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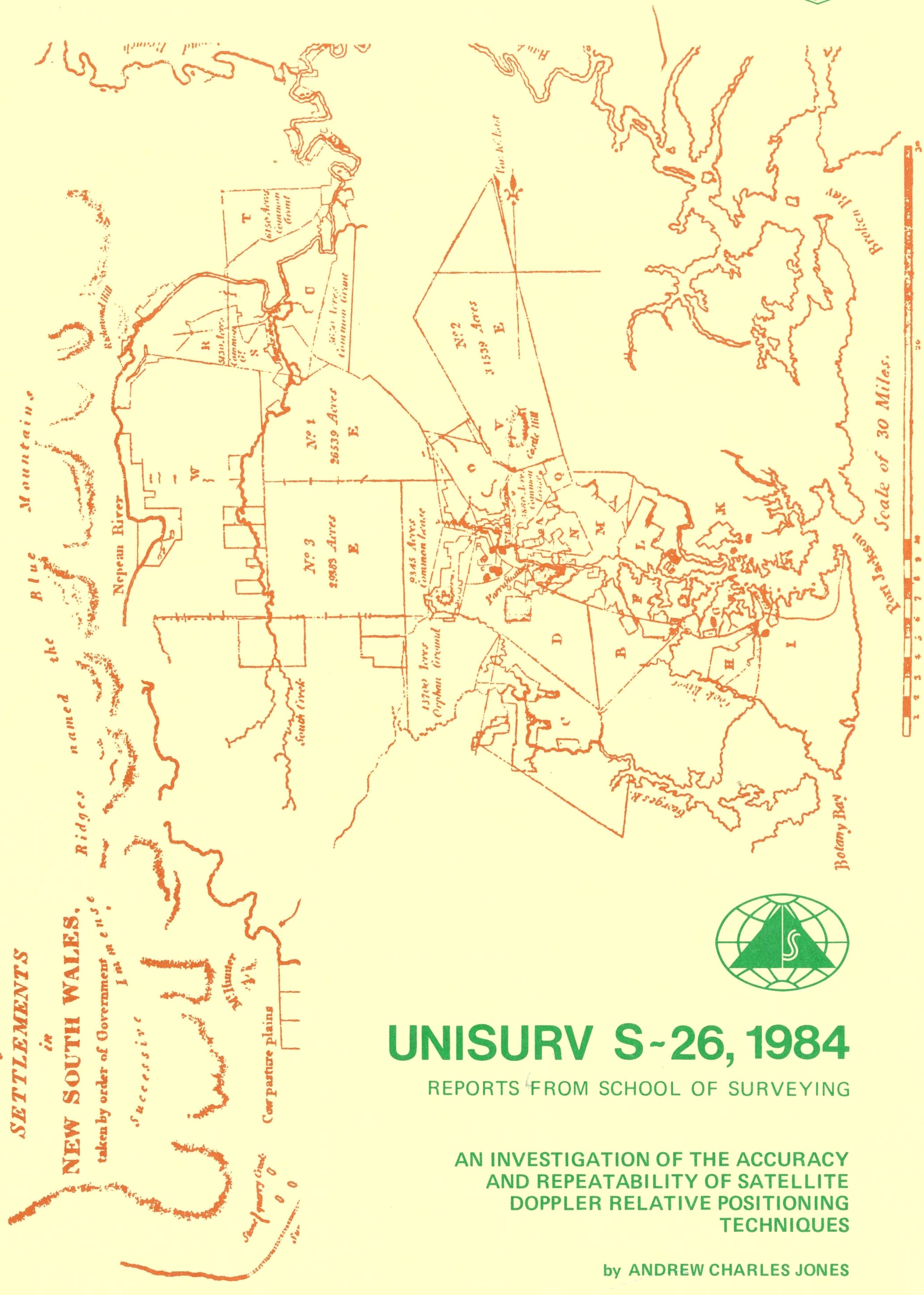

UNISURV S-26, 1984

REPORTS FROM SCHOOL OF SURVEYING

AN INVESTIGATION OF THE ACCURACY AND REPEATABILITY OF SATELLITE DOPPLER RELATIVE POSITIONING TECHNIQUES

by ANDREW CHARLES JONES

UNISURV REPORT S26, 1984

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SCHOOL OF SURVEYING
THE UNIVERSITY OF NEW SOUTH WALES
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To Ursula , Stephanie
and Nicholas.

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My extended family have provided wonderful encouragement this year and I am deeply indebted to all of them for it. However one person stands out. Without the unquestioning support that was provided by my wife , Ursula , my period at the University would never have eventuated. Her task in having to bring up two small children single-handed was I think , harder than mine. My gratitude and admiration know no bounds.

ABSTRACT

The use of satellite Doppler techniques for geodetic positioning has become increasingly common in recent years. The relative mode of operation has become particularly popular due to assertions regarding its accuracy capabilities. Unfortunately , at the present time , meaningful accuracy tests are still relativly few in number. Moreover , such tests are generally restricted to distances of less than one hundred kilometres. The assessment of accuracies over longer distances is most desirable , particularly in Australia where the remoteness of much of the country makes Doppler positioning a very attractive technique.

This aim of this project is to add to the growing body of data that is becoming available regarding relative positioning accuracies. An analysis is undertaken of a multi-station Doppler figure which was observed for inclusion in the GMA82 adjustment. The effects of processing with different reduction programs are examined as are the influences of network size , ephemeris type and ephemeris constraint. An examination of repeatability is also undertaken , this involving data from a second multi-station figure.

Data analysis is principally carried out through the intercomparison of chord distances. All Doppler derived

distances are compared with those computed from AHD elevations , Fryer geoidal undulations and preliminary GMA82 coordinates. Comparisons with results obtained by other authors are performed where possible.

CHAPTER 1

INTRODUCTION

The Navy Navigation Satellite System (NNSS) or TRANSIT system was commissioned by the United States Government in 1963. Originally conceived as a navigation system for military vessels , it was made available for civilian use in 1967.

The operation of the system is based on a principle first noted by Christian Johann Doppler , an Austrian , in 1842. Doppler drew attention to the fact that the frequency of a signal as perceived by an observer is dependent upon the relative motion between the observer and the signal source. Thus for example , if a noise producing object , such as a train , approaches a stationary observer , the observer will hear a higher pitched noise until the train reaches him , the actual pitch of the noise as the train becomes level with him , and a lower pitched noise as the train moves away. This phenomenon is known as the Doppler effect and applies to waveforms in general. The difference between the perceived frequency and its actual value is known as the Doppler shift (Resnick and Halliday 1966 A).

The TRANSIT system is based entirely upon the Doppler effect. The system utilises a series of satellites which are located in polar orbits. Each of these satellites transmits two modulated tones at approximatly 400 MHz. and 150 MHz.

As a satellite approaches and passes a receiving station,Doppler shifts in both of the emitted signals become perceivable. An analysis of these shifts coupled with sufficiently accurate ephemeris data enables the receivers position to be determined.

By the begining of 1969 , the accuracy available from the TRANSIT system had improved to the point where it became viable as a tool for geodetic positioning. Two modes of operation quickly emerged. The first became known as the point positioning mode , and involved the determination of the absolute values of latitude , longitude and spheroidal height at a single receiver station. This technique relies heavily upon the integrity of the satellite positions as defined by an ephemeris. As neither the broadcast nor precise ephemeris are perfectly known , point positions always contain ephemeris-induced errors. At the present time it is estimated that horizontal positions computed using the broadcast or operational ephemeris have an accuracy of ten metres (1σ) after fifteen passes while those computed using the precise ephemeris have a corresponding accuracy of 0.7 of a metre after forty passes (Hoar 1982a A).

The second mode of operation is known as the relative positioning mode. Techniques in this category require that two or more receivers occupy different stations during the same period of time and simultaneously receive data from each satellite pass. The determined absolute positions of the receivers will thus be subject to the same ephemeris biases. However , as the errors at each station are correlated , the relative station positions will remain more or less bias free. By employing suitable modelling techniques , the relative positions of the receivers may be recovered to a significantly higher degree of accuracy than their absolute positions.

Relative positioning has increasingly become the main mode of operation in recent years. The vast majority of Doppler software currently available facilitates the use of such techniques. Indeed some manufacturers are now incorporating relative positioning software into their geodetic receivers. The reason for this popularity is the assessment that relative positioning techniques can produce accuracies from the broadcast ephemeris which are equivalent to those obtainable from the precise ephemeris (Stansell 1978 A). (An explanation of the differences between the ephemerides is given in Chapter 4.) It is the purpose of this report to investigate the veracity of such claims. Clearly the report cannot hope to be exhaustive in its investigations due to the extensive nature of the subject area. It is intended however that it should at least contribute to the growing body of data which is becoming available regarding attainable accuracies from relative positioning.

The report commences by giving an overview of the TRANSIT system in Chapter 2. It then proceeds to discuss theoretical aspects in Chapters 3 and 4 . These aspects include geometric modelling , time recovery , atmospheric effects and the ephemerides. It is intended that these chapters should highlight the essential components of Doppler processing , thus providing a background against which the results may be discussed.

In Chapter 5 , the test data itself is presented. Relative positioning solutions involving two , five and eleven stations are tabled , these having been processed using both the broadcast and precise ephemeris. Chord distances , when computed , are compared with those determined from the best available 'ground truth'. All Doppler reductions were performed using the Geodetic Survey of Canada program , GEODOP , and the on-board translocation facilities of the Magnavox MX1502 Satellite Surveyor.

The conclusions are presented in Chapter 6.

CHAPTER 2

THE NAVY NAVIGATION SATELLITE SYSTEM - AN OVERVIEW

This chapter is intended to provide an introduction to the TRANSIT system. It commences by briefly describing the history of the system's development. It then proceeds to describe the three principal system components. Finally it concludes with an elaboration on the techniques of relative positioning.

2.1. History

The TRANSIT navigation system was patented by F.T.McClure in 1958 (Hoskins 1982 C). Its history began in 1957 with the launching of the first artificial earth satellite , Sputnik 1. Interest in this satellite was very high and it was tracked world-wide by both optical and radio techniques. As had been anticipated , the signals radiated by the satellite displayed a Doppler shift as the satellite moved past each observing station. However it was not until 1958 that the information content of that shift was fully recognised.

It was found that ephemerides computed for Sputnik 1 using only radio Doppler measurements were as accurate as those determined from optical and other radio techniques

(Stone and Weiffenbach 1961 A). As a result , the principle was investigated further. R.R.Newton demonstrated that the technique could be adapted to the tracking of inter-planetary vehicles to ranges of 150 - 200 million miles using a relatively low-power on-board transmitter. The method was used to track Pioneer 5 for eight million miles before a malfunction prevented further observations.

It quickly became apparent that if satellite ephemerides could be generated to an adequate accuracy from an independent source , the technique could be inverted and used to determine positions on earth. Thus ideas for a two-stage system were developed along the following lines. In the first stage,the Doppler shift from a satellite would be detected at a number of tracking stations whose positions were known. This signal would be used to determine the satellite's ephemeris. In the second stage,a receiver at an unknown locality would track the same satellite and measure its perceived Doppler shift. This , together with the ephemeris determined in stage one would enable the unique definition of the receiver's position in a geocentric coordinate system.

The real impetus for the development of such a system came from the United States Navy. With the introduction of their POLARIS nuclear submarines , the Navy required a position fixing system that would enable the frequent updating of inertial navigation systems at any point in the world. As a result the TRANSIT system came into being. The system was developed at the Applied Physics Laboratory of the John Hopkins University between 1958 and 1963. It became operational in 1964. Initially the system was only available to military users. However in 1967 , by Presidential directive , it was made generally available to the civilian population.

Since its inception , the TRANSIT system has been operated and maintained by the U.S. Navy Strategic Systems Project Office (SSPO). In addition to POLARIS, the system is used on both POSEIDON and TRIDENT submarines. However the number of civilian users now far exceeds that of their military counterparts. In 1981 it was estimated that the U.S. Government operated approximatly 1000 receivers. At the same time, it was estimated that the civilian community operated in excess of 15000 receivers and that this would rise to 45000 by the end of 1982 (Hoskins 1982 C).

The system has been progressivly improved over the years. The introduction of superior gravity models in 1968 and 1975 resulted in the accuracy improving to the point where it could be used for geodetic positioning. TRANSIT has proved to be superbly reliable , having achieved a 99.9% system reliability/availability rating on all currently operational satellites. The SSPO will continue to support the TRANSIT system until after the installation of the GPS system. It is anticipated that this will take at least until 1992. The future of the TRANSIT system beyond that time is not certain. However its continued operation by another agency is a possibility.

2.2. System Components

The TRANSIT system may be considered as consisting of three components. They are -

1. The Space Segment
2. The Control Segment
3. The User Segment

Each will now be considered in turn.

2.2.1. The Space Segment

The space segment consists of the satellites themselves. Currently there are six satellites in the TRANSIT configuration although only five of them are operational. Four of the six satellites (30130 , 30140 , 30190 , 30200) are of the original design and are known as OSCAR type satellites. Two of these have been operational since 1967 and have demonstrated better than 99% reliability/availability. The fifth satellite , 30110 , has a specialised dual function. It has not been operational since April 1981 although it is expected that it will be returned to service in mid-1984 (Hoar 1982b A). The remaining satellite , 30480 , is of a newer design. It is known as a NOVA type satellite and is designated NOVA 1.

All of the satellites are in polar orbits approximatly 1100 km. above the earth. The period of their orbits is approximatly 106 minutes (Hoskins 1982 C). The average pass frequency with five operational satellites varies between 35 and 100 minutes depending on latitude. The pass geometry of the TRANSIT system repeats at two day intervals (Boal and Vamosi 1981 A).

All TRANSIT satellites (also known as NAVSAT satellites) are launched from the Vandenberg Air Force Base in California aboard Scout Boosters. Unfortunatly the pointing accuracy of the Scout Booster is slightly imperfect (σ = 0.43 degrees) , with the result that most of the satellites have been placed into slightly out-of-polar orbits. This imperfection has caused the orbital planes to precess , resulting in sub-optimal satellite coverage , and at times , interference between the signals from two satellites. The latter effect may occur if the orbital planes of two satellites become approximatly coincident. It can only be overcome by switching off one of the satellites until the period of coincidence has passed. It was for this reason that satellite 30110 was switched off in 1981.

Each of the TRANSIT satellites contains the following systems.

1. A power supply system.
2. A highly stable 5 MHz. frequency oscillator.
3. A clock.
4. 150 MHz. and 400 MHz. transmitters.
5. A core memory for storing ephemeris data.
6. A telemetry system for monitoring satellite performance.
7. A command system.

All of the satellites are sustained by solar power. Each has four solar panels attached to the main body. These supply energy to charge internal batteries.

The 5 MHz. frequency oscillator powers the clock and generates the two transmitted signals. This component is vital to the integrity of a satellite's performance. Any oscillator instability will result in the detection of spurious Doppler shifts at the receiving stations , resulting in erroneous position fixes. The development of an ultra-stable frequency source which could survive a satellite's launch and then operate satisfactorily for years afterwards was a major achievment (Decca Survey Sat-Fix A).

Despite their high stability characteristics , the frequency generated by the 5 MHz. oscillators varies and drifts with both time and satellite. In the OSCAR satellites , the resulting timing errors are compensated by the introduction of 9.6 microsecond time delay steps to the broadcast message (Vide Section 3.3.2.). In the NOVA 1 satellite , a Digital Frequency Adjustment System known as I.P.S. has been incorporated to control the operating frequency of the master oscillator within a narrow range of values. As a result , the NOVA's oscillator frequency is capable of being held at a constant value , regardless of the oscillators aging characteristics. It is anticipated that by incorporating the I.P.S. module into a closed loop system with a ground based monitoring station , it will be possible to maintain frequency and timing accuracy at a level of a few parts in 10^{12} . It was reported by Hoar (1982b A) that the I.P.S. system used since launch had failed. The satellite is now using a backup I.P.S. system.

Each satellite transmits two modulated signals. The higher frequency is nominally set at 399.968 MHz. although this is subject to drift as discussed above. The lower frequency is precisely 3/8 of the higher frequency at all times. The signals are coherent at the time of transmission (Smith et al 1976 A). Both signals are phase modulated by ephemeris data in the satellite's memory. The exact nature of this data will be discussed in Chapter 4. The signals are transmitted by an antenna which always points to earth , this orientation being achieved by an elongated boom which naturally aligns itself with the earths gravity field.

The core memory on board the OSCAR satellite has sufficient capacity to store sixteen hours of ephemeris data. The memory is updated every twelve hours,this leaving a safety margin of four hours in the event of a missed injection. In the NOVA 1 satellite , the core memory was increased to enable the storage of eight days of ephemeris data , thus reducing the need for frequent data injections. However the realisation of the full potential of the extra memory was only made possible by the incorporation of the Disturbance Compensation System (DISCOS) into the new satellite.

The effects of drag on the accuracy of predicted satellite ephemerides will be discussed more fully in Chapter 4. At this stage it suffices to mention that drag effects are caused by the passage of the satellite through the earth's atmosphere and by solar radiation pressure. The errors resulting from these effects increase quadratically with time. They can become as large as 170 metres over an eighteen hour prediction period during periods of high solar activity. Clearly the prediction of several days of ephemeris data would be pointless with such a high error growth rate.

The DISCOS system is designed to detect and compensate for drag forces which act in the along track direction. Compensation is achieved by firing a pair of along track thrusters. The worth of such a system has been demonstrated by comparitive testing with the OSCAR satellites. Wheras the errors in the predicted ephemeris for the NOVA 1 satellite varied between 5 - 15 metres (RMS) over the test period,those for the OSCAR satelite varied between 5 - 70 metres (RMS) (Eisner et al 1982 C).

It is apparent therefore that the use of an enlarged memory unit is entirely appropriate on the NOVA type satellite and will reduce that satellites dependence on the control segment for frequent updates of ephemeris data. An expanded OSCAR memory however would be of little advantage due to the presence of uncompensated drag effects.

Other features of the OSCAR satellites which were improved upon in the NOVA 1 satellite include the boosting of the signal strength on both transmitted frequencies,an increase in the number of channels in the telemetry system and an improvment in the stability of the vertical alignment of the transmitting antenna. In addition , the NOVA 1 satellite carries an on-board computer.

At the present time there are thirteen OSCAR type satellites in storage to replace those in orbit should they fail. In addition it is planned that two other NOVA type satellites be produced and placed in orbit by the end of 1984 (Hoar 1982b A). Eight SCOUT Boosters have been procured for the launching of these satellites. As the SCOUT Booster program is due to be terminated in 1988 , and as the TRANSIT satellites are not easily adaptable for launches via the Space Shuttle , plans are currently being made to store the satellites in space by launching two OSCAR type satellites on each booster. It is anticipated that the first of these launches will take place in 1984.

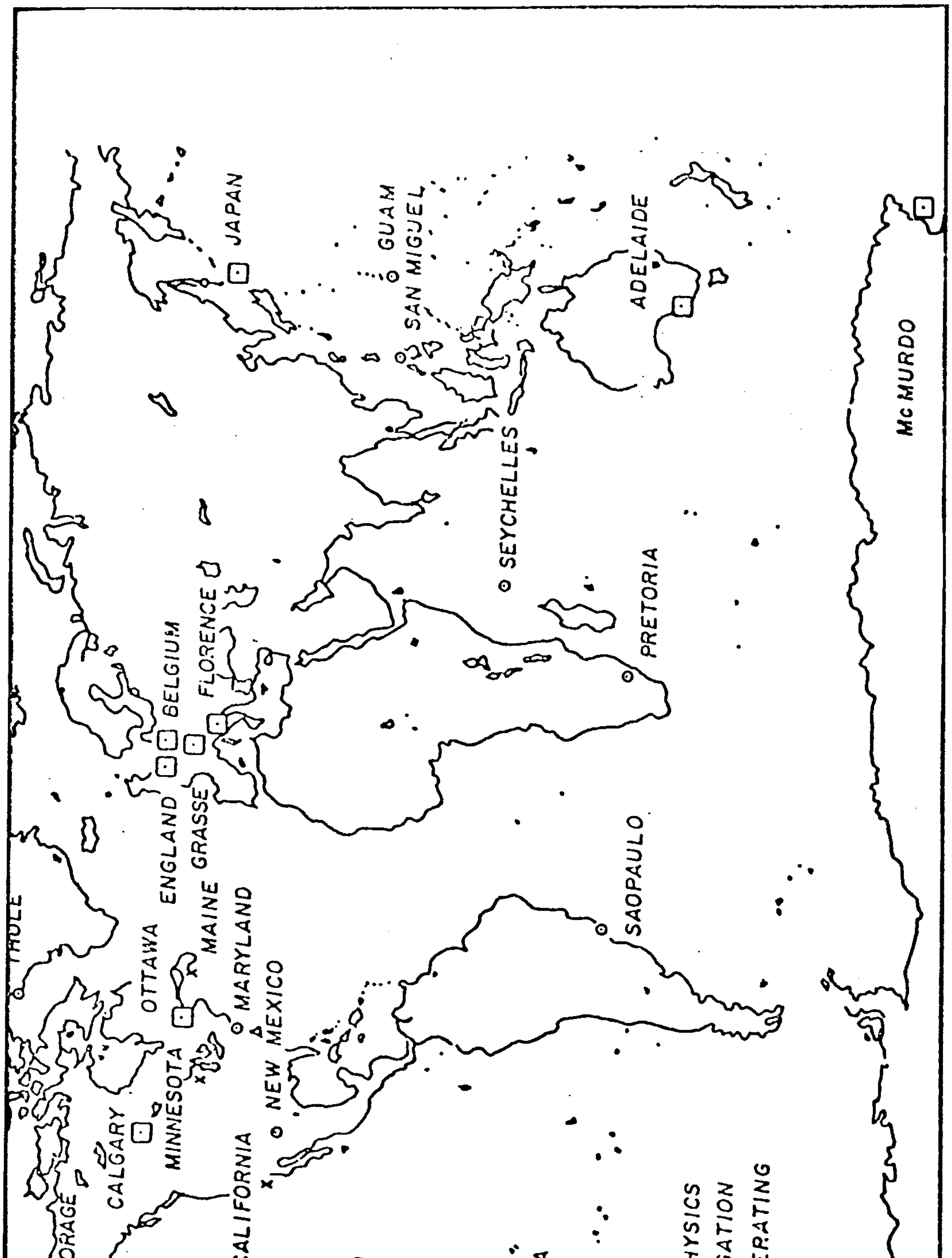
2.2.2. The Control Segment

The TRANSIT system is operated and maintained by the U.S. Navy Astronautics Group (NAG). This group has their headquarters at Pt. Mugu , California.

The NAG operates four tracking stations. These are located in Maine , Minnesota , California and Hawaii (Vide Diagram 1). They are collectivly known as the OPNET stations. These stations track the 150 MHz. and 400 MHz. signals transmitted by each satellite , measuring and recording the Doppler shifts as they pass overhead. The data thus recovered is transmitted to a computing center at Pt. Mugu. Here an orbit is computed for each satellite from the

DIAGRAM 1
TRACKING STATION LOCATIONS

(ANDERLE 1976a A)



preceeding thirty-six hours data. This is then extrapolated forward to produce the broadcast or operational ephemeris for the following thirty hours (Jenkins and Leroy 1979 B , Hoskins 1982 C).

The broadcast ephemeris is uploaded to the OSCAR satellites twice per day and to the NOVA satellite once per day. The orbit determination is generally repeated every twenty-four hours. However during periods of high solar activity , the computation may be repeated every twelve hours (Jenkins and Leroy 1979 B).

Facilities for uploading information to the satellites exist at the Minnesota and California tracking stations. In the period between January 1964 and April 1977,only seven out of a total of 32,389 attempts at message injection failed for any reason. In each case the upload was successfully carried out during the next satellite pass. These figures provide further clear evidence of the reliability of the system (Stansell 1978 A).

In addition to the OPNET stations , there exists a second network of receivers known as the TRANET network. These stations have only a tracking capacity and are not able in any way to upload information to the satellites. The NAG are the sole managers of the satellite constellation.

The TRANET network consists of at least twenty permanent receivers and several portable receivers distributed worldwide (Vide Diagram 1). Like the OPNET stations , the TRANET stations track the satellite orbits using Doppler techniques. The information thus recovered is used to determine the precise ephemeris , this being used purely for post-processing operations rather than orbit prediction.

Until the end of April 1975 , the precise ephemeris was computed by the U.S. Naval Surface Weapons Center. Since May 1975 , this role has been taken over by the Defence Mapping Agency Hydrographic/Topographic Centre (Hotham 1979 B). The ephemeris is computed on alternate days based on forty-eight hours of observations (Anderle 1976a A). It is made available on a government to government basis.

2.2.3. The User Segment

Broadly speaking the user segment consists of all portable TRANSIT receivers. For the purposes of this discussion , the user segment will be restricted to include only those instruments designed for geodetic positioning. Navigation receivers will not be considered.

Portable TRANSIT receivers have been developed by a number of companies during the past sixteen years , the most notable being Magnavox , JMR , Canadian Marconi and Motorola. Ignoring the various microprocessor based features which most of these organisations have introduced , all receivers contain the same basic components. They are

1. A Reference Oscillator.
2. 150 MHz. and 400 MHz. Receivers.
3. An Antenna and Preamplifier.
4. A Data Recording Device.
5. An Energy Supply.

The reference oscillator is the 'heart' of the instrument. It is used to drive an internal clock and generate reference signals at frequencies of 150 MHz. and 400 MHz. These frequencies differ from those transmitted at the satellite by a nominal 80 ppm. , this being 32KHz. at 400 MHz. (The satellite transmits at 399.968 MHz.). The received signal and reference signal are mixed to produce a beat frequency , the cycles of which are accumulated to form the Doppler counts. As the Doppler shift in the beat frequency never exceeds 20 ppm. of the transmitted frequency , the 80 ppm. offset ensures that the Doppler counts always retain the same sign , simplifying receiver design by avoiding the need to count positive and negative cycles (Smith et al 1976 A , Hatch 1982 C).

The oscillators used in Doppler receivers (usually of the quartz-crystal type) exhibit excellent short-term stability characteristics , this being essential for the accurate accumulation and timing of Doppler counts. However both the satellite and receiver oscillators are subject to linear drifts over extended periods of time , a fact which should be taken into account during the modelling of positioning solutions (Brown 1976 A).

A receiver's oscillator needs to be allowed to warm up prior to the receiver being used. Magnavox recommends a warm up period of twenty-four hours for the MX1502 to enable the oscillator to stabilise (MX1502 Field Translocation Satellite Surveyor - Operation and Service Manual 1980 A). It should be noted that the oscillator is sensitive to instrument mal-treatment. Brunell (1979 B) states that a blow to the instrument will excite the oscillator for a period of time afterwards. In addition , a jump discontinuity in the frequency may occur if the orientation of the oscillator relative to the local vertical is at all changed. Such disturbances would make the achievment of high precision measurements impossible.

The antenna/preamplifier assembly contains the electrical centre of the instrument. This is the point to which derived Doppler positions are referenced. The locations of antenna sites need to be selected with some care. Satellite signals travel along a direct line of sight to the instrument. Consequently it is desirable that receiver sites have the benefit of an unobstructed horizon. In addition it is important that the antenna be located such that it avoids receipt of reflected signals and other spurious radio interference.

Modern receivers generally record their acquired data onto tape cassettes for later post-processing. Some older instruments however , notably the AN/PRR-14 Geoceiver use paper tape.

Geodetic receivers invariably possess an internal twelve volt power source to maintain oscillator temperature during instrument transportation. An external power source , usually a twelve volt car battery has to be used during tracking operations. A single fully charged car battery will normally supply adequate power for periods of twenty-four hours or longer.

2.3. Data Reduction Techniques

As was noted in the introduction , Doppler positioning techniques can be catagorised into two modes of operation - absolute and relative.

The absolute mode is also known as point positioning.

It can be defined as follows.

> 'Point positioning is the process of collecting data from multiple satellite passes at one location , along with an ephemeris , to determine the independent station position referenced to the earth centered coordinate system'
>
> (Hoar 1982a A)

At their most basic level , point positioning computations assume that the positions of the satellites as defined by the ephemerides are error free. The reduction process solves simply for the three station coordinates plus the offset of the 400 MHz. frequency from its nominal value. More sophisticated programs , in particular those which were principally developed for relative positioning , recognise the presence of satellite errors and include such biases among the parameters of the solution. Frequently these programs also include other undetermined quantities among their unknowns , such as the tropospheric refraction bias. The use of these programs in the point positioning mode requires some care , particularly in the assignment of appropriate variances to constrain the orbital parameters (Hoar 1982a A).

Relative positioning techniques can be subdivided into several different categories. At the most basic level , relative positioning may be accomplished by displacement translocation (Vide Diagram 2). This involves the assumption that ephemeris-induced errors in two simultaneously determined point positions are identical and thus that the inter-station vector is error free. If accurate coordinates are known for one of the stations , then the coordinates of the other may be obtained by the addition of the inter-station vector. Clearly this technique depends heavily on there being a very high degree of correlation between the satellite-induced errors at the two stations. Consequently it should be used with caution and only over relativly short distances.

The following terminology for relative positioning techniques was recommended for adoption by The Workshop on Doppler Data Reduction and Analysis at the First International Geodetic Symposium on Satellite Doppler Positioning. The text is taken directly from Hoar (1982a A).

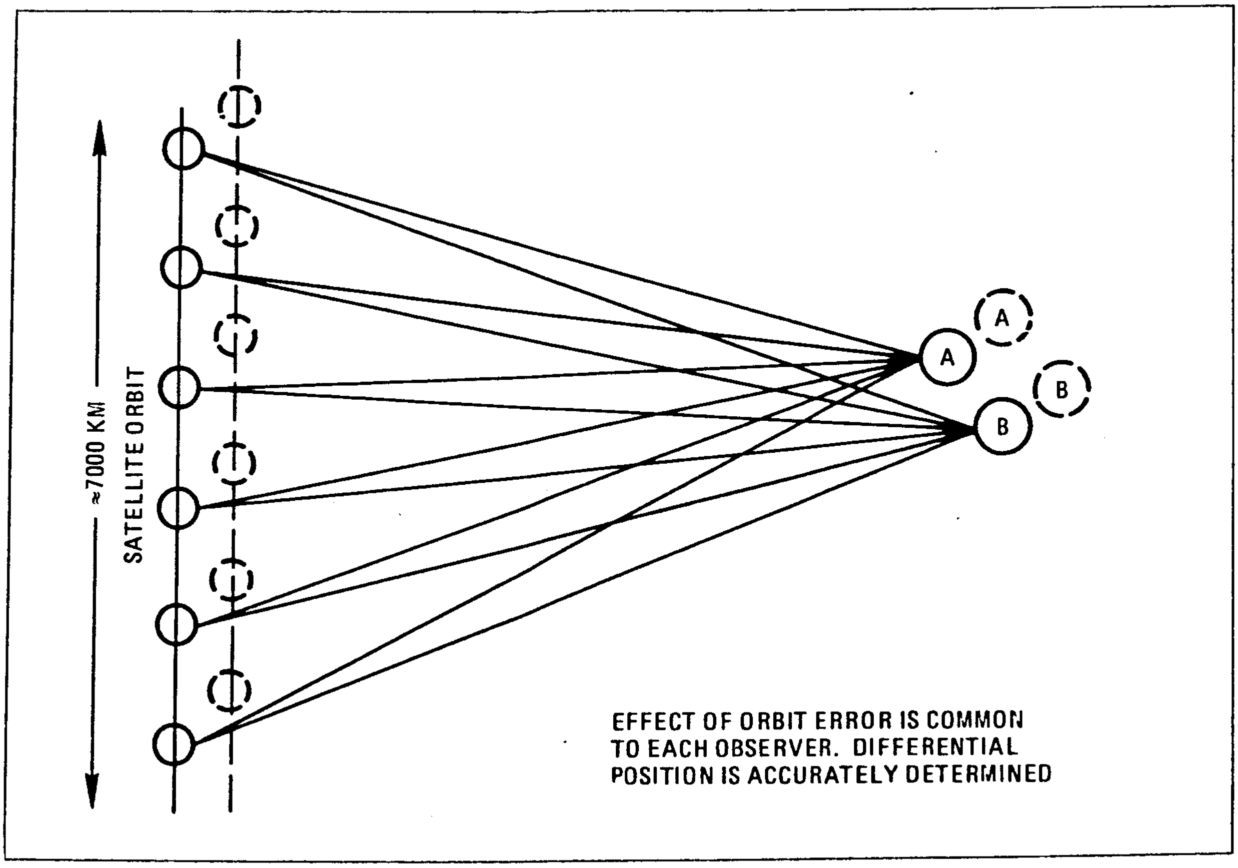
a. 'Short Arc' refers to methods in which the apriori ephemeris is given at least six degrees of freedom.

b. 'Semi-Short Arc' refers to methods in which the apriori ephemeris is given between one and five degrees of freedom.

c. 'Rigorous Translocation' refers to methods in which only common data points from passes

DIAGRAM 2
PRINCIPLE OF TRANSLOCATION

(POINT POSITIONING AND TRANSLOCATION PROGRAM 1979 A)



simultaneously tracked at all stations are used in the data reduction.

d. 'Translocation' refers to methods in which receivers are operated simultaneously , although the data points may not be identical.

Reference to these methods will be made again in later sections. At this point it is worth noting that program GEODOP and the MX1502 translocation software are both classified as being semi-short arc programs (Kouba and Boal 1975 A , Hoar 1982 A). In this report , relative positioning solutions which involve only two stations will be referred to as translocation solutions. Solutions which involve more than two stations will be referred to as multi-station solutions.

CHAPTER 3

THE NATURE , ACQUISITION AND CORRECTION OF DOPPLER DATA

In Chapter 1. , it was noted that Doppler positioning is dependent on the interaction of two data sets , these being

1. The set implicit in the Doppler curve.
2. The set provided by the satellite ephemeris.

The latter of these will be considered in Chapter 4. The former will be discussed in the following subsections.

Section 3.1. is principally concerned with the geometric modelling of Doppler solutions. Section 3.1.1. commences by investigating the characteristics of the Doppler curve. This is followed in Sections 3.1.2. and 3.1.3. by definitions of the satellite and receiver time frames , and by the derivation of the range-rate equations. Section 3.1.4. then outlines a solution technique which is commonly used for data editing and which may also be used for position or ephemeris computation. Finally Section 3.1.5. considers the modelling of multi-station solutions.

In the remaining subsections , the emphasis is placed on the Doppler counts themselves. Section 3.2. considers count accumulation and the resolution of correlation problems. Section 3.3 investigates the influence of time recovery errors , these being caused by both the receiver (3.3.1.) and the satellite (3.3.2.). Finally Section 3.4. considers the corrections for atmospheric effects.

3.1. Geometric Modelling

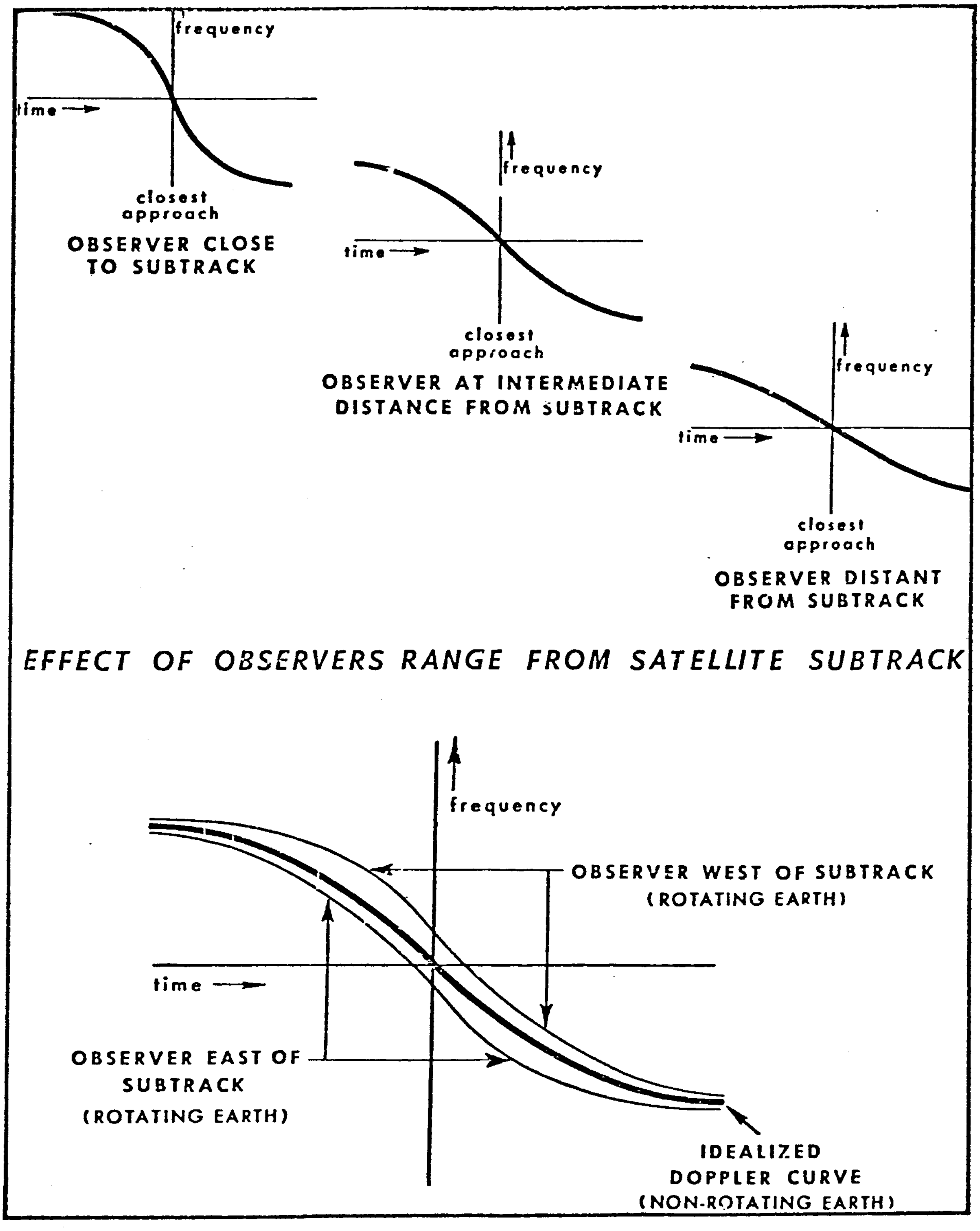
3.1.1. The Doppler Curve

The curves in Diagram 3 (Stone and Weiffenbach 1961 A) illustrate the variations in the Doppler shift perceived at a receiving station during a satellite pass. The curves are characterised by two long 'tails' and a relativly abrupt frequency change , this occuring around the point of closest approach. The exact shape of a Doppler curve is determined by the magnitude of the satellite's velocity in the direction of the receiver. Thus a satellite which passes nearly directly overhead displays a very pronounced frequency shift whereas signals from a low elevation pass display a more gradual change. A signal from a satellite orbiting at constant distance from a receiver would exhibit no shift at all. (This is not possible in the TRANSIT system.)

