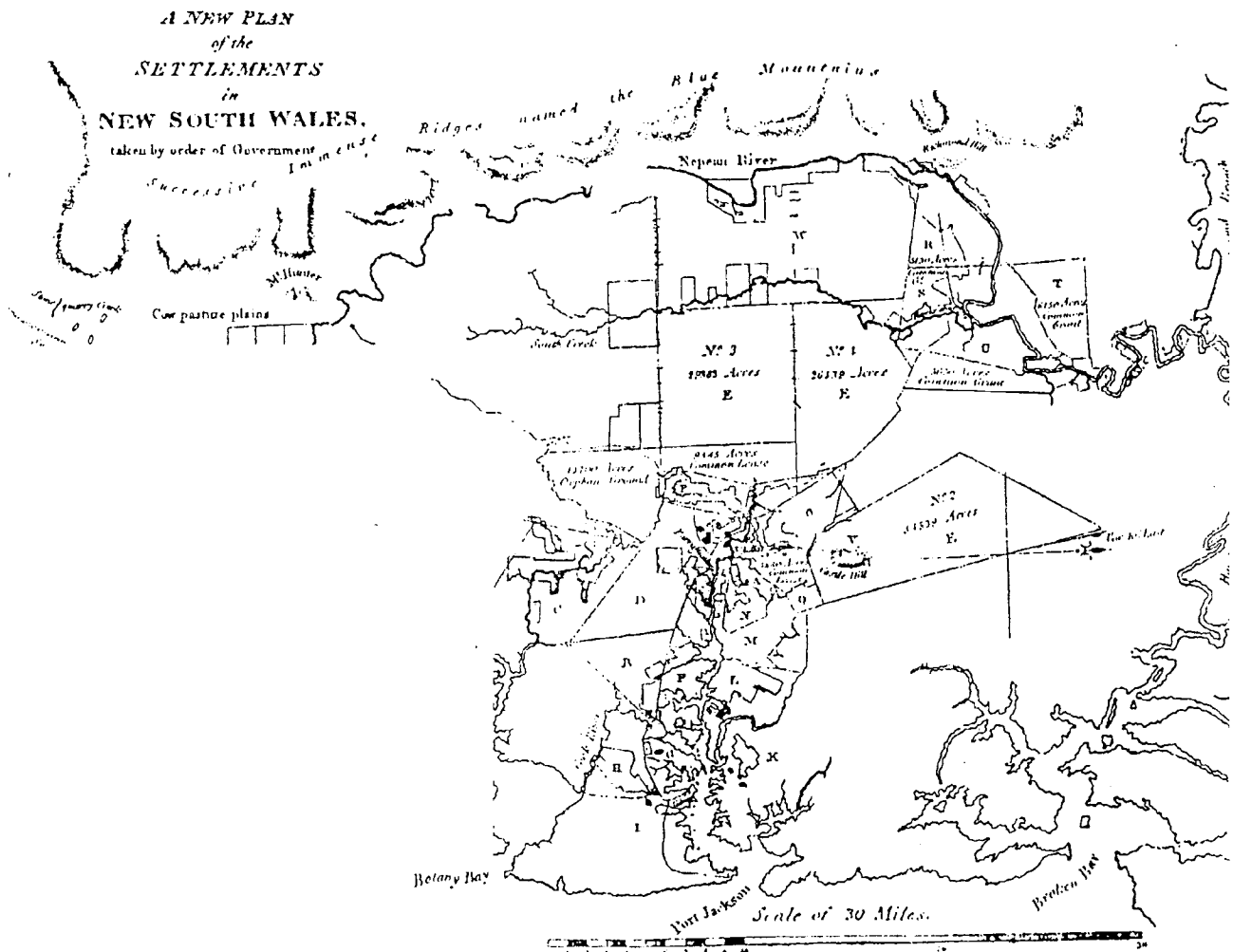


REFRACTIVE INDICES OF LIGHT, INFRARED AND RADIO WAVES IN THE ATMOSPHERE

JEAN M. RÜEGER



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FOREWORD

This collection of papers is the product of work carried out in the context of the Ad-Hoc Working Party of the IAG Special Commission SC3 (on Fundamental Constants, SCFC) on Refractive Indices of Light, Infrared and Radio Waves in the Atmosphere. The first contribution (on the radio refractive index) is an original article whereas the other contributions are reprints of reports or conference papers.

This form of publication became necessary since the first paper would have been too long for publication in a journal. It is hoped that the reprints in this volume will provide a wider circulation for the papers concerned. Also, the reprints should be free of any errors caused by conversion to the IAG's web page and the FIG's congress proceedings on CD.

The readers of this volume are reminded that a number of other members of the IAG working party have published papers on the refractive index in journals. Because of copyright aspects, these papers are not reprinted here. The following papers have been published by members of the group in support of the work of the group:

Ciddor, P. E., Hill, R. J. 1999. Refractive Index of Air. 2. Group Index. *Applied Optics (Lasers, Photonics and Environmental Optics)*, **38**(9): 1663-1667.

Ciddor, P. E. 1996. Refractive Index of Air: New Equations for the Visible and Near Infrared. *Applied Optics (Lasers, Photonics and Environmental Optics)*, **35**(9): 1566-1573.

Galkin, Y. S., Tatevian, R. A. 1997a. The Problem of Obtaining Formulae for the Refractive Index of Air for High-Precision EDM. *Journal of Geodesy*, **71**:483-485.

Galkin, Y. S., Tatevian, R. A. 1997b. Influence of Resonances on the Phase and Group Refractive Indices of Air. *Journal of Geodesy*, **71**:680-684.

J. M. Rüeger

December 2001

REFRACTIVE INDEX FORMULAE FOR ELECTRONIC DISTANCE MEASUREMENT WITH RADIO AND MILLIMETRE WAVES

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ABSTRACT

The radio wave refractive index formula, adopted by the International Union of Geodesy and Geophysics (IUGG) in 1960 and 1963 and based on Essen & Froome (1953), is no longer appropriate. The new developments in refractive index equations for radio wave and millimetre waves over the last thirty years are summarised. State-of-the-art formulae for precision EDM for routine as well as precision applications are reviewed. It is noted, that the coefficients of Thayer's formula of 1974, adopted by many authors since, are inappropriate for a number of reasons and should not be used. The use of compressibility factors in radio refractivity formulae requires further evaluation.

Two formulae for hand calculations of the radio refractive index N_r (in ppm, for infinite wavelengths) have been designed, one based on 'best available' coefficients and one based on 'best average' coefficients. For air with 0.0375% (375 ppm) content of CO_2 , the latter is:

$$N_r = 77.6890 \frac{p_d}{T} + 71.2952 \frac{p_w}{T} + 375463 \frac{p_w}{T^2}$$

where $p_d (= p_{\text{tot}} - p_w)$ is the dry air (including 375 ppm carbon dioxide) pressure (in hPa), p_w is the partial water vapour pressure (in hPa), and T is the temperature (in K). The accuracy of the dry air refractivity component N_d (first term on right hand side) is, conservatively, 0.02% of N_d . A realistic value for the accuracy of the water vapour component N_w (sum of second and third term on the right hand side) is 0.2% of N_w . A four-term formula with a separate input for the CO_2 partial pressure is given in the text.

For frequencies above 1 GHz and, particularly, for frequencies close and beyond the relevant resonance lines (e.g. between 53.59 and 66.30 GHz and at 118.75 GHz for oxygen, 22.23 GHz, 67.81 GHz and 119.99 GHz for water vapour) anomalous refractivity has to be modelled. Liebe's Millimetre-Wave Propagation Model (MPM) for frequencies below 1000 GHz is, presently, the most accessible computer routine that models anomalous refractivity (oxygen and water vapour only). The review carried out showed that MPM might require some fine tuning as far as the coefficients of the continuum formula are concerned. Hill's software routines (IR_N) for a much more elaborate model of resonance lines are also available, but require some FORTRAN programming by the user.

1. Introduction

The last resolutions of the International Union of Geodesy and Geophysics (IUGG) on refractive indices date back to 1960 and 1963. Because of more recent determinations of the radio wave refractivity of air, the 1960 and 1963 resolutions do no longer satisfy the needs of geodesy and surveying. In consequence, the radio wave formula adopted by IUGG in 1963 is rarely (if ever) used in connection with the GPS (Global Positioning System) and VLBI (Very Long Baseline Interferometry). Even in microwave EDM, the 1963 IUGG resolutions were not universally followed.

In 1991, at the 20th General Assembly of the International Association of Geodesy (IAG) in Vienna, it was suggested that new IUGG resolutions on refractive indices be prepared for adoption at a future General Assembly of IUGG. Eventually an ad-hoc working party was formed in 1993 under the umbrella of the International Association of Geodesy (IAG) Special Commission SC3 – Fundamental Constants (SCFC). Present members of the working party are: P. E. Ciddor (Australia), M. G. Cotton (USA), Y. S. Galkin (Russia), R. J. Hill (USA), J. Hinderling (Switzerland), R. Langley (USA), H. Matsumoto (Japan), V. de B. Mendes (Portugal), J. M. Rieger (Australia), R. A. Tatevian (Russia). The working party tabled an interim report at the General Assembly of the IAG in 1999. Two recommendations on the refractive index of visible and infrared waves were adopted by IAG in 1999.

So far, the working party has concentrated on the refractive index in the visible and near infrared spectrum, because of an apparent lack of interest from geodesists working with GPS and VLBI. However, some proposals for the radio and millimetre waves have been made and are discussed in this paper. Although terrestrial microwave distance measurement ($\lambda = 8 - 30$ mm, $f = 10 - 35$ GHz) is no longer used on a routine basis in most parts of the world, very long baseline interferometry (VLBI: $\lambda = 13 - 210$ mm, $f = 1.4 - 23$ GHz) is and the Global Positioning System (GPS: $\lambda = 190 - 250$ mm, $f = 1.2 - 1.6$ GHz) even more so. A re-evaluation of the radio wave refractive index in geodesy and surveying is, thus, warranted. This also allows to consider the change of the CO₂ content of the atmosphere. Traditionally, the carbon dioxide content of air was taken as 0.03 percent or 300 ppm (by volume). The new IAG resolutions of 1999 for the refractive index of visible and infrared waves adopted a revised default carbon dioxide content of 375 ppm (0.0375 percent).

Although this paper is based on the work of the working party, the views expressed are those of the author and might not be shared by the working party as a whole. The working party will report again on its work at the next IUGG General Assembly in 2003.

Unless stated otherwise, the following formula is used for the conversions between the pressure units of mm Hg (or Torr) and hPa (hectopascal): p [hPa] = (1013.25/760) p [mm Hg]. A temperature in Kelvin (K) is computed from $T = 273.15 + t$, where t is the temperature in degrees Celsius.

