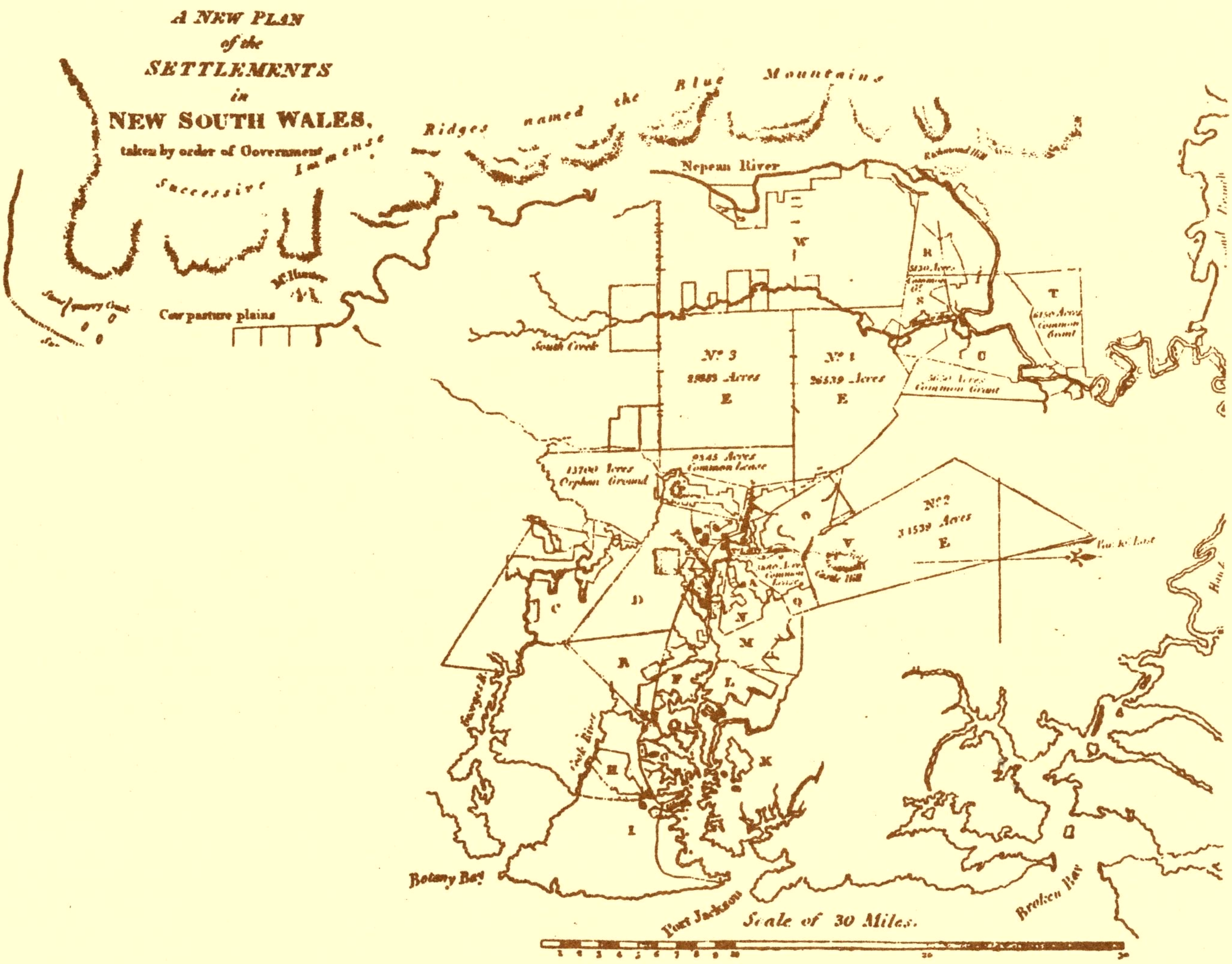
THE RESOLUTION OF MEAN SEA LEVEL ANOMALIES ALONG THE NSW COASTLINE USING THE GLOBAL POSITIONING SYSTEM

RODERICK T. MACLEOD



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ACRONYMS

AGD84	Australian Geodetic Datum 1984
AGU	American Geophysical Union
AHD	Australian Height Datum
AUSLIG	Australian Land Information Group
BDOP	Bias Dilution of Position
BMR	Bureau of Mineral Resources
CMA	Central Mapping Authority (NSW)
DGI	Department of Geographic Information (Qld)
DMA	Defense Mapping Agency (USA)
DoD	Department of Defense (USA)
DTM	digital terrain model
EDM	Electronic Distance Measurement
FFT	Fast Fourier Transform
GDOP	Geometric Dilution of Position
GMM	Gauss Markov Model
GPS	Global Positioning System
GRS67	Geodetic Reference System 1967
GRS80	Geodetic Reference System 1980
IAG	International Association of Geodesy
IGSN71	International Gravity Standardisation Net 1971
IOC	Intergovernmental Oceanographic Commission
IUGG	International Union of Geodesy and Geophysics
LIC	Land Information Centre (NSW)
LMSL	local mean sea level
MHW	mean high water
MMSL	monthly mean sea level
MSL	mean sea level
MTL	mean tide level
NAVSTAR	Navigation Satellite Timing and Ranging
NGS	National Geodetic Survey (USA)
NLRS	National Laser Ranging Station
NOAA	US National Oceanic and Atmospheric Administration
PRN	Pseudo Random Noise
PWD	Public Works Department (NSW)
RINT	Ring Integration
RMIT	Royal Melbourne Institute of Technology (VIC)
SLH	sea level height
SLR	Satellite Laser Ranging
SST	sea surface topography
TEC	total electron content
TOGA	Tropical Ocean and Global Atmosphere project
UNSW	University of New South Wales
UTC	Coordinated Universal Time
VCE	Variance Component Estimation
VLBI	Very Long Base Interferometry
WGS84	World Geodetic System 1984
WOCE	World Ocean Circulation Experiment
WVR	Water Vapour Radiometer

ABSTRACT

The Public Works Department of New South Wales (PWD) is responsible for the operation of the coastal tide monitoring system and the information obtained therefrom is used in the determination of Mean High Water (MHW). Due to intensified development along the coast, information such as flood heights and MHW (which are used to define flood prone lands, cadastral boundaries and in extensive numerical modelling of rivers) must be accurately related to a common datum such as the Australian Height Datum (AHD). Anomalies have been found between determinations of Mean High Water from ocean gauges and that derived from spirit levelling between tide gauges along the entire New South Wales coast.

It was therefore considered necessary to investigate the existing NSW coastal levelling network by undertaking an independent levelling traverse using the new satellite technology of the Global Positioning System (GPS). In May 1987, a 40 station GPS survey was carried out from Lakes Entrance in Victoria to Caloundra in Queensland. In September of the same year this network was strengthened when the then Central Mapping Authority of NSW (now known as the Land Information Centre of the Department of Lands) carried out a survey using TI4100 dual frequency receivers which connected the coastal survey to three VLBI stations and a large single frequency GPS network in the New England region.

This thesis discusses the project, firstly by highlighting the problems presently encountered when relating geodetic levelling to mean sea level determinations. The principles and concepts of the GPS are then explained and by reviewing the project, this study tries to illustrate the need to plan thoroughly for all aspects of such a large

logistical exercise. Processing strategies for the reduction the GPS data are outlined by analysing existing methods. These methods cover two distinct areas; observations within a session and combining sessions into a network. Since GPS heights refer to a satellite reference system a convenient method to convert these values to orthometric heights which are related to the geoid was required. A review of existing methods is given and the combined ring integration technique (RINT) is adopted. These GPS derived orthometric heights are then compared to the existing New South Wales coastal levelling, and Mean Sea Level calculated at the tide gauges. Conclusions are made on the validity of the GPS technique and a solution to the Public Works Department's river datum problem is outlined.

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1. INTRODUCTION

1.1 BACKGROUND

The Public Works Department of New South Wales (PWD) has over many years invested heavily in the purchase, installation and maintenance of water level recorders in rivers and estuaries along the NSW coastline. These recorders have been placed to monitor the water levels in the ocean and rivers during cyclones and floods, and to obtain tidal data necessary for a number of the Department's investigations and projects. Some of these recorders have been integrated into the Australian tide gauge network which is overseen by the Permanent Tides Committee, and data is also forwarded to the Permanent Service of Mean Sea Level at Bidston Observatory, in the United Kingdom.

Since the early 1970's there have been increasing statutory and social requirements placed on land development in New South Wales. Legislation such as the Local Government Act, the Coastal Protection Act, and the Environmental Planning and Assessment Act have placed restrictions on the type of development allowed in many areas. These decisions to restrict certain developments have been made on the basis of physical topography, health and safety, social attitudes, environmental considerations, political pressures, and economics. Statistical information from various sources (including the Public Works Department) forms an important part in finalising these restrictions and the accompanying legislation.

The introduction of the New South Wales Government's Flood Prone Lands Policy (PWD, 1987b) and the apparent increase in the rate of coastal erosion, has created in recent years an increased demand for accurate water level data from the Department. The information required, consists of the heights of flood levels and the Mean High Water (MHW) accurately related to a datum such as the Australian Height Datum (AHD). These heights are necessary for identifying flood affected lands and the legal definition of cadastral boundaries abutting tidal waters. The information is used by both state and local authorities to help control the increasing amount of development now occurring along the coast.

To accomplish the above mentioned aims and to obtain a better understanding of the effects of tides and floods in rivers and estuaries, the Department is undertaking extensive numerical modelling (ie. creating mathematical models to study flooding in rivers and estuaries) that requires accurate water level records. Engineers and scientists require this water level data to be accurately related to a common datum not only within the river or estuary system being studied, but also in relation to other river systems along the coast. This allows comparative studies of similar flood recurrence intervals in each river system. These river datums have been connected via orthometric heights derived from spirit levelling.

However the process of numerical modelling and a study of the Mean Sea Level (MSL) along the coast has found that there are significant variations in the position of MSL calculated at each tide gauge when these tide gauges were connected to the AHD. A study of geodetic levelling and MSL is described in Chapter 2, with specific reference to the situation along the New South Wales coast.

1.2 THE GPS SOLUTION

From the above investigation and because of the increasing use of Departmental advice on matters of development policy, it was considered necessary to undertake a review of the existing coastal levelling which presently relates the tide gauges to the AHD. After discussions with various groups within the Public Works, a recommendation was made that the Department's Survey and Property Branch undertake an independent differential levelling traverse of the coast using the new satellite technology of the Global Positioning System (GPS).

RUSH (1986) had used doppler observations to locate errors in the Queensland levelling network and suggested the use of satellite ranging techniques to improve the accuracy of the existing levelling. HOLLOWAY (1988) and KEARSLEY (1988a) also suggest this approach by using GPS and gravimetry to supplement the terrestrial data. Experience gained by the Survey and Property Branch in May 1986 using GPS equipment to carry out control surveys for photogrammetric mapping (ROBINSON & WATTERSON, 1987), indicated it was possible to obtain accurate differential heights rapidly and cheaply. This technology offered a method of height determination which was independent of conventional spirit levelling. This thesis discusses the methods used to determine accurate ellipsoid and orthometric heights from GPS and its application to the Public Works Department coastal datum problem. The basic concepts of GPS surveying are outlined in Chapter 3.

As a bonus, this GPS network would have the capability of connecting the tide gauges into the International Oceanographic Commission's GLOSS program (see figure 2.3). This program connects tide gauges around the world for such projects as monitoring regional and global increases of sea level and the control of sea surface topographic models (PUGH, 1987a & 1987b; BOUCHER, 1987).

Many individuals, organisations and institutions were approached by the Public Works Department to provide input into the planning, execution and post-processing phases of this project. The survey took place over 17 days in May 1987 and occupied 40 stations (38 coastal and 2 inland). The campaign used 5 Trimble 4000SX single frequency receivers. The success of this GPS survey depended on its thorough planning. This planning and the campaign's execution are outlined in Chapter 4.

After the survey was completed, the data collected in the campaign required processing. The determination of coordinate values from GPS observations is a least squares process which is, in many ways, analogous to determining coordinates by terrestrial geodetic survey methods. Both require observations to be modelled, parameters estimated and their quality analysed. Generally a GPS campaign can be divided into two distinct parts. The first part is to determine the coordinates of the sites within one observation session. The second is to combine these sessions into a network. The nature of correlations is of particular importance in this process when studying error propagation through the system, since it affects both the precision and accuracy of the final solution. Chapter 5 explains the concept of correlations and discusses it in relation to the GPS data reduction process, and in particular its effect on the different methods presently available to process carrier phase observations. For example, as there were more than 2 receivers operating simultaneously in the PWD coastal survey, both multi-station and multi-baseline solutions are possible. BATCH-PHASER (JONES et al, 1987) and TRIMVEC™ (TRIMBLE, 1986) reduction software are used on the Public Works Department's Trimble 4000SX survey to test different methods of solution.

Once the parameters of each session have been estimated, the next task is to combine the sessions together into a network adjustment. In Chapter 6 both the multi-station and multi-baseline session solutions are analysed to form the best network solution. Existing network adjustment software packages NEWGAN (ALLMAN, 1988a) and SHAKE (ECKELS, 1987) are used to study the effects of these different GPS session processing strategies in a final network solution. The problem of combining various GPS receiver networks together is discussed and the optimal combination of the Public Works Department's Trimble 4000SX network with the Central Mapping Authority's Texas Instruments TI4100 dual frequency network (measured in September 1987) is developed.

Once the GPS network has been adjusted, the ellipsoid heights related to the earth centred satellite datum (WGS84) can be determined using established transformation parameters. However these ellipsoid heights have to be converted to orthometric heights that are related to the geoid, before they are of any use in water engineering studies. This requires the determination of the geoid-spheroid separation (N). Various methods have been employed in recent years and these are outlined in Chapter 7. The combined approach using a higher degree geopotential model and the gravimetric ring integration technique (RINT) advocated by HOLLOWAY (1988) and KEARSLEY (1985;1988b) has been adopted and the GRAV series of programs developed at the School of Surveying at the University of New South Wales have been used to calculate N values for the coastal network. Since the absolute values for the WGS84 ellipsoid heights and the geoid-spheroid separations have systematic errors due in part to the uncertainty of their origins, the differential values between stations will minimise this error and accuracies in the range of 2-4 ppm can be expected in all values (KEARSLEY, 1988b; HOLLOWAY, 1988). These ΔN values have be subtracted from the differences in the GPS ellipsoid heights (Δh) to produce differential orthometric heights (ΔH).

In Chapter 8 the GPS/RINT derived orthometric heights are compared to AHD values at the tide gauge benchmarks. These heights are then transferred to the Mean Sea Level calculated at each tide gauge and analysed.

Chapter 9 draws some conclusions on the merits of the GPS technique for accurate height determination. It also suggests a solution to the Public Works Department's coastal datum problem.

2. HEIGHTS, MEAN SEA LEVEL AND GEODETIC LEVELLING

2.1 INTRODUCTION

The science of geodesy involves the study of the earth's shape and requires a coordinate reference system to compute and record the position of points on or near the earth's surface. This coordinate system must firstly have its datum defined.

The word "datum" is defined by WELLS et al (1986) as "a surface of constant coordinate values". These coordinates are defined by imposing constraints, ie. by holding an appropriate number of parameters fixed, that allow points to be uniquely described. While geodesy provides position in three dimensions, this study is primarily interested in the vertical component.

2.1.1 Vertical Datum Definitions

Returning to the definition by WELLS et al (1986), a vertical datum can be defined as "a surface of zero heights". The height of a point can be defined as "the distance between a surface (usually an equipotential surface) through the point in question and a reference surface, measured along a line of force or along its tangent" (MUELLER & ROCKIE, 1966). Under this definition, there can exist a number of height systems with different measuring techniques, datum concepts and definitions. HOLLOWAY (1988) suggests that when quoting the height of a point, the strengths and weaknesses of the implied height system should be understood. This is particularly important if two or more height systems are to be accurately compared.

Therefore in geodesy, heights can be conveniently described above the vertical datum defined by an ellipsoid, through a set of geometric parameters. These parameters may have been determined for a region such as the Australian Geodetic Datum (AGD), or for a global system defined from satellite tracking stations such as the GPS WGS84 reference system. However if the approach of oceanographers is taken, there are no geometric parameters defined, but instead the water column density is used to determine height as a function of the pressure differential between the point and a reference surface that is assumed to be of constant pressure. Here the vertical datum of "zero heights" is based on the assumption that the pressure is constant along any geopotential surface deep down in the ocean (over 1000 metres) (GODFREY, 1973). Therefore this can be described as a "barometric" and not geometric method of defining the vertical datum and heights. Similarly when defining topographic relief, the heights on the map are related to a vertical datum that is defined by a gravitational equipotential surface that best equals mean sea level.

One of the most important concepts to be understood in the definition of heights and height systems is that of "absolute" and "relative" heights. This is particularly important when looking at how accurately the heights and reference surface (or datum) can be defined. **Absolute heights** are defined as heights above the reference surface calculated from direct and independent measurements such as barometric pressure or GPS codes (see Chapter 3). The errors in such a height system are site dependent and generally physically correlated by the measurement techniques and instrumentation used, as well as how accurately the datum can be defined. **Relative heights** are heights determined by measurements made relative to one or more fixed points (ie. bench marks) that have some known relationship to the reference surface. The measurement techniques used, such as spirit levelling and trigonometric heighting, cannot access the reference surface (or datum) directly so their measurements are said to be made "relative" to the datum. Errors are propagated through the system such that they become larger the further from a known point. Additional error sources are introduced by the accuracy of the known points in relation to the reference datum. So for example, while the ellipsoid heights calculated directly from the GPS codes can be defined as an absolute height system, the definition of GPS heights become more complicated when the GPS carrier phase measurements are used in a differential or "relative" mode from certain fixed stations in a network, as described in Chapters 5 and 6. This means that GPS heights can also be described in a relative height system.

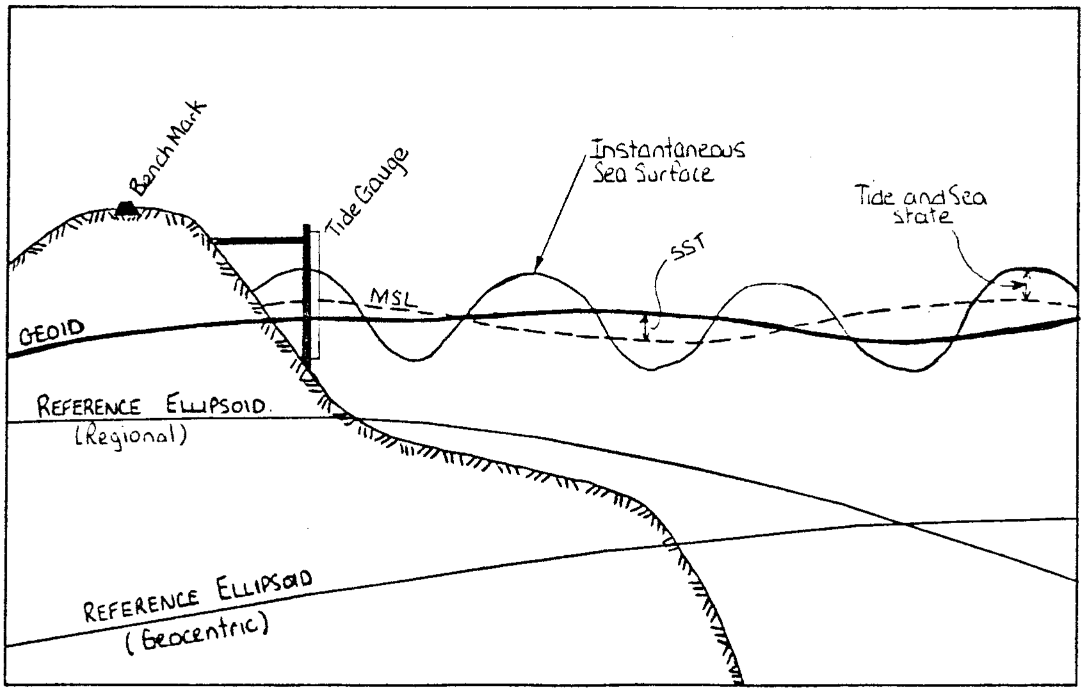
Furthermore, these height systems and their datum definitions (both absolute and relative) can be considered "artificial" or "natural". Some are "artificial" because they are developed as mathematical approximations of the earth's complex shape. These heights will refer to a reference surface usually defined by an ellipsoid. Others are considered "natural" since they are based on the earth's physical mensuration (HOLLOWAY, 1988). These natural systems are important in engineering and scientific studies, because they have the inherent attribute of being able to show in which direction water will flow. However, these natural height systems are "time specific", as for example, time variations of the gravity field may change the height reference surface position in space. This further complicates height comparison studies over long periods (TORGE, 1986).

The ability of the free water surface to define a level or "natural" geopotential surface was recognised as far back as around 3000 B.C. when the Egyptians developed irrigation systems for their crops along the Nile River. In recent times, investigations such as by OTWAY (1986) have accurately monitored vertical crustal movements in the Taupo Fault Belt in New Zealand by using lake level observations. An extension of this concept is to use the ocean surface, but the ocean is influenced by many factors which place it in continual motion.

If the oceans were comprised of a homogeneous body of water in hydrostatic equilibrium, then under the influence of the earth's gravity field, it would coincide with a surface of constant geopotential. Unfortunately the oceans are not homogeneous nor are they in hydrostatic equilibrium, hence the sea surface deviates with time and position from the gravitational equipotential surface called the geoid.

Oceanographers explain the deviations between sea level and the geoid as being caused by various physical effects on the sea such as the tidal potential, atmospheric pressure and temperature, and salinity variations in the ocean. As a result sea level measurements have been averaged to derive a "Mean Sea Level" surface usually assumed to be coincident with the geoid. Figure 2.1 depicts the relationship of the geoid and ocean surface.

Figure 2.1 Vertical datums



While the geoid is conventionally defined as the equipotential surface of the earth's gravity field corresponding to Mean Sea Level, this definition is only adequate for geodetic considerations that require resolution to ±1-2 metres. Significant inconsistencies arise if it is extended to instances where the required precision is ±0.1 metres (MATHER, 1975). A more specific definition of the geoid which is acceptable to both oceanographers and geodesists is (MATHER, 1978):

"The geoid for a specified epoch, is the surface of the earth's gravity field which best fits MSL as sampled over the global oceans on an equal area basis."

Evidence indicates that local MSL estimates do not define a geopotential surface by an amount equal to the quasi-stationary sea surface topography (SST) (MATHER, 1978). SST can have a magnitude of up to about two metres (MATHER, 1975; 1976; RIZOS, 1980; COLEMAN, 1981; CHELTON & ENFIELD, 1986; PUGH, 1987b) and is due to established differences in the influences that are responsible for ocean dynamics, such as the prevailing wind patterns, barometric pressure, salinity and thermal structure of the ocean. Up to now this evidence has come from three independent sources:

a) the comparison of free net adjustments of geodetic levelling with MSL as estimated at tide gauges along the coastlines.

b) the estimates of sea surface topography in open oceans from oceanographic considerations using steric levelling techniques.

c) the estimates of sea surface topography using satellite altimetry.

While the first method is fairly well known to geodesists, and has led to a large amount of research (see 2.1.2), the steric levelling method is primarily used by oceanographers in open ocean regions were tide gauges are not available. It utilises the difference in pressure potentials that exist in the oceans, to develop dynamic heights related to some reference surface (usually deep within the ocean), where it is assumed the temperature and salinity of the ocean are constant. Simply the surface of low density ocean water will be elevated with respect to the surface of high density ocean water. This method is discussed further in 2.2.2.

The third method uses the remote sensing technique of satellite altimetry which relates the sea surface to a satellite derived datum. Although recent technological advances in satellite altimetry allow measurements of sea level to a sub-decimetre precision and offers many advantages over steric levelling in sea surface studies (CHELTON & ENFIELD, 1986), its specific application to comparisons with spirit levelling (ie. at the coastline) is far from straightforward and is beyond the scope of this study. Therefore further discussions will be primarily limited to tide gauging, geodetic levelling and steric levelling.

2.1.2 The Sea Slope Problem

The investigation carried out by the PWD along the NSW coast in 1987 uncovered significant variations in the trend between MSL determined at each gauge and that implied by the existing spirit levelling that relates the tide gauges to the Australian Height Datum (see section 2.5). The deviation of the datum surface defined by mean sea level at the tide gauges from that defined by geodetic levelling is commonly referred to as the "sea slope problem". Found along coasts that run broadly north-south rather than east-west (DONE, 1984), the magnitude of this slope varies greatly around the world and has been investigated by various authors including

ROSSITER (1967)(Europe), MITCHELL (1973)(Australia), FISCHER (1976) and BALAZS & DOUGLAS (1979)(West Coast of USA), CASTLE & ELLIOTT (1982)(Australia & USA) and BAKER (1989)(British Isles).

There is much debate on the reasons why this sea surface slope varies around the world. Geodesists believe that the quasi-stationary SST is reflected in attempts to relate geodetic levelling to local mean sea level (LMSL) defined at tide gauges. However, by using steric measurements between the gauges, oceanographers generally believe that while these deviations are latitudinally dependent (CASTLE & ELLIOTT, 1982), the deviations are too large to be explained by oceanographic effects alone (LENNON, 1988) and must be largely due to systematic errors in the geodetic levelling. For example, BAKER (1989) showed mean sea surface slopes around the British Isles of less than 1cm/100kms using three different oceanographic methods as compared to 5.3cms/100kms defined by the third order levelling network.

The purpose of this chapter is therefore to investigate whether MSL and geodetic spirit levelling can be related accurately and assess the discrepancies that exist along the NSW coast. CASTLE & ELLIOTT (1982) point out that to understand the problem, one must study the sources and magnitude of the errors in the three basic measurements. These are:

1) the determination of local mean sea level at the tide gauges

2) the spirit levelling height differences between tide gauges

3) the sterically determined estimates of height differences between tide gauges.

2.2 MEASURING SEA LEVEL HEIGHTS (SLH) AND MEAN SEA LEVEL

While Mean Sea Level (MSL) can be defined as the average level (or height) of the sea surface taken from a series of readings usually over a long period of time, the accuracy of such a definition depends largely on the measurements used, instrumentation and the relationship to a specific datum. These readings are normally taken from tide gauges and this average level of the sea surface must be monitored relative to some fixed and accessible datum (PUGH, 1987a; 1987b). However other methods such as using pressure measurements and satellite altimetry are also available to calculate a mean position of the sea surface. To compound the problem, the sea has forces acting upon it and their effects must be taken into account.

2.2.1 Tide Gauges

The most popular method of measuring sea level heights (SLH's) is to place a gauge on a stable platform and record the subsequent sea level movements. In most cases this tide gauge will be attached or adjacent to the land so that it can be monitored relative to stable bench marks. This monitoring is generally carried out on a regular basis using conventional spirit levelling techniques.

Historic and economic factors have influenced the location of most of the established tide gauges around the world. The longest series of sea level observations are found in Amsterdam and Stockholm (since 1774) (EKMAN, 1986a). These observations provided the information to predict water depths that assisted mariners in the safe passage of their ships and were made because of the economic importance of shipping to the two cities. In Australia tide gauges have mainly been operated by port authorities, again for practical purposes such as assisting navigation (LENNON, 1987), with the earliest automatic gauge installed in Williamstown, Victoria, in 1859 (ROELSE, 1975).

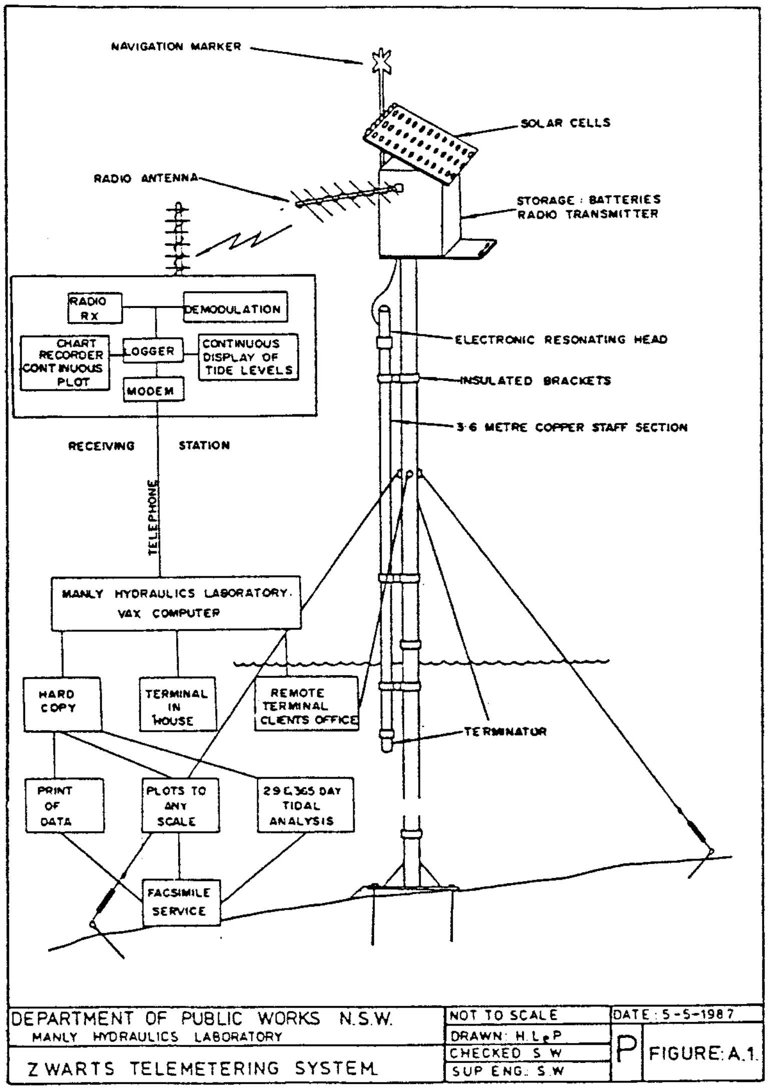
Recently, various groups have used sea level data from tide gauges for intense scientific research. Geophysists have used apparent secular changes in MSL, calculated from tide gauges measurements, to analyse vertical tectonic movements. YAMAGUTI (1971) used exaggerated changes in yearly mean sea levels to show increased vertical movements of the earth's crust before great earthquakes. EKMAN (1986b) used long term mean sea level records from 20 stations in Sweden to calculate the apparent postglacial uplift of the land over the past 100 years.

Surveyors have used the mean sea level information extracted from tide gauge measurements in conjunction with spirit levelling, to define local and national height datums such as the Australian Height Datum (AHD). Engineers and scientists are using increases in sea level heights to warn of coastal flooding as in the North Sea (PUGH, 1987a) and to help determine coastal erosion rates (GORDON, 1987).

The original tide gauges of the 18th and 19th century such as those of Stockholm and Amsterdam required someone to manually record the heights at regular intervals. Advances in technology saw the introduction of analogue tide recorders which allowed automatic recording of sea level heights through the use of a mechanically driven pen that plotted the heights on graphed paper that in turn was driven by a timing mechanism. Many of the permanent tide gauges around Australia and the world are still of this type. Unfortunately analogue gauges require continual maintenance to replace paper, pens and adjust the timing mechanisms. Records from these gauges are notoriously unreliable and exhibit low precisions.

More recently, the use of silicon chip technology has produced digital recorders. These recorders utilise electronic recording into solid state memory. Some gauges have a direct data link through telephone lines, with a computer such as that installed at the PWD's Manly Hydraulics Laboratory in Sydney. This type of recorder has eliminated many of the problems associated with the older analogue recorders. They are very reliable with a stated precision (1 sigma) of a few centimetres and require only limited maintenance and calibration. An example of a typical digital tide recorder developed by the PWD is illustrated in Figure 2.2.

Figure 2.2 Zwarts Telemetering System after PWD (1987a)



In NSW, the Maritime Services Board's of Sydney and Newcastle take measurements of local sea level heights for the purpose of providing safe navigation within their respective ports. The remainder of the NSW coastline and tidal waters are administered by the PWD and these tide gauges are predominantly used for engineering studies (PWD, 1987a).

2.2.1.1 Tide Gauge Networks

Relating tide gauges together into a network with a single vertical datum has the advantage of being able to analyse the sea level movements over various areas, from a local estuary to the size of oceans. There are various ways that these gauges and their local datums can be related.

a) No Mathematical Connection - here MSL is assumed to be the geoid. It is calculated at each gauge and movements of the sea surface against each local MSL are assumed to be absolute movements of the ocean. WALLACE (1981) uses this method to transfer hydrographic survey datum between tide recorders. One major problem is that sea surface topography (see 2.2.3) will affect the MSL determinations.

b) By Geodetic Levelling - here the tide gauges and their adjacent bench marks are connected by spirit levelling. This network allows the tide gauges to be accurately related to the geoid. Problems occur in the possible systematic errors and undetected blunders which may be present in the geodetic levelling (see section 2.3 and 2.4). Further, spirit levelling requires intervisibility between levelling staves to transfer height. This limits the technique to within continents, and to islands that are close together.

c) By Steric Levelling - this technique is used by oceanographers and is based on determining pressure potentials, where the absolute height is calculated from a reference surface defined by a certain pressure. This reference surface is defined at large depths (> 500 metres) where it is assumed the pressure is constant (see section 2.2.2). Problems exist when this technique is applied to coastal areas where the effect of the coast and seabed topography disturb this reference pressure. CASTLE & ELLIOTT (1982) believed that steric estimates could only be expected to have accuracies of at best 5-10 centimetres.

d) By Satellite Geodesy - the advent of measurement techniques such as VLBI and GPS allow tide gauges to be connected to an absolute height system based on an earth centred datum, eliminating problems of local and regional tectonic movements that contaminate the tide gauge observations. By using gravity data to convert these ellipsoid heights to orthometric heights, the sea level movements can be related to the geoid (see Chapter 7).

2.2.1.2 <u>The GLOSS Network</u>

MACLEOD et al (1988) lists some important applications in which regional and global sea level measurements are being presently utilised. These include predictions of climatic events such as El Nino, monitoring the "Greenhouse Effect" and calibration of satellite altimeters. The Intergovernmental Oceanographic Commission's (IOC) GLOSS Network (Global Level OF the Sea Surface) is one attempt at establishing a sea level monitoring system on a global basis (see Figure 2.3). It acknowledges the need to use space geodesy techniques such as VLBI and GPS to define a global datum which can relate all sea level measurements together. GLOSS is expected to help such projects as the scientific

Figure 2.3 The GLOSS network

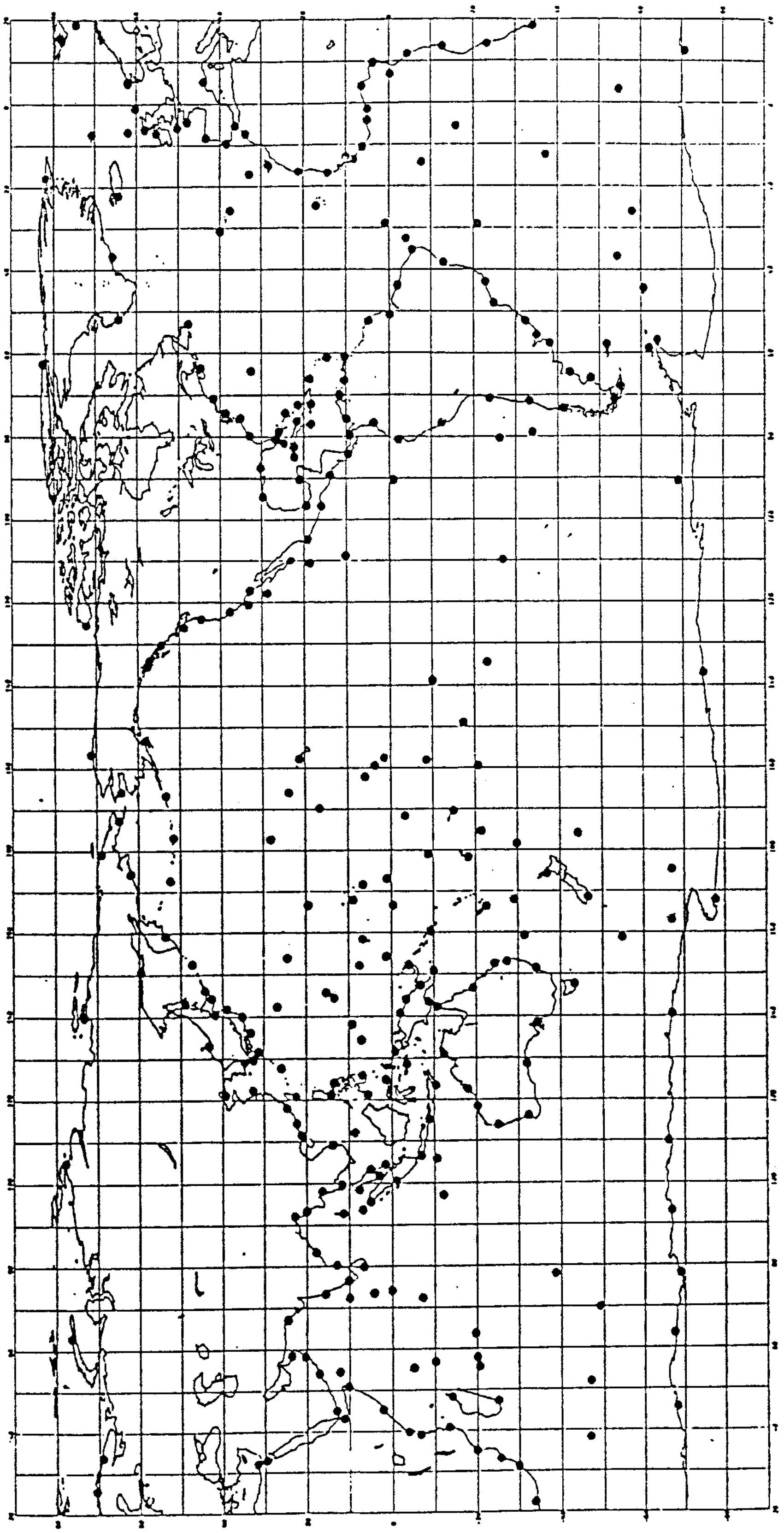


Fig 2.3: Gauges proposed for inclusion in the GLOSS Network

experiments of Tropical Ocean & Global Atmosphere (TOGA) and World Ocean Circulation Experiment (WOCE) (PUGH, 1987a).

The GLOSS proposal was approved by the IOC in March, 1985 with the implementation expected to take 5 years. Details of the proposal can be found in the IOC's official GLOSS documentation. PUGH (1987a) summarises the major points as:

1) Set up a primary network of about 250 stations around the world. They are to be separated by about 1000 kilometres along continents and 500 kilometres in island groups.

2) Gauges are to be accurate to 10 millimetres in height and 1 minute in time.

3) All gauges are to be linked to stable bench marks and be monitored on a regular basis.

4) The data collection format and acquisition procedures should be standardised and monthly means are to be sent to the Permanent Service of Mean Sea Level (PSMSL) at Bidston, United Kingdom.

5) Assist in training to establish and maintain sea level recorders.

6) Encourage the placement of regional networks that would provide more concentrated information than the GLOSS network. These regional networks would provide analysis of local problems such as storm surges.

At present 50 countries (including Australia) have agreed to contribute to the GLOSS network. The Public Works Department of NSW already supplies information to the PSMSL and has recently endorsed the aims of GLOSS. It will assist the GLOSS program by utilising the regional network in place along the NSW coast.

2.2.2 Steric Levelling

While the absolute and relative SLH's around coastal waters are easily measured by tide gauges, relative SLH measurements in open oceans are difficult to obtain because of the lack of islands on which tide gauges can be installed. The usual method of estimating open ocean SLH's is to measure the "steric" sea level computed from the vertical distribution of temperature and salinity. This is based on the hydrostatic equation, where the pressure (p) at a point P is given by

$$p = \bar{\rho}\,\bar{g}\,h \qquad (2.1)$$

where p = pressure
$\bar{\rho}$ = mean water density along the vertical above P
$\bar{g}$ = mean gravity above P
h = height

The steric levelling equation can be developed from this hydrostatic equation using dynamic heights (based on pressure potentials) with the positive z axis upward, and is given by (MITCHELL, 1973; CASTLE & ELLIOTT,1982; CHELTON & ENFIELD, 1986) such that

$$\frac{dp}{dz} = -g\,\rho\,(z) \qquad (2.2)$$

Integrating from a reference pressure surface p_0 at a specific depth z_0 which is assumed to be of constant density and pressure, to the sea surface which has been corrected for the inverse barometric effect (see section 2.2.3.2) where the water pressure is zero, gives the steric height of the sea surface relative to z_0.

$$z_s - z_0 = -\frac{1}{g} \int_{p_o}^{0} \alpha \, \delta p \tag{2.3}$$

where $\alpha = \frac{1}{\rho}$ = specific volume

Oceanographers including LENNON (1987, 1988) and WYRTKI et al (1988) are using variations in sea level measurements determined by steric techniques to help monitor temperature changes in the oceans. These temperature changes can be related to ocean currents and climate patterns such as El Nino (PARIWONO et al, 1986).

The general problem with this approach is that the hydrostatic equation requires equilibrium which is said to never exist in the oceans. The specific volume is a function of temperature, salinity and pressure. Only where the reference pressure p_0 coincides with a gravitationally equipotential surface will steric heights correspond to absolute topography of the sea surface. In deep water (500 m to 2000 m) this most nearly occurs with accuracies of 5-10 cm achievable (GODFREY, 1973; CHELTON & ENFIELD, 1986). However the effect of the sea bed near coastlines on p_0 and other factors in ocean/coastal boundary conditions make this method difficult to use in coastal waters with the result that the accuracies are severely degraded when extrapolated to the tide gauges (CASTLE & ELLIOTT, 1982; PARIWONO et al, 1986).

Another problem highlighted by CHELTON & ENFIELD (1986) is that the steric levelling technique generally uses temperature and salinity observations that have been taken from ships. This data is built up over many years and therefore cannot adequately resolve temporal variability of the sea surface except over long time scales.

DONE (1984) notes that the relationship between MSL as measured at tide gauges and MSL in the open oceans is problematic. The types of measurements are quite different, as are the sea surface models that are developed from them. Tide gauge measurements being coastal, local and usually long term, vary greatly from steric measurements that are comparatively sparse, often seasonally effected with salinity and temperature variations, taken at various depths below a vessel which is anything but stationary. From the accuracies achievable in deep water and the degradation of steric results toward the coast, it can be seen that steric levelling may not be the most suitable method of relating tide gauge datums.

2.2.3 Physical Effects on Measured Sea Level

While accurately relating the tide gauges together is an important aspect in the resolution of the sea slope problem, the problem is compounded by the tide gauge measurements themselves. There are different physical phenomena that cause both time and position deviations of the sea surface. These influences, and estimates of their magnitude are given by various authors including MITCHELL (1973), CASTLE & ELLIOTT (1982), MERRY & VANICEK (1983), CHELTON & ENFIELD (1986), PARIWONO et al (1986) and PUGH (1987b). The physical effects on the sea surface measured from tide gauges can be broadly divided into 7 groups:

1) Tides
2) Atmospheric Effects such as temperature, pressure and wind
3) Ocean water density and currents
4) Seabed and coastal topography
5) Secular variations
6) Tectonic forces
7) Other effects eg. Tsunamis, Tide Gauge Faults, Run-Off

2.2.3.1 Tides

The simplest of all signals to detect in tide gauge records are astronomical tides (CHELTON & ENFIELD, 1986). The tides are primarily controlled by the gravitational attraction of the moon and sun, and since their motions are known precisely, it is possible to compute predictions of theoretical heights of the tide at any time. DOODSON (1922) decomposed the potential into discrete frequencies and amplitudes and defined 389 components.

However, the earth is not covered by a uniform layer of water due to continental boundaries, complex sea bottom topography in near shore regions, and the earth's rotation. The discrepancies between the simplified ocean models and real oceans are so large that these dynamic predictions are essentially useless (MITCHELL, 1973; CHELTON & ENFIELD, 1986).

A number of methods to reduce the effect of tides on sea level measurements are explained in MITCHELL (1973, pp 84-85) and listed below.

1) Planimetric Integration of the area under the curve obtained from tide gauge records.

2) Mean Tide Level (MTL) - the mean of a number of successive high and low tides. Not as accurate as conventional MSL.

3) Mean Sea Level (MSL) - the mean of regular measurements of sea level heights take over a period of time. The commonly used MSL's used by the Department of Public Works are:

a) 15 minute or hourly readings calculated from 1 minute gauge readings.
b) Daily MSL calculated from 15 minute or hourly mean gauge heights.
c) Monthly MSL calculated from hourly mean gauge heights (MMSL).
d) Annual MSL from MMSL's

4) Numerical Filters (for the theory see MITCHELL, 1973, pp.78-83).

5) Harmonic Analysis - where tidal constituents of a gauge are determined empirically, by observing over a sufficient time period to resolve the principal constituents - usually a few months to more than a year (CHELTON & ENFIELD, 1986).

While CHELTON & ENFIELD (1986), LENNON (1988), and other oceanographers used harmonic analysis and other numerical filters to eliminate tidal effects, the MSL method from meaning MMSL's has been used in most comparisons with geodetic levelling including MITCHELL (1973); BALAZS & DOUGLAS (1979) and CASTLE & ELLIOTT (1982). The method of using MMSL eliminates tidal fluctuations which have a period of less than one month, however tides with a period longer than a month will still be present. The known tides with amplitudes of over a millimetre are the solar (semi-annual) (Ssa) tide, the solar annual (Sa), the nodal tide and the pole tide. Their periods are 182.6211 days, 365.2422 days, 18.6 years and 428 days respectively (MITCHELL, 1973, p.157). The nodal tide is the tidal effect created by the "regression of the nodes" being a lunar phenomenon which has a period of 18.6 years. The "Metonic Cycle" created by lunar and solar fluctuations has a period of 19 years and hence "long term" Mean Sea Level readings are taken over 19 years to account for these influences (DOODSON & WARBURG, 1941). The magnitude of this nodal tidal is estimated in Australia by MITCHELL (1973, p.158) to be up to 5 mm. The pole tide is caused by perturbations in the earth's rotational axis and its magnitude is estimated to be 5 mm. The semi-annual and annual tides are difficult to estimate as they are masked by millimetre sea level variations of the same frequency which are of meteorological and oceanographic origin, but theoretically they have negligible amplitude (EASTON, 1970).

Therefore the selection of the time period over which MSL is to be determined presents some problems. It should include the most important tidal constituents comprising the full tidal spectrum. Although determining local mean sea level over extended time scales ideally improves the accuracy by increasing redundancy, the accuracy will in fact diminished due to the likelihood that the MSL estimate includes eustatic movements and sharp transitions in the dynamic configuration of the ocean (CASTLE & ELLIOTT, 1982). Therefore this study along the NSW coast has utilised the MMSL values over a 19 year epoch.

2.2.3.2 Atmospheric Effects

The most significant contribution to the deviation of the sea surface from the geoid after tides is the result of atmospheric influences. They can by divided into the following groups:

a) **Atmospheric Pressure**
b) **Wind stress**
c) **Temperature**

a) Atmospheric Pressure - This creates what CHELTON & ENFIELD (1986) call the "Static Inverted Barometer Effect". The downward pressure force on the sea surface due to the mass of the overlying atmosphere results in a horizontal redistribution of the water mass. Assuming the total mass of air integrated over an ocean remains constant, the changes in SLH can be define using the hydrostatic equation (2.1):

$$\Delta h = -\frac{1}{\rho g}\,\Delta p \qquad (2.4)$$

where Δh = change in height (in cms)
Δp = change in pressure (in millibars)
ρ = water density
g = gravity

Therefore the theoretical inverted barometer response is -1.01 cm/mbar. However this factor is affected by the coastline and seabed topography, which can force SLH changes induced by atmospheric pressure to dynamically propagate poleward (CHELTON & ENFIELD, 1986). This means that gauges on the east coast of Australia will not only be affected by atmospheric pressure changes over the specific sites but also from the pressure changes occurring to the north.

Difficulty in separating the inverted barometer response from other causes of SST has been experience. MITCHELL (1973) used the theoretical value of -1.01 cm/mbar to determine corrections to tide gauge readings in Australia, but had to use interpolated data of air pressure from distant barometers. He concluded that air pressure was not a direct cause of significant variations with time. However, CHELTON & ENFIELD (1986) believed that removal of the inverted barometer effects typically accounted for 10-15% of the variance in daily SLH south of San Francisco and 50-60% to the north. Typical values of 1-10 centimetres, depending primarily on the latitude could be expected over weekly scales.

b) Wind Effects - CHELTON & ENFIELD (1986) found after correcting for atmospheric pressure in Newport, USA, that large fluctuations of 2-10 days duration remained. These fluctuations corresponded to synoptic wind events from travelling atmospheric pressure systems. The wind stress created by these pressure systems can, near the coast, be divided into onshore and alongshore components. Over the open sea, wind stress helps to create ocean currents (see 2.2.2.3).

Over short time scales, onshore winds tend to create "pile up" when the water is pushed against an obstruction (ie. the coast). However, over longer time scales this is relatively unimportant compared to alongshore winds. Alongshore winds create dynamic height changes on the sea surface due to the phenomena of "Ekman Transport". EKMAN (1905) showed that water moves 90° to the direction of the wind, due to the Coriolis force from the earth's rotation. The movement is to the right in the northern hemisphere and the left in the southern hemisphere. This Ekman Transport is compensated by a corresponding "upwelling" or "downwelling" of the coastal SLH's.

Wind stress can also cause coastally trapped waves or storm surges which propagate out along the coast from the initial wind disturbance similarly to that induced by pressure changes described in (a). Amplitudes of up to 10-20cm are estimated for wind effects but this is difficult to quantify (CHELTON & ENFIELD, 1986; MERRY & VANICEK, 1983). Since these events are typically of 2 to 10 days duration, their signals in tide gauge observations can be greatly reduced by constructing simple MMSL's.

c) Temperature - Seasonal variations in SLH can be seen in data from many tide gauges and is explained by the changes in the thermal structure of the upper ocean from heating and cooling associated with temperature changes. Temperature is also associated with wind stress that helps to form ocean currents (see 2.2.2.3) and the "Greenhouse Effect". The Greenhouse Effect is supposedly causing a global increase in sea level, primarily due to the increase in the melting rates of the polar caps and increased volume of the heated upper layers of the oceans.

Meaning SLH data over many years (or seasons) will effectively eliminate the seasonal variations caused by these three effects. It will not however remove the long term dynamic movements.

2.2.3.3 Ocean Water Density and Currents

Ocean current flow and ocean water density are closely related as they are affected by the same parameters. It is based on the hydrostatic equation outlined in section 2.2.2.2. Current flow occurs due to variations in density of the oceans. Wind stress will redistribute the water-mass as the wind moves the less dense upper layers. Variations in temperature and salinity will also change the density of the ocean water. The coriolis effect of the earth rotation modifies the current movements, forcing them to turn. As low density water is a high point, the current flows anticlockwise and perpendicular to this flow and creates a slope which is upward from right to left when looking in the direction of current movement in the southern hemisphere.

CHELTON & ENFIELD (1986), stated that sea level slopes associated with geostrophic currents are generally very small, with a current of 2 m/s (being a very rapid flow in an ocean), at 30 degrees latitude being associated with a sea level slope of 74 cm over 100 km, while typical currents of 10 cm/sec along eastern ocean boundaries in the USA give slopes of 10 cm over 200-300 kilometres. LENNON (1988) predicts these heights along the east coast of Australia to be rather small, being of the order of 5 cm over hundreds of kilometres.

The SLH's from the tide gauges along the NSW coast have not been adjusted for ocean current/density effects in this study. This is because although current/density effects would seem to be one of the most significant aspects of sea level variations with position and time, the practical problems in measuring due to the dynamic topography correction theory being imperfect make estimates inaccurate (MITCHELL, 1973). The MSL results used in this study should therefore exhibit a geostrophic gradient equivalent to that suggested by LENNON (1988) when compared to the geoid defined by spirit levelling.

2.2.3.4 Seabed and Coastal Topography

From the above discussions, the existence of coastal and seabed topography influences the magnitude and frequency of variations in sea level height from other physical effects. Winds create surges which have a resonance depending on the shape of the bay or coastline, but are usually a high frequency signal (LENNON, 1987) and more easily eliminated by meaning readings over a long period. The shape of the seabed near the coastline causes variations in the geostrophic gradient as well as causing "pile up" as the sea is forced against the land mass.

2.2.3.5 Secular Variations

The measurement of sea level at a tide gauge is made relative to adjacent bench marks. Relative sea level changes from these gauges are assumed to indicate eustatic changes. Because the secular trends are small, record lengths of 10 years or longer are necessary for this signal to stand out above background variations. Secular changes in sea level are of great scientific and economic importance, since any increase in the current rate of sea level rise would cause the increased erosion of many sensitive and highly populated coastal areas (RAPP, 1988; CEI et al, 1987). GORDON (1987) has shown some correlation between recent increases in sea level and increased coastal erosion along the New South Wales coast.

Global movements in the volume of the oceans are currently creating a rate of eustatic increase in MSL estimated to be between 0.1 mm and 1.5 mm per year (GORNITZ, 1988; CEI et al, 1987; LAMBECK et al, 1989). EKMAN (1986a) used sea level observations from Stockholm (since 1774) to estimate that the eustatic changes in mean sea level since 550 AD have only been between -1 and +2 mm per year. LAMBECK et al (1989) suggests that the magnitude of secular rises are latitude dependent, varying from about 1.5 mm per year in equatorial regions to zero in high latitudes. However accurate monitoring of this component is needed as predictions of future annual increases of 10 mm or more are estimated due to ice melting resulting from the "Greenhouse Effect". Unfortunately it is difficult to separate the oceanographic contribution in secular variations from non oceanographic contributions such as tectonic movements as was found by HICKS & CROSBY (1975) when analysing yearly mean sea levels along the US west coast over an 80 year period.

2.2.3.6 Tectonic Forces

If sea level measurements are made in tectonically active regions such as the west coast of the USA, then the results will include movements of the land (to which the tide gauge is attached) relative to the sea, making the actual secular variations in sea level difficult to estimate.

Vertical tectonic movements require accurate monitoring so that scientists can warn of impending earthquakes and estimate their magnitude. EMERY & AUBREY (1986) used tide gauge records and found that long term vertical tectonic movements of between -9 mm to +13 mm per year were occurring on the west coast of the USA. Movements up to 30 mm per year have been noted by YAMAGUTI (1971) in Japan and Chile for a few years prior to large earthquakes. CHELTON & ENFIELD (1986) stated that datum shifts of up to 20 cm occurred just before earthquakes in the Aleutian Islands. These were followed by a steady return to normal over a period of years. Areas away from fault lines may still produce large systematic errors in MSL results due to phenomena such as post-glacial rebound, which since 1895 has caused land movements of up to 9 mm per year in Sweden (EKMAN, 1986b). In all the above studies, the separation of the eustatic component from the tectonic component has not been attempted but the eustatic movement of the ocean can be expected to be only a small component of these total movements, based on present estimates stated in 2.2.2.5.

