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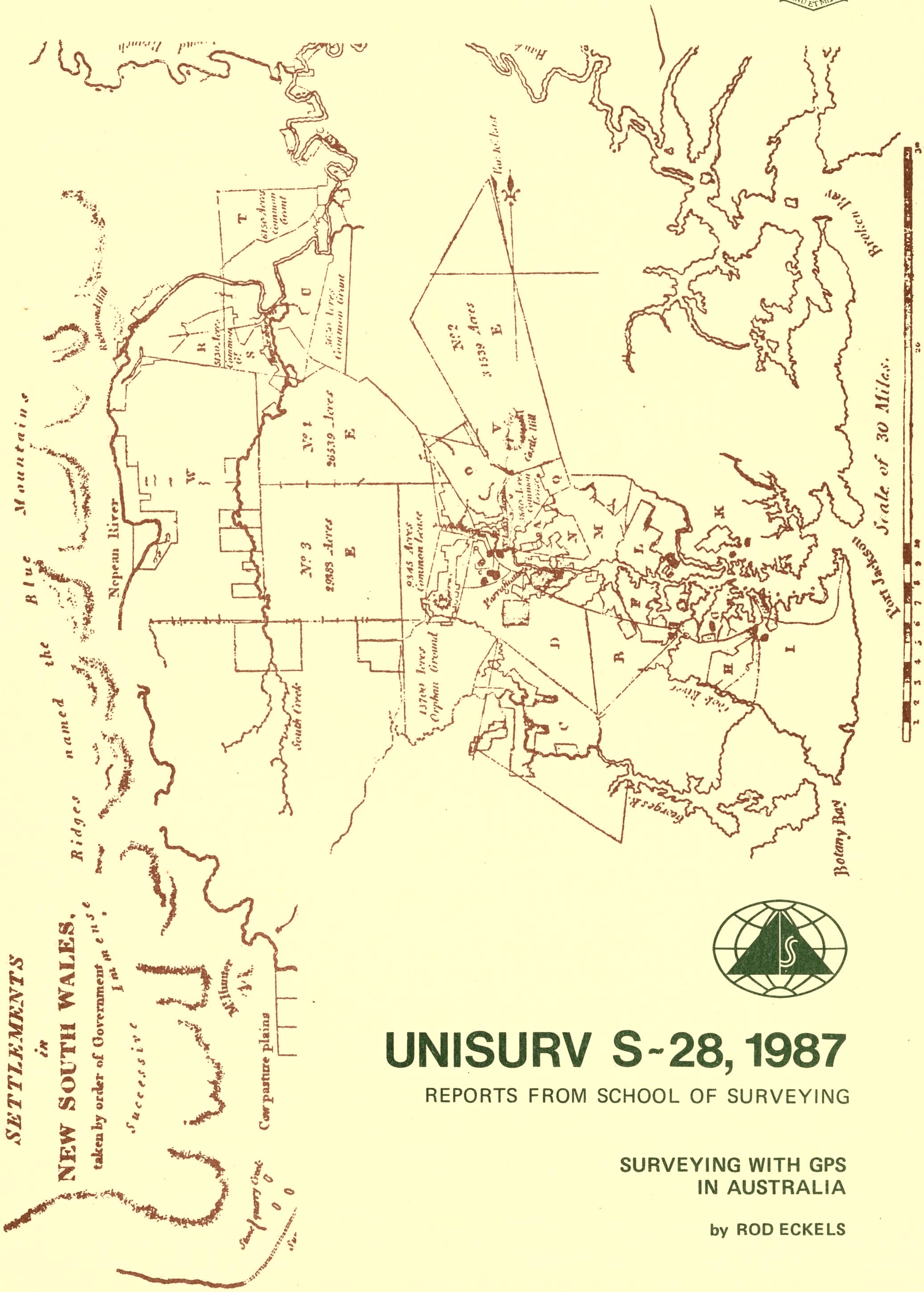


# UNISURV S-28, 1987

REPORTS FROM SCHOOL OF SURVEYING

## SURVEYING WITH GPS IN AUSTRALIA

by ROD ECKELS

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# SURVEYING WITH GPS IN AUSTRALIA

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## ABSTRACT

Over the last few years, the Global Positioning System (GPS) has demonstrated that it is the first space based technique that is capable of competing with existing terrestrial measurement technologies. GPS is a new system that is still under development. As the system becomes more fully configured, it will have an enormous impact on the surveying and positioning industries of Australia. It is important that surveyors have an understanding of how GPS works so that they know the system's capabilities and limitations. If surveyors do not make an effort to understand this new technique, they will become mere receiver operators. Knowledge of GPS allows this powerful technique to be used to its best advantage. If the basic principles of GPS measurement are understood, the technique may also be used for a range of positioning tasks that traditionally lie outside the scope of surveyors. The thesis describes the GPS system, and how it can be used for surveying in Australia. The basic component parts of the system, and the role they play are outlined. The satellite signal measurement process is described, and presently available receivers are reviewed. Field procedures and processing techniques that should be followed to achieve surveying accuracies are discussed. As GPS surveys refer to a satellite reference datum, the relationship between this datum and the Australian datum is given. The thesis provides a basic knowledge and understanding of GPS. With this knowledge, the surveyor should be able to use GPS confidently and incorporate this technique into his measurement arsenal.

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I would also wish to thank the late Grahame Sands for his imagination, knowledge and encouragement. He was an inspiration to us all. This thesis is a tribute to him.

# 1. INTRODUCTION

In the last few years the Global Positioning System (GPS) has emerged as a revolutionary measurement technology. GPS is the first space based measurement system that can compete favourably with existing techniques over distances ranging from tens of metres to hundreds of kilometres.

Although GPS is not yet fully configured, results from surveys carried out both in Australia and overseas show that GPS already provides accurate, fast and cost effective positioning. The results from a number of GPS surveys are discussed in Chapter 3.

One of the best documented GPS control densification surveys, is the 'Eifel' network, which was completed in 1983 in the Eifel region of West Germany. This survey aimed to tie a number of second order points into a first order geodetic network, and to show how GPS could be used for this purpose. The network consisted of 21 stations, that were fixed by measuring 43 independent baselines.

At the completion of the Eifel network survey a comparison of the cost, time taken and resulting accuracy of the survey was made between GPS and conventional techniques. This comparison, which is shown in Table 1.1, is typical of the GPS surveys that have been completed in the last few years.

Table 1.1 demonstrates the advantages that GPS already has over conventional techniques and indicates that GPS will have a significant impact on the Australian survey and navigation industries.

Table 1.1 : Comparison of Conventional Surveying Techniques with GPS for the Eifel Network (BOCK, 1984)

| | Conventional | GPS |
|---|---|---|
| Costs | 10K/Point | 1.5K/Point |
| Time Taken | 3 Months | 1 Month |
| Accuracy | 2-10 ppm | 1-2 ppm |

Note : K = (US)$1000

In order to assess the effect GPS will have on Australian surveying, it is necessary to evaluate how surveys are currently carried out. As there was little appropriate documentation available on present survey techniques, a questionnaire was distributed in January 1985 (ECKELS & STOLZ, 1985). The aim of the questionnaire was to indicate the areas of surveying where GPS will have its greatest impact, how the profession can prepare for GPS and what assistance may be required to help future users of the system in Australia.

The questionnaire was sent to a cross section of potential GPS users. Academic institutions were surveyed, as were private organisations and government departments involved in cadastral, engineering, control, mapping, offshore positioning and hydrographic surveys. In all 180 questionnaires were distributed and 80 were returned.

One of the most important conclusions drawn from analysis of the responses to the questionnaire was that surveyors were largely unaware of the enormous potential of GPS for routine surveying. 90% of the respondents, however, indicated that they would have liked to know more about the GPS system and the available instrumentation. It was also interesting that approximately 50% of respondents considered that GPS would raise the level of expertise required by their employees, and that the introduction of GPS would change survey methodology.

Since the questionnaire was distributed the general level of awareness of GPS has increased in the survey community, as more information has been disseminated through journal articles, conferences and seminars. Although surveyors are now becoming more aware of the potential of GPS, there is a need for surveyors to understand how GPS operates, and how the system can be used for routine survey work.

This thesis aims to address the introduction of GPS into Australian surveying. By understanding the operating principles of the system, the confusion and uncertainty that often heralds the introduction of new ideas and methods can be overcome. Ignorance of equipment, reference frames and transformations contributed to the confusion that was apparent with the introduction of the TRANSIT Doppler system.

Understanding GPS and its operating principles has other benefits for the survey community. Planning for data acquisition and archiving can commence, networks of 'base' stations (Chapter 3) can be established and guidelines for the use of the system and standards and specifications can be introduced. The completion of these tasks will ensure that surveyors are able to use GPS to their best advantage in the future.

GPS was originally designed for use by the U.S. military for real time navigation. Chapter 2 describes how the system has been adapted to surveying. This chapter describes the development of the system since its inception in the early 1970's and outlines the three major segments that make up the system.

Chapter 3 compares current survey techniques with GPS to highlight the impact that GPS will have on the Australian surveying community. Information about present survey methods were taken from the questionnaire distributed in 1985. Comparisons of survey accuracy, cost and speed are made using both overseas and Australian examples.

An outline of the various characteristics of currently available GPS hardware is given in chapter 4. This chapter reviews the four major categories of GPS receivers, describes the different satellite signal observing techniques and considers design features that are common to all GPS receivers.

Chapters 5, 6 and 7 describe how GPS can be used for surveying. A GPS survey is a two step process. The first step involves the planning and field procedures required to collect the satellite data. The second, and equally important step, is the processing of the collected data, to produce the final station coordinates.

Chapter 5 describes the planning and execution of a GPS survey. GPS can be used in a number of different modes, each of which has different accuracy limitations. These observation modes are discussed. Consideration is also given to draft standards and specifications that have been published both in Australia and overseas for the field use of GPS.

Chapter 6 describes some of the techniques for processing field data. The processing component of GPS is vital, for it determines the final positioning accuracy of the survey. Several processing options are discussed, as well as the techniques that are necessary to achieve surveying accuracies.

The result of GPS processing is either a baseline vector between two stations or the three coordinates for each station. These are analogous to the reduced distances and directions obtained from a conventional field survey. The final processing step, therefore, involves combining these baselines or sets of coordinates to produce a network and then transforming the network to a local datum. Programs were developed by the author to carry out these tasks. Chapter 7 reviews the network adjustment process, and describes the procedures that are used to transform this adjusted network onto a local datum.

The software package developed by the author, can adjust processed GPS baselines into a network, solve for the set of transformation parameters relating the GPS reference datum to the local datum and then apply these parameters to the entire GPS network. The software package is described in Appendix B, and an example of its use is given in Chapter 8.

The opportunity will arise for all surveyors to increase their current productivity and to expand into many other areas where positioning is required. The future of the surveying profession is bright if surveyors endeavour to educate themselves about GPS and accept this technology as one of the measuring tools of the future.

# 2. <u>THE GLOBAL POSITIONING SYSTEM</u>

## INTRODUCTION

The Global Positioning System (GPS) was established in the early 1970's by the U.S. military for defense purposes. Although GPS was designed to provide continuous, worldwide, accurate, real time positioning to the defense forces of the U.S. and her allies, it also has a wide range of civilian applications. These include navigation on the ground, at sea and in the air, as well as positioning for surveying, geophysics and geodynamics.

GPS is a satellite system that, when fully configured, will consist of 18 satellites orbiting the earth in six planes. In order to determine a position on earth it is necessary to range to three satellites simultaneously. These satellite range measurements can be thought of as a resection by distances, where the satellites take the place of known ground control. From the three range equations the three unknown position coordinates (X, Y, Z) can be obtained. A range measurement to a fourth satellite is required, however, for accurate positioning. This is explained later in the chapter.

This measurement method requires knowledge of the satellite orbits at each measurement epoch. Each orbit is calculated using data from a number of tracking stations located around the globe. This orbit information is regularly uploaded to each satellite and is continually broadcast as part of a transmitted signal.

The ranges to the satellites are computed by measuring the time taken for a signal to travel from the satellite to the receiver and then multiplying this time by the speed of light.

The measurement of this time lag is made with coded signals simultaneously generated at the receiver and the satellites. These codes have random characteristics, and are referred to as pseudorandom signals. They are, however, binary codes generated by complex mathematical algorithms. The codes are generated by a 'clock' or 'oscillator'. The terms 'clock' and 'oscillator' are used synonymously and will be freely interchanged in this thesis.

When the coded signal transmitted from a satellite is correlated with the code generated in a receiver, a time lag is measured. Figure 2.1 illustrates the measurement of this time lag. By multiplying the measured time lag by the speed of light a range to the satellite is obtained. This measurement is subject to a number of error sources and therefore is not the true range between the satellite and the receiver. For this reason it is known as the pseudorange.

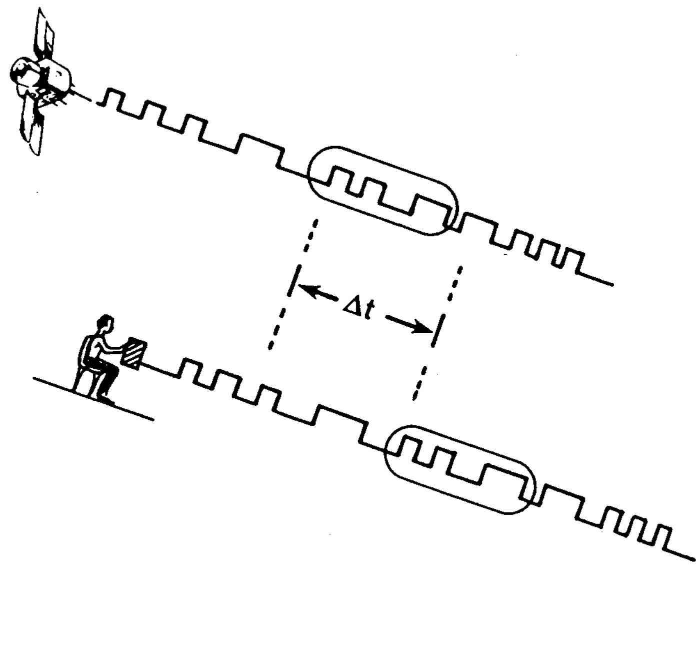



Figure 2.1 : Measurement of the Time Lag ($\Delta t$) of a Coded Signal Travelling between a Satellite and a Receiver (THOMPSON, 1985)

To compute the true range to a satellite, the satellite and receiver clocks would have to be synchronised to the nanosecond level. This level of synchronisation would require the use of atomic clocks, which are very expensive. Another way of overcoming this problem of clock synchronisation, is to solve for the clock error (known as the 'clock bias') that exists between the receiver and the satellite clocks. The main contribution to the clock bias term comes from the receiver clock as the cesium oscillators in the satellites are considerably more stable than the crystal oscillators found in most receivers. The clock bias represents the difference between the receiver time and the time kept by the satellites, known as GPS time.

The true range between the satellite and the receiver at each measurement epoch, therefore, consists of the pseudorange and a clock error range. This is illustrated in Figure 2.2.

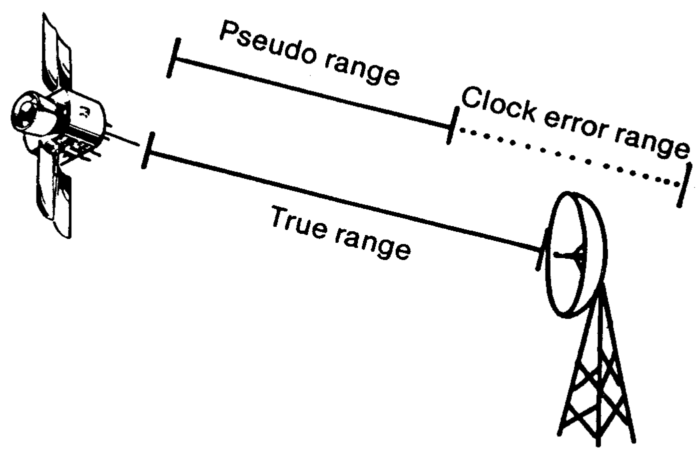



Figure 2.2 : Relationship between True Range and Pseudorange. (THOMPSON, 1985)

In order to determine the receivers coordinates, therefore, a receiver clock bias must be determined, in addition to the three receiver coordinates (X, Y, Z). To solve for these four unknowns in real time it is necessary to observe four different satellites simultaneously. When the GPS system is fully configured, with 18 orbiting satellites, there will be at least four satellites above the horizon, at any time, in all parts of the world.

GPS consists of the space, control and user segments. This chapter describes how GPS is configured and what role each segment plays in the system.

**THE BASIC GPS CONFIGURATION**

In 1973 the United States Defense System Acquisition Review Board authorized the introduction and development of GPS in three phases. Phase I was the concept validation program, that evaluated the effectiveness of the GPS concept for positioning. Phase II was the full scale development and system test phase, that involved launching the Block I prototype satellites and testing the system. At the time of writing, Phase II is complete, for the system has been fully tested and evaluated and the production satellites are ready to be launched. The Space Shuttle disaster of January, 1986 has delayed Phase III which is the final production and development phase.

The three major component parts of GPS are the space segment consisting of the satellites, the control segment that monitors the movement of the satellites and updates the information broadcast from them, and the user segment which comprises the receivers, and the necessary hardware and software to process received signals.

## SPACE SEGMENT

The space segment consists of the satellites that orbit the earth, and the signals they emit. When the system is fully configured there will be 18 satellites in six orbital planes, 20,200km above the earth. The satellites have periods of twelve sidereal hours which means that they pass over the same point on earth four minutes earlier each day. GPS was designed so that at least four satellites will be above the horizon at anytime, anywhere on earth. This enables users to determine station coordinates and the receiver clock error in real time.

Each satellite is equipped with a high precision oscillator (Rubidium or Cesium). The oscillator has a fundamental frequency of 10.23 MHz. This oscillator is used to generate two L band carrier waves. The L1 carrier wave has a frequency of 1575.42 MHz which is 154 times the fundamental frequency, and the L2 carrier has a frequency of 1227.60 MHz which is 120 times the fundamental frequency.

The L1 carrier is modulated by both a CA (Clear Acquisition) code and a P (Precise) code, while the L2 carrier is modulated with the P code only. The CA code modulation has a frequency of 1.023 MHz (equivalent to a tenth of the fundamental frequency), which corresponds to a 300 m wavelength, and the code sequence has a repetition period of one millisecond. The P code is generated at the fundamental frequency, which corresponds to a wavelength of 30 m, and has a code repetition period of 267 days. As the P code has a shorter wavelength than the CA code, the range to the satellite can be determined to a higher accuracy than with the CA code. The P code, therefore, yields a more accurate position.

In addition to these codes a navigation message common to both the P and the CA codes is broadcast at 50 Hz. This message contains information including the status of each satellite, the satellite clock correction to GPS time, the satellites ephemeris and the corrections for delays in the propagation of the signal through the atmosphere. The navigation message is updated and uploaded daily into the satellites. Figure 2.3 illustrates the nature and type of signal that is generated by each satellite.

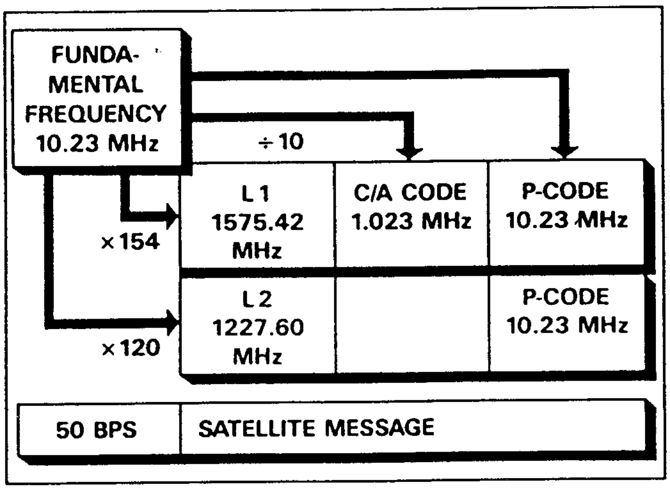



Figure 2.3 : The Satellite Message (SCHERRER, 1985)

The accuracy of absolute positioning using the CA and P code and the Block I satellites has been found to be approximately ±10m (MASTERS, 1986, SANDS, 1985, JONES, 1986). This level of accuracy is expected of the P code, but far exceeds the expectation of the CA code. Once the production satellites have been launched, however, it is expected that both the ephemeris information and the carrier frequencies transmitted from the satellites will be subject to dithering. The dithering is expected to reduce the inherent accuracy of the CA code to the ±100 m level.

It is generally accepted that with the launching of the production satellites, the P code will only be available to the U.S. military, its allies and several approved civilian users. The CA code should be available to the general user community.

Presently there are 6 operational Block I satellites in orbit, which provide a four hour observation window in Australia. It was originally planned to launch the production (Block II) satellites between 1986 and 1989. The space shuttle disaster of January 1986, however, has delayed the commencement of launching until 1988 (STEIN, 1986). Figure 2.4 shows the delayed launching schedule caused by the shuttle disaster.

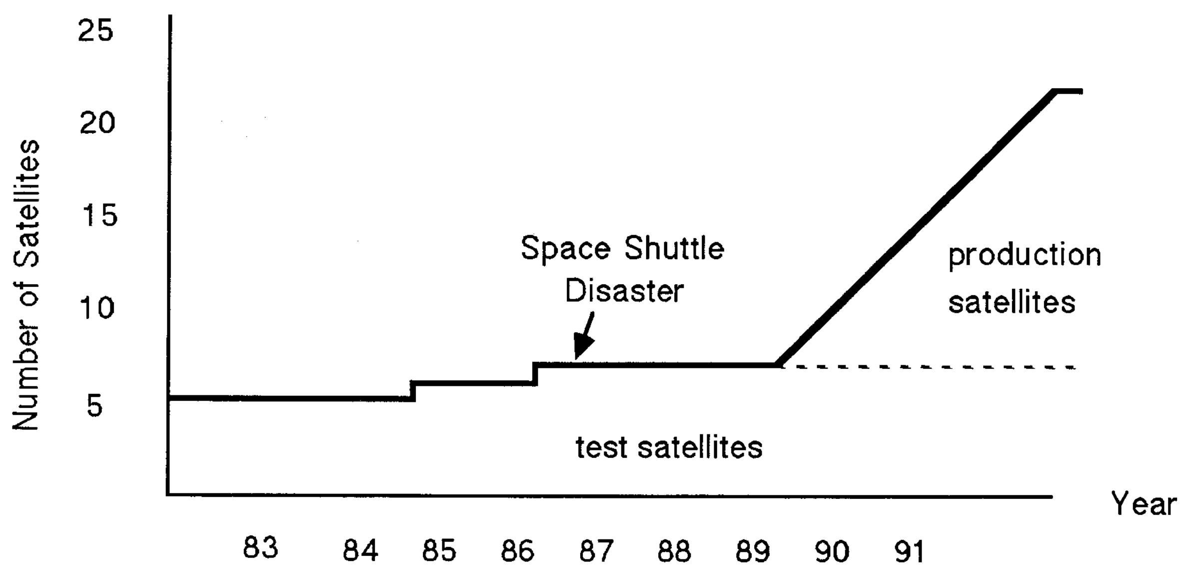



Figure 2.4 : Satellite Availability

The broadcast ephemeris transmitted via the navigation message is used by the receiver to determine the position of each satellite at the measurement epoch. The computation of this ephemeris is the responsibility of the Control Segment.

## CONTROL SEGMENT

The function of the control segment is described by WOODEN (1985).

> " The control segment consists of equipment and facilities required for satellite monitoring, telemetry, tracking, commanding, and control, uploading, and navigation message generation. The monitor stations passively track the satellites, accumulate ranging data from their signals, and relay them to the Master Control Station (MCS) where they are processed to determine satellite position and signal data accuracy. The MCS updates the navigation message of each satellite and relays this information to the ground antennas which transmit it to the satellites. The ground antennas are also used for transmitting and receiving satellite control information."

The broadcast ephemeris provides the satellites' position in orbit, computed using tracking data from the previous two weeks. It consists of a set of hourly elements describing the motion of the satellite for each hour of that day. At the end of each day a new ephemeris solution with new hourly elements is calculated. GPS operators may also have the choice of using a post processed precise ephemeris, or an ephemeris generated from local tracking data. The advantages and disadvantages of each of these types of ephemerides is discussed in Chapter 6.

The final operational control segment consists of a MCS at Colorado Springs, Colorado, three ground antennae and monitor stations located at Kwajalein in the Phillipines, Diago Garcia and Ascension Is. and an additional monitor station at the MCS and in Hawaii. The Defense Mapping Agency (DMA) plans to establish three additional ground antennae located in the United Kingdom, Australia and Argentina, as well as 16 other monitor stations located around the world for computation of the precise ephemeris. Figure 2.5 shows the location of the Control Segment

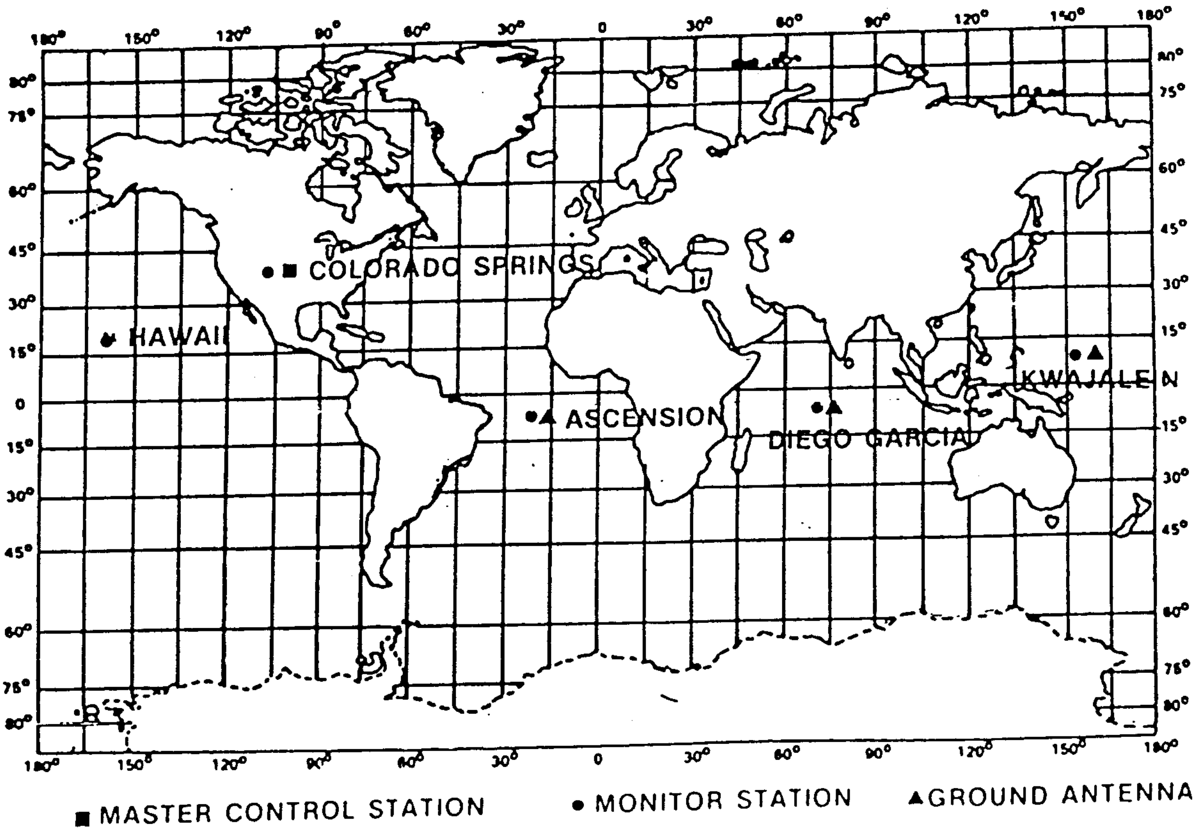



Figure 2.5 : Location of the Control Segment Stations (HARDCASTLE, 1986)

The broadcast ephemeris generated by the control segment is presently in a reference system known as World Geodetic System

1972 (WGS72). When the production satellites are launched, however, it is planned to change to the World Geodetic System 1984 (WGS84). Not only are these systems different from each other, they are also different from the Australian Geodetic Datum 1984 (AGD84). The reference systems can be related through transformation parameters. The relationship between WGS72, WGS84 and AGD84 is given in Chapter 7.

## USER SEGMENT

The user segment involves the use of GPS, and is described in detail in various chapters of this thesis. This segment consists of the satellite receivers (Chapter 4), observation procedures (Chapter 5), processing methods for data reduction (Chapter 6), network adjustments and transformations (Chapter 7).

GPS was designed for navigation, and not for survey measurement. Navigation requires real time absolute positioning to the ±20 m level, while surveying aims to achieve a high accuracy of relative positioning (1:10,000-1:100,000). Consequently, the design accuracy for GPS positioning is much lower than that required to achieve surveying accuracies. Surveying accuracies can be achieved, however, by using the system in a way that was not originally intended.

The idea of the user segment is to achieve surveying accuracies from GPS by reducing the magnitude of the error sources that affect the system. These include errors in the broadcast ephemeris, the instability of the satellite and receiver clocks and the propagation delay to the transmitted signal caused by atmospheric refraction. Surveyors need to understand how these errors can be reduced through hardware improvements, special observation procedures and more rigorous processing techniques.

GPS can be used in either point positioning mode, where one receiver is used to determine absolute position, or differential mode where two or more receivers are used to determine relative positions. Point positioning currently provides ±10 m positions, which are suitable for most navigation requirements and reconnaissance surveys. To achieve surveying accuracies, however, it is necessary to use differential mode. Higher relative positioning accuracies are possible as many of the errors common to the receivers either cancel or are greatly reduced. In the differential mode, the GPS data must be brought together from all receivers before it can be processed. This makes real time relative positioning difficult.

As the differential mode is mandatory for most surveying applications, the surveyor must operate at least two receivers simultaneously. In practice, one of these receivers is located on an established station while the other is placed on a new survey mark.

GPS receivers are currently very expensive (see Chapter 4). Thus the concept of the 'base station' has been proposed (TEXAS STATE, 1984). Base stations are GPS receivers permanently set up on a number of established marks throughout a state, or country to continuously observe the GPS satellite data. Surveyors would therefore only have to purchase one receiver, for they could establish new marks relative to the already occupied base stations. The base stations may perform a number of tasks, including tracking the satellites for orbit determination and disseminating the ephemerides (Chapter 6) and geoid-spheroid data (Chapter 7).

# 3. IMPACT OF GPS ON AUSTRALIAN SURVEYING

## INTRODUCTION

Within the next few years, GPS will have an enormous impact on surveying in Australia. This technology has been accepted by large companies such as BHP and by government departments (eg. South Australian and Queensland Department of Mapping and Surveying) as the surveying tool of the future. GPS has already been used on several surveying projects in Australia, including a control densification survey of South Australia, the establishment of photogrammetrical control on the New South Wales north coast and for a gas pipeline control survey in the Northern Territory.

In order to understand the advantages GPS has over traditional positioning technology, this chapter reviews measurement technologies currently used in Australia and compares them with GPS. Some advantages of GPS are highlighted, and the best current applications of the system are identified.

Data concerning existing techniques were obtained from a questionnaire that was distributed to the surveying profession in January, 1985 (ECKELS & STOLZ, 1985). The capability of GPS, has been amply demonstrated by GPS surveys carried out both in Australia and overseas.

## EXISTING MEASUREMENT TECHNOLOGIES

This section of the chapter describes the advantages and disadvantages of terrestrial surveying with electronic distance measurement equipment (EDM), inertial systems and the TRANSIT Doppler system, taking particular note of the accuracies achievable and cost.

## ELECTRONIC DISTANCE MEASUREMENT

The questionnaire showed that a large percentage (approximately 80%) of surveyors presently own an EDM instrument. EDM is used widely in engineering, cadastral, control, mapping, mining and hydrographic surveys and represents the principal distance measuring tool of the surveying profession.

The major users of the short and long range EDM were found to be surveyors involved in engineering, cadastral and control work. Mapping organisations are also major users of long range instruments. The components and features of EDM instruments are described by RUEGER (1978) and BURNSIDE (1971).

Short range EDM can measure distances up to about 2km with an accuracy of ±(5mm+5ppm). This type of instrument has a number of advantages. Most of the equipment is light weight and does not require highly trained operators. Distances are measured quickly (within a few seconds) which allows these instruments to be competitive with a steel band for distances less than 100m and in most cases measurements can be performed faster than with a steel band over 100m. Corrections to allow for the additive constant of the instrument, its cyclic error and atmospheric refraction can be entered into the instrument in the field.

Many short range instruments are compact and 'user friendly' and have been specifically designed to be integrated with theodolites. The instruments can perform many tasks, including reduction of measured distance to the horizontal, making atmospheric corrections, and computing coordinates from measurements. The concept of the total station is now widely accepted, and many organisations are moving toward a totally computerised office, where data is collected and recorded in the field with a total station and is then brought back to the office to be processed and plotted. Each year the collection of field data for surveying becomes easier with this type of equipment.

Long range EDM can measure distances up to 70km in ideal conditions with an accuracy of $\pm$(5mm+1ppm). The operating procedures of these instruments may be slightly more involved than that of the short range instruments. Many long range EDM instruments have not been integrated with theodolites, and are therefore used separately. In some of the older instruments the power supply was an external battery, and this impeded portability. More recently, power packs have been included as part of the instrument itself.

The major drawback of all EDM instruments is that intervisibility is required between stations. This implies that control surveys are limited to using high points, and in many cases require extensive clearing to meet line of sight requirements. The limitation imposed by intervisibility must be a major consideration in the design of any survey network. Error propagation throughout the network can be minimised by maintaining good network geometry, and ensuring sufficient direction and distance measurements are taken to satisfy accuracy requirements.

Accuracy of EDM is also limited by atmospheric refraction. Errors of 1° C in the measurement of the atmospheric temperature results in approximately 1ppm baseline errors, while a 1mb error in the pressure measurement contributes 0.3ppm to each baseline (RUEGER, 1984). To achieve surveying accuracies 'environmental corrections' can be made to the measured distance using an atmospheric model built into the instrument and meteorological data recorded at the base station.

EDMs are compared with other measurement technologies in Table 3.1.

Table 3.1 : Comparison of Measurement Techniques

| Characteristic | GPS[1] | Terrestrial | ISS[2] | Doppler[3] |
|---|---|---|---|---|
| Typical uncertainty for relative positions | 1-20cm | dist/$10^5$ | 20-40cm | 20-50cm |
| Typical operating range | .5-1000km | .1-20km | 5-100km | 50-1000km |
| Measurement Time/baseline | 1-2hr | mins-hrs | 5-20mins | 2 days |
| Number of persons required | 1-2 | 2-4 | 2 | 1-2 |
| Intervisibility requirement | no | yes | no | no |
| Back-packable ? | yes | yes | no | yes |

References: (1) Counselmann (1982)
(2) Babbage (1981)
(3) Kouba (1983)

## INERTIAL SURVEYING SYSTEMS

The questionnaire showed that very few Australian surveyors (6 respondents) had used inertial surveying techniques. Inertial systems have been used in Queensland for control densification purposes (GRIEP & CLERICI, 1984), and in the Northern Territory (CHUDLEIGH, 1984) and South Australia (LARDEN & WARHURST, 1985) for establishing photogrammetrical control. The system can be mounted in either a car or a helicopter and is capable of rapidly establishing decimetre-level control. In North America inertial systems have been widely used to provide data for mapping, and control for route selection and construction of pipelines, transmission lines, dams, roads and railways. Inertial techniques have also been used for petroleum exploration and the positioning of seismic and gravity data (BABBAGE, 1981). The reader is referred to BRITTING (1971) and RUEGER (1983) for an explanation of the principles of inertial systems.

Inertial surveying systems have many advantages. Inertial systems have the ability to position many points rapidly, which can lead to time savings and reduced costs on many jobs. However, more time is spent in planning, reconnaissance and marking a survey than on performing the measurements themselves.

The accuracy of the system depends on the way the network is measured. The inertial system survey for control densification, recently carried out in Queensland gave double run accuracies of $\sigma_x$= 4.6cm, $\sigma_y$= 4.4cm, and $\sigma_z$= 2.1cm respectively (GREIP & CLERICI, 1984). RUEGER (1983) claims that the system is accurate to approximately ±1m on single run traverses between two fixed points. This accuracy increases to between ±.3-.5m for a double run traverse between two fixed points. The highest accuracy is achieved in a grid type network, with several fixed points, as well as crossover points. Typical accuracies for these type of surveys are ±.1-.3m. This accuracy was achieved in the Queensland project mentioned earlier.

In the navigation mode, inertial instruments may be used to guide the surveyor to the next control station. The inertial system gives total surveying capability, by allowing the operator to measure change in position, change in velocity, deflections of the vertical and gravity anomalies. Intervisibility between points is not necessary and the requirement for establishing stations is dependent on accessibility rather than visibility.

There are, however, some drawbacks to the inertial system. The system relies on the existence of ground control in the area to be surveyed. If existing control is scarce, new stations must be established. This has often been done with the TRANSIT Doppler system. Inertial equipment is difficult to operate and requires highly trained operators to deal with any problems as they arise. The systems have not been found to be reliable in the past, and users (eg. The Geodetic Survey of Canada, the Canadian Nortech Company) have purchased several instruments to ensure that spare parts are available, and that at least one instrument is working at any time!

Another disadvantage of inertial systems is that their cost (approximately $300,000) is beyond the budget of most surveyors. The cost of instrumentation tends to limit the use of inertial equipment to large jobs, that require rapid control at decimetre-level accuracy. Problems have also been encountered with the software for real time processing, especially in incorporating gravity anomaly data with the inertial measurement.

Finally, one must consider topography and vegetation of the area to be surveyed. Many projects demand the thorough preparation of the site which includes reconnaissance, clearing landing pads for helicopters and marking. Thick vegetation and difficult access can slow this process considerably, thus increasing the cost of the survey. Inertial navigation systems are best suited to flat, open country.

The features of inertial systems are shown in Table 3.1.

## SATELLITE SYSTEMS

Satellite positioning systems, such as the TRANSIT Doppler system and GPS have several advantages over ground based techniques. Station intervisibility is not required, as the high altitude of the satellites allows them to be 'seen' from large areas of the earth's surface. This enables receivers hundreds of kilometres apart to measure to the same satellite without experiencing any of the terrain interference that commonly limits ground based systems. To allow satellite signals to be tracked, however, it is necessary that the occupied stations are cleared.

As satellite systems are global in nature, they can be used to establish new stations anywhere on earth to the same level of accuracy. Stations fixed by satellite techniques can be positioned anywhere, and do not require the good network geometry needed to minimise the propagation of measurement errors that affect traditional direction and distance surveys.