The data content of the Doppler curve is described by three parameters,these being -

DIAGRAM 3
DOPPLER CURVE CHARACTERISTICS

(STONE AND WEIFFENBACH 1961 A)



1. The time of closest approach.
2. The receiver to satellite range at closest approach.
3. The frequency offset between the receiver and the satellite signals.

The time of closest approach occurs when the slope of the Doppler curve is a maximum. At that time the receiver-to-satellite range vector is orthogonal to the direction of the satellite's motion. The magnitude of the range vector is inversely related to the magnitude of the maximum slope (i.e. a steep slope represents a short range). In addition , as the satellite-receiver velocity component is zero at closest approach , the received frequency at that time represents the true value of the transmitted frequency. Thus if the satellite's ephemeris were known at the time of closest approach , (and if the height of the receiving station could be determined from other sources) , the data provided by the Doppler curve could be used to determine the receiver's coordinates and the true value of the frequency offset (Vide Section 3.1.4.).

The determination of the closest approach parameters in isolation is an inherently difficult task. For this reason,data from all points on the Doppler curve are used in a least squares fit to determine the desired quantities.

Finally it must be remembered that the perceived Doppler shift results from the overall relative motion between the satellite and the receiver. Consequently earth rotation will contribute a component to the curve as illustrated in Diagram 3 (Stone and Weiffenbach 1961 A). The different curves for the east and west sub-tracks enable resolution of the east-west ambiguity.

All variations not directly caused by the motion of the satellite will contaminate the results derived from the acquired data. It is essential that such variations be removed , either by apriori correction or through the modelling of undetermined parameters.

3.1.2. The Satellite and Receiver Time Frames

The 150 MHz. and 400 MHz. signals are phase modulated by a 6103 binary bit message. This message is transmitted continually over every two minute period. The bits are organised into 26 lines and 6 columns of 19 and 39 bit words as illustrated in Diagram 4.

The modulated message serves three purposes. First of all it propagates the parameters of the broadcast ephemeris to receiving stations , thereby facilitating the computation of receiver station coordinates. These orbital

DIAGRAM 4
SATELLITE MESSAGE FORMAT

(POINT POSITIONING AND TRANSLOCATION PROGRAM 1979 A)

TIME MARK T_i — MESSAGE SET A

LINE 1	3	4	5	6	7	8	$t_i - 3$	VARIABLE PARAMETERS - DEVIATION FROM ELLIPTICAL ORBIT AT INDICATED TIMES
LINE 2	9	10	11	12	13	14	$t_i - 2$	
LINE 3	15	16	17	18	19	20	$t_i - 1$	
LINE 4	21	22	23	24	25	26	t_i	
LINE 5	27	28	29	30	31	32	t_{i+1}	
LINE 6	33	34	35	36	37	38	t_{i+2}	
LINE 7	39	40	41	42	43	44	t_{i+3}	
LINE 8	45	46	47	48	49	50	t_{i+4}	
LINE 9	51	52	53	54	55	56	TIME OF PERIGEE t_p	
LINE 10	57	58	59	60	61	62	MEAN MOTION	
LINE 11	63	64	65	66	67	68	ANGLE OF PERIGEE AT t_p	
LINE 12	69	70	71	72	73	74	PRECESSION RATE OF PERIGEE	
LINE 13	75	76	77	78	79	80	ECCENTRICITY	
LINE 14	81	82	83	84	85	86	SEMI-MAJOR AXIS	
LINE 15	87	88	89	90	91	92	ANGLE OF ASCENDING NODE AT t_p	
LINE 16	93	94	95	96	97	98	PRECESSION RATE OF NODE	
LINE 17	99	100	101	102	103	104	COSINE OF INCLINATION	
LINE 18	105	106	107	108	109	110	LONGITUDE OF GREENWICH AT t_p	
LINE 19	111	112	113	114	115	116	SATELLITE NUMBER	
LINE 20	117	118	119	120	121	122	MESSAGE INJECTION TIME	
LINE 21	123	124	125	126	127	128	SINE OF INCLINATION	
LINE 22	129	130	131	132	133	134	FREQUENCY OFFSET	
LINE 23	135	136	137	138	139	140	INJECTION FLAG	
LINE 24	141	142	143	144	145	146	INJECTION FLAG	
LINE 25	147	148	149	150	151	152	INJECTION FLAG	
LINE 26	153	154	155	156	157	1	2	TIME MARK T_{i+1}

← 4.601016 SECS →

WORD		TIME
1	111000100101110001001000011101101XXXX00	766.836 mSEC
2	XXXXXXXXXXXXXXX01111111111111111111111110	766.836 mSEC
3	400 Hz Beep Tone	766.836 mSEC
4 - 156	39 Bit Words	
157	19 Bit Word	

parameters will be discussed further in section 4.4. Secondly, it enables the recovery of time , absolute time being required for ephemeris interpolation and relative time for Doppler count accumulation. Finally it provides marks for the commencement and completion of count accumulations , these being known as integration gates.

Each message begins and ends at an even two minute epoch of Universal Time. These epochs are referred to as two minute marks. They are immediately preceeded in the message by a twenty-five bit sychronisation word (0111111111111111111111110). Recognition of this word enables a receiver to become synchronised with the satellite , thus permitting the identification of specific message words (Stansell 1978 A).

The constant transmission of the binary bits enables the recovery of time at epochs between the two minute marks (However Vide Section 3.3.2.). In particular it permits the definition of time intervals for the accumulation of Doppler counts. Short Doppler counts are usually gated by the receipt of the last bit in each message line. Consequently the counts may be gated and timed by the same marks. The accumulation period for lines 1 to 25 is -

$$(6103 \times 6 \times 39) / 120 = 4.601016 \text{ Seconds.}$$

In the case of line 26 it is 4.974603 seconds.

Receivers which time their Doppler counts in this manner are said to be operating in the satellite time frame. The use of this time frame is very popular in navigation receivers as it avoids the need for a receiver clock. Consequently the Doppler curve and the modulated message provide all the information that is required to compute a position fix.

However , operating in the satellite time frame has its disadvantages. Unmodelled delays in the propogation of satellite signals through both the atmosphere and the receiver result in the time base provided by the bit pattern becoming irregular. This in turn causes errors in the Doppler counts and a general degradation of the positioning accuracy. Errors from this source are generally not significant in navigation type receivers. However they are significant in geodetic receivers and necessitate the use of a more precise form of time recovery.

Most geodetic receivers (e.g. MX1502, JMR 4) use their own internal clocks to time integration intervals. As such,they are said to operate in the receiver time frame. Some of these receivers still use the modulated bit pattern to provide the integration gates. Others , notably the Canadian Marconi instruments , dispense with the bit pattern altogether and integrate over internally generated time periods. These periods are given by

$$(234 \times 7\ 865\ 000\ ) \ /\ f_g = 4.601025 \text{ Seconds}$$

where f_g is the internally generated 400 MHz. frequency. Note that the 4.601025 figure is nominal and is subject to drifts in f_g.

3.1.3. The Range-Rate Equation

The derivation of the range-rate equation is included in several references. The version reproduced here is from Ashkenazi and Gough (1975 A).

First it is desirable to define the following terms.

f_g - the reference frequency generated by the receiver.

f_s - the frequency transmitted by the satellite.

f_r - the Doppler shifted satellite frequency recovered at the receiver.

t_1, t_2 - epochs of time at the satellite corresponding to the start and finish of a counting period.

$\Delta t_1, \Delta t_2$ - the time delays between the signal leaving the satellite and arriving at the receiver.

r_1, r_2 - the distances between the receiver and the satellite at times t_1 and t_2.

c - the propogation speed of microwaves in vacuo.

As was noted in section 2.2.3. , the integrated Doppler count N is obtained by accumulating cycles of the beat frequency $f_g - f_r$. The accumulation process may be expressed mathematically by the formula -

$$\int_{t_1+\Delta t_1}^{t_2+\Delta t_2} (f_g - f_r)\, dt \qquad (3\text{-}1)$$

Integrating the two terms individually produces -

$$N = f_g(t_2 - t_1) + f_g(\Delta t_2 - \Delta t_1) - \int_{t_1+\Delta t_1}^{t_2+\Delta t_2} f_r\, dt \qquad (3\text{-}2)$$

Now the number of cycles of f_r received between $t_1 + \Delta t_1$ and $t_2 + \Delta t_2$ must equal the number of cycles of f_s transmitted between t_1 and t_2. Therefore -

$$\int_{t_1+\Delta t_1}^{t_2+\Delta t_2} f_r\, dt = \int_{t_1}^{t_2} f_s\, dt = f_s\,(t_2 - t_1) \qquad (3\text{-}3)$$

Substituting (3-3) into (3-2) gives

$$N = f_g(t_2 - t_1) + f_g(\Delta t_2 - \Delta t_1) - f_s(t_2 - t_1)$$

$$= (f_g - f_s)(t_2 - t_1) + f_g(\Delta t_2 - \Delta t_1) \qquad (3-4)$$

Now the time delay due to signal propagation is simply given by -

$$\Delta t = r / c$$

Therefore $\Delta t_2 - \Delta t_1 = (r_2 - r_1) / c$ (3-5)

Substituting (3-5) into (3-4) gives -

$$N = (f_g - f_s)(t_2 - t_1) + f_g\{(r_2 - r_1) / c\} \qquad (3-6)$$

This is regarded as being the basic integrated Doppler count formula for the <u>satellite time frame</u>. Note that all quantities are known except $r_2 - r_1$

The derivation for the receiver time frame is as follows.

Let $\tau_1 = t_1 + \Delta t_1$

and $\tau_2 = t_2 + \Delta t_2$

Then the integration period in the receiver time frame is defined as being $\tau_2 - \tau_1$. The accumulation of the Doppler count (equation (3-1)) can be written as -

$$N = \int_{\tau_1}^{\tau_2} \{(f_g - f_s) - (f_r - f_s)\}\, dt$$

$$= (f_g - f_s)(\tau_2 - \tau_1) - \int_{\tau_1}^{\tau_2} (f_r - f_s)\, dt \qquad (3\text{-}7)$$

But from (3-3)

$$\int_{\tau_1}^{\tau_2} f_r\, dt = f_s\,(t_2 - t_1) \qquad (3\text{-}8)$$

Substituting (3-8) into (3-7) and completing the integration gives

$$N = (f_g - f_s)(\tau_2 - \tau_1) - \{f_s(t_2 - t_1) - f_s(\tau_2 - \tau_1)\}$$

$$= (f_g - f_s)(\tau_2 - \tau_1) + f_s\,(\Delta t_2 - \Delta t_1) \qquad (3\text{-}9)$$

Noting equation (3-5) leads to

$$N = (f_g - f_s)(\tau_2 - \tau_1) + f_s\{(r_2 - r_1) / c\} \qquad (3\text{-}10)$$

This is the range-rate equation in the <u>receiver time</u>

frame. As was the case for the satellite time frame , all quantities are known except $r_2 - r_1$.

Equations (3-6) and (3-10) both assume that all quantities are error free. In reality this is not the case. Consequently the range-rate equation must be modified to include two error terms as follows.

$$N + \varepsilon = (f_g - f_s)(t_2 - t_1) + s + f_g((r_2 - r_1) / c \qquad (3\text{-}11)$$

$$N + \varepsilon = (f_g - f_s)(\tau_2 - \tau_1) + s + f_s((r_2 - r_1) / c \qquad (3\text{-}12)$$

where ε = the random errors of observation

and s = the undetermined systematic errors.

3.1.4. The Guier Plane

It was noted in section 3.1.1. that the Doppler curve enables solution for two geometrical parameters and one nuisance parameter. The geometric parameters lie very close to the Guier plane , this being defined as follows.

> 'The Guier plane is .... the plane which passes through the receiver coordinates (estimated) and contains the velocity vector of the satellite at closest approach.'
>
> (Hatch 1976 A)

The Guier plane forms the x-y plane of a local 3D cartesian coordinate system such that -

a. The origin is at the tracking station.
b. The x axis coincides with the range vector at closest approach.
c. The y axis is parallel to the direction of the velocity vector at closest approach.
d. The z axis completes a right handed triad.

The Guier plane is eminently suitable for the two dimensional processing of Doppler data. The geometric relationship between the orbital plane and the Guier plane is such that acquired Doppler data are largely insensitive to the satellite's z coordinate. Thus out-of plane errors do not significantly contaminate the results. In addition , Guier plane processing enables the separation of systematic positioning errors from the random errors of observation. This situation is summarised in the following version of Guier's theorem.

> 'An adjustment in which the observations are Doppler data from a single pass , and the unknown parameters are the closest approach range and time (and the frequency offset) , or equivalently the coordinates of the satellite at closest approach in the Guier

plane (and the frequency offset), or equivalently the Navigators coordinates in the Guier plane (and the frequency offset), will result in an estimated variance factor which is, to first order, uncontaminated by satellite orbit errors, and a vector expressing the satellite orbit errors resolved into range (x) and along track (y) components'

(Guiers Theorem (D), Wells 1974 A)

Consequently computations in the Guier plane may be used for three purposes, these being -

a. To compute an improved receiver position given the receiver height and the satellite ephemeris.
b. To compute an improved ephemeris (x and y shifts) given the receiver position.
c. To edit the Doppler data using the uncontaminated variance factor.

The processing of Doppler data in the Guier plane involves the use of the range-rate formulae derived in Section 3.1.3. In the Guier plane system, the ranges r_i at time t are given by

$$r_i(t) = [(x_G(t)-x_R)^2 + (y_G(t)-y_R)^2 + (z_G(t)-z_R)^2]^{0.5}$$

where $x_G(t)$, $y_G(t)$, $z_G(t)$ are the satellite coordinates in the Guier plane system. Assume that the satellite coordinates , satellite signal frequency and receiver height are perfectly known. Then given approximate values for x_R , y_R and f_g , refined values may be determined through a parametric least squares adjustment , the observation equations having the general form

$$L = F(X)$$

These are linearised to give

$$A\ X + W = V$$

which in turn produce the normal equations

$$A^T P\ A\ X + A^T P\ W = 0$$

where L is the vector of observations,

A is the design matrix,

P is the inverse weight coefficient matrix,

X is the vector of unknown parameters,

W is the vector of absolute terms.

V is the vector of observational residuals

As a combined frequency offset for both the satellite and receiver has to be recovered , it is convenient to replace f_g and f_s in equations (3-6) and (3-10) by $f_0(1+\Delta f_g)$ and $f_0(1+\Delta f_s)$ where

$$\Delta f_g = (f_g - f_0) / f_0$$

$$\Delta f_s = (f_s - f_0) / f_0$$

f_0 = the nominal frequency

The elements of the design matrix are then determined by partially differentiating the range-rate equations , thus producing the following.

$$\delta F/\delta x_R = f_0(1+\Delta f_g)(\delta r(t_2)/\delta x_R - \delta r(t_1)/\delta x_R) / c$$

$$\delta F/\delta y_R = f_0(1+\Delta f_g)(\delta r(t_2)/\delta y_R - \delta r(t_1)/\delta y_R) / c$$

$$F/\delta \Delta f_g = f_0(t_2 - t_1) + (r(t_2) - r(t_1)) f_0 /c$$

where

$$\delta S(t)/\delta x_R = -(x_G(t) - x_R) / S(t)$$

$$\delta S(t)/\delta y_R = -(y_G(t) - y_R) / S(t)$$

Prior to computation , the satellite and approximate receiver coordinates have to be transformed from a geocentic system into that of the Guier plane. The transformation depends on the determination of the time of closest approach t_{CA} , this being needed to relate the two systems. A Doppler count which is centered about t_{CA} may be expressed mathematically as follows.

$$N_{CA} = (f_g - f_r)\Delta t$$

A theoretical value for N_{CA} may be determined from apriori information. By comparing this value with the observed values , counts N_A and N_B may be identified , these lying either side of the theoretical value. The variation of the Doppler shift with time is very nearly linear about the point of closest approach. As the Doppler counts are usually accumulated over short time periods , it is possible to construct a linear function to determine the time of closest approach from epochs t_A and t_B , these being the centres of integration intervals N_A and N_B. Thus

$$t_{CA} = t_B - (N_B - N_{CA})(t_B - t_A)/(N_B - N_A)$$

The x and y axes of the Guier plane system are then defined in the Geocentric system by the range vector

$$X_G = (X_S(t_{CA}) - X_R)$$

and the satellite velocity vector at closest approach.

$$Y_G = \dot{X}_S(t_{CA})$$

Having defined the orientation of the Guier plane axes , the transformation proceeds in the standard fashion. The origins of the two systems are first bought into coincidence by the translation of the geocentric origin to the tracking station. The transformation is completed by applying a rotation matrix to the terrestrial system such that

$$x_G(t) = R(X(t) - X_R)$$

$$\text{where } R = \begin{bmatrix} U_1^T \\ U_2^T \\ U_3^T \end{bmatrix}$$

Precise formulae for the computation of the row vectors of R are given by Wells (1974 A) and Ashkenazi and Gough (1975 A).

Hatch (1976 A) discusses an alternative to the Guier plane for the editing of Doppler data. Called the plane of least movement , it is determined in such a manner that it minimises the sensitivity of the correction vector to out of plane errors. Hatch et al (1979 B) note that the difference between editing in the Guier plane and editing in the plane of least movement is not very significant as both techniques will detect doubtful counts. However the planes have been found to differ significantly at low altitudes due to increased sensitivity to height. The plane of least movement is used extensivly in modern Magnavox software.

3.1.5. Multi-Station Modelling

The processing of Doppler data in the Guier plane reference system is not without its disadvantages. The technique is restrictive in that it prevents the simultaneous adjustment of both satellite and receiver positions. Consequently , short arc and semi-short arc programs are invariably modelled in terms of geocentric coordinates , the Guier plane being utilised only for data editing.

In its most basic form , a Doppler reduction program solves for the coordinates (X,Y,Z) of the receiver together with a frequency offset term. More advanced programs , in particular the multi-station programs , seek also to remove systematic biases by including other parameters among the unknowns. One such program is GEODOP . This will now be used to illustrate the modelling of a typical multistation Doppler program.

The mathematical model used in GEODOP was described by Kouba and Boal in 1975 (A). The program generates range-rate equations for each Doppler count (28 or 32 second) , these having the general form of (3-11) or (3-12) . The ranges r_1 and r_2 are defined by the formula

$$r_k = ([X_s(t_k)-\bar{X}_g]^2 + [Y_s(t_k)-\bar{Y}_g]^2 + [Z_s(t_k)-Z_g]^2)^{0.5} \qquad (3\text{-}13)$$

where $\bar{X}_g = X_g - \omega_\lambda Y_g$

$\bar{Y}_g = Y_g + \omega_\lambda X_g$

$\omega_\lambda = \omega\ (\ \delta t + r_i\ /\ c\ )$

ω = the mean rotation rate of the earth

δt = the receiver delay

X_g , Y_g , Z_g are the coordinates of the receiver

$X_s(t_k)$, $Y_s(t_k)$, $Z_s(t_k)$ are the coordinates of the satellite at time t_k.

Note that equation (3-13) takes into account the distortion caused to the Doppler curve by earth rotation.

At this point it is worth mentioning that not all programs use range-rate equations as their principle modelling formulae. In particular , the short arc program SAGA III (Brown 1976 A) formulates its solution using range equations.

The range-rate equations (3-11) and (3-12) incorporate a term s_i to describe a systematic error model. In GEODOP this model takes the following form.

$$s_{12} = -\Delta N_{TR}\, dk/100 + [\delta(r_2 - r_1)/\delta t]dt + [\delta(r_2 - r_1)/\delta b]db$$

where dk is a percentage correction to the nominal tropospheric refraction correction,

dt is the correction to the nominal receiver delay plus the synchronisation error,

db represents the vector of orbital biases in the along-track , across-track and out-of-plane directions.

Note that GEODOP is a semi-short arc program , solving only for three orbital bias parameters. Short arc programs solve for six such parameters , although the parameter type varies from program to program. For example , SAGA III is formulated in terms of the satellite's along-track , across-track and out of plane biases plus the three geocentric velocity components $\dot{X},\dot{Y},\dot{Z}$ (Brown 1976 A). GEODOP V on the other hand , works with Keplerian elements (Schenke 1982 C).

Station positions are computed using a least squares technique. The range-rate equations are linearised using a Taylors series which is truncated after the first term. The resulting observation equations take the following form

$$A\,X + C\,Y + W = V$$

where X is the vector of the unknown station coordinates

Y is the vector of the unknown terms in the systematic error model,

A and C are coefficient matrices

W is the misclosure vector

V is the vector of the residuals of observation

The elements of the A and C matrices are obtained by differentiating the range-rate equations with respect to the unknowns. Thus the elements of the n^{th} row of the A matrix pertaining to the i^{th} station in the network have the form

$$a_{n,3i-2} = ([\ X_s(t_2) - x_i\ ]/\ r_2 - [\ X_s(t_1) - x_i\ ]\ /\ r_1)/\lambda$$

$$a_{n,3i-1} = ([\ Y_s(t_2) - y_i\ ]/\ r_2 - [\ Y_s(t_1) - y_i\ ]\ /\ r_1)/\lambda$$

$$a_{n,3i} = ([\ Z_s(t_2) - z_i\ ]/\ r_2 - [\ Z_s(t_1) - z_i\ ]\ /\ r_1)/\lambda$$

where each Doppler count is accumulated between t_1 and t_2.

The elements of the C matrix which relate to the orbital biases are expressed relative to the orbital plane. Consequently the chain rule has to be applied to determine these terms. The relevent expressions are -

$$[c_{n1}, c_{n2}, c_{n3}]$$
$$=([\delta r_k/\delta X_s.\delta X_s/\delta U.\delta U/\delta b]_{t=t2} - [\delta r_j/\delta X_s.\delta X_s/\delta U.\delta U/\delta b]_{t=t1})/\lambda$$
$$=[(X_s(t_2)-X_i)/r_2.R_{XU}(t_2).dU/db - (X_s(t_1)-X_i)/r_1.R_{XU}(t_1).dU/db]/\lambda$$

where $\delta r/\delta X$ represents the Geometic Partials

$\delta X/\delta U$ represents the Rotation to Inertial Partials, these being expessed by the rotation matrix R_{XU}

$\delta U/\delta b$ represent the Variational Partials.

The terms $\delta U/\delta b$ are evaluated using special formulae defined for the TRANSIT system (Vide Section 4.5.).

The remaining three elements in each row of the C matrix are given by

$$c_{n4} = t_2 - t_1$$

$$c_{n5} = -(t_2 - t_1)\ \Delta N_{TR} / 100$$

$$c_{n6} = ([\{X_S(t_2) - X_i\}/r_2].\dot{X}_S(t_2) - [\{X_S(t_1) - X_i\}/r_1].\dot{X}_S(t_1))$$

These relate to the frequency offset , tropospheric refraction and receiver delay corrections respectivly.

Between them the X and Y vectors contribute 3n + m(3 + 3n) unknown parameters to the solution where m is the number of passes and n is the number of stations. In most situations, this is too many to accommodate in a single adjustment. Conseqently , a sequential technique has to applied.

A sequential adjustment as applied to Doppler positioning processes one satellite pass at a time. Following the adjustment of each pass , the influence of it and preceding passes are introduced into the normal equations for the following pass. This is done through the coordinate correction vector X and its variance-covariance matrix G. By proceeding in this manner , the Y vector parameters for each pass are eliminated after the pass has been processed. Thus the parameter set never gets larger than 6n+3 unknowns.

The solution vectors are given by

$$X = -G^{-1}[A^TPW-A^TPC(P_y+C_TPC)^{-1}C^TPW]$$

$$Y = -(P_y+C^TPC)^{-1}(C^TPW+C^TPAX)$$

where

$$G = [(P_x+A^TPA)-A^TPC(P_y+C_TPC)^{-1}C^TPA]$$

P is the inverted apriori weight coefficient matrix of the Doppler counts

P_x and P_y are the inverted apriori weight coefficient matrices corresponding to X and Y respectivly.

The variance-covariance matrix for the coordinate corrections is given by

$$\Sigma_X = \bar{\sigma}_2 N^{-1}$$

where $\bar{\sigma}^2$ is the variance factor.

3.2. Doppler Counts and Correlation

Doppler counts are accumulated on both frequencies to enable corrections to be computed for the effects of the Ionosphere (Vide Section 3.4.1). In most , if not all geodetic receivers , one count terminates and the next begins at the first positive going zero crossing of the beat signal following the receipt of a time mark. As this process prevents the loss of any partial cycles between successive counts , the counts are referred to as Continuously Integrated Dopplers (CID's) (Smith et al 1976 A, Ashkenazi et al 1978 A).

In general,the basic integration interval for modern geodetic receivers is 4.6 seconds (Vide Section 3.1.2.). During the reduction process it is normal to accumulate these short counts into longer counts. Several studies have been made over a period of years to determine the optimum integration interval. One of these was published by Ashkenazi et al in 1978 (A). The report noted that computing costs rose steeply for integration intervals below twenty seconds. It also noted that integration intervals above thirty to forty seconds resulted in an unnecessary loss of data. It was therefore concluded that in general,an interval of twenty to forty seconds should be utilised,although the choice of an interval for a particular task would depend on the accuracy required and the resources available.

There is considerable evidence that this criterion is used in practice. Program PREDOP (Lawnikanis 1976 a) , the preprocessor to GEODOP , accumulates the twenty-six short counts from each two minute interval into groups of six and seven , these lasting approximatly twenty-eight seconds and thirty-two seconds respectivly. Program DOPPLR does likewise with Geoceiver data (Smith et al 1976 A). Recent Magnavox programs accumulate four groups of five counts and one group of six , these lasting approximatly twenty-three seconds and twenty-eight seconds respectivly (Point Positioning and Translocation Program 1979 A).

Continuously integrated Doppler counts are generally accumulated over a two minute period. The cumulative totals at each time mark are later subtracted to obtain the counts for the desired integration periods. While the cumulative totals may be considered to be uncorrelated quantities , the derived differences most certainly cannot and this correlation must be taken into account in the mathematical modelling of any reduction program.

At least three techniques have been developed for accomodating correlation , all of which are mathematically equivalent. The first of these was introduced by Duane C. Brown of DBA Systems in 1970. His technique was to use range instead of range-rate equations in the mathematical model. By determining cumulative Doppler counts for each time mark subsequent to the first , he effectivly defined the range differences between every Doppler point and the lock on point. He then included the range from the receiver to the lock on point as an unknown in the solution. The use of range rather than range-rate equations effectivly removed the correlation problem as it involved the use of independent quantities (cumulative Doppler counts) instead of related quantities (differences between cumulative Doppler counts).

This technique works well if the data set for each pass is complete. However if short Doppler counts are lost or edited out , then the resulting blocks of data have to be considered separately , resulting in the need to solve for additional unknown ranges. Thus the Brown approach is likely to prove unworkable on noisy passes.

The second method has been described as the Kirkham technique by Hatch (1976 A). The method involves the introduction of the full apriori variance-covariance matrix for the correlated Doppler counts into the reduction process. If the independent accumulated Doppler counts at successive time marks are denoted N_i , and the correlated Doppler counts for each integration interval are denoted $N_{i,i+1}$ then their relationship may be expressed by the equation -

$$\begin{bmatrix} N_{1,2} \\ N_{2,3} \\ N_{3,4} \end{bmatrix} = \begin{bmatrix} -1 & +1 & 0 & 0 \\ 0 & -1 & +1 & 0 \\ 0 & 0 & -1 & +1 \end{bmatrix} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix}$$

or

$$\Delta\bar{N} = R\ N$$

The weight coefficient matrix for the vector N may be shown to be -

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

This may be inverted to give -

$$W = 0.25 \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

which is then introduced into the reduction process. Hatch (1976 A) dismissed this technique as being unworkable if the solution involved thirty or more range-rate equations. He considered that the size of the variance-covariance matrix to be inverted would be too large to permit efficient implementation. However Ashkenazi et al (1976 A) noted that the inverted weight coefficient matrix itself had the following general form -

$$
W = 1 / (n+1) \begin{bmatrix} n & n-1 & n-2 & n-3 & \cdots & 1 \\ n-1 & 2(n-1) & 2(n-2) & 2(n-3) & \cdots & 2 \\ n-2 & 2(n-2) & 3(n-2) & 3(n-3) & \cdots & 3 \\ n-3 & 2(n-3) & 3(n-3) & 4(n-3) & \cdots & 4 \\ \cdot & \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & \cdot & & \cdot \\ 1 & 2 & 3 & 4 & & n \end{bmatrix}
$$

where n = the number of cumulative Doppler counts.

As such the elements of the matrix can be generated using -

$$
w_{i,j} = w_{j,i} = (n + 1 - i) \times [j / (n + 1)]
$$

where $w_{i,j}$ is the matrix element in the ith row and jth column of W. This avoids the storage and inversion problem altogether.

These formulae were used to accommodate the correlation problem in the JMR short arc program GP-2S (Brunell et al 1982 C).

The third technique for dealing with correlation was advocated by Hatch (1976 A) and is known as the pseudorange process. This method is very similar to that of

Brown. However it avoids the need to solve for the unknown satellite-receiver distance(s) by subtracting the average of the range equations from each individual range equation. This cancels the unknown nuisance range component while still producing statistically uncorrelated pseudo-measurements. Examples of the technique are given in Hatch (1976 A). The method has been incorporated into the Magnavox program, MAGNET.

3.3. Time Recovery Errors

3.3.1. Receiver Delay and Time Jitter

When a signal arrives at a receivers antenna , two distinct events take place. These are -

1. The mixing of the carrier frequency with the reference signal to produce the beat frequency.
2. The processing and identification of the information modulated onto the carrier signal.

The time delay between signal arrival and the first event is negligable , typically being less than ten microseconds. However the delay between the first and second events may be considerably longer due to the retarding effects of the receiver circuitry.

This second delay may be considered as consisting of a systematic component and a random component. The systematic component is known as the receiver delay. Its value for each instrument is determined by the manufacturer from a series of time delay measurements (Brunell 1979 B). Values ranging between 500 and 1500 microseconds are often quoted for most geodetic receivers. However one instrument , the Magnavox MX1502 , is claimed to have a negligable delay , this having been bought about by improvements to the time recovery loop and by passing both signal components through the same circuitry (Kouba and Wells 1976 A , Brunell 1979 B , Hatch 1982 C). The random component is known as time jitter and may reach magnitudes of up to 100 microseconds depending on the receiver used.

Doppler receivers in general do not exhibit a direct sensitivity to receiver delay. Doppler counts are related to time intervals rather than absolute time values. Thus a constant systematic error in the timing marks over a counting interval will not adversly affect the data recovered. Receiver delay however does have an indirect effect. In order for Doppler data to be reduced , it is necessary to know the position of the satellite at each instant of gating. As the detection of the modulated timing marks is being delayed by the receiver , the satellite's position at the start and finish of each Doppler count will always be further along-track than the ephemeris suggests.

The magnitude of the errors from this source may be gauged by the fact that a satellite will travel approximatly six metres every millisecond. Such errors will principally affect the latitude determinations.

Timing jitter may be considered to comprise of two components. The first is the truly random component which occurs in any measuring system. The second reflects the fact that the receiver delay is not constant (as is often assumed) but is subject to variations dependent on temperature and signal strength. As signal strength varies during a satellite pass in response to antenna gain characteristics , it is apparent that the receiver delay will change during each pass as well as from pass to pass.

Time jitter causes the same indirect effect that was noted for time delay. Thus a jitter of fifty microseconds will result in a satellite position error of thirty centimetres. However on this occasion there is also a direct effect as the jitter at each time mark is not constant. The magnitude of jitter-induced errors may be estimated by examining the basic Doppler count formulae (Equations (3-11) or (3-12) depending on the time frame). It is apparent that any errors in the time interval term t_2-t_1 will be amplified by the 32KHz. frequency offset (Hatch 1982 C). Thus a fifty microsecond time error due to jitter will produce a 1.6 cycle error in the accumulated count. As the wavelength of each cycle is approximatly 75 centimetres , this translates into a 120 centimetre error in the range difference. This is four times the magnitude of the indirect effect and principally affects the height and longitude determinations.

Time delay and time jitter errors will adversly influence the results from both point positioning and relative positioning solutions. In the point positioning mode the effects may be reduced by observing a large number of passes to randomise jitter errors , and by balancing north and south going satellites to cancel receiver delay errors. In the relative positioning mode,the effects of time delay will cancel completely if all receivers have exactly the same bias. Generally this will not be the case. In addition, the different time jitter values at each receiver will produce random errors (known as noise) which will be significant at the level of accuracy achievable from relative positioning. Consequently it is desirable that both the direct and indirect effects of timing errors be reduced to a minimum , either by apriori correction or through inclusion in the solution as unknown parameters.

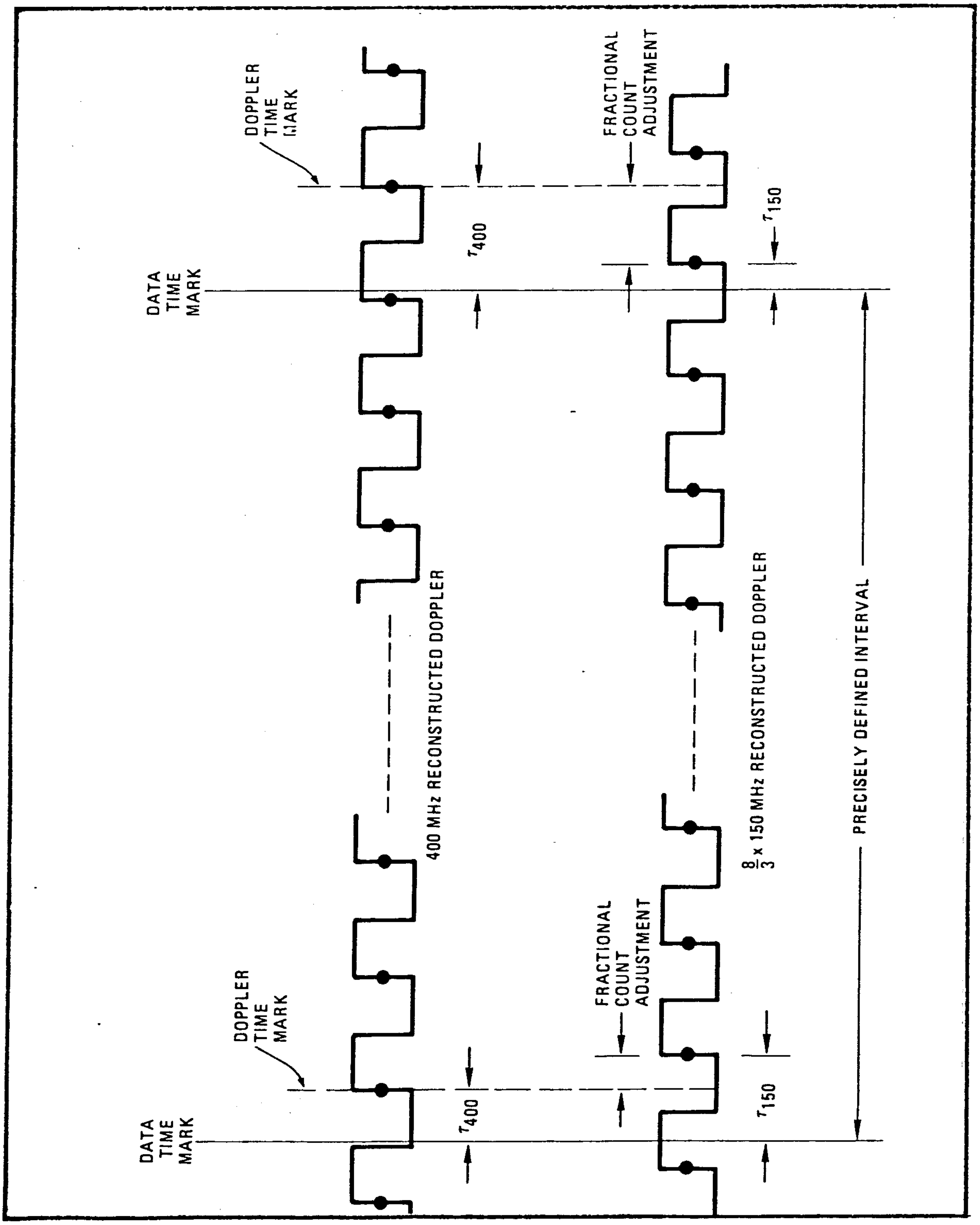
It is apparent from equation (3-11) and (3-12) that the effects of time jitter could be attenuated by reducing the 32 KHz. offset between the satellite and reference frequencies (Hatch 1982 C). However system design considerations make this undesirable. Instead , geodetic receivers minimise the effects of undetermined jitter by operating in the receiver time frame and using the local clock to precisely time the counting intervals.

