geoid to geodetic levelling networks through estimates of mean sea level at coastal tide gauges. Local mean sea level at the 30 tide gauges connected to the Australian levelling survey does not appear to lie on the same level surface (see Table 6.4).

TABLE 6.4

Heights of MSL at Tide Gauges in Australia - Referred to the Jervis Bay Datum Level Surface

Gauge Identifier	Height(m)	Gauge Identifier	Height(m)
Port Kembla	1.05	Broome	1.27
Camp Cove	1.05	Port Hedland	1.39
Coffs Harbour	1.06	Carnarvon	1.20
Brisbane	1.24	Geraldton	1.27
Bundaberg	1.43	Fremantle	1.59
Mackay	1.73	Bunbury	1.75
Townsville	1.89	Albany	1.58
Cairns	2.12	Esperance	1.16
Cooktown	2.59	Eucla	0.92
Bamaga	2.81	Thevenard	0.89
Weipa	2.65	Port Lincoln	0.77
Karumba	2.08	Victor Harbour	1.03
Centre Island	1.66	Port Macdonnell	1.12
Darwin	2.04	Port Fairy	1.20
Wyndham	1.92	Port Lonsdale	1.15

It is therefore most desirable that the Australian levelling survey be treated as a free network for the computation of gravity anomalies. For this purpose, the Australian free network is referred to the tide gauge at Jervis Bay. The latter, called the Jervis Bay Datum Level Surface, was adopted on the recommendation of the Division of National Mapping, Canberra (LEPPERT 1976).

Although connections to the 30 tide gauges around the coastline of Australia played a major role in the definition of the AHD (GRANGER 1972), MITCHELL (1972) has made the observation that the conditions and operating methods of many gauges left much to be desired. No single authority was responsible for the maintenance of all the gauges and many have since ceased operation. There are very few tide gauges in Australia which have an acceptable location for serving as a National Datum Level Surface for geodetic elevations. On the east coast of Australia, only the Jervis Bay tide gauge satisfies the criteria for a geodetic elevation datum:

- ease of access;
- acceptable site location, free from effects of river water flow and other aberrations likely to unfavourably influence the calculation of mean sea level; and
- guaranteed continuity of operation over long periods of time by a national agency.

The Jervis Bay tide gauge is owned and operated by the Division of National Mapping. The gauge was installed in 1970 and was not included in the AHD.

Levelling connection between the zero mark of the tide gauge (defining the Jervis Bay Datum Level Surface) and an adjacent point of the AHD was carried out by the Central Mapping Authority of New South Wales. The height of the 30 tide gauges with respect to the Jervis Bay Datum Level Surface is given in Table 6.4.

The differences between free net orthometric elevations and AHD heights provided by the Division of National Mapping, were computed after conversion of the former to the Jervis Bay Datum Level Surface. These differences δh as computed at 497 junction points in the Australian levelling survey were analysed by a two dimensional Fourier series of the form:

$$\delta h^{(m)} = \sum_n \sum_m \sum_\alpha c_{\alpha nm} F_{\alpha nm} \tag{6.25}$$

The quantities $c_{\alpha nm}$ in equation 6.25 are Fourier coefficients, while:

$$F_{1nm} = \cos(n\,\Delta\phi + m\,\Delta\lambda) \;;\quad F_{2nm} = \sin(n\,\Delta\phi + m\,\Delta\lambda) \tag{6.26}$$

$\Delta\phi, \Delta\lambda$ are differences in latitude and longitude in relation to some convenient point of reference. The error of fit obtained by restricting $n,m < 8$ and neglecting the higher order interaction of harmonics (i.e. 161 coefficients) at the 497 junction points was $\pm 8\mu$gal.

The correction δg_h was then defined at the general point (ϕ,λ) by using the relation:

$$\delta g_h^{(\mu gal)} = 308.6^{(\mu gal)} \sum_n \sum_m \sum_\alpha c_{\alpha nm} F_{\alpha nm} \tag{6.27}$$

where the constant in equation 6.27 is the approximate vertical free air gravity gradient (equation 2.48). The distribution of δg_h with position is illustrated in Figure 6.3.

6.2.2.4 *The Correction from Free Air Anomalies to Gravity Anomalies*

The required correction δg_f is obtained from equations 6.1, 6.3 and 6.18 as:

$$\delta g_f^{(\mu gal)} = -10^9 \times 2\,\Delta W^{(kgalm)} \left(1 + f + m - 2f\sin^2\phi + \frac{\Delta W}{2a\gamma}\right)/a^{(m)} - 308.6^{(\mu gal)} H^{(m)} \tag{6.28}$$

where all the above quantities have been defined in § 6.1.2.1. Values of δg_f were computed at the 497 junction points of the Australian levelling survey from free net adjustment elevations H and values of geopotential differences ΔW as computed by MITCHELL (1972) and referred to the Jervis Bay Datum Level Surface (§ 6.2.2.3).

A two dimensional Fourier series of the type described by equations 6.25 and 6.26 was fitted to this data using equation 6.28. The resulting coefficients were used to generate values of δg_f at the general point (ϕ,λ) in the Australian region. The fit of the series to the values of δg_f at the 497 junction points was not as close as that obtained in § 6.2.2.3. This was due to the strong correlation of the correction δg_f with topography. The root mean square residual for comparisons at the 497 junction points was $\pm 130\mu$gal. However, it was estimated that all wavelengths greater than about 5×10^2km were represented in the Fourier

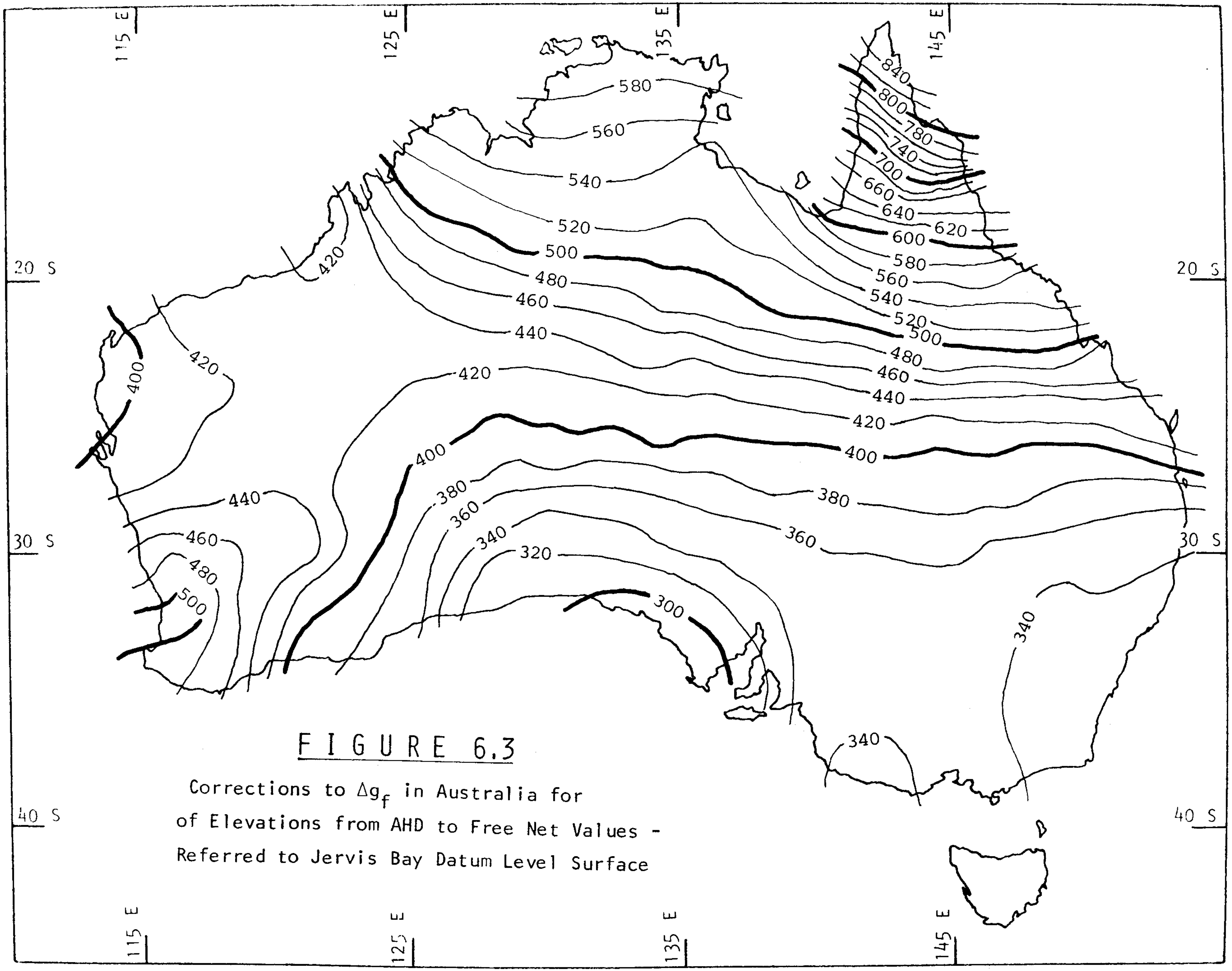


FIGURE 6.3

Corrections to Δg_f in Australia for of Elevations from AHD to Free Net Values - Referred to Jervis Bay Datum Level Surface

Source: *MATHER ET AL (1976b)*

Contour Interval - 20 μgal

model generated. The residual effect on sea surface topography determinations is estimated as ±5cm. This is not considered to be of significance unless detailed studies take place close to the south-eastern seaboard of Australia where a more detailed representation of δg_f will be necessary. This could be obtained by computing δg_f at individual bench marks instead of the more widely spread junction points in regions of considerable topographic variation.

Figure 6.4 illustrates this correction for Australia when considering effects with wavelengths in excess of 5 x 10^2km.

6.2.2.5 *Corrections Required to Obtain the Computable Part* Δg_c *of the Pseudo Gravity Anomaly* $\Delta g''$

The required correction is defined by equation 6.17. The correction is made up of two distinct constituents:

- the atmospheric correction δg_a; and
- the correction to the "higher" reference model defined by equation 6.12.

The atmospheric correction δg_a was investigated in section 4.5 for the Smithsonian model of the atmosphere (ANDERSON ET AL 1975). This ellipsoidal atmospheric model has an irregular lower boundary coincident with the topography and oceans. The values of this correction have a magnitude (0.85 - c) mgal where c takes values in the range $0 \leq c$ (mgal) ≤ 0.4 as a function of elevation (for elevations up to 5km). It was concluded that apart from the dominant effect of zero degree (0.85 mgal), the effect of the atmosphere could be adequately represented by low degree harmonic models of δg_a. This is especially true in the case of Australia where the topographic variations are minimal in all regions except those adjacent to the Pacific littoral. Figure 6.5 shows values of δg_a computed using the approximate methods described in section 4.5. The neglected higher degree terms are likely to have amplitudes of up to 0.1mgal and wavelengths of up to 3 x 10^2km. The resultant effect on determinations of sea surface topography is estimated to be less than ±3cm.

Figure 6.6 shows the computable part Δg_c of the pseudo gravity anomaly $\Delta g''$ referred to the "higher" reference model defined by Goddard Earth Model (GEM) 9 (LERCH ET AL 1977), incorporating all the corrections described above. The resulting data bank is called the Australian Gravity Anomaly Data Bank 1977 (AUSGAD 77). The "higher" reference model for the Australian Gravity Anomaly Data Bank 1976 (AUSGAD 76) was defined by GEM 7 (WAGNER ET AL 1976).

6.2.3 RESIDUAL ERRORS IN THE AUSTRALIAN GRAVITY ANOMALY DATA BANK 1977 (AUSGAD 77)

6.2.3.1 *Preamble*

The residual sources of error in AUSGAD 77 are those which, though in themselves small, could affect results because they occur with significant wavelength. The effect of any error e_g in the gravity anomaly Δg which holds its magnitude over a n^o x n^o area but behaves as an accidental error over larger extents, has been shown to have a probable cumulative effect e_ζ on quadrature determinations of the geoid governed by a relation of the

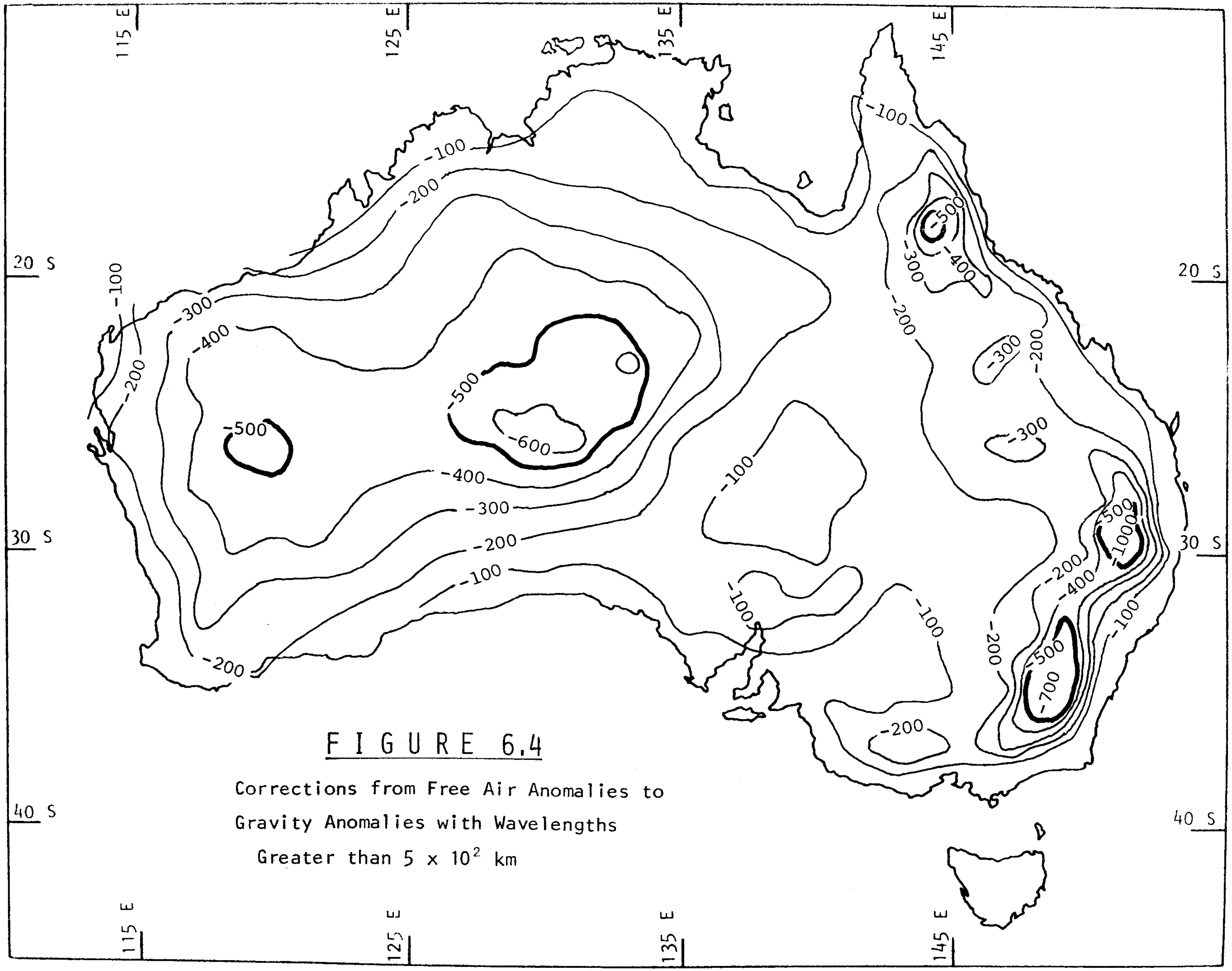


FIGURE 6.4

Corrections from Free Air Anomalies to Gravity Anomalies with Wavelengths Greater than 5 x 10^2 km

Source: *MATHER ET AL (1976b)*

Contour Interval - 100 μgal

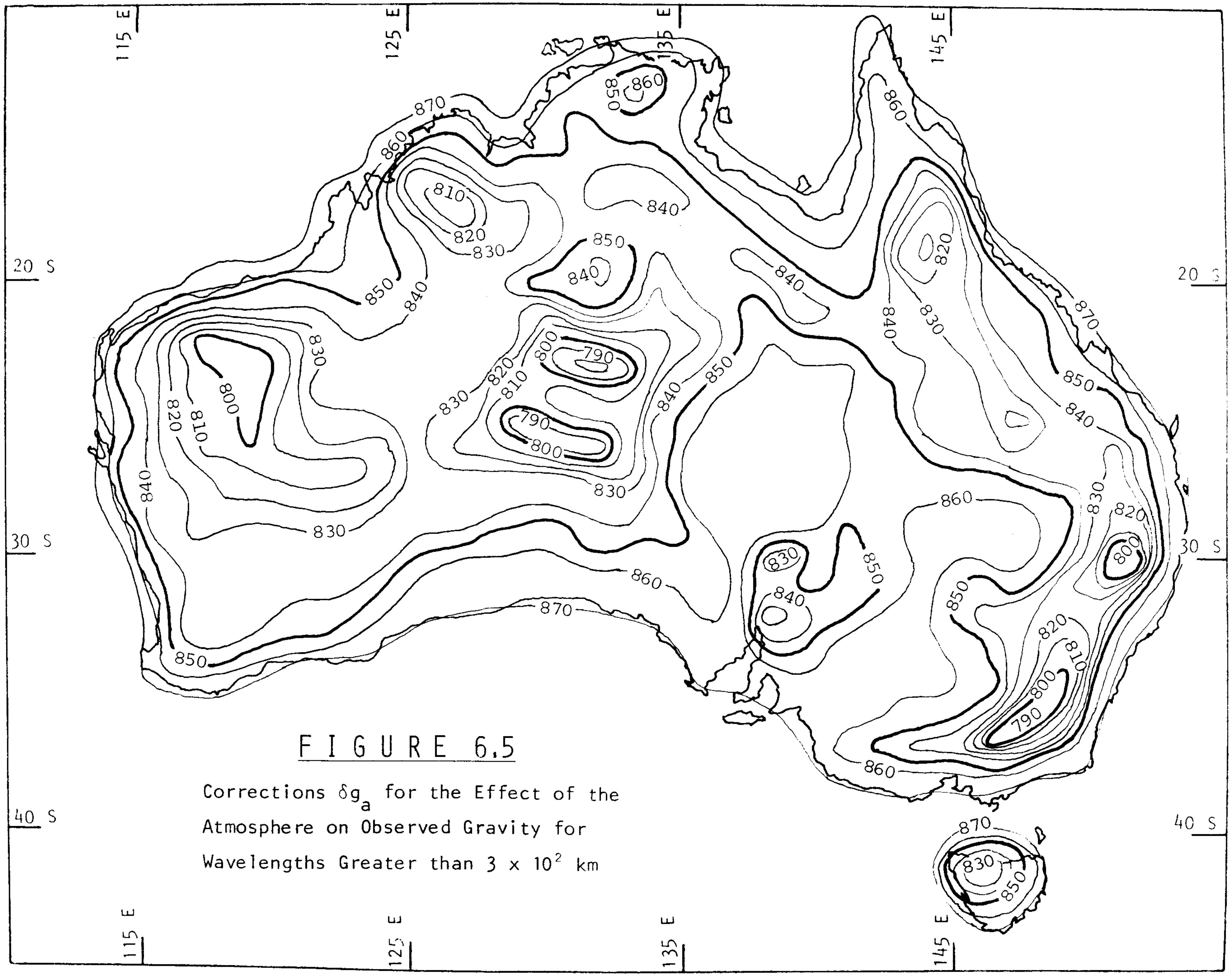


FIGURE 6.5

Corrections δg_a for the Effect of the Atmosphere on Observed Gravity for Wavelengths Greater than 3×10^2 km

Source: *MATHER ET AL (1976b)*

Contour Interval - 10 μgal

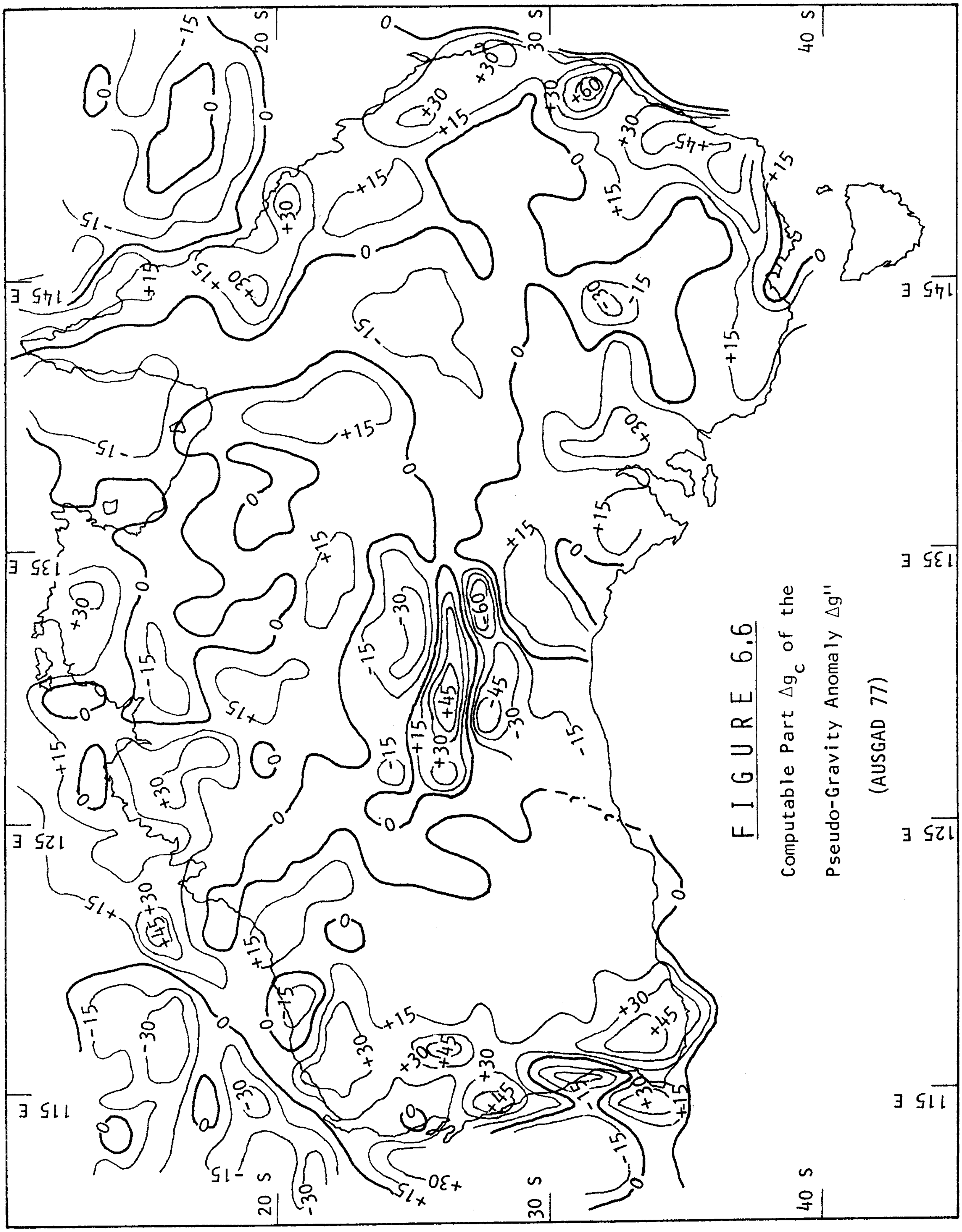


FIGURE 6.6

Computable Part Δg_c of the Pseudo-Gravity Anomaly $\Delta g''$

(AUSGAD 77)

(Contour Interval - 15 mgal)

form (equation 2.68):

$$e_\zeta = \pm o\{K\, e_g\, n\} \qquad (6.29)$$

where $K \simeq 10^{-2}$ for e_g in μgal, n in degrees and e_ζ in cm. Thus, the greater the wavelength of an error type, the smaller would its magnitude have to be, to produce the same effect in the computed value for the geoid height.

Four sources of error which need to be considered are the following:

(1) the error in the value adopted at the Sydney A datum to which all Isogal values are referred;

(2) errors in the gravity anomaly data bank itself;

(3) errors of long wavelength in the Isogal network; and

(4) the nature of errors in gravity station elevations apart from those already taken into account.

6.2.3.2 *The Gravity Datum for Australia*

The basis for the adoption of the gravity datum for Australia was dealt with in § 6.2.1.2. The ISGN 71 value for the National Gravity Base Station at Sydney has a standard deviation (σ) of ±0.21mgal calculated from the internal statistics of the network and is confirmed to ±0.02mgal by the Soviet OVM pendulum tie which was not included in the network calculations. Nevertheless, the IGSN 71 value differs from the value resulting from the absolute determination at an adjacent location by -0.14mgal. This difference is twice as large as the σ of that determination. An unknown systematic error is more likely to have affected the result of the absolute determination. However, the possibility of such an error in the IGSN 71 value cannot be ruled out. It would be reasonable to assess the uncertainty in the value adopted as ±0.06mgal.

Considering that such an error would have the same sign and magnitude over an area equivalent to a thirty five degree square, the cumulative effect, if it were to occur in all other land areas, on determinations of geoid height and ultimately sea surface topography, would be about ±6cm on allowing for a 30% representation of the Earth's surface by land gravity data. In practice, such distortions are more likely to be about ±4cm as the Australian continental margins are approached if distant areas were represented by models of the disturbing potential which are not prone to such errors (see "combination solutions" - sub-section 2.3.6).

6.2.3.3 *Gravity Scale in Australia*

As mentioned in § 6.2.1.2, it is estimated that the gravity scale has been established to an accuracy (σ) of better than 2.5 parts in 10^5 along the ACL. This is equivalent to an error amplitude of 0.06mgal in the gravity interval Hobart-Port Moresby, a half wavelength distance of 3.5 x 10^3km. It should be noted that the adopted scale differs from that established by IGSN 71 along the ACL by 15 parts in 10^5 which is equivalent to 0.34mgal in the above interval. Scale has not been independently established for the other two north-south lines of the ANGN; these lines depend on the east-west traverses of the Isogal network for scale. There is evidence that the intervals Darwin-Perth and Darwin-Adelaide (with half wavelength distances of 3-4 x 10^3km) may have an error amplitude of about 0.2mgal. Without further measurements, it is difficult to separate scale and loop closure effects, but it seems likely that systematic error amplitudes over wavelengths of about 7 x 10^3km do not exceed 0.3mgal.

Differences between GAG-2 and La Coste gravity meter results on ACL suggest that scale error amplitudes of about 0.1mgal and wavelength of about 2×10^3km may be present along the ACL (BOULANGER ET AL 1973, Figure 3). The magnitude of shorter wavelength contributions in the other north-south lines cannot be estimated.

6.2.3.4 *Accumulation of Errors in the ANGN*

The internal precision of the Isogal network can be assessed from the network adjustment by McCRACKEN (1977) and the comparison of results obtained from different types of gravity meters during precise multi-meter surveys along the ACL. The standard error of an air tie between adjacent base stations is of the order of ±0.04mgal. There seems to be normal accumulation of error along traverses and the expected error at node stations between north-south and east-west lines is less than ±0.07mgal. As mentioned previously in § 6.2.1.2, a few portions of the network near Perth and Darwin, and in Tasmania seem to be in error by amounts in excess of ±0.2mgal. Error amplitudes in excess of 0.1mgal over distances up to 1.5×10^3km are also indicated in these areas. Errors over shorter distances can be expected to be even smaller.

The possibility of systematic errors in the Isogal network cannot be discounted. External checks on the precision of the network away from the ACL are available only from IGSN 71 values at a few widely distributed stations. Errors of the order of ±0.1mgal are indicated but they are within the range of experimental error. It has been found in the laboratory that quartz-type gravity meters such as the Worden, change their readings when the ambient pressure is altered. Much of the Isogal network was measured using two quartz-type and one La Coste & Romberg gravity meters. A systematic error which correlates with elevation is possible but it is unlikely to exceed 0.1mgal per 2000m elevation change.

6.2.3.5 *Effect of Gravity Station Elevation Errors*

The effect of gravity station elevation errors on sea surface topography studies is a function of the wavelength and amplitude of the constituent errors in both the Australian levelling survey as well as the nature of the connections between this network and the individual gravity stations. The latter are described in § 6.2.1.2, being established by barometric levelling during the course of helicopter gravity surveys. It is estimated that while individual station heights could have errors of up to ±10m, any tendency towards correlation of errors between neighbouring gravity stations is more likely to have amplitudes of between 1 and 2m and wavelengths of less than 4×10^2km.

Consequently, the errors in AUSGAD 77 due to those in establishing gravity station elevations can be estimated as being the following:

(i) Random errors with 3mgal amplitude and 20km wavelength due to uncertainties in individual gravity station elevations.

(ii) Intermediate error components with amplitudes of around 0.4mgal and wavelength less than 4×10^2km, due to the nature of the helicopter gravity surveys.