2. History of Formulae for the Refractive Index of Radio and Millimetre Waves

The 12th General Assembly of IUGG (Helsinki, 26 July - 6 August 1960) passed a resolution that adopted the following equation (after Essen and Froome 1951) for the reduction of microwave electronic distance measurements (IUGG 1960, Edge 1962):

$$N_r = (n_r - 1) \times 10^6 = \frac{103.49}{T} (p_{\text{tot}} - p_w) + \frac{86.26}{T} \times \left(1 + \frac{5748}{T}\right) p_w \quad (1)$$

where T = temperature (in K), p_{tot} = total atmospheric pressure (in mm Hg), p_w = partial water vapour pressure (in mm Hg), n_r = refractive index of radio waves in air at ambient conditions, N_r = refractivity (in ppm) of radio waves in air at ambient conditions. The same formula was confirmed in another resolution (IUGG 1963) at the 13th General Assembly of IUGG (Berkley, 19 - 31 August 1963). After conversion to hPa (hectopascal) as the pressure unit, the above equation becomes (Rüeger 1990 1996, Eq. 5.17):

$$N_r = \frac{77.624}{T} (p_{\text{tot}} - p_w) + \frac{64.700}{T} \times \left(1 + \frac{5748}{T}\right) p_w \quad (2a)$$

or, in the more usual form (see Eq. (3)):

$$N_r = (n_r - 1) \times 10^6 = 103.49 \frac{p_d}{T} + 64.700 \frac{p_w}{T} + 371896 \frac{p_w}{T^2} \quad (2b)$$

The accuracy of Eqs. (1), (2) and (2a) was perceived to be ± 0.1 ppm under 'normal' conditions and better than ± 1.0 ppm under 'extreme' conditions (Edge 1962). According to Deichl (1984), the simplifications made in the above equation to the

original Essen and Froome equations of 1951 cause systematic errors of about 0.35 ppm (refractive index too small).

As the 1963 IUGG Resolution is based on Essen & Froome (1951), a brief summary of that work is warranted. At 24 GHz, these authors measured the refractive index of dry carbon-dioxide-free air, nitrogen, oxygen, argon and carbon dioxide at laboratory conditions and reduced the measurements to 20 °C and 1013.25 hPa. They also measured the refractive index of water vapour between 15°C and 25°C and water vapour pressures from 8.0 hPa to 18.7 hPa and clearly state that the 'extrapolation formula (to other conditions) cannot be given the same confidence as that of the other gases'. They give a precision of ± 0.10 ppm (0.035% of the refractivity value) for their dry air value (at 0°C and 1013.25 hPa) and of ± 0.1 ppm (0.165% of the refractivity value) for their water vapour value (at 20°C and 13.3 hPa). Essen & Froome (1951) state that their radio wave dry air refractivity 'differs appreciably' (0.35 ppm) from the extrapolated optical dry air refractivity and note that 'it is not certain that extrapolation (from visible to the radio wavelengths) is completely valid' and that 'the radio frequency value should in any case be slightly greater than the extrapolated optical value because of the magnetic dipole resonant absorption by oxygen at wavelengths near 5 mm' (60 GHz). Because of the restricted temperature range of their measurements, Essen & Froome (1951) had to predict the $1/T$ term of the water vapour refractivity from visible values. Only the $1/T^2$ term (last term in Eqs. (1) and (2)) of the water vapour refractivity is based on their measurements. Hill et al. (1982, p. 1256) noted that Essen & Froome's use of the optical frequency water vapour coefficient (K_2 in Eq. (3) below) was 'an inaccurate assumption'. In addition, the water vapour resonance line at 22.2 Ghz would have to be considered.

Essen & Froome (1951) published two refractivity equations for moist air. Their equation for highest precision (Eq. (11), p. 872 of their paper) is valid between -20°C and +60°C and for partial water vapour pressures of less than 133 hPa within the precision of their experimental data. Their simplified equation (Eq. (12), p. 873 of their paper), that has been adopted by IUGG in 1963 (after omission of the CO₂ term), introduces errors in refractivity amounting to about 0.5 ppm at the extremes of the specified range of temperatures (-20°C, +60°C) and at normal water vapour pressures (13 hPa). The authors did not predict the errors at higher values of partial water vapour pressure. Using their precision (in percent, at 20°C and 13.3 hPa) of 0.165% would suggest an uncertainty of at least ± 0.8 ppm at 60°C and 133 hPa. The fact that they

define $T = 273 + t$ introduces further errors. Hill et al. (1982, p. 1256) noted that the experiments by Essen and Froome covered only a very limited temperature range.

Essen (1953) published the results of new measurements at 9 GHz. The first coefficient K_1' of the three-term refractivity equation can be calculated follows from the measured refractivity of dry and carbon-dioxide-free air at 0°C and 760 mm Hg as $K_1' = 288.10 (273.15/1013.25) = 77.6654$. The water vapour refractivity was measured between 16°C and 30°C and for water vapour pressures of 5 to 15 mm Hg. Using the same simplifications as used by Essen and Froome (1951), the two water vapour coefficients are obtained as $K_2 = 75.1682$ ppm K/hPa and $K_3 = 369226$ ppm K²/hPa. These two values differ considerably from those of Eq. (2a) since 'polarisation due to infrared absorption' has been added when extrapolating the K_2 value from optical wavelengths. It should be noted that Essen (1953) states the precision of the measured refractivities, but not of the derived coefficients. In 1955, Froome published the results of further refractivity measurements of water vapour and carbon dioxide free air, amongst other gases, at 72 GHz and found that 'the refractive indices for air, oxygen and water vapour are modified in much the manner to be expected from dipole theory' when comparing the new data with those of 1951 and 1953. (There is a water vapour resonance line at 22.2 GHz and an oxygen resonance line at 60 GHz.) Froome (1955) followed the methods used in the previous experiments and lists measured refractivities at reference conditions, with their respective precisions. No values for the coefficients K_2 and K_3 are given. The comments by Hill et al. (1982, p. 1256) on the limitations of the Essen & Froome (1951) data apply equally to the Essen (1953) and Froome (1955) data.

Since the work of Essen and Froome, new experimental values for the radio refractive index were published by a number of researchers, such as Birnbaum & Chatterjee (1952), Boudouris (1963), Newell & Baird (1965), Zhevakin & Naumov (1967), Liebe (1975a) and Liebe et al. (1977b). These contributions are briefly discussed.

Birnbaum & Chatterjee (1952) measured the dielectric constant of water vapour between 32°C and 103°C and up to 90% of saturation water vapour pressure at 9.28 GHz and for 24.5°C at 24.8 GHz and derived the constants of the Debye equation for molar polarisation. The measured pressure was converted to 'the pressure the vapour would exert were it an ideal gas' using Van der Waals' constants. They noted a maximum deviation from ideal gas behaviour of 0.5% at 1 atmosphere and +100°C. The Debye constants (A and B) were derived by weighted least squares curve fits as

3.84 ± 0.72 and $2.092 \times 10^4/T \pm 0.024 \times 10^4/T$ for the pressure unit mm Hg. Multiplication by 24.05 (Boudouris, 1963, Eq. 3.4, p. 659) and conversion to hPa gives the following K_2 and K_3 terms in Eq. (3): $K_2 = 69.270 \pm 12.99$ ppm K/hPa and $K_3 = 377380 \pm 4330$ ppm K²/hPa. These converted values agree with those quoted by Walter (1990, Table 3-3, p.102).

Boudouris (1963), before reporting the results of his new refractivity measurements of atmospheric gases, air and water vapour, discussed his experimental set-up in detail and gave an in-depth review of the theory of the refractive index, absorption and dispersion. The spectroscopic measurements were carried out between 7 and 12 GHz, 1-1013.25 hPa (except water vapour) and 0-50°C. He noted that the precision was basically limited by the resolution of the pressure readings. (The temperature was stabilised to ± 0.1 °C.) He tested the dependency of refractivity on pressure (linear, see his Fig. 18) and absolute temperature (linear with inverse of absolute temperature). Boudouris compared his 1958 refractivities of dry air, argon, nitrogen, carbon dioxide, helium, hydrogen, krypton, neon, oxygen and xenon with those of other researchers (Boudouris 1963, Table 1, p. 658). His results refer to 0°C, 1013.25 hPa, 'after the Van der Waals correction for real gases'. The precision of his measurements compares favourably with those of the other data. The Essen & Froome data are often better, however. It seems that the values in brackets refer to derived refractivities from measurements of the dielectric constant. In doing so, the magnetic permeability is considered for air and oxygen only. Since the dry air used did not contain carbon dioxide, the author 'increased by 0.01 percent the measured index to take into account the presence of 0.03 percent CO₂' (Boudouris 1963, p. 661). The numerical correction is not shown. (According to the calculations in Eq. (17) below, this should have been 0.022 percent.)

Boudouris (1963) also computed, for each gas and for dry air, a weighted mean of the refractivity measurements obtained at frequencies above 1 GHz and from data with given precisions. (Values without precision tags and data flagged as unreliable were omitted.) All of Boudouris values agree with the weighted averages within the accuracy limits given.

The refractivity of water vapour was measured by Boudouris (1963) between 0°C and 63°C and between 0 hPa and 200 hPa (at 63 °C) water vapour pressure. 'The observed pressure was reduced to the pressure corresponding to vapour treated as an ideal gas', using Van der Waals' constants (Boudouris 1963, Eq. (3.2), p. 659). The constants K_2

and K_3 are derived by linear regression techniques. I was unable to completely reproduce Boudouris' results on the basis of the data given in his Figs. 19 and 20. (Two important values are missing in Fig. 19 and the use of weights or otherwise is not discussed.) My repeat calculation of Boudouris' curve fit did nevertheless provide a valuable estimate of the correlation coefficient between the two constants K_2 and K_3 ($\rho = -0.995$), which will be used later in Table 8. Using the hypothesis of the additivity of the contributions of two constituents, Boudouris (1963, Eq. (3.9), p. 660) gives the following equation for the refractivity of moist air:

$$N_r = K_1 \frac{p_d}{T} + K_2 \frac{p_w}{T} + K_3 \frac{p_w}{T^2} \quad (3)$$

where p_d is the (partial) pressure of the dry air ($= p_{\text{tot}} - p_w$), p_w is the partial water vapour pressure and where the K_i are constants and T is the temperature (in K). K_1 includes the CO_2 component. Boudouris' values, after conversion to hPa are: $K_1 = 77.594 \pm 0.075$ K/hPa, $K_2 = 71.968 \pm 10.5$ K/hPa, $K_3 = 375406 \pm 3000$ K²/hPa.

$$N_r = 77.594 \frac{p_d}{T} + 71.968 \frac{p_w}{T} + 375406 \frac{p_w}{T^2} \quad (4)$$

It is not clear, how Boudouris got his K_1 value; his weighted mean dry air refractivity is 287.98 that, after multiplication with 273.15/1013.25, gives $K_1 = 77.633$. (A better agreement could be obtained by multiplying with 273/1013.25.) Boudouris then quotes 'the best values to be used in practice' proposed by Smith & Weintraub (1953). After conversion to hPa, the formula reads (after Smith & Weintraub: Boudouris 1963, Eq. (3.12), p. 660)

$$N_r = 77.631 \frac{p_d}{T} + 72.006 \frac{p_w}{T} + 375031 \frac{p_w}{T^2} \quad (5)$$

According to Boudouris (1963, p. 661), this formula should be accurate to within 0.5% between -50°C and $+40^\circ\text{C}$, a total pressure between 187 hPa and 1013 hPa, a partial water vapour pressure from 0 to 27 hPa and for frequencies between 1 Hz and 30 GHz. To test the Eq. (4), Boudouris measured the refractivity of moist air directly, between 0°C and 49°C and for partial water vapour pressures of up to 20 percent of total pressure. His Figure 21 (p. 661) shows good agreement (about 6 ppm in refractivity at a total pressure of 700 hPa). Boudouris (1963, p. 680-681) concluded that his work

contributed to clarify the confusion concerning the coefficients to be used in the formula (his Eq. (3.12) = Eq. (5) above) which expresses the refractive index of moist air.