As was noted in Section 3.1.2. , the exact mechanism for defining integration intervals varies from manufacturer to manufacturer. Canadian Marconi dispenses with the modulated timing marks altogether and uses the internal clock to regulate the the integration intervals. This virtually overcomes the jitter problem altogether as the interpretation of externally generated integration gates is no longer necessary. On the other hand , both JMR and Magnavox continue to use the modulated bit pattern to define the integration periods. However in these cases the internal clock is used to measure the time interval between the interpreted time marks , thus including jitter in the determined period. The Magnavox MX1502 accumulates 9.6 microsecond clock pulses over the integration interval, these being applied as corrections to a coarse estimate of the interval at the time of post-processing.

A further source of error in the Doppler count may also be removed by means of receiver timing. It was noted in Section 3.2. that integrated Doppler counts start and finish at positive going crossings of the beat frequency. (This of course implies that observed Doppler counts are integer quantities.) These crossings rarely coincide with the occurance of time marks. Consequently the start/finish of each count will occur after the detection of the corresponding integration gate (Vide Diagram 5). The resulting timing errors in the Doppler counts translate into

DIAGRAM 5
PARTIAL CYCLE CORRECTION

(STANSELL 1979 B)



a twenty-five centimetre RMS error in the range-rate measurements.

Canadian Marconi overcame this problem by multiplying the beat frequency by a factor of one hundred. As a result the errors due to the partial cycle were reduced by two orders of magnitude. Magnavox and JMR on the other hand used the internal clock to measure the delay between the time mark and the zero crossing. These intervals were then applied as corrections during post-processing.

While use of the receiver time frame overcomes the direct effects of time jitter , it has no such influence on the indirect effects. The satellite positions as defined by the ephemerides still relate to the time marks at the instant of transmission. The epochs of time which define the Doppler counts differ from the transmission epochs because of propagation delays , jitter and the partial cycle corrections. It is necessary that the satellite positions be adjusted to reflect the new gating epochs.

For Canadian Marconi data this is a complex process , an iterative technique being required to recover the corrected ephemeris values. The problem is considerably simpler for instruments which derive their timing marks from the modulated signal. This tie with the satellite time frame makes it possible to compute and apply differential corrections to the satellite positions as defined by the broadcast marks. Alternativly,the time intervals associated with the Doppler counts may be mapped into the satellite time frame , equation (3-11) then being used to model the solution (Hatch et al 1979 B). Such techniques will account for errors caused by the indirect effects of both time delay and time jitter.

One further error source should be considered at this point. An apparent additional receiver delay will occur if the receiver clock is not correctly set to absolute time. In several post-processing programs , including GEODOP , the required sychronisation is achieved through the two minute timing mark at the instant of lock-on. However this mark will invariably contain an error due to time jitter , thus introducing a synchronisation error into the reductions. The effect of this is to cause latitude biases , particularly if a multi-station data set has been gathered using the receivers of different manufacturers (Boal and Vamosi 1981 A).

The problem may be overcome by including synchronisation as an unknown parameter in the solution. Its value may then be determined from all timing marks instead of only the first , resulting in a significant improvment in time recovery. This procedure has already been adopted by Magnavox (Hatch 1982 C) and is likely to be included in PREDOP/GEODOP (Boal and Vamosi 1981 A).

3.3.2. Satellite Induced Time Errors

In order to counter the effects of oscillator drift on the stability of the OSCAR satellite time transmissions , provision was made in the original design of the TRANSIT system for the inclusion of 9.6 microsecond time-delay steps. These were incorporated by designating the thirty-ninth bit of each message word to be a time correction bit. If one of these bits is set , transmission of the next bit is delayed by 9.6 microseconds. Thus the effects of oscillator frequency variations on the precision of the time interval between the two minute time marks may be compensated by activating sufficient of these bits.

On average , fifty-five time correction bits are set for each two minute time period. Unfortunately these bits adversly affect Doppler positioning as their distribution is not uniform throughout the broadcast message. It can be seen from Diagram 6 that several words are not permitted to contain time correction bits. In particular they are prohibited in the interval between word 135 and word 3. (Data injection takes place during the period between words 135 and 156. Time corrections included here would be lost during the injection causing considerable difficulty) (Brunell 1979 B)

DIAGRAM 6
PROHIBITED MESSAGE WORDS

(BRUNELL 1979 B)

LINE NUMBER	GROUP A					GROUP B	
1	BEEPER	4	5	6	7	T-6	VARIABLE DATA
2	9	10	11	12	13	T-4	
3	15	16	17	18	19	T-2	
4	21	22	23	24	25	T	
5	27	28	29	30	31	T+2	
6	33	34	35	36	37	T+4	
7	39	40	41	42	43	T+6	
8	45	46	47	48	49	T+8	
9	51	52	53	54	55	t_p	FIXED DATA
10	57	58	59	60	61	η	
11	63	64	65	66	67	ω_o	
12	69	70	71	72	73	$\dot{\omega}$	
13	75	76	77	78	79	ϵ	
14	81	82	83	84	85	A_o	
15	87	88	89	90	91	Ω_o	
16	93	94	95	96	97	$\dot{\Omega}$	
17	99	100	101	102	103	C_i	
18	105	106	107	108	109	Λ_G	
19	111	112	113	114	115	LAST INJ	
20	117	118	119	120	121	SAT ID	
21	123	124	125	126	127	S_i	
22	129	130	131	132	133	$\Delta\gamma_S$	
23	135	136	137	138	139	ZERO	INJECTION
24	141	142	143	144	145	ZERO	
25	147	148	149	150	151	ZERO	
26	153	154	155	156	+19	1 | 2	TIMING MARK

TIME CORRECTION BITS NOT ALLOWED IN SHADED WORDS

NOTE - EPHEMERIS DATA OCCURS ONLY IN GROUP B

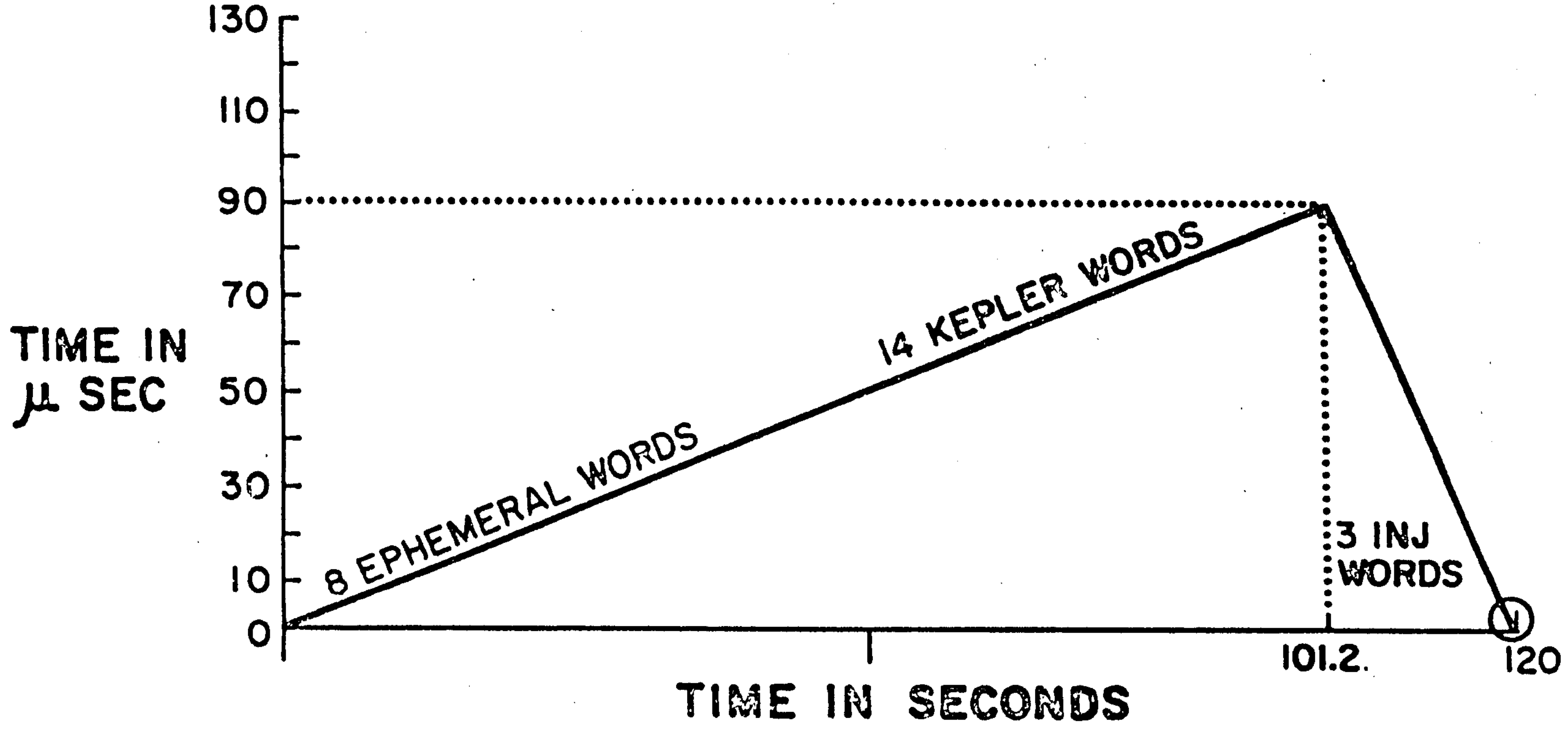
As a result of the non-availability of the prohibited blocks , all correction bits have to be set in the interval between words 4 and 133. Consequently the effect of the frequency drift on timing is progressivly overcompensated in the interval up to word 133 , the overcompensation then diminishing to the two minute mark.

Overcompensation can commonly exceed 100 microseconds and has been known to reach 120 to 130 microseconds (Vide Diagram 7). Consequently , if one is operating in the satellite time frame , the 4.6 second accumulation periods will be too long from lines 1 to 22 (by 5 to 6 μsecs.) and too short from lines 23 to 26 (by 20 to 30 μsecs.) This will result in increased noise and a time bias in the reduced Doppler data , the average bias being approximatly thirty microseconds (twenty centimetres) (Hatch 1982 C).

The effects of satellite time errors may be avoided by working in the receiver time frame. Alternativly , corrections may be applied to the observations in the satellite time frame. Information notifying the receiver of the existance of the time steps is included in the broadcast message. At least one program , MAGNET , uses this information (together with the transmitted satellite frequency parameter) to compute and remove the satellite time errors.

DIAGRAM 7
TIMING GRADIENT

(BRUNELL 1979 B)



3.4. Atmospheric Effects

3.4.1. Ionospheric Refraction

The Ionosphere forms a layer extending from fifty kilometers to four hundred kilometers above the surface of the earth. It consists of positively and negatively charged particles , these having been ionised by ultra-violet radiation from the Sun. Its effect on transmitted satellite signals is to cause their wavelengths to stretch in a manner which is almost inversly proportional to the signal frequencies. By considering the effect of the ionosphere on two radio frequencies , it is possible to determine an appropriate correction (Stansell 1978 A). It is for this reason that the TRANSIT satellites broadcast both the 150MHz. and 400MHz. signals.

The correction to the observed Doppler count N_{OBS} which gives the Doppler count in vacuo is given by

$$\Delta N = N_{VAC} - N_{OBS} = a_1/f_g + a_2/f_g^2 + \ldots.$$

where a_1 , a_2 ..... are coefficients (independent of frequency)

and f_g is the reference frequency

Ignoring second and higher order terms,corrections to the observed Doppler counts N_{OBS}^{400} and N_{OBS}^{150} are given by

$$N_{VAC}^{400} - N_{OBS}^{400} = a_1/f_{400}$$

$$(3/8)(N_{VAC}^{400}) - N_{OBS}^{150} = a_1/f_{150} = (8/3)a_1/ f_{400})$$

Combining the above equations produces the desired expression for the 400 MHz. count in vacuo.

$$N_{VAC}^{400} = (24/55)((8/3) N_{OBS}^{400} - N_{OBS}^{150})$$

It is estimated that the error in the ionospheric correction due to the neglect of higher order terms is less than 1% (Ashkenazi and Gough 1975 A).

3.4.2. Tropospheric Refraction

The Troposphere extends to an altitude of fifty kilometers above the earth. Unfortunately its effects on satellite transmissions do not have the same mathematical simplicity as those of the Ionosphere. The model is complicated by the nonhomogenity of the atmosphere and its constituents , as well as by the dispersive nature of the waves.

Tropospheric correction models usually depend on the measurement of atmospheric pressure,dry bulb temperature and wet bulb temperature at the receiver station. A popular atmospheric model is that attributed to Hopfield. However it suffers from the disadvantage that its equations produce high roundoff errors if integrated in their closed form. Yionoulis overcame this problem by deriving two series expressions for the integrand , one being suitable for high elevation passes and the other for low passes. In practice however , a simplified Hopfield model is employed , this being described by the equations

$$\Delta r = \Delta r_d + \Delta r_w$$

$$\Delta r_d = k_d/\mathrm{Sin}(E^2 + \theta_d)^{0.5}$$

$$\Delta r_w = k_w/\mathrm{Sin}(E^2 + \theta_w)^{0.5}$$

where Δr = the range correction

E = the elevation

θ_d = 2.5°

θ_w = 1.5°

$k_d = 77.6\ P_0(h_d - h_0)10^{-6}/(5\ T_0)$

$k_w = 77.6\ (\ 4810\ )\ e\ (h_w - h_0)\ 10^{-6}/(5\ T_0^2)$

$h_d = 40136 + 148.72\ (\ T_0 - 273.16)$

$h_w = 11000$

T_0 = Temperature at receiver station

P_0 = Atmospheric Pressure at receiver station

e = Vapour Pressure at receiver station

h_0 = Orthometric height of receiver station

(Ashkenazi and Gough 1975 A)

The correction to an integrated Doppler count $\bar{N}_{12}$ is then obtained from the following relationship.

$$\bar{N}_{12} = N_{12} [\Delta r(E_1) - \Delta r(E_2)] / \lambda$$

where λ = the carrier wavelength in vacuo

and E_1 , E_2 are the elevations at the epochs of gating.

(Kouba and Boal 1975 A)

A standardised model for the atmosphere may be used in place of the surface meteorological values indicated above. Such models typically adjust the atmospheric pressure in accordance with station elevation.

CHAPTER 4

REFERENCE SYSTEMS AND ORBIT RECOVERY

4.1. The Ephemerides

An ephemeris is a table of astronomic positions. When used in conjuction with the TRANSIT system , an ephemeris may refer to predicted or observed satellite positions , as both types are encountered.

The precise ephemeris is computed by the DMAH/TC. It is post fitted to data observed by at least twenty tracking stations (TRANET stations) distributed globally. Inertial coordinates (X,Y,Z) and velocity components are computed at one minute intervals of Universal Time , these having component resolutions of 1 metre and 1 mm./sec. respectivly. Usually the precise ephemeris is only available for one or two satellites.

The broadcast ephemeris is computed by the NAG. It is a predicted ephemeris and is available in real time at two minute intervals for all satellites. It is computed using data from four tracking stations (OPNET stations) , all of which are in the United States. The ephemeris is transmitted as a set of fixed and variable orbit parameters , the resolutions of which are 0.0001 degrees (13 Metres) in the along track component and 10 metres in both the radial and out of plane components (Anderle 1976a A , Kouba and Wells 1976 A).

The ephemerides represent two different estimates of satellite position in two different Earth centered coordinate systems. For this reason , receiver coordinates are frequently referred to as being in either the broadcast datum or the precise datum. The intention has been to maintain these datums so that they remain as close to each other as possible. Indeed Jenkins and Leroy (1979 B) undertook a study , the aim of which was to make the systems identical to within one metre. However, at the present time , the datums remain distinctly different (Kumar 1982 C).

Both ephemerides are computed to similar degrees of accuracy. However their solutions are not identical due to differences in the generating programs. Furthermore the broadcast ephemeris is prone to corruption by the action of unmodelled drag forces. For this reason the repeatability from the broadcast ephemeris will never be as good as that from the precise ephemeris (Jenkins and Leroy 1979 B).

In the following subsections,various aspects of the ephemerides are considered. Section 4.2. examines ephemeris computation , focusing particularly on coordinate systems , gravity models and the mathematical solution. Section 4.3.

examines ephemeris accuracy as described by apriori orbital biases. It also considers the effects of pass balancing and compares further the broadcast and precise systems. In Section 4.4. the transmission of the ephemerides is considered , this principally being concerned with the broadcast ephemeris orbital parameters. Section 4.5 examines the recovery of satellite orbits from ephemeris data and Section 4.6. considers transformations between geocentric coordinate systems.

4.2. Ephemeris Computation

4.2.1. Gravity Models

Mathematical models of the earths gravity field are formulated in terms of spherical harmonics. Typically the coefficients of these harmonics are determined as part of a general geodetic solution. The accuracy of the models has improved dramatically during the life of the TRANSIT system due to the inclusion of vast quantities of satellite data in the general solutions. This has resulted in corresponding improvments to the quality of both ephemeris and positioning computations.

A major improvement in the accuracy of orbit determination occurred in 1968 with the introduction of APL4.5 geodesy. This was a spherical harmonic model which included terms through to J_{15}^{15}. A further improvement occurred with the introduction of the WGS72 gravity model in 1975. This contained terms through to J_{19}^{19} plus higher order resonance terms (Hoskins 1982 C). WGS72 was determined from a general solution involving over one thousand parameters , approximatly four hundred of which described the gravity field (Anderle 1976a A).

The NAG broadcast ephemeris has been generated using both of the above mentioned coefficient sets , WGS72 superceding APL4.5 in December 1975 (Holland et al 1977 A). The precise ephemeris however has been computed using APL4.5 and WGS72 'derivatives' , the most recent of which are listed in Table 1 (Anderle 1976a A , Kumar 1982 C).

The gravity models currently in use are WGS72 for the broadcast ephemeris (Hoar 1982a A) and NSWC 10E-1 for the precise ephemeris (Kouba 1983 A). The differences between the models are not great. The WGS72 and NSWC 10E-1 coefficients were both determined using the same Doppler normal equations. However the WGS72 solution included gravimetric data which was omitted from NSWC 10E-1. The difference between the two models only becomes significant in the twenty-eighth order resonance term. It is however geographically correlated. Ephemerides computed using the two models will differ by up to one metre (Jenkins and Leroy 1979 B).

TABLE 1

PRECISE EPHEMERIS GRAVITY MODELS

Date of Revision	Gravity Field
20 February 1967	NWL 8E
18 April 1968	NWL 8H
13 February 1970	NWL 9B
2 January 1973	NWL 10E
15 June 1977	NWL 10E-1

Information for this table was taken directly from Anderle (1976a A) and Kumar (1982 C)

TABLE 2

PRECISE EPHEMERIS COORDINATE SYSTEMS

Date of Revision	Coordinate System
20 February 1967	NWL 8E
19 January 1968	NWL 8F
20 December 1970	NWL 9C
18 October 1971	NWL 9D
15 June 1977	NSWC 9Z-2

Information for this Table was taken directly from Andele (1976a A) and Kumar (1982 C)

Recent experiments involving the NOVA 1 satellite have indicated that its ephemeris could be significantly improved by modifying the odd zonal coefficients and the twenty-sixth degree and order resonant coefficient (Malyevac and Anderle 1982 C). It is conceivable that further improvments to the gravity models will eventuate from continued experimentation with this satellite.

4.2.2. Coordinate Systems

A list of the coordinate systems that have been associated with the precise ephemeris since 1967 are provided in Table 2 (Anderle 1976a A , Kumar 1982 C). In general the adoption of each new system has been precipitated by a change to the gravity model. Anderle (1976a A) describes the changes that have occurred since 1967.

The NWL 8E coordinates were determined during a general geodetic solution. This solution utilised data acquired from seven satellites over a period of more than a year. The NWL 8F coordinates were obtained by transforming the NWL 8E coordinates from the mean pole of 1966.7 to the CIO pole. The NWL 9C coordinates were again found from a general solution , this time using data from a forty day period in 1970. The solution held the pole fixed at preliminary BIH values and produced coordinates which were about three metres different from NWL 8F values. The NWL 9D values altered the heights of three of the NWL 8F stations (Anderle 1976a A).

A comparison of interstation distances derived from NWL 9D coordinates , terrestrial measurements and other extra-terrestrial techniques indicated that the scale of the NWL 9D system was 1 ppm long. The reasons for this could not be determined at the time. To correct the scale anomoly and to make longitude values consistent with gravimetric data in North America ,the NWL 10F system was introduced. This was related to NWL 9D by the following transformations.

$$\lambda_{10F} = \lambda_{9D} + 0.26''$$

$$\phi_{10F} = \phi_{9D}$$

$$H_{10F} = H_{9D} - 5.27m$$

(Anderle 1976b A)

At this point , confusion often arises. The <u>spatial</u> coordinates (X,Y,Z) described by the NWL 10F values are consistent with the WGS72 coordinate system. This has often been misinterpreted as meaning that ϕ_{10F} , λ_{10F} , H_{10F} are coordinates on the WGS72 ellipsoid. This is not the case. NWL 10F coordinates are simply NWL 9D coordinates which have been rotated in longitude and which have had their heights adjusted by -5.27 metres <u>at all points</u> to effect a scale change. When converting NWL 10F coordinates to the X,Y,Z system , NWL 9D ellipsoidal parameters must be used. If the WGS72 parameters are used , a bias of approximatly ten metres will be introduced into the spheroidal heights. Note however NWL 9D (and thus NWL 10F) coordinates may be transformed to the WGS72 figure using Seppelins formulae.

$$\phi_W = \phi_{9D} - 0.0232 \text{ Sin}2\phi$$

$$\lambda_W = \lambda_{9D} + 0.26'' \qquad (\lambda \text{ +ve East})$$

$$H_W = H_{9D} + 4.73 - 0.717 \text{ Sin}^2\phi$$

(Meade 1982 C)

As a post-script to the above , Hothem (1979 B) reported that a 0.6 ppm scale error had been introduced into the distance comparisons by a height error of 4 metres at a key station. He considered that the other 0.4 ppm. was caused by an outdated GM value in the processing software , and by a 0.7 metre offset between the centre of mass of the satellite and the phase centre of its antenna.

Anderle (1976c A) forshadowed that the NWL 9D coordinate system would shortly be revised to NWL 9Z1. The purpose of this revision was to remove discrepancies in the relative poitions of the TRANET observing stations , these having been detected over a twelve year period. The NWL 9Z1 system was apparently never introduced for general use. However in 1977 , the NSWC 9Z-2 system came into being,this having been determined from a solution in which the Love numbers were changed from 0.26 to 0.28 .

At the present time , the precise ephemeris is produced in the NSWC 9Z-2 system (Kumar 1982 C). This system has an ellipsoid associated with it which has the same parameters as that associated with NWL 9D (a=6378145.0 1/f=298.25) (Hoar 1982a A).

The history of the broadcast coordinate system prior to 1975 is not readily available. However Jenkins and Leroy (1979 B) describe its developement since that time. Following the introduction of the WGS72 gravity model in 1975 , evidence of a continuing scale bias between Doppler and astronomical observations remained apparent. At the request of the Applied Physics Laboratory of the John Hopkins University , Anderle determined a set of coordinates for the OPNET stations which were the optimum values for Doppler tracking in conjunction with the new gravity model. These coordinates became known as the NWL 10D system. They were geocentric positions uncorrected for any scale bias and containing no transformation to the WGS72 spatial system. Prior to adoption , these coordinates were rotated by $5x10^{-7}$ radians in longitude to preserve the longitude origin of the previous broadcast ephemeris system. The resulting system became known as 'modified' NWL 10D. Apart from a small longitude bias,its values were close to those of the NWL 9D system. The broadcast ephemeris coordinate system is currently defined as being 'modified' NWL 10D (Kumar 1982 C). It should be noted that although these coordinates are used in conjunction with the WGS72 geopotential model,they are not coordinates on the WGS ellipsoid (Jenkins and Leroy 1979 B).

4.2.3. The Equations of Motion

The NAG broadcast ephemeris and the DMAH/TC precise ephemeris are computed using programs OIP-II and CELEST respectivly. The two programs are close to being identical as regards their mathematical modelling. However , they differ significantly in some important respects and consequently produce different solutions (Jenkins and Leroy 1979 B).

The force models in both programs have the following general form.

$$A_S = A_{CB} + A_H + A_M + A_S + A_D + A_{SR} \qquad (4\text{-}1)$$

where A_S = the sum of the accelerations acting on a satellite

A_{CB} = accelerations due to the central body ,

A_H = accelerations due to gravity harmonics ,

A_M = accelerations due to the gravitational attraction of the Moon,

A_S = accelerations due to the gravitational attraction of the Sun,

A_D = accelerations due to atmospheric drag,

A_{SR} = accelerations due to solar radiation pressure.

Equation (4-1) represents the equations of motion of a satellite.

A_{CS} and A_H describe the effects of the earths gravity field. These have already been discussed in Section 4.2.1.

The Sun and the Moon affect a satellite's trajectory in two ways. First of course , there are the direct attractions between the bodies and the satellite. These are small but significant and must be taken into account. Secondly , there is a diurnal periodic distortion of the earth's gravity field due to the occurance of earth tides. This causes sympathetic perturbations in a satellite's orbit which have to be modelled into the solution.

The effects of earth tides are modelled in terms of Love numbers. In 1976 , the CELEST program formulated its solution using a Love number of 0.26 , this having been determined in a general geodetic solution (Anderle 1976a A). At the time it was thought that this value was too low due to the neglect of ocean tides. As the NSWC 9Z-2 coordinate system was derived using a Love number of 0.28 , it is possible that the value used in CELEST has now been revised.

The atmospheric drag model in CELEST describes the atmospheric density as varying exponentially with altitude. Scale factors for drag are normally determined as a parameter in each ephemeris determination.

Solar radiation pressure was determined in the cone angle formed by the Sun and the Earth. The scale factor for solar radiation was determined simultaneously with the Love number in a general geodetic solution.

Atmospheric drag and solar radiation pressure are the most difficult parameters to model as they are both inherently erratic in nature. The NOVA-1 satellite was designed to reduce these adverse influences by detecting and compensating for along-track drag forces. Consequently , the equations of motion for the NOVA satellite have to modelled differently to those for the OSCAR satellites.

Zeigler (1982 C) reported on the generation of the precise ephemeris for NOVA-1. He stated that the compensation for atmospheric drag was taken into account by constraining the drag model coefficient to zero in CELEST. Similarly the compensation for solar radiation pressure was accommodated by eliminating the pressure model altogether.

The NOVA-1 ephemeris that was subsequently generated produced results which were significantly better than corresponding results from an OSCAR satellite.

Zeigler noted that the NOVA solar radiation model needed to be developed to reflect the fact that compensation was only occuring in the along-track direction. Experiments by Malyevac and Anderle (1982 C) to determine an appropriate force model for NOVA-1 included this model in the solution.

4.2.4. Mathematical Solution

Equation (4-1) models the accelerations which act on an orbiting satellite. It can be considered to be the generic form of three such equations , expressing the components of the accelerations in the x , y and z directions. These equations are second order differential equations in each of the satellite's coordinates. By integrating them with respect to time , the trajectory of the satellite may be computed.

Both numerical and analytical techniques are available for solving the equations of motion. While analytical techniques have some desirable attributes as regards the theoretical analysis of an orbit , the numerical techniques are considerably quicker and more accurate.

Consequently it is the latter group which are used for the routine production of ephemeris data.

A popular numerical technique for integrating the equations of motion is that attributed to Cowell. This method draws much of its popularity from the fact that it is simple and is able to handle highly perturbed and eccentric orbits. Its main disadvantage is that it requires a small step size due to the inclusion of the central body term. Enche's method overcomes this problem by omitting the term and introducing a reference orbit in it's place. However Enche's method tends to lose accuracy when used in conjunction with near-earth satellites , due to the rapid departure of the actual orbit from that of the reference model. Consequently the Cowell approach remains popular for near earth satellites.

Program CELEST uses a tenth order Cowell technique to integrate both the equation of motion and the variational partials. (The variational partials are first partial derivatives describing the rate of change of satellite observations with respect to satellite position. They are used in the formation of observation equations (Vide Section 3.1.5.)). The integration is carried out in an Earth-centered inertial coordinate frame defined by the mean equator and the equinox of 1950. As the tracking data is determined in an earth-centered fixed frame,the tracking station coordinates must first be transformed into the inertial frame before residuals of observation can be formed. This transformation involves rotations due to precession , nutation and hour angle , plus a correction from the CIO pole to the instantaneous pole (Anderle 1976a A).

Program CELEST solves for the following parameters.

6 Constants of Integration.
1 Drag Scaling Factor.
2 Components of Polar Motion.
1 Frequency Bias for each satellite pass.
1 Refraction Bias each satellite pass.
3 Components of station position for any station for which precise coordinates have not been determined.

Solutions for other parameters , including the solar radiation constant , may be specified if desired (Anderle 1976a A).

Over one hundred good passes are used in each orbit fit for the precise ephemeris (Anderle 1976a A). The variances associated with the observations in each pass are determined during filtering. Apriori data such as the frequency bias , orbit constants , the drag scaling factor and the positions of new tracking stations are given very high variances so that they are essentially unconstrained. The scale factor for the tropospheric correction parameter assumes a nominal value of one but is assigned an apriori variance which corresponds to a 10% uncertainty in refraction.

4.3. Ephemeris Accuracy

4.3.1. Apriori Biases

The accuracy of the broadcast ephemeris was investigated by Wells in 1974 (A). Three different techniques were used , these being

1. A comparison with the precise ephemeris
2. A comparison of fresh and stale broadcast ephemeris data.
3. A statistical analysis of 2877 pairs of Guier plane coordinates obtained at eight stations.

The first technique produced results which indicated that the broadcast and precise ephemerides were closely parallel. This finding has considerable significance in that it justifies the assumption used in semi-short arc reduction programs that the ephemeris is only translated from the correct values.

The comparison of fresh and stale broadcast ephemeris data , together with the broadcast/precise comparison , produced the conclusion that drag and gravity modelling errors contributed fourteen metres and seventeen metres respectively to the uncertainty in the broadcast ephemeris. It was further concluded that the rounding off of the transmitted parameters introduced a further six metres of uncertainty into the ephemeris.

The Guier plane testing provided accuracy results which supported the fresh/stale comparisons more than the broadcast/precise comparison.

From these investigations , Wells concluded that the errors in the satellite positions as described by the broadcast ephemeris were consistent with the following standard deviations.

Along Track	26 Metres
Across Track	5 Metres
Out of Plane	10 Metres

These values have endured remarkably well over the years as apriori broadcast ephemeris constraints in semi-short arc reduction programs. This is a little surprising in some respects as the introduction of WGS72 geodesy in 1975 reduced the gravity model contribution to the ephemeris uncertainty. Indeed Brown (1976 A) , on the basis of results obtained using short arc program SAGA III , suggested that the along track value be revised downwards to between 15 and 20 metres. [Brown used apriori constraints of 25,8,17 metres and 0.02 metres/second for the three velocity components]. Recently however , gains made through the adoption of the WGS72 model have been more than neutralised by the onset of the peak period of a thirteen year solar radiation cycle. Boal and Vamosi (1981 A) had to multiply the Wells values by three to accomodate the large ephemeris errors induced by solar radiation pressure.

The accuracy of the precise ephemeris has generally been assessed through the results of Guier plane editing. The precision of the two components of satellite position available from a single pass is better than seventy-five centimetres if the satellite is greater than thirty degrees above the horizon (Anderle 1976a A). Such results however are misleading as they only consider internal random errors and ignore completely external error sources. In order to obtain a realistic accuracy estimate , a statistical analysis of a considerable number of passes , such as that under taken by Wells , has to be performed.

Anderle (1976a A) produced results which indicated that the actual residuals resulting from the use of the precise ephemeris were approximatly 2.5 metres in slant range and 1.6 metres along-track for satellite elevations above thirty degrees. Consequently , he concluded that the RMS periodic error for the precise ephemeris should be set at two metres in each component.

Kouba and Wells in 1976 (A) initially used apriori standard deviations of 3 , 1.5 , 3 metres in their precise ephemeris computations. These were later revised to 1.5 , 0.6 , 1.5 metres following an anlysis of orbital biases.

Zeigler (1982 C) analysed passes obtained over a 142 day period from the NOVA 1 satellite and OSCAR satellite 30190. Both satellites produced the same results. The slant range component varied between 1.5 and 2.0 metres RMS while the along track component varied between 2.0 and 2.5 metres. Peak errors were noted to occur during periods of high solar activity.

It can be seen that there is a good deal of consistency in the assessments of the accuracy of the precise ephemeris. Boal and Vamosi (1981 A) used orbital constraints of 2 , 1 , 2 metres during their experiments in Canada. These values fall neatly into the middle of the range suggested by other authorities. For this reason they have been adopted for precise ephemeris computations during the course of this project.

4.3.2. Balancing of Passes

The largest errors in the broadcast ephemeris occur in the along track direction. They are principly caused by atmospheric drag and solar radiation pressure. The Solar radiation component can be very unpredictable and has been known to attain exceptionally large values in recent years (Boal and Vamosi 1981 A , Allman 1983b A).

The accepted procedure for minimising the effects of along track errors is to balance the number of north and south going passes. However Jenkins and Leroy (1979 B) point out that this will only work well under certain conditions.

The broadcast ephemeris is generated by a forward extrapolation of an orbit fitted to 1.5 days of satellite observations. Because it is computed on a daily basis , the ephemeris will age twelve hours between the observation of a satellite's north and south going geometries. As it is probable that the along track error will grow monotonically in the intervening period , it is clear that the resulting effects will not cancel for the north and south going passes of a single satellite.

This situation could be considerably improved if the broadcast ephemeris were recomputed at twelve hour intervals instead of twenty four hour intervals. Under such circumstances , the ages of the north and south going ephemerides would be approximatly the same and thus the errors would be more likely to cancel. The NAG has been known to undertake twice-per-day orbit predictions during periods of high solar activity.

Despite the above mentioned short-coming , the balancing of north and south going passes is the most practical method by which to minimise latitude biases. An experiment by Jenkins and Leroy (1979 B) produced differences of $\Delta\phi$ = 12.7m , $\Delta\lambda$ = 2.4m , Δh = 2.9m between balanced and unbalanced data sets. Serious pass imbalances were shown to be most detrimental in small data sets.

It was noted in Section 3.3.1. that along track errors are effectivly introduced by receiver delay and time jitter. Errors from these sources affect both broadcast and precise ephemeris solutions , necessitating the balancing of passes in both cases (Hatch 1982 C). It was demonstrated by Jenkins and Leroy (1979 B) that balanced passes would cause a constant twenty metre along-track error to be absorbed into the time delay parameter during solution , resulting in a net positioning error of twenty centimetres. However , with a 15 - 20 pass imbalance , the positioning error would increase to approximatly three metres. This clearly demonstrates that a balanced pass geometry will largely remove the effects of a <u>constant</u> along track bias.

In addition to north-south balancing , it is highly desirable that there be as many passes to the east of a receiving station as to the west. The range component of TRANSIT Doppler observations contains information about both longitude and altitude. Errors in the across-track component of the satellite ephemeris , or in the apriori height estimate of the receiver station , will produce corresponding errors in the longitude determination , these increasing with satellite elevation. The effects of such errors may be substantially reduced by observing equal numbers of passes to the east and west of an observing station and by ensuring that pairs of east and west passes are at approximatly the same elevation.

Finally , during computation , it is desirable that the Doppler counts extracted from each pass be balanced about the point of closest approach. Shenke (1982 C) reported that solutions undertaken without such balancing displayed larger latitude and longitude biases than those in which balancing took place.

4.3.3. The Effect of Tracking Station Distribution

There has been some debate in recent years as to the effect of tracking station distribution on ephemeris accuracy. As was noted in section 2.2.2. , the OPNET stations are all located within the United States whereas the TRANET stations are distributed worldwide. Brown (1976 A) speculated that this factor might account for a positive along-track bias that had been found to persist in Europe and Africa but not in North America.

Holland et al (1977 A) investigated this question by comparing ephemerides generated from OPNET and TRANET data. It was concluded that the ephemerides were essentially identical worldwide and therefore independent of the tracking station network. It was also commented that this conclusion had been reached in previous experiments and that it could be regarded as a firmly established fact.