The favorable error propagation of satellite systems allows them to establish control stations in areas where they are required, independent of either network design or line of sight requirements. The almost unlimited range of the satellite systems makes them suitable for establishing control in previously unsurveyed areas.

As the accuracy achievable with satellite systems is equal to or better than that claimed by short range radio location systems, they provide ideal navigation tools. Moreover, satellite systems do not require shore based transmitters for offshore work.

Satellite systems determine station coordinates in a three dimensional coordinate system. The height given by the satellite system is an ellipsoidal height in the satellite reference frame. This height can be transformed into a local orthometric height with knowledge of the transformation parameters relating the satellite datum to the local datum, and knowledge of the corresponding geoid-spheroid separation. Height considerations of GPS surveys are discussed in Chapter 7.

Another advantage of satellite systems is that they are designed to be used at any time during the day, in any sort of weather condition.

**TRANSIT System**

The TRANSIT system, like GPS, uses satellite signals for positioning. Although the system was designed for military purposes, the U.S. government made available information concerning the system which allowed it to be used commercially as a navigation and survey tool in the late 1960s. It was the first time surveyors had the opportunity to use artificial satellites for positioning.

Principles of Operation

The TRANSIT system uses a Doppler shift measurement for positioning. This measurement allows the velocity of a space vehicle to be determined from the change in frequency of a transmitted signal from the satellite with time. By knowing the velocity of the vehicle and the location of the satellite at any instant, the coordinates of the ground station can be computed. This is done by intersecting the hyperboloids of revolution formed at each time interval $\Delta t$, whose focal points correspond to the satellite positions at the beginning and end of that time interval (see Figure 3.1).

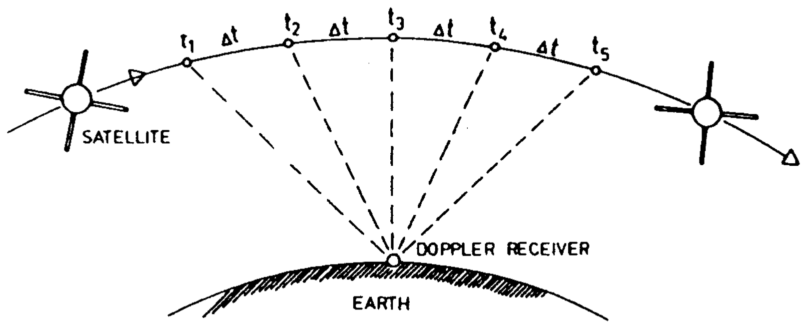



Figure 3.1 : Doppler Measurements (RUEGER, 1984)

The TRANSIT satellites transmit highly stable coherent carrier frequencies at approximately 150 MHz and 400 MHz. The transmitted signal contains a constant reference frequency, from which the measurements are made, a navigation message, giving the satellites position and a timing signal. The navigation message and timing signal are revised and updated periodically from a ground control network established in North America.

Each of the six satellites in the TRANSIT system pass quickly overhead, in order to create good satellite/receiver geometry (see Figure 3.1) and to produce a pronounced Doppler effect. Consequently, the TRANSIT satellites orbit only 1075km above the earth, circling every 107 minutes. These low orbits, however, are difficult to model as they are perturbed by high-degree harmonics in the earth's gravity field which are not precisely known.

The satellites are tracked by four ground stations operated by the U.S. Navy Astronautics Group. These stations are responsible for determining the satellites present orbit, and predicting the orbit into the future. The updated orbit information is then injected into the satellites navigation message which is transmitted with the reference frequency.

The TRANSIT system, like GPS, gives surveyors the choice of using the broadcast ephemeris transmitted from the satellite, or a post processed precise ephemeris for their data reductions. The broadcast ephemeris is not as accurate as the precise ephemeris, but it is available at the time of measurement. For most surveying control work the precise ephemeris is preferred, as higher accuracy is more important than real time processing. As the precise ephemeris is computed from the tracking data from twenty stations around the world, as opposed to the four U.S. based stations used for the broadcast ephemeris, it provides much better orbit data for all stations, especially those in the southern hemisphere. The disadvantage of the precise ephemeris is the excessive time taken to collect the tracking data, compute the orbits and distribute this information to the user. The precise ephemeris is also protected, and may not be made available to all users.

TRANSIT can be used in the point positioning or translocation mode. The point positioning accuracy that can be achieved, using 25 satellite passes and the broadcast ephemeris is ±5m, while ±1.5m can be obtained using the precise ephemeris (STANSELL, 1979). A number of errors that occur in the point positioning mode can be eliminated by using differential positioning. In this mode the relative position between stations can be determined to ±2m with the broadcast ephemeris, and to less than a metre using the precise ephemeris.

A joint U.S. Department of Defense / Department of Transport statement concerning the U.S. government policy regarding the funding of radionavigation systems was released in November 1984. The policy statement addressed the future of radionavigation systems in the light of GPS. One of the policies was the planned replacement of the TRANSIT system with GPS by 1994. As the shuttle disaster of January 1986 has delayed the launching of the GPS production satellites, the scheduled replacement of the TRANSIT system may be similarly delayed.

The TRANSIT Doppler system is compared with other measurement techniques in Table 3.1.

## TRANSIT use in Australia

The questionnaire showed that Australian surveyors use the TRANSIT system for photogrammetric and geodetic control surveys, geophysical positioning (closely related to mineral exploration) and offshore positioning. The technology is used to establish sub-metre level control for large scale surveys in areas where only sparse survey control exists. Geodetic accuracies can be achieved if the stations are sufficiently widely spaced (ie .5m relative positioning is equivalent to 1ppm over 500km).

The survey showed that TRANSIT was used for photogrammetric control at the 50ppm level, and geodetic control at the 10ppm level, while geophysical and offshore surveys only required accuracies at the 1000ppm level. In order to achieve the accuracies required for photogrammetrical and geodetic control the TRANSIT system is used in the differential mode. Point positioning is adequate, however, for many low accuracy geophysical surveys.

Although the TRANSIT system has many advantages over other existing technologies, several of its disadvantages were highlighted in response to the survey. Approximately 70% of respondents indicated that the observations took too long, while 30% felt the equipment was too expensive to buy or hire and did not provide results that were accurate enough. Another drawback of TRANSIT equipment was that the receivers did not directly provide heights above sea level. This problem, which is common to all satellite techniques, is addressed in Chapter 7.

## PRESENT GPS CAPABILITY

The operation of GPS has been described Chapter 2. GPS is the first space based measurement technique able to compete with terrestrial measurement over short distances (<10 km) and able to improve on many terrestrial measurements over longer distances (>10 km).

GPS has a number of advantages over existing terrestrial techniques, including the existing TRANSIT satellite system. GPS receivers observe the range to each satellite, while the TRANSIT Doppler receivers observe the range rate to the satellite. GPS satellites have high (20200km) orbits, so they pass overhead slowly and can be observed for long periods. TRANSIT satellites have low (1075km) orbits, so they pass overhead quickly and provide good determinations of the range rate observable. The high GPS satellite orbits are more easily modelled than the TRANSIT satellites orbit, as they are less affected by the gravitational field and the atmospheric drag of the earth.

The 18 GPS satellites will ensure continually good satellite/receiver geometry, while the 6 TRANSIT satellites can only provide this intermittently.

Table 3.1 shows that GPS already has many advantages over existing technologies.

Over the past few years there have been numerous examples of successfully completed GPS surveys. Although GPS has mainly been used for navigation and geophysical exploration, it has also been used to densify existing control networks, and to establish control for mapping and engineering projects.

The Eifel network, already mentioned in the introduction, is an example of a control densification survey. A diagram of this network is shown in Figure 3.2. The network, which consisted of 21 stations, was surveyed by measuring 43 independent baselines in 25 days. Progress for the survey was hindered by delays in the shipment of some of the equipment, and by equipment malfunctions. During the survey there were 2 instruments in operation for 7 days, 3 instruments for 9 days and 4 instruments for the other 7 days. Two days were completely lost due to equipment malfunction.

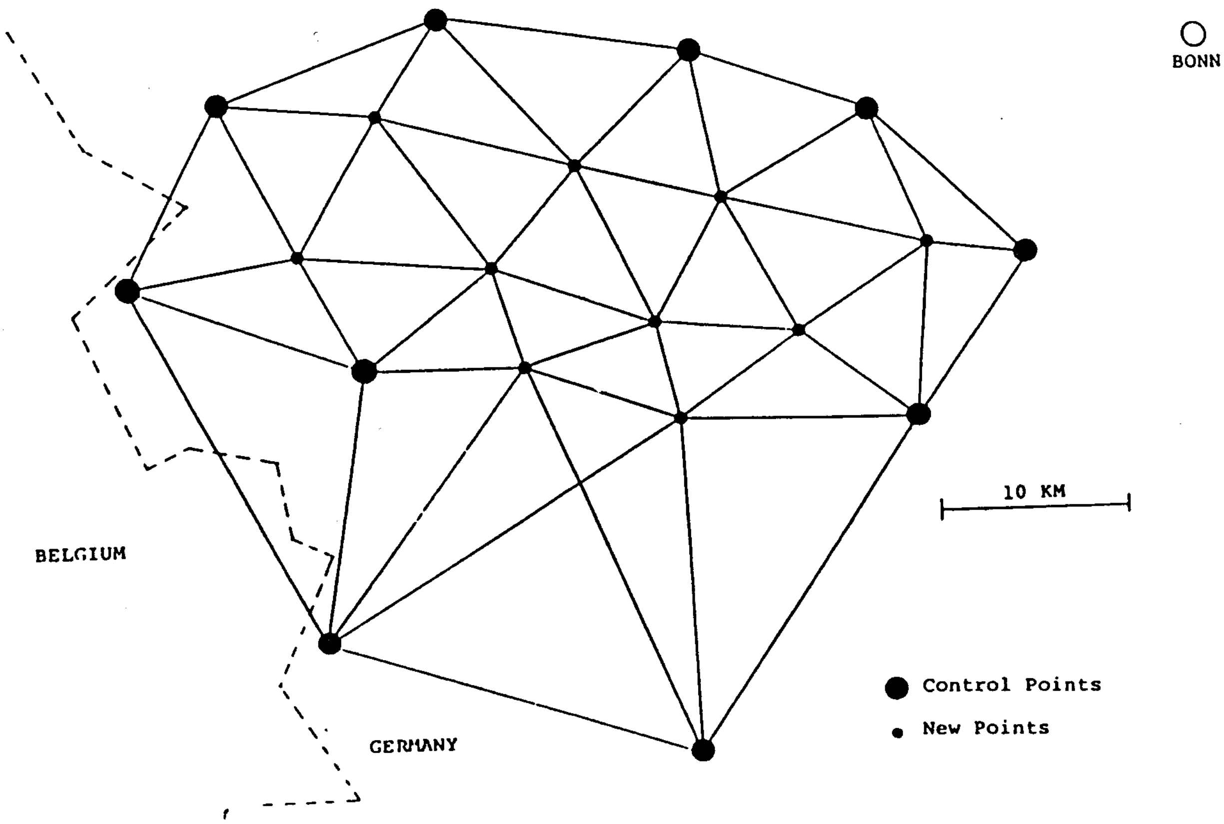



Figure 3.2: The Eifel Network (BOCK et al., 1984)

Each Macrometer GPS receiver (see Chapter 4) tracked 5-6 satellites over a 3.5 hr observation period. Of the 43 baselines measured, 37 were of excellent accuracy (1-2ppm), 5 were of good accuracy (3-7ppm) and one was of relatively poor quality (>10ppm). One of the major accuracy limitations of the survey was the poor satellite geometry during the observations (see Chapter 5). All of these baselines, however, were included in the final network adjustment.

The network adjustment found the baseline lengths to be consistent with each other to approximately 1ppm for the entire network, and the baseline vector components were consistent to 1-2ppm. The root mean square of the standard deviations for the adjusted baselines was about 1cm for the horizontal components, and 2cm for the height component of the baseline vectors.

In November and December of 1985 the U.S. survey firm Geo/Hydro Inc. carried out a control densification survey for the South Australian Department of Lands (SALD). This survey established 90 new survey stations adjoining the Victorian border, south east of Adelaide. The survey was carried out with Macrometer V-1000 instruments and the data was processed as single baselines by MACROMETRICS software (see Chapter 6). LARDEN (1986) reported that the horizontal components (latitude and longitude) of new stations were established to an accuracy of 2ppm, while the height was determined to 4ppm.

The positioning accuracy achieved by GPS in the Eifel network (Table 1.1) and in the South Australian control densification survey is typical of other completed GPS surveys.

Table 3.2 lists the accuracies achieved in several GPS surveys carried out in 1982 and 1983 with the Macrometer V-1000 receiver (see Chapter 4). Items 1-9 of Table 3.2 are 'short' baseline results using 2-3 hour observing sessions, while items 10-11 are 'long' baseline results using 10 hour observing sessions. The table shows that accuracies of 2ppm over a range of distances are possible.

Table 3.2 : Surveys with the Macrometer V-1000 (BOCK et al, 1983)

| Item | Date | Place | Average Distance | No of Lines | Lines/ Day | Worst Coord. Error | Notes |
|---|---|---|---|---|---|---|---|
| 1 | 12/82 | Mass. | 30 km | 5 | 1 | 1:500,000 | (1) |
| 2 | 1/83 | FGCC(2) | <1 km | 4 | 1 | millimetres | (3) |
| | " | " | 20 km | 5 | 1 | 1:500,000 | (3) |
| 3 | 2/83 | Florida | 12 km | 8 | 2 | 1:200,000 | (1) |
| 4 | 2/83 | Alabama | 11 km | 3 | 2 | 1:700,000 | (1) |
| 5 | 3/83 | Texas | 24 km | 5 | 1 | 1:285,000 | (3) |
| 6 | 3/83 | Germany | 11 km | 2 | 1 | 1:230,000 | (3,4) |
| 7 | 4/83 | Ohio | 5 km | 4 | 1 | 1:450,000 | (1) |
| | " | " | 793 km | 18 | 2 | 1:275,000 | (1) |
| 8 | 4/83 | Colorado | 5 km | 12 | 2 | 1:500,000 | (1) |
| 9 | 4/83 | Maryland | 748 km | 1 | 1 | 1mm | (5) |
| | " | " | 32 km | 3 | 1 | 1:500,000 | (3,5) |
| 10 | 4/83 | Mass. - W. Va. | 845 km | 1 | 1 | 1:500,000 | (3) |
| 11 | 4/83 | Mass. - W. Va. Arizona | 2400 km | 3 | 1 | 1:500,000 | (1) |

Notes:

(1) Misclosure of polygon whose sides were measured at different times.

(2) Federal Geodetic Control Committee test network near Washington D.C.

(3) Accuracy tested by comparison with independent survey by other methods.

(4) Accurate independent information on ellipsoidal height not available.

(5) Difference from previous measurement by different Macrometer V-1000's and operators is shown.

Although Table 3.2 indicates that GPS is capable of high accuracy measurement, the bottom line for surveyors is the cost effectiveness of the system. If the system cannot produce results quickly and cheaply it will have little impact on current technology. It has been shown, however, that GPS can reduce survey time and produces coordinates of higher or equivalent accuracy to conventional techniques for less cost per station.

The comparison of the cost and speed of survey, made between GPS and conventional techniques for the Eifel network (Table 1.1) demonstrates the savings that were achieved.

Another example of a control densification survey that used GPS to improve speed and reduce cost was done in Montgomery County, Pennsylvania in 1984. The survey was contracted by the Pennsylvania Department of Transportation (Penn DoT) as part of an ongoing control densification program. Initially Penn DoT planned to use conventional steel tower triangulation techniques to complete the survey. They changed their plans, however, and decided to use GPS. According to COLLINS (1985):

> ' Two factors precluded the use of steel tower triangulation. First this conventional method would have cost five times more than the GPS method, and secondly the liability associated with constructing 100 foot towers in an urban area is unacceptable.'

The Montgomery County survey which consisted of coordinating 42 stations using 68 independent baseline measurements, resulted in a vertical and horizontal position accuracy of 1:800,000 and 1:500,000 respectively.

In a discussion on the cost for establishing geodetic control points with GPS, COLLINS (1984) remarks:

> 'The cost of establishing a first-order horizontal control point ranges from (US)$6000 to (US)$10000. This price includes monumentation and an azimuth mark at each point. A point of the same (or better) accuracy established by Macrometer is about (US)$2000...The cost of establishing second- or third-order control is naturally a fraction of the cost of first order control since greater equipment utilization is possible. For example, we (Geo/Hydro) have been able to establish closely spaced (2-3mile) second order points for about (US)$500 per point'

The South Australian Control survey demonstrated the cost effectiveness of GPS under Australian conditions. The final cost of establishing new control stations with GPS (including research and development costs) was approximately $AUS 3000 per station (LARDEN, 1986). This figure is substantially less than the estimated cost of carrying out the same survey with conventional techniques, which was expected to be $AUS 4000 -10,000 per station. The South Australian survey also demonstrated the time savings through using GPS. The GPS survey took six weeks in the field and four months to process, while it was expected that the same survey with conventional techniques would have taken six months in the field and twelve months to process.

The Texas State Department of Highways and Public Transportation has also realised the potential benefits of using GPS. In 1984 they proposed a plan to establish permanent satellite receiver base stations throughout Texas. When these base receivers are used in conjunction with remote 'roving' receivers it is expected that new stations could be coordinated anywhere in Texas to an accuracy of 2-5cm (TEXAS SDOHAPT, 1984). It is claimed that the introduction of GPS into the Highways Department would reduce present survey costs by a factor of five, reduce manpower costs by a factor of six and improve present survey accuracies by a factor of two. Moreover, the introduction of GPS would reduce survey time drastically. It is suggested that each receiver would only need approximately 15 minutes to coordinate new stations to an accuracy of 1:200,000. When GPS is fully configured it is expected that remote receivers could be used to establish 3-30 new stations per day, depending on distance and accessibility considerations.

The examples of surveys and proposals given above show that GPS will have enormous impact on surveying. Figure 3.3 shows the approximate accuracy and range capabilities of GPS, terrestrial techniques (EDM), inertial systems (ISS) and the TRANSIT satellite doppler system. Inspection of Figure 3.3 shows that

1) The accuracy of differential GPS is better than the accuracy of both TRANSIT translocation and inertial systems.
2) Differential GPS measurement will compete with EDM.

Figure 3.3 shows that GPS can provide accuracies equal to, or better than other existing measurement techniques for distances greater than a few kilometres. For short distances, it may presently be quicker and more cost effective to use a wire or an EDM. As GPS does not require intervisibility, however, it would have advantages in thickly vegetated or rugged terrain.

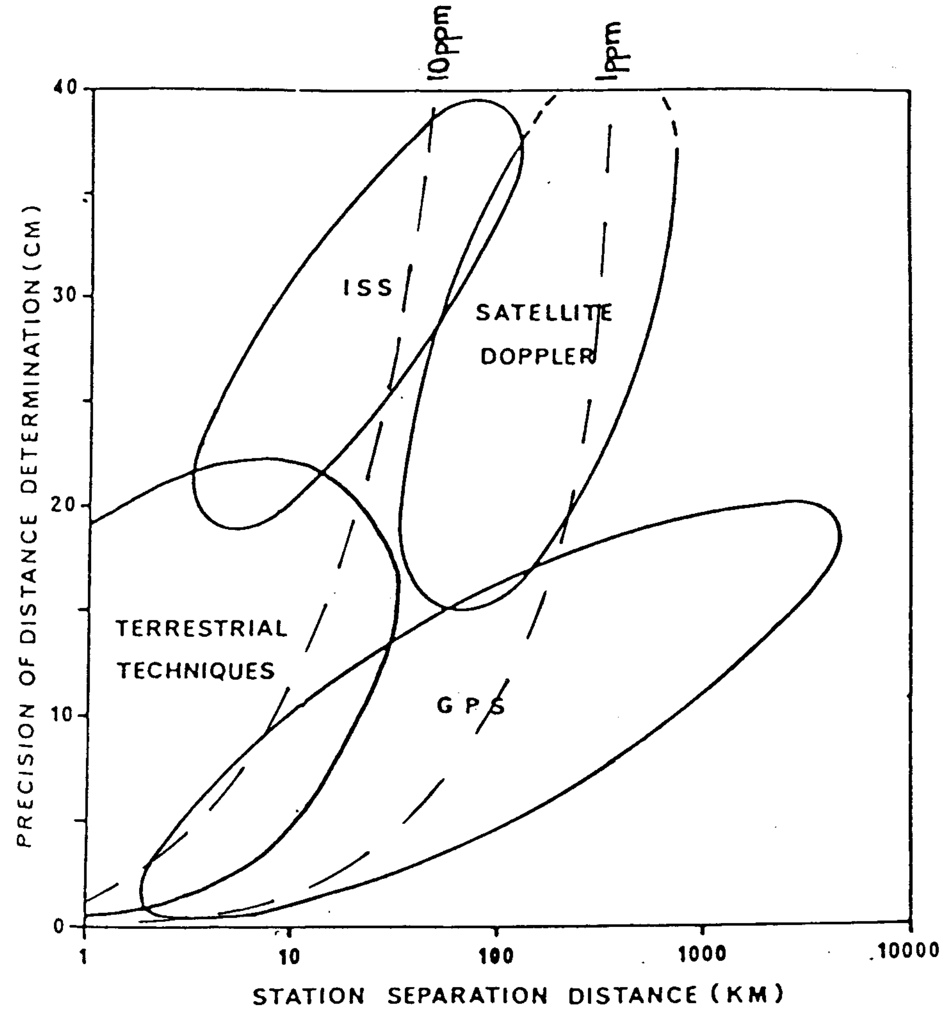



Figure 3.3 : The range and accuracy capability of terrestrial techniques (EDM), inertial systems (ISS), TRANSIT system (satellite doppler) and GPS.

## CURRENT APPLICATIONS OF GPS

GPS presently has a range of applications as both a navigation and a surveying positioning tool. The major market for GPS receivers will always be in the real time navigation area. There are many potential users of the system on land, at sea and in the air.

GPS has an immediate role to play in surveying. The questionnaire distributed to surveyors in 1985 indicated that the first three major users of GPS in Australia would be those involved in offshore positioning, establishing and densifying photogrammetric and geodetic control, and geophysical exploration. This finding is supported by recent GPS activity in Australia.

Offshore positioning using GPS is currently being carried out by Broken Hill Pty. Ltd. and Woodside Offshore Petroleum in the Timor sea, and by the Bureau of Mineral resources in the Great Barrier Reef. Offshore rig positioning involves using two CA code GPS receivers in the differential mode, while for navigation at sea, one receiver has proved sufficient. CHISHOLM (1986) reported that accuracies at the 5m level were consistently achieved using differential CA code receivers. It is expected that the offshore industry will closely embrace GPS, as the technique will provide continuous, accurate positioning at a fraction of the cost of existing radionavigation systems.

The geophysical exploration industry has also used GPS in Australia. The first privately owned GPS receiver in Australia was purchased by Aerodata Pty Ltd, for geophysical exploration. The coded receivers that are now available provide an adequate accuracy level for most geophysical surveys ($\approx$10m) at a price that is considered very reasonable.

GPS has already been used to densify and upgrade the control network in South Australia, and to provide control for a Land Information System established in Queensland. These projects were organised by the mapping departments of South Australia and Queensland respectively and are aimed to use the cost effectiveness of GPS and to provide an opportunity for local surveyors to learn about this new system. It is significant, however, that both the South Australian and Queensland Lands Departments have realised the potential of GPS, and have accepted it as the most effective surveying tool presently available for establishing and densifying control networks.

Another important application of GPS lies in the monitoring of crustal dynamics. The New Zealand government is particularly aware of this problem, as the country lies on the boundary of two tectonic plates. The Department of Lands and Survey regularly carry out Earth Deformation Surveys (EDS) to monitor the magnitude of the crustal movement. HANNAH (1985) considers the vast potential of GPS in this type of measurement, and describes the cost and time savings that may be achieved using GPS. HANNAH (1985) proposes that GPS will provide the same accuracy as conventional techniques, but at approximately one half the cost of present EDS surveys. By using GPS it will be possible to establish EDS stations in more accessible locations than on mountain tops, as intervisiblity will no longer be required. Once these GPS stations are connected to the established EDS network, they can be used to act as a bench mark for future GPS crustal dynamic measurement.

# 4. GPS RECEIVERS

## INTRODUCTION

GPS is capable of providing positioning for a range of surveying and navigation purposes. The accuracy and speed of a position fix by GPS depends on the type of equipment used, the observing procedures followed (see Chapter 5) and the processing method (see Chapter 6).

This chapter describes the observables measured by GPS receivers and outlines the different techniques used to collect and record them. The design characteristics of GPS receivers are then discussed and the GPS receivers that are either presently available or are under development are reviewed.

## GPS OBSERVABLES

The signals that are emitted from each satellite were described in Chapter 2. The signals consist of the L1 carrier wave which is modulated by both the P and the CA code, and the L2 carrier wave which is modulated by the P code only. Both carrier waves also contain a navigation message. GPS receivers use a number of techniques to record these satellite signals. The collected data are known as the observables, and can be processed to determine a receivers position or the baseline vector between two receivers.

The five different observables that may be recorded by GPS receivers are the pseudorange, the carrier wave phase, the integrated Doppler count, the Doppler count and the transition phase.

## PSEUDORANGE

GPS was originally designed to use the coded signals, that were modulated on the carrier waves, to measure ranges to the satellites. The range measurement using the codes is known as the 'pseudorange' observable. The pseudorange observable can be described as the time shift required to align a replica of the GPS code generated in the receiver with the GPS code transmitted by the satellite, scaled into distance by the speed of light. By measuring the pseudorange to four satellites simultaneously the receiver can solve for the three station coordinates and the receiver clock offset to GPS time.

The resolution of the pseudorange measurement depends on the accuracy with which the transmitted and the generated code can be aligned. As the P code's wavelength ($\lambda$=30m) is ten times shorter than the CA code ($\lambda$=300m), more accurate signal alignment is possible and hence more accurate positions are obtained. The P code was designed for point positioning to the 10m level while the CA code was designed to achieve ±100m point positioning.

## CARRIER WAVE PHASE

The positioning accuracy achieved by pseudorange measurement is insufficient for surveying purposes. The required accuracy can be achieved, however, if the range to the satellite is measured using the L band carrier waves which have a wavelength of approximately 20cm. It can be assumed that the range between the satellite and the receiver consists of an integer number of carrier waves (ambiguous term) and some fraction of a cycle of one carrier wave.

Providing the observations are made over long periods to allow the geometry of the satellite to change in relation to the receiver, the unknown ambiguity term can be determined using post-processing techniques,. GPS receivers are designed to record the remaining fraction of the carrier wave cycle between the satellite and the receiver. In this way, ranges to the satellite can be measured to the millimetre level, and relative positioning to surveying accuracies can be achieved.

The modulation of the codes on the carrier wave (see Figure 4.1) cause instantaneous phase changes in the wave. These phase changes result in a discontinuous signal being emitted from the satellite. In order to measure phase of the carrier wave, the signal must first be reconstructed to its original state. Signal reconstruction is carried out in the software component of each receiver channel (see next section) by either demodulating the signal or by using a squaring technique.

Once the carrier wave is reconstructed, its phase is measured electronically. This is done by beating the carrier signal with another receiver generated signal of the same frequency. This technique results in the carrier beat phase observable.

## INTEGRATED DOPPLER COUNT

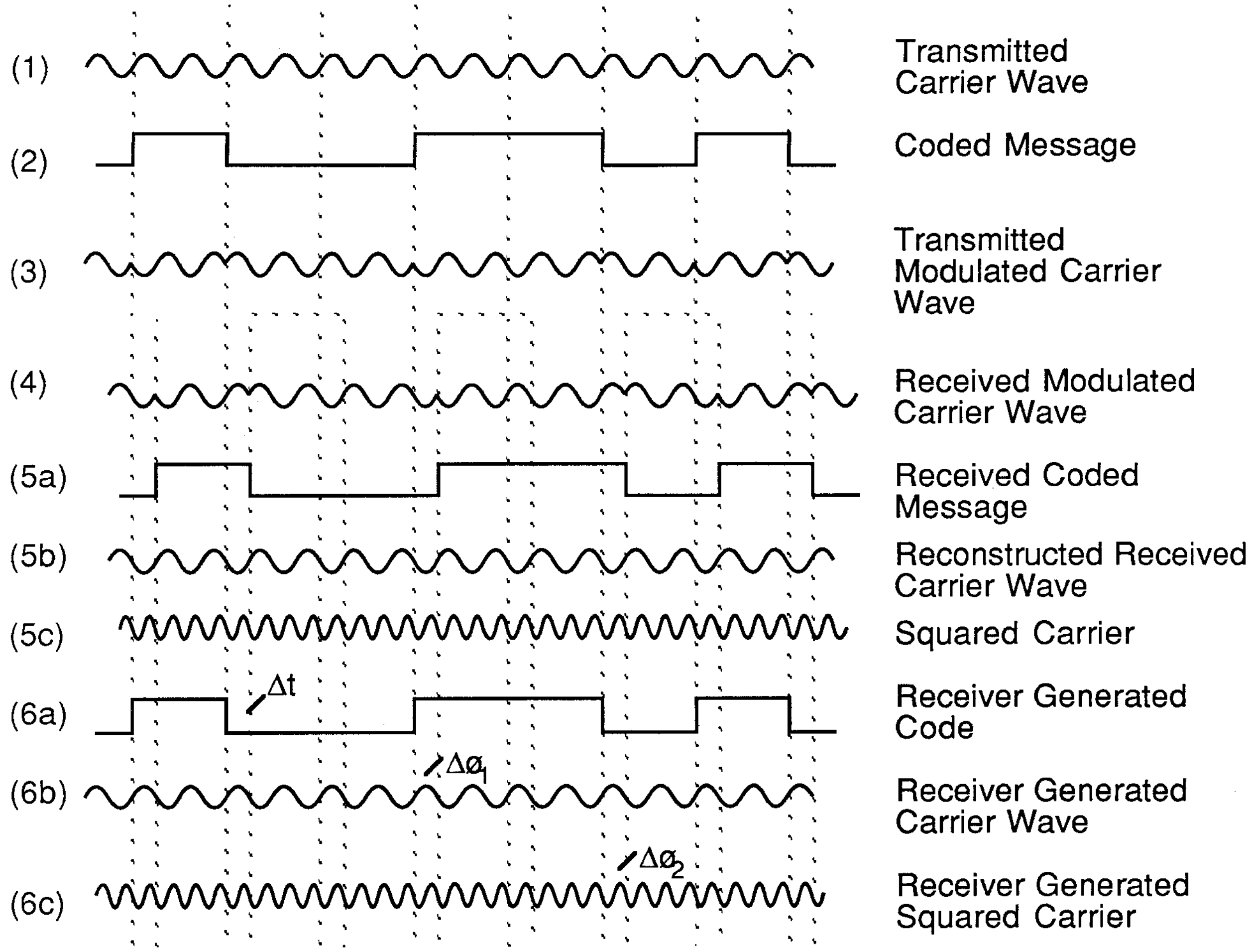
Several receivers record a cycle count once the satellite signals are acquired, so that a constant ambiguity term between the satellite and the receiver is maintained throughout the observing session. This observation is known as the integrated Doppler count. Although the integrated Doppler count is equivalent to the carrier phase measurement, it may be used as a separate type of observable.

DOPPLER COUNT

The Doppler count is the measurement of the carrier beat phase over a short time interval. The observable can be processed in exactly the same way as the measurements of the Transit Doppler system. Presently the Doppler count observable is only used by some instruments (eg. Trimble 4000A, Sercel TRS5) to smooth the measurements of the pseudoranges. This technique does not increase the accuracy of the pseudorange measurement, but uses the Doppler count to reduce the noise of the signal, which results in a more precise position determination.

TRANSITION PHASE

Another observable that may be recorded is the P code transition phase observable. The transition phase is the phase of the basic bit pattern or sub-carrier of the incoming P code. This observable is measured in the same way as the carrier wave phase observable. This measurement technique is only capable of decimetre level differential positioning as the wavelength of the P code subcarrier is approximately 30m. This observable permits the determination of pseudoranges without the use of codes.




$\Delta t$ = Time Lag of Signal Measured from Coded Message

$\Delta \phi_1$ = Phase Observable of Carrier Wave

$\Delta \phi_2$ = Phase Observable of Squared Carrier Wave

Figure 4.1 : Observation of the Pseudorange and Carrier Wave Phase

Figure 4.1 shows how the pseudorange and the carrier wave phase observables are measured. In a perfect system carrier waves are generated simultaneously at the satellite (1) and at the receiver (6b). The satellite carrier is then modulated with a coded message (2). The resulting modulated carrier wave is shown as (3). For a coded receiver, the same code is being generated at the same time in that receiver (6a).

The modulated carrier is observed at the receiver some time,

$\Delta t$, after it has been transmitted (4). The coded signal (5a) can be reconstructed using knowledge of the modulated carrier. The time lag for the satellite coded message to reach the receiver can be measured by correlating the incoming coded message (5a) with the receiver generated coded message (6a) to give the pseudorange observable.

The carrier wave can be either reconstructed using knowledge of the codes (5b), or a 'squared' signal can be created by squaring the modulated carrier, to produce an unmodulated wave, with one half the wavelength (5c). This process will be described in more detail below. GPS observables can then be measured by comparing these received signals (5b, 5c) to similar signals generated in the receiver (6b, 6c).

If the receiver loses lock on the satellite signal during an observation session, the count of the whole number of cycles is lost. This problem, known as a cycle slip, may occur with electronic or software failure in the receiver or when the line of sight between the receiver and the satellite is obstructed. When a cycle slip occurs the integrated Doppler count contains a new ambiguity bias, which must be accounted for at the processing stage (Chapter 6).

## SATELLITE SIGNAL OBSERVING TECHNIQUES

Various techniques are used by GPS receivers to acquire the satellite signals. These techniques are common to all receivers and are independent of the type of observable that is measured. The differences between observing the coded signal, the carrier wave phase or the transition phase were outlined in the previous section.

In order to determine positions a GPS receiver must be able to observe several satellites simultaneously. This is done by using receiver observing channels that consist of a digital hardware component and a software processor.

THE HARDWARE COMPONENT

Three different types of tracking channels are used in GPS receivers. They are *continuous*, the *multiplex*, and the *fast switching* channels. These hardware channels can be be used independently or combined together.

Some receivers (eg. Trimble 4000S) use several continuous channels that each track and process the signal from one satellite. These receivers are known as *multichannel* receivers. Once observed, the satellite information is passed on to a software processor before it is stored on either disk or cassette tape. Figure 4.2 shows the configuration of a multichannel receiver using continuous hardware channels.

```
                   ___ HARDWARE    _________   SOFTWARE
                  |    CHANNEL 1               PROCESSOR 1
                  |
                  |___ HARDWARE    _________   SOFTWARE
                  |    CHANNEL 2               PROCESSOR 2
ANTENNA ----------|
                  |___ HARDWARE    _________   SOFTWARE
                  |    CHANNEL 3               PROCESSOR 3
                  |
                  |___ HARDWARE    _________   SOFTWARE
                       CHANNEL N               PROCESSOR N
```

Figure 4.2 : Multichannel Receiver using Continuous Channels (adapted from PARADISSIS & WELLS, 1984)

An alternative to the multichannel receiver is the <u>multiplex</u> receiver (eg. TI4100). This type of receiver uses a single multiplex channel that quickly sequences through a number of different satellite signals. Each of these signals is then processed by its own software processor. A multiplex channel sequences through the satellite signals at a rate that is synchronous with the satellite message bit rate (50 bits/sec., 20ms/bit). In this manner a multiple of sequences of satellite signals is completed in 20ms. If a multiplex channel observes four satellite signals for 5ms each, it must lose lock on the carrier beat phase from each satellite for 15ms before it observes it again. This observation technique requires receiver resident software to predict the movement of the carrier beat wave during that period. Figure 4.3 shows the configuration of a single channel multiplex receiver.

```
                                          ____ SOFTWARE
                                         |     PROCESSOR 1
                                         |
                                         |____ SOFTWARE
ANTENNA _________ MULTIPLEX _____|     PROCESSOR 2
                  CHANNEL                |
                                         |____ SOFTWARE
                                         |     PROCESSOR 3
                                         |
                                         |____ SOFTWARE
                                               PROCESSOR N
```

Figure 4.3 : Configuration of a Single Channel Multiplex Receiver (adapted from PARADISSIS & WELLS, 1984)

Multichannel receivers may also use fast switching hardware channels (eg. WM101), which switch through a number of satellite signals at a much slower rate than the multiplex hardware channels. These receivers dedicate one channel to record the navigation message, and then may use several other channels to record the satellite signals. This allows each of the channels to observe satellite signals for a couple of seconds before switching to the next satellite. This type of GPS receiver must also use instrument resident software to predict the carrier wave phase while it is not being observed. Figure 4.4 shows the configuration of a multichannel receiver using fast switching channels.

```
                                           ___ SOFTWARE
                                          |    PROCESSOR 1
                                          |
                ___ FAST SWITCHING  ___|___ SOFTWARE
               |    CHANNEL 1             |    PROCESSOR 2
               |                          |
               |                          |___ SOFTWARE
               |                               PROCESSOR 3
ANTENNA ---|
               |
               |                           ___ SOFTWARE
               |                          |    PROCESSOR 4
               |                          |
               |___ FAST SWITCHING  ___|__  SOFTWARE
                    CHANNEL 2             |    PROCESSOR 5
                                          |
                                          |___ SOFTWARE
                                               PROCESSOR 6
```

Figure 4.4 : Configuration of Multichannel Receiver using Fast Switching Channels

Figure 4.5 illustrates the difference between satellite signals passed to software processors from the different receiver types.

## SOFTWARE PROCESSORS

Once the satellite signals have been acquired, the carrier wave can be recreated in two different ways. If the receiver measures the coded message transmitted by the satellite, a code correlation processing technique can be used to reconstruct the

L1 carrier or P code subcarrier wave. This reconstructed wave can then be compared with a receiver generated wave so that the carrier beat phase can be observed.