Newell & Baird (1965) directly determined the refractivity (at temperatures around 20°C and at pressures between 800 hPa and 1013.25 hPa) of a number of gases at 47.7 GHz to about 5 part in 10⁸. The following gases were measured: dry CO₂-free air, argon, carbon dioxide, helium, hydrogen, nitrogen and oxygen. The accuracy is better than in earlier determinations and the measurements executed at a higher frequency than before. The authors used an experimental approach similar to that of Essen & Froome whilst aiming at a better accuracy and repeated the measurements with a different type of resonator. An (optical) equation by Barrel & Sears was, unfortunately, used to reduce the dry-air measurements to 0°C and 1013.25 hPa, and an equation by Berthelot for the reduction of the CO₂ measurements (Newell & Baird, 1965, Eqs. (8) and (14)). Newell & Baird (1965) give the refractivity of dry carbon-dioxide-free air as 288.13±0.05 ppm and that of carbon dioxide as 495.16±0.08 ppm at 1013.25 hPa and 0°C. The K₁' term (see Eq. (10)) can be derived from 288.13×(273.15/1013.25) = 77.6735 ±0.0135 K/hPa and the K₄-term from 495.16×(273.15/1013.25) = 133.484 ±0.022 K/hPa. The authors note that 'there is no reason to expect any dispersive effects for any of the measured gases except oxygen' in the frequency range from 1 Hz to 47.7 GHz.

In 1974, Thayer proposed an 'improved' three-term equation for the radio refractive index for precise geodetic and laboratory use that included compressibility factors for the first time for the radio refractive index. The author noted that the omission of compressibility factors leads to errors in the radio wave refractivity of 0.04 ppm in the dry term and 0.1 ppm in the wet term at high humidities. He computed a first K₁ value (see Eq. (3)) by converting the visible dry air refractivity into the radio wave refractivity with the aid of the magnetic permeability at radio frequencies. (A similar approach was used by Essen and Froome (1951) to convert the visible water vapour refractivity into the radio wavelengths.) Hill (1996, p. 267) used a similar procedure in his Table 1 considering that the infrared dispersion resonances of air are insignificant but notes that 'the great accuracy of dispersion measurements in the visible and near-infrared is lost in such an extrapolation'. Like Essen & Froome (1951) and Thayer (1974) before, Hill (1996) points out that the measured radio refractive index is larger than the extrapolated visible one. Essen & Froome (1951) cautioned that 'it is not certain' that the

extrapolation of the dry air refractivity to the radio wavelengths 'is completely valid'. In the end, Thayer (1974) derived his final K_1 from the refractivity value published by Smith & Weintraub (1953), which in turn obtained their value from a conversion of three published values of the dielectric constant of dry air, one being measured at optical wavelengths and one at 24 GHz. The claimed accuracy claimed by Thayer (1974) and Smith & Weintraub (1953) is, essentially, that of the optical measurements. The better of the two microwave measurements is worse by a factor two.

Thayer (1974) also 'extrapolates' the 'visible' water vapour refractivity to radio wavelengths and then computes the K_2 (Eq. (3)) value from it. He then uses Boudouris' (1963) water vapour measurements to derive the K_3 value. Hill (1996) states that 'the contributions to refraction by water vapour cannot be extrapolated to the infrared and radio regions because of the strong contribution by the infrared resonances of water vapour'. Hill et al. (1982) add that 'Thayer's hypothesis is unfounded' and 'suggest that one should use entirely empirical coefficients'. In consequence, the coefficients K_1 , K_2 and K_3 proposed by Thayer (1974) should **not** be used. Unfortunately, Thayer's coefficients were used later by many authors, particularly in the field of geodesy (e.g. Herring 1992, Jarlemark 1994, Mendes 1999).

Thayer (1974) is correct in pointing out that the precision quoted by Boudouris for his K_2 and K_3 coefficients cannot be used directly to estimate the precision of the derived water vapour refractivity because of the strong mathematical correlation between K_2 and K_3 . (It is unfortunate that Bevis et al. (1994) missed that comment when assessing the accuracy of their compiled coefficients.) Thayer (1974) stated the standard deviation of Boudouris' ($N_w T/p_w$) data about the linear regression fit as ± 2.2 ppm K/hPa. The precision of the water vapour radio refractivity N_w can be estimated by multiplication with p_w/T (hPa/K). Using the maximum water vapour pressures measured by Boudouris at his lowest, one of his intermediate and his highest temperature, the following precision estimates for N_w are obtained: ± 0.03 ppm (0.14%) at 0°C , $p_w = 4$ hPa (46% R.H.), ± 0.36 ppm (0.174 %) at 37.6°C , $p_w = 50.7$ hPa (78 % R.H.), ± 0.83 ppm (0.185%) at 63°C , $p_w = 126.7$ hPa (56% R.H.). Based on world records of temperature and humidity (his Fig. 2, p. 806), Thayer (1974) estimates the likely maximum error in N_w at realistic extreme combinations of temperature and humidity (37°C and 75% R.H.) as ± 0.3 ppm. Based on a comparison of his with the two-term Smith and Weintraub formula, Thayer estimates the actual overall error of the latter as 0.14 ppm (0.05% on N_r) to 0.5 ppm (0.1% of N_r) for very dry and very moist

air, respectively. This is five times better than the 0.5% quoted by Smith and Weintraub in 1953!

Hasegawa & Stokesberry (1975) gave a good summary of available K_1 , K_2 , and K_3 values that are based on measurement. They computed weighted means for each of the three coefficients from the standard deviations listed by the respective authors. Hill et al. (1982) criticised these authors for including the Essen & Froome and Essen data in their means because the latter were established over a very limited temperature range. In addition, these authors did not list the frequency at which the experiments were carried out and, thus, ignored the aspect of anomalous refractivity. The authors also ignored the correlation between the K_2 and K_3 terms that, unnecessarily and significantly, degraded some data, including those of Boudouris.

The International Radio Consultative Committee (CCIR) of the International Telecommunication Union (ITU), at the 16th Plenary Assembly, Dubrovnik 1986, in Recommendation 453-1, considering the necessity of using a single formula, unanimously recommended the following formula for the refractive index of radio waves:

$$N_r = (77.6/T) (P + 4810 (e/T)) \quad (6)$$

with e , P in hPa, T in K. Report 563-3 gives an equivalent formula (after Bean and Dutton), with an error of less than 0.5% (2.0 ppm) for frequencies of less than 100 GHz. The formula adopted by CCIR does not comprise compressibility factors and only one 'dry' term and one 'wet' term. The second (K_2) term of Eq. (2) is missing. Bean & Dutton (1968) derived Eq. (6) from the two-term Smith and Weintraub (1953, Eq. (10), p. 1036) formula, also quoted by Thayer (1974, Eq. (14), p. 806), for example. The second coefficient should be 4806.7 (= 373000/77.6); it was, most likely, rounded to the three significant digits like the first coefficient. Also, Eq. (6) assumes that T (K) = 273 + t (°C), a fact not mentioned in the ITU recommendation.

Zhevakin & Naumov (1967a) extended the usual formula for the refractivity to wavelengths longer than 10 μm by accounting for the rotational part of the dielectric permittivity ϵ_{rot} of water vapour and the magnetic permeability μ of molecular oxygen. In our notation, the Zhevakin & Naumov (1967a, Eq. (7), p. 887) formula reads

$$N_r = 77.607 \frac{P_{d-c}}{T} + 71.6 \frac{P_w}{T} + 0.5 \times 10^6 (\epsilon_{rot} - 1) + 129.34 \frac{P_c}{T} + 0.5 \times 10^6 (\mu - 1) \quad (7)$$

where p_{d-c} ($= p_d - p_c = p_{tot} - p_w - p_c$) is the (partial) pressure of the dry carbon dioxide free air, p_d is the (partial) pressure of the dry air ($= p_{tot} - p_w$), p_w is the partial water vapour pressure and p_c is the partial carbon dioxide pressure. The authors note that the constants K_1 (77.607 ± 0.013 K/hPa) and K_2 (71.6 ± 0.8 K/hPa) are practically constant for wavelengths longer than 10 μ m. The same applies for K_4 (carbon dioxide term, 129.34 ± 0.02 K/hPa) outside the wavelengths 12.9 to 17.1 μ m. Only the terms with ϵ_{rot} and μ are dispersive at wavelengths longer than 10 μ m. A formula for the computation of the dielectric permittivity ($\epsilon_{rot} - 1$) is given in Zhevakin & Naumov (1967b, Eq. (3), p. 1069). Formulae for the computation of $0.5 \times 10^6 (\mu - 1)$ are given elsewhere. The authors note, however, that the term $0.5 \times 10^6 (\mu - 1)$ does not exceed 0.43 ppm and constitutes less than 0.15 percent of N_r away from the resonant regions of oxygen (50 to 75 GHz or 4 to 6 mm, 116 to 126 GHz or 2.38 to 2.59 mm) in normal atmospheric conditions. A table lists the values of $0.5 \times 10^6 (\epsilon_{rot} - 1)$ and $0.5 \times 10^6 (\mu - 1)$ at $p_w = 10.14$ hPa and $T = 293$ K for wavelengths between 600 mm and 2.4 mm. At a wavelength of 600 mm, the N_r contribution is 43.09 ppm and 0.1234 ppm for the former and the latter, respectively. Zhevakin & Naumov (1967, Eq. (9), p. 890) also modelled the temperature dependence of the dielectric permittivity of water vapour for centimetre waves for the temperature range -50°C to $+70^\circ\text{C}$ as

$$(\epsilon_{rot} - 1) = K_3'' \frac{P_w}{T^{1.97}} \quad (8)$$

Using the value of $0.5 \times 10^6 (\epsilon_{rot} - 1)$ listed for $\lambda = 600$ mm, $p_w = 10.14$ hPa and $T = 293$ K, the K_3 constant can be derived as $307658 \text{ K}^{1.97}/\text{hPa}$. This coefficient is constant to 0.16 percent ($492 \text{ K}^{1.97}/\text{hPa}$) within the specified conditions. Substitution of this constant and Eq. (7) in Eq. (6) gives

$$N_r = 77.607 \frac{P_{d-c}}{T} + 71.6 \frac{P_w}{T} + 307658 \frac{P_w}{T^{1.97}} + 129.34 \frac{P_c}{T} + 0.5 \times 10^6 (\mu - 1) \quad (9)$$

Please note that the exponent of the temperature in this K_3 term is 1.97 and not 2 exactly, as in all other formulae.