The location of the NSW coastline on the Australian plate, well to the west of the plate boundary passing through New Zealand and New Guinea, means that it can be considered very stable. Therefore tectonic signals in the tidal data would be expected to be minimal. However the levelling used to connect the tide gauges may exhibit intrasurvey movement at junction points caused by the compaction of poorly consolidated surface material (CASTLE & ELLIOTT, 1982).

2.2.3.7 Other Effects

These effects can be classified as being caused by phenomena which are functions of location. They include river discharge, tidal gradients, tsunamis and tide gauge faults. Variations in river discharge will effect gauges which are positioned in or near rivers. Flooding will raise water levels, while fresh water will alter the density (salinity) of the near surface waters. Their resultant effects on measured sea level are again difficult to determine. MERRY &

VANICEK (1983) carried out an analysis of a number of gauges in the Maritime Provinces of Canada and found significant contributions to SST from river discharge at two sites. They where both situated at or near the mouths of rivers and the contributions where 0.085 and 0.246 metres respectively.

Tidal gradients also affect tide gauges that are situated inside rivers and bays by producing position and time variations in measured SLH's from those experienced in the adjacent open ocean. The magnitude of these variations depend on the position of the gauge with respect to the open ocean and the shape of the river or bay. The magnitude of the gradients along the NSW coastline are approximated by HARVEY (1988) and shown in Table 2.1, with the largest effect being 0.05 metres.

Tsunamis which are often incorrectly called "tidal waves", are long waves which are generated from coastal or submarine tectonic activity. These waves travel in the open ocean at velocities of between 150 and 800 km hour^{-1}, with periods from 15 to 60 minutes (MITCHELL, 1973). Their amplitude increases as they approach the coastline with sometimes devastating results. However tsunamis can be neglected as a significant effect on MMSL's because it is a periodic wave form of short duration (less than a day) and therefore substantially eliminated in the meaning process. MITCHELL (1973) pointed out that even an aperiodical tsunami of 15 cm amplitude for a full day, would affect the MMSL by only 5 mm.

Tide gauges will not necessarily accurately record the times and heights of mean sea level. The causes of faulty tide gauge readings are varied but may be divided into two groups.

1) Time errors - where the gauge clock malfunctions.
2) Height errors - where the level of water surface is not correctly recorded. This can be caused by incorrect calibration, the failure of the recording mechanism, or changes in level of the gauge due to the movement of the support structure. It may also occur when the well is clogged and the monitor cannot operate successfully.

While all tide recorders are affected by these problems from time to time, the advent of digital recorders with solid state memory and direct data linking has reduced many of these problems. Continual monitoring of the data allows problems to be solved quickly, while regular maintenance provides extended reliability.

2.2.3.8 The Net Effect

It can been seen from the above discussion that the sea surface deviates both spatially and temporally from a level surface. This is because the oceans are not homogeneous or static, so that they cannot satisfy the hydrostatic equation by which the surface of the sea would be expected to correspond to a level surface of the earth's gravitational field. Even averaging the sea surface to obtain what is known as Mean Sea Level will not produce a surface coincident with the geoid, as smaller spatial and temporal deviations in the order of +/-1 to 2 metres for the former and the latter at the sub-metre level (RIZOS, 1980, p.19) will remain. In practice, if the averaging period is sufficiently long (usually 19 years) the temporal variations can be effectively eliminated. However the spatial deviations from the geoid will remain. The deviation between the sea surface, whether instantaneous or averaged, and the geoid can be described as the "sea surface topography" or "SST" at a specific tide gauge.

Modelling of the SST to give a corrected MSL with respect to the geoid is possible with a knowledge of tidal and oceanographic parameters describing salinity, temperature, currents, barometric pressure, etc, but are most difficult to determine closest to the coastline and are only accurate to at best +/-0.2 metres (MITCHELL, 1989). More specifically, tides can be removed by harmonic analysis, numerical filtering or meaning SLH's over an extended time period. Water density as well as most atmospheric and minor effects can be modelled or minimised by averaging SLH's over long periods. Seabed and coastal topographic effects, as well as some minor effects are harder to quantify. However up until now the separation of secular variations from tectonic movements has been the most difficult to accomplish.

In Sweden EKMAN (1986a) showed a deviation of Mean Sea Level from the geoid between -5 and +15 cm. CHELTON & ENFIELD (1986) give estimates of 10 to 20 centimetres for most components that make up the SST. MITCHELL (1973) states corrections to MSL in Australia due to all influences on SST from steric levelling estimates is in the order of 10 centimetres, although the accuracy of such estimates must be questioned due to the technique's limitations and the data used. MERRY & VANICEK (1983) calculated total corrections of between -13 to +10 centimetres using a least squares spectral analysis to model the SST components of atmospheric pressure, air temperature, wind stress and river discharge for selected eastern Canadian ports. While these estimates of SST are significantly smaller than the magnitude of the spatial deviations stated earlier, MATHER (1973b) states that the while the maximum amplitude of long term SST is about 2 metres, it has wavelengths of up to 8000 kilometres. The above studies were only carried out over specific regions and would therefore be expected to show regional correlation. The smaller values of SST could also be due to systematic errors in the measurements used, and the limitations of these SST studies to calculate all SST effects.

The long term SST in Australia (after taking at least a 19 year mean) is such that MSL deviates from the geoid by slopes of the order of 1 in 2000000 (MITCHELL, 1989). This is predominantly in a north-south direction (LENNON, 1988) and a maximum separation of the order of a metre or so must be expected. However this does not explain the large discrepancies between MSL at some tide gauges and the Australian Height Datum (AHD) that has been determined from spirit levelling which is suppose to define the geoid. More particularly, along the NSW coastline the SST estimates of CASTLE & ELLIOTT (1982) would seem to still show discrepancies with that determined from spirit levelling. Therefore the spirit levelling technique should be investigated.

2.3 SPIRIT LEVELLING

The process of defining elevations on the earth's surface by spirit levelling is outlined in any treatise on surveying including BOMFORD (1977). Briefly described, it involves the setting up of a levelling instrument so that the line of sight is tangential to the equipotential surface of the earth's gravity field at the instrument. Observations are made to graduated staves that stand vertically at points between which the elevation difference is required. The process can be extended over the area required and if the height of one or more points are related to a specific datum that is defined by absolute measurements eg MSL, then the height differences obtained can be propagated from these points to define a system of relative heights.

A study of this operation reveals that it contains assumptions which are not completely correct, especially when applied over a large area. In particular, since levelling combines the method of defining linear displacements along the vertical with a method of defining an equipotential surface at the instrument, then the assumption that the equipotential surfaces are separated by an amount which is directly proportional to their potential differences (ie. equipotential surfaces are parallel) is not correct (MITCHELL, 1973). An equipotential level surface can be considered a level surface at which the gravitational potential is constant. This means that these surfaces will convergence toward the poles due to the earth's oblatness in the global case and can converge or diverge due to mass irregularities inside the earth and the earth's rotation in the regional case (NIEMEIER, 1986). Therefore, summing spirit level height differences around a loop, will not necessarily produce a zero closure. A correction has to be applied to the height increments to convert them to orthometric heights.

2.3.1 The Orthometric Height System

HOLLOWAY (1988, p.4), notes that the various height systems related to the geoid, differ only by how they scale the geopotential number. The geopotential number (C) is a method to define a height of a point (P) with potential (W_P) relative to a reference potential surface (in this case the geoid) denoted as W_0 and is given by (HEISKANEN & MORTIZ, 1967, p.162),

$$C = W_0 - W_P \qquad (2.5)$$

$$= -\int_0^P dW \qquad (2.6)$$

$$= \int_0^P g.dh \qquad (2.7)$$

where g = gravity
h = height (normal to the potential surface)

Unfortunately the geopotential number is in units of kGal metre and has no geometrical meaning. Other height systems such as Dynamic, Orthometric and Normal heights have been developed, in part, to overcome this problem. Definitions of these systems can be found in HOLLOWAY (1988) and NIEMEIER (1986). However for surveyors, it is the orthometric height system which is "geometrically and physically significant" (HEISKANEN & MORTIZ, 1967, p.172) since the heights are the natural heights above sea level (more correctly the geoid).

The orthometric height is defined as:

$$H^0 = \frac{C}{\bar{g}} = \frac{1}{\bar{g}} \int_0^P g.dh \qquad (2.8)$$

where $\bar{g}$ is the mean actual gravity along the curved plumbline between the geoid and the point P on the earth's surface.

HOLLOWAY (1988) points out that $\bar{g}$ is difficult to measure but some methods can be used to approximate $\bar{g}$ which lead to "a special kind of orthometric height". One such method calculates the mean normal gravity ($\bar{\gamma}$) instead of the mean gravity ($\bar{g}$). This height system is called Normal Orthometric Height (BOMFORD, 1977) and expressed as:

$$H^{NO} = \frac{C}{\bar{\gamma}} \qquad (2.9)$$

2.3.2 Orthometric Correction

As mentioned earlier, because of the non parallelism of the equipotential surfaces, the observed height increments (dh) from spirit levelling require an orthometric correction to convert them to orthometric height differences.

$$\Delta H^{O}_{AB} = dh^{meas}_{AB} + OC_{AB} \tag{2.10}$$

where OC_{AB} is the orthometric correction between points A and B on the earth's surface and is given by HEISKANEN & MORTIZ (1967, p.168) as:

$$OC_{AB} = \int_{A}^{B} \frac{g - \gamma_\phi}{\gamma_\phi} . dh + \int_{0}^{A} \frac{g - \gamma_\phi}{\gamma_\phi} . dH_A + \int_{0}^{B} \frac{g - \gamma_\phi}{\gamma_\phi} . dH_B \tag{2.11}$$

$$OC_{AB} = \sum_{A}^{B} \frac{g - \gamma_\phi}{\gamma_\phi} . dh + \frac{\bar{g}_A - \gamma_\phi}{\gamma_\phi} . H_A + \frac{\bar{g}_B - \gamma_\phi}{\gamma_\phi} . H_B \tag{2.12}$$

where $\bar{g}_A$ and $\bar{g}_B$ is the mean gravity along the plumbline at the point A and B on the earth's surface and the geoid.

γ_ϕ is the normal gravity at a reference latitude.

Once again a knowledge of the mean value of gravity on the plumbline is required. However errors caused by an inexact knowledge of the density inside the earth or the mean gravity value will be small (HOLLOWAY, 1988).

2.4 THE AUSTRALIAN HEIGHT DATUM (AHD)

The AHD was formed as a national height datum to alleviate the problems within the surveying community, caused by the multitude of levelling datums in existence before 1972. It is described in GRANGER, (1972); ROELSE et al, (1975); MITCHELL, (1973); and HOLLOWAY, (1988). Briefly, the National Mapping Council of Australia adopted the AHD in 1972 as the primary national datum over mainland Australia. It is a relative height system made up of 97,320 km of primary levelling observed between 1945 and 1970, which was simultaneously constrained to 30 tide gauges fixed at the their MSL values for the January 1966 to December 1968 epoch.

A minimally constrained adjustment was also carried out, holding Johnston Geodetic Station in the centre of Australia at an arbitrary height. A comparison of the constrained and minimally constrained adjustments is given in ROELSE et al (1975, Annex D) and shows strain in the levelling mathematical model, introduced by the height constraints of the tide gauges. The worst result was along the north Queensland coast with a discrepancy of 1.7 metres.

These discrepancies between the heights from geodetic levelling and MSL determined at the tide gauges can be attributed to the "sea slope problem" outlined in section 2.1. The possible error sources in the levelling network are examined below with particular interest given to the NSW coast (Section 2.5) while MSL has been examined in detail in section 2.2.

2.4.1 Errors in the Spirit Levelling Network

The accuracies of the observed levelling may vary greatly depending on the techniques and instruments employed. Standards have been defined to assess these levelling surveys. These standards are in the form of tolerances based on the agreement between a forward and backward level run. In Australia they are defined in the specifications of the NATIONAL MAPPING COUNCIL (1970) as:

1st order $4\sqrt{k}$ mm

2nd order $8.4\sqrt{k}$ mm

3rd order $12\sqrt{k}$ mm

where k is the distance in kilometres

The levelling making up the AHD was primarily third order standard, apart from some first order levelling along eastern NSW, and parts of Victoria and southern Western Australia. The tolerances above, were used to weight the a priori variances in the adjustment. After its unconstrained adjustment the levelling network was estimated to have an overall accuracy of $8.1\sqrt{k}$ mm and to have a maximum precision on the height difference at any tide gauge of 0.34 from the centre of the continent (GRANGER, 1972; MITCHELL, 1973; ROELSE, 1975). However these are only internal estimates of the quality of the data and do not take into account unmodelled external errors. Errors in levelling, like all surveying measurements, can be defined as gross, systematic and random errors.

Random errors are assumed to follow a "normal probability distribution function" and are generally accounted for in the least squares process.

Gross errors in the observations and reductions can be detected using geometric checking procedures before processing, such as loop closures, unless compensating errors occur. In 1975 the Department of National Mapping reobserved the lines along the north Queensland coast and differences with the original data of 0.727 between Rockhampton and Mackay and 0.539 between Mackay and Townsville where discovered. However, these new results had little effect on the resultant network loop closures since the loops were very large (NATIONAL MAPPING, undated internal report). The AHD again showed its susceptibility to compensating gross errors when RUSH (1986) used doppler observations to highlight gross errors of up to 0.4 m in the AHD network in central Queensland. From these examples, it must be assumed that there are further blunders in the AHD network which have not yet been detected.

Systematic errors in levelling can be categorised into **physical** and **mathematical** errors. A number of physical systematic errors are well known and have been documented in ANGUS-LEPPAN (1979), and PRIJANTO (1987). They can be divided into instrumental, environmental and personal errors (HOLLOWAY, 1988).

Instrumental errors include:
* compensator error in an automatic level
* misalignment of the parallel plate micrometer
* collimation error
* staff calibration error

Environmental errors include:
* the earth's curvature
* refraction effects

Personal errors include:
* reading the incorrect position on the staff

These systematic errors can be minimised by calibration and laboratory testing, and adhering to specific levelling procedures such as using equal backsight and foresight over short distances. However there are residual effects and undetected systematics that will still remain. ZILKOSKI & KUMAR (1989) point out that in 1981, the existence of a magnetic error in some automatic compensator-type levels was detected and subsequent corrections for this systematic error reduced the large loop misclosures in the 1978 southern Californian relevelling program to within first order levelling tolerances. In the case of the AHD network, a resultant systematic error of only 0.07 mm for each levelling set up, assuming a staff separation of 100 m, will result in a 2 metre difference in height over the distance equal to that of the north Queensland coastline (MITCHELL, 1973, p.205).

Mathematical systematic errors are introduced when orthometric corrections are made to the observed height differences to produce orthometric heights. In the case of the AHD network, the orthometric correction is applied as in (2.10). However it uses a formula that does not depend on a knowledge of the mean value of gravity along the plumbline as in (2.11) and (2.12). The orthometric correction used is given in RAPP (1961) and modified for Australian conditions (ROELSE, 1975) such that:

$$OC = (A.H + B.H^2)\ \delta\phi \tag{2.13}$$

where H = mean height of the two end points
$\delta\phi$ = difference in latitude of the end points
A and B are coefficients that vary with latitude and are defined by RAPP (1961) and given in HOLLOWAY (1988, p.15) using parameters based on the Geographic Reference System, 1967 (GRS67)

HOLLOWAY (1988, p.16) states that using these formulae means that the AHD cannot be defined as an orthometric height system, but is in fact a **Normal Orthometric Height** system defined by equation (2.9). This is because the orthometric correction depends upon normal gravity referred to the GRS67 ellipsoid at mean height and not measured gravity. These two systems are conceptually quite different, however they produce similar results. This is shown in investigations by MITCHELL (1973) when observed gravity was used instead of normal gravity in the orthometric correction. It was found to have a negligible effect over the Australian continent, being 11 mm north/south and 19 mm east/west.

2.5 INVESTIGATING MSL ALONG THE NSW COASTLINE

In early 1987 the PWD investigated the relationship between levelling along the NSW coastline from Eden to Tweed Heads and MSL defined at various tide gauges. This was in response to the difficulties the Department's engineers were experiencing in relating various river systems together (outlined in Chapter 1). The PWD investigation differed from that carried out by MITCHELL (1973) and ROELSE (1975) because it dealt with only the NSW coast and included both new levelling and additional tide gauge connections. Furthermore, this study used the 19 year epoch 1951-1969 to determine MSL, instead of the 1966-1968 epoch used in the determination of the AHD.

2.5.1 MSL determination

MSL values along the NSW coastline have been calculated from the SLH's measured at tide gauges installed in ports, rivers and estuaries by port authorities, the federal government's Australian Land Information Group (AUSLIG) and PWD. They are listed in Table 2.1, while further details can be found in PWD (1987a).

The Fort Denison tide gauge contains the longest set of continuous sea level data in NSW and it has been adopted as the reference gauge. Figure 2.4 shows the values of the annual MSL at Fort Denison in Sydney between 1897 and 1987 (courtesy of the Maritime Services Board, Sydney, NSW).

The overall mean of these annual MSL's is 0.915 with a standard deviation of 0.033. The range of the measurements is between -0.059 to 0.075 of this mean. A sharp rise in MSL occurs around the late 1940's and corresponds to an increase in the rate of coastal erosion along the NSW coastline (GORDON, 1987). If it is assumed that MSL has been rising at a constant rate since 1897 then we can calculate a line of

Figure 2.4 Annual MSL at Fort Dension, Sydney (1897-1987)

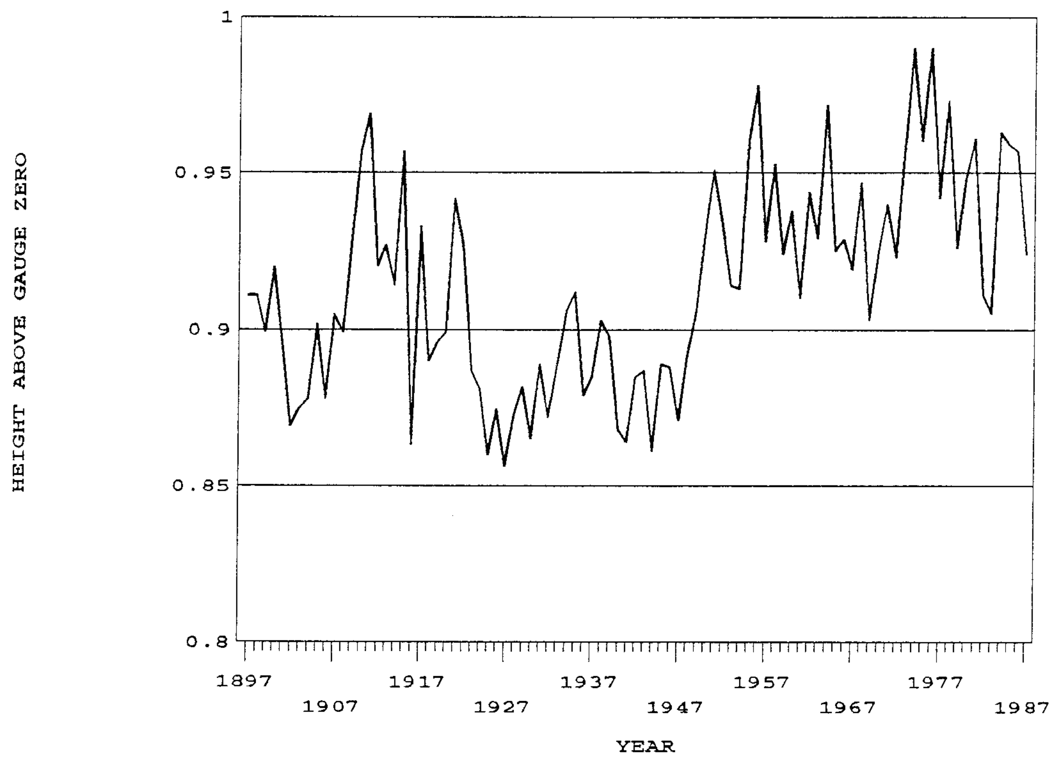
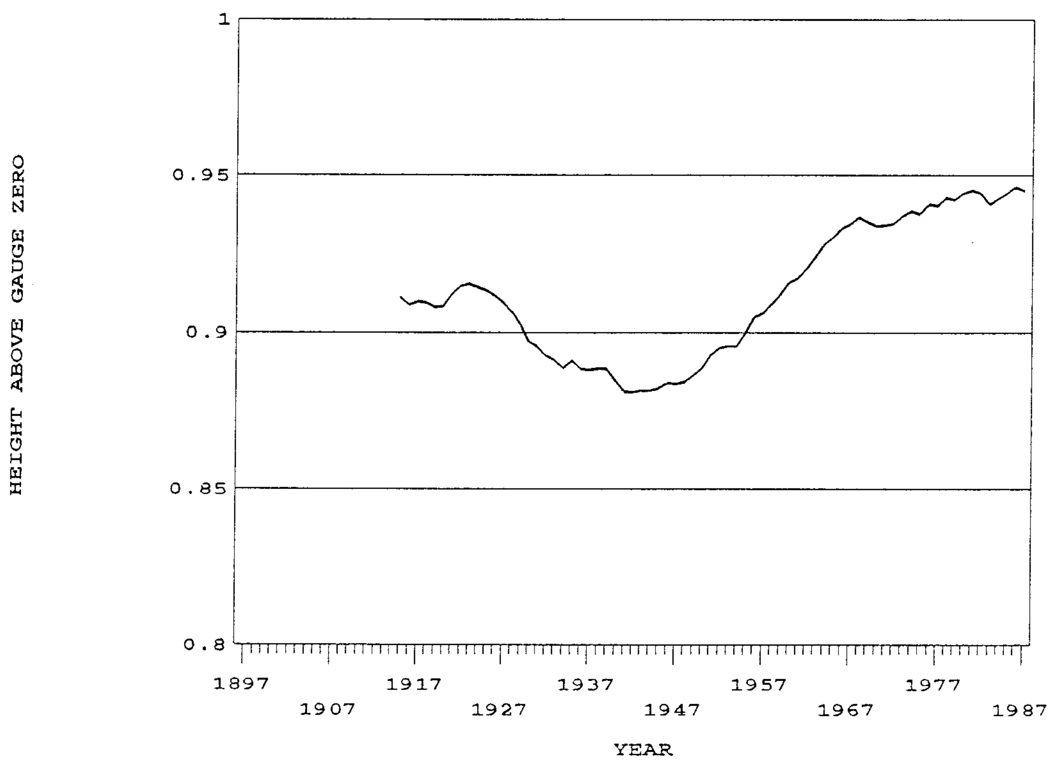


Figure 2.5 19 year MSL at Fort Denison, Sydney (1897-1987)



best fit using the linear regression technique. This results in an estimated increase in sea level of 0.6 millimetres per year with a standard deviation of 0.029 and is consistent with estimates from other studies including EKMAN (1986a), CEI et al (1987), GORNITZ (1988) and LAMBECK et al, (1989).

Table 2.1 Tide Gauges used in MSL Investigation in NSW

TIDE GAUGE LOCATION (town)	PRESENT GAUGE TYPE	GAUGE POSITION	**TIDAL GRADIENT
Eden *	Digital	Ocean Gauge	0
Bermagui	Digital	500m inside	2 cm
Moruya	Removed		
Narooma	Removed		
Batemans Bay	Digital	Ocean Gauge	0
Jervis Bay	Digital	Ocean Gauge	0
Huskisson	Digital	Ocean Gauge	0
Port Kembla	Analogue	Ocean Gauge	0
Sydney (Ft Denison)*	Analogue	1000m inside	0.3 cm
Swansea	Digital	300m inside	5 cm
Newcastle	Analogue	1000m inside	3 cm
Nelsons Bay	Digital	Ocean Gauge	0
Forster	Digital	100m inside	2 cm
Crowdy Head	Digital	Ocean Gauge	0
Port Macquarie	Digital	800m inside	2 cm
Coffs Harbour *	Digital	Ocean Gauge	0
Iluka	Digital	1000m inside	2 cm
Evans Head	Removed		
Ballina	Digital	1000m inside	2 cm
Brunswick Heads	Digital	600m inside	3 cm
Tweed Heads *	Digital	300m inside	2 cm

* used in MITCHELL (1973) and ROELSE et al (1975)
** approximate estimates by HARVEY (1988)

The MSL value at reference gauge at Fort Denison used in this study has been derived from the 19 year epoch (1951-1969). The three reasons for using the 1951-1969 epoch in this study are:

1) This 19 year epoch includes the full tidal effects created by the lunar and solar fluctuations (section 2.2.2.1) and minimises SST effects.

2) Since 1951 the annual MSL values at Fort Denison have been in a narrow range, with a mean of 0.941 and a standard deviation of 0.025, so any MSL determination after 1951 and longer than 1 year will be within a range of -0.051 to 0.049 of the adopted 1951-1969 epoch. This is consistent with other MSL studies such as VANICEK (1978) and CASTLE & ELLIOTT (1982) that showed that generally, the annual MSL's differed by only a few centimetres with the 19 year epoch. The 1951-1969 epoch at Fort Denison also falls near the middle of all 19 year epochs as shown in Figure 2.5.

3) MSL information used to constrain the Australian spirit levelling network to form the AHD, was taken from monthly means of tide gauges between January 1966 and December 1968. Therefore if an analysis of this network is to be carried out, then MSL including this same period should be used.

Many of the other gauges have not been operating over this full period, therefore their calculated MSL's have been adjusted to the Fort Denison 1951-1969 epoch. This has been done by calculating the equivalent MSL for the time period, beginning after 1951 (dt), at both the Fort Denison reference gauge and the newer gauge. Since we know the 1951-1969 epoch value at Fort Denison then the correction to the newer gauge is assumed to be the same as the reference gauge such that:

$$Z = z + (H - h) \tag{2.14}$$

where Z = the corrected (1951-1969) MSL value at the new gauge

z = the MSL value calculated over the period dt at the new gauge

H = the (1951-1969) MSL value at Fort Denison

h = the MSL value over the period dt at Fort Denison.

This formula assumes that the influences on the ocean are the same at each gauge. This assumption is not strictly correct, as each gauge experiences varying SST due to its location. There has been no attempt to adjust the MSL's for SST effects. PARIWONO et al (1986) described the SST along the NSW coast as predominately "anti-cyclone warm core eddies" that move southward and last up to a year, and there is very strong regional coherence in these SST effects along the majority of the east coast of Australia. So for simplicity, this analysis has assumed similar SST characteristics at each gauge. Therefore the resulting corrections from (2.14) have only been of a few centimetres.

No adjustment of the annual MSL values for the secular movement (estimated at Fort Denison to be 0.6 mm per year) has occurred in the analysis. This corresponds to a systematic error of less than 6 mm in the 1951-1969 epoch and is therefore considered insignificant compared to the remaining SST effects.

2.5.2 The NSW Coastal Levelling

The levelling data was taken from records at the NSW Department of Lands that had been used in the original AHD determination and subsequent revisions. Its stated accuracy is predominantly of 1st order standard. In addition, 3rd order levelling connections measured by the PWD were used to relate a number of the tide gauges to the 1st order network. The area between Eden and Coffs Harbour was made up of conventional two-way 1st order levelling carried out in the 1960's and used in the 1971 AHD adjustment. The area from Coffs Harbour to Tweed Heads includes the "one-way double run rapid 1st order levelling" carried out by the then Division of National Mapping in 1976 (now known as AUSLIG). This levelling method is described in the internal report by NATIONAL MAPPING (undated). The area north of Coffs Harbour also includes precise levelling carried out by the NSW Department of Lands in the late 1970's.

Figure 2.6 shows the relationship of the 1951-1969 MSL at each gauge as derived by the corrected normal orthometric levelling (using equation (2.10)). The reference datum is taken as a horizontal level surface through the 1951-1969 MSL value (0.936) at the Fort Denison gauge.

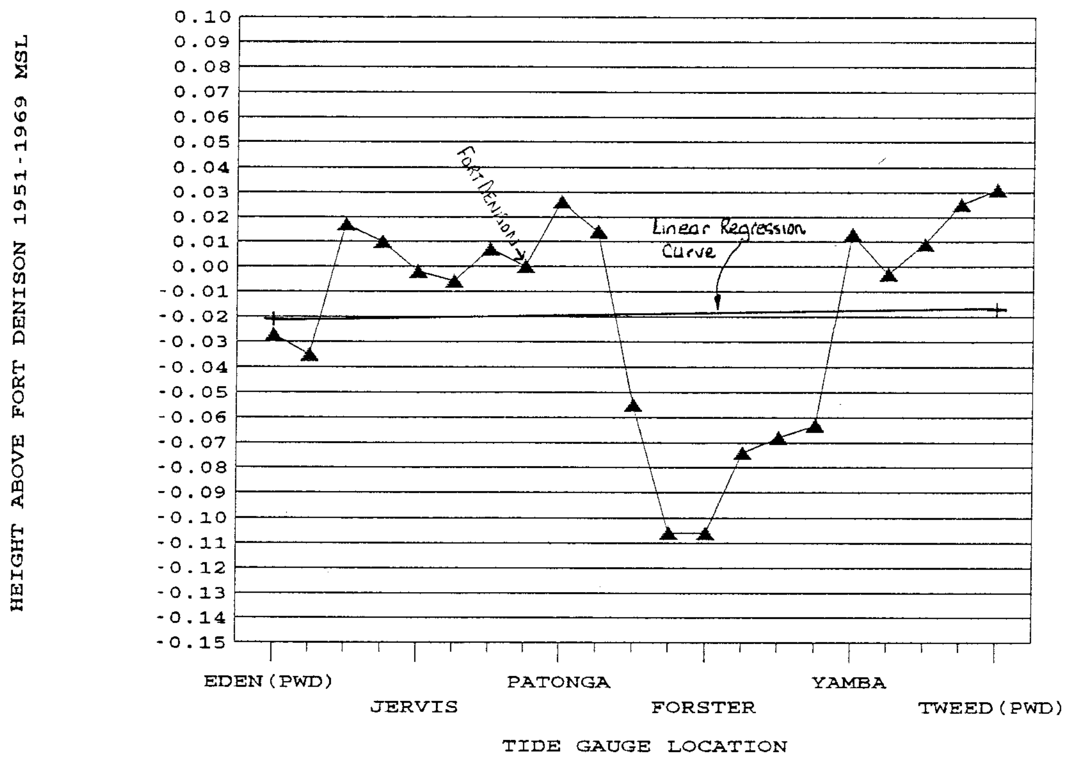
In addition AHD values for each of these MSL determinations are shown in Figure 2.7. By definition the AHD when RL=0 should be the same as MSL, but differences of between 8 mm at Fort Denison, up to 0.1 metres can be seen in the project region. These differences are disturbing, since the Department's engineers and scientists presently rely on the AHD to relate one river to another. Even an error of 0.1 metres in the estimation of flood heights can make a large error in the water volume determinations and their comparisons.

As expected the two diagrams show substantial correlation, since the AHD is determined from the spirit levelling. The exception is at Eden, where the AHD shows a difference of about 0.1 metres compared to the unadjusted levelling. The main problems highlighted by the long term MSL defined by both the unadjusted spirited levelling and AHD are:

1) the dip around Forster/Crowdy Head
2) the step of approximately 0.1 metres around Coffs Harbour.

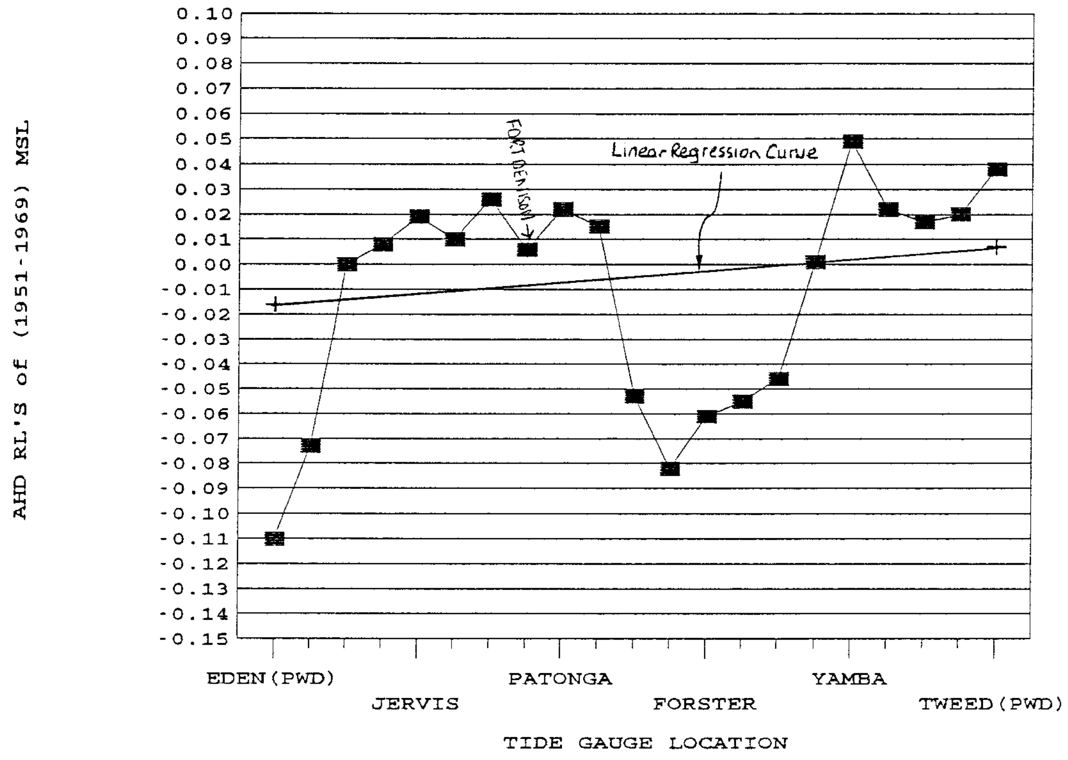
MITCHELL (1989, p.5) notes that the slopes encountered between AHD and the geoid should be fairly smooth and be modellable by linear functions. If we extend this to MSL (as AHD attempts to represents MSL), then by using linear regression techniques the slope of MSL can be modelled.

Figure 2.6 1951-1969 MSL at each NSW tide gauge defined by the original spirit levelling



It is interesting to note that the general slope of MSL as defined by the normal orthometric levelling between Bermagui and Coffs Harbour is consistent with evidence found in America (FISCHER, 1976) and the United Kingdom (ROSSITER, 1967) by sloping downward toward the equator. However if the linear regression is taken through all the gauges in the project area, both diagrams show a trend that is similar to that predicted by LENNON (1988). Like most oceanographers he believed that from steric estimates, the geostrophic slope along this part of the eastern coast of Australia is of the order of a few centimetres over 100 kilometres and should slope upward toward the equator. However, the uncertainties in both the steric and geodetic estimates would tend to make this agreement between the two separately developed height systems unrealistically optimistic. None the less, the upward trend toward the equator predicted by both approaches, while at different gradients, is encouraging. These results

Figure 2.7 1951-1969 MSL at each NSW tide gauge defined by the AHD



presented in Figures 2.6 and 2.7 differ from those originally presented in MACLEOD et al (1988) due to the addition of further information and the detection of errors in the initial data.

2.6 SOLUTIONS TO THE PROBLEM

Possible reasons for the differences between MSL and the spirit levelling used to form the AHD along the NSW coast can be placed into the following general categories:

The incorrect assumption that the MSL derived from a 19 year epoch defines the geoid. From the discussions in sections 2.1 and 2.2 the determination of MSL from tide gauge SLH's will not be coincident with the geoid by an amount equivalent to the SST at each gauge. Furthermore, the tidal characteristics in Australia are more complex than any other known region (LENNON, 1987) and may exhibit long term effects that cannot be eliminated in the 19 year epoch. This can be seen by the trend in the 19 year epochs at Fort Denison shown in Figure 2.5.

The residuals after applying a linear regression to the New South Wales tide gauge network shown in Figures 2.6 and 2.7 exhibit values that are within the estimates of SST as described in section 2.2. This is particularly true when the residuals are compared to the tidal gradients predicted by HARVEY (1988) and shown in Table 2.1. The large residuals exhibited by the gauges around Foster/Crowdy Head may be partially explained by the oceanographic phenomena of the beginning of the south-eastern Australian Current in this region (COLEMAN, 1989). Therefore forcing the levelling network to the 1951-69 epoch MSL values at the tide gauges will introduce distortions into the network. Clearly, an approach such as MERRY & VANICEK (1983) that models the SST effects would produce a mean sea surface that is closer to the geoid. The techniques necessary to correct MSL for the SST are beyond the scope of this study. From various regional studies it can be assumed that although SST varies at each tide gauge, SST effects on MSL will exhibit some regional correlation and be about the size of the anomalies being experienced along the NSW coast.

The errors that exist in the levelling network. Section 2.4.1 highlighted the possible errors sources in spirit levelling. The most important are systematic errors and compensating gross errors. Gross errors have been found in various investigations of the AHD and NATIONAL MAPPING (undated) conceded that large levelling errors probably still existed in the network and that a 1st order levelling traverse around the whole continent was necessary. CASTLE & ELLIOTT (1982) believed the levelling that made up the AHD to be "a quality inappropriate for comparison" to the MSL. RUSH (1986) pointed out that under the specifications for third order levelling, a blunder of 0.1 metres could be hidden in a levelling run of 30 kilometres.

While the levelling along the NSW coast is primarily 1st order, the possibility of compensating gross errors still exists. Moreover, the presence and magnitude of all types of systematic errors, such as the predominantly north-south magnetic error recently discovered in the automatic compensator-type levelling (ZILKOSKI & KUMAR, 1989), will remain undetected until connected to tide gauges or an independent technique such as GPS is introduced into the system.

Furthermore, the variance of the gradient of the MSL slope derived by the spirit levelling may in fact be due to the adjustment process itself. MITCHELL (1989, p.5) points out that the MSL slope is flatter in the centre of the continent and is probably a result of attenuated influences on the warped adjustment by the coastal gauges.

The incorrect position of many of the tide gauges used as datum points. Tide gauge location in rivers and estuaries will not reflect true ocean conditions due to such effects as tidal gradients and river discharge. While these effects can only be approximated, the errors in tide gauge heights in general tend to increase the further the tide gauge is away from the ocean.

The Department wished to investigate whether the variations in MSL as defined by the AHD were in fact due to the errors in the relative height system definition. The possible presence of systematic errors in the 1st order network and the 3rd order accuracy of most tide gauge connections has lead to the Department questioning the integrity of the spirit levelling. Without reliable height connections between the tide gauges all other analyses and conclusions would have little merit. The satellite navigation system called the Global Positioning System (GPS) was used to investigate the existing tide gauge connections derived by spirit levelling. The principles of this technology are outlined in the following chapter.

CASTLE & ELLIOTT (1982) suggested that looking at the "sea slope problem", was a local problem and restricted to specific physically defined regimes and its resolution was almost certainly multifaceted. Furthermore, the expected agreement between geodetic and steric levelling methods used to relate the tide gauges together was unrealistically optimistic given the estimated accuracies of both. Therefore, an accurate and independent method, that can be used to relate the MSL at each tide gauge to an absolute datum, will only help in solving the "sea slope problem". MATHER (1976; 1978) tried to reconcile the problem by using the space technique of satellite altimetry to prove the results of the oceanographic (steric) technique. Now, another space technique in the form of the Global Positioning System may be able to prove the results of the spirit levelling technique.

3. CONCEPTS OF THE GLOBAL POSITIONING SYSTEM

3.1 BACKGROUND

The Global Positioning System (GPS) is the latest in a line of satellite positioning technologies stretching back to the beginning of the space age, when SPUTNIK I was launched on the 4th October, 1957. Systems for determining position on the earth's surface developed since this date have utilised a number of measuring techniques. These include optical tracking (where satellites are photographed against a star background), satellite laser ranging (SLR), and various radar and microwave tracking systems including NASA's Minitrack and the US Army's SECOR (SEquential Collation Of Range) systems. Unfortunately, these early systems required equipment that was expensive and not very portable, while obtaining accuracies that were often not suitable for the required positioning applications. It was not until the mid 1960's that the use of satellites for positioning became commercially viable.

Experiments using Doppler shifts from satellite radio transmissions led to the development of the United States Navy Navigation Satellite System, commonly known as NAVSAT or TRANSIT. The first prototype TRANSIT satellite was launched in 1961, with the system becoming operational in 1964. In 1967 it became the first generally available satellite navigation system when it was opened to civilian use. TRANSIT had a major impact on geodetic surveying, offering surveyors a relative positioning technique that could produce geodetic accuracies without the need for station intervisibility, while being independent of weather and terrestrial network design considerations. However to achieve these geodetic accuracies required the stations to be sufficiently widely spaced, since the relative position between stations could at best be determined to a few decimetres. Furthermore, the nature of the doppler observations meant many satellite passes were required, leading to long observation sessions (up to 4 days). These constraints, combined with significant equipment costs at the time, limited the system's surveying applications to large scale geodetic control surveys and lower accuracy geophysical and offshore surveys (ECKELS, 1987). TRANSIT is still being used today and the U.S. military plans to maintain its operation until 1995. Further details can be found in HOAR (1982) and WELLS et al (1986).

Work on the NAVSTAR (NAVigation Satellite Time And Ranging) Global Positioning System (GPS) began in 1973 as a result of the military's need to replace the aging TRANSIT system. It is a satellite based radio-navigation technology that is revolutionising surveying and navigation throughout the world. As with the case of TRANSIT, GPS was designed specifically as a navigation system to satisfy the United States of America's military needs for real-time positioning and its operation is controlled by the U.S. Department of Defense (DoD). However, the system has been utilised by civilian users for a variety of purposes. In its final form, GPS will allow a quick and accurate 24 hour location capability without the need for such considerations as good weather and intervisibility between stations which conventional surveying ground methods require. The GPS system and the principles of GPS surveying will be briefly described in the remainder of this chapter.

3.2 COMPONENTS OF THE GLOBAL POSITIONING SYSTEM

The basic principles of NAVSTAR/GPS are well described in various publications including WELLS et al (1986), KING et al (1987) and ECKELS (1987). Only a brief review of the system's components is given here.

Developed as a response to overcome various shortcomings in the TRANSIT system, the first GPS prototype satellite was launched on the 22nd February, 1978. A total of 11 prototype (Block I) satellites were launched between 1978 and 1985. At the time of writing, 6 were still operational. Extensive testing has occurred with these Block I satellites, and the DoD has begun deployment of the operational (Block II) satellites with the launch of satellite PRN 14 on the 13th of February, 1989 and currently has seven of these new satellites in orbit.

3.2.1 The Satellites

In the final Block II constellation it is presently planned to have 24 satellites evenly distributed in six orbital planes, inclined by 55 degrees to the equator and travelling in approximately circular orbits at an altitude of about 20,000 kilometres, giving each satellite an orbital period of 12 sidereal hours (DORFLER, 1988; ANANDA, 1988). Three of these will be "active" spare satellites, while a further 4 units will be kept on the ground, ready for launch to quickly replace a disabled satellite.

When the PWD carried out the GPS surveys described in this report in 1987, there were seven experimental Block I satellites in operation. This allowed use of the system during limited times of the day. Table 3.1 gives details of the status of the Block I satellites at the time of survey. The deployment of the Block II satellites was to begin in 1986, but due to the Challenger Space Shuttle disaster the launch program was delayed. All but a few satellites will now be launched by the Delta II expendable launch vehicle. The complete constellation is scheduled for deployment by 1993.

Table 3.1 Status of Block I GPS Satellites at May 1987

Sat No	PRN No	Launch Date	Status	
1	4	22/02/78	not operating	
2	7	13/05/78	not operating	
3	6	07/10/78	operational	
4	8	11/12/78	operational	*
5	5	09/02/80	not operating	
6	9	26/04/80	operational	
7		18/12/81	launch failed	
8	11	14/07/83	operational	
9	13	13/06/84	operational	
10	12	08/09/84	operational	
11	3	09/10/85	operational	

* PRN 8 operating with only a quartz crystal oscillator

3.2.2 The Signals

A detailed explanation of the GPS signal structure can be found in WELLS et al, (1986), KING et al, (1987) and ECKELS (1987). Simply explained, the signals are designed to provide information that can allow instantaneous position to be determined. They are made up of codes, to calculate ranges to the satellites and a navigation message giving information about the system, all modulated on L band carrier waves.

Each satellite transmits a unique signal on two L-band frequencies generated by an onboard oscillator (referred to here as the satellite clock). The L1 carrier has a frequency of 1575.42 MHz while the L2 carrier is at 1227.60 MHz. These carriers are modulated with pseudo random noise (PRN) sequences that are in fact binary codes generated by a mathematical algorithm. Known as C/A (clear acquisition) code and a P (precise) code, the chipping frequencies of these signals are 1.023 MHz and 10.23 MHz respectively. The L1 carrier is modulated with both codes, while the L2 carrier is modulated only with the P code. In addition, both carriers have a navigation message modulated at a frequency of 50 Hz, which contains the satellite ephemerides, satellite clock correction parameters, corrections for delays in the propagation of the signal through the atmosphere and general health of the system. This navigation information is monitored and updated regularly through a network of DoD tracking stations around the world.

3.2.2.1 Codes

The codes are accurate time marks which allow the time of transmission of the signal to be determined. The signal is collected at the receiver and related to its own receiver clock. If all the clocks in the system (satellite and receiver) are synchronised and the identical PRN code is generated in the receiver as broadcast by the satellite, the time shift required to correlate the two signals can be calculated. Scaling this time shift by the speed of light produces the range between the receiver and satellite. It is called the **pseudo-range** because in reality it is biased by, among other things, both receiver and satellite clock synchronisation errors. Code range measurements involving the P code (wavelength = 30 metres and in a sequence that is repeated once every 267 days) can resolve the range more accurately than the C/A code (wavelength = 300 metres and a sequence repeated every millisecond). The standard observation procedure is to initially "lock on" to the simpler C/A code, decode the navigation message and when code and carrier phase loops are synchronised within the receiver, hand over to the P code for precise pseudo-range measurements. However, the DoD plans to restrict the use of the P code when the Block II satellites are operational (HOTHEM & ELLETT, 1988).

The position of the receiver can be determined by measuring the pseudo-ranges to four satellites simultaneously. This is analogous to a resection by distance in terrestrial surveying, but in this case it is a 4 parameter solution; being the three spatial parameters (X,Y,Z) and the receiver clock error (or offset) with respect to the satellite time system. This assumes that satellites coordinates are accurately known, and that the L band signals can be aligned accurately. Because of these errors, as well as the effect of the satellite geometry, the estimated point positioning accuracies are generally about ±20 metres or better using P code ranging and sometimes as bad as ±100 metres using C/A code if selective availability is invoked by the DoD on the satellite signals (ANANDA, 1988). Selective availability (known as SA) is a method whereby the GPS signals from Block II satellites are dithered and ephemeris is degraded to produce less accurate navigation solutions to those general users without access to the necessary decoding electronics in the receiver. Less than 4 satellites can be used if one or more of the unknowns are constrained eg: an accurate external atomic clock is added to the receiver or on a ship at the sea, the height is assumed to be zero (sea level).

Simultaneously observing a minimum of four satellites at two receivers, reduces many of the errors that exist in the single point solution. This "differential pseudo-ranging" can achieve accuracies at the 2 to 3 metre level (CHISHOLM, 1987). While these accuracies may be adequate for navigation, they are generally still unacceptable for surveying. As a consequence the principal observation used in GPS surveying is the integrated carrier beat phase. However, pseudo-ranges can be used to provide extra information such as the approximate coordinates of receiver stations and to synchronise the receiver clocks to the μsecond level (GRANT, 1989).

3.2.2.2 Carrier Beat Phase

Geodetic positioning accuracy can be achieved if the range to the individual satellites is measured using the L band carrier waves, with two or more receivers deployed in a differential or relative mode. The range between the satellite and the receiver can be considered as an integer number of carrier wavelengths, plus a fractional part of a cycle of the carrier wave. The fractional part of the carrier wave cycle is determined by "beating" or comparing the received carrier wave (after the PRN and navigation message modulations have been removed) with that generated by the local receiver oscillator at the same nominal frequency. The resulting carrier beat phase observation when multiplied by the wavelength of L1 (19 cm) or L2 (24 cm) gives the fractional part in distance units to an accuracy of millimetres. This is 2 orders of magnitude more precise than P code and 3 orders of magnitude better than C/A code ranging. The integer number of cycles (or integer ambiguity) in the range is indeterminate in this measurement and must be accounted for in some way during data processing. The carrier beat phase can therefore be considered as an "ambiguous pseudo-range" (GRANT, 1989) because it is biased by the same errors as pseudo-ranges plus the integer ambiguity.

3.2.2.3 The Navigation Message

Information about the GPS satellite's position and status is necessary if the receiver positions are to be determined. The GPS Control Segment carries out the function of monitoring the GPS satellites and is operated by the US Air Force Space Command headquartered at Falcon Air Force Base in Colorado Springs. The Master Control Station (MCS) collects tracking data to all satellites from monitor stations at Colorado Springs, Hawaii, Ascension Island, Diego Garcia, and Kwajalein. The data is used to determine satellite ephemerides, the satellite clock calibrations and the system's health. This information is formulated into the navigation message and regularly uploaded into each satellite via three antenna stations at Ascension Island, Diego Garcia and Kwajalein.

This navigation message, which transmitted to the user, includes the broadcast ephemeris which provides the satellites' position in space, made up of hourly elements describing the motion of the satellite for a number of hours in the future. The broadcast ephemeris is therefore a prediction of the position of the satellites. GPS users may also use precise ephemeris information, a post-processed ephemeris calculated from data acquired at a number of monitoring stations. Various organisations offer this post-processed ephemeris, including the Defense Mapping Agency (DMA) and Litton Aeroservices Inc. The relative merits of each type of ephemerides are examined in Chapter 5. The Block IIR satellites (replacement satellites due for launch after 1993) will be equipped with facilities that will provide cross-linking (or ranging) between satellites and onboard orbit determination capability. This will allow "autonomous navigation" (ANANDA, 1988) for up to 6 months free from ground control.

The clock error parameters in the navigation message give the relationship between the individual satellite clocks and the GPS Time system (GPST). GPST has the same rate as Coordinated Universal Time (UTC) but differs by an integer number of seconds (at present 25 seconds).

The broadcast ephemeris generated by the control segment is presently expressed in terms of the geocentric reference system known as the World Geodetic System 1984 (WGS84). To relate this system to a regional reference system such as the Australian Geodetic Datum (AGD) requires the use of transformation parameters, such as those derived in HIGGINS (1987) (see section 7.1).

3.2.3 The Equipment

The quality of a position fix by GPS depends on the type of equipment used, the observing procedures followed (Chapter 4) and the processing methodology (Chapters 5 & 6). A detailed description of the equipment requirements for surveying applications can be found in WELLS et al (1986) and ECKELS (1987). In short, the geodetic surveyor requires an antenna to collect the signals transmitted from the satellites and a receiver that can extract the carrier wave from these incoming signals and record the data. The carrier wave can be obtained using either of two methods.

1) **Code-correlation** - if the receiver can duplicate the P or C/A codes, it is possible to remove the code modulations from the incoming signal and reconstruct the original phase signal. The advantages of having access to the codes is that the navigation message can also be obtained.

Receivers with the P code capability such as the Texas Instruments TI4100 can do this on both frequencies. Many manufacturers have developed C/A code receivers such as the Trimble 4000SX and the Wild WM101 that can only use this technique to make measurements on the L1 carrier. This was partly in response to the DoD intention to restrict access to the P code with the Block II constellation. Some new receivers including the Ashtech LD-XII and Trimble 4000SST have incorporated L2 carrier measurement capability (primarily to eliminate ionospheric effects) without the use of the P code. The methods used by these new receivers to obtain access to the L2 carrier are still proprietary information but are probably variations of the "squaring" technique.

2) **Squaring** - Another technique used to recover the carrier wave is to square the received signal thus generating a "codeless" signal with twice the frequency of the original

carrier wave (KING et al, 1987) and can be used on both L1 and L2. The technique is used by the Macrometer II and Mini-Mac receivers. An advantage of this method is that it is independent of the knowledge of the PRN codes. Disadvantages of this technique include the need to obtain ephemeris data from an outside source. Also the receivers are required to be physically brought together at the start and finish of each observation session in order to synchronise them to GPS time and calibrate the stability of the oscillators (ECKELS, 1987). The Mini-Mac receiver has attempted to overcome these problems by also having a code-correlating capability and hence is able to extract the navigation message.

In the following section, the basic concepts of GPS surveying are briefly presented.

3.3 PRINCIPLES OF GPS SURVEYING

The accuracies achievable in GPS surveying are dependent on many factors, including the amount, type and quality of data observed, the satellite geometry and corresponding ephemeris, and the mathematical modelling employed in the processing software. While GPS has been specifically designed to provide navigation position in real time, primarily using simultaneously measured pseudo-ranges to at least 4 satellites (section 3.2.2.1), there are in fact a number of measurement modes available to the user. In GPS surveying it is generally the carrier beat phase measurement that is converted to a very precise but ambiguous range. It is ambiguous because the total distance to the satellite is not measured, just the fractional part of the wavelength of the L band carrier wave, plus the whole cycles detected by the receiver hardware since the initial "lock on" to the satellite signal. This measurement is known as the **"integrated carrier phase"**.

Post-processing of GPS observables follow least squares techniques (see Appendix B for a review) that relate the observables to a set of parameters through a mathematical model. This mathematical model attempts to describe the physical situation and can be divided into two parts, the functional model that describes the deterministic properties and the stochastic model that takes into account the statistical properties of the measurements (see Appendix B for a detailed explanation). The mathematical model is linearised to form observation equations and the parameters (including receiver coordinates) are then solved.

In an ideal physical environment, the functional model adequate to describe the integrated carrier phase measurement would be:

$$\theta_R^S(t) = \frac{1}{\lambda}\sqrt{(x^S(t) - x_R(t))^2} - n_R^S \qquad (3.1)$$

where θ_R^S is the integrated carrier phase

x is the position vector of the satellite or receiver

n_R^S is the initial number of cycles of the carrier phase

λ is the nominal wavelength of the GPS signal

In fact it is the change in the receiver-satellite range that is actually measured by the receiver. This is because of the presence of n, the ambiguity (or bias) term that is equal to the unknown number of carrier wavelengths that were present at the first measurement, and which is constant throughout the entire observation period, if no "cycle slips" have occurred. A **cycle slip** is caused when the receiver fails to track the signal correctly and a "loss of lock" in the carrier phase loop of the receiver produces a change in the ambiguity between epochs.