(iii) Long wave errors in the Australian levelling survey which are estimated to have amplitudes of 0.15mgal in AUSGAD 77 and half wavelengths of 3.5×10^3km on the basis of the internal statistics of the free net adjustment (ROELSE ET AL 1971, Annexure F).

The effects of (i), (ii) and (iii), if repeated on a global scale, on determinations of the geoid are estimated to give errors with magnitudes of order ±3cm, ±5cm and ±30cm respectively on taking the distribution of the error with position and on using equation 6.29. It follows that the most detrimental source of error in gravity station elevations when attempting to use AUSGAD 77 in sea surface topography studies comes from the residual errors in the Australian levelling survey which is only of third order standard. The precision of the control station elevations would have to be increased by a factor of three if the data were to be adequate for ±10cm determinations. Such a task would be impracticable in the foreseeable future.

It must be emphasised however, that even with a gravity anomaly data bank free of the long wavelength systematic errors mentioned earlier, the use of such data in high precision gravimetric solutions in support of sea surface topography studies still suffers from the problem that the elevation datum is related to local MSL and not the geoid. The effect of SST on the determination of the ±10cm geoid has been discussed in § 2.3.5.5 & 4.2.3. The possibility of estimating the height of the MSL datum above the geoid was discussed in section 5.3 and preliminary results using AUSGAD 77 and the North American Gravity Anomaly Data Bank are given in section 8.2.

6.2.4 SUMMARY

The Australian Gravity Anomaly Data Bank 1977 (AUSGAD 77), prepared on the lines described in the preceding sections, has been corrected for all the obvious sources of systematic error which could be reliably modelled. These include allowance for the following factors:

(a) The formula for normal gravity is correct to ±4μgal.

(b) The latitudes used in such computations refer to a geocentric and not the regional ellipsoid.

(c) The data set is a gravity anomaly data bank and not a set of free air anomalies.

(d) The gravity anomalies are computed in relation to zero at the Jervis Bay Datum Level Surface on the basis of a free net adjustment of the levelling (and not the datum for elevations based on the AHD).

(e) The effect of the atmosphere has been allowed for in evaluating the computable part Δg_c of the spherically harmonic pseudo gravity anomaly $\Delta g''$.

It is also estimated that the following sources of error may bias the data bank so produced in the context of its use for sea surface topography determinations to ±10cm:

- Errors of scale in the central and western part of Australia are likely to produce systematic error amplitudes of 0.2mgal and half wavelengths of 3.5×10^3km.
- Residual errors in the Australian levelling survey, which is of third order standard, could affect the data bank producing errors with amplitude 0.15mgal and half wavelengths of 3.5×10^3km.

The resulting errors, if repeated globally, would result in the precision of geoid determinations using data banks of quality similar to AUSGAD 77, being limited to ±30cm. This is unlikely to be so in the case of levelling network errors, though the other sources of error are likely to be as great elsewhere. It would appear that AUSGAD 77 would not be adequate if a precision better than ±30cm were sought in determinations of sea surface topography.

AUSGAD 77 could be made more compatible with the requirements for ±10cm sea surface topography studies if the following supplementary data were obtained:

(i) Absolute gravity determinations with a precision of ±20μgal were established at 10^3km intervals and used to refine the Isogal network datum, scale and adjustment.

(ii) The additional semi-detailed gravity data already available in Australia, were added to the gravity bank to eliminate most of the need for prediction.

(iii) Three dimensional positions were established on a 10^3km grid on the Australian mainland as part of a ±10cm global geodetic system. Such data would strengthen the Australian elevation datum.

6.3 A CENTRAL NORTH AMERICAN GRAVITY ANOMALY DATA BANK FOR SEA SURFACE TOPOGRAPHY STUDIES (CNAGAD 77)

The gravity data for the region of North America lying between 28°N and 50°N was compiled by the Defense Mapping Agency Aerospace Center, using data from a variety of sources. The datum for gravity values was that provided by IGSN 71. The resulting $1^\circ \times 1^\circ$ free air gravity anomaly data based on the Geodetic Reference System 1967 (IAG 1971) was prepared by RAPP (1977) and rounded off to the nearest mgal. This data set forms the basis of the Central North American Gravity Anomaly Data Bank 1977 (CNAGAD 77).

The free air anomalies Δg_f were obtained according to the relation:

$$\Delta g_f = g - \gamma_o + 0.3086^{(mgal)}\ H^{(m)} \qquad (6.30)$$

where γ_o is normal gravity computed for the GRS 67 equipotential ellipsoidal model, defined by equations 6.10 and 6.11, while H is the orthometric height in metres. As far as can be ascertained the latitudes used for the computation of γ_o are based on values referred to a geocentric reference ellipsoid. The free air anomaly data bank is illustrated in Figure 6.7.

Without performing a major re-examination of the data, it has not been clearly established whether or not the heights of the gravity stations H are in any way related to the regional geodetic levelling network in a continental context. As a starting point, it was decided to adopt the suggestion that the tide gauge at Galveston be adopted as a suitable datum for this levelling network (HOLDAHL 1978).

The gravity anomaly Δg was obtained from the quantity Δg_f using equation 6.28:

$$\Delta g^{(mgal)} = \Delta g_f^{(mgal)} - 0.3086\ H^{(m)} - \frac{2\Delta W}{a}\left(1 + f + m + \frac{\Delta W}{2a\gamma} - 2f\sin^2\phi\right) \qquad (6.31)$$

where all quantities are defined in § 6.1.2.1.

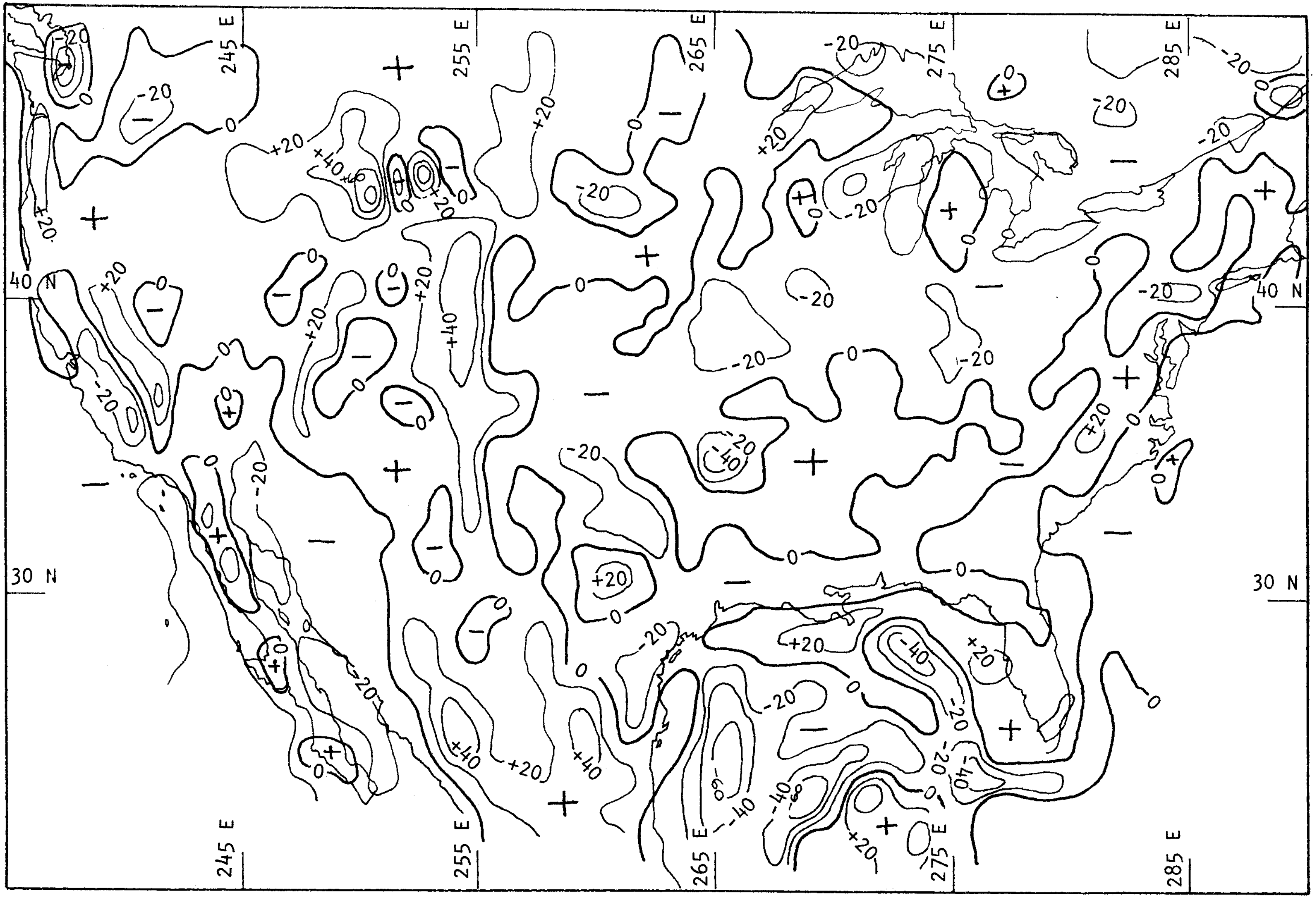
The geopotential difference ΔW, in mgalm, is related to increments of geodetic levelling dn by the relation (equation 6.2),

$$\Delta W = - \int_{\text{MSL Datum}}^{P} g \, dn \qquad (6.32)$$

Equation 6.32 was used to obtain $1^o \times 1^o$ geopotential differences ΔW_p from the $1^o \times 1^o$ mean elevation and gravity data banks in relation to the elevation value in the $1^o \times 1^o$ square containing the Galveston MSL datum.

The difference in elevation between contiguous $1^o \times 1^o$ areas was assumed to define the quantity dn, while the mean gravity for the square was an estimate of g. The resultant set of geopotential differences ΔW were assumed to relate to the Galveston tide gauge and were used to evaluate the quantity Δg in equation 6.31.

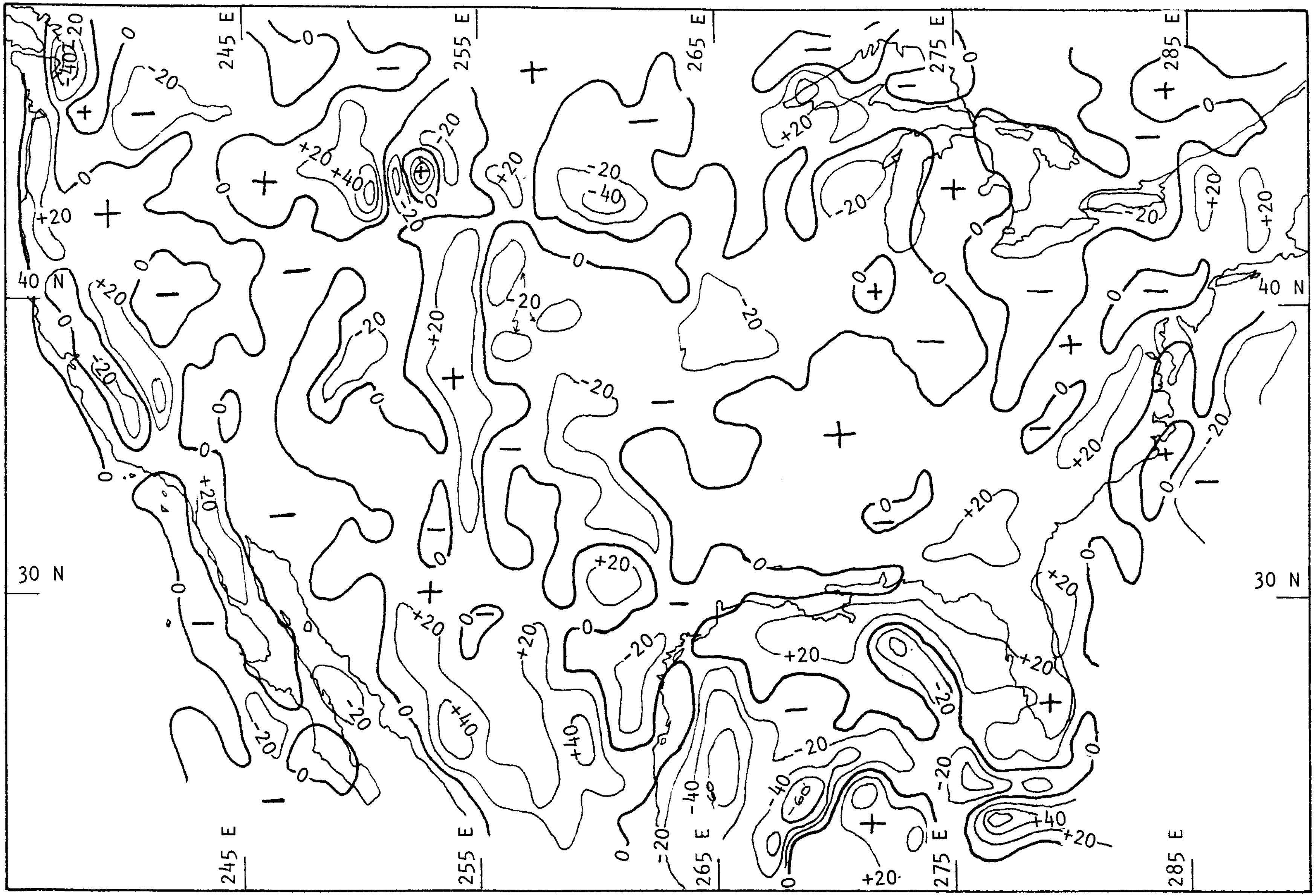
All gravity anomalies were finally referred to the "higher" reference model defined by GEM9 (LERCH ET AL 1977) and corrected for the atmospheric effect to give the computable part Δg_c of the spherically harmonic pseudo gravity anomaly $\Delta g''$, as described in § 6.2.2.5. Figure 6.8 illustrates the final values in CNAGAD 77.



(Contour Interval - 20 mgal)

FIGURE 6.7

Free Air Anomalies for Central North America - Geodetic Reference System 1967



(Contour Interval - 20 mgal)

<u>F I G U R E 6.8</u>

Computable Part Δg_c of the Pseudo-Gravity Anomaly $\Delta g''$ (CNAGAD 77)

7

DETERMINATION OF SOME DOMINANT PARAMETERS OF THE GLOBAL STATIONARY SEA SURFACE TOPOGRAPHY FROM GEOS-3 ALTIMETRY

7.1 INTRODUCTION

7.1.1 PREAMBLE

It was established in section 5.2 that satellite altimetry data has a crucial role to play in determinations of stationary and time-varying components of the SST, through both long and short wavelengths. However, at first glance, it would appear that the signal-to-noise problems preclude the recovery of information on the SST from GEOS-3 altimeter data. The principal difficulties being:

(1) The GEOS-3 satellite has an inclination of 115° and cannot collect data at latitudes north of 65°N and south of 65°S. This fact, together with the absence of on-board recording facilities limits the available surface coverage to that shown in Figure 7.1, approximately 80% of the oceanic regions below the 65° parallel.

(2) The distribution of altimetry measurements in space and time is random (see Figures 7.3, 7.4 and 7.6), the data, for the most part, were collected on a <u>regional</u> rather than a global basis. The acquisition areas and periods being governed by the location of transportable telemetry units. GEOS-3 altimetry was recorded in the form of discrete passes never exceeding 9 x 10^3km in length. No attempt was made to obtain near-simultaneous global coverage.

(3) The precision of the orbit determination, and hence the values of ζ, is variable. The principal check on the quality of orbit integration is the crossover discrepancy d. The radial error can then be estimated as $d/\sqrt{2}$. The analysis of GEOS-3 orbits based on Doppler tracking data indicates a root mean square radial error of ±1.3m after exclusion of approximately 9% of the passes (see sub-section 7.1.2). This figure reduces to ±0.9m in the case of a GEOS-3 ephemeris based on laser tracking data (MATHER ET AL 1979b, p.11). As mentioned earlier, the major source of error

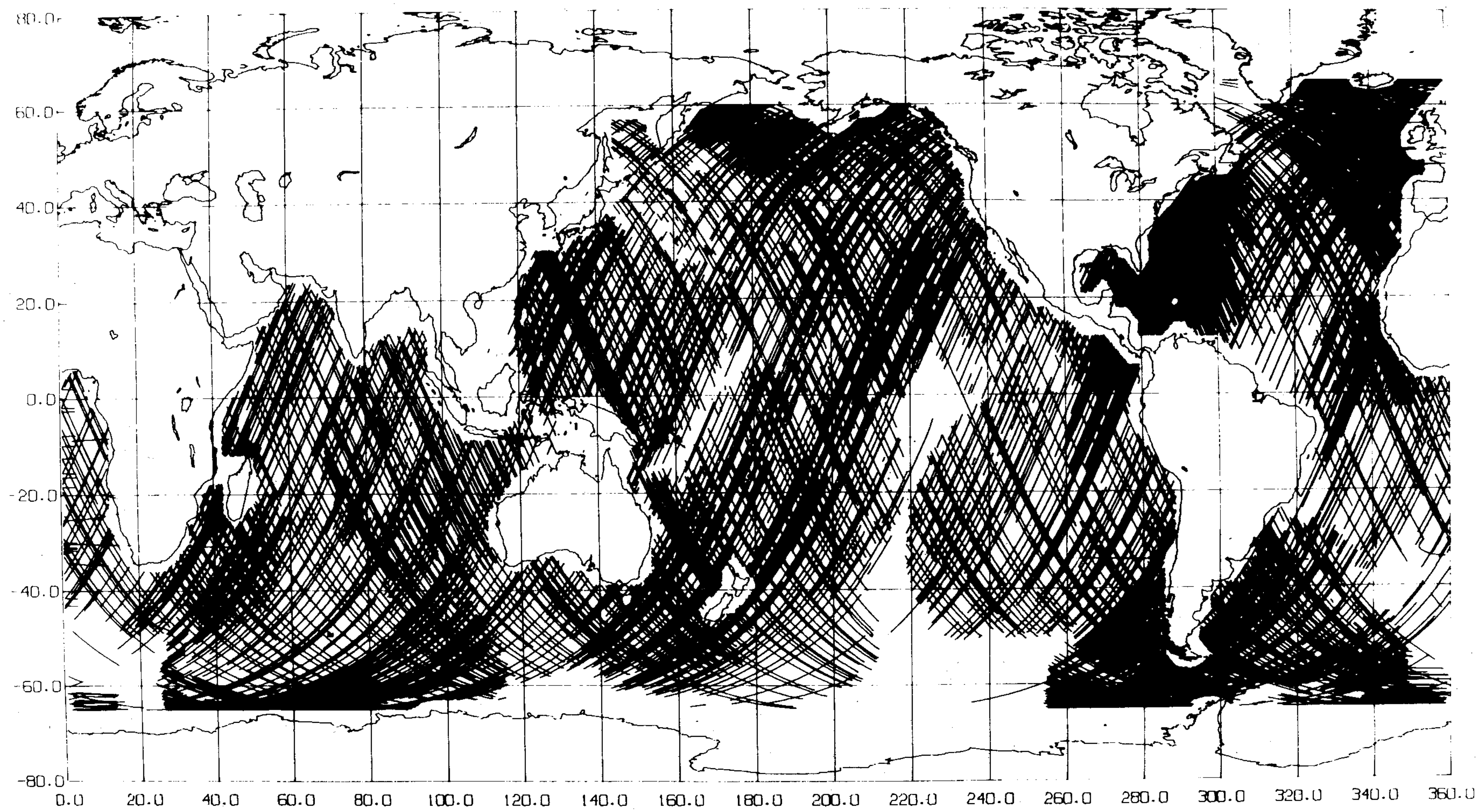


FIGURE 7.1

Complete GEOS-3 Altimetry Data Bank 1975-76 appears to be the model of the earth's gravity field used in orbit integration.

(4) The geoid needs to be defined in ocean areas with a precision of the order of ±10cm through all wavelengths of interest. The determination of the marine geoid from surface gravity data, satellite altimetry and gravity field models has been discussed in section 5.1. It was concluded that neither surface gravity observations nor satellite altimetry data on their own could provide information on the geoid with a precision adequate for SST studies because all measurements are made either at or in relation to the sea surface and not the geoid.

Despite these adverse factors, it is possible to obtain information on selected wavelengths of the stationary SST from GEOS-3 altimetry, under the following conditions (MATHER ET AL 1978b, p.4):

(i) From orbits which have a radial uncertainty of ± b cm, there should be sufficient GEOS-3 data available for the average representation of any particular wavelength so that the resolution is $\pm b/\sqrt{n}$ cm, where n is the number of samples.

(ii) The amplitude of the SST feature is greater than $b/\sqrt{n}$ cm.

(iii) The error in the geoidal model with wavelengths comparable to that of the feature, are significantly less than $\pm b/\sqrt{n}$ cm.

It is not difficult to conjure up a scenario in which features of the SST can be determined with a precision of ±10cm. For example, it has been observed that the second degree of the SST is significantly larger than other terms in its representation by a surface harmonic series such as that in equation 5.59 (MATHER 1975a, p.67). Therefore a basis exists for the recovery of this term from the GEOS-3 data bank, despite the problems due to inadequate satellite tracking support and the irregular manner in which the data were collected in both space and time. This study concentrates on the <u>definition of parameters of the global stationary SST through those wavelengths for which the earth's gravity field model is known with an acceptable level of precision</u>.

It was established in sub-section 5.2.2 that (equation 5.52):

$$T'' = (W_o - U_o) - W_A + \gamma\zeta' - \gamma\zeta_s \tag{7.1}$$

where T'' is the disturbing potential of the solid earth and oceans on the "higher" reference model, W_o is the potential of the geoid, U_o is the potential of the irregular equipotential surface implied in the "higher" reference system (see section 4.4) and W_A is the potential of the atmosphere. ζ' is the height anomaly on this reference model and ζ_s is the SST.

ζ' is defined by the relation:

$$\zeta' = \zeta - \frac{GM}{R_o\gamma} \sum_{n=2}^{n'} \left(\frac{a}{R_o}\right)^n \sum_{m=0}^{n} \sum_{\alpha=1}^{2} C^*_{\alpha nm} S_{\alpha nm} \tag{7.2}$$

where $C^*_{\alpha nm}$ are the spherical harmonic coefficients defining the "mapping" of the "higher" reference surface in relation to an ellipsoid of revolution (see equations 4.84 and 4.85).

On adopting surface harmonic models for ζ_s (coefficients $\zeta_{s\alpha nm}$ - equation 5.59), W_A (coefficients $V_{g\alpha nm}$ - equation 5.34), T'' (coefficients $dC_{\alpha nm}$ - equation 4.87) and ζ' (coefficients $\zeta'_{\alpha nm}$), it follows that (MATHER 1978e, Section 5):

$$\bar{\zeta}_{s\alpha nm} = \bar{\zeta}'_{\alpha nm} - \frac{1}{\gamma}\left\{\frac{GM}{R_o}\left(\frac{a}{R_o}\right)^n \overline{dC}_{\alpha nm} + \bar{V}_{g\alpha nm}\right\} \tag{7.3}$$

where R_o is the geocentric distance to the sea surface. The overbar refers to normalised values (HEISKANEN & MORITZ 1967, Section 1-14).

It has been shown in § 5.1.3.2 that for all practical purposes, the net effect of the terms within parentheses should be zero (see discussion on coefficients $dC_{\alpha nm}$ in section 4.4) if the value of $C'_{\alpha nm}$ used in forming both $C^*_{\alpha nm}$ (equation 4.84) and $dC_{\alpha nm}$ (equation 4.87) are free from error. In such a case:

$$\bar{\zeta}_{s\alpha nm} = \bar{\zeta}'_{\alpha nm} \tag{7.4}$$

Table 7.1 (based on Solution 3, Table 7.3) lists the normalised coefficients $\bar{\zeta}_{s\alpha nm}$ ($n < 5$) in the surface spherical harmonic representation of the oceanographically-determined stationary SST, based on data confined to the oceans lying between the parallels 65°S and 65°N (see Figure 3.3). Also listed are errors $e_{c\alpha nm}$ in the coefficients $C'_{\alpha nm}$ of GEM9 (LERCH ET AL 1977, p.52) with their linear equivalents. A study of the signal-to-noise ratio ($\bar{\zeta}_{s\alpha nm}/e_{c\alpha nm}$) shows that conditions are favourable only for the recovery of the coefficients $\bar{\zeta}_{s100}$, $\bar{\zeta}_{s110}$, $\bar{\zeta}_{s111}$, $\bar{\zeta}_{s120}$, $\bar{\zeta}_{s130}$, $\bar{\zeta}_{s140}$ and possibly $\bar{\zeta}_{s160}$. Table 5.2 lists the rms error per degree in GEM9, which indicates that there is no possibility of recovering any information on SST from present-day gravity field models with wavelengths less than 10^4km ($n > 4$). In other words, only seven coefficients can be recovered from a perfect determination of ζ' and the GEM9 gravity field model. Other coefficients can be estimated but the level of uncertainty is much greater, being a function of:

- the *uncertainty* in the value of the GEM9 coefficient; and
- the *magnitude* of the coefficient $\bar{\zeta}_{s\alpha nm}$ (Table 5.2 and 7.1).

Numerical solutions for the recovery of these dominant parameters of the long wavelength stationary SST are described in section 7.2.

The analyses for the non-zero degree coefficients are described in sub-section 7.2.3, the zero degree coefficient $\bar{\zeta}_{s100}$ is analysed in section 7.3 and section 7.4 presents the conclusions drawn from these studies.

The GEOS-3 data bank, the system of reference, procedures for the modelling of SST and the effect of the permanent earth tide on numerical solutions are described below.

7.1.2 THE GEOS-3 ALTIMETER DATA BANK

In the search for the dominant features of the SST, the primary objective is determination of the geometry of the sea surface in Earth space as defined in relation to a geocentric coordinate system. Such a model is directly comparable to the geoid in any determination of ζ_s. This would also apply to any satellite-determined gravity field model used in lieu of

TABLE 7.1

FACTORS INFLUENCING DETERMINATIONS OF STATIONARY SST
FROM SATELLITE ALTIMETRY

- The Signal ($\zeta_{s\alpha nm}$), The Noise per Coefficient ($e_{c\alpha nm}$)
For GEM9, and The Signal-To-Noise Ratio ($\zeta_{s\alpha nm}/e_{c\alpha nm}$)

DEGREE	ORDER	THE SIGNAL ($\zeta_{s\alpha nm}$)+ (cm) $\alpha=1$	$\alpha=2$	GEM 9 NOISE ($e_{c\alpha nm}$) (± kGal cm) $\alpha=1$	$\alpha=2$	SIGNAL-TO-NOISE $\lvert\zeta_{s\alpha nm}/e_{c\alpha nm}\rvert$ $\alpha=1$	$\alpha=2$
0	0	114.5	-	-	-	-	-
1	0	+6.9	-	-	-	-	-
1	1	-21.8	+2.4	-	-		-
2	0	-46.2	-	0.4	-	115.5	-
2	1	-4.0	+4.4	1.7	1.6	2.4	2.8
2	2	-0.7	-0.2	2.2	2.2	0.3	0.1
3	0	+6.7	-	1.0	-	6.7	-
3	1	-4.1	-5.3	3.4	3.3	1.2	1.6
3	2	-0.7	-2.4	5.1	4.4	0.1	0.6
3	3	-3.0	+1.4	6.1	5.9	0.5	0.2
4	0	-9.5	-	0.8	-	11.9	-
4	1	+2.1	+2.8	3.2	3.0	0.7	0.9
4	2	-0.5	+1.2	3.0	3.1	0.2	0.4
4	3	+1.2	-0.2	2.9	2.7	0.3	0.1
4	4	-1.8	-0.9	4.0	4.0	0.5	0.2
5	0	+0.9	-	1.0	-	0.9	-
5	1	-3.7	-0.8	4.1	4.1	0.9	0.2
5	2	-0.6	+2.4	6.1	5.9	0.1	0.4
5	3	+0.6	+0.2	5.7	6.1	0.1	0.0
5	4	-0.3	+1.1	5.6	5.8	0.1	0.2
5	5	-0.3	-0.8	9.0	9.0	0.0	0.1
6	0	+4.4	-	1.2	-	3.7	-
7	0	-0.5	-	1.4	-	0.4	-
8	0	-0.8	-	1.3	-	0.6	-

+ BASED ON AN ANALYSIS IN OCEAN AREAS ONLY BETWEEN 65°S and 65°N

Source: MATHER (1978e)

the latter in the search for dominant parameters defining the global distribution of ζ_S.

The orbits used in reducing GEOS-3 altimeter data were computed at the National Aeronautics and Space Administration's Wallops Flight Center (WFC) or at the U.S. Naval Surface Weapons Center, Dahlgren, Virginia, under WFC supervision. These orbits were prepared using predominantly Doppler tracking data and are called Wallops orbits in this study. A smaller set of orbits provided by F.J. Lerch of Goddard Space Flight Center were based primarily on high precision laser tracking data.