Liebe (1969) calculated the dispersion and attenuation caused by water vapour resonance at 22.235 GHz. In doing so, the rotational H₂O spectrum was reduced to a single resonant term and a residual term. Liebe (1969, Eq. (4), p. 622) gave the frequency-independent refractive index as

$$N_r = \sum (x_i K_i) \frac{p_d}{T} + K_2 \frac{p_w}{T} + K_3 \frac{p_w}{T^2} \quad (10)$$

where p_d is the (partial) pressure of the dry air ($= p_{\text{tot}} - p_w$), p_w is the partial water vapour pressure and where the x_i are the fractions of the dry air components, K_i the corresponding displacement polarisation terms (constants) and T the temperature (in K). K_2 and K_3 are the displacement polarisation and dipole orientation terms of water vapour. After conversion to hectopascal, the water vapour constants given by Liebe (based on previous work by Zhevakin & Naumov 1967 and Liebe 1966) are: $K_2 = 71.6009$ K/hPa and $K_3 = 374656$ K²/hPa. The author lists the constants K_i for nitrogen, oxygen, argon and carbon dioxide. These were converted from Newell & Baird's (1965) measured refractivity values. No value is given for the sum $\sum(x_i K_i) = K_1$. Using the listed x_i (for clean dry air near sea level) and the listed K_i values (and using K_{nitrogen} for the unlisted K_i), one obtains $K_1 = 77.666$ K/hPa. This value of K_1 includes a CO₂ content of 314 ppm (US Standard Atmosphere at June 1962).

Liebe et al. (1977b) investigated the microwave spectrum of oxygen between 53.5 and 63.5 GHz. Strength and width parameters of 21 lines were determined (at less than 27 hPa) and the dispersion and attenuation measured between 133 hPa and 1066 hPa. As the spectrometer could also operate as a refractometer, the frequency-independent refractivities of 13 atmospheric gases were 'checked' at 61 GHz. These refractivities were determined at 61.16 GHz, 8°C, 0-67 hPa and relative to Newell & Baird's (1965) nitrogen refractivity. (Oxygen was measured at 59.59 GHz, where the anomalous refractivity is less than 0.02 ppm.) The coefficients determined experimentally by Liebe et al. (1977b, Table II) are $K_1 = 77.676 \pm 0.023$ [K/hPa] and $K_4 = 133.51 \pm 0.15$ [K/hPa]. Liebe et al. (1977b) quote values for K_2 and K_3 as follows: $K_2 = 71.631$ [K/hPa] and $K_3 = 374656$ [K²/hPa]. Noting that these values are listed in brackets, one has to assume that the coefficients were not actually measured. (The K_2 value is slightly different from that quoted by Liebe in 1969; the K_3 value is exactly the same.) The authors give, however, a measured value of the $(K_2 + K_3/T)$ term at 300 K as 1320.48 ± 1.88 [K/hPa]. This is consistent with the given K_2 and K_3 terms. It differs by

2.84 K/hPa from the $(K_2 + K_3/300)$ value of 1323.32 K/hPa computed from Boudouris' constants, by 0.030 K/hPa from the equivalent value of 1320.454 K/hPa computed from Liebe's 1966 constants and by 1.625 K/hPa from the equivalent value of 1322.103 K/hPa computed from the Smith-Weintraub constants quoted by Boudouris. An error of 2.84 K/hPa in the $(K_2 + K_3/300)$ term causes an error of 0.3 ppm in the refractive index at 27°C (300 K) and at a saturation water vapour pressure of 35.6 hPa. The following formula follows from the coefficients quoted by Liebe et al. (1977b)

$$N_r = 77.676 \frac{P_d}{T} + 71.631 \frac{P_w}{T} + 374656 \frac{P_w}{T^2} \quad (11)$$

In connection with his **Millimetre-Wave Propagation Model** (MPM, see Section 5 for details) and the associated experiments, Liebe did revise the coefficients of the radio wave refractivity formula over the years. Table 1 gives a summary of the changes in Liebe's dry air coefficient K_1 and the water vapour coefficients K_2 and K_3 . The table shows that Liebe (and Zhevakin et al. 1967b) initially started with the K_2 and K_3 coefficients proposed by Bean (1962) which, in turn, are based on a weighted mean of four experimental results listed by Birnbaum & Chatterjee (1952). Table 1 also gives a summary of the K_1 coefficients used by Liebe et al. over the years; the coefficient K_1 is quoted as 77.6314 ppm K/hPa by Liebe (1975b) and as 77.6764 ppm K/hPa by Liebe (1975a) and Liebe et al. (1977b). Since 1985, Liebe used $K_1 = 77.6400$ ppm K/hPa (e.g. Liebe 1985, 1996). The coefficients quoted by Liebe (1996) lead to the following equation for the radio refractivity:

$$N_r = 77.6400 \frac{P_d}{T} + 71.700 \frac{P_w}{T} + 374670 \frac{P_w}{T^2} \quad (11a)$$

Bevis et al. (1994) revisited the data used by Hasegawa & Stokesberry. They plotted the data, eliminated outliers and computed mean values for K_1 , K_2 and K_3 . Again, anomalous refractivity was not considered nor the appropriateness of averaging the K_2 and K_3 values separately, considering their high correlation. Like Hasegawa & Stokesberry (1975) before, the authors also included the Essen and Froome and the Essen data despite the cautionary remarks of Essen & Froome (1951). They also included Barrell's 1951 value that was extrapolated from the visible. In consequence, their standard errors (of the unweighted mean values) of ± 0.05 k/hPa for K_1 , ± 2.2 K/hPa for K_2 and ± 1200 K²/hPa for K_3 should be treated with care. The analysis by Bevis et al. (1994) is discussed further in Section 4.2.

| Source | K_1 | K_2 | $10^3 \times K_3$ | Ref(s) cited in paper |
|--|------------------------------|---------------------------|--------------------------|---|
| | ppm K/hPa | ppm K/hPa | ppm K ² /hPa | |
| Refractivity coefficients published by Liebe et al.: | | | | |
| Liebe (1996) | 77.6400 | 71.700 | 374.670 | none |
| Liebe et al. (1993) | 77.6400 | 71.700 | 374.670 | none |
| Liebe (1989) | 77.6400 | 71.700 | 374.670 | 'from microwave experiments at frequencies where dispersive contributions are negligible' |
| Liebe et al. (1987) | 77.6400 | 71.700 | 374.400 | 'from microwave experiments at frequencies where dispersive contributions are negligible' |
| Liebe (1985) | 77.6400 | 71.700 | 374.400 | Boudouris (1963), Liebe et al. (1977b) |
| Liebe et al. (1977b) | 77.6764 | 71.6309 | 374.656 | Liebe (1975b), Liebe (1977c) |
| Liebe (1977c) | 77.6764 | 71.6309 | 374.281 | |
| Liebe (1975b) | 77.6314 | 71.6309 | 374.656 | Newell & Baird (1965), Liebe (1975a) |
| Liebe (1975a) | 77.6764 | 71.6309 | 374.656 | none |
| Liebe (1969) | n/a | 71.6009 | 374.656 | Liebe (1966) |
| Liebe (1966) | n/a | 71.6009 | 374.686 | |
| Refractivity coefficients referred to by Liebe et al.: | | | | |
| Zhevakin et al. (1967b) | 77.607 | 71.6000 | 374.700 | Bean (1962) |
| Newell & Baird (1965) | 77.6735 (± 0.014) | n/a | n/a | |
| Boudouris (1963) | n/a | 71.9684 (± 10.5) | 375.406 (± 3.0) | (Boudouris' measurements only) |
| Bean (1962) | 77.607* ($\pm 0.013^*$) | 71.6000 (± 8.6) | 374.700 (± 3.1) | * derived from 3 dielectric const. (incl. 1 optical), assumed permeability and use of $T = 273 + t$ |

Table 1: Coefficients K_1 , K_2 and K_3 used by Liebe and collaborators, and cross references given.

For shorter (millimetre) wavelengths, Hill et al. (1980) published diagrams of anomalous refractivity caused by water vapour monomers and continuum covering the wavelength ranges 1.0 m to 5.682 μm and 5.102 μm (60 THz) to 4.854 μm . Liebe et al. (1991) modelled anomalous refractivity due to water vapour and oxygen from infinity to 0.3 mm (1 Hz to 1 THz). The work by Hill and Liebe and their coworkers has shown that (millimetre) radio waves are not free of dispersion (as previously assumed in

microwave distance measurement) and that line-by-line integration of the effects of resonance lines does not entirely reproduce the experimental refractivity values and must be corrected to the latter. The Millimetre-Wave Propagation Model (MPM, e.g. Liebe 1996, 1989) has been developed to account for the anomalous refractivity of oxygen and water vapour. This model is discussed later in some detail.

In 1996, Hill published an excellent review on the refractive index of atmospheric gases. Methane and nitrogen oxides have significant absorption lines that are, however, of no consequence at wavelengths larger than UV. So-called 'absorption windows' permit the derivation of simple and accurate refractive index formulae for specific ranges of wavelengths such as the visible, the infrared and the radio wave windows. The dependence of refractivity on the density of gases is described. It is noted that dispersion can be measured more accurately than absolute refractivity. The author lists the dispersion equations of Edlén (1966) and Peck & Reeder (1972) for visible and NIR waves. The effect on refractivity by water vapour cannot be neglected in the infrared. Line-by-line summation of the resonances of atmospheric molecules may be used between the visible and radio frequency windows. However, the line-by-line summation must be augmented by an extrapolation to the IR of the refractivity in the visible window to account for the resonances in the UV. The error in refraction calculations by line-by-line summation 'is probably due to errors in the compilation of resonances parameters'. In a table, the radio refractivities of nitrogen, carbon dioxide, water vapour, dry air, and oxygen are compared with extrapolated values from the visible. Dry air, nitrogen and oxygen have insignificant infrared dispersion resonances. Carbon dioxide and water vapour however have. The water vapour resonances in the infrared fully explain the difference between the extrapolated visible and the radio wave refractivity. The carbon dioxide difference is accounted for by the CO₂ rotational resonance near 15 μm . The CO₂ dispersion formula used by Edlén (1966) is not appropriate for extrapolation to longer wavelengths. The formula by Old et al. (1971) is better for this purpose.