In this mathematical model defining the change in the receiver-satellite range, the initial ambiguity is considered as an unknown to be solved. In all post-processing software packages the initial solution for n is estimated as a real number. This is known as a **float** or **ambiguity free** solution. However from the above definition this value should be an integer number of cycles. If the ambiguities are well determined in the adjustment, it may be possible to hold them fixed at integer values in a new adjustment (ie. n becomes a known parameter or constant in the observation equation). This is considered the most accurate solution (if the correct integer values have been found) and is known as an **ambiguity fixed** solution. GRANT (1989, p.122) notes that while this is also often referred to as "bias fixing", this name is itself ambiguous as the set of integer ambiguities is only one of a number of different sets of biases in the GPS observations.

Unfortunately the ideal physical situation described in equation (3.1) does not occur in practice as there are number of physical effects that cause both random and systematic errors in the observations. Therefore the ability to correctly estimate the integer ambiguities depends on modelling or eliminating these errors, coupled with the geometric strength of the observed satellite configuration (in the case of integrated carrier phase it is the change of the satellite geometry and therefore the length of the observation period (GRANT, 1989, p.123)). The random errors are a result of the observation process itself and their magnitude is unknown. However, repeated observations will allow an estimate to be made of the error source which can statistically be propagated into the final solution through the stochastic model (HOLLOWAY, 1988, p.31). Biases however have the same magnitude and sign and will not be eliminated by repeated observations. They can be eliminated or at least minimised by modelling the error in the solution, or eliminating the bias by making linear combinations of observations. Errors may still directly affect the ambiguity parameter estimation.

These physical effects are discussed in various publications including WELLS et al (1986), KING et al (1987) and GRAFAREND et al (1985) and the errors that affect the change in integrated carrier phase can be categorised as:

SYSTEMATIC ERRORS (BIASES)

1) **Satellite specific biases**
   - satellite orbit errors (through the broadcast and precise ephemeris)
   - satellite clock instabilities
2) **Receiver specific biases**
   - receiver clock errors
   - station coordinates (eg centring errors)
3) **Satellite-receiver paired specific biases**
   - Ionospheric delay effects on the signals
   - Tropospheric delay effects on the signals
   - Carrier phase ambiguity

RANDOM ERRORS

- multipath (delays introduced by signal reflections from nearby structures)
- receiver noise
- cycle slips
- antenna phase centre movement
- random observation error
- residual biases

These errors are examined in more detail in chapter 5. Therefore the functional model that more accurately defines the physical situation for the integrated carrier phase from one satellite to one receiver would be:

$$\theta_R^S(t) = \frac{1}{\lambda}\sqrt{(x^S(t) - x_R(t))^2} - f.c_R(t) - f.c^S(t) - n_R^S$$

$$+ (\theta_{ION})_R^S + (\theta_{TROP})_R^S \qquad (3.2)$$

where θ_R^S is the integrated carrier phase measurement

t is the measurement time or epoch

f is the nominal frequency of the carrier wave

x is the position vector of the satellite or receiver

c^S is the satellite oscillator (clock) error

c_R is the receiver oscillator (clock) error

n_R^S is the initial number of cycles of the carrier phase

θ_{ION} is the phase delay due to the ionosphere

θ_{TROP} is the phase delay due to the troposphere

λ is the nominal wavelength of the GPS signal

By adding a θ_{NOISE} term to the functional model which considers the effects due to measurement noise and unmodelled influences, both of which are assumed to be random (WELLS et al, 1986), equation (3.2) becomes the well known one-way phase observation equation.

Because of the uncertainties in the satellite positions, clock behaviour, and propagation delays, absolute (single point) positioning will at best produce accuracies of only a few metres. However if two or more receivers are operated simultaneously and observe the same satellites, then many of these systematic errors will exhibit an amount of physical correlation among the GPS signals collected at each receiver. This fact allows techniques to be used to eliminate (for example, the satellite clock biases) or at least significantly reduce the effects of these errors (as in the case of the orbital and atmospheric propagation errors) when relative positions between the receivers are calculated.

Precise carrier phase applications therefore utilise mathematical models which use carrier phase data in a relative mode to obtain accuracies of the order of a few parts per million of the baseline length. Higher accuracies (up to parts in 10^8) may be achieved by using better satellite ephemerides, water vapour radiometers to reduce the tropospheric effects on the phase signal, and both L_1 and L_2 frequencies to reduce the ionospheric refraction.

Taking advantage of the physical correlation between signals received at each station, leads to a variety of processing strategies utilising relative positioning. Although the potential of the one-way phase observation equation (equation 3.2) has been discussed by various authors including GRAFAREND et al (1985) and GOAD (1985) and LINDLOHR & WELLS (1985), it is the basis for only a few GPS data reduction software packages. The best known program using the one-way phase observable is PHASER (GOAD, 1985; JONES et al, 1987). The most common processing strategy implemented in GPS post-processing software, is where the ambiguous ranges measured simultaneously from several stations to several satellites are **differenced** between satellites to eliminate or reduce receiver dependent biases, and between receivers to eliminate or reduce satellite dependent biases. What remain are "double differenced" observations that can be modelled in terms of the receiver and satellite coordinates and double differenced ambiguity parameters. Further differencing between measurement epochs allows the elimination of the integer ambiguity.

However as the level of differencing increases, there is a loss of information, since for example the resulting differenced observations may be processed in a sub-optimum fashion by ignoring part of the stochastic component of the mathematical model being shown in the measurement covariance (HATCH, 1988). In fact the triple difference observable is used in final processing, where accuracy requirements are less stringent (WANLESS & LACHAPELLE, 1987) or only to assist in the detection and repair of cycle slips before further processing (TRIMBLE, 1987). Therefore the differencing observation equations are only equivalent to the original one-way phase observation equation if the stochastic model of the reduced or differenced observations are correctly determined.

3.3.1 Precise Static GPS Surveying

Generally, a GPS campaign will set out to determine a set of coordinates for those stations within a network, and possibly adjust the satellite orbits at the same time. A number of receivers will be deployed at sites simultaneously tracking carrier phase data from a number of satellites over a period of time. This is commonly referred to as a **session.** The receivers are deployed over several sessions until all sites in the **network** have been occupied. Some authors such as BOCK et al (1986), GURTNER et al (1987) and GRAFAREND, (1989) advocate that these two steps can be combined into one adjustment process. This **"multi-session"** model has advantages if orbits are being estimated, as the orbital parameters are common to several observation sessions (GRANT, 1989, p 132). However in large networks that contain many sessions, the number of parameters and observations become very large and a computational burden for most computers. Therefore in terms of this discussion, it is easier to consider the GPS campaign as being composed of two distinct phases. Chapter 5 will discuss the determination of positions within a session, while Chapter 6 will discuss the combination of the results of these sessions in an optimal way.

In the precise **static** case, many repeated observations are made to several satellites, meaning that there are many more observations than parameters creating an overdetermined solution that can be solved by least squares techniques. More recently **kinematic** GPS surveying using the integrated carrier phase measurement has been made commercially available. The solution consists of a number of instantaneous "fixes" with little or no redundancy (at least 4 simultaneous observations are needed). The resultant string of fixes still use the one-way phase observation equation that can be post-processed using modified differencing techniques. This thesis will concentrate on the precise static case and the reader is referred to MERMINOD (1989) for code phase kinematic surveying and REMONDI (1988) for further information on carrier beat phase kinematic processing.

The software available for processing carrier phase observations (or their derivatives using differencing) can provide station coordinates and/or vectors between stations within a network. Because of the computation considerations such as the size of the matrices, many commercial packages use differencing algorithms that only calculate these parameters and appropriate variance-covariances between two stations. The process is repeated until a number of baselines have been determined between all occupied stations. This solution is commonly known as a **multi-baseline** solution. However if more than 2 stations are deployed within a session, these baseline solutions may ignore the correlations between the differenced observations between baselines. Those that rigorously determine this covariance between all stations within a session can be defined as **multi-station** solutions. They can use both undifferenced (original carrier phase) observations (GOAD, 1985) or differenced observations (BEUTLER et al ,1986), and their output is generally a set of coordinates and the associated variance covariance matrices.

3.3.2 Software

The nature of the GPS observations requires the use of computer software in all phases of a GPS survey. When planning the survey, the time and positions of the satellites need to be predicted. Once the observations have been collected, the computations to determine the position of the receiver need to be performed. This may include point positioning, baseline or coordinate determinations, as well as datum transformations.

From the previous discussions an optimal geodetic GPS software package should therefore be capable of the following tasks:

1) **Satellite predictions** - including satellite availability, azimuths and elevations, and geometry (DOP values).

2) **Network Simulation** - to determine the minimum number of stations and observation sessions needed to obtain the required accuracy. This exercise saves on the final fieldwork and data processing costs.

3) **Data preprocessing** - where the data from any receiver is decoded, corrections and additional information applied (eg. pseudo-range positioning, cycle slip detection, introduce precise ephemerides, etc) and reformatted ready for processing.

4) **Data processing** - using a rigorous method to determine the position of each station in an observation session with a possible orbit determination option, in a baseline or multi-station mode.

5) **Network adjustment** - combining the station solutions from various sessions together. In addition it should

be able to combine doppler, terrestrial and other geodetic measurements.

6) **Datum Transformations** - be able to transform the adjusted network into other coordinate systems.

Processing software has been developed by either academic institutions, government organisations or by the receiver manufacturers. Up until recently each receiver manufacturer supplied software that was receiver dependent eg. TRIMVEC™ (TRIMBLE, 1986). Some manufactures have recently developed post-processing software packages that reduced data from a variety of different receivers eg. NOVAS (WANLESS & LACHAPELLE, 1987). While these packages are microcomputer based, some are not user friendly and all fail in some way to fulfil all the requirements listed above. On the other hand, the academic and government packages tend to be large and complex. An exception is PC-PHASER (TALBOT, 1988) which is a microcomputer based program developed at RMIT, Melbourne from the mainframe programs BATCH-PHASER (JONES et al, 1987), and PHASER (GOAD, 1985). However again this package does not fulfil all the user's requirements.

However before processing the observations, they must firstly be collected. Chapter 4 discusses the planning and execution of a large scale GPS campaign by highlighting the NSW coastal campaign.

4. THE SURVEY

4.1 INTRODUCTION

In response to the continuing difficulties experienced by PWD engineers in relating NSW coastal river systems together and the 1987 investigation highlighted in Section 2.5, discussions were held within the Department in early 1987, in an attempt to resolve the situation. These discussions involved members of the Survey and Property Branch, the Coastal Engineering Branch, the Rivers and Ports Branch and the Manly Hydraulics Laboratory. While it was agreed that the problems were in part due to spirit levelling errors between the existing tide gauges, these problems could not be isolated to any one area of the coastline. Therefore an analysis of the entire NSW coastal levelling network was proposed.

The Survey and Property Branch believed attempting this analysis by repeating the spirit levelling would be too expensive, time consuming and could not guarantee any improved accuracies. An independent levelling method was needed. Therefore the Survey and Property Branch recommended the use of GPS. The Branch had used GPS previously and had attained accurate heights. RUSH (1986) suggested the use of satellite ranging techniques to supplement and strengthen the existing levelling information making up the AHD. Various authors including ENGELIS et al (1984,1985), KEARSLEY (1985,1988a,1988b), and HOLLOWAY (1988) discuss the methods that can achieve accurate orthometric heights from GPS and these are outlined in Chapter 7.

After defining the survey's objectives, its planning could commence. While a "planning component" is necessary when carrying out any type of survey, the importance of extensive planning in a GPS campaign as outlined in WELLS et al (1986) and ECKELS (1987) cannot be underestimated. In a GPS survey there is a direct relationship between the observing procedures used and the accuracies achieved. Increased user accuracy requirements demand an increased understanding of the possible error sources that can affect GPS surveying. These error sources can be reduced by improved field procedures and equipment, and special post-processing techniques (Chapters 5 & 6).

4.2 EARLY EXPERIENCE

The Public Works Department was the first NSW government department to use GPS technology, when in May 1986 the Survey and Property Branch carried out two control surveys to support aerial mapping. The first was required for sewerage design in an area on the north coast of the state. The second was necessary for monitoring the degradation of the coastal alignment on the south coast (ROBINSON & WATTERSON, 1987). These exercises demonstrated the potential of GPS techniques to satisfy survey requirements quickly and independently of existing terrestrial methods, for both horizontal position and height.

4.3 PLANNING THE SURVEY

Because the cost of GPS equipment is still high and the full satellite constellation is currently not available, careful planning is critical for a successful GPS campaign. While certain observation procedures are common for all GPS surveys; the need for increased accuracy (at the few ppm level) requires an increased understanding of the technology. Consideration must be given to the type and number of receivers available, the most efficient network configuration for these receivers, the satellite availability and geometry, the amount of redundancy required in the network, manpower needs and logistics of the campaign. Computing facilities and efficient post-processing software are also very important. However in the case of the PWD coastal survey, additional planning was required to determine the best strategies for the determination of orthometric heights from the GPS observations (see Chapter 7).

Planning of a campaign is not completed until the reconnaissance of all proposed sites has been carried out. This ensures efficient data collection during field operations. It includes selection and monumentation of cleared sites, checking vehicle access, estimating travelling times between sites and subsequent documentation. The following discussion of the planning and execution of the coastal survey carried out by the PWD emphasises these various aspects.

4.3.1 Receiver Hire

To obtain the most accurate results in GPS, dual frequency observations would be required in order to minimise the effect of the ionosphere on the height results (see 5.2.1.4). Unfortunately in Australia, the sources of hire of GPS receivers are generally limited to a handful of instrument distributors and more recently a few government and academic institutions. In 1987 these receivers had been almost exclusively of the single frequency variety.

A total of five single frequency Trimble 4000SX receivers were used. Three receivers were hired from Amalgamated Wireless Australia (AWA) Ltd in Sydney, the fourth from the Royal Melbourne Institute of Technology, and the fifth from the Victorian Division of Survey and Mapping. The Trimble 4000 series receivers had been developed from earlier navigation instruments and had proven their reliability when the earlier model 4000S receivers were used on the two small projects conducted by the Department in May

1986. The distributors network of agents offered back up services on the job that were to prove invaluable.

4.3.2 Network Design

Having secured five receivers, the Branch began planning the network and associated logistics. Since the purpose of the survey was to locate existing levelling anomalies along the coast, the network would be designed to give the most accurate determination of ellipsoid heights, and eventually orthometric heights. In recent years, proposed geodetic networks have been designed using simulation/optimisation software. However the network was not designed by this method because the station positions were constrained by the positions of the tide gauges. Generally the nearest bench mark (or where possible a state permanent mark) related to each tide gauge was used as the GPS station. This was to minimise the possibility of levelling errors being introduced when transferring height from the mark to the tide gauge.

The 5 receiver configuration was set up so that each observation period had a common baseline with the previous period (ie. 2 common stations). This repeat baseline approach would (1) allow a stronger least squares adjustment process through higher redundancy, (2) check the overall accuracy of the data reduction by comparing common baselines on different days and (3) reduce the possibility of systematic errors in height determination (GRANT, 1989).

Additional constraints were applied to the final network design. The survey teams operating the receivers were expected after each day's observations to travel to their next station, therefore the distances between setups had to be within reasonable driving times. Also the distances between adjacent stations should be kept to a minimum (around 30 kilometres or less) so the strongest possible baseline solution could be obtained. This is because the resolution of the cycle ambiguities to integers, is less likely for distances over 30 kilometres (KING et al, 1987; ECKELS, 1987; MACLEOD & RIZOS, 1988). As a result it was decided to place stations at Stanwell Park and Bulahdelah where there are no tide gauges. The final network of 40 stations is shown in Figure 4.1.

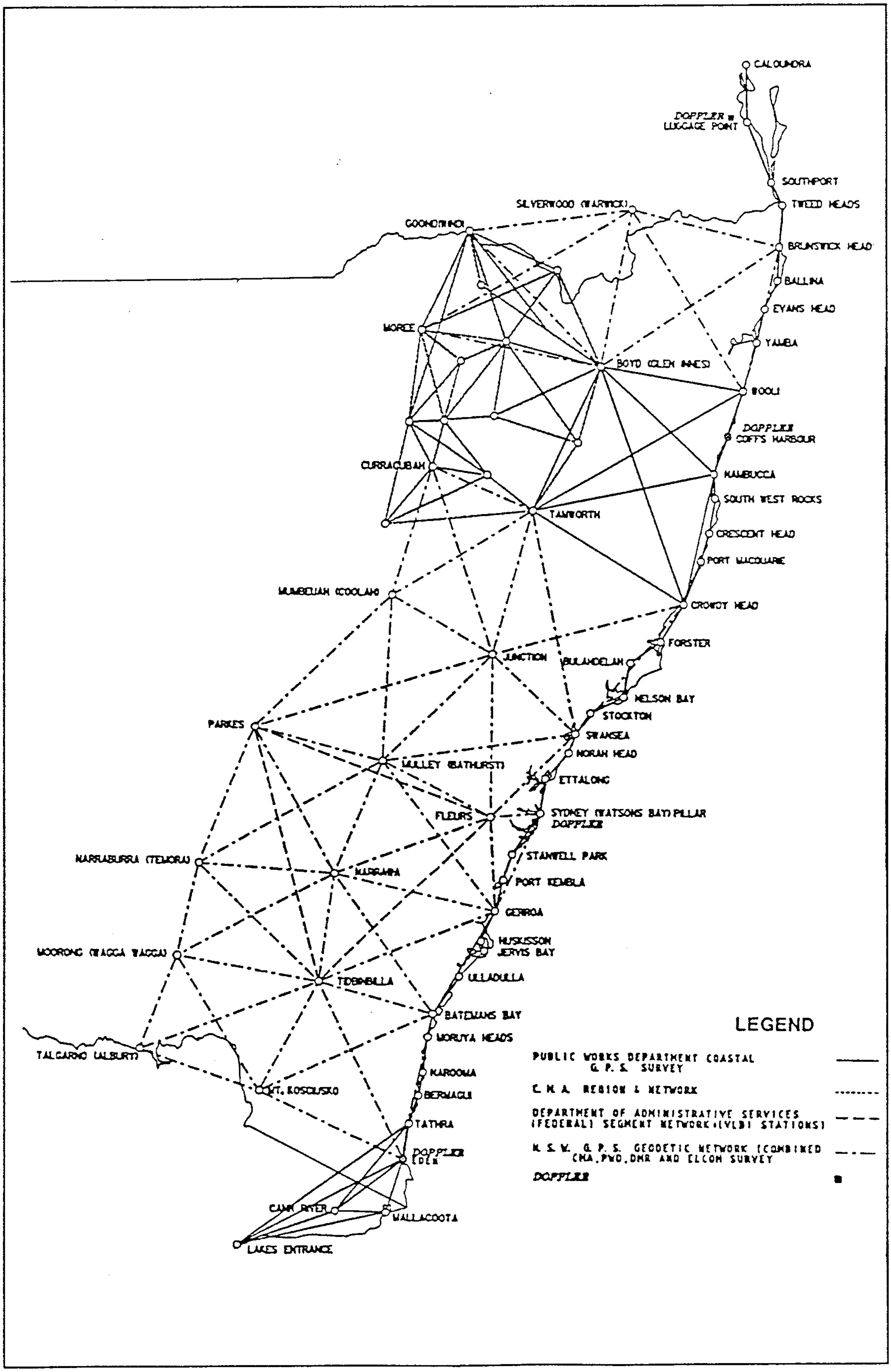
4.3.3 Involvement by Other Organisations

Meetings were held with the NSW Lands Department and the then Department of Mapping and Surveying in Queensland (DMS)(now known as the Division of Geographic Information (DGI)) to acquire both the existing levelling information and AHD values, for later comparisons with the GPS results. During these discussions, the DMS decided to set up 3 Trimble 4000S receivers on the last two nights of the coastal survey to connect the Queensland GPS network into the PWD project.

The Central Mapping Authority of NSW (CMA) and BHP Ltd had expressed interest in acquiring field experience with the receivers and each organisation supplied a vehicle and personnel, while the other three survey teams were from the PWD. BHP were also interested in the reduction techniques and post-processing software and were later given a sample of the data to reduce. The CMA requested that the coastal survey be connected to their Region 4 network in the New England area, which had been observed using Wild-Magnavox WM101 receivers earlier in 1987. If time permitted this connection was planned for the day after the completion of the coastal measurements as the survey teams returned from Queensland.

Through discussions with the School of Surveying, University of New South Wales, the Australian Army Survey Corps became interested in the project and decided to deploy two of their newly acquired Texas Instruments TI4100 receivers at the Sydney and Newcastle stations. These were set up on eccentric marks, at the same time as the Public

Figure 4.1 The GPS network in NSW at October 1987



Works Department's observations so a comparison of the baseline measurement could be made. Being dual frequency receivers, it was hoped that the TI4100 data could help investigate the ionospheric effects on the single frequency observations.

4.3.4 Ionospheric Effects and Modelling

Because the Trimble 4000SX is only a single frequency receiver, the ionospheric delay in the carrier phase observations cannot be directly determined. While this delay will tend to cancel in the double difference phase observable (MERMINOD, 1988a; MACLEOD & RIZOS, 1988), any residual error will translate into the height difference between the stations (Chapter 5). Therefore if the ionospheric delay could be determined by some other method, it would increase the overall accuracy of the derived heights. The ionospheric delay is one of the main reasons why ambiguities cannot be resolved to integers for lines of over 30 kilometres when using single frequency data (ECKELS, 1987; GRANT, 1989). However improving ambiguity resolution, does not necessarily in itself lead to more accuracy GPS heights. Chapter 5 gives an analysis of these findings in greater detail.

Investigators at the School of Surveying planned to model the ionosphere. The model would be developed by using TRANSIT satellite dual frequency observations from four Magnavox MX1502 receivers operating at the same time as the GPS campaign at Eden, Sydney, Coffs Harbour, and Brisbane. By using the doppler data from the 150 MHz and 400 MHz frequencies of the TRANSIT satellites, the Total Electron Content (TEC) could be estimated. The ionospheric delay is a function of TEC (see 5.2.1.4) and could then be calculated for each epoch in a TRANSIT pass. A model to correct the GPS carrier phase observations could then be incorporated before processing. A detailed discussion of this research and the subsequent results can be found in PARTIS & BRUNNER (1988).

4.3.5 Satellite Predictions

The next step was to predict the satellite availability window. These predictions were produced using the Trimble prediction software called TRIMSAT which uses broadcast ephemeris that should not be more than a few weeks old. Figure 4.2 shows the satellites available for a minimum elevation mask of 15 degrees. The reason for not observing satellites at these lower altitudes is because atmospheric effects become intolerably large (KING et al, 1987).

The survey was planned for May and since the Trimble receivers could track up to 5 satellites simultaneously the plan was to track a minimum of 4 satellites for the longest period of time without switching constellations. This allowed an observation window of about three and half hours, beginning at about 4.30am with the starting time receding by approximately 4 minutes per day.

One of the important factors in obtaining accurate results from GPS observations is the relative geometry of the satellites with respect to the ground stations. Up until recently, the Geometric Dilution of Position (GDOP) has been the recommended indicator of precision for all types of GPS positioning. GDOP gives the geometric strength of the instantaneous 4 satellite pseudo-range point position solution. However the geometric strength of the integrated carrier phase is quite different to pseudo-range (GRANT, 1989, Appendix E). NORTON (1987) showed that the best results in differential carrier phase solutions in fact came when the GDOP was changing rapidly. MERMINOD (1988a) developed a new indicator of precision based on the partial trace of the variance-covariance matrix of the differential carrier phase ambiguity free solution, called the Bias Dilution of Position (BDOP1).

Figure 4.2 Satellite predictions for May 1987

SATELLITE AVAILABILITY FOR SYDNEY, NSW
Lat: S 34° 00' Long: E 151° 00'
Date: 17/05/87
Elevation mask: 15°

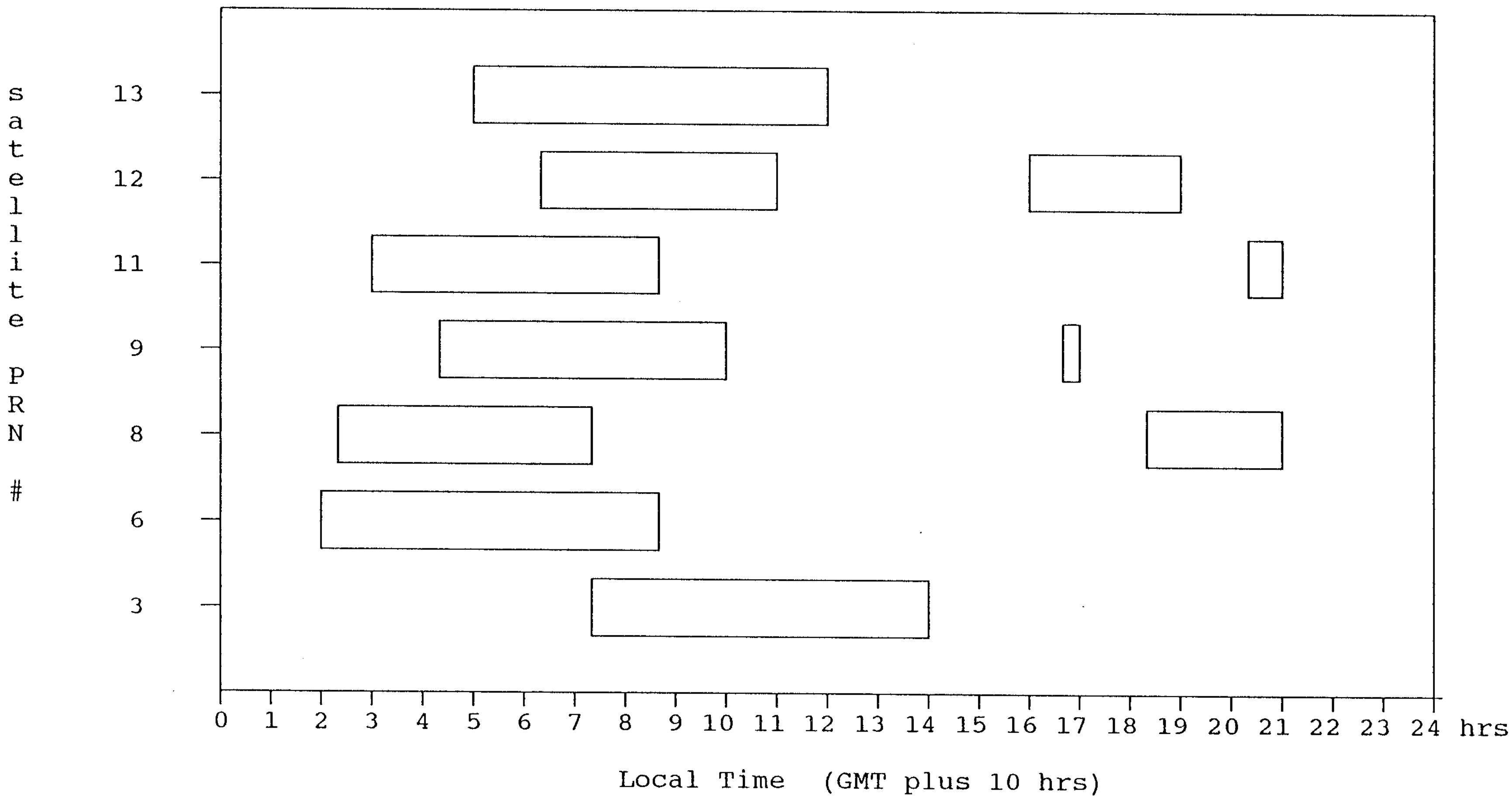
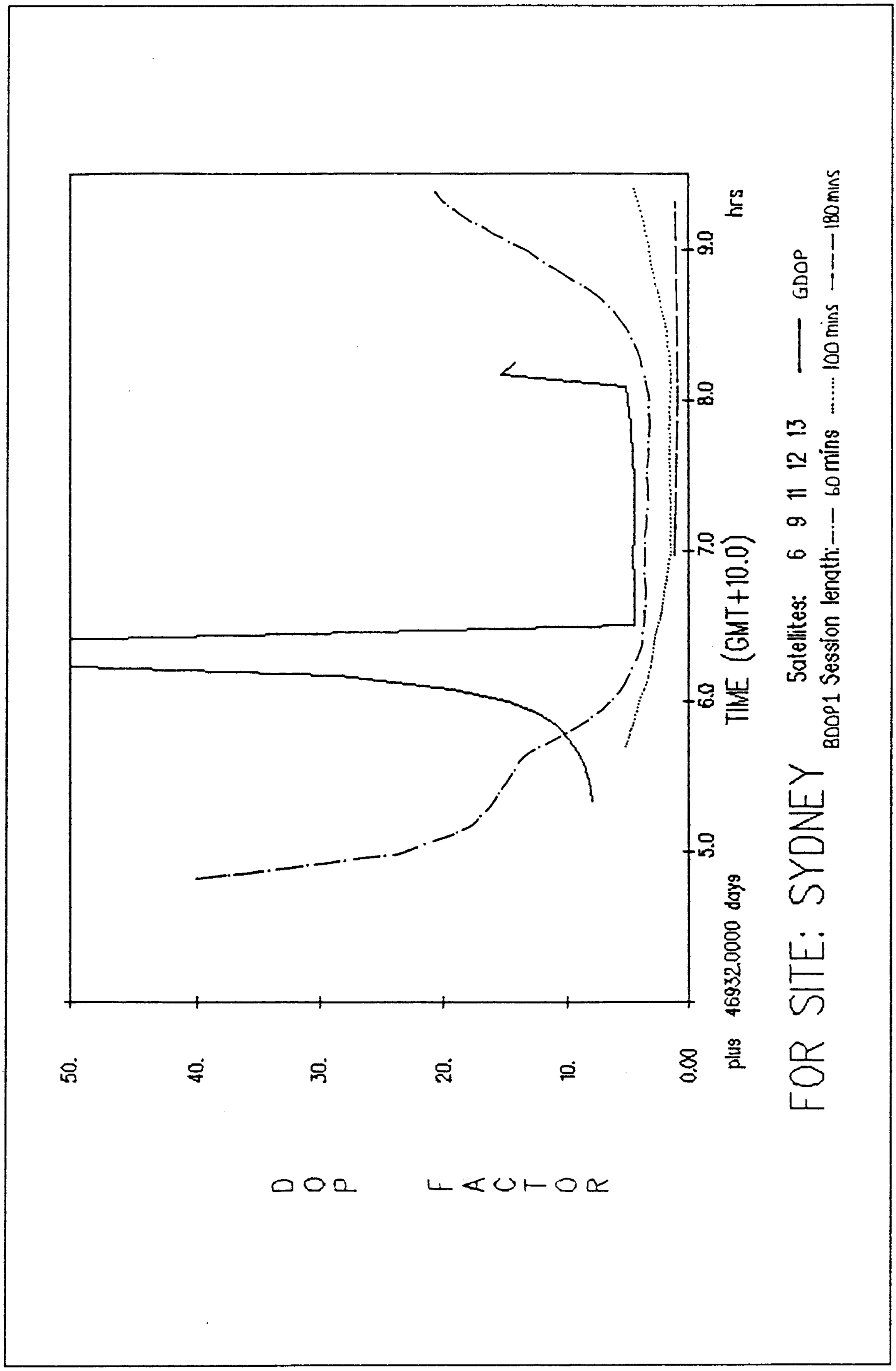


Figure 4.3 DOP estimates for Sydney at the 17/05/87



The GDOP and BDOP1 values for Sydney (17/05/87) have been calculated using the University of NSW's PREDICT program (MERMINOD, 1988a) and are shown in Figure 4.3. BDOP1 values are based on observation sessions of 60, 100 and 180 minutes. While the site experiences a GDOP "outage" near 6am, the BDOP1 values show that even observing for only 1 hour should produce good results over most of the observation window.

4.3.6 Reconnaissance

After the stations and satellites had been selected and the observation times determined, a reconnaissance of all sites was undertaken. This was a very important part of the planning as anything that looked like a potential problem could be noted and rectified. At each station visited a site plan was drawn to be analysed with a sky plot of the satellite orbits to make sure all relevant areas were clear of obstructions that could block the satellite signals. Examples of the sky plots are shown in figure 4.4 (Tathra) and 4.5 (Sydney). Sites with nearby objects such as iron roofs which could cause multipathing were avoided. Since the survey teams would be arriving on site around 3.30am. access to the marks was checked, particularly for their ease of location in the dark. Driving times between stations were also noted. Photographs were taken of each mark and copies made and distributed to each survey team. As a result of this reconnaissance exercise, new stations were needed in some areas. At other locations, alternative marks were added in case of possible problems. The important thing was that these tasks were carried out only a month before the start of the survey, to minimise the possibility of the conditions changing between the time of the reconnaissance and the time of survey.

Figure 4.4 Sky plot at Tathra station

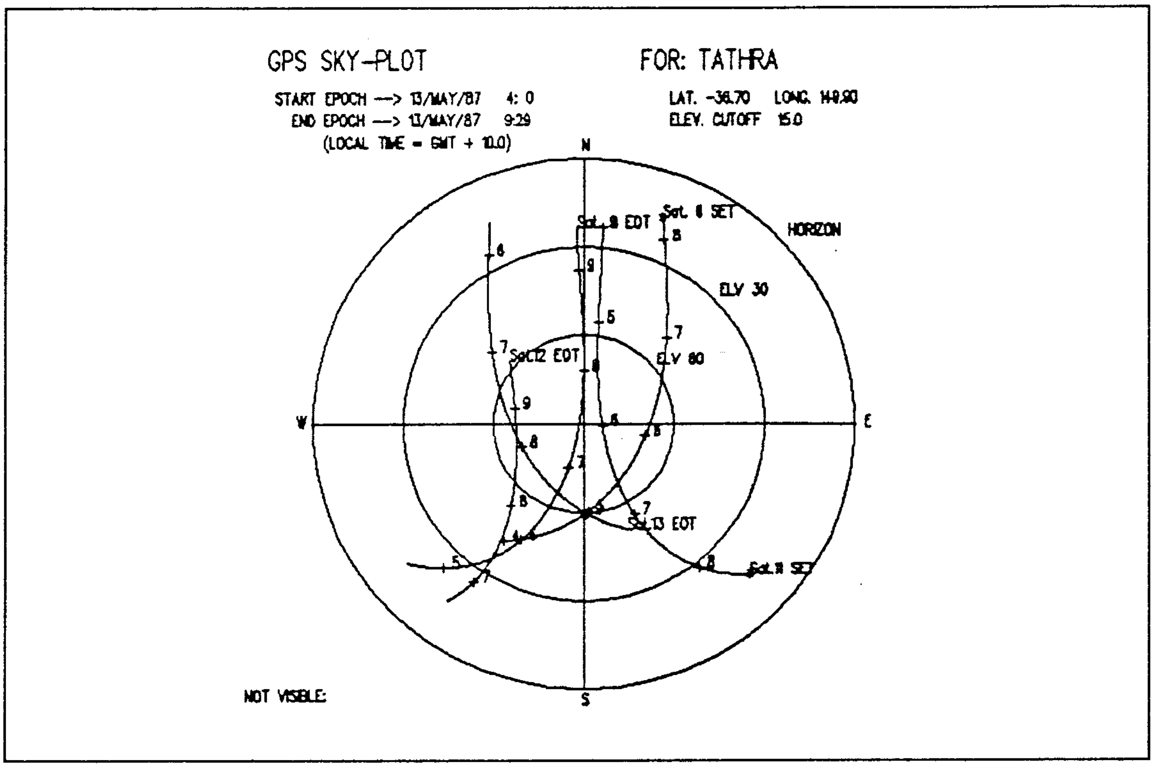
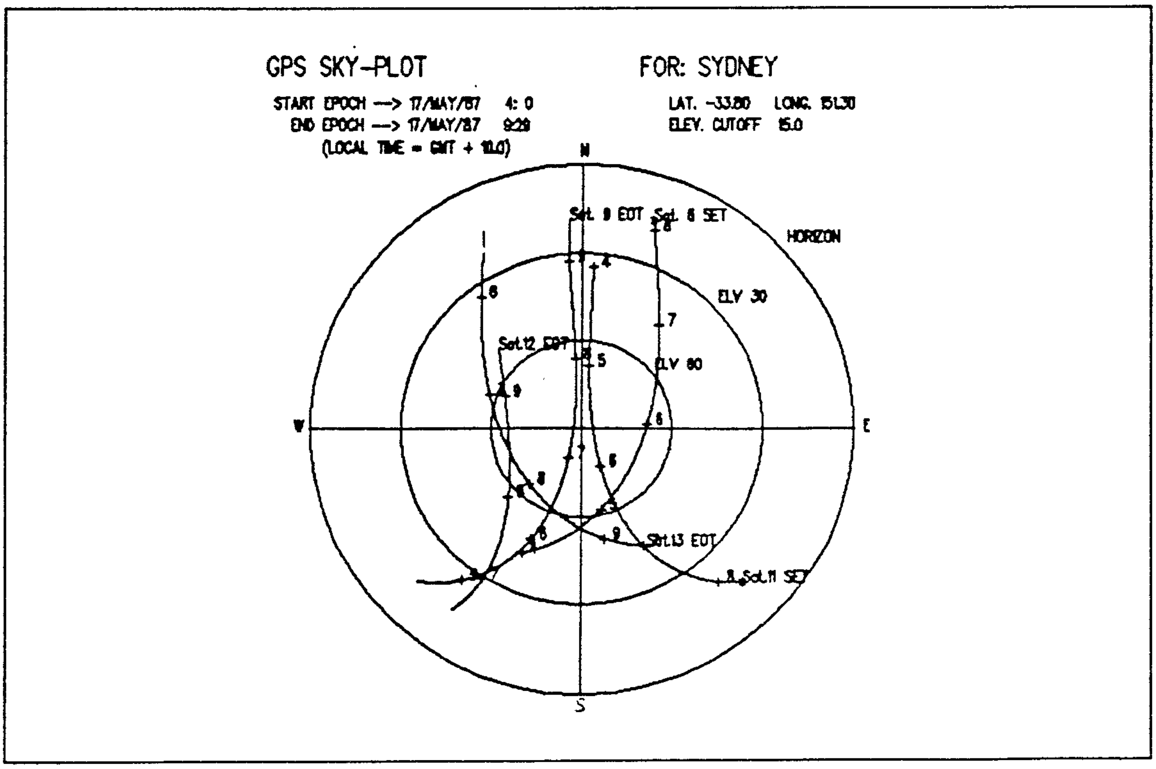


Figure 4.5 Sky plot at Sydney (Watson Bay pillar)



4.3.7 Logistics and Documentation

Locality sketches of the marks to be used were collected. Maps of the surrounding areas (eg 1:25,000 topographic series, road maps) were marked with the location of the stations and suggested access. Operating instructions for the receivers, the proposed itinerary for all survey teams and information on methods of reporting to a control centre set up at the Department's office in Sydney were documented. A copy of all the above information was placed in folders and given to each survey team. The proposed itinerary for each crew was based on 5 receivers functioning throughout the whole exercise and is shown in Table 4.1.

Table 4.1 Proposed itinerary for each observation team - GPS MSL investigation. (May 1987)

RECEIVER			A	B	C	D	E
DEDICATED PERSONNEL			R.MacLeod (PWD)	W.Baldock (PWD)	C.Falzon (PWD)	M.Campbell (BHP)	D.Burnett (PWD & CMA)
DAY	DATE	DAY #	STATION LOCATION				
WED	13	132	Lakes Entrance	Cann River	Mallacoota	Eden	Tathra
THU	14	133	Bermagui	Narooma	Moruya	Eden	Tathra
FRI	15	134	Jervis Bay	Narooma	Moruya	Batemans	Ulladulla
SAT	16	135	Jervis Bay	Huskisson	Gerroa	Port Kembla	Ulladulla
SUN	17	136	Sydney	Ettalong	Gerroa	Port Kembla	Stanwell
MON	18	137	Sydney	Ettalong	Norah Head	Swansea	Newcastle
TUE	19	138	Nelson Bay	Bulahdelah	Forster	Swansea	Newcastle
WED	20	139	Crescent Head	Bulahdelah	Forster	Crowdy Head	Port Macquarie
THU	21	140	Crescent Head	S.W. Rocks	Nambucca Heads	Coffs Harbour	Port Macquarie
FRI	22	141	Yamba	Evans Head	Nambucca Heads	Coffs Harbour	Wooli
SAT	23	142	Yamba	Evans Head	Ballina	Brunswick Heads	Tweed Heads
SUN	24	143	Southport	Luggage Point	Caloundra	Brunswick Heads	Tweed Heads
MON	25	144	Nambucca Heads	Wooli	Glen Innes	Crowdy Head	Moonbi

* Dedicated Personnel are those who will use one particular receiver unit throughout the duration of the project.

** Observation Time approx 05.00 - 08.30 EST

Adjustments to this program could be made as necessary. Driving times were kept to a minimum, except at the beginning and the end of the project, with the average being 4 hours. The itineraries were designed to allow teams time to visit the stations before dark to familiarise themselves with the access and mark location. The inclusion of repeat baselines gave the unforeseen advantage of a rest day for each crew every third or fourth day. Because of the early starts and subsequent long drives to the next station, fatigue was perceived as a problem. Therefore its control would be a major factor in the success of the project. The repeat baselines meant these crews could relax as they would not have to drive to the next station.

In addition to the 5 crews, 2 extra vehicles were to be used. One was to be used as a "runner", to help where ever needed. On many occasions this vehicle was to become the communications link, by driving to a high spot and relaying messages between the crews on the coast via two way radios installed in each vehicle. The second vehicle was used by the person who would collect the data stored on 3.5 inch computer disks from each crew after every observation session. Using the Trimble post processing software (TRIMVEC™), the analyst would reduce adjacent station baselines to validate the data. The collection of the data disks was to be made at a convenient point along the road as the crews were driving to their next stations.

4.3.8 Equipment

A list of equipment for each vehicle and crew was drawn up and issued to every organisation involved. Four wheel drive vehicles were used to ensure access to all stations. Beside the receiver, antenna and computer logging facilities, essential equipment included a sufficient power supply in the form of at least 4 heavy duty car batteries, meteorological instruments and additional lighting to allow setting up and data recording at night. Some interesting equipment used on the survey included extension poles (to lift the antenna over adjacent obstacles such as a sign at Gerroa), and barbecuing utensils (since the observation time spanned breakfast)!

4.3.9 Pre-survey Briefing

On the Friday before the survey was to commence, all available personnel involved in the campaign assembled at the Department's Survey and Property Branch office in Sydney for a final briefing and to discuss possible problems and their solutions. After the briefing all the prospective receiver operators travelled to AWA where they were instructed on the use of the Trimble 4000SX receivers. Although some of the Public Works personnel had experience in the use of the older 4000S receivers, the updated data logging software and enhanced hardware of the 4000SX made it essential for all operators to attend.

4.4 THE SURVEY

After the extensive planning phase was completed, the actual data acquisition component could begin. Unfortunately even the most extensive planning cannot account for all problems that may arise. However, the number of problems and the time lost in rectifying them are a function of how well the planning has been carried out. While problems were encountered on the PWD survey, they were generally addressed with a minimum of inconvenience.

The survey began on the 11th of May with all crews heading south to stations from Tathra in NSW to Lakes Entrance in Victoria. The first observation session began on the morning of the 13th (Day 132) with the crews heading north each day until the 25th (Day 144), while the final session was completed on the 27th of May (Day 146). This was two days longer than originally scheduled due to a failure to collect useable data at two sites on both the 18th (Day 137) and 22nd (Day 141) for a variety of reasons. The fieldwork was however completed within the budget estimate because provision had been made in the planning stages for an extension of the project if problems occurred.

4.4.1 Problems

Problems experienced while carrying out the project, could be divided into the following categories;

1) Power supply problems
2) Equipment malfunction and failure
3) Multipath

4.4.1.1 Power supply problems

The first major problem to arise was a loss of data on the first day at three stations. This was due to a failure of power supplies (probably due to the long observation period of over 3.5 hours). By connecting additional batteries in parallel, two of these crews had no further problems, but the third continued to lose power on the following few days. It was determined that the problem was due to the external Kaypro computer. The computer was replaced and the problem did not occur again.

4.4.1.2 Equipment malfunction or failure

For such a large scale survey where equipment reliability was critical for a successful result, the Trimble receivers performed well. Only two problems were encountered. The first was when on days 137 and 138, the receiver at Stockton encountered continual "cycle slipping". This is where the receiver temporarily loses the satellite signal. The second was at Yamba on day 141, when the receiver completely failed after only one hour of logging. In both cases AWA repaired the problem quickly, in the first instance by replacing an internal switching system, the second by replacing "shorted out" boards caused by loose carry case screws lodging in the circuitry. The second problem required the receiver to be sent from Coffs Harbour to Sydney, reducing the number of stations observed on day 142 from 5 to 4.

Problems also occurred with the RS232 cables connecting the receivers to the Kaypro computers. Loose or broken connections caused the loss of varying amounts of data, the worst being complete failure at Evans Head on day 141. This problem has been solved with the introduction of the internal logging capabilities of the newer 4000SL and 4000ST receivers, thus dispensing with the need for the external computer.

4.4.1.3 Multipath

Multipath is where some portion of the transmitted signal is reflected from the ground or other nearby structures and arrives at the antenna after the main signal, causing interference (see 5.2.1.7).

The original Norah Head site experienced what seemed to have been severe multipathing. Although the area seemed clear, on both day 137 and 146, the receiver could not lock onto any satellites. When on day 146 the receiver was moved to a new location nearby, data logging was carried out without any further problems.

4.4.2 The Debriefing

After the survey was completed, those involved met in Sydney. Each survey team was asked to outline the problems they had experienced and discuss improvements that could be made for future surveys. All believed the project was well planned and executed, while the problems encountered were solved quickly. The debriefing information was to become beneficial when later in 1987, the Survey and Property Branch was asked to assist in the execution of another large scale GPS campaign in NSW, in conjunction with the Central Mapping Authority.

4.5 CMA STRENGTHENING OF THE NETWORK

In August 1987 the then Central Mapping Authority of NSW (CMA) (now known as the Land Information Centre (LIC)) decided that the availability in September of four Texas Instruments TI4100 dual frequency receivers from Nortech Inc (Canada), should be used to connect all the substantial GPS surveys in NSW together. Those to be connected were the Public Works Department's coastal project, the Region 4 network (New England Area) observed with Wild-Magnavox WM101 receivers by the CMA and the Segment network which includes the Very Long Base Interferometry (VLBI) stations of Parkes, Fleurs and Tidbinbilla, that had been measured by the Federal Government's Survey and Land Information Group (AUSLIG) (formerly the Division of National Mapping). In the process the survey would provide a first order GPS geodetic network over the eastern third of the state. The network and its connections to the other surveys is shown in figure 4.1

The CMA with assistance from the state government's Public Works Department, the Department of Main Roads, and Electricity Commission (ELCOM), successfully carried out the survey between the 9th and 21st September. The project was jointly funded by the above organisations with each organisation supplying a vehicle and the necessary personnel to assist the Canadian and Australian operators hired through Geospectrum Pty Ltd.

The benefit in this project to the Public Works Department was that 8 stations in the coastal network would be included in the more accurate dual frequency network, thus "bracing" the single frequency network. In addition the connection into the VLBI network would supposedly give very accurate global coordinates to the coastal network. In the future, this could allow the NSW tide gauge network to be accurately related to other tide gauging systems around the world through the GLOSS network described in 2.2.1.2.

The TI4100 data was processed for the CMA by Nortech personnel using NOVAS software (WANLESS & LACHAPELLE, 1987). These results would be added to the Trimble results and placed in a network adjustment. The accuracy of the combined network was expected to be higher than the single frequency coastal network.

4.6 CONCLUSIONS

The planning and execution of a large scale GPS survey such as the PWD's coastal network requires a vast amount of planning in many different areas. However the acquisition of the data is only part of the GPS process and the factors affecting the reduction of the data need to be understood so that the most accurate results can be derived. Chapters 5 and 6 discuss the post-processing component of the survey.

5. SESSION PROCESSING METHODOLOGIES

5.1 INTRODUCTION

It is generally agreed that the acquisition of the data is the quickest and easiest part of a GPS survey. The enormous number of observations generated in a GPS campaign requires computer hardware and software that can provide an efficient method of data reduction. Although the post-processing techniques presently being developed are streamlining the data reduction process, it is still the most time consuming phase for large surveys.

It has been shown in section 3.3 that the GPS observations are biased by a number of physical effects that must be eliminated or minimised before accurate estimates of the geodetic parameters can be obtained. How well the physical situation is explained by the mathematical model is critical in the overall processing strategies and final results. It can be divided into two separate models; the functional model and the stochastic model (CASPARY, 1987; MACLEOD & RIZOS, 1988). The function model is the means by which the observations are combined to produce estimates of the selected parameters. The stochastic model contains information of the quality of the observations and the estimated parameters and this information is contained in the covariance matrices. A detailed explanation of the mathematical model is found in Appendix A.

The following discussions of the modelling and processing strategies used in the coastal survey will be taken from the approach of correlations. This chapter attempts to discuss how "correlations" are used to provide modelling and estimation techniques to improve the overall accuracies of the final solution within a session while Chapter 6 looks at the overall network accuracies as applied to the GPS coastal survey. Both multi-baseline and multi-station processing strategies are used on the coastal survey to supply empirical information in this study.

5.2 CORRELATIONS IN GPS

Appendix C provides a formal definition of correlations and their effect on GPS surveying. Briefly, correlation describes the interdependence of one entity upon another, and can be defined by how the amount of change in one correlates with changes in the other. It can be described in terms of both the actual physical situations and in terms of those mathematical models that approximate these physical situations. Going further, the mathematical models developed can have both functional and stochastic correlations.

5.2.1 Physical Effects in GPS and Their Correlations

Looking firstly at the operation of just one receiver within a session, we can visualise the observations, whether code or carrier beat phase, as a range between the satellites and the receiver. These receiver-satellite ranges are "contaminated" with a variety of errors listed in section 3.3. The accuracy of calculating the absolute position of a single point is therefore a function of the accuracies of the observed quantities and the mathematical modelling that attempts to remove these errors. The uncertainties in the satellite positions, satellite and receiver clock errors and propagation delays means that the "absolute" point position will only be accurate to no better than a few metres.

These systematic error sources will exhibit some physical correlation among the signals when received at several stations that are simultaneously tracking the same set of satellites. This means that for most survey applications it is necessary to use GPS in a "relative" mode in which two or more receivers observe the same satellites simultaneously. These physical effects can then be explicitly modelled as bias terms in the functional model. Looking more closely at these error sources, they can be divided into clock derived and geometric biases. Using mathematical operations such as "differencing" these effects that show physical correlation between stations can be eliminated or greatly reduced, giving relative positioning accuracies at the centimetre level.

CLOCK TERMS

5.2.1.1 Satellite and Receiver Clocks

The largest error sources in the satellite-receiver range measurements are due to the satellite and receiver clock instabilities. The total carrier phase error caused by these clock errors can be separated into variations in the receiver oscillator frequency (clock phase error) and the error in the observation time tag (time tag error) (GRANT, 1989, p.103). The time tag error (ie the error in the calculated signal emission time) need only be resolved to the millisecond level as the satellites travel at much slower velocities than light (ie around 4 km/sec) and this error is reduced when converted to a satellite-receiver range error. Usually the time tag error is determined to a suitable accuracy using the pseudo-range (code phase) solution. Accurate resolution of the clock phase error is the more important since an error of 1 μsecond gives a direct range error of 3 metres. This is most accurately determined using the phase information, where it can be estimated or eliminated using various mathematical models.

So the clock biases in differential positioning will have three components (HOLLOWAY, 1988, p.42)

1) an epoch offset from GPST
2) an epoch difference between the receivers
3) a time rate difference between the receivers and the satellite

An epoch offset from GPST will result in the satellite ephemerides being interpolated at the incorrect time and will be common to both receivers. The error is a combination of the time bias from GPST and the satellite along-track velocity (4 km/sec) commonly known as the time tag error. To minimise this bias effect, the receiver clocks need to be synchronised to GPST within 7 milliseconds for a baseline error below 1 ppm (KING et al, 1987) and if both the receiver quartz clocks are synchronised to each other within 3 microseconds then the error reduces below 1 cm (ibid).

The time rate difference between receiver and satellite depends on the frequency stabilities of the clocks used. While the satellites use atomic standards, the receivers are usually equipped with less stable quartz oscillators. A common model that describes the clock behaviour (and that is used in the broadcast message) in the form of coefficients of a second order polynomial is:

$$f(t) = f_0 + a_1 + a_2(t - t_0) + a_3(t - t_0)^2 \tag{5.1}$$

where f_0 is the nominal frequency
a_1 is the frequency offset
a_2 is the fractional frequency offset or frequency drift
a_3 is the fractional frequency drift or frequency acceleration
t_0 is the reference epoch

Generally the frequency offset, drift and acceleration will change for each receiver over a few hours while the satellite clocks are more stable. The stabilities of various clocks used in GPS are given in Table 5.1.

Table 5.1 Oscillator stability (KING et al, 1987)

OSCILLATOR TYPE	FRACTIONAL FREQUENCY STABILITY (over 6 hours) a_2/f_0	FRACTIONAL DRIFT STABILITY (over 6 hours) a_3/f_0
Cesium	10^{-13}	$10^{-15}s^{-1}$
Rubidium	10^{-12}	$10^{-14}s^{-1}$
Quartz	10^{-10}	$10^{-12}s^{-1}$

The 7 Block I satellites existing at the time of the coastal survey used various atomic clocks shown in Table 5.2. There are clock corrections estimated by the control network for each satellite clock and these are transmitted in the navigation message the accuracies of the corrected clocks is around 30 nanoseconds (WELL, et al, 1986).

Table 5.2 Satellite clocks of Block I satellites used in the coastal survey, May 1987 (after KING et al, 1987)

SV NUMBER	CLOCK TYPE	
3	Rubidium	
6	Rubidium	
8	Quartz	*
9	Cesium/Rubidium	
11	Rubidium	
12	Cesium	
13	Cesium	

* The original Rubidium clock had failed

The residual errors in the modelled corrections of each satellite clock and the receiver clock errors are unwanted biases that can be dealt with in the mathematical modelling in various ways using differential GPS observations (GRANT, 1989, p.197; WELL et al, 1986; ECKELS, 1987, p.107).

1) observation differencing (explicit differencing)
   Between satellite differencing
   (eliminates receiver clock bias)
   Between receiver differencing
   (eliminates satellite clock bias)
2) eliminating independent clock biases at each epoch (implicit differencing which is equivalent to (1) - see section 5.3)
3) estimation throughout the observation session using polynomials in the functional model (eg equation 5.1)
4) Stochastic modelling using Weighted Parameters (eg. a random walk process in a Kalman filter.)