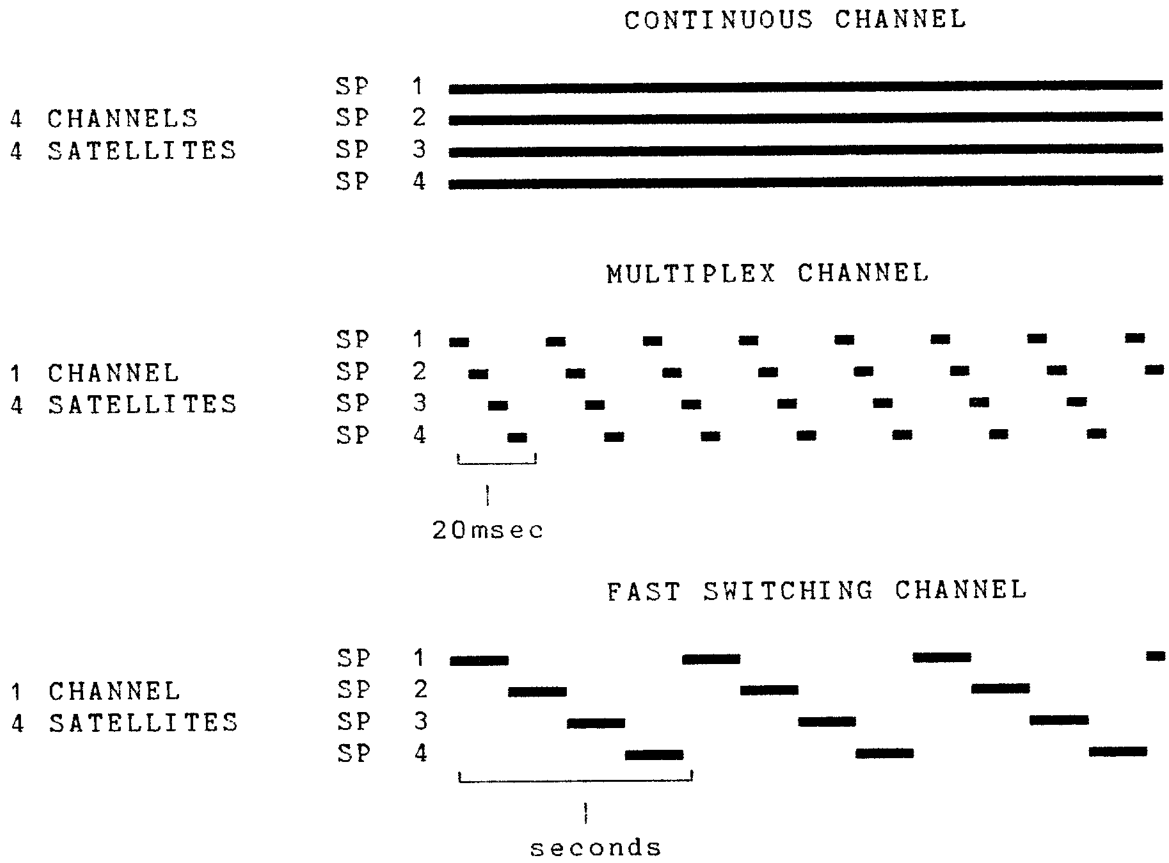



Figure 4.5: Observed Signals from Multiplex, Continuous and Fast Switching Hardware Channels to Signal Processors (SP) (adapted from WELLS, 1985)

A code correlation processor uses a delay lock loop to maintain an alignment between the incoming satellite coded message and the receiver generated code. Figure 4.6 shows a typical code correlation processor.

Figure 4.6 illustrates that the incoming carrier wave is first reduced in frequency by combining with a local carrier (A). The signal resulting from multiplying this incoming signal by the local code replica (B) will remove the code if the two codes are aligned. The correlation peak detector tests for the presence of the code and corrects the delay (C) of the locally generated code replica to maintain alignment, completing the delay lock loop. Figure 4.6 shows that the pseudorange and the navigation message as well as the carrier wave phase can also be obtained from such a software processor.

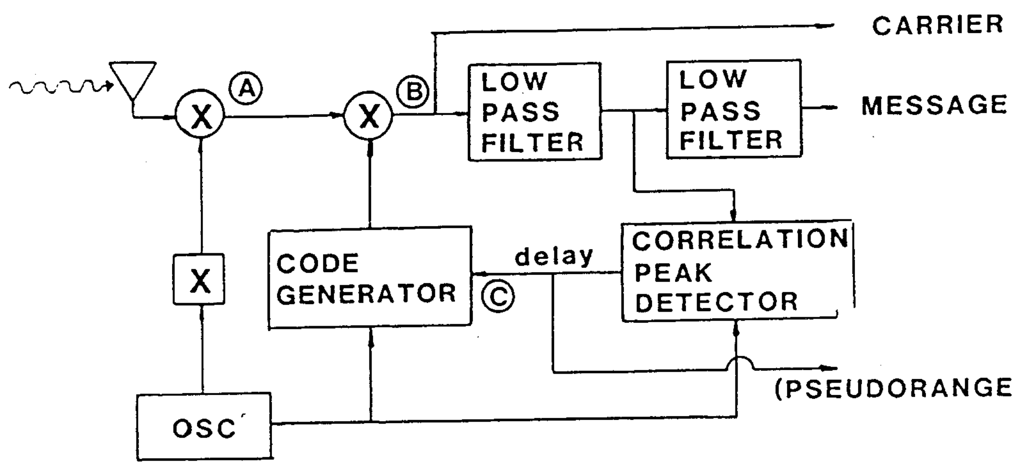



Figure 4.6 : Correlation Type Channel (PARADISSIS & WELLS, 1984)

If the receiver cannot measure the transmitted coded message the incoming modulated carrier wave can be 'squared', which results in a reconstructed carrier that can be compared with a 'squared' wave generated in the receiver.

The squaring software processor multiplies the received signal by itself to obtain a second harmonic of the carrier which does not contain code modulation. The incoming modulated carrier wave can be modelled by the following equation

$$y = A \cos ( \omega t + \Phi ) \qquad (4.1)$$

where $y$ = incoming modulated carrier

$A$ = amplitude function (a series of +1 and -1 values that represent 0° and 180° phase changes

$\omega$ = frequency of the carrier

$t$ = time

$\Phi$ = initial phase

If this carrier is squared we obtain

$$y^2 = A^2 \cos^2 ( \omega t + \Phi ) \quad (4.2)$$

$$= A^2 [1 + \cos (2\omega t + 2\Phi)] / 2 \quad (4.3)$$

Since A is a sequence of +1 or -1 values representing the code, $A^2$ is always equal to +1 and may be dropped from the equation 4.3. The resulting signal $y^2$ is then pure carrier at twice the original frequency. An example of a squaring type software processor is shown in Figure 4.7.

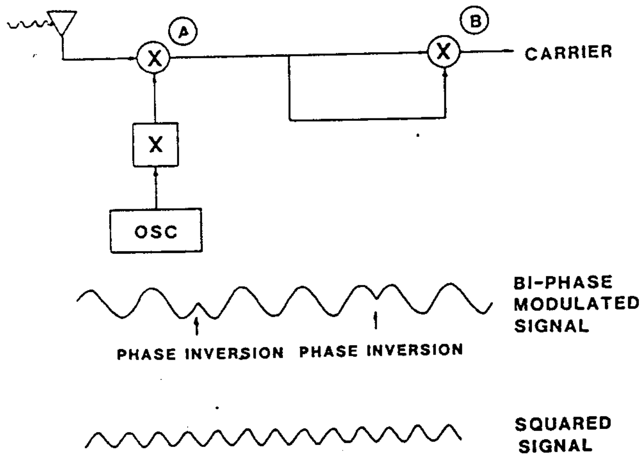



Figure 4.7 : Squaring Type Channel (PARADISSIS & WELLS, 1984)

## THE INTERFEROMETRIC APPROACH

ISTAC Inc. have developed receivers that do not record the coded message or the carrier wave phase directly, but record all of the radio waves at a certain frequency range as noise (eg. ISTAC 2002). As interferometric measurement is a proprietary technique it is uncertain how these signals are collected. It is assumed that all the signals of a certain frequency are collected on a single continuous channel and the stored on disk or cassette.

Once the data has been collected, it is processed using proprietary techniques developed as part of the SERIES (Satellite Emission Range Inferred Earth Surveying) program (WHITCOMB et al, 1986). The success of this technique relies on comprehensive software packages that are able to recognise patterns in the recorded noise signals, align the patterns to compute time delays, and then to process this data to give relative station positions.

## RECEIVER DESIGN CONSIDERATIONS

Apart from the different observing techniques used by GPS receivers, a number of other considerations must be taken into account when comparing available instruments. These include antenna design, number of satellites tracked, number of frequencies observed, power supply requirement, receivers cost, availability, ruggedness, portability and flexibility as well as the service back up provided by the manufacturer or distributor.

### ANTENNA DESIGN

A major source of GPS observation error is due to the effect of multipath. Multipath interference of the satellite signals occur when some fraction of the transmitted signal is reflected off the ground or from some other object onto the antenna. The effect that multipath interference has on the observed satellite signal depends on the design of the antenna, the proximity of large reflective objects and the height of the antenna above the ground. As a rule of thumb, multipath effects are equivalent to about 10% of the signal wavelength, and hence the coded signals are significantly more affected than the carrier wave measurements.

The three basic types of antenna that are presently used in GPS receivers are the helix, the dipole and the microstrip. As COLLINS (1986) states :

> " The dipole antenna used by the Macrometer instrument is probably the most accurate one in use and tests show its electrical center is virtually identical to its geometric center. A number of helix antennas are presently being used. The type used by Trimble Navigation for their 4000A receiver has demonstrated a variability of about one centimeter between electrical and geometric centers "

The accuracy variability of helix antennae was tested by SIMS (1985) who found that the electrical and geometric centres of the TI4100 helix antenna varied by 1-2cm depending on the orientation of the antenna and the elevation and azimuth of the satellite at the time of observation. This variation in the centre of an antenna may be critical if the receivers are to be used to determine azimuth over short distances. ASHJAEE (1986) has found that the phase centre of the Trimble Navigation instruments may also vary the length of a baseline by about 2cm if the receivers are aligned in opposite directions (180°). ASHJAEE (1986) claims that aligning the antenna at each station reduces this baseline error to about 2mm.

An alternative to the dipole and the helix antenna is the microstrip antenna which has been tested independently (COLLINS, 1986) and found to have an electrical centre that is stable to within a couple of millimetres.

## NUMBER OF SATELLITES TRACKED

Navigation GPS receivers must observe four satellites simultaneously to solve for the four position unknowns (X, Y, Z, receiver clock bias) in real time. If the receiver clock error can be minimised by using more stable oscillators (eg. rubidium instead of the usual crystal) only three position unknowns remain, which can be obtained by observing three satellites. If height information is also available and input into the position solution, only two satellites need be observed for real time positioning.

Receivers that can track more than four satellites provide the surveyor with greater processing flexibility. Satellite positioning is analogous to carrying out a resection by conventional methods. Increasing the number of satellites that are observed is the same as increasing the number of stations observed, for both result in a stronger positioning solution.

The number of satellites that can be tracked depends on the number and type of receiver channels within each instrument (see previous section). Receivers that only track four satellites must select the satellites that they will observe before each observing session commences. This can be done with the aid of skyplots, which will be explained in the next chapter. Careful selection of the satellites is necessary to ensure that satellites will be visible for the entire observing period. If one of the satellites drops below the horizon during the observation period, another satellite must be tracked. This is an undesirable situation as the introduction of a new satellite increases the number of parameters (see Chapter 6) in the final positioning processing without increasing the number of observations.

If more than four satellites are observed simultaneously, the number of observations in the positioning solution will be increased as well as the number of parameters. This allows the surveyor to choose which satellite signals are processed and gives him the option to disregard any observations that have either been obstructed or transmitted from an unhealthy satellite. At present this is a valuable option as the health status of some Block I satellites (eg SV8, SV11) is poor, and observations to those satellites must be closely monitored.

## DUAL FREQUENCY RECEIVERS

Several GPS receivers presently available are capable of measuring both the L1 and the L2 carrier wave phases. The advantage of measuring both the carrier wave phases is that the delay in the satellite signal introduced by the ionosphere can be calibrated and corrected.

In absolute terms the ability of a GPS receiver to measure dual frequency improves the measured accuracy of a baseline by approximately 1ppm (KLEUSBERG et al, 1986). Although the improved accuracy, provided by dual frequency is not required to achieve surveying accuracies, it must be used in high precision measurement for plate tectonic movement surveys or for earth deformation surveys. Dual frequency receivers are also used at tracking network stations to ensure good satellite orbit determination.

Presently, single frequency GPS receivers are considerably cheaper than the dual frequency receivers. It is expected that with the ever increasing investment into research and development that the cost of dual frequency receivers (whether they use a squaring technique, or P code correlation) should soon decrease, and may even become a standard feature of all future GPS receivers.

## FIELD CONSIDERATIONS

When operating in the field it is necessary to have an instrument that is designed to take everyday wear and tear. The surveyor should also consider the size, weight and portability of the equipment and the power supply that is required for operation.

First generation receivers like the Macrometer V-1000 and SERIES were developed as laboratory instruments and were not designed to be taken in the field. Consequently they were bulky, heavy and difficult to transport. Recent receivers are 'ruggedised' for field use, and have been reduced to the size of a suit case (eg. Trimble 4000S, Sercel TRS5).

The amount of power required to run most instruments has also decreased from a generator to one or two car batteries. Advances made in receiver design may soon see the introduction of shoe box size instruments capable of running off internal batteries. The Sony GTT 2000 which is presently being developed in Japan has these design aims.

## FLEXIBILITY OF INSTRUMENTATION

When comparing GPS receivers one must consider the versatility of the receiver. This includes the receivers' suitability for a range of surveys and the ability of the receiver to be interfaced with other measuring instrumentation, microcomputers, printers etc.. Code correlating receivers, for example, can use the pseudorange measurement for reconnaissance work and the carrier beat phase measurement for surveying accuracy work.

For the equipment to be flexible, the hardware and software design as well as the receiver output must be well documented. Good documentation of the equipment allows easy instrument repair and upgrading of the receiver capabilities. Good output documentation, allows surveyors to develop their own logging software to observe and format the satellite signal to the advantage of their processing software. An examples of a well documented instrument is the Trimble Navigation GPS receiver. Flexibility is paramount when the receiver is being used in the dynamic mode, along with other data logging equipment.

## SERVICING

Servicing and other manufacturer back up of GPS instrumentation is important to ensure that the receivers are kept running during a survey. Down time caused by either repairing or upgrading can be costly. If the instrumentation is well documented it may be possible that minor repairs to the equipment may be carried out by the operators themselves.

To ensure quick reliable service it is preferable that the instrumentation is repaired by trained technicians in Australia, and that expert advice is readily available from the manufacturer or distributor if problems develop in the field.

## GPS RECEIVERS

The development of GPS receivers has progressed markedly over the last few years. GPS receivers that are presently available can be divided into four categories. These are the pseudoranging navigation receivers and the code correlating, squaring and code phase surveying receivers.

## NAVIGATION RECEIVERS

As navigation receivers only measure pseudoranges, they cannot produce surveying accuracies over short distances. Navigation receivers have been designed to provide position in real time at an accuracy that is suitable for most navigation requirements. These receivers, however, may also be used in differential mode for surveys requiring low accuracy like exploration and reconnaissance surveys. There is a huge market for these GPS receivers which can be used for navigation on land, at sea and in the air.

Navigation receivers were designed to observe the CA code as the P code will be unavailable to civilian users in the future. Presently the CA code can achieve real time point positioning to the $\pm$ 10m level (SANDS, 1985), and allow metre level differential positioning (RUSH et al., 1986). The point positioning accuracy is expected to downgrade to the $\pm$100m level with the introduction of the production (Block II) satellites.

Several different navigation receivers are listed in Table 4.1, which shows that most of these receivers only require 24V DC to run. The navigation receivers that are presently available are priced at about $AUS 40K. This price is approximately half the cost of available surveying receivers. Each receiver tracks either 4 or 5 satellites. If more than 4 satellites are above the horizon at any one time the receiver observes the satellites that have the strongest geometrical constellation (see Chapter 5).

Table 4.1 demonstrates how the navigation receivers have developed over time. The earliest receiver listed is the Magnavox T-Set which is slightly bigger than the second generation Trimble 4000A and Polytechnic XR1. Both the Trimble and the Polytechnic instrument were designed to be rack mounted, which is desirable for installation in vehicles. The latest addition to the list is the Sony instrument which is only shoe box size. The Sony GTT-2000 is an example of the design of future GPS instrumentation.

All navigation receivers provide the operator with RS232 or similar interfacing ports, so that a range of data can be logged, and so that the instrument can be connected to printers, plotters, tape drives and other logging instrumentation. The receivers usually have a menu driven display which gives the operator access to information such as the latitude, longitude

Table 4.1

## Navigation GPS Receivers

| RECEIVER | CODE | NUMBER OF CHANNELS | CHANNEL TYPE | ANTENNA TYPE | POWER SUPPLY | SATELLITES TRACKED | COMPONENT SIZE | | | COST $AUS X1000 | AVAIL -ABILITY |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | RECORDER | DISPLAY | ANTENNA | | |
| TRIMBLE 4000A | CA | 4 | MULTICHANNEL CONTINUOUS | HELIX | DC 20-35V AC 110/240V | 4 | 178 x 457 x 483 mm 20 kg | | 178x153mm 1.5 kg | 34 | NOW |
| SONY GTT-2000 | CA | 4 | MULTICHANNEL CONTINUOUS | HELIX | DC 18-32V | 4 | 160 x 205 x 295 mm 7 kg | | 100x130mm .5 kg | NA | END 1986 |
| POLYTECHNIC XR1 | CA | 1 | SINGLE CHANNEL MULTIPLEX | MICRO-STRIP | DC 19-32V AC 110/220V | 5 | 480 x 135 x 435 mm 12 kg | | NA | 40 | NA |
| MAGNAVOX T-SET | CA | 5 | MULTICHANNEL CONTINUOUS | HELIX | AC 90-128V/ AC 180-256 | 5 | 368 x 457x 362mm 16kg | 89 x 457x 203mm 2kg | 160x203mm 1.3 kg | NA | NOW |
| PRAKLA-SEISMOS | CA | 1 | SINGLE CHANNEL MULTIPLEX | HELIX | DC 24V | 4 | 493 x 259x 271mm 25kg | 250 x146 x 181mm 8kg | 200x400mm 4 kg | NA | NA |

and height of the receiver in the satellite reference system, the speed and the direction of the vehicle, the distance and direction to a way point, the Greenwich Mean Time and the health of all the satellites.

## SURVEYING RECEIVERS

Surveying GPS receivers observe either the transmitted carrier wave phases or the transition phase. The measurement of the transmitted carrier wave phase can be accomplished by using either squaring or code correlation techniques. Presently only the ISTAC receivers use the transition phase observable.

The accuracy that can be achieved by each receiver type depends on the wavelength of the observed signal. Receivers that observe the L1 carrier wave ( $\lambda$ = 19cm) are inherently more accurate than the instrument that observes the transition phase ( $\lambda$ = 30m) and both are more accurate than navigation instruments that measure the CA code ( $\lambda$ = 300m).

### Code Correlating Receivers

The code correlating receivers use the coded measurements to reconstruct the transmitted carrier wave. These receivers use code correlating software processors which output the navigation message, the coded signal and the carrier beat phase information. These receivers are most suited for surveying purposes. They can be used for many types of surveys ranging from reconnaissance and high accuracy geodetic accuracy work.

There are two advantages in having access to the codes. The codes allow the operator to interrogate the navigation message which contains the ephemeris data, the satellite clock corrections to GPS time and the health status of each satellite. The other advantage is that the receiver clock error can be solved for at each measurement epoch, eliminating the need to synchronise the receiver oscillators before each measurement session and monitor the behaviour of the oscillators throughout the session. The significance of these advantages will become apparent in chapter 5 which describes the planning and execution of a GPS survey.

The code correlating receivers presently available are listed in Table 4.2. Their main distinguishing feature is the degree to which they were designed for use by surveyors in the field.

The **Trimble Navigation 4000S** receiver was a natural progression from the **4000A** which was a navigation receiver. Although these receivers are only suitcase size they have been designed to be rack mounted to be compatible with other existing navigation equipment. The instruments can be installed in all sorts of vehicles, but are not suitable for backpacking in the field. The Trimble company has not yet developed a complete survey processing package (see Chapter 6) that can be used with the receiver. Processing of baselines can currently be carried out with program TRIMVEC that is supplied with the receiver. In order to carry out multistation processing, network adjustments and transformations, however, the user is required to develop his own software package, or adopt existing software packages.

Although the Wild Magnaox **WM101** is not yet available it is

Table 4.2 Code Correlating GPS Receivers

| RECEIVER | CODE | NUMBER OF CHANNELS | CHANNEL TYPE | ANTENNA TYPE | POWER SUPPLY | SATELLITES TRACKED | COMPONENT SIZE | | | COST $AUS X1000 | AVAIL -ABILITY |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | RECORDER | DISPLAY | ANTENNA | | |
| TRIMBLE 4000S | CA | 4 or 5 | MULTICHANNEL CONTINUOUS | HELIX or MICRO-STRIP | DC 20-35V AC 110/240V | 5 | 178 x 457 x 483 mm 20 kg | | 178x153mm 1.5 kg | 72 | NOW |
| TEXAS INSTS TI 4100 | CA or P | 1 | SINGLE CHANNEL MULTIPLEX | HELIX | DC 22-32V | 4 | 373 x 445x 211mm 24kg | 178 x 109x 53mm .5kg | 168x279mm 1.7 kg | 205 | NOW |
| WILD - MAGNAVVOX WM 101 | CA | 1 | MULTICHANNEL FAST SWITCHING | HELIX | DC 10.5-15V | 6 | 510 x 390 x 170 mm 14.4 kg | | 180x210mm 1.5 kg | 120 | LATE 1986 |
| SERCEL TRS5 | CA | 5 | MULTICHANNEL CONTINUOUS | HELIX | DC 24V | 5 | 370 x 525x 445mm 30kg | 285 x 250x 200mm 5kg | 160x203mm 1.3 kg | 80 | LATE 1986 |
| LITTON SURVEY SET | CA | NA | NA | HELIX | DC 20-30V AC 113-230V | 8 | 410 x 230 x 200 mm 5 kg | | NA | 54 | NA |
| NORSTAR 1000 | CA | 5 or 7 | MULTICHANNEL CONTINUOUS | NA | DC 11-32V AC 100-250V | 5 or 7 | 180 x 450 x 550 mm 15 kg | | NA | NA | NA |

presented as a complete survey package. The instrument is specifically designed for field use as it is light and rugged and only runs on internal batteries. The software provided with the instrument is a comprehensive package designed to guide the operator through the generation of sky plots, collecting data in the field, processing data and transforming the final adjusted GPS data into the local reference frame. The WM101 is a total package as it offers the surveyor all of the hardware and software that is necessary to complete all survey tasks (1:10,000 - 1:100,000 accuracy range).

The **Texas Instruments TI4100** was the first code correlating GPS receiver. It was developed with funding from the Defense Mapping Agency, the National Geodetic Survey and the Geological Survey. This receiver was designed to satisfy all positioning requirements. As the instrument was designed to be used for the military, it has access to the P code of the Block I satellites. The **TI4100** could therefore be used to evaluate the positioning accuracy achievable with the CA code, the P code and the carrier beat phase. P code access allows the dual frequency measurements to take place using code correlation techniques. Although the TI4100 can presently observe the P code, thereby enabling it to carry out real time positioning to the ±10m level, this facility will not be available in the future. The encryption of the P code for the Block II satellites will restrict the P code to military users. This will limit the TI4100 to a CA code, single frequency receiver.

The **Sercel** receiver is aimed at the offshore positioning community. Although the instrument reconstructs and outputs carrier wave phase, software has not yet been developed to process this data. Doppler count observables are used, however, to smooth the navigation solution provided by the pseudorange measurement. This is done by integrating the carrier wave phase over 0.6 second intervals to track the range rate of each satellite. This 'Doppler' measurement is then used to predict the future position of the receiver. By using this method the pseudorange precision for both point positioning and differential positioning is increased. Sercel intends to market receivers capable of real time differential pseudorange positioning over distances of approximately 1000km for offshore positioning. The data transmission technique linking the two receivers is still proprietary information and will not be available to the public before the receiver is released.

Little is known about the workings and the results achieved with the **Litton Survey Set**, and **Mini-Mac**, at the time of writing (October, 1986).

**Squaring Instruments**

The first commercially used GPS surveying instruments were the **Macrometer V-1000** receivers. These receivers were designed to be independent of the codes and used the squaring technique to observe the carrier wave phase. Theoretically, this observation technique could be used with either the L1 or the L2 carrier waves. Independence of the codes may be advantageous if the codes are downgraded or dithered in the future.

The Macrometer V-1000 has a proven field record for it has been used in many surveys where positioning accuracies of 1-2ppm were achieved. As the Macrometer V-1000 is a first generation instrument it is heavy and bulky and must be vehicle mounted. The instrument requires 115V AC to run, which must be supplied by a generator.

GPS receivers that depend on squaring software processors, however, have certain disadvantages. In order to process recorded data, ephemeris information must be available from an outside source. This information may be obtained from a cheap coded navigation receiver, or through the Defense Mapping Agency in the U.S.. The receivers must also be physically brought together before and after measurement sessions in order to synchronise them to GPS time and to calibrate the stability of their clocks. This is necessary as the receiver clock offset to GPS time cannot be determined without access to the codes. The disadvantages of the squaring software processor make the GPS instrument less convenient to use than the code correlation software processor. To date all but one receiver manufacturer has opted for the code correlation software processor.

These disadvantages of squaring receivers may be overcome when the **Mini-Mac** receiver is released. This receiver is designed to measure both the L1 and L2 carrier waves using squaring techniques, as well as recording the navigation message on the CA code. As yet, there is little more information available this instrument (October, 1986).

The second generation Macrometer II instrument is smaller, lighter and cheaper than the Macrometer V-1000 and also requires less power to run. As the instrument is a squaring type receiver, however, the same observing and processing procedures as the Macrometer V-1000 must be followed.

One of the greatest drawbacks of the Macrometer V-1000 is the poor documentation of the instrument hardware, the observed signals and the associated processing software provided with the instrument. This is an undesirable situation as the surveyor is dependent on the manufacturer to carry out all repairs, which is a costly and time consuming procedure. Lack of information on the type and format of receiver output makes interfacing this receiver with other instrumentation nearly impossible. Accordingly, the receiver cannot be adapted to a range of uses and is limited to carrying out surveying measurements. This situation is undesirable but may change with the introduction of the Macrometer II. Table 4.3 shows the features of the squaring receivers.

**Code Phase Instruments**

The Istac **Land Surveyor Model 1991** and **Model 2002** are the only available GPS receivers that use the transition phase observable. These receivers provide decimetre level positioning accuracy - which is better than using the coded signals but is less accurate than the code correlating or squaring receivers.

The Istac receivers use the interferometric observing approach developed for the **Series** and **Series-X** receivers (WHITCOMB et al, 1986). Series receivers measured data from all the satellites at a frequency equivalent to that of the L1 and L2 carrier waves. The Series receivers furnished the same accuracy as the code correlating and squaring receivers, but had the advantage of measuring all satellites simultaneously, independent of the coded signals. Unfortunately, these receivers are

Table 4.3 **Code Squaring GPS Receivers**

| RECEIVER | L1/L2 | NUMBER OF CHANNELS | CHANNEL TYPE | ANTENNA TYPE | POWER SUPPLY | SATELLITES TRACKED | COMPONENT SIZE | | | COST $AUS X 1000 | AVAIL -ABILITY |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | RECORDER | DISPLAY | ANTENNA | | |
| MACROMETER V-1000 | L1 | 6 | MULTICHANNEL CONTINUOUS | DIPOLE | AC 115V | 6 | 690 x 530 x 640 mm 45 kg | | 910x 910x 150mm16kg | 150 | NOW |
| MACROMETER II | L1 & L2 | 6 | MULTICHANNEL CONTINUOUS | MICRO-STRIP | DC 24V | 6 | 27 kg | | 910x 910x 150mm16kg | NA | 1986 |
| MINI-MAC | L1 & L2 | 4 | MULTICHANNEL FAST SWITCHING | HELIX or MICRO-STRIP | DC 12V | 8 | NA | | | NA | NA |

Table 4.4 **Code Phase GPS Receivers**

| RECEIVER | CARRIER | ANTENNA TYPE | POWER SUPPLY | SATELLITES TRACKED | COMPONENT SIZE | | | COST $AUS X1000 | AVAIL -ABILITY |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | RECORDER | DISPLAY | ANTENNA | | |
| ISTAC MODEL 1991 | L1 | MICROSTRIP | DC 24V | ALL | 660 x 360 x 230mm 23kg | 460 x 360 x 150mm 14kg | 280 x 280 x 80mm 5kg | 140 | NOW |
| ISTAC MODEL 2002 | L1 / L2 | MICROSTRIP | INTERNAL BATTERIES | ALL | 230 x 420 x 420mm 7kg | 420 x 280 x 260mm 18kg | Combined in Recorder | 82 | NA |

unsuitable for surveying as they were extremely bulky and had to be truck mounted.

The interferometric method can be used to give near real time positioning if a good apriori position of the origin is given, the receivers do not lose lock on the transmitted signals and if a broadcast ephemeris is available from a coded receiver in real time. The Istac receivers, which only measure the transition phase, seem to have sacrificed the accuracy of the Series instruments, to reduce the size and weight of the equipment. To date the Istac receivers have not made a big impact on the market place mainly due to their low accuracy performance.

The interferometric instruments are suitable, however, for tracking stations, since they track all the satellites simultaneously, they are independent of the code and they can observe both the L1 and the L2 carrier waves. The features of the code phase instruments are shown in Table 4.4.

**RECEIVER COMPARISON**

In the 'Interagency Test' of 1984 (GOAD et. al., 1984) the Texas Instruments TI4100, the Macrometer V-1000, and the Series and Series-X GPS receivers were compared. The test locations were chosen to provide baseline lengths ranging from 13km to 1300km. Several of the baselines had previously been measured with VLBI.

Some baselines were not measured due to equipment failures. These were expected particularly as the Series and Series-X receivers were being tested for the first time. This experiment showed that all instruments were performing at the 1ppm level in length determination and only slightly worse in the baseline components.

In March, 1985, further comparisons of available instrumentation were carried out (STEPHENS et al., 1986) in California. At the time of writing, however, the results from this test had not been published.

## CHOOSING A RECEIVER

Presently there are many GPS packages (consisting of receivers and processing software) available on the market. The development of GPS packages is expected to continue at a fast rate, as many manufacturers are investing substantial amounts into the research and development of new equipment and processing techniques. Over the next few years it is expected that receivers will be smaller and more flexible, and that software packages will become more comprehensive.

Each GPS package has advantages and disadvantages for a range of uses. The surveyor must therefore select the receiver and software package that best suits his needs.

Receiver manufacturers and distributers usually offer GPS packages for lease. Leasing equipment is presently an attractive alternative to purchasing for many surveyors. Leasing provides an opportunity to assess the capability and suitability of GPS with only a limited investment.

After each surveyor selects the GPS receiver that best suits his needs, he must ensure that it is used correctly. The observing procedures that should be followed are described in the next chapter.

# 5. FIELD SURVEY PROCEDURE

## INTRODUCTION

This chapter outlines the observing procedures for a GPS survey. These observing procedures can be used on all GPS surveys, whether they require low accuracies (1:5,000) or very high accuracies (1:1,000,000). Increased user accuracy requirements, however, demands increased understanding of the possible error sources that limit GPS surveying. These error sources can be reduced by better field survey procedures, by improving receiver hardware (Chapter 4) and through special processing techniques (Chapter 6). The observing procedures used by the surveyor and the accuracy that is achieved by the GPS survey are interdependent. Special attention is given to the observing procedures that should be followed to achieve surveying accuracies (1:10,000-1:100,000).

Table 5.1 summarises the field procedure required for GPS surveys. The first step of any survey is to consider the design of the network. The surveyor must consider the purpose of the survey, the required accuracy of the survey, the extent and spacing of existing control stations and the number of GPS receivers that are available. At this pre-survey stage the surveyor determines the current observation window, and the number and length of station occupations that will enable the accuracy requirements of the survey to be achieved.

After these pre-survey office computations are completed, the reconnaissance and marking of new stations can begin.

Reconnaissance is vital to ensure smooth operation of the survey during the observation stage. Reconnaissance involves selecting cleared station sites in positions that best suit the nature of the survey, estimating the travel time between stations and drawing up access diagrams. The reconnaissance survey also provides an opportunity to monument new stations and, if required, place azimuth marks.

Table 5.1 : GPS Survey Procedure

| SURVEY STAGE | PROCEDURE | CONSIDERATION/TASK |
|---|---|---|
| PRE-SURVEY | NETWORK DESIGN CONSIDERATIONS | Accuracy of survey<br>Purpose of survey<br>No. of receivers<br>Station spacing |
| | OFFICE COMPUTATIONS | Simulations<br>Skyplots<br>Almanac files |
| | RECONNAISSANCE & MONUMENTATION | Select cleared sites<br>Access Diagrams<br>Recovery sketches |
| | OBSERVING SCHEDULE | |
| SURVEY | OBSERVATIONS | Pre-observation<br>Observation<br>Post-observation |
| | DATA PROCESSING | Data collection<br>Processing<br>Presentation |

The **observation procedure** varies according to the nature of the survey and the type of GPS receiver that is used. The field procedures that are necessary to achieve 'surveying' accuracies are outlined below and the requirements of the various receiver types are discussed.

Once the data has been recorded in the field, it is brought back to the office and processed. This produces either station coordinates or vector baselines between stations in the WGS72 datum. The baselines are then combined into a network providing coordinates of all stations in the WGS72 datum. Finally, the coordinates are transformed to the local datum (eg Australian Geodetic Datum).

The options available for processing GPS field data are described in Chapter 6. The network adjustment procedure and the transformation of the adjusted coordinates into the local datum are outlined in Chapter 7.

Before GPS data is collected in the field, attention must be given to planning and preparing the survey. Proper planning ensures the smooth running of the field observations, and that the requirements of the survey can be met.

## PRE-SURVEY PLANNING

Planning a survey is important for two reasons.

(1) GPS receivers are expensive and the user must ensure that they are properly utilised. Presently the best possible observation window (which is location dependent) is approximately 4 hours. It is therefore important that the receivers are operative during this period to ensure maximum productivity.

(2) To ensure that the accuracies required for the survey are achievable.

The pre-survey stage involves the consideration of the network design and office computations, reconnaissance and monumentation and production of an observation schedule.

## NETWORK DESIGN

When designing a network for a GPS survey, the shape of the network is not considered, as GPS and other satellite techniques do not propagate 'observation' errors throughout the network. The four design considerations are the purpose of the survey, the required accuracy, the number of receivers available and the distance between and accessibility to each station.

### Purpose of the Survey

The purpose of the survey dictates the number of local control stations that must be occupied in the GPS survey. If GPS is used for control densification, the resulting station coordinates must fit existing control. These surveys require at least three common stations to solve for the seven transformation parameters which relate the satellite datum (WGS72) to the local datum (see Chapter 7). The accuracy of the transformation parameters is improved by occupying a larger number of established stations.

Some surveys only require high accuracy relative positions, and are not concerned with fitting in to local control. Many types of engineering surveys are included in this category. These surveys do not require established local stations to be occupied, as station coordinates are given in relation to one station, held fixed in the WGS72 datum. The pseudorange solution can be used to fix this stations coordinates to about ±10m. If required, national or regional transformation parameters can be used to transform the survey into the national or regional datum.

If GPS is used to establish height, several leveled stations may need to be occupied throughout the network. The ellipsoidal height from the GPS survey combined with the known orthometric height of the occupied stations can then be used to determine the local geoid undulations. The knowledge of the shape of the local geoid can be used to determine the orthometric height of the other GPS stations. A full explanation of how to solve for and apply transformation parameters, and how to convert measured GPS height into a local orthometric height is given in Chapter 7.

The purpose of the survey also dictates the amount and type of monumentation that is required on the job. The monumentation considerations of GPS surveys are similar to those of conventional surveys. The surveyor must consider the most suitable markers for the type of survey, the local conditions, transportation, the materials available, the equipment available for setting marks and the cost.

As GPS does not require newly established stations to be intervisible, it may also be necessary to establish a number of intervisible marks to provide a reference "azimuth" for subsequent control breakdown. The azimuth can be calculated at the processing stage from the adjusted station coordinates.

**Accuracy**

The accuracy requirements of a GPS survey dictate how the survey is carried out, and what sort of hardware and software is required.

Code correlating or squaring GPS receivers must be used to achieve surveying accuracies. Surveying receivers, which were reviewed in Chapter 4, require slightly different observation procedures. The observing procedures are described below. HOTHEM & WILLIAMS (1985) state that single frequency receivers are sufficient to achieve surveying accuracies.

The accuracy requirements of each survey also determine the station occupation time and the number of station occupations that are needed. HOTHEM & WILLIAMS (1985) recommend that at least one hours observation of each baseline is necessary to achieve surveying accuracies. COLLINS & LEICK (1985) showed that this estimate was realistic for a typical control densification survey in Montgomery County Pennsylvania. In this survey, Geo/Hydro Inc. found the horizontal and vertical coordinates to be consistent with local control to 1:800,000 and 1:500,000 respectively, with two hours observations on each station. Although this is twice the recommended minimum station occupation time, the accuracy achieved was more than an order of magnitude greater than that required for most engineering and cadastral surveys.

HOTHEM & WILLIAMS (1985) also suggest that for surveys requiring accuracies of 1:50,000 or less, each network station need only be occupied once in the observation campaign. This implies that each station may be connected to the network by a single baseline. Accuracies are improved as the number of station occupations, and hence baselines measured from each station increases. To achieve accuracies around the 1:100,000 level, HOTHEM & WILLIAMS (1985) suggest that each station be occupied at least twice, and that at least three baselines be measured from each station.

Other methods of improving survey accuracy include using more stable receiver oscillators (ie. Rubidium), acquiring more accurate ephemeris data for processing and improving the accuracy of the meteorological observations. For routine surveying applications none of these improvements are necessary. By using differential measurement techniques and comprehensive processing packages, the above mentioned error sources can either be eliminated or much reduced. The magnitude of these error sources, and the methods of overcoming them are discussed in Chapter 6.

**Number of Receivers**

Increasing the number of GPS receivers used on each job can increase productivity. For example, if three GPS receivers are used instead of two, productivity will double. The ideal number of receivers used on each job depends on the size of the survey, the time limits imposed and the accuracy required.

The highest survey accuracy can be achieved by simultaneously occupying as many stations as possible. This procedure eliminates or minimises many of the error sources that limit GPS accuracy. Higher coordinate accuracies are achieved as there are fewer unknown parameters like satellite clock terms, epoch biases etc. (Chapter 6) to be estimated at the processing stage. To achieve surveying accuracies, however, two receivers are sufficient.

Unfortunately, the ideal number of receivers that should be used on a survey are not always available. Extra receivers may be obtained by leasing, or purchasing additional equipment from the manufacturers and receiver distributors.