Mendes (1999) reviewed a number of radio refractivity formula and used them to predict the total delay in measurements to GPS satellites. The dry air delays (in zenith direction) computed with the Boudouris and the Smith and Weintraub formulae differ by only 0.6 mm (being 0.026 percent of the dry zenith delay of 2.3 m). Mendes also compared different water vapour refractivity formulae but found that the differences in the wet zenith delay were negligible, that is smaller than 0.1 mm (Mendes 1999,

personal communication). Omission of the compressibility factor for dry air does not change the dry (zenith) delay. Omission of the water vapour compressibility factor changes the wet zenith delay by 0.1 to 0.2 mm. The omission of the enhancement factor (when computing the water vapour pressure in moist air) changes the wet zenith delay by a maximum of 1.3 mm at 30° latitude and in summer (being 0.5 percent of the wet delay of 0.258 m). Mendes also noted that the wet zenith delay can change by as much as 3 mm depending on the formula used for the computation of the saturation water vapour pressure. The wet zenith delays computed with saturation water vapour pressures after Wexler (Mendes, Eq. 5.6, p. 159) and Goff & Gratch (Mendes, Eq. 5.5, p. 158) agree within 0.1 to 0.2 mm. The wet zenith delays based on saturation water vapour pressures computed with the Magnus-Tetens formula (Mendes, Eq. 5.3, p. 158; Rüeger 1996 Eq. 5.25) agree to 0.3 mm with those obtained using the Wexler formula. To relate this information to terrestrial measurements, we note that similar dry and wet delays are experienced for a (one-way) horizontal path of 8.58 km length at sea level, 20°C and 28% relative humidity. For low lying satellites (elevation angle of 15°, zenith angle of 75°) the delays are 3.86 times larger, giving a value of 8.9 m for the dry delay and 1.00 m for the wet delay (at 30° latitude and in summer). The corresponding (one-way) horizontal sea level path length would be 33.1 km.

3. Specific Aspects

3.1 Accuracy

Boudouris (1963) and Hartmann (1993) quoted an accuracy of the radio refractivity of 0.5% (for moist air, $p_w = 27$ hPa) whereas Thayer (1974) claimed 0.02% (0.05 ppm) for dry air and 0.05% (0.21 ppm) for 'extremely moist air'. Walter (1990 p. 101, quoting Liebe 1983) states an accuracy of 0.05 ppm. Ignoring anomalous refractivity, Bevis et al. (1994) estimated a conservative accuracy of K_1 of 0.05 K/hPa (0.064 % of K_1 , 0.17 ppm at 20°C and 1013 hPa) for a mean value of 17 independent K_1 determinations. They rejected another three K_1 values. Rather than follow Thayer's approach and consider the correlation between K_2 and K_3 terms, Bevis et al. (1994) computed straight averages of seven independent determinations of K_2 and K_3 , including the Essen and Essen & Froome values, that are partly based on an extrapolation from the visible. Their accuracy values for K_2 and K_3 , thus, do not give a true indication of the accuracy of the computed wet refractivity.

The accuracy value of 0.5% (of N_T) for moist air radio refractivity, that is widely quoted in textbooks, is usually associated with simple two-term formulae used in telecommunications. For geodesy, a more realistic figure based on real data is required. As the accuracy of the radio wave refractivity is heavily dependent on the water vapour content, it is best to quote the accuracy of the dry and the wet terms separately.

3.1.1 Accuracy of the Refractivity of Dry Air

Considering the widely differing views on the accuracy of the radiowave refractive index equations, I made a new assessment of the accuracy based on experimental radio wave data. Two approaches were chosen. The first is based on the variability and the precision of the published values of directly measured 'dry' and 'wet' refractivity. The second looks at the variability and the precision of published refractivity coefficients K_1 , K_2 and K_3 that were derived from experimental data.

Boudouris (1963) and Newell & Baird (1965) tabled measured dry air radio refractivity (N_d) at 0°C and 1013.25 hPa. Computing (from Boudouris' Table 1) a mean of seven directly measured refractivity values, measured at 48 GHz or below, gives a standard deviation of ± 0.12 ppm of a single determination (about the mean of seven) and a standard deviation of ± 0.05 ppm for the mean (of seven), if uncorrelated. The former and latter are equivalent to 0.042% and 0.016 % of refractivity, respectively. Newell & Baird (1965) listed six dry air refractivity values, with individual standard deviations between 0.05 ppm and 0.2 ppm. A pooled standard deviation across the six determinations gives ± 0.12 ppm (or 0.042 % of N_d). Depending on the correlation between, and the common systematic errors inherent to the different determination, the accuracy of a mean dry air radio refractivity would be better than the pooled accuracy of ± 0.12 ppm (at 0°C and 1013.25 hPa, 0.042 % of N_d) of an individual determination.

Even though the values of the refractivity coefficients K_1 , K_2 , and K_3 are tabled by a large number of authors such as Birnbaum & Chatterjee (1952), Smith & Weintraub (1953), Bean (1962), Bean & Dutton (1968), Hasegawa & Stokesberry (1975), Hartmann & Leitinger (1984), Bevis et al. (1994) and Mendes (1999), only few compilations quote the precision of the coefficients. Hasegawa & Stokesberry (1975, Table II, p. 870) quote precisions of all the K_2 and K_3 coefficients that are listed, even though a number of the source papers do not list these. No indications are given by Hasegawa & Stokesberry (1975) as to how these precisions were obtained. The most

recent list is given by Bevis et al. (1994), based on the compilations by Boudouris (1963) and Hasegawa & Stokesberry (1975). After omitting three values for K_1 , they obtained the standard deviation of the (unweighted) mean from the variation of 17 values about the mean as ± 0.05 K/hPa (0.064 % of K_1). At 0°C and 1013 hPa, this translates into a precision of the dry refractivity of ± 0.19 ppm. This is a conservative estimate as some values used have very large standard deviations attached to them, one value was converted from optical wavelengths and two were based on measurements above 9 GHz. I did recompute a mean of the best 11 of the 20 values listed by Bevis et al. (1994, Table A1), ignoring 1, 3, 4, 6, 9, 11, 15, 16, 20, and obtained the standard deviation of the mean as ± 0.016 K/hPa and that of one independent value as ± 0.053 K/hPa. The former amounts to 0.021% of K_1 or ± 0.06 ppm at 0°C and 1013 hPa, the latter to 0.068% of K_1 or ± 0.20 ppm. The accuracy of 0.02% of K_1 (± 0.05 ppm) suggested by Thayer (1974, derived from the accuracy of optical measurements) is of similar magnitude as are the accuracy values of $\pm 0.017\%$ and $\pm 0.012\%$ stated by Newell & Baird (1965) for their K_1 coefficient and quoted for the newly computed weighted mean K_1 of Table 4, respectively.

| Coefficient K_1 | Precision of Dry Air Coeff K_1 | | | Sample Size n_i | Sample Reference |
|---|--|--|---|--------------------|--|
| | ppm K/hPa | % of K_1 (relative) | ppm (of n_r) (absolute)* | | |
| Best Published Value | ± 0.013 | ± 0.017 | ± 0.05 | (1) | Newell & Baird 1965 |
| Variation of one publ. K_1 value about mean of a number n_i of publ. K_1 | ± 0.03 ± 0.007 ± 0.05 ± 0.053 | ± 0.042 ± 0.009 ± 0.064 ± 0.068 | ± 0.12 ± 0.025 ± 0.18 ± 0.20 | 7 5 17 11 | Boudouris 1963 Newell&Baird 1965 # Bevis et al. 1994 this paper, Bevis subset |
| Standard dev. of arithm. K_1 mean from a number n_i of published K_1 values | ± 0.013 ± 0.003 ± 0.012 ± 0.016 | ± 0.016 ± 0.004 ± 0.016 ± 0.021 | ± 0.05 ± 0.011 ± 0.04 ± 0.06 | 7 5 17 11 | Boudouris 1963 Newell&Baird 1965 # Bevis et al. 1994 this paper, Bevis et al. subset |
| New weighted K_1 mean | ± 0.0094 | ± 0.012 | ± 0.035 | 10 | this paper, Table 4 |

Table 2: Summary information on the experimental precision of the dry air radio refractivity coefficient K_1 . * = The absolute precision (of the radio refractive index n_r) is obtained by multiplying the measurement precision (in Col. 2) with (1013.25 hPa/273.15 K). # = Froome's 1955 data not considered when taking the mean since they are affected by the 60 GHz oxygen line.

Table 2 shows a summary of the previously discussed precision values for the dry air refractivity and refractivity coefficient. The relative precisions, in percent of K_1 or, for that matter, of N_d , are the most useful numbers. The accuracy of 0.02% (± 0.05 ppm) suggested by Thayer (1974) might be an appropriate estimate (even though it was derived from optical measurements), since it can be realised by either a mean of several independent determinations or a very precise single experiment (as shown in Table 2).

3.1.2 Accuracy of the Refractivity of Moist Air

Boudouris (1963, Table 2) listed twelve water vapour refractivity values for radio waves at 20°C and 13.33 hPa water vapour pressure. (N_w is about 60.8 ppm at these conditions.) Computing a mean value from six independent measurements (with listed accuracy values and frequencies of less or equal than 9 GHz) gives a standard deviation of ± 0.38 ppm (0.62% of N_w) of the individual values about the mean and a standard deviation of ± 0.15 ppm (0.24% of N_w) of the mean (of six values). (The same result (± 0.15 ppm) is being obtained for the precision of a weighted mean.) The computation of a pooled value of all but the worst two accuracy values listed in Boudouris' Table 2 gives ± 0.58 ppm (0.95% of N_w) for the accuracy of an independent determination of water vapour refractivity. So, the independent determinations ($\pm 0.62\%$ of N_w) agree better than expected by the respective authors ($\pm 0.95\%$ of N_w). This is to be expected since most authors ignored the strong correlation between the K_2 and K_3 coefficients when calculating the precision of N_w . The accuracy of a mean water vapour radio refractivity would have to be better than the pooled accuracy of an individual determination (± 0.58 ppm at 20°C and 13.33 hPa, 0.95 % of N_w). The best determination (at a frequency of 9 GHz or less) listed by Boudouris (1963) has a precision of ± 0.2 ppm ($\pm 0.33\%$ of N_w). Thayer (1974) claimed an accuracy 0.05% (0.21 ppm) for 'extremely moist air'. This value cannot be compared directly with the values listed above, as it refers to the total refractivity N_T and not just the water vapour component. At 20°C and 1013 hPa, 0.05% of total radio refractivity N_T translate into 0.16 ppm, which are 0.27% of the water vapour refractivity N_w (at $p_w = 13.33$ hPa). This agrees with the accuracy of a mean quoted above but is 3.6 times better than the pooled accuracy of an independent determination (± 0.58 ppm at $p_w = 13.33$ hPa and 20°C).

Thayer (1974) assessed the accuracy of Boudouris' (1963) water vapour data on the basis of the deviation of Boudouris' ($N_w T/p_w$) data about a linear regression line. The

accuracy of the curve fit was ± 2.2 [ppm K/hPa]. To get the standard deviation of N_w one has to multiply this value (± 2.2) with (p_w/T) . At the most critical temperature-humidity combination of 37°C and 75 % relative humidity, the water vapour refractivity after Boudouris has an accuracy of ± 0.33 ppm, which is 0.17% of N_w or 0.076% of the total radio refractivity N_r . At 20°C and $p_w = 13.33$ hPa (57% R.H.) the accuracy of Boudouris' water vapour refractivity is ± 0.10 ppm or 0.16% of N_w or 0.03% of N_r .