5.2.1.2 Carrier Phase Ambiguity

While not actually a direct physical effect on the measurements, it is in fact part of the receiver-satellite range that is directly related to the satellite and receiver clocks. Described in section 3.3, once all other errors are eliminated or modelled, it is the ambiguity that has to be correctly resolved to accurately determine the range. Residual biases left after modelling will be incorporated in this ambiguity and if they are small, then the calculated ambiguity to each satellite will be close to an integer. Treating the ambiguity terms as a constant, by holding them to their integer values in a second adjustment can in many cases, more accurately determine the relative horizontal position of the unknown station. However using covariance analyses, HOLLOWAY (1988) and GRANT (1989) found that there is usually only minimal improvement in the determination of the height component and in some cases the height accuracy is actually degraded by ambiguity resolution. Of course if the ambiguities are fixed to incorrect values, then the solution will be biased.

GEOMETRIC TERMS

5.2.1.3 Ephemeris (Orbits)

The satellite ephemeris is a set of parameters describing the position of a satellite with respect to the earth's centre of mass as a function of time. While the satellites' orbits generally follow Kepler's three laws of planetary motion, they are "perturbed" by gravitational and non-gravitational disturbing effects acting on the satellite. The gravitational derived perturbations are caused by density irregularities within the earth, the third body effects of the mass of the sun and moon, and ocean tidal effects. Non-gravitational perturbations are caused by atmospheric drag and solar radiation.

Therefore the procedure used to develop an orbit generation program for the broadcast ephemeris in the navigation message (see 3.2.1.3) is to track the satellites from a number of ground stations around the world and use an appropriate force model that reflects the perturbing effects on the satellites, then from this information predict future orbits. Errors in the broadcast ephemeris are caused by (HOLLOWAY, 1988, p.41)

1) errors in the dimensions of the reference ellipsoid (WGS84)
2) errors in the positions of the tracking stations
3) errors in the position and velocity of the satellite (the initial state) caused by tracking errors
4) errors in the force model of the perturbing effects

Errors in the satellite's orbit may be described in the Height, Cross-track and Long Track (HCL) system. The Height is the radius vector in the direction of the satellite to the earth, the Cross-track is normal to the orbital plane and the Long-track is perpendicular to both H and C. GRANT (1989) notes that the height component is best determined, while the along-track being the worst determined (L having an error 2 to 4 times worse than H). The corresponding velocity errors ($\dot{H}\dot{C}\dot{L}$) are also in the same proportion.

Most GPS literature states the typical accuracy of the broadcast ephemeris is about 20 metres, with occasional errors up to 80 metres. KING (1987) however, has noticed cross track errors up to 80 metres and long-track errors occasionally up to 150 metres. The DoD policy is to maintain the orbits to an accuracy of better than 200 metres. This presently allows up loading of new ephemeris data on the Block I satellites every 4 to 6 hours, while the Block II satellites with their increased protection against solar radiation only require uploads every 24 hours (DoD, 1989). In addition the DoD's policy of selective availability that "degrades" the broadcast ephemeris and satellite clock parameters to non-military users is now in place. Therefore the 20 metres accuracy stated for the extrapolated broadcast ephemeris may be optimistic.

Conversely using a globally determined post processed ephemeris based on pseudo-range observations, this accuracy of 20 metres may be conservative. The precise ephemeris can be requested from such organisations as the Defense Mapping Agency (DMA) in the USA, and since it interpolates actual tracking data, it should reduce the effect of some errors inherent in the broadcast ephemeris. This may not always be the case, as tests in 1985 by the South Australian Lands Department showed that the precise ephemeris supplied by the DMA was in fact worse than the broadcast ephemeris in the Australian region (JONES, 1988a). One of the reasons was that the tracking network used by the DMA at that time did not include a site in Australia. Presently the DMA's precise ephemeris is computed by the U.S. Naval Weapons Center (NSWC) using a similar operation to the broadcast ephemeris computation with the five monitor stations being augmented by information from sites in Australia, Seychelles, England and Argentina. Regional tracking networks using dual frequency carrier phase observations have produced post-computed orbits with accuracies of a few metres (BEUTLER et al, 1987; MASTERS & STOLZ, 1985) but require widely spaced base stations with very accurate station coordinates. This was not available in Australia at the time of the coastal and subsequent TI4100 surveys in 1987.

Ephemeris errors will directly translate to the same magnitude errors in receiver single point positions, scaled by the satellite geometry. Whilst differential techniques reduce the effect on a measured baseline, the error will not be eliminated. As a general rule of thumb, for a given error in the satellite's orbit, the error in the baseline will obey the approximate relation (KING et al, 1987):

$$\Delta b \text{ (ppm)} = \frac{\Delta e}{a} \tag{5.2}$$

where Δb is the baseline error in ppm
Δe is the ephemeris error
a is the altitude of the satellite

On this basis an ephemeris error of 20 metres translates into an error in a baseline of about 1 ppm.

Modelling orbital biases is a problem, with the simplest solution being to ignore their existence and hold the orbit fixed. This method was adopted for the PWD coastal network, because the commercial software used had no facility for modelling orbital parameters. However, both the broadcast and precise ephemerides have been used with surprisingly similar results (see 5.5).

5.2.1.4 Ionosphere

Electromagnetic signals passing through the earth's atmosphere will experience bending and various "delays" as they travel through a number of physically different layers. The excess path length from the ideal geometric straight line between satellite and receiver and that travelled by the signal (being the minimum electrical path length) can be approximated as (DODSON, 1988):

$$\Delta P_{ION} = \int_{LOS} (n - 1)\, ds \qquad (5.3)$$

where n is the (varying) refractive index
LOS is the line of sight

This propagation delay is more commonly defined in terms of the refractivity (N) of the atmosphere, through the relationship:

$$N = (n - 1) \times 10^6 \qquad (5.4)$$

Therefore

$$\Delta P_{ION} = \int_{LOS} N.ds \qquad (5.5)$$

The atmosphere can generally be divided into the ionosphere (altitude of 70-1000 kilometres) and neutral atmosphere (altitude up to 70 kilometres). In the ionospheric region the X-ray and ultra-violet radiation from the sun ionises some of the gas molecules, releasing free electrons that cause the region to be negatively charged. The non-linear characteristics of this dispersion within the medium affect the GPS signals and these effects vary with the frequency.

While GPS code signals are dependent upon the group refractive index and are retarded through the ionosphere, the carrier phase is governed by the phase refractive index and is in fact the same magnitude but opposite sign, that is to say the phase is actually advanced when passing through the ionosphere (DODSON, 1988). By relating the refractive index to the frequency, the change in the propagation path length for the carrier phase travelling through the ionosphere (known as the ionospheric phase delay) in metres (PARTIS & BRUNNER, 1988; WELLS et al 1986) is:

$$\Delta P_{ION} = \int_{LOS} (n_p - 1)\, ds = \frac{40.3}{f^2} \times SEC \tag{5.6}$$

where n_p is the phase refractive index
SEC is the Slant Electron Content

The Slant Electron Content is the total number of electrons along the propagation path and is dependent on the elevation of the satellite and will increase as the satellite moves away from the zenith. The magnitude of the SEC can be approximated by the function:

$$SEC = \frac{TEC}{\cos(z)} \tag{5.7}$$

where z is the zenith angle from receiver to the satellite
TEC is the Total Electron Content

TEC is the total number of free electrons in the zenith direction, expressed as the number of free electrons per square metre and typically ranging from 2000 x $10^{15}m^{-2}$ down to

$10 \times 10^{15} m^{-2}$. The variations in the magnitude of the TEC depend on:

1) Latitude - TEC is a minimum at the poles and maximum at low to mid-latitudes.
2) Diurnal cycle - TEC is generally a minimum at night
3) Annual cycle - TEC can vary between seasons
4) Sunspot activity - solar activity has an 11 year cycle
5) Magnetic storms

Figure 5.1 shows typical diurnal zenith ionospheric delay results, and Figure 5.2 monthly sunspot numbers that indicate the monthly and 11 year solar cycle effects.

Figure 5.1 Diurnal Variation in Zenith Ionospheric Delay at L1 Frequency at latitude 39° N (CAMPBELL & LOHMAR, 1985)

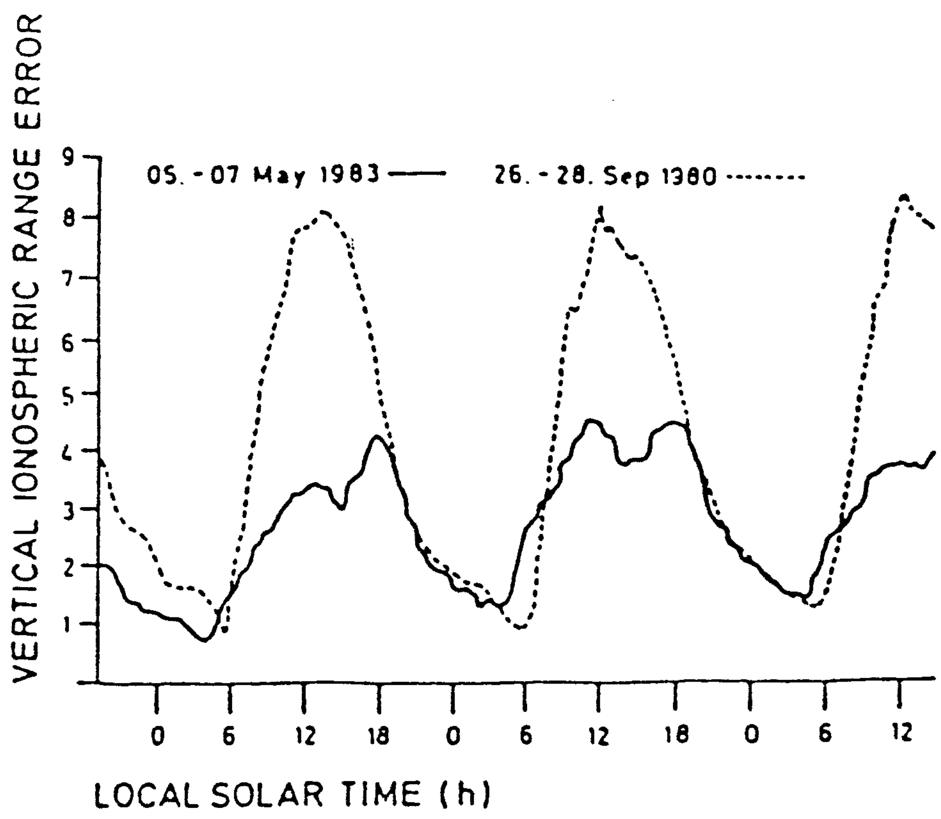
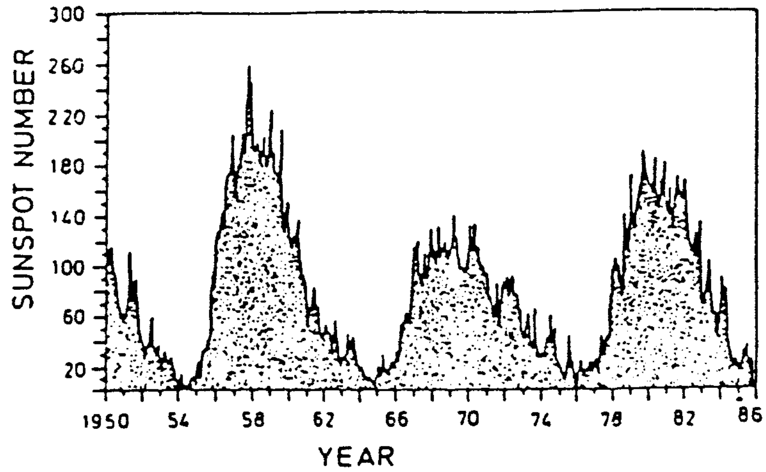


Figure 5.2 Monthly Sunspot Numbers 1950-1985
(CAMPBELL & LOHMAR, 1985)



At GPS frequencies the ionospheric delay may vary from more than 150 metres (near the horizon at midday at a period of maximum sunspot activity) to less than 3 metres (minimum sunspot activity, night observations and at the zenith). A number of methods have been devised to correct the carrier phase observations with varying degrees of success.

Since the ionospheric delay is frequency dependent, where measurements are made simultaneously on both the L_1 and L_2 frequencies the ionospheric delay can be quantified or eliminated using combinations of the L_1 and L_2 frequencies (REMONDI, 1984; GOAD, 1985; LINDLOHR & WELLS, 1985; WELLS et al, 1986) before being considered in the functional model. The phase correction equation (WELLS et al, 1986) is given as

$$\Delta\theta_{ION}(L1) = \frac{(f_2^2 - f_1^2)}{f_2^2}[\theta(L1) - \frac{f_1}{f_2}\theta(L2) - (N(L1) - \frac{f_1}{f_2}N(L2))] \qquad ...(5.8)$$

where N(L1) = cycle ambiguity on the L1 frequency
N(L2) = cycle ambiguity on the L2 frequency

Another way is to produce a linearly combined (LC) "ionospheric free" phase observable θ_{LC} that can be used in the processing (KING et al, 1987), where:

$$\theta_{LC} = \theta_{L1} - 1.984(\theta_{L2} - 0.779\theta_{L1}) \qquad (5.9)$$

However any dispersive noise in the original phase observations such as multipath will be amplified in the LC observable (SCHAFFRIN & BOCK, 1987). While improved antenna design and filtering help reduce the problem (BISHOP et al, 1985) this will cause particular problems in resolving the integer ambiguities on short lines and it may be preferable to treat L1 and L2 separately, since the GPS signals will pass essentially through the same part of the atmosphere and will be effectively cancelled in between-station differencing. However, for long baselines where the atmospheric (including the ionosphere) conditions are physically uncorrelated at each end of the baseline, then it is preferable to form LC and eliminate θ_{ION} as differencing alone will not eliminate the bias. From research DODSON (1988) notes that the residual error or bias in the absolute ionospheric delay after modelling propagated into the baseline more as an overall scale error. This is because the baseline will be shortened since the apparent ranges are themselves shortened when using carrier phase observations. However a residual error in the relative ionospheric delay produces proportional errors in the heights, particularly those oriented north-south (HOLLOWAY, 1988, p.35).

When using single frequency receivers such as in the PWD coastal survey, the measurements cannot be "preprocessed" in this way and the delay must be modelled or ignored in the mathematical model. Table 5.3 shows some common models that have been developed.

Table 5.3 Expected residual errors in the zenith from various ionospheric correction methods (After HOLLOWAY, 1988)

METHOD USED	RESIDUAL IONOSPHERIC RANGE ERROR
Without correction	2 - 50 m
Klobuchar model	1 - 25 m
Bent model	0.5 - 10 m
Bent model + updating	0.25 - 3 m
Dual frequency observations	< 0.1 m

Many suggest the use of an ionospheric model such as that broadcast in the navigation message. Developed by Klobuchar in 1982, it is a simplified global model that is estimated to have an average accuracy of 50% globally. The fact is that due to significant daily variations in TEC, this model has sometimes been shown to produce worse results than if no model was used.

More recently the idea of regional models has been proposed. One such method to develop such a model is to use dual frequency Doppler data transmitted from TRANSIT satellites at 150 and 400 MHz to estimate the TEC and then calculate the ionospheric delay at the GPS frequencies. The advantage is that the GPS and TRANSIT observations are taken over the same period in the same region, so a realistic model can be developed. This approach was suggested by the University of NSW for the initial PWD coastal survey when only single frequency Trimble 4000SX receivers were used. TRANSIT receivers (MX1502's) were located at Eden, Sydney, Coffs Harbour and Brisbane and operated throughout the GPS campaign. PARTIS & BRUNNER (1988) describe the methods used to calculate the TEC and tabulate the initial results over 3 days as compared to local ionosonde data. While the results were very encouraging, further work was unfortunately not continued into creating a model to correct the Trimble observations for ionospheric delay.

5.2.1.5 Tropospheric Delay

At radio frequencies the neutral atmosphere that includes the troposphere (0-11 kilometres) is a non-dispersive propagation medium and so affects all GPS signals similarly. It therefore cannot be eliminated by the L1/L2 combination. As with the ionospheric delay, the basic requirement is to estimate the integral of the refractive index or refractivity along the ray path and this is commonly divided in to the dry or hydrostatic component (N_D) and wet component (N_W). Applying equation (5.5):

where $N = N_D + N_W$

$$\Delta P_{TROP} = 10^{-6} \int_{LOS} (N_D + N_W).ds \tag{5.10}$$

and where SMITH & WEINTRAUB (1953) define:

$$N_D = 77.6 \frac{P}{T} \tag{5.11}$$

$$N_W = 3.73 \times 10^5 \frac{e}{T^2} \tag{5.12}$$

where P is the total atmospheric pressure in mbar
T is the temperature in degrees Kelvin
e is the partial water vapour pressure in mbar

The tropospheric range delay in the direction of the satellite can be considered in similar terms to the ionospheric delay, being the zenith delay and a mapping function that is dependent on the elevation and azimuth of the satellite. In the zenith direction the hydrostatic component is of the order of 2.3 metres and can be modelled within 2-3 millimetres using a more comprehensive version of equation (5.11) with measured surface pressure, temperature and knowing the station height above sea level and gravity (HENDY, 1990). This is approximately 90% of the total delay. However, the wet component that has a magnitude of 0.1 to 0.3 metres is difficult to model accurately because it depends on the atmospheric conditions along the total signal path. The distribution of the partial water vapour pressure can vary both spatially and temporally, meaning surface measured meteorological values may have little relation to the actual situation at higher altitudes. This causes variability of the modelled zenith wet delay component equivalent to about 6 centimetres (HENDY, 1990).

Methods to correct or minimise the tropospheric delay in the GPS carrier phase measurements include:

- No correction: Here the error is assumed to be the same at each station and will be eliminated in the between station differencing. This is valid when the baseline is short and there is high correlation in the atmospheric conditions at each station.

- Modelling the delay using one of a number of empirical models that use standard or surface measured meteorological values. Careful application of these models summarised in Table 5.4, will in most cases reduce the error in the delay to a few centimetres. However the residual errors become large at lower elevations and it is common practice to restrict observations to a minimum "cut off" elevation (normally 15 degrees). This method is used extensively in commercial software packages such as TRIMVEC™ (Modified Hopfield Model).

- The use of a priori information such as radiosonde data, and atmospheric measurements from weather balloons to produce statistical models and profiles.

These can be improved by information from meteorological satellites (ASKNE & NORDIUS, 1987).

Table 5.4 Tropospheric correction models
(After COCO & CLYNCH, 1982)

MODEL	NOTES
HOPFIELD	Based on a quartic model of the refractivity
MODIFIED HOPFIELD	Improved empirical values of model's constants.
BLACK	Empirical model with no surface measurement inputs, depends only on latitude
BERMAN	Empirical model with different parameters for day and night cases
CHAO 73	Assumes adiabatic law rather than perfect gas law, semi-empirical model
SAASTAMOINEN	Assumes a linear decreasing temperature as a function of altitude
SAASTAMOINEN/ MARINI	A variation of the Saastamoinen model

- Use Water Vapour Radiometer (WVR) measurements along the line of sight. Here microwave emissions of the atmosphere, at frequencies around the water vapour spectral line centred at 22.235 GHz, are accurately measured. The wet component can be reduced to about 1 centimetre but the cost and size of the equipment is prohibitive.

- Functional and stochastic modelling of the residual tropospheric delay error (see GRANT, 1987).

The difference between atmospheric conditions at network stations is the most difficult error source to handle. The factors affecting the wet delay are the most difficult to determine and the total residual delay error after corrections are applied is the major source of height errors in GPS, as a relative residual delay error of 1 mm in the zenith transposes into an error of 2.9 mm in the relative height (BEUTLER et al, 1987b). Modelling this error, for example by using a "residual tropospheric delay" bias term at each station over the whole session or on an epoch by epoch basis using a Kalman filter and dynamic model, have been shown in simulations carried out by GRANT (1987) to reduce the bias and increase the accuracy of the heights.

5.2.1.6 Station Coordinates

The one-way observation equation includes the station coordinates. Normally treated as unknowns to be estimated, they however may be constrained in situations including:

- non positioning applications such as time transfer or orbit determination.
- differential carrier phase observations due to the datum defect in the solution (5.3.2).

The accuracies of these "constrained" stations depend on the methods used to determine the WGS84 coordinates. Weighted averages of C/A code pseudo-range positions may be accurate to about 20 metres, while VLBI coordinates can produce absolute accuracies of greater than 5 metres and relative accuracies in parts in 10^8 of the baseline length. Another method is to use established transformation parameters to calculate WGS84 coordinates from local geodetic coordinates. The resultant accuracies are dependent on the accuracy of the transformation parameters in the area.

For non positional applications, absolute errors in station coordinates will transfer directly into the unknowns to be estimated (eg. the orbital parameters). In the case of differential applications the resulting error is proportional in nature. For example an error of the "fixed" base station or origin coordinates of 20 metres causes a baseline uncertainty of about 1 ppm, similar to that generated by a corresponding satellite ephemeris error of the same magnitude.

5.2.1.7 Noise

Noise is a general term used to describe other observational errors that are assumed random in nature. These include multipath effects and antenna phase centre movements, and errors due to the limitations of the receivers themselves.

Multipath

This is a form of signal interference where the incoming signal arrives at the receiver antenna via two or more paths. Usually caused by part of signal being reflected from nearby surfaces, the different path lengths causes the signals to interfere at the antenna. Antenna designs to minimise this problem have been developed through antenna beam shaping that discriminate against signals arriving from certain directions. However the effects cannot be totally eliminated and proposed sites should be checked for possible objects and surfaces which could cause multipathing such as a metal roof or a body of water.

Being antenna and site dependent, these effects will not cancel when using differencing techniques. Multipath effects in highly reflective environments may limit the accuracies of the receiver's code phase measurements to about 10 metres, and average out to a few centimetres in static carrier phase observations when made over an extended session. However in the most severe situations, a code correlating receiver may not be able to synchronise ("lock on") the incoming signal with that generated in the receiver. This is what is thought to have happened in the coastal survey at Norah Head on the 18/5/87, since no satellites could be tracked. Because multipathing effects are repeatable on a day to day basis for the same satellite - antenna geometry, a second attempt to observe was made on the 27/5/87. Again no satellites were tracked, even though the area was clear of obstructions. The receiver was moved 1.5 kilometres away and the satellite signals were acquired and tracked normally. The original area did however have a house with a galvanised iron roof only 50 metres away from the station that is thought to be the cause of the multipathing.

Antenna Phase Centre Movement

Another error source that is specifically related to the antenna design is that of "phase centre movement". Defined as the difference between the electrical phase centre (where the signals are measured) and the geometrical phase centre (the coordinate origin) of the antenna, its magnitude will vary with azimuth and elevation of the satellite. Again good antenna design can overcome this bias and to this end, microstrip antennas such as those used in the Trimble receivers reduce its magnitude to a few millimetres.

Random Observational Errors

Additional observational errors generally come from the limitations in the receiver's electronics but are usually of magnitudes less than 1% of the signal's wavelength ie. 3 metres for C/A code phase and 2 millimetres for carrier phase (WELLS et al, 1986).

5.2.1.8 <u>Residual Biases</u>

While differencing eliminates some errors (clock errors), the variation in spatial and temporal coherence in other errors (orbits errors and propagation delays) will create residual biases after modelling. Also known as model errors (WELLS et al, 1986), they remain in the satellite-receiver ranges. Caused when the mathematical model used does not completely or correctly account for the physical situation, these residual biases are systematic in nature and their minimisation is the subject of a lot of research that is producing more complex models in an attempt to provide higher accuracies.

5.2.2 The GPS Functional Model and Functional Correlation

The functional model described in equation (3.2),

$$\theta_R^S(t) = \frac{1}{\lambda}\sqrt{(x^S(t) - x_R(t))^2} - f.c_R(t) - f.c^S(t) - n_R^S$$
$$+ (\theta_{ION})_R^S + (\theta_{TROP})_R^S + \theta_{NOISE}$$

defines the classic observation equation for the carrier phase from one satellite to one receiver. The $c_R.f$, $c^S.f$, θ_{ION} and θ_{TROP} terms in this (undifferenced) one-way phase functional model are the biases introduced to reflect the physical correlation in the observations. In addition the carrier beat phase model includes the $n_R{}^S$ term, which is the initial ambiguity term. This is analogous to an orientation parameter in the adjustment of horizontal directions. Introducing this parameter creates functional correlation, but eliminates stochastic correlation by creating independence between the observations.

Other functional models can be derived using linear combinations of this one-way carrier beat phase observable. The process of differencing takes advantage of the physical correlations which occur in the measurements when two or more receivers are tracking satellites simultaneously by mathematically eliminating their equivalent functional correlations. The original one-way phase measurements can be differenced using various differencing operators between-receivers, between-satellites and between-epochs, or in combination.

Defined another way, taking linear combinations between satellites, between receivers, between epochs or in combination, allows systematic errors common to the range measurements to be removed or severely reduced. While many differencing combinations are possible, the most common approach is to difference in the above order. When differencing occurs, the original carrier phase observations and their one-way phase mathematical model are modified. This means both the functional and stochastic models change. Using the generalised matrix form of the GPS functional model given in Appendix B, a differencing operator D is applied to the system:

$$DAx = Db + Dv \tag{5.13}$$

This leads to a new functional model:

$$\acute{A}x = \acute{b} + \acute{v} \tag{5.14}$$

The new observations in $\acute{b}$ are $\acute{l} = Dl$

In the least squares process, a measure of the quality of the measurements is generally known as the variance covariance matrix (VCV) of the observations and it will be changed when the original observations are altered by a mathematical operator such as that introduced in differencing. The new "differenced observations" will be accompanied by a change in the VCV of the observations through the law of the propagation of variances (MIKHAIL, 1976; CROSS, 1983):

$$C_{Dl} = D\,C_l\,D^T \tag{5.15}$$

where C_l is the variance-covariance matrix of the original carrier phase observations
D is the differencing operator
C_{Dl} is the variance-covariance matrix of the "differenced" phase observations

5.2.2.1 "Differencing" Functional Models

Single Differences

Considering two receivers; the differences between simultaneous one-way phase observables at both receivers to the same satellite can be represented as:

$$\Delta\theta = D_\Delta \theta \tag{5.16}$$

Replacing the one way phases with their functional model, the single differences have the form:

$$\Delta\theta = \Delta p^S(t) + \Delta fc_R - \Delta n^S \tag{5.17}$$

where p is the range defined by $\frac{1}{\lambda}\sqrt{(x^S(t) - x_R(t))^2}$

The functionally correlated satellite clock terms have cancelled. The assumption that the atmospheric conditions at both receivers are the same means the atmospheric delay terms can also be considered to have vanished. This assumption is reasonably valid for short baselines (under 30 km) (see 5.2.1.1). However decorrelation of the environmental influences occurs as the distance between receivers increases. MERMINOD (1988a) and WELLS et al (1986) point out that from experience with single frequency GPS surveys, we know that even after differencing, it's usually not possible to resolve ambiguities for baseline lengths over 30 km, irrespective of session length, without appropriate modelling of the atmospheric delay. A scale error proportional to the baseline length, caused by the error in the orbit of the satellite will also remain (see 5.2.1.3).

Double Differences

At each epoch the single differences can be differenced between satellites. A differencing operator can be predefined such that:

$$\nabla\Delta\theta = D_{\nabla}\,\Delta\theta \tag{5.18}$$

The choice of an operator is not straight forward and several possibilities are available. They include:

1) fixed reference, say the satellite with the lowest PRN number or highest elevation (eg TRIMVEC™)
2) sequential with respect to the previous satellite
3) sequential and uncorrelated - ie the stochastic model of the observations is decorrelated by a mathematical process called the Gram-Schimidt orthogonalisation (see MERMINOD (1988a)).
4) reduction by the mean

If we replace single differences with their functional model, the double differences have the form:

$$\nabla\Delta\theta = \nabla\Delta p(t) - \nabla\Delta n \tag{5.19}$$

The receiver clock bias terms, independent of the satellites have cancelled. So building doubled differences is a convenient way to avoid modelling the oscillator (clock) errors by taking advantage of their functional correlation in the single differences.

Triple Differences

A simple technique for computing differences between epochs is to considered two successive epochs. Thus the end epoch of a double differenced observation is also the start epoch of the next one. Therefore, the triple differences are correlated pairwise. Applying the triple difference operator:

$$\delta\nabla\Delta\theta = D_\delta \nabla\Delta\theta \tag{5.20}$$

The differencing operator acting on the double differences is identical to that used to create double differences by the sequential differencing option. There are other ways to build triple differences such as using a fixed reference. However, the sequential approach presents an advantage if lock is lost on a satellite between one epoch and the next, only the corresponding triple difference will be affected.

Replacing the double differences by their functional model, the triple difference becomes:

$$\delta\nabla\Delta\theta = \delta\nabla\Delta p(t) \tag{5.21}$$

The integer terms, invariant with time (and therefore functionally correlated in the double difference model) have cancelled.

Triple differences at different epochs are stochastically correlated. One approach is to ignore the correlation between successive triple differences. However, a rigorous solution requires this correlation to be taken into account. A method of accounting for pairwise correlation between epochs is to decorrelate the observations with a recursive algorithm (see for example MERMINOD, 1988a, p.39). However, triple differences are correlated pairwise only if the double differences themselves are uncorrelated.

Neglecting the correlation between epochs results in a weaker solution. On the other hand the solution is also less sensitive to errors in a particular measurement, because the error in one triple difference are not propagated to subsequent ones. This is why triple differences, when their stochastic correlation is ignored, are commonly used to identify cycle slips in GPS data processing.

5.2.3 The GPS Stochastic Model and Stochastic Correlation

So the single, double and triple difference operators, progressively eliminate functional correlations between observations. However as the functional correlation is decreased, the stochastic correlation increases. MERMINOD (1988b) states

"The word "correlation" carries several concepts and this is leading to numerous misunderstandings in the GPS community. As a typical example, "uncorrelated" triple differences are not uniquely defined. Some people will believe that non-diagonal terms of the covariance matrix are simply neglected. Others will consider that the covariance matrix has been made diagonal through an appropriate transformation of the set of observations."

Generally we assume all original carrier phases observations are independent and have the same variance, producing a covariance matrix that is diagonal.

$$C_l = \sigma^2 Q_l = \sigma^2 I \qquad (5.22)$$

where σ^2 is the population variance

Q_l is the co-factor matrix related to the observations

As we are usually interested in error propagation rather than in absolute values, this variance can be set to unity. Therefore:

$$C_l = I \tag{5.23}$$

However TALBOT (1988) has proposed to weight each observation by a factor related to the signal to noise ratio. Tests have shown a marked smoothing of the residuals after adjustment using this method, although clearly erroneous observations should be eliminated before the weighting is applied. BOCK et al (1986) suggests that the covariance matrix is not diagonal, since propagation effects on the phase measurements are highly physically correlated over tens of kilometres, while orbital errors are also proportional to the baseline length. Furthermore, cancellation (or modelling) of environmental effects is likely to be incomplete for longer baselines, mainly due to differing environmental conditions at each station. To express the residual effect between stations i and j, terms relative to the baseline length could be introduced into the covariance matrix of the phase observations. In the approach the residual effects can be grouped as constant (a) or length dependent (b) such that:

$$\sigma^2 = a^2 + b^2 L_{ij}^2 \tag{5.24}$$

If at a certain epoch two stations observed two satellites simultaneously such that the 4 carrier beat phases can be formed, then the observation matrix becomes:

$$l^T = [\theta_{11}\ \theta_{12}\ \theta_{21}\ \theta_{22}] \tag{5.25}$$

and the covariance matrix for the carrier beat phase observations becomes:

$$C_l = \begin{bmatrix} a^2 & 0 & -b^2 L_{12}^2 & 0 \\ 0 & a^2 & 0 & -b^2 L_{12}^2 \\ -b^2 L_{12}^2 & 0 & a^2 & 0 \\ 0 & -b^2 L_{12}^2 & 0 & a^2 \end{bmatrix} \qquad (5.26)$$

C_l now has off diagonal terms and therefore exhibits stochastic correlation. This method can be used for both differenced and undifferenced approaches. In effect, as receiver separation increases, the inability of the functional model to reflect the physical decorrelation in atmospherics and orbital errors can be compensated by adding terms in the stochastic model of the observations.

When differencing occurs the resulting covariance matrix will generally show stochastic correlation due to the law of the propagation of variances and is of the general form shown in equation (5.15). It can be shown that the single differencing between only 2 receivers reduces both the number of observations and unknowns by the same amount, without introducing stochastic correlations if the original carrier phase observations are assumed independent. MERMINOD (1988a) shows for a single baseline the variance of single differences are twice as large as that of one-way phases. However, not all single differences can be independent if more than 2 receivers are considered, since data from some receivers must be used several times to build a set of single differences that include all receivers in a session. Therefore stochastic correlation is introduced into the single difference mathematical model.

On the other hand, double and triple differences exhibit stochastic correlation no matter what differencing operator is used. The covariance matrix of the double and triple difference observables can be made diagonal using the recursive algorithm in MERMINOD (1988a) but it is computationally cumbersome. It is simpler to take the correlations into account in the reduction process and BEUTLER et al (1987c) shows efficient methods to compute the inverse of these differenced covariance matrices.

5.3 POST-PROCESSING THE CARRIER PHASE OBSERVABLE

5.3.1 Differenced versus One-way phase Mathematical Models

From section 5.2 it has been shown that a number of linearly independent observations can be formed by differencing between satellites, receivers and epochs from the original one-way carrier phase observations. The "differenced" models and original one-way phase model (or undifferenced model) will show equivalence, based on the fundamental differencing theorem. The fundamental differencing theorem as described by LINDLOHR & WELLS (1985) states:

"Linear biases can be accounted for either by reducing the number of observations so that the biases cancel, or by adding an equal number of unknowns to model the biases. Both approaches give identical results."

Therefore the effect of the clock derived phase errors or biases (c_R, c^S, $n_R{}^S$) may be eliminated either by differencing them all out (ie. Triple differencing), or explicitly modelling them all (ie. undifferenced model), or a combination of the two (ie. single and double differencing). However, this statement is only correct if,

1) the variance-covariance matrix of the differenced observables takes into account the stochastic correlation introduced in the differencing models.
2) the clock error parameter models are equivalent

While (1) has been discussed in detail in section (5.2.3) a brief outline of (2) is given here, since both undifferenced and differenced models are used in the coastal survey, and the differences in the results obtained from these models requires some explanation. A more detailed discussion is found in GRANT (1989) and GRANT et al (1990).

There are a number of parameter modelling options available when using explicit clock bias modelling (in undifferenced or single differenced mathematical models) as noted in section (5.2.1.1). The parameter can be modelled as a constant for the whole session, a independent constant each epoch, or something between the two, such as a truncated power series in time. The method most commonly used with one-way phase observations is to model the integer ambiguity as a constant bias term and the clock errors are estimated at each epoch. While there are a large number of clock biases to be estimated, they can be eliminated from the solution at each epoch using matrix partitioning techniques (GOAD, 1985) such as "Helmert-blocking" (see CROSS, 1983).

In this epoch by epoch modelling there are no assumptions made as to the structure and stochastic behaviour of the errors and therefore are considered as "white noise" and uncorrelated. This may not be true if other models are used (eg polynomials) since they have a predictable structure and place constraints on the bias estimation that may leave residual clock biases. Hence different clock models will generally produce different coordinate results. In fact exact equivalence between explicit clock error modelling and clock error elimination through differencing is only possible when the clock error is modelled as an independent bias at each epoch. In the undifferenced approach, this epoch by epoch modelling is called **implicit differencing** (KING et al, 1987) to separate it from other clock error modelling options.

So what mathematical model should be used? The matrix manipulations used in the undifferenced model to eliminate the epoch by epoch clock biases is an additional computational burden that is not required in the double difference model. However the $D C_l D^T$ covariance matrix must be formed and inverted for a rigorous double difference adjustment and while efficient methods are available to do this (BEUTLER et al, 1987c) the two methods would seem computational similar. Often the conditions for exact equivalence are not met primarily because the stochastic correlations in the differencing approaches (usually double differencing) are neglected when more than two receivers operate simultaneously and hence produce sub-optimal solutions. The undifferenced mathematical model presents a number of advantages over other models, including (GRANT et al, 1990):

1) It uses physically meaningful GPS measurements instead of mathematically derived "differenced" observables.
2) The basic geodetic parameters remain the receiver and satellite vectors, rather than receiver baselines.
3) There is no need to make decisions about what observation differences should be formed.
4) The adjustment of undifferenced phase data is always rigorous
5) The undifferenced model allows the flexibility to implement various clock modelling options, whereas differencing can only eliminate the clock errors.

In its favour, differencing reduces the number of parameters to be estimated that in turn can reduce the computational space required in the computer, particularly if baselines are computed separately. This is why it is commonly used in commercial software that run on micro-computers. Furthermore GOAD (1988) acknowledged that the base station-base satellite undifferenced approach may suffer when attempting to solve the integer ambiguities, since they are referenced to this base station and ambiguities to stations close to the base station are affected by the difficulty in resolving ambiguities over longer lines in the session. Using differencing, the minimum vector separations between adjacent stations need only be processed and their ambiguities resolved.

5.3.2 Rank Defect and Datum Definition

The fundamental differencing theorem provides GPS users with a variety of "observables" by which to process GPS carrier phase measurements. However if an attempt is made to estimate all geodetic parameters and biases the normal equation matrix will be singular (GRANT, 1989). This indicates that some information is required through the observations or the model. The resulting **rank defect** in the matrix is in fact the result of a problem in datum definition where the datum cannot be completely defined and therefore a **datum defect** exists. If the normal matrix is partitioned into geodetic parameter and clock bias submatrices then rank deficiencies exit in both and these datum defects have been discussed by LINDLOHR & WELLS (1985), GRAFAREND et al, (1985) and GRANT et al, (1990). It can be summarised as a **coordinate datum defect** that is found in all geodetic adjustments and in this case related to the geometric ranges (or their linear combinations), being the origin and orientation (6 components) and a **phase datum defect** since there is an attempt to estimate "absolute" clock phase errors from observations that are only relative measurements between satellite and receiver oscillators. There are number of methods to overcome a rank defect but the simplest method adopted by most software packages is to hold some of the parameters fixed. This results in the base station-base satellite approach, and in the undifferenced model, an additional base receiver clock.

5.4 RESULTS

5.4.1 Previous Experience

There is an abundance of literature published about various processing algorithms and their results, however specific comparisons of different algorithms and their treatment of correlations has been generally limited to small data sets (ASHKENAZI & YAU, 1985) or confined to discussions on the resultant coordinates and have disregarded the stochastic estimates of these results (eg. BOCK et al, 1985). ASHKENAZI & YAU (1985) verified the fundamental differencing theorem when they found that for various baseline lengths the undifferenced, single and double differenced observables lead to identical results so long as the correct stochastic correlation matrices were used. BOCK et al (1985) found that not taking inter-baseline correlations into account lead to differences of the order of only 0.5 ppm compared to when correlations were taken into account.

JONES et al (1987) carried out an analysis of the 90 station Phase 1 geodetic network in South Australia measured by Macrometer V-1000 receivers and reduced using Macrometrics proprietary baseline software and BATCH-PHASER multi-station software. Although both producing similar session results, the multi-station technique with its rigorous treatment of the stochastic model produced better results in the network adjustment stage. This applied to both the free net adjustment and when terrestrial data was introduced to constrain the adjustment (see Chapter 6 for further discussions on network adjustments).

Results of the PWD coastal survey not only highlight the differences between multi-station and multi-baseline solutions but also the bias modelling employed. Three post-processing software packages were used, being TRIMVEC™ and BATCH-PHASER to process the Trimble 4000SX data, while

NOVAS processed the TI4100 data collected in September 1987. A general summary of the data type and modelling used is shown in Table 5.5.

Table 5.5 GPS software used in the PWD coastal network

	TRIMVEC	BATCH-PHASER	NOVAS
ORIGIN	Trimble	NGS/SA Lands	Nortech
RECEIVER	Trimble	Trimble	TI4100
MEASUREMENT	L1 only	L1 only	L1 and L2
OBSERVABLES	Double/triple differences	Undifferenced Triple diff	Double differences
TROPOSPHERIC MODEL	Modified Hopfield	Saastamoinen	Hopfield
IONOSPHERIC MODEL	not used	not used	L1/L2 combination
COORDINATES BASE STATION	Weighted mean pseudorange (FIXED)	Weighted mean pseudorange (CONSTRAINED)	Weighted mean pseudorange (FIXED)
EPHEMERIS	Broadcast	DMA Precise	Broadcast
OUTPUT	Single baseline	Multi-station	Multi-baseline

5.4.2 TRIMVEC™ - A Commercial Example of the Differencing Observation Equation in a Baseline Approach

5.4.2.1 Background

Trimble Navigation is a company based in California that has been involved in navigation products for over 10 years. As an extension to their GPS navigation product range, they introduced their first GPS survey receiver called the 4000S in 1986. This 4 channel receiver was a modification of the 4000A navigation receiver and measured code and carrier phase information to a maximum of 4 satellites. With this new receiver came the need for post processing software. The "ICC-4 baseliner" software carried out single baseline computations and was used with the 4000S receivers in the Public Works Department coastal surveys carried out in May 1986 (ROBINSON & WATTERSON, 1987). A prototype for later processing software, it was not very user friendly and the introduction of the upgraded 5 channel 4000SX in 1987 was accompanied by the new TRIMVEC™ post-processing software developed by Goad.

TRIMVEC™ is an example of a commercial post-processing program that has been designed to conform to certain specifications. It is instrument specific, run on a micro-computer, and processes data from only two receivers at a time using double and triple differenced observables formed between the two receivers and a maximum of five satellites. Obtaining specific information about the software was made difficult because I am now employed with a rival GPS receiver company and the exact details of processing remain proprietary information. Versions 87.030 and 87.086 were used in the coastal network reductions.

5.4.2.2 <u>Data Acquisition</u>

The Trimble 4000SX receivers are 5 channel receivers with each channel dedicated to one satellite. In the coastal survey the data was recorded at 15 sec intervals. Four files are produced by each receiver for each observation session. They are:

1) Filename.DAT
2) Filename.MES
3) Filename.EPH
4) Filename.ION

Filename.DAT includes the integrated carrier phase measurement and code phase measurement for each satellite tracked, plus the calculated pseudo-range point position and PDOP at each epoch. Filename.EPH contains the orbital parameters from the broadcast ephemeris used to calculate the pseudo-range solution and used later in the carrier phase post-processing. Extracted from the navigation message of each satellite tracked, this data is only recorded at the start of the observation session. This presents a problem over long observation periods as the orbital parameters have to be extrapolated over a long period and may lead to less accurate solutions. This differs with such receivers as the TI4100 and WM101 which allow the broadcast ephemeris to be recorded each hour and to used later in both the pseudorange and carrier phase solutions, by smoothing the broadcast orbital parameters over the whole observation session. The ionospheric correction data broadcast as part of the navigation message is extracted separately into Filename.ION but seems to have no specific use in the software. Filename.MES is an ascii file giving details of the site.

5.4.2.3 Processing

In the preprocessing phase the data is set up to be processed by the double difference algorithm. The pseudo-ranges are used to provide initial station coordinates and accurately synchronise the observation time tags to GPS time. The triple difference algorithm is then used to provide an accurate a priori baseline vector and to locate and repair cycle slips.

If no more accurate positions are available (eg by transformation from a local geodetic datum) then a least squares estimation of the coordinates using the pseudoranges in an observation session is required. The individual fixes are placed in a least squares solution and weighted by their PDOP. An iterative process occurs until the solution has converged. TRIMBLE (1987) claim that this solution is within 10 metres of the true value in all 3 components. Tests against the pseudorange values derived by the TI4100 receivers using both L1 and L2 frequencies (ie: both C/A and P codes) at the 8 common points on the coast show that the Trimble data agrees very well with the more accurate TI4100 data.

The Saastamoinen (Version 87.030) or Modified Hopfield (Version 87.086) tropospheric models are used to correct the integrated carrier phase measurements and use meteorological readings that can be entered as default values of 20° C, 1020 mb, 50% humidity or as observed. As stated in section 5.2.1.5 care should be taken to ensure that the observed meteorological data is representative of the conditions of the medium in which the GPS signal is travelling. In the case of the coastal survey, meteorological data was taken every 15 minutes and the mean value for the session at each station was entered. Because the sessions were around sunrise, the relative humidity readings often reflected the foggy conditions and showed 100% values. This only indicates the conditions on the ground and up to say 300 metres, so were not a true measurement of the conditions being experienced by the GPS signal along the total signal path. (KING, 1987) suggested that a nominal value of 50% be used in all calculations and this was entered into the modified Hopfield model used in version 87.086.

Once the base station (or station one) coordinates have been determined, the data is checked for continuity. There have been a variety of methods developed to attempt to locate and repair cycle slips with varying degrees of automation and success. TRIMVEC™ calculates a preliminary triple difference baseline vector (using sequential techniques) that, while not taking the stochastic correlations into account, should still be close the actual baseline length. As defined in section 5.2.2.1 it is insensitive to biases and can be used to isolate blunders and cycle slips. The successive differences of the double difference observables generated are studied for unacceptable "jumps" in the residuals as these "jumps" are equivalent to cycle slips. They are "repaired" by adding an integer which corrects the residual so that it fits the existing residual trend on both sides of the cycle slip. After the cycle slips have been repaired, another triple difference solution is calculated to provide an accurate a priori baseline vector to be used in the double difference solution.

The primary adjustment in TRIMVEC™ uses the double differenced observable. It uses the functional model in equation (5.19) and so it is necessary to determine the double differenced integer ambiguities. The stochastic model is rigorous only if two receivers are operating in the session, otherwise between baseline correlations are ignored. To overcome the rank defect in the double differenced normal matrix, the fixed base station - base satellite approach is used. Where 5 satellites are observed, the processing involves the estimate of the 4 double differenced integer ambiguities (k_{12}^{ab}) where:

$$k_{12}^{12} = n_1^1 - n_1^2 - n_2^1 + n_2^2 \tag{5.27}$$

$$k_{12}^{13} = n_1^1 - n_1^3 - n_2^1 + n_2^3 \tag{5.28}$$

$$k_{12}^{14} = n_1^1 - n_1^4 - n_2^1 + n_2^4 \tag{5.29}$$

$$k_{12}^{15} = n_1^1 - n_1^5 - n_2^1 + n_2^5 \tag{5.30}$$

The base satellite is the "pivot" in the double differencing process. If observations are no longer available from this base satellite, say, because the satellite has moved below the horizon, then a new base satellite is required. GRANT et al (1990) show that these new double difference ambiguities can be modelled as a combination of 2 of the originally defined double difference ambiguity parameters. For example, if satellite 2 becomes the new base satellite then a new ambiguity parameter would be:

$$k_{12}^{23} = n_1^2 - n_1^3 - n_2^2 + n_2^3 \tag{5.31}$$

However

$$k_{12}^{23} = k_{12}^{13} - k_{12}^{12} \tag{5.32}$$

Therefore this method that is employed in TRIMVEC™ provides an efficient solution to the problem of rising and setting satellites. In this way TRIMVEC™ can adopt the satellite with the highest elevation as its base satellite, that will generally have the smallest atmospheric biases during the computations.

An ambiguity free (or float solution) is initially carried out. A number of different sets of hypothesised integer values for all ambiguities are created and tested for the set with the lowest $v^T P v$. If the best integer combination has a weighted sum of the residuals two times smaller than the next best choice, it is adopted and these ambiguity parameters are held fixed in an additional adjustment to provide an "ambiguity fixed" solution. As a housekeeping tool, when studying the double differenced residuals particularly in the integer search, an integer offset is introduced to keep the biases close to zero for easier analysis (GOAD, 1985).

The output of the triple, double (float) and double (fixed) solutions provides station coordinates and baseline vectors in both WGS84 geographic and cartesian coordinate systems, plus variance-covariance and correlation matrices of the estimated parameters (ie. related to the baseline vector).

5.4.2.4 Results

After the completion of the Trimble 4000SX survey in May 1987, the data was reduced using TRIMVEC™ (Version 87.030) (TRIMBLE, 1987). Version 87.086 was obtained in early 1988 and the coastal data was recalculated using the updated software. More consistent results were obtained due to the adoption of improved processing techniques. These included the modification of the atmospheric data used in the earlier processing to incorporate a 50% relative humidity instead of the ground measured data. In addition the base station coordinates for each baseline were propagated through the network from origin at Sydney, instead of taking pseudorange solutions at each station. The coordinates of Sydney were taken from the mean of pseudorange solutions over three sessions. The estimated absolute accuracy of this solution was found to be within 15 metres in each spatial component of that finally adopted from a network adjustment using the accurate coordinates of Orroral SLR station (see Chapter 6). This "seeding" or propagating the coordinates through the network, increases the accuracy of the "differential" baseline vector determinations because a more accurate (fixed) station in the baseline solution is always used.

All baseline combinations were calculated and checked for processing consistency using a set of geometric tests by closures of the polygons formed on each day. This means for a 5 receiver session configuration a total of 10 baselines were determined per session. Appendix D shows the results of polygon closures for each day using Version 87.086. Large misclosures would indicate any outliers, which could then be analysed and reprocessed if necessary. This method had been used by the CMA in their Region 4 network in NSW (LETHABY et al, 1987) and by JONES et al (1987) in the South Australian network. A total of 168 baselines were calculated and a summary of the results is shown in Table 5.6.

The reasons for the low number of fixed integer ambiguity solutions are the large residual biases remaining after the double difference float adjustment, due to the length of the baselines and length of the observation session. The average length between adjacent stations was over 30 kilometres, at which large decorrelation in the tropospheric biases occurs. Add this to the rapid changes in TEC over the long observation period shown by PARTIS & BRUNNER (1988) and this would explain the difficulty of resolving the ambiguities to integer values.

Table 5.6 Summary of TRIMVEC™ baseline results - May 1987

No of DD Fixed solutions	6	Distances 22km-67km
No of DD Float solutions	162	Distances 10km-275km

Problems with excessively noisy data, indicated by high RMS values and a large percentage of observation rejection (rejection criteria being over 3.5 times the RMS value) in the baseline solutions on days 137, 141 and 145 have being investigated. Reasons for the problems are difficult to pinpoint although again PARTIS & BRUNNER (1988) showed there was large ionospheric activity on day 137, and this can be extended to the assumption that this was the reason for this noisy data on all three days.

The baselines observed in more than one session were then compared in all 3 dimensions to show the system's overall consistency. These inter-session comparisons are shown in Figure 5.3 and Appendix D, with the best result being 0.2 ppm and the worst being 4.2 ppm. Since the purpose of this project was to determine accurate heights, the repeated differential ellipsoid heights are shown in Figure 5.4 and the results show similar consistency to Figure 5.3.

The baseline vectors were finally placed in a 3 dimensional least squares network adjustment to further check for outliers and provide provisional coordinates. The results are discussed in Chapter 6.

5.4.3 BATCH-PHASER - An Example of a Multi-Station Approach Using the One-way Phase Observation Equation

5.4.3.1 Background

A multi-station reduction package which treated stochastic correlations rigorously within a session was needed to improve the results. The same procedure was used in the South Australian Department of Lands adjustment of their first order GPS network using BATCH-PHASER and this software was used on the PWD Trimble data.

The original program named PHASER (Phase reduction) was written by GOAD (1985) at the United States National Geodetic Survey but has been substantially modified by the South Australian Department of Lands (JONES et al, 1987). Processing of the Trimble data was initially carried out with the help of JONES (1988a), using the South Australian Land Department's Data General MV4000 mini-computer, in July 1988. The precise ephemeris obtained from the Defense Mapping Agency (DMA) in the USA was included for the first time. TALBOT (1988) modified this program at the Royal Melbourne Institute of Technology to run on a micro computer and on returning to Sydney, subsequent refinements to the processing were attempted with this version called PC-PHASER.

5.4.3.2 Processing

BATCH-PHASER uses the same preprocessing techniques as in TRIMVEC™ by utilising the triple difference solution to carry out cycle slip repair and provide accurate a priori coordinates. However the principal observation model used is the undifferenced or one-way phase model. Again the base station/base satellite technique is used to overcome the rank defect in the normal matrix. However the base station is not fixed but constrained by placing a priori weights through the variance-covariance matrix of the parameters, rendering the normal matrix non-singular by giving some of the parameters a higher weight (lower variance) than others. Furthermore because the undifferenced model is used, not only a number of integer ambiguities but also a receiver and satellite clock must be held fixed. The processing carries out explicit differencing by partitioning the normal matrix and estimating the receiver clock biases on an epoch by epoch basis (for details see GOAD (1985)). No ionospheric modelling is used and this may provide scale errors to the session solutions. The Saastamoinen tropospheric model with standard meteorological parameters is used. Precise ephemeris from the DMA was used, however session 146A was also processed using the Trimble recorded broadcast ephemeris as a comparison to the precise ephemeris results and TRIMVEC™.

The output of BATCH-PHASER solutions provides a set of station coordinates (including the base station) and baseline vectors in both WGS84 geographic and cartesian coordinate systems, plus variance-covariance (VCV) and correlation matrices of the estimated parameters (ie. related to the station coordinates). The combined set of estimated station coordinates and their corresponding VCV matrix for each session are known as the position equations. These were entered directly into the network adjustment program NEWGAN to check for further outliers (see Chapter 6).