### Spacing between Stations

The final consideration of GPS network design is the station accessibility and spacing. As there is presently a limited observing window of about four hours in Australia it is important to maximise the number of baselines that can be measured in each session. In order to do this the GPS user must consider the travel time between stations, the transportation that is available and the length of station occupation time. Severe survey time limits, large distances between stations or difficult access may require alternative forms of transport, like helicopters.

## OFFICE COMPUTATIONS

Pre-survey office computations include network simulations to ensure that the desired accuracy of the survey is achievable, the generation of skyplots for reconnaissance and final planning purposes and possibly the creation of ephemeris files that are required for codeless receivers.

### Network Simulations

Network simulations are performed to ensure that the required accuracy of the survey can be met. The simulations are similar to network adjustment computations. Data input into the simulation program is generated by the computer to give realistic estimates of expected data that is collected in the field. These generated observations are then adjusted in a network, in the same way as 'real' observations, which results in station coordinates and their variance-covariance matrix (VCV). If the accuracy of the simulated network is below the required accuracy of the survey, extra observations can be included until the required accuracy is achieved. In this way the surveyor can develop an observing schedule that will meet the accuracy requirements of the survey.

In order to carry out simulations the new and existing network stations are plotted on a map, and the approximate coordinates and baseline lengths between stations are scaled off, These values, along with their expected VCV are then entered into the simulation program. The approximate VCV depends on the time of occupation at each station, and the Geometric Dilution Of Precision (GDOP) of the satellites at the observation time.

The GDOP of the satellites is a function of the relative geometry of the satellites and the user. The concept of GDOP is familiar to surveyors, as it is analogous to positioning a station by intersection. The strength of the fix is weakest when the intersecting lines are nearly parallel, and strongest when the lines are perpendicular. In order to improve the strength of the position solution, therefore, it is desirable to observe satellites that are in each quadrant of the sky. In this case the value of GDOP will be low. A high GDOP will result if all the satellites are clustered in one quadrant at the time of observation.

If more than four satellites can be observed simultaneously, the GDOP of different combinations of four satellites indicates which satellites should be observed. The GDOP will tend to indicate one satellite that is high in the sky, and three others that are 120° apart on the horizon, as this configuration provides the best geometry. Some satellites that are on the horizon, however, may be just about to set. These satellites are unsuitable for observing, as the prime requirement of a GPS survey is that the same satellites are observed, by both receivers, throughout each session.

Network simulations are only a guide to the surveyor as to the number of baselines that need be measured to meet accuracy requirements. Field operations can be delayed by equipment malfunction, vehicular problems and an array of other unforeseen events that cannot be entered into the simulation program. It is therefore necessary to plan the survey as it progresses. If problems arise the surveyor should be able to overcome them with a number of different observing scheme options.

**Skyplots**

Skyplots are representations of the satellites' positions in the sky, in relation to the survey site at the time of observation. Skyplots are used to compute the time and the duration of the observation window for a survey area. This observation window occurs when there are at least four satellites above the horizon. The window occurs four minutes earlier each day, as the satellites have a twelve siderial hour period.

In order to compute skyplots the user must have approximate coordinates for his stations and ephemeris information for the satellites. The approximate coordinates can be taken from a map, and need only be accurate to a few tens of kilometres. The ephemeris information may be harder to acquire. If the user has a coded receiver the ephemeris elements can be read out of the navigation message. Otherwise ephemeris elements may be acquired through academic institutions such as the University of New South Wales or government departments like the Division of National Mapping who are in touch with GPS users and receiver distributors.

Skyplots are used during the reconnaissance of a survey to ensure that the satellite signals can be acquired by the receiver. If obstructions to the satellite signals are found, these must either be removed or the station must be shifted to a more suitable position.

Skyplots are also useful during the observation process. If a signal is obstructed while observations are taking place the skyplot can be used to indicate the satellites' position, and to identify the possible source of interference. This information can then be recorded, and subsequently used to tag erroneous observations at the processing stage.

Measurements to the satellites are not usually taken below an elevation of 15° due to the large atmospheric propagation error that affects them. The error occurs as satellite signals received at low elevation angles travel further through the atmosphere than those signals from higher elevations, and thus have a lower signal to noise ratio.

A number of different skyplot formats are used. Figure 5.1

shows one method of representing the satellites path across the sky. The paths of the satellites are not shown below 15° elevation.

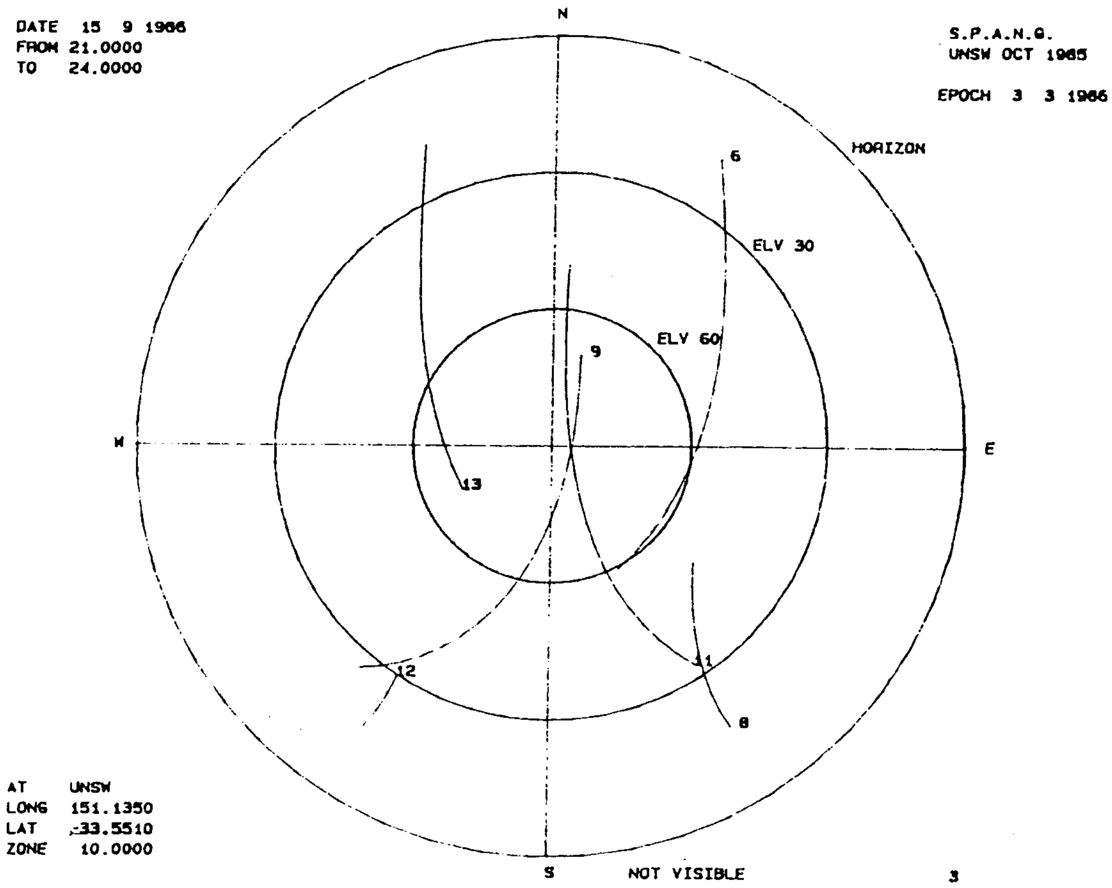



Figure 5.1 : Skyplot showing Satellite Movements at the University of NSW on 15th Sept 1986

**Ephemeris Generation**

With codeless GPS receivers (Macrometer V-1000) it is necessary to generate an 'almanac' file before observations commence. This file contains the approximate position of each satellite and the expected frequencies of each of the satellites signals at a number of epochs during an observation period. This information is required so that the receiver can search out each satellite signal at every epoch during the observation session.

The almanac file is computed using the approximate coordinates of the station, and satellite ephemeris data. The ephemeris data may be obtained from coded receivers or from previously completed surveys. The file can be generated in the office and then loaded onto a cassette. This cassette is then taken into the field and loaded into the instrument prior to the commencement of the observations.

## RECONNAISSANCE

At the completion of the office computations the reconnaissance and monumentation stage can commence. The main aim of the reconnaissance survey is to select points on the ground suitable for GPS stations.

Ideally GPS control stations should be accessible by car or helicopter. Accessibility is desirable as all receivers need to be brought within an antenna cable length of the station. As many of the early receivers like the Macrometer V-1000 and the TI 4100, are heavy, it is easier for the surveyor to bring them to the site by vehicle. Portability poses less of a problem for more recently released instruments, like the Trimble 4000S, as they are more compact and much lighter. The cable linking the receiver with the antenna should be as short as possible. This is necessary as antenna cables delay the signal travelling between the antenna and the receiver by an amount that is proportional to their length. Although this cable delay term can be modelled and corrected for, it is preferable to take precautionary measures in the field by keeping the cable short.

During the observation period, the GPS station should be free from obstruction in parts of the sky where the satellites are located. The skyplots that were generated in the office are used during the reconnaissance to indicate the satellites' path across the sky. If the satellites signals are obstructed, the surveyor can choose to move the station, clear the site or to mount the GPS antenna on high poles or tall tripods.

The site selection procedure depends on the purpose of the survey. For network densification purposes it is desirable to place the new stations on public property such as parks where they are unlikely to be disturbed. It may also be desirable to establish intervisible stations for azimuth marks.

The reconnaissance stage of the survey provides an opportunity for the monumentation of new stations, and reference and azimuth marks.

The final step in the reconnaissance process is to prepare recovery sketches for all the network stations. These should show the access to the station, existing or newly constructed monumentation, connections to reference marks and comments on the suitability of the site for GPS surveys.

## OBSERVING SCHEDULE

The final step of pre-survey preparation is the drawing up of the observing schedule for each receiver party. This schedule should include :-

a) Observing session number;

b) Date of observation, and the start and stop time of the observations;

c) Names of the stations;

d) Parties to occupy each station and the duration of the observing period;

e) Travelling details and access diagrams;

f) Recovery diagrams;

g) A list of equipment necessary for observations including power supply, chainsaw, tripods etc.;

h) Satellites to be observed; and

i) Other relevant remarks eg. owner of the property on which the station is located etc..

## THE FIELD SURVEY

The field survey procedure depends predominantly on the type of receiver used. Most receivers that are presently available, like the Trimble 4000S, are designed to observe both the phase of the transmitted carrier wave and the coded message, while codeless receivers like the Macrometer V-1000 can only observe the carrier wave phase. If a receiver can observe the coded message the observation procedure is greatly simplified.

The field observation procedure consists of the pre-observation stage, in which the receivers are prepared for observation, and the observation stage in which the data transmitted by the satellites are recorded.

### PRE-OBSERVATION

Pre-observation procedures are only required for codeless receivers. The two pre-observation tasks that must be carried out are: -

1) Set and synchronise the receivers' oscillators to Universal Coordinated Time (UTC); and

2) Read the almanac file.

The accuracy of a survey with codeless receivers depends on how well the receiver oscillators are synchronised with each other, and with an external source. The receiver oscillators should be synchronised to UTC to within a few milliseconds in order to achieve 1ppm baseline accuracies. This can be done by using an external time source such as an atomic clock, or a coded GPS receiver that can acquire time information from the satellites. All receivers are synchronised with each other to the microsecond level by actually physically bringing them together and linking their oscillators.

While the receivers are being powered up and synchronised the almanac files that were prepared in the office prior to survey can be entered into the receiver. These files tell the receiver where to observe the satellite signals during the observation period.

The Istac GPS receivers use interferometric techniques to observe the satellite signals. These instruments record all of the available satellite signals at the time of observation and therefore do not require an almanac file to be loaded. However, the Istac receivers need to be synchronised with UTC in the same manner as the Macrometer V-1000.

Coded receivers, like the Trimble 4000s and the WM101, need not be synchronised with each other or any outside source as they determine GPS time through the measurement of the pseudoranges.

Almanac files are also not required as coded receivers have access to the ephemeris information through the navigation message.

OBSERVATIONS

On arrival at the new GPS station, the GPS receiver is placed somewhere near the station, while the antenna is mounted on a tribrach and tripod and set over the station. The antenna height above the ground station is measured and recorded.

The receiver is then powered up. The Macrometer V-1000 has a power requirement of 350 watts, which needs to be supplied by small generator. More recent instruments, however, like the Trimble 4000s are able to run off smaller power supplies like car batteries, and the WM101 will be able to use internal nickel cadnium batteries.

The observing procedure now depends on the type of GPS receiver being used. The Macrometer V-1000 is almost immediately ready for observation once the "A" or almanac file is loaded, and some site data is entered. Different values are displayed on the instrument to indicate that the observations are being collected satisfactorily. The observer must then wait until the sixty epochs have been recorded onto a bubble memory. This data is then transferred to cassette before turning the power off and heading to the next site.

The Land Surveyor Model 1991 must be connected to either a rubidium oscillator or a GPS receiver capable of determining GPS time. Moreover, the start and stop times of the observation period must be entered. A rubidium oscillator is standard equipment for the Istac 2002 receiver. A few function buttons are then pressed and the instrument works automatically from then on. The instrument gives off a sound signal to indicate that the observations are being collected satisfactorily. The observations from this instrument are written onto cassette tape every 15 seconds.

It may take a few seconds to initialise coded receivers. Firstly site information is entered into the equipment. Then receivers like the TI4100 search the sky to look for and lock onto a satellite signal. Once one satellite signal has been acquired, it is interrogated for the ephemeris data of the other satellites, which can then be found and the observations can commence. The observation period varies from less than half an hour to several hours depending on the required accuracy of the survey.

There are a number of ways in which the recorded data is stored. The observations from the WM101 and the TI4100 are loaded directly onto cassette. The Trimble 4000s, however, downloads the recorded data through the RS232 port, into a minicomputer, where it is stored on disk.

While the receiver is performing measurements on the satellite signal, the surveyor completes the field log book. This log should contain site photographs that record the weather conditions, the station position, and a field report that includes information about equipment failures, obstructions to signals, clock offsets, meteorological data and any other relevant information. The observer should ensure that the equipment is functioning properly and that there are no power failures.

When the observations are complete, the cassette or disk containing the observed data must be labelled and stored for safekeeping. The data is returned to the office where it is processed to determine either station coordinates, or baseline vectors between stations. Before processing can commence it may be necessary to transfer this data to 9-track tape or some other medium. GPS data processing is dealt with in Chapter 6.

With codeless receivers, like the Macrometer V-1000 or the Land Surveyor Model 1991, their oscillators need to be calibrated at the conclusion of each observing session in order to determine clock drift between receivers. This clock data is required at the processing stage.

## STANDARDS and SPECIFICATIONS

Many surveyors are now starting on the GPS learning curve. Lack of experience leads to uncertainty about the correct procedures required for different types of surveys. As surveyors use GPS and become familiar with its limitations, correct and efficient field procedures will emerge.

The learning process can be accelerated through the involvement of government departments, like the Division of National Mapping, and academic institutions, like the University of NSW. All aspects of the system need to be tested and evaluated, especially receiver hardware, processing packages and the field procedures used.

Responsibility for evaluating the system is already accepted by various organisations. The Division of National Mapping has established the 'SEGMENT' network, designed to evaluate receiver hardware over a range of distances. To date the Macrometer V-1000 and the TI4100 receivers have been used to measure this network.

The National Mapping Council is in the process of establishing acceptable field procedures. These procedures are currently based on the results of overseas experience. As Australian surveyors become more familiar with the system, however, the procedures will evolve to suit the needs of the Australian survey scene.

The University of New South Wales, and the Division of National Mapping are developing GPS processing software. This software will be used to calibrate other software packages, and to assess the error sources that affect the system.

These developments show that the Australian survey community has realised the potential and the impact of GPS. To ensure the success of GPS in Australian surveying, further evaluation of the system needs to be encouraged and supported by all sectors of the survey industry.

# 6. PROCESSING GPS DATA

## INTRODUCTION

After GPS data has been collected in the field it is brought back to the office for processing. At the processing stage the recorded GPS data is used to compute either station coordinates or baseline vectors between stations. The GPS processing software is a vital component of the overall system, as the accuracy of positioning depends on the techniques and algorithms that are applied.

GPS is designed to provide navigation positioning in real time to a range of users. Navigation positioning requires the simultaneous measurement of pseudoranges to four different satellites. This measurement process is described in Chapter 2. At present, point positions derived from CA code pseudorange measurements are accurate to approximately ±10m and relative positions can be determined to the ±2m level. Higher relative accuracy is achieved as many of the errors that affect the pseudorange measurement are common to both receivers, and hence cancel out. The limiting factor of pseudorange measurement is the ability of the receiver to measure the phase of the incoming coded signal with respect to its own generated signal. This measurement is only accurate to the metre level.

In order to achieve surveying accuracies, receivers must be able to measure either the carrier beat phase or the transition phase (see Chapter 4). Presently the only receivers using transition phase are the Istac Model 1991 and Model 2002. These Istac instruments are only capable of decimetre level relative positioning, and will not be described in this chapter for this reason.

At the receiver, the incoming carrier waves are aligned with a receiver generated wave in the same way as the coded signals are aligned for pseudorange measurement. As the carrier and transition waves have shorter wavelengths than the CA code, they can be aligned more accurately with the receiver generated wave. Carrier phase observables allow ranges between the satellite and the receiver to be measured to the millimetre level permitting surveying accuracies to be achieved.

Although the surveying receivers measure the phase shift to the millimetre level, a number of other error sources affect the accuracy of the determination of the range. These error sources are largely eliminated with differential techniques and can, moreover, be further reduced by special of processing techniques.

The processing of carrier beat phases is a complex task. A multitude of papers have been written on optimum processing techniques, and on the different algorithms that can be used. This chapter describes some of the basic processing concepts, and explains the options available to the surveyor for eliminating or reducing the error sources affecting each measurement. The characteristics of an ideal software package and the advantages and disadvantages of currently available processing packages are also discussed.

## PROCESSING TECHNIQUES

The aim of processing is to achieve accurate receiver coordinates or the baseline vector between two receivers. Ideally, the receiver coordinates would be the only unknowns in the least squares solution. There are, however, error sources that affect the accuracy of the position determination. Some of the error sources can be estimated in the solution, while others can be modelled. Different mathematical models may also be used to eliminate these error sources from the solution.

Processing of the carrier beat phase, the transition phase and the squared carrier wave follow the same principles. All available processing packages use least squares techniques (see Appendix A for a review). In the least squares technique, a mathematical model is used to relate the recorded GPS observables (see Chapter 4) to a number of parameters. This model is linearised to form observation equations, which are in turn solved to obtain improvements in the model parameters (eg. receiver coordinates).

Each GPS observation campaign consists of a number of observation sessions. During each session, the receiver records time tagged GPS observables at a number of epochs determined by the receiver clock. At each epoch, an observation equation is formed to relate these new observables to the parameters.

A number of mathematical models and processing techniques have been developed for processing this data. The advantages and limitations of each technique is discussed below.

## MODELLING THE PHASE OBSERVABLE

Mathematical models that relate the carrier beat phase observable to the receiver position are given in KING et al (1985) and REMONDI (1985). SCHERRER (1985) models the instantaneous phase difference measured at each receiver as (see Figure 6.1)

$$\Phi_j^i = \frac{F_j^i}{c} \left| r^i - R_j \right| + F_j^i \left[ \Delta t^i(t_T) - \Delta T_j(t_R) \right] + N_j^i + \Phi_{noise} \qquad \ldots. \quad (6.1)$$

where

$\Phi_j^i$ = instantaneous carrier beat phase observation of the signal from the ith satellite measured at the jth receiver

$F_j^i$ = frequency of the signal from the ith satellite measured at the jth receiver

$c$ = speed of light

$r^i$ = position vector of the ith satellite

$R_j$ = position vector of the jth ground station

$\Delta t^i(t_T)$ = satellite clock offset at transmission time, T

$\Delta T_j(t_R)$ = receiver clock offset at receiver time, R

$N_j^i$ = ambiguity term

= unknown number of integer cycles at first measurement epoch

$\Phi_{noise}$ = noise of the signal

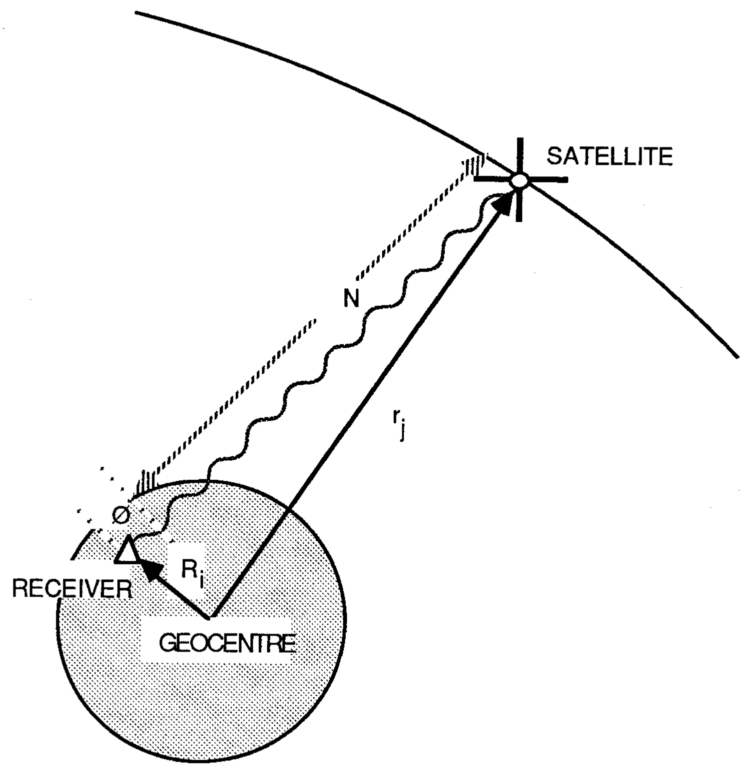



Figure 6.1 : Instantaneous Phase Observable

The first term in equation 6.1 is the geometric term describing the measured range between satellite, i, and receiver, j, at a measurement epoch, $t_R$ in cycles. The accuracy of this term depends on the accuracy of the satellite ephemeris data, the accuracy of the apriori receiver coordinates and the model that describes the atmospheric propagation delay experienced by the satellite signal. Table 6.1 shows the approximate effect of these error sources on the measurement of the satellite-receiver range. Methods of reducing these terms by modelling are given below.

The second term in Equation 6.1 describes the effect of instabilities in both the receiver clock, $t_R$, and the satellite clock, $t_T$ on the range measurement. Table 6.1 shows the effect of these clock instabilities on the position solution is quite large. The clock errors must therefore be estimated in the solution or eliminated completely from the model. In order to eliminate them, observations can be differenced between receivers and satellites. Clock errors and methods of overcoming them are described below.

Table 6.1 : GPS Range Error Budget

| SOURCE | PSEUDORANGE | | CARRIER PHASE | |
|---|---|---|---|---|
| | Absolute | Relative | Absolute | Relative |
| Satellite Position | 100m | 5ppm | 100m | 5ppm |
| Satellite Clock | 10m | - | 10m | -* |
| Ionospheric Refraction | 50m | 2ppm | 50m | 2ppm |
| Tropospheric Refraction | 10m | 1ppm | 10m | 1ppm |
| Receiver Position | - | -* | - | -* |
| Receiver Clock | 1000m | -* | 1000m | -* |
| Multipath | 1m | 1m | 0.05m | 0.05m |
| Ambiguity | - | - | 0.2m | -* |
| Observation Error | 10m | 10m | 0.001m | 0.001m |

* estimated

The ambiguity term in Equation 6.1 represents the integer number of cycles between the satellite and the receiver. When the receiver first acquires the satellite signal, the integer number of cycles in the carrier beat phase is not known. However, the ambiguity will remain constant throughout the observation session as subsequent phase observations will include the integer number of cycles contained in the previous observation. Table 6.1 shows that the ambiguity term only affects the carrier beat phase observable. The ambiguity term is estimated as part of the solution, or is eliminated using between epochs differencing (see below).

The final term in Equation 6.1 is a noise term that includes receiver error and other error sources, like multipath. These cannot be modelled or estimated in the solution. Table 6.1 shows the magnitude of the noise for carrier beat phase measurement is at the 5cm level, which is significantly less than the effect on pseudorange measurement.

Once the observation equation has been formed for each measurement, the partial derivatives of the observations with respect to the unknown parameters are computed. These define the design matrix (see Appendix A) which is used in a least squares procedure to obtain the satellite and receiver positions, the satellite and receiver clock offset to GPS time at each measurement epoch and a bias term made up of the ambiguity and the initial satellite and receiver clock offset. As the bias is a combined term, the integer ambiguity can not be estimated explicitly using this processing technique. This is referred to as the undifferenced or One-Way phase model.

## MODELLING THE GEOMETRIC TERM

The accuracy of the geometric term depends on the accuracy of the model for the satellite ephemeris, the apriori coordinates of the origin station and the atmospheric propagation delay. Ignoring the effect of these parameters is analogous to not correcting EDM distances for atmospheric propagation delay :- in other words, the resulting distances may be precise, but not accurate.

Poor modelling of these error sources introduces systematic errors into the solution which cannot be removed or minimised by increasing the number of observations.

## THE SATELLITE EPHEMERIS

The satellite ephemeris is a set of coordinates describing the position of a satellite with respect to the earth as a function of time. The contribution of the satellite ephemeris to the mathematical model is shown as $R_j$ in equation 6.1. As the GPS satellites are effectively orbiting control stations, improving the values of the satellite coordinates will lead to an improved model of the range from the satellites to the receivers.

The effects of errors in the broadcast ephemeris are reduced by differential positioning. The effects on baseline determinations can be further reduced by using more accurate ephemeris models.

The broadcast ephemeris is predicted by the Master Control Station from tracking data collected by the other monitor stations. The Master Control Station updates each satellites' navigation message daily. This new ephemeris is then broadcast as part of the navigation message to be used in real time by the GPS receivers on earth.

At each measurement epoch the receiver time is recorded along with the carrier phase and pseudorange observables. Time is required to determine the position of the satellite when the measurement was made. For surveying applications the receiver clocks need to be synchronised with each other to the microsecond level, and with GPS time to the millisecond level.

The broadcast ephemeris is usually given in GPS time. Therefore, we need to determine the receiver clock offset from GPS time at each measurement epoch. As the satellite moves at approximately 4km/sec, these time tags need be accurate to .001 secs in order to obtain absolute satellite position within the general accuracy of the ephemeris (10-20m).

Microsecond synchronisation between receivers ensures that the satellite positions used to determine the range are the same, to within a millimetre, for each receiver.

Code correlating instruments estimate the clock offset from GPS time along with receiver positions at each epoch. With codeless receivers the problem is overcome by synchronising receivers before and after each observation session and calibrating them against UTC. This calibration allows the surveyor to monitor the receivers clock drift, which is then used at the processing stage to correct the receiver time tags to GPS time.

The broadcast ephemeris is presently given in the WGS72 reference system (see Control Segment), but this will soon change to WGS84. Test measurements have shown that the present accuracy of the broadcast ephemeris is at the ±10m level. This accuracy will be downgraded to about ±100m with the launching of the production satellites. Ephemeris errors enter receivers point positions on a 1:1 basis. Ephemeris errors of ± 10m, however, only contribute a baseline error of .5ppm at the processing stage. KING et. al.(1985) state :

> ".. a given error in the satellite ephemeris introduces an error in relative positioning (baseline) reduced by the ratio of the baseline length to the satellites' altitude."

Expressed mathematically

$$\Delta b = \frac{b}{a} . \Delta e \qquad (6.2)$$

where $\Delta b$ = change in baseline length

$b$ = baseline length

$a$ = altitude of the satellite

$\Delta e$ = ephemeris error

The error contribution of the ephemeris on GPS positioning can be reduced by using a precise ephemeris. This is a post-processed ephemeris calculated from the five monitor stations and three ground antennae around the globe. The precise ephemeris is a post-processed ephemeris calculated from tracking data from 16 monitor stations around the world (DMA, 1986). The precise ephemeris is more accurate than the broadcast ephemeris as it uses actual tracking data to determine the satellites position at each epoch. The broadcast ephemeris uses algorithms to predict the expected orbit in time.

The precise ephemeris gives the satellites position to better than ±10m thereby making relative positioning to better than 0.5ppm possible. The disadvantage of the precise ephemeris is that it is only available to selected users and usually many months after the survey has been completed. At present an agreement between the DMA, the Australian Army and the Division of National Mapping has left the responsibility for the distribution of the precise ephemeris with Division of National Mapping. KING et al (1985) describe the nature of the precise ephemeris, and explain how it can be used for GPS processing.

An alternative source of ephemeris data may become available if a local satellite tracking network is established. This alternative was explored by RIZOS et al (1985). A local tracking scheme has several advantages over both the precise and broadcast ephemerides. These include :-

1) Australian GPS users would become independent of the broadcast and precise ephemeris. Accordingly, if access to the ephemerides is denied or if accuracy is seriously downgraded, this will have little effect on the capability of system in Australia.

2) As a local tracking network is better able to determine the satellites path over a local region an improved ephemeris would result. This would permit the determination of highly accurate ephemerides (<10m) suitable for crustal movement surveys.

3) Control can be exercised over the ephemeris reference system (see Chapter 7). Presently the Naval Surface Weapons Centre (NSWC), which is responsible for computing the ephemerides, can change any of the parameters defining the reference system without warning.

The establishment of a permanent local tracking network would benefit the high accuracy users of GPS. It is possible, however, to establish such a network on a temporary basis to coincide with a specific measurement campaign, for instance, for studying local tectonic plate movement. However, for most surveying applications the broadcast ephemeris has proved adequate.

The establishment of a tracking network is of concern to future surveyors, however, as the accuracy of the satellite ephemerides will be downgraded with the launching of the production satellites. If a regional tracking network is not established when the ephemeris information is downgraded, the possible positioning accuracy that can be obtained from GPS will be reduced.

COORDINATES OF ORIGIN STATION

When processing GPS data in the differential mode, one receiver station is held fixed (origin station). The uncertainty in the baseline measurement produced by the errors in the coordinates of the origin station is similar to that caused by the uncertainty of the accuracy of the satellite ephemeris. The geometry between two receivers and the satellite is distorted by the same amount if either the satellite position is in error or the coordinates of the origin station are held fixed at the wrong values. Figure 6.2 shows these effects.

There are two possible ways in which a surveyors can obtain the coordinates of the origin station in the WGS72 datum. The simplest is to use coded receivers, which can presently achieve ± 10m point positioning accuracy (SANDS, 1985) from the CA code. At this level of accuracy, the position error contributes only .5ppm error to each baseline determination.

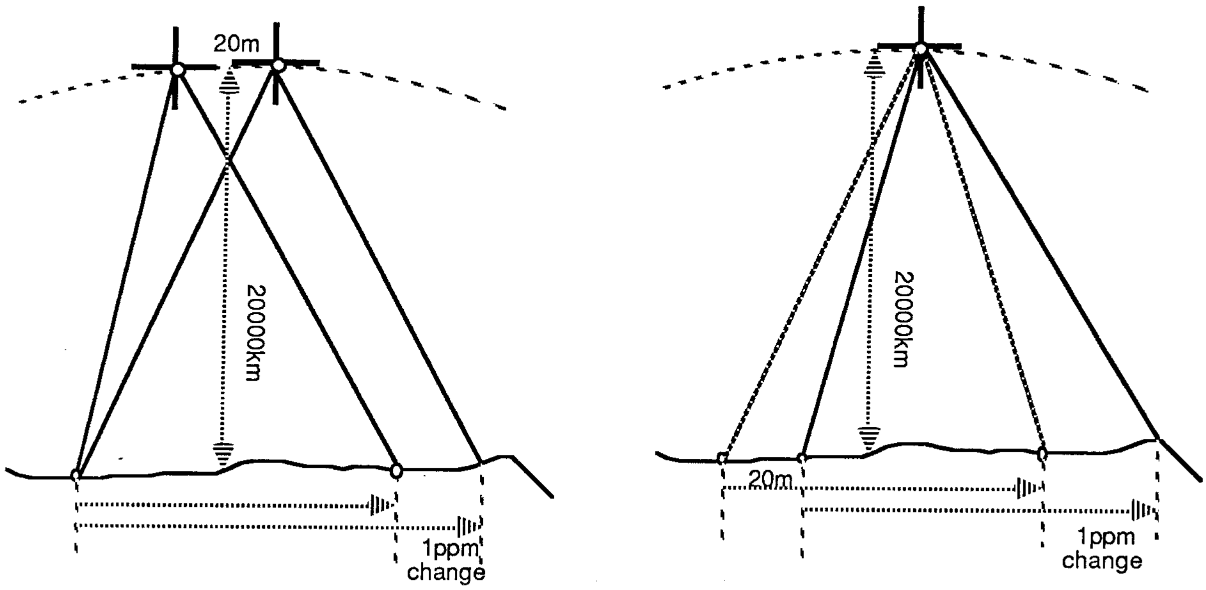



Figure 6.2 : Baseline Uncertainty Paused by (a) Ephemeris Error and (b) Origin Station Coordinate Errors

A second method of establishing the WGS72 coordinates for the origin station is by means of the transformation parameters relating the precise ephemeris of the TRANSIT Doppler system to the Australian Geodetic Datum (AGD), and the mathematical relationship between this datum and the WGS72 datum developed by Seppelin (SEPPELIN, 1972). A detailed explanation of this technique appears in Chapter 7.

## ATMOSPHERIC PROPAGATION ERROR

The atmosphere can be divided into the ionosphere which extends from approximately 80-1000km above the earth and the troposphere which extends from 0-70km above the earth (LANDAU & EISSFELLER, 1985). The carrier waves broadcast by the satellite are delayed as they propagate through the atmosphere. The delay increases the transit time of the signal from the satellite to the receiver, and hence affects the range determination in the mathematical model.

The atmospheric delay errors are largely cancelled by differential positioning. For short lines (say <30km) the atmosphere affects the transmitted satellite signal nearly equally at each receiver, and hence the error is much reduced. For longer lines, however, the atmospheric conditions at either end of the line may be completely different and in order to achieve high accuracies it is necessary to model or measure the atmospheric delay terms.

The effect of the ionosphere and the troposphere on the pseudorange determination is shown in Figure 6.3. This figure shows that the ionospheric delay is frequency dependent, while the tropospheric delay term is not. It is also evident that the ionospheric correction is greater during maximum sun activity (ie the ionospheric effect is smaller at night than during the day).

The propagation delay due to the ionosphere depends on the electron content of the ionosphere at the zenith and the elevation angle to the satellite. The ionospheric delay changes throughout the day from a minimum early in the morning (up to 0900hrs) to a maximum at 2000hrs before it starts dropping again. The delay affects the pseudorange by up to 10's of metres, depending on the time of year, sunspot activity and the latitude of the observation. The delay is a minimum at the zenith and increases to about three times this value at the horizon.

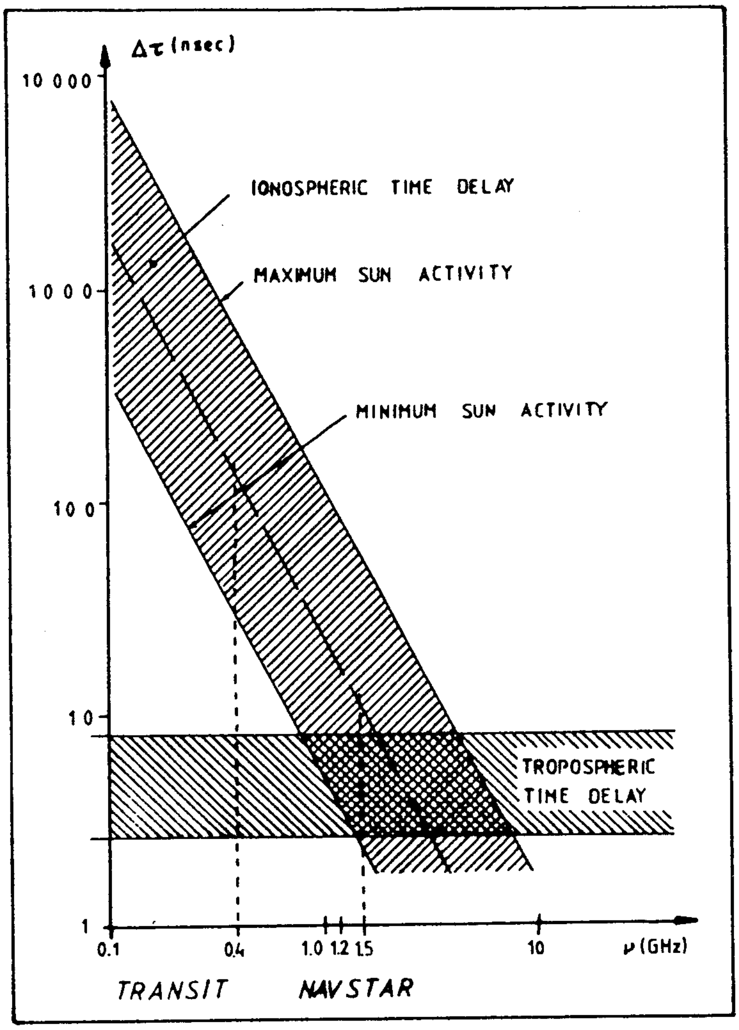



Figure 6.3 : Ionospheric and Tropospheric Delay of TRANSIT and GPS Satellite Signal (LANDAU & EISSFELLER, 1985)

The effect of the ionospheric delay can be modelled using the formula (KING et.al., 1986, SPILKER, 1978)

$$\tau_{ion} = \frac{\Phi_{ion}}{f} = \frac{1.35 * 10^{-7} Ne}{f^2} \qquad (6.3)$$

where

$\tau_{ion}$ is delay in seconds

f is frequency in Hertz

Ne is integrated electron content over the signal path in electrons/$m^2$

For high accuracy work, a dual frequency receiver (eg

Macrometer II, TI4100) is used to remove the ionospheric delay. This process is fully explained in KING et.al. (1985).

The other major contributor to the atmospheric delay comes from the troposphere. The tropospheric delay is a function of air pressure, the elevation angle to the satellite and the height of the observer above sea level. Minimum tropospheric delays are found at high elevations, when the satellite is at zenith.

Tropospheric delay can also be modelled as (KING et. al., 1985)

$$\tau_{atm} = 7.595 * 10^{-12} \sec z \left[ P + \left[\frac{1255}{T} + 0.05\right] e - \tan^2 z \right] \quad (6.4)$$

where

$\tau_{atm}$ = atmospheric delay in seconds

z = zenith angle of the satellite

P = pressure in millibars

T = temperature in degrees Kelvin ( =°C + 273.16)

e = partial pressure of water vapour in millibars.