Timing is critical in correlating the GEOS-3 ephemeris with the time tag in the altimeter sensor data record as dh/dt can be as large as 20 m s^{-1}. For example, a constant timing bias of θ millisecond causes sea surface height errors of 2θcm forcing south-to-north passes to have an error which is equal and opposite to that in a north-to-south pass at the same location. This has serious implications when attempting to enforce crossover constraints, discussed below.

The difference between values of ζ obtained by this method at crossover points should not exceed ±2m (due to the time-varying SST). Nevertheless, the data originally supplied by WFC was of variable quality (differences due to time tag errors were sometimes in excess of ±10m), and it was necessary to attempt an orbit improvement so that radial orbital errors could be reduced:

(a) from values in excess of ±10m (e.g. MATHER ET AL 1977a, p.30) to values smaller than ±2m; and

(b) from values less than ±2m to the relative precision of ±30cm implied in the analysis of overlapping passes (MATHER 1977, p.25).

The following is a development from MATHER ET AL (1978b, Section 2).

While it is possible to achieve a median value of ±2m for the crossover discrepancy (Table 7.2), the improvement at (b) cannot be obtained from the GEOS-3 tracking data alone. The best tracking data available at the present time is laser data taken from the network of stations shown in Figure 7.2. The laser ranging precision varies from around ±10cm for the NASA lasers to around ±1m for other systems. The resulting orbit obtained from integration is in the form of the instantaneous satellite position $X_{is}(t)$ at time t on a geocentric coordinate system X_i, with the X_3 axis passing through the CIO pole and the X_1X_3 plane being that of zero longitude (λ). The instantaneous position $X_{iss}(t)$ of the sea surface at the point with geodetic coordinates (ϕ,λ) is obtained from the altimeter range h(t) using the relations (equations 5.11 and 5.12):

$$X_{iss}(t) = X_{is}(t) - \ell_i h(t) + o\{10^{-8}h\} \qquad (7.5)$$

where ℓ_i are defined by the equations:

$$\ell_1 = \cos\phi\cos\lambda \quad ; \quad \ell_2 = \cos\phi\sin\lambda \quad ; \quad \ell_3 = \sin\phi \qquad (7.6)$$

The geocentric radial distance to the sea surface $R_s(t)$ is obtained from

T A B L E 7.2

The Analysis of Crossovers for twelve Ten Day Arcs Used in the Equinox Experiment (Wallops Orbits)

ARC	DURATION		NO. OF ALTIMETER PASSES	CROSSOVER STATISTICS					
				UNADJUSTED			CONSTRAINED+		
	FROM	TO		NO. OF XO's	MEAN DISCREPANCY (m)	ROOT MEAN SQUARE RESIDUAL (± m)	NO. OF XO's	MEAN DISCREPANCY (m)	ROOT MEAN SQUARE RESIDUAL (± m)
1	9.1.75	9.10.75	65	111	+0.76	4.4	95	+0.35	1.6
2	9.11.75	9.20.75	51	40	+5.05	19.7	28	+0.07	2.5
3	9.21.75	9.30.75	63	98	+3.49	10.6	89	+0.66	1.8
4	10.1.75	10.10.75	49	48	+2.19	3.6	39	+0.47	1.8
5	10.11.75	10.20.75	61	61	+1.01	2.1	59	+0.47	1.5
6	10.21.75	10.31.75	51	62	+1.20	2.7	55	+0.21	2.0
7	3.1.76	3.10.76	83	38	-2.08	1.9	35	-1.17	1.9
8	3.11.76	3.20.76	52	53	+0.10	2.4	53	-0.07	1.7
9	3.21.76	3.31.76	46	28	+0.10	2.4	27	-0.35	1.7
10	4.1.76	4.10.76	34	30	+0.25	1.5	30	-0.01	1.2
11	4.11.76	4.20.76	29	10	-0.62	2.7	10	0.00	1.8
12	4.21.76	4.30.76	40	63	-1.15	2.1	62	-0.34	1.8

Source: MATHER ET AL (1978b)

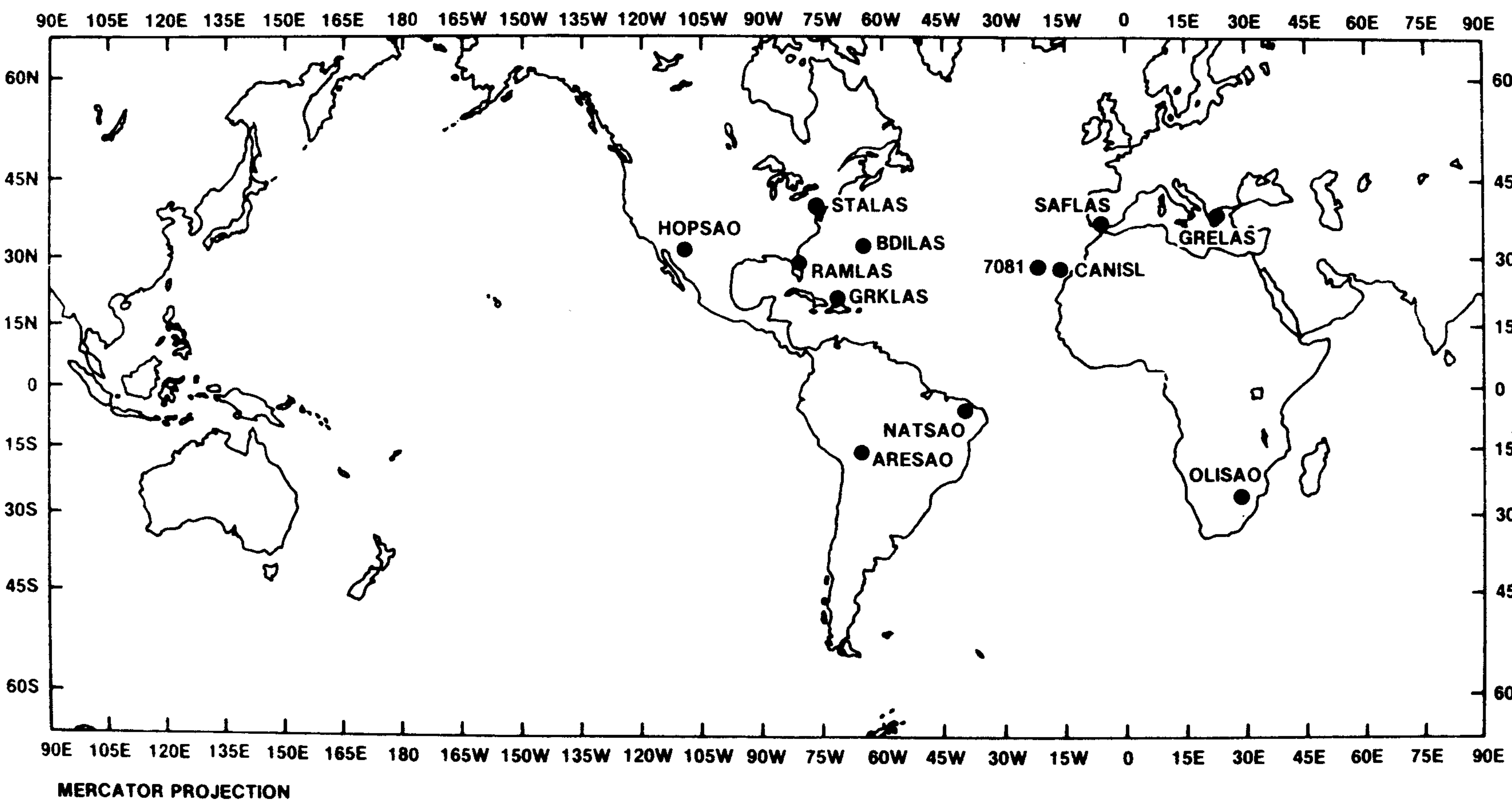


FIGURE 7.2

Laser Tracking Station Complement for GEOS-3 Mission the coordinates $X_{iss}(t)$ using the relation:

$$R_s(t) = \left[\sum_{i=1}^{3} \left(X_{iss}(t)\right)^2 \right]^{\frac{1}{2}} \qquad (7.7)$$

R_s as evaluated at each altimeter data point during a 5 day arc, can be examined at crossover points for the propagation of the radial component of the orbital error as a function of time, using observation equations of the form:

$$R_s(t_2) - R_s(t_1) + \sum_{i=1}^{n} C_i \left(F_i(t_2) - F_i(t_1)\right) = v \qquad (7.8)$$

where C_i are the coefficients required in the solution and F_i are functions of time. Observation equations of this form could, in theory, include the ocean tide. In practice, however, the number of observation equations is rather small (less than 40 for a 5 day arc of GEOS-3 data), precluding this possibility. It is important that crossover constraints be used only to eliminate orbital errors due to unmodelled force field effects and not result in the removal of any of the oceanographic signal. The former are estimated as having predominant periods of one half revolution, one revolution, 14 revolutions, a resonance effect with period of approximately 4.70 days and any linear drifts with time.

Twelve ten day arcs of Wallops data originally selected for the Equinox Experiment (sub-section 7.2.1) were analysed using crossover constraints per ten day arc based on the following model for the orbital error:

$$\sum_{i=1}^{9} C_i F_i(t) = C_1 t + C_2 \cos 4\pi\frac{t}{t_o} + C_3 \sin 4\pi\frac{t}{t_o} + C_4 \cos 2\pi\frac{t}{t_o} + C_5 \sin 2\pi\frac{t}{t_o} + \\ C_6 \cos\frac{2\pi}{14}\frac{t}{t_o} + C_7 \sin\frac{2\pi}{14}\frac{t}{t_o} + C_8 \cos\frac{2\pi}{66.21}\frac{t}{t_o} + C_9 \sin\frac{2\pi}{66.21}\frac{t}{t_o} \qquad (7.9)$$

where t is the time in days from the start of the 10 day arc and t_o the orbital period in the same units. The results obtained per 10 day arc are set out in Table 7.2. These results show that the rms residual of crossover discrepancies (XO) per 10 day arc averaged ±6.8m. This figure can be reduced to ±6.1m by using equation 7.9 to represent the unmodelled orbital errors (i.e. 18 percent of the power). A study of Table 7.2 shows that this average crossover discrepancy can be reduced significantly to ±1.8m by rejecting data associated with 9 percent of the XO points, on enforcing the criteria that any pass subject to an average XO discrepancy in excess of ± 5m was subject to an unknown source of error, tentatively associated with time tag problems.

Data sets referred to as "constrained" in this context are based on Wallops orbits as amended by XO constraints using equation 7.9 and the ±5m cut-off limit explained above. The significance of the coefficients C_i obtained by the use of equation 7.9 is not apparent at this stage.

It therefore appears doubtful whether there is any means of obtaining profiles of sea surface heights from the orbital ephemeris of GEOS-3 using equation 7.5 with an absolute radial precision of better than ±1.3m in global terms at present. It must be emphasised that such an estimate is probably optimistic as the XO analysis is carried out over 10 day periods. This estimate is worse if the whole data bank is analysed in one step.

LERCH ET AL (1978c) reports a figure of ±1.3m for the radial error of precise laser GEOS-3 orbits from such an analysis. Nevertheless, the internal precision per pass has been shown to be around ±30cm from overlapping pass comparisons (see earlier discussions).

Data of this type can play a role in determining parameters of the global stationary SST under the following conditions:

(a) The error of ±1.3 metres mentioned above and which can be modelled by two parameters (a bias and a tilt) is randomly distributed as a function of position.

(b) The parameters sought should not be much less than ±15cm. It should have a sufficiently long wavelength to enable its recovery from a global bank of data where satisfactory solution procedures can be devised for recovery of the stationary SST signal under conditions where the signal-to-noise ratio approaches 0.1.

If the sea surface topography were modelled using equation 5.59, the analysis of the data used to construct Figure 3.3 and summarized in Table 7.3 (Solution 3) shows that only the coefficients ζ_{S111} and ζ_{S120} satisfy the above criteria. However, if the orbital errors could in some way be brought to below ±60cm there is some chance that the coefficients ζ_{S110}, ζ_{S130} and ζ_{S140} can also be recovered.

Note that these probabilities are assessed only on the basis of an analysis of the GEOS-3 altimeter data bank and do not take any other factors into consideration.

The "constraining" defined by equation 7.9 was not attempted on laser orbits. The results obtained using laser orbits are compared with those obtained from "constrained" and "unconstrained" Wallops solutions.

7.1.3 THE SYSTEM OF REFERENCE

The principal role of a system of reference is the removal of systematic effects in the data which can be eliminated prior to analysis, thereby reducing the signal-to-noise ratio. The model used in reducing the satellite altimetry data is that described as the "higher" reference model in section 4.4.

In the first stage, the height anomaly ζ' on the "higher" reference system is computed using the relation:

$$\zeta' = \zeta - \zeta_M \tag{7.10}$$

where (equation 7.2):

$$\zeta_M = \frac{GM}{R_o\gamma}\sum_{n=2}^{n'}\left(\frac{a}{R_o}\right)^n\sum_{m=0}^{n}\sum_{\alpha=1}^{2} C^*_{\alpha nm} S_{\alpha nm} \tag{7.11}$$

where (ϕ,λ,R_o) are the geocentric spherical coordinates of the sea surface and $C^*_{\alpha nm}$ are obtained from satellite-determined harmonic coefficients $C'_{\alpha nm}$ of GEM9 (LERCH ET AL 1977) to degree 30. They are given by:

$$C^*_{\alpha nm} = C'_{\alpha nm} \tag{7.12}$$

except when α = 1, m = 0 and n = 2, 4 and 6.

In these three special cases (see § 5.1.2.3),

$$C^{*}_{120} = 0$$
$$C^{*}_{140} = C'_{140} - \tilde{C}_{140} \qquad (7.13)$$
$$C^{*}_{160} = C'_{160} - \tilde{C}_{160}$$

The use of the height anomaly ζ in relation to a reference ellipsoid (equatorial radius a, flattening f) in equation 7.10, requires that C^{*}_{120} = 0 if the value of f is to be consistent with the coefficient C'_{120} of GEM9. If this is the case, the incorporation of this reference ellipsoid as part of the "higher" reference model results in the corrections $\tilde{C}_{140}$ and $\tilde{C}_{160}$ taking specific values which are functions of f (see equations 5.23 - 5.27).

The reference system used for stationary SST experiments was defined by the following set of constants:

$$c = 2.997\,924\,58 \times 10^{8} \text{ m s}^{-1}$$
$$GM = 398\,600.47 \text{ km}^{3}\text{s}^{-2} \qquad (7.14)$$
$$C'_{120} = -1.082\,627\,5 \times 10^{-3}$$
$$\omega = 7.292\,115\,146\,7 \times 10^{-5} \text{ rad s}^{-1}$$

These values have been incorporated into the Geodetic Reference System 1980 (GRS 80) (MORITZ 1979).

The dependent constants have the following values:

$$f^{-1} = 298.257\,316 \quad \text{(from equation 5.29)}$$
$$\tilde{C}_{140} = 2.370\,897\,9 \times 10^{-6} \quad \text{(from equation 5.23)} \qquad (7.15)$$
$$\tilde{C}_{160} = -6.083\,393\,7 \times 10^{-9} \quad \text{(from equation 5.23)}$$

7.1.4 MODELLING THE STATIONARY SEA SURFACE TOPOGRAPHY

Special problems are involved in modelling data which does not continuously cover the surface of the Earth. Oceanic phenomena cover only about 70 percent of the Earth's surface. The data for GEOS-3 only provides data for those parts of the oceans which lie between 65°S and 65°N. On using the 1° x 1° global elevation data bank as a mask for the oceans, this defines the ocean area in terms of 33902 such squares between the parallels defined above.

The obvious model for representing the long wave features of the SST is a surface spherical harmonic series (equation 5.59):

$$\zeta_{s\ell} = \sum_{n=1}^{\infty} \sum_{m=0}^{n} \sum_{\alpha=1}^{2} \zeta_{s\alpha nm} S_{\alpha nm} + o\{f\zeta_{s}\} \qquad (7.16)$$

The values obtained for the coefficients $\zeta_{s\alpha nm}$ are dependent on the method of solution. The harmonic coefficients can be obtained by two different methods if the data is uniformly distributed over a sphere.

(1) By least squares, using observation equations of the form:

$$AX - K = V \tag{7.17}$$

from equation 7.16, where K is the column matrix of known values of $\zeta_{s\ell}$. Values of $\zeta_{s\alpha nm}$ are obtained by minimising:

$$\Phi = V^T W V \tag{7.18}$$

where W is the matrix of weight coefficients which in this set of computations is $\cos\phi$, where ϕ is the latitude of the equi-angular square in which the data was sampled.

The solution is then:

$$X = (A^T W A)^{-1} A^T W K \tag{7.19}$$

The array X being composed of the coefficients $\zeta_{s\alpha nm}$.

(2) Alternatively, given a distribution of $\zeta_{s\ell}$, the normalised coefficients $\bar{\zeta}_{s\alpha nm}$ can be obtained using the relation:

$$\bar{\zeta}_{s\alpha nm} = \frac{1}{4\pi} \iint \zeta_{s\ell} S_{\alpha nm} d\sigma \tag{7.20}$$

$d\sigma$ being the element of surface area on the unit sphere.

The error introduced by the spherical assumption is less than ± 0.5cm as the SST does not have a magnitude in excess of ± 2m.

The question of modelling the long wave features of the SST has been discussed earlier (sub-section 5.3.2). The following is based on a development by MATHER ET AL (1978b, Section 6). Two distinct possibilities are open:

(a) Solve for $\zeta_{s\alpha nm}$, sampling $\zeta_{s\ell}$ in ocean areas only.

(b) Solve for $\zeta_{s\alpha nm}$ but replacing $\zeta_{s\ell}$ by:

$$\zeta_{s\ell} = k_o(\phi,\lambda) \sum_{n=1}^{\infty} \sum_{m=0}^{n} \sum_{\alpha=1}^{2} \zeta_{s\alpha nm} S_{\alpha nm} \tag{7.21}$$

where $k_o(\phi,\lambda)$ is the ocean function defined by (equation 5.91):

$$k_o(\phi,\lambda) = \begin{cases} 0 & \text{if } (\phi,\lambda) \text{ on land} \\ 1 & \text{if } (\phi,\lambda) \text{ is oceanic} \end{cases} \tag{7.22}$$

Table 7.3 sets out the normalised coefficients $\bar{\zeta}_{s\alpha nm}$ obtained from the analysis of the stationary SST represented in Figure 3.3 by four different methods. Solution 1 gives the results obtained by using the least squares method in ocean areas only. Solution 2 gives values of $\zeta_{s\alpha nm}$ obtained by using equations 7.20, 7.21 and 7.22. The results for Solution 2 change negligibly between equivalent least squares solutions and values obtained by quadratures, (i.e. when the least squares method is applied to land areas with zero $\zeta_{s\ell}$ on land). The results in Solution 1 for a determination to (8,8) vary considerably when the analysis is carried out by least squares to differing degrees due to a high level of correlation between the solved coefficients as a result of the incomplete representation of data on the sphere.

A third type of solution was obtained by constraining the coefficients $\overline{\zeta}_{s\alpha nm}$ by imposing the condition

$$\Phi = V^T W V + X^T W_c X = \text{Minimum} \tag{7.23}$$

instead of equation 7.18 when obtaining a solution. The constraints were imposed through the array W_c, using coefficients of the form (a crude Kaula's rule expectation for the magnitude of the coefficients $\overline{\zeta}_{s\alpha nm}$):

$$W_{cnn} = \left(\frac{10^{-5}R}{n^2}\right)^2 \times 10^3 \tag{7.24}$$

for answers in metres, where n is the degree of the harmonic and R the mean radius of the Earth.

Solutions obtained in this manner changed only slightly as a function of the degree to which the data was analysed. A solution of this type is shown as Solution 3 of Table 7.3.

The constrained solution in ocean areas so obtained was checked by interpolation of values in land areas along a parallel using a cubic polynomial in longitude difference $d\lambda$ from the eastern sea/land boundary, of the form:

$$\zeta_{s\ell} = \sum_{i=0}^{4} a_i (d\lambda)^i \tag{7.25}$$

The coefficients a_i are estimated by least squares fitting to between 4 and 8 data points in oceans along the same parallel on either side of the land mass to be bridged.

The results so obtained from both least squares and quadratures are listed as Solution 4 in Table 7.3

The results in Table 7.3 emphasize the care needed in selecting a system for modelling the SST. No two models in Table 7.3 are exactly equivalent, though the models in the last two sets of columns differ only in that zero has been assumed for values of $\zeta_{s\ell}$ outside the parallels 65°N and 65°S in one evaluation by quadratures.

The coefficients in unconstrained solutions are heavily correlated and change dramatically with the maximum degree to which analysis is carried out. The quadratures solutions using zero on land are not directly comparable with harmonic solutions obtained from sets of observation equations formed in ocean areas only. The values obtained from Solution 3 however will be a useful yardstick for the comparison of results obtained for $\overline{\zeta}_{s\alpha nm}$ in section 7.2.

The variation in the coefficients for solutions of type 3 and 4 indicate the strong influence of areal representation in solutions for $\zeta_{s\alpha nm}$ from altimetry data. It is therefore considered essential that this factor be

TABLE 7.3

Surface Spherical Harmonic Analysis of Oceanographically Determined Stationary SST to (5,5) from Data Restricted to Between Parallels 65°N and 65°S

Coefficients Normalised | Units: cm

SOLUTION TYPE n	m	(1) UNCONSTRAINED LEAST SQUARES OCEAN ONLY		(2) QUADRATURES ZERO ON LAND		(3) CONSTRAINED LEAST SQUARES OCEAN ONLY		(4) QUADRATURES USING LAND BRIDGE+	
α		1	2	1	2	1	2	1	2
0	0	110.2		84.3		114.5		113.5	
1	0	5.3		- 8.5		6.9		3.4	
1	1	-27.9	- 2.7	-34.0	- 4.1	-21.8	+2.5	-20.6	+3.5
2	0	-43.7		-39.8		-46.2		-48.7	
2	1	-13.1	- 3.4	- 8.9	- 2.6	- 4.0	+4.4	- 3.5	+4.6
2	2	+ 0.8	- 4.6	+ 5.7	+ 1.0	- 0.7	-0.2	- 0.1	+0.1
3	0	+18.5		+ 6.5		+ 6.7		+ 1.8	
3	1	-13.2	-11.6	+ 4.2	- 7.5	- 4.1	-5.3	- 2.0	-3.6
3	2	+ 1.8	- 9.9	+ 8.5	-13.6	- 0.7	-2.4	- 0.0	-1.9
3	3	- 0.7	- 0.4	- 3.2	-10.4	- 3.0	+1.4	- 2.3	+1.7
4	0	2.6		- 5.8		- 9.5		-11.9	
4	1	- 4.4	+ 2.4	+ 7.2	+ 3.6	+ 2.1	+2.8	+ 1.9	+2.7
4	2	+ 2.1	- 6.8	+ 7.8	- 0.9	- 0.5	+1.2	+ 2.7	-0.6
4	3	+ 4.6	- 3.3	- 2.0	+ 0.7	+ 1.2	-0.2	+ 1.5	-0.4
4	4	- 0.7	- 2.1	+ 0.6	-15.3	- 1.8	-0.9	- 1.9	-0.4
5	0	+14.7		+ 2.7		+ 1.0		- 3.2	
5	1	- 7.6	+ 3.1	- 1.0	+ 5.5	- 3.7	-0.8	- 1.0	+0.4
5	2	+ 1.9	- 3.0	+ 2.5	+ 7.9	- 0.6	+2.4	- 1.2	+2.6
5	3	+ 4.8	- 2.5	- 4.0	+ 0.1	+ 0.6	+0.2	+ 0.8	+0.1
5	4	+ 1.7	+ 0.4	-14.3	+ 4.9	- 0.3	+1.1	- 0.4	+1.8
5	5	+ 0.6	- 0.1	- 0.8	- 6.3	- 0.3	-0.8	- 0.4	-0.6
Analysis to		8,8				16,16			
RMS Fit (± cm)		9				9			

+Zeros Assumed for Polar Regions Outside Parallels 65°S and 65°N

Source: MATHER ET AL (1978b)

taken into account when assessing the relative quality of solutions obtained from different incomplete data banks. The dominant surface spherical harmonic coefficients for the stationary SST used in the present study to represent the fully sampled region between the two sixty-fifth parallels in ocean areas is given in Column 1 of Table 7.4. Column 2 of this table gives the values obtained had the analysis been done using data sampled only in the areas where altimetry data were available for the Equinox experiment (see § 7.2.1.2).

7.1.5 THE EFFECT OF THE PERMANENT EARTH TIDE

The geoid for a *static* earth, as opposed to an earth subject to tidal deformation (§ 2.1.1.8), is defined as the level surface of the earth's gravity field due to gravitation and rotation, which best fits the mean sea surface for the epoch of observations. In this context, the term "geoid" is restricted to a geometric surface coinciding with the level surface $W = W_o$ for the undeformed Earth. Such a surface will reflect the gravity field model defined by the coefficients $C'_{\alpha nm}$ in equations 7.10, 7.11 and 7.12, but not the earth tide, which is separately modelled as part of the tidal effect on orbits in the solution procedure for the coefficients $C'_{\alpha nm}$ (SMITH 1972).

At any instant (τ=t) the "true" geopotential $W_g(t)$ at the general point P on the geoid is given by:

$$W_g(t) = W_o + \delta W(t) \tag{7.25}$$

where $\delta W(t)$ is the change in geopotential at P due to the motion of the Sun and Moon. A consequence of this is that the spatial location of the "instantaneous" geoid varies due to the earth tide.

The effect of neglecting earth tides in SST determinations was investigated by MATHER (1978f) and is based on the following premises.

Sea surface heights as determined from satellite altimetry need to be referred to an instantaneous level surface in order to derive information on the global distribution of ζ_s that have ocean surface circulation relevance (see e.g. MATHER 1976 for a discussion on the recovery of oceanographic information from SST).

The change in geopotential $\delta W(t)$ due to the luni-solar attraction has diurnal, semi-diurnal and long period components. The diurnal and semi-diurnal and components are of short period and are thus not considered in this discussion. The long period components of the earth tide influence SST determinations if not properly taken into account because the tide producing bodies (the Sun and Moon) are banded in declination ($\pm 28^o$), the effect having characteristics of a second degree zonal harmonic. The long period geopotential variation $\delta W_L(t)$ is expressed as (MATHER 1978f, equation 10):

$$\delta W_L(t) = \frac{GM}{R}\sum_{i=1}^{2}\left(\frac{R}{R_i}\right)^3\left(\frac{M_i}{M}\right)(1 + k_2 - h_2)P_{20}(\sin\delta_i) \tag{7.26}$$

where M_i is the mass of the ith tide producing body (Sun or Moon) at a distance R_i from the geocentre and δ_i is the declination of the body. h_2 and k_2 are the second degree Love numbers.

If C'_{120} is the second degree zonal harmonic of the earth's gravity field model consistent with the "static" geoid and $C'_{120}(t)$ that of the instantaneous datum level surface at time t, then:

$$C'_{120}(t) = C'_{120} + dC_{20}(t) \tag{7.27}$$

where

$$dC_{20}(t) = \frac{R}{GM}\,\delta W_L(t) \tag{7.28}$$

As illustrated in IBID (Table 1), the long period second degree zonal component of the earth tide $dC_{20}(t)$ causes the flattening of the datum level surface to vary, being in all cases greater than that of the undeformed earth.

The average value of $dC_{20}(t)$ is -2.13×10^{-8} whereas the variation in the long period tide can be up to 0.7×10^{-8} (IBID, Table 1).

The equivalent change df in the flattening f of the ellipsoid which best fits the geoid is (MATHER 1978e, equation A-33):

$$df = -\frac{3}{2}\,dC_{20} + o\{f\,df\} \tag{7.29}$$

and the change $d\bar{\zeta}_{s120}$ in the second degree zonal harmonic of the SST is given by:

$$d\bar{\zeta}_{s120} = a\,\overline{dC}_{20} + o\{f\,d\bar{\zeta}_{s120}\} \tag{7.30}$$

the overbar referring to normalised quantities.

The permanent constituent of the earth tide causes the second degree zonal harmonic of the "static" geoid C'_{120} to be different from that of the actual deformed gravity field. The magnitude of the correction $d\bar{\zeta}_{s120}$ to the normalised coefficient $\bar{\zeta}_{s120}$ due to the permanent earth tide is (equation 7.26, 7.28 and 7.30):

$$d\bar{\zeta}_{s120} = \frac{a}{\sqrt{5}}\,M\,\{\sum_{i=1}^{2}\left(\frac{R}{R_i}\right)^3\left(\frac{M_i}{M}\right)(1 + k_2 - h_2)P_{20}(\sin\delta_i)\} \tag{7.31}$$

where M{ } refers to the mean value over the period of altimeter data acquisition. The value obtained for $d\bar{\zeta}_{s120}$ is +6.1cm.

Analyses of altimetry data in restricted time spans will be influenced by the seasonal variation in the long period tide. However, as this value has a maximum magnitude of 2cm (smaller than the estimated precision of SST determinations), this factor is ignored in practical considerations.