Jenkins and Leroy (1979 B) undertook a similar investigation. They concluded that the tracking station networks produced negligable ephemeris differences when all other conditions were identical. However , when the different GM values used by the NAG and DMAH/TC were considered , a radius difference of about one metre and a z-axis translation of two metres were detected in the solutions. In addition there occured a longitude shift of 3×10^{-7} radians which , coincidently , had been cancelled out during the preceding years by the rate of change of UTC - UT1. The authors recommended that NAG and DMAH/TC use the same program constants in their ephemeris generation programs.

Thus it can be concluded that there may be minor systematic differences between the ephemerides due to the interaction of tracking station coordinates with the GM values.

4.4. Ephemeris Distribution

Distribution of the precise ephemeris is very simply described. It is distributed on a nine track tape (Archinal and Mueller 1982 C) on a government to government basis. The ephemeris data consists of Earth-centered inertial coordinates (X,Y,Z) and velocity components ($\dot{X},\dot{Y},\dot{Z}$) at one minute intervals of Universal Time.

The propagation of the broadcast ephemeris is considerably more involved. As was described in Section 3.1.2. , the ephemeris parameters are contained in the 6103 binary bit message which is modulated onto the 150 MHz. and 400 MHz. signals. These are grouped into fixed and variable parameters as illustrated in Diagram 6.

The fixed parameters remain constant until changed by a new data injection. They define a reference orbit for the satellite , this being a precessing ellipse. The fixed parameters are described in Table 3. and are illustrated in Diagrams 8 and 9. The interpretation of the code recovered from the satellite is well described in Ashkenazi and Gough

TABLE 3

BROADCAST EPHEMERIS FIXED PARAMETERS

Message Word		Parameter
56	tp	- Time of first satellite perigee after last data injection.
62	n	- Mean motion
68	ω_{tp}	- Argument of perigee (at tp)
74	$\lvert\dot{\omega}\rvert$	- Absolute value of precession rate of perigee
80	e	- Eccentricity of orbit ellipse
86	$\bar{a}$	- Mean semi-major axis of orbit
92	Ω_{tp}	- Right ascension of ascending node (at tp)
98	$\dot{\Omega}$	- Precession rate of ascending node
104	Cos i	- Cosine of inclination
110	$GAST_{tp}$	- Greenwich apparent siderial time (at tp)
116		- Satellite identification number
122		- Day and time of last data injection
128	Sin i	- Sine of inclination
134		- Fractional satellite frequancy offset $(f_o-f_s)/f_o$

DIAGRAM 8
ELLIPTICAL ORBIT

(WELLS 1974 A)

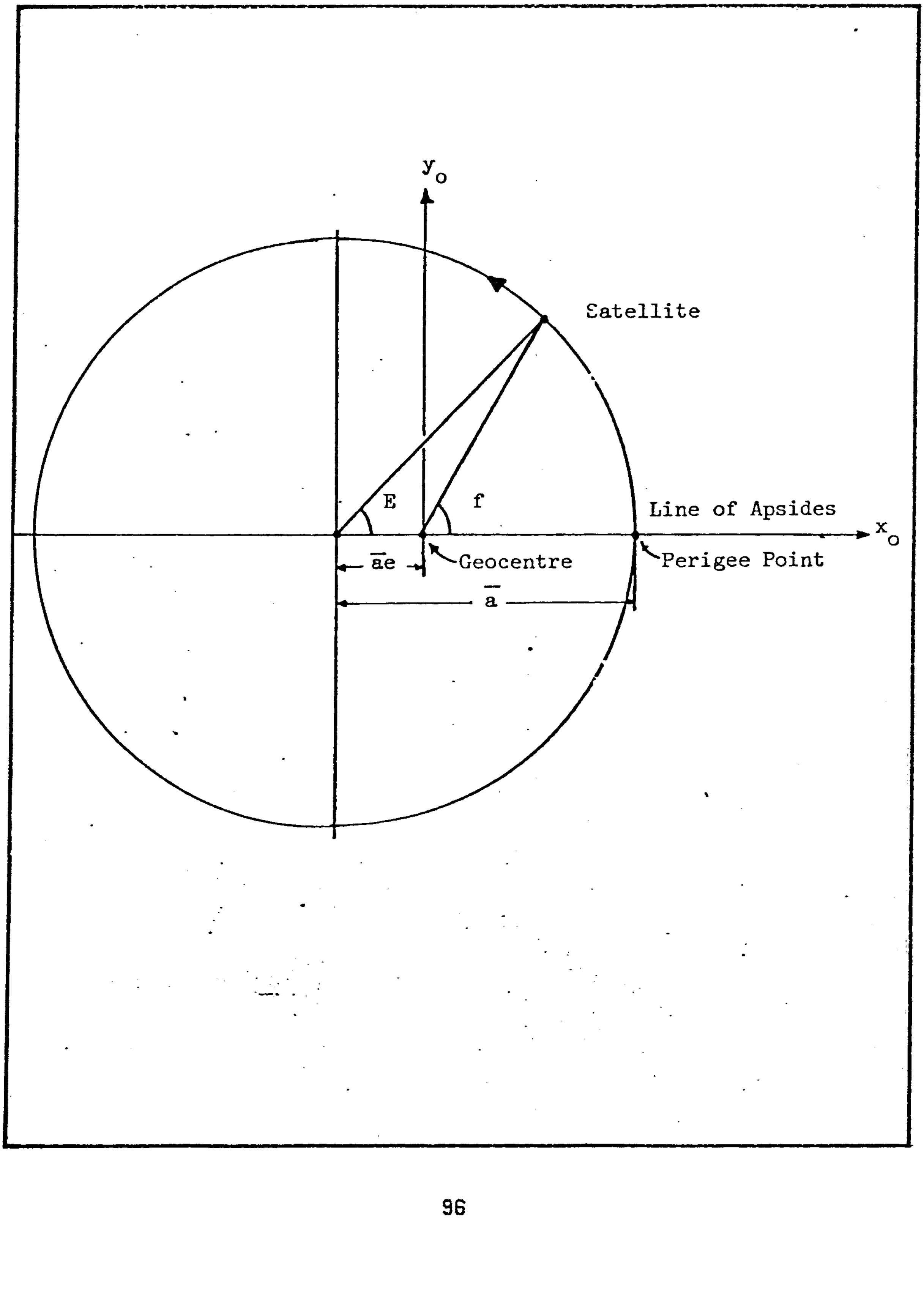
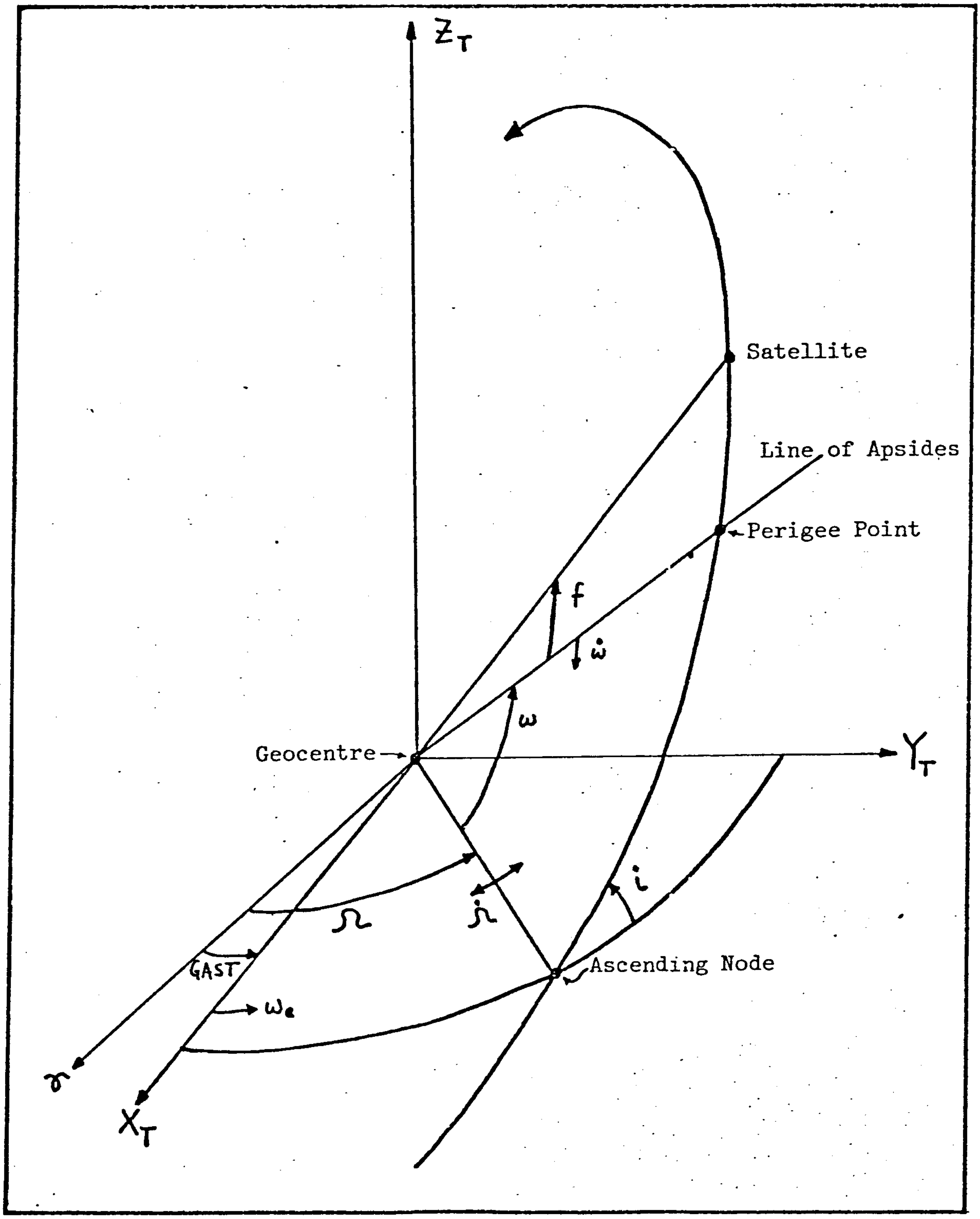


DIAGRAM 9
TRANSIT FIXED ORBITAL PARAMETERS

(WELLS 1974 A)



(1975 A) and Stansell (1978 A) and will not be elaborated upon here.

The departure of the satellite from its reference orbit is described by along track (ΔE) , across track (Δa) and out-of-plane (η) position corrections. Every two minute message contains correction data for eight even minute epochs of Universal Time. Thus if the transmission of a message commences at epoch t_k , the information words in lines 1 to 8 contain correction data for epochs t_{k-6} , t_{k-4} , t_{k-2} , t_k , t_{k+2} , t_{k+4} , t_{k+6} and t_{k+8}. As the same format is broadcast every two minutes , each parameter will steadily progress from line 8 up to line 1.

A variable parameter word contains four elements of data , these being -

1. A 'Q' number
2. The along-track correction
3. The across-track correction
4. One figure of the out-of-plane correction.

The 'Q' number identifies the epoch of the corrections relative to the preceding half hour. Thus a 'Q' number of 07 means that the epoch occurs fourteen minutes after the half hour. Time uncertainties of less than fifteen minutes may be resolved from this information (Stansell 1978 A).

The along-track and across-track corrections are coded in degrees and kilometres respectively. These occur in every variable parameter word and are thus available for each two minute epoch of Universal Time. The out-of-plane correction is also coded in kilometres. However , due to the length of the message words , only a single figure of this correction is included in each of the variable parameters. The out-of-plane correction therefore has to be generated using the information from two message words and is thus only available at four minute intervals.

The interpretation of the data from the variable parameter words is again well covered in Stansell (1978 A) and Ashkenazi and Gough (1975 A).

Prior to decoding the broadcast ephemeris , the received data has to be Majority Voted. The purpose of Majority Voting is to minimise the effects of data corruption caused by radio interference during signal propagation. The process involves the digit by digit comparison of each information word with the corresponding words in at least two succeeding messages. If differences are detected , the digit specified in the majority of words is adopted. In cases where three message words are not available , (e.g. the first variable parameter word at the epoch of lock-on) , the words transmitted on the 150 MHz.

and 400 MHz. signals may be compared. Data received by older instruments has to be majority voted on an external computer at the completion of the observing period. More modern instruments , notably the Magnavox MX1502 , have the capacity to perform the majority voting task during data collection.

Once decoded , the instantaneous satellite positions as defined by the ephemeris parameters have to be transformed into geocentric coordinates. The following description of the transformation process comes from Ashkenazi and Gough (1975 A).

The position of a satellite in an elliptical orbit may be described by the eccentric anomoly and the orbit's semi-major axis. The instantaneous value of the semi-major axis a_{IN} at a two minute epoch t_{IN} is given by

$$a_{IN} = a + \Delta\bar{a} \qquad (4-2)$$

In the TRANSIT system , the instantaneous value of the eccentric anomoly E_{IN} is defined as being

$$E_{IN} = M_{IN} + e \operatorname{Sin} M_{IN} + \Delta E \qquad (4-3)$$

where

$$M_{IN} = n \left(t_{IN} - t_{tp} \right) \qquad (4-4)$$

t_{tp} = the time of perigee passage,

n = the mean motion.

(Note that equation (4-3) is a modified version of Kepler's equation for use in conjunction with the TRANSIT system only. It should not be used in classical orbit computations.)

Consider a local cartesian coordinate system whose x-y plane coincides with the plane of the instantaneous ellipse and whose z axis is perpendicular to that ellipse. It is clear from Diagram 8 that the position of a satellite in this system is given by

$$\begin{bmatrix} x_{IN} \\ y_{IN} \\ z_{IN} \end{bmatrix} = \begin{bmatrix} a_{IN}(\text{Cos } E_{IN} - e) \\ a_{IN} \text{ Sin } E_{IN} \\ \eta \end{bmatrix}$$

where e is the eccentricity of the orbit.

The problem of determining geocentric coordinates now becomes one of transforming from one cartesian system to another. As the origins of both systems lie at the earth's centre of mass , only three rotations have to be considered ,these being (in order) -

1. A rotation about the z_{IN} axis to bring the x axis of the orbital system into the geocentric

X-Y plane (Rotation angle = argument of perigee). The rotated axes become x′ , y′ , z′.

2. A rotation about the x′ axis to bring the x′-y′ plane into the geocentric X-Y plane (Rotation angle = inclination angle). The rotated axes become x" , y" , z".

3. A rotation about the z" axis to bring the x" and y" axes into coincidence with the geocentric X and Y axes (Rotation angle = right ascension of the ascending node + GAST).

The transformation may be described mathematically as follows.

$$\begin{bmatrix} X_{IN} \\ Y_{IN} \\ Z_{IN} \end{bmatrix} = \bar{R}_{IN} \begin{bmatrix} x_{IN} \\ y_{IN} \\ z_{IN} \end{bmatrix}$$

where

$$\bar{R}_{IN} = R_3(\alpha_3)\ R_1(\alpha_2)\ R_3(\alpha_1)$$

$$\alpha_1 = -\omega_{IN} = -\omega_{tp} + |\dot{\omega}|(t_{IN} - t_{tp})$$

$$\alpha_2 = -i = -\arctan(\sin i / \cos i)$$

$$\alpha_3 = -\Omega_{IN} + GAST_{IN}$$

$$= -\Omega_{tp} + GAST_{tp} + (\dot{\omega}_e - \dot{\Omega})(t_{IN} - t_{tp})$$

ω_{IN} = the instantaneous argument of perigee ,

Ω_{IN} = the instantaneous Right Ascension of the ascending node

R_i = the rotation matrix for a rotation about axis i in a right handed system.

$\dot{\omega}_e$ = rotation rate of the earth

= $7.29211583 \times 10^{-5}$ rad / sec.

$GAST_{IN}$ is the instantaneous Greenwich Apparent Siderial Time.

The symbols ω_{tp} , $|\dot{\omega}|$, i , $GAST_{tp}$, Ω_{tp} and $\dot{\Omega}$ are defined in Table 3.

4.5. Orbit Recovery

The use of 20 - 30 second Doppler counts in post-processing operations necessitates the determination of satellite positions at epochs other than those provided by an ephemeris. Several schemes have been devised for satisfying this requirement.

The crudest method is to simply interpolate polynomials between the supplied ephemeris positions. In the case of the broadcast ephemeris , the polynomials interpolate the values of the variable parameters , these being combined with the fixed parameters to give the required information. The disadvantage of this approach is that it constrains the polynomials to pass the through the ephemeris values. Consequently it makes no allowance for errors which have been introduced through the rounding of the orbital parameters. The method also suffers from the deficiency that it is not based on any mathematical model of the satellites motion. A least squares polynomial fit also suffers from the latter short-coming.

Wells (1974 A) noted that the broadcast ephemeris variable parameters were not based on a Keplarian analysis of the orbit. Rather they were the residuals from a least squares fit of the reference orbit to numerically determined cartesian coordinates. Furthermore the definition of the ephemeral parameters differed from the equivalent Keplarian parameters as follows.

Broadcast Ephemeris	$E(t)-e\mathrm{Sin}M(t)=M(t)$
Perturbed Keplerian	$E(t)-e\mathrm{Sin}E(t)=M(t)$
Broadcast Ephemeris	$y_0(t)=(a+\Delta a(t))\mathrm{Sin}(E(t)+\Delta E(t))$
Perturbed Keplerian	$y_0(t)=(a+\delta a(t))(1-e^2)^{0.5}\mathrm{Sin}(E(t)+\delta E(t))$
Broadcast Ephemeris	$GAST(t)=GAST(t_p)+\omega_e(t-t_p)$
Perturbed Keplerian	$GAST(t)=GAST(t_p)+\omega_e(t-t_p)+.....$

Wells therefore concluded that the elements $\Delta E(t)$, $\Delta a(t)$ and η would display the influence of forces which caused the satellite to be perturbed from its reference orbit , plus the effects of the formula approximations. Many of these components (including equatorial oblateness , lunar effects and solar effects) display a sinusoidal characteristic , the frequency of which is double that of the orbital frequency. In other cases , notably that of solar radiation pressure , the effect is to cause long period or secular perturbatons which may be considered linear over a twelve hour period. Consequently , Wells proposed and subsequently verified that a function of the form

$$a_1 + a_2 t + a_3 \mathrm{Cos}(2nt) + a_4 \mathrm{Sin}(2nt) \qquad (4\text{-}5)$$

would effectivly describe the variations of the variable parameters with time.

Interpolation functions of this form were successfully incorporated into program PREDOP (Lawnikanis 1976 A). The unknown coefficients are determined for each parameter by a least squares fit.

Clearly the Wells method is an improvement over simple polynomial fitting as it does at least take orbital modelling into account. However a method discussed by Hatch (1976 A) goes one step further by mathematically removing some of the disturbing effects. This algorithm proceeds as follows.

1. Corrections for the effects of gravitational oblateness and formula approximation are computed for the eccentric anomoly and the semi-major axis at two minute intervals.

2. The corrections are subtracted from the broadcast values of ΔE and Δa to obtain residuals.

3. The residuals together with the out-of-plane component are fitted to a curve of the form

$$b_1 + b_2 \operatorname{Sin}(nt) + b_3 \operatorname{Cos}(nt) \qquad (4-6)$$

Note that this curve has only three terms. Note also that the period of the residuals is now equal to that of the orbital frequency.

4. Determination of the satellite position is then achieved by

   a. Evaluating the equations (4-6) at the desired time ,

   b. Adding corrections for gravitational oblateness and formula approximation to the values determined in a. ,

   c. Applying the total corrections determined in b. to the values for the reference orbit.

This technique requires more computing time than that of Wells. However it has the advantage that it permits one extra degree of freedom in each orbit fit. The method has been used extensivly by Magnavox in recent years. It has been incorporated into both the MX1502 on-board software and program MAGNET (Ross 1982 C)

The most rigorous way to recover a satellite's orbit is by the short arc method. This involves a precise numerical integration of the equations of motion and the variational partials over the period of a satellite's pass. The method introduces six degrees of freedom into the orbit determination by letting both the velocity and position components adjust. Consequently both the shape and location of the orbit are permitted to alter.

A short arc program called SAGA III is discussed by Brown (1976 A). The orbit recovery algorithm in this program is as follows.

1. Inertial coordinates (X,Y,Z) are computed for up to eight two minute marks recorded during a pass. Polynomials (fifth or sixth order) are fitted to the coordinate values.

2. The polynomials and their derivatives are evaluated at a selected initial epoch t_0 , thus generating an inital approximation to the state

vector $(X_0, Y_0, Z_0, \dot{X}_0, \dot{Y}_0, \dot{Z}_0)$.

3. An approximation to the satellite orbit is generated by integrating the equations of motion. The integration commences from the approximate state vector.

4. The orbit computed in 3. is fitted to the X,Y,Z coordinates in 1. , providing revised values for the elements of the state vector.

5. Steps 3. and 4. are iterated to convergence. The final residuals in X , Y and Z resulting from the orbit fit are examined for acceptability . RMS values of 3 to 5 metres indicate good data.

By comparison with the methods of Wells and Hatch , this technique is relativly expensive in terms of computing time. Kouba (1983 A) notes that a short arc computation which uses a gravity model truncated as low as degree and order eight , can take up to ten times as long as a semi-short arc technique using an externally generated orbit. SAGA III contained a gravity model described by spherical harmonics up to degree and order sixteen. However Brown (1976 A) found that from a practical point of view , it was sufficient to truncate the model at degree and order four. This would have significantly reduced the computing time.

Kouba (1983 A) demonstrated that a shape accuracy comparable to that of the precise ephemeris could not be obtained from a model truncated below order eight. To overcome the computing penalty that this imposed , he determined accelerations (A_x ,A_y ,A_z) at the two minute marks using the equations of motion , and then interpolated using Chebychev polynomials. These polynomials were then used in conjunction with six initial conditions to determine a least squares approximation to the orbit.

This technique is utilised in GEODOP V and works well for relative positioning over long distances. Kouba notes that the improvment in broadcast orbit shape produces results which are 10% to 30% more accurate than semi-short arc solutions over 2000 Km. distances. Below 1000 Km. however , the short and semi-short arc techniques give comparible results.

4.6. Transformation Parameters

The reference system for terrestrial geodetic surveys in Australia is known as the Australian Geodetic Datum (AGD). The AGD is a local datum , representing the best fit of a reference ellipsoid to the Geoid over the Australian continent. In addition to being non-geocentric , Allman and Steed (1980 A) have demonstrated that its semi-major axis is not parallel to the Z axes of the satellite datums. Consequently , it is not orientated to the CIO pole

The satellite datums , NSWC 9Z-2 and 'modified' NWL 10D , are both geocentric to within the precision permitted by the determining observations. Unlike local datums , a reference ellipsoid is not a necessary part of the datum definition , although one is often included for convenience. A satellite datum is defined in terms of the adopted coordinates of the tracking stations (i.e. OPNET , TRANET) , a gravity model , the speed of light , the earths GM value and the earths rotation rate (with respect to the instantaneous equinox) . In addition , ephemeris computations which are performed with respect to the datum are influenced by the clock corrections and the oscillator drift rates at the participating tracking stations (Hoar 1982 A).

In order to use Doppler observations in conjunction with terrestrial networks , it is necessary that they be transformed into the appropriate datum. The question then arises as to whether a 3 , 4 , 5 or 7 parameter transformation should be applied.

A number of authors have favored the use of transformations involving less than seven parameters. Such transformations depend on the assumption that the Z axes of both datums are parallel , thus implying that there are no rotations about the X and Y axes. If it is further assumed that both datums have the same scale and the same longitude origin , then a simple three translation transformation relating the two spatial systems may be determined. This approach was favored by Ashkenazi et al (1978 A) . Meade (1982 C) also favored the use of three parameters to transform between the broadcast and precise datums. In this case , the parameters were a change in scale , a rotation about the Z axis and a translation to the Z coordinate.

In general however , a seven parameter transformation is the most desirable, this accommodating all possible degrees of freedom between two spatial systems (3 translations , 3 rotations and a scale factor) . Two formulations for such transformations are commonly used , these being the Bursa-Wolf model

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + (1 + \Delta) \begin{bmatrix} 1 & -\Omega z & \Omega y \\ \Omega z & 1 & -\Omega x \\ -\Omega y & \Omega x & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_1 \\ Z_1 \end{bmatrix}$$

and the Molodensky-Badekas model

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} + \begin{bmatrix} \Delta & -\Omega z & \Omega y \\ \Omega z & \Delta & -\Omega x \\ -\Omega y & \Omega x & \Delta \end{bmatrix} \begin{bmatrix} X_1 - X_m \\ Y_1 - Y_m \\ Z_1 - Z_m \end{bmatrix}$$

where X_1, Y_1, Z_1 are the local datum coordinates

X_2, Y_2, Z_2 are the satellite datum coordinates

$\Delta X, \Delta Y, \Delta Z$ are the translation elements

$\Omega x, \Omega y, \Omega z$ are the rotation elements

Δ is a scale factor

X_m, Y_m, Z_m is the average position of the common points in the local datum

The Bursa-Wolf model works well in a situation where data is available on a global scale. However it is less suited to a local datum-satellite datum situation due to correlations which arise between the various parameters. While no adverse effects will be noticed in the area contained by the common points , serious discrepancies may occur if the parameters are used outside the determining area. The Molodensky-Badekas model seeks to overcome the correlation problem through the introduction of a fundamental point. Computations are then performed in terms of differences relative to this point (Hoar 1982a A).

The use of a seven parameter transformation becomes particularly necessary in situations where the Z axes of the spatial systems do not coincide (Allman and Steed 1980 A) . Accordingly Allman (1983a A) determined three sets of transformation parameters during the GMA82 adjustment , these being between -

1. The AGD and the Broadcast datum ,

2. The AGD and the Precise datum ,

3. The Broadcast and Precise datums.

Preliminary values for these parameters are recorded in Appendix 1.

Other notable seven parameter transformations have been performed by Jenkins and Leroy (1979 B) and by Boucher , Parquet and Wilson (Meade 1982 C) , both being between the broadcast and precise datums. (These are listed together with the corresponding preliminary Allman parameters in Table 4 .) The former transformation used global data in its determination whereas the latter used only European stations .

TABLE 4

BROADCAST - PRECISE TRANSFORMATION PARAMETERS

Parameter	Set 1	Set 2	Set 3
T_X	-0.1 ±0.9	-0.8 ±0.5	-5.9 ±3.5
T_Y	-0.6 ±1.3	0.2 ±0.5	-0.3 ±3.4
T_Z	-8.8 ±0.9	-2.6 ±0.5	-1.6 ±4.1
θ_X	0.04±0.04	-0.05±0.02	-0.03±0.12
θ_Y	-0.04±0.03	0.02±0.02	0.11±0.12
θ_Z	0.08±0.03	-0.01±0.07	0.12±0.11
ε	0.85±0.12	0.22±0.07	-0.41±0.20

LEGEND

Set 1 were determined by Boucher , Parquet and Wilson (Meade 1982 C).

Set 2 were determined by Jenkins and Leroy (Meade 1982 C).

Set 3 are provisional values determined by Allman (1983a A).

T_X , T_Y , T_Z are in metres.

θ_X , θ_Y , θ_Z are in seconds.

ε is in parts per million.

CHAPTER 5

INVESTIGATION OF RELATIVE POSITIONING ACCURACIES

5.1. Introduction and Background

Considerable enthusiasm for Doppler relative positioning techniques has become evident in recent years. This has been encouraged by the general proposition that such techniques can provide relative accuracies using the broadcast ephemeris which are equivalent to those available from the precise ephemeris (Stansell 1978 A). Experiments have largely supported this assertion. However these tests have usually been performed over distances of less than one hundred kilometers (Hothem 1980 A , Boal and Vamosi 1981 A , Schenke 1982 C , Larden et al 1983 A) , very few having considered longer lines (Kouba and Wells 1976 A , Videla et al 1982 C).

Investigations into Doppler relative positioning techniques are usually impeded by two factors. First of all there is frequently an absence of adequate ground truth against which to compare the Doppler derived quantities , thus preventing a thorough analysis. Secondly,the economics of locating Doppler receivers on known stations for prolonged periods of time are such that they effectively prevent the sustained acquisition of suitable data. Consequently accuracy investigations tend to be relatively few in number.

The steady growth in the use of Doppler receivers , in particular those with on-board translocation software , make it imperative that reliable accuracy estimates for relative positioning be determined. The need for such estimates over long distances is especially acute in Australia where the remote nature of much of the country makes Doppler positioning a particularly desirable technique.

In 1981 , the National Mapping Council of Australia passed a resolution to initiate a new adjustment of the Australian Primary Control Network. This adjustment was subsequently performed by Associate Professor J.S. Allman at the University of New South Wales and became known as GMA82. At the instigation of the Queensland Department of Mapping and Surveying , and later , Professor Allman and the Division of National Mapping , a series of multi-station Doppler networks were observed for inclusion in the adjustment. These networks,which were observed using Magnavox MX1502's , interconnected to span the entire continent. In August 1982 , the South Australian Department of Lands and the Victorian Division of Surveys and Mapping observed a further multi-station figure covering south-eastern South Australia and and the whole of Victoria. This network contained eleven stations , nine being occupied by

MX1502 receivers and two by JMRs . It subsequently became known as Figure E6 .

The GMA82 project therefore produced a considerable amount of Doppler data suitable for use in the assessment of relative positioning accuracies. Not only did the multi-station figures span long distances (in excess of 1000 Km. in many cases) but they involved primary control stations whose coordinates now represent the best available ground truth.

Part of the GMA82 Doppler data set was used for the investigations which follow. The project set out to determine the effects of the following on relative positioning accuracies.

1. The use of different reduction programs.
2. The influence of network size.
3. The use of broadcast and precise ephemerides.
4. The effect of different ephemeris constraints.

In addition it was desired to obtain some idea of the repeatability that could be achieved from the TRANSIT system.

Testing was greatly facilitated by the fact that the precise ephemeris was available for all five operational satellites during the period of the GMA82 Doppler projects.

The geographic locations of the stations involved in the investigation are illustrated in Section 5.2. This is followed by a discussion of the computing procedure in Section 5.3. The results themeselves are presented in the subsections of 5.4. , an analysis being provided in Section 5.5.

5.2. The Test Area

The data set used for this project was drawn predominently from multi-station Figure E6 (Observed 10-15/8/82). However a small quantity of data was also drawn from Figure E3 (Observed 3-8/3/82). The geographic localities of the stations involved are illustrated in Diagram 10. The number of passes observed at each station are summarised in Table 5.

Much of the testing was done using data from five of the E6 stations. The localities of these stations are illustrated in Diagram 11. They will in future be referred to as the subset stations.

The E6 figure spans an area which is served by a good quality first order terrestrial network. The stations are , in general , connected by triangulation and trilateration chains , although one station (Bambadin) is connected by

DIAGRAM 10

STATION LOCATIONS - E6 FULL NETWORK

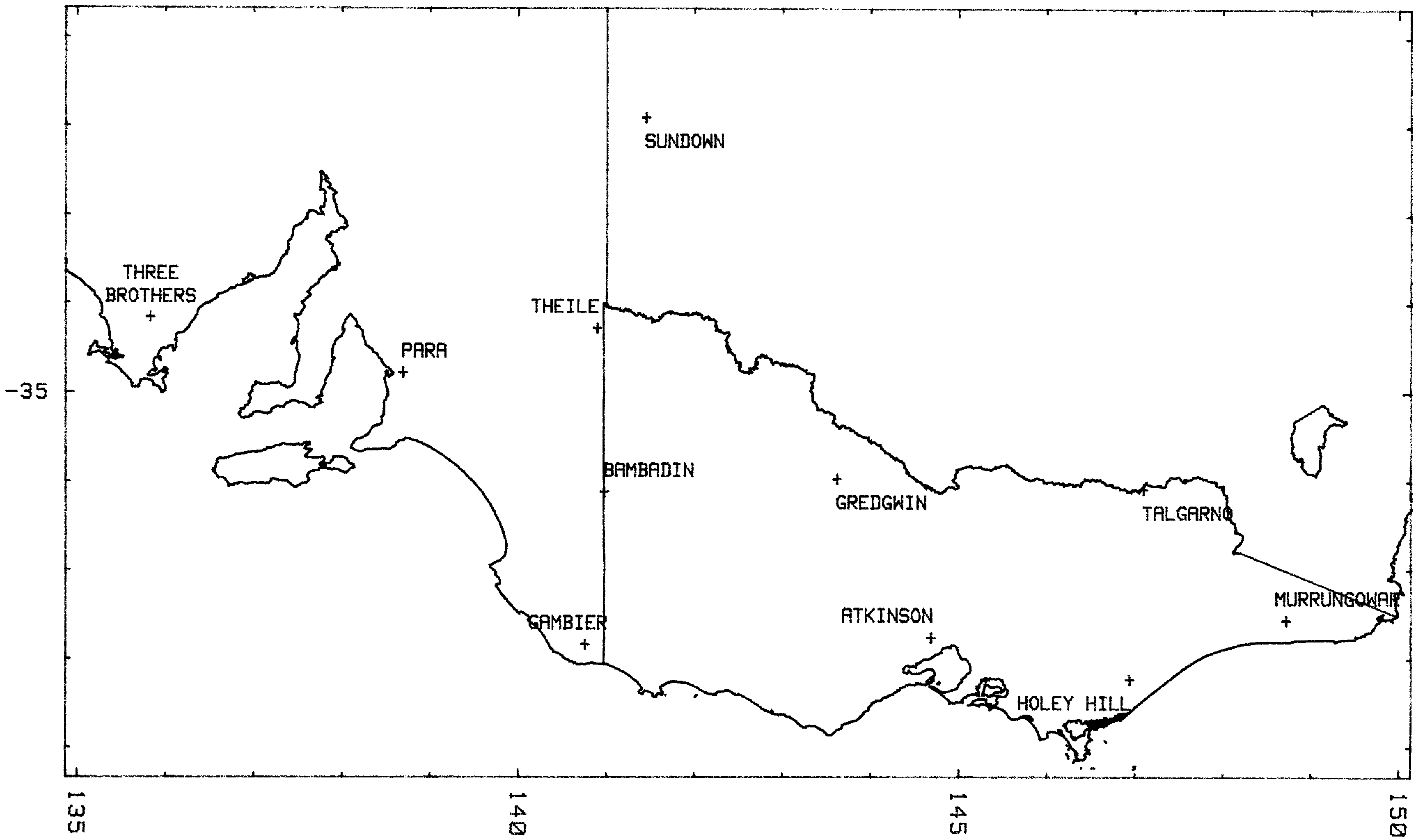


DIAGRAM 11
STATION LOCATIONS
E6 SUBSET NETWORK

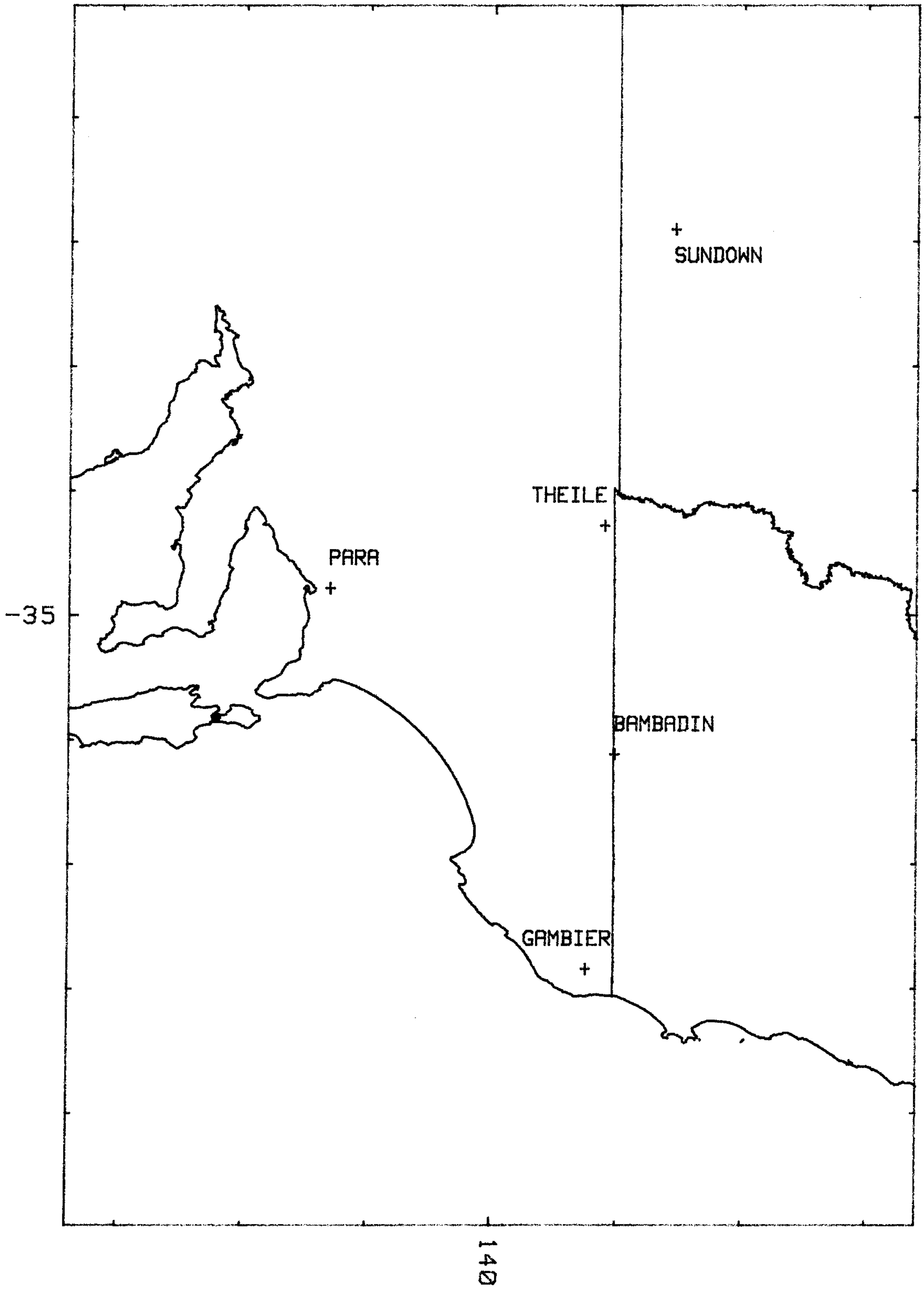


TABLE 5

SATELLITE PASS DISTRIBUTION - FIGURE E6

Station	Passes	SW	SE	NW	NE
Para	65	17	15	17	16
Sundown	76	20	15	21	20
Theile	81	20	20	20	21
Bambadin	68	17	17	16	18
Gambier	67	16	16	17	18
Three Brothers	72	19	15	19	19
Atkinson	85	21	20	21	23
Gredgwin	76	20	19	17	20
Talgarno	79	21	21	19	18
Murrungowar	66	18	15	18	15
Holey Hill	30	8	9	6	7

The above statistics were determined during the GEODOP multi-station reduction of the full network using the precise ephemeris. (Orbital constraints = 2 , 1 , 2) traversing only. Inter-station distances range from 173 Km. to 1220 Km.