5.4.3.3 Results

No session polygon closures are necessary as it is a multi-station adjustment. Ambiguities were kept at real number values (equivalent to a session "float" solution)

Figure 5.3 Comparisons of TRIMVEC™ and BATCH-PHASER repeat baselines

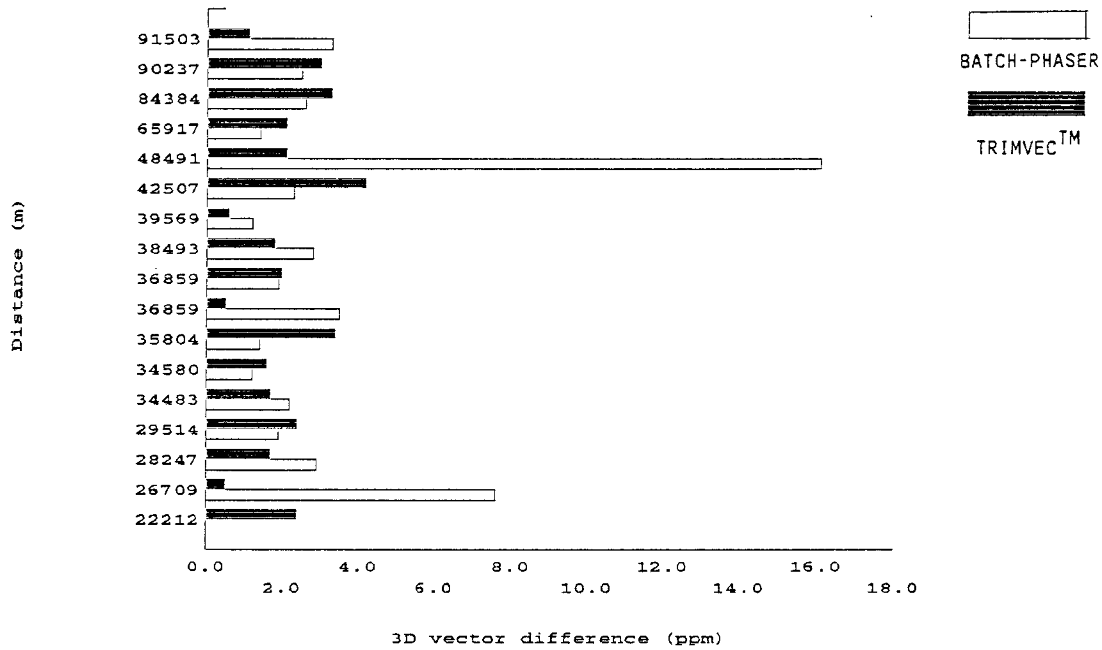
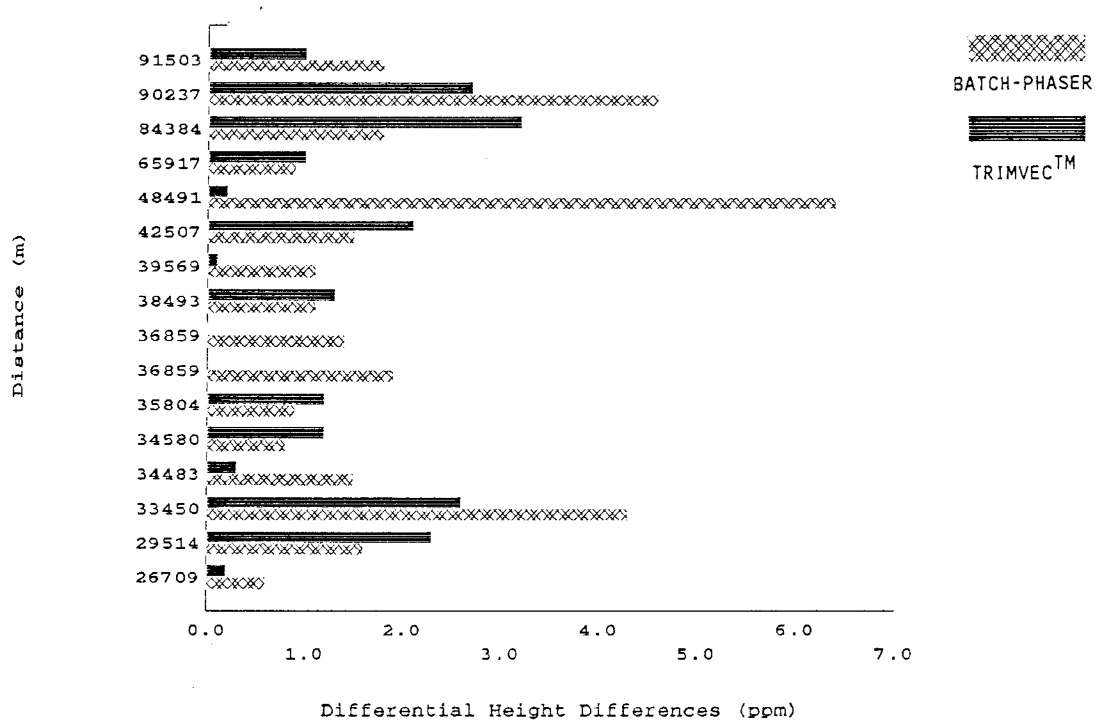


Figure 5.4 Comparisons of TRIMVEC™ and BATCH-PHASER repeat dh_{AB} results



because of the large distances between stations. Repeated baseline vectors are compared to the TRIMVEC comparisons and shown in Figure 5.3 and ellipsoid heights in Figure 5.4. Results showed large differences in comparisons that included session 141A (22/5/87). An extreme outlier was Wooli-Yamba at 17.4 ppm. Discussions with TALBOT (1989) indicated that if little or no overlap occurred in the phase observations taken at some stations within the session, then the normal matrix may become unstable. In addition, the BATCH-PHASER solution showed similar high RMS values to the TRIMVEC solutions of that day. This indicates the possible influences of external factors such as ionospheric and tropospheric delay effects on the observations. By removing Yamba (1032) with only 1.25 hours of data at the beginning of the session and Evans Head (1033) with only 50 minutes of data at the end of the session the remaining three stations could be reprocessed. However using PC-PHASER to reprocess the data could not supply the complete set of position equations need for NEWGAN because it holds fixed the base station and cannot constrain it. As an alternative the original session solution (including Yamba and Evans Head) has been adopted but has been assigned a low weight accordingly in the network adjustment (see Chapter 6).

Another large difference was found with the line Port Macquarie (1026) to Crescent Head (1027). Analysis of the baseline solutions on the 20/05/87 (139A) and 21/05/87 (140A) show no apparent reasons for the large difference between the solutions. Therefore both have been included in the network adjustment.

5.4.4 NOVAS - An Example of a Differencing Model in a Multi-baseline Solution Using Dual Frequency Observations

5.4.4.1 Background

NOVAS (NOrtech Vector Adjustment Software) was written by Nortech Surveys Inc, Canada in 1986, with the objective being to provide a flexible processing package that eliminated operator subjectivity and tedious data editing requirements. The software was originally designed to process data from TI4100 and NORSTAR receivers, however it has since been extended to handle other receivers types including the Trimble 4000 series and Wild-Magnavox WM101. In the coastal project NOVAS was used to process the combined government TI4100 campaign carried out between the 9th and 21st of September 1987 (Days 252 to 264). Details of the logistics and data acquisition phase can be found in DICKSON (1989). For details of the algorithms used the reader is referred to WANLESS & LACHAPELLE (1987).

Briefly, each session collected data from 4 Texas Instruments TI4100 receivers. L1 and L2 data was recorded on Nortech developed tape drives that allowed up to 8 hours of observation without changing tapes. The TI4100 tracks a maximum of 4 satellites, and a predetermined satellite observation schedule was used that required changes in the observed constellation to maintain good satellite geometry. The observation period spanned 7 hours, from mid evening to early morning. The navigation message including the broadcast ephemeris was recorded every hour and meteorological data was taken every 15 minutes. The data tapes were later transferred to computer discs, ready for processing.

5.4.4.2 Processing

The preprocessing in NOVAS uses the pseudoranges to provide time tagged carrier phase observations on both L1 and L2 and performs a single point solution that can be used as the base receiver coordinates in the final phase processing. Broadcast or precise ephemeris can be used, with the NSW project using the broadcast ephemeris. Importantly the detection of cycle slips and high observation noise is not performed in the preprocessing stage but is executed automatically in the primary adjustment.

The double difference phase observable is used in the primary adjustment. In the NSW project, these double differenced observations have been developed from the ionospheric free or θ_{LC} observations as determined by equation (5.9). Like TRIMVEC™ it determines baselines and the stochastic correlations introduced are included rigorously with respect to the baseline only. Up to 7 satellites can be processed simultaneously and changes in the constellation introduced by the switching of satellites in the TI4100 are handled similarly to that of rising and falling satellites in TRIMVEC™. The differencing between satellites is operator selectable and can be relative to a base satellite (as in TRIMVEC™) or formed sequentially, ie in the case of 4 satellites the double difference ambiguity terms are:

$$k_{12}^{12} = n_1^1 - n_1^2 - n_2^1 + n_2^2 \tag{5.33}$$

$$k_{12}^{23} = n_1^2 - n_1^3 - n_2^2 + n_2^3 \tag{5.34}$$

$$k_{12}^{34} = n_1^3 - n_1^4 - n_2^3 + n_2^4 \tag{5.35}$$

The adjustment algorithm employs an adaptive or "self learning" Kalman filter that produces a sequential adjustment with the ability to isolate the integer ambiguities on short lines using large scale multiple hypothesis testing. The

Kalman filter equations developed in NOVAS (see WANLESS & LACHAPELLE, 1987, p.5) are executed recursively, processing one epoch at a time, enabling automatic inspection of the double difference residual before the observation is included in the baseline solution and eliminating data with high noise. A type of triple difference residual is also formed during the double difference processing to sequentially monitor and repair cycle slips.

The stochastic model assumes a default observation variance depending on the original carrier phase frequency being processed. The resultant VCV is scaled by this a priori value and the degrees of freedom in the adjustment.

In the NSW survey, the processing was carried out by Nortech using sequential satellite selection, broadcast ephemeris smoothed using a 9th order Lagrangian interpolator (REMONDI, 1984) and pseudorange base station coordinates "seeded" from one station throughout the quadrilateral.

5.4.4.3 Results

Here the results from the LIC's TI4100 dual frequency network are published in LIC (1989) and discussed in DICKSON (1989). Since the processing was not carried out by the author, only general comments are made here. 13 sessions were processed, using all baseline combinations, making a total of 78 baselines. Appendix D shows daily polygon closures and Figures 5.5 and 5.6 shows the repeat baseline and ellipsoid height comparisons respectively. Integer ambiguities were not fixed due the use of the LC observable and long lines. The improved consistency of these results over longer lines compared to TRIMVEC™ and BATCH-PHASER is a direct effect of using dual frequency observations. While the ionospheric error had been effectively removed, determining integer values for the ambiguities was difficult because the long lines (over 110 km) had large decorrelation of the

Figure 5.5 Comparisons of NOVAS repeat baselines

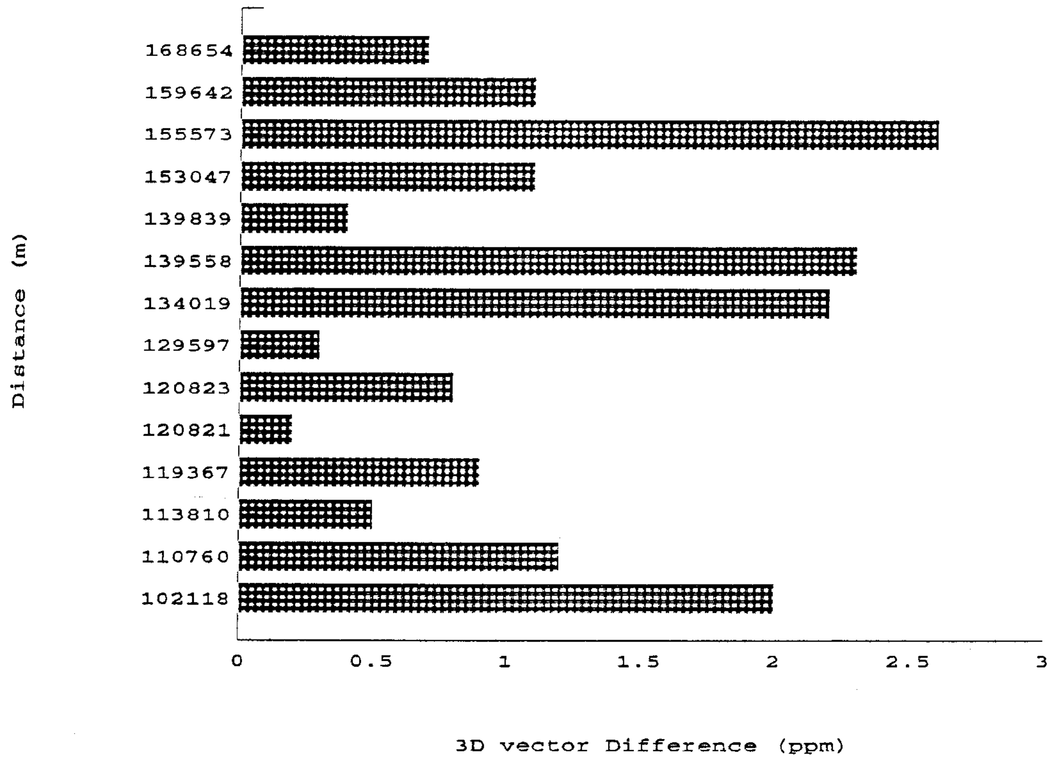
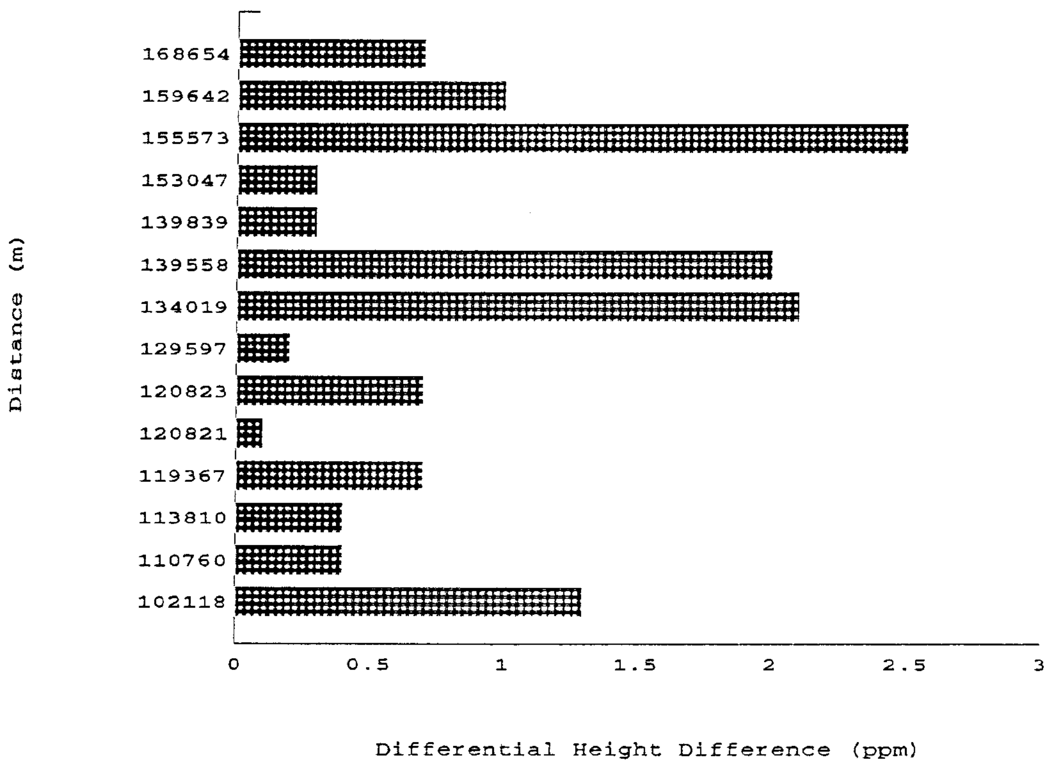


Figure 5.6 Comparisons of NOVAS repeat dh_{AB} results



tropospheric effects at each site.

5.5 CONCLUSIONS

The role of correlations both physical and mathematical in forming the model and its results is very important. The assumption that the physical effects on the GPS signals are correlated between receivers allows techniques to be developed such as differencing, that provide high precision and easier computation. As the inter-station distances become larger, a decorrelation of the physical effects of orbit error and atmospheric delay will occur and other methods such as placing terms in the stochastic model of the observations are required to maintain the accuracy of the mathematical model.

The results of both BATCH-PHASER and TRIMVEC™ showed similar consistency in their repeated baseline comparisons, however the BATCH-PHASER solutions differed from the TRIMVEC™ solutions by an average of 3 ppm. Because BATCH-PHASER estimates the clock bias parameters on an epoch by epoch basis, effectively the only major differences between it and the double difference TRIMVEC™ results that could be expected to create significant baseline vector differences of this magnitude are the treatment of the stochastic correlations, the use of precise ephemeris and differences in the tropospheric bias modelling (eg. TRIMVEC™ allowed the use of observed meteorological data in its tropospheric model while BATCH-PHASER used a standard model).

Looking more closely at reasons for the large discrepancies in the solutions, session 146A was processed using PC-PHASER with broadcast ephemeris and compared to the results of TRIMVEC™ (broadcast) and BATCH-PHASER (Precise) results. The baseline vectors from PC-PHASER (broadcast) fell between BATCH-PHASER and TRIMVEC™ as can be seen in Tables 5.7 and 5.8.

Table 5.7 Comparisons of PHASER results using precise and broadcast ephemeris - session 146A (27/5/87)

BASELINE	Dist(m)	δX	δY	δZ	$\delta 3D$	ppm	δht	ppm
1017-1018	36859	-.024	-.013	.004	.028	0.7	.010	0.2
1017-1019	68738	-.045	-.025	-.006	.052	0.7	.018	0.2
1017-1020	91503	-.058	-.025	-.009	.063	0.6	.030	0.3
1017-1021	113501	-.072	-.029	-.005	.078	0.6	.038	0.3

Table 5.8 Comparisons of PHASER and TRIMVEC™ using broadcast ephemeris - session 146A (27/5/87)

BASELINE	DIST(m)	δX	δY	δZ	$\delta 3D$	ppm	δht	ppm
1017-1018	36859	.052	.087	.033	.107	2.8	.022	0.5
1017-1019	68738	-.106	.149	.046	.190	2.7	.111	1.6
1017-1020	91503	.038	.283	.056	.291	3.1	.110	1.2
1017-1021	113501	-.050	.349	.079	.361	3.1	.132	1.1

Surprisingly the PHASER results show an average scale difference of only 0.7 ppm between the precise and broadcast ephemeris, meaning the two ephemerides are within 20 metres of each other. There is also a small systematic difference in the ellipsoid heights of about 0.3 ppm. As a result, the BATCH-PHASER solutions using the precise ephemerides have been adopted. However there still remains an average 2.9 ppm scale error and 1.1 ppm systematic height error between TRIMVEC™ and BATCH-PHASER which must be due to the variations in the stochastic model and the tropospheric bias modelling.

The difference in the tropospheric modelling can be assumed to be minimal, as tests using TRIMVEC™ versions 87.030 (Saastamoinen model) and 87.086 (Modified Hopfield) returned millimetre changes in the baseline vectors. Therefore the remaining variations between TRIMVEC™ and BATCH-PHASER must be due to the differences in the rigour in the stochastic modelling. Previous experience has shown session results between baseline and multi-station solutions that indicate differences of around 0.5 ppm to 2 ppm (eg. BOCK et al, 1985). The 2.9 ppm found in the coastal survey is higher than other research and may be due the larger line lengths and the integer ambiguities not being fixed. This means that for general surveying purposes "baseline" software is adequate, but to obtain optimal results a rigorous treatment of stochastic correlations must be made.

From experience gathered in actual GPS differential carrier phase surveys, the accuracies expected in the baseline vectors have been stated as 1-3 ppm for the horizontal components and 1-4 ppm for heights (eg. HUSTI et al, 1988; LETHABY et al, 1987). The coastal survey reflects these findings with an averaged baseline repeatability for single frequency Trimble observations of 2.1 ppm to 3.7 ppm depending on the software used, reducing to 1.2 ppm when using TI4100 dual frequency observations. The repeatability of the differential ellipsoid heights was of the same magnitude, however the systematic nature of the differences in the results of the multi-station and multi-baseline solutions and when using the precise ephemeris as compared to the broadcast ephemerides would indicate some problems. This systematic trend would become more important when these heights were converted to orthometric heights and applied to the MSL observations at the coastal tide gauges in Chapter 8.

6. COMBINING OBSERVATION SESSIONS INTO A NETWORK ADJUSTMENT

6.1 INTRODUCTION

Once GPS carrier phase observations have been collected (Chapter 4) and processed to obtain session results (Chapter 5), the sessions can be combined into a network adjustment. Some authors such as EREN (1987), HEIN et al (1985) and GRAFAREND, (1989) advocate that all sessions be included in one "multi-station/multi-session" least squares adjustment. However, the more common approach is to adjust the sessions in a second and separate step. This is the best approach here, because the output obtained from GPS data processing varies with the processing package used.

From Chapter 5 it has been shown that the session processing falls into two distinct categories. The baseline output from TRIMVEC™ and NOVAS consists of a number of baseline vectors and their associated covariance matrices. The multi-station output from BATCH-PHASER produces a set of station coordinates and the associated full covariance matrix. While these processing methods may be considered equivalent, it has been shown in section 5.3.1 that they are only truly equivalent if the stochastic model of the (differenced or undifferenced) observations rigorously accounts for stochastic correlations and the clock error parameter models are equivalent. Since this is generally not the case, particularly with respect to the lack of stochastic rigour in the baseline approach, the session output of each method should be examined separately.

The station coordinates or baseline vectors obtained from GPS data processing of an observation session, can be equated to that of reduced field data in conventional terrestrial surveys. The fact that the GPS carrier phase observations have already gone through a least squares process to determine these coordinates or baseline vectors makes little difference to the development of a network adjustment model, except that the stochastic model of these "reduced" session observations is more complex in structure when compared to a conventional terrestrial network.

Various procedures can be used to combine this session data into a network using the general observation equation in (B.2).

$$Ax_{net} = b_{net} + v_{net} \tag{6.1}$$

This time the required parameters (x_{net}) are the final coordinates, while the observations (in b_{net}) are the baseline vectors or the multi-station session coordinates. A description of the theory of how baselines & multi-station figures may be combined into a network adjustment can be found in ECKELS (1987, pp.131-136). The covariance matrix of the observations ($C_{l_{net}}$) is the sum of the a posteriori covariance matrices of the sessions where each session is assumed independent, such that:

$$C_{l_{net}} = C_{x_1} + C_{x_2} + \ldots. + C_{x_n} \tag{6.2}$$

where C_{x_n} is the a posteriori covariance matrix of the estimated session parameters for sessions 1 to n and is defined as:

$$C_{x_n} = \sigma_o^2 Q_{x_n} = \sigma_o^2 [A^T P_n A]^{-1} = \sigma_o^2 [A^T Q_{l_n}{}^{-1} A]^{-1} \tag{6.3}$$

where σ_o^2 is the "population" variance parameter

Q_{x_n} is the co-factor matrix for session "n"

A is the design matrix

P_n is the inverse of the observation co-factor matrix of the carrier phase observations Q_{l_n} in session n.

In this discussion, the word "session" also defines the baseline approach.

When carrying out any network adjustment, it is prudent to first carry out a preliminary adjustment to test for significant measurement outliers. This is called a **"minimally constrained"** adjustment, where the coordinates of only one station are held fixed, preventing the normal matrix from becoming singular and effectively eliminating the coordinate datum defect as outlined in section 5.3.2. Where a number of stations in the network have coordinates that can be related to a local ellipsoid, then these may then be held fixed in a **"constrained"** adjustment and transformation parameters can be applied or determined to relate the GPS (WGS84) coordinates to the local datum. Details can be found in ECKELS (1987) and HARVEY (1986). In the NSW coastal survey, only minimally constrained adjustments were carried out because the aim of the project was to uncover differences in the AHD and MSL using GPS and not to constrain the GPS results to the "suspect" terrestrial data.

It should be stressed that the findings in this analysis are generally based on a sample size of one. The best approach would be to stockpile similar projects, prepare a histogram and then draw conclusions. Therefore, where relevant results were available, such as the research by JONES (1988b) that included the PWD coastal data, then they have been incorporated in the following discussions.

6.1.1 Statistical Testing

Both before and after a least squares computation it is usual to ask a number of questions regarding the significance of certain aspects of the observed and computed data. The answers to such questions are investigated by means of the process of statistical testing. Questions are asked as to whether some estimator function is consistent with a hypothesis. This hypothesis may be, for example, that a sample was drawn from a population with specific parameter values, such as a normal distribution with a given standard deviation, where a

Variate - is a random variable with specific range and probability

Population - is a "near infinite" collection of objects having in common a particular variate.

Sample - is a group of individual members or subset from the population.

A **null hypothesis** (H_0) is a statement about the probability distribution of the variate to be tested, and in the case of an adjustment compares the probability distribution of the estimated parameters with the probability distribution of the population. Tested against this null hypothesis is an **alternative hypothesis** (H_A) and if the null hypothesis is accepted then the estimated parameters are said to represent a sample from the population. A detailed explanation of this process can be found in MIKHAIL (1976) and CROSS (1983).

In a least squares network adjustment, statistical tests can be applied to assess the quality of the observations and allow outlier detection methods to be developed. These tests also allow a method of analysing the quality of the adjusted parameters and the validity of the mathematical model hypothesised by the adjustment data set. It should be noted that a number of models can be developed for a single data set, for example, the model could be based on planar, spherical, ellipsoidal or geoidal considerations.

The adjustment process provides residuals which are a mixture of all error types defined in section 2.4 (ie. random, systematic and gross errors). Certain assumptions about the stochastic properties of the residuals are required in order to deal with the problem of isolating the gross errors. An outlier is therefore defined as a residual which according to some test, is in contradiction to the assumption (CASPARY, 1987, p.68). The approach to outlier detection is based on the well known Gauss-Markov model (GMM). The first step is usually a global test of the model and if this test fails, procedures to flag the erroneous measurements must be used. While a variety of tests can be carried out on the observations, their a priori VCV's and the adjustment's results, the network adjustment software used in the coastal GPS survey has specifically used Chi-squared testing for the global model test, and Baarda and Tau tests for outlier detection.

Global Model Test

The criterion of the null hypothesis of the global test is (CASPARY, 1987, p.6),

"The model is correct and complete".

The GMM is defined by the equations (B.1) to (B.3) and the least squares estimates that are dependent on the geodetic datum (see section 6.2) are defined by equations (B.4) to (B.11). Since all tests are based on statistical distributions, a reasonable stochastic assumption must be introduced. Therefore the observations are assumed to have a valid distribution and have an absence of systematic and gross errors. The global model test questions the assumptions by comparing the a posteriori estimate of the variance factor with the a priori variance factor σ_o^2. If S_o^2 is the variance of a random sample from a normal population with mean u and variance σ_o^2 then χ_r^2 represents a Chi-squared distribution with r degrees of freedom. Therefore the null hypothesis that can be tested is:

$$H_0 : S_o^2 = \sigma_o^2 \qquad H_A : S_o^2 > \sigma_o^2$$

H_0 is rejected when:

$$T > \chi^2_{\alpha,r}$$

where in a least squares network adjustment the test statistic (T) is :

$$T = \frac{v^T P v}{\sigma_o^2} = r \cdot S_o^2 = v^T P v \qquad (6.4)$$

since in a network adjustment σ_o^2 = unity

and $\chi^2_{\alpha,r}$ is the Chi-squared distribution value at α significance level and r degrees of freedom.

Testing is usually performed at the 95% confidence level (ie α = 5%) and if the null hypothesis is rejected, then the sample is not consistent with mathematical model. In GPS adjustments it may indicate the presence of one or more of the following problems:

1) the observation residuals are significantly larger or smaller than those implied by the a priori standard deviations, ie. a priori covariance matrix is inappropriate and may require re-scaling by S_o^2
2) the functional and/or stochastic models used in the adjustment are incomplete.
3) significant errors exist in one or more of the observations, for example,

i) blunders in the field measurements such as incorrect antenna height.

ii) systematic errors that affect all observations and that were not eliminated at the session processing stage (eg. residual tropospheric errors).

4) the calculations are incorrect, eg. there is a "bug" in the reduction program.

Outlier detection

As explained above, one of the reasons why the network adjustment will not pass the global model test is that one or more blunders (gross errors) are present in the observations. Additional tests can be applied to detect these outliers by using "data snooping" techniques that screen the computed residuals.

Baarda's Data Snooping

A method developed by BAARDA (1968), assumes that only **one** gross error exists and the residuals are normally distributed. The scaled standard deviations of the residuals of the adjusted observations are calculated;

$$\mu_i = \frac{v_i}{\sigma_o \sqrt{q_{vv}}} \tag{6.5}$$

where μ_i is the normalised standard deviation of the residual

v_i is the residual

σ_o is the a priori estimate of the standard deviation that is often equal to 1

q_{vv} is the corresponding trace element from the VCV matrix of the residuals

An outlier is identified by comparing the u_i value with a critical value (u_{α_c}) of the standardised normal distribution at a specific significance level α_0 and the outlier will exceeded this critical value with a probability of $1-\beta_o$. β_o is known as the power of the test. Baarda recommends the probabilities of β_o=20% and α_o=0.1% be used for a single test and therefore in this case if $\mu_i \leq 3.29$, the residual lies within that expected by the model. Larger μ_i values indicate possible gross errors and the corresponding observations should be examined and either reprocessed or removed from the adjustment.

Tau Test

In practice, unless there is external evidence or the stochastic information is taken from a very large sample, the true population variance factor σ_o is unknown. Hence a slightly different philosophy to Baarda's data snooping technique is followed. POPE (1976) used studentized residuals where:

$$T_i = \frac{v_i}{M_2 \sqrt{q_{vv}}} \tag{6.6}$$

where T_i = studentized standard deviation of the residual

v_i = residual

M_2 = the second moment about zero

$= \sqrt{\frac{v^T Q^{-1} v}{n}}$ where n is the number of observations

q_{vv}= corresponding trace element from the VCV matrix of the residuals

Some texts use the a posteriori estimate of the standard deviation (S_o) instead of M_2 in equation 6.6, (ie replace n with the redundancy (r)) but this is incorrect (ALLMAN, 1990). The studentized residual (Tau statistic) is governed by a τ (Tau) distribution with r degrees of freedom from the least squares computation (CASPARY, 1987, p.76). Related to the t distribution (see CROSS, 1983, p.79) the Tau statistic is compared to the τ-distribution at a level of significance α (usually 5%) and if the Tau statistic is greater, then the observation is rejected.

6.1.2 Network Adjustment Software

Originally the PWD had no adjustment software suitable to combine GPS results. Two network adjustment software packages were subsequently obtained for this study. Briefly, SHAKE written by ECKELS (1987) was initially used to combine the TRIMVEC™ baselines together. Its features include:

1) A 3 dimensional least squares adjustment combining uncorrelated baselines using the WGS84 cartesian coordinate system.
2 modified by the PWD to take up to 70 stations and 250 baselines.
3) It provides a global model test using Chi-squared testing on the a posteriori variance factor.
4) outputs normalised residuals for Baarda data snooping.
5) However it is limited to GPS baselines and cannot process multi-session GPS outputs or incorporate terrestrial measurements.

In July 1988, NEWGAN (ALLMAN, 1988) was purchased after reprocessing the Trimble data with the multi-station BATCH-PHASER software. Some of its features include:

1) A 3 dimensional adjustment program that can input both terrestrial and satellite derived observations.

2) Provides comprehensive statistical testing in the form of Chi squared, Tau and F tests.
3) It forms its parametric equations in terms of local or WGS84 latitude, longitude and height rather than cartesian coordinates making all diagnostic information produced in the adjustment process easily related to the physical situation.
4) Importantly NEWGAN has been designed to handle both baseline and multi-station GPS session output.
5) In the case of BATCH-PHASER multi-station results, each position equation set is determined relative to a fixed or tightly constrained base station. By adding extra translation parameters and applying appropriate stochastic constraints, NEWGAN permits block shifts to be applied to the multi-station session figures to reflect the uncertainty in the absolute positions of each base station, as well as allowing individual station adjustment.

The results of the GPS carrier phase post-processing packages have been analysed separately to get some indication of the achievable precisions of the results, then the Trimble 4000SX and Texas Instruments TI4100 results have been combined into one network solution.

6.2 GPS DATUM DEFINITION

The datum of any geodetic network can be defined by holding the appropriate number of parameters fixed. Minimum constraints will estimate translation, orientation and perhaps scale without imposing any strain. In a 3 dimensional terrestrial network this means that the origin may be defined by holding the coordinates of one station fixed, while the orientation is defined by zenith distance and azimuth observations or by fixed coordinates. The scale can be defined by either measured distances or a fixed coordinate.

A gravimetric datum may defined by fixed deflections of the vertical at a point and either a fixed geoid height or a value for the geopotential at some point. In this case the datum definition is easy to see and it is a simple matter to ensure that it is not overdetermined.

In GPS, the datum definition is more complex and generally over-constrained. The reference system is uniquely defined (presently WGS84) and its realisation through GPS, is in part by the tracking stations that are used to generate the satellite ephemeris. This immediately leads to problems as these tracking stations are situated on tectonic plates that move slowly and therefore their relationship with the originally defined WGS84 will vary. For example, the transformation parameters between WGS84 and $WGS84_{GPS}$ at the end of 1988 are shown in Table 6.1.

Table 6.1 7 parameter transformation (WGS84 to $WGS84_{GPS}$)

	X	Y	Z
Translation (m)	0.026	-0.006	0.093
Rotation (sec)	0.001	0.000	0.002
Scale (ppm)	-0.128		

With no direct observations between these tracking stations and the survey network, the datum is transferred via the satellite ephemerides. This is usually a two step process with an orbit adjustment that produces the satellite ephemerides, followed by a session adjustment of the survey points where the orbits are held fixed. Both the station coordinates and satellite ephemerides can be estimated in a one step adjustment, but in the context of this study only the session adjustment with fixed orbits will be briefly examined.

Therefore, the fixed ephemerides and the accompanying force model define a datum. While the elements of one satellite would be sufficient to define the origin, orientation and scale, GRANT (1989, p.128) notes that holding more than one satellite fixed will over-constrain the datum and the discrepancies between these datum definitions will be magnified if the survey is distant from the tracking stations (eg. in the southern hemisphere). A further over-constraint occurs when one station is held fixed in the carrier phase processing to prevent the normal matrix from becoming unstable (see 5.3.2). Large errors in this fixed station's coordinates with respect to the satellite reference frame will introduce distortions in the network (see 5.2.1.6). So combining the sessions (whether multi-station or multi-baseline) into a network adjustment must make allowance for the standard practice of holding an arbitrarily chosen station fixed in each carrier phase adjustment.

What is generally assumed in a GPS network adjustment is that all session solutions have the same orientation and scale. From the above discussion, this may not be the case and highlights the complexity of the datum definition in GPS adjustments, for while a terrestrial network has its datum defined once for the whole network, there is a different datum definition for each session solution that is combined in a GPS network adjustment.

In this study, the datum definition problem is magnified when two independent GPS networks are combined. The datum in the BATCH-PHASER results has scale and orientation defined by the post-processed DMA precise ephemeris that presently uses 9 globally well spaced tracking stations. Additional scale in introduced by the single frequency observations. This datum could be expected to have scale and rotation differences compared to that defined by the NOVAS processed dual frequency observations that has used the broadcast ephemeris. It will be assumed that there is no significant scale or rotation differences between the sessions measured with the same type of receiver and processed with the same post-processing software using the same ephemeris information.

Why is the geodetic datum definition so important? Simply, the precisions in the computed covariance matrices are dependent on the geodetic datum definition. In a network adjustment of homogeneous session results, the GPS vectors (or coordinate differences) that already have an over-constrained datum, will determine the size and orientation of the each station's error polyhedron. Therefore, only 3 constraints are required (ie. hold one station fixed) to define the translatory components of the geodetic datum for the network. However the choice of this fixed point effects the coordinates in vector $\hat{x}_{net}$ as well as the corresponding computed VCV matrix $C_{\hat{x}_{net}}$. They are not invariant quantities of the model and therefore depend on the selection of the geodetic datum.

The fact that these results are not unique causes problems in assessing the precision of the adjusted positions of the points. Comparisons of VCV matrices, and importantly, the combination of two or more geodetic networks, must be carried out using the same geodetic datum. In this study, the various GPS networks have used the same fixed station, but the scale and orientation components of the geodetic datum for each network may vary due to the different ephemeris, observations and processing software used.

6.3 MODIFYING THE STOCHASTIC MODEL

Baseline and multi-station session processing software produce a variance-covariance matrix (VCV) that provides precisions of the estimated session parameters. When combined in the network adjustment, the resultant VCV of the estimated network parameters can be defined as:

$$C_{\hat{x}_{net}} = \sigma_o^2 Q_{\hat{x}_{net}} = \sigma_o^2 [A^T P_{net} A]^{-1} = \sigma_o^2 [A^T Q_{l_{net}}^{-1} A]^{-1} \qquad (6.7)$$

$$S_o^2 = \frac{v_{net}^T P_{net} v_{net}}{r} \qquad (6.8)$$

where σ_o^2 is the "population" variance parameter from which the "sample" a posteriori variance factor s_o^2 defined is drawn.

$Q_{\hat{x}_{net}}$ is the co-factor matrix for the network

A is the network design matrix

P_{net} is the inverse of the observation co-factor matrix $Q_{l_{net}}$ from equation (6.2)

v_{net} are the residuals of the network observations

In current GPS analysis, the statistical testing of S_o^2 against the assumed population parameter will in many cases be unsuccessful, and the hypothesis implicit in the mathematical model should be re-evaluated, particularly with regard to the accuracy estimates used to weight the original observations. For example, in this study the NOVAS baseline sessions produce a posteriori variance factors after adjustments of the southern and northern networks of 3 and 18 respectively.

In most present GPS processing software, the resultant variances produced by equation (6.3) are very small as compared to the inter-station separations and are basically only an indication of the internal precision of the carrier phase measurements. More specifically while the stochastic correlation between the estimated parameters is a function of the satellite and station geometry defined in the A matrix, the magnitude of the components of the computed VCV are primarily a function of the number of observations and datum definition. Its magnitude is, however, limited by the signal quality, associated noise of the receiver, and residual tropospheric and ionospheric effects. In general the more observations taken then the lower the variance of the results. This is not a good indication of the true precision and accuracy of the results. For example, the TRIMVEC™ (Version 87.030) baseline outputs in this study have observation sessions of over 3 hours and have average standard deviations of a few centimetres and correspondingly low rms values. The problem is compounded when the covariance matrices of the estimated session parameters are combined in a network adjustment using equation (6.2).

When there is a new type of observation and the errors are not well known a priori such as in GPS, the use of the a posteriori variance factor to scale the computed VCV of the adjusted parameters is a "standard" technique in least squares analysis. The technique is based on the premise that the mathematical model is the best approximation of reality, which based on the GMM assumes that the observations have random errors and an unknown variance. Therefore equation (6.8) can be modified to:

$$C_{\hat{x}_{net}} = S_o^2\, Q_{\hat{x}_{net}} = S_o^2\,[A^T P_{net} A]^{-1} = S_o^2\,[A^T Q_{l_{net}}^{-1} A]^{-1} \qquad (6.9)$$

CASPARY (1987, p.40) suggests using this method only if the model has sufficient degrees of freedom, such that:

$$\frac{n - u}{u} > 0.5 \qquad (6.10)$$

where n = the number of observations
u = the number of unknowns

In GPS this test should be accompanied by an evaluation with respect to the type of observations and amount of redundancy in the field observations (ie. the time taken and number of satellites recorded within the observation sessions).

However, in GPS, the random component of the observational error is small and the systematic errors play a major role. Therefore standard statistical procedures become unreliable. GRANT (1989) showed that a constant scale factor applied to the VCV matrix will result in very optimistic variances for some coordinates and very pessimistic variances for others. He suggested that for statistical testing, a VCV determined from covariance analysis be used instead of a scaled computed VCV matrix. The problem with covariance analysis in a normal GPS surveying environment, is that the analysis needs specialized software that up to now has only been used with powerful computers. Further problems stem from the uncertainty in the accuracy of the assumptions made in the analysis about the behaviour of the systematic errors sources affecting the observations.

So if scaling the computed VCV by S_o^2 is invalid, and the facilities for covariance analysis are not available, then other practical methods must be found to produce a VCV which is representative of the physical situation. Variance component estimation (VCE) is described, for example, in CASPARY (1987, pp.97-110) and has been used by WELSCH & OSWALD (1985) to combine satellite and terrestrial networks. It can be used on both homogeneous and heterogeneous sets of observables to determine the relative uncertainties in the various components that make up the total stochastic model. In practice, the total covariance matrix of the observations can be defined as:

$$C_l = \sigma_{o_1}^2 Q_1 + \sigma_{o_2}^2 Q_2 + \ldots\ldots + \sigma_{o_k}^2 Q_k \tag{6.11}$$

where k = number of variance components with unknown k variances to be estimated.

In the case of homogeneous observations, the model may have additive effects (CASPARY, 1987, p.100) where the total error in the single observation type can be divided into various error sources that contribute in different relationships to produce the VCV of the observations. In GPS session observations, this could be a constant error plus an error that is distance dependent due to say orbital errors. In the case of heterogeneous observations, each observational group has its own error characteristics and stochastic behaviour.

Networks can consist of sets of observations with both group and additive variance components. The method to determine these components uses the best invariant unbiased estimator σ_o^2 of the variance components S_o^2. It calls for a iterative numerical procedure starting with reasonable a priori estimates of the variance components, until all variances converge.

The technique would be valid if strongly geodetic datum affected observations or observation weights (eg. baseline observations and their associated VCV matrices) are used in the network adjustment. However, VCE relies on the GMM assumptions that all significant errors are randomly represented in the data set. It will often assume that single scale factor is sufficient to account for the errors in one observation type, particularly where observations may seem consistent and yet still be affected by undetected systematic errors. This is usually true when the propagation of errors into the observation types are not well known, as in GPS. Therefore it is recommended that methods such as VCE be accompanied by attempts to evaluate the effects of systematic errors.

GRANT (1989, p.92) discusses an approximation of VCE that while not the same, is based on similar principles. It has been used in terrestrial networks, where one observation type (eg triangulation) is adjusted so the a priori weights are scaled to give an a posteriori variance (S_o^2) of unity. The next observation type (eg distances) is then added and their a priori weights scaled to give an S_o^2 of unity again. In the above method, any inconsistency between the two observational groups will be accommodated entirely by an increase in the a priori uncertainties of the second group and the results would probably be different if the sequence was reversed. However it does produce a method that is simpler to implement than VCE which can require a large amount of computation time and has been shown to lead to optimistic estimates of σ_o^2.

An empirical method has been developed in this study, that uses the knowledge that session observational errors are distant dependent (see section 6.3.1), to apply suitable a priori scaling factors to the session observations. Each GPS observational type (or network) has then been combined so that the a posteriori variance factor of the each network adjustment is near unity.

6.3.1 Determining the A Priori Scaling Factor

In surveying, the accuracy estimates of the results are affected by error sources that can broadly be divided into internal and external errors. Traditionally, internal errors are defined as those inherent in the data acquisition procedures, while external errors can be considered as those resulting from influences not connected with the measuring process. However when analysing GPS surveying results, these definitions become complex since the use of relative positioning cancels or substantially reduces the influence of many of the errors during data processing. It is more correct to consider internal error sources in GPS surveying to be those directly affecting the internal precision of the session results. External errors can be defined as those to which the internal solution appears largely insensitive, but which directly affect the accuracy of the final coordinates. This suggests that two population parameters require estimation. One to verify the internal consistency, the second to indicate external consistency and by default the accuracy of the results.

The method of applying such internal and external population estimates to the weight coefficient matrix is a matter of some conjecture. Because GPS is such a new technique, there is little previous experience to use as a reference for an indication of the expected precision or accuracy of each type of receiver or software reduction package. So the internal precision of a GPS relative positioning solution could in fact be assessed by its a posteriori variance factor, as it is derived directly from the residuals and will reflect the effects of errors such as receiver noise, and residual tropospheric and ionospheric effects. However from section 6.3, scaling the weight coefficient matrix by this factor (or more correctly its population parameter) will produce accuracy estimates that only reflect the internal error sources.

On the other hand, the presence of external error sources in the GPS reductions can only be detected through their consistency as compared to other data. This is sometimes rather difficult, as comparing GPS results to say existing terrestrial networks, will show differences that on examination may in fact be due to errors inherent the terrestrial network. A more appropriate method could be to compare repeated session "measurements" of GPS baselines at widely different times of the year for consistency. Again systematic errors that effect all session results with a similar magnitude will not be detected using this technique.

A popular method of described the total error (ie. combined internal and external errors) in GPS session results is to modify the $C_{l_{net}}$ by relative weighting the session observations based on the inter-station separations. Used by, among others, BOCK et al (1985; 1986), VINCENTY (1987) and SCHAFFRIN & BOCK (1987), the errors can be grouped as constant (a) and length dependent (b) and a suitable population variance can be developed by:

$$\sigma^2 = a^2 + b^2 L_{ij}^2 \tag{5.24}$$

This method can be used as an alternative to the proposal for weighting the phase observations as a function of the baseline distances outlined earlier in section 5.2.3. The assumption that the errors behave this way is based on the knowledge that their are residual ionospheric and tropospheric errors in the carrier phase observations, as well as the positional errors of the satellite orbits and base station. These have been shown in section 5.2 to map into the session results with magnitudes that increase with increased inter-station separation.

Importantly, JONES (1988b) carried out tests on 154 Trimble 4000SX BATCH-PHASER multi-session results (including the PWD coastal network) to determine the population variances for both the internal and external error sources. By carrying out Chi-squared tests on the a posteriori variance factors then placing the results in a linear regression, he found that these results did follow this length dependent pattern for the internal population, producing a slightly different definition for the internal population parameter to that in equation (5.24),

$$\sigma^2 = 0.000308 + 0.000041\, L_{ij} \tag{6.12}$$

Furthermore, by comparing baselines that were measured more than once, an estimate of the external population parameter was obtained,

$$\sigma^2 = 0.004855 + 0.001017\,L_{ij} \qquad (6.13)$$

While this analysis was taken over a small sample (in statistical terms) and is receiver and software specific that includes the use of DMA precise ephemeris, some conclusions can be made for the coastal survey. The external population factor overwhelmed the internal factor (by a factor of 5). Interestingly, both population parameters are dependent on inter-station length, and it can be assumed that other GPS receivers and software will produce similar results with only the magnitudes of the factors varying depending on how well the internal and external errors have been contained.

The use of length dependent population parameters to scale the session co-factor matrices has been used extensively in recent GPS network adjustments. VINCENTY (1987) used the technique when designing the FILLNET GPS network adjustment program for control densification. Interestingly, the computed VCV's of the initial baseline reductions are ignored, and replaced by a diagonal $C_{\ell_{net}}$ matrix with empirically derived values for a and b. These empirical values have been developed from data sets of the United States National Geodetic Survey (NGS), and range from a=10mm and b=2ppm to a=40mm and b=6ppm depending on the type of observable used. HUSTI (1987) found a need to scale Trimble 4000S covariance matrices by 1 ppm (using TRIMVEC™ version 87.030) to obtain realistic statistics from the adjustment model. In an attempt to solve this problem, TRIMVEC™ (version 87.086) had an external factor of 5 ppm incorporated in the processing. When this was applied to the coastal network it produced a minimally constrained network adjustment which failed the Chi-squared test with an a posteriori variance factor of around 0.07, indicating that this external factor was too large to be valid. However it may be valid as an external estimate if the network were to be constrained to terrestrially derived control points.

6.4 NETWORK ADJUSTMENT OF MULTI-BASELINE SESSIONS.

TRIMVEC™ (Version 87.086) and NOVAS baseline results have been placed into network adjustments and are analysed separately. Firstly some general comments are made on the combination of GPS baseline session data.

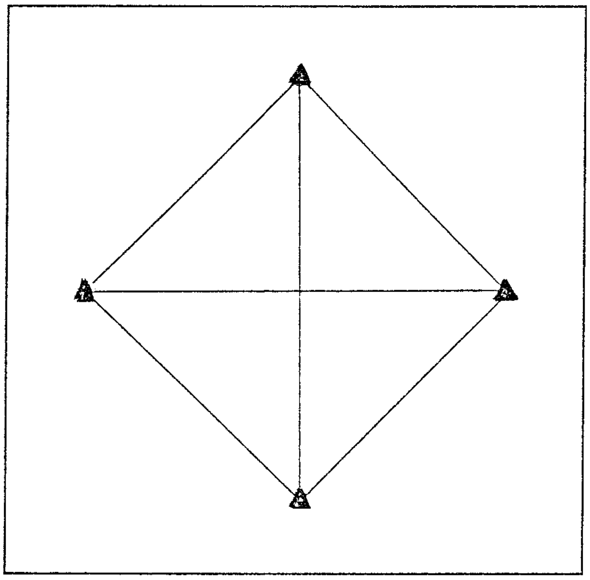
As outlined in Chapter 5, while current baseline software treat mathematical correlations rigorously for a single baseline, they neglect stochastic correlation between baselines within a session. From the general least squares equations (B.4) to (B.6) the single baseline approach will therefore provide sub-optimal estimates of the a posteriori network coordinates as well as their quality ($C_{\hat{x}_{net}}$) since they rely on $C_{l_{net}}$. From equation (6.2) this is made up of the a posteriori covariance matrices of the baselines becoming the a priori stochastic information for the network adjustment. Producing a GPS network that is less than optimal in both results and statistics would be expected to lead to problems, particularly in larger networks where the covariance matrix of the adjusted parameters is used in determining transformation parameters.

Another problem with the single baseline approach is to decide on the optimal method of combining the baselines into the network adjustment. There are two common methods; the first used by researchers such as LETHABY (1987), suggests the use of all possible baseline combinations within a session (see Figure 6.1) where:

$$b = \frac{n(n-1)}{2} \qquad (6.14)$$

where b is the number of baselines in the session
n is the number of receivers operating simultaneously

Figure 6.1 All baseline combinations



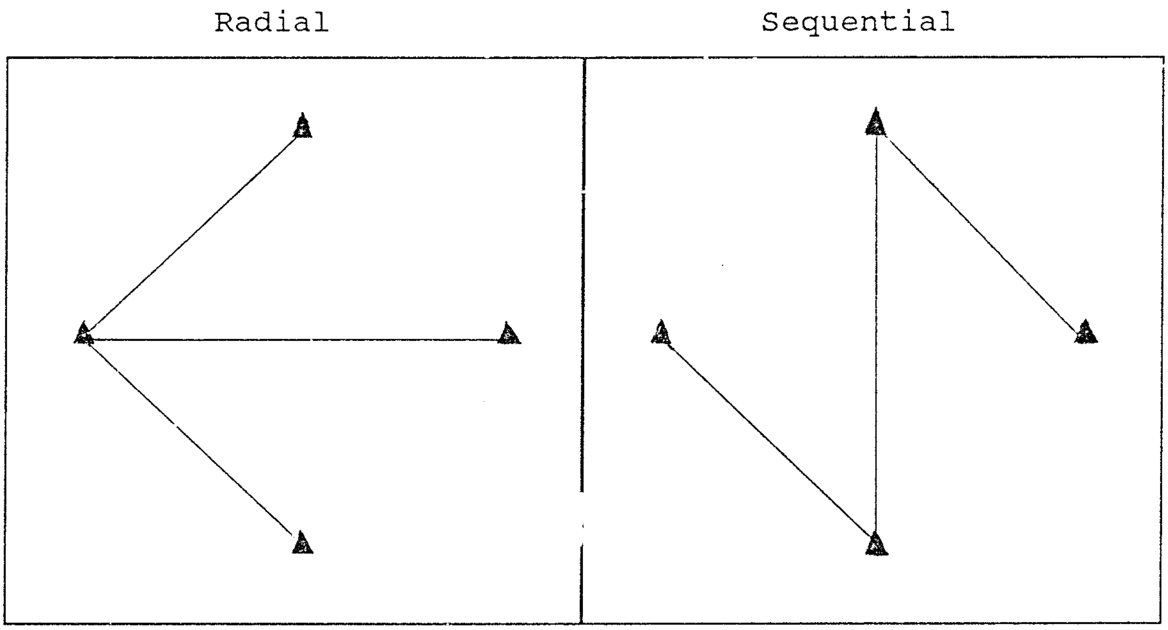
The second method is to select the number of "quasi-independent" baselines per session, equal to one less than number of receivers. This is the minimum requirement to describe the position of receivers in a session and the rest are rejected as "trivial" (see Figure 6.2) where:

$$b = (n - 1) \tag{6.15}$$

The remaining baselines are considered trivial because they provide no independent information. BOCK et al (1985) used these "quasi-independent" baselines in adjusting the Eifel network, and MORGAN et al (1986) followed this approach in their analysis of Phase 1 of the South Australian GPS network.

Method 2 is appealing because it produces the correct number of baseline "observations" but are called "quasi-independent" because they ignore the correlation between the baselines. In most cases, rejecting the "trivial" vectors means that some of the carrier phase data may be ignored. That is to say, some of the carrier phase observations used in the determination of the "trivial" vector(s) will be unique to these vectors and not necessarily be used in the determination of the other vectors. Furthermore, the selection of these "quasi-independent" baselines is essentially arbitrary and has its problems as the high number of different combinations produce different results.

Figure 6.2 "Quasi-independent" baselines



The problem of assuming the baselines are stochastically independent remains if the first method is used. However this method does reflect the geometric strength of the contributions of all carrier phase observations and results in a unique solution. Furthermore, using all possible baseline combinations tests for the processing consistency within the session by checking that the polygons formed, close to reasonable accuracies. The network adjustment will have an artificially high redundancy since some of the baselines can be considered "trivial". MACARTHUR et al (1985) suggests a third method that uses equation (6.14) in a session adjustment and then combines these session results of (n-1) sets of coordinate differences in an overall network adjustment. This is similar to method 1 but appears more rigorous with the correct number of degrees of freedom, however the stochastic model of the modified session observations is still incorrect.

Table 6.2 Comparison of baseline network adjustment methods (TRIMVEC™ results)

STN	METHOD 1 - METHOD 2			STN	METHOD 1 - METHOD 2			STN	METHOD 1 - METHOD 2		
	δLat	δLong	δHt		δLat	δLong	δHt		δLat	δLong	δHt
1	-0.065	-0.126	-0.071	15	0.001	0.029	0.003	28	0.029	-0.091	0.007
2	-0.056	-0.098	-0.055	16	0.000	0.020	-0.001	29	0.026	-0.033	0.013
3	-0.052	-0.077	-0.055	17	0.000	0.000	0.000	30	0.026	-0.025	0.021
4	-0.008	-0.043	-0.001	18	0.000	-0.009	0.005	31	0.027	-0.024	0.034
5	-0.004	-0.019	-0.004	19	0.005	-0.033	-0.002	32	0.039	-0.021	0.041
6	-0.002	-0.118	-0.016	20	0.001	-0.024	-0.012	33	0.040	-0.023	0.056
7	-0.002	-0.117	-0.018	21	0.000	-0.011	-0.011	34	0.045	-0.025	0.059
8	0.002	-0.045	-0.012	22	0.003	-0.007	0.005	35	0.044	-0.020	0.051
9	0.002	-0.053	-0.005	23	0.041	-0.139	0.007	36	0.038	-0.003	0.053
10	0.003	-0.061	0.000	24	0.047	-0.155	0.003	37	0.039	0.011	0.052
11	0.004	-0.058	-0.001	25	0.034	-0.134	-0.003	38	0.024	0.078	0.053
13	0.000	-0.004	-0.001	26	0.030	-0.116	0.004	39	0.035	0.105	0.045
14	0.001	0.020	0.002	27	0.030	-0.100	0.012				

A comparison of method 1 and 2 (radial method) was made using the TRIMVEC™ processed Trimble network and the results are summarised in Table 6.2. Method 2 produced the highest a posteriori standard deviations due to the loss of geometric strength, but there was no statistically significant differences in the methods. It was therefore decided, because of its unique solution and geometric strength, to use all possible baselines in the following network adjustments.