The inclusion of the correction $d\bar{\zeta}_{s120}$ will enable the satellite altimetry data to be referred to the instantaneous datum level surface rather than some "static" geoid, thereby ensuring that the aliasing effect of the permanent earth tide is satisfactorily removed in determinations of SST. Note that oceanographically-determined SST values (see § 3.3.3.2) are in relation to the permanently deformed level surfaces of the earth's gravity field and, consequently, ambiguities can result if the quantities being compared, the results of steric levelling on the one hand and satellite altimetry on the other, have not been related to either a deformed or static geoid model.

7.2 SOLUTIONS FOR DOMINANT COEFFICIENTS OF THE LONG WAVELENGTH STATIONARY SEA SURFACE TOPOGRAPHY

The following procedures and results are based on MATHER ET AL (1978b).

7.2.1 GLOBAL SOLUTIONS

Global solutions for the stationary SST are based on equation 7.4 and although they are noisier than regional solutions - see § 5.2.3.4 (average radial error of ±1.3m in Wallops orbits and ±0.9m for laser orbits), they have the advantage of being related to the orbits. In addition, no information on low degree contributions to the shape of the sea surface has been removed by bias/tilting passes.

These solutions require a set of stationary ζ values from GEOS-3 altimetry in the form of $1^o \times 1^o$ equi-angular area means. The tidal signal has been largely eliminated in the process of averaging the data. The geoid model defined by GEM9 was then differenced from the grid of sea surface heights (see equation 7.2). These residual heights would in fact be the stationary component of the SST as a continuous field if the GEM9 "higher" reference model were free from error for all wavelengths. The residual heights however, do allow a determination of the dominant harmonic coefficients $\overline{\zeta}_{s111}$, $\overline{\zeta}_{s120}$, $\overline{\zeta}_{s110}$, $\overline{\zeta}_{s130}$ and $\overline{\zeta}_{s140}$ (normalised values), as they do not contain errors equivalent in wavelength and amplitude to the SST (see sub-sections 7.1.1 and 7.1.2 for discussion).

7.2.1.1 *Complete Wallops Data Set*

A global solution was attempted with the complete Wallops data set of over 875000 data points recorded during the period from April 1975 to August 1976. Over half of this data, recorded during the earlier part of the period, were subject to high levels of noise as indicated by the rms variation of values of ζ within a $1^o \times 1^o$ square in Table 7.4 (Row 6). This data set provides an 80.1% coverage of the oceans between 65^oN and 65^oS, as illustrated in Figure 7.1. The results are given in Solution 1 of Table 7.4.

7.2.1.2 *Equinox Data Set*

The occurrence of seasonal variations in sea surface height ζ (see § 3.3.2.3) is of considerable significance in the establishment of sampling techniques for the determination of stationary SST, as sea level tends to be at its lowest in northern mid-latitudes during January - February and highest in July - August. It is obvious that determinations of the global SST from data collected during a northern summer will give different results to that collected during a northern winter (the terms affected will probably be the zonal harmonics, three of which contribute almost 80% of the spectrum of SST - MATHER 1978e, Section 4).

A separate determination of the dominant terms in the global SST was therefore attempted from data which excluded samples recorded during the summer and winter months. This set is called the Equinox data set as only the intensive mode altimetry data collected during the months of September - October 1975 and March - April 1976 were considered in its compilation. It was originally intended to use only the twelve 10 day spans of data which fell within the four month period mentioned. The selected GEOS-3 altimeter data distribution for the September equinox in 1975 is shown in Figure 7.3 while that for the March 1976 equinox is shown in Figure 7.4. Each distribution on its own is inadequate for determination of parameters of the global SST. However the combination of the two, when supplemented by four additional five day spans on either side of the two periods (Figure 7.5), provided a coverage of 49.3% of the 33902 $1^{\circ} \times 1^{\circ}$ equi-angular blocks within the parallels 65°S and 65°N which constitute the "global ocean areas" for the definition of the geoid used in this series of studies (see sub-section 7.1.4).

The equinox data set was used both in the original and the "constrained" form to prepare models of the sea surface as a first stage in the analysis for those long wave components of the SST which have a favorable signal-to-noise in relation to the errors in the GEM9 gravity field model. The results are listed in Solutions 2 and 3 of Table 7.4. The data derived from the original Wallops data has a higher level of noise than that obtained after the application of the error model at equation 7.9 in the analysis of crossovers. Laser orbits were also used for these time periods to obtain Solution 6 (Table 7.4).

7.2.1.3 *Wallops 1976 Data Set*

The analysis of crossover discrepancies for the entire GEOS-3 altimeter data set in 10 day spans indicated that the orbits in 1976 were significantly better than those in 1975 (Table 7.2). It was therefore decided to compile a separate data set based on over 350000 data points collected during the first 8 months of 1976 (Figure 7.6). The same two types of solutions using Wallops orbits as attempted for the Equinox data set were repeated and the results listed as Solutions 4 and 5 in Table 7.4.

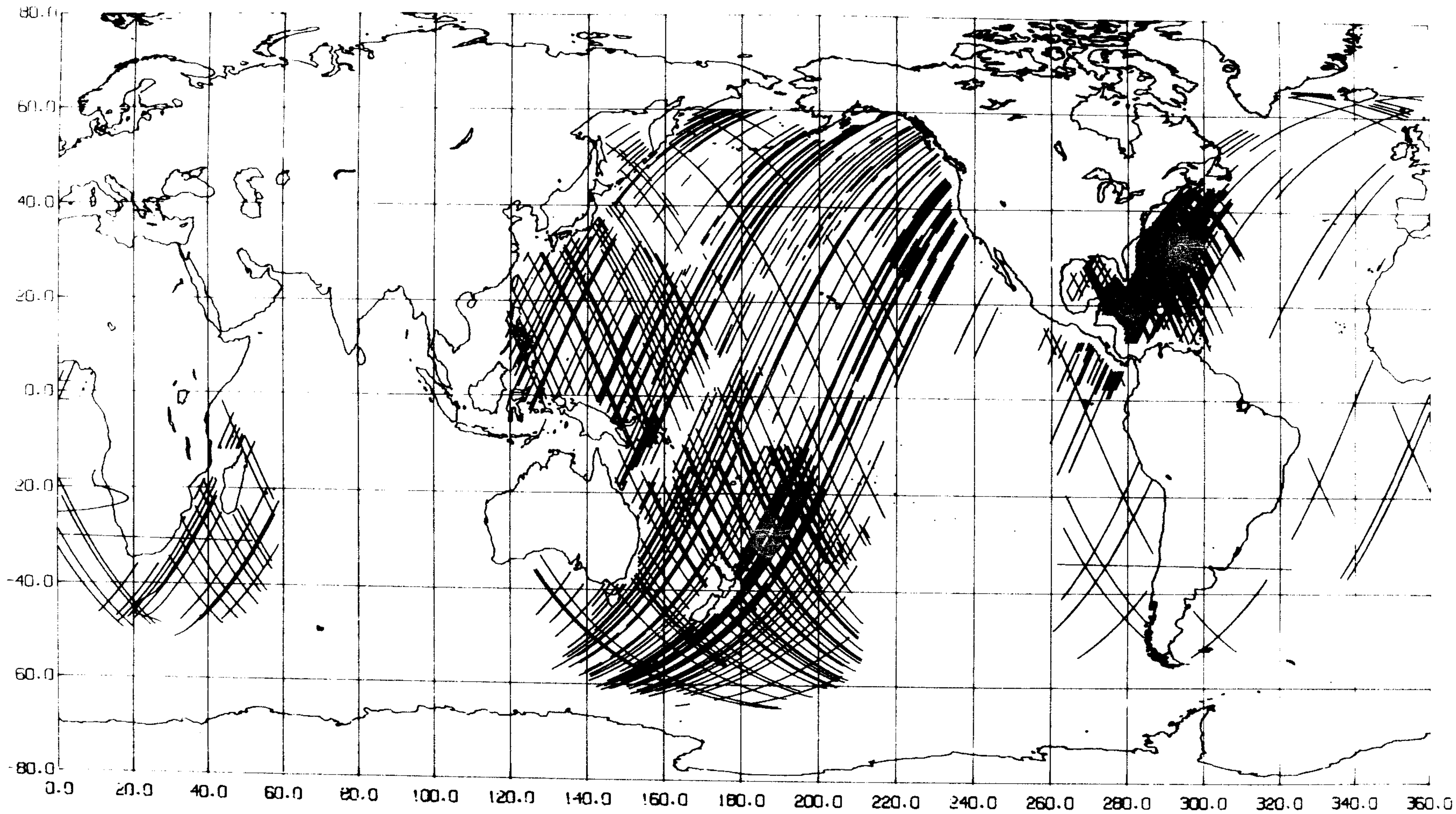
7.2.2 LONG PASS SOLUTIONS

A study of overlapping passes in the Tasman and Coral Seas (MATHER ET AL 1977a, p.36) showed that the root mean square discrepancy between sea surface heights on two passes of length greater than 3000km dropped on average to around ±30cm when the radial orbital error was modelled by a bias (b) and a tilt (c). This practice has been widely used by more than one GEOS-3 investigator when attempting to refine the altimeter data for specific studies (see discussion in § 5.2.3.4).

In attempting to reduce the high level of orbital noise per pass from ±1.3m to ±30cm by the introduction of corrections for tilt and bias, the goal is a means of determining the corrections c and b without losing any information about the geometry of the sea surface. The introduction of the corrections c and b per pass effectively removes all information with wavelengths greater than twice the length of the pass unless some special conditions are imposed.

The longest passes of GEOS-3 altimetry are of the order of 9000km. Information on the shape of the sea surface which would be lost in tilting and bias correcting such a pass will be of wavelength 18000km (i.e. harmonics less than degree 3 in a spherical harmonic representation of the SST). However, any contribution to the sea surface topography which is of degree 2 and symmetrical about the equator will not be lost on introducing corrections for tilt and bias; see discussion in MATHER ET AL (1978b, Section 3).

It was established in IBID (1978b) that a basis existed for recovering the SST from GEOS-3 altimetry from long passes under the following conditions:



F I G U R E 7.3

The Distribution of 1975 GEOS-3 Data in the Equinox Data Set for Epoch 1976.0 : September-October, 1975

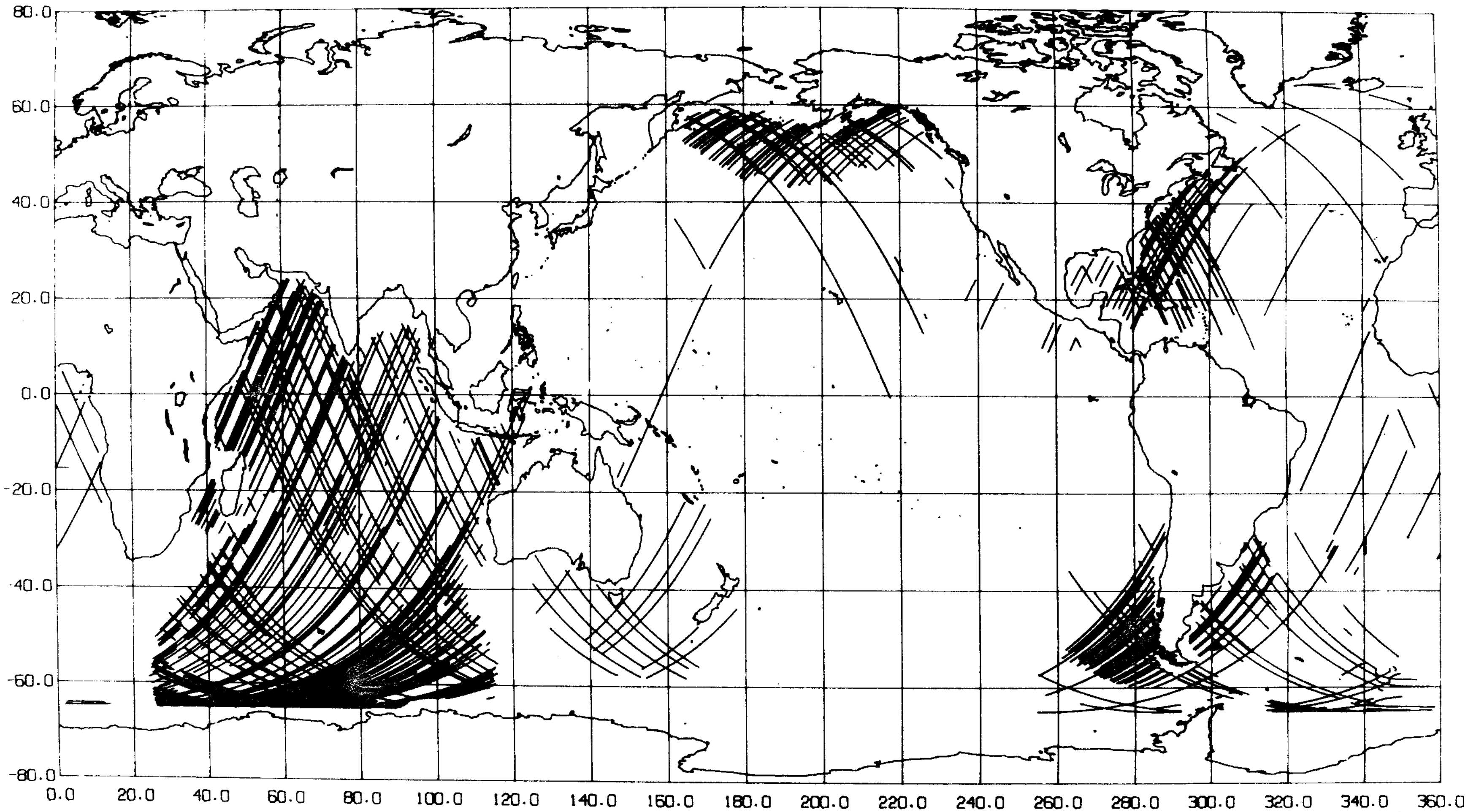


FIGURE 7.4

The Distribution of 1976 GEOS-3 Data in the Equinox Data Set for Epoch 1976.0 : March-April, 1976

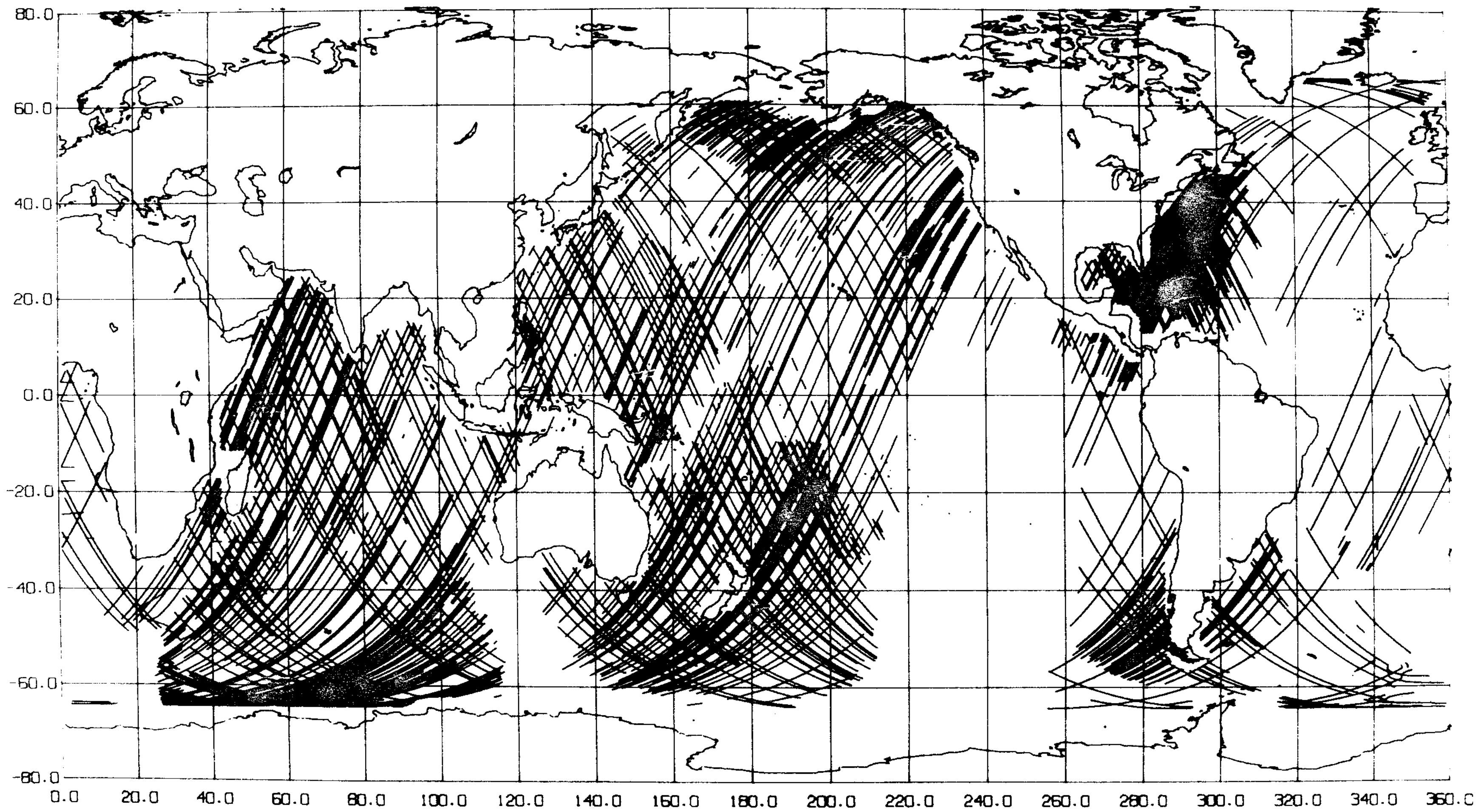
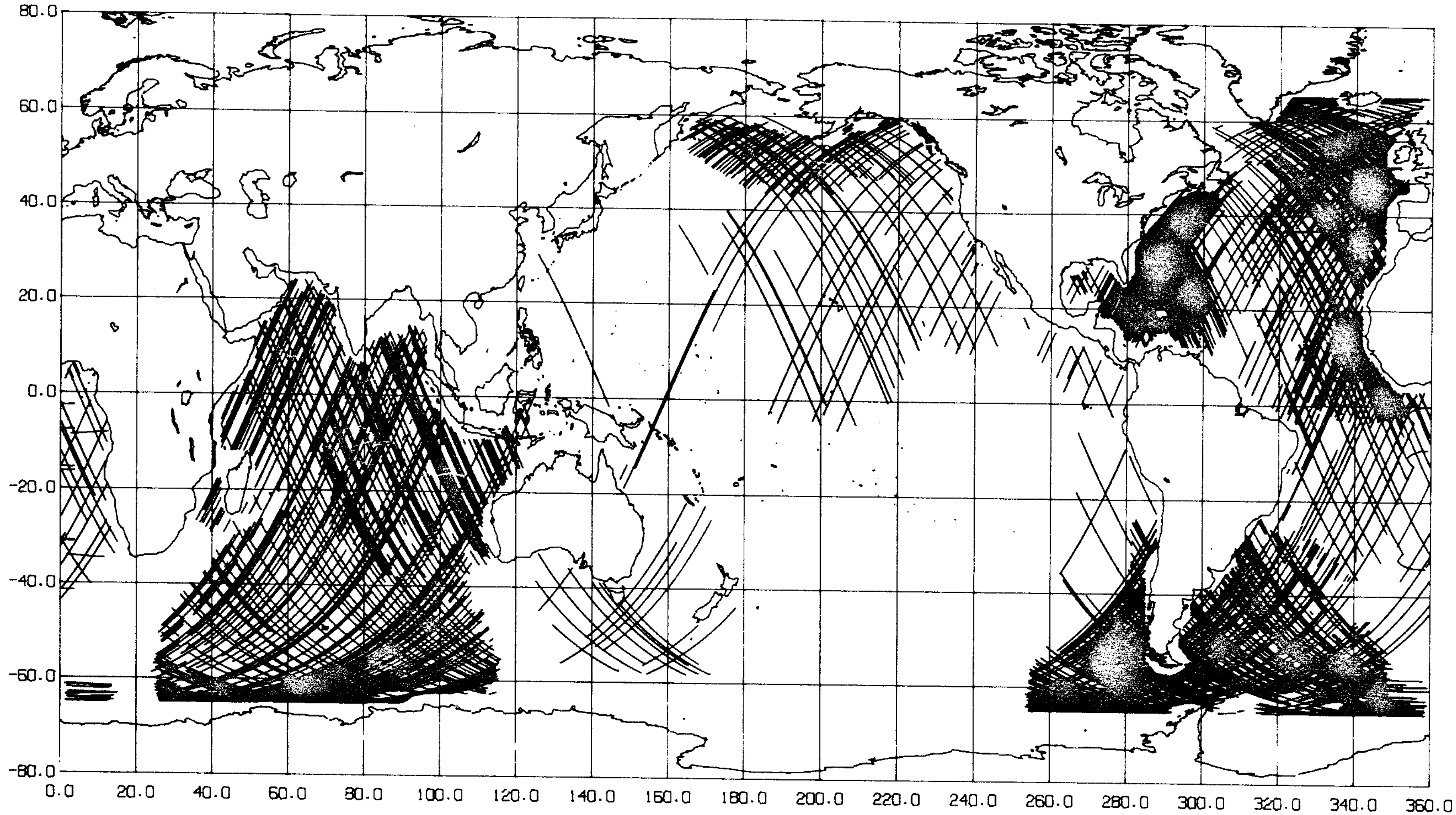


FIGURE 7.5

The Equinox Data Set for Epoch 1976.0



F I G U R E 7.6

The Distribution of GEOS-3 Altimetry in the 1976 Data Set

(i) There should be sufficient passes of the maximum possible length, symmetrical about the equator, for the recovery of even degree constituents of the sea surface shape.

(ii) Sufficient passes with a tolerance of about 500km from equatorial symmetry should be included to provide an even coverage of the Indian Ocean region.

(iii) It is desirable to obtain a coverage of the entire oceans to avoid aliasing effects due to incomplete sampling.

Under such circumstances, there are several means of obtaining the necessary set of tilt and bias corrections. The simplest means is by fitting the set of long arcs to the best available geoid model (GEM9), arc by arc, and solving for the shape of the sea surface, ensuring that the three conditions set out above are enforced. The following problems arise when using this method:

(a) The resulting sea surface model will not have information on harmonics which are not symmetrical about the equator and with wavelengths greater than twice the length of the pass.

(b) The harmonics of the SST deduced from a global set of such long passes of GEOS-3 altimetry are limited to those terms for which the error in the gravity field model is below at least 1 part in 10^8 (±6kgalcm).

Solutions based on the method of long passes allow the bias and tilt parameters to be solved <u>simultaneously</u> with the SST coefficients in the following manner.

For each data point within a long pass (which has been selected according to the three criteria described above), an observation equation of the following form is constructed:

$$v = \zeta - \zeta_M - (b + c(t - t_o) + \delta h) \qquad (7.32)$$

where ζ is the sea surface height obtained from GEOS-3 altimetry, ζ_M is the height anomaly implied by the GEM9 model (equation 7.11), (t, t_o) the parameters quantifying the tilt in terms of the time elapsed since the start of the pass and δh is the departure of the sea surface from the geoid modelled by the relation:

$$\delta h = \frac{GM}{R_o\gamma} \sum_{n=2}^{n'} \left(\frac{a}{R_o}\right)^n \sum_{m=0}^{n} \sum_{\alpha=1}^{2} \overline{\zeta}'_{\alpha nm} S_{\alpha nm} \qquad (7.33)$$

The coefficients $\overline{\zeta}'_{\alpha nm}$ obtained from the resulting least squares solution of the set of equations 7.32 represent the geometrical distortion of the average sea surface from the level surface implicit in GEM9. The results for the dominant harmonics $\overline{\zeta}_{S120}$, $\overline{\zeta}_{S130}$ and $\overline{\zeta}_{S140}$ of the SST from equation 7.33 (using the assumption implicit in equation 7.4) are set out in Table 7.4 (Solution 7). For more details on the long pass solutions see IBID (1978b, Section 3).

7.2.3 THE ANALYSIS OF THE RESULTS

Table 7.4 presents, in summary, the results obtained to date in determinations of the dominant parameters of the SST (the zero degree term

TABLE 7.4

Estimation of the Five Dominant Coefficients of the Stationary SST from GEOS-3 Altimetry

Normalised Coefficient			Oceanographic¶ (from 16,16) Solution		Global Solutions						Long Pass Solution
			Global	Equinox Representation	Wallops Complete	Wallops Equinox	Wallops Constrained Equinox†	Wallops 1976	Wallops Constrained 1976†	Laser Equinox	
n	m	α	(a)	(b)	1	2	3	4	5	6	7
1	0	1	+6.9	+9.6	-137*	-127*	-136*	-161*	-141*	-22*	-
1			-21.8	-20.6	-5	-1*	+6*	+7*	-1*	-72*	-
2			-46.2	-43.6	-47+	-46+	-35+	-47+	-35+	-54+	-45+
3			+6.7	+7.0	+10	+14	-1	+13	-11	+10	+2
4			-9.5	-9.9	-12	-11	-38	-18	-28	-10	-7
rms variation per 1^{o} x 1^{o} square			±5	-	±205	±174	±149	±178	±161	±133	±80
No. of 1^{o} blocks			-	-	27 171	16 707	13 275	14 782	12 074	16 575	-
No. of Data § Points			-	-	875 273	310 920	232 687	367 762	234 929	308 401	27 674

* Includes non-geocentricity of the reference coordinate system.
+ These values should be corrected by +6 cm for the permanent earth tide (sub-section 7.1.5).
¶ Between 65°N and 65°S (sub-section 7.1.4).
† Using equation 7.9 and ±5 m average XO limit.
§ The number of data points in the "constrained" solutions is significantly lower than the unadjusted data sets because all the altimeter data had not been processed at the time the "constrained" data sets were being compiled for the analyses reported in MATHER ET AL (1978b, Table 5).

is discussed in section 7.3). The results listed, with the exception of those for the long pass and "constrained" global solutions, have been obtained subsequent to those presented in MATHER ET AL (1978b, Table 5).

For global solutions, the "rule of thumb" adopted has been to assume that it is possible to derive long wave signals whose amplitudes are ten times smaller than the noise level, provided the background noise does not contain errors of equivalent wavelength. The analysis of crossovers (XO) to model short period orbital error, as described in sub-section 7.1.2, reduced the XO discrepancies on average by ±3m (Table 7.2). However, the resulting "constrained" solutions (3 and 5) produce values of $\overline{\zeta}_{s120}$ which are significantly smaller than the oceanographic value. This casts a shadow over any ad hoc modelling of terms in the enforcement of XO constraints by equation 7.9. On the other hand, the value of $\overline{\zeta}_{s120}$ obtained from the laser equinox data set (Solution 6) is too large.

7.2.3.1 *The Second Degree Harmonic*

In view of the reservations about the "constrained" solutions, the preferred value is the unweighted mean of Solutions 1, 2, 4, 6 and 7. This value of -48cm for $\overline{\zeta}_{s120}$ is only a preliminary estimate and is obtained by referring the sea surface to a static geoid model which is free of earth tide effects. A value of $\overline{\zeta}_{s120}$ comparable to the oceanographic value (-46cm globally) is obtained by correcting the above value by +6cm (see sub-section 7.1.5). In addition, the equinox results need to be corrected by -2cm due to the irregular data sampling as was discussed in sub-section 7.1.4 (Table 7.4, Columns (a) and (b)).

7.2.3.2 *The First Degree Harmonic*

It had been hoped to recover the second largest contribution to the SST ($\overline{\zeta}_{s111}$ - Table 7.4) which has a value -21cm. Being a first degree harmonic term, it can only be obtained from the analysis of the orbit-related sea surface height data bank (i.e. global solutions). Solutions for this harmonic being meaningful only if the GEOS-3 orbits refer to the geocentre without error. This is not the case. As the first degree harmonic of ζ_s has a degree variance of 530cm^2 (Table 5.2), it follows that the centre of the ellipsoid of best fit to the sea surface does not depart from the geocentre by more than 25cm. The degree variance of the value obtained from the Wallops orbits in all cases exceeds 16000cm^2. The only inference that can be drawn is that the origin of the system of reference used in integrating the Wallops orbits is displaced from the geocentre by not more than ±1.5m.

The largest contribution by far comes from the first degree zonal harmonic (Table7.4), which indicates a southward displacement of 1.40 ± 0.13m. However, the results from the laser orbits are significantly different. As the sources from which orbital information were obtained varied, no specific conclusions can be drawn from these numbers at the present time.

7.2.3.3 *The Zonal Harmonics of Degree 3 and 4*

The magnitudes of $\overline{\zeta}_{s130}$ and $\overline{\zeta}_{s140}$ are smaller than one-tenth the estimated noise in the case of the global solutions. The signal-to-noise ratio approaches the 0.1 level in the case of the long pass solutions. However, it is debatable whether the long arc solutions can provide estimates of ζ_{s130} (MATHER ET AL 1978b). Disregarding values from the suspect "constrained" solutions, the average result from Table 7.4 is:

$$\overline{\zeta}_{s130} = +10 \pm 5 \text{ cm} \qquad (7.34)$$

The standard deviation is significantly smaller than expected from the input data noise levels and is in good agreement with the oceanographically-determined value. If the standard deviations are to be taken at face value, the seasonal effect on $\bar{\zeta}_{s130}$ should be absent in the result from the Equinox Experiment.