The GMA82 coordinates for the stations were obtained using a combined data set which included terrestrial data , Doppler point positions , VLBI , Satellite Lasar Ranging and the multi-station Doppler figures. It could legitimatly be argued that , by comparing Doppler relative positioning results against a coordinate set which was partially derived from the same data , one is comparing correlated quantities and thus acquiring misleading results. An investigation of the impact of the introduction of the multi-station figures into GMA82 indicated that they had very little effect in south-eastern Australia due to the inherent strength of the terrestrial network (Allman 1983b A). Consequently , while there may be some correlation between the Doppler quantities and the ground truth estimates in the following results , it will be very small and may be safely neglected.

The coordinates used for the ground truth comparisons were derived from the GMA82 Stage 2 adjustment dated 2nd June 1983. (Coordinates were determined for eccentric antenna stations when required.) The spheroidal heights of the electrical centres were computed using AHD elevations , vertical eccentric data and geoidal undulations based on the Fryer geoid model (Fryer 1971 A). The Fryer model was used in preference to that of GEM 8 as it was considered that it would be less subject to oversmoothing in localised areas. Fryer's values were adjusted by 10.9 metres to make them consistent with the gazetted AGD definition.

The horizontal and vertical coordinates of the electrical centres are listed in Table 6.

5.3. Computing Procedure

The processing of data was done in three stages. These were as follows.

1. The formation of the data decks.

2. The computation of the relative positioning solutions.

3. The transformation of those solutions into a common spatial coordinate system for comparative purposes.

These will now be discussed in turn.

5.3.1. Data Deck Formation

This section is not applicable to data processing

TABLE 6

RECEIVER STATION COORDINATES

STATION	LATITUDE	LONGITUDE	HEIGHT
PARA	-34 47 11.4855	138 41 28.7379	230.60
SUNDOWN	-31 54 2.5152	141 26 55.6147	407.80
THEILE	-34 16 58.7785	140 53 34.0598	63.62
BAMBADIN	-36 7 9.5213	140 58 36.9403	169.22
GAMBIER	-37 50 32.9209	140 45 18.9700	191.99
THREE BR.	-34 9 37.7903	135 49 43.7501	225.70
ATKINSON	-37 45 32.2330	144 40 53.6086	158.72
GREDGWIN	-35 58 21.9672	143 37 6.6309	168.00
TALGARNO	-36 5 6.6620	147 5 43.3776	662.91
MURRUNGOWAR	-37 33 48.2534	148 42 53.3301	745.86
HOLEY HILL	-38 14 .1208	146 56 19.3863	235.25

undertaken using the Magnavox MX1502 on-board software (Vide section 5.3.2.1.).

In general , suitable data decks were already available for processing , these having been created at the time the GMA82 project was undertaken. The decks consisted of the raw cassette data for each station coded in hexadecimal format. The data for each station was stored in a separate file.

In order to perform the repeatability tests , data decks containing twenty passes each had to be extracted from the parent decks. As only passes common to pairs of stations were required , a program had to be written which would selectivly recover the desired passes.

This program was written in FORTRAN 77 and was called SELECT. It was installed on the Cyber 171 computer at the University of New South Wales and run in interactive mode. A listing of the program is included as Appendix 4.

5.3.2. Computation of Relative Positioning Solutions

5.3.2.1. MX1502 Software

The Magnavox MX1502 on-board software is contained on a circuit board within the satellite receiver itself. The user operating procedure is explained in the MX1502 Field Translocation Satellite Surveyor - Operation and Service Manual (1980 A). The following points should be noted.

First of all , the MX1502 can only compute relative positions between two points using the broadcast ephemeris. (The coordinates of the Master station are held fixed.) Multi-station solutions involving three or more points still have to be performed on external computers as do solutions involving the precise ephemeris. Secondly , the MX1502 solutions are performed using common passes only. Passes which are observed by one receiver but not the other are specifically excluded. Thirdly , data entry is achieved directly from the cassettes via the tape cassette transport. This enables post-processing to be done in the field , resulting in rapid solution availability if few passes are involved. Finally options are available which enable translocation solutions to be determined in real time , the participating receivers being linked by radio or land line. However such options are of no interest as regards the aims of this project.

Post-processing with the MX1502 is slow if a considerable number of passes are involved. The receivers memory capacity prevents the input of more than seventeen passes at any one time. Consequently , operator attendance is required at least once per hour to change cassettes. The

MX1502 solution for line Para-Theile contained 60 common passes and took approximatly six hours to process.

An example of the data output during translocation is included as Appendix 3.

5.3.2.2. PREDOP/MERGE/GEODOP

PREDOP , MERGE and GEODOP are a family of programs which were written at the Canadian Department of Energy Mines and Resources. The versions of PREDOP and GEODOP which run at the University of New South Wales were revised by Shi in 1981 . In addition , PREDOP was further modified by Allman in 1982. All programs are installed on the Cyber 171 computer at the University of New South Wales. Their operation is explained by Lawnikanis (1976 A) , Kouba and Boal (1975 A) and Shi (1982 A).

Program PREDOP is a preprocessing program which is used to edit and filter raw Doppler data. It is run for each network station individually , computations being performed in the Guier plane. Program MERGE is used to consolidate the output PREDOP data into a single multi-station data deck. In addition , it may be used to incorporate the precise ephemeris , although this has to be done during a second run of the program. Program GEODOP then performs the actual multi-station computations.

The program family has to be run on a main frame computer and consequently lacks the portability of the MX1502 software. However it does have the advantage of being able to process multistation figures relativly quickly. It is also capable of processing with the precise ephemeris. Finally of course , it provides the user with considerably more flexability than the MX1502 by enabling the selection of processing options. (During processing with GEODOP , no stations were held fixed.)

5.3.3. Intercomparison of Solutions

Processing involving the broadcast and precise ephemerides produces coordinates in two different datums , these being the 'modified' NWL 10D and NSWC 9Z-2 systems respectivly. GMA82 coordinates are defined in a third system , this being the Australian Geodetic Datum (AGD) . To enable a comparison of results to take place , all values must be converted to a common datum. Consequently transformations were performed to bring all results into the AGD , this being the datum of the ground truth measurements.

The transformations were performed using a computer program called CHORD7 . This was written in FORTRAN 77 and was installed on the Cyber 171 at the University of New South Wales. A listing of the program is included as Appendix 5.

Program CHORD7 uses a seven parameter transformation and the Bursa-Wolf model. It enables the transformation and comparison of up to four different coordinate sets for a group of stations. The comparisons involve both the coordinates directly and the derived chord distances. The intercomparisons between multi-station Doppler coordinate sets were all performed using this program. However as chord distances are sensitive only to the transformation scale factor , their intercomparisons were achieved through the simple application of the appropriate scale factors.

The transformation parameters used in CHORD7 were the preliminary values determined for Australia by Allman (1983a A) during the GMA82 project. These are listed in Appendix 1. Some consideration was given as to whether local parameters should be computed and applied instead of the national parameters. It was decided however that as the area in question would have provided relativly few degrees of freedom for the parameter determination , it would be more satisfactory to use the national parameters and remain alert for any local systematic effects. It was also decided to use chord distances for most intercomparisons , as these would be free of residual transformation systematics in all but scale.

5.4. Results

5.4.1. Reduction Program Comparison

In order to investigate the overall effects of different modelling procedures , translocation solutions were computed for ten lines using

1. The Magnavox MX1502 on-board translocation software
2. Programs PREDOP , MERGE and GEODOP using the broadcast ephemeris.

The lines were those connecting every possible station pair in the five station subset. The computed chord distances are presented in Tables 7 and 8. In addition , the differences between the chords and the ground truth (ground truth residuals) are plotted in Diagram 12.

The chord distances in each data set are correlated. Consequently it is not meaningful to determine their sample mean and standard deviation. However the following points can be noted from the diagram.

The MX1502 and GEODOP ground truth residuals appear to be biased by approximatly -0.7 metres and -0.5 metres respectivly. In nine out of ten cases , the magnitudes of

TABLE 7

CHORD DISTANCES FOR

TRANSLOCATION - MX1502

LINE	OBSERVED	SCALE	CORRECTED	GROUND	DIFF
PS	410191.889	-.587	410191.302	410192.721	-1.419
PT	209666.307	-.300	209666.007	209667.733	-1.726
PB	254770.702	-.364	254770.338	254771.282	-.944
PG	386378.957	-.553	386378.404	386379.474	-1.070
ST	269251.467	-.385	269251.082	269250.748	.334
SB	469881.186	-.672	469880.514	469879.595	.919
SG	661942.963	-.947	661942.016	661942.707	-.691
TB	203869.444	-.292	203869.152	203868.667	.485
TG	395100.530	-.565	395099.965	395101.183	-1.218
BG	192245.028	-.275	192244.753	192246.617	-1.864

LEGEND

OBSERVED - TRANSLOCATION - MX1502
SCALE - SATELLITE DATUM TO GMA82 SCALE CORRECTION
CORRECTED - SATELLITE DISTANCE TRANSFORMED TO GMA82
GROUND - CHORD DISTANCE FROM GMA82 COORDINATES
DIFF - CORRECTED-GROUND

PS - PARA TO SUNDOWN
PT - PARA TO THEILE
PB - PARA TO BAMBADIN
PG - PARA TO GAMBIER
ST - SUNDOWN TO THEILE
SB - SUNDOWN TO BAMBADIN
SG - SUNDOWN TO GAMBIER
TB - THEILE TO BAMBADIN
TG - THEILE TO GAMBIER
BG - BAMBADIN TO GAMBIER

TABLE 8

CHORD DISTANCES FOR

TRANSLOCATION - GEODOP,BROADCAST,(26,5,10)

LINE	OBSERVED	SCALE	CORRECTED	GROUND	DIFF
PS	410192.408	-.587	410191.821	410192.721	-.900
PT	209667.116	-.300	209666.816	209667.733	-.917
PB	254771.775	-.364	254771.411	254771.282	.129
PG	386379.972	-.553	386379.419	386379.474	-.055
ST	269251.107	-.385	269250.722	269250.748	-.026
SB	469880.146	-.672	469879.474	469879.595	-.121
SG	661942.091	-.947	661941.144	661942.707	-1.563
TB	203868.993	-.292	203868.701	203868.667	.034
TG	395100.708	-.565	395100.143	395101.183	-1.040
BG	192246.128	-.275	192245.853	192246.617	-.764

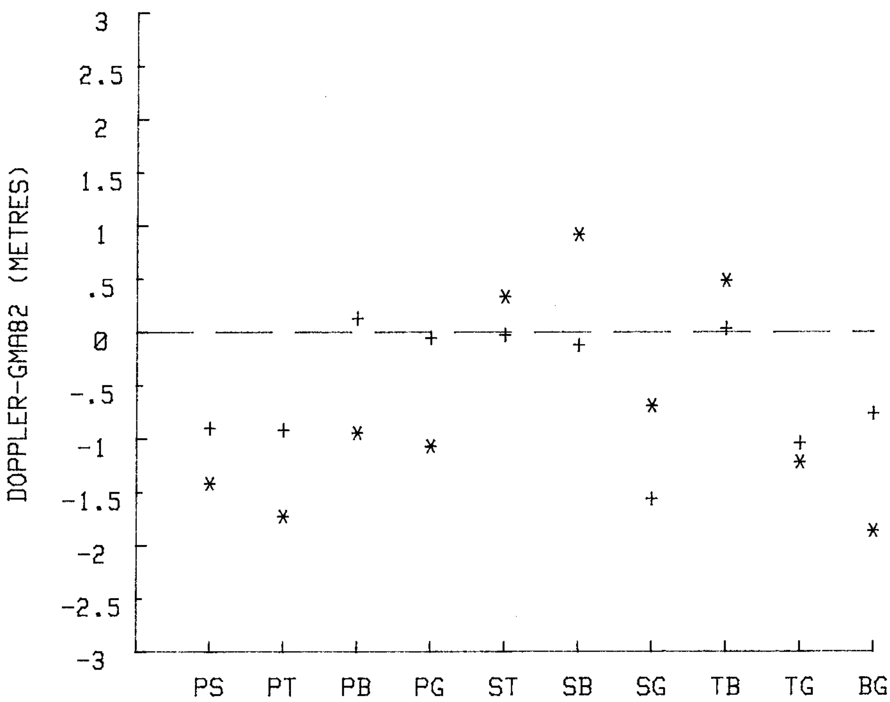
LEGEND

OBSERVED - TRANSLOCATION - GEODOP,BROADCAST,(26,5,10)
SCALE - SATELLITE DATUM TO GMA82 SCALE CORRECTION
CORRECTED - SATELLITE DISTANCE TRANSFORMED TO GMA82
GROUND - CHORD DISTANCE FROM GMA82 COORDINATES
DIFF - CORRECTED-GROUND

PS - PARA TO SUNDOWN
PT - PARA TO THEILE
PB - PARA TO BAMBADIN
PG - PARA TO GAMBIER
ST - SUNDOWN TO THEILE
SB - SUNDOWN TO BAMBADIN
SG - SUNDOWN TO GAMBIER
TB - THEILE TO BAMBADIN
TG - THEILE TO GAMBIER
BG - BAMBADIN TO GAMBIER

DIAGRAM 12

REDUCTION PROGRAM COMPARISON



NOTE - Raw Doppler data is from Multi-Station Figure E6
Figure E6 was observed during the period 10-15/8/82
All distances have been transformed to the AGD

LEGEND

* - TRANSLOCATION - MX1502
\+ - TRANSLOCATION - GEODOP, BROADCAST, (26,5,10)

PS - PARA TO SUNDOWN
PT - PARA TO THEILE
PB - PARA TO BAMBADIN
PG - PARA TO GAMBIER
ST - SUNDOWN TO THEILE
SB - SUNDOWN TO BAMBADIN
SG - SUNDOWN TO GAMBIER
TB - THEILE TO BAMBADIN
TG - THEILE TO GAMBIER
BG - BAMBADIN TO GAMBIER

the GEODOP residuals relative to ground truth are smaller than their MX1502 counterparts. The differences between the two sets range from 0.36 m. (along the line Sundown-Theile) to 1.10 m. (along the line Bambadin-Gambier) and appear to be independent of line length. In addition , whereas five of the MX1502 residuals exceed 1.0 metres , only two of the GEODOP residuals do likewise.

Thus the results as presented suggest that MX1502 software produces significantly inferior relative accuracies when compared with those from GEODOP in translocation mode.

5.4.2. Repeatability

Translocation solutions were computed for the line Para-Sundown using four subsets of twenty passes drawn from the data sets for multi-station figures E3 and E6. These two figures were observed five months apart. The passes were selected on the basis that they had successfully participated in multi-station precise ephemeris solutions during the data reductions for GMA82. No attempt was made to ensure that all passes would be accepted during the repeatability tests. Instead , the number of rejected passes were made a parameter of the investigations.

Each of the subsets was processed using -

1. The Magnavox MX1502 on-board translocation software.
2. Program GEODOP using broadcast ephemeris.
3. Program GEODOP using precise ephemeris.

The results are presented in Table 9 and are plotted in Diagram 13. As the data sets are independent of each other , sample statistics have been computed for each mode of reduction , these being presented in Table 11.

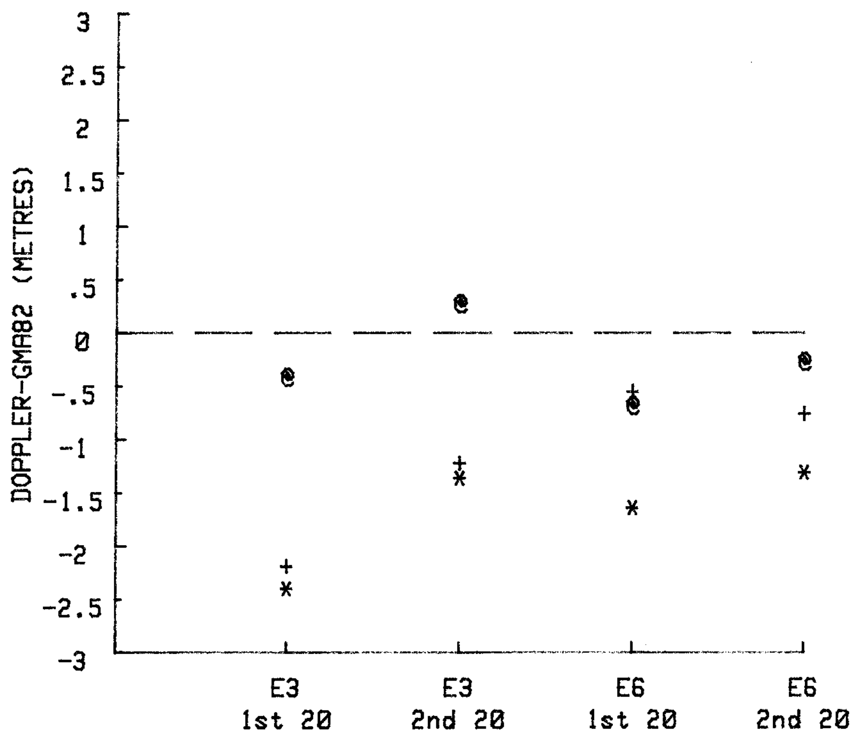
The results show that the MX1502 and the precise ephemeris solutions have comparaible repeatability. The ranges of the two sets of ground truth residuals are almost identical , although the precise ephemeris solutions have a slightly superior standard deviation. The broadcast ephemeris solutions however display inferior repeatability , their range and standard deviation being significantly larger than those of the other two modes.

The observations for figure E3 were undertaken during a period of very high solar radiation pressure. This is reflected by the fact that while the MX1502 and GEODOP broadcast solutions rejected fourteen and two passes respectivly , the GEODOP precise solutions did not reject any. The difference between the rejection rates of the MX1502 and GEODOP broadcast solutions probably explains the difference in repeatability. The GEODOP solutions were

DIAGRAM 13

REPEATABILITY TEST

PARA TO SUNDOWN



NOTE - All distances have been transformed into the AGD.
Figure E3 was observed during the period 3-7/3/82
Figure E6 was observed during the period 10-15/8/82

LEGEND

* - TRANSLOCATION - MX1502
+ - TRANSLOCATION - GEODOP,BROADCAST,(26,5,10)
@ - TRANSLOCATION - GEODOP,PRECISE,(2,1,2),10)

computed using an apriori along-track standard deviation of twenty six metres. As the along-track corrections were often in excess of one hundred metres , this constraint was clearly too tight. By including the passes that the MX1502 rejected , the solutions and the repeatability were significantly degraded.

The experiment was repeated along the line Para-Theile using E6 data only. The results are presented in Tables 10 and 11 and are plotted in Diagram 14. On this occasion there was little solar radiation pressure and few rejected passes. The GEODOP broadcast solution showed the best repeatability , closely followed by the MX1502. In both cases , the sample ranges and standard deviations were vastly superior to those for the Para-Sundown line. Surprisingly the GEODOP precise solutions displayed the worst repeatability. However , as the GEODOP precise sample statistics were almost identical for both test lines , it may be concluded that they are typical for the mode of reduction.

While not being directly concerned with repeatability , the sample means do deserve comment. This will be made in Section 5.6. during a general discussion of systematic biases.

TABLE 9

REPEATABILITY TEST - LINE PARA-SUNDOWN

** GROUND TRUTH DISTANCE = 410192.721 METRES **

TRANSLOCATION - MX1502

SET	AC	OBSERVED	SCALE	CORRECTED	DIFF
E3 1st 20	13	410190.909	-.587	410190.322	-2.399
E3 2nd 20	13	410191.945	-.587	410191.358	-1.363
E6 1st 20	19	410191.669	-.587	410191.082	-1.639
E6 2nd 20	19	410192.000	-.587	410191.413	-1.308

TRANSLOCATION - GEODOP,BROADCAST,(26,5,10)

SET	AC	OBSERVED	SCALE	CORRECTED	DIFF
E3 1st 20	18	410191.118	-.587	410190.531	-2.190
E3 2nd 20	20	410192.085	-.587	410191.498	-1.223
E6 1st 20	20	410192.757	-.587	410192.170	-.551
E6 2nd 20	20	410192.548	-.587	410191.961	-.760

TRANSLOCATION - GEODOP,PRECISE,(2,1,2)

SET	AC	OBSERVED	SCALE	CORRECTED	DIFF
E3 1st 20	20	410192.613	-.308	410192.305	-.416
E3 2nd 20	20	410193.304	-.308	410192.996	.275
E6 1st 20	20	410192.347	-.308	410192.039	-.682
E6 2nd 20	20	410192.758	-.308	410192.450	-.271

NOTE - AC = NUMBER OF ACCEPTED PASSES.

TABLE 10

REPEATABILITY TEST - LINE PARA-THEILE

** GROUND TRUTH DISTANCE = 209667.733 METRES **

TRANSLOCATION - MX1502

SET	AC	OBSERVED	SCALE	CORRECTED	DIFF
E6 1st 20	19	209666.626	-.300	209666.326	-1.407
E6 2nd 20	17	209666.120	-.300	209665.820	-1.913
E6 3rd 20	19	209666.082	-.300	209665.782	-1.951

TRANSLOCATION - GEODOP,BROADCAST,(26,5,10)

SET	AC	OBSERVED	SCALE	CORRECTED	DIFF
E6 1st 20	20	209667.448	-.300	209667.148	-.585
E6 2nd 20	20	209667.349	-.300	209667.049	-.684
E6 3rd 20	20	209667.012	-.300	209666.712	-1.021

TRANSLOCATION - GEODOP,PRECISE,(2,1,2)

SET	AC	OBSERVED	SCALE	CORRECTED	DIFF
E6 1st 20	20	209667.416	-.157	209667.259	-.474
E6 2nd 20	20	209668.083	-.157	209667.926	.193
E6 3rd 20	20	209667.235	-.157	209667.078	-.655

NOTE - AC = NUMBER OF ACCEPTED PASSES.

TABLE 11

REPEATABILITY TEST STATISTICS

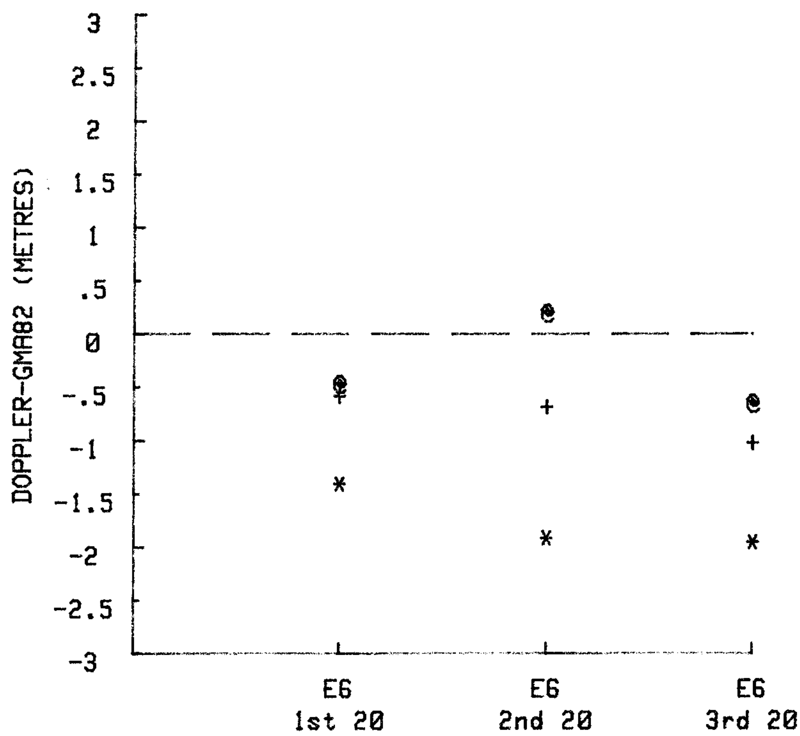
PARA - SUNDOWN

	MX1502	GEODOP BROADCAST	GEODOP PRECISE
MEAN	1.677	1.181	0.274
S.D.	0.502	0.729	0.403
RANGE	1.090	1.639	1.098

PARA - THEILE

	MX1502	GEODOP BROADCAST	GEODOP PRECISE
MEAN	1.757	0.763	0.312
S.D.	0.304	0.229	0.447
RANGE	0.544	0.436	0.848

DIAGRAM 14
REPEATABILITY TEST
PARA TO THEILE



NOTE - All distances have been transformed into the AGD.
Figure E6 was observed during the period 10-15/8/82

LEGEND

* - TRANSLOCATION - MX1502
+ - TRANSLOCATION - GEODOP,BROADCAST,(26,5,10)
@ - TRANSLOCATION - GEODOP,PRECISE,(2,1,2)

5.4.3. Network Size

In order to distinguish between the effects of receiver timing biases and satellite along-track errors , it is necessary to operate in the relative positioning mode. The satellite component , which is common to all data sets , may then be separated from the receiver component through its inclusion as an unknown parameter in a short arc or semi-short arc multi-station solution.

It is not unreasonable to expect that the isolation of along-track errors will be achieved more effectivly in large multi-station networks than in small ones due to the inclusion of the extra observations. It might also be expected therefore that the relative accuracies of multi-station networks would improve with network size. In order to test this hypothesis , chord distances were computed for the ten lines in the subset network using the following reduction modes.

1. Individual translocations between station pairs.
2. A multistation solution using the subset stations only.
3. A multi-station solution using all eleven stations in E6.

All reductions were undertaken using program GEODOP. The investigation was performed using both the broadcast and precise ephemerides. The chord distance comparisons are presented in Tables 8 and 12 to 16 and are plotted in Diagrams 15 and 16.

It is clear from the diagrams that network size had little significant impact on the relative accuracy of the translocation and multi-station solutions , regardless of which ephemeris was used. In general , the chord distances varied by less than ten centimetres between the three network configurations. The only exceptions involved two broadcast ephemeris translocation solutions into station Gambier , these differing from the multi-station results by approximatly 0.33 metres.

The results will be commented on further in Section 5.5.

TABLE 12

CHORD DISTANCES FOR

E6 SUBSET MULTI-STATION - GEODOP,BROADCAST,(26,5,10)

LINE	OBSERVED	SCALE	CORRECTED	GROUND	DIFF
PS	410192.475	-.587	410191.888	410192.721	-.833
PT	209667.113	-.300	209666.813	209667.733	-.920
PB	254771.715	-.364	254771.351	254771.282	.069
PG	386379.939	-.553	386379.386	386379.474	-.088
ST	269250.998	-.385	269250.613	269250.748	-.135
SB	469880.174	-.672	469879.502	469879.595	-.093
SG	661942.419	-.947	661941.472	661942.707	-1.235
TB	203869.008	-.292	203868.716	203868.667	.049
TG	395100.677	-.565	395100.112	395101.183	-1.071
BG	192245.795	-.275	192245.520	192246.617	-1.097

LEGEND

OBSERVED - E6 SUBSET MULTI-STATION - GEODOP,BROADCAST,(26,5,
SCALE - SATELLITE DATUM TO GMA82 SCALE CORRECTION
CORRECTED - SATELLITE DISTANCE TRANSFORMED TO GMA82
GROUND - CHORD DISTANCE FROM GMA82 COORDINATES
DIFF - CORRECTED-GROUND

PS - PARA TO SUNDOWN
PT - PARA TO THEILE
PB - PARA TO BAMBADIN
PG - PARA TO GAMBIER
ST - SUNDOWN TO THEILE
SB - SUNDOWN TO BAMBADIN
SG - SUNDOWN TO GAMBIER
TB - THEILE TO BAMBADIN
TG - THEILE TO GAMBIER
BG - BAMBADIN TO GAMBIER

TABLE 13

CHORD DISTANCES FOR

E6 FULL MULTI-STATION - GEODOP,BROADCAST,(26,5,10)

LINE	OBSERVED	SCALE	CORRECTED	GROUND	DIFF
PS	410192.352	-.587	410191.765	410192.721	-.956
PT	209667.105	-.300	209666.805	209667.733	-.928
PB	254771.632	-.364	254771.268	254771.282	-.014
PG	386380.067	-.553	386379.514	386379.474	.040
ST	269250.898	-.385	269250.513	269250.748	-.235
SB	469880.131	-.672	469879.459	469879.595	-.136
SG	661942.379	-.947	661941.432	661942.707	-1.275
TB	203869.043	-.292	203868.751	203868.667	.084
TG	395100.741	-.565	395100.176	395101.183	-1.007
BG	192245.796	-.275	192245.521	192246.617	-1.096

LEGEND

OBSERVED - E6 FULL MULTI-STATION - GEODOP,BROADCAST,(26,5,10
SCALE - SATELLITE DATUM TO GMA82 SCALE CORRECTION
CORRECTED - SATELLITE DISTANCE TRANSFORMED TO GMA82
GROUND - CHORD DISTANCE FROM GMA82 COORDINATES
DIFF - CORRECTED-GROUND

PS - PARA TO SUNDOWN
PT - PARA TO THEILE
PB - PARA TO BAMBADIN
PG - PARA TO GAMBIER
ST - SUNDOWN TO THEILE
SB - SUNDOWN TO BAMBADIN
SG - SUNDOWN TO GAMBIER
TB - THEILE TO BAMBADIN
TG - THEILE TO GAMBIER
BG - BAMBADIN TO GAMBIER

TABLE 14

CHORD DISTANCES FOR

TRANSLOCATION - GEODOP,PRECISE,(2,1,2)

LINE	OBSERVED	SCALE	CORRECTED	GROUND	DIFF
PS	410192.828	-.308	410192.520	410192.721	-.201
PT	209667.485	-.157	209667.328	209667.733	-.405
PB	254772.116	-.191	254771.925	254771.282	.643
PG	386380.238	-.290	386379.948	386379.474	.474
ST	269251.271	-.202	269251.069	269250.748	.321
SB	469880.416	-.352	469880.064	469879.595	.469
SG	661942.782	-.496	661942.286	661942.707	-.421
TB	203868.966	-.153	203868.813	203868.667	.146
TG	395100.736	-.296	395100.440	395101.183	-.743
BG	192245.934	-.144	192245.790	192246.617	-.827

LEGEND

OBSERVED - TRANSLOCATION - GEODOP,PRECISE,(2,1,2)
SCALE - SATELLITE DATUM TO GMA82 SCALE CORRECTION
CORRECTED - SATELLITE DISTANCE TRANSFORMED TO GMA82
GROUND - CHORD DISTANCE FROM GMA82 COORDINATES
DIFF - CORRECTED-GROUND

PS - PARA TO SUNDOWN
PT - PARA TO THEILE
PB - PARA TO BAMBADIN
PG - PARA TO GAMBIER
ST - SUNDOWN TO THEILE
SB - SUNDOWN TO BAMBADIN
SG - SUNDOWN TO GAMBIER
TB - THEILE TO BAMBADIN
TG - THEILE TO GAMBIER
BG - BAMBADIN TO GAMBIER

TABLE 15

CHORD DISTANCES FOR

E6 SUBSET MULTI-STATION - GEODOP,PRECISE,(2,1,2)

LINE	OBSERVED	SCALE	CORRECTED	GROUND	DIFF
PS	410192.810	-.308	410192.502	410192.721	-.219
PT	209667.459	-.157	209667.302	209667.733	-.431
PB	254772.109	-.191	254771.918	254771.282	.636
PG	386380.223	-.290	386379.933	386379.474	.459
ST	269251.220	-.202	269251.018	269250.748	.270
SB	469880.338	-.352	469879.986	469879.595	.391
SG	661942.736	-.496	661942.240	661942.707	-.467
TB	203868.949	-.153	203868.796	203868.667	.129
TG	395100.754	-.296	395100.458	395101.183	-.725
BG	192245.953	-.144	192245.809	192246.617	-.808

LEGEND

OBSERVED - E6 SUBSET MULTI-STATION - GEODOP,PRECISE,(2,1,2)
SCALE - SATELLITE DATUM TO GMA82 SCALE CORRECTION
CORRECTED - SATELLITE DISTANCE TRANSFORMED TO GMA82
GROUND - CHORD DISTANCE FROM GMA82 COORDINATES
DIFF - CORRECTED-GROUND

PS - PARA TO SUNDOWN
PT - PARA TO THEILE
PB - PARA TO BAMBADIN
PG - PARA TO GAMBIER
ST - SUNDOWN TO THEILE
SB - SUNDOWN TO BAMBADIN
SG - SUNDOWN TO GAMBIER
TB - THEILE TO BAMBADIN
TG - THEILE TO GAMBIER
BG - BAMBADIN TO GAMBIER

TABLE 16

CHORD DISTANCES FOR

E6 FULL MULTI-STATION - GEODOP,PRECISE,(2,1,2)

LINE	OBSERVED	SCALE	CORRECTED	GROUND	DIFF
PS	410192.786	-.308	410192.478	410192.721	-.243
PT	209667.361	-.157	209667.204	209667.733	-.529
PB	254772.016	-.191	254771.825	254771.282	.543
PG	386380.116	-.290	386379.826	386379.474	.352
ST	269251.187	-.202	269250.985	269250.748	.237
SB	469880.339	-.352	469879.987	469879.595	.392
SG	661942.660	-.496	661942.164	661942.707	-.543
TB	203868.994	-.153	203868.841	203868.667	.174
TG	395100.723	-.296	395100.427	395101.183	-.756
BG	192245.874	-.144	192245.730	192246.617	-.887

LEGEND

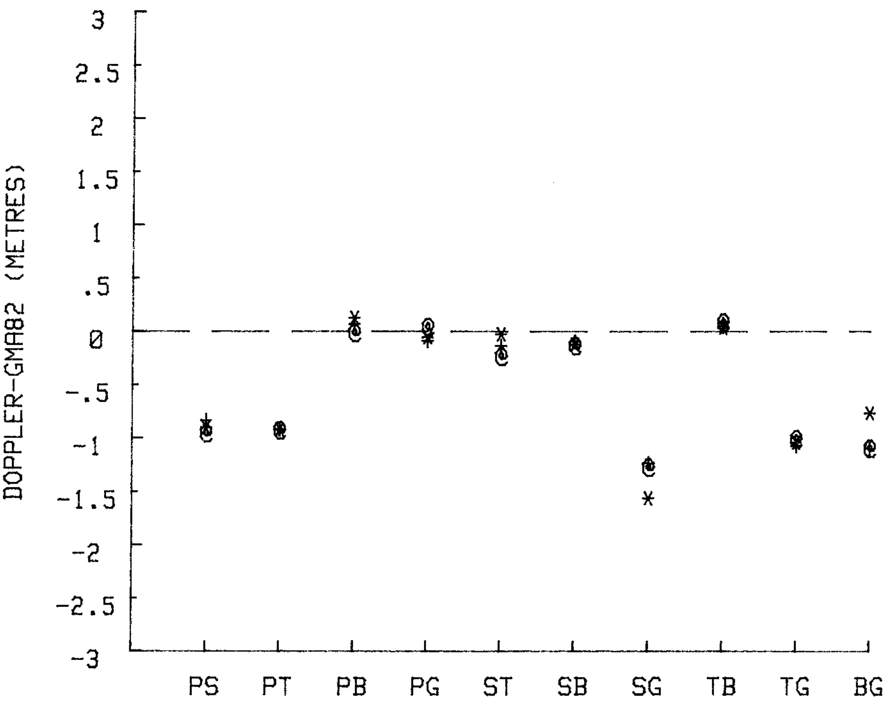
OBSERVED - E6 FULL MULTI-STATION - GEODOP,PRECISE,(2,1,2)
SCALE - SATELLITE DATUM TO GMA82 SCALE CORRECTION
CORRECTED - SATELLITE DISTANCE TRANSFORMED TO GMA82
GROUND - CHORD DISTANCE FROM GMA82 COORDINATES
DIFF - CORRECTED-GROUND

PS - PARA TO SUNDOWN
PT - PARA TO THEILE
PB - PARA TO BAMBADIN
PG - PARA TO GAMBIER
ST - SUNDOWN TO THEILE
SB - SUNDOWN TO BAMBADIN
SG - SUNDOWN TO GAMBIER
TB - THEILE TO BAMBADIN
TG - THEILE TO GAMBIER
BG - BAMBADIN TO GAMBIER

DIAGRAM 15

NETWORK SIZE COMPARISON

BROADCAST EPHEMERIS SOLUTION



NOTE - Raw Doppler data is from Multi-Station Figure E6
Figure E6 was observed during the period 10-15/8/82
All distances have been transformed to the AGD

LEGEND

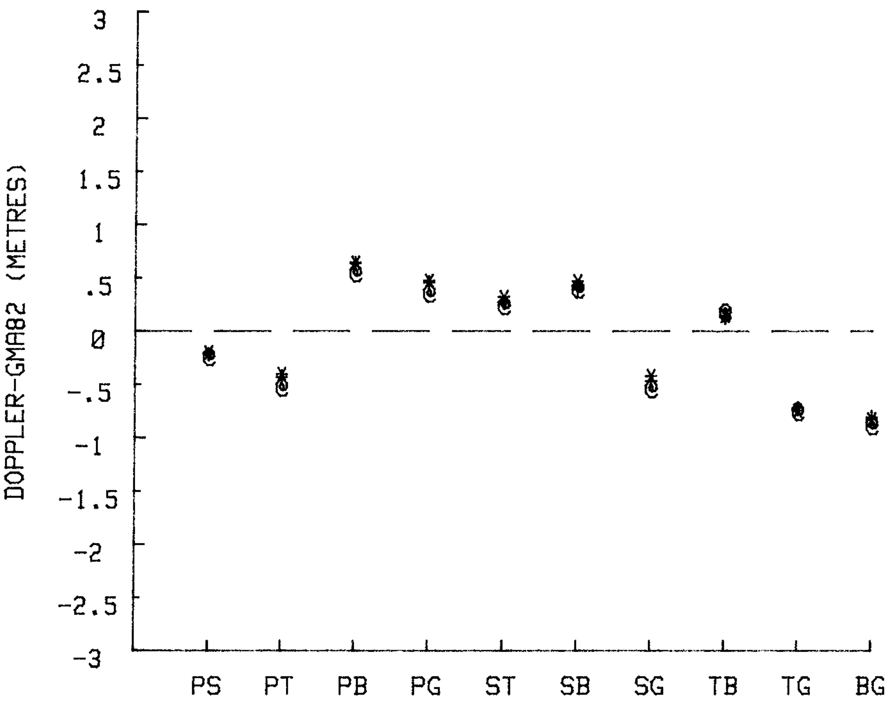
* - TRANSLOCATION - GEODOP,BROADCAST,(26,5,10)
+ - E6 SUBSET MULTI-STATION - GEODOP,BROADCAST,(26,5,10)
@ - E6 FULL MULTI-STATION - GEODOP,BROADCAST,(26,5,10)

PS - PARA TO SUNDOWN
PT - PARA TO THEILE
PB - PARA TO BAMBADIN
PG - PARA TO GAMBIER
ST - SUNDOWN TO THEILE
SB - SUNDOWN TO BAMBADIN
SG - SUNDOWN TO GAMBIER
TB - THEILE TO BAMBADIN
TG - THEILE TO GAMBIER
BG - BAMBADIN TO GAMBIER

DIAGRAM 16

NETWORK SIZE COMPARISON

PRECISE EPHEMERIS SOLUTION



NOTE - Raw Doppler data is from Multi-Station Figure E6
Figure E6 was observed during the period 10-15/8/82
All distances have been transformed to the AGD

LEGEND

* - TRANSLOCATION - GEODOP,PRECISE,(2,1,2)
+ - E6 SUBSET MULTI-STATION - GEODOP,PRECISE,(2,1,2)
@ - E6 FULL MULTI-STATION - GEODOP,PRECISE,(2,1,2)

PS - PARA TO SUNDOWN
PT - PARA TO THEILE
PB - PARA TO BAMBADIN
PG - PARA TO GAMBIER
ST - SUNDOWN TO THEILE
SB - SUNDOWN TO BAMBADIN
SG - SUNDOWN TO GAMBIER
TB - THEILE TO BAMBADIN
TG - THEILE TO GAMBIER
BG - BAMBADIN TO GAMBIER

5.4.4. Ephemeris Constraints

In order to determine the influence of apriori orbital constraints on relative positioning solutions , the following tests were performed. First , using the precise ephemeris with constraints of -

26 metres along-track

5 metres across-track

10 metres out-of-plane

chord distances were computed for the ten lines in the subset network from -

1. Individual translocations between all station pairs,
2. A multi-station solution involving the subset stations only,
3. A multi-station network involving the full E6 network.