6.4.1 TRIMVEC™

Initially the 168 TRIMVEC™ (Version 87.086) baselines were placed in the SHAKE adjustment software. A minimally constrained adjustment was executed by only holding Sydney (Watson Pillar) fixed to obtain provisional coordinates and ellipsoid heights, and to test for outliers. The SHAKE adjustment model failed the global test with S_o^2 of 0.07. This indicated that the 5 ppm external scaling of the computed VCV had made the variances too pessimistic, since others had encountered the same problem, such as MRSTIK (198ɩ) in a 50 station survey in Ottawa, Canada.

Re-scaling of the original computed session VCV matrices in $C_{l_{net}}$ was needed to obtain an network with S_o^2 near unity. With the knowledge that the session observations have errors that have magnitudes defined by equation (5.24) and using previous Trimble 4000S results by HUSTI (1988) of a=0 and b=1.0ppm as a starting point, the adjustment produced an a posteriori variance factor of 1.35. Four baselines were detected as outliers using Baarda snooping techniques (with α=0.1 and β=10). Since the technique assumes only one gross error, the baseline with the largest normalised residual was analysed and found to have substantially large rms values due to poor quality carrier phase observations. The baseline was removed from the adjustment and the remaining 167 baselines readjusted. All four lines flagged in the first adjustment were subsequently removed due to a lack of carrier phase observations caused by receiver malfunctions (3 lines in session 141A, 1 line in session 137A). The a posteriori variance factor was reduced to 0.97.

6.4.2 NOVAS

A network adjustment of the NOVAS baselines was carried out in two sections. The most northern two quadrilaterals made up one adjustment, holding station Boyd (1041) fixed. The remaining 11 sessions south of Crowdy Head made up the second, where Tidbinbilla (1053) was held fixed. The calculations were initially done by the CMA using the adjustment software GEOLAB™. This adjustment was checked by the PWD using SHAKE and later NEWGAN, returning a posteriori variance factors of 18.8 and 3.3 respectively. The adjustments both failed the Chi-squared global model test and this time indicated that the internal VCV of the baseline observations were possibly too optimistic for the adjustment model. Using NEWGAN, an external variance factor was applied to the diagonal elements of the baseline's internal VCV only. Because NOVAS produces non-rigorous stochastic modelling within a session, maintaining the existing stochastic correlations was thought not to be important.

An external variance factor of a=0 and b=0.5 ppm was used to obtain an a posteriori variance factor near unity. Due to a receiver malfunction at station 1056, the VCV for the lines emanating from this station were additionally scaled by a factor of 3 to indicate the reduced data sets of only 2 hours. The adjustment produced a posteriori variance factor of 0.96 for the top two quadrilaterals, and a slightly worse 0.42 for the bottom 11 sessions. The results highlight the problem in small data sets, of the sensitivity of the a posteriori variance factor to the degrees of freedom. By introducing more observations into the adjustment model (and increasing the degrees of freedom) it will generally produce a lower variance factor. So the use of S_o^2 as a valid statistic, say, as an estimate of the σ_o^2 must be in doubt in adjustments with small observation sets and low degrees of freedom.

6.5 NETWORK ADJUSTMENT OF MULTI-STATION SESSIONS

NEWGAN was used because of its multi-station adjustment capacity and a minimally constrained adjustment was carried out using the BATCH-PHASER Trimble session results. Again Watson's Pillar (1017) in the middle of the network was fixed and at the suggestion of JONES (1988a) scaling of the all computed session covariance matrices was implemented to reflect the influence of unmodelled and residual external errors. At the time, the research by JONES (1988b) had not been published, so a constant scaling factor was used. An initial external factor of 10 was applied as this had been used successfully by the South Australian Lands Department with the Trimble 4000SX data collected in phases 2 and 3 of the state's primary geodetic network and was consistent with values applied in other parts of the world (JONES et al, 1987). This proved to be within the population values estimated in section 6.2.1 for the BATCH-PHASER results except session 145A that with an average baseline length of 180 km would need additional scaling.

The results suggest sessions 137A, 141A and 145A be investigated. Figure 137A had shown data processing problems with two receiver's malfunctioning and large carrier phase residuals. Measurements of the high ionospheric activity on this day by PARTIS & BRUNNER (1988) would account for these poor results. The stations were reobserved (session 146A), and this data was therefore redundant and so was deleted. Session 141A had also shown a high S_o^2 value in the initial processing due to the limited data sets at certain stations (see 5.4.3), however the session was included in the adjustment but was substantially de-weighted. This was to allow continuity in the network. Interestingly, three lines that were flagged as outliers in the TRIMVEC™ network adjustment were from this session. Session 145A was the connection to the LIC's region 4 network and involved distances of up to 250 km. This figure was suspected of large unmodelled external errors (particularly ionospheric) and based on JONES (1988b) empirical tests, would require a further re-scaling of the VCV matrix nominally chosen at 10 to reflect the longer distances involved. Furthermore, an error in locating the antenna over the point at Jervis Bay meant that two separate points were occupied and had to be considered separately in the adjustment. However the height was the same in both sessions and NEWGAN allowed the heights of these two stations to be constrained to one value. The adjustment was repeated and returned a network a posteriori variance factor of 1.02.

6.6 COMBINING VARIOUS GPS RECEIVER NETWORKS TOGETHER.

The problem of determining how efficiently different GPS carrier phase algorithms have modelled the physical situation, through their functional and stochastic models, is magnified when the results of two or more "session" models are placed in the same network adjustment. This primarily occurs when two or more types of receivers are used in a project and their observations are reduced separately using proprietary software. The relative accuracies of each receiver type may be masked by variations in the mathematical models used and their geodetic datum definitions.

From the initial network adjustments of the three GPS software session solutions, it was decided that the BATCH-PHASER single frequency Trimble network would be combined with the dual frequency TI4100 NOVAS baseline solutions. using NEWGAN. BATCH-PHASER's rigorous treatment of the stochastic model of the phase measurements, and its use of the DMA precise ephemerides was expected to be more accurate than the TRIMVEC™ results in a combined adjustment.

Combining various GPS receivers, or more particularly their post-processing software solutions (one can assume that most of the present geodetic GPS receivers obtain phase measurements with similar precisions) will certainly be a problem in the future as GPS networks proliferate. Up until now, researchers have discussed the adjustment of one type of GPS session software into a network then combine it with terrestrial data. Little or no literature is available on the methods employed to combine various GPS receivers and their reduction software solutions together into one network. DICKSON (1989) joined the combined government measured TI4100 (NOVAS), CMA's WM101 (PoPs) Region 4 network's and PWD's Trimble 4000SX (TRIMVEC™) coastal network to the state's existing first order terrestrial network by first analysing each network in isolation. Then using a method where firstly each GPS network is added together then constrained to the terrestrial network, a total network a posteriori factor near unity was obtained. Again this is similar to that highlighted by GRANT (1989) and it approximates VCE techniques. Others such as SHRESTHA & STANISLAWKI (1988) have used various receivers over the same network (WM101, Trimble 4000SX and Istac 2002) yet have analysed each receiver separately.

The relative weighting of each solution type in the combined network adjustment presented a dilemma. One must take into account such things as the receiver type, the observables, processing algorithms and ephemeris used. Comparisons between solutions with different receiver types (and their software) gives some indication of their relative accuracies. Unfortunately the coastal project has only a very small sample, nearly all taken on one day (26/05/87). Therefore no definite conclusions can be made, although the results do tend to show that the receivers and their post processing software produce similar results at the few parts per million level (see Appendix D).

While the NOVAS (TI4100) solutions used dual frequency observations that would reduce the residual ionospheric delay, the BATCH-PHASER (4000SX) multi-station solutions used precise ephemeris and rigorously maintained stochastic correlations within the session. Importantly the multi-session results showed a much better relationship to MSL determined at each tide gauge when the GPS WGS84 heights were converted to orthometric heights (see Chapter 8). Without any other external evidence to favour one solution or the other, the final combined solution assumed each mathematical model was equivalent and combined the two network solutions accordingly. Using NEWGAN the combined network held Tidbinbilla fixed with the coordinates defined via GPS to Orroral NLRS and gave an overall S_o^2 value of 2.02. Applying the Tau test, 2 dual frequency baselines were flagged as outliers. These lines were examined and found to be in the Stockton and Gerroa regions where direct measurements of Trimble 4000SX and TI4100 vectors had been made. This showed some strain, with the relative scaling of each receiver/software package. The Trimble network needed re-scaling to reflect the larger errors inherent over longer lines. The PWD Trimble 4000SX and CMA TI4100 networks were re-scaled separately to obtain individual a posteriori variance factors closer to 0.8. The PWD Trimble network was scaled by an average factor of 20 (with session 145A scaled at 200), while the CMA TI4100 networks were scaled by 0.8 ppm. The final network adjustment was minimally constrained at Tidbinbilla and had 57 stations with 271 degrees of freedom. This produced a S_0^2 of 1.07, that will allow for further observations to be added without failing the chi squared test. A summary of the results can be found in Appendix E.

Interestingly if the TRIMVEC™ solutions are used instead of BATCH-PHASER, then scaling of the Trimble network by 1.35 ppm is required and produces a combined a posteriori variance factor of 0.97. This is consistent with the findings of HUSTI et al (1988) and scaling of the TI4100 and Trimble 4000SX roughly at a 2 to 1 ratio is consistent with overseas experience, which quotes accuracies of around 1 ppm for dual frequency networks and 2 ppm for single frequency networks (ECKELS, 1987).

While these results indicate a balanced unconstrained network, the network has not been constrained to existing external control. If the accuracies of the average comparisons of the repeat baselines from Appendix D are used, the stochastic model in the GPS network adjustment becomes too pessimistic. However using some of the PWD coastal survey, the TI4100 survey and Region 4 network, the LIC have constrained the GPS data to the state's first order control network. The resultant a priori standard deviations developed for the receivers and software were larger (DICKSON, 1989, p.7) in similar proportions to the comparisons in Appendix D. That is,

Trimble 4000SX (TRIMVEC)	= 3 ppm
TI4100 (NOVAS)	= 1.5 ppm

This is the same ratio as that developed in this study, but reflects the need to de-weight or "loosen" the more accurate GPS observations to fit the less accurate existing terrestrial network.

7. THE DETERMINATION OF ORTHOMETRIC HEIGHTS FROM GPS

7.1 OBTAINING HEIGHTS FROM GPS SURVEYS

One of the major advantages of GPS is that it can be used to calculate baselines or station coordinates in three dimensions. A three dimensional reference system may take many different forms, depending on what is most convenient. These may include (BOMFORD, 1977):

1) Cartesian coordinates (x,y,z) - with the datum minimally defined by an origin at the earth's centre of gravity, with axes orientation and scale.

2) Geocentric coordinates (r,θ,λ) - with the datum equivalent to (1).

3) Ellipsoidal coordinates (ϕ,λ,h) - determined for an arbitrary reference ellipsoid that has a defined origin, major axis and flattening. The third (height) component may be measured above the ellipsoid or recorded relative to the earth's gravitational field.

The cartesian coordinate system used in GPS has its origin, orientation and scale defined by the "fixed" coordinates of the tracking stations that monitor and compute the GPS satellite orbits (sections 3.1.3 and 6.2). Furthermore, a WGS84 geocentric ellipsoid that has a precise mathematical relationship to the cartesian system has been developed. Once the network has been successfully adjusted (Chapter 6), the WGS84 XYZ coordinates of each station can be transformed into geodetic latitude, longitude and ellipsoid height on an ellipsoid whose position relative to the WGS84 geocentric cartesian coordinate system is known. This reference ellipsoid may have its origin at the geocentre, such as the WGS84 ellipsoid or be arbitrarily located to fit a specific region. Using published parameters or those determined by the network adjustment (see ECKELS, 1987, pp.140-155), the GPS WGS84 ellipsoid coordinates can be defined in terms of the local coordinates.

The absolute accuracy of a network's coordinates, including station ellipsoid heights, will depend amongst other things, on the accuracy of the coordinates of the fixed station(s) in the network adjustment. This means that any error in the origin of a minimally constrained adjustment will propagate through the network. However the relative accuracy in station coordinates will be largely free of this error, as for example in the ellipsoid heights:

$$\Delta h_{AB} = (h_A + e) - (h_B + e) = h_A - h_B \qquad (7.1)$$

where Δh_{AB} is the difference in ellipsoid height between stations A and B
h_A is the ellipsoid height of station A
h_B is the ellipsoid height of station B
e is the constant (or near constant) error in height in the network caused by the error in the network's origin.

The coordinates of the origin of the combined solution adopted in Chapter 6 at Tidbinbilla have been defined via a GPS connection to Orroral NLRS, Mirror 7 which has Centre of Space Research (CSR) LAGEOS LONG ARC 8511 adjusted values that are considered to be better than 5 metres absolute, ie. its position on the earth's surface is known to within 5 metres in the WGS84 datum (LIC, 1989).

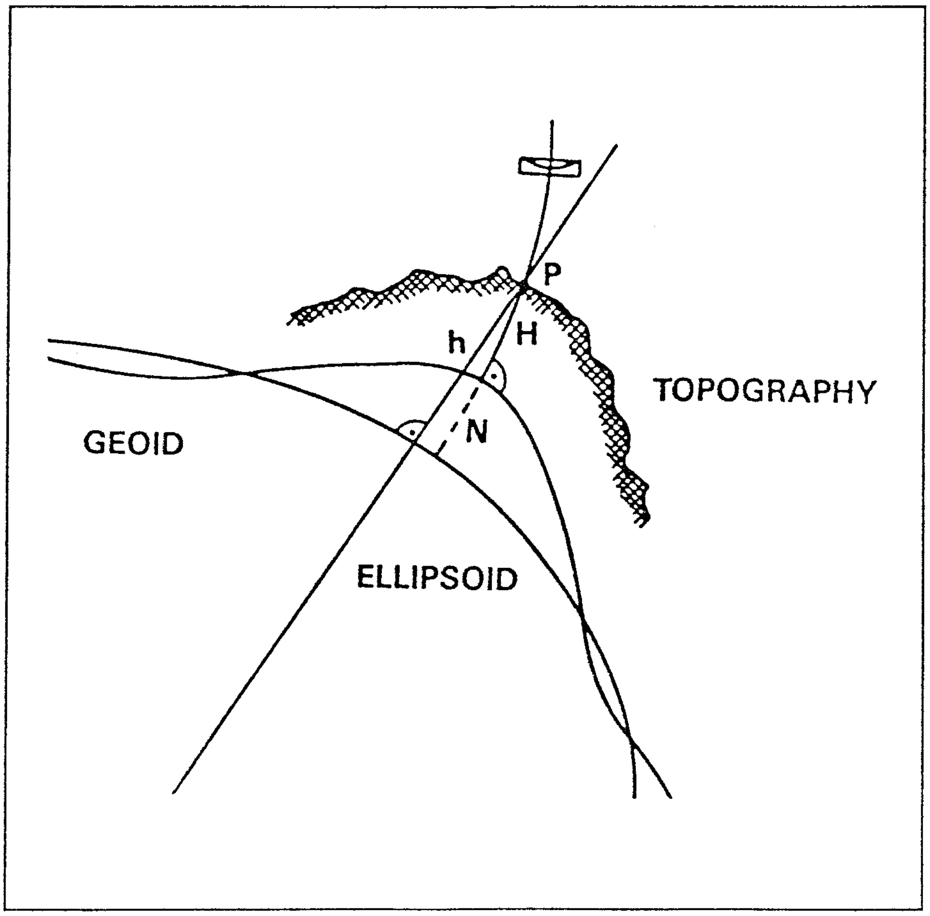
These heights must not be confused with the "natural relative height system" defined by spirit levelling that is connected to MSL determinations at the tide gauges as defined in 2.5. To relate the two, the GPS ellipsoid heights must be converted to orthometric heights.

7.1.1 Orthometric Heights from GPS Heights

A problem in applying GPS to engineering studies is that the GPS ellipsoid heights are essentially "mathematical" whereas orthometric heights have greater "physical" meaning because they are related to the geoid and therefore show the direction water will flow (see 2.3). The difference between the orthometric and ellipsoid heights at a point is referred to as the geoid-ellipsoid separation, geoid height or geoid undulation N and is shown in Figure 7.1, with the relationship being:

$$H = h - N \tag{7.2}$$

Figure 7.1 - Orthometric and ellipsoid heights



H is measured from the geoid along the curved plumbline and h is measured normal to the ellipsoid. The difference in the direction is the deflection of the vertical and for small deflections of the vertical, the two can be considered coincident and equation (7.2) is valid.

Generally the systematic errors in both h (derived for example from GPS) and N (derived by the techniques described in section 7.2) at points A and B will be similar, and it is preferable when seeking the highest precisions to adopt a differential technique, ie. to use line elements Δh and ΔN. For this reason, we therefore adopt the approach:

$$\Delta H_{AB} = \Delta h_{AB} - \Delta N_{AB} \tag{7.3}$$

to propagate orthometric height differences through the tide gauge network. So to maintain the precision of ΔH, the desired precision of ΔN should at least match that of Δh (KEARSLEY, 1984 & 1988a).

7.2 DETERMINATION OF THE GEOIDAL HEIGHT DIFFERENCES (ΔN)

There are various methods for determining N and ΔN, although some are not particularly suitable for high precision determinations using GPS. KING et al (1987), ECKELS (1987) and HOLLOWAY (1988) discuss these methods and they are only briefly outlined below.

7.2.1 Astro-geodetic Methods

The relationship between geodetic coordinates that are referenced to a ellipsoid and observed astronomical coordinates observed in, and hence referred to the local gravity field, is described by the deflection of the vertical (see Figure 7.1). The two components of the deflection of the vertical in the meridian (ξ) and prime vertical (η) are defined by:

$$\xi = \Phi - \phi \tag{7.4}$$

$$\eta = (\Lambda - \lambda)\cos\phi \tag{7.5}$$

The geodetic latitude (ϕ) and longitude (λ) can be established using GPS survey methods, while the astronomic latitude (Φ) and longitude (Λ) are calculated from observations to the stars. The deflection component (ϵ) between surfaces A and B, whose azimuth is α becomes (HEISKANEN & MORTIZ, 1967):

$$\epsilon = \xi\cos\alpha_{AB} + \eta\sin\alpha_{AB} \tag{7.6}$$

The geoid-ellipsoid separation at a point B with respect to the value at the point A, and separated by a distance ds is (ibid):

$$\Delta N_{AB} = -\int_A^B \epsilon . ds \tag{7.7}$$

The method of geoidal height determination using equation (7.7) is known as astro-geodetic levelling. A map of geoidal heights can be determined if there is a sufficient density of stations at which both geodetic and astronomic coordinates are available. Historically, astronomic observations have been taken at a number of points throughout large terrestrial geodetic networks to control azimuth errors. These observations were used, for example to determine N, referenced to a local datum such as the Australian Geodetic Datum (AGD) (FRYER, 1971).

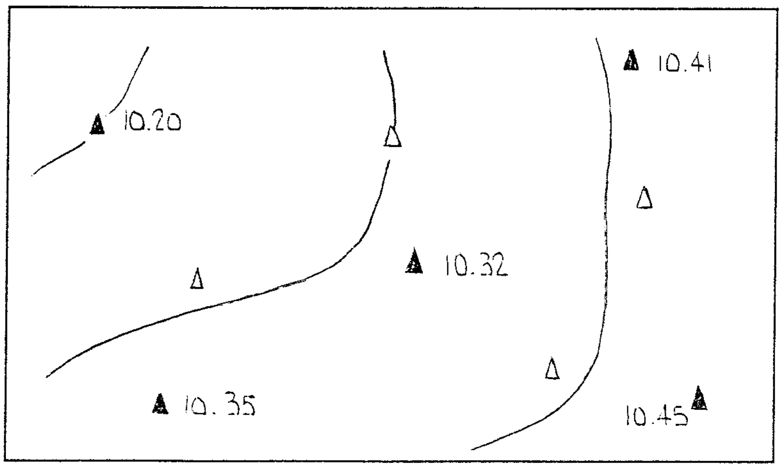
However the astro-geodetic method is generally unsuitable for transforming GPS heights to orthometric heights because astronomical observations are time consuming, expensive and labour intensive. In addition, the geoid information is not in relation to a geocentric ellipsoid.

7.2.2 Geometric Methods

Geometric methods are probably the most practical way of determining geoidal heights and have been used in several overseas GPS surveys to achieve decimetre accuracies (ECKELS, 1987). If a number of stations with known orthometric height are included in the GPS network, then their geoidal heights can be determined directly using (7.2). Three or more non-collinear control stations will define a surface of geoidal heights from which N values can be calculated for other stations in the network.

The surface of N values can be defined graphically or analytically. The graphical method uses the differences between the two height systems to form the basis of a contour map of the N values in the survey area. The map is then used to interpolate N values at the other points in the network as shown in Figure 7.2.

Figure 7.2 - Contouring method to determine N values (After King et al (1987))



The second approach is to define the surface analytically, with the simplest model being to assume a first order plane surface. The analytical function of the geoid will be in the form (KING et al, 1987):

$$N = AX + BY + C \tag{7.8}$$

where N is the Geoid-Ellipsoid Separation at a Station with easting and northing coordinates (X,Y)
A is the East-West Tilt of the Geoid
B is the North-South Tilt of the Geoid
C is the Geoid-Ellipsoid Separation at the Centroid of the Network

If there are more than three points with both orthometric and ellipsoid heights, then the coefficients A,B,C can be solved by the use of least squares, where the variance-covariance matrix of the observations is identity (HOLLOWAY, 1988):

$$\hat{x} = (A^T A)^{-1} (A^T l) \tag{7.9}$$

where $\hat{x}$ is the least squares estimate of A,B,C
A is the design matrix of the parameter coefficients
l is the geoid-ellipsoid separation at each control point.

HOLLOWAY (1988) applied this technique to networks in South Australia and Western Australia with some success. COLLINS & LEICK (1985) used this method in Montgomery County and achieved standard deviations of ±3 cm on the adjusted heights.

Although it is possible to generate higher order surfaces to interpolate N, these methods become unstable when the point data is irregularly spaced (HOLLOWAY, 1988, p.61). KEARSLEY (1985) noted that using higher order surfaces to model topography led to cases where some calculated heights were hundreds of metres above the highest point on a mountain!

ECKELS (1987, p.161) and VINCENTY (1987) suggest that the plane fit parameters can be determined using a seven parameter topocentric transformation model, where the rotations in the northerly and easterly axes include the slope of the geoid in these directions and the geoid-ellipsoid separation is absorbed in the scale factor. PASCOE (1989) has carried out tests using this method but the results are inconclusive.

The main advantage of these geometric methods is that they are conceptually simple and quick to implement. However there are problems in these techniques, particularly in their use of over-simplified models. Firstly, there are few checks that can be applied to detect errors in either the GPS or orthometric heights that determine N at the control points and these errors will propagate into the N surface.

Secondly, the assumption that the geoid is regular between the data points that define the interpolation surface may be incorrect. GILLILAND (1986, p.279) warns that "linear interpolation over distances as small as 25 km could result in errors much greater than 10 cm" (ie. > 4 ppm). Therefore the technique is only recommended for "areas up to 50 * 50 km where the geoid is smooth" (KING et al, 1987, p.94). As a consequence, HOLLOWAY (1988) and ZILKOSKI & HOTHEM (1988) suggest that during the planning stage, a prior knowledge of the nature of the geoid in the area to be surveyed would be advantageous in estimating achievable orthometric height precisions.

Another problem involves the position of the control stations within the network. Where the control stations surround those stations with unknown N then the process is one of interpolation. However if there are unknown stations outside the control stations, then the process is one of extrapolation and this may produce assumptions concerning the shape of the geoid that make it dangerous to implement.

With these problems in mind, it would not be possible to apply this technique to the PWD's coastal datum problem. This is because the orthometric heights needed at the control stations to determine N in equation (7.2) are in fact the heights being investigated. Therefore an independent method is required that does not need existing orthometric heights. Furthermore, the size of the survey (1300 km long) and the distance between stations (up to 60 km) makes it difficult to assume the geoid will be regular and therefore difficult to interpolated accurately.

7.2.3 Higher Order Geopotential Models

From Newton's third law it can be shown that the gravitational potential from a point (P) is a function of the inverse of the distance and therefore satisfies Laplace's Equation in space (TORGE, 1980, pp.26-31). The functions which satisfy Laplace's Equation are "Harmonic Functions" and when these are expressed in spherical coordinates (r,θ,λ), they are known as "Spherical Harmonics". These functions can be used to model various aspects of the earths gravitational field that determine global geoid models and predict perturbations in satellite orbits.

The geoid height (N_L) and gravity anomaly (Δg_L) at a point can be expressed in the form of a truncated spherical harmonic series (RAPP, 1982):

$$N_L = \frac{kM}{\gamma R} \sum_{n=2}^{n_{max}} \left(\frac{a}{R}\right)^2 \sum_{m=0}^{n} P_{nm}(\cos\theta)\,(C_{nm}\cos m\lambda + S_{nm}\sin m\lambda) \quad (7.10)$$

$$\Delta g_L = \frac{kM}{R^2} \sum_{n=2}^{n_{max}} (n-1) \sum_{m=0}^{n} P_{nm}(\cos\theta)\,(C_{nm}\cos m\lambda + S_{nm}\sin m\lambda) \quad (7.11)$$

where kM = the gravitational constant & earth's mass

R = the geocentric radius

γ = the normal gravity defined by the International Gravity Formula 1980

θ = the polar distance

λ = the geocentric longitude

a = the equatorial radius of the reference ellipsoid

C_{nm} and S_{nm} = the fully normalised potential coefficients

n and m = the degree and order of the coefficients

$P_{nm}(\cos\theta)$ = the associated Legendre functions of the first kind

In such models, the coefficients C_{nm} and S_{nm} to a maximum degree n_{max} are determined by combining the analysis of orbit perturbations of satellites, with terrestrial gravity and satellite altimetry measurements of N over the oceans. The satellite orbital analysis provides the information on the long wavelength geoidal features (described in the lower degree harmonics of the model). The higher degree harmonics are obtained from the terrestrial gravity and satellite altimetry and contain the high frequency, short wavelength information.

The past few years have seen the development of gravity field models of very high degree and order (up to n_{max}=360°). The most commonly used high order geopotential models and the data used to determine them is given in Table 7.1.

TABLE 7.1 Earth gravity models
after HOLLOWAY, (1988, p.63)

MODEL	DEGREE	ORIGIN	DATE	INPUT DATA
OSU81	180	Rapp	1981	GEOS 3 data + 1° surface gravity + SEASAT altimetry
GRIM 3	36	Reigber	1983	Satellite tracking + surface gravity
GEM-L2	20	Lerch	1984	SLR + GEM9 data
GPM2	200	Wenzel	1985	GEM-L2 + 1° surface gravity + altimetry
OSU86C,D	250	Rapp,Cruz	1986	as above
OSU86E,F	360	Rapp Cruz	1986	as above + 30' surface gravity + Δg from sat altimetry over oceans
OSU89A,B	360	Rapp	1989	GEM-T2 to 36x36 + 30' surface gravity + Δg from sat altimetry over oceans

Geopotential models to degree 180 have the ability to detect geoidal features with a half wavelength of 1°, or about 110 km, to an accuracy of ± 0.2 m (KEARSLEY, 1984, p.94). However the computations require a computer as there are 16471 (C_{nm} & S_{nm}) coefficients for a geopotential model with n_{max}=180°. For a geopotential model of n_{max}=360° the number increases to 65431 coefficients (HOLLOWAY, 1988, p.64).

The accuracy of these geoidal heights varies throughout the world. The principal error sources in the higher degree terms of the geopotential coefficients come from the gravity data itself and the sampling and smoothing techniques used to determine Δg. Therefore, while the higher the maximum degree of a geopotential model may more accurately portray the local geoid, the extent to which this is true depends on the quality of the data used. RAPP & CRUZ (1986, p.10) warn that "just because we have a high degree field, it does not mean we have a highly accurate high degree field" and estimate that due to the noise in the data, there is a 100% uncertainty in the coefficients above degree 175 for models OSU86E,F. However these values are still the best estimates for these coefficients.

The advantage of these geopotential models over other heighting techniques is that the models can be used anywhere on the world. Being merely a set of coefficients permits the geoid heights to be evaluated quickly. Importantly, these models are geocentric in nature and N can easily be related to the GPS (WGS84) geocentric datum (KING et al, 1987). For instance the OSU81 model is based on the Geodetic Reference System (GRS80) where a=6378137, f=1/298.257223. These are essentially the same parameters as those adopted for the geometric model used for the GPS WGS84 coordinate system. Furthermore, if there are small differences in the ellipsoid parameters between the geometric model (used to find h) and geopotential model (used to find N), then using equation (7.3) will not introduce a significant error in ΔN over short lines (KEARSLEY, 1988a, p.13).

7.2.4 Gravimetric Methods (using Stokes' Theorem)

In 1849 G.G. Stokes formulated a practical method for computing geoidal heights using surface gravity measurements. An outline of the method can be found in, for example, HEISKANEN & MORTIZ (1967). The disturbing potential (T) can be defined:

$$T_p = W_p - U_p \qquad (7.12)$$

where W_p is the actual gravity potential
U_p is the potential referenced to the ellipsoid.

The solution for the disturbing potential (T) can be accomplished by Stokes' integral (TORGE, 1980, p.157). It is however, more common to use it to solve for the geoidal height in the form:

$$N_P = \frac{R}{4\pi\gamma_m}\iint_\sigma \Delta g\, S(\psi)\, d\sigma \qquad (7.13)$$

and

$$S(\psi) = \operatorname{cosec}\frac{\psi}{2} - 6\sin\frac{\psi}{2} + 1 - 5\cos\psi - 3\cos\psi \ln\left(\sin\frac{\psi}{2} + \sin^2\frac{\psi}{2}\right) \qquad ...(7.14)$$

where R = the radius of the spherical model of the earth
γ_m = the mean gravity of the earth
Δg = the free air gravity anomaly associated with $d\sigma$
ψ = the spherical distance between P and $d\sigma$
$S(\psi)$ = the Stokes' function being equation (7.14)
$d\sigma$ = is the element of the surface area σ over which the integration is performed

Theoretically, the integration is performed over the whole earth and requires a perfect knowledge of the continuous gravity field. However, this is not possible as the gravity field data is defined at discrete points and limited to only some of the ocean and land areas (HOLLOWAY, 1988). So in practice, the surface integrals are replaced by a finite summation which is restricted to an area defined by geographical blocks or a cap defined by rings and radial lines. More recently this summation has been carried out using the technique of Fast Fourier Transforms (FORSBERG, 1989).

7.2.4.1 Geographic Blocks

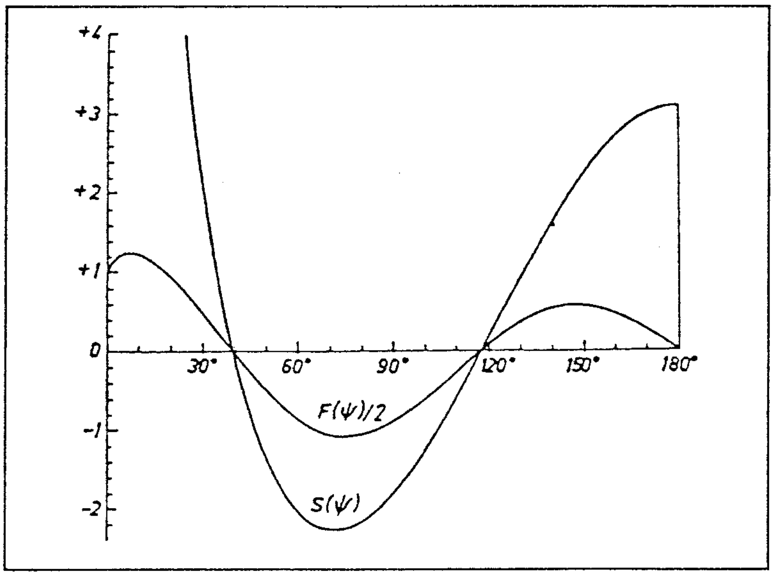
The method of subdivision into geographical blocks is used by GILLILAND (1982, 1983) and ENGELIS et al (1984, 1985), where the gravity anomalies (Δg_i) are replaced by the mean gravity anomaly ($\Delta\overline{g}_i$), for the midpoint of the compartment i.

So equation (7.13) becomes:

$$N_p = \frac{R}{4\pi\gamma_m} \sum_i S(\psi)_i \, \Delta\overline{g}_i \, d\sigma \qquad (7.15)$$

Unfortunately the behaviour of the Stokes' function $S(\psi)$ is non linear and its value tends to infinity close to the computation point (See Figure 7.3). This means that errors in Δg_Q will propagate strongly in N_p as $\psi \to 0$ (ie. as $Q \to P$). Special computational techniques are adopted to overcome this problem, such as by GILLILAND (1982, p.50), where the compartment size is reduced to a finer and finer mesh of mean gravity anomalies as $\psi \to 0$.

Figure 7.3 Behaviour of $S(\psi)$ and $F(\psi)/2$
(HEISKANEN & MORTIZ 1967, p.96)



7.2.4.2 Ring Integration (RINT)

The second method of computing N using Stokes' formula is to integrate over a spherical cap centred on the computation point (P) with a maximum radius ψ_0. This involves subdividing the integration surface into compartments formed by concentric rings of increasing radius from $0 \rightarrow \psi_0$ and radial lines with azimuth α.

We can define:

$$d\sigma = \sin\psi \, d\psi \, d\alpha \tag{7.16}$$

So if

$$F(\psi) = S(\psi)\sin\psi \tag{7.17}$$

Then equation (7.16) becomes:

$$N_p = \frac{R}{4\pi\gamma_m}\int_{\psi=0}^{\pi}\int_{\alpha=0}^{2\pi}\Delta g\, F(\psi)\, d\psi d\alpha \tag{7.18}$$

$F(\psi)$ can be alternatively defined as (KEARSLEY 1985, p.80):

$$F(\psi) = 2\cos\frac{\psi}{2} - \sin\psi(6\sin\frac{\psi}{2} - 1 + \cos\psi(5 + 3\ln(\sin\frac{\psi}{2} + \sin^2\frac{\psi}{2}))) \qquad \ldots(7.19)$$

and its behaviour is far more stable than $S(\psi)$ as $\psi \rightarrow 0$. Therefore errors in the terrestrial gravity data or the derived mean gravity anomalies for a compartment (particularly near the computation point) will be contained.

HOLLOWAY (1988, p.75) points out that the ring integration method (RINT) has the great advantage of being both flexible and simple. Computers can generate rings centred at each point quickly and calculate mean gravity anomalies for each compartment without the need to precompute mean gravity values for different sized zones as required in the first method.

7.2.4.3 Fast Fourier Transforms

A third method is to use equation (7.16) in planar approximation, and evaluate it using Fast Fourier Transforms (FFT's). In this approach a grid of observed gravity is derived over the area of interest, then Δg is transformed to the geoid undulations at the same grid points, by using the spectrum of the gravity field, in convolution with the Stokes' function expressed in a system of plane polar coordinates. The N values at the required points are then obtained by interpolation from the grid (KEARSLEY, 1988a, p.14). When there are a number of points to be calculated then the quickest approach is FFT's. FORSBERG (1989b) showed that this method can determine accurate geoids over large areas such as Scandinavia.

7.2.5 Collocation

Another approach used to determine N is by least squares collocation. This is a method that combines geodetic measurements of different types into one least squares adjustment to determine any quantity of the earth's gravity field.

Details can be found in HOLLOWAY (1988, p.76) and the technique has been used by various researchers (HEIN, 1985; TSCHERNING & FORSBERG, 1986) to determine geoidal heights. While this method has the advantage of allowing heterogeneous data such as deflections of the vertical, gravity anomalies and satellite altimetry to be combined in one estimate, it may result in instabilities if a large number of data points are close together. Furthermore, topography must be taken into account in rugged terrain, through the use of Digital Terrain Models (DTM) if satisfactory results are to be obtained (TSCHERNING & FORSBERG, 1986; KEARSLEY & FORSBERG, 1989).

7.2.6 A Combined Solution

Section 7.2.4 discussed Stokes' integral as a method to determine N. It is neither feasible nor necessary to integrate over the whole earth's surface with Stokes' integral. A combined solution using both gravimetric techniques and higher order geopotential models has been shown by various authors including KEARSLEY (1984; 1986; 1988b) and ENGELIS (1984; 1985) to determine ΔN to similar precisions to those of GPS ellipsoid heights (Δh). The contribution to equation (7.18) from gravity anomalies of area remote to the point P decrease rapidly with increased and is relatively small for $\psi > 30^\circ$ (KING et al, 1987). Therefore to ease the computational burden of Stokes' integral, the integration is made over an inner zone (say $\psi = 1.5^\circ$) to pick up the short wavelength contribution (N_S) and combined with a geopotential model to provide the longer wavelength (lower frequency) features of the gravity field (N_L). The total geoid height is the sum of the two components (KEARSLEY, 1987; 1988b):

$$N = N_S + N_L \qquad (7.20)$$

The general rule is that features whose half wavelengths are greater than $180^\circ/n$ are modelled by the nth term in the spherical harmonic series of the remote zone geoid contribution (N_L) using equation (7.10).

$$N_L = \frac{kM}{\gamma R} \sum_{n=2}^{n_{max}} (\frac{a}{R})^2 \sum_{m=0}^{n} P_{nm}(\cos\theta)(C_{nm}\cos m\lambda + S_{nm}\sin m\lambda)$$

Theoretically, this means that it should only be necessary to integrate Stokes' integral over a spherical cap with maximum radius ψ_0=180°/n to determine the short wavelength characteristics of the geoid N_S from equation (7.21). Therefore if OSU81 is used to n_{max}=180, the half wavelength features up to 1° will be modelled by the geopotential model. If OSU86E is used to n_{max}=360°, the half wavelength feature of up to 0.5° will be modelled. Thus, using RINT as an example:

$$N_S = \frac{R}{4\pi\gamma_m} \int_{\psi=0}^{\psi_0} \int_{\alpha=0}^{2\pi} \Delta g' \, F(\psi) \, d\psi d\alpha \qquad (7.21)$$

The residual gravity anomaly ($\Delta g'$) is found by subtracting the modelled gravity anomaly (Δg_L) generated by the geopotential model using equation (7.11) from the terrestrial gravity anomaly (Δg) determined by:

$$\Delta g = \acute{g} - \gamma \qquad (7.22)$$

where $\acute{g}$ is the gravity reduced to the geoid by applying the free air correction and γ is the normal gravity

$$\Delta g' = \Delta g - \Delta g_L \qquad (7.23)$$

Using the residual gravity anomalies avoids duplication of the signal when the inner zone is combined with the remote zone contribution that would cause a "double counting" effect (HOLLOWAY, 1988, p.79).

The computation of N is carried out at each end of a GPS baseline and then differenced to calculate ΔN_{AB}. This differencing technique tends to cancel systematic errors. These errors include (HOLLOWAY, 1988, p.79):

* the assumption of a spherical earth adopted for the Stokes' formula

* the reference surface of the levelling network not being coincident with the geoid introducing an error in the reduction of the gravity data to the geoid

* any errors in the low to medium degree terms of the geopotential model.

This technique is strengthened if some computation points within the network have both known ellipsoidal and orthometric heights. This allows a method of checking the combined solution within the computation area (GILLILAND, 1986; KEARSLEY, 1988a). In other words, it allows the user to determine the optimum reference field for the region, and the optimum inner zone (ψ_0), for the Stokes' integration.

7.3 SUMMARY

Reviewing the possible methods of determining the geoidal heights within the NSW coastal network as outlined in this Chapter, the accuracy limitations and high cost of the astro-geodetic method make it unsuitable for transforming GPS ellipsoid heights to orthometric heights. While geometric methods are computationally simple and easy to use, the methodology requires knowledge of a certain number of orthometric heights throughout the network and a regular geoid. This causes problems in this project, because its aim is to locate errors in the orthometric levelling that connect the tide gauge datums, through its analysis of all the AHD heights. A method independent of using established orthometric heights is required.

While high degree geopotential models derived N independently using equation (7.10), their accuracies depend on the sampled gravity data used in determining the higher degree coefficients, the location of the area of computation and the upper limit of summation of the spherical harmonic expansion (n_{max}). Gravimetric methods have provided accurate geoidal heights (eg GILLILAND (1982;1983)) using Stokes' integral, but prove computationally cumbersome when taken to a large ψ. In the same way collocation requires a large amount of data processing, using a variety of measurements that require stochastic estimation, usually through covariance analysis. However the combined solution, using the ease of calculation offered by the high degree geopotential model to determine the long wavelength (lower frequency) features of the geoid (N_L) and the accurate gravimetric approach over a small region around the point to provide the short wavelength (high frequency) geoidal features (N_S), provides an efficient and accurate method. Because of its computational easy compared to using geographic blocks, the combined RINT approach has been adopted for the GPS coastal survey.

8. ANALYSIS OF THE NSW TIDE GAUGE DATUMS

8.1 ANALYSIS PROCEDURE

The combined solution as outlined in 7.2.6, where the geoidal height at a point is defined as:

$$N = N_S + N_L$$

has been adopted for the coastal survey using the "ring integration technique" to solve the short wavelength contribution (N_S) and high degree geopotential model to calculate the medium and long wavelength contribution (N_L). If a differential approach is adopted, KEARSLEY (1984) has shown that this method can determine ΔN to a similar precision as Δh is determined by GPS.

This **combined RINT solution** has been implemented by using the "GRAV" suite of programs developed at the School of Surveying at the University of New South Wales, and that are run on a VAX mainframe computer. These programs have been used successfully to calculate ΔN for GPS networks in New Mexico and Ohio, USA (KEARSLEY, 1985), Ontario and Manitoba, Canada (KEARSLEY, 1988b), and Western Australia and South Australia (HOLLOWAY, 1988).

Where points within the GPS network have existing orthometric heights derived from spirit levelling, then $\Delta N_{GPS-levelling}$ values can be calculated by:

$$\Delta N_{GPS-levelling} = \Delta h_{GPS} - \Delta H_{levelling} \qquad (8.1)$$

Equation (8.1) can then be used as a control to compare the ΔN_{grav} values derived from the combined RINT method, such that:

$$\delta N = \Delta N_{GPS-levelling} - \Delta N_{grav} \tag{8.2}$$

This difference (δN) can be expressed as a fraction of the baseline length (s), in parts per million (ppm), such that:

$$p_i = \left| \frac{\delta N_{AB}}{s_{AB}} \right| \times 10^6 \tag{8.3}$$

and the mean and rms of the n baselines in the network are calculated to give an estimate of the overall agreement between the RINT solution and the control for different ψ values, by:

$$m = \sum_{i=1}^{n} \frac{p_i}{n} \tag{8.4}$$

$$rms = \sum_{i=1}^{n} \left(\frac{p_i^2}{n} \right)^{\frac{1}{2}} \tag{8.5}$$

KEARSLEY (1988b) and HOLLOWAY (1988) use all measured baseline combinations (as defined equation (6.14)) to give a indication of the accuracy of the technique within the network. It can be argued that in any analysis of the combined technique, only independent combinations of GPS baselines produced within a session (using equation (6.15)) should be used. More specifically, using all the possible baseline combinations measured in each session in the coastal survey with such a (north-south) linear network shape, will tend to reduce the effect of the geoidal height difference (δN) at a station on (m) in equation (8.4) by dividing by a larger i value. This analysis has therefore only used the independent vectors calculated between adjacent stations.

In this study there are various combinations that can be analysed using equation (8.1). More specifically, the differential GPS ellipsoid heights (Δh_{GPS}) can be calculated between coastal stations using the results of the different processing strategies (ie. multi-baseline, multi-station solutions) incorporated in the various post-processing software packages and network adjustments outlined in chapters 5 and 6. However unless otherwise stated, the combined PHASER-NOVAS combined GPS network solution outlined in section 6.5 is adopted to determine Δh_{GPS}. The existing orthometric heights differences can be calculated between adjacent stations using AHD values (ΔH_{AHD}). Therefore equation (8.2) can been modified to:

$$\delta N = \Delta N_{GPS-AHD} - \Delta N_{grav} \tag{8.6}$$

After this ΔN analysis, an optimum integration cap size (ψ_0), where m in equation (8.4) is a minimum, can be determined for each of the solutions generated from equation (8.6) and any large values in δN_{AB} can be analysed.

A GPS/RINT orthometric height system can then be defined using this ψ_0 and compared to the AHD at the tide gauges. The orthometric heights at the GPS coastal stations derived by the GPS/RINT method have been transferred via spirit levelling connections to the tide gauges. MSL estimates (1951-1969) at these gauges are then defined in the GPS/RINT and AHD height systems and compared. The GPS/RINT system will contain a systematic error compared to the AHD because it is biased by the errors in the GPS height (h) and geoidal height (N) determinations at the origin of the unconstrained network as defined in equation (7.1). Therefore to make the analyses easier, a datum shift is applied to allow the MSL defined by the GPS/RINT orthometric height system to coincide with AHD value of MSL at Fort Denison. At the same time a original spirit levelling height system has been propagated from this same point at Fort Denison.

8.2 THE COMBINED RINT SOLUTION

Using the "ring integration" technique (RINT) to solve for the short wavelength contribution (N_S) and a high degree geopotential model to calculate the medium and long wavelength contribution (N_L) to solve the combined solution given in equation (7.20) such that:

$$N_S = \frac{R}{4\pi\gamma_m} \int_{\psi=0}^{\psi_0} \int_{\alpha=0}^{2\pi} \Delta g' \, F(\psi) \, d\psi d\alpha$$

$$N_L = \frac{kM}{\gamma R} \sum_{n=2}^{n_{max}} \left(\frac{a}{R}\right)^2 \sum_{m=0}^{n} P_{nm}(\cos\theta) \, (C_{nm}\cos m\lambda + S_{nm}\sin m\lambda)$$

was first proposed by MATHER (1973a). If a differential approach is adopted, this method has been shown, in theory, to be capable of producing a relative error in ΔN_{AB} ($\sigma\,\Delta N_{AB}$) of 2-3 ppm of the line length (KEARSLEY 1985; 1986). Tests have confirmed this precision to be possible, particularly in areas were the geopotential model recovers the geoid signal well (4-6 ppm) (KEARSLEY, 1988b). This is providing certain conditions for the accuracy of the gravity field are met, which would result in the mean gravity anomaly for a 10km * 10km block to be known to 3 mGal (MACLEOD et al, 1988).

The "GRAV" series of programs used in this analysis to determine N_S and N_L are outlined in HOLLOWAY (1988, pp.80-82) and are summarised in Figure 8.1. Details of each program are found in SCHOOL OF SURVEYING (1988). Summarising the process,

GRAV00 - generates gravity anomalies (Δg_L) or geoid heights (N_L) using the geopotential model and equations (7.10) and (7.11) on a user specified grid covering the GPS network.

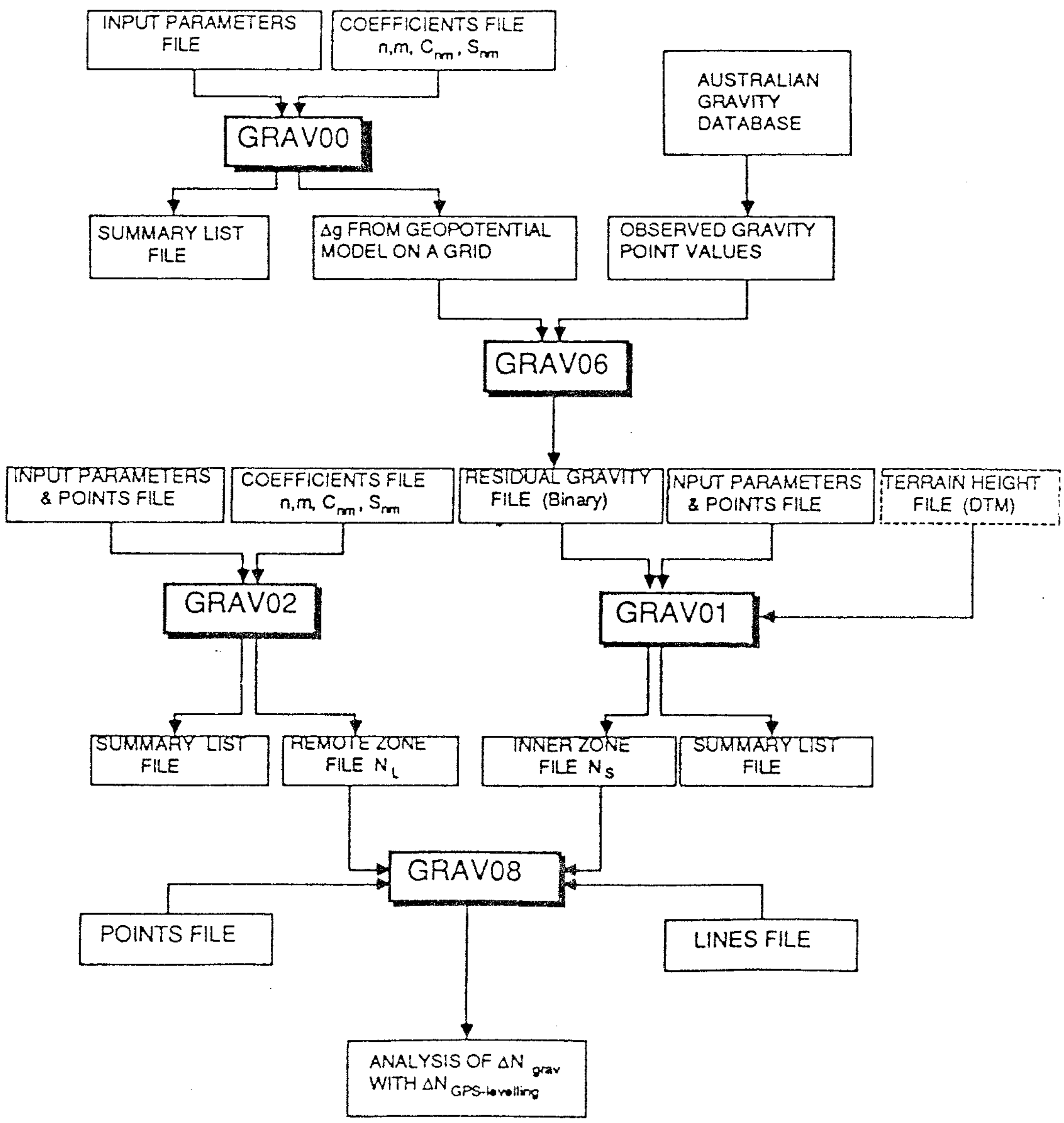
GRAV06 - extracts the gravity data from the gravity database for the GPS network area, converts it to the IGSN 71 system (see section 8.2.1) and reduces it to the geoid and the normal gravity is subtracted to give point gravity anomalies (Δg). A point gravity anomaly from the geopotential model (Δg_L) is interpolated from the grid and the residual gravity anomalies ($\Delta g'$) for all points using equation (7.23) is then calculated.

GRAV01 - generates the rings and performs the ring integration with the residual gravity anomaly field output from GRAV06 using Stokes formula with modified Stokes function in equation (7.21). The number of rings can be varied and the accumulated contribution to N_S at each ring is output at each GPS point.

GRAV02 - calculates the remote zone contribution (N_L) at each point using the same geopotential model as GRAV00.

GRAV08 - computes δN between for the combined RINT method (GRAV01 + GRAV02) and all points in the network with $\Delta N_{GPS-levelling}$ using equation (8.2), and generates statistics such as those in equations (8.4) and (8.5).

Figure 8.1 GRAV series of programs



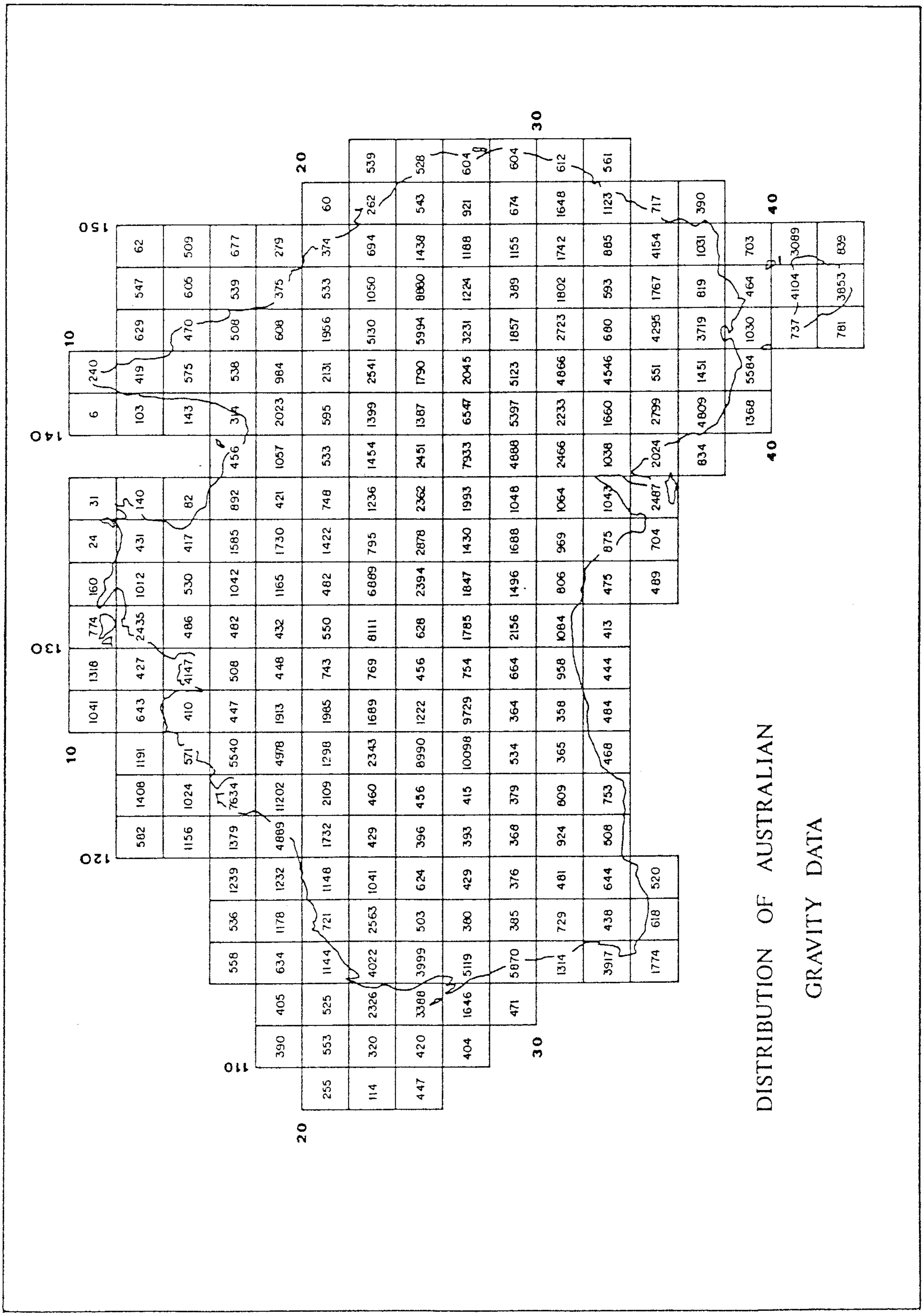
8.2.1 Gravity Database

The gravity data used in the gravimetric component (N_S) of this combined technique is an edited version of the Australian Gravity Database administered by the Australian Bureau of Mineral Resources (BMR). The Australian Gravity Database contains gravity stations, observed both onshore and offshore, by the BMR, their contractors, State Mines Departments and private companies (GILLILAND, 1987). This point gravity database has been edited at the School of Surveying, University of NSW (UNSW), to remove duplicated and obviously erroneous data, such as stations with ground heights higher than Mount Kosciusko. The UNSW version contains 443,754 gravity points in 7 files ordered in approximately 5° latitude bands across Australia (HOLLOWAY, 1988, p.66). The gravity data has been converted from the Isogal 65 gravity datum (based on the Potsdam system) to the International Union of Geodesy and Geophysics (IUGG) gravity reference system known as the International Gravity Standardisation Net, 1971 (IGSN 71). Figure 8.2 shows the number of gravity data points in the UNSW version, defined in 2° x 2° blocks over the continent and near offshore areas.