Bevis et al. (1994) used another approach to assess the accuracy of the published water vapour refractivity constants (K_2 and K_3). They listed the constants published by seven authors, together with the standard deviations (after Hasegawa & Stokesberry 1975) assigned to the K_2 and K_3 constants and then computed an unweighted mean K_2 and an unweighted mean K_3 value as well as the standard deviation of the input values about these means. Since the K_2 and K_3 constants are strongly (mathematically) correlated, their precisions cannot be used to estimate the precision of derived water vapour refractivities without considering the applicable covariance. The standard deviations quoted by Bevis et al. in their Table A3 are, thus, too conservative. The mean values of K_2 and K_3 listed by these authors include Essen and Froome coefficients, which have been found to be inappropriate. See Section 2, Essen & Froome (1951) and Hill et al. (1982, p. 1256) for a discussion of this aspect. It is suggested that neither the mean values of K_2 and K_3 derived by Bevis et al. (1994), nor their quoted precisions, be used.

Without published correlation coefficients (or covariances) between K_2 and K_3 pairs, another approach for the assessment of the variation of the N_w computed with different published values for K_2 and K_3 had to be found. I computed the water vapour refractivity from the pairs of K_2 and K_3 coefficients listed by Bevis et al. (1994) and Liebe et al. (1977b) at 100 % relative humidity and temperatures between -20°C and $+50^\circ\text{C}$ (and for the worst practical temperature-humidity combination). In doing so, I omitted the Essen & Froome (1951) and the Essen (1953) data in Bevis et al.'s (1994) data set (see Section 2 for reasons) but added the Liebe et al. (1977b) data. The standard deviations of the six remaining water vapour refractivity values about their mean at each temperature were always between 0.29% and 0.33% of the N_w value computed. At the worst *practical* temperature-humidity combination (37°C , $p_w = 47.07$ hPa (75% R.H.), $p_d = 966$ hPa) used by Thayer (1974), the absolute error of the refractivity computed with one of the six formulas about the mean refractivity was ± 0.6 ppm (0.32% of N_w). This is significantly better than the precision (± 1.28 ppm) that follow under these atmospheric conditions from the precision of the mean K_3 quoted by

Bevis et al. (1994). As Table 3 shows, the value of ± 0.6 ppm ($\pm 0.32\%$ of N_w) determined here for six independent experiments is only the double of the value computed above for the Boudouris formula.

| Description | Precision of Water Vapour Refractivity N_w | | |
|---|--|---------------------------------------|---------------------------------------|
| | absolute [ppm] | relative to N_w [% of N_w] | relative to N_T [% of N_T] |
| At Moderate Humidity: (20°C, $p_w = 13.33$ hPa (57% R.H.), $p_d = 1000$ hPa, $N_w = 60.8$ ppm, $N_T = 326$ ppm) | | | |
| Evaluation of the Accuracy of the Boudouris (1963) data (Thayer 1974) | ± 0.10 | ± 0.16 | ± 0.03 |
| Variability of N_w computed from six equations (with K_2 and K_3 terms) about mean of six values | ± 0.19 | ± 0.31 | ± 0.06 |
| Accuracy of N_w from weighted mean coefficients [Eqs. (19a), (19b), Table 5] | $r = -0.5 \rightarrow$ | ± 0.10 | ± 0.17 |
| | $r = 0 \rightarrow$ | ± 0.13 | ± 0.22 |
| At Highest Practical Humidity: (37°C, $p_w = 47.07$ hPa (75% R.H.), $p_d = 966$ hPa, $N_w = 194$ ppm, $N_T = 425.8$ ppm) | | | |
| Evaluation of the accuracy of the Boudouris (1963) data (Thayer 1974) | ± 0.33 | ± 0.17 | ± 0.08 |
| Variability of N_w computed from six equations (with K_2 and K_3 terms) about mean of 6 values | ± 0.62 | ± 0.32 | ± 0.15 |
| Accuracy of N_w from weighted mean coefficients [Eqs. (19a), (19b), Table 5] | $\rho = -0.5 \rightarrow$ | ± 0.32 | ± 0.17 |
| | $\rho = 0 \rightarrow$ | ± 0.42 | ± 0.22 |

Table 3: Summary information on the expected precision of the water vapour refractive index N_w for moderate and highest practical humidity on the Earth's surface. The precision is quoted in absolute terms as well as relative to N_w and the total radio refractivity N_T . The precision (of N_T) relative to N_T is obtained by dividing the value in the ppm column by the listed N_w value. Similarly, the precision relative to the total radio refractivity is obtained by dividing the value in the ppm column by the listed N_T value.

Table 3 also lists the precisions that can be derived, with and without the application of a negative correlation coefficient, from the precisions of the best mean water vapour refractivity coefficients shown in Table 5. When using a correlation coefficient of -0.5, the precisions obtained agree with the precisions of the experiments (± 0.14 - 0.25% of N_w , see Tables 3 and 5). For general purposes, it is suggested to use the slightly more conservative value of 0.2% for the estimation of the precision of water vapour refractivity. This precision is met by most experiments, including those by Boudouris

(1963), Essen & Froome (1951) and Liebe et al. (1977b), is delivered by the new 'best average' formula (Eqs. 19a and 19b, see Tables 3 and 5) and can also be thought to apply to the variability of a mean of three (of the six) equations reported in Table 3 (since $\pm 0.32\% / \sqrt{3} = \pm 0.18\%$).

Naturally, the accuracy of the refractivity of moist air can be computed directly, using the propagation law of variances, from the quoted accuracy of the K_2 and K_3 coefficients, if the covariance (or correlation coefficient) between K_2 and K_3 is also known. The effectiveness of this approach will be demonstrated in Table 8 (Column 11), using the correlation coefficient of -0.995 that was obtained from a repetition of Boudouris' curve fit.

3.2 Compressibility Factors

Thayer (1974, Eq. (1)) suggested a closed formula (with compressibility factors Z) of the following form:

$$N_r = K_1 \frac{P_d}{T} Z_d^{-1} + K_2 \frac{P_w}{T} Z_w^{-1} + K_3 \frac{P_w}{T^2} Z_w^{-1} \quad (12)$$

The same author noted that the omission of compressibility factors leads to errors in the radio wave refractivity of 0.04 ppm in the dry term and 0.1 ppm in the wet term at high humidities. The former is of the same magnitude as the accuracy of the dry terms whereas the latter is half of the wet term accuracy claimed by Thayer and a twentieth of the accuracy quoted by others. In consequence, it is suggested to propose a closed formula for general use in geodesy and surveying without compressibility factors.

Some authors did use compressibility factors (or equivalent corrections) when reducing their measured refractivities to standard conditions whereas others did not. So, one could argue that coefficients K_i that were reduced with compressibility factors can be used directly in formulae with compressibility factors. On the other hand, coefficients K_i that were determined without compressibility factors can be used directly in formulae without compressibility factors. Considering Eq. (2), it becomes clear that the coefficients K_i in formulae without compressibility factors really cannot be numerically the same as those in formulae with compressibility factors. This aspect must yet be verified in all known formulae of interest in this context.

At this point it might be worthwhile to consider the significance of compressibility in the context of experiments. Liebe (1975a, pp. 109-117) reported on some anomalies found in his water vapour studies and made some general remarks on the difficulties of experiments with water vapour. The latter include the strong adsorption onto the surfaces of the cavity, the long periods required to reach equilibrium conditions and that water layers alter the detection capability of the spectrometer used. Figure 1 (Liebe 1975a, Fig. 39d, p. 114) shows the discrepancies between computed and measured refractivity close to saturation pressure for two experiments, one by Liebe (1975a) and one by Mrowinski (1970, p. 9). Liebe (1975a, p. 113) notes that he attributes the deviations in Fig. 1 to the 'real gas law' and that 'there is no satisfactory theory to account for the experimentally observed excess'. The compressibility corrections shown in Fig. 1 were computed by myself, using the compressibility factors quoted by Thayer (1974, Eq. 3) and Owens (1967, Eq. 31). (Thayer's correction is different because he 'simplified' and rearranged Owens' equation.) Figure 1 shows clearly that the anomalies found in the experimental data cannot be modelled by the compressibility factors. It could also be argued that, in the case of the experimental data shown in Fig. 1, the application of a compressibility correction would have introduced additional errors and that Owens' (1967) water vapour compressibility correction is not supported

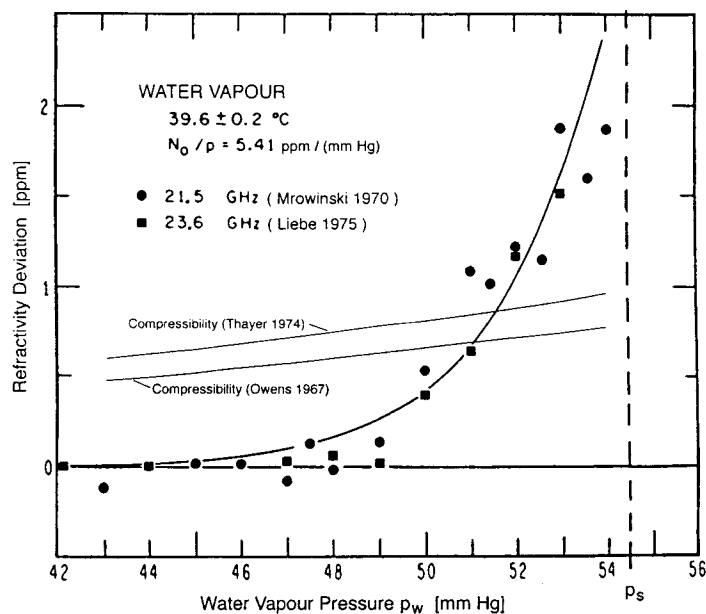


Figure 1: Deviations of the measured refractivity N from the computed refractivity (N_0) with $K_2 = 95.5$ (ppm K/mm Hg) and $K_3 = 499500$ (ppm K²/mm Hg) or with $K_2 = 71.631$ (ppm K/hPa) and $K_3 = 374655.8$ (ppm K²/hPa) at 39.6°C, for two experiments.

by the data shown in Fig. 1. Liebe (1975a) stated that the 'sketchy evidence of anomalies in the microwave spectrum of water ... warrant more systematic studies ... over a wider range of temperatures, frequencies and close to saturation'. One might add that the validity of the water vapour compressibility equation should be verified at the same time.

It might be desirable that precision formulae use compressibility factors Z_i and the matching K_i constants for dry (carbon dioxide free) air, water vapour and carbon dioxide terms even though the compressibilities change refractivity by less than the claimed accuracy of the formulae. To be consistent with the recommended formulae for the visible and NIR waves, the compressibility factors should be computed with the BIPM formulae. See Ciddor (1996) for reference. However, considering the poor agreement of experimental data with computed water vapour compressibility corrections in Fig. 1, it might be appropriate to investigate the validity of the compressibility factors used for visible and near infrared waves for use with radio waves.