The partial water vapour pressure is computed from the relative humidity by

$$e = 6.108 \text{ RH} \exp \left[ \frac{(17.15T - 4684)}{T - 38.45} \right] \quad (6.5)$$

and the pressure P at height above sea level h (km) is given in terms of the surface pressure $P_s$, and the temperature $T_s$ by

$$P = P_s \left[ \frac{T_s - 4.5h}{T_s} \right]^{7.58} \quad (6.6)$$

Although the dry component of the troposphere can be accurately modelled, the wet component is more difficult to model as ground measurements do not accurately reflect the distribution of water vapour along the signal path.

## CLOCK INSTABILITIES

Table 6.1 shows that the biggest error sources in the satellite-receiver range measurement are due to the satellite and receiver clock instability. The three approaches that can be used to account for this clock error are :-

(1) estimate the clock offsets at each epoch;

(2) estimate the behaviour of the clocks throughout the observing session; and

(3) eliminate the effect of the clocks completely by using differencing techniques.

## CLOCKS

A common model describing clock behaviour is

$$f(t) = f_o + a_i + b_i(t-t_o) \qquad (6.7)$$

where $f_o$ = nominal frequency

$a_i$ = frequency offset

$b_i$ = drift coefficient

$t_o$ = reference epoch

In general, the frequency offset $a_i$ and the drift coefficient $b_i$ will change for each receiver over a few hours. Several types of oscillator are in use, each with characteristic offset and drift stabilities. Table 6.2 shows the typical stability values for Rubidium, Cesium and Crystal oscillators over a six hour period.

The production satellites will be equipped with cesium oscillators which contribute approximately 10m to the range measurement between the satellite and the receiver over 6 hours (see Table 6.1).

Table 6.2 : Oscillator Stability (KING et al, 1985)

| Oscillator | Fractional frequency stability over 6 hrs ai/fo | Fractional drift stability over 6 hrs bi/fo |
|---|---|---|
| Cesium | $10^{-13}$ | $10^{-15}s^{-1}$ |
| Rubidium | $10^{-12}$ | $10^{-14}s^{-1}$ |
| Crystal | $10^{-10}$ | $10^{-12}s^{-1}$ |

The satellite clock drift terms are modelled by the Control Segment, and these are broadcast daily as part of the navigation message. It can, therefore, be expected that the clock corrections will be less accurate at the end of the daily period. As codeless receivers do not have access to the navigation message, the satellite clock drift will have a greater error contribution (10m) to the range measurement, than the coded receivers (<1m) that can correct for the drift.

Presently most receivers have crystal oscillators, however, it is usually possible to connect an external rubidium or a cesium oscillator. Over an hours observation, the instability of a crystal oscillator may contribute 100's of metres to the measurement of the range to a satellite (KING et al, 1985).

ESTIMATING CLOCK TERMS

The clock offsets to GPS time can be determined at each measurement epoch. Alternatively they can be obtained by estimating the clock behaviour throughout the observing session. Chapter 2 described how the receiver clock offset to GPS time was determined at each measurement epoch for the pseudorange solution. If this is not done (ie. included in the model) the range measurement would be in error, and so would the estimate of receiver position.

There GPS observables can be modelled in a number of ways :

**One Way Phase Model**

The mathematical model for one way phase was given in equation 6.1. This model, which is depicted in Figure 6.4 represents the instantaneous phase measurement at receiver 1, with respect to the signal emitted by satellite j at epoch $t_i$ : ie:

$$\Phi_1^j = \frac{F_1^j}{c}\left| r^j - R_1 \right| + F_1^j[\Delta t^j(t_i) - \Delta T_1(t_i)] + N_1^j + \Phi_{noise}$$

The characteristics of the geometric term were described in the previous section.

The satellite clock offset, $\Delta t$, and the receiver clock offset, $\Delta T$, from GPS time are estimated at each measurement epoch. In the one way phase model its assumed that the clock offsets are independent from one epoch to the next. Thus, mathematical techniques like partitioning can be used for data processing. In the partitioning method the dimensions of the design matrix are reduced since each epoch clock term is estimated independently and then disregarded (see KING et. al. 1985). Partitioning therefore significantly reduces the computer memory requirement.

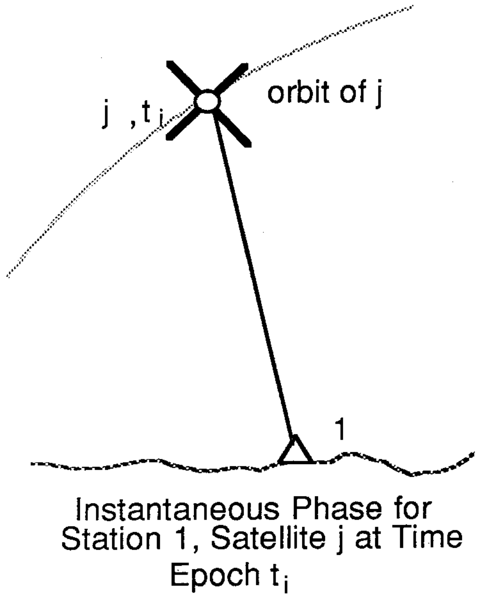



Figure 6.4 : One Way Phase Model

Fewer parameters are needed to describe the stability of the satellite and receiver clocks if their behaviour over an observing session is modelled by a polynomial function (eg eqn 6.7). More parameters are needed to describe the behaviour of crystal oscillators than the more stable rubidium oscillators.

The one way phase method of processing cannot be used to detect small cycle slips in the observed data as these are swamped by the satellite and receiver clock errors. In other words, the effect of a few slipped cycles on the range measurement is insignificant compared to the error in range caused by the receiver clock instability. Measurement data must, therefore, be preprocessed to ensure that it is free of cycle slips.

ELIMINATING CLOCK TERMS

A range of differencing techniques is used to eliminate the effect of satellite and receiver clock errors on the positioning solution. Any parameter which is common to the observations being differenced is eliminated. The differenced observations, however, can still be used to solve for the relative positions of the receivers. Measurements can be differenced between epochs, between receivers, between satellites and any combination of these. This process is analogous to using angles instead of directions in a conventional survey network to eliminate the unknown orientation.

**Between Stations Differencing**

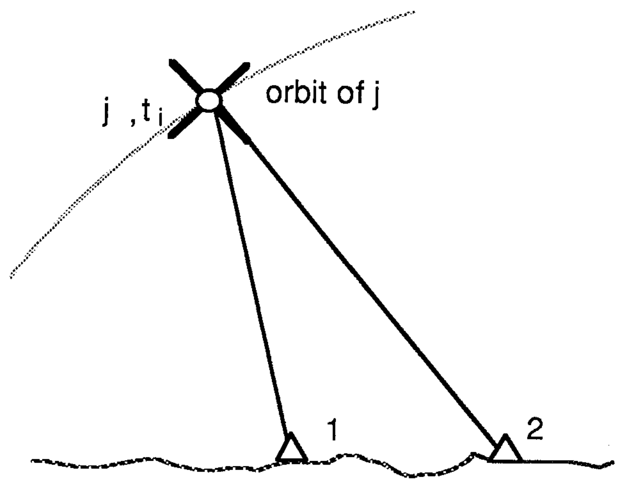
The between stations difference from one satellite (j) is the difference between the carrier phase observed at two stations (1,2) at the same epoch ($t_i$). This measurement, which is known as the single difference is shown in Figure 6.5.

The observation equation is

$$\Delta\Phi^j_{12} = \Phi^j_1 - \Phi^j_2 \qquad (6.8)$$

$$= \frac{F^j_1}{c}\left|r^j - R_1\right| - \frac{F^j_2}{c}\left|r^j - R_2\right| + F^j_1\ \Delta T_1(t_i) + F^j_2\ \Delta T_2(t_i) - N^j_1 - N^j_2 + \Phi_{noise} \qquad (6.9)$$

The first line of this equation is the geometric term which is the calculated from the difference in ranges to the satellite from each station. This term depends on the apriori values of the satellite and receiver coordinates and the model for the atmospheric propagation delay.




Single Difference for Stations 1 and 2,
Satellite j, Time Epoch $t_i$

Figure 6.5 : Between Stations Differences

The second line of equation (6.9) describes the receiver clock offsets from GPS time at stations 1 and 2. The third line represents the difference in ambiguity terms between the stations and the noise component of the observed signal.

Unknown satellite clock parameters are removed in the between station differenced observable. The between stations difference algorithm may be used to solve for relative positions between stations, a differenced bias (combined differenced cycle ambiguity and the initial differenced receiver clock offset) and the difference between the receiver clock terms.

**Double Differencing**

The double differenced observable is obtained by differencing between stations differences. Two receiver stations

(1,2) observe two satellites (j,k) simultaneously. The double difference model is shown in Figure 6.6. The double difference observable is of the form

$$\Delta\Delta\Phi_{12}^{ij} = \Delta\Phi_{12}^{j} - \Delta\Phi_{12}^{i} \tag{6.10}$$

$$\begin{aligned} &= \frac{F_1^j}{c}\left|r^j - R_1\right| - \frac{F_2^j}{c}\left|r^j - R_2\right| - \\ &\quad \frac{F_1^i}{c}\left|r^i - R_1\right| + \frac{F_2^i}{c}\left|r^i - R_2\right| + \\ &\quad N_1^j - N_2^j - N_1^i + N_2^i + \Phi_{noise} \end{aligned} \tag{6.11}$$

The geometric effect is represented by the first two lines of equation (6.11). The remaining terms of the equation represent the unknown ambiguity parameters between the receivers and the satellites.

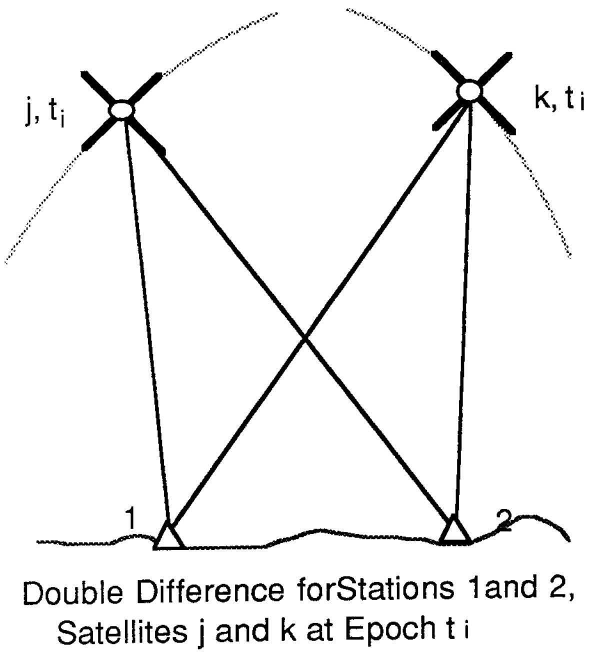



Figure 6.6 : Double Difference Observation

This double difference model eliminates the effect of both the receiver and satellite clocks on the position. The model may be used to solve for the relative position between receivers, relative orbits and the differenced bias. The bias term consists of the ambiguity terms from each receiver to each satellite and is therefore an integer. The double difference observation equation is suitable for detecting cycle slips as the effect of unstable satellite and receiver oscillators on the observed bias term are removed. Double differencing is therefore suitable for preprocessing of the data for one way phase processing.

If more than two receivers are used simultaneously, all of the processed baseline data will be correlated, which should be taken into account in the network adjustment. Complex algorithms are needed to account for these correlations. Processing is more efficient and easier if all the observations are uncorrelated (eg. the partitioned one way phase model) or if the correlations are ignored.

One-way phase and the double difference observables provide the most accurate positioning determinations and are, therefore, preferred for surveying.

## AMBIGUITY RESOLUTION

The third term in equation 6.1 is the unknown ambiguity between the satellite and the receiver.

The ambiguity of a carrier beat phase measurement is the integer number of cycles between the satellite and the receiver when the receiver first acquires the signal. Once a satellite signal is acquired, the range between the satellite and the receiver can be considered to be made up of the ambiguity and the carrier wave phase. The ambiguity does not change during the observing session as the receiver records the integer number of carrier waves that pass from this initial epoch. However, there will be more than one ambiguity per satellite-receiver pair if a cycle slip occurs or the receiver loses lock during an observation session.

The ambiguity term is an integer which can either be estimated as part of the solution, or eliminated using triple differencing. In order to achieve the most accurate results from GPS, the ambiguity should be estimated. The GPS data can then be reprocessed holding the ambiguity term fixed. This improves the accuracy of the position determination, as the uncertainty in the ambiguity term does not contribute to the position solution.

The double difference technique can be used to solve for the ambiguities explicitly. The bias term in the observation equations, which represents the difference of ambiguity between two satellites and sites, should be an integer value. Unfortunately, this is not the case, as measurement noise and theoretical model errors contaminate this value. Ideally, however, this bias value will be close to an integer, with an uncertainty of less than one cycle. Although this may be the case for short lines, increased measurement noise over longer lines makes it difficult to choose the correct integer value for the ambiguity. When this happens the ambiguity values that fall within the confidence interval must be scanned by least squares techniques to select the most probable integer values.

If the ambiguity terms are held fixed at the wrong value in the subsequent solution, the relative position between stations may be in error by several decimetres.

The ambiguity term can also be eliminated from the observation equation by differencing the double differenced observable between epochs. This mathematical model is known as the triple or between epoch difference.

### Between Epochs or Triple Differencing

This observable is obtained by differencing between double differences at two separate epochs. Figure 6.7 shows the triple difference observation between stations 1 and 2, satellites j and k at epochs $t_i$ and $t_{i+1}$.

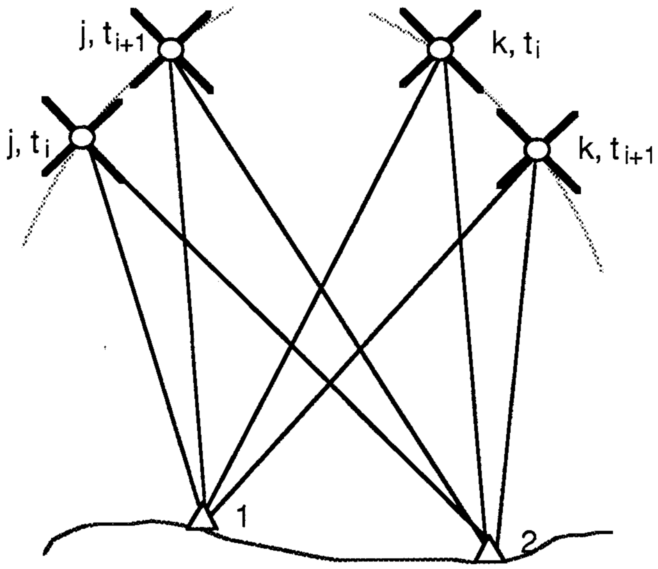
The mathematical model is

$$\Delta\Delta\Delta\Phi_{12}^{ij} = \Delta\Delta\Phi_{12}^{ij}(t_i) - \Delta\Delta\Phi_{12}^{ij}(t_{i+1}) \qquad (6.12)$$

When the correct values are substituted in Equation 6.12, the ambiguity terms of the double difference model cancel, leaving only the geometric terms. Triple differencing is, therefore, easy to automate, and is suitable for fast batch processing.

As some information content is removed from the data, the triple difference model provides less accurate positioning results than either the one way phase or double differencing model. Moreover, as the triple difference model does not provide the ambiguity, the uncertainty in the ambiguity term contributes to the determined station position.

Triple differencing is suitable for providing a preliminary approximation of the baseline (10-20ppm), which may be used as a good apriori estimate for other processing algorithms.




Triple Difference for Stations 1 and 2, Satellites j and k, and Epochs $t_i$ and $t_{i+1}$

Figure 6.7 : Triple Difference Observation

The ambiguity will change if a cycle slip occurs in the data. Once a cycle slip is detected the new ambiguity must be solved for at the data processing stage.

**CYCLE SLIP EDITING**

Cycle slips occur when the satellite signal is lost momentarily during the observation session. Although the fractional phase measurement, after reacquisition of the signal, is the same as if tracking had been maintained, the ambiguity to the satellite will have changed. As these ambiguity terms are eliminated with the triple difference algorithm, cycle slips will show up as outliers in the plot of the position residuals. These cycle slips are therefore easily detected in this processing technique. If other processing techniques are used, however, the cycle slips must first be removed before processing can commence.

One method of detecting cycle slips is to first process the data with a triple difference technique thereby gaining a good first approximation of the baseline vector. Once the baseline has thus been determined, the station coordinates can be held fixed and the data run through double difference processing. Cycle slips can then be identified as discontinuities in the double difference residuals. Once the cycle slips have been detected, the data can be corrected before reprocessing commences. Errors in cycle slip detection will affect the final positioning determination. BEUTLER et al (1984), REMONDI (1985) and HILLA (1986) describe automatic cycle slip editors.

NOISE

The noise term includes measurement error in the receiver, multipath effects and the unknown phase centre of some antennae. Antenna design has been discussed in Chapter 4.

MEASUREMENT ERROR

This error concerns the measurement of the incoming signal. The measurement error indicates the ability of the receiver to correlate receiver generated signals with those transmitted from the satellite. The measurement error is a function of the stability of the receiver oscillator. The more stable this is, the better the signal can be correlated. The pseudorange measurement can be correlated to approximately the one metre level for both the P and CA codes, while the carrier beat phase can be measured to the millimetre level.

MULTIPATH

A major source of errors that occurs when observing a satellite signal is due to the effect of multipath. Multipath interference of the satellite signals occur when some fraction of the transmitted signal is reflected off the ground or some other object before reaching the antenna. The effect on the observed satellite signal depends on the design of the antenna, the proximity of large reflective objects and the height of the antenna above the ground. As a rule of thumb, multipath effects are proportional to the signal wavelength. Table 6.1 shows that multipath effects can contribute approximately 1m to pseudorange measurements and approximately 0.05m to carrier beat phase measurements.

## PROCESSING OVERVIEW

In order to summarize the GPS data processing procedure, a flowchart showing the necessary steps is included (MASTERS et al, 1985)

1) Preprocessing
   - Decide on differencing to be applied (see Processing Techniques)
   - Read ephemeris files, site files and any apriori information

2) Processing

Start loop over measurement epochs

Start loop over sites and satellites in each epoch

- Read in phase observation and time tag
- Obtain apriori station position, ephemeris information and transformations between various reference systems
- Calculate the theoretically modelled phase or differenced observable using the time tag and relevant apriori information
- Correct observations for ionospheric and tropospheric refraction
- Obtain the difference between the observed and the modelled phase
- Evaluate the partial derivatives of the theoretical observable with respect to the parameters (to form the design matrix).

End the site-satellite loop.

- Obtain differences of the residuals and partials.
- Increment the normal equation matrix for the least squares solution.

End measurement epoch loop.

- Increment normal equation matrix for effect of any apriori information for the parameters.
- Solve normal equations
- Update parameters and update statistical information

- Calculate and plot the residuals using the updated parameters
- Edit any gross errors from the observations
- Search for cycle slips and repeat solution if required
- Resolve ambiguities if possible and repeat solution with ambiguity biases fixed

Output site coordinates and variance covariance matrix.

Adjust the processed results into a network (see Chapter 7)

Transform the adjusted network into a local reference system (see Chapter 7)

## SOFTWARE PACKAGES

A number of software packages are presently available for GPS processing. These packages have either been developed by universities and government departments for research purposes or by the receiver manufacturers in order to promote the use of their instruments. Currently available software packages are compared below.

The ideal surveying software package must not only contain a complete range of software but must be user friendly, automated, and microcomputer based. The software should also use efficient algorithms to produce fast accurate results, and be presented as an integrated package able to process data from different instruments, for any number of receivers over a number of observing sessions. As yet no such package exists.

The major components of any package include software to aid pre-survey planning, decision making and field observations, data preprocessing and checking, network adjustments and the transformation of processed data into a local reference system.

The pre-planning and decision making software includes the generation of skyplots for reconnaissance purposes, the calculation of GDOP values necessary to select the satellites to be observed and software to simulate networks.

Skyplot packages are generally available for all code correlating instruments (eg. Trimble 4000S, WM101) and are a common feature of many currently available software packages (eg. PoPs, Magnet, Baseliner). The GDOP gives the surveyor a guide on which satellites to select to obtain the strongest solution. Once the satellites are selected for the observing session, each receiver can be programmed accordingly. This is not important for receivers capable of observing more than five satellites simultaneously (eg. WM101). The importance of generating and simulating networks was emphasized in Chapter 5.

Another constituent of a complete package is the ability to read the collected data from the recording medium. For some instruments, like the WM101 or the TI4100, this data is collected on cassette. The software must then interface with a cassette reader (eg. Memtec Cassette terminal 5450 XL) to read the data from the cassette into memory or onto disk. The Macrometer V-1000 collects data on a bubble memory, from which it is downloaded onto cassette for processing. If microcomputers are used to log data in the field (eg. Trimble 4000S) the data is already stored on disk ready to be processed.

The preprocessing component of the software package prepares the collected data for processing. At this stage the raw observations, meteorological and ephemeris data, and apriori station information are read, and the collected data is formatted ready for processing. With code correlating receivers, the pseudorange station positions and the receiver clock offsets from GPS time are calculated at the preprocessing stage, that is, if these have not been previously determined in the receiver.

Some packages provide the operator with a choice of processing techniques. An ideal package would provide a range of processing options, and be able to process a network either one baseline at a time, or by multistation techniques for one or more observation sessions.

Once the data has been processed either as baselines or point positions, the software should be available to rigorously combine this data into a network using least squares techniques. The final component of a complete software package is the transformation of the adjusted network data to a local reference system. This is achieved using published values of the transformation parameters or values computed from a knowledge of both the local and the WGS72 coordinates of several common stations. Network adjustments and the procedures used to transform data are described in the next chapter.

## CURRENT SOFTWARE PACKAGES

To date most of the program development has been carried out either by academic and government institutions or by receiver manufacturers as already mentioned. The packages from the academic and government institutions tend to be large and complex, requiring a high degree of analyst skill to run. These programs (eg. Phaser, Nibble & Crunch) have been written for mainframe computers, and are designed to carry out most of the abovementioned functions. Manufacturer developed software like PoPS and GEOMARK, on the other hand, are usually receiver dependent and are designed with user friendliness in mind. These packages are usually microcomputer based, but provide the operator with only a limited range of processing options. Some manufacturers (eg. Texas Insts) however, provide additional software to complement the processing package.

Table 6.3 shows some of the GPS software packages that are currently available. Important features of each package are listed. As GPS receivers continue to develop and more research is done, it is expected that many more software packages will be introduced. Moreover, the existing packages will undoubtedly be upgraded.

TABLE 6.3 **Software Packages**

| SOFTWARE | SOURCE | USER FRIENDLINESS | MULTI SESSION | MULTI STATION | MULTI INSTS | MICRO BASED | LIMITATIONS |
|---|---|---|---|---|---|---|---|
| NIBBLE & CRUNCH | SPANG UNSW | Partly | YES | YES | YES | NOT YET | Requires large Computer |
| POPS | WILD MAGNAVOX (Bern) | Very | YES | YES | NO | YES | Proprietry Soft. Receiver Dependent |
| MAGNET 4100 | MAGNAVOX (Hatch) | Not Very | NO | YES | YES | YES | Proprietry Soft. Receiver Dependent |
| TRIMVEC | TRIMBLE (Goad) | Partly | NO | NO | NO | YES | Proprietry Soft. Receiver Dependent |
| PHASER | NGS (Goad) | Not Very | NO | YES | YES | NO | Requires large Computer |
| MACROMETER | LITTON (MIT) | Partly | NO | NO | NO | YES | Proprietry soft. Receiver Dependent |
| GEOMARK | TEXAS INSTS | Very | NO | NO | NO | YES | Proprietry Software |
| BERNESE GPS SOFTWARE | UNI OF BERN Beutler | Partly | YES | YES | YES | NO | Requires large Computer |

| SOFTWARE | PRE PROCESSING | | | PROCESSING ALGORITHMS | POST PROCESSING | |
|---|---|---|---|---|---|---|
| | SIMULATIONS | SKYPLOTS | GDOP | | NETWORK ADJUSTMENTS | TRANSFORM |
| NIBBLE & CRUNCH | YES | Additional Software Available | | UNDIFFERENCED SINGLE DOUBLE | Additional Software Available | |
| POPS | NO | YES | YES | DOUBLE | NOT YET IMPLEMENTED | NOT YET IMPLEMENTED |
| MAGNET 4100 | NO | NO | NO | UNDIFFERENCED | NO | NO |
| TRIMVEC | NO | YES | YES | DOUBLE TRIPLE | NO | NO |
| PHASER | NO | NO | NO | UNDIFFERENCED TRIPLE | NO | NO |
| MACROMETER | Additional Software Available | | | DOUBLE | Additional Software Available | |
| GEOMARK | NO | Additional Software Available | | DOUBLE | NO | NO |
| BERNESE GPS SOFTWARE | NO | Not Available | | DOUBLE | YES | YES |

# 7. NETWORK ADJUSTMENT AND TRANSFORMATION

## INTRODUCTION

The station coordinates or the baseline vectors obtained from GPS data processing are the equivalent of reduced field data in conventional surveys. Once this data has been collected and reduced, a network adjustment is carried out and the adjusted data can be transformed into the local reference datum.

Several basic concepts must be understood before the surveyor can transform adjusted GPS measurements into a local reference system.

The shape of the earth can be described by an equipotential surface known as the geoid. The geoid can be approximated by Mean Sea Level (MSL) around the globe. For convenience, satellite based survey networks are computed on a geocentric ellipsoid, which is the geometric figure that best approximates the shape of the geoid. As measurement technology and the understanding of the geoid improve, the parameters that define the best-fit ellipsoid are refined. A number of reference ellipsoids have thus been computed over the years. The WGS72 reference ellipsoid, which is presently used in GPS surveying, is one such 'best-fit' geocentric ellipsoid.

The WGS72 reference datum is defined by the WGS72 ellipsoid which has a defined orientation and location in space. The WGS72 datum is not an ideal reference datum for Australia as the geoid-spheroid separation for geocentric ellipsoids is large and variable across the continent (Figure 7.1).

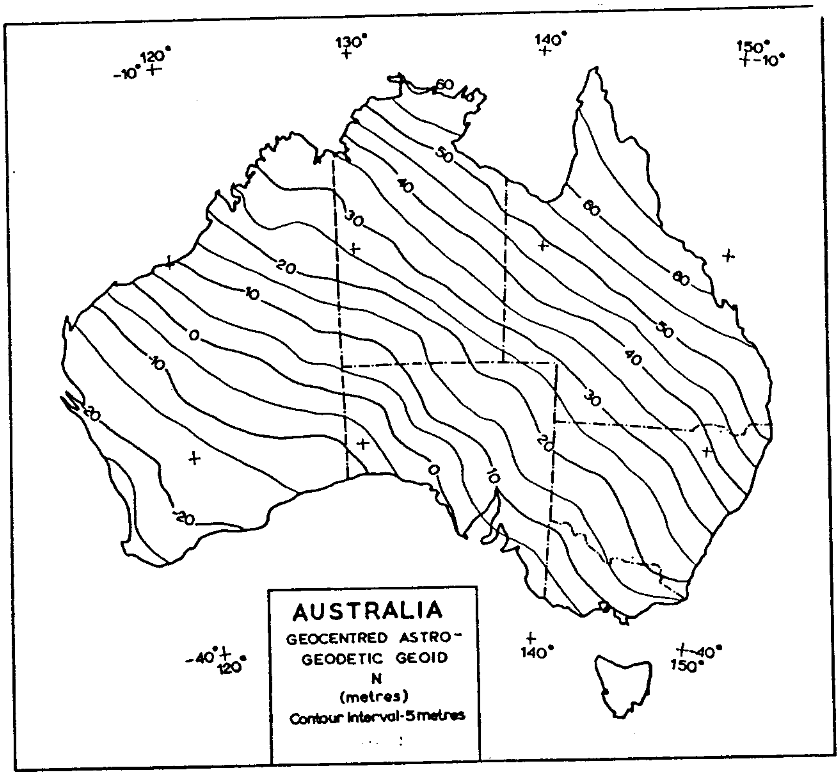



Figure 7.1 : Australian Geoid Mapped referred to a Geocentric Ellipsoid, (MATHER and FRYER, 1970)

If the WGS72 datum were accepted as the Australian datum, large corrections would need to be applied to measured distances to reduce them to the ellipsoid. For an ideal local datum, the values of geoid-spheroid separation over a region should be small and uniform, thereby, eliminating the need to reduce distances to the ellipsoid. This is the classical approach of defining geodetic datum. If the geoid does not deviate by more than six metres from the ellipsoid, distances on both the geoid and the ellipsoid can be considered equal to within one part per million. In Australia the reference datum is known as the Australian Geodetic Datum (AGD). This datum, which is not geocentric, comprises the Australian National Spheroid (ANS) orientated and located in such a manner as to 'best-fit' the geoid over the Australian continent.

Figure 7.2 shows the relationship between the geoid, the WGS72 reference system and the AGD.

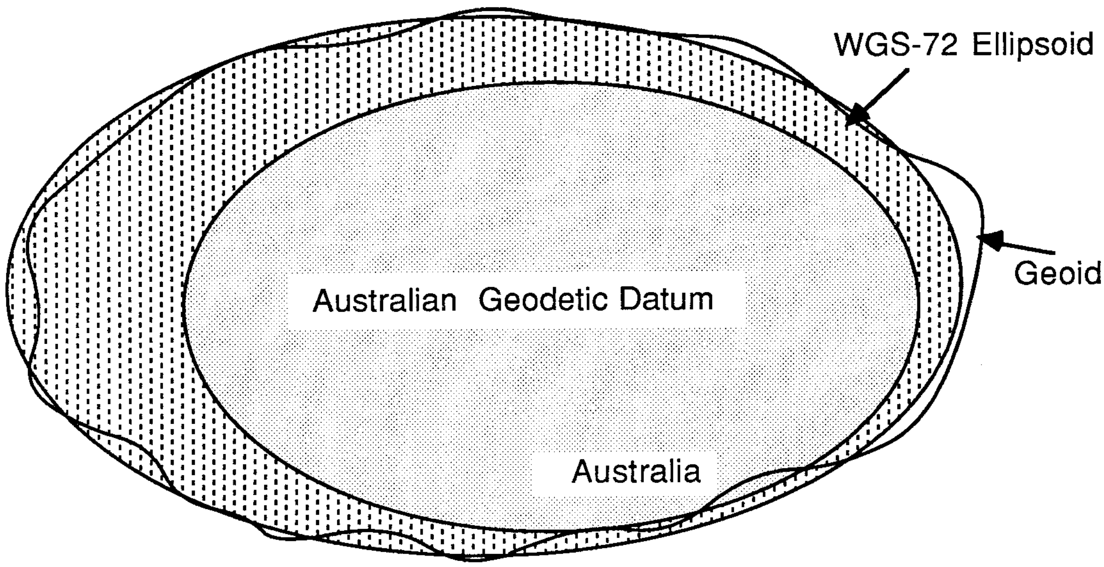



Figure 7.2 : WGS72, AGD and the Geoid

In this chapter satellite reference systems and datums are described. Methods of network adjustment for different types of GPS data are presented, and ways in which the network adjustment can be used to check for erroneous data are discussed. The procedure for transforming the station coordinates obtained from the network adjustment to a local reference system is also described. The latter is accomplished through a set of transformation parameters. If no transformation parameters are available the user must derive them as part of the observation campaign. Three commonly used transformation models are described below and methods of solving and applying the parameters are given.

When transforming the coordinates, it may be convenient to transform the GPS ellipsoidal height measured with respect to the

WGS72 datum to the local levelling datum. This requires knowledge of the Australian Height Datum (AHD) and an understanding of the various methods of determining the geoid-spheroid separation (N). This chapter, therefore, also describes height datums and gives the transformation techniques surveyors can use.

A case study of a network adjustment and transformation is presented in Chapter 8. The GPS data used is part of the Phase I control densification survey completed by the South Australian Lands Department (SALD) in late 1985 (JONES, 1986).

## SATELLITE REFERENCE SYSTEMS

A satellite reference system defines a framework in which the satellites' motion can be described. It is defined by a number of disparate models, which include the adopted coordinates of the tracking stations, and a geopotential model of the earth together with a set of constants, such as the product of the Gravitational constant and earth's mass (GM), the velocity of light (C) and the earth's mean rotation rate ($\Omega_e$). The reference system is constrained to be geocentric by excluding certain low degree and order harmonics of the geopotential.

The satellite reference <u>datum</u> is the spatial coordinate system <u>implied in the satellite reference system</u>. It is the datum to which GPS surveys are related and is defined by the satellite ephemeris in the same way that a geodetic datum is defined by the geodetic control stations in a continental network. Although a reference ellipsoid is not necessary to describe the satellite motion and hence the satellite datum, an ellipsoid is often implied.

Confusion arose over the TRANSIT Doppler system for which the precise and broadcast ephemeris referred to different satellite systems : the GM (Precise - $3.98601 m^3 s^{-1}$, Broadcast - $3.986008 m^3 s^{-1}$), geopotential models (Precise - NSWC10E-1, Broadcast - WGS72) and the tracking stations (Precise - NSWC9Z-2, Broadcast - NWL10D) each were different. The different reference systems produced differences in satellite datums of $\pm$ 2-5m in parts of the earth (JENKINS & LEROY 1979).

Further confusion arose over the adoption of WGS72 by the U.S. Defense Mapping Agency for mapping and positioning purposes. Some surveyors assumed that the TRANSIT broadcast ephemeris datum and the WGS72 were one and the same, when in fact only the geopotential model of the WGS72 system was used to determine the broadcast ephemeris. The TRANSIT broadcast ephemeris datum is actually referred to as the NWL10D datum. Although this datum has the same ellipsoidal parameters as the WGS72 ellipsoid it is rotated with respect to the WGS72 datum by .26" in longitude.

In the near future the GPS broadcast ephemeris datum will be changed from the WGS72 datum to the WGS84 datum which is a new reference system. When this happens, GPS coordinates of previously established stations, derived from point positioning will appear to change. The change of reference system, and hence datum, will mean that new transformation parameters relating the WGS84 reference system and the AGD need to be derived.

## NETWORK ADJUSTMENT

The type of output obtained from GPS data processing varies with the processing package used. For single baselines the output will consist of the baseline vector components and their variance covariance matrix (VCV). With "multi-station" packages, a set of station coordinates and associated VCV matrix will be obtained. These outputs are basically equivalent, for if one is known the other can be easily computed.

There are a number of ways in which least squares techniques may be used to combine data in a network. This section describes how baselines can be combined in a network, and how multi-station coordinate sets can be combined. An example of a network adjustment of baselines is given in Chapter 8.

### NETWORK ADJUSTMENT OF MULTI-STATION COORDINATE SETS

When two or more sets of GPS multi-station coordinate sets are determined separately they can be combined in a network using sequential least squares techniques. These coordinate sets may be obtained from software packages designed for multi-station processing (eg. packages such as PoPs, MAGNET, PHASER described in Chapter 6), or may have been obtained from measurements taken at different epochs.

Sequential least squares is a simple method of adjusting this type of data. In the technique described here, it is assumed that there is no correlation between each set of observations. The method is based on theory presented by MIKHAIL (1976).

We consider two sets of equations :

$$A_{i-1}v_{i-1} + B_{i-1}\Delta = f_{i-1} \qquad (7.1)$$

describes a one multi-station coordinate set with unknown coordinate parameters ($\Delta$), and

$$A_i v_i + B_i \Delta + b_i \delta = f_i \qquad (7.2)$$

represents additional observations pertaining to both the unknown parameters in (7.1) $\Delta$, and to some previously unobserved station coordinates $\delta$

where $i$ = current epoch

$i-1$ = previously adjusted result

$A$ = coefficient matrix for the observations

$B$ = coefficient matrix for the parameters

$v$ = residuals

$\Delta$ = unknown parameters (station coordinates)

$b$ = coefficient matrix for new parameters

$\delta$ = vector of new parameters

Define

$l$ = observations

$Q$ = variance-covariance matrix of the observations

$W$ = $Q^{-1}$ = weight of the observations

$f$ = $-l$ = constant vector

In this case, both the observations ($l$) and the parameters ($\Delta$, $\delta$) are the station coordinates.

Equations (7.1) and (7.2) can be combined to give

$$\begin{bmatrix} A_{i-1} & 0 \\ 0 & A_i \end{bmatrix} \begin{bmatrix} v_{i-1} \\ v_i \end{bmatrix} + \begin{bmatrix} B_{i-1} & 0 \\ B_i & b \end{bmatrix} \begin{bmatrix} \Delta \\ \delta \end{bmatrix} = \begin{bmatrix} f_{i-1} \\ f_i \end{bmatrix}$$

In the case where the observations are the station coordinates, A is an identity matrix, and can be omitted. The reduced normal equations become.

$$N_i = \begin{bmatrix} (B^TWB)_{i-1} + (B^TWB)_i & (B^TWb)_i \\ (b^TWB)_i & (b^TWb)_i \end{bmatrix} \tag{7.3}$$

$$= \begin{bmatrix} N_{i-1} + \delta N_i & \bar{n}_i \\ \bar{n}_i^T & n_i \end{bmatrix} \tag{7.4}$$

$$t_i = \begin{bmatrix} (B^TWf)_{i-1} + (B^TWf)_i \\ (b^TWf)_i \end{bmatrix}$$

$$= \begin{bmatrix} t_{i-1} + \delta t_i \\ \bar{t}_i \end{bmatrix} = \begin{bmatrix} t_i' \\ \bar{t}_i \end{bmatrix} \tag{7.5}$$

Where $N_{i-1}$ and $t_{i-1}$ are saved from the previous adjustment. The inverse of $N_i$ may be evaluated from partitioning as

$$N_i^{-1} = \begin{bmatrix} \bar{N}_i & \bar{n}_i \\ \bar{n}_i^T & n_i \end{bmatrix}^{-1} = \begin{bmatrix} F_i & G_i \\ G_i^T & H_i \end{bmatrix} \tag{7.6}$$

where $F_i = (\bar{N}_i - \bar{n}_i n_i^{-1} \bar{n}_i^T)^{-1}$

$G_i = -F_i \bar{n}_i n_i^{-1}$

$H_i = n_i^{-1} - n_i^{-1} \bar{n}_i^T G_i$

If $G_i$ and $F_i$ are replaced by their expressions, $H_i$ would become

$$H_i = (n_i - \bar{n}_i^T \bar{N}_i^{-1} \bar{n}_i)^{-1} \tag{7.6a}$$

Using the matrix relationship A69 given by MIKHAIL (1976), $F_i$ can be shown to be equivalent to

$$F_i = \bar{N}_i^{-1} + \bar{N}_i^{-1}\bar{n}_i(n_i - \bar{n}_i^T\bar{N}_i^{-1}\bar{n}_i)^{-1}\bar{n}_i^T\bar{N}_i^{-1}$$

Which in view of eqn 7.6a becomes

$$F_i = \bar{N}_i^{-1} + \bar{N}_i^{-1}\bar{n}_i H_i \bar{n}_i^T \bar{N}_i^{-1}$$

And since from eqn 7.4 and 7.6

$$\bar{N}_i = N_{i-1} + \delta N_i = N_{i-1} + B_i^T W_{ei} B_i \qquad (7.6b)$$

The application of eqn A69 MIKHAIL (1976) to eqn 7.6b leads to

$$\bar{N}_i^{-1} = N_{i-1}^{-1} - N_{i-1}^{-1}B_i^T(Q_i + B_i N_{i-1}^{-1}B_i^T)^{-1}B_i N_{i-1}^{-1} \qquad (7.7)$$

$$\bar{n}_i = B_i^T W_i b_i \qquad (7.8)$$

$$n_i = b_i^T W_i b_i \qquad (7.9)$$

and the t vector (see eqn 7.5) is given by

$$t_i' = t_{i-1} + B_i^T W_i f_i \qquad (7.10)$$

$$t = b_i^T W_i f_i \qquad (7.11)$$

The new values of the parameters (station coordinates) are

$$\begin{bmatrix} \Delta \\ \delta \end{bmatrix} = N_i^{-1} t_i \qquad (7.12)$$

The residuals of the adjusted observations are

$$\begin{bmatrix} v_{i-1} \\ v_i \end{bmatrix} = \begin{bmatrix} f_{i-1} \\ f_i \end{bmatrix} - \begin{bmatrix} B_{i-1} & 0 \\ B_i & b \end{bmatrix} \begin{bmatrix} \Delta \\ \delta \end{bmatrix} \qquad (7.13)$$

The adjusted observations are

$$\hat{l} = l + v \qquad (7.14)$$

The aposteriori variance of unit weight is

$$\hat{\sigma}_0^2 = v^T W v \; / \; r \tag{7.15}$$

where $r$ = redundancy

The variance covariance matrices of the parameters, the adjusted observations and the residuals are respectively

$$Q_{\Delta\Delta} = N^{-1} \tag{7.16}$$

$$Q_{\hat{l}\hat{l}} = Q - Q_{vv} = BN^{-1}B^T \tag{7.17}$$

$$Q_{vv} = Q - BN^{-1}B^T \tag{7.18}$$

These equations are used to incorporate any new processed coordinate data into an existing network. New observations made on existing stations are used to update the normal matrix, and this results in improved estimates of those stations coordinates. Observations to new stations will increase the dimensions of the normal matrix and t vector.