The density of the gravity stations varies, with an average spacing between stations of 11 km except in South Australia and Tasmania where the average spacing is 7 km (MATHER et al, 1976). However due primarily to a lack of gravity data off the south east coast of Australia, particularly south of Nelson Bay, it can be expected that the precision of ΔN calculated by the combined RINT method will be degraded within the GPS coastal network.

The gravity data in the national database is estimated to have a standard error of +0.2 to +0.5 mGal, relative to the network as a whole, while the estimated precision of marine surveys is given as +2.6 mGal (HOLLOWAY, 1988, p.67). The $\sigma\Delta g$ will however, be much higher due to the

Figure 8.2 Distribution of the Australian gravity data



uncertainties in the heights measured at the gravity stations, as these heights are used to determine the free air correction which reduces the gravity to the geoid. These estimated accuracies still fulfil the requirements for the computation of geoid height differences by the combined RINT technique to an accuracy of 5 cm over 100 km, as outlined in KEARSLEY (1986, p.9200).

8.2.2 What Geopotential Model to Use?

The ability of high order geopotential models to define the geoid varies according to the model used, its maximum degree and order and the frequency of the perturbations of the geoid in the region where the computation is made. While it should be expected that some areas will be better modelled than others, tests have shown significant differences in the ability of geopotential models to recover ΔN. In fact the best fit of the geoid in an area may not necessarily occur when the model is taken to its maximum degree and order. For example, KEARSLEY (1988, p.6561) found that OSU81 recovered ΔN to an average of better than 4 ppm of baseline length in GPS test networks in Ontario, Canada, and South Australia when summed to 180°. However the same model gave the best fit at n_{max}=90° in Manitoba, Canada, while in Western Australia, n_{max}=60° gave the best results. Therefore the accuracy to which the model can recover ΔN is primarily a function of the survey area's location. This is particularly important since tests have shown that the mean fit of the geopotential model to the local region places a limit on the precision obtainable from the full combined solution, since the contribution of the local gravity field used in determining N_S is reduced to the geopotential model (KEARSLEY & HOLLOWAY, 1988).

Looking more closely at the NSW coast, an attempt was made to assess the accuracy of the OSU81 and OSU86E higher order geopotential models to determine orthometric heights for the coastal GPS survey. In the Australian region, investigations by HOLLOWAY (1988) and KEARSLEY & HOLLOWAY (1988) used computed gravity anomalies from geopotential models (using equation (7.11)), and compared them to the terrestrial gravity anomalies. The comparisons were made using all the gravity points in the UNSW database and group in the same $2^\circ \times 2^\circ$ blocks as Figure 8.2.

HOLLOWAY (1988, p.69) describes the procedure. A mesh of gravity anomalies Δg_L from each geopotential model was calculated, based on equation (7.11) and using the computer program by RIZOS (1979) and the coefficients were taken to the model's n_{max} and calculated on a $0.1^\circ \times 0.1^\circ$ grid. The terrestrial gravity points were reduced to the geoid by applying the free air correction and then the free air anomaly Δg was computed using equation (7.22). The model gravity anomaly (Δg_L) was then interpolated from the gravity anomaly mesh (developed by equation (7.11)) for the same position as the gravity point. The residual gravity anomaly ($\Delta g'$) was then found by subtracting the two using equation (7.23). The procedure was repeated for all gravity points in each $2^\circ \times 2^\circ$ block and the mean residual gravity anomaly $\Delta \bar{g}'$ was calculated. These results can be related to differences in geoidal height determinations using equation (8.1), expressed as of fraction of baseline length in parts per million. They have been placed in four categories and shown in Table 8.1.

Table 8.1 Geopotential model and terrestrial gravity anomaly agreement

Category	$m_{\Delta\bar{g}}$	Equivalent $m_{\Delta N}$
1	0 - 5 mGal	0 - 5 ppm
2	5 - 13 mGal	5 - 10 ppm
3	13 - 21 mGal	10 - 15 ppm
4	> 21 mGal	> 15 ppm

These geometric results for OSU81 and OSU86E are shown in Figure 8.3 and 8.4. The improvement of the OSU86E model over OSU81 can be seen quite clearly in areas such as south eastern Western Australia, central Australia and the coastal areas of south eastern Australia. This is because the OSU86E geopotential model has a higher frequency signal compared to OSU81 and can pick up shorter wavelength features in the geoid.

While this analysis tends to show that OSU86E is the better of the two models in the survey region, Figure 8.4 still indicates that OSU86E is generally in poorer agreement with the geoid in the project area when compared to the Australian Region as a whole. This may be in part due to localised high frequency perturbations in the gravity field that could not be picked up by the geopotential model. FORSBERG (1989) noted that satellite altimetry within the area had shown that the gravity field was complex, in that the land-ocean boundary was complicated by the narrow continental shelf and the mountain range that is adjacent to the east coast.

8.2.3 The Ring Integration Technique

The Stokes' integral (using the modified Stokes' function $F(\psi)$ in equation (7.19)) is evaluated from the computation point by subdividing the surface into compartments delineated by concentric rings with $0 < \psi_0 < \psi_{max}$, incremented in 0.1° steps and radial lines with a constant change of azimuth ($d\alpha$) of 10° (KEARSLEY 1985, p.83). The RINT method offers flexibility by way of analysing all ring combinations from 0 (where only the geopotential model is being tested) to ψ_{max}, to determine the smallest mean differences (m) and hence the optimum cap size. The gravity data inside the rings should contain a sufficient number of gravity points with an even distribution so that the mean

Figure 8.3 Gravity anomalies OSU81 minus observed

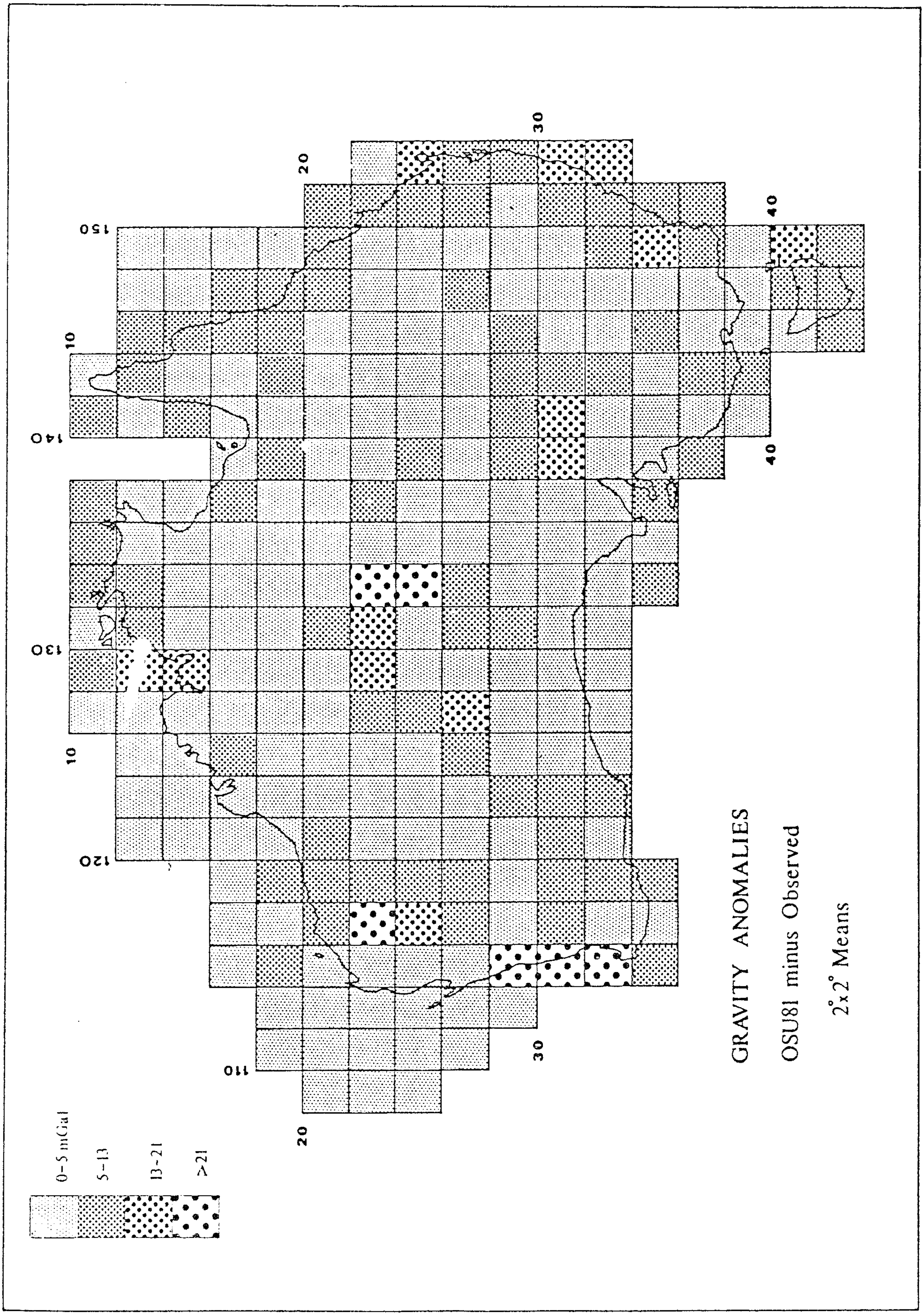
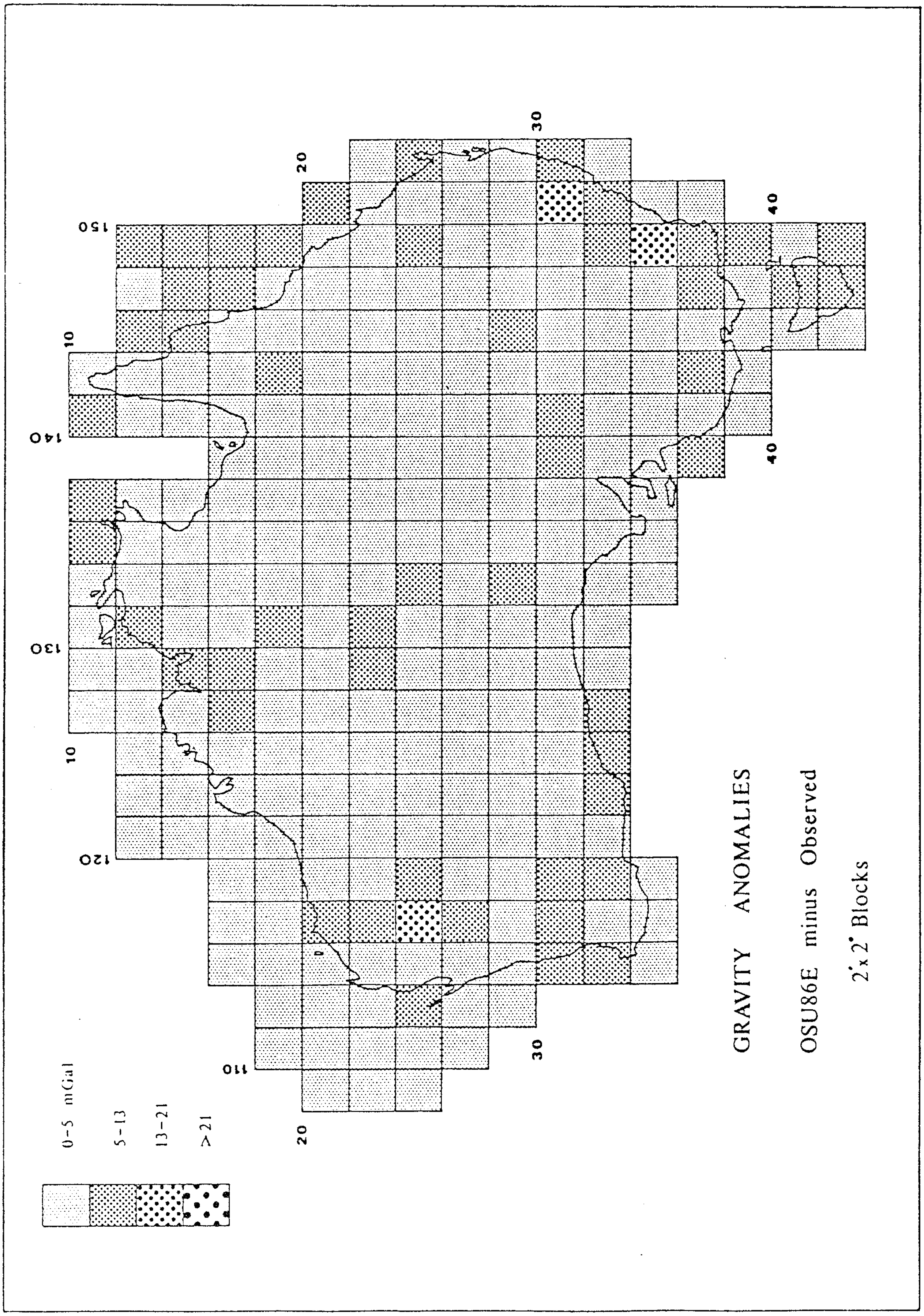


Figure 8.4 Gravity amonalies OSU86E minus observed



gravity anomalies calculated for each compartment are truly representative of the gravity field in the compartment.

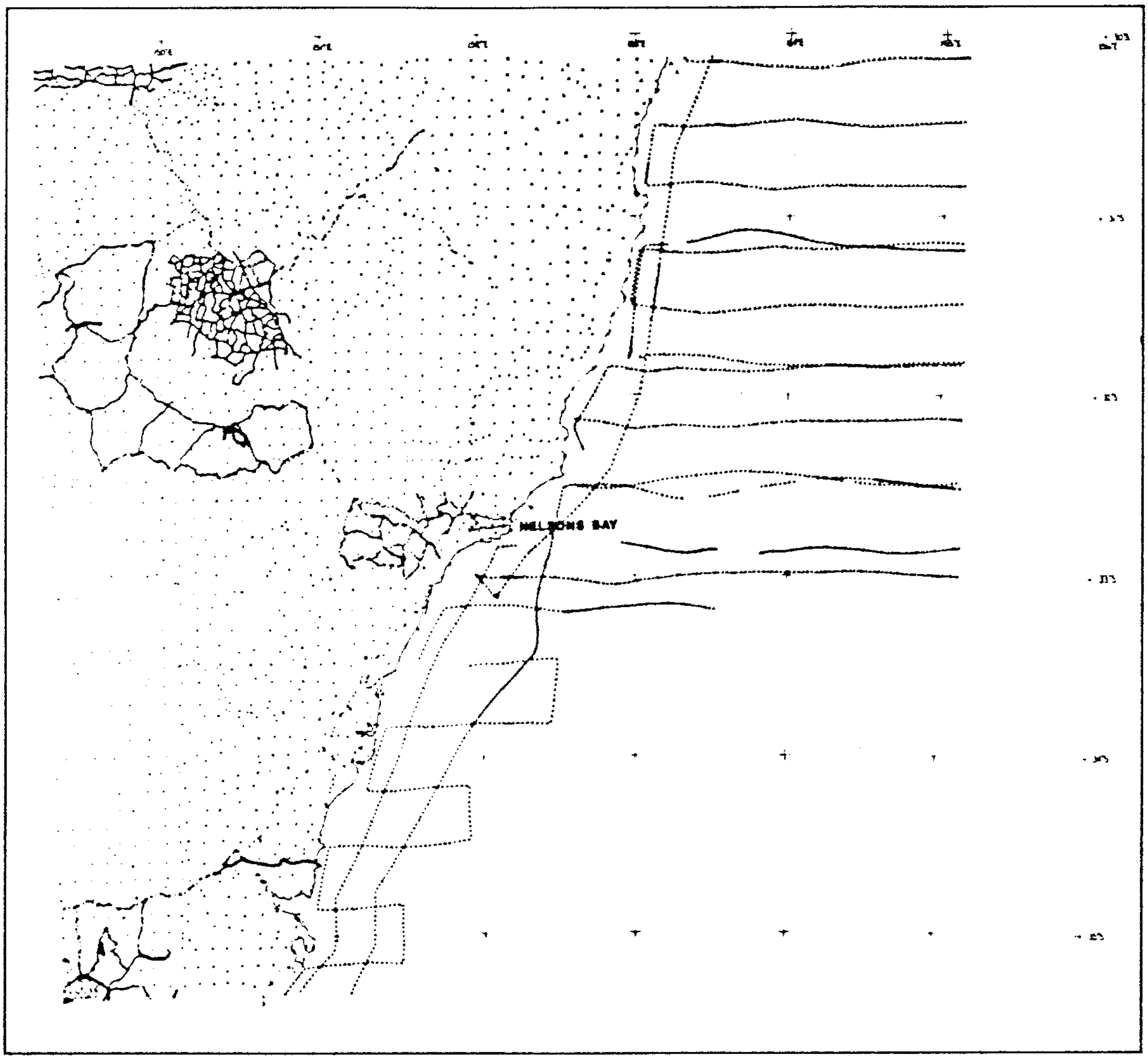
Figure 8.5 shows the point gravity data (marked as crosses), that were used to calculated the inner cap at Nelson Bay. It can be seen that the gravity point coverage in this region is well distributed on the land mass, with gravity traverses and concentrations of gravity data in the north west, probably carried out for coal mining activities. The ship's tracks show the offshore gravity data, however there are few, if any, tracks south of Nelson Bay. This lack of offshore gravity data exists for all stations south of this point. Where no gravity data existed in the compartment, the geopotential model's estimate was used (ie. $\Delta g' = 0$) to allow the RINT algorithm to continue, particularly at larger ring numbers. The maximum number of rings was taken to 16 ($\psi_{max} \approx 1.5°$) as the contribution to N decreases as the rings increase and little or no point gravity data exists in the eastern half of the area after this cap size.

8.3 RINT VERSUS CONTROL

The results of this analysis are shown in Figure 8.6 and have been calculated using both OSU81 and OSU86E geopotential models. The x axis shows the number of rings included in the Stokes' integration of the inner component, while the y axis shows the mean error calculated by equation (8.4), for all adjacent lines in the network.

Both figures show that at ring 0, where only the geopotential models are used, the mean error is quite large while as expected OSU86E provides the better solutions. The inner cap integration of the gravity field data improves all solutions when compared to the control, with the optimum cap size resulting in agreement at the 2 to 4 ppm level.

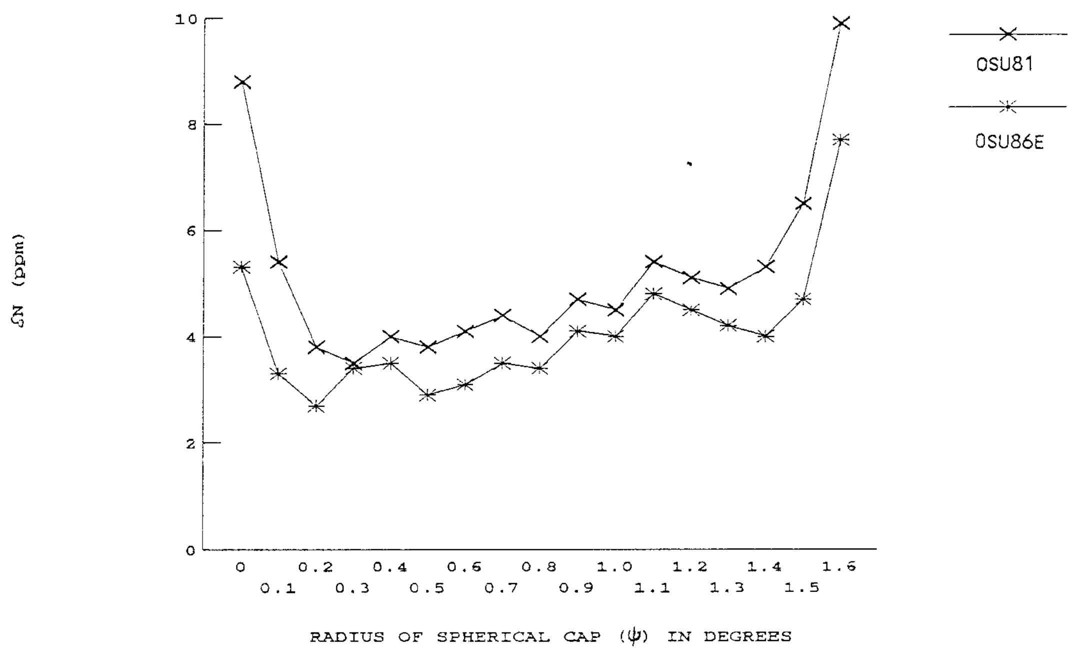
Figure 8.5 Point gravity data around Nelson Bay, NSW



As expected, the optimum cap size is smaller when OSU86E is used because of its larger n_{max}. Furthermore, the bimodal characteristics exhibited by the W shape curves in the investigations by KEARSLEY (1988b) and HOLLOWAY (1988) are, while not so pronounced, still present in the NSW coastal solutions.

This W shape of the plot is primarily a result of small cyclic errors in the geopotential model used to determine $\Delta g'$ in the gravimetric solution. KEARSLEY (1988b) found that a decreasing wavelength and amplitude in this W curve occurred with an increase in n_{max} of the geopotential model.

Figure 8.6 δN comparisons for RINT using both OSU81 and OSU86E (all stations) - NSW coastal network



Furthermore the wavelength of the curve approximated the half wavelength recovered (in theory) by the geopotential model when summed to n_{max} (ie. half wavelength = π/n_{max}). Additionally, as n_{max} increased, the W curve became flatter and the selection of an optimum ψ_0 became less critical. The lack of a pronounced W curve in the coastal network is probably due to a degradation in the solution caused by the lack of gravity data out to sea as ψ_0 increases. In fact it can be seen that as ψ_0 increases from 0.5°, the mean error steadily increases for both models.

To further highlight the problems of the lack of gravity data offshore, the network was divided into two areas, being the areas north and south of Crescent Head. Figures 8.7 and 8.8 show these sub-networks. If we look at the density of the gravity points along the coast, there is little or no offshore gravity data, south of Nelson Bay (see Figure 8.5).

North of Nelson Bay there have been substantial offshore gravity surveys, particularly nearer the Queensland boarder.

Therefore it would be assumed that the northern network would produce a more accurate solution than the southern network as it can be expected that Stokes' integral will break down due to this lack of gravity in the eastern half of the southern network. This was in fact the case, with OSU86E again providing better solutions. At ring 0, the OSU86E model agrees well in both the north and south sub-networks, while using OSU81 in the south shows poor agreement. It should be noted that by using the OSU86E geopotential model with the increased gravity offshore, the northern network exhibits the W curve. Conversely the southern network reflects the lack of gravity off the coast, with the solution "fading away" from $\psi_0 = 0.5°$ (at about 50 kilometres). However at this ψ_0 and using OSU86E, the combined solution still returns a mean error of about 4 ppm. Interestingly, using OSU86E in both sub-networks means that the choice of ψ_0 is not critical, since the curve is rather flat after $\psi_0 = 0.2°$. The lack of a pronounced W curve in the southern network may also be due to errors in the established orthometric heights used as control in this analysis. This was found in tests carried out in South Australia by HOLLOWAY (1988).

Large values in δN are highlighted in Appendix F, and are analysed further in section 8.4. Briefly, with respect to equation (8.4), the optimum cap sizes using the OSU86E geopotential model are between of $\psi_0 = 0.2°$ and $0.5°$, and return RMS values at about the 15 cm level. Significant δN values above this RMS level occur around Nelson Bay, where SST may effect the AHD adjustment in the area, and Cann River, Ballina and Stanwell Park where δN is over 20 cm.

Figure 8.7 δN comparisons north of Crescent Head

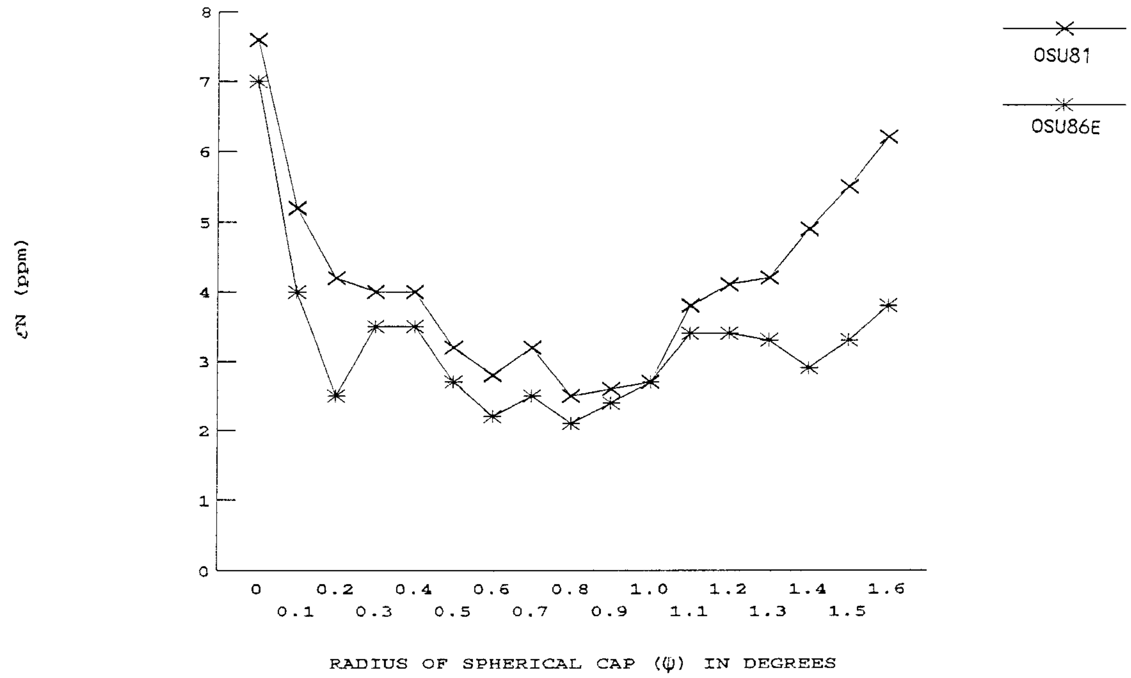
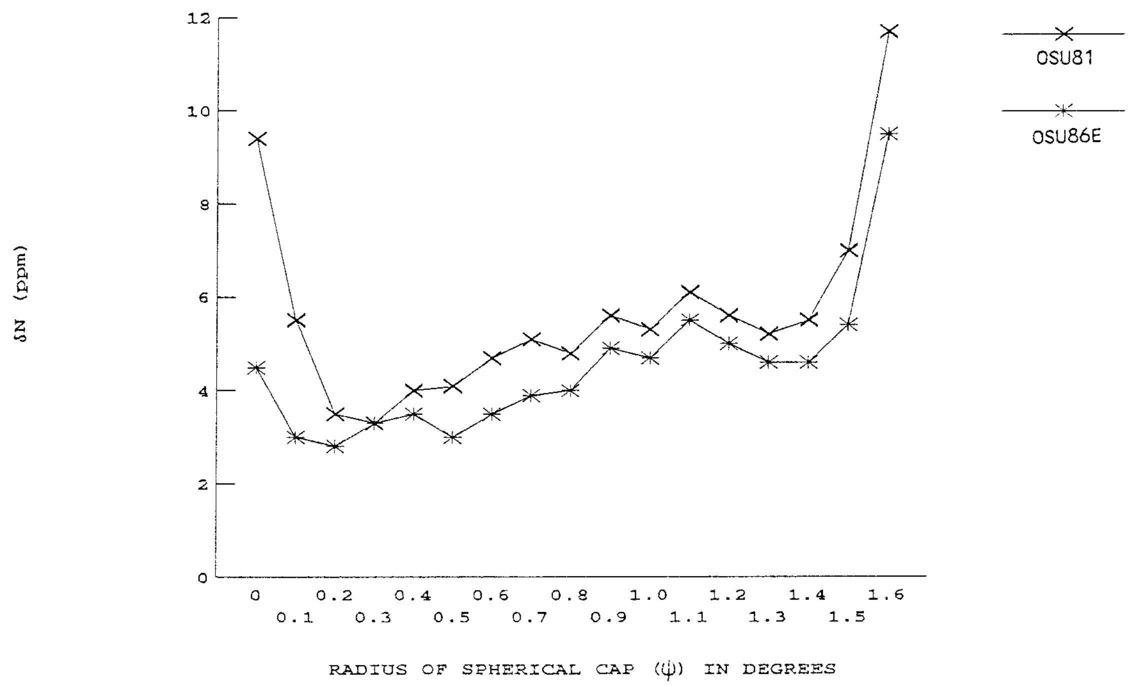


Figure 8.8 δN comparisons south of Crescent Head



In summary, the results from this network span distances between 10 and 100 kilometres and are similar to those achieved by KEARSLEY (1986,1988) and HOLLOWAY (1988) with accuracies of 2-3 ppm using OSU86E as compared again to GPS-AHD. KEARSLEY (1986) noted that the size of the formal errors in the inner zone contribution resulting from random errors in Δg increases with increased line lengths and with increased cap size. However further research (KEARSLEY, 1988b) found the optimal cap size for various line lengths difficult to calculate and was location dependent. While it is believed that the inner solution stabilised at 2-3 ppm with $0.7^\circ < \psi_0 < 1.7^\circ$ regardless of line length, results from the NSW coastal study show this is only true when a high degree and order geopotential model is taken to a large n_{max} and sufficient gravity is even spaced through out the GPS network. The sufficient gravity criteria was not fulfilled along most of the coastal network and therefore the solutions have failed to stabilise at larger cap sizes.

8.4 GPS/RINT VERSUS MSL

Appendix G shows the MSL related to the GPS/RINT orthometric height system using OSU86E with an optimal integration of the spherical cap of about 50 km as determined in section 8.3. This cap size was adopted because it was equivalent to the half wavelength recovered by the OSU86E geopotential model, and allowed sufficient gravity information to be incorporated in the inner cap integration. The coastal GPS/RINT heights determined for the GPS network stations have been transferred to the tide gauges using short spirit levelling connections carried out to third order levelling standards. It is evident that a bias and positive south to north systematic trend exists.

The most substantial cause of the bias is the error in the absolute position of the origin of the local network of both the WGS84 satellite ellipsoid heights and geopotential model used. In the case of the coastal network, the minimally constrained adjustment had the origin of the WGS84 ellipsoid coordinates determined from the SLR measurements at Orroral and transferred to Tidbinbilla, and has an expected absolute accuracy in WGS84 height datum of about 5 metres (LIC, 1989). The estimated error in the WGS84 ellipsoid height at Watson's Bay station (Sydney) can therefore be estimated at this level, as the network adjustment carried out using NEWGAN in Chapter 6 gives a estimated standard error in height from Tidbinbilla to Watson's Bay of only 0.05 metres (1 sigma). The estimated accuracy of the N values generated from the OSU86E geopotential model at any point when taken to their maximum degree and order is 0.5 metres and the transfer of height to Fort Denison could be estimated with an accuracy of less than a centimetre. The total error estimates fit the bias experienced in the coastal network. This GPS/RINT orthometric height network can therefore be called a "relative height system" as it has been developed using the differential approach in calculating both h and N, and cannot eliminate errors in the datum definition of the origin (see section 2.1.1). A method to remove this bias in the datum definition has been employed here for comparison purposes. Assuming the 19 year epoch of MSL at the Fort Denison tide gauge represents the geoid, a datum shift of 0.99 has been applied to the GPS/RINT height system to make this MSL height equivalent to, and coincident with the AHD at the gauge (as AHD RL=0 and MSL (1951-1969) are within 8 mm, at this gauge) (see section 2.5 and Appendix G).

Figure 8.9 shows MSL as defined by GPS/RINT derived orthometric height system and with the bias removed. The GPS (WGS84) ellipsoid heights have been derived from the combined Trimble (BATCH-PHASER) and TI4100 (NOVAS) network, and a linear regression curve has been fitted to the results. The results show the GPS/RINT approach produces orthometric heights of MSL that have a positive south to north systematic

Figure 8.9 MSL (1951-1969) defined by combined GPS/RINT orthometric height system

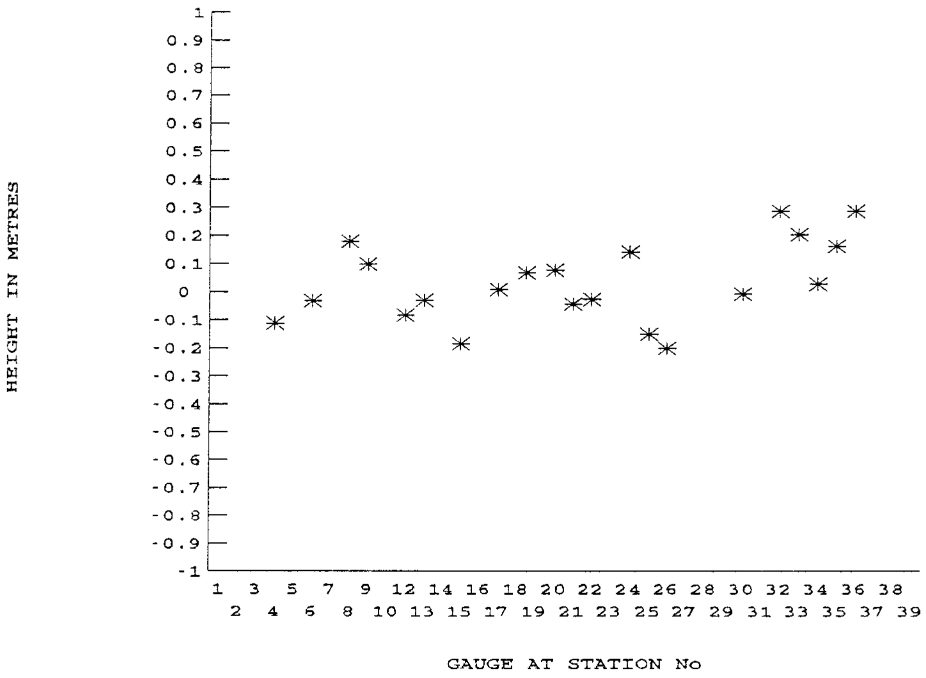
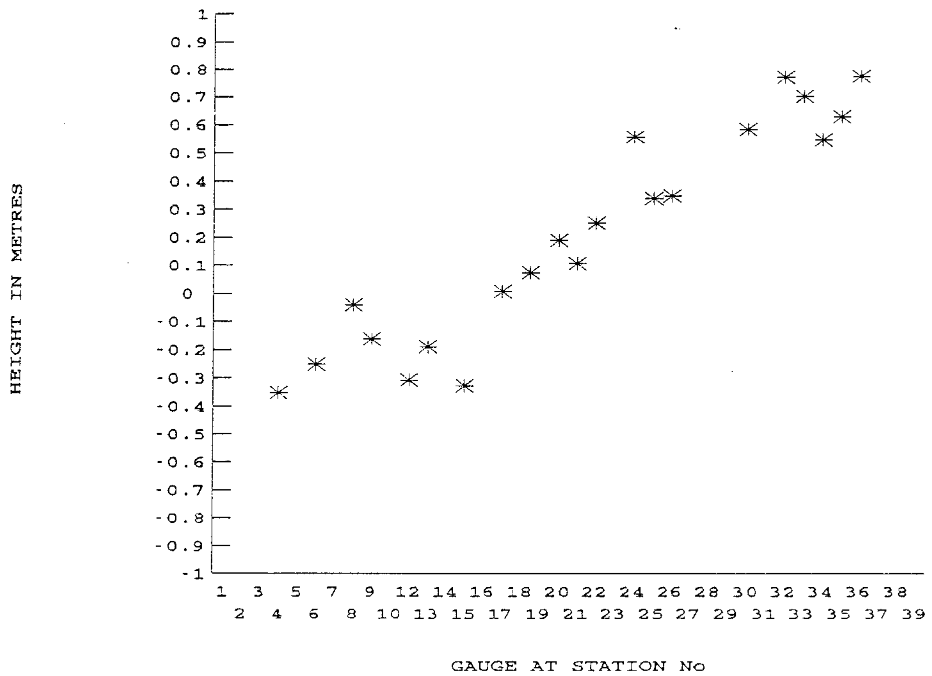


Figure 8.10 MSL (1951-1969) defined by TRIMVEC™ GPS/RINT orthometric height system



trend with a magnitude of 0.1 m/1400 km. While being consistent with oceanographic predictions on Australia's east coast (LENNON, 1988), this may also be a product of unmodelled systematic errors in the GPS reductions, particularly in relation to the satellite ephemerides. Interestingly, if only the Trimble network (Figure 8.10) (using TRIMVEC™ and broadcast ephemeris) is used, the resultant north-south slope is much larger at 1.1 m/1400 km, yet the ΔH values still agree with the terrestrial levelling data to third order levelling specifications.

Reasons for this larger systematic trend in the GPS/RINT solutions would seem to point to a bias in the broadcast ephemeris that transfers into the GPS ellipsoid height determination. This systematic trend may be due to the Trimble ephemeris being only logged at the start of the session, causing errors to propagate as the observation session becomes longer. However investigations show that the same magnitude systematic trend occurs if only the NOVAS TI4100 network solutions. These are derived from supposedly more accurate dual frequency observations, and the broadcast ephemeris is updated each hour and a polynomial fitted through it (WANLESS & LACHAPELLE, 1987). Furthermore, from Table 5.7, the difference between broadcast ephemeris and precise ephemeris, while systematic, was only 0.3 ppm. Removing this from the comparisons between the solutions, still leaves about 0.6 metres unaccounted.

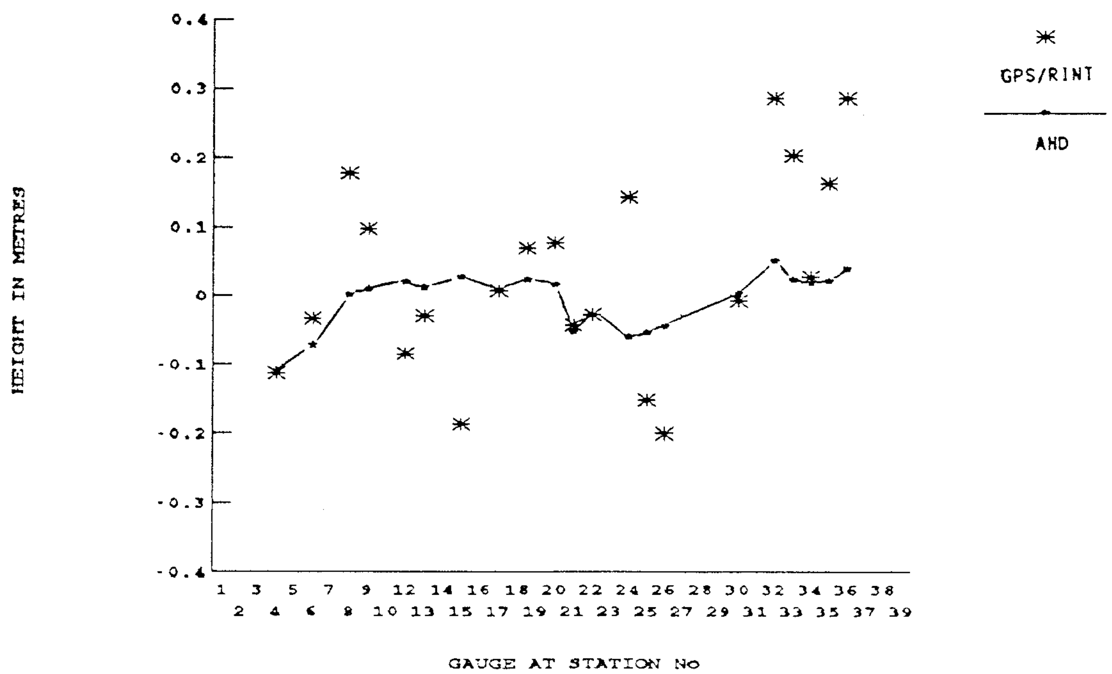
Therefore it is more likely that this south-north trend also includes a systematic error introduced by the use of incomplete stochastic session modelling used in the TRIMVEC™ and NOVAS baseline solutions. Table 5.8 showed an average 1.2 ppm difference in ellipsoid heights when comparing baseline and multi-station solutions for session 146A.

Other reasons for the overall systematic trend could be (KEARSLEY, 1989; FORSBERG, 1989a):

1) The non-coincidence of the satellite reference ellipsoid (WGS84) and the gravity (geopotential model) reference ellipsoid (GRS80), even though the same parameters are given for both as a = 6378137.0 f = 1/298.257223563. These "errors" in the geopotential solution can be considered as a type of "datum error" (SCHWARZ and SIDERIS, 1989).

2) The systematic error in the satellite altimetry data used in the determination of the geopotential model. Geoid models of the east coast of Australia show large perturbations at the coast, indicating either significant geoid undulation or a systematic error in the satellite altimetry used to map the geoid offshore.

Figure 8.11 MSL (1951-1969) by AHD and GPS/RINT



The orthometric height system developed by the combined (PHASER & NOVAS) GPS network using the combined RINT (with $\psi_0 = 0.5^\circ$ and OSU86E) was compared to the calculated AHD heights of (1951-1969) MSL at each tide gauge. Figure 8.11 shows some correlation between the two sets of data. Looking at the AHD results, the variations in MSL heights at certain gauges are of the same magnitude as the tidal gradients predicted by HARVEY (1988) in Table 2.1. While larger in magnitude, the GPS/RINT values generally follow the same trend between tide gauges.

These results indicates that the terrestrial levelling and AHD are in fact generally reliable, since the AHD values fit within the variations of the independent GPS/RINT results. More detailed results can be found in Appendix G, but it seems that the GPS/RINT results are not as accurate as the existing levelling along the NSW coast. The project however, has shown that the GPS/RINT approach can be used to provide an independent analysis of orthometric heights to third order standards. Finally, using the unadjusted levelling data in place of the AHD values makes little difference to the results shown in Figure 8.11, since the AHD and original levelling agree within a few centimetres of each other at most gauges.

9. CONCLUSIONS

9.1 REVIEW

The Public Works Department of NSW through its tidal analysis and numerical modelling, had encountered a number of height anomalies along the NSW coast that were manifested in the relationships of various river datums to Mean Sea Level and the AHD at specific tide gauges. An independent method was sought to accurately relate these various tide gauges together, to verify the integrity of the spirit levelling network that makes up the AHD. Through its Survey and Property Branch, the Department successfully carried out a GPS survey to connect various tide gauges along the NSW coastline using Trimble 4000SX receivers in May 1987. In September 1987, the Department assisted the then Central Mapping Authority of NSW in connecting various GPS campaigns together using dual frequency TI4100 observations, that in turn strengthen the single frequency Trimble coastal network. Various processing strategies were explored to firstly derive accurate geocentric ellipsoid heights, then convert them to orthometric heights and produce a relative orthometric height system that could be compared to the AHD and mean sea level estimated at the tide gauges. A number of points were raised while carrying out this study.

With respect to Mean Sea Level along the NSW coastline and its relationship to spirit levelling and the geoid:

* The sea surface deviates both spatially and temporally from a level surface. This is because the oceans are not homogeneous or static and cannot satisfy the hydrostatic equation that relates the surface of the sea to a level surface of the earth's gravitational field. The deviation between the sea surface, whether instantaneous or averaged, and the geoid can be described as the sea surface topography

or SST at a specific tide gauge. The mean sea level created by averaging the sea surface over a long period such as the 19 year epoch (1951-1969) adopted in this study at each NSW tide gauge, will still have small spatial deviations at the sub metre level.

* SST is caused by a number of local and regional physical effects, such as the tide gauge location, ocean density and currents, atmospherics and secular variations. Its magnitude will vary with location and this make its total removal through techniques such as modelling, difficult to achieve. Along the NSW coast (approx. 1400 km) the SST will exhibit a large amount of regional correlation, however local variations of a few decimetres could be expected.

* Analysing MSL at these tide gauges by connecting the gauges together into one height system is dependent on the accuracy of the definition of the vertical datum and the method by which the height system is developed. For example, both the AHD and steric levelling have their datums based on assumptions about the earth's gravity field. The AHD is a relative height system developed by spirit levelling from specific datum points defined by MSL, while steric levelling is an absolute height system based on the hydrostatic equation that relates height to the pressure and density within the oceans. Both height systems have different error sources and will therefore be expected to produce different trends when related to MSL. Interestingly, along the NSW coastline, both methods do indicate an upward MSL slope toward the equator.

AHD along the NSW coastline is essentially made up of first order spirit levelling with an estimated accuracy of $4\sqrt{k}$ mm. However, it may still contain unknown systematic errors in the levelling measurements and distortions due to the nature of the assumptions made in the constrained adjustment that held 30 tide gauge stations fixed at their

1966-1968 MSL epochs. The relationship of the 1951-1969 MSL epoch to the AHD at the NSW tide gauges indicates a small upward slope of MSL toward the equator in the order of 2cm/1000km, with local SST variations of up to 0.1 metres. Steric levelling also indicates an upward slope toward the equator of the order of a few centimetres over the NSW coastline, however steric results are degraded toward the coast and its accuracy can only be expected to be 10-15 cm.

With respect to the planning and logistics of such a large scale GPS survey as that undertaken in this study:

* The success of the GPS fieldwork depends on the thorough planning of all aspects of these surveys, particularly on such a large scale as the coastal survey. While field operations are becoming easier as the receivers become more automated, logistics planning is still an important component of the survey, particularly when maximum use of the presently limited satellite "window" is required.

* Experience is invaluable and should be sought before stepping out into the unknown. Large organisations should work together to supply resources and impart knowledge to the benefit of all. Joint projects that may include public, academic and private sectors, are the way to carry out these large surveys until the capital costs of GPS decrease (WARHURST, 1988). The PWD's previous experiences with this technology helped in the coastal project, as did the advice give by such organisations as the School of Surveying, at the University of NSW. This joint approach has been followed by the LIC, who have sort to train personnel from various government departments in exchange for manpower and cost sharing when carrying out various GPS control surveys.

With respect to the GPS carrier phase processing in obtaining accurate ellipsoid heights, using in particular,

the TRIMVEC™ and NOVAS baseline processing software and BATCH-PHASER multi-station processing software:

* Current commercial GPS differential carrier phase processing is divided into session and network adjustments. The session mathematical models fall into two distinct categories. The baseline approach calculates a vector between two stations in isolation and ignores stochastic correlation between baselines within a session. This leads to incorrect a posteriori stochastic information and sub-optimal results. Multi-station session adjustments maintain fully stochastic modelling and are recommended if the highest accuracies are required.

* While differential techniques eliminate or substantially reduce many error sources in the GPS carrier measurements, the accuracy of the results will be affected primarily by the residual tropospheric and ionospheric delay, and errors in the a priori fix station coordinates and satellite orbits. These will in turn affect the resultant ellipsoid heights, generally increasing the errors in proportion to the inter-station distance.

* The error due to the delay in signal caused by the troposphere can be minimised by keeping inter-station distances to a minimum, and using a tropospheric model to pre-process the carrier phase observations, when direct measurements of the path delay with a WVR are not available.

* The error caused by the ionospheric delay is generally eliminated by dual frequency observations. If dual frequency observations are not available, the residual delay in the differential processing can be minimised by observing at night over short inter-station distances.

* In this study, the long observation sessions exhibited difficulties in resolving the cycle ambiguities to integers.

While in the single frequency network this can be explained by the ionospheric residual errors caused by the large inter-station distances, the fluctuations in the tropospheric conditions throughout the session must also be considered. The observations were taken for 3.5 hours over sunrise and the standard tropospheric modelling used in the processing can only take a mean of these widely varying conditions. With respect to the dual frequency network, ambiguity resolution was again difficult due to the residual tropospheric effects over the long lines. However simulations by GRANT (1989) and HOLLOWAY (1988) have indicated that the resolution of ambiguities to integers does not substantially increase the accuracy of GPS ellipsoid heights.

* The a priori base station coordinates that need to be constrained to eliminate the datum defect in the normal matrix of the mathematical model should be as accurate as possible. "Seeding" or carrying through the accurate base station coordinates from session to session improves the consistency of the carrier phase processing.

* In this limited study, the increased accuracy of the precise ephemeris (supplied by the DMA), over the broadcast ephemeris, seems to be produce marginal improvements in the differential ellipsoid height of around 0.3 ppm. However these differences exhibit a systematic south to north trend. This indicates that the precise ephemeris increases the accuracy of the radial component of the predominantly north-south orbits in Australia. This is mainly due to the inclusion of a Australian tracking station in the precise ephemeris computations.

* Differencing and undifferenced mathematical models will produce the same results only if the stochastic correlations are treated rigorously and the receiver clock parameter modelling is equivalent. If the stochastic correlations are ignored, the results will be sub-optimal,

exhibiting variations of over 1 ppm in the differential ellipsoid heights as compared to rigorous solutions.

* Repeat baseline comparisons provide an estimate of accuracy and while being software specific, in this study indicate that single frequency line repeatability at about the 3 ppm level, while dual frequency results reflect the elimination of the ionospheric delay with an average of 1.2 ppm.

* Difficulty was experienced in combining the sessions solutions together into a network adjustment. Due to the existence of systematic errors in the GPS observations, the assumptions in the Gauss Markov Model do not hold, so scaling the network by its a posteriori variance factor will not be valid. Rescaling of the stochastic model of the network observations (combined session results) is required to reflect both the internal and external errors. One method is based on the inter-station distance where the population variance parameter can be defined in terms of $a^2 + b^2$ ppm. These parameters depend on the observations, the accuracy of the ephemerides used, the base station coordinates and post-processing software.

* Combining separate GPS networks together produces further difficulties. They must have the same datum definition, and this presents problems when these networks have been calculated using different instruments, observations, ephemeris and post-processing algorithms. In this study, the two network solutions have had their geodetic datums separately defined. The first network was defined by precise ephemeris, Trimble single frequency carrier phase observations in a BATCH-PHASER multi-station solution, while the second network had its datum defined by dual frequency TI4100 carrier phase observations, using broadcast ephemeris and NOVAS multi-baseline processing. This hindered the effectiveness of September TI4100 campaign to increase the

accuracy of the coastal campaign. This is because the increased accuracy inherent in the dual frequency observations (that effectively eliminates the ionospheric delay), were offset by the use of broadcast ephemeris and a processing software that processed in a baseline by baseline mode, therefore ignoring the physical and stochastic correlation between baselines within a session. It is recommended that to improve the results and reduce the datum definition problem, the TI4100 observations should be processed using BATCH-PHASER with precise ephemeris.

The accuracy of geoid heights (N) used to convert GPS ellipsoid heights to orthometric heights is dependent on various factors:

* While various methods can be used to determine N, this study used the combined RINT method. This is because it was found to be the conceptually simple, easy to implement through the GRAV series of programs developed at the UNSW, and truly independent of existing spirit levelling being analysed.

* Both OSU81 and OSU86E geopotential models were used to their maximum degree and order in the project area to determine the long wavelength component ΔN_L. Both exhibiting agreement with $\Delta N_{GPS-AHD}$ to 7 ppm in the northern part of the project area, but OSU86E showing much better agreement in the south at 5 ppm compared to 10 ppm for OSU81, reflecting its increased resolution of the gravity field at $n_{max}=360°$.

* The ring integration technique (RINT) was used to determine the short wavelength contribution ΔN_S of the combined solution. With only a small inner zone integration (<0.4°), it produces a significant improvement in the overall solution, even when the geopotential model was in poor agreement.

* The optimum cap size used for the inner zone integration is dependent on the geopotential model used and the amount of point gravity data available. This is shown by looking at the northern and southern sub-networks. While OSU86E produced better results in both networks, the northern network, where there is sufficient gravity offshore, showed an optimal cap size at $\psi_0 = 0.2^\circ$ and again at $\psi_0 = 0.8^\circ$ producing agreement at about 2 ppm. The southern network had an optimal cap size at $\psi_0 = 0.2^\circ$ producing agreement at 2.8 ppm. The accuracy of the solution in both networks diminished as the density of gravity data offshore decreased.

Combining the GPS ellipsoid height differences with geoidal height differences to create a GPS/RINT relative height system, produced some interesting results when related to the MSL determined at the tide gauges:

* Using different carrier phase modelling in the GPS ellipsoid component of the GPS/RINT determined orthometric heights, produces varying magnitudes of a positive south to north systematic trend in the MSL at the tide gauges. Those solutions that used broadcast ephemerides and baseline processing showed the largest south to north trend, while using the BATCH-PHASER multi-station solutions with precise ephemeris reduced this bias and agreed more closely with oceanographic and spirit levelling estimates of MSL along the NSW coastline. This indicates that precise ephemeris and rigorous stochastic modelling reduce systematic errors in GPS ellipsoid heights.

* The MSL surface defined by the GPS/RINT shows a large correlation to that MSL surface defined by the AHD. This would indicate that, with the exception of the area around Eden, the existing spirit levelling along the NSW coastline is of first order standard as stated in NATIONAL MAPPING (undated). In summary, this combined GPS/RINT solution would

seem to have produced heights that have accuracies that are better than those derived by third order spirit levelling. Furthermore, the variations of up to 0.1 metres between the MSL estimates and AHD values at each tide gauge are predominantly due to the variations of SST.

The MSL peaks at Ettalong and Patonga are due to the tidal gradient in Brisbane Water, while the 0.1 difference between AHD and MSL at Eden shows a problem in the AHD adjustment around the area. Investigations of the levelling around the Eden area by HARVEY (1988) show that there was an 0.1 correction to the levelling between Eden and Tathra in the 1971 AHD adjustment, without any obvious reason. There still exists a dip in MSL between Newcastle and Yamba and this is reflected in the GPS/RINT solution. This is probably due to the Sea Surface Topography (SST) causes by the start of the southern current. If this is the case, then SST would seem to have a small range over the NSW coast, since MSL and AHD determinations at all other gauges agree substantially with MSL in Sydney. The slightly higher values around Brunswick Heads and Tweed Heads are consistent with the estimates of tidal gradients in these rivers as outlined by Harvey (1988) and shown in Table 2.1. These relationships determined through this analysis now allow the Department's engineers to relate their river studies to AHD and provide accurate comparisons between rivers systems. To improve these results in the future, further recommendations are made.