The average value of $\bar{\zeta}_{s140}$ obtained without enforcing crossover constraints is:

$$\bar{\zeta}_{s140} = -12 \pm 4 \text{ cm} \tag{7.35}$$

This is also in good agreement with the oceanographic value obtained in sub-section 7.1.4.

7.3 THE BEST FITTING ELLIPSOID TO THE GLOBAL MEAN SEA LEVEL FROM GEOS-3 ALTIMETRY

Eight data sets based on two separate GEOS-3 ephemerides were considered in this study. Four of the data sets were based on Wallops orbits while the remaining four were based on a set of laser orbits. The laser ranging systems have a higher precision than other tracking systems but do not have a global distribution at present (see Figure 7.2) and are prone to weather limitations. As seen in Row 4 of Table 7.5, the resulting sea surface heights had the lowest level of noise amongst the data sets used in this series of solutions.

The Wallops orbits in 1975 were principally determined using NASA's orbit determination system GEODYN (MARTIN 1972) with the Goddard Earth Model (GEM) 7 (WAGNER ET AL 1977) based on a value of the geocentric gravitational constant GM equal to 398600.80 km^3s^{-2}. The majority of Wallops orbits in 1976 were prepared by the Naval Surface Weapons Center using a specially tailored gravity field model and a value of 398601.00 km^3s^{-2} for GM. In view of the differences reported above, the analysis for the best fitting ellipsoid to the shape of MSL was carried out on subsets of data for 1975 and 1976 as well as for the complete data set.

In an effort to minimise the effect of seasonal changes in MSL on the result, a solution was performed on a fourth Equinox data set (see § 7.2.1.2). Altimeter data used in this solution were collected between August 25 and November 5 1975 and February 25 and May 5 1976. Another four data sets based on the same time periods were prepared from a laser ephemeris which was obtained using the GEM10 gravity field model (LERCH ET AL 1977) with a GM value of 398600.64 km^3s^{-2}. None of these data sets were "constrained" by the procedure referred to in sub-section 7.1.2.

The quantities a_o and f_o in this development are the equatorial radius and meridional flattening of the ellipsoid of revolution which best fits MSL between latitude 65°N and 65°S for the epoch 1975-76. These can be obtained by analysing the altimeter-derived sea surface heights ζ which are related to an adopted reference surface whose equatorial radius and flattening are a and f respectively. The quantities da and df are defined by:

$$a_o = a + da \tag{7.36}$$

and

$$f_o = f + df \tag{7.37}$$

where df is the change in flattening between the sea surface and the value of f obtained from C'_{120} (equation 7.5), and is directly related to the normalised second degree zonal coefficient of the stationary SST $\bar{\zeta}_{s120}$ by (MATHER 1978e, equation A-33):

$$df = -\frac{3\sqrt{5}}{2a}\bar{\zeta}_{s120} \tag{7.38}$$

For a value of $\bar{\zeta}_{s120}$ = -42cm the corresponding quantity df is 2.2×10^{-7}. The value for f_o is estimated at $(298.2377)^{-1}$.

The change da in a can be obtained from the zero degree coefficient $\bar{\zeta}_{s100}$ in the surface spherical harmonic representation of $\zeta_{s\ell}$ defined by equation 5.59. However, da may also be estimated from (see MATHER & RIZOS 1979):

$$da = M\{\zeta_i\} \tag{7.39}$$

where ζ_i are estimates of ζ from a particular data set.

Such an analysis was carried out for all eight data sets. The results obtained for a_o are set out in Table 7.5 (IBID, Table 1). Note that such estimates assume that there is no uncertainty in the value of GM (see § 5.1.3.1).

The results in Table 7.5 indicate that some correlation exists between the values obtained for a_o and the type of orbital ephemeris that was used. In addition, the most anomalous values of a_o were obtained using both the laser and Wallops 1975 data sets.

The mean motion m of a near-earth satellite is related to its average orbital radius a_s and GM by Kepler's third law,

$$m = \left(GM/a_s^3\right)^{\frac{1}{2}} \tag{7.40}$$

On assuming error free timing in the maintenance of the GEOS-3 ephemerides and a lack of space distortion between the tracking station coordinates and the tracking data, Newtonian considerations allow a correction δa for the radial component of the orbital position (and hence of the the heights of the sea surface) on changing GM by d(GM), according to:

$$\delta a = \frac{d(GM)}{3\ GM}\, a_s \tag{7.41}$$

The change in a_o computed from GEOS-3 orbits (equivalent to a change in a_s - as the altimeter measurement remains unaltered) is 6cm for every 0.01 $km^3 s^{-2}$ change in GM. The values of a_o obtained after correction from the estimated values of GM used in the orbit integration, to the value of 398600.64 $km^3 s^{-2}$ is given in Row 3 of Table 7.5. This value of GM is the best available estimate based on the velocity of light of 2.997925×10^8 m s^{-1}. It should be emphasised that the values of GM given in Row 1 for Columns 1 to 4, are only estimates as the orbits were not integrated in a uniform manner during the periods studied. The average value of a_o

TABLE 7.5

Solutions for the Equatorial Radius of the Ellipsoid of Best Fit to the Global MSL from GEOS-3 Altimetry

Orbits / Data Set		Wallops 1975	Wallops 1976	Wallops Full	Wallops Equinox	Laser Equinox	Laser Full	Laser 1976	Laser 1975
GM ($Km^3\ s^{-2}$)	398 600.00 +	.80**	1.00**	.90**	.90**	.64	.64	.64	.64
a (m)	6 378 100.0 +	39.5	40.1	40.0	39.8	38.0	37.6	37.7	37.2
Corrected a* (m)	6 378 100.0 +	38.5	37.9	38.2	38.4	38.0	37.6	37.7	37.2
RMS Variation per $1^{\circ} \times 1^{\circ}$ square (± cm)		284	178	205	174	133	167	120	177
Extent of Coverage (%)		54	44	80	49	49	69	32	54
No. of Data Points ($\times 10^5$)		4.1	3.7	8.8	3.1	3.1	6.1	2.0	4.1

* For change in GM in row 1 to 398 600.64 $Km^3\ s^{-2}$

** Estimates only as orbit integration was not performed in a uniform manner during the entire period obtained is 6378137.9 ± 0.3m. In view of the uncertainties associated with the value of GM in the case of the four solutions using Wallops orbits, the preferred value of a_o is 6378138.0 ± 0.3m.

Values of a_o obtained from GEOS-3 data are directly dependent on the calibration of the altimeter. The only calibration available is pseudo-geometric in nature (MARTIN & BUTLER 1977) and is based on a single overhead pass of altimetry at the Bermuda laser tracking station, and has not been verified. However, the value of a_o obtained is not time dependent at the ±50cm level. Nevertheless some doubts remain concerning the reliability of the value of a_o given above.

For GM equal to 398600.47 $km^3 s^{-2}$, the average value of a_o obtained from the above results is 6378137.0m. This value for GM is consistent with the adopted value of 2.99792458 x 10^8 $m\ s^{-1}$ for the velocity of light (LERCH ET AL 1978d). Such an adjustment in GM is equivalent to shrinking of the GEOS-3 orbit by 1 metre. In addition, the changing of the value of the velocity of light from 2.997925 x 10^8 $m\ s^{-1}$ to the presently adopted value causes a decrease in range distances (for a 1600km journey to the satellite and back to the tracking station) of 25cm or an approximate decrease of 0.1m in the average orbital radius a_s. The total correction of -1.1m is therefore equivalent to adopting a new length standard.

This value of a_o is approximately 1 metre more than the value obtained from an analysis of Doppler tracking stations (ANDERLE 1979) and 2 metres less than the value obtained from laser tracking station coordinates (LERCH ET AL 1978a, Section 4.2). Such estimates are based on an analysis of geocentric tracking station coordinates in combination with estimates of MSL obtained from levelling data connections to coastal tide gauges. The ellipsoid obtained from GEOS-3 altimetry data is one of best fit to the global oceans and can therefore be expected to lie somewhat lower than one which fits MSL in coastal areas, due to the positive contribution by the topography and its isostatic compensation to the shape of the geoid in such areas. A value of 6378137m for the semi-major axis from an analysis of GEOS-3 altimetry has also been reported by RAPP (1979b). From an inspection of recent results for a_o (see MORITZ 1979), it appears that Doppler results tend to give values which are lower than 6378137m, while laser tracking station coordinates imply values which are higher. In all cases the sampling is non-uniform with GEOS-3 altimetry having the best coverage. The value of 6378137m has therefore been adopted as the representative value for the Geodetic Reference System 1980 (IBID).

7.4 CONCLUSIONS

The following conclusions can be reached:

(1) A basis exists for determining three out of five dominant parameters of the stationary SST from GEOS-3 altimetry despite the signal-to-noise problems and the fact that the data acquisition patterns did not have global oceanographic objectives in view.

(2) On the basis of this preliminary investigation, the best values for the dominant harmonics in the SST other than

those of degree one, are the following:

$$\bar{\zeta}_{s120} \text{ (second degree zonal)} = -42 \pm 4 \text{ cm } (-46)$$
$$\bar{\zeta}_{s130} \text{ (third degree zonal)} = +10 \pm 5 \text{ cm } (+7) \quad (7.42)$$
$$\bar{\zeta}_{s140} \text{ (fourth degree zonal)} = -12 \pm 4 \text{ cm } (-10)$$

the values in brackets being the oceanographic estimates (Table 7.3, Solution 3). The value for $\bar{\zeta}_{s120}$ has taken into account the contribution of the permanent earth tide (subsection 7.1.5).

(3) A possible improvement in these results can be expected with a revision of the "constraining" procedure (equation 7.9).

(4) The parameters of the ellipsoid of revolution that is a best fit to the global MSL, are:

$$a_o \text{ (equatorial radius)} = 6\,378\,137.0 \pm 0.3 \text{ m}$$
$$f_o^{-1} \text{ (meridional flattening)} = 298.2377 \; (298.2358) \quad (7.43)$$

The value in brackets for f_o is that implied by the oceanographic estimate of $\bar{\zeta}_{s120}$.

(5) The value of a_o is consistent with the value of 2.99792458 x 10^8 m s^{-1} for the velocity of light, a value of 398600.47 km^{3}s^{-2} for GM and a single calibration of the altimeter. This value of a_o results in the following estimate of the potential of the geoid W_o (see § 5.1.3.2):

$$W_o = 6\,263\,686.0 \pm 0.4 \text{ kgalm} \quad (7.44)$$

(6) An improved gravity field model for the low degree tesseral terms aiming for a resolution of 2 parts in 10^9 is an important prerequisite to further progress in SST studies. In addition, a global network of high precision tracking systems to reduce the radial component of orbital error of the altimeter-equipped spacecraft is essential for recovering, with confidence, SST information through all wavelengths.

8

LEVELLING DATUM DEFINITION FOR SEA SURFACE TOPOGRAPHY AND VERTICAL CRUSTAL MOTION STUDIES

8.1 VERTICAL REFERENCE SYSTEMS IN SPACE AND TIME

The existence of sea surface topography poses problems in geodetic terms due to the reliance on Mean Sea Level as a means for providing the datum of vertical reference on a global scale for first order geodetic levelling.

It was noted in sub-section 3.2.2 that first order geodetic levelling at the present time has the capacity to provide a higher *relative* resolution than any of the other geodetic techniques. The superior resolution of first order levelling is used primarily for sophisticated engineering projects, which are of *local relevance*, and for the establishment of continental gravity anomaly data banks (see Chapter 6) or vertical crustal motion studies where *global considerations*, which should be of consequence, are not resolved with an equivalent degree of certainty due to the levelling being referred in all cases to the regional definition of MSL (see Figure 5.2). On the basis of coastal estimates of SST (see section 3.4) and the consequent lack of continuity in the definition of the datum for levelling (MATHER 1974c), an uncertainty of up to ±2m is anticipated in the inter-connection between any two regional gravity anomaly data banks or two vertical crustal motion studies. This figure is an order of magnitude greater than the internal precision expected from a network of first order geodetic levelling.

It must be emphasised that, without any "effort", MSL provides geodesists with transoceanic links of an accuracy adequate for the unification of global elevation datums in the context of 1 part in 10^6 geodesy. It is the definition of a global vertical datum to a resolution substantially better than ±2m that is one of the most challenging tasks of modern geodesy. This is most easily done by referring displacements to a global datum level surface - the geoid. It could be argued that such a unification can be provided in purely geometrical terms by relating such level datums to three dimensional positional systems. However, the operations of geodetic levelling have practical utility by virtue of their direct relation to level surfaces of the earth's gravity field (see discussion in sub-section 3.2.2). COLOMBO (1979) describes a technique for the establishment of a World Vertical Network by a combination of three dimensional fixes, gravity and levelling data. Position fixes at the levelling datums allow the regional levelling networks to be unified with respect to the reference ellipsoid. Information on the potential *differences* between various datums is provided by satellite-determined gravity field models, supplemented with surface gravity in the near zones (see sub-section 2.3.6). However, such a technique does not make use of the vast quantity of satellite altimetry data available and does not have the utility of being related to sea level, and hence the geoid, in any way. The determination of coastal SST at tide gauges, in relation to a unique geoid with a precision of ± 10cm is most effectively done by using a global data bank of satellite altimetry.

Section 8.2 investigates the conditions under which satellite altimetry, in combination with other geodetic data, can provide the means for:

- the unique definition of a *particular* level surface of the earth's gravity field as the geoid, with a resolution of at least ±10cm.
- the establishment of the heights of MSL above the geoid so defined at:
  - (i) tide gauges which serve as levelling datums for specific regions; and
  - (ii) tide gauges to which no levelling data is related.

Geodetic levelling has always played a significant role in vertical crustal motion studies (see e.g. MEAD 1973, VANICEK & CHRISTODOULIDIS 1974), however, relating levelling datums to a unique geoid is a necessary prerequisite to studying the stability of vertical reference systems with time. Present practice in the measurement of vertical crustal motion relies on the resolution of the phenomenon in terms of changes in orthometric elevations. The latter are referred to a datum usually provided by a convenient tide gauge. The MSL datum so defined cannot be considered desirable for basing extended studies over long time spans for the following reasons:

(a) The geoid may change shape and spatial location in relation to the earth's surface as a function of time (see section 2.4).

(b) The spatial position of the MSL datum may undergo variations of a cyclic, random or secular nature.

Section 8.3 describes how the required geometric displacements can be recovered from geodetic levelling combined with gravity and MSL data.

8.2 VERTICAL DATUM DEFINITION USING SATELLITE ALTIMETRY

8.2.1 PREAMBLE

Although the adjustment of first order geodetic levelling networks over continental extents should provide data at bench marks which are internally consistent to better than ±20cm, serious doubts currently exist concerning the reliability of such networks. Such evidence comes primarily from the comparison of the heights of MSL at tide gauge locations obtained from freely adjusted levelling networks, with those obtained from tidal analysis, which indicate significant discrepancies in the deduced slopes of coastal SST with those estimates obtained from hydrostatic considerations. This has been discussed in detail in section 3.4.

In the latter technique (§ 3.3.3.2), isobaric and level surfaces are assumed to coincide at great depth in the oceans (> 10^3m). Changes dT in temperature T, dp_a in atmospheric pressure p_a and dS in the salinity S of sea water (density ρ_w), produce changes dh in the dynamic height of the sea surface in relation to that of a standard column of sea water at temperature T_o (273°K), pressure p_o (1 atmosphere) and salinity S_o (35 parts per thousand), up to a depth of no motion h_o, according to the relation (see equation 3.35):

$$dh = \frac{1}{g}\left(\int_{p_a}^{p_o} \left(\frac{\partial\alpha}{\partial T}\right) dT dp + \int_{p_a}^{p_o} \left(\frac{\partial\alpha}{\partial S}\right) dS dp - \frac{dp_a}{\rho_w} \right) \tag{8.1}$$

where dp is the incremental change in pressure and α is the specific volume of sea water.

Possible reasons for the discrepancies between coastal comparisons of sea surface slopes from geodetic and oceanographic levelling techniques have been discussed in sub-sections 3.4.2 and 3.4.3. It was mentioned that a potentially significant source of uncertainty is the need for extrapolating values of SST (ζ_s) determined by the method of steric levelling (equation 8.1) in deep oceans to coastal tide gauge sites using the principles of geostrophic levelling (§ 3.3.3.3). The method of geostrophic levelling is based on the Lagrangian equations (developed from equation 3.36 - derived for an arbitrary orientation of the x direction):

$$\ddot{x}_1 - f\dot{x}_2 = -g\frac{\partial\zeta_s}{\partial x_1} - \frac{1}{\rho_w}\frac{\partial p_a}{\partial x_1} + F_1$$

and (8.2)

$$\ddot{x}_2 + f\dot{x}_1 = -g\frac{\partial\zeta_s}{\partial x_2} - \frac{1}{\rho_w}\frac{\partial p_a}{\partial x_2} + F_2$$

where $(\dot{x}_1,\dot{x}_2)$, $(\ddot{x}_1,\ddot{x}_2)$ and (F_1,F_2) are components respectively of surface velocity, acceleration and frictional forces of the ocean along the axes (x_1,x_2) of a two dimensional Cartesian coordinate system in the local horizon plane with the X_1 axis oriented east and the X_2 axis oriented north, f the Coriolis parameter being defined by equation 3.28.

The use of this technique assumes that current meter measurements of $\dot{x}_1$, $\dot{x}_2$ are available, along with observations of horizontal atmospheric pressure gradients and data for the evaluation of the frictional forces. Practical calculations are performed by assuming a non-accelerated system (i.e. $\ddot{x}_1$, $\ddot{x}_2 = 0$). Except in abnormal conditions, $\dot{x}_\alpha < 10^2$ cm s^{-1} and $\partial\zeta_s/\partial x_\alpha =$ o{0.2"}. It follows that F_1, F_2 must be estimated to $\pm 10^{-4}$ cm s^{-2} (±0.1mgal) if extrapolation errors are to be held to below ±1cm. Physical oceanographers have maintained that extrapolation of values of ζ_s over distances up to 300km from deep oceans to coastal sites are unlikely to introduce errors of more than ±10cm in the result (HAMON & GREIG 1972, p.7160). However, a recent examination of this extrapolation process by COLEMAN ET AL (1979, Section 3.1) has indicated that this error estimate appears to be optimistic and an error two or three times greater may be more realistic.

In view of the uncertainties surrounding the estimates of coastal SST from oceanographic considerations (sub-section 3.4.2) and the doubts cast on the validity of geodetic levelling networks (sub-section 3.4.3), it is necessary that an independent means be established to achieve the following objectives:

- Determination of the height of MSL at each tide gauge linked to a geodetic levelling network, with a precision of at least ±10cm in the first instance.
- Definition of the universal datum level surface to which each of these MSL heights is referred, with an equivalent precision.

The resolution of coastal SST to a precision of ±10cm is of interest in high precision geodesy for the following reasons:

(1) To facilitate the implementation of geoid definitions that require estimates of non-tidal SST for land areas. These include the "geodetic" definition (§ 2.2.5.2), the "oceanic/geodetic" definition (§ 2.2.5.4) and the "GBVP" definition (§ 2.2.5.5) for the geoid.

(2) Provide an additional set of constraints for controlling the propagation of systematic error in the adjustment of continental level networks. The "heights" of MSL above a level surface would provide control at the peripheries of such networks.

(3) To facilitate the unification of geodetic levelling datums by relating each MSL datum to a uniquely defined geoid (see sub-section 8.2.3).

(4) Provide valuable information on the stability of the levelling datum for vertical crustal motion studies (see sub-section 8.3.2).

The problems in using altimetry data directly for the determination of coastal SST are discussed in sub-section 8.2.2. Practical considerations indicate that as a result of deficiencies in GEOS-3 altimetry and present day geoid models only the levelling datum displacements from the geoid can be estimated, as opposed to the determination of SST along coastlines as a continuous ribbon. The role that continental gravity anomaly data banks can play in determining such displacements is described in sub-section 8.2.3 and preliminary numerical results are presented in 8.2.4. The results are discussed in sub-section 8.2.5. The determination of coastal SST from satellite altimetry data has been investigated by MATHER ET AL (1978c) and MATHER & RIZOS (1978). The following sections are developed from IBID (1978c).

8.2.2 CONDITIONS INFLUENCING THE DETERMINATION OF HEIGHTS OF MSL AT COASTAL SITES FROM SATELLITE ALTIMETRY

Satellite altimetry data in coastal regions has been acquired by the radar altimeter on board the GEOS-3 spacecraft since 1975. The analysis of data in the Tasman and Coral Seas (MATHER ET AL 1977a) in continental shelf areas off the east coast of Australia indicates the following:

- the sea surface appears to rise relatively steeply over sea mounts and the continental shelf slope; and
- non-oceanic readings and, hence, the transition from ocean to land, are clearly recognisable at the ±1m level between successive data records.

On the basis of these figures it can be conservatively estimated that GEOS-3 satellite altimetry can provide data of quality up to 20km from the coastline. This is also true of the ±10cm radar altimeter on board the SEASAT-A spacecraft launched in mid-1978, which has a "footprint" of 2-12km

(NAGLER & McCANDLESS 1975, p.2).

The basic data is in the form of heights ζ' of the instantaneous sea surface above the adopted reference figure. The sequence of operations to convert such data into values of the heights of MSL at the regional tide gauges sites is the following:

(a) Determine the heights ζ_s of the stationary SST in the adjacent continental shelf areas. This presumes that the geoid has already been defined through short wavelengths ($< \ell$, where ℓ has the same definition as in sub-section 5.2.1).

(b) Extrapolate the resulting values of ζ_s in the shallow continental shelf ocean to the coastal site using equation 8.2.

Values of the stationary height of MSL deduced from satellite altimetry should be the average of at least one year's readings. As ζ_s is not greater than ± 2m, values of ζ' should be computed from orbits which have a resolution of at least ± 10cm in the radial component of position. The oceanographic surveys for current velocities, atmospheric pressure gradients and frictional forces can only be carried out on a few occasions, possibly just once. However, the continuous monitoring of local ground truth during the period of altimetry should provide a basis for accurate extrapolation using equation 8.2, in most areas.

It was established in sub-section 5.1.3 that satellite altimetry data has the potential to select a particular level surface of the earth's gravity field as the geoid by defining a magnitude for $(W_o - U_o)$. The difficulties likely to be encountered in determining the *shape* of the geoid through short wavelengths in oceanic areas have been described at length in section 5.1. In summary, on assuming the data to be of adequate quality, the principal problems to be overcome are the following:

(1) No complete coverage exists globally for either ζ' or $\Delta g'$. The former is probably subject to systematic errors of long wavelength, while the precision of oceanic gravity data is at least an order of magnitude worse than that of land gravity data. It has been shown in section 6.2 that even a homogeneous gravity field determination like that available for Australia is only adequate for geoid determinations with a precision of ± 30cm.

(2) All data are measured in relation to the sea surface, either instantaneous or MSL, and not the geoid.

(3) Local MSL approximates the geoid to no better than ± 2m.

In view of these difficulties it was established in section 5.2 that the most favourable procedure for determining the stationary SST ζ_s is the following:

(i) Define the long wavelength components of the SST $\zeta_{s\ell}$ from a combination of altimetry data and satellite-determined gravity field model information (described in sub-section 5.2.2 and attempted in section 7.2).

(ii) Having accomplished (i), define the short wavelength components of SST ζ_{ss} by using information contained in values of height anomalies and gravity anomalies on a regional basis in the solution of the "inverse" of the conventional GBVP (this technique is described in § 5.2.3.2). This calls for a knowledge of the gravity anomalies within 500km of the point of computation with a precision better than ± 0.1mgal through "wavelengths of interest". It also requires the definition of ζ' on land with a resolution of ± 10cm by a combination of high precision position fixes and first order geodetic levelling. This has been discussed in § 5.2.3.3.

The determination of ζ_{ss} in the continental shelf areas, in conjunction with the collection of current meter data, atmospheric data and the development of models for frictional forces for the extrapolation of ζ_{ss} (equation 8.2) would permit the geodetic goals listed in sub-section 8.2.1 to be satisfied. However, in view of the inadequate quality and distribution of either ζ' or $\Delta g'$ at the present time, it was concluded in § 5.2.3.3 that the determination of ζ_{ss} is unlikely to be successful in the foreseeable future. It is therefore desirable to look for alternate techniques for the determination of the height of MSL at a coastal site. This is discussed in the following section.

8.2.3 PRACTICAL CONSIDERATIONS

8.2.3.1 *Preamble*

A method has been described in sub-section 5.3.2 for determining the height of MSL at regional levelling datums which serve areas greater than ℓ^2 km^2 if the regional gravity anomaly data bank is computed using levelling data related to this datum. The area covered by the datum is larger in extent (ℓ^2 km^2) than the highest full harmonic of the gravity field model which is known free of error.

RELATION 2 (equation 5.54) describes the relationship between gravity anomalies and the unknown SST. In land areas the effect of SST is represented by ζ_{sd} at the regional levelling datum and the fundamental relation is (equation 5.95):

$$\Delta g_c = \Delta g'' + \frac{2}{R}(W_o - U_o) - \frac{2\gamma}{R}\zeta_{sd} \tag{8.3}$$

where $\Delta g''$ is the gravity anomaly on the "higher" reference system (section 4.4), W_o is the potential of the geoid and U_o is the potential of the reference system and R is, for all practical purposes, the mean radius of the earth. The computable part of the gravity anomaly $\Delta g''$ is, to sufficient accuracy, given by (equation 6.17):

$$\Delta g_c = \Delta g - \delta\gamma + \delta g_a \tag{8.4}$$

The surface gravity anomaly is evaluated from gravity measurements and geodetic levelling data according to equation 5.88, $\delta\gamma$ is the effect of using the "higher" reference model (equation 4.93) and δg_a is the effect of the atmosphere on the gravity anomaly (sub-section 4.5.1).

The height of MSL at the regional datum (ζ_{sd}) can be obtained by forming observation equations of the form (equation 5.96):

$$v_{\Delta g} = (\Delta g_c - \delta g_a) + \frac{2\gamma}{R}\zeta_{sd} - \frac{2}{R}(W_o - U_o) \tag{8.5}$$

The derivation of this equation is found in § 5.2.2.2.

The successful recovery of ζ_{sd} by the solution of a system of observation equations using data ($\Delta g_c - \delta g_a$) collected in relation to the regional levelling datum is dependent on three factors:

(1) There should be no other sources of constant bias in the gravity anomaly data bank apart from the effect of the height of MSL above the geoid at the datum.

(2) The formation of the observation equations should extend over an area at least as large as the highest full harmonic in the "higher" reference model which is free from error.

(3) The solution procedure adopted should ensure that the gravity field model used in computing $\delta\gamma$ introduces no aliasing effect on the result. This is discussed below.

In view of the fact that ζ_{sd} is about an order of magnitude smaller than $v_{\Delta g}$ in equation 8.5 (see discussion on equation 5.96) the stability of the solution is likely to improve by using the largest possible block means when forming observation equations, say two degree or five degree area means. As the values of such area means are strongly correlated with position (see e.g. MATHER 1975b) and in view of the adverse signal-to-noise ratio, it is prudent to model these variations in $v_{\Delta g}$. Any two dimensional model should suffice for the task, assuming that the gravity data is evenly distributed about the datum. Thus equation 8.5 can be written for land areas on the same levelling datum in the form:

$$v_{\Delta g} = (\Delta g_c - \delta g_a) + \frac{2\gamma}{R}\zeta_{sd} + \sum_n \sum_m \sum_\alpha K_{\alpha nm} F_{\alpha nm}(\phi,\lambda) - \frac{2}{R}(W_o - U_o) \qquad (8.6)$$

where $K_{\alpha nm}$ are coefficients of the Fourier functions $F_{\alpha nm}$, defined by (equation 5.75),

$$F_{1nm} = \cos(n\,\Delta\phi + m\,\Delta\lambda) \quad ; \quad F_{2nm} = \sin(n\,\Delta\phi + m\,\Delta\lambda) \qquad (8.7)$$

n and m not being equal to zero simultaneously. $\Delta\phi$, $\Delta\lambda$ in equation 8.6 are differences of geocentric surface coordinates of the centres of the area means from some convenient point of reference in the region. The most important wavelengths which need to be modelled in order that the resulting value of ζ_{sd} is not aliased, are the following (MATHER ET AL 1978c, p.11):

(a) Those equal to 4 times the smallest dimension (d) of the region served by the datum arising from errors in the assumption that all wavelengths longer than ℓ in the "higher" reference model are free of error (these would be manifest as a "tilt" of the regional gravity anomaly data Δg with respect to those implied by the "higher" reference model $\delta\gamma$).

(b) Those equal to twice d, due to residual errors in the gravity and levelling networks (manifest as a "bow" in the values of $(\Delta g - \delta\gamma)$ at the centre of the area served by the levelling datum).

In summary, the quality of the determination is therefore dependent on:

- the extent of the area served by the levelling datum, represented in the solution; and
- whether all wavelengths longer than ℓ have been sampled in the determination.