NETWORK ADJUSTMENT OF BASELINE VECTORS

Some GPS processing packages, which use double or triple difference algorithms (eg. MACROMETRICS, TRIMVEC), process each baseline separately.

The number of independent baselines that can be used in the network adjustment is one less than the number of receivers used for each observation session. For example, if three receivers observe simultaneously, only two independent baselines result. The third baseline is dependent on the other two as it uses the same data that has been previously processed in the other two lines.

The correlations that exist between baselines, when three or more receivers are used simultaneously, are not calculated with single baseline processing techniques. The VCV matrix of the observations is, therefore, block diagonal with each block element consisting of the VCV matrix of each measured baseline.

The correlations between baselines should be included in a rigorous adjustment. The results that have been achieved without taking correlations into account, however, are of survey accuracy (1:500,000). Geo/Hydro Inc. has used this method to adjust a number of large GPS surveys, including the Eifel network in Germany (BOCK, 1984) and the control densification survey of South Australia (LARDEN, 1986).

Least squares adjustment of condition equations is ideally suited to combining baselines, as each condition equation contains only one observation. The method is also easy to program. In such a scheme the general linear function equation is

$$l + v + B\Delta = d \tag{7.19}$$

or

$$v + B\Delta = (-l+d) = f \tag{7.20}$$

where

$l$ = observations

$v$ = residuals

$B$ = coefficient matrix for the parameters

$\Delta$ = unknown parameters (station coordinates)

$f, d$ = constant vectors

The weight matrix for the observations, l, are block diagonal. Once the observation equations are formed the unknown parameters are obtained from

$$(B^T W B)\ \Delta = B^T W f \tag{7.21}$$

$$\Delta = (B^T W B)^{-1}\ B^T W f \tag{7.22}$$

$$= N^{-1} t$$

The residuals, the adjusted observations and aposteriori variance of the unit weight are calculated in the same way as for the sequential least squares techniques (see equations 7.13-7.18).

The abovementioned technique is better suited for combining baselines in a network, while the sequential adjustment technique described earlier is more suited for combining already adjusted networks.

STATISTICAL TESTS

Statistical tests are used to assess the quality of the measurements, and the adjusted parameters. MIKHAIL (1976) states:

> "Statistical tests are used to compare the results with previous ones or with given standards. In testing, one seeks a judgement as to whether some estimator function is consistent with the assumption (hypothesis) that the sample was drawn from a population with specified parameter values, such as a normal distribution with a given standard deviation."

A hypothesis is a statement about the probability distribution of a random variable. The null hypothesis ($H_0$) is a statement which compares the probability distribution of the estimated parameters with the probability distribution of a population. The null hypothesis is accepted if the probability distribution of both the new parameters and the population are in agreement.

The tests involve a decision between the null hypothesis and an alternative hypothesis ($H_1$). A multitude of testing can be done, but the most often used is the Chi-squared test.

## The Chi-squared Test

The Chi-squared test operates on the aposteriori estimate of variance factor $\hat{\sigma}_0^2$. If $\hat{\sigma}_0^2$ is the variance of a random sample from a normal population with mean $\mu$ and variance $\sigma_0^2$ then $\chi_r^2$

represents a Chi-squared distribution with r degrees of freedom. This property is used to test if the estimated parameters belong to the expected population.

One possible hypothesis that can be tested is

$$H_0 \ : \ \hat{\sigma}_0^2 = \sigma_0^2 \qquad H_1 \ : \ \hat{\sigma}_0^2 > \sigma_0^2$$

$H_0$ is rejected when

$$\chi_0^2 > \chi_{\alpha,r}^2$$

where $$\chi_0^2 = \frac{v^T W v}{\sigma_0^2} = r\,\hat{\sigma}_0^2 = v^T W v \qquad (7.24)$$

when $\sigma_0^2 = 1$

$\chi_{\alpha,r}^2$ = Chi-squared value at $\alpha$ significance level, and r degrees of freedom

If the value of $\chi_0^2$ obtained from the network adjustment exceeds the expected $\chi_{\alpha,r}^2$ value, then the sample is not consistent with a normal distribution. This may be due to a number of factors which include :

a) blunders made in the field (eg. wrong height of antenna)

b) systematic errors affecting all observations, that were not accounted for at the processing stage (eg. atmospheric propagation errors).

c) estimates of the standard deviations of the observations that are too optimistic.

If the value of $\chi_0^2$ obtained is significantly less than the expected $\chi_{\alpha,r}^2$ value, then the standard deviations of the observations are probably pessimistic.

## Outlier Detection

One of the reasons why data will not pass a Chi-squared test is that a blunder was made in some of the data included in the network. An additional statistical test can be applied to detect if outliers are present. This test uses Baarda's data snooping technique (CASPARY, 1985, BAARDA, 1968), whereby the normalised standard deviations of the residuals of the adjusted observations are calculated from

$$\mu_i = \frac{v_i}{\sigma_0 \sqrt{Q_{vv}}} \tag{7.25}$$

where
$\mu_i$ = normalised standard deviation of the residual
$v_i$ = residual
$\sigma_0$ = apriori estimate of the variance = 1
$Q_{vv}$ = trace element from the variance covariance matrix of the residuals

As the $\mu_i$ are normally distributed, outliers are identified by comparing the $\mu_i$ values with the $\mu$ values of the normal distribution density function. If $\mu_i$ is within two or three standard deviations, the observation will lie within a value that is expected for 95% and 99.7% of the population respectively. High $\mu_i$ values indicate the presence of outliers. These values should be examined for blunders and then either reprocessed or disregarded.

## TRANSFORMATION OF COORDINATES

After a GPS network has been adjusted, the next step in the processing procedure is to transform the adjusted coordinates to a local reference datum. This requires a set of transformation parameters, that may either be determined by the surveyor from the common station information or obtained from another source.

This section reviews the transformation procedure, and describes three transformation models that can be used. Presently there are no transformation parameters readily available to directly relate the WGS72 reference datum (used by GPS) to the AGD. An alternative method is therefore given. The determination of transformation parameters as part of the GPS survey, and their application will be demonstrated in the final chapter.

### TRANSFORMATION

The most frequently used transformation in surveying is the similarity transformation (HARVEY, 1986). A similarity transformation preserves the shape of the network (ie. angles stay the same). However, the lengths of baselines and the positions of the stations may vary due to the uniform scale change allowable in the model. One form of a similarity transformation is the Molodensky-Badekas model (Figure 7.4) which is given by

$$\vec{N} = \vec{U}_0 + (1+s)\ R\ (\ \vec{O} - \vec{U}_0\ ) + \vec{T} \qquad (7.26)$$

where $s$ = scale difference

$R$ = rotation matrix

$\vec{T}$ = translation terms between datum origins

$\vec{N}$ = coordinates in the new datum (Net B)

$\vec{O}$ = coordinates in the old datum (Net A)

$\vec{U}_0$ = coordinates of the centroid of the network in the new datum

The scale factor (1 + s) is the ratio of Net B baseline lengths to the Net A baseline lengths.

The matrix, R, describes the rotations of the points about the Net B coordinate axes. These rotations are

$$Rz(k) = \begin{bmatrix} \cos k & \sin k & 0 \\ -\sin k & \cos k & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (7.27)$$

$$Ry(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \qquad (7.28)$$

$$Rx(w) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos w & \sin w \\ 0 & -\sin w & \cos w \end{bmatrix} \qquad (7.29)$$

where k, θ, w are the rotation angles about the X, Y and Z axes, respectively

As these matrices are not commutative they must be applied in the correct order. A common order of multiplication is Rz(k) Ry(θ) Rx(w). If the rotation angles k, θ and w are less than 3", this matrix can be approximated by

$$\begin{bmatrix} 1 & k & -\theta \\ -k & 1 & w \\ \theta & -w & 1 \end{bmatrix} \qquad (7.30)$$

k, θ and w being in radians

For larger rotation angles the full rotation matrices should be used. The translation terms are the coordinates of the origin of Net A in the Net B reference datum.

TRANSFORMATION MODELS

Two frequently used transformation models are those of Bursa-Wolf and Molodensky-Badekas. The Bursa-Wolf model (Figure 7.3) is a special case of the seven parameter similarity transformation model described in equation (7.26).

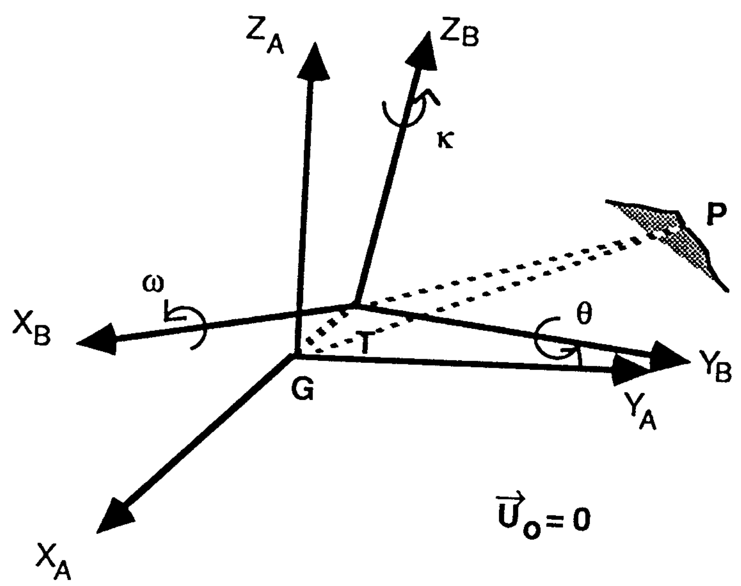



Figure 7.3 : Bursa-Wolf Model

In the Bursa-Wolf model the coordinates of the centroid of the network in the new datum, $\vec{U}_0$, are zero. The model is of the form

$$\begin{bmatrix} X_B \\ Y_B \\ Z_B \end{bmatrix} = (1+s)\ R \begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix} + \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix} \tag{7.31}$$

where $X_A$, $Y_A$, $Z_A$ = Station coordinates in the old datum

$X_B$, $Y_B$, $Z_B$ = Station coordinates in the new datum

$T_X$, $T_Y$, $T_Z$ = Translation components between datum origins

The transformation parameters are highly correlated in the Bursa-Wolf model, when the data covers a small area of the earth's surface. The rotations about the datum axes, therefore, have much the same affect on the network coordinates as the translations applied. For example, an easterly rotation about the Z axis will shift the network in the same way as a combined translation along the Y axis and a translation along the X axis.

The Molodensky-Badekas model (Figure 7.4) is a more general transformation model, where the rotations are related to the centroid of the network in the new datum. This model removes the high correlations that exist between the rotations and translations.

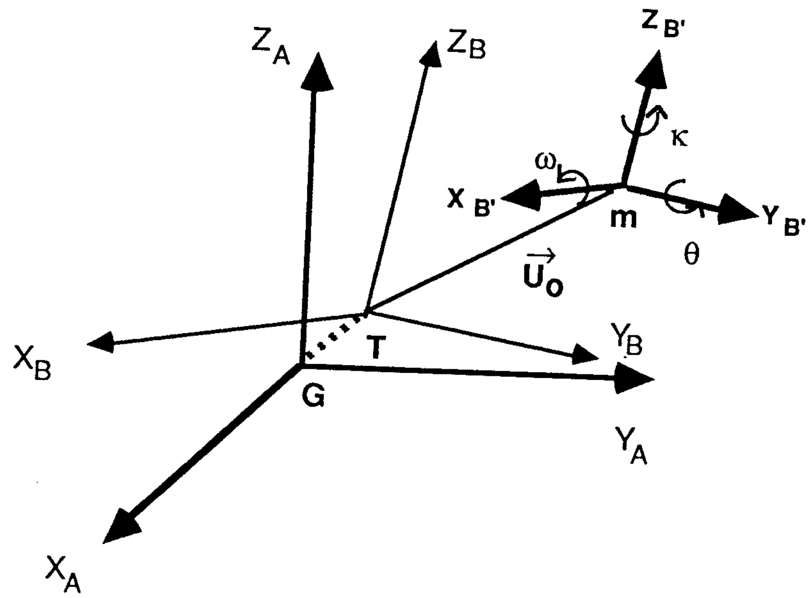



Figure 7.4 : Molodensky-Badekas Transformation Model

The Molodensky-Badekas model is of the form

$$\begin{bmatrix} X_B \\ Y_B \\ Z_B \end{bmatrix} = \begin{bmatrix} X_m \\ Y_m \\ Z_m \end{bmatrix} + \begin{bmatrix} T_{X'} \\ T_{Y'} \\ T_{Z'} \end{bmatrix} + (1+s)\ R \begin{bmatrix} X_A - X_m \\ Y_A - Y_m \\ Z_A - Z_m \end{bmatrix} \qquad (7.32)$$

where $X_m$, $Y_m$, $Z_m$ = Coordinates of the centroid of the network

$T_{X'}$, $T_{Y'}$, $T_{Z'}$ = Molodensky Badekas translation terms

Although this model gives the same values for the scale and rotation parameters as the Bursa-Wolf model, the translation parameters are different and have smaller aposteriori precisions.

The topocentric model shown in Figure 7.5 may also be used for transformations. In Figure 7.5 the 'Zenith' represents the direction of the ellipsoidal normal through the network. The Zenith could pass through a network station or through the centroid. The north direction is orthogonal to to the ellipsoidal normal and lies in the meridional plane, and east is orthogonal to both the ellipsoidal normal and the meridional plane. The angles $\alpha$, $\xi$, $\eta$ represent rotations about zenith, east and north directions respectively.

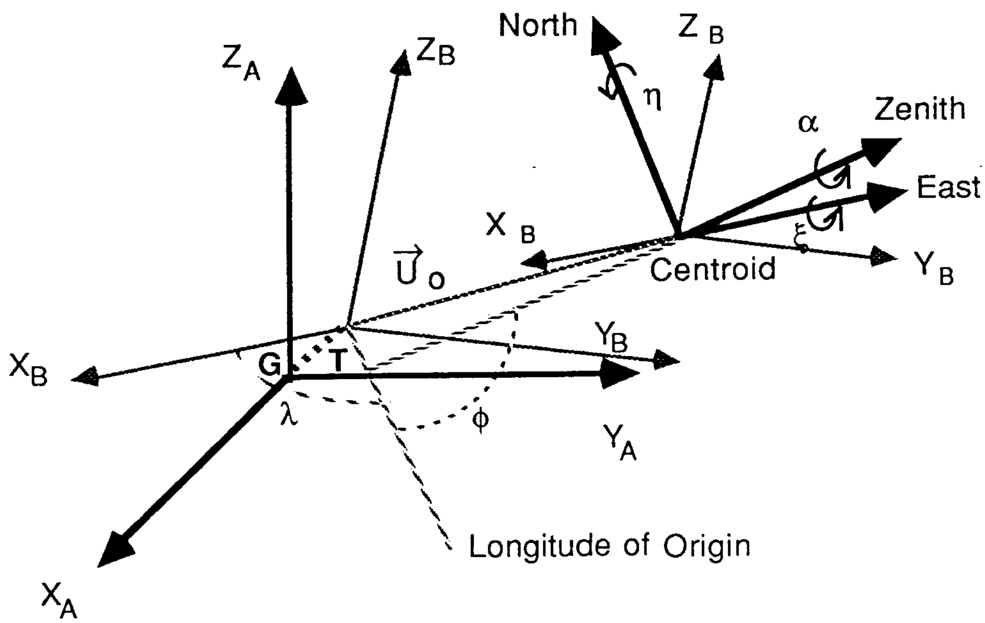



Figure 7.5 : Topocentric Model

The main advantage of the topocentric model is that the seven parameters have some physical meaning in the local datum. The angle $\alpha$ is the azimuth swing between the two datums. The angles $\xi$ and $\eta$ are the mean deflections of the vertical in the area of the survey.

THE AUSTRALIAN SCENE

Presently no transformation parameters are available that directly relate the WGS72 reference datum to the AGD. This leaves surveyors two options : they can determine their own parameters for each survey, or they can use the known transformation parameters relating the TRANSIT precise ephemeris to the AGD and the mathematical relationship between this datum and WGS72.

**TRANSIT Transformation Parameters**

The transformation parameters between the precise TRANSIT ephemeris datum (NSWC-9Z2), and the AGD were published by ALLMAN and VEENSTRA (1984). The parameters were obtained from 156 common stations evenly distributed throughout Australia.

SEPPELIN (1972) derived equations relating the WGS72 datum to the precise TRANSIT ephemeris datum. Seppelin's formulae allow for the change in ellipsoidal parameters, a scale change and a rotation of 0.26" in longitude between the two datums.

These relationships can be used to transform GPS station coordinates (WGS72 datum) to AGD coordinates. Seppelin's formulas convert WGS72 coordinates in the NSWC-9Z2 system. TRANSIT transformation parameters can be used to transform these to coordinates on the AGD.

The absolute accuracy of the final coordinates is limited by both the absolute accuracy of the WGS72 coordinates of the origin station (±10m if point positioning is used with the CA code), and the accuracy of the transformation parameters. ALLMAN & VEENSTRA (1984) show that transformed multi-station Doppler positions agree to within a couple of metres with AGD84, which is the level of accuracy that the parameters allow. However, the relative positions of transformed network points should be much more accurate.

## Deriving Transformation Parameters

If transformation parameters are not available, or if GPS is used for control densification purposes, the transformation parameters can be determined by the surveyor using least squares techniques. At least three common stations with coordinates on each datum are required to solve for seven transformation parameters. Three common stations will provide nine observations for each datum (three coordinates per station) resulting in a redundancy of two in the least squares solution. The respective VCV matrices in both reference datums are also required.

As there are two measurements of each baseline (one in each reference datum), the least squares adjustment produces adjusted coordinates of the common stations in both datums as well as the transformation parameters relating the datums. The adjusted common station coordinates in each reference system have the same weighted scale, orientation and location as their apriori coordinates. As HARVEY (1986) states :

'If a transformation adjustment is computed between the apriori and adjusted versions of a net, it will be found that the estimated rotation angles, scale difference and scale will all equal zero.'

The solution procedure is the same for the Bursa-Wolf, the Molodensky-Badekas or the topocentric models. However the terms of the first design matrix A, and the misclosure vector W are different (see Appendix B, HARVEY, 1985).

The input data required to determine transformation parameters includes the latitude, longitude and ellipsoidal height for each common station (see Appendix B). Ellipsoidal height on the WGS72 datum can be determined with good precision. However, height is the least well known coordinate on the local datum (see Heighting with GPS). Consequently, the local ellipsoidal height used to determine transformation parameters has a larger standard deviation than latitude and longitude. This in turn adversely affects the precision of all the parameters.

For the topocentric model, the uncertainty in the height data is reflected predominantly in the rotations about the north and east directions and in the scale. When the rotations are found to be statistically insignificant (see next section), they can be held fixed at zero to create the so-called five parameter topocentric transformation model. This model provides a more numerically stable solution of transformation parameters, although it produces unrealistically small standard deviations of the transformed coordinates.

When the five parameter transformation was applied to the

South Australian control densification network (see Chapter 8), the latitude and longitude values of the transformed network stations were found to be the same as from the seven parameter solution, but the heights were less accurate. The ellipsoidal heights, however, provided sufficient information to compute the orthometric heights for the stations. This height reduction process is described in the next section of this chapter.

When transformation is required to a lower accuracy, a three parameter transformation (using only the translation terms) may be sufficient. These translations can be readily determined from the difference of the average X, Y and Z coordinates of the common stations in each reference datum.

The least squares adjustment procedure given in Appendix B requires Cartesian coordinates to be entered. If only the ellipsoidal coordinates and corresponding VCV matrix are available, these should first be converted to a Cartesian coordinates using the formula given in Appendix C and in HARVEY (1985).

## STATISTICAL TESTS

Once the transformation parameters have been computed, statistical tests should be applied to determine whether they are significantly different from their apriori values.

One method of testing for significance is described in VANICEK and KRAKIWSKY (1982), based on the following formula

$$t = (U-X)^{T} Cx^{-1} (U-X) \tag{7.33}$$

where $t$ = test statistic

X = parameters to be tested

U = apriori transformation parameters with which X is compared (usually null vector)

Cx = estimated VCV matrix of the parameters being tested

If a subset of parameters are tested, the corresponding portion of the VCV matrix must be used in the test.

The hypothesis that $X_i \risingdotseq U_i$ should be rejected if

$$t < \chi^2_{k,\alpha}$$

where k = number of parameters being tested

$\alpha$ = significance level (which should be 5% or lower)

If the parameters differ from their apriori values, reasons for accepting them should be found. For example a significant scale change between datums may be due to a large geoid-spheroid separation that is not taken into account in the local datum.

Statistically insignificant parameters may be held fixed in the solution of the transformation parameters, especially if there are only a few observation points. By holding the stations fixed, the solution may become more numerically stable. This may be necessary for small computers. The final transformed coordinates will appear more precise when the parameters are held fixed. These improved precisions, however, are unrealistically high as they do not include the error contribution of the fixed parameters.

All seven transformation parameters should be estimated to obtain the most accurate results.

APPLICATION

The different transformation models are just different ways of accomplishing the same task. All models should produce exactly the same result if the appropriate transformation parameters and their full VCV matrix are used to transform a set of coordinates and their full VCV matrix. However, if no information is available on the covariances of the transformation parameters it is best to use the Molodensky-Badekas model. Although the resulting VCV matrix of the transformed coordinates will be wrong in both models, the Molodensky-Badekas model will give better VCV values as the transformation parameters are less correlated.

The VCV matrix of the transformed coordinates can be calculated from the law of propagation of variances (MIKHAIL, 1976). The Bursa-Wolf transformation model is of the form

$$\begin{bmatrix} X_B \\ Y_B \\ Z_B \end{bmatrix} = (1+s) \begin{bmatrix} 1 & k & -\theta \\ -k & 1 & w \\ \theta & -w & 1 \end{bmatrix} \begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix} + \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix} \quad (7.34)$$

Application of the law of propagation of variances gives

$$VCV_{XYZB} = J \begin{bmatrix} VCV_{XYZA} & 0 \\ 0 & VCV_P \end{bmatrix} J^T \quad (7.35)$$

where $VCV_{XYZB}$ = VCV matrix of the Net B coordinates

$VCV_{XYZA}$ = VCV matrix of the Net A coordinates

$VCV_P$ = VCV matrix of the transformation parameters

$J$ = Jacobian matrix

The Jacobian matrix is calculated from the partial derivatives of the transformation model (eqns 7.31, 7.32) with respect to $X_A$, $Y_A$, $Z_A$, s, w, θ, k, $T_X$, $T_Y$, $T_Z$. For the Bursa-Wolf model the Jacobian matrix is (HARVEY, 1986)

$$J = \begin{bmatrix} s & sk & -s\theta & X_A+kY_A-\theta Z_A & 0 & -sZ_A & sY_A & 1 & 0 & 0 \\ -sk & s & sw & -kX_A+Y_A+wZ_A & sZ_A & 0 & -sX_A & 0 & 1 & 0 \\ s\theta & -sw & s & \theta X_A-wY_A+Z_A & -sY_A & sX_A & 0 & 0 & 0 & 1 \end{bmatrix}$$

The Jacobian matrix for the Molodensky-Badekas model is obtained by replacing $X_A$, $Y_A$, $Z_A$ in the above with $(X_A-X_m)$, $(Y_A-Y_m)$, $(Z_A-Z_m)$.

**Options**

The method used to transform coordinates depends on the purpose of the survey. If the survey is carried out to upgrade local coordinates or to establish a set of regional transformation parameters, the full VCV matrix of the coordinates of the common stations in both the GPS and the local datum should be used when solving for the transformation parameters. The least constrained transformation parameters relating the datums will be obtained. However, the coordinates of the common stations will change in both datums.

This method could be used by mapping and surveying agencies around Australia to determine the relationship between WGS72 (the current GPS satellite datum) and the AGD84. The transformation parameters should be available to surveyors, once they have been determined.

The full VCV matrices of the common points should be employed when estimating the transformation parameters. If the correlations between stations are not taken into account, erroneous parameters and VCV matrix will result.

A different approach can be used, however, if the surveyor wishes to densify established control in a small region. In this case, national transformation parameters may not be the most appropriate as they are smoothed values and, therefore, may not account for regional distortions. Furthermore, the local geodetic datum may not be AGD84. In this case, the surveyor needs to determine transformation parameters for his own use. He has several options :-

(1) Local station coordinates can be held virtually fixed by assigning small variances (eg a few mm) to their coordinates in the local datum when determining the transformation parameters. When these parameters are subsequently applied, the GPS network is distorted to fit into established local control.

(2) To maintain the relative accuracy of the GPS survey, say for engineering works, the GPS coordinates can be assigned small variances in the solution. However, the adjusted coordinates of the common stations in the local datum may change significantly with this method.

(3) The transformation parameters can be derived by using the full VCV matrix of each network to calculate the least constrained set of parameters. These parameters are then applied to transform the local coordinates back into the GPS datum. The transformed common stations are now held fixed in a new network adjustment of the GPS data. The same set of transformation parameters can be applied to

transform the adjusted network back into the local datum. This method has the advantage of using the least constrained set of transformation parameters, without changing the local coordinates of the common stations.

Established transformation parameters can be used to transform GPS coordinates to the local system if the network contains only one common station. Although the inherent accuracy of the GPS positions is maintained, the transformed coordinates will only fit into the local system, if the parameters apply to the region and there are no distortions in the local reference system. Discrepancies between established coordinates and transformed coordinates may arise should national parameters, or parameters from another region be used.

The task of transforming coordinates is simple if one is not too concerned with fitting into established control. In this case national transformation parameters can be used to transform GPS coordinates into the local datum after a GPS network has been adjusted. If the network was initially adjusted holding one station fixed, and the coordinates of that fixed station were in error or the transformation process introduces an error, then the whole GPS network and the transformed coordinates would be in error by this amount. However, the method is convenient in areas of little or no control, or for surveys for which only good relative accuracies are required.

## DISCUSSION

Below we mention a number of matters that the surveyor should consider before deriving and applying transformation parameters.

1. The transformed coordinates and VCV matrix may be unreliable if the stations being transformed lie outside the region spanned by the common stations.

2. Transformation parameters represent mean values of scale rotation and translation, and will tend to smooth out distortions and systematic errors present in either network. If network distortions are known to exist, it is better to solve for a number of sets of local parameters which better represent those areas.

3. A scale difference between the two nets may be due to systematic errors in the ground heights as well as errors in either GPS or ground distance scale.

4. Uniform geographic distribution of common points is desirable to avoid the solution being biased toward the areas of high common point density.

5. Errors in the station coordinate or distortions of either network produce erroneous transformation parameters. One way of detecting these errors is to use "check" points. These points are common points left out of the solution of the transformation parameters. The estimated parameters are used to transform the coordinates of the "check" points from one system, so they can be compared with the known coordinate values in the other system.

## HEIGHTING WITH GPS

The transformation of the coordinates of the adjusted GPS network to the local datum produces ellipsoidal heights on that datum. Generally surveyors do not work with ellipsoidal heights, but with orthometric height - which is height above the geoid. The geoid-spheroid separation (N) is required to convert these heights to orthometric heights. (see Figure 7.6). The geoid-spheroid separation is computed from

$$N = h - H \qquad (7.36)$$

where $h$ = ellipsoidal height

$H$ = orthometric height

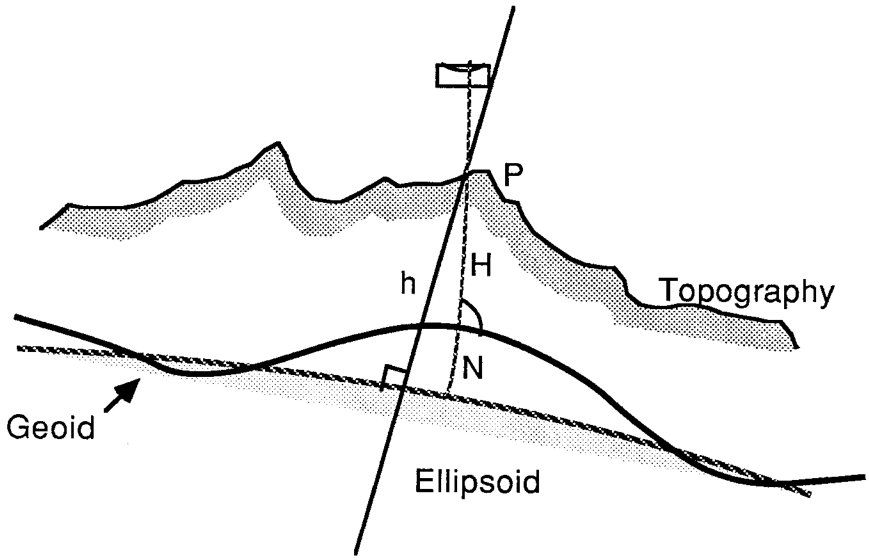



Figure 7.6 : Relation between Geoid-Spheroid Separation (N), Ellipsoidal Height (h), Orthometric Height (H)

## DETERMINATION OF N

The accuracy of the orthometric height of a station depends on the accuracies of the ellipsoidal height of the station and the geoid-spheroid separation. N can change significantly over relatively short baselines. KING et al (1985) state

> "... in geologically disturbed areas it is not uncommon to have geoid slopes of the order of 10 metres in 100km"

N can be determined a number of ways : astrogeodetic levelling, geopotential models, gravimetry and geometric techniques. The change in geoid-spheroid separation, $\Delta N$, over a baseline can be determined quite accurately using these techniques.

**Astrogeodetic Methods**

In astrogeodetic levelling astronomical observations of latitude and longitude are used to determine the deflection of the vertical at a number of stations throughout the network. Figure 7.7 shows that the deflection of the vertical at a point is equivalent to the slope of the geoid.

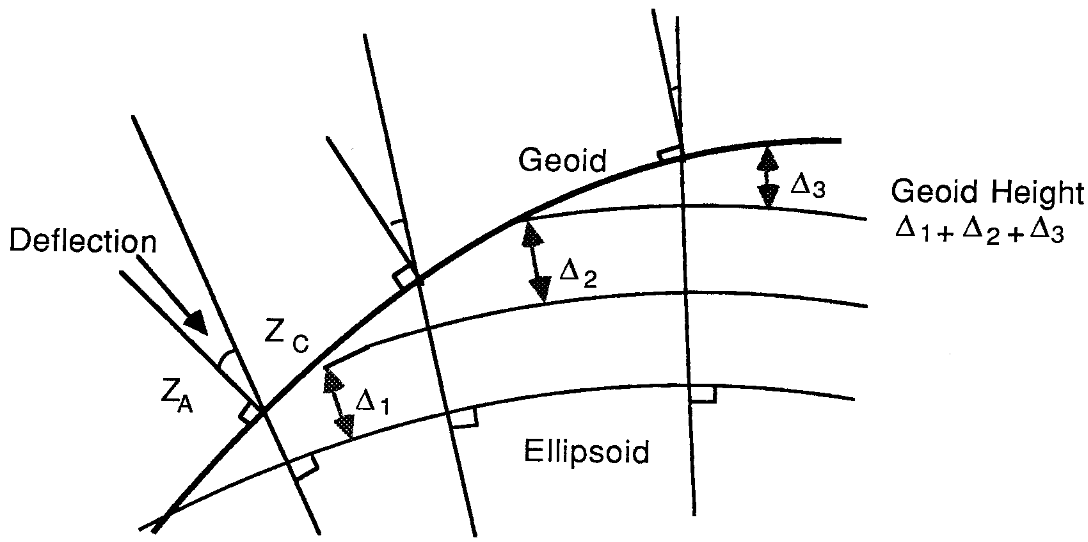



Figure 7.7 : Geoid from Astrogeodetic Methods

The astrogeodetic method is only capable of determining the deflection of the vertical with an accuracy of a few arcseconds. The method, moreover, is costly and labour intensive (BOMFORD, 1971).

**Geopotential Models**

Geopotential models of the geoid are computed from satellite tracking data and surface gravity data.

The orthometric height of any point with known ellipsoidal coordinates is computed from a truncated spherical harmonic series (see KING et al, 1985). The accuracy and resolution of the orthometric height depends on the degree and order of the coefficients used in the model.

The satellite tracking data provides the low order harmonics of the geopotential model. The higher order harmonics are obtained from surface gravity data and altimeter derived marine geoid heights. Geopotential models are, however, oversmoothed interpretations of geoid undulations that do not account for local geoid anomalies. The high order geopotential models have half wavelengths of 100km and can be used to determine relative values of N to the ±2m level. Although this level of accuracy may be adequate for reconnaissance and low order surveys, it is not high enough for most surveying purposes.

Geopotential models have many advantages over other heighting techniques for low order GPS surveys. The models can be used around the world to the same level of accuracy. Moreover, the model consists merely of a set of coefficients which are easy the model consists merely of a set of coefficients which are easy to store in a computer. These models may even be stored in the GPS receiver to correct the ellipsoidal height obtained.

### Gravimetry

Gravimetric methods of determining N utilise observed gravity over the entire earth. This method places stringent requirements on the gravity coverage in and around the survey area, is computationally extensive and requires advanced geodetic knowledge. Gravimetric methods, however, are suitable for most surveying purposes as they can be used to determine the relative change in N to the decimetre level or better (KEARSLEY, 1984).

The contribution of the gravity data to the height determination decreases rapidly with distance from the survey site. Beyond a certain spherical cap, with radius $\psi_0$, geopotential models may be used to determine gravity contributions to the geoid height. This method is currently being investigated by KEARSLEY (1984) and shows promise for the future of levelling with GPS techniques.

The gravity coverage for Australia is already quite dense (KEARSLEY, 1986). Gravimetric techniques combined with geopotential models may, hence, be used for determining N throughout the continent. These could be stored in gridded form on a computer where they could be accessed by surveyors.

To date, gravimetry has not been widely used by surveyors to determine N as it requires gravity data and processing software that is not readily available. However, as the GPS system develops it is expected that methods and procedures for using gravimetry for determining N will become more widespread.

## Geometric Methods

The geometric method is presently the most practical way of determining the geoid. This method has been used in several overseas GPS surveys (COLLINS & LEICK, 1985) to achieve decimetre accuracy. The method requires that stations of known orthometric height be included in the GPS network. Once the GPS ellipsoidal heights of these stations have been determined, the orthometric heights of the known stations are subtracted. The differences between the two height systems forms the basis of a contour map of the geoid in that area. This map is then used to determine the value of N throughout the survey area. Figure 7.8 is an example of such a geoid map.

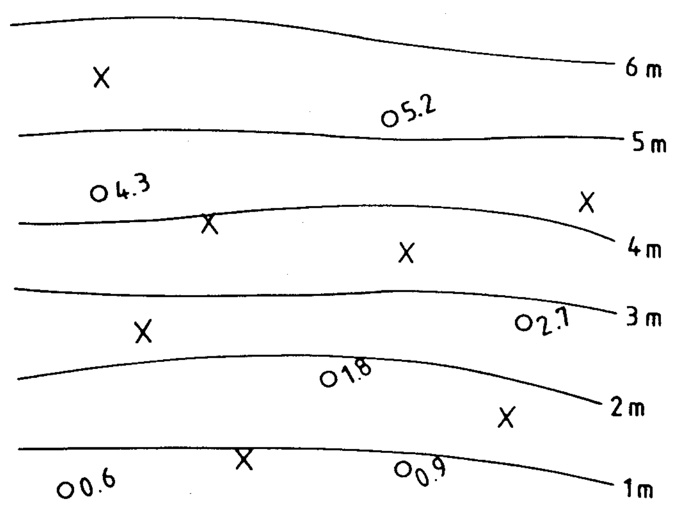



Figure 7.8 : A Geoid Contour from Point-Geoid Height Values (KING et al, 1985)

o Points of known orthometric and geoid height
X Points of GPS survey where geoid heights are required

A second approach is to assume that the geoid and the ellipsoid are related by a mathematical surface. By far the simplest approach is to assume a plane surface for the geoid in the local area. In this case, the ellipsoidal coordinates of each station are projected onto a plane and the east-west tilt, north-south tilt and geoid-spheroid separation at an arbitrary origin is calculated. The geoid is modelled by

$$N = AX + BY + C \qquad (7.37)$$

where N = geoid-spheroid separation at a station (X, Y)

A = east-west tilt of the geoid

B = north-south tilt of the geoid

C = ellipsoid-spheroid separation at the centroid of the network

The least squares solution of this model can be given as

$$D\Delta = f \qquad (7.38)$$

where D = matrix of parameter coefficients, consisting of the change in projected coordinates of each common station from the origin

$\Delta$ = parameters A, B, and C

f = constant vector of the difference between ellipsoidal height and orthometric height of the common stations

It is assumed in this case that the weight matrix of the observations is an identity. Therefore the parameters A, B and C can be calculated from

$$\Delta = (D^T D)^{-1} D^T f \qquad (7.39)$$

A minimum of three stations must be occupied to solve for the three unknowns A, B, and C. If more than three stations were occupied, least squares techniques are used to determine the "best" values of these coefficients.