9.2 RECOMMENDATIONS

9.2.1 Use of Super Tide Gauges to Remove SST from MSL

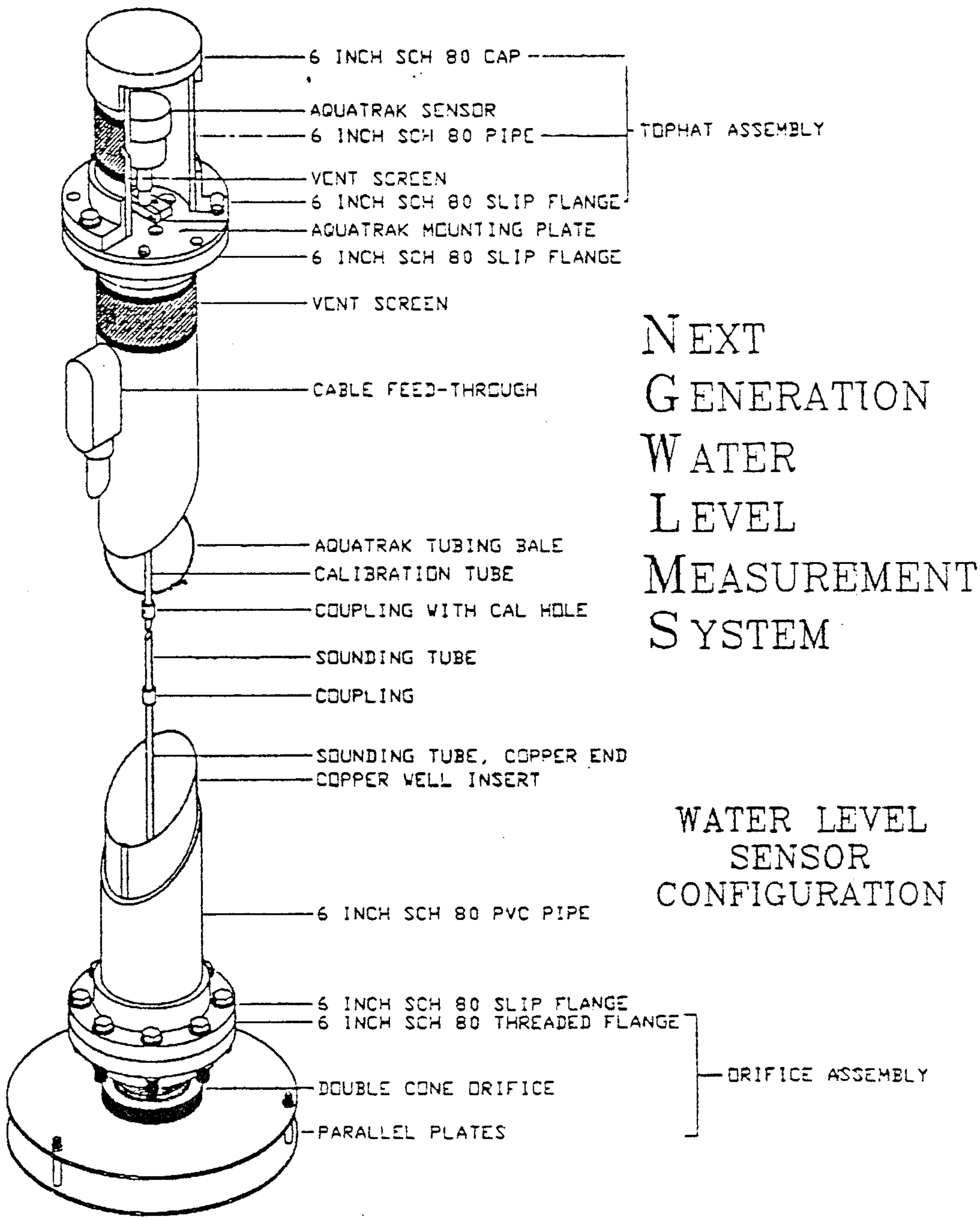
Recently there have been moves to place a limited number of super tide gauges around Australia, to in part help a world effort to monitor sea level effects caused by the global warming of the oceans. Closely related to the GLOSS network, these super gauges will have sophisticated equipment to monitor sea level, local winds, currents, salinity, temperature and atmospheric pressure. Using this accurate data, more sophisticated modelling techniques could be used to eliminate SST effects and a prototype gauge is shown in Figure 9.1. The gauge position will be monitored in relation to the WGS84 ellipsoid from very accurate coordinates at VLBI stations and connected by dual frequency GPS measurements (BOUCHER, 1986; MACLEOD et al, 1988). A pilot study has been progress since 1987 in the western Pacific ocean and the reader is referred to CARTER et al (1986 & 1987) for details. Importantly, these gauges could become fixed points to which the AHD could be readjusted. **It is recommended that the PWD invest in a number of these gauges, to improve the sea level measurements, and help determine the magnitude of SST at each gauge.**

9.2.2 Satellite Orbit Improvement

As stated in Chapter 5, one of the major error sources in many GPS solutions is that of the GPS satellite coordinates. Most commercially available GPS post-processing software packages treat the satellite coordinates as known, using the orbital parameters transmitted in the broadcast ("predicted") ephemeris, that are usually accurate to no better than 20 metres. The use of a precise ephemeris (such as from the DMA) produces some improvement in the results as seen by this study. However the remaining systematic trend in the MSL heights indicates some possible remaining bias in the ephemeral information in southern hemisphere, since the MSL trend is definitely latitude dependent and the GPS satellite orbits are in the same north-south direction as the network.

It is recommended that the possibility of using orbit relaxation techniques on the TI4100 network be examined. Any further GPS surveys should use the fiducial network concept to determine the satellite coordinates using the short or

Figure 9.1 Proposed super tide gauge



long arc methods (WELLS et al, 1986). Using these techniques, the VLBI stations at Parkes, Tidbinbilla and Fleurs could be used as fixed stations in these methods to recalculate the TI4100 network, improving the satellite positions to create baseline accuracies in the order of a few parts in 10^7 (KING, 1987). Accuracies of the order of 10^8 can be achieved but require the fiducial stations to be widely spaced, in combination with very accurate modelling of the troposphere. Unfortunately the software and computing facilities were not available at the time the survey was processed.

9.2.3 Use the Same Multi-Station Software

It is recommended that the use of the same processing software on the various GPS receiver observations, would allow a more confident analysis. This is because the adjustment would be use the same geodetic datum and correct population parameters for dual and single frequency could be estimated. The multi-station approach is the preferred session processing strategy, as it is stochastically rigorous.

9.3 COST/BENEFIT ANALYSIS

With the implementation of new technologies, come the questions of their performance in relation the task required and to the overall project costs. Efficient management practices tend to continually look toward advanced technologies to save time. However, one must also look at the cost efficiencies to fully assess the benefit of new technologies such as GPS.

The total cost of the 17 day fieldwork component of the coastal GPS survey was $73,500 of which $25,000 was for the receiver hire. The remaining $48,500 for salaries, expenses and vehicle costs was shared with the other organisations involved (CMA and BHP), who paid for their personnel and vehicles, making it even more cost effective to the Public Works Department. In addition, the combined TI4100 survey for the CMA consisted of $7,200 for the PWD's share of the receiver hire and the costs of supplying a vehicle and personnel. Thus the total fieldwork cost to the Department of the GPS campaigns to be used to assess the coastal levelling problems was less than $90,000.

The post-processing took 22 months at an estimated cost of $60,000. This period included a large amount of training and research and development, as new concepts, software and various computer systems had to understood and the best strategies implemented. Therefore the overall cost of the project has been estimated at $150,000.

This compares favourably with existing methods. Had third order levelling been used, then the cost is estimated to be well over $250,000 and would have taken over two years to complete. So the GPS/RINT method proved to be about half the cost of conventional levelling. If higher order levelling was to be used the costs would have been even greater. Furthermore spirit levelling would not have been a truly independent method of locating and analysing the coastal height anomalies. The GPS solution had the added advantage of developing a new technology that would have cost benefits in the future for other fields of surveying as well as engineering and other sciences.

9.4 CLOSING REMARKS

Allowing for SST effects, this study shows that the combination of GPS ellipsoid heights and gravimetric techniques produce orthometric heights with larger precisions than first order spirit levelling. Concurrent research by groups such as CARTER et al (1987) in the western Pacific region and BASKER (1989) in the United Kingdom have indicated difficulties in producing very accurate GPS ellipsoid heights (<.01 m/1000 km) necessary to monitor sea level observations over large distances. Even with orbit improvement, dual frequency observations, and using water vapour radiometer measurements to reduce the tropospheric effects, carrier phase mathematical modelling errors particularly residual tropospheric errors limit the repeatability of the height determinations.

Relating these heights to global and regional studies of the sea level creates further problems when trying to determine the SST. Super tide gauges are now planned to overcome this problem by providing not only water level measurements, but also accurate atmospheric, water salinity and temperature measurements to allow comprehensive SST modelling. It is now left to researchers' to develop the valid models required to produce "absolute" MSL that will equal the geoid.

Converting the GPS ellipsoidal heights to orthometric heights through the use of the gravimetric methods promises to revolutionise the definitions of such height networks as the AHD. It is still in its infancy but with the advent of more powerful micro computers, its every day application by the GPS surveying community is eminent. Some problems exist in the system, particularly with regard to the availability of accurate gravity and height data used to calculate $\Delta g'$, that with GPS ellipsoid heights place restrictions on the accuracy of ΔN to about third order levelling standards. Further research is required to produce equivalent first order levelling results.

With the first of the new Block II operational satellites, launched on the 13th February 1989 and the full 24 satellite constellation expected to be operational by 1993, allowing 24 hour coverage, this technology has huge potential. Continuing developments in both software and hardware will produce an even better and a more cost effective technology, that should be monitored by the those intending to use GPS to determine heights.

A major benefit of the GPS survey is the data can give 3 dimensional results which can be used for various purposes. In the context of this study the sea level measurements at the gauges can be related to the global mathematical WGS84 ellipsoid and can therefore be used for worldwide scientific studies such as secular movements of the sea due to the greenhouse effect. More importantly, when combined with gravimetric techniques, a truly independent method of developing orthometric heights has been developed.

GPS is here and not since the introduction of the electronic distance measurement machines (EDM) in the 1950's has a technology created such a change in the thinking of the way we measure. The use of GPS as a tool in the solution of various complex positioning problems now seems well established.

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11. APPENDIX A

THE METHODOLOGY OF POSITION FIXING

CROSS (1983) points out that no matter what surveying method is used to obtain the position of points on the earth's surface (eg. GPS, Doppler, Traversing, Triangulation, etc.) the process can be divided into the following steps:

1) **Design** - where the type of measurements to be made are selected. These quantities include, angles and distances (traditional terrestrial surveys), frequency shifts (satellite doppler), vehicle acceleration (inertial surveying) and time delay (GPS and VLBI).

2) **Measurement** - this is the process of capturing the data from the various measurement systems being used. For example in a traverse, distances are used and can be measured by EDM or metal chain depending on decisions made in the design phase.

3) **Mathematical Modelling** - a mathematical model which relates the measurements (or observations) to the required coordinates must be developed. Here the physical processes of the measurements are converted to mathematical relationships.

4) **Estimation** - also known by surveyors as "adjustment", is the phase during which the estimation of the parameters of interest (eg. coordinates) is carried out using the measurements and mathematical model as input. Improvements in the quality of GPS results in the past few years has been nearly entirely due to the advances in processing methods, since instrumentation hardware architecture has hardly changed. This can be attributed to the availability of

more powerful, yet inexpensive computers that permit the use of more sophisticated processing techniques.

5) **<u>Analysis</u>** - is possibly the most important phase of the survey process. The coordinates are of little value without some estimate of their quality. The quality of a position fix is a measure of the size of an error that can be expected with a specific probability. In surveying these errors are classed under three headings; random, gross, systematic. The terms precision and reliability (or accuracy) are often used to describe measures of quality of the solution.

We can say that GPS data processing involves two equally important operations, the first being the mathematical combination of the observations to produce estimates of position, the second being the study of the errors in the measurements and their propagation through the computational procedure in order to yield the quality of the estimated positions (CROSS, 1983). Since the mathematical model is central to both operations, a more detailed discussion of this component is required.

The Mathematical Model

MIKHAIL (1976) describes the mathematical model as the theoretical system by which one describes a physical situation or set of events. The model replaces the physical situation, but the same physical system can be described by a number of different mathematical models. The mathematical model can be divided into two parts; the **functional model** and the **stochastic model**.

Functional Model

This describes the "deterministic" properties of the physical situation. In geodesy and surveying we deal with both geometric and dynamic models.

> eg. Geometric model - (in surveying) a plane triangle with angles, sides and orientation with respect to a coordinate system.
>
> Dynamic model - (in geodesy) the gravity field of the earth.

The functional model is linked to the physical situation by the measurements. The functional model not only refers directly to the measurements but also to the properties of the measurement process.

> eg. when measuring phase observations in GPS, the troposphere through which the signal passes must be considered and a decision made as to whether it should be taken as known, ignored or estimated in the adjustment.

The mathematical models must be as complete as possible, yet not be so complex as to become unmanageable. For example it is often a procedure that direct readings from the measurement operation are preprocessed before they are introduced into the model (eg directions from a theodolite are "reduced" or GPS phase measurements are "compressed" by meaning them or formed into linear combinations in a process known as differencing). However when the functional model is altered in anyway, the stochastic model must also change accordingly.

Stochastic Model

Since measurements are subject to influences, both random and systematic, which cannot be totally controlled, there is an uncertainty in the outcomes if measurements are repeated. This statistical variation (expressed in the form of variance and covariance) is important in determining the quality of the measurements and of the final results. It may be difficult to assess these variations since observations may be physically correlated and these correlations difficult to quantify. One approach is to use estimates from previous experience with similar observations. The totality of assumptions on the statistical properties of the measurements (and parameters if a priori information is available) is called the stochastic model (MIKHAIL, 1976).

The functional and stochastic models must be considered as coupled systems at all times, since there may be several alternatives for each model which can represent a certain physical situation. By modifying one model, one also changes the other model.

12. APPENDIX B

A REVIEW OF LEAST SQUARES

It is important to ensure that the best possible "treatment" is given to high precision surveying measurements in order that accurate and reliable results are obtained. This is achieved by using the estimation theory "least squares", which allows not only the "optimum" estimation of the desired parameters but also gives an estimate of their quality. The technique is commonly used when there are more observations than are strictly necessary to solve the problem.

The basic mathematical model can be written as a vector function (CROSS, 1983),

$$F(\bar{x}, \bar{l})$$

where $\bar{x}$ = the true parameters required
$\bar{l}$ = the true observed quantities

If we linearise the model and replace $\bar{x}$ and $\bar{l}$ with estimates such that:

$$\bar{x} = x_o + x \qquad\qquad \bar{l} = l + v$$

where x_o= approximate value of $\bar{x}$
x = correction to the parameter x_o
l = estimate of $\bar{l}$ given by the field observations.
v = observation residual

then the matrix form of the functional model becomes:

$$Ax + Rv - b = 0 \tag{B.1}$$

where $A = dF/dx$ at (x,l); is the design matrix made up of the partial derivatives of the function relating the observations and the parameters with respect to the parameters.

$R = dF/dl$ at (x,l); is the design matrix made up of the partial derivatives of the function relating the observations and the parameters with respect to the observations.

$-b = F(x_0,l)$

and in the simplified case when $R = -I$ then:

$$Ax = b + v \tag{B.2}$$

This is the standard "observation equation" and is the linearised functional model used in GPS.

This mathematical model will contain m equations, where u parameters are estimated from n observations. The number of observations is usually much larger than the number of parameters, which leads to an overdetermined system of equations. Therefore, certain statistical criteria are introduced to obtain optimal estimates of the parameters. An estimate is considered optimal in the statistical sense, if it is unbiased and has minimum variance (CASPARY, 1987).

Therefore in the "least squares" estimation method we apply the condition that the weighted sum of the squared residuals be a minimum, such that:

$$v^T P v = \text{minimum} \qquad P = Q_l^{-1} \tag{B.3}$$

where Q_l^{-1} is the co-factor matrix of the observations which represents the stochastic model of the observations.

The solution is defined by explicit expressions known as normal equations (CROSS, 1983),

$$\hat{x} = (A^T P A)^{-1} A^T P b \tag{B.4}$$

$$\hat{v} = A\hat{x} - b \tag{B.5}$$

$$\hat{l} = l - \hat{v} \tag{B.6}$$

Note that the notation $\hat{x}$, $\hat{v}$ and $\hat{l}$ are used to denote the least squares estimates of the values x, v and l respectively.

An a posteriori variance factor is used to test the validity of the statistical assumptions of the mathematical model by comparing it with the a priori variance factor σ^2 and is defined as,

$$S_o^2 = \frac{v^T P v}{r} \tag{B.7}$$

where r = (n-u) is known as the degrees of freedom.

Obtaining the estimates of the position is only half the process. Equally important is the estimation of the quality of the measurements and the estimated positions. Through the stochastic model their quality can be derived using covariance matrices. While the quality of the observations are defined by the a priori co-factor matrix Q_l, the a posteriori covariance matrices of the least squares estimates of the parameters $Q_{\hat{x}}$, the observations $Q_{\hat{l}}$ and residuals $Q_{\hat{v}}$ are obtained using the propagation of variances principle:

$$Q_{\hat{x}} = (A^T P A)^{-1} \tag{B.8}$$

$$Q_{\hat{v}} = Q_l - A(A^T P A)^{-1} A^T \tag{B.9}$$

$$Q_{\hat{l}} = A(A^T P A)^{-1} A^T = Q_l - Q_{\hat{v}} \tag{B.10}$$

Authors such as KRAKIWSKY (1976) include the unit variance or a posteriori variance factor estimated in equation (B7) in these expressions, for example

$$C_{\hat{x}} = \sigma_o^2 Q_{\hat{x}} = \sigma_o^2 (A^T P A)^{-1} \tag{B.11}$$

where σ_o^2 is the population variance of the sample a posteriori estimate S_o^2.

This should not be used without first applying the relevant statistical test of the a posteriori variance factor against the population parameter representing all measurements, to determine whether the data set is indeed been drawn from the population.

It is important to note the influence of both the functional model (through the matrix A) and stochastic model (through the co-factor matrix Q_l) on the estimated parameters and their covariance matrices.

13. APPENDIX C

TYPES OF CORRELATION

The word "correlation" is used extensively in literature to describe the relationship between entities. However its formal definition is rather more complex and can lead to confusion in discussions on the effects of correlations in physical processes and mathematical models. This is particularly true in GPS surveying where correlations are often mentioned in publications but are seldom explained. Statements such as "correlations have been taken into account" or "the adjustment of independent baselines" should be critically appraised as these generalised comments can sometimes lead to incorrect assumptions by the reader.

Correlations give an indication of the interdependence between variables. For two variables x and y in a linear relationship, the correlation between them is defined as

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \, \sigma_y} \tag{C.1}$$

where σ_x = the standard deviation of x
where σ_y = the standard deviation of y
where σ_{xy} = the covariance of x and y

Correlations by definition vary between -1 and +1 and when the correlation of two variables is near +/- 1 this means the variations in their observations have a lot in common. This does not necessarily mean that the variations of one are caused by the variations of the other, although it may be possible. In many cases, external influences may be affecting both variables in a similar fashion.

Various authors have used different terms to describe the types of correlation that exist in GPS. HATCH (1988) uses "real correlations" and "apparent correlations" while SCHAFFRIN & GRAFAREND (1986) refer to "interactions". The dilemma in defining the appropriate terminology can be overcome by returning to the modelling process described in Appendix A. Since correlations describe the interdependence between variables it follows that correlation can be described in the mathematical modelling process. Therefore correlations can be defined as;

1) Physical Correlation

2) Mathematical Correlation - Functional Correlation
- Statistical (or stochastic correlation)

Physical Correlation

Physical correlation refers to the correlations between actual field observations. It arises from the nature of the observations as well as their method of collection. If different observations or sets of observations are affected by common external influences, they are said to be physically correlated. Examples of such correlation include:

- measurements of vertical angles from one point to two targets for trigonometric heighting are taken throughout the day. Individual observations to both targets exhibit very similar trends, with the observations at sunrise being the largest then dropping to the lowest at midday before increasing again during the afternoon. Therefore the observation sets to the two targets are physically correlated due to the similar affects on the line of sights caused by refraction.

- a look at the plots of the carrier phase data from different GPS satellites observed by the same receiver will vary in a similar manner showing physical correlation (LINDLOHR & WELLS, 1985). This is because the measurements are made using the same GPS receiver clock.

Mathematical Correlation

This is related to the parameters in the mathematical model. It can therefore be divided into two types which correspond to those two components of the mathematical model.

Functional Correlation

Here the physical correlations can be taken into account by introducing adequate terms in the functional model of the observations. In other words, functionally correlated quantities have the same parameter in the observation model. Examples include:

- placing a refraction parameter in the observation model involving vertical angles from which the trigonometric heights are to be calculated.

- a clock bias parameter can be placed in the GPS observation equation to take into account the physical correlation introduced into the measurements by the receiver or satellite clocks.

Stochastic Correlation

Stochastic correlation (also known as statistical correlation) will occur between observations when off-diagonal elements are introduce into the covariance matrix of the observations (Q_l). This correlation also appears when functions of the observations are considered, due the law of the propagation of variances. Even if the covariance matrix of observations is diagonal (no stochastic correlation), the covariance matrices of the resultant least squares estimates of the parameters ($Q_{\hat{x}}$), residuals ($Q_{\hat{v}}$) and observations ($Q_{\hat{l}}$) defined in (B.8) to (B.10), will generally be full covariance matrices, and therefore exhibit stochastic correlation.

14. APPENDIX D

RESULTS OF GEOMETRIC TESTS

Daily polygon closures - Trimble 4000SX TRIMVEC™ solutions

DAY no	POLYGON (stations used)	δX(m)	δY(m)	δZ(m)	δ3D	DIST (kms)	δ3D ppm	
132A	01-02-03-04-05-01	.011	-.064	.024	.069	455.7	0.2	
133A	04-05-07-08-04	.004	.022	.028	.036	264.3	0.1	
134A	07-08-09-10-11-07	-.093	-.117	-.027	.152	268.7	0.6	
135A	10-12-13-14-15-10	-.047	-.071	.001	.085	215.3	0.4	
136A	14-17-18-16-14	.015	.041	-.008	.044	295.4	0.2	
137A	17-18-20-17	.028	.006	.085	.090	185.4	0.5	
138A	20-21-22-23-24-20	-.060	.024	-.060	.088	264.3	0.3	
139A	23-24-25-27-23	.159	.059	-.015	.170	305.3	0.6	
140A	26-27-28-29-30-26	-.005	-.151	.006	.151	259.8	0.6	
141A	29-30-31-33-29	.233	.465	-.535	.746	349.1	2.1	*
142A	31-32-33-31	.016	.030	-.008	.035	168.8	0.2	
143A	33-34-35-33	-.030	-.032	-.010	.045	133.9	0.3	
144A	35-36-37-38-39-35	-.081	-.240	.101	.273	393.7	0.7	
145A	25-29-31-41-40-25	.091	-.138	.044	.171	710.3	0.2	
146A	17-18-19-20-21-17	.056	.035	.001	.066	230.1	0.3	

MEAN = 0.5
RMS = 0.5

Daily polygon closures - TI4100 NOVAS solutions

DAY no	POLYGON (stations)	δX(m)	δY(m)	δZ(m)	δ3D	DIST (kms)	δ3D ppm
251A	41-42-43-44-41	.097	-.070	.037	.125	638.4	0.2
252A	41-42-35-31-41	.145	-.051	.028	.156	688.0	0.2
253A	40-25-20-47-40	-.003	-.041	-.027	.049	624.6	0.1
254A	47-40-48-49-47	.020	.046	.033	.060	536.5	0.1
255A	47-49-50-45-47	-.007	.072	.012	.073	579.5	0.1
256A	47-45-46-20-47	-.024	-.001	-.002	.024	512.6	0.0
257A	45-46-14-51-45	-.004	.001	-.022	.022	500.5	0.0
258A	45-53-52-50-45	.071	.027	-.008	.076	538.6	0.1
259A	54-53-52-55-54	-.036	.052	-.014	.065	504.0	0.1
260A	54-53-14-09-54	.059	.083	.041	.110	502.4	0.2
261A	54-09-04-56-54	-.001	.016	-.051	.053	564.0	0.1
262A	54-56-57-55-54	.333	.058	.013	.338	511.5	0.7
263A	46-17-14-53-46	.034	-.030	-.027	.053	480.8	0.1

MEAN = 0.2
RMS = 0.2

Repeat baseline comparisons - Trimble 4000SX TRIMVEC™ solutions

LINE	DAYS	δX(m)	δY(m)	δZ(m)	δ3D	LENGTH(m)	ppm
1005-1004	132A-133A	.028	-.061	.013	.068	38493.0	1.8
1008-1007	133A-134A	.033	.047	.013	.059	34483.0	1.7
1015-1014	135A-136A	.045	.015	.029	.056	34580.0	1.6
1017-1018	136A-137A	.032	.065	.006	.073	36859.0	2.0
1017-1018	137A-146A	-.009	-.018	-.002	.020	36859.0	0.5
1017-1020	137A-146A	-.082	-.004	-.057	.100	91503.0	1.1
1020-1021	138A-146A	-.026	.029	-.038	.054	22212.0	2.4
1023-1024	138A-139A	.099	.071	-.007	.122	35804.0	3.4
1026-1027	139A-140A	-.003	-.011	.008	.014	26709.0	0.5
1029-1030	140A-141A	-.157	-.089	-.007	.181	42507.0	4.2
1029-1031	141A-145A	.257	.072	.044	.270	90237.0	3.0
1031-1032	141A-142A	.041	.090	.030	.103	48491.0	2.1
1031-1033	141A-142A	.185	-.107	.172	.274	84384.0	3.3
1033-1034	142A-143A	-.041	.030	-.048	.070	29514.0	2.4
1035-1036	143A-144A	-.017	-.017	.003	.024	39569.0	0.6
1035-1037	143A-144A	.114	.081	.016	.141	65917.0	2.1

MEAN = 2.1
RMS = 1.0

Repeat baseline comparisons - Trimble 4000SX BATCH-PHASER solutions

LINE	DAYS	δX(m)	δY(m)	δZ(m)	δ3D	LENGTH(m)	ppm
1005-1004	132A-133A	-.064	-.074	-.047	.109	38493.0	2.8
1008-1007	133A-134A	.063	.035	.019	.075	34483.0	2.2
1015-1014	135A-136A	-.033	-.017	-.020	.042	34580.0	1.2
1017-1018	136A-137A	.096	.071	.051	.130	36859.0	3.5
1017-1018	136A-146A	-.056	.029	-.040	.074	36859.0	2.0
1017-1020	136A-146A	-.229	.159	-.112	.300	91503.0	3.2
1023-1024	138A-139A	-.047	.011	-.009	.049	35804.0	1.4
1026-1027	139A-140A	-.092	-.178	.031	.203	26709.0	7.6
1029-1030	140A-141A	-.091	-.032	-.014	.097	42507.0	2.3
1029-1031	141A-145A	.214	.068	.023	.226	90237.0	2.5
1031-1032	141A-142A	.534	.504	.264	.780	48491.0	16.
1031-1033	141A-142A	.216	.000	.035	.219	84384.0	2.6
1033-1034	142A-143A	-.015	.042	-.035	.057	29514.0	1.9
1035-1036	143A-144A	-.033	.024	-.021	.046	39569.0	1.2
1035-1037	143A-144A	.015	.087	-.022	.091	65917.0	1.4

MEAN = 3.5
RMS = 3.6

Repeat baseline comparisons - TI4100 NOVAS solutions

LINE	DAYS	$\delta X(m)$	$\delta Y(m)$	$\delta Z(m)$	$\delta 3D$	LENGTH(m)	ppm
1041-1042	252A-253A	.278	.092	.129	.320	139558.0	2.3
1040-1047	254A-255A	.110	.049	.013	.121	168654.0	0.7
1047-1049	255A-256A	.083	.029	.036	.095	120823.0	0.8
1045-1047	256A-257A	.040	.160	.029	.167	153047.0	1.1
1047-1020	254A-257A	.080	.035	.070	.112	119367.0	0.9
1046-1045	257A-258A	.018	.006	.006	.020	120821.0	0.2
1014-1046	258A-264A	.133	.118	.097	.203	102118.0	2.0
1050-1045	256A-259A	.023	.035	.017	.045	129597.0	0.3
1053-1052	259A-260A	.044	.032	.028	.061	139839.0	0.4
1054-1053	260A-261A	.045	.118	.022	.128	110760.0	1.2
1054-1009	261A-262A	.043	.040	.006	.059	113810.0	0.5
1053-1014	261A-264A	.161	.045	.016	.168	159642.0	1.1
1056-1054	262A-263A	.284	.067	.073	.301	134019.0	2.2
1054-1055	260A-263A	.386	.063	.068	.397	155573.0	2.6

MEAN = 1.2
RMS = 0.8

Baseline comparisons -
Trimble 4000SX(PHASER) vs TI4000(NOVAS)

LINE	DAYS	$\delta X(m)$	$\delta Y(m)$	$\delta Z(m)$	$\delta 3D$	LENGTH(m)	ppm
1017-1014	136A-264A	.010	-.493	-.031	.494	111222.0	4.4
1041-1031	145A-253A	-.407	-.047	.202	.457	140246.0	3.3
1040-1025	145A-254A	-.569	-.225	.220	.650	182457.0	3.6

MEAN = 3.8
RMS = 0.6

Baseline comparisons -
Trimble 4000SX(PHASER) vs WM101(PoPs)

LINE	DAYS	$\delta X(m)$	$\delta Y(m)$	$\delta Z(m)$	$\delta 3D$	LENGTH(m)	ppm
1025-1029	145A-018A	-.213	.399	.113	.466	135124.0	3.5
1029-1031	145A-019A	-.129	.096	.232	.282	90237.0	3.1

MEAN = 3.3
RMS = 0.3

Repeat differential ellipsoid height comparisons
Trimble 4000SX

LINE	DAYS	TRIMVEC		BATCH-PHASER		LENGTH(m)
		δh(m)	PPM	δh(m)	PPM	
1005-1004	132A-133A	0.050	1.5	0.042	1.1	38493.0
1008-1007	133A-134A	0.012	0.3	0.051	1.5	34483.0
1010-1011	134A-135A	0.087	2.6	0.144	4.3	33450.0
1015-1014	135A-136A	0.043	1.2	-0.029	0.8	34580.0
1017-1018	136A-137A	0.000	0.0	-0.071	1.9	36859.0
1017-1018	137A-146A	0.000	0.0	-0.051	1.4	36859.0
1017-1020	137A-146A	-0.090	1.0	0.166	1.8	91503.0
1020-1021	138A-146A	-0.051	2.3	-0.020	0.9	22212.0
1023-1024	138A-139A	0.043	1.2	-0.033	0.9	35804.0
1026-1027	139A-140A	0.005	0.2	0.015	0.6	26709.0
1029-1030	140A-141A	-0.090	2.1	-0.065	1.5	42507.0
1029-1031	141A-145A	0.244	2.7	-0.418	4.6	90237.0
1031-1032	141A-142A	0.012	0.2	0.310	6.4	48491.0
1031-1033	141A-142A	0.270	3.2	0.152	1.8	84384.0
1033-1034	142A-143A	0.068	2.3	0.046	1.6	29514.0
1035-1036	143A-144A	-0.005	0.1	-0.045	1.1	39569.0
1035-1037	143A-144A	0.065	1.0	0.060	0.9	65917.0

MEAN = 1.3 1.9
RMS = 1.0 1.0

Repeat differential ellipsoid height comparisons
TI4100 (NOVAS)

LINE	DAYS	δh(m)	PPM	LENGTH(m)
1041-1042	252A-253A	0.278	2.0	139558.00
1040-1047	254A-255A	0.110	0.7	168654.00
1047-1049	255A-256A	0.083	0.7	120823.00
1045-1047	256A-257A	0.040	0.3	153047.00
1047-1020	254A-257A	0.080	0.7	119367.00
1046-1045	257A-258A	0.018	0.1	120821.00
1014-1046	258A-264A	0.133	1.3	102118.00
1050-1045	256A-259A	0.023	0.2	129597.00
1053-1052	259A-260A	0.044	0.3	139839.00
1054-1053	260A-261A	0.045	0.4	110760.00
1054-1009	261A-262A	0.043	0.4	113810.00
1053-1014	261A-264A	0.161	1.0	159642.00
1056-1054	262A-263A	0.284	2.1	134019.00
1054-1055	260A-263A	0.386	2.5	155573.00

MEAN = 0.9
RMS = 0.8

15. APPENDIX E

COMBINED NETWORK ADJUSTMENT RESULTS

FINAL RESULTS OF COMBINED TI4100(NOVAS) & TRIMBLE 4000SX (PHASER) NETWORK
Minimally Constrained at Tidbinbilla in the WGS84 datum

STN No		STN NAME	LATITUDE	LONGITUDE	ELL HGT	Point Error ellipsoid maj	min	orient	σ ht
6000	1042	SILVER	S 28 21 43.757038	E 151 58 50.454006	875.455	0.186	0.085	86.3	0.106
6000	1041	BOYD	S 29 36 55.918581	E 151 50 45.922194	1296.878	0.179	0.066	86.4	0.093
6000	1031	WOOLI	S 29 51 42.371664	E 153 16 4.590066	45.736	0.177	0.064	86.2	0.092
6000	1035	BRUNS	S 28 31 29.656138	E 153 32 17.335654	135.016	0.185	0.066	86.2	0.095
6000	1043	MOREE	S 29 28 44.941575	E 149 50 20.717648	270.786	0.197	0.114	85.9	0.132
6000	1044	GOONDI	S 28 32 17.950302	E 150 17 58.279997	279.200	0.196	0.110	86.0	0.128
6000	1020	REID	S 33 5 27.253034	E 151 39 46.528487	59.509	0.063	0.046	86.4	0.051
6000	1046	FLEURS	S 33 51 33.878237	E 150 45 49.540776	65.890	0.057	0.052	82.8	0.056
6000	1047	JUNCTION	S 32 24 47.072029	E 150 40 24.522166	189.890	0.066	0.059	85.4	0.063
6000	1045	MULLEY	S 33 25 46.861130	E 149 34 1.455817	756.415	0.058	0.055	80.9	0.058
6000	1040	MOONBI	S 30 56 48.033957	E 151 8 42.507685	1356.692	0.082	0.059	86.7	0.073
6000	1025	CROWDY	S 31 50 36.634263	E 152 45 12.350700	83.497	0.081	0.048	85.8	0.058
6000	1048	CURRACUB	S 30 42 27.496784	E 149 59 23.196165	445.744	0.090	0.084	83.8	0.091
6000	1049	MUMBEDAH	S 31 51 20.062991	E 149 34 24.164074	782.921	0.075	0.071	83.3	0.075
6000	1050	PARKES	S 32 59 55.842474	E 148 16 29.957438	398.333	0.069	0.067	7.0	0.071
6000	1051	NARRAWA	S 34 23 39.947753	E 149 6 43.334222	836.711	0.081	0.078	15.8	0.084
6000	1014	GERROA	S 34 46 43.419771	E 150 49 18.277385	37.798	0.051	0.044	87.7	0.047
6000	1052	NARRABUR	S 34 16 27.627946	E 147 37 44.962866	513.176	0.059	0.055	4.8	0.060
6000	1053	HOLLY	S 34 24 29.701464	E 149 8 24.805149	653.904	0.045	0.044	0.0	0.047
6000	**1054**	**TIDBIN**	**S 35 23 51.450538**	**E 148 58 42.839221**	**663.121**	**0.010**	**0.008**	**180.0**	**0.010**
6000	1055	MOORONG	S 35 6 51.931540	E 147 18 15.561673	312.024	0.059	0.055	26.2	0.061
6000	1009	BATEMAN	S 35 42 10.303193	E 150 10 36.541927	18.450	0.046	0.044	84.6	0.046
6000	1056	KOS	S 36 27 21.017141	E 148 15 48.567059	2246.803	0.069	0.062	76.0	0.069
6000	1004	EDEN	S 37 4 27.413851	E 149 54 28.475855	17.015	0.065	0.047	90.5	0.053
6000	1057	TALGARNO	S 36 5 1.131418	E 147 5 47.925339	657.711	0.085	0.075	75.7	0.081
6000	1017	WATSON-Sydney	S 33 51 10.731885	E 151 17 9.103387	108.060	0.058	0.045	86.2	0.049
6000	1003	Malacoota	S 37 34 16.009670	E 149 45 29.554079	31.378	0.152	0.054	99.4	0.082
6000	1005	Tathra	S 37 43 57.284915	E 149 58 55.426523	77.582	0.077	0.047	90.2	0.054
6000	1001	Lakes Entr	S 37 52 7.199248	E 148 0 54.700986	46.729	0.174	0.058	98.1	0.090
6000	1002	Cann River	S 37 34 9.815216	E 149 8 19.184805	100.573	0.158	0.055	98.4	0.084
6000	1006	Bermagui	S 36 25 28.834709	E 150 4 25.990914	23.240	0.078	0.047	90.0	0.054
6000	1008	Moruya	S 35 54 4.029999	E 150 8 44.401604	20.323	0.072	0.046	92.6	0.052
6000	1007	Narooma	S 36 12 42.350304	E 150 8 4.842292	31.692	0.062	0.044	88.0	0.048
6000	1010	Ulladulla	S 35 21 25.959774	E 150 28 39.561388	20.003	0.057	0.044	87.7	0.047
6000	1011	Jervis B	S 35 7 19.923587	E 150 42 28.210295	19.928	0.068	0.045	87.7	0.047
6000	1013	Huskisson	S 35 2 16.868760	E 150 40 17.027111	20.776	0.070	0.045	88.5	0.050
6000	1012	Jervis A	S 35 7 19.928710	E 150 42 28.208968	19.928	0.073	0.045	86.8	0.047
6000	1015	Kembla	S 34 28 32.187253	E 150 54 34.924404	23.224	0.060	0.045	87.2	0.048
6000	1016	Stanwell	S 34 13 23.319912	E 150 59 53.786979	211.387	0.069	0.045	86.9	0.050
6000	1018	Ettalong	S 33 31 22.113927	E 151 19 51.247071	29.471	0.063	0.045	86.6	0.050
6000	1022	Nelson Bay	S 32 43 4.351734	E 152 8 33.477395	28.144	0.084	0.048	86.1	0.056
6000	1023	Bulladelah	S 32 23 21.195306	E 152 13 25.222660	42.517	0.081	0.048	85.2	0.056
6000	1024	Forster	S 32 10 35.594876	E 152 30 34.770541	46.028	0.079	0.048	86.0	0.055
6000	1026	Pt Macquarie	S 31 25 36.736726	E 152 54 40.166035	31.823	0.088	0.048	86.1	0.058
6000	1027	Crescent	S 31 11 35.631830	E 152 58 46.129181	121.139	0.087	0.049	85.6	0.058
6000	1028	SW Rocks	S 30 52 57.937942	E 153 2 31.742807	41.061	0.106	0.050	84.1	0.062
6000	1029	Nambucca	S 30 38 45.117628	E 153 0 58.111885	74.163	0.099	0.050	85.4	0.061
6000	1030	Coffs Harbour	S 30 16 42.555193	E 153 8 34.199202	77.723	0.098	0.049	86.0	0.061
6000	1032	Yamba	S 29 25 56.693894	E 153 21 49.986333	62.601	0.188	0.066	85.9	0.095
6000	1033	Evans	S 29 6 55.469563	E 153 26 21.714500	89.420	0.184	0.065	86.2	0.094
6000	1034	Ballina	S 28 52 10.740035	E 153 33 21.461910	38.942	0.185	0.065	86.2	0.094
6000	1037	Southport	S 27 56 17.756574	E 153 25 38.433930	45.971	0.188	0.066	86.2	0.095
6000	1036	Tweed	S 28 10 5.052060	E 153 33 5.853007	53.513	0.187	0.066	86.2	0.095
6000	1039	Caloundra	S 26 48 10.393756	E 153 8 48.487502	53.493	0.193	0.066	86.3	0.096
6000	1038	Luggage Pt	S 27 22 40.815482	E 153 9 36.771818	44.809	0.192	0.066	86.3	0.096
6000	1019	Norah	S 33 16 46.420102	E 151 34 0.038643	45.649	0.112	0.050	78.6	0.058
6000	1021	Stockton	S 32 55 10.704612	E 151 47 10.168624	27.935	0.073	0.046	86.4	0.052

16. APPENDIX F

δN COMPARISONS - NSW COASTAL SURVEY
(Combined RINT using the OSU86E geopotential model)

SCHOOL OF SURVEYING, U.N.S.W. DELTA N ANALYSIS
GRAV08 Ver 1.4 adpwdcb.dat
PUBLIC WORKS TIDE GAUGE GPS HEIGHT NETWORK
ROD MACLEOD

Control File: PWDPTSCB.090 Lines File: ADPWDLNS.DAT
Inner Zone File: PWD01_86.SUM Remote Zone File: PWD02_86.OUT

Number of Rings in Inner Zone: 2

LINE NO.	FROM	TO	DIST (km)	GRAVITY REMOTE	INNER	SUM	G.P.S.- LEVEL	DIFF (cm)	PPM
1	1	2	104.47	227.0	-12.6	214.4	235.2	20.8	2.0
2	2	3	54.73	-70.0	24.9	-45.1	-60.6	-15.5	2.8
3	3	4	56.71	198.0	9.8	207.8	200.3	-7.5	1.3
4	4	5	38.49	116.0	-6.5	109.5	108.2	-1.3	.3
5	5	6	35.14	90.0	37.0	127.0	112.0	-15.0	4.3
6	6	7	24.25	72.0	-3.4	68.6	70.6	2.0	.8
7	7	8	34.48	172.0	-49.7	122.3	130.0	7.7	2.2
8	8	9	22.18	114.0	-29.5	84.5	65.2	-19.3	8.7
9	9	10	47.07	89.0	26.9	115.9	97.9	-18.0	3.8
10	10	12	33.45	24.0	7.3	31.3	27.8	-3.5	1.1
11	12	13	9.91	68.0	-11.0	57.0	63.6	6.6	6.6
12	13	14	31.88	49.0	19.0	68.0	84.6	16.6	5.2
13	14	15	34.58	98.0	-7.3	90.7	99.7	9.0	2.6
14	15	16	29.17	76.0	14.9	90.9	72.5	-18.4	6.3
15	16	17	48.90	77.0	12.0	89.0	118.1	29.1	5.9
16	17	18	36.86	132.0	-14.9	117.1	118.3	1.2	.3
17	18	19	34.77	63.0	-.9	62.1	76.1	14.0	4.0
18	19	20	22.77	55.0	5.7	60.7	55.3	-5.4	2.4
19	20	21	22.21	51.0	3.2	54.2	53.1	-1.1	.5
20	21	22	40.19	59.0	-4.9	54.1	55.8	1.7	.4
21	22	23	37.23	132.0	-30.2	101.8	116.0	14.2	3.8
22	23	24	35.80	42.0	23.5	65.5	59.2	-6.3	1.7
23	24	25	43.52	78.0	20.4	98.4	101.7	3.3	.7
24	25	26	48.56	65.0	6.5	71.5	79.6	8.1	1.7
25	26	27	26.71	8.0	28.2	36.2	36.9	.7	.3
26	27	28	34.94	70.0	6.2	76.2	66.3	-9.9	2.8
27	28	29	26.38	137.0	-66.0	71.0	84.7	13.7	5.2
28	29	30	42.51	140.0	5.3	145.3	136.3	-9.0	2.1
29	30	31	47.74	144.0	13.7	157.7	145.0	-12.7	2.7
30	31	32	48.49	174.0	-60.2	113.8	103.6	-10.2	2.1
31	32	33	35.89	72.0	17.8	89.8	94.8	5.0	1.4
32	33	34	29.51	24.0	62.8	86.8	82.5	-4.3	1.5
33	34	35	38.25	110.0	-11.4	98.6	126.1	27.5	7.2
34	35	36	39.57	107.0	-9.4	97.6	102.8	5.2	1.3
35	36	37	28.25	79.0	5.4	84.4	83.7	-.7	.2
36	37	38	67.45	230.0	-65.8	164.2	152.6	-11.6	1.7
37	38	39	63.74	154.0	29.3	183.3	175.5	-7.8	1.2

MEAN: .2 S.D. 12.4 R.M.S. 12.2 MEAN PPM: 2.7

SCHOOL OF SURVEYING, U.N.S.W. DELTA N ANALYSIS
GRAV08 Ver 1.4 adpwdcb.dat
PUBLIC WORKS TIDE GAUGE GPS HEIGHT NETWORK
ROD MACLEOD

Control File: PWDPTSCB.090 Lines File: ADPWDLNS.DAT
Inner Zone File: PWD01_86.SUM Remote Zone File: PWD02_86.OUT

Number of Rings in Inner Zone: 5

LINE NO.	FROM	TO	DIST (km)	GRAVITY REMOTE	INNER	SUM	G.P.S.-LEVEL	DIFF (cm)	PPM
1	1	2	104.47	227.0	-2.4	224.6	235.2	10.6	1.0
2	2	3	54.73	-70.0	57.5	-12.5	-60.6	-48.1	8.8
3	3	4	56.71	198.0	13.2	211.2	200.3	-10.9	1.9
4	4	5	38.49	116.0	-11.6	104.4	108.2	3.8	1.0
5	5	6	35.14	90.0	21.6	111.6	112.0	.4	.1
6	6	7	24.25	72.0	-2.8	69.2	70.6	1.4	.6
7	7	8	34.48	172.0	-54.6	117.4	130.0	12.6	3.6
8	8	9	22.18	114.0	-39.9	74.1	65.2	-8.9	4.0
9	9	10	47.07	89.0	21.2	110.2	97.9	-12.3	2.6
10	10	12	33.45	24.0	9.3	33.3	27.8	-5.5	1.6
11	12	13	9.91	68.0	-9.1	58.9	63.6	4.7	4.7
12	13	14	31.88	49.0	41.0	90.0	84.6	-5.4	1.7
13	14	15	34.58	98.0	13.5	111.4	99.7	-11.7	3.4
14	15	16	29.17	76.0	3.8	79.8	72.5	-7.3	2.5
15	16	17	48.90	77.0	12.6	89.6	118.1	28.5	5.8
16	17	18	36.86	132.0	-18.4	113.6	118.3	4.7	1.3
17	18	19	34.77	63.0	5.9	68.9	76.1	7.2	2.1
18	19	20	22.77	55.0	5.8	60.8	55.3	-5.5	2.4
19	20	21	22.21	51.0	1.8	52.8	53.1	.3	.2
20	21	22	40.19	59.0	-2.0	57.0	55.8	-1.2	.3
21	22	23	37.23	132.0	-53.9	78.1	116.0	37.9	10.2
22	23	24	35.80	42.0	40.2	82.2	59.2	-23.0	6.4
23	24	25	43.52	78.0	53.7	131.7	101.7	-30.0	6.9
24	25	26	48.56	65.0	20.5	85.5	79.6	-5.9	1.2
25	26	27	26.71	8.0	32.5	40.5	36.9	-3.6	1.3
26	27	28	34.94	70.0	-16.8	53.2	66.3	13.1	3.8
27	28	29	26.38	137.0	-59.5	77.5	84.7	7.2	2.7
28	29	30	42.51	140.0	-1.7	138.3	136.3	-2.0	.5
29	30	31	47.74	144.0	-14.8	129.2	145.0	15.8	3.3
30	31	32	48.49	174.0	-79.1	94.9	103.6	8.7	1.8
31	32	33	35.89	72.0	28.2	100.2	94.8	-5.4	1.5
32	33	34	29.51	24.0	75.6	99.6	82.5	-17.1	5.8
33	34	35	38.25	110.0	2.9	112.9	126.1	13.2	3.5
34	35	36	39.57	107.0	-14.8	92.2	102.8	10.6	2.7
35	36	37	28.25	79.0	.6	79.6	83.7	4.1	1.4
36	37	38	67.45	230.0	-106.5	123.5	152.6	29.1	4.3
37	38	39	63.74	154.0	31.9	185.9	175.5	-10.4	1.6

MEAN: .0 S.D. 16.0 R.M.S. 15.8 MEAN PPM: 2.9

17. APPENDIX G

GPS/GRAVIMETRY ORTHOMETRIC HEIGHTS OF 1951-1969 MSL EPOCH
GPS NETWORK SOLUTION USED = COMBINED PHASER (TRIMBLE 4000SX), NOVAS (TI4100)
COMBINED RINT = OSU86E and RING INTEGRATION TO RING 5 (0.5 degrees)
Tide gauge information from Victoria and Queensland not provided

LOCATION INFORMATION			DATA														
STN	LOCATION	MARK	BASELIN DIST (Km)	CHAINAGE (Km)	h (GPS) PHASER-NOVAS	RL AHD (H_{AHD})	REMOTE N (OSU86)	INNER N RING(5)	H_{GPS} = h+TOT.N	TIDE GAUGE	GAUGE OP-ERATION STATUS	dH_o STN to 0 of GAUGE	MSL on GAUGE 1951-69	dH_{MSL} = dH_o+MSL	H_{MSL} by AHD	H_{MSL} by GPS	H_{MSL} GPS+.99
1	LAKES ENTR	BM 116	104.47	0	46.729	38.707	8.91	-0.381	38.2	VIC	19??-NOW						
2	CANN RIVER	BM 15	54.73	104.47	100.573	90.199	11.18	-0.404	89.797								
3	MALLACOOTA	TRIG PLUG	56.71	159.2	31.378	21.61	10.48	0.171	20.727								
4	EDEN	PILLAR	38.49	215.91	17.015	5.244	12.46	0.303	4.252	PWD CSIRO	19??-NOW REMOVED	-6.168	0.814	-5.354	-0.11	-1.102	-0.112
5	TATHRA	PM 12810	35.14	254.4	77.582	64.729	13.62	0.187	63.775	NONE							
6	BERMAGUI	SSM 35110	24.25	289.54	23.240	9.267	14.52	0.403	8.317	PWD	19??-NOW	-9.984	0.644	-9.34	-0.073	-1.023	-0.033
7	NAROOMA	SSM 30606	34.48	313.79	31.692	17.013	15.24	0.375	16.077								
8	MORUYA HEADS	PM 5	22.18	348.27	20.323	4.344	16.96	-0.17	3.533	PWD	19??-NOW	-5.216	0.872	-4.344	0	-0.811	0.179
9	BATEMANS BAY	PWD 540	47.07	370.45	18.45	1.819	18.1	-0.569	0.919	PWD	19??-NOW	-2.708	0.897	-1.811	0.008	-0.892	0.098
10	ULLADULLA	DH&W (STN 3)	33.45	417.52	20.003	2.393	18.99	-0.357	1.37	NONE							
12	JERVIS BAY	NMV/C/502	9.91	450.97	19.928	2.04	19.23	-0.265	0.963	NAVY	19??-NOW	-3.159	1.122	-2.037	0.019	-1.074	-0.084
13	HUSKISSON	DH&Ws WHARF	31.88	460.88	20.776	2.252	19.91	-0.356	1.222	PWD	19??-NOW	-3.152	0.91	-2.242	0.01	-1.02	-0.03
14	GERROA	PM 46715	34.58	492.76	37.798	18.428	20.4	0.054	17.344	NONE							
15	PT. KEMBLA	PWD 51	29.71	527.34	23.224	2.857	21.38	0.189	1.655	PWD	19??-NOW	-3.729	0.898	-2.831	0.026	-1.176	-0.186
16	STANWELL TOP	PIN IN CONC.	48.9	557.05	211.387	190.29	22.14	0.227	189.02	NONE							
17	SYDNEY	WATSON P.	36.89	605.95	108.060	85.787	22.91	0.353	84.797	F.DENISON CAMP COVE	1866-NOW 19??-NOW	-86.72 -86.72	0.936 0.934	-85.78 -85.78	0.008 0.006	-0.982 -0.984	0.008 0.006
18	ETTALONG	PWD 458		642.84	29.471	6.015	24.23	0.169	5.072	ETTALONG	19??-NOW						

19	NORAH HEAD	PM 16445	34.77	677.61	45.649	21.432	24.86	0.228	20.561	PATONGA NONE	19??-NOW	-6.064	0.071	-5.993	0.022	-0.921	0.069
20	SWANSEA	REID MISTAKE	22.77	700.38	59.509	34.739	25.41	0.286	33.813	PWD		-35.62	0.895	-34.72	0.015	-0.912	0.078
21	NEWCASTLE	PM 13431	22.21	722.59	27.935	2.634	25.92	0.361	1.654	NEWCASTLE	19??-NOW	-3.641	0.954	-2.687	-0.053	-1.033	-0.043
22	PT. STEPHENS	PWD 477	40.19	762.78	28.144	2.285	26.51	0.284	1.35	TOMAREE	19??-NOW	-3.229	0.862	-2.367	-0.082	-1.017	-0.027
23	BULADELAH	SSM 33593	37.23	800.01	42.517	15.498	27.83	-0.254	14.941	NONE							
24	FORSTER	PM 15795	35.8	835.81	46.028	18.417	28.25	0.147	17.631	PWD	19??-NOW	-19.48	1.002	-18.48	-0.0[illegible]	-0.847	0.143
25	CROWDY HEAD	PILLAR	43.52	879.33	83.497	54.869	29.03	0.684	53.783	PWD	19??-NOW	-55.79	0.865	-54.92	-0.055	-1.141	-0.151
26	PT MACQUARIE	SSM 10544	48.56	927.89	31.823	2.399	29.68	0.888	1.255	PWD	19??-NOW	-2.401	-0.044	-2.445	-0.046	-1.19	-0.2
27	CRES. HEAD	DH&W IN CONC	26.71	954.6	121.139	91.346	29.76	1.213	90.166	NONE							
28	S-W ROCKS	PWD 159	34.94	989.54	41.061	10.605	30.46	1.045	9.556	NONE							
29	NAMBUCCA	PILLAR	26.38	1015.9	74.163	42.86	31.85	0.45	41.883	NONE							
30	COFF HARBOUR	PILLAR	42.51	1058.4	77.723	45.057	33.23	0.434	44.059	PWD	19??-NOW	-45.92	0.869	-45.06	0.001	-0.997	-0.007
31	WOOLI	PILLAR	47.74	1106.2	45.736	11.62	34.67	0.285	10.781	NONE							
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