It follows that the quality of the determination will diminish as a function of the shortfall below $\ell^2 km^2$ of the area served by the regional levelling datum. The use of equation 8.5 cannot be expected to give stable results if the area sampled is less than $\frac{1}{4}\ell^2 km^2$, even if the modification at equation 8.6 is used.

The numerical value for ζ_{sd} is dependent on the value of the other zero degree term in equation 8.6, namely the value of $(W_o - U_o)$. The only role played by the GEOS-3 altimetry data is in defining the quantity $(W_o - U_o)$ for the epoch 1976.0 - attempted in § 8.2.3.2. The preparation of the gravity anomaly data sets for Australia and the United States is described in § 8.2.3.3.

8.2.3.2 *The Geoid for Epoch 1976.0*

As abnormal conditions may prevail in coastal areas, it is preferable to select the datum level surface on the basis of data sampled in ocean areas alone (see § 2.2.5.3). The GEOS-3 altimetry used for the definition of the geoid for epoch 1976.0 was the total data available for the periods September - October 1975 and March - April, 1976 (the original Equinox data set - see § 7.2.1.2). The distribution of data is as shown in Figures 7.3 and 7.4. This data was used to derive a geometrical model of the sea surface which was considered to be minimally affected by seasonal variations in sea level. The representation obtained, however, is less than ideal due to the irregular data distribution.

The determination of the value of W_o from satellite altimetry data has been described in § 5.1.3.2. The sea surface positions deduced from the altimetry are expressed in geocentric spherical coordinates (ϕ, λ, R_o) and the geopotential W_{ss} at the sea surface is represented by (equation 5.45):

$$W_{ss} = \frac{GM}{R_o} \sum_{n=0}^{n'} \left(\frac{a}{R_o}\right)^n \sum_{m=0}^{n} \sum_{\alpha=1}^{2} C'_{\alpha nm} S_{\alpha nm} + \tfrac{1}{2} R_o{}^2 \omega^2 \cos^2\phi + o\{2\text{kgalm}\} \qquad (8.8)$$

where $C'_{\alpha nm}$ are coefficients of the best available gravity field model to degree n' (it was shown in § 5.1.3.2 that the effect of the earth's atmosphere on the downward continuation of the geopotential to the earth's surface is less than 1kgalcm for the low degree terms). $S_{\alpha nm}$ are surface spherical harmonic functions of degree n and m defined by equation 5.17. In the current series of calculations, $C'_{\alpha nm}$ are the coefficients of GEM9 (LERCH ET AL 1977).

The potential of the geoid is then given by (equation 5.48):

$$W_o = M\{W_{ss}\} \qquad (8.9)$$

where $M\{\ \}$ is the global mean, which in this case is restricted to ocean areas between parallels 65°S and 65°N. The Equinox data set was not fully representative of the whole ocean, the most serious omission in the context of evaluating W_o being the large gap in the south Pacific. The result obtained from the data in 39.6% of the 33902 1° x 1° area means classified as ocean (see sub-section 7.1.4), and based on the Bermuda calibration of the altimeter (MARTIN & BUTLER 1977), is

$$W_o = 6\ 263\ 682.76 \pm 0.4 \text{ kgalm} \qquad (8.10)$$

The rms residual representing variations within a 1° x 1° square was ± 4.4m.

A summary of results is given in Table 8.1. The value of W_o was based on the following constants (see sub-section 7.1.3):

$$\begin{aligned} GM &= 398\ 600.47 \ \ km^3s^{-2} \\ \omega &= 7.292\ 115\ 1467 \times 10^{-5} \ rad\ s^{-1} \\ a &= 6\ 378\ 140.0 \ \ m \\ c &= 2.997\ 924\ 58 \times 10^{8} \ \ m\ s^{-1} \end{aligned} \qquad (8.11)$$

T A B L E 8.1

The Potential of the Geoid (W_o) from GEOS-3 Altimetry

Data Source	Wallops	Wallops
Epoch	Sep-Oct '75 Mar-Apr '76	Feb-Aug '76
No. of Passes	634	882
No. of 1^o Sq. Sampled	13,499	12,349
rms $(W_{ss} - W_o)$ ± kgalm	5.8	5.1
W_o (kgalm)	6 263 682.76	6 263 682.39

Source: MATHER ET AL (1978c)

The flattening of the reference ellipsoid which is consistent with the velocity of light at (8.11) and the second degree zonal harmonic coefficient C'_{120} of the GEM9 gravity field model is (equation 7.15):

$$f^{-1} = 298.257\ 316 \qquad (8.12)$$

It follows that the potential U_o on the surface of the rotating equipotential ellipsoid defined by the parameters at 8.11 and 8.12 is (equation 5.36):

$$U_o = \frac{GM}{a} \frac{\alpha}{\sin\alpha} + \frac{1}{3} a^2\omega^2 \qquad (8.13)$$

where α is defined by equations 5.25 and 5.26. U_o for the system of reference adopted in the present series of calculations is

$$U_o = 6\ 263\ 682.67 \ \ kgalm \qquad (8.14)$$

This is equivalent to the value of the spheropotential of the "higher" reference model, whose shape closely approximates the geoid.

The value of the term $(W_o - U_o)$ is given by:

$$W_o - U_o \;=\; +0.09 \pm 0.4 \;\; \text{kgalm} \qquad (8.15)$$

8.2.3.3 *The Computation of the Gravity Anomaly*

The correct procedure for preparing gravity data for high precision geodetic computations is described in sub-section 6.1.2. The quantity required is Δg_c, defined by equation 8.4. The preparation of the gravity anomaly data banks for Australia (AUSGAD 77) and central North America (CNAGAD 77) is described in sections 6.2 and 6.3 respectively.

These gravity anomalies however, are based on the Geodetic Reference System 1967 (GRS 67) (IAG 1971) defined by the parameters (equation 6.19):

$$\begin{aligned} GM &= 398\ 603.0 \;\; km^3s^{-2} \\ \omega &= 7.292\ 115\ 1467 \times 10^{-5} \;\; rad\ s^{-1} \\ a &= 6\ 378\ 160.0 \;\; m \\ f^{-1} &= 298.247\ 167\ 427 \end{aligned} \qquad (8.16)$$

Normal gravity on the ellipsoid for GRS 67 is computed using the formula (equation 6.10):

$$\gamma_o = \gamma_e(1 + \beta_3 \sin^2\phi + \beta_4 \sin^4\phi) + o\{\pm 4\mu gal\} \qquad (8.17)$$

where ϕ is geodetic latitude and the value of the constants γ_e, β_3 and β_4 are (equations 6.9 and 6.11):

$$\gamma_e = 978\ 031.85 \;\; mgal \;\; ; \;\; \beta_3 = 5.278\ 895 \times 10^{-3} \;\; ; \;\; \beta_4 = 2.3462 \times 10^{-5} \qquad (8.18)$$

For the present series of computations, the AUSGAD 77 and CNAGAD 77 data sets have been referred to the reference system defined by the parameters at equations 8.11 and 8.12. These values, with the exception of a, define GRS 80 (MORITZ 1979). The value of a adopted for GRS 80 is 6378137m (see section 7.3). The gravity anomalies were therefore adjusted by means of the following relation:

$$\Delta g_c^{NEW} = \Delta g_c^{GRS\ 67} + \gamma_o - \gamma_o' \qquad (8.19)$$

where γ_o', normal gravity on the "new" reference ellipsoid is defined by an expression of the form at equation 8.17 but with constants:

$$\gamma_e' = 978\ 031.68 \;\; mgal \;\; ; \;\; \beta_3' = 5.278\ 93 \times 10^{-3} \;\; ; \;\; \beta_4' = 2.3461 \times 10^{-5} \qquad (8.20)$$

All gravity anomalies were finally referred to the "higher" reference model defined by GEM9. The quantity Δg_d was then obtained by (equation 5.62):

$$\Delta g_d \;=\; \Delta g_c \;-\; \delta g_a \qquad (8.21)$$

8.2.4 NUMERICAL RESULTS FOR THE COMPUTATION OF DATUM LEVEL SURFACE DISPLACEMENTS USING GRAVITY ANOMALY DATA BANKS

8.2.4.1 *The Jervis Bay Datum Level Surface*

All Australian gravity data in AUSGAD 77 is related to the freely adjusted Australian Levelling Survey of 1970 and referred to the Jervis Bay Datum ($\phi = 35.1^{o}$S, $\lambda = 150.7^{o}$E) - see § 6.2.2.3.

A solution procedure based on equation 8.6 will be subject to considerable aliasing of the value of ζ_{sd} if the errors in the "higher" reference model with wavelengths greater than the shortest dimension (d) of the area served by the datum, were not modelled in the computations. It is estimated that the error in the GEM9 coefficients to (4,4) on models at the surface of the earth is $\pm 1.4 \times 10^{-8}$ (LERCH ET AL 1977, p.52), equivalent to approximately ±9kgalcm in disturbing potential. These estimated errors increase rapidly with increase in degree to around ±60kgalcm for degree 20.

The value of ζ_{sd} can, in principle, be obtained by the analysis of either the $1^{o} \times 1^{o}$, $2^{o} \times 2^{o}$ or $5^{o} \times 5^{o}$ data banks. The results obtained are influenced by the following factors:

- The signal-to-noise. ζ_{sd} is not larger than ±2m while the variability of the data increases with decrease of square size (Table 8.2, Row 3).
- Departures from the assumption that the gravity field model is error free. The existence of a large non-zero value for the regional mean ($\overline{\Delta g_d}$) of Δg_d over Australia emphasises the need for Fourier modelling the long wavelengths errors in the gravity field. The large positive values of $\overline{\Delta g_d}$ for Australia (Table 8.2, Row 2) indicate the net high of surface gravity in the region. These values are highly correlated with position showing net highs in the east and west of the continent with a band of lows in the centre (e.g. Figure 6.6). This type of effect has a wavelength two-thirds that of the east-west dimension of the continent and should be modelled when using equation 8.6.
- Errors in the area means. These arise primarily due to inadequate sampling.

It was therefore decided to model the following wavelengths in the Fourier series when effecting a solution:

$$\frac{2}{3}d, \ \frac{4}{3}d, \ 2d, \ \frac{8}{3}d \ldots\ldots$$

The values adopted for d in the Australian calculations were $d_\phi = 30^{o}$ in latitude and $d_\lambda = 45^{o}$ in longitude.

TABLE 8.2

Statistics From Area Mean Values of Δg_d in the Australian Gravity Data Bank (AUSGAD 77), Based on the Freely Adjusted Level Network for Australia Referred to the Jervis Bay Datum Level Surface

$d_\phi = 30^o$ $d_\lambda = 45^o$ (Units mgal)

Square Size	$1^o \times 1^o$	$2^o \times 2^o$	$5^o \times 5^o$
No. of Blocks	722	181*	30* (15)
Mean Value ($\overline{\Delta g}_d$)	3.65	3.81	3.41 (-0.18)
rms	16.6	13.3	8.3 (6.5)
Expected $\lvert\zeta_{sd}/\overline{\Delta g}_d\rvert$	0.02	0.02	0.04 (0.05)

* Minimum Representation = 40 percent

(Figures within brackets for $5^o \times 5^o$ squares are based on a sample which includes only square where mean is computed from 25 $1^o \times 1^o$ values; i.e., 100% representation.)

Source: MATHER ET AL (1978c)

In view of the unfavourable signal-to-noise it was necessary to constrain the solution to an a priori assessment of the magnitudes of the corrections. For example, the term $\gamma\zeta_{sd}/R$ in equation 8.6 will not exceed ±0.3mgal while the coefficients $K_{\alpha nm}$ should on the average, not be significantly larger than $\overline{\Delta g}_d/N$, where N is the total number of harmonics modelled. Consequently, the solutions shown in Tables 8.2 and 8.3 were obtained by minimising:

$$\Phi = \sum_{i=1}^{N'} w_i v^2_{\Delta g_i} + \sum_{i=1}^{N} w_{ci} (K_{\alpha nm})^2 \tag{8.22}$$

where

$$w_i = 1/\cos\phi_i \quad ; \quad w_{ci} = \begin{cases} 100 & \text{if } \alpha=1,\ n=m=0 \\ N/(\overline{\Delta g}_d)^2 & \end{cases} \tag{8.23}$$

The solutions obtained for Australia using AUSGAD 77 and the GEOS-3 altimeter-determined geoid for 1976.0 are set out in Table 8.2. The preferred result is obtained using fully represented $5^o \times 5^o$ values as the area means are probably more reliable, being less affected by irregularities in gravity field sampling. The number of observation equations is limited, reducing to 15 if only fully represented squares (i.e. only $5^\circ \times 5^\circ$ squares based on 25 $1^\circ \times 1^\circ$ values) were considered (Table 8.2, Row 1).

On the basis of a value of $\overline{\Delta g_d}$ of -0.18mgal and $(W_o - U_o)$ equal to +0.09kgalm the preferred value for the height of MSL at the Jervis Bay Datum is

$$(\zeta_{sd})_{\text{Jervis Bay}} = +0.21 \pm 0.4 \text{ m} \tag{8.24}$$

The equivalent value as extrapolated from the deep oceans using oceanographic data is +0.3 ±0.2m (see Figure 3.3) - note that a zero degree effect of +1.10m has been eliminated. The figure at 8.24 is referred to epoch 1968.0. The variation of the height of MSL with time at Sydney is estimated at less than +1mm per year (Figure 2.6). Thus there is less than 1cm discrepancy introduced into the result due to the non-coincidence of epochs of the levelling and the altimetry. The error in the datum for the Australian gravity is estimated at ±0.06mgal (see § 6.2.3.2), introducing an uncertainty of ±0.2m in the results at 8.24. For estimates of other sources of error, see comments on the results in § 8.2.4.2.

8.2.4.2 *Estimating the Effects of Zero Degree in the Gravity Data Bank for Central North America*

The region covered by this study was the North American continent bounded by the parallels 28°N and 50°N. This included a small part of Mexico and the south-eastern part of Canada. Gravity values on the North American continent are, as best as possible, referred to the the International Gravity Standardization Network (IGSN 71) (MORELLI ET AL 1971). The basic network was assembled by the Defense Mapping Agency Aerospace Center. It would be difficult to assess, without a major re-examination of the data, whether the pattern of errors in the United States Levelling Network are reflected in the resulting $1^\circ \times 1^\circ$ free air anomaly data bank compiled by RAPP (1977). The data used in this study had been rounded off to the nearest mgal. Its characteristics are summarized in Table 8.3. Parts of the Canadian gravity data bank were also included in this study. The same comments made about the elevations of gravity stations in the United States apply to those in Canada, there being a variable systematic difference between common junction points of the two levelling systems which is about ±10cm on the average (LACHAPELLE 1978b). This has not been considered significant in the present study, which is of an exploratory nature.

It is therefore not clear whether the analysis of the gravity anomaly data bank for central North America, prepared as described in section 6.3, will contain any information on the height of MSL at the datum level surface for the region, as implied in the computation of free air anomalies. Geopotential differences were computed using the $1^\circ \times 1^\circ$ mean square elevation and gravity data banks in relation to the value in the $1^\circ \times 1^\circ$ square ($\phi = 29.5^\circ$N; $\lambda = 261.5^\circ$E) containing the Galveston tide gauge.

The discrepancy between the GEM9 model and the surface gravity data, as embodied in the value of $\overline{\Delta g_d}$ for the region is five times smaller than that for Australia (Tables 8.2 and 8.3, Row 2). This is probably a reflection of the better tracking coverage available in the North American area when compiling the GEM9 model.

If it were assumed that all the gravity data in the North American study were

- based on a regional standardisation network of the same quality as IGSN 71; and

- converted to gravity anomalies based on a network of elevations substantially controlled by the freely adjusted regional levelling network,

it can be said that for $\overline{\Delta g_d}$ equal to -0.25mgal (Table 8.3, Row 2), the height of MSL at Galveston is given by

$$(\zeta_{sd})_{\text{Galveston}} = +0.14 \pm 0.4 \text{ m} \qquad (8.25)$$

It is only possible to obtain a very approximate oceanographic value for ζ_{sd} at Galveston as +0.1 ± 0.3m (LEVITAS & DORT 1977, p.1283), allowing for the zero degree effect. The sources of uncertainty in the result at 8.25, provided the above assumptions were valid, are the following:

(1) 20cm due to errors in the gravity standardisation network. This figure is a guess, compatible with the more carefully assessed figure for the Australian national network, quoted in § 8.2.4.1.

(2) 12cm due to aliasing as a result of using too few coefficients in the Fourier modelling - an inevitable consequence when using larger area means for improving the signal-to-noise.

(3) The value of W_o obtained was not based on a full coverage of the oceans between 65°S and 65°N. As shown in Table 8.1, the result may require revision by up to ±30cm as the altimeter data set is varied.

It is not unreasonable to conclude that the values of ζ_{sd} given in equations 8.24 and 8.25 have uncertainties at the ±0.4m level. The level of agreement obtained with oceanographic values is much better, being about one-quarter this value.

It could be argued that the result at 8.25 could also be obtained from a set of heights which had a randomly established datum (e.g., obtained from maps). However, the stability of the results obtained warrant a closer look at the technique under less speculative conditions.

TABLE 8.3

Statistics from Area Mean Values of Δg_d in the Gravity Data Set for Central North America, Based on Geopotential Estimates Related to the Galveston Datum Level Surface (Units mgal)

$d_\phi = 20^o$ $d_\lambda = 45^o$

Square Size	$1^o \times 1^o$	$2^o \times 2^o$	$5^o \times 5^o$
No. of Blocks	835	218*	34*
Mean Value ($\overline{\Delta g}_d$)	-0.75	-0.67	-0.25
rms	15.9	10.4	6.5
Expected $\lvert\zeta_{sd}/\overline{\Delta g}_d\rvert$	0.02	0.03	0.05

* Minimum Representation = 40 percent

Source: MATHER ET AL (1978c)

8.2.5 DISCUSSION OF RESULTS

The results presented in § 8.2.4.1 and 8.2.4.2 are based on the following data:

- A geoid for epoch 1976.0 based on data in the 1977 GEOS-3 altimeter data bank. This data base is being added to and in the process of further revision. It is not expected that the value of W_o given in § 8.2.3.2 will change by more than ±0.3kgalm when the representation increases from the 39.6 percent coverage used in the present study and when orbit improvement has been completed.
- The gravity anomaly data bank for Australia specially prepared for sea surface topography determinations (AUSGAD 77).
- The $1^o \times 1^o$ free air anomaly data set for central North America originally compiled by the DMA Aerospace Center and provided by RAPP (1977) in the form of values rounded off to the nearest mgal.
- The GEM9 gravity field model.

The last named data set is not critically involved in the determination though weaknesses in the model cause additional signal-to-noise problems.

While some doubt exists about the practical significance of the results, the following observations can be made:

(a) The analysis of the data for Australia (Table 8.2) indicate the extent of the aliasing influence of $5^o \times 5^o$ area means which were not based on a full representation of surface gravity data (i.e., twenty-five $1^{o} \times 1^{o}$ values). Restriction of the analysis to fully represented areas reduces the ratio of unknowns to observation equations. This is offset by the reduction of noise in the observational data and results in an improved solution. Stability of solution is enhanced by restricting the Fourier modelling to the same range of longitude per parallel sampled.

(b) The results for Australia indicates that the use of this technique in regions not providing heavy tracking coverage for the development of the satellite-determined gravity field model, will produce conditions where ζ_{sd} has to be determined in the presence of adverse levels of noise. Subsequent computational instability can be avoided by studying the nature of the distribution of Δg_d over the region before the selection of wavelengths for Fourier modelling.

(c) The results given in this study for the MSL datum at Galveston are based on the assumption that the gravity anomaly data bank for central North America was based on the geodetic levelling. There is no assurance that this is the case. It is most desirable that this experiment be repeated with a gravity data set whose elevations are known to be related to the continental levelling network based on the Galveston Datum Level Surface.

(d) The results presented in this analysis establish the potential of this method for defining the height of MSL at the regional levelling datum serving areas larger than the square of the minimum wavelength in the satellite- determined gravity field model. Ideally, the model should be free from error through these wavelengths. However, slightly degraded results can be obtained even if this condition is not satisfied, as seen from the results given above.

(e) This study shows that gravity anomalies computed from levelling data related to either the Jervis Bay or Galveston Datums can be assumed to refer to the geoid to 0.1mgal.

(f) This technique also provides a test of the value of GM used to compute normal gravity, as described in the following section.

8.2.6 SIGNIFICANCE OF THE RESULTS AS A TEST FOR THE VALUE OF THE GEOCENTRIC GRAVITATIONAL CONSTANT (GM)

The value of the equatorial radius of the ellipsoid which best fits the geoid at sea between 65^{o}N and 65^{o}S ($a = 6378140$m) used in § 8.2.3.2 to compute the potential of the geoid W_o was approximate. It was noted in section 7.3 that the GEOS-3 ephemeris (and hence the sea surface heights above a unique reference ellipsoid) was based on orbit integrations performed by more than one agency using different computer programs and Earth models. The principal inconsistency when dealing with scale was the use of different values of GM in orbit integration (see Table 7.5). On standardising the orbits for this effect, it was found that the ellipsoid of revolution which best fits MSL between 65^{o}N and 65^{o}S as sampled between September - October 1975 and March - April 1976 (Table 7.5, Column 4, Row 3) has an equatorial radius of:

$$a = 6\,378\,137.3 \pm 0.3 \text{ m} \qquad (8.26)$$

Note that a correction of -1.1m has been applied to make the value consistent with the velocity of light of 2.99792458 x 10^8m s^{-1} - see discussion in section 7.3. The estimate obtained in this manner is dependent on the value used for GM and an unverified calibration of the GEOS-3 altimeter (MARTIN & BUTLER 1977). This value of a, together with the remaining quantities in 8.11 and 8.12, has been adopted into the Geodetic Reference System 1980 (GRS 80) (MORITZ 1979). The value of GM used in the set of computations to obtain the result at 8.26 was:

$$GM = 398\ 600.47\ km^3s^{-2} \tag{8.27}$$

This is the same value as used in the computations for W_o in § 8.2.3.2. However, this scaling effect on the GEOS-3 ephemeris was not allowed for in those computations. The Equinox altimetry data set was used to obtain a value of

$$W_o = 6\ 263\ 682.8 \text{ kgalm} \tag{8.28}$$

which was based on an assumed value of GM = 398600.9 km^3s^{-2}. Standardising the GEOS-3 orbits during the Equinox data acquisition period for this effect, i.e. correcting the value of a used in deducing R_o in equation 8.8 by -2.5m (see Table 7.5, Column 4, Rows 2 and 3), the value of W_o obtained is given by (MATHER 1978g, Section 2):

$$W_o = 6\ 263\ 685.3 \pm 0.4 \text{ kgalm} \tag{8.29}$$

The value of U_o based on the constants in equations 8.11 and 8.12 but using the value of a in 8.26 is:

$$U_o = 6\ 263\ 685.3 \text{ kgalm} \tag{8.30}$$

Obviously, the changing of the length standard does not alter the value of the quantity (W_o- U_o) as the reference ellipsoid is one of best fit to the geoid. The condition that the geoid is the level surface of best fit to global MSL is enforced through the use of altimetry data to define the quantity W_{ss} in equation 8.9.

However, the use of a value of a = 6378140m in the computation of normal gravity for the preparation of regional gravity anomaly data sets (see § 8.2.3.3) means that the results obtained for the height of MSL at the levelling datums (8.24 and 8.25) are referred to the geoid for epoch 1976.0 defined by the value of W_o at 8.28. Therefore there is doubt concerning the parameters defining the geoid for the following reasons:

(i) The value of W_o used in the datum level displacement studies (i.e. the value at 8.28) is estimated as being too small by about 2.5kgalm due to the positions of the mean sea surface used in these pilot studies being computed from orbits which were integrated using too great a value of GM.

(ii) The use of GM at 8.27 for the preparation of the GEOS-3 ephemeris would result in a larger value for W_o (equation 8.29) and hence, a greater height ζ_{sd} above the geoid (now assumed a best fit to the ellipsoid of revolution with an equatorial radius at 8.26).

This would result in significant disagreement with oceanographic estimates of ζ_{sd} given at 8.24 and 8.25 (based on Figure 3.3, whose best fitting ellipsoid has the value of a at 8.26). The latter agree well with GEOS-3 altimetry estimates of sea surface heights based on orbits integrated using a higher value of GM (398600.9 km^3s^{-2}) and a dependent value of a = 6378140m. This discrepancy can be due to one or a combination of three factors. If it were assumed that the total discrepancy were due to only one of the factors, the source could be:

(a) an error of approximately 2.5m in calibrating the GEOS-3 altimeter.

(b) The value of GM used in the computation of W_0 being larger than that given at 8.27.

(c) The results at 8.24 and 8.25 have a greater uncertainty due to the gravity datum in AUSGAD 77 and CNAGAD 77 being of poorer quality than was assumed in the study described in sub-section 8.2.4.

Recent investigations seem to indicate that the altimeter calibration may be in error, causing the altimeter to read too long (see e.g. EISNER 1979), however it is not expected to be as large as at (b).

If (a) were discounted, the value of GM implied by the surface gravity values on the datums studied, in agreement with oceanographic estimates of the heights of MSL at coastal sites, is

$$GM = 398\,600.9 \;\; km^3s^{-2} \qquad (8.31)$$

The factor at (c) is a possibility that cannot be ruled out.

An unambiguous definition of the geoid to better than ± lkgalm cannot therefore be achieved at the present time. Results from the SEASAT-1 altimeter should have a higher accuracy as the altimeter has been calibrated on a regular basis (TAPLEY ET AL 1979).

8.3 THE SYSTEM OF REFERENCE FOR VERTICAL CRUSTAL MOTION (VCM) STUDIES

8.3.1 THE ROLE OF GEODETIC LEVELLING IN VCM STUDIES

Geodetic levelling techniques in use at present for the determination of VCM can at best be classified as regional in concept, rather than global. The basic principles in use, more from necessity rather than by design, can be summarised as follows. Points of reference (bench marks) are linked during a selected epoch ($\tau = t_1$) by geodetic levelling operations to some datum of reference, usually a convenient tide gauge which is assessed as lying "outside" the area affected by crustal motion. The observed quantity provided by the levelling operation is a height increment dz_i between adjacent bench marks P_i and P_{i+1}. This elemental data can be treated in a number of ways to form a holonomic system of increments which is amenable to loop adjustment procedures (see sub-section 3.2.1). The most desirable system for VCM studies is based on geopotential differences. Here, dz_i is converted to an equivalent difference in geopotential dW_i using the relation (equation 3.8),

$$dW_i = - g\, dz_i \qquad (8.32)$$

where g is the local value of gravity.

This holonomic system of increments can be adjusted to form a network of geopotential differences ΔW_i at the bench mark P_i in relation to the adopted datum level surface.

To establish vertical crustal movement, the levelling and ideally, the gravity survey (see sub-section 8.3.2), is repeated at a subsequent epoch ($\tau = t_2$) and the "elevation" H_i (usually orthometric height - see § 3.2.1.1) of the bench mark P_i is re-established. The difference δh_i given by

$$\delta h_i = H_i(t_2) - H_i(t_1) \tag{8.33}$$

is considered to define vertical crustal movement at P_i. This quantity, the linear equivalent to the differences in geopotential between the two levellings, is given by

$$\delta h_i = \frac{1}{\gamma} \left(\Delta W_i(t_1) - W_i(t_2)\right) \tag{8.34}$$

where, for all practical purposes, γ is the global mean value of normal gravity.

On the basis of levelling and gravity data alone it is only possible to infer a constant velocity of elevation change for the model of VCM defined by equation 8.34. The computational maintainence of a geopotential network from a collection of time-inhomogeneous observations and scattered relevellings is beyond the scope of this study. Such a mathematical problem is treated in e.g. VANICEK & CHRISTODOULIDIS (1974), HOLDAHL & HARDY (1977).

Equation 8.34 is valid only if (MATHER 1974c, Section 4):

(1) The earth space location of the datum for elevations has not changed between epochs.

(2) The change in geopotential obtained between epochs at any benchmark, aside from the effect at (1), is a measure of the spatial displacements along the local vertical.

The validity of the first assumption is partly dependent on the definition of the elevation datum and the monitoring techniques. For example, the amount of vertical movement of the datum tide gauge is not necessarily detectable from tide gauge records in the event of secular changes in the volume of sea water (see sub-section 2.4.2). However, it is possible to define a three dimensional geocentric Cartesian system of reference X_i for position whereby the differences between the coordinates $X_i(\tau = t_1)$ and $X_i(\tau = t_2)$ gives the true displacement vector of the point with time (see discussion in sub-section 3.1.2). Elevation datums which were incorporated into such a world-wide network of geodetic control through the use of space techniques would provide space characteristics of global relevance for determinations of VCM using levelling techniques only if there were no change in the shape and spacing between adjacent geops (this follows from the nature of the levelling process were the level is always set tangential to the instantaneous surface geop - see sub-section 3.2.1).