Secondly , using the broadcast ephemeris with contraints of

2 metres along-track

1 metre across-track

2 metres out-of-plane

chord distances were computed for the ten lines in the subset network by means of a multi-station solution involving the full E6 network.

The results are presented in Tables 17-20. In addition they are illustrated on comparitive plots , these being Diagrams 17 to 20.

Consider first the precise ephemeris solutions. From the diagrams it is apparent that the effects of constraint relaxation vary from line to line. In the case of the translocation solutions , the differences between the relaxed and unrelaxed values range from 0.002 metres to 0.426 metres , the relaxed values being short in nine out of ten cases. The two multi-station solutions produce almost identical diagrams , the patterns of the corrections being the same as for the translocation case. The shortening varies from 0.002 metres to 0.253 metres in the instance of the full network.

The significance of these differences lies not so much in their magnitude as in the fact that they are systematically short. The diagrams seem to indicate that the correct orbital constraints produce a more balanced distribution of ground truth residuals about zero. A similar effect will be noted in section 5.4.5.

TABLE 17

CHORD DISTANCES FOR

TRANSLOCATION - GEODOP,PRECISE,(26,5,10)

LINE	OBSERVED	SCALE	CORRECTED	GROUND	DIFF
PS	410192.484	-.308	410192.176	410192.721	-.545
PT	209667.444	-.157	209667.287	209667.733	-.446
PB	254771.815	-.191	254771.624	254771.282	.342
PG	386380.032	-.290	386379.742	386379.474	.268
ST	269250.998	-.202	269250.796	269250.748	.048
SB	469880.078	-.352	469879.726	469879.595	.131
SG	661942.356	-.496	661941.860	661942.707	-.847
TB	203868.964	-.153	203868.811	203868.667	.144
TG	395100.634	-.296	395100.338	395101.183	-.845
BG	192246.017	-.144	192245.873	192246.617	-.744

LEGEND

OBSERVED - TRANSLOCATION - GEODOP,PRECISE,(26,5,10)
SCALE - SATELLITE DATUM TO GMA82 SCALE CORRECTION
CORRECTED - SATELLITE DISTANCE TRANSFORMED TO GMA82
GROUND - CHORD DISTANCE FROM GMA82 COORDINATES
DIFF - CORRECTED-GROUND

PS - PARA TO SUNDOWN
PT - PARA TO THEILE
PB - PARA TO BAMBADIN
PG - PARA TO GAMBIER
ST - SUNDOWN TO THEILE
SB - SUNDOWN TO BAMBADIN
SG - SUNDOWN TO GAMBIER
TB - THEILE TO BAMBADIN
TG - THEILE TO GAMBIER
BG - BAMBADIN TO GAMBIER

TABLE 18

CHORD DISTANCES FOR

E6 SUBSET MULTI-STATION - GEODOP,PRECISE,(26,5,10)

LINE	OBSERVED	SCALE	CORRECTED	GROUND	DIFF
PS	410192.681	-.308	410192.373	410192.721	-.348
PT	209667.331	-.157	209667.174	209667.733	-.559
PB	254771.904	-.191	254771.713	254771.282	.431
PG	386380.012	-.290	386379.722	386379.474	.248
ST	269251.017	-.202	269250.815	269250.748	.067
SB	469880.124	-.352	469879.772	469879.595	.177
SG	661942.477	-.496	661941.981	661942.707	-.726
TB	203868.953	-.153	203868.800	203868.667	.133
TG	395100.718	-.296	395100.422	395101.183	-.761
BG	192245.911	-.144	192245.767	192246.617	-.850

LEGEND

OBSERVED - E6 SUBSET MULTI-STATION - GEODOP,PRECISE,(26,5,10)
SCALE - SATELLITE DATUM TO GMA82 SCALE CORRECTION
CORRECTED - SATELLITE DISTANCE TRANSFORMED TO GMA82
GROUND - CHORD DISTANCE FROM GMA82 COORDINATES
DIFF - CORRECTED-GROUND

PS - PARA TO SUNDOWN
PT - PARA TO THEILE
PB - PARA TO BAMBADIN
PG - PARA TO GAMBIER
ST - SUNDOWN TO THEILE
SB - SUNDOWN TO BAMBADIN
SG - SUNDOWN TO GAMBIER
TB - THEILE TO BAMBADIN
TG - THEILE TO GAMBIER
BG - BAMBADIN TO GAMBIER

TABLE 19

CHORD DISTANCES FOR

E6 FULL MULTI-STATION - GEODOP,PRECISE,(26,5,10)

LINE	OBSERVED	SCALE	CORRECTED	GROUND	DIFF
PS	410192.704	-.308	410192.396	410192.721	-.325
PT	209667.233	-.157	209667.076	209667.733	-.657
PB	254771.763	-.191	254771.572	254771.282	.290
PG	386379.910	-.290	386379.620	386379.474	.146
ST	269251.048	-.202	269250.846	269250.748	.098
SB	469880.194	-.352	469879.842	469879.595	.247
SG	661942.454	-.496	661941.958	661942.707	-.749
TB	203868.992	-.153	203868.839	203868.667	.172
TG	395100.673	-.296	395100.377	395101.183	-.806
BG	192245.814	-.144	192245.670	192246.617	-.947

LEGEND

OBSERVED - E6 FULL MULTI-STATION - GEODOP,PRECISE,(26,5,10)
SCALE - SATELLITE DATUM TO GMA82 SCALE CORRECTION
CORRECTED - SATELLITE DISTANCE TRANSFORMED TO GMA82
GROUND - CHORD DISTANCE FROM GMA82 COORDINATES
DIFF - CORRECTED-GROUND

PS - PARA TO SUNDOWN
PT - PARA TO THEILE
PB - PARA TO BAMBADIN
PG - PARA TO GAMBIER
ST - SUNDOWN TO THEILE
SB - SUNDOWN TO BAMBADIN
SG - SUNDOWN TO GAMBIER
TB - THEILE TO BAMBADIN
TG - THEILE TO GAMBIER
BG - BAMBADIN TO GAMBIER

TABLE 20

CHORD DISTANCES FOR

E6 FULL MULTI-STATION - GEODOP,BROADCAST,(2,1,2)

LINE	OBSERVED	SCALE	CORRECTED	GROUND	DIFF
PS	410192.375	-.587	410191.788	410192.721	-.933
PT	209667.394	-.300	209667.094	209667.733	-.639
PB	254771.616	-.364	254771.252	254771.282	-.030
PG	386379.308	-.553	386378.755	386379.474	-.719
ST	269250.822	-.385	269250.437	269250.748	-.311
SB	469880.542	-.672	469879.870	469879.595	.275
SG	661941.604	-.947	661940.657	661942.707	-2.050
TB	203869.438	-.292	203869.146	203868.667	.479
TG	395099.973	-.565	395099.408	395101.183	-1.775
BG	192244.565	-.275	192244.290	192246.617	-2.327

LEGEND

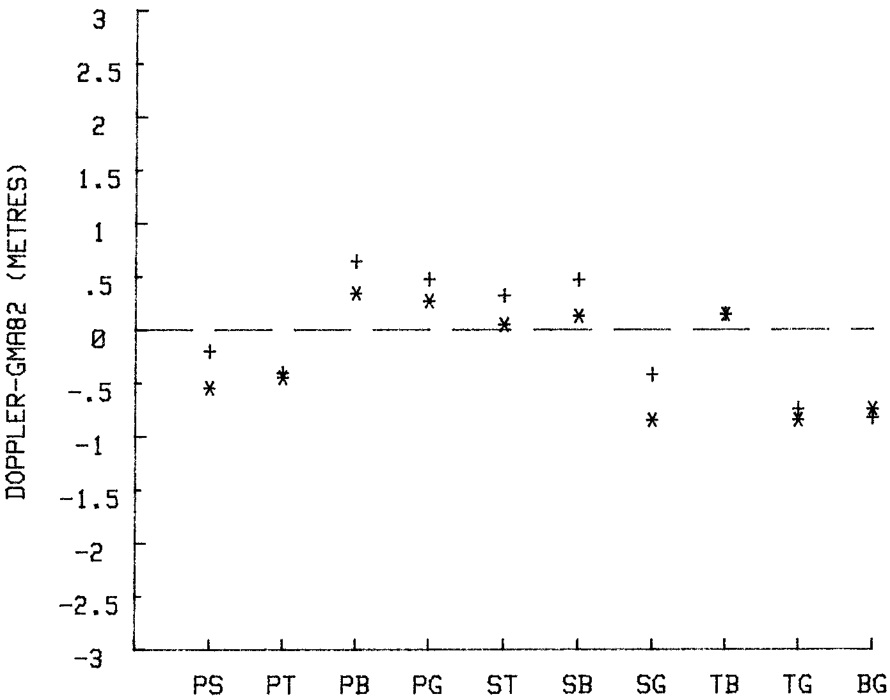
OBSERVED - E6 FULL MULTI-STATION - GEODOP,BROADCAST,(2,1,2)
SCALE - SATELLITE DATUM TO GMA82 SCALE CORRECTION
CORRECTED - SATELLITE DISTANCE TRANSFORMED TO GMA82
GROUND - CHORD DISTANCE FROM GMA82 COORDINATES
DIFF - CORRECTED-GROUND

PS - PARA TO SUNDOWN
PT - PARA TO THEILE
PB - PARA TO BAMBADIN
PG - PARA TO GAMBIER
ST - SUNDOWN TO THEILE
SB - SUNDOWN TO BAMBADIN
SG - SUNDOWN TO GAMBIER
TB - THEILE TO BAMBADIN
TG - THEILE TO GAMBIER
BG - BAMBADIN TO GAMBIER

DIAGRAM 17

EPHEMERIS CONSTRAINT COMPARISON

TRANSLOCATION SOLUTIONS



NOTE - Raw Doppler data is from Multi-Station Figure E6
Figure E6 was observed during the period 10-15/8/82
All distances have been transformed to the AGD

LEGEND

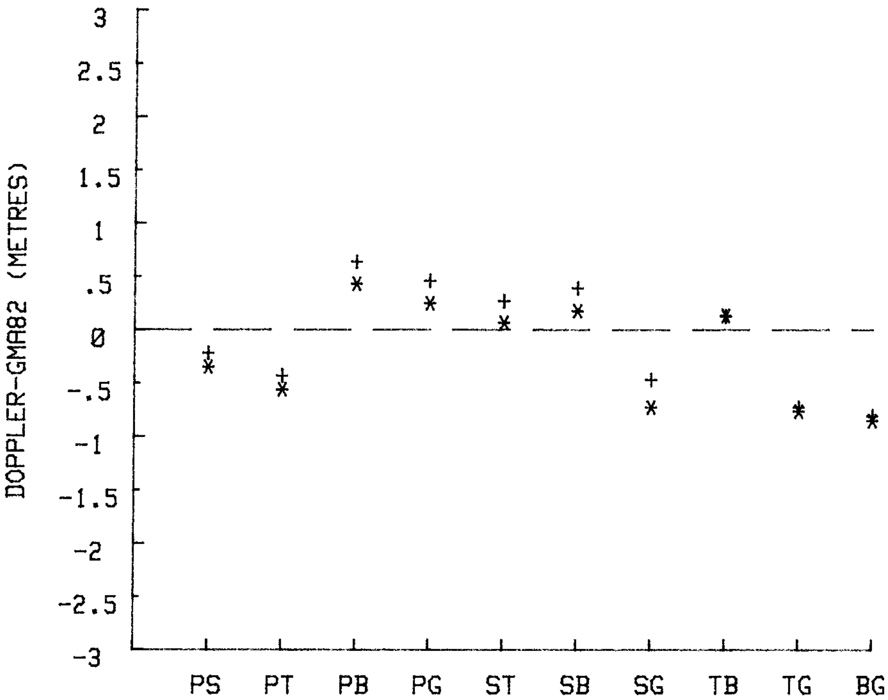
* - TRANSLOCATION - GEODOP,PRECISE,(26,5,10)
\+ - TRANSLOCATION - GEODOP,PRECISE,(2,1,2)

PS - PARA TO SUNDOWN
PT - PARA TO THEILE
PB - PARA TO BAMBADIN
PG - PARA TO GAMBIER
ST - SUNDOWN TO THEILE
SB - SUNDOWN TO BAMBADIN
SG - SUNDOWN TO GAMBIER
TB - THEILE TO BAMBADIN
TG - THEILE TO GAMBIER
BG - BAMBADIN TO GAMBIER

DIAGRAM 18

EPHEMERIS CONSTRAINT COMPARISON

E6 SUBSET SOLUTIONS



NOTE - Raw Doppler data is from Multi-Station Figure E6
Figure E6 was observed during the period 10-15/8/82
All distances have been transformed to the AGD

LEGEND

* - E6 SUBSET MULTI-STATION - GEODOP,PRECISE,(26,5,10)
\+ - E6 SUBSET MULTI-STATION - GEODOP,PRECISE,(2,1,2)

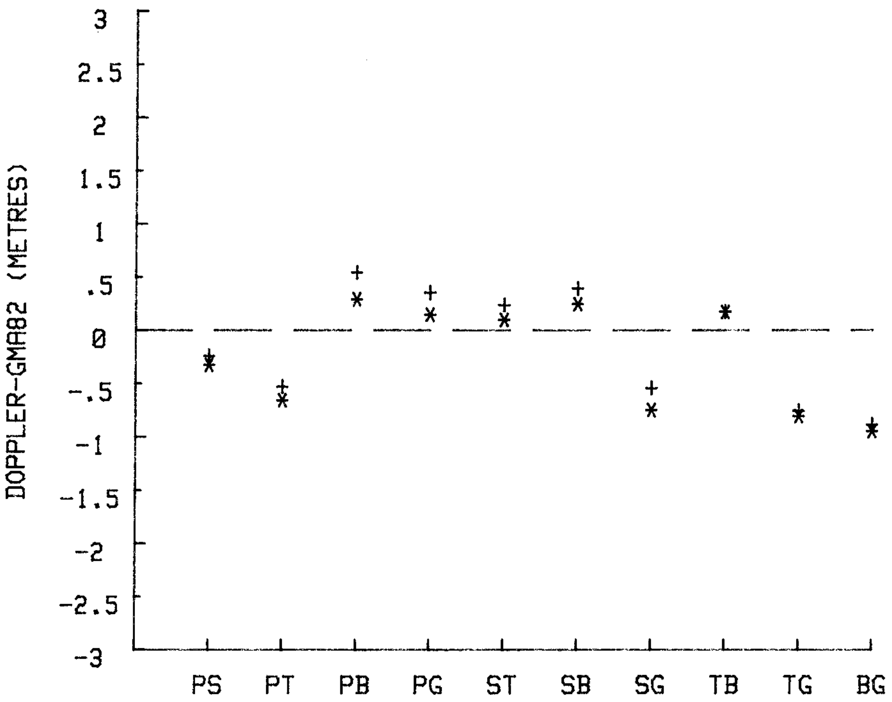
PS - PARA TO SUNDOWN
PT - PARA TO THEILE
PB - PARA TO BAMBADIN
PG - PARA TO GAMBIER
ST - SUNDOWN TO THEILE
SB - SUNDOWN TO BAMBADIN
SG - SUNDOWN TO GAMBIER
TB - THEILE TO BAMBADIN
TG - THEILE TO GAMBIER
BG - BAMBADIN TO GAMBIER

DIAGRAM 19

EPHEMERIS CONSTRAINT COMPARISON

E6 NETWORK SOLUTIONS

(A)



NOTE - Raw Doppler data is from Multi-Station Figure E6
Figure E6 was observed during the period 10-15/8/82
All distances have been transformed to the AGD

LEGEND

* - E6 FULL MULTI-STATION - GEODOP,PRECISE,(26,5,10)
\+ - E6 FULL MULTI-STATION - GEODOP,PRECISE,(2,1,2)

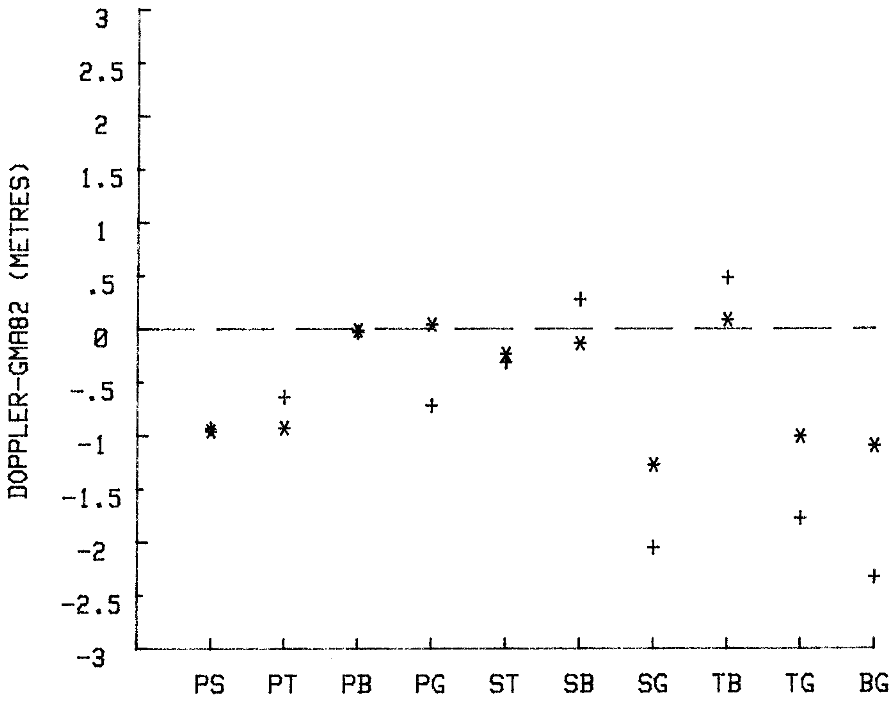
PS - PARA TO SUNDOWN
PT - PARA TO THEILE
PB - PARA TO BAMBADIN
PG - PARA TO GAMBIER
ST - SUNDOWN TO THEILE
SB - SUNDOWN TO BAMBADIN
SG - SUNDOWN TO GAMBIER
TB - THEILE TO BAMBADIN
TG - THEILE TO GAMBIER
BG - BAMBADIN TO GAMBIER

DIAGRAM 20

EPHEMERIS CONSTRAINT COMPARISON

E6 NETWORK SOLUTIONS

(B)



NOTE - Raw Doppler data is from Multi-Station Figure E6
Figure E6 was observed during the period 10-15/8/82
All distances have been transformed to the AGD

LEGEND

* - E6 FULL MULTI-STATION - GEODOP,BROADCAST,(26,5,10)
\+ - E6 FULL MULTI-STATION - GEODOP,BROADCAST,(2,1,2)

PS - PARA TO SUNDOWN
PT - PARA TO THEILE
PB - PARA TO BAMBADIN
PG - PARA TO GAMBIER
ST - SUNDOWN TO THEILE
SB - SUNDOWN TO BAMBADIN
SG - SUNDOWN TO GAMBIER
TB - THEILE TO BAMBADIN
TG - THEILE TO GAMBIER
BG - BAMBADIN TO GAMBIER

The broadcast ephemeris solutions (Diagram 20) contrast markedly with those discussed above. The effects of constraint tightening again vary from line to line. However , on this occasion , both lengthening and shortening takes place.

It is interesting to note that all of the lines out of Gambier were substantially shortened , the remainder being either lengthened slightly or remaining significantly the same. Indeed the results in general suggest that the observations at Gambier were of poor quality , probably because of multi-path interference from a nearby lake. It would seem that the solution involving realistic orbital biases (26,5,10) was able to absorb a proportion of the error from this source. However the tightening of the constraints effectivly inhibited orbit relaxation , forcing the error back into the station coordinates. This in turn caused the solution to degrade. Diagram 20 therefore clearly illustrates the extent to which orbit relaxation can improve solution accuracy , even at a poorly observed station.

5.4.5. Ephemeris Type

It was mentioned in the introduction that the popularity of relative positioning had largely been brought about by the assumption that it could produce accuracies from the broadcast ephemeris which compared favorably with those from the precise ephemeris. In order to assess the validity of this proposition , the results of the network size tests were replotted , this time to enable the broadcast and precise ephemeris solutions to be compared. These plots are included as Diagrams 21 , 22 and 23.

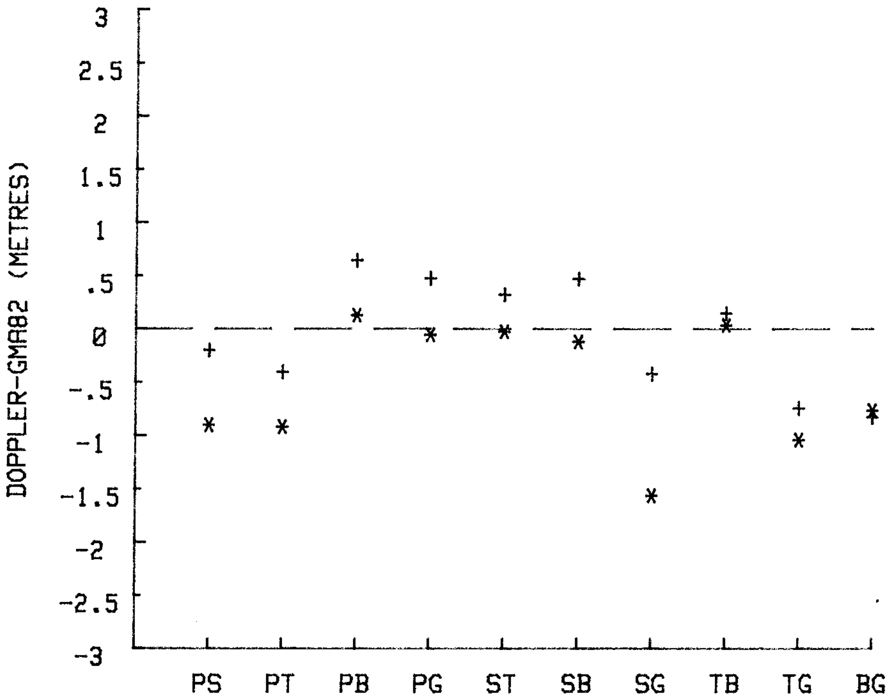
All three diagrams are very similar . The differences between the broadcast and precise ephemeris solutions follow the same pattern in each case. The magnitudes of the differences range between 0.063 metres and 1.142 metres for the translocation solutions , becoming slightly smaller for the multi-station solutions.

Interestingly enough , the pattern of corrections is the same as that which became apparent during the precise ephemeris constraint investigations. Indeed there is little to distinguish the two sets of diagrams apart from the magnitudes of the residuals involved. The systematic shortening is again in evidence and again appears to be unrelated to line length. In general it would appear the the broadcast ephemeris solutions are biased by approximately

DIAGRAM 21

EPHEMERIS TYPE COMPARISON

TRANSLOCATION SOLUTIONS



NOTE - Raw Doppler data is from Multi-Station Figure E6
Figure E6 was observed during the period 10-15/8/82
All distances have been transformed to the AGD

LEGEND

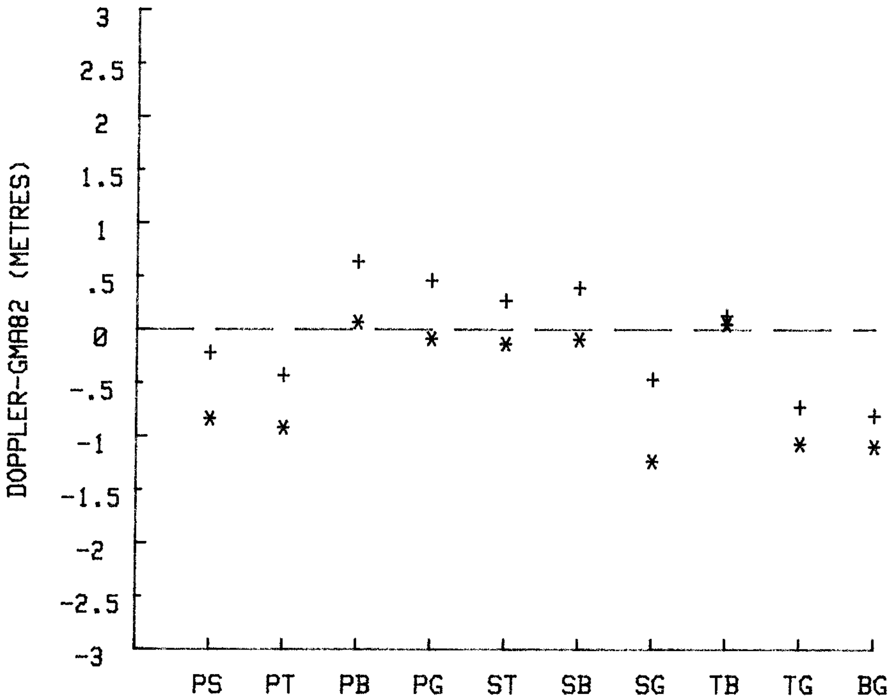
* - TRANSLOCATION - GEODOP,BROADCAST,(26,5,10)
\+ - TRANSLOCATION - GEODOP,PRECISE,(2,1,2)

PS - PARA TO SUNDOWN
PT - PARA TO THEILE
PB - PARA TO BAMBADIN
PG - PARA TO GAMBIER
ST - SUNDOWN TO THEILE
SB - SUNDOWN TO BAMBADIN
SG - SUNDOWN TO GAMBIER
TB - THEILE TO BAMBADIN
TG - THEILE TO GAMBIER
BG - BAMBADIN TO GAMBIER

DIAGRAM 22

EPHEMERIS TYPE COMPARISON

E6 SUBSET SOLUTIONS



NOTE - Raw Doppler data is from Multi-Station Figure E6
Figure E6 was observed during the period 10-15/8/82
All distances have been transformed to the AGD

LEGEND

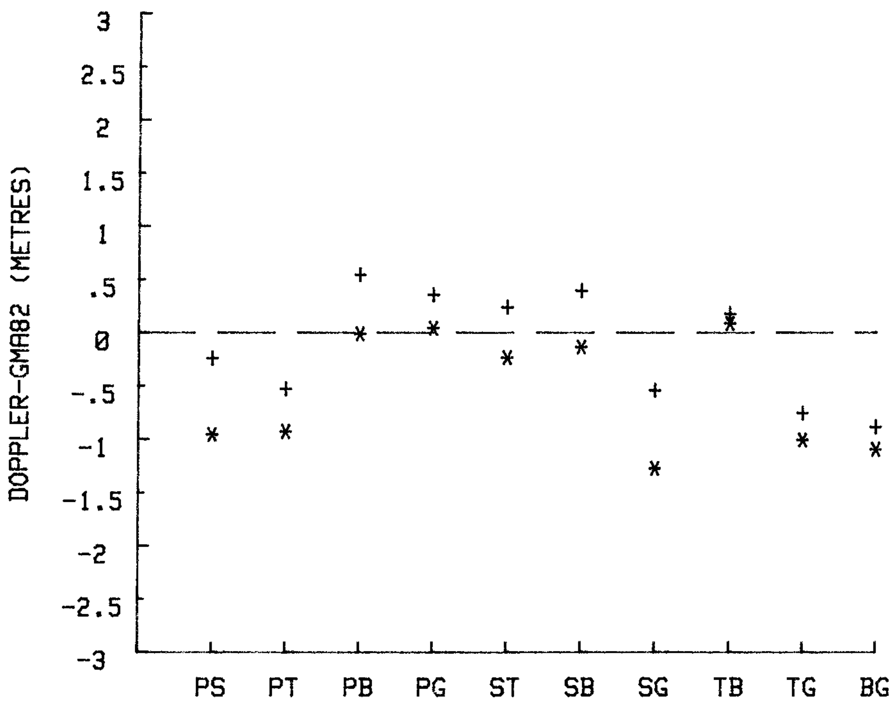
* - E6 SUBSET MULTI-STATION - GEODOP,BROADCAST,(26,5,10)
+ - E6 SUBSET MULTI-STATION - GEODOP,PRECISE,(2,1,2)

PS - PARA TO SUNDOWN
PT - PARA TO THEILE
PB - PARA TO BAMBADIN
PG - PARA TO GAMBIER
ST - SUNDOWN TO THEILE
SB - SUNDOWN TO BAMBADIN
SG - SUNDOWN TO GAMBIER
TB - THEILE TO BAMBADIN
TG - THEILE TO GAMBIER
BG - BAMBADIN TO GAMBIER

DIAGRAM 23

EPHEMERIS TYPE COMPARISON

E6 NETWORK SOLUTIONS



NOTE - Raw Doppler data is from Multi-Station Figure E6
Figure E6 was observed during the period 10-15/8/82
All distances have been transformed to the AGD

LEGEND

* - E6 FULL MULTI-STATION - GEODOP,BROADCAST,(26,5,10)
\+ - E6 FULL MULTI-STATION - GEODOP,PRECISE,(2,1,2)

PS - PARA TO SUNDOWN
PT - PARA TO THEILE
PB - PARA TO BAMBADIN
PG - PARA TO GAMBIER
ST - SUNDOWN TO THEILE
SB - SUNDOWN TO BAMBADIN
SG - SUNDOWN TO GAMBIER
TB - THEILE TO BAMBADIN
TG - THEILE TO GAMBIER
BG - BAMBADIN TO GAMBIER

-0.5 metres whereas the precise solutions have a bias value approaching zero. The distribution of the residuals about the bias values is approximately the same in each case. Their ranges vary between 1.30 metres and 1.70 metres but are in general below 1.50 metres. This suggests that the ephemerides produce equivalent precisions but different accuracies from the same data sets. It further suggests that the results are consistent with precision values in the range 0.3 to 0.5 metres.

In order to investigate the shortening effect further , two additional tests were performed. In the first , chord distances from the broadcast and precise multi-station solutions of the full E6 network were compared. Their differences were found to vary from 1.024 ppm to -3.116 ppm , the broadcast ephemeris distances being longer on only 6 of the 55 lines. The second test involved the comparison of coordinate values and was performed as follows.

The multi-station coordinate sets (broadcast and precise) for both the full and subset networks were transformed into the AGD (GMA82 coordinate system) using program CHORD7. Residual differences between the transformed and ground truth coordinates were subsequently computed following the removal of average block shifts. The transformed coordinates and residuals have been tabulated and are included as Appendix 2. In addition , the latitude and longitude residuals are plotted as displacement vectors in Diagrams 24 and 25 while the height residuals are plotted in Diagrams 26 and 27.

Consider first Diagram 25 . It can be seen that the displacement vectors are predominently orientated in an east-west direction and that the broadcast ephemeris vectors are longer than those of the precise ephemeris. This is consistent with the chords in that it implies a shortening in the broadcast ephemeris relative to the precise. A second systematic trend is also apparent , this being the tendancy of the vectors to point towards the centre of the network. This will be discussed further in Section 5.5.