This method was used by COLLINS and LEICK (1985) in the Montgomery County survey. In this survey a standard deviation of ± 3cm was achieved for the adjusted height difference for each station. Surveying accuracies have been achieved for areas up to 50km square, where the geoid is smooth.

The plane fit parameters may also be obtained from the seven parameter topocentric transformation model. If orthometric heights are used instead of ellipsoidal heights for the local datum common stations to solve for the transformation parameters, the rotations about the northerly and easterly axes include the slope of the geoid in these directions. The geoid-spheroid separation is absorbed into the scale term. When these parameters are subsequently applied to the GPS network stations, the geoid-ellipsoid separation is automatically taken into account. In this way, the transformation of the horizontal coordinates and height can be accomplished in a one step process.

This method was tried over an 100km by 60km area of South Australia, and was found to give the same results using approximate ellipsoidal heights and the plane fit geoid. More research is required, however, to test if this one step approach is suitable in areas where the value of N at the centroid of the network is greater than 50m, and where the geoid slopes change by more than 20mm/km.

AUSTRALIAN HEIGHT DATUM

The Australian Height Datum (AHD) is defined by the Mean Sea Level (MSL) at 30 tide gauges around the continent. Although MSL is in fact a close approximation to the geoid, it is incorrect to assume that it coincides with it exactly. Changes in the salinity and the temperature of water, combined with ocean currents can cause MSL to deviate from the geoid by a few metres worldwide. The AHD, therefore, does not define a true geoidal height system.

In 1971 a geoid-spheroid separation map was compiled and published by the Division of National Mapping. (FRYER, 1971). This map, however, shows the values of N referred to an ellipsoid, that best fits the geoid over Australia. Although the parameters that describe the ellipsoid are the same as those of the Australian National Spheroid (ANS), it is not the ANS defined by the Australian Geodetic Datum. This matter needs clarification. Accordingly the series of events leading to the publication of Fryers map is summarized.

The origin of the Australian Geodetic Datum 1966 (AGD66) is the Johnston Geodetic Station, the coordinates of which are

| | |
|---|---|
| Latitude | S $25^{\circ}$ 56' 54.5515" |
| Longitude | E $133^{\circ}$ 12' 30.0771" |
| Spheroidal Height | 571.2 m |

At the time of the AGD66 adjustment it was assumed that the geoid and the spheroid coincided at Johnston.

In 1972 the AHD was established. Levelling networks around Australia were adjusted holding the values of thirty tide gauges fixed. In the course of this adjustment the AHD height of Johnston was determined to be 566.3m, which was 4.9m lower than the orthometric height assumed in the 1966 adjustment.

In 1971, Fryer used astrogeodetic observations to determine the geoid in Australia. He then fitted the ANS to the geoid model that he determined, finding a best fit orthometric height of Johnston to be 577.2m. This implies that the orthometric height of Johnston is 6m above the gazetted value, and 10.9m above the AHD. Fryer gives the geoid-spheroid separation values relative to his best-fit ANS.

In the latest adjustment (GMA82) of Australian survey data, the measured distances were reduced to the spheroid (defined by the previously gazetted coordinates of Johnston) using AHD heights. Hence the difference in orthometric height defined by Fryer and AHD height at Johnston, is of the order of 10.9m. Although Fryer's ellipsoid has been raised 10.9m from the AGD at Johnston, Figure 7.9 shows that the effect on the ellipsoidal height in Sydney, measured on the normal axis will be of the order of 10.1m.

The correction to Fryers N value decreases with distance away from the Johnston origin. From spherical trigonometry this correction can be approximated by

$$\Delta F = 10.9 * ( \cos(\lambda_j - \lambda) * \cos(\Phi_j - \Phi) ) \qquad (7.40)$$

where $\Delta F$ = correction to Fryers N value at survey site

$\lambda_j$, $\Phi_j$ = longitude and latitude of Johnston

$\lambda$, $\Phi$ = longitude and latitude of survey site

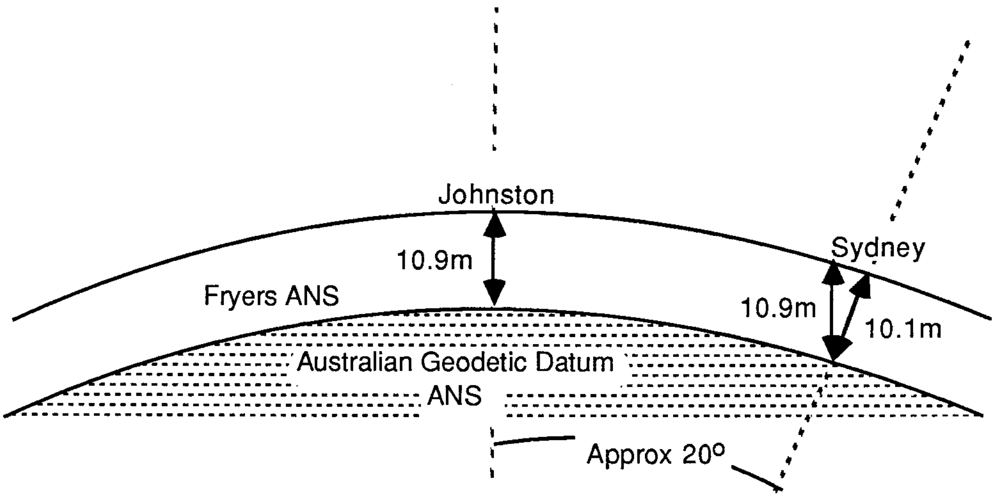



Figure 7.9 : Effect of Height Datum Shift across Australia

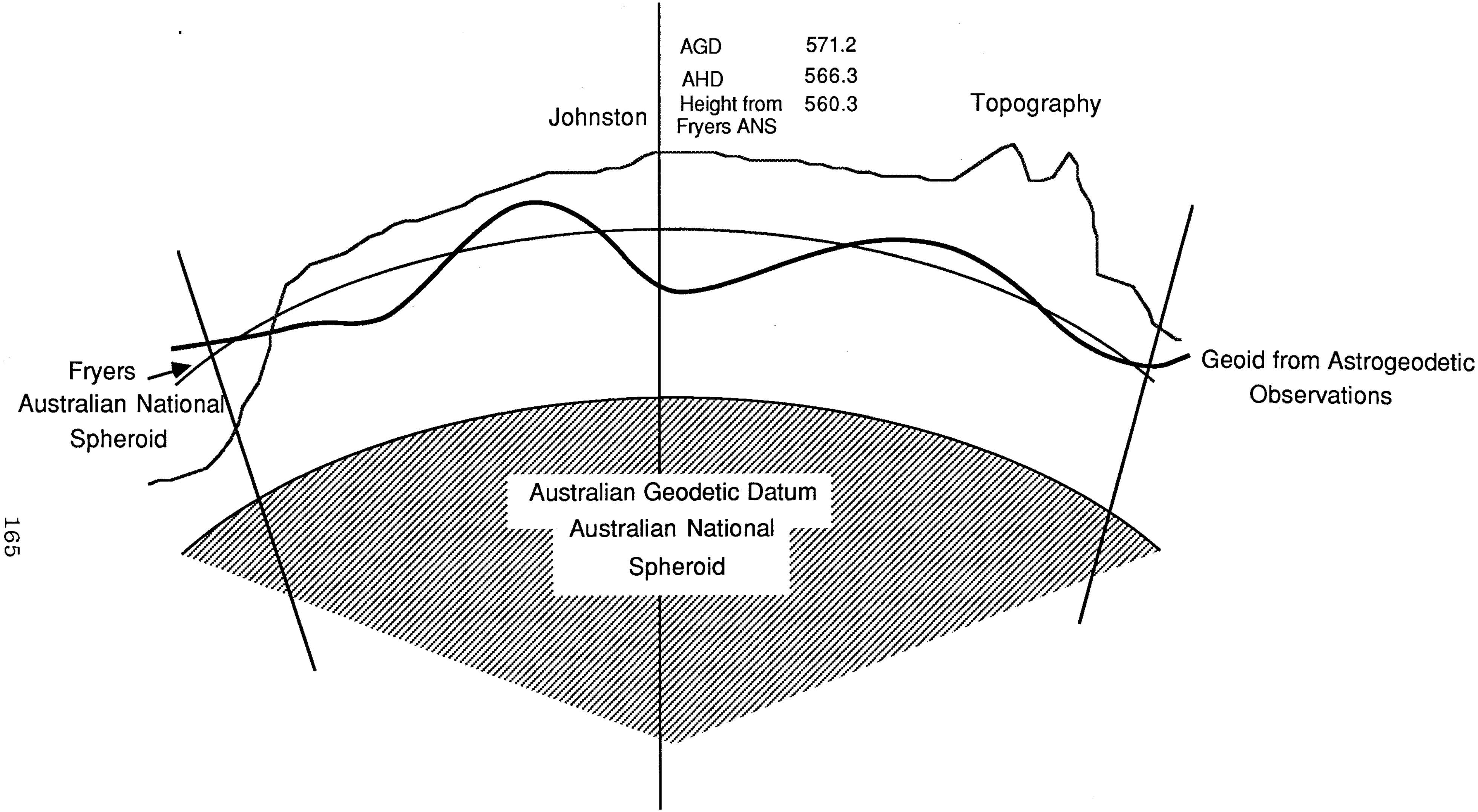
Corrections to Fryer's N values were calculated at several points around Australia using equation 7.40 (Table 7.1).

Table 7.1 : Corrections to Fryers N Values

| Position | Approximate $\phi$ | $\lambda$ | $\Delta F$ (m) | N (Fryer) | Correct N (m) |
|---|---|---|---|---|---|
| Sydney | $-34^{0}$ | $151^{0}$ | 10.1 | 3.8 | 13.9 |
| Adelaide | $-35^{0}$ | $139^{0}$ | 10.7 | 2.0 | 12.7 |
| Perth | $-32^{0}$ | $116^{0}$ | 10.4 | 9.5 | 19.9 |
| Alice Springs | $-24.5^{0}$ | $133.5^{0}$ | 10.9 | -4.0 | 6.9 |

Figure 7.10 shows the relationship between the geoid, the ANS defined by the AGD, and Fryers best fit ANS. In the figure the relationship between these height systems is given at the Johnston origin.

Figure 7.10 : AGD, the Geoid and Fryers best-fit ANS



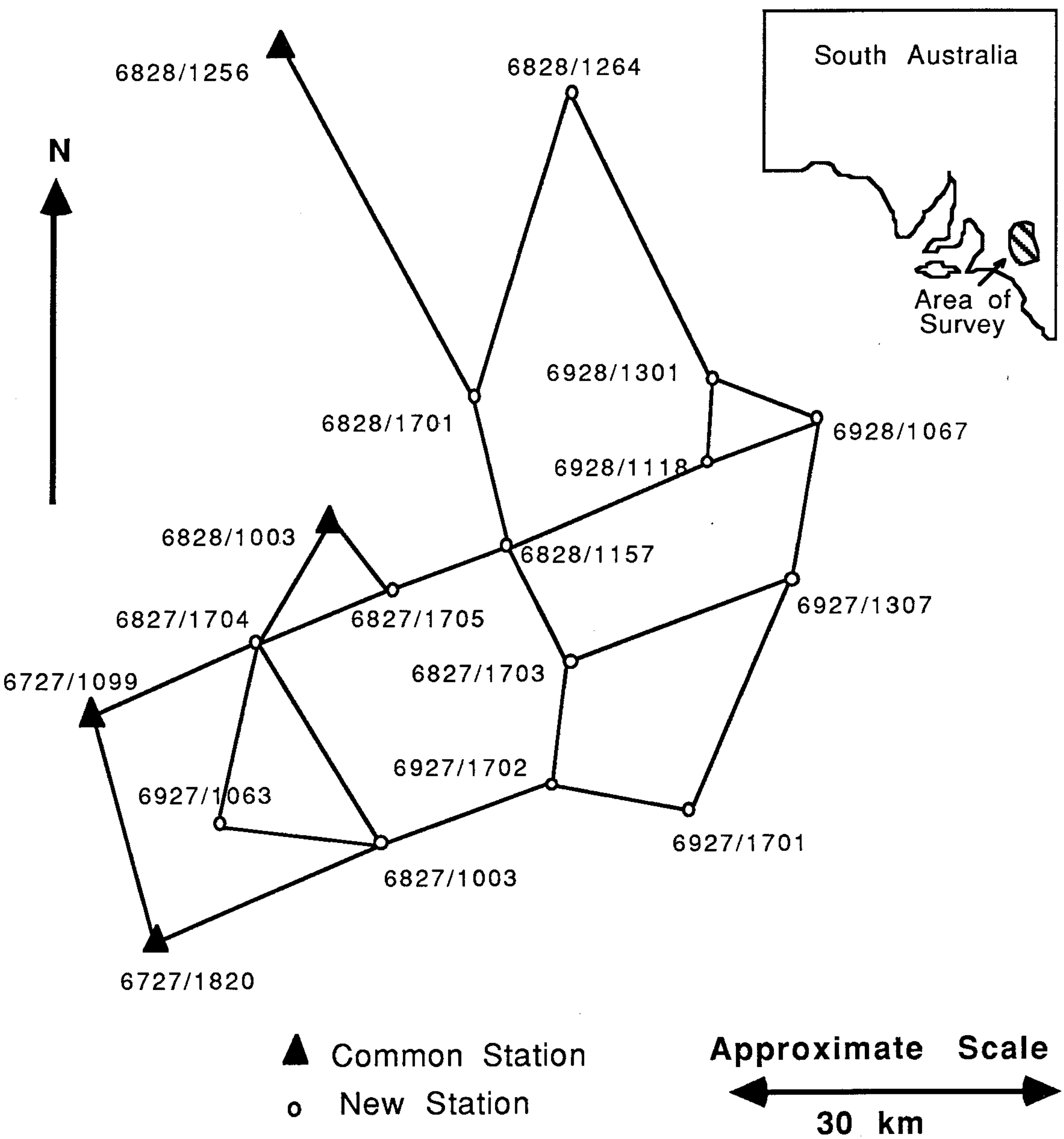
# 8. THE SOUTH AUSTRALIAN GPS SURVEY

In November and December of 1985 the U.S. survey firm Geo/Hydro Inc. carried out a control densification survey for the South Australian Lands Department (SALD). The survey area was located south east of Adelaide, in the region adjoining the Victorian border. A total of 107 stations were occupied, 90 of which were new stations. The survey was carried out using Macrometer V-1000 receivers and the measurements were processed as single baselines using MACROMETRICS software.

A portion of this processed data, which included 25 baseline measurements between 18 stations was kindly supplied by SALD (see Figure 8.1). These baselines were combined in a network adjustment, and transformed to the local datum using programs developed at the University of New South Wales by Bruce Harvey, Don Grant and the author. The programs and their capabilities are described in Appendix B.

The data included several baselines with standard deviations of about 50mm. These baselines were obtained by estimating the ambiguity term in the solution. For the remaining baselines the ambiguity term was held at an integer value, yielding standard deviations of about 10mm. The smaller standard deviations of some lines arose through fewer parameters being estimated with the fixed biases.

Figure 8.1 : A SECTION OF THE SOUTH AUSTRALIAN GPS CONTROL DENSIFICATION SURVEY



NETWORK ADJUSTMENT

The initial step in processing and presenting GPS baseline data is the network adjustment. This is performed with program SHAKE (see Appendix B).

A minimally constrained network adjustment was carried out holding station 6828/1003 fixed (origin). The initial adjustment indicates the overall consistency of the network. Observational blunders that may have occurred in the measurement campaign will show up at this stage.

The WGS72 coordinates of the origin station were obtained from the AGD84 latitude and longitude, and the AHD height. The transformation parameters published by ALLMAN and VEENSTRA (1984) and Seppelins formula were used to transform the AGD coordinates to the WGS72 datum (see Chapter 7). The ellipsoidal height of the origin was obtained by adding the geoid-spheroid separation to the AHD height. The ellipsoidal height was read off Fryer's geoid map, and a correction was made using equation 7.40 (see Chapter 7). The calculation of N is shown in Table 8.1.

Table 8.1 : Calculation on N at Station 6828/1003

| Station | Approximate $\Phi$ | Approximate $\lambda$ | N (Fryer) | Corrn. to Fryer (m) | Correct N |
|---|---|---|---|---|---|
| 6727/1099 | $-35^{0}$ | $139.5^{0}$ | 2.0 | 10.7 | 12.7 |

Although the $\Delta X$, $\Delta Y$ and the $\Delta Z$ components of each baseline were given, the standard deviations and correlations of each baseline were given in terms of spherical latitude, longitude and radius from the centre of the earth. To be consistent with the baseline information, the standard deviation and correlation data were transformed to a Cartesian system, using the law of propagation of variances (MIKHAIL, 1976) ie.

$$VCV_{XYZ} = J \left[ VCV_{\Phi\lambda R} \right] J^{T}$$

where $J = \dfrac{\partial_{XYZ}}{\partial_{\Phi\lambda R}}$

$$= \begin{bmatrix} -R\sin\Phi\cos\lambda & -R\cos\Phi\sin\lambda & \cos\Phi\cos\lambda \\ -R\sin\Phi\sin\lambda & R\cos\Phi\cos\lambda & \cos\Phi\sin\lambda \\ R\cos\Phi & 0 & \sin\Phi \end{bmatrix}$$

$R$ = radius from the centre of the earth

$\Phi$ = spherical latitude

$\lambda$ = spherical longitude

The minimally constrained network adjustment failed the Chi-squared test. This could have been due to a number of reasons, including processing blunders, observational errors or apriori standard deviations that were too optimistic. Inspection of the normalised standard deviations of the residuals showed that the line joining stations 6828/1157 and 6827/1703 was an outlier.

For the second network adjustment the standard deviation of each baseline component was increased by 2ppm of the component length, to allow for the ionospheric propagation errors that were not taken into account when the baselines were processed. The aposteriori estimate of the variance factor also failed the Chi-squared test at the 95% significance level. Inspection of the normalised standard deviations of the residuals indicated that the same line was still suspect. The baseline would have less affect on the network if its components were assigned larger standard deviations. It is difficult to justify increasing the standard deviations of this line, however, as there is no evidence to suggest that they are in error.

At this stage in the network adjustment procedure, the processing of this line should be checked. Possible error sources include blunders, like the incorrect measurement of the height of the antenna above the station, or systematic errors that may occur with atmospheric anomalies. However, as the raw data is not in the public domain, the baseline could not be reprocessed.

The results of the second network adjustment, which includes a 2ppm allowance for systematic errors in each baseline component, are shown in Appendix D. The original baseline data, the adjusted baselines, the residuals of each baseline component and the adjusted coordinates and standard deviations of all the network stations are shown .

## SOLVING FOR THE TRANSFORMATION PARAMETERS

Four common stations were used to determine the transformation parameters relating the GPS network to the local datum (see Figure 8.1). The program described in Appendix B was used for this purpose. As the common stations were nearly collinear and were confined to one side of the network, the data set was not ideal for calculating transformation parameters. The GPS network was transformed to the local datum using the seven parameter Molodensky-Badekas model (see Chapter 7) described below.

## Seven Parameter Molodensky-Badekas Transformation using Ellipsoidal Heights

The seven parameters relating the adjusted GPS network to the local geodetic datum were solved using software described in Appendix B. The input data consisted of the GPS coordinates and the VCV matrix of the common stations, and the AGD latitude and longitude, and the AHD height of the stations. It was assumed that there was a constant geoid-spheroid separation of 12.7m over the entire network. This may not be a valid assumption in areas where the geoid is undulating with respect to the ellipsoid. The latitudes and longitudes of the common stations were assigned standard deviations of 20cm which was recommended by SALD. Standard deviations of 1.5m were assigned to the ellipsoidal heights. This allowed for errors in the determination of N. The adjusted transformation parameters are shown in Table 8.1.

The high correlations between the translation terms and the rotations about the X and Z axes is explained by the large (1.5m) standard deviations that were assigned to the heights on the local datum. The highly correlated parameters are all influenced by the adjustment of the height. If the local heights had been assigned smaller standard deviations, the parameters would be less correlated.

The scale and rotation terms were tested (eqn 7.33) to see whether they were statistically significantly different from their apriori values. Table 8.2 shows the tested parameters, their computed test statistic and the Chi-squared values for the appropriate degrees of freedom. The correlations were taken into account, if more than one parameter was tested.

Table 8.1 : Solution of 7 Molodensky-Badekas Parameters

COORDINATES FOR THE CENTROID OF THE NETWORK (6828/1003)

| X | Y | Z |
|---|---|---|
| -3989130.343 | 3389243.746 | -3632271.888 |

THE ADJUSTED PARAMETERS

| PARAMETER | | ADJUSTED VALUE | SIGMA |
|---|---|---|---|
| SCALE FACTOR | (ppm) | -2.44 | +/- 3.58 |
| ROTN ABT X AXIS | (secs) | 2.92 | +/- 18.96 |
| ROTN ABT Y AXIS | (secs) | .95 | +/- 4.81 |
| ROTN ABT Z AXIS | (secs) | -4.69 | +/- 19.35 |
| TRANS ALONG X AXIS | (m) | 119.21 | +/- .89 |
| TRANS ALONG Y AXIS | (m) | 42.49 | +/- .77 |
| TRANS ALONG Z AXIS | (m) | -148.95 | +/- .82 |

CORRELATIONS OF ADJUSTED PARAMETERS

| SC | RX | RY | RZ | TX | TY | TZ |
|---|---|---|---|---|---|---|
| 1.000 | | | | | | |
| -.013 | 1.000 | | | | | |
| .022 | .383 | 1.000 | | | | |
| .010 | -.976 | -.190 | 1.000 | | | |
| -.079 | .824 | .151 | -.842 | 1.000 | | |
| -.036 | -.822 | -.170 | .845 | -.973 | 1.000 | |
| -.031 | .823 | .145 | -.838 | .983 | -.969 | 1.000 |

Table 8.2 : Statistical Tests on Solved Parameters

| Parameters Tested | Test Statistic | Chi-Squared Value 90% | 95% |
|---|---|---|---|
| Rx | 0.02 | 2.71 | 3.84 |
| Ry | 0.04 | 2.71 | 3.84 |
| Rz | 0.06 | 2.71 | 3.84 |
| Rx, Ry | 0.05 | 4.61 | 5.99 |
| Rx, Rz | 0.20 | 4.61 | 5.99 |
| Ry, Rz | 0.08 | 4.61 | 5.99 |
| Rx, Ry, Rz | 2.00 | 6.25 | 7.81 |
| Sc, Rx, Ry | 0.52 | 6.25 | 7.81 |
| Sc, Rx, Rz | 0.68 | 6.25 | 7.81 |
| Sc, Ry, Rz | 0.55 | 6.25 | 7.81 |
| Sc, Rx, Ry, Rz | 2.70 | 7.78 | 9.49 |

Table 8.2 shows that the rotation and scale terms are not significantly different from their apriori values (null values). Therefore, these parameters could be held fixed at zero without affecting the resulting transformed coordinates significantly (ie. the coordinate values would change by an amount less than one standard deviation). These parameters were applied to transform the entire GPS network to the local datum.

HEIGHT TRANSFORMATION

In order to achieve third order orthometric heighting in small GPS networks (say of 100km extent) COLLINS and LEICK (1985) showed that it may be sufficient to approximate the geoid as a plane. This technique assumes that the geoid is smooth in the area where the survey is being carried out. However, as this assumption may not be valid in some areas, the surveyor must have some idea of the variability of the geoid before applying this technique. The method, which was described in the previous chapter, was used to compute orthometric height for a number of the transformed stations in the South Australian GPS network. Table 8.3 shows the information that was used in the model. The parameters estimated in the simplified least squares adjustment were :-

A = east-west tilt of the geoid = 3.142 mm/km

B = north-south tilt of the geoid = -1.957 mm/km

C = ellipsoid-spheroid surface separation at the origin = 12.668 m

Stations used to calculate the relationship between the geoid and the spheroid should ideally be distributed throughout the entire network. As these parameters were determined from stations located on the west side of the network, they were only applied to stations in the same area. The resulting orthometric heights are shown in Table 8.3.

Table 8.4 shows the transformed coordinates of the GPS network, and compares these to the known coordinates of the common stations. The standard deviations of the transformed latitudes and longitudes vary from ±.1m in the west to about ±.3m in the east. The standard deviations of the heights vary from ±1m in the west to ±6m in the east. This is due to the common stations all being located on the western side of the network.

TABLE 8.3 : ORTHOMETRIC HEIGHT COMPUTATION OF WESTERN NETWORK STATIONS

| STATION | E | N | ΔE | ΔN | h | H | ΔH | Calc N | Calc H |
|---|---|---|---|---|---|---|---|---|---|
| 6828/1003 | 376432.866 | 6132873.340 | 0.000 | 0.000 | 152.972 | 140.286 | 12.686 | 12.668 | 140.304 |
| 6727/1099 | 351548.225 | 6118736.162 | 24884.641 | 14137.178 | 95.409 | 82.653 | 12.756 | 12.721 | 82.688 |
| 6828/1256 | 384278.546 | 6173441.185 | -7845.680 | -40567.845 | 83.209 | 70.499 | 12.710 | 12.724 | 70.485 |
| 6727/1820 | 351261.318 | 6105773.108 | 25171.548 | 27100.232 | 77.068 | 64.412 | 12.656 | 12.695 | 64.373 |
| 6927/1063 | 359601.438 | 6109592.905 | 16831.428 | 23280.435 | 111.179 | NA | NA | 12.676 | 98.503 |
| 6827/1704 | 368196.748 | 6120873.072 | 8736.118 | 12000.268 | 141.149 | NA | NA | 12.671 | 128.478 |
| 6828/1701 | 395861.163 | 6144771.750 | -19428.297 | -11898.410 | 108.943 | NA | NA | 12.628 | 96.315 |

TABLE 8.4 : COMPARISON OF KNOWN AND TRANSFORMED AGD ELLIPSOIDAL COORDINATES

| STATION | KNOWN LATITUDE | TRANSFORMED LATITUDE | SD | ΔLAT | KNOWN LONGITUDE | TRANSFORMED LONGITUDE | SD | ΔLON |
|---|---|---|---|---|---|---|---|---|
| 6727/1099 | -35 03 47.09089 | -35 03 47.09230 | .00399 | -.00141 | 139 22 19.16816 | 139 22 19.16877 | .00433 | -.00061 |
| 6727/1820 | -35 10 47.57643 | -35 10 47.57584 | .00476 | .00059 | 139 21 59.45249 | 139 21 59.45235 | .00620 | .00014 |
| 6827/1003 | | -35 10 43.01140 | .00487 | | | 139 36 40.33126 | .00679 | |
| 6927/1063 | | -35 08 47.94526 | .00509 | | | 139 27 31.41833 | .00727 | |
| 6827/1704 | | -35 02 46.07199 | .00372 | | | 139 33 17.47198 | .00604 | |
| 6828/1003 | -34 56 20.37148 | -34 56 20.37096 | .00501 | .00052 | 139 38 48.90062 | 139 38 48.89872 | .00796 | .00190 |
| 6827/1705 | | -35 01 05.15905 | .00466 | | | 139 42 28.77571 | .00681 | |
| 6828/1157 | | -34 58 44.02288 | .00615 | | | 139 50 08.39227 | .00833 | |
| 6827/1703 | | -35 07 02.50045 | .00651 | | | 139 49 29.05269 | .00823 | |
| 6927/1702 | | -35 11 03.63961 | .00633 | | | 139 45 47.07996 | .00812 | |
| 6927/1701 | | -35 14 00.99434 | .00767 | | | 139 51 54.15404 | .00914 | |
| 6927/1307 | | -35 06 08.33667 | .00868 | | | 140 01 06.25331 | .00993 | |
| 6928/1067 | | -34 59 06.69952 | .00951 | | | 140 06 46.73883 | .01098 | |
| 6928/1118 | | -34 58 49.67837 | .00822 | | | 140 00 40.86967 | .00997 | |
| 6928/1301 | | -34 52 20.99708 | .00823 | | | 140 00 54.03442 | .01076 | |
| 6828/1264 | | -34 40 30.70916 | .00816 | | | 139 57 10.11068 | .01384 | |
| 6828/1701 | | -34 50 2.04542 | .00653 | | | 139 51 39.96097 | .00968 | |
| 6828/1256 | -34 34 27.03409 | -34 34 27.03421 | .00751 | -.00012 | 139 44 18.18302 | 139 44 18.18558 | .01712 | -.00256 |

COMPARISON OF KNOWN AND TRANSFORMED AHD HEIGHTS

| STATION | ELLIPSOIDAL HEIGHT | SD | ORTHOMETRIC HEIGHT KNOWN | TRANSFORMED HEIGHT | ΔHT |
|---|---|---|---|---|---|
| 6727/1099 | 95.409 | 1.180 | 82.653 | 82.688 | -.035 |
| 6727/1820 | 77.068 | 1.096 | 64.412 | 64.373 | .039 |
| 6827/1003 | 126.371 | 2.700 | | | |
| 6927/1063 | 111.179 | 1.214 | | 98.503 | |
| 6827/1704 | 141.149 | 1.282 | | 128.490 | |
| 6828/1003 | 152.972 | 1.423 | 140.286 | 140.304 | -.018 |
| 6827/1705 | 108.765 | 2.515 | | | |
| 6828/1157 | 146.678 | 3.537 | | | |
| 6827/1703 | 129.018 | 4.386 | | | |
| 6927/1702 | 145.266 | 4.239 | | | |
| 6927/1701 | 144.904 | 5.627 | | | |
| 6927/1307 | 102.100 | 6.271 | | | |
| 6928/1067 | 109.225 | 6.446 | | | |
| 6928/1118 | 98.672 | 5.358 | | | |
| 6928/1301 | 109.115 | 4.677 | | | |
| 6828/1264 | 84.179 | 2.863 | | | |
| 6828/1701 | 108.943 | 2.858 | | 96.315 | |
| 6828/1256 | 83.209 | 1.465 | 70.499 | 70.485 | .014 |

# 9. CONCLUSIONS

Although GPS is still under development, it has demonstrated a capability to compete with any other terrestrial measurement technology. As the system develops, it will have an enormous impact on the surveying and positioning industries. GPS will prove to be a cost effective and productive positioning technique.

In the future, the capability and cost effectiveness of GPS will improve for several reasons. These include :-

(1) Receivers will improve in design and will become cheaper.

As more manufacturers enter the GPS market, receivers will improve in design and flexibility. Manufacturers will ensure that the receivers are easy to use, portable, rugged, flexible and require small power supplies. Future receivers may have dual frequency capability and be able to track all the satellites.

Only two receivers were commercially available three years ago : the TI4100 and the Macrometer V-1000. At that time the receivers cost approximately US$150,000 and US$120,000, respectively. Since then, the price of the equipment has dropped dramatically. The Trimble 4000S currently retails at AUS$85,000 and the WM101, which should be available in December, 1986, will retail at US$90,000. As research continues into receiver development, and the competition between receiver manufacturers increases the costs will continue to fall.

(2) Satellite coverage will improve

The launching schedule, which was revised after the Shuttle disaster of January, 1986, shows that the launching of GPS satellites should recommence in February, 1989. The full constellation should be available by the end of 1991. This will enable GPS surveys to carried out anywhere in the world for 24 hours every day. This improved satellite coverage will allow surveyors to use their receivers more productively.

(3) Processing software will become more flexible and user friendly

The currently available GPS processing software is limited in some degree. The comprehensive and flexible packages require mainframe computers and skilled operators, while the packages developed by receiver manufacturers are user friendly but are restricted in their flexibility. As GPS develops, more versatile, user friendly software will become available, which will enable surveyors to perform a wider variety of positioning tasks : for example, real time differential positioning software will enable instantaneous accurate positioning for a range of surveys.

These future GPS developments will impact on the navigation and surveying professions. Once the system is fully operational, with 24 hour coverage, GPS will provide an ideal navigation tool. GPS will be used to provide accurate, cheap and continuous positioning for a wide range of moving platforms. By installing GPS receivers on these platforms (eg. ships, trains, aircraft, taxis etc.) it will be possible to navigate in unfamiliar regions, monitor their movement, optimise routing and improve scheduling. GPS will also be beneficial in search and rescue operations, as it will allow fast and accurate position reporting and coordination of searching operations.

GPS will be used for a wider range of static positioning tasks. Presently the system is best suited to geodetic control surveys, photogrammetrical control and for large engineering works. However, GPS will impact on many facets of surveying in the future. GPS could be employed in rural cadastral surveys, small scale engineering works, land information systems, geographic data bases, exploration and reconnaissance surveys. GPS could be used as a cost effective method of connecting boundary corners to geodetic control.

GPS is an ideal tool for surveyors, as they have the experience and training in measurement techniques, reference frames, geometry and computing to enable them to apply this technology to a range of applications. In order to gain maximum benefit from the system, however, the surveying profession should now prepare for GPS. Preparation is important to :-

(1) Avoid future confusion

Surveyors should understand capabilities and limitations of the GPS system, so that they can apply it to their daily work. Surveyors should also know how to transform their GPS survey to the AGD and the AHD.

(2) Plan an infrastructure

Standards and procedures concerning the use of GPS should be established, so that there is some level of control over the use of the system. These standards should cover observing procedures, software capability and receiver design. Procedures should be established for collecting gravity and geoid-spheroid separation data from completed GPS surveys, to improve the determination of N. The problems of ephemerides accuracy and availability should be addressed, and consideration given to the establishment of a tracking network.

Presently a number of groups have been set up (GPS User Group, Qeensland User Group) to discuss the present use of GPS, the problems encountered in GPS surveys and how they can be overcome. These discussions indicate that the educational, government and private sectors of the survey community have a role to play in ensuring the successful introduction of GPS into Australian surveying. Clearly, the roles of these sectors are :-

Academic

Academic institutions are responsible for the ongoing education of the profession. Universities and technical colleges must master new technologies and encourage students to learn about them. They should educate students to keep an open mind about the endless opportunities arising within the profession.

Academic institutions should also be responsible for GPS research. The system should be fully understood, so problems can be analysed, improvements can be made and new applications can be developed.

Government

The role of the government is to provide guidelines within which surveyors can operate. These guidelines would be in the form of specifications of survey procedure and processing techniques. Standards should also be created in processing packages, to allow data from any receiver to be processed by any software package.

Government should also provide a number of services to the community. Geoid-spheroid separation data from completed surveys should be archived. This data would provide valuable information in reducing GPS height information to the geoid. Government should also be considering establishing a satellite tracking network. This would enable the generation of Australia's own ephemeris data, which would be more accurate than the broadcast and precise ephemeris, and could be distributed to Australian users, free of selective availability.

Private

The private sector is responsible for educating themselves about GPS, and participating in the establishment of user guidelines.

GPS provides the surveying industry an opportunity to become involved in a wide range of positioning tasks. Surveyors are ideally suited to take up the GPS challenge, and if properly trained and educated, should look forward to a bright and interesting future.

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# APPENDIX A

## REVIEW OF LEAST SQUARES

Least squares techniques are commonly used to compute overdetermined parameters. The technique involves relating a set of observed data (L) with the theoretical values of that data determined by apriori parameters (X) through a mathematical model or function.

The linearized mathematical model is (from Krakiwsky, 1981).

$$A\Delta X + Bv + W = 0$$

where $A = \frac{\partial F}{\partial X}$ at $(X, L)$

= first design matrix, made up of the partial derivatives of the function relating the observations and the parameters with respect to the parameters.

$\Delta X$ = Corrections to the apriori parameters

= $X' - X$

= Adjusted parameters - apriori parameters

$B = \frac{\partial F}{\partial L}$ at $(X, L)$

= second design matrix, made up of the partial derivatives of the function relating the observations and the parameters with respect to the observations.

$v$ = Observation Residuals

= $L' - L$

= Adjusted observations - observations

$W$ = Misclosure vector = $F(X, L)$

Once the design matrices and the misclosure vector have been calculated, the final adjusted parameters (X) can be computed. As the computation of the adjusted parameters is not a linear process, it may take several iterations for the solution to converge. The values of X are obtained by minimising the weighted sum of the squared residuals $v^T P v$, where $P = Q^{-1}$, and Q is the variance covariance matrix of the observations.

The results of the adjustment are obtained from

$$\Delta X = -(A^T(BQB^T)^{-1}A + Q_x^{-1})^{-1}A^T(BQB^T)^{-1}W$$

where $Q$ = VCV matrix of the observations

$Q_x$ = VCV matrix of the apriori parameters

$$V = -QB^T(BQB^T)^{-1}(A\Delta X+W)$$

$$\hat{\sigma}_0^2 = (v^TQ^{-1}v+\Delta X^TQ_x^{-1}\Delta X)/(n-u+u_x)$$

where $\hat{\sigma}_0^2$ = a posteriori variance factor

$n$ = number of observation equations

$u$ = number of unknowns

$u_x$ = number of weighted parameters

$$Q'_x = \sigma_0^2(A^T(BQB^T)^{-1}A + Q_x^{-1})^{-1}$$

where $Q'_x$ = VCV of the adjusted parameters

$$Ql' = \sigma_0^2\{Q+QB^T(BQB^T)^{-1}A(Q_x'/\sigma_0^2)A^T(BQB^T)^{-1}BQ-QB^T(BQB^T)^{-1}BQ\}$$

where $Ql'$ = VCV of the adjusted observations.