3.3 Carbon Dioxide Content of Air

At Mauna Loa in Hawaii, the average carbon dioxide content of air was 338 ppm (0.0338%) in 1980 and 360 ppm (0.0360%) in 1994, with annual cycles of 6 ppm peak-to-peak (Heimann 1996). In the short term, this annual increase of 1.57 ppm per year extrapolates to a CO₂ content of 366 ppm in 1998, 369 ppm in 2000 and 375 ppm in 2004. In the long term, the CO₂ content will depend on the result of international agreements for the reduction of greenhouse gases. Depending on the adopted Intergovernmental Panel on Climate Change (IPCC) reduction scenarios, Taylor (1994) predicts CO₂ contents of 385–430 ppm in 2030, 385–485 ppm in 2050 and 365–610 ppm in 2100. It is suggested to use a carbon dioxide content of 0.0375% (375 ppm) in all new fixed CO₂ content formulae for outdoor measurements. Precision formulae should have an input for the ambient CO₂ content. (The new IAG resolutions (IAG 1999) on the refractive index of air for visible and near infrared waves are based on a CO₂ content of 375 ppm.)

These values naturally apply to outdoor measurements. It should be noted, however, that vegetation can locally increase the CO₂ concentration. McCoy et al. (1969) reported on a nocturnal 900 ppm CO₂ content near vegetation. In 1988, Birch & Downs

(1988) reported CO₂ contents of 400 to 800 ppm in laboratories. A maximum of 1200 ppm was noted on another occasion (Birch & Downs 1989). They adopted a default value of 450 ppm in their 1993 (laboratory) equation (Birch & Downs 1993).

3.4 Upper Frequency Limit for Closed Refractivity Formula

Different authors quote different upper limits for the non-dispersive region of radio wave refractivity. For example, Hartmann (1991) quoted 5 GHz, Thayer (1974) 20 GHz and Boudouris (1963) 30 GHz. These limits vary because of the magnitude of spurious effects tolerated and the accuracy of formulae assumed by these authors. Rüeiger (1996b) suggested 1 GHz as cut-off as Liebe (1996) suggests additional terms for dispersive refractivity above 1 GHz. This limit is sufficiently removed from the nearest resonance frequencies of water vapour and oxygen. Liebe (1983) and Hartmann (1991) list, for example, H₂O resonances at 22.23 GHz, 67.81 GHz, 119.99 GHz, 321.22 GHz, and O₂ resonances between 53.59 GHz and 66.30 GHz and at 118.75 GHz. Further evaluation of the magnitude of the effect of absorption lines might permit to extend the validity range of simple closed formulae to 5 GHz or 10 GHz.

3.5 Dispersive Refractivity (to 1 THz)

As Hill et al. (1980) and Liebe (1989) did, dispersive refractivity can be combined with experimental refractivity. Liebe's **Millimetre-Wave Propagation Model (MPM)** is the only operational model to account for 44 oxygen and 30 local water resonance lines plus an empirical water vapour continuum to offset experimental discrepancies (e.g. Liebe 1996). The CO₂ resonance at 15 μm should be sufficiently distant to cause anomalous refractivity between 1 Hz and 1 THz. The MPM is discussed in detail in Section 5. MPM93 can be downloaded from the web site of the Institute of Telecommunication Sciences (<http://www.its.blrdoc.gov> -> Resources -> Anonymous ftp -> pub/ -> mpm93/ -> refrac/).

In connection with a JPL/NASA research project, Hill (2000) prepared FORTRAN77 software routines (IR_N) for the calculation of the phase and group refractive indices of air and its gaseous constituents. The subroutines are applicable to wavelengths from the visible through to infinite (radio waves). The effects of absorption resonances of H₂O, CO₂, O₂, O₃, CH₄, CO and N₂O are modelled using the HITRAN data base. The effects of N₂ resonances were found to be negligible. Some other gases (without infrared

resonances) are also included; these are Ar, Ne, He, Kr, Xe and H₂. The new software is particularly useful for wavelengths between 0.002 mm and 0.3 mm where 'the refractive index of air was previously poorly known' (Hill 2000). Because of insufficient funding, there is presently no manual for nor a description of IR_N. The source code, however, has been released and can be obtained (on CD) from J. M. Rüeger. It should be noted that some programming by the user is required before IR_N can be used.

3.6 Phase, Group and Signal Velocity

In geodesy and surveying, it is generally assumed that the refractive index of radio waves is not dependent on the carrier frequency. In consequence, only the phase refractive index is considered. Microwave electronic distance measurements (EDM) used frequency modulation techniques to derive the distance measurements. GPS signals are code modulated but reconstructed carrier waves are usually used for the actual measurement in geodesy. It could be argued that, since some of the radio wave spectrum is dispersive, the concept of group or signal refractivity might have to be introduced where the propagation of modulated waves is used for measurements. If the visible spectrum can be taken as a guide then the anomalous group refractivity can be 100'000 times larger than the anomalous phase refractivity.

The question now arises if the concept of group velocity or signal velocity has to be introduced in the dispersive regions of the radio wave spectrum. According to Hufford (1998), 'signal velocity' is already used for the wave propagation in wave guides. Hufford (1987) did compute the millimetre-wave pulse distortion by a single absorption line simulating the terrestrial atmosphere and, therein, gives a reference to Trizna & Weber (1982), which also discuss the signal velocity for pulse propagation in a medium with resonant anomalous dispersion.

Further investigation and quantification of the difference between phase and signal velocity in the frequency range 1 Hz to 1 GHz (non-dispersive) and 1 GHz to 1 THz (dispersive) are clearly required.

3.7 Anomalous Refractivity Effects in Historic Data

Most radio wave refractivity data were measured above 1 GHz, where dispersive refractivity starts to have an effect. For example: Birnbaum & Chatterjee (1952; 9.28 GHz, 24.8 GHz), Boudouris (1963; 7 GHz to 12 GHz), Newell & Baird (1965; 47.7 GHz), Liebe (1969; 22.235 GHz), Liebe et al. (1977; 53.5 to 63.5 GHz). The last two determinations accounted for anomalous refractivity whereas the first three did not.

To get consistent data for the derivation of a dispersion-free radio wave refractivity, it would be of great benefit if anomalous refractivity were removed from experimental data. In particular, it would be worthwhile to remove anomalous refractivity from the values in Newell & Baird's Table III (1965) for dry air and oxygen. The poor agreement of Froome's oxygen value could be entirely due to dispersive oxygen refractivity.

4. Formula for Hand Calculations (1 Hz to about 1 GHz, ∞ m to 0.3 m)

It is valuable to have a relatively simple closed solution for the refractive index of radio waves for easy calculation with pocket calculators and personal computers. The equations given here are empirical, based on experiment and ignore the non-ideal gas behaviour (compressibility) of air. However, some constants have been derived after taking the real gas behaviour into account (e.g. Birnbaum & Chatterjee (1952) and Boudouris (1963)). Thayer (1974) stated that omission of the (dry air?) compressibility factor leads to errors of about 0.04 ppm at 500 hPa and that of the water vapour compressibility factor to 0.1 ppm at high humidities. Owens' (1967, p. 55) noted that 'the partial density of CO₂ is always so small that ideal gas behaviour may be assumed' for carbon dioxide.

The refractivity N_r of radio waves (in ppm) can be expressed as:

$$N_r = K_1 \frac{p_{d-c}}{T} + K_2 \frac{p_w}{T} + K_3 \frac{p_w}{T^2} + K_4 \frac{p_c}{T} \quad (13)$$

where p_{d-c} ($= p_d - p_c = p_{tot} - p_w - p_c$) is the (partial) pressure of the dry carbon dioxide free air, p_d is the (partial) pressure of the dry air ($= p_{tot} - p_w$), p_w is the partial water vapour pressure and p_c is the partial carbon dioxide pressure and where the K_i are

constants and T the temperature (in K). K_1' is the constant K_1 without the CO_2 component. Because of its polar nature, water vapour has a density and a density-temperature term (see Eqs. (6) and (7)).

Hartmann (1991 Eq.(7), with reference to Kruspe 1965) and Hartmann & Leitinger (1984) state that such a frequency independent formula assumes the validity of geometrical optics and applies to a non-ionised atmosphere and to radio frequencies below 5 GHz. Liebe (1996), however, suggests additional terms for the dispersive refractivity for frequencies above 1 GHz. This limit between the non-dispersive and the dispersive region is adopted here.

Over the years, a number of authors have assembled experimental refractivity coefficients of moist air, e.g. Birnbaum & Chatterjee (1952), Bean (1962), Newell & Baird (1965), Bean & Dutton (1968), Hasegawa & Stokesberry (1975), Hill et al. (1982), Hartmann & Leitinger (1984; note that their ' K_2 ' = $K_2 - K_1$), Walter (1990), Hartmann (1991, 1993; note that his ' K_2 ' = $K_2 - K_1$) and Bevis et al. (1994). A formula for hand calculations can be derived from either selecting the best possible values for each of the refractivity coefficients involved or by calculating 'mean' values for each coefficient from the most appropriate data sets available for each coefficient. To allow for a comparison, both approaches are followed here.

4.1 Equation Based on 'Best Available' Coefficients

After pointing out some erroneous assumption in Thayer's derivations, Hill et al. (1982) support the use of the coefficients by Boudouris (1963) and Birnbaum & Chatterjee (1952). (It should be noted that the agreement between the 1952 and 1963 data is much less favourable in the summary by Walter (1990)). Following a later suggestion by Hill (1995), the K_2 and K_3 terms are taken from Boudouris (1963) and the K_1' term (CO_2 free dry air) as well as the carbon dioxide term (K_4) from Newell & Baird (1965).

$$K_1' = 77.674 \pm 0.013 \text{ [K/hPa]} \quad (14a)$$

$$K_2 = 71.97 \pm 10.5 \text{ [K/hPa]} \quad (14b)$$

$$K_3 = 375406 \pm 3000 \text{ [K}^2\text{/hPa]} \quad (14c)$$

$$K_4 = 133.484 \pm 0.022 \text{ [K/hPa]} \quad (14d)$$

The precisions listed against K_1' and K_4 are one half of the 2σ values quoted by Newell & Baird (1965). Boudouris (1963) did derive the coefficients K_2 and K_3 from a linear regression solution from his measurements with temperatures from 0°C to $+63^\circ\text{C}$ and (water vapour) pressures from 0 to 127 mm hPa. A measured value for K_4 is also given by Liebe et al. (1977b) as 133.5 ± 0.15 [K/hPa]. Zhevakin & Naumov (1967) quoted a significantly different value of 129.30 ± 0.02 K/hPa for K_4 . Considering that the omission of the CO_2 -term would lead to an error of 0.02% only (Hartmann 1991, Hartmann & Leitinger 1984) and that the carbon dioxide content of air is rarely measured by geodesists and surveyors, it is often appropriate to adopt a current value for the CO_2 content and to merge the terms K_1' and K_4 to give the 'dry-air' K_1 term:

$$K_1 \frac{p_d}{T} = K_1' \frac{p_{d-c}}{T} + K_4 \frac{p_c}{T} = K_1' \frac{p_d - p_c}{T} + K_4 \frac{p_c}{T} = K_1' \frac{p_d}{T} + (K_4 - K_1') \frac{p_c}{T} \quad (15a)$$