DIAGRAM 24
DISPLACEMENT VECTORS
E6 SUBSET NETWORK
Scale of Displacement Vectors
1 Cm.= 0.5 Metres

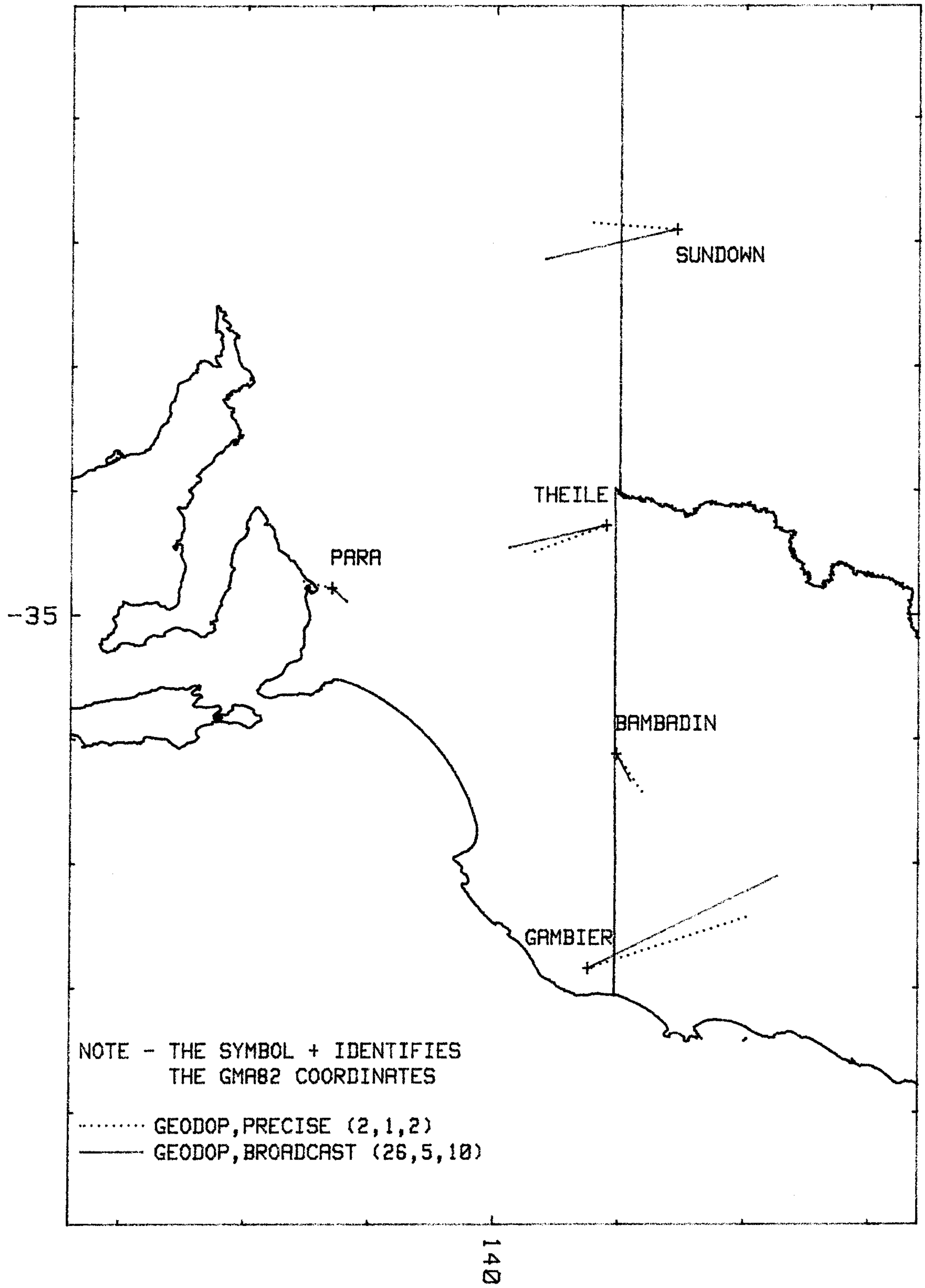


DIAGRAM 25

DISPLACEMENT VECTORS - E6 FULL NETWORK

Scale of Displacement Vectors - 1 Cm.= 0.5 Metres

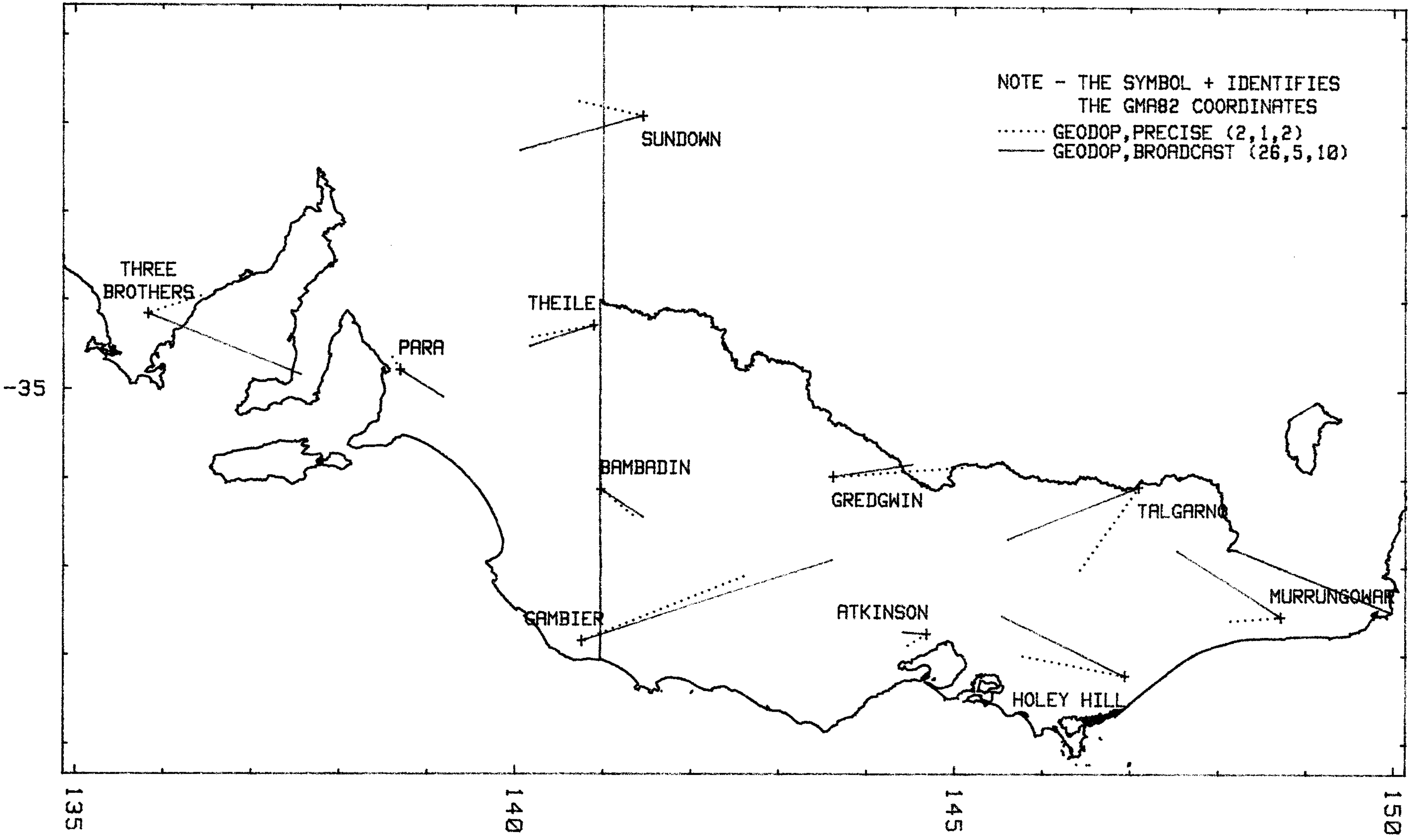
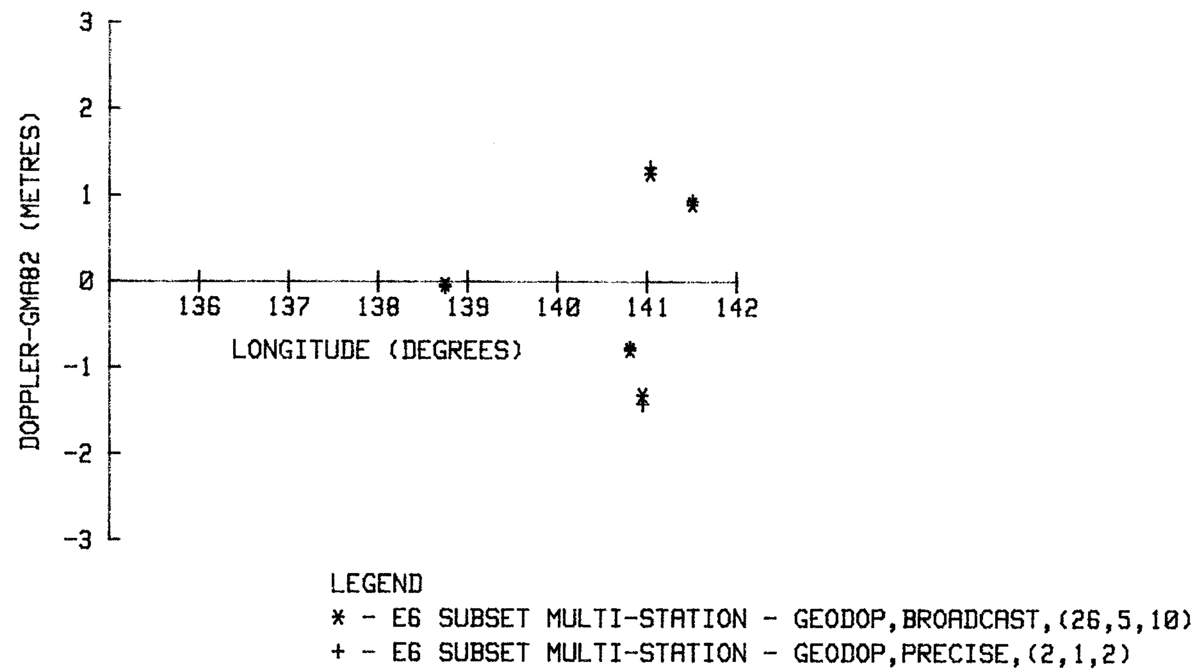


DIAGRAM 26

MULTI-STATION HEIGHT RESIDUALS

E6 SUBSET SOLUTIONS



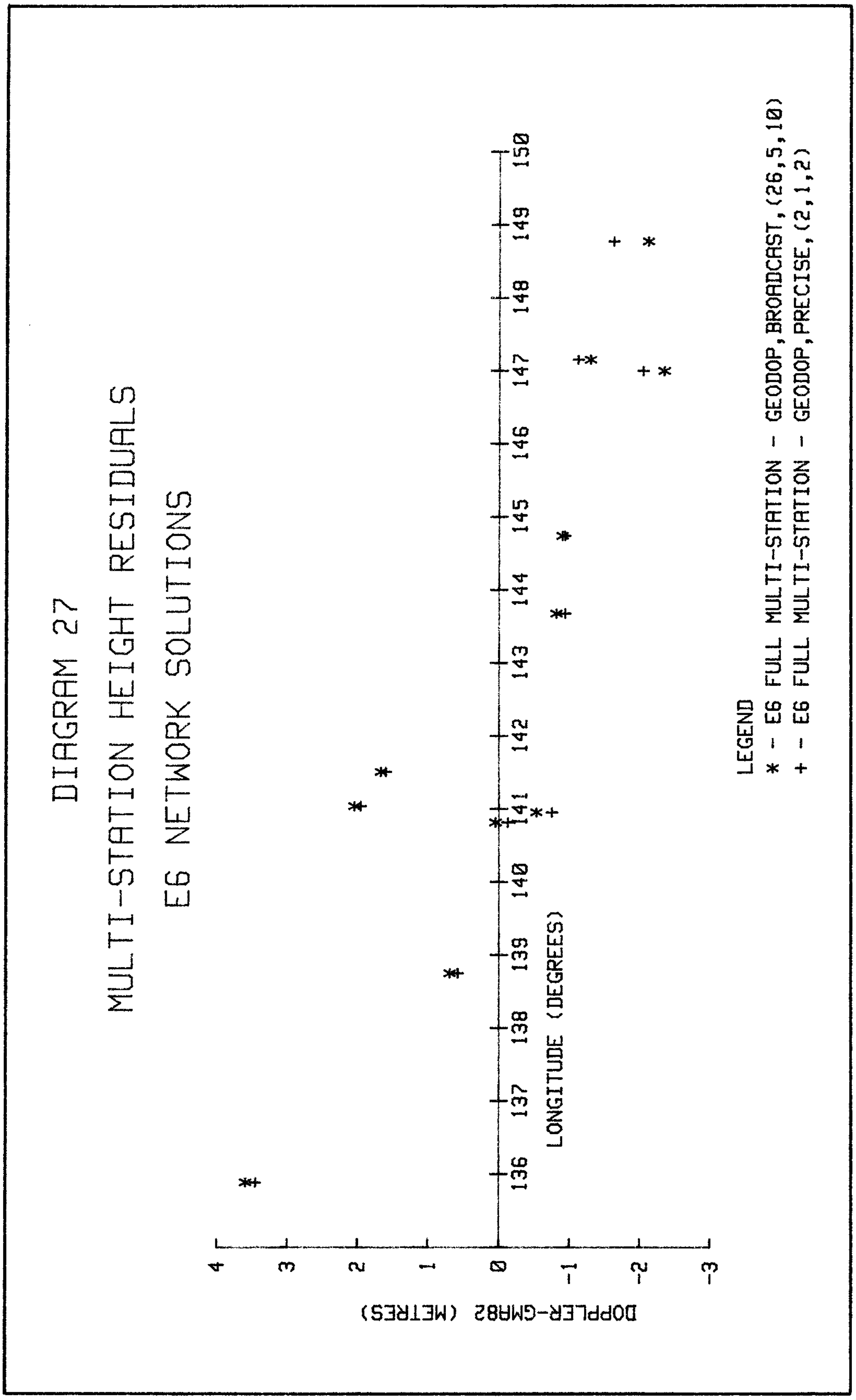


DIAGRAM 27
MULTI-STATION HEIGHT RESIDUALS
E6 NETWORK SOLUTIONS

5.5. Analysis

Consider first the comparison between the MX1502 and the GEODOP broadcast translocation solutions. In 1976 , Kouba and Wells supported the proposition that all statistically acceptable passes be included in relative positioning solutions , regardless of whether they were common or not. They argued that this would result in the retention of a sound balance of data at each station as in point positioning. They went on to say that by proceding in this manner -

> ' ..... all stations are biased consistently (the translations , rotations and scale at each station , with respect to the Terrestrial system are similar) ; the relative accuracy is significantly increased , (more so than for translocations or simultaneous point positioning) and proper correlation between stations is preserved'
>
> (Kouba and Wells 1976 A)

The modelling in program GEODOP is consistent with the above statements. The MX1502 however , processes only common passes. The reduction program comparison therefore contrasts not only the effects of different modelling techniques but also two different philosophies of computation.

The MX1502 and GEODOP results were contrasted during both the reduction program comparison and the repeatability tests. The MX1502 consistently produced the larger residuals relative to ground truth. On both occasions , the differences between the two solution sets were at the one metre level. This is consistent with results obtained by Videla et al (1982 C). Following field operations in the Venezualan Guayana , translocation solutions were computed using both the MX1502 software and program MAGNET. The solutions involved the processing of between twenty-two and forty-one passes , the exact number varying from line to line. Differences of between 0.78 metres and 1.70 metres were noted between the solutions over distances ranging between 226 Km. and 461 Km. Unfortunately no ground truth was available against which to compare the results. Consequently , it is not possible to determine whether one program was consistently producing inferior results.

In 1980 Magnavox was claiming in its advertising material (e.g. Bulletin Geodesique , Vol 54 , No 4 , 1980) that the MX1502 on-board software would provide relative positionin accuracies with a standard deviation of one metre over distances of up to 1000 Km. The above results suggest that this figure is realistic for distances over 200 Km. They also suggest that the statement by Kouba and Wells regarding program philosophy may be correct in practice.

The repeatability tests have already been discussed in some detail in section 5.4.2. and will not be reconsidered here. Three points should be noted however. First of all , the repeatability obtainable from the MX1502 compares very favorably with that obtainable from GEODOP. This is clearly illustrated in Diagrams 13 and 14. Secondly it would appear that the MX1502 is more sensitive to the detection of poor passes than GEODOP. This is reflected in the results for line Para-Sundown. The improvement in repeatability that the tighter editing apparently produces suggests that the criterion for pass rejection in PREDOP and GEODOP should be reconsidered. Finally , it must be commented that repeatability effectivly provides a measure of the precision of a measuring techniques. Consequently it may be concluded that the standard deviations for the line Para-Sundown which are presented in Table 10 are representative of the precisions available from translocation techniques.

The results of the network size test agree in some respects with those suggested by Ross (1982 C). Ross suggests that after fifty passes , the relative accuracy of a two station solution will be 21 cm. whereas that of a ten station solution will be 7.5 cm. It is impossible to agree with these figures as regards the accuracy of multi-station networks given the results stated above. However , they do support the proposition that network size has no significant effect on accuracy if a large number of passes (>50) are processed. It must be added of course that this conclusion does not necessarily hold for networks with a lesser number of passes.

The ephemeris constraint and the ephemeris type tests provide the most curious results of all. The apparent systematic shortening between the precise ephemeris solutions , the relaxed precise ephemeris solutions and the broadcast ephemeris solutions requires an explanation , as does the predominent east-west orientation of the displacement vectors. Four possible theories come to mind , two which are best explained by reference to the vectors diagrams.

The first theory is that there is a modelling problem in GEODOP which is causing a 'simple' shortening in longitude. An examination of Diagrams 24 and 25 suggest that this in unlikely to be the case. The vectors generated from the subset network are substantially the same as those from the full net. They show no tendancy to pull towards the centre of the network. Indeed if anything they suggest a network rotation. Thus a program-induced shortening in longitude appears improbable.

The second theory is that the program constants which define the scale of the network are inconsistent with those which were used in the generation of the ephemerides. Consequently there are two scale definitions conflicting with each other in the multi-station computations. If tight orbital constraints are used , as is the case with the precise ephemeris , then the scale as defined by the ephemeris will be very influential. However , as the constraints are relaxed , the scale as defined by the program will dominate the computations. The failing in this argument is that both the MX1502 and GEODOP broadcast solutions produced approximately the same biases, suggesting that the problem is not program based (Vide Diagram 12). In addition the systematic changes do not appear to be dependent on line length and thus cannot be simple scale biases. However as the modelling of a Doppler program is inherently complex , the theory could only be proved by altering the program constants and noting the effect.

The third theory is that the vectors reflect systematic errors in the ephemerides. Note that the vectors in Diagram 25 suggest rotations about two points , one being east of Theile and the other south of Murrungowar. In addition the systematic slope of the height residuals as displayed in Diagram 27 strongly suggests a tilt between the satellite and terrestrial datums. Malyevac and Anderle (1982 C) noted secular along-track errors at Southern Hemisphere tracking stations when investigating force field modifications using the NOVA satellite. These errors were removed when odd zonal harmonics were introduced as part of the solution , indicating a possible deficency in the current ephemeris gravity models. This theory could best be investigated by reobserving the multi-station network or by re-reducing the existing data using a short-arc program with an improved gravity model.

The final theory is that the vectors have achieved their values through statistical chance and that a second data set would produce completely different results. This is a possibility that cannot be ignored. It is certainly true that there is a random component in the displacement vectors as displayed. It is therefore quite probable that a reobservation of the network would produce a significantly different vector pattern. However the shortening of the chord distances has resulted from the processing of the same Doppler data sets with different ephemeris parameters. This suggests that influences other than random chance are at work. The argument against the theory is further strengthened by the results of Kouba and Wells (1976 A) which also showed a predominent shortening of broadcast chord distances relative to precise chord distances. These tests involved a multi-station network in Canada which contained lines of up to 1268 Km. in length. The number of passes involved were comparible with those of this project. Interestingly , a significant east-west component was again evident between the two systems.

Tenuous evidence against the fourth theory is also provided by the repeatabilty tests for the line Para-Sundown. The precise ephemeris and MX1502 biases determined from data sets acquired five months apart show good repeatability.

It is possible that a combination of the above theories is responsible for the results which have been obtained. Certainly they deserve further investigation as the magnitudes of the biases effectivly limit the accuracy that can be obtained from relative positioning techniques. The biases also have significant implications for the incorporation of Doppler data into terrestrial networks. In particular they provide an argument for the generation of local or regional transformation parameters. The apparent systematic biases were only detected because national parameters were used to transform the Doppler data. Had local parameters been generated , the biases would largly have been absorbed. Clearly such absorption would be very desirable in many situations as it would effectivly improve the accuracy of the measurements.

However , any enthusiasm for local parameters must be tempered by the realisation that the results as presented have been derived from a single data set. Proof that these vectors could be repeated would be a necessary prerequisite for local parameter generation. At the present time , this proof does not exist. It is of course possible to improve multi-station results by generating local parameters for each job that is undertaken. However in general , this not an economic proposition as it necessitates the occupation of at least three known stations during every project. Consequently , it must be concluded that at the present time , the GMA82 parameters are the best available for datum transformations.

CHAPTER 6

CONCLUSIONS

It was stated in the introduction that it was intended that the above investigations should contribute to the growing body of data which is becoming available regarding relative positioning techniques. Indeed this is all they can do. When all is said and done , most of the testing done during this project was performed using a statistical sample of one. Consequently while the results were backed by external evidence whenever possible , they cannot on their own be considered representative of the accuracies achievable from Doppler positioning.

Having said that , the following conclusions may be drawn from the results and analysis as presented.

1. The MX1502 on-board software appears capable of computing relative positions to an accuracy of one metre (1σ) over distances in excess of 200 Km.

2. The repeatability achievable from the MX1502 on-board software is competitive with that achievable from GEODOP using the precise ephemeris , even during periods of high solar radiation pressure. The repeatability of solutions derived from GEODOP using the broadcast ephemeris is severely degraded during such periods. All three modes of

reduction give similar repeatability during less extreme conditions. However each is subject to different biases relative to the ground truth value. The most consistent repeatabilty may be obtained using the precise ephemeris.

3. Relative positioning precisions do not appear to be highly dependent on the number of stations in the multi-station network if fifty or more passes are involved in the processing.

4. Systematic biases between broadcast and precise ephemeris solutions have been noted. It is not certain whether these are program or ephemeris based , or indeed whether they are a chance occurance. Further testing is required to resolve these points.

5. A comparison of solutions suggests that similar residual distributions will be obtained from both broadcast and precise ephemeris processing. The results obtained from GEODOP are consistent with a standard deviation of 0.3 - 0.5 metres relative to a mean bias value. Thus it may be concluded that the ephemerides produce equivalent precisions when used in either the translocation or multi-station mode.

6. The apparent detection of systematic biases prevents the making of any positive statement about accuracy at this stage. However it may be very cautiously concluded

from the bias values that the best accuracy appears to be given by processing with the precise ephemeris.

Finally , a few comments should be made about areas of further research. First of all , a considerable amount of the Doppler data gathered during the GMA82 project still has to be analysed. It is recommended that this be done in the future to confirm or confound the results which have been presented here. It is important to know whether the apparent systematic biases are geographically correlated. Very little research has been done into Doppler positioning in the Southern Hemisphere. It is desirable that this situation be changed.

Secondly , the conclusions regarding network size were based on a data set containing over sixty passes. Experiments involving smaller data sets would be very useful in determining the optimum number of passes required to obtain a specified accuracy from a networks of various sizes.

Finally , the body of data which is available regarding relative positioning accuracies is steadily growing. A complete analysis of this collective data set would make a very worthwhile project. Only through such research will a realistic assessment be made of the accuracies attainable from Doppler positioning.

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APPENDIX 1

TRANSFORMATION PARAMETERS - GMA82 PRELIMINARY VALUES

Parameter	Precise to GMA82	Broadcast to Precise	Broadcast to GMA82
T_X	116.47±1.4	-5.9±3.5	112.29±3.46
T_Y	50.25±1.3	-0.3±3.4	50.56±3.17
T_Z	-138.87±1.7	-1.6±4.1	-142.73±3.94
θ_X	0.21±0.05	-0.03±0.12	0.16±0.11
θ_Y	0.36±0.05	0.11±0.12	0.37±0.12
θ_Z	-0.47±0.05	0.12±0.11	-0.40±0.12
ε	-0.75±0.03	-0.41±0.20	-1.43±0.35

The information in this table was taken from Allman (1983a A).

The translations are in metres.

The rotations are in seconds of arc.

The scale factor ε is in parts per million.

Stated precisions are at the 1σ level.

APPENDIX 2

MULTI-STATION COORDINATES AND RESIDUALS

TABLE A2-1

E6 SUBSET NETWORK

MULTI-STATION BROADCAST (26,5,10)

STATION	LATITUDE	LONGITUDE	HEIGHT
PARA	-34 47 6.3365	138 41 32.9066	214.42
SUNDOWN	-31 53 57.2127	141 26 59.4768	404.51
THEILE	-34 16 53.5378	140 53 38.0755	50.97
BAMBADIN	-36 7 4.3188	140 58 41.0906	154.52
GAMBIER	-37 50 27.7383	140 45 23.2929	170.46

TABLE A2-2

E6 SUBSET NETWORK

MULTI-STATION PRECISE (2,1,2)

STATION	LATITUDE	LONGITUDE	HEIGHT
PARA	-34 47 6.3437	138 41 32.8261	215.94
SUNDOWN	-31 53 57.2239	141 26 59.4171	406.12
THEILE	-34 16 53.5533	140 53 38.0101	52.48
BAMBADIN	-36 7 4.3307	140 58 41.0223	156.24
GAMBIER	-37 50 27.7526	140 45 23.2121	172.16

TABLE A2-3

E6 SUBSET NETWORK

TRANSFORMED MULTI-STATION BROADCAST (26,5,10)

STATION	LATITUDE	LONGITUDE	HEIGHT
PARA	-34 47 11.4709	138 41 28.8118	229.92
SUNDOWN	-31 54 2.5050	141 26 55.6438	408.06
THEILE	-34 16 58.7664	140 53 34.0984	61.66
BAMBADIN	-36 7 9.5104	140 58 37.0141	169.84
GAMBIER	-37 50 32.8789	140 45 19.1011	190.55

TRANSFORMATION PARAMETERS

DX = 112.29 DY = 50.56 DZ = -142.73

RX = -.16 RY = -.37 DZ = .40

SC = -1.43 PPM.

TABLE A2-4

E6 SUBSET NETWORK

TRANSFORMED MULTI-STATION PRECISE (2,1,2)

STATION	LATITUDE	LONGITUDE	HEIGHT
PARA	-34 47 11.4623	138 41 28.7324	230.83
SUNDOWN	-31 54 2.4922	141 26 55.5923	409.03
THEILE	-34 16 58.7646	140 53 34.0399	62.49
BAMBADIN	-36 7 9.5102	140 58 36.9529	170.84
GAMBIER	-37 50 32.8861	140 45 19.0268	191.50

TRANSFORMATION PARAMETERS

DX = 116.47 DY = 50.25 DZ = -138.87

RX = -.21 RY = -.36 DZ = .47

SC = -.75 PPM.

TABLE A2-5

E6 FULL NETWORK

MULTI-STATION BROADCAST (26,5,10)

STATION	LATITUDE	LONGITUDE	HEIGHT
PARA	-34 47 6.4019	138 41 32.8958	214.67
SUNDOWN	-31 53 57.2762	141 26 59.4565	404.80
THEILE	-34 16 53.5992	140 53 38.0643	51.28
BAMBADIN	-36 7 4.3809	140 58 41.0798	154.83
GAMBIER	-37 50 27.8014	140 45 23.2979	170.83
THREE BR.	-34 9 32.8115	135 49 48.0162	209.59
ATKINSON	-37 45 26.9794	144 40 57.6505	142.69
GREDGWIN	-35 58 16.7124	143 37 10.6549	155.18
TALGARNO	-36 5 1.3009	147 5 47.1644	654.48
MURRUNGOWAR	-37 33 42.8286	148 42 57.1211	734.92
HOLEY HILL	-38 13 54.7763	146 56 23.3058	219.76

TABLE A2-6

E6 FULL NETWORK

MULTI-STATION PRECISE (2,1,2)

STATION	LATITUDE	LONGITUDE	HEIGHT
PARA	-34 47 6.3470	138 41 32.8289	215.94
SUNDOWN	-31 53 57.2264	141 26 59.4174	406.14
THEILE	-34 16 53.5543	140 53 38.0082	52.51
BAMBADIN	-36 7 4.3331	140 58 41.0214	156.23
GAMBIER	-37 50 27.7527	140 45 23.2133	172.17
THREE BR.	-34 9 32.7481	135 49 47.9406	210.71
ATKINSON	-37 45 26.9311	144 40 57.5848	144.29
GREDGWIN	-35 58 16.6657	143 37 10.6093	156.63
TALGARNO	-36 5 1.2622	147 5 47.1160	656.34
MURRUNGOWAR	-37 33 42.7965	148 42 57.0594	737.16
HOLEY HILL	-38 13 54.7334	146 56 23.2400	221.77

TABLE A2-7

E6 FULL NETWORK

TRANSFORMED MULTI-STATION BROADCAST (26,5,10)

STATION	LATTITUDE	LONGITUDE	HEIGHT
PARA	-34 47 11.5363	138 41 28.8009	230.17
SUNDOWN	-31 54 2.5685	141 26 55.6235	408.35
THEILE	-34 16 58.8278	140 53 34.0872	61.97
BAMBADIN	-36 7 9.5725	140 58 37.0033	170.15
GAMBIER	-37 50 32.9420	140 45 19.1061	190.92
THREE BR.	-34 9 37.8503	135 49 43.8494	228.18
ATKINSON	-37 45 32.2761	144 40 53.6490	156.70
GREDGWIN	-35 58 22.0074	143 37 6.7065	166.07
TALGARNO	-36 5 6.7202	147 5 43.3817	660.49
MURRUNGOWAR	-37 33 48.2787	148 42 53.3427	742.62
HOLEY HILL	-38 14 .1480	146 56 19.3919	231.78

TRANSFORMATION PARAMETERS

DX = 112.29 DY = 50.56 DZ = -142.73

RX = -.16 RY = -.37 DZ = .40

SC = -1.43 PPM.

TABLE A2-8

E6 FULL NETWORK

TRANSFORMED MULTI-STATION PRECISE (2,1,2)

STATION	LATITUDE	LONGITUDE	HEIGHT
PARA	-34 47 11.4656	138 41 28.7352	230.83
SUNDOWN	-31 54 2.4947	141 26 55.5926	409.05
THEILE	-34 16 58.7656	140 53 34.0380	62.52
BAMBADIN	-36 7 9.5126	140 58 36.9520	170.83
GAMBIER	-37 50 32.8862	140 45 19.0280	191.51
THREE BR.	-34 9 37.7691	135 49 43.7677	228.80
ATKINSON	-37 45 32.2204	144 40 53.6008	157.44
GREDGWIN	-35 58 21.9482	143 37 6.6752	166.71
TALGARNO	-36 5 6.6689	147 5 43.3572	661.44
MURRUNGOWAR	-37 33 48.2381	148 42 53.3099	743.88
HOLEY HILL	-38 14 .0990	146 56 19.3500	232.85

TRANSFORMATION PARAMETERS

DX = 116.47 DY = 50.25 DZ = -138.87

RX = -.21 RY = -.36 DZ = .47

SC = -.75 PPM.

TABLE A2-9

E6 SUBSET NETWORK

LATITUDE COMPARISON

MULTI-STATION BROADCAST (26,5,10) VERSUS GMA82 ADJUSTMENT

	1	2	BS	2S	2S-1
PARA	-34 47 11.4855	11.4709	-.0179	11.4889	-.0034
SUNDOWN	-31 54 2.5152	2.5050	-.0179	2.5230	-.0078
THEILE	-34 16 58.7785	58.7664	-.0179	58.7843	-.0058
BAMBADIN	-36 7 9.5213	9.5104	-.0179	9.5283	-.0070
GAMBIER	-37 50 32.9209	32.8789	-.0179	32.8968	.0241

LEGEND

1 - GMA82 ADJUSTMENT

2 - TRANSFORMED MULTI-STATION BROADCAST (26,5,10)

BS - BLOCK SHIFT

2S - 2 MINUS THE BLOCK SHIFT

2S-1 - TRANSFORMED MULTI-STATION (2S) MINUS GROUND TRUTH

TABLE A2-10

E6 SUBSET NETWORK

LONGITUDE COMPARISON

MULTI-STATION BROADCAST (26,5,10) VERSUS GMA82 ADJUSTMENT

	1	2	BS	2S	2S-1
PARA	138 41 28.7379	28.8118	-.0693	28.7424	.0045
SUNDOWN	141 26 55.6147	55.6439	-.0693	55.5745	-.0402
THEILE	140 53 34.0598	34.0985	-.0693	34.0291	-.0307
BAMBADIN	140 58 36.9403	37.0141	-.0693	36.9448	.0045
GAMBIER	140 45 18.9700	19.1012	-.0693	19.0318	.0618

LEGEND

1 - GMA82 ADJUSTMENT

2 - TRANSFORMED MULTI-STATION BROADCAST (26,5,10)

BS - BLOCK SHIFT

2S - 2 MINUS THE BLOCK SHIFT

2S-1 - TRANSFORMED MULTI-STATION (2S) MINUS GROUND TRUTH

TABLE A2-11

E6 SUBSET NETWORK

HEIGHT COMPARISON

MULTI-STATION BROADCAST (26,5,10) VERSUS GMA82 ADJUSTMENT

	1	2	BS	2S	2S-1
PARA	230.600	229.927	.635	230.562	-.038
SUNDOWN	407.800	408.068	.635	408.703	.903
THEILE	63.620	61.663	.635	62.298	-1.322
BAMBADIN	169.220	169.844	.635	170.479	1.259
GAMBIER	191.990	190.554	.635	191.189	-.801

LEGEND

1 - GMA82 ADJUSTMENT

2 - TRANSFORMED MULTI-STATION BROADCAST (26,5,10)

BS - BLOCK SHIFT

2S - 2 MINUS THE BLOCK SHIFT

2S-1 - TRANSFORMED MULTI-STATION (2S) MINUS GROUND TRUTH

TABLE A2-12

E6 SUBSET NETWORK

LATITUDE COMPARISON

MULTI-STATION PRECISE (2,1,2) VERSUS GMA82 ADJUSTMENT

	1	2	BS	2S	2S-1
PARA	-34 47 11.4855	11.4623	-.0212	11.4835	.0020
SUNDOWN	-31 54 2.5152	2.4922	-.0212	2.5134	.0018
THEILE	-34 16 58.7785	58.7647	-.0212	58.7858	-.0073
BAMBADIN	-36 7 9.5213	9.5103	-.0212	9.5314	-.0101
GAMBIER	-37 50 32.9209	32.8861	-.0212	32.9073	.0136

LEGEND

1 - GMA82 ADJUSTMENT

2 - TRANSFORMED MULTI-STATION PRECISE (2,1,2)

BS - BLOCK SHIFT

2S - 2 MINUS THE BLOCK SHIFT

2S-1 - TRANSFORMED MULTI-STATION (2S) MINUS GROUND TRUTH

TABLE A2-13

E6 SUBSET NETWORK

LONGITUDE COMPARISON

MULTI-STATION PRECISE (2,1,2) VERSUS GMA82 ADJUSTMENT

	1	2	BS	2S	2S-1
PARA	138 41 28.7379	28.7324	-.0043	28.7281	-.0098
SUNDOWN	141 26 55.6147	55.5923	-.0043	55.5880	-.0267
THEILE	140 53 34.0598	34.0399	-.0043	34.0356	-.0242
BAMBADIN	140 58 36.9403	36.9529	-.0043	36.9486	.0083
GAMBIER	140 45 18.9700	19.0268	-.0043	19.0225	.0525

LEGEND

1 - GMA82 ADJUSTMENT

2 - TRANSFORMED MULTI-STATION PRECISE (2,1,2)

BS - BLOCK SHIFT

2S - 2 MINUS THE BLOCK SHIFT

2S-1 - TRANSFORMED MULTI-STATION (2S) MINUS GROUND TRUTH

TABLE A2-14

E6 SUBSET NETWORK

HEIGHT COMPARISON

MULTI-STATION PRECISE (2,1,2) VERSUS GMA82 ADJUSTMENT

	1	2	BS	2S	2S-1
PARA	230.600	230.833	.296	230.537	-.063
SUNDOWN	407.800	409.034	.296	408.738	.938
THEILE	63.620	62.492	.296	62.196	-1.424
BAMBADIN	169.220	170.842	.296	170.546	1.326
GAMBIER	191.990	191.509	.296	191.213	-.777

LEGEND

1 - GMA82 ADJUSTMENT

2 - TRANSFORMED MULTI-STATION PRECISE (2,1,2)

BS - BLOCK SHIFT

2S - 2 MINUS THE BLOCK SHIFT

2S-1 - TRANSFORMED MULTI-STATION (2S) MINUS GROUND TRUTH

TABLE A2-15

E6 FULL NETWORK

LATITUDE COMPARISON

MULTI-STATION BROADCAST (26,5,10) VERSUS GMA82 ADJUSTMENT

	1	2	BS	2S	2S-1
PARA	-34 47 11.4855	11.5363	.0436	11.4927	-.0072
SUNDOWN	-31 54 2.5152	2.5685	.0436	2.5249	-.0097
THEILE	-34 16 58.7785	58.8278	.0436	58.7842	-.0057
BAMBADIN	-36 7 9.5213	9.5725	.0436	9.5289	-.0076
GAMBIER	-37 50 32.9209	32.9420	.0436	32.8984	.0225
THREE BR.	-34 9 37.7903	37.8503	.0436	37.8067	-.0164
ATKINSON	-37 45 32.2330	32.2761	.0436	32.2325	.0005
GREDGWIN	-35 58 21.9672	22.0075	.0436	21.9638	.0034
TALGARNO	-36 5 6.6620	6.7202	.0436	6.6766	-.0146
MURRUNG..	-37 33 48.2534	48.2787	.0436	48.2351	.0183
HOLEY HILL	-38 14 .1208	.1481	.0436	.1044	.0164

LEGEND

1 - GMA82 ADJUSTMENT

2 - TRANSFORMED MULTI-STATION BROADCAST (26,5,10)

BS - BLOCK SHIFT

2S - 2 MINUS THE BLOCK SHIFT

2S-1 - TRANSFORMED MULTI-STATION (2S) MINUS GROUND TRUTH

TABLE A2-16

E6 FULL NETWORK

LONGITUDE COMPARISON

MULTI-STATION BROADCAST (26,5,10) VERSUS GMA82 ADJUSTMENT

	1	2	BS	2S	2S-1
PARA	138 41 28.7379	28.8010	-.0488	28.7522	.0143
SUNDOWN	141 26 55.6147	55.6236	-.0488	55.5748	-.0399
THEILE	140 53 34.0598	34.0873	-.0488	34.0385	-.0213
BAMBADIN	140 58 36.9403	37.0033	-.0488	36.9546	.0143
GAMBIER	140 45 18.9700	19.1062	-.0488	19.0574	.0874
THREE BR.	135 49 43.7501	43.8495	-.0488	43.8007	.0506
ATKINSON	144 40 53.6086	53.6490	-.0488	53.6003	-.0083
GREDGWIN	143 37 6.6309	6.7066	-.0488	6.6578	.0269
TALGARNO	147 5 43.3776	43.3817	-.0488	43.3330	-.0446
MURRUNG..	148 42 53.3301	53.3427	-.0488	53.2940	-.0361
HOLEY HILL	146 56 19.3863	19.3919	-.0488	19.3431	-.0432