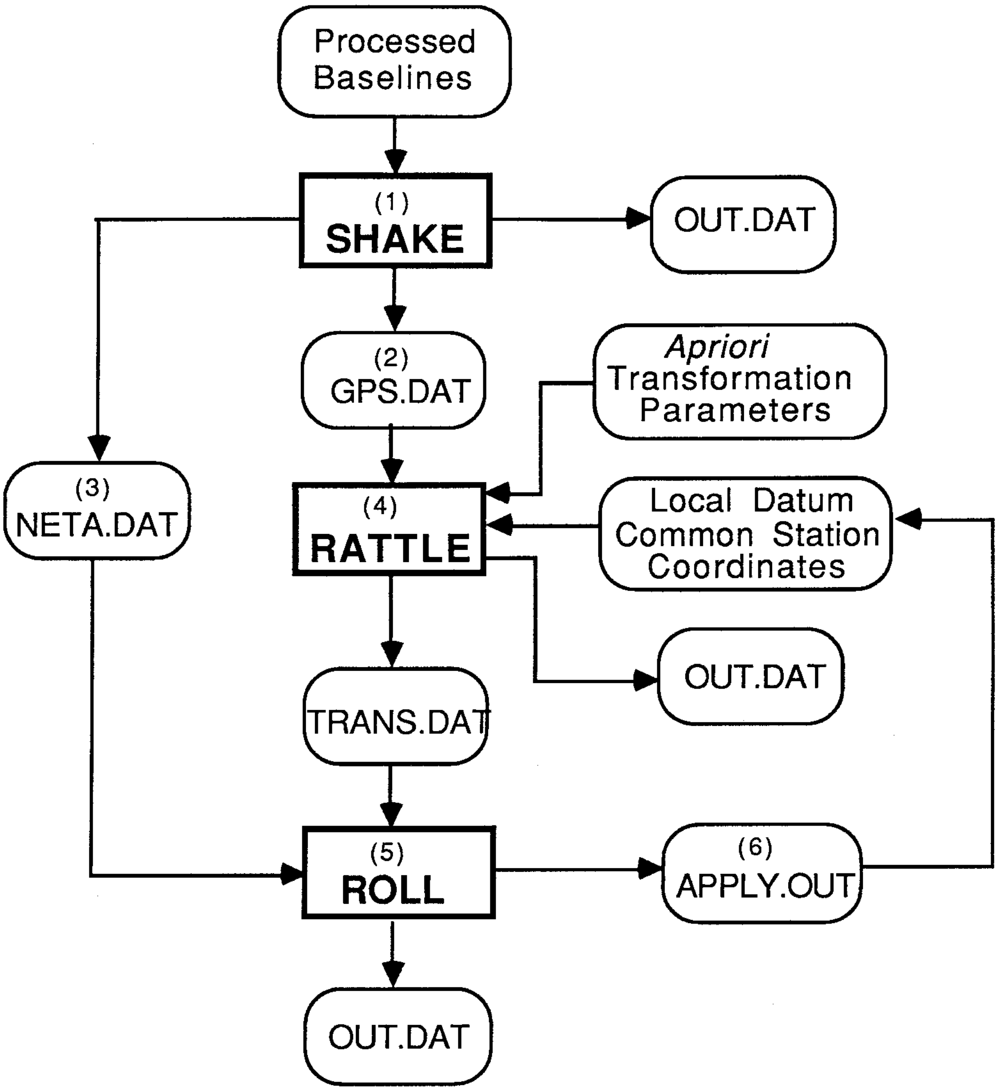
# APPENDIX B

## SOFTWARE PACKAGE

Programs SHAKE, RATTLE and ROLL are designed to combine GPS baselines in a network, solve for the transformation parameters relating this network with a local datum and apply these parameters to the adjusted network. These programs are an important component of a GPS software package, as they are designed to present processed GPS data in a form that is meaningful to surveyors.

The programs were written in FORTRAN under the MS-DOS operating system. They were designed to run on the NEC APCII personal computer. The presently small data storage capacity of the APCII (64k) limits the programs capabilities, in terms of the amount of data that can be processed. Listings of the programs can be obtained from the author.

Figure A.1 outlines how programs SHAKE, RATTLE and ROLL interact to produce an adjusted, transformed survey network from processed GPS baselines.




| | | |
|---|---|---|
| Note : | (1) SHAKE | - Network adjustment |
| | (2) GPS.DAT | - Contains common stations coordinates and VCV matrix |
| | (3) NETA.DAT | - Contains coordinates and VCV matrix for whole network |
| | (4) RATTLE | - Estimates transformation parameters |
| | (5) ROLL | - Applies transformation parameters |
| | (6) APPLY.OUT | - Transformed coordinates that may be re-entered in RATTLE |

## SHAKE

Program SHAKE combines uncorrelated baselines, resulting from software packages like MACPOMETRICS and TRIMVEC (see Current Software Packages, Chapter 6) in a network using the adjustment of indirect observations technique (see Network Adjustment, Chapter 7).

SHAKE can adjust a network holding any number of stations fixed. On the NEC APC II microcomputer, SHAKE can combine up to 50 baselines, measured between 20 stations. SHAKE carries out a Chi-squared test and detects outliers using Baardas data snooping technique (see Chapter 7).

SHAKE was tested using a data set from the Eifel network GPS survey, carried out in Germany in 1983. This data had been processed using MACROMETRICS software and then adjusted in a network using program 'XMAC', developed by Alfred Leick when he was on sabatical at Massachusetts Institute of Technology (MIT). The results from the Eifel were subsequently published in BOCK (1984).

SHAKE includes two features that were not present in XMAC.

(1) SHAKE outputs the normalised standard deviations of each of the adjusted observation vectors for outlier detection. A description of the algorithms used to compute the normalised standard deviations is given in Chapter 7 (eqn 7.25).

(2) SHAKE allows the operator to increase the standard deviations of each of the observed vectors. This feature allows for systematic errors, like ionospheric effects

that are not accounted for at the processing stage. With this option the new standard deviation of each baseline is computed from

$$SD_n = \sqrt{SD_p^2 + (A_{ppm} * BV)^2}$$

where $SD_n$ = newly computed standard deviation

$SD_p$ = standard deviation resulting from processing

$A$ = magnitude of the systematic correction

$BV$ = measured baseline vector component

SHAKE reads in baseline data which consists of the baseline terminal stations, the vector components and standard deviations of each baseline, and the appropriate correlations between these vector components.

Input data can be enetered from the screen or from a file. If data is read in from the screen, it is correctly formatted into file EDIT.DAT, which is then read into SHAKE. In this way the operator may edit input information merely by editing file EDIT.DAT.

SHAKE outputs three files.

(1) OUT.DAT shows the results of the network adjustment. This file lists the results of statistical tests, the adjusted coordinates of all network stations and their standard deviations and the adjusted baseline vectors.

(2) GPS.DAT contains the adjusted coordinates and the VCV matrix of the common stations. This file is correctly formatted to be read into program RATTLE.

(3) NETA.DAT contains the final adjusted coordinates and VCV

matrix of the entire GPS network. This file may be used later in program ROLL.

## RATTLE

Program RATTLE is designed to solve for the transformation parameters relating two datums. RATTLE can solve both Bursa-Wolf and Molodensky-Badekas parameters (see Chapter 7). If fewer than seven transformation parameters are required (eg. in a block shift),, the parameters that are not estimated can be held fixed at null values in the solution.

RATTLE reads in the coordinates and the associated VCV matrix of common stations in the two datums, as well as a set of apriori transformation parameters and their VCV matrix. A possible set of apriori parameters for Australian surveyors are the transformation parameters published by ALLMAN and VEENSTRA (1984) relating the precise ephemeris TRANSIT datum to AGD84. The common station coordinates may be either Cartesian, ANS ellipsoidal or WGS72 ellipsoidal. With the NEC APC II microcomputers, RATTLE can process up to five common stations.

Program RATTLE allows the user to enter the coordinate sets and variances of the common stations in each datum from the screen or from a file. File GPS.DAT output from program SHAKE is correctly formatted for direct input into RATTLE.

Program RATTLE has two output files.

(1) OUT.DAT shows the input data, the apriori and adjusted baseline vectors between the common stations in each datum, the adjusted transformation parameters and the

adjusted coordinates of the common stations and their VCV matrix in each datum.

(2) TRANS.DAT contains the adjusted transformation parameters and VCV matrix relating the refence frames from the final iteration of the solution.

## Solution of Transformation Parameters

RATTLE uses least squares techniques to solve the transformation parameters. The linearized mathematical model is (from KRAKIWSKY, 1981).

$$A\Delta X + Bv + W = 0 \qquad (B.1)$$

where $A = \frac{\partial F}{\partial X}$ at $(X, L)$

= first design matrix, made up of the partial derivatives of the function relating the observations and the parameters with respect to the parameters.

This matrix is an (N,7) matrix where N is the number of coordinates in the adjustment - which is usually three times the number of common stations.

The A matrix takes the form

$$A = \begin{bmatrix} X_1+kY_1-\theta Z_1 & 0 & -sZ_1 & sY_1 & 1 & 0 & 0 \\ -kX_1+Y_1+wZ_1 & sZ_1 & 0 & -sX_1 & 0 & 1 & 0 \\ \theta X_1-wY_1+Z_1 & -sY_1 & sX_1 & 0 & 0 & 0 & 1 \\ X_2+kY_2-\theta Z_2 & 0 & -sZ_2 & sY_2 & 1 & 0 & 0 \\ . & . & . & . & . & . & . \end{bmatrix}$$

For the Molodensky-Badekas model the X, Y, and Z values are replaced by $X-X_m$, $Y-Y_m$ and $Z-Z_m$ respectively.

$\Delta X$ = Corrections to the apriori transformation parameters which are s, w, $\theta$, k, $T_x$, $T_y$ and $T_z$.

= X' - X

= Adjusted parameters - apriori parameters

B $= \frac{\partial F}{\partial L}$ at (X, L)

= second design matrix, made up of the partial derivatives of the function relating the observations and the parameters with respect to the observations.

This B matrix is a (N, N2) matrix where N is the number of coordinates in one reference frame, and N2 is twice that number (ie the total number of coordinates from both networks in the adjustment).

The B matrix takes the form

$$\begin{bmatrix} s & sk & -s\theta & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & . & . \\ -sk & s & sw & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & . & . \\ s\theta & -sw & s & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & . & . \\ 0 & 0 & 0 & 0 & 0 & 0 & s & sk & -s\theta & -1 & 0 & 0 & . & . \\ 0 & 0 & 0 & 0 & 0 & 0 & -sk & s & sw & 0 & -1 & 0 & . & . \\ 0 & 0 & 0 & 0 & 0 & 0 & s\theta & -sw & s & 0 & 0 & -1 & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . \end{bmatrix}$$

V = Residuals of the observations in both reference frames

= L' - L

= Adjusted observations - observations

W = Misclosure vector = F(X, L)

The misclosure vector can be calculated from

$$W = \begin{bmatrix} sX_{1_A} + skY_{1_A} - s\theta Z_{1_A} + T_X - X_{1_B} \\ -skX_{1_A} + sY_{1_A} + swZ_{1_A} + T_Y - Y_{1_B} \\ s\theta X_{1_A} - swY_{1_A} + sZ_{1_A} + T_Z - Z_{1_B} \\ sX_{2_A} + skY_{2_A} - s\theta Z_{2_A} + T_X - X_{2_B} \\ . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \end{bmatrix}$$

The VCV matrix of the observations for the solution is formed by combining the VCV matrices from the two different networks in the following manner.

$$Q = Q_A + Q_B$$

$$= \begin{bmatrix} VCV_{1_A} & 0 & VCV_{12_A} & 0 & . & . \\ 0 & VCV_{1_B} & 0 & VCV_{12_B} & . & . \\ VCV_{12_A} & 0 & VCV_{2_A} & 0 & . & . \\ 0 & VCV_{12_B} & 0 & VCV_{2_B} & . & . \\ . & . & . & . & . & . \end{bmatrix}$$

where VCV = submatrix from the respective VCV matrix of network A or B.

The results of the adjustment are obtained from

$$\Delta X = -(A^T(BQB^T)^{-1}A + Q_x^{-1})^{-1}A^T(BQB^T)^{-1}W \qquad (B.2)$$

where $Q$ = VCV matrix of the observations

$Q_x$ = VCV matrix of the apriori parameters

$$V = -QB^T(BQB^T)^{-1}(A\Delta X+W) \qquad (B.3)$$

$$\hat{\sigma}_0^2 = (v^TQ^{-1}v+\Delta X^TQ_x^{-1}\Delta X)/(n-u+u_x) \qquad (B.4)$$

where $\hat{\sigma}_0^2$ = a posteriori variance factor

$n$ = number of observation equations

$u$ = number of unknowns

$u_x$ = number of weighted parameters

$$Q'_x = \sigma_0^2(A^T(BQB^T)^{-1}A + Q_x^{-1})^{-1} \qquad (B.5)$$

where $Q'_x$ = VCV of the adjusted parameters

$$Q1' = \sigma_0^2(Q+QB^T(BQB^T)^{-1}A(Q_x'/\sigma_0^2)A^T(BQB^T)^{-1}BQ-QB^T(BQB^T)^{-1}BQ) \qquad (B.6)$$

where $Q1'$ = VCV of the adjusted observations.

ROLL

ROLL applies the transformation parameters obtained in RATTLE to transform the survey network into the new datum. This program requires the user to input the full VCV matrix of both the transformation parameters and the GPS survey network in order to compute the full VCV matrix of the transformed coordinates.

On the NEC APC II's, ROLL is capable of transforming 25 stations and their VCV matrix.

File NETA.DAT resulting from program SHAKE, and TRANS.DAT resulting from program RATTLE, are correctly formatted to directly read into program ROLL. Program ROLL also allows the user to enter network station coordinates, and their associated variances from the screen.

Program ROLL outputs two files.

(1) OUT.DAT consists of the input information and the transformed coordinates and VCV matrix of the network stations. The transformed values may be output as

Cartesian coordinates, or as latitude, longitude and height on the ANS or the WGS72 ellipsoid.

(2) APPLY.OUT contains only the transformed coordinates and associated VCV matrix correctly formatted to be read into program RATTLE. This file is output mainly for research purposes.

## APPENDIX C

## COORDINATE CONVERSION BETWEEN CAETESIAN and ELLIPSOIDAL SYSTEMS

The transformation formulae given in Chapter 6 require coordinates to be entered in a Cartesian (XYZ) system. If a set of coordinates and its associated VCV matrix are in an ellipsoidal system, they can be converted to a cartesian system using the following formula (HARVEY, 1986)

$$X = (N+h)\cos\Phi\cos\lambda \qquad (C.1)$$

$$Y = (N+h)\cos\Phi\sin\lambda$$

$$Z = \{(1-e^2)N+h\}\sin\Phi$$

where $N = a / \sqrt{1 - e^2\sin^2\Phi}$

$e^2 = 2f - f^2$

The converted VCV matrix can be calculated from

$$VCV_{XYZ} = J\ VCV_{\Phi\lambda h}\ J^T \qquad (C.2)$$

where $J = \frac{\partial F}{\partial \Phi\lambda h}$ = Jacobian matrix

$$= \begin{bmatrix} \frac{Ne^2\sin\Phi\cos^2\Phi\cos\lambda}{(1-e^2\sin^2\Phi)}-(N+h)\cos\lambda\sin\Phi & -(N+h)\cos\Phi\sin\lambda & \cos\Phi\cos\lambda \\ \frac{Ne^2\sin\Phi\cos^2\Phi\sin\lambda}{(1-e^2\sin^2\Phi)}-(N+h)\sin\lambda\sin\Phi & (N+h)\cos\Phi\cos\lambda & \cos\Phi\cos\lambda \\ \frac{\{Ne^2\sin^2\Phi\cos\Phi}{(1-e^2\sin^2\Phi)}+N\cos\Phi\}(1-e^2)+h\cos\Phi & 0 & \sin\Phi \end{bmatrix}$$

The Jacobian matrix given above is used to transform the VCV of a single point. For the conversion of the VCV of more than one point the full Jacobian matrix is -

$$J = \begin{bmatrix} J_1 & 0 & 0 & 0 & . & . \\ 0 & J_2 & 0 & 0 & . & . \\ 0 & 0 & J_3 & 0 & . & . \\ 0 & 0 & 0 & J_4 & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & J_n \end{bmatrix}$$

If Cartesian coordinates and VCV matrix are output from the transformation formula, ellipsoidal coordinates can be calculated from :-

$$\Phi = \arctan\{(Z+e^2N\sin\Phi)/R\} \quad [\text{iterate}] \qquad (C.3)$$

$$\lambda = \arctan(Y/X)$$

$$h = R/\cos\Phi - N$$

where $R = \sqrt{(X^2 + Y^2)}$

The VCV matrix of these ellipsoidal coordinates can be calculated from -

$$VCV_{\Phi\lambda h} = J\ VCV_{XYZ}\ J^T \qquad (C.4)$$

where the Jacobian matrix J is -

$$\begin{bmatrix} A & YA/X & B \\ -Y/R^2 & X/R^2 & 0 \\ \{X/R\cos\Phi\}+AC & \{Y/R\cos\Phi\}+YAC/X & BC \end{bmatrix}$$

where

$$A \approx X\tan\Phi \ / \ R^2(e^2 - \sec^2\Phi)$$

$$B \approx 1/\{R\sec^2\Phi - e^2N\cos\Phi\}$$

$$C = R\sin\Phi/\cos^2\Phi - Ne^2\sin\Phi\cos\Phi/(1-e^2\sin^2\Phi)$$

## APPENDIX D

```
OUTPUT FROM SHAKE - SOUTH AUSTRALIAN CONTROL DENSIFICATION SURVEY


  THESE RESULTS HAVE NOT PASSED THE CHI SQUARED TEST ( VALUE =   1.52) WITH
24 DEGREES OF FREEDOM, AT A 95% SIGNIFICANCE LEVEL

   PLEASE CHECK THAT INPUT DATA IS CORRECT !
   Errors could be due to :-
   (1) Wrong station numbers
   (2) Station numbers in wrong order
   (3) Incorrectly entered baseline data
   (4) Bad observations
   (5) Observation Variances Too Optimistic

  PLEASE CHECK EDIT.DAT FILE


  NUMBER OF OBSERVATIONS          75
  NUMBER OF PARAMETERS            51
  DEGREES OF FREEDOM              24
  A-POSTERIORI VARIANCE OF UNIT WEIGHT         4.235
  VPV (WEIGHTED SUM OF RESIDUALS SQUARED)    101.644
```

## ADJUSTED STATION POSITIONS

| NO | STATION | X | STD DEV | Y | STD DEV | Z | STD DEV |
|---|---|---|---|---|---|---|---|
| 1 | 6727/1099 | -3966792.302 | | 3403172.496 | | -3643515.805 | |
| 2 | 6727/1820 | -3960798.084 | .061 | 3398686.953 | .060 | -3654104.526 | .038 |
| 3 | 6827/1003 | -3975369.451 | .079 | 3381820.365 | .079 | -3654018.127 | .036 |
| 4 | 6927/1063 | -3967897.778 | .136 | 3393706.514 | .084 | -3651110.188 | .072 |
| 5 | 6827/1704 | -3978484.355 | .078 | 3391220.549 | .077 | -3642002.792 | .030 |
| 6 | 6828/1003 | -3989130.343 | .143 | 3389243.746 | .090 | -3632271.888 | .077 |
| 7 | 6827/1705 | -3988875.965 | .085 | 3381712.575 | .081 | -3639437.798 | .034 |
| 8 | 6828/1157 | -3998331.886 | .111 | 3374445.294 | .087 | -3635896.651 | .056 |
| 9 | 6827/1703 | -3990936.772 | .114 | 3369507.236 | .089 | -3648463.054 | .056 |
| 10 | 6927/1702 | -3984052.262 | .112 | 3371044.802 | .089 | -3654548.730 | .054 |
| 11 | 6927/1701 | -3987635.908 | .114 | 3361918.138 | .093 | -3659014.537 | .057 |
| 12 | 6927/1307 | -4003022.527 | .120 | 3356601.500 | .093 | -3647082.187 | .059 |
| 13 | 6928/1067 | -4014287.431 | .116 | 3354777.725 | .094 | -3636447.964 | .058 |
| 14 | 6928/1118 | -4008554.006 | .116 | 3362080.299 | .095 | -3636012.062 | .058 |
| 15 | 6928/1301 | -4014029.484 | .118 | 3366236.629 | .098 | -3626197.310 | .063 |
| 16 | 6828/1264 | -4019913.150 | .165 | 3378611.235 | .145 | -3608203.497 | .093 |
| 17 | 6828/1701 | -4006842.765 | .116 | 3378583.949 | .091 | -3622683.143 | .074 |
| 18 | 6828/1256 | -4012098.848 | .241 | 3397745.733 | .235 | -3598980.706 | .150 |

ADJUSTED BASELINE VECTORS

| DAY | ST1 | ST2 | DELX (ST2-ST1) | DELY (ST2-ST1) | DELZ (ST2-ST1) | SX CRXY | SY CRXZ | SZ CRYZ | LENGTH | SL | PPM SL/L |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 334 | 2 | 3 | 14571.367 | 16866.588 | -86.399 | 87.5 | 87.7 | 36.7 | 22289.325 | 121.1 | 5.4 |
| | | | | | | .92 | .87 | .72 | | | |
| 329 | 3 | 10 | 8682.811 | 10775.563 | 530.603 | 79.5 | 42.8 | 42.7 | 13848.665 | 74.1 | 5.3 |
| | | | | | | .64 | -.84 | -.85 | | | |
| 326 | 10 | 11 | 3583.646 | 9126.664 | 4465.807 | 21.0 | 29.3 | 19.0 | 10774.133 | 15.7 | 1.5 |
| | | | | | | -.67 | .63 | -.81 | | | |
| 340 | 3 | 4 | -7471.672 | -11886.149 | -2907.939 | 116.8 | 46.2 | 63.4 | 14337.452 | 80.0 | 5.6 |
| | | | | | | .68 | -.85 | -.81 | | | |
| 340 | 4 | 5 | 10586.576 | 2485.965 | -9107.396 | 111.2 | 35.0 | 66.6 | 14184.508 | 125.3 | 8.8 |
| | | | | | | .65 | -.82 | -.83 | | | |
| 334 | 5 | 3 | -3114.904 | 9400.184 | 12015.335 | 28.6 | 30.4 | 23.3 | 15570.304 | 20.1 | 1.3 |
| | | | | | | .53 | .45 | -.13 | | | |
| 334 | 2 | 1 | 5994.218 | -4485.543 | -10588.721 | 61.0 | 60.2 | 37.6 | 12968.106 | 22.4 | 1.7 |
| | | | | | | .92 | .84 | .68 | | | |
| 334 | 1 | 5 | 11692.053 | 11951.947 | -1513.013 | 77.7 | 77.2 | 29.5 | 16788.160 | 104.9 | 6.2 |
| | | | | | | -.93 | -.87 | -.72 | | | |
| 340 | 5 | 7 | 10391.610 | 9507.974 | -2564.993 | 35.5 | 27.3 | 16.8 | 14316.645 | 22.6 | 1.6 |
| | | | | | | -.53 | .48 | -.72 | | | |
| 338 | 5 | 6 | 10645.989 | 1976.803 | -9730.904 | 119.5 | 46.5 | 71.2 | 14557.998 | 135.1 | 9.3 |
| | | | | | | .66 | -.85 | -.86 | | | |
| 338 | 6 | 7 | -254.379 | 7531.171 | 7165.910 | 116.0 | 45.2 | 70.5 | 10398.726 | 26.0 | 2.5 |
| | | | | | | .79 | -.90 | -.90 | | | |
| 338 | 7 | 8 | 9455.922 | 7267.282 | -3541.148 | 81.0 | 34.0 | 44.8 | 12440.561 | 86.3 | 6.9 |
| | | | | | | .60 | -.77 | -.83 | | | |
| 338 | 8 | 9 | -7395.115 | 4938.058 | 12566.403 | 36.4 | 41.3 | 29.8 | 15394.370 | 17.1 | 1.1 |
| | | | | | | .45 | .59 | -.10 | | | |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 326 | 9 | 10 | -6884.510 | -1537.566 | 6085.676 | 27.2 | 11.1 | 21.4 | 9316.439 | 16.7 | 1.8 |
| | | | | | | -.65 | .56 | -.78 | | | |
| 326 | 9 | 12 | 12085.756 | 12905.735 | -1380.866 | 42.1 | 30.3 | 25.7 | 17735.002 | 39.9 | 2.3 |
| | | | | | | .14 | -.34 | -.74 | | | |
| 326 | 12 | 11 | -15386.620 | -5316.637 | 11932.349 | 46.3 | 22.7 | 29.3 | 20184.045 | 46.6 | 2.3 |
| | | | | | | .12 | -.32 | -.73 | | | |
| 338 | 8 | 14 | 10222.120 | 12364.995 | 115.412 | 35.2 | 42.2 | 17.8 | 16043.633 | 38.4 | 2.4 |
| | | | | | | -.06 | .20 | -.77 | | | |
| 339 | 14 | 13 | 5733.425 | 7302.573 | 435.901 | 24.1 | 26.1 | 8.9 | 9294.608 | 15.6 | 1.7 |
| | | | | | | -.64 | .63 | -.79 | | | |
| 340 | 13 | 12 | -11264.903 | -1823.775 | 10634.224 | 46.1 | 50.3 | 31.0 | 15598.426 | 35.8 | 2.3 |
| | | | | | | .57 | .40 | -.14 | | | |
| 339 | 15 | 13 | 257.947 | 11458.904 | 10250.654 | 37.3 | 31.5 | 26.7 | 15376.896 | 18.6 | 1.2 |
| | | | | | | .10 | -.01 | -.63 | | | |
| 339 | 15 | 14 | -5475.478 | 4156.330 | 9814.752 | 32.0 | 37.4 | 27.6 | 11982.709 | 20.1 | 1.7 |
| | | | | | | .18 | .08 | -.63 | | | |
| 340 | 15 | 16 | 5883.666 | -12374.606 | -17993.813 | 118.3 | 119.0 | 71.5 | 22616.934 | 81.1 | 3.6 |
| | | | | | | .89 | .83 | .63 | | | |
| 335 | 16 | 17 | -13070.385 | 27.287 | 14479.645 | 123.1 | 114.3 | 70.2 | 19506.302 | 46.4 | 2.4 |
| | | | | | | .91 | .86 | .68 | | | |
| 338 | 17 | 8 | -8510.879 | 4138.655 | 13213.508 | 36.1 | 27.1 | 48.6 | 16253.010 | 30.7 | 1.9 |
| | | | | | | -.35 | .46 | -.79 | | | |
| 335 | 17 | 18 | 5256.083 | -19161.784 | -23702.437 | 211.0 | 216.6 | 130.3 | 30929.046 | 184.5 | 6.0 |
| | | | | | | .93 | .89 | .75 | | | |

ADJUSTED BASELINE VECTORS

| DAY | AT | TO | OBSERVATION | RESIDUAL | NORM SD | ADJUSTED OB |
|---|---|---|---|---|---|---|
| 334 | 2 | 3 | 14571.377 | -.010 | -.21 | 14571.367 |
| | | | 16866.598 | -.010 | -.22 | 16866.588 |
| | | | -86.394 | -.005 | -.30 | -86.399 |
| 329 | 3 | 10 | 8682.802 | .009 | .31 | 8682.811 |
| | | | 10775.592 | -.029 | -2.02 | 10775.563 |
| | | | 530.593 | .010 | .61 | 530.603 |
| 326 | 10 | 11 | 3583.641 | .005 | .69 | 3583.646 |
| | | | 9126.684 | -.020 | -1.55 | 9126.664 |
| | | | 4465.803 | .004 | .61 | 4465.807 |
| 340 | 3 | 4 | -7471.674 | .002 | .03 | -7471.672 |
| | | | -11886.169 | .020 | .85 | -11886.149 |
| | | | -2907.938 | -.001 | -.03 | -2907.939 |
| 340 | 4 | 5 | 10586.630 | -.054 | -.96 | 10586.576 |
| | | | 2485.973 | -.008 | -.62 | 2485.965 |
| | | | -9107.435 | .039 | 1.25 | -9107.396 |
| 334 | 5 | 3 | -3114.878 | -.026 | -.51 | -3114.904 |
| | | | 9400.227 | -.043 | -.83 | 9400.184 |
| | | | 12015.340 | -.005 | -.16 | 12015.335 |
| 334 | 2 | 1 | 5994.208 | .010 | .51 | 5994.218 |
| | | | -4485.552 | .009 | .49 | -4485.543 |
| | | | -10588.730 | .009 | .52 | -10588.721 |
| 334 | 1 | 5 | 11692.049 | .004 | .06 | 11692.053 |
| | | | 11951.943 | .004 | .06 | 11951.947 |
| | | | -1513.015 | .002 | .09 | -1513.013 |
| 340 | 5 | 7 | 10391.645 | -.035 | -2.01 | 10391.610 |
| | | | 9507.920 | .054 | 3.43 | 9507.974 |
| | | | -2564.977 | -.016 | -2.13 | -2564.993 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 338 | 5 | 6 | 10645.976 | .013 | .11 | 10645.989 |
| | | | 1976.788 | .015 | .43 | 1976.803 |
| | | | -9730.817 | -.087 | -1.16 | -9730.904 |
| 338 | 6 | 7 | -254.318 | -.061 | -1.88 | -254.379 |
| | | | 7531.192 | -.021 | -1.55 | 7531.171 |
| | | | 7165.897 | .013 | .72 | 7165.910 |
| 338 | 7 | 8 | 9455.984 | -.062 | -1.37 | 9455.922 |
| | | | 7267.281 | .001 | .04 | 7267.282 |
| | | | -3541.169 | .021 | .92 | -3541.148 |
| 338 | 8 | 9 | -7395.383 | .268 | 5.71 | -7395.115 |
| | | | 4937.759 | .299 | 6.11 | 4938.058 |
| | | | 12566.245 | .158 | 5.24 | 12566.403 |
| 326 | 9 | 10 | -6884.469 | -.041 | -3.79 | -6884.510 |
| | | | -1537.581 | .015 | 3.68 | -1537.566 |
| | | | 6085.710 | -.034 | -3.61 | 6085.676 |
| 326 | 9 | 12 | 12085.746 | .010 | .17 | 12085.756 |
| | | | 12905.719 | .016 | .60 | 12905.735 |
| | | | -1380.871 | .005 | .17 | -1380.866 |
| 326 | 12 | 11 | -15386.615 | -.005 | -.08 | -15386.620 |
| | | | -5316.631 | -.006 | -.44 | -5316.637 |
| | | | 11932.314 | .035 | 1.01 | 11932.349 |
| 338 | 8 | 14 | 10222.135 | -.015 | -1.11 | 10222.120 |
| | | | 12364.956 | .039 | 2.13 | 12364.995 |
| | | | 115.425 | -.013 | -1.97 | 115.412 |
| 339 | 14 | 13 | 5733.418 | .007 | .85 | 5733.425 |
| | | | 7302.578 | -.005 | -.52 | 7302.573 |
| | | | 435.900 | .001 | .54 | 435.901 |
| 340 | 13 | 12 | -11265.104 | .201 | 3.00 | -11264.903 |
| | | | -1823.969 | .194 | 3.22 | -1823.775 |
| | | | 10634.145 | .079 | 2.47 | 10634.224 |
| 339 | 15 | 13 | 257.993 | -.046 | -.76 | 257.947 |
| | | | 11458.908 | -.004 | -.16 | 11458.904 |
| | | | 10250.637 | .017 | .43 | 10250.654 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 339 | 15 | 14 | -5475.439 | -.039 | -.46 | -5475.478 |
| | | | 4156.402 | -.072 | -.85 | 4156.330 |
| | | | 9814.764 | -.012 | -.31 | 9814.752 |
| 340 | 15 | 16 | 5883.653 | .013 | .20 | 5883.666 |
| | | | -12374.627 | .021 | .30 | -12374.606 |
| | | | -17993.826 | .013 | .38 | -17993.813 |
| 335 | 16 | 17 | -13070.378 | -.007 | -.14 | -13070.385 |
| | | | 27.286 | .001 | .01 | 27.287 |
| | | | 14479.643 | .002 | .08 | 14479.645 |
| 338 | 17 | 8 | -8510.872 | -.007 | -.82 | -8510.879 |
| | | | 4138.657 | -.002 | -.24 | 4138.655 |
| | | | 13213.501 | .007 | .43 | 13213.508 |
| 335 | 17 | 18 | | | | |

THIS OBSERVATION IS A RADIATION - CHECK NETWORK DESIGN

INPUT DATA

NUMBER OF STATIONS : 18
NUMBER OF FIXED STATIONS : 1
NUMBER OF OBSERVATIONS : 25

| NO. | STATION | X COORD | Y COORD | Z COORD |
|---|---|---|---|---|
| 1 | 6727/1099 | -3966792.302 | 3403172.496 | -3643515.805 |
| 2 | 6727/1820 | | | |
| 3 | 6827/1003 | | | |
| 4 | 6927/1063 | | | |
| 5 | 6827/1704 | | | |
| 6 | 6828/1003 | | | |
| 7 | 6827/1705 | | | |
| 8 | 6828/1157 | | | |
| 9 | 6827/1703 | | | |
| 10 | 6927/1702 | | | |
| 11 | 6927/1701 | | | |
| 12 | 6927/1307 | | | |
| 13 | 6928/1067 | | | |
| 14 | 6928/1118 | | | |
| 15 | 6928/1301 | | | |
| 16 | 6828/1264 | | | |
| 17 | 6828/1701 | | | |
| 18 | 6828/1256 | | | |

| NO. | DAY | AT | TO | OBSERVATION (ST2-ST1) | STD DEV | CORRELATION MATRIX | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 334 | 2 | 3 | 14571.377 | .063 | 1.00 | .94 | .91 |
| | | | | 16866.598 | .064 | | 1.00 | .79 |
| | | | | -86.394 | .024 | | | 1.00 |
| 2 | 329 | 3 | 10 | 8682.802 | .048 | 1.00 | .71 | -.88 |
| | | | | 10775.592 | .025 | | 1.00 | -.89 |
| | | | | 530.593 | .026 | | | 1.00 |
| 3 | 326 | 10 | 11 | 3583.641 | .012 | 1.00 | -.78 | .75 |
| | | | | 9126.684 | .019 | | 1.00 | -.87 |
| | | | | 4465.803 | .011 | | | 1.00 |
| 4 | 340 | 3 | 4 | -7471.674 | .079 | 1.00 | .72 | -.88 |
| | | | | -11886.169 | .032 | | 1.00 | -.88 |
| | | | | -2907.938 | .044 | | | 1.00 |
| 5 | 340 | 4 | 5 | 10586.630 | .078 | 1.00 | .69 | -.86 |
| | | | | 2485.973 | .022 | | 1.00 | -.88 |
| | | | | -9107.435 | .045 | | | 1.00 |
| 6 | 334 | 5 | 3 | -3114.878 | .052 | 1.00 | .94 | .91 |
| | | | | 9400.227 | .053 | | 1.00 | .79 |
| | | | | 12015.340 | .033 | | | 1.00 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 7 | 334 | 2 | 1 | 5994.208 | .036 | 1.00 | .93 | .88 |
| | | | | -4485.552 | .034 | | 1.00 | .74 |
| | | | | -10588.730 | .025 | | | 1.00 |
| 8 | 334 | 1 | 5 | 11692.049 | .045 | 1.00 | .93 | .89 |
| | | | | 11951.943 | .044 | | 1.00 | .74 |
| | | | | -1513.015 | .016 | | | 1.00 |
| 9 | 340 | 5 | 7 | 10391.645 | .025 | 1.00 | -.76 | .73 |
| | | | | 9507.920 | .021 | | 1.00 | -.85 |
| | | | | -2564.977 | .011 | | | 1.00 |
| 10 | 338 | 5 | 6 | 10645.976 | .128 | 1.00 | .83 | -.93 |
| | | | | 1976.788 | .042 | | 1.00 | -.92 |
| | | | | -9730.817 | .082 | | | 1.00 |
| 11 | 338 | 6 | 7 | -254.318 | .065 | 1.00 | .82 | -.91 |
| | | | | 7531.192 | .026 | | 1.00 | -.91 |
| | | | | 7165.897 | .039 | | | 1.00 |
| 12 | 338 | 7 | 8 | 9455.984 | .060 | 1.00 | .75 | -.88 |
| | | | | 7267.281 | .022 | | 1.00 | -.90 |
| | | | | -3541.169 | .032 | | | 1.00 |
| 13 | 338 | 8 | 9 | -7395.383 | .050 | 1.00 | .88 | .91 |
| | | | | 4937.759 | .053 | | 1.00 | .69 |
| | | | | 12566.245 | .033 | | | 1.00 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 14 | 326 | 9 | 10 | -6884.469 | .017 | 1.00 | -.78 | .74 |
| | | | | -1537.581 | .007 | | 1.00 | -.86 |
| | | | | 6085.710 | .014 | | | 1.00 |
| 15 | 326 | 9 | 12 | 12085.746 | .061 | 1.00 | .70 | -.86 |
| | | | | 12905.719 | .031 | | 1.00 | -.90 |
| | | | | -1380.871 | .031 | | | 1.00 |
| 16 | 326 | 12 | 11 | -15386.615 | .060 | 1.00 | .71 | -.86 |
| | | | | -5316.631 | .018 | | 1.00 | -.89 |
| | | | | 11932.314 | .038 | | | 1.00 |
| 17 | 338 | 8 | 14 | 10222.135 | .022 | 1.00 | -.37 | .42 |
| | | | | 12364.956 | .027 | | 1.00 | -.86 |
| | | | | 115.425 | .011 | | | 1.00 |
| 18 | 339 | 14 | 13 | 5733.418 | .014 | 1.00 | -.75 | .73 |
| | | | | 7302.578 | .015 | | 1.00 | -.85 |
| | | | | 435.900 | .005 | | | 1.00 |
| 19 | 340 | 13 | 12 | -11265.104 | .071 | 1.00 | .93 | .89 |
| | | | | -1823.969 | .065 | | 1.00 | .75 |
| | | | | 10634.145 | .035 | | | 1.00 |
| 20 | 339 | 15 | 13 | 257.993 | .064 | 1.00 | .74 | -.87 |
| | | | | 11458.908 | .030 | | 1.00 | -.90 |
| | | | | 10250.637 | .041 | | | 1.00 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 21 | 339 | 15 | 14 | -5475.439 | .087 | 1.00 | .94 | .89 |
| | | | | 4156.402 | .086 | | 1.00 | .76 |
| | | | | 9814.764 | .039 | | | 1.00 |
| 22 | 340 | 15 | 16 | 5883.653 | .087 | 1.00 | .94 | .89 |
| | | | | -12374.627 | .089 | | 1.00 | .76 |
| | | | | -17993.826 | .050 | | | 1.00 |
| 23 | 335 | 16 | 17 | -13070.378 | .078 | 1.00 | .93 | .89 |
| | | | | 27.286 | .071 | | 1.00 | .75 |
| | | | | 14479.643 | .043 | | | 1.00 |
| 24 | 338 | 17 | 8 | -8510.872 | .019 | 1.00 | -.40 | .45 |
| | | | | 4138.657 | .015 | | 1.00 | -.85 |
| | | | | 13213.501 | .029 | | | 1.00 |
| 25 | 335 | 17 | 18 | 5256.083 | .103 | 1.00 | .93 | .89 |
| | | | | -19161.784 | .105 | | 1.00 | .75 |
| | | | | -23702.437 | .063 | | | 1.00 |

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