This latter contention is therefore based on the same foundation as the assumption at (2) above. Such a notion however, is inconsistent with the concept of a non-rigid Earth undergoing deformation, as the latter implies mass redistribution on various scales. The following factors cause changes in the shape of level surfaces of the earth's gravity field as a function of time:

(a) Earth expansion.

(b) Geocentre motion.

(c) Changes in the earth's figure.

(d) Mass transfers due to plate motion.

The effect at (a) was discussed in sub-section 2.4.2, while (b) and (c) were briefly described in sub-section 2.4.3 (more details can be found in MATHER ET AL (1979a)). The effect of tectonic plate motions on the shape of the geoid was described in sub-section 2.4.4 (see Figures 2.7 and 2.8). The importance of combining levelling data and gravity survey information for the definition of the spatial location of a bench mark network at each epoch has long been recognised (see e.g. WHITCOMB 1976). However, this information on its own can only define changes in orthometric height not in the desired ellipsoidal height changes.

It is therefore most desirable that studies of vertical crustal motion be based on observations referred to a system of geodetic reference with an unambiguous location in space.

Three dimensional Cartesian coordinates X_i obtained in the manner referred to earlier, for all bench marks in the region of VCM determination, can be converted to positional parameters (ϕ,λ,h) with respect to a reference ellipsoid, if desired, using procedures defined by equations 3.3 and 3.4. The changes

$$\delta h_i = h_i(t_2) - h_i(t_1) \tag{8.35}$$

obtained at any bench mark P_i using such a system of reference would be a measure of VCM which is free from the errors of assumption given at (1) and (2) in this section.

In addition, three dimensional position techniques would completely define the crustal motion vector. The precision requirements for the geocentric coordinates in such a solution would be of the order of ±10cm in each coordinate. While no such achievements have been realised on a regular basis, there is promise that technological developments could result in this happening in the foreseeable future, by the use of global networks of either laser ranging system to satellite and the moon, or interferometric techniques (see e.g., FLINN 1979, COUNSELMAN 1979, SILVERBURG 1979, SMITH 1979, etc.) The main obstacles to implementation appear to be logistic and it therefore is important to develop transportable systems (including satellite-borne laser systems) to complement fixed observatory-type stations.

The advantages of three dimensional position systems over the traditional geodetic levelling procedures are (MATHER 1974c):

(i) The total definition of the crustal motion vector, as opposed to only the vertical component;

(ii) The global relevance of the results; and

(iii) Independence from temporal variations in the shape and spatial location of equipotential surfaces of the earth's gravity field.

However, the adoption of equation 8.35 for evaluating VCM is of practical relevance only if the resulting estimate of the phenomenon has the same resolution as the results obtained from first order geodetic levelling. At the present time, geodetic levelling retains the capacity to provide the highest relative resolution over regional extents (say $o\{10^4 km^2\}$) and therefore satisfies the requirements for VCM studies in, for example, areas of subsidence due to mining operations or tectonically active zones such as the San Andreas fault in California. Consequently it is important to monitor the motion of equipotential surfaces of the earth's gravity field and the relationship of the levelling datum to a unique geoid over long periods of time. Such a scheme is developed in the following section and is based on MATHER ET AL (1979a, Appendix).

8.3.2 THE INFLUENCE OF THE STABLILITY OF THE LEVELLING DATUM ON VCM STUDIES

The relationship between the datum for elevation D, the mean sea surface, the geoid, the reference ellipsoid and the general bench mark P in the regional network of levelling related to D is shown in Figure 8.1 for epochs $(\tau = t_1)$ and $(\tau = t_2)$.

The observed data necessary to unambiguously determine the vertical crustal movement δh at P consists of the following:

(1) Levelled differences of geopotential $\Delta W(t_1)$ at epoch $(\tau = t_1)$ between the locations of $P(t_1)$ and $D(t_1)$, and $\Delta W(t_2)$ between $P(t_2)$ and $D(t_2)$ at $(\tau = t_2)$. Such information is provided by levelling and gravity surveys carried out during the two epochs.

(2) The apparent rise in mean sea level $\delta\zeta_{sr}$ between epochs in relation to the "zero" height mark at D. This is defined by the quantity:

$$\delta\zeta_{sr} = dh(t_1) - dh(t_2) \tag{8.36}$$

where dh is the "height" of the zero mark above MSL.

(3) The global change $\delta\zeta_{so}$ of mean sea level between epochs, defined by:

$$\delta\zeta_{so} = \zeta_{MSL}(t_2) - \zeta_{MSL}(t_1) \tag{8.37}$$

where ζ_{MSL} is the "average" height of MSL above the reference ellipsoid, as determined at an ensemble of observatory-type tide gauge stations and/or from the results of satellite altimetry (see section 7.3).

(4) The change $\delta\zeta_{sd}$ in the sea surface topography at D between epochs, determined from a global analysis of satellite altimetry and surface gravity data as described in section 8.2. This is given by:

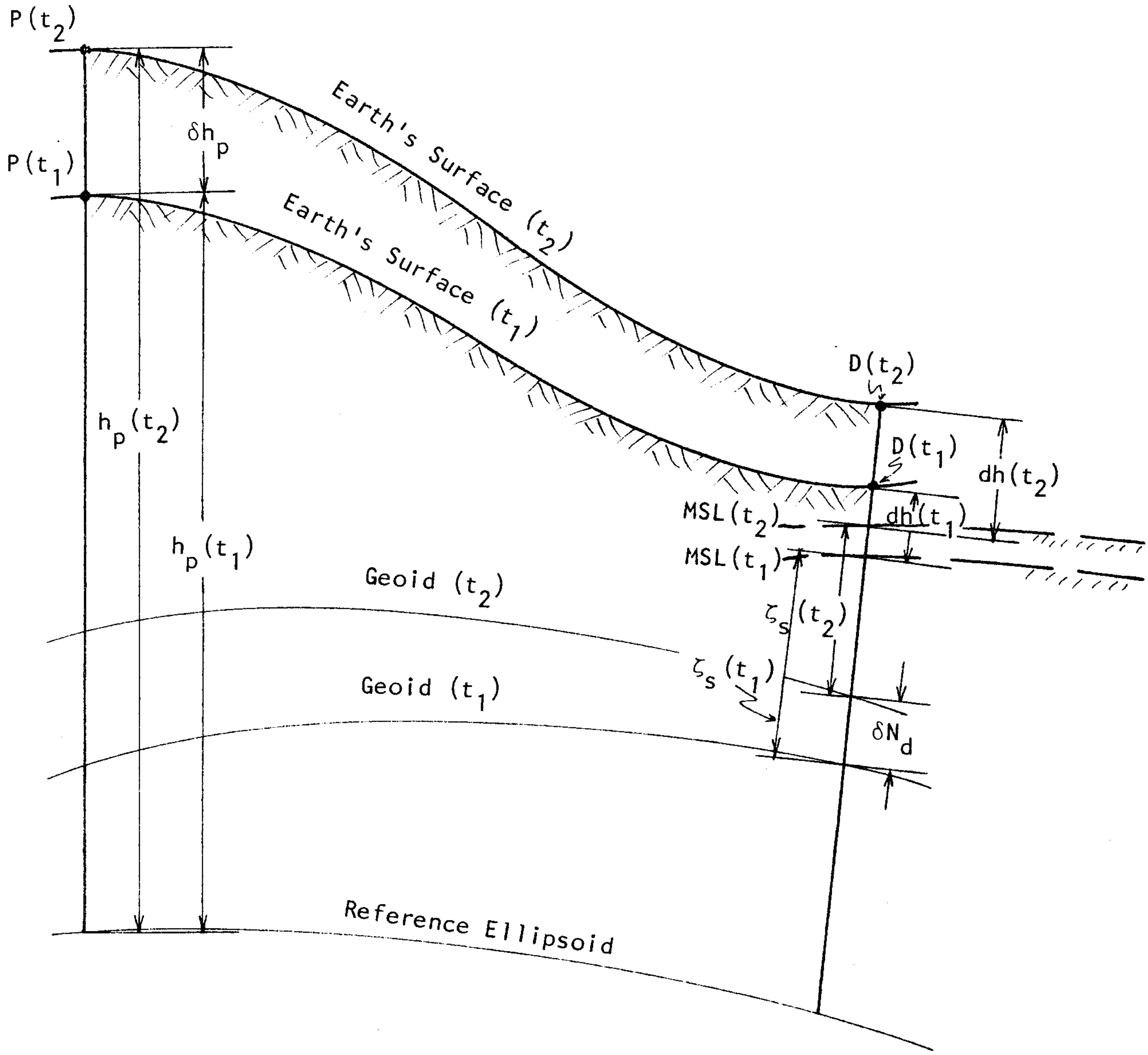
$$\delta\zeta_{sd} = \zeta_{sd}(t_2) - \zeta_{sd}(t_1) \tag{8.38}$$

where ζ_{sd} is the SST at D in relation to a geoid uniquely defined for each epoch.

(5) Determinations of the height anomaly at D with values $N_p(t_1)$ at epoch $(\tau = t_1)$ and $N_p(t_2)$ at epoch $(\tau = t_2)$.

F I G U R E 8.1

Relationships Between the Levelling Datum, Mean Sea Level, the Geoid and the Ellipsoid and the General Bench Mark



If the reference ellipsoid were not changed between epochs, the height h_p of P above the ellipsoid is given by:

$$h_p = H^*_p + \zeta_p \tag{8.39}$$

H^*_p being the normal height given by (equation 3.17):

$$H^* = -\frac{\Delta W'}{\gamma_o}\left(1 - \frac{\Delta W'}{a\gamma_o}(1 + f + m - 2f\sin\phi_c) + \left(\frac{\Delta W'}{a\gamma_o}\right)^2 + o\{f^3\}\right) \tag{8.40}$$

where γ_o is the value of normal gravity on the ellipsoid defined by the parameters (a,f), ϕ_c is the geocentric latitude and

$$m = a\omega^2/\gamma_e \tag{8.41}$$

All other quantities having been defined in § 2.3.5.2.

In equation 8.40, $\Delta W'$ is the difference in geopotential between P and the geoid <u>at the epoch of measurement</u>. The latter can be defined as the level surface of the earth's gravity field which corresponds to MSL as sampled globally (§ 8.2.3.2). The geoid height at D for epoch $(\tau = t_1)$ can differ from that of epoch $(\tau = t_2)$ due to the following factors:

(a) Secular change in global MSL $(\delta\zeta_{so})$ causing a different level surface to be selected as the geoid. The "new" geoid will be higher than the "old" geoid by the positive quantity $\delta\zeta_{so}$ at D.

(b) The shape of the geoid can change (δN) due to mass redistribution (see sub-sections 2.4.3 and 2.4.4). Such radial changes can be computed, for all practical purposes, by Stokes' integral (equation 2.42) as corrected for the contribution of zero degree (sub-section 4.2.2):

$$\delta N = \frac{R}{4\pi\gamma}\iint f(\psi)\,\delta\Delta g\,d\sigma - \frac{R}{\gamma}M\{\delta\Delta g\} \tag{8.42}$$

where ψ is the angular distance between the element of surface area $d\sigma$ at which the gravity change is $\delta\Delta g$ and the point of computation, $f(\psi)$ is Stokes' function (equation 2.43) and $M\{\delta\Delta g\}$ is the global mean value of $\delta\Delta g$. The term of zero degree in equation 8.41 is independent of the geoid <u>definition</u> change at (a). The quantity $\delta\Delta g$ is related to values of observed gravity at the two epochs by the relation:

$$\delta\Delta g = g(t_2) - g(t_1) \tag{8.43}$$

<u>if the equipotential ellipsoidal gravity model is unchanged between epochs</u>.

As computations using equation 8.41 are insensitive to harmonics of degree 1 in $\delta\Delta g$, the position of the reference ellipsoid at each epoch is located with its centre at the instantaneous geocentre.

The total change in the height anomaly at a point P is for all practical purposes given by (see equation 2.57):

$$\delta\zeta_p = \delta\zeta_{so} + \delta N_p + \varepsilon_p H_p^* \qquad (8.44)$$

where $\delta\zeta_{so}$, the change in geoid definition, is defined at equation 8.37 and incorporated into the quantity $\delta\zeta_{sr}$ (equation 8.36), and δN_p is the change in geoid shape at P defined by equation 8.42. ε_p is the change in the mean gravity along the vertical between the geoid and the earth's surface. This is likely to be negligible and can be ignored.

However, the required quantity $\Delta W'$ in equation 8.40 is related to the observed difference in geopotential ΔW by the relation:

$$\Delta W' = \Delta W - \gamma\zeta_d \qquad (8.45)$$

where ζ_d is the height of D above the geoid and is related to the height of D above MSL dh and the SST ζ_{sd} by:

$$\zeta_d = dh + \zeta_{sd} \qquad (8.46)$$

It follows that the change $\delta\zeta_d$ in ζ_d between epochs $(\tau = t_1)$ and $(\tau = t_2)$ is given by (equations 8.36 and 8.38):

$$\delta\zeta_d = - \delta\zeta_{sr} + \delta\zeta_{sd} \qquad (8.47)$$

Note, if there is a local change in sea level, e.g. due to regional uplift (i.e. the geoid does not change position with respect to the geocentre), $\delta\zeta_d$ is zero. In the case of a global change in sea level, $\delta\zeta_d$ is non-zero due to the quantity $\delta\zeta_{sr}$ being non-zero. $\delta\zeta_{sd}$ in such a case is, for all intents and purposes, zero and $\delta\zeta_{so} = -\delta\zeta_{sr}$.

As the effect of geocentre motion is effectively filtered out in vertical crustal motion studies on including a term (δN) based on the solution of the GBVP, the consideration of equations 8.39, 8.40, 8.42, 8.45 and 8.47 defines the true vertical crustal movement δh_p at P (ϕ,λ) as:

$$\delta h_p = \frac{1}{\gamma}\left(\Delta W(t_1) - \Delta W(t_2)\right) + \delta N_p - \delta\zeta_{sr} + \delta\zeta_{sd} \qquad (8.48)$$

The subscript p referring to values at the point of computation.

Equation 8.48 therefore defines the VCM with respect to a reference ellipsoid whose shape and orientation in space is unchanged, but which is insensitive to geocentre motion. This is due to the fact that the correction terms δN and $\delta\zeta_{sd}$ are based on an evaluation of Stokes' integral and satellite orbital behaviour respectively, and consequently the reference ellipsoid is centred at the mean geocentre position for each epoch. It is seldom of interest in VCM studies to define a change in the ellipsoidal height of a point P arising from a translation X_{io} (equation 3.4) of the geocentre from epoch $(\tau = t_1)$ to that of epoch $(\tau = t_2)$.

The change in normal height δH_g due to geocentre motion is given by (MATHER ET AL 1979a, Section 3):

$$\delta H_g = \delta r_g (t_2 - t_1) \sum_{i=1}^{3} \ell_i \ell_{ip} \qquad (8.49)$$

where δr_g is the rate of geocentre motion in the direction (ϕ_g, λ_g) from the geocentre for the period between epochs $(\tau = t_1)$ and $(\tau = t_2)$. The quantities ℓ_i and ℓ_{ip} are defined by:

$$\ell_1 = \cos\phi_g \cos\lambda_g \;;\; \ell_2 = \cos\phi_g \sin\lambda_g \;;\; \ell_3 = \sin\phi_g$$

and (8.50)

$$\ell_{1p} = \cos\phi_p \cos\lambda_p \;;\; \ell_{2p} = \cos\phi_p \sin\lambda_p \;;\; \ell_{3p} = \sin\phi_p$$

Equation 8.48 provides the absolute definition of height change not available in equation 8.34. Although the correction terms in equation 8.48 have varying significance for regional studies, it is not difficult to include an analysis of data provided by a global network of absolute gravity stations engaged in a programme of monitoring secular changes in gravity (MATHER ET AL 1977b) and thereby minimise the uncertainty on this account. However, it must be emphasised that equation 8.48 is insensitive to geocentre motion and as a result the VCM δh is determined with respect to a reference ellipsoid centred at the mean position of the geocentre for the epoch under study. If the change in ellipsoidal height of a bench mark P in relation to a reference ellipsoid defined for some reference epoch (see subsection 3.1.2) is required, equation 8.48 should also include the effect of the term δH_{gp} defined by equations 8.49 and 8.50.

9

CONCLUSIONS AND RECOMMENDATIONS

9.1 PREAMBLE

Current understanding of the phenomenon of sea surface topography is almost exclusively based on data collected during the course of sporadic surface ship surveys (sub-section 3.3.3). Consequently the best that can be hoped for from such traditional oceanographic techniques is an estimate of the stationary "atlas-type" features (Figure 3.3). The variation of SST on a variety of time scales (related to the dynamics of the ocean surface layer - see section 1.1) makes it imperative that remote sensing techniques be developed for improving the rate of data acquisition. This would, in addition, enable a synoptic monitoring capability to be established.

One such technique which requires a geodetic basis for its use is satellite altimetry, where the distance from the satellite to the instantaneous sea surface is measured along the local vertical. This development in space technology has resulted in the activities of geodesy no longer being land based (as, for example, in support of mapping operations), but to encompass the oceans as well - a domain that has been hitherto the exclusive preserve of physical oceanographers.

Satellite altimetry data has the capability of defining the geometry of the instantaneous sea surface in relation to an earth-centred Cartesian coordinate system to sub-metre precision. The extraction of such information is only possible through the efforts of specialists in high precision orbit determination - a geodetic activity. Geodesy also is concerned with defining various characteristics of the earth's gravity field - in particular the shape of the geoid (Chapter 2). Therefore the geodetic task of determining the SST in the open oceans from satellite altimetry is, *in principle*, quite straightforward. For the accomplishment of this task the desired resolution in the geocentric position of both the geoid and the altimeter-equipped satellite is ±10cm.

However, at the present time, the difficulties encountered by specialists in satellite orbit determination and those defining oceanic geoids are the following:

(1) A model of the earth's disturbing potential, and hence the geoid, is unknown at better than the 1% level. An order of

magnitude improvement is required.

(2) The geocentric position of the altimeter-equipped spacecraft cannot be defined at the decimetre level.

It has been the aim of this thesis to investigate the role that present gravity field information can play in sea surface topography studies utilising GEOS-3 altimetry data (NASA 1972a), and establish what geodetic activities need to expand in order to exploit the full potential of satellite altimetry from the future SEASAT missions (NAGLER & McCANDLESS 1975). The findings of this study can be summarised under the following headings.

9.2 A HIGH PRECISION MARINE GEOID

9.2.1 A GEOID DEFINITION ADEQUATE FOR SEA SURFACE TOPOGRAPHY STUDIES

An adequate conceptual definition of the geoid is essential for the unambiguous combination of satellite tracking data, satellite altimetry and surface gravity measurements to obtain, ultimately, the sea surface topography. The choice of a particular level surface of the earth's gravity field as the geoid is quite arbitrary although a definition acceptable to oceanographers and which has great utility in the context of SST studies is the following:

> "The geoid, for a selected epoch of measurement, is that level surface of the earth's gravity field in relation to which the average non-tidal sea surface topography is zero as sampled on an equi-area basis in ocean regions".

The practical implementation of this definition from a near-complete global sampling of MSL (restricted to regions between 65°N and 65°S) by GEOS-3 altimetry for the epoch 1976.0 has been attempted (§ 5.1.3.2 and 7.4). The geopotential of the geoid for 1976.0 was found to be (equation 7.44)

$$W_o = 6\ 263\ 686.0 \pm 0.4 \text{ kgalm}$$

Such an estimate is based on the geopotential model defined by GEM9 (LERCH ET AL 1977) and the following values of the geocentric gravitational constant and equatorial radius of the best fitting ellipsoid to global MSL (see section 9.4):

$$GM = 398\ 600.47 \text{ km}^3\text{s}^{-2}$$

$$a = 6\ 378\ 137.0 \text{ m}$$

and is scaled by the velocity of light,

$$c = 2.997\ 924\ 58 \times 10^8 \text{ m s}^{-1}$$

For sea surface topography studies over geodetic time scales ($< 10^2$ years), the definition for the marine geoid should be capable of being maintained to accuracies of 1 part in 10^8 in the presence of:

(1) changes in sea water volume or changes in scale.

(2) mass redistribution within the earth associated with tectonic

plate motion.

It has been established (sub-section 2.4.2) that on the basis of present estimates for earth expansion/MSL increase (which are at the noise level of geodetic concepts for time spans of the order of 10^2 years), the definition of the geoid is unaffected at the ±10cm level.

Changes in geoid shape over geodetic time scales due to global mass transfer have been shown to be too small to be of concern (sub-section 2.4.4). This contention may require review if it were established that a steady rate of mass transfer substantially greater than that modelled in this study were taking place from the shrinking lithospheric plates to the expanding ones and if the latter were concentrated in a particular part of the globe. A global network of absolute gravity stations would assist considerably in defining the mass transfers that have the characteristics of low degree spherical harmonics.

9.2.2 DETERMINATION OF THE SHAPE OF A HIGH PRECISION GEOID

The central role that the geoid plays in sea surface topography determinations has revived interest in developing very accurate solutions of the Geodetic Boundary Value Problem (sub-section 2.3.5). For high precision solutions of the GBVP, it has long been recognised that it is unsatisfactory to make any assumptions concerning the crustal density distribution exterior to the geoid. This problem, although not critical for gravimetric geoid determinations in ocean areas, is easily circumvented by adopting the approach suggested by Molodenskii (§ 2.3.5.3). A solution to Molodenskii's problem gives the height anomaly which, to all intents and purposes, is assumed to be equivalent to the geoid height in ocean areas. A solution with ±5cm resolution is presented in section 4.2 and is essentially an upgrading of Stokes' solution to take into account the following factors:

(a) Geometrical differences between a spherical and ellipsoidal reference model used in formulating the fundamental boundary condition.

(b) The effect of the topography exterior to the geoid.

(c) The use of orthogonal properties of surface harmonics to be restricted to the surface of a sphere and only where such expansions exist. The minimum sphere to which this applies is the Brillouin sphere concentric with the reference ellipsoid such that all topography is included within it.

(d) The use of spherical harmonic representations to be restricted only to those quantities that satisfy Laplace's equation at the surface of measurement and exterior to it.

The factor at (d) requires that the GBVP be formulated in terms of the disturbing potential of the solid earth and oceans. Consequently, a solution for the geoid height requires that the potential and attraction of the atmosphere be treated within the framework of the BVP. It has been shown in section 4.5 that the atmospheric effect must be correctly allowed for in the definition of the gravity anomaly (sub-section 4.1.4) in order to ensure that the solution of the GBVP is not biased at the decimetre level. In addition, gravity anomaly data banks must be prepared with due regard to the effect of the atmosphere prior to combination with satellite tracking data for gravity field model determination.

Error accumulation in quadrature evaluations for the integral solution of the GBVP is largely a function of the magnitude and wavelength of errors in the gravity anomaly data. The criteria to be satisfied by the gravity anomaly in high precision geodesy are the following (see § 2.3.5.4):

(1) Gravity station spacing should be such that the error of representation is approximately ±3mgal (equivalent to a 10km spacing in non-mountainous areas).

(2) Gravity values should be based on a consistent global gravity standardisation network with a station spacing of 10^3km in continental areas and absolute errors held to less than ±0.1mgal without significant error correlation.

(3) Geopotential heights of land gravity stations should be based on levelling data with correlated errors held to below 0.15kgalm, although individual station elevations can have uncorrelated errors an order of magnitude greater.

(4) The contribution of long wavelength systematic errors in gravity anomalies due to (2) and (3) should be kept to below 50μgal for wavelengths of 4 x 10^3km. The longer the wavelength of the systematic error, the smaller must be its magnitude.

(5) Normal gravity should be computed from geocentric, rather than regional geodetic coordinates.

(6) Atmospheric attraction should be allowed for in the final gravity anomaly values.

An attempt has been made to prepare an Australian gravity anomaly data bank that meets these requirements (section 6.2). It is estimated that the precision of the resulting data bank (AUSGAD 77) is only adequate for geoid determinations to ±30cm. This is primarily due to residual errors in the Australian levelling network, which is only of third order standard.

Further, a solution of the GBVP to a precision of ±10cm requires that all gravity data is related to the geoid with at least equivalent precision. As it is more realistic to accept that the data is taken at or in relation to the instantaneous sea surface or MSL, which deviates by up to ±2m from the geoid, quadrature solutions of the GBVP are subject to errors of the order of 15-60 cm on this account. An additional consequence of the existence of SST is that the geometry of the Earth-Telluroid system (§ 2.3.5.3) requires modification (see § 2.3.5.5). The effect of such a change is to ensure that the height anomaly in the oceans is equivalent to the altimeter-derived sea surface height ζ (sub-section 2.3.4). Therefore a gravimetric determination of the height anomaly to a precision of ±10cm requires that the SST at the computation point be known to this level of precision. As a corollary, the geoid height deduced implicitly from solutions of the GBVP as formulated by Molodenskii does not require a knowledge of the SST at the computation point, although errors of ±15-60cm can still be expected from the "indirect" effect on gravity anomalies of the SST through the quadratures evaluation.

On land, the discrepancy between the height anomaly and the geoid height is the sum of the uncertainty in the knowledge of the displacement from the geoid of the regional elevation datum (to which the gravity anomalies are referred) and any errors in the assumptions concerning the vertical crustal density between the surface point and the geoid. In oceanic areas, the discrepancy between the height anomaly (equivalent to the height of the sea surface above the reference ellipsoid) and the geoid height is the sea surface topography. Consequently the oceanic height anomaly cannot be determined from gravimetric solutions to better than the magnitude of the

SST. Therefore, if the data were of adequate quality (see § 2.3.5.4) the geoid height in ocean areas can be obtained from solutions of the GBVP to an ultimate precision of ±15-60cm, whereas the limiting precision of estimates of the height anomaly from satellite altimetry is only the uncertainty in the orbit determination for the altimeter-equipped spacecraft.

It is therefore not possible to determine the geoid to a precision adequate for global SST studies from solutions of the GBVP because none of the data can be unambiguously related to the geoid at the desired level of precision without making unwarranted assumptions about the magnitude and distribution of the SST. However, combination solutions (sub-section 2.3.6) incorporating geoidal information from satellite-determined gravity field models are prone to smaller errors on this account (see § 5.1.2.3). Gravity field models are the only source of data on the gravity field, and hence the geoid, which is potentially uncontaminated by data referenced to the ocean surface provided such models are based solely on satellite tracking data (see discussion in sub-section 5.2.4 on the problems of gravity field model improvement from "surface" data such as gravimetry and satellite altimetry). The resolution obtainable is a function of the noise level of the tracking and of spacecraft altitudes. For sub-decimetre accuracy laser systems tracking satellites at 10^3km altitudes it is possible to improve present day gravity field models to the desired precision (±0.1kgalm in the disturbing potential) for wavelengths of 10^3km and greater.

The gravimetric determination of high precision relative ocean geoids for regional SST studies is more promising. This is possible because of the fact that the "indirect" effect of SST through the quadratures procedure is essentially of long wavelength while the phenomenon being studied is of relatively short wavelength, e.g. the western Atlantic Ocean. Similarly, high precision continental geoids can be computed from surface gravity data because the effect of the SST at the MSL levelling datum is indistinguishable from a datum error in the national gravity network.

9.3 THE ROLE OF FIRST ORDER GEODETIC LEVELLING IN HIGH PRECISION GEODESY

On the assumption that geodetic levelling data is capable of defining an equipotential surface of the earth's gravity field, the comparison of the heights of MSL at tide gauge locations, obtained from freely adjusted levelling networks, with heights obtained from tidal analysis, indicate discrepancies significantly larger than expected from the internal statistics of the network adjustment (see sub-section 3.4.1). In addition, the deduced slopes of this coastal SST disagree with estimates obtained from oceanic levelling data (sub-section 3.3.3). The uncertainties surrounding the estimates of coastal SST from oceanographic considerations on the one hand and geodetic levelling on the other have been investigated in section 3.4.

A review of the procedures of oceanic and first order geodetic levelling and a study of the available information on geodetic levelling/MSL comparisons in various parts of the world, raises serious doubts concerning the reliability of continental first order levelling networks. The most likely candidate for the "negligible" systematic error that appears to affect geodetic levelling results is that due to refraction (sub-section 3.4.3). Although it is small, it tends to accumulate in a north-south direction. However the existence of such a systematic effect does not account for the time-varying discrepancies (as seen in several relevellings along the Californian and east Australian coast) nor does it explain the discrepancies observed in levelling surveys carried out in an east-west direction.

First order geodetic levelling networks should therefore only be used with caution for scientific investigations. This does not imply that the results of oceanic levelling, as extrapolated into the coastline, are free from error (see sub-section 3.4.2). The doubts expressed about the reliability of continental levelling networks does not necessarily affect their utility for such practical purposes as mapping operations.

The only alternative source of data upon which to base a vertical reference system is from three dimensional positional systems. However, at the present time, the density of determinations with ±10cm precision is very low, and consequently first order levelling will, in the short term, continue to provide "height" information for such geodetic purposes as the establishment of gravity anomaly data banks.