LEGEND

1 - GMA82 ADJUSTMENT

2 - TRANSFORMED MULTI-STATION BROADCAST (26,5,10)

BS - BLOCK SHIFT

2S - 2 MINUS THE BLOCK SHIFT

2S-1 - TRANSFORMED MULTI-STATION (2S) MINUS GROUND TRUTH

TABLE A2-17

E6 FULL NETWORK

HEIGHT COMPARISON

MULTI-STATION BROADCAST (26,5,10) VERSUS GMA82 ADJUSTMENT

	1	2	BS	2S	2S-1
PARA	230.600	230.177	.111	231.288	.688
SUNDOWN	407.800	408.358	.111	409.469	1.669
THEILE	63.620	61.973	.111	63.084	-.536
BAMBADIN	169.220	170.154	.111	171.265	2.045
GAMBIER	191.990	190.924	.111	192.035	.045
THREE BR.	225.700	228.180	.111	229.291	3.591
ATKINSON	158.720	156.708	.111	157.819	-.901
GREDGWIN	168.000	166.071	.111	167.182	-.818
TALGARNO	662.910	660.497	.111	661.609	-1.301
MURRUNG..	745.860	742.624	.111	743.736	-2.124
HOLEY HILL	235.250	231.782	.111	232.893	-2.357

LEGEND

1 - GMA82 ADJUSTMENT

2 - TRANSFORMED MULTI-STATION BROADCAST (26,5,10)

BS - BLOCK SHIFT

2S - 2 MINUS THE BLOCK SHIFT

2S-1 - TRANSFORMED MULTI-STATION (2S) MINUS GROUND TRUTH

TABLE A2-18

E6 FULL NETWORK

LATITUDE COMPARISON

MULTI-STATION PRECISE (2,1,2) VERSUS GMA82 ADJUSTMENT

	1	2	BS	2S	2S-1
PARA	-34 47 11.4855	11.4656	-.0163	11.4819	.0036
SUNDOWN	-31 54 2.5152	2.4947	-.0163	2.5110	.0042
THEILE	-34 16 58.7785	58.7657	-.0163	58.7820	-.0035
BAMBADIN	-36 7 9.5213	9.5127	-.0163	9.5290	-.0077
GAMBIER	-37 50 32.9209	32.8862	-.0163	32.9025	.0184
THREE BR.	-34 9 37.7903	37.7692	-.0163	37.7854	.0049
ATKINSON	-37 45 32.2330	32.2205	-.0163	32.2367	-.0037
GREDGWIN	-35 58 21.9672	21.9482	-.0163	21.9645	.0027
TALGARNO	-36 5 6.6620	6.6690	-.0163	6.6853	-.0233
MURRUNG..	-37 33 48.2534	48.2382	-.0163	48.2545	-.0011
HOLEY HILL	-38 14 .1208	.0990	-.0163	.1153	.0055

LEGEND

1 - GMA82 ADJUSTMENT

2 - TRANSFORMED MULTI-STATION PRECISE (2,1,2)

BS - BLOCK SHIFT

2S - 2 MINUS THE BLOCK SHIFT

2S-1 - TRANSFORMED MULTI-STATION (2S) MINUS GROUND TRUTH

TABLE A2-19

E6 FULL NETWORK

LONGITUDE COMPARISON

MULTI-STATION PRECISE (2,1,2) VERSUS GMA82 ADJUSTMENT

	1	2	BS	2S	2S-1
PARA	138 41 28.7379	28.7352	-.0001	28.7351	-.0028
SUNDOWN	141 26 55.6147	55.5926	-.0001	55.5925	-.0222
THEILE	140 53 34.0598	34.0380	-.0001	34.0380	-.0218
BAMBADIN	140 58 36.9403	36.9520	-.0001	36.9520	.0117
GAMBIER	140 45 18.9700	19.0280	-.0001	19.0279	.0579
THREE BR.	135 49 43.7501	43.7677	-.0001	43.7676	.0175
ATKINSON	144 40 53.6086	53.6008	-.0001	53.6007	-.0079
GREDGWIN	143 37 6.6309	6.6753	-.0001	6.6752	.0443
TALGARNO	147 5 43.3776	43.3573	-.0001	43.3572	-.0204
MURRUNG..	148 42 53.3301	53.3100	-.0001	53.3099	-.0202
HOLEY HILL	146 56 19.3863	19.3501	-.0001	19.3500	-.0363

LEGEND

1 - GMA82 ADJUSTMENT

2 - TRANSFORMED MULTI-STATION PRECISE (2,1,2)

BS - BLOCK SHIFT

2S - 2 MINUS THE BLOCK SHIFT

2S-1 - TRANSFORMED MULTI-STATION (2S) MINUS GROUND TRUTH

TABLE A2-20

E6 FULL NETWORK

HEIGHT COMPARISON

MULTI-STATION PRECISE (2,1,2) VERSUS GMA82 ADJUSTMENT

	1	2	BS	2S	2S-1
PARA	230.600	230.833	.342	231.175	.575
SUNDOWN	407.800	409.054	.342	409.396	1.596
THEILE	63.620	62.522	.342	62.864	-.756
BAMBADIN	169.220	170.832	.342	171.174	1.954
GAMBIER	191.990	191.519	.342	191.861	-.129
THREE BR.	225.700	228.808	.342	229.150	3.450
ATKINSON	158.720	157.440	.342	157.782	-.938
GREDGWIN	168.000	166.715	.342	167.057	-.943
TALGARNO	662.910	661.446	.342	661.788	-1.122
MURRUNG..	745.860	743.886	.342	744.228	-1.632
HOLEY HILL	235.250	232.852	.342	233.194	-2.056

LEGEND

1 - GMA82 ADJUSTMENT

2 - TRANSFORMED MULTI-STATION PRECISE (2,1,2)

BS - BLOCK SHIFT

2S - 2 MINUS THE BLOCK SHIFT

2S-1 - TRANSFORMED MULTI-STATION (2S) MINUS GROUND TRUTH

APPENDIX 3
MX1502 TRANSLOCATION PRINT-OUT

```
PASS   72
2D   18    4  25  200
N-E          0.062
LT  S  34  47  05.965
LN  E138  41  33.167
GMT   01  28  00
DATE  15   8  1982
SD   0.626   0.502
```

INFORMATION FROM MASTER INSTRUMENT CASSETTE

```
PASS   88
2D   21    3  26  200
N-E          0.072
LT  S  34  16  53.129
LN  E140  53  38.114
GMT   01  28  00
DATE  15   8  1982
SD   0.543   0.476

TR      60      0.022
LT  S  34  16  53.538
LN  E140  53  37.974
HT       58.95
SD   0.034   0.093
SDH     0.089
N-WE    13    14
S-WE    18    15
DLT       -0.31
DLN       -1.60
DHT       -3.28
DREF       3.67
```

INFORMATION FROM REMOTE INSTRUMENT CASSETTE

APPENDIX 4

LISTING OF PROGRAM SELECT

```
SELECT
      PROGRAM SELECT(INPUT,OUTPUT,TAPE7,TAPE9)
C
C
C
C     PURPOSE     - THIS PROGRAM ENABLES THE SELECTION OF PARTICULAR
C                   SATELLITE PASSES FROM A DOPPLER DATA SET WHICH
C                   HAS BEEN RETRIEVED FROM 9 TRACK TAPE.
C                   THE SELECTED PASSES ARE OUTPUT TO LOCAL FILE TAPE9
C                   MAGNAVOX FORMAT IS ASSUMED.
C
C                   **** NOTE - THE PROGRAM RUNS INTERACTIVELY ****
C
C
C     AUTHOR      - ANDREW JONES
C
C
C     DATE        - AUGUST 1983
C
C
C     INPUT FILE - TAPE7 (9 TRACK TAPE DATA)
C     OUTPUT FILE- TAPE9 (SELECTED PASSES)
      CHARACTER *1,ANS
      DIMENSION IRECORD(86)
C
C
C**** HEADING
      PRINT 10
   10 FORMAT(/////,30X,'PASS SELECTION PROGRAM',
     1/,30X,22('='),//,
     210X,'INFORMATION FOR EACH PASS IN TURN WILL BE DISPLAYED ON'/,
     310X,'THE SCREEN.'/
     410X,'ACCEPT OR REJECT THE PASS AS REQUIRED.'/,
     510X,'DONT FORGET TO SAVE THE OUTPUT FILE (TAPE9)'////)
C
C
C**** INPUT INITIAL PASS NUMBER
      PRINT 30
   30 FORMAT(10X,'INPUT INITIAL PASS NUMBER I.E. 1 , 2 ...(I3 FORMAT)'
      READ 31,ISTART
   31 FORMAT(I3)
C
C
C**** INPUT MAGNAVOX RECORD
   50 READ(7,100)(IRECORD(I),I=1,43)
C     PRINT 100,(IRECORD(I),I=1,43)
      IF(EOF(7).NE.0)GO TO 900
```

```
      READ(7,100)(IRECORD(I),I=44,86)
C     PRINT 100,(IRECORD(I),I=44,86)
  100 FORMAT(43Z2)
C
C
C**** CHECK IF DATA RECORD
      IF(IRECORD(1).GT.1.AND.IRECORD(1).LT.31)THEN
C
C
C****      CHECK IF FIRST RECORD OF PASS
           IF(IRECORD(1).EQ.2)THEN
C
C
C****           IF PASS IS LESS THAN REQUIRED INITIAL PASS,READ NEXT PA
                IF(IRECORD(4).LT.ISTART)GO TO 50
C
C
C****           SET STORE SWITCH OFF.WRITE PASS DETAILS TO SCREEN.
                ISS=0
                ISITE=IRECORD(2)*256+IRECORD(3)
                ISAT=INT(FTMX(IRECORD(57)))/10
                JAHR=INT(FTMX(IRECORD(37)))
                IDAY=INT(FTMX(IRECORD(41)))
                MMIN=INT(FTMX(IRECORD(33)))
                MHR=MMIN/60
                MMIN=MMIN-MHR*60
                IMIN=INT(FTMX(IRECORD(61)))
                IHR=IMIN/60
                IMIN=IMIN-IHR*60
                PRINT 200,ISITE,IRECORD(4),ISAT,JAHR,IDAY,IHR,IMIN
     1          ,MHR,MMIN
  200           FORMAT(10X,'SITE',2X,I4,/,
     1          10X,'PASS',2X,I4,5X,'SATELLITE',2X,I4,5X,'YEAR',2X,I4,/
     2          10X,'DAY',2X,I4,5X,'LOKON TIME',2X,I4,I4,
     3          5X,'TIME MAX ELEV.',2X,2I4)

C
C****           DETERMINE IF PASS IS TO BE ACCEPTED.
  205           PRINT 210
  210           FORMAT(//,10X,'ACCEPT/REJECT/END? (A/R/E) ')
                READ 250,ANS
  250           FORMAT(A1)
                IF(ANS.NE.'A'.AND.ANS.NE.'R'.AND.ANS.NE.'E')GO TO 205
                IF(ANS.EQ.'A')ISS=1
           ENDIF
C
C****      IF AN ACCEPTED PASS WRITE TO OUTPUT FILE
           IF(ISS.EQ.1)THEN
                WRITE(9,100)(IRECORD(I),I=1,43)
                WRITE(9,100)(IRECORD(I),I=44,86)
           ENDIF
      ENDIF
C
C
C**** CHECK IF FINISHING.
```

```
      IF(ANS.EQ.'E')GO TO 900
      GO TO 50
  900 PRINT 910
  910 FORMAT(10X,'**** DONT FORGET TO SAVE OUTPUT FILE ****')
      STOP
      END
      FUNCTION FTMX(ID)
C
C       NOTE - THIS FUNCTION WAS EXTRACTED FROM PROGRAM DPPPP (SHI 19
C
      DIMENSION ID(4)
      IF(ID(1)+ID(2)+ID(3)+ID(4).EQ.0)GO TO 10
      IT=SHIFT(ID(1),16)+SHIFT(ID(2),8)+ID(3)
      IF(IT.GT.8388607)IT=IT-16777216
      FTMX=FLOAT(IT)*2.**(ID(4)-151)
      RETURN
   10 FTMX=0
      RETURN
      END
```

APPENDIX 5

LISTING OF PROGRAM CHORD7

```
      PROGRAM CHORD7(INPUT,OUTPUT,TAPE3,TAPE9,TAPE10,TAPE11,TAPE12,
     1TAPE13,TAPE14)
C
C
C**** PURPOSE     - THIS PROGRAM:
C                           1. ENABLES TRANSFORMATIONS BETWEEN GMA82,
C                              BROADCAST EPHEMERIS AND PRECISE EPHEMERIS
C                              USING TRANSFORMATION PARAMETERS COMPUTED
C                              DURING THE GMA82 ADJUSTMENT.
C                           2. ENABLES TRANSFORMATIONS BETWEEN NWL9D,
C                              WGS72 AND A.N.S SPHEROIDS.
C                           3. COMPUTES CHORD DISTANCES BETWEEN ALL PAIRS
C                              OF POINTS IN THE TRANSFORMED SYSTEM.
C                           4. ENABLES UP TO FOUR SETS OF STATION
C                              TO BE TRANSFORMED TO A COMMON DATUM
C                              AND COMPARED.
C
C
C            INPUT FILE   - TAPE3
C            INPUT FORMAT - THE DATA DECKS ARE ORGANISED IN THE FOLLOWI
C                           MANNER
C
C                             JOB HEADER CARD
C
C                             SET HEADER CARD (1ST SET)
C
C                             COORDINATE CARD (1ST SET)
C
C                             COORDINATE CARD (1ST SET)
C
C                             COORDINATE CARD (1ST SET)
C
C                                 "        "     "    "
C
C                                 "        "     "    "
C
C                             COORDINATE CARD (1ST SET)
C
C                             SET HEADER CARD (2ND SET)
C
C                             COORDINATE CARD (2ND SET)
C
C                                      ETC.
C
C
C            NOTE - STATIONS MUST BE MAINTAINED IN THE SAME ORDER IN E
C            COORDINATE SET
```

```
C
C              THE FORMAT OF THE THREE CARD TYPES IS AS FOLLOWS
P                   JOB HEADER CARD - NO. OF COORINATE SETS (I5)
C                                     JOB NAME (A20)
C
C                   SET HEADER CARD - NUMBER OF STATIONS (I10)
C                                     DESIRED DATUM  E.G. PRECISE (A10)
C                                     DESIRED SPHEROID  E.G. WGS72 (A10)
C                                     COORD SET IDENT. (A40)
C
C                   COORDINATE CARD - DATUM OF COORDS (A10)
C                                     SPHEROID OF COORDS (A10)
C                                     STATION NAME (A10)
C                                     LATTITUDE (I4,I3,F9.5)
C                                     LONGITUDE (I4,I3,F9.5)
C                                     SPHEROIDAL HEIGHT (F9.5)
C
C
C         OUTPUT FILES - TAPE 9  - INPUT COORDINATES
C                        TAPE 10 - LATTITUDE COMPARISONS
C                        TAPE 11 - LONGITUDE COMPARISONS
C                        TAPE 12 - HEIGHT COMPARISONS
C                        TAPE 13 - TRANSFORMED COORDS
C
C
C                        OUTPUT IS ALSO SET TO THE PRINTER
C
C
C
C
C**** AUTHOR      - ANDREW JONES.
C
C
C**** DATE        - AUGUST 1983.
C
C
      DIMENSION TR(7,3),NAME(7),IDAT(50),ISPH(50),ISTN(50)
     1,SPH(6),X(2,50),Y(2,50),Z(2,50),ITR(3,3)
     2,DIST(5,100),XLAT2(5,15),XLON2(5,15),INTOCO(5),INTOSP(5)
     3,XMEANLA(5),XMEANLO(5),XMEANHT(5),D(3),DM(3)
     4,SPHT2(5,15)
      CHARACTER*20 NAMEJB
      CHARACTER*40 NAMED(5)
      COMMON/TRA/TR1(7)
      DATA TR/116.47,50.25,-138.87,-0.21,-0.36,0.47,-0.75
     1,-5.9,-0.3,-1.6,0.03,-0.11,-0.12,-0.41
     2,112.29,50.56,-142.73,-0.16,-0.37,0.40,-1.43/
      DATA NAME/'BROADCAST','PRECISE','GMA82','WGS72'
     1,'NWL9D','ANS','MAGVOX'/
      DATA ITR/0,2,3,-2,0,1,-3,-1,0/
      DATA SPH/6378135.0,298.26,6378145.0,298.25,6378160.0,298.25/
C
C
      PI=3.1415926535898
      ICOUNT=1
```

```
C
C
C**** READ HEADER CARD
      READ(3,90)NODECK,NAMEJB
   90 FORMAT(I5,A20)
C
C
C**** INITIALISE ACCUMULATORS FOR XMEANS
      DO 95 I=1,NODECK
   95 XMEANLA(I)=XMEANLO(I)=XMEANHT(I)=0.0
C
C
C**** ENTER LOOP TO TRANSFORM EACH COORDINATE SET
C**** PRINT HEADINGS
  105 PRINT 100
  100 FORMAT(1H1,23X,'COORDINATE TRANSFORMATION PROGRAM',/
     1,24X,33('='),/)
C
C
C**** INPUT OPTION CARD AND PRINT
      READ(3,110)NO,INTOCO(ICOUNT),INTOSP(ICOUNT),NAMED(ICOUNT)
  110 FORMAT(I10,2A10,A40)
      PRINT 120,NAMEJB,ICOUNT,NAMED(ICOUNT),NO
  120 FORMAT(10X,A20,/,10X,'TRANSFORMATION OF COORDINATE SET ',I2
     1,' - ',A40,//,10X,
     2'NUMBER OF POINTS TO BE INPUT = ',I2)
      IF(INTOCO(ICOUNT).EQ.NAME(1).AND.INTOSP(ICOUNT).EQ.NAME(4))THEN
          PRINT 130
  130     FORMAT(10X,'TRANSFORMATION INTO MAGVOX DATUM'//)
      ELSE
          PRINT 140,INTOCO(ICOUNT),INTOSP(ICOUNT)
  140     FORMAT(10X,'TRANSFORMATION INTO ',A10,' DATUM.',/
     1    ,28X,A10,'SPHEROID.'//)
      ENDIF
C
C
C**** OUTPUT TO TABLE FILE 9.
      WRITE(9,1100)NO,NAMEJB,NAMED(ICOUNT)
      WRITE(13,1101)NO,NAMEJB,INTOCO(ICOUNT),NAMED(ICOUNT)
 1100 FORMAT(I2,A10,A40)
 1101 FORMAT(I2,A10,'TRANSFORMED INTO ',A10,A40)
C
C
C**** IDENTIFY TRANSFORMATION AND ELLIPSOIDAL PARAMETER CODE
C     FOR 'NEW' SYSTEM.
      DO 145 K=1,3
          IF(INTOCO(ICOUNT).EQ.NAME(K))KI=K
  145     IF(INTOSP(ICOUNT).EQ.NAME(K+3))KM=K
C
C
C**** ENTER LOOP FOR TRANSFORMATION OF STATIONS,ONE BY ONE.
      DO 500 I=1,NO
C
C
C****     INPUT STATION DATA
```

```
      READ(3,150)IDAT(I),ISPH(I),ISTN(I),LADEG,LAMIN,SECLA
     1 ,LODEG,LOMIN,SECLO,SPHT1
  150 FORMAT(3A10,2(I4,I3,F9.5),1X,F9.5)
       WRITE(9,1110)ISTN(I),LADEG,LAMIN,SECLA,LODEG,LOMIN,
     1  SECLO,SPHT1
 1110  FORMAT(15X,A10,2(2X,I4,I3,F9.5),2X,F7.3)
      LADEG1=IABS(LADEG)
      XLAT1=(LADEG1+LAMIN/60.+SECLA/3600)*LADEG/LADEG1
      XLON1=LODEG+LOMIN/60.+SECLO/3600
      XLAT1=XLAT1*PI/180.0
      XLON1=XLON1*PI/180.0
C
C
C****  DETERMINE TRANSFORMATION AND SPHEROIDAL PARAMETER CODE FOR
C      'FROM' SYSTEM
      DO 160,K=1,3
          IF(ISPH(I).EQ.NAME(K+3))KN=K
  160     IF(IDAT(I).EQ.NAME(K))KJ=K
C
C
C****  EXTRACT TRANSFORMATION PARAMETERS REQUIRED
      ICODE=ITR(KI,KJ)
      K2=KN*2
      K1=K2-1
      A=SPH(K1)
      F=1.0/SPH(K2)
      IF(ICODE.NE.0)THEN
          DO 170 K=1,7
              IICODE=IABS(ICODE)
              TR1(K)=TR(K,IICODE)
              IF(ICODE.LT.0)TR1(K)=-TR1(K)
  170     CONTINUE
          ELSE
              DO 175 K=1,7
  175         TR1(K)=0.0
      ENDIF
C
C
C****  CONVERT TO X,Y,Z ('FROM' DATUM)
      CALL GEOXYZ(A,F,XLAT1,XLON1,SPHT1,X(1,I),Y(1,I),Z(1,I))
C
C
C****  TRANSFORM TO 'TO' DATUM.
      IF(ICODE.NE.0)THEN
          CALL ROTA(X(1,I),Y(1,I),Z(1,I),X(2,I),Y(2,I),Z(2,I))
      ELSE
          X(2,I)=X(1,I)
          Y(2,I)=Y(1,I)
          Z(2,I)=Z(1,I)
      ENDIF
C
C
C****  CONVERT X,Y,Z, TO LAT,LONG AND HT ('TO' SYSTEM)
      K2=KM*2
      K1=K2-1
```

```
            A=SPH(K1)
            F=1.0/SPH(K2)
            CALL XYZGEO(A,F,XLAT2(ICOUNT,I),XLON2(ICOUNT,I)
     1      ,SPHT2(ICOUNT,I),X(2,I),Y(2,I),Z(2,I))
            CALL RADDMS(XLAT2(ICOUNT,I),JADEG,JAMIN,SECJA)
            CALL RADDMS(XLON2(ICOUNT,I),JODEG,JOMIN,SECJO)
C
C
C****  ACUMULATE VALUES FOR COMPUTATIONS OF XMEANS
            XMEANLA(ICOUNT)=XMEANLA(ICOUNT)+XLAT2(ICOUNT,I)
            XMEANLO(ICOUNT)=XMEANLO(ICOUNT)+XLON2(ICOUNT,I)
            XMEANHT(ICOUNT)=XMEANHT(ICOUNT)+SPHT2(ICOUNT,I)
C
C
C****       PRINT TRANSFORMED COORDINATES
  500       PRINT 200,ISTN(I),IDAT(I),ISPH(I),LADEG,LAMIN,SECLA,LODEG
     1      ,LOMIN,SECLO,SPHT1,ISTN(I),INTOCO(ICOUNT),INTOSP(ICOUNT)
     2      ,JADEG,JAMIN,SECJA,JODEG,JOMIN,SECJO,SPHT2(ICOUNT,I)
     3       ,(TR1(MN),MN=1,7)
  200       FORMAT(2(5X,3A10,     I3,I3,F9.5,1X,     2I3,F9.5,1X,F9.5/),
     1      ,5X,'TRANSFORMATION PARAMETERS ',7F7.2//)
       DO 1130 KM=1,NO
            CALL RADDMS(XLAT2(ICOUNT,KM),JADEG,JAMIN,SECJA)
            CALL RADDMS(XLON2(ICOUNT,KM),JODEG,JOMIN,SECJO)
 1130       WRITE(13,1140)ISTN(KM),JADEG,JAMIN,SECJA,JODEG,JOMIN
     1      ,SECJO,SPHT2(ICOUNT,KM)
 1140       FORMAT(15X,A10,2(2X,I4,I3,F9.5),2X,F7.3)
      WRITE(13,1150)(TR1(MN),MN=1,7)
 1150 FORMAT(7F7.2)
C
C
C**** COMPUTE XMEAN LATITUDE,LONGITUDE AND HEIGHT
      XMEANLA(ICOUNT)=XMEANLA(ICOUNT)/NO
      XMEANLO(ICOUNT)=XMEANLO(ICOUNT)/NO
      XMEANHT(ICOUNT)=XMEANHT(ICOUNT)/NO
C
C
C**** COMPUTE CHORD DISTANCES
      PRINT 550,INTOCO(ICOUNT),NAMED(ICOUNT)
  550 FORMAT(1H1,5X,'CHORD DISTANCES - ',A10,'DATUM - ',A40,//)
      ICOUNT1=0
      NN=NO-1
      DO 600 K=1,NN
           KK=K+1
           DO 600,L=KK,NO
                DELX=X(2,K)-X(2,L)
                DELY=Y(2,K)-Y(2,L)
                DELZ=Z(2,K)-Z(2,L)
                ICOUNT1=ICOUNT1+1
                DIST(ICOUNT,ICOUNT1)=SQRT(DELX*DELX+DELY*DELY+DELZ*DELZ
                PRINT 560,K,ISTN(K),L,ISTN(L),DIST(ICOUNT,ICOUNT1)
  560           FORMAT(10X,2(I3,2X,A10),5X,F12.3)
  600 CONTINUE
C
C
```

```
C**** DETERMINE IF ANOTHER DATA SET IS TO BE PROCESSED
      ICOUNT=ICOUNT+1
      IF(ICOUNT.LE.NODECK)GO TO 105
C
C
C**** IF ONLY ONE COORDINATE SET,TERMINATE PROGRAM
      IF(NODECK.EQ.1)GO TO 999
C
C
C**** COMPARISON SECTION
C
C
C**** COMPARISON OF LATITUDE,LONGITUDE AND HEIGHT VALUES.
      NODECK1=NODECK-1
      DO 999 MM=1,NODECK1
      DO 800,KK=MM+1,NODECK
          PRINT 700,NAMEJB,NAMED(MM),NAMED(KK),NAMED(MM),NAMED(KK)
     1    ,NAMED(KK)
  700     FORMAT(1H1,/
     1    ,/,41X,'COMPARISON OF LATITUDE,LONGITUDE'
     1    ,' AND HEIGHT VALUES',/,41X,50('='),//,10X,A20,/,10X
     2    ,A40,/,10X,'VS ',A40,//,10X,'LEGEND FOR,THIS PAGE',/
     3    ,15X,'1     - ',A40,/,15X,'2     - ',A40,/
     4    ,15X,'BS    - BLOCK SHIFT',/
     5    ,15X,'2S    - SHIFTED ',A40,/,
     6    ,15X,'2S-1  - DIFFERENCE BETWEEN 2S AND 1'
     7    ,//,2X,'STATION',30X,'LATITUDE',51X,'LONGITUDE'
     4    ,/,12X,2(11X,'1',13X,'2',8X,'BS',8X,'2S',7X,'2S-1'1X))
C
C
C****     HEADINGS FOR FILE TABLES
          DO 1250 KKK=10,12
              GO TO(1160,1180,1200),KKK-9
 1160         WRITE(KKK,1170)
 1170         FORMAT('LATITUDE COMPARISON')
              GO TO 1220
 1180         WRITE(KKK,1190)
 1190         FORMAT('LONGITUDE COMPARISON')
              GO TO 1220
 1200         WRITE(KKK,1210)
 1210         FORMAT('HEIGHT COMPARISON')
 1220         WRITE(KKK,1230)NO,NAMED(KK),NAMED(MM)
 1230         FORMAT(I2,/,A40,/,A40)
 1250     CONTINUE
C
C
C****     DETERMINE BLOCK SHIFTS
          SHLA=XMEANLA(MM)-XMEANLA(KK)
          SHLO=XMEANLO(MM)-XMEANLO(KK)
          SHHT=XMEANHT(MM)-XMEANHT(KK)
C
C
C****     UNDERTAKE COMPARISONS FOR EACH STATION
          DO 740 KJ=1,NO
              CALL RADDMS(XLAT2(MM,KJ),JADEG,JAMIN,SECJA)
```

```
                CALL RADDMS(XLON2(MM,KJ),JODEG,JOMIN,SECJO)
                CALL RADDMS(XLAT2(KK,KJ),IDUM1,IDUM2,SECKA)
                CALL RADDMS(XLON2(KK,KJ),IDUM1,IDUM2,SECKO)
                CALL RADDMS(SHLA,IDUM1,IDUM2,SHLAP)
                CALL RADDMS(SHLO,IDUM1,IDUM2,SHLOP)
                IF(SHLA.LT.0)SHLAP=-SHLAP
                IF(SHLO.LT.0)SHLOP=-SHLOP
                XLA2S=XLAT2(KK,KJ)+SHLA
                XLO2S=XLON2(KK,KJ)+SHLO
                CALL RADDMS(XLA2S,IDUM1,IDUM2,XLA2SP)
                CALL RADDMS(XLO2S,IDUM1,IDUM2,XLO2SP)
                XLA12S=XLA2S-XLAT2(MM,KJ)
                XLO12S=XLO2S-XLON2(MM,KJ)
                CALL RADDMS(XLA12S,IDUM1,IDUM2,XLA12SP)
                CALL RADDMS(XLO12S,IDUM1,IDUM2,XLO12SP)
                IF(XLA12S.LT.0)XLA12SP=-XLA12SP
                IF(XLO12S.LT.0)XLO12SP=-XLO12SP
                PRINT 750,ISTN(KJ),JADEG,JAMIN,SECJA,SECKA
     1          ,SHLAP,XLA2SP,XLA12SP,JODEG,JOMIN,SECJO,SECKO
     2          ,SHLOP,XLO2SP,XLO12SP
                THOPE=ISTN(KJ)
  750           FORMAT(1X,A10,2X,2(4X,2I3,2(F8.4,2X),F7.4,3X,
     1          F8.4,2X,F7.4,1X))
                WRITE(10,1260)ISTN(KJ),JADEG,JAMIN,SECJA,SECKA
     1          ,SHLAP,XLA2SP,XLA12SP
 1260           FORMAT(13X,A10,2X,2I3,2(F8.4,2X),F7.4,2X,F8.4,2X
     1          ,F7.4)
  740           WRITE(11,1260)ISTN(KJ),JODEG,JOMIN,SECJO,SECKO
     1          ,SHLOP,XLO2SP,XLO12SP
            PRINT 760
  760       FORMAT(//,2X,'STATION',26X,'HEIGHT',//
     1      ,20X,'1',8X,'2',8X,'BS',7X,'2S',6X,'1-2S')
            DO 800 KJ=1,NO
                SHT2S=SPHT2(KK,KJ)+SHHT
                SHT12S=SHT2S-SPHT2(MM,KJ)
                WRITE(12,771)ISTN(KJ),SPHT2(MM,KJ)
     1          ,SPHT2(KK,KJ),SHHT,SHT2S,SHT12S
  800           PRINT 770,ISTN(KJ),SPHT2(MM,KJ)
     1          ,SPHT2(KK,KJ),SHHT,SHT2S,SHT12S
  770           FORMAT(1X,A10,6X,2(F7.3,2X),F6.3,3X,2(F7.3,2X))
  771           FORMAT(15X,A10,1X,2(F7.3,2X),F6.3,3X,2(F7.3,2X))
C
C
C**** COMPARISON OF CHORD DISTANCES
      PRINT 810,NAMEJB
  810 FORMAT(1H1,35X,'COMPARISON OF CHORD DISTANCES'
     1,' FROM ALL DATA SETS IN',A20,//,
     2 10X,'LEGEND FOR THIS PAGE')
      DO 820 J=1,NODECK-MM+1
  820     PRINT 830,J,NAMED(J+MM-1)
  830     FORMAT(15X,I1,' - ',A40)
      IF(NODECK-MM+1.LT.3)PRINT 832
  832 FORMAT(15X,'3 - NOT USED.')
      IF(NODECK-MM+1.LT.4)PRINT 834
  834 FORMAT(15X,'4 - NOT USED.')
```

```
      PRINT 840
  840 FORMAT(/,14X,'LINE',22X,'1',11X,'2',11X,'3',11X,'4',4X
     1,2(7X,'2-1',4X,'3-1',4X,'4-1'),/,112X,3('PPM',4X))
C
C
C**** ENTER LOOP TO COMPARE CHORD DISTANCES
      NN=NO-1
      KM=0
      DO 940 K=1,NN
           KK=K+1
           DO 940 L=KK,NO
                KM=KM+1
                DO 910 KJ=1,NODECK-MM
                     KJ1=MM+KJ
                     D(KJ)=DIST(KJ1,KM)-DIST(MM,KM)
  910                DM(KJ)=D(KJ)*1E6/DIST(MM,KM)
                PRINT 930,K,ISTN(K),L,ISTN(L),DIST(MM,KM)
  930           FORMAT(1X,2(I3,2X,A10),3X, F12.3)
                PRINT 931,(DIST(I,KM),I=1+MM,NODECK)
  931           FORMAT(1H+,46X,3F12.3)
                NODECK1=NODECK-MM
                PRINT 932,(D(I),I=1,NODECK1)
  932           FORMAT(1H+,84X,3(1X,F6.3))
  940           PRINT 933,(DM(I),I=1,NODECK1)
  933           FORMAT(1H+,108X,3(1X,F6.3))
C
C
  999 CONTINUE
      END
      SUBROUTINE GEOXYZ(A,F,XLAT,XLON,SPHT,X,Y,Z)
C
C
C**** PURPOSE       - THIS SUBROUTINE CONVERTS GEOGRAPHICAL
C                     COORDINATES TO X,Y,Z.
C
C
C**** AUTHOR        - ANDREW JONES
C
C
C**** DATE          - AUGUST 1983
C
C
      E=2*F-F*F
      RAD=(A/SQRT(1.0-E*SIN(XLAT)*SIN(XLAT)))
      X=(RAD+SPHT)*COS(XLAT)*COS(XLON)
      Y=(RAD+SPHT)*COS(XLAT)*SIN(XLON)
      Z=(RAD*(1.0-E)+SPHT)*SIN(XLAT)
      RETURN
      END
      SUBROUTINE ROTA(A,B,C,D,E,F)
C
C
C
C**** PURPOSE       - THIS SUBROUTINE PERFORMS A 7-PARAMETER
C                     TRANSFORMATION BETWEEN TWO COORDINATE SYSTEMS.
```

```
C
C
C**** AUTHOR        - ANDREW JONES
C
C
C**** DATE          - AUGUST 1983
C
C
      DIMENSION RO(3,3),XOLD(3),XNEW(3)
      COMMON/TRA/TR1(7)
      DATA RO(1,1),RO(2,2),RO(3,3)/3*1.0/
      RHO=206264.8062
      XOLD(1)=A
      XOLD(2)=B
      XOLD(3)=C
C
C
C**** SET UP ROTATION MATRIX
      RO(2,1)=TR1(6)/RHO
      RO(1,3)=TR1(5)/RHO
      RO(3,2)=TR1(4)/RHO
      RO(1,2)=-RO(2,1)
      RO(3,1)=-RO(1,3)
      RO(2,3)=-RO(3,2)
C
C
C**** PERFORM TRANSFORMATION
      DO 100 I=1,3
          SUM=0.0
          DO 90 J=1,3
   90         SUM=SUM+RO(I,J)*XOLD(J)
  100     XNEW(I)=TR1(I)+(1.0+TR1(7)*1E-6)*SUM
C
C
      D=XNEW(1)
      E=XNEW(2)
      F=XNEW(3)
      RETURN
      END
      SUBROUTINE XYZGEO(A,F,XLAT,XLON,SPHT,X,Y,Z)
C
C
C**** PURPOSE       - THIS SUBROUTINE CONVERTS X,Y,Z TO
C                     GEOGRAPHICAL COORDINATES.
C
C
C**** AUTHOR        - ANDREW JONES
C
C
C**** DATE          - AUGUST 1983
C
C
      B=A*(1.0-F)
      E2=(A*A-B*B)/(A*A)
      E2D=(A*A-B*B)/(B*B)
```

```
      P=SQRT(X*X+Y*Y)
      THETA=ATAN(Z*A/(P*B))
      XLAT=ATAN((Z+E2D*B*SIN(THETA)**3)/(P-E2*A*COS(THETA)**3))
      XLON=3.14159265358 98+ATAN(Y/X)
      SPHT=(P/COS(XLAT))-A/SQRT(1-E2*SIN(XLAT)*SIN(XLAT))
      RETURN
      END
      SUBROUTINE RADDMS(ANG,IDEG,MIN,SEC)
C
C
C**** PURPOSE      - THIS SUBROUTINE CONVERTS RADIANS TO
C                    DEGREES,MINUTES AND SECONDS.
C
C
C**** AUTHOR       - ANDREW JONES
C
C
C**** DATE         - AUGUST 1983
C
C
      PI=3.14159265358 98
      ANG1=ABS(ANG)
      DEG=ANG1*180.0/PI
      IDEG=INT(DEG)
      MIN=INT((DEG-IDEG)*60)
      SEC=(DEG-IDEG-MIN/60.0)*3600
      IF(ANG.LT.0)IDEG=-IDEG
      RETURN
      END
```