The gravity anomaly is computed using the relation (equation 5.88):

$$\Delta g_p = g_p - \gamma_o - \frac{2\Delta W}{a}\left(1 + f + m + \frac{\Delta W}{2a\gamma_o} - 2f\sin^2\phi + o\{f^2\}\right) \qquad (9.1)$$

where normal gravity (γ_o) is computed on the surface of the equipotential reference ellipsoid (equatorial radius a, flattening f) rotating with angular velocity ω, g_p is observed gravity at the point P with geodetic latitude ϕ and

$$m = a\omega^2/\gamma_e \qquad (9.2)$$

γ_e being normal gravity at the equator and ΔW the difference in geopotential with respect to the regional MSL datum for levelling, given by (sub-section 3.2.2):

$$\Delta W = -\int_{\text{MSL Datum}}^{P} g\, dz \qquad (9.3)$$

where dz are levelling increments.

Furthermore, geodetic levelling is a dynamic operation related to the earth's gravity field, and therefore defining free flow. Equivalent information for sophisticated engineering projects cannot be obtained from purely geometric height determinations, even if they were available at the required density and precision (sub-section 3.2.2).

Geodetic levelling also has a role to play in vertical crustal motion studies, as first order levelling has the highest relative resolution of any of the other geodetic processes for distances of the order of 100-300km (for distances greater than this, satellite positioning techniques are expected to be more competitive in the near future). Vertical crustal movement information from geodetic levelling surveys is not exactly equivalent to geometrical displacements unless appropriately combined with gravity and mean sea level data determined as a function of time (see sub-section 8.3.2).

The true change in ellipsoidal height δh_p at the general point P is related to the measured geopotential differences $\Delta W(t_1)$ and $\Delta W(t_2)$ (obtained from independent levelling and gravity surveys) at the two epochs of measurement by the relation (equation 8.48):

$$\delta h_p = \frac{1}{\gamma}\left(\Delta W(t_1) - \Delta W(t_2)\right) + \delta N_p - \delta\zeta_{sr} + \delta\zeta_{sd} \qquad (9.4)$$

where $\delta\zeta_{sr}$ is the apparent rise in MSL at the regional elevation datum, $\delta\zeta_{sd}$ is the change in the height of the SST between epochs as determined from independent global solutions (section 5.3) and δN_p is the change in the height anomaly (to all intents and purposes representing the geoid height change) at the point P. These three correction terms have varying significance for regional VCM studies. For example, both $\delta\zeta_{sr}$ and $\delta\zeta_{sd}$ are constant for data related to the same elevation datum. Furthermore, it is unlikely that the high frequency end of the spectrum of mass changes as a function of position will generate sufficient power to cause significant variations in δN at the bench marks comprising the local crustal motion control network. Nevertheless, the incorporation of data from a global network of absolute gravity stations engaged in studies of secular geodynamics would minimise the uncertainty on this account.

9.4 THE BEST FITTING ELLIPSOID TO GLOBAL MSL FROM GEOS-3 ALTIMETRY

The parameters of the ellipsoid of revolution that is a best fit to MSL between 65°S and 65°N as sampled by GEOS-3 between April 1975 and August 1976 are:

$$\begin{aligned} &\text{Equatorial radius} &- a_o &= 6\ 378\ 137.0 \pm 0.3 \text{ m} \\ &\text{Meridional flattening} &- f_o^{-1} &= 298.236 \pm 0.002 \end{aligned} \qquad (9.5)$$

The value of f_o quoted above is in good agreement with the second degree zonal harmonic of the oceanographically-determined stationary SST (see sub-section 7.1.4).

The value of a_o is dependent on the value of GM (see sub-section 9.2.1) used in integrating the GEOS-3 orbits, and is consistent with the velocity of light given by:

$$c = 2.997\ 924\ 58 \times 10^8 \text{ m s}^{-1} \qquad (9.6)$$

Upon adopting the conceptual definition of the geoid given in section 9.2, the value of the equatorial radius of the ellipsoid of revolution that is a best fit to the geoid is therefore given by:

$$a = 6\ 378\ 137.0 \pm 0.3 \text{ m} \qquad (9.7)$$

This is the recommended value of a for the rotating equipotential ellipsoid known as Geodetic Reference System 1980 (GRS 80) (MORITZ 1979). Other recommended parameters for GRS 80 are:

$$\begin{aligned} f^{-1} &= 298.257\ 316 \\ GM &= 398\ 600.5 \text{ km}^3\text{s}^{-2} \\ \omega &= 7.292\ 115 \times 10^{-5} \text{ rad s}^{-1} \end{aligned} \qquad (9.8)$$

The value of f is equivalent to the second degree zonal coefficient C'_{120} of the GEM9 gravity field model (see sub-section 7.1.3), excluding the effect of the permanent tidal deformation (sub-section 7.1.5).

9.5 A GEODETIC BASIS FOR SST STUDIES

9.5.1 OPEN OCEAN SST

The principal difficulties encountered in using GEOS-3 altimeter data for the recovery of SST are (sub-section 7.1.1):

(1) The data were collected only between the parallels 65°N and 65°S. This, together with the absence of on-board recording facilities, limits the data coverage available to investigators to approximately 80% of the oceanic regions below the 65° parallels, for the epoch April 1975 - August 1976.

(2) The altimetry measurements were recorded in the form of discrete passes not exceeding 9 x 10^3km in length. The data were, for the most part, collected on a regional rather than a global basis, the acquisition areas and periods being governed by the location of transportable telemetry units.

(3) The precision of the orbit determination was variable, always worse than the desired ±10cm precision. The analysis of GEOS-3 orbits based on Doppler tracking data indicated a root mean square radial error of ±1.3m from crossovers in a 10 day arc (see sub-section 7.1.2). This figure reduces to ±0.9m in the case of an ephemeris based on laser tracking data. The major source of error is the model of the earth's gravity field used in orbit integration necessary to maintain orbital dynamics during periods of no direct tracking.

(4) The geoid needs to be defined in ocean areas with a precision of ±10cm through wavelengths of interest. It has been conventional to assume that such determinations are obtained from solutions of the GBVP (see sub-section 9.2.2) using surface gravity and elevation data which are related to the geoid. It was mentioned earlier that the desired precision cannot be obtained because:

  (i) the gravity data in ocean areas are not of sufficient quality or density to allow reliable computations of a gravimetric geoid; and

  (ii) the relevant data in the form of gravity anomalies, even if assumed to be of adequate quality, are nevertheless related to the sea surface and not the geoid.

The difficulties described in (1) - (3) are not expected to affect the data from the SEASAT-A mission to the same degree. The difficulty at (4) however is not easily surmounted as it is required to geometrically map the geoid in ocean areas where no measurements have been made in relation to it. The only potential source of data able to fulfill this role is a satellite-determined gravity field model. Consequently, the task to be tackled initially is the definition of the SST ζ_s through certain ranges of wavelengths. ζ_s can be considered to occur in five distinct constituents according to the relation:

$$\zeta_s = \zeta_{s\ell} + \zeta_{ss} + \zeta_{v\ell} + \zeta_{vs} + \zeta_t \tag{9.9}$$

where the subscripts have the following significance:

Primary	Secondary
s = Stationary or time invariant	ℓ = Contributions with long wavelength
v = Non-tidal temporal variations	s = Contributions with short wavelength
t = Tidal variations	

Contributions of long wavelength (> ℓ) are defined as those equivalent to wavelengths in the gravity field model which cause perturbations of the altimeter-equipped satellite above the noise level of the tracking data.

9.5.1.1 *Recovery of the Stationary Component* $\zeta_{s\ell}$

Despite the signal-to-noise problems with GEOS-3 altimeter data (factors (1) to (4) above) it has been possible to obtain some information on the stationary components of the SST with very long wavelength ($\zeta_{s\ell}$) for which gravity field components are known to better than 1 part in 10^8, i.e. ±0.1kgalm in the disturbing potential (§ 5.2.2.2). On adopting a surface harmonic model for $\zeta_{s\ell}$ (coefficients $\zeta_{s\alpha nm}$) and for the sea surface heights ζ (coefficients $\zeta_{\alpha nm}$), it follows that:

$$\zeta_{s\alpha nm} = \zeta_{\alpha nm} - \frac{GM}{R_o\gamma}\left(\frac{a}{R_o}\right)^n C^*_{\alpha nm} \tag{9.10}$$

where $C^*_{\alpha nm}$ are spherical harmonic coefficients of degree n and order m defining the mapping of the geoid in relation to the reference ellipsoid (equatorial radius a) in terms of a representation of the earth's disturbing potential provided by the GEM9 gravity field model (see section 4.4). R_o is the geocentric radius to the sea surface, GM is the geocentric gravitational constant and γ is normal gravity.

A study of the power spectrum of the stationary SST (the signal) as estimated by oceanographers, in relation to that of the estimated errors in the GEM9 model (the noise) (see Table 7.1) shows that the signal-to-noise ratio is favourable for the recovery of the seven dominant normalised harmonic coefficients in the global representation of $\zeta_{s\ell}$ <u>if the satellite altimetry data are free of error</u> (i.e. $\zeta_{\alpha nm}$ are known to ±10cm). These dominant coefficients are ζ_{s100}, ζ_{s110}, ζ_{s111}, ζ_{s120}, ζ_{s130}, ζ_{s140} and possibly ζ_{s160}.

However, the altimetry data are not free of error and any solution procedure would fail if errors in the altimetry had the same wavelengths as those sought in the SST. On the basis of a ratio of the magnitude of the parameter sought to the average radial orbit error in GEOS-3 of 0.1 it appears that only the coefficients ζ_{s111} and ζ_{s120} satisfy this criterion. However, if the orbital errors could in some way be brought to below ±60cm there is some chance that the coefficients ζ_{s110}, ζ_{s130} and ζ_{s140} can also be recovered. (The recovery of the zero degree coefficient ζ_{s100} has been attempted in section 7.3 and the results discussed in section 9.4).

On the basis of the results obtained in section 7.2, the best values for the dominant harmonics in the SST, other than those of degree one, are the following:

$$\bar{\zeta}_{s120}\ \text{(second degree zonal)} = -42 \pm 4\ \text{cm}\ (-46)$$

$$\bar{\zeta}_{s130}\ \text{(third degree zonal)} = +10 \pm 5\ \text{cm}\ (+7) \qquad (9.11)$$

$$\bar{\zeta}_{s140}\ \text{(fourth degree zonal)} = -12 \pm 4\ \text{cm}\ (-10)$$

(normalised values)

the values in brackets being the oceanographic estimates (Table 7.3, Solution 3). The value for $\bar{\zeta}_{s120}$ takes into account the contribution of the permanent earth tide (sub-section 7.1.5)

The three zonal harmonics of degrees 2, 3 and 4 are in good agreement with the oceanographic values. Furthermore, they contain approximately 75% of the estimated strength of signal of the stationary SST. Significant gravity field model improvements and more precise ephemerides are needed before further advances are possible in this area. No evidence currently exists for any widespread discrepancies between satellite and surface oceanographic determinations of the stationary SST, therefore oceanographically-determined global field of $\zeta_{s\ell}$ will, in the short term, be useful yardsticks for such studies.

9.5.1.2 *Recovery of the Stationary Component* ζ_{ss}

In principle, with the completion of the task described in § 9.5.1.1, the residual SST signal is essentially of short wavelength. The recovery of ζ_{ss} can then be attempted as a subsequent phase.

It was established in § 5.2.3.3 that the most viable solution procedure is one based on the inverse of the GBVP, formulated to take into account the departures of the sea surface from the geoid (considered known for wavelengths greater than ℓ).

The relations comprising this solution are the following (equation 5.79):

$$\overline{\Delta g}_{cp} + \frac{1}{\bar{R}}\left(\gamma \bar{N}_{cp} - (W_o - U_o)\right) - \frac{\gamma}{4\pi \bar{R}}\iint M_1(\psi)\left(\bar{N}_c - \bar{N}_{cp}\right) d\sigma +$$

$$\frac{\gamma}{\bar{R}}\sum_{n=2}^{n'}(n+1)\sum_{m=0}^{n}\sum_{\alpha=1}^{2}\zeta_{s\alpha nm}S_{\alpha nm} = -\frac{\gamma}{\bar{R}}\left[\zeta_{ssp} + \frac{1}{4\pi}\iint M_1(\psi)\left(\zeta_{ss} - \zeta_{ssp}\right)d\sigma\right] \qquad (9.12)$$

where γ is normal gravity, $(W_o - U_o)$ is a term of zero degree, $\zeta_{s\alpha nm}$ are harmonic coefficients of the long wavelength SST (§ 9.5.1.1) and $S_{\alpha nm}$ are spherical harmonic functions. The surface integrations are taken over the Brillouin sphere (radius $\bar{R}$). All the terms on the left hand side of the equation are assumed known and a solution for ζ_{ss} can therefore be attempted. The kernel function $M_1(\psi)$ is:

$$M_1(\psi) = \sum_{n=2}^{\infty} n(2n+1)P_{n0}(\cos\psi) \qquad (9.13)$$

ψ being the geocentric angular distance of the element of surface area $d\sigma$ from the point of computation P. The computable gravity anomaly is defined by (equation 4.94):

$$\overline{\Delta g}_c = \Delta g + \delta g_a - \tfrac{1}{2} g \varepsilon^2 + \delta \Delta g' \qquad (9.14)$$

where Δg is the conventional gravity anomaly, δg_a is the atmospheric effect (sub-section 4.5.1), ε is the deflection of the vertical and $\delta\Delta g'$ is the correction necessary to upward continue Δg_c to the Brillouin sphere (equation 4.38).

The quantity $\overline{N}_c$ is defined by (equation 4.78):

$$\overline{N}_c = \zeta - \frac{1}{\gamma}\left(W_A - \delta T'\right) \qquad (9.15)$$

where ζ is the height anomaly, W_A is the potential of the atmosphere and $\delta T'$ is the change in ζ on upward continuation to the Brillouin sphere (equation 4.53).

However, the principal problem with using this approach is the requirement of oceanic gravity data of adequate quality, i.e. correct to ± 0.3mgal per metre of precision sought in the definition of ζ_{ss}, through wavelengths of interest. In addition, a near-complete global coverage of height anomalies with ± 10cm precision is necessary as the use of the $M_1(\psi)$ kernel in oceanic areas close to the coast requires reliable values of N_c to be computed in adjacent land areas (within approximately 500 km of the point of computation). This is unlikely to be economically feasible if ζ were determined in land areas using three dimensional position fixes.

It therefore appears optimistic to expect results from this technique in the foreseeable future as:

(i) surface gravity data with a precision of ± 0.03mgal for wavelengths sought in ζ_{ss} will not be available for the tasks; and

(ii) Useful values of $\overline{N}_c$ from expensive three dimensional position fixes are unlikely to be available in land areas within 500km of the coastline.

An alternate means for obtaining information on short wavelength stationary SST is to improve the definition of the gravity field for such wavelengths from the analysis of satellite-to-satellite tracking of low flying drag-free satellites by a network of spacecraft in synchronous orbits and use the technique described in §9.5.1.1. In other words, it is required to make the limiting wavelength ℓ as short as possible.

9.5.1.3 *Non-tidal Variations in the SST* $\zeta_{v\ell}$, ζ_{vs}

The spectrum of variations covers all possible wavelengths, with amplitudes between 1cm and 10^3cm, and frequencies from less than one day to over a year (see sub-section 3.3.2).

In the ideal situation, non-tidal variations could be recovered with a precision equivalent to the noise level of the altimeter if the radial component of the orbital position were known at all times with an equivalent resolution. However, variations in SST can be treated as being equivalent to variations in the radial component of sea surface position only if the shape of the geoid can be considered time invariant at the ± 5 cm level. It has been estimated that the ocean tides produce geopotential variations as large as 15kgalcm (HENDERSHOTT 1973, p.81). In addition, the existence of seasonal variations in sea level with markedly zonal harmonic characteristics and amplitudes approximately half that of the tides, could produce changes in geoid height in excess of ±5cm.

The true variation in the SST is given by (see § 5.2.3.3):

$$\delta\zeta_s = \delta\zeta_{sa} + \delta N \tag{9.16}$$

where δN is the change in the geoid height as a consequence of mass redistribution in the oceans and $\delta\zeta_{sa}$ is the change in the radial position of the sea surface between epochs $(\tau = t_1)$ and $(\tau = t_2)$, obtained from:

$$\delta\zeta_{sa} = \zeta(t_2) - \zeta(t_1) \tag{9.17}$$

ζ being the altimeter-derived height anomaly, after elimination of the tidal signal. δN can be computed from a global field of $\delta\zeta_{sa}$ according to the relation:

$$\delta N = \frac{G\rho_W R}{2\gamma} \iint \operatorname{cosec}(\psi/2)\,\delta\zeta_{sa}\,d\sigma \tag{9.18}$$

where G is the universal gravitational constant, ρ_W is the density of sea water, R is the mean earth radius and ψ is the angle between the geocentric radii to the element of volume and the point of computation.

This computation is only valid if the variations in the SST were produced by mass transport (as opposed to density changes brought about by the loading effect of the atmosphere). The use of such a procedure will give non-tidal time variations $\zeta_{v\ell}$ and ζ_{vs} with a precision equivalent to the radial component of orbital position and the resolution of the altimeter. In addition, it is assumed that a tidal model of adequate precision is already available (see § 9.5.1.4).

However, radial orbit errors in GEOS-3 are at least an order of magnitude greater than the desired precision. Such orbital errors are the result of insufficient laser tracking of the GEOS-3 spacecraft and an inadequate gravity field model (particularly in the southern oceans) to compensate for any gaps in the tracking coverage. Radial orbital error for altimeter passes with lengths in excess of 10^3km can be characterised by a bias and tilt to the profile of sea surface heights ζ. Consequently orbital errors will mask any time variations in ζ (i.e. $\delta\zeta_{sa}$) that can also be modelled by a bias and tilt.

Therefore in the case of GEOS-3, all information in the time-varying spectrum of the SST with wavelengths greater than twice the pass length will be lost. Hence the recovery of long wavelength non-tidal variations in SST ($\zeta_{v\ell}$) is not possible from GEOS-3 altimetry data at the present time.

A basis exists for studying short wavelength variations ζ_{vs} on a *regional basis* if:

(i) the corrections for bias and tilt can be determined; or

(ii) the technique of analysis does not require that the bias and tilt be known explicitly.

Two techniques that have been successfully used are described in § 5.2.3.4.

They are:

(a) The method of *regional models*.

(b) The analysis of *overlapping passes*.

GEOS-3 ground tracks repeat themselves approximately every 526 revolutions. The comparison of pairs of such overlapping passes shows that the rms discrepancy between any pair can be reduced to around ±30cm if one profile is biased and tilted onto the other pass. This figure represents the noise in the altimeter data (a proportion of which may be due to sea surface roughness). This principle can be used to define regional models of the sea surface in limited areas for the study of time variations in ζ_s, making it feasible to study the motion of eddies. The expected variations ζ_{vs} due to eddy formation and decay can be as large as $\pm 10^2$cm with half wavelengths of 10^2km.

The area most densely surveyed during the GEOS-3 mission is in the vicinity of the Gulf Stream and Sargasso Sea to the east of the Gulf Stream in the western North Atlantic. Altimetry data has been processed by MATHER ET AL (1978a) to obtain monthly regional models of the Sargasso Sea for the periods July 1975 to November 1975 and April 1976 to July 1976. Such models appear to be adequate for the study of variations ζ_{vs} associated with eddies with amplitudes larger than 50cm, wavelengths between 50 and 500km and periods greater than one month (the minimum sampling interval).

The variation of ζ_s with time has also been studied along selected profiles of overlapping passes in the western North Atlantic (IBID).

These analyses have demonstrated the potential of the GEOS-3 altimeter as a tool for the study of the dynamics of the sea surface. Intensive tracking support is not needed for eddy studies, provided approximately 30 passes of data are available each month per 10 km in the region of interest. A resolution of ±30cm can be expected in such a case. However, the recovery of variations in the long wavelength SST requires that the geocentric position of the altimeter-equipped spacecraft be known to an accuracy commensurate with the amplitude of $\zeta_{v\ell}$.

9.5.1.4 *Tidal Variations* ζ_t

Open ocean tides have amplitudes of the order of ±50cm and as a result of the large distances between amphidromes (zero tide height), it appears that a large part of the tidal signature is removed in the process of bias and tilting altimeter passes. Consequently, it would appear that the analysis of regional models or overlapping passes (§ 9.5.1.3) would be incapable of discriminating the tidal signal. However this contention ignores the fact that the frequencies of the major tidal constituents M_2, S_2, K_1, O_1 and P_1 are related to the luni-solar ephemeris and are therefore well known.

Studies have shown that a particular tidal constituent can be successfully recovered with 80% reliability if the noise in the altimetry data is no larger than the amplitude of the former (BRETREGER, 1976, p.94). Nevertheless, the analysis of tidal signals in satellite altimetry data presents different problems from those encountered in the analysis of conventional tidal records. Consecutive records at a point are obtained after a lapse of days and not hours. In the case of GEOS-3 a 100km square, after being sampled by a north-south altimeter pass, is again sampled by a south-north pass within a half day. However, the same 100km square is not sampled again for another 25 days.

Tidal studies in the Sargasso Sea by MASTERS ET AL (1979) have shown that the sampling rate and level of orbital error for GEOS-3 are such that only the phase and amplitude of the dominant tidal constituent M_2 in a $12^o \times 12^o$ region could be recovered.

The SEASAT-A orbit was planned to have a daily offset of approximately 20km with the ground tracks repeating themselves every 4 months or so. Consequently, the greater data acquisition rate and the improved signal-to-noise ratio of the SEASAT altimetry data would make the task of recovering regional ocean tide models more promising.

9.5.2 COASTAL SST

The term "coastal SST" refers to the displacement of the sea surface from the geoid at stations on the land/ocean boundary where MSL is sampled by tide gauge equipment. Such coastal tide gauges may, in addition, be connected to the continental geodetic levelling network whose elevations are based on one or more datum tide gauge(s). Geodetic levelling networks in different regions of the world are based on separate MSL datums.

In view of the uncertainties surrounding the estimates of coastal SST from oceanographic considerations (sub-section 3.4.2) and the doubts cast on the validity of large geodetic levelling networks (sub-section 3.4.3), it is desirable to develop an independent technique for determining coastal SST. One such means is through the use of satellite altimetry which, in addition, has the advantage of being able to establish the SST in relation to a four dimensional geodetic reference system having global relevance.

The determination of coastal SST to a precision of ±10cm, <u>in the global context</u>, is of interest for the following reasons (sub-section 8.2.1):

(1) To facilitate the realisation of a high precision geoid definition (sub-section 2.2.5).

(2) To facilitate the unification of geodetic levelling datums by relating each MSL datum to a uniquely defined geoid.

(3) Provide an additional set of constraints for controlling the propagation of systematic errors in the adjustment of continental levelling networks.

(4) Provide valuable information on the stability of the levelling datum for long term vertical crustal motion studies (sub-section 8.3.2).

It was established in sub-section 5.3.2 that the principal difficulties in using satellite altimetry directly to determine the heights of MSL above the geoid at coastal sites are:

(i) As a result of the finite altimeter "footprint" (approximately 15km diameter for GEOS-3), the stationary SST ζ_s has to be determined in the adjacent continental shelf areas. This presumes that the geoid has already been defined at the ±10cm level for wavelengths as short as 20km.

(ii) The extrapolation of the resulting values of ζ_s in the shallow continental shelf ocean to the coastal site requires the undertaking of oceanographic surveys to determine the current velocities, atmospheric pressure gradients and frictional forces operating in the shelf areas.

An alternate technique for the determination of the height (ζ_{sd}) of MSL at a coastal site is through the use of continental gravity anomaly data banks (see sub-section 5.3.2). However, this method is only useful for determining ζ_{sd} at regional levelling datums which serve areas greater than $\ell^2 km^2$, *if* the regional gravity anomaly data bank was computed using levelling data related to this datum and the satellite-determined gravity field model has a ±10cm resolution through wavelengths greater than ℓ. For example, only one estimate of coastal SST for the Australian continent is possible from such a method : the value of ζ_{sd} at the Jervis Bay Datum Tide Gauge (§ 6.2.2.3). The only role played by satellite altimetry is in defining a unique geoid (see sub-section 9.2.1).

In simplified form, the method can be summarised as follows (§ 8.2.3.1). Compute a reduced gravity anomaly Δg_d given by:

$$\Delta g_d = \Delta g - \delta\gamma \tag{9.19}$$

where Δg is as defined in section 9.3 and $\delta\gamma$ is the gravity anomaly implied by the GEM9 gravity field model. The height of MSL at the regional datum is obtained by forming observation equations of the form:

$$v = \Delta g_d + \frac{2\gamma}{R}\zeta_{sd} - \frac{2}{R}(W_o - U_o) \tag{9.20}$$

where $(W_o - U_o)$ is defined by satellite altimetry (sub-section 9.2.1), R and γ are, for practical purposes, the mean radius of the earth and mean normal gravity respectively. These equations can be formed over five degree area means with additional harmonic models used to remove aliasing effects arising from systematic errors in both the gravity and levelling networks.

Preliminary investigations were performed using gravity data sets on the Australian continent (AUSGAD 77) and the area covered by the United States (CNAGAD 77).

The value of ζ_{sd} estimated at the Jervis Bay Datum is (§ 8.2.4.1):

$$\left(\zeta_{sd}\right)_{\text{Jervis Bay}} = +0.2 \pm 0.4 \text{ m} \tag{9.21}$$

The equivalent value obtained by extrapolation from the surrounding deep oceans using oceanographic data is +0.3 ± 0.2m.

An analysis of a bank of gravity anomalies for central North America, based on the assumption that all elevations were related to MSL at Galveston, gave (§ 8.2.4.2):

$$\left(\zeta_{sd}\right)_{\text{Galveston}} = +0.1 \pm 0.4 \text{ m} \tag{9.22}$$

The equvalent oceanographic value was +0.1 ± 0.3m.

While these preliminary results are in good agreement with oceanographic estimates, some doubts exist about the practical significance of these results, primarily as a result of gravitational scale uncertainties - see sub-section 8.2.6.

9.6 RECOMMENDATIONS

Considerable work remains to be done in developing reliable techniques for extracting SST information from combinations of satellite altimetry data, surface gravity measurements and models of the earth's disturbing potential. With the successful completion of this task, the quantifying of ocean dynamics through the use of satellite remote sensing techniques will be a real possibility.

Further progress in Sea Surface Topography Studies will only be possible if the following requirements are satisfied:

(a) Definition of the radial component of the altimeter-equipped satellite to at least ±10cm on a global basis.

(b) Refinement of models of the earth's geopotential to a resolution of 0.2 - 1 part in 10^8 (i.e. 1 - 6kgalcm) in each harmonic.

In theory, a global network of at least 25 laser tracking systems capable of ±10cm precision is a necessary prerequisite for satisfying both these requirements. Although a purely geometric definition of the altimeter's position to ±10cm is not possible with such a tracking network, the appropriate analysis of tracking data provided by such a density of coverage should define a gravity field model to degree n' (≃20) with a precision equivalent to that of the tracking data. In the short term however, satellite altimetry and surface gravity data can play a role in gravity model improvement. However, given the choice, it is preferable to improve the density and distribution of high precision laser tracking systems rather than to carry out extensive surface gravity surveys.

A high precision gravity field model, in addition to maintaining the orbital position of the altimeter-equipped satellite in the absence of continuous tracking, will define those features of the geoid with wavelengths greater than ℓ (=o{10^3km} for n' = 20) and amplitudes in excess of the noise level of the tracking, with an equivalent precision.

An alternate means, in principle, for obtaining an improved definition of the gravity field for wavelengths shorter than ℓ is by satellite-to-satellite tracking of a low-flying satellite like the proposed GRAVSAT (NASA 1972b).

The analyses of satellite altimetry from the GEOS-3 mission described in this thesis demonstrate the potential of radar altimeter data. However, the SEASAT-A mission has promised an improved basis for the recovery of ocean dynamic information for several reasons:

(i) a more complete data coverage in both space and time (although no data beyond the 72° parallel is collected);

(ii) the complement of high precision tracking systems is better distributed than for the GEOS-3 mission;

(iii) the improved gravity field models obtained from the analysis of GEOS-3 data would provide more precise orbits; and

(iv) the additional on-board instrumentation is expected to provide information on wind and thermal emissions from the air/sea interface that could be coalesced with altimeter data to construct ocean circulation models.

Despite the fact that SEASAT-A had a lifetime of less than 100 days before its premature failure in October 1978, it has amassed an altimetry data set of comparable size to that of the 1975-76 GEOS-3 data bank. This altimetry data, in conjunction with the GEOS-3 data, will continue to provide valuable information to investigators in the fields of geodesy and oceanography.

However, there is no active satellite altimeter in operation at present and, furthermore, there is no firm schedule for launching such a satellite. As much work remains to be done on the recovery of oceanographic information from satellite altimetry it is desirable that another altimeter-equipped spacecraft be launched as soon as possible. A resolution to this effect has been passed by the International Association of Geodesy at the XXVIIth General Assembly of the International Union of Geodesy and Geophysics, 3-14 December 1979, Canberra, Australia.

Nevertheless, the concept of a geodetic basis for ocean dynamical studies, although in its infancy, is an exciting prospect.

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