Assuming initially the traditional 300 ppm (0.03%) content of carbon dioxide, p_c can be taken as $0.0003 p_d$.

$$K_1 \frac{p_d}{T} = K_1' \frac{p_d}{T} + 0.0003 (K_4 - K_1') \frac{p_d}{T} \quad (15b)$$

$$K_1 \frac{p_d}{T} = \{K_1' + 0.0003 (133.484 - 77.674)\} \frac{p_d}{T} \quad (15c)$$

$$K_1 \frac{p_d}{T} = \{77.674 + 0.017\} \frac{p_d}{T} = 77.691 \frac{p_d}{T} \quad (15d)$$

Using the propagation law of variances, it can be shown that the precision of K_1 is the same as that of K_1' . The final three-term equation (after Boudouris, Newell and Baird) for air with 0.03% (300 ppm) content of CO_2 then is:

$$N_r = 77.691 \frac{p_d}{T} + 71.97 \frac{p_w}{T} + 375406 \frac{p_w}{T^2} \quad (16a)$$

where the dry air (including carbon dioxide) pressure $p_d (= p_{\text{tot}} - p_w)$ and the partial water vapour pressure p_w are taken in hPa and the temperature T in K. The accuracy of an equivalent equation is estimated by Boudouris (1963, p. 661) to be within 0.5% for temperatures between -50°C and $+40^\circ\text{C}$, (total) pressures between 187 and 1013.25 hPa, partial water vapour pressures between 0 and 27 hPa and frequencies between 1 Hz and 30 GHz. A comparison of this with other formulae can be found in Tables 4 and 5.

In comparison, the coefficients determined experimentally by Liebe et al. (1977b, Table II) are $K_1 = 77.676 \pm 0.023$ [K/hPa], $K_2 = 71.631$ [K/hPa] and $K_3 = 374656$ [K²/hPa]. K_1 agrees within one standard deviation. Liebe et al. (1977b) do not quote individual precision values for K_2 and K_3 . Instead, they give the value and the precision of the (K_2+K_3/T) term at 300 K as 1320.48 and ± 1.88 , respectively. This differs by 2.84 K/hPa from the equivalent value of 1323.32 K/hPa computed from the Eq. (18), or 1.5 times the precision quoted by Liebe et al. (1977b).

Recomputing the K_1 term for the carbon dioxide content of 375 ppm (0.0375%) expected around the year 2004 gives the final form of the formula for the non-dispersive radio wave refractivity N_r (after Boudouris-Newell-Baird, in units of K and hPa)

$$N_r = 77.695 \frac{P_d}{T} + 71.97 \frac{P_w}{T} + 375406 \frac{P_w}{T^2} \quad (16b)$$

4.2 Equation Based on 'Best Average' Coefficients

Some of the authors that compiled known experimental values of refractivity coefficients (see introduction to this section) have also computed mean values of the coefficients for general usage, e.g. Bean (1962), Hasegawa & Stokesberry (1975) and Bevis et al. (1994). The use of 'best average' rather than 'best available' coefficients provides a certain robustness against unmodelled systematic errors and increases the reliability of the values, particularly if data from different laboratories and researchers can be averaged. As an alternative to the formula based on the 'best available' coefficients (see above), the available data have been revisited and new 'best average' coefficients has been computed.

Hasegawa & Stokesberry (1975) and Bevis et al. (1994) listed 20 values for the CO₂-free dry air refractivity coefficient K_1' . Both papers give the same values. The precisions of the K_1' coefficients were computed from the precisions given by Hasegawa & Stokesberry (1975) for the measured refractivities of dry air at 0°C and 1013.25 hPa. Boudouris (1963, Table I (1), p. 658) listed already 16 of the 20 data sets and gives more information on the source of the data. In particular, Boudouris (1963) stated the frequency at which the measurements were obtained and if the original publications stated the results in terms of refractive index or the dielectric constant. Hasegawa &

Stokesberry (1975) added the data sets 12, 14, 19, 20 (numbers as in Bevis et al. 1994) to Boudouris' list. Boudouris (1963) gives conflicting information on the conversion between the dielectric constant and the refractive index: In the text he suggests a magnetic permeability of $(\mu - 1) = 0.4 \times 10^{-6}$ for air but in his Table I he uses $\mu = 1$ for the conversion of the table values. Since the refractive index cannot be derived from the dielectric constant without an assumed magnetic permeability, all values of K_1' derived from the measurement of the dielectric constant are, here, not considered further. This applies to the data sets numbered 2, 3, 4, 5, 6, 9, 11, 16 by Bevis et al. (1994). Also omitted from the list of Bevis et al. (1994) are the data sets 2, 4, 6 (no precision given), Nos. 3, 4, 6, 11, 20 (outliers, as flagged by Bevis et al. (1994)), No. 1 (derived from optical wavelengths) and No. 15 (measured at 72 GHz and, thus, affected by the 60 GHz oxygen line).

Table 4 shows the remaining data from the lists of Hasegawa & Stokesberry (1975) and Bevis et al. (1994) as well as one added value (No. 21), by Liebe et al. (1977b). This gives 10 experimental values for K_1' . Boudouris (1963, top of page 661) states that 'measurements on dry air are made without CO₂'. Hasegawa & Stokesberry (1975, above their Table IV) confirm this statement for their K_1' values. Since the precisions stated for the experimental K_1' values in Table 4 are not affected by unknown mathematical correlations (as those of water vapour below) computing a weighted mean (with weights inversely proportional to the square of the listed standard deviations) is appropriate. The same approach was chosen by Bean (1962) and Hasegawa & Stokesberry (1975). (Bevis et al. (1994) computed the arithmetic mean, the weighted mean and the median, with the three values agreeing within 0.01 ppm K/hPa for their 20 input values.) The weighted mean for K_1' is obtained as 77.668₁ ppm \pm 0.0094 K/hPa. The variance factor (variance of observation of unit weight) of the adjustment (for the weighted mean) is 1.047 that indicates that the experimental precisions listed by the respective authors are consistent with the variability of the experimental data about the weighted mean.

A subset (of five) of the seven water vapour refractivity coefficients listed by Bevis et al. (1994, Table A3, p. 386) and Hasegawa & Stokesberry (1975, Table II, p. 870) is shown in the upper part of Table 5. The first four sets coincide with those that Bean (1962, Table II) derived from Birnbaum & Chatterjee's (1952) Debye constants (A, B) for molar polarisation. The three articles use the same K_3 values and their associated standard deviations. The K_2 values (and associated precisions) of Bean (1962) differ

slightly from those listed by Hasegawa & Stokesberry (1975) and Bevis et al. (1994). Hasegawa & Stokesberry (1975) may have used a slightly different conversion from the Debye constant A to the refractivity coefficient K_2 . The seven data sets used by Bevis et al. (1994) and Hasegawa & Stokesberry (1975) are themselves a subset of the 13 experimental water vapour refractivity values mentioned by Boudouris (1963, Table 2, p. 660). One optical value and three sets with no stated precision have been omitted from the analysis by Hasegawa & Stokesberry (1975) as have Froome's 1955 measurement at 72 GHz (no K_2 , K_3 stated) and a 1957 measurement by Battaglia, Boudouris and Gozzini. (See Boudouris (1963) for the full references to all papers.)

| No. | Author(s) | Date | Freq. [GHz] | K_1' [ppm K/hPa] | St. Dev. [ppm K/hPa] | Comments |
|---------------|----------------------------------|-------|----------------|-----------------------|-------------------------|--------------------------------------|
| 7 | Birnbaum, Kryder, Lyons | 1951 | 9 | 77.612 | 0.19 | |
| 8 | Essen & Froome | 1951 | 24 | 77.679 | 0.027 | |
| 10 | Ziemann | 1952 | 9 | 77.612 | 0.27 | |
| 12 | Gabriel | 1952 | ? | 77.585 | 0.03 | |
| 13 | Essen | 1953 | 9 | 77.682 | 0.027 | |
| 14 | Jansinski & Berry | 1954 | ? | 77.719 | 0.05 | |
| 17 | Battaglia, Boudouris, Gozzini | 1957 | 9 | 77.638 | 0.08 | |
| 18 | Boudouris | 1958 | 9 | 77.638 | 0.08 | |
| 19 | Newell & Baird | 1965 | 47.7 | 77.674 | 0.013 | |
| 21 | Liebe, Gimmestad, Hopponen | 1977b | 61 | 77.676 | 0.023 | Freq. indep. refractivity only |
| Weighted Mean | | | | 77.6681 | ± 0.0094 | ($\pm 0.012\%$ of K_1) |

Table 4: Experimental refractivity coefficients K_1' for dry air, without water vapour and without carbon dioxide. Apart of the data set No. 21, the numbers of the data sets and the standard deviations are those given by Bevis et al. (1994, Table 1A, p. 383). The K_1' values have been recomputed from the N values at 0°C and 1013.25 hPa given by Hasegawa & Stokesberry (1975, Table I, p. 870) by multiplication with 273.15/1013.25.

The primary quantity measured (through capacitive means) by Stranathan (1935), Hurdis & Smith (1942) and Groves & Sudgen (1934) was the dielectric constant. Stranathan (1935) listed the Debye constants (A, B) and their precisions as obtained by a least square fit of the product of molecular polarisation and temperature versus temperature. Groves & Sudgen (1935) quoted the water vapour dipole moment (and its

precision) as final result, but listed the pressure, polarisation and dipole moment for different temperatures. These authors made certain assumptions when deriving the dipole moments. Hurdis & Smyth (1942) also listed the polarisation and the dipole moment for a number of temperatures. No Debye constants nor precisions were given in the source paper. The Debye constants and their precisions for the Groves & Sudgen (1935) and the Hurdis & Smyth (1942) data were computed later by Birnbaum & Chatterjee (1952), and the K_2 and K_3 values listed below by Bean (1962) and Hasegawa & Stokesberry (1975). The later data by Birnbaum & Chatterjee (1952), Boudouris (1953), Essen & Froome (1951, 1953) and Liebe et al. (1977b) were based on cavity measurements of the refractive index at microwave frequencies. The instrumentation and the measurement techniques involved are discussed in Liebe (1975c), for example.

The experimental data shown in the upper part of Table 5 are those that have a precision stated and have been obtained over a wide range of temperatures (and, thus, allowed the experimental determination of both coefficients). In consequence, the Essen & Froome (1951) and the Essen (1953) data in Bevis et al. (1994, Table A3) selection were not considered for this new analysis. The K_2 and K_3 values and their associated uncertainties are those given by Bevin et al. (1994) and Hasegawa & Stokesberry (1975). The precision of N_w (last column, upper part of Table 5) was calculated from the N_w values (and their associated precisions) quoted by Boudouris (1963, Table 2, p. 660). It has been pointed out before that Boudouris (1963) did ignore the (mathematical) correlation between K_2 and K_3 when computing the precision of N_w at 20°C and 13.33 hPa. Other researchers might have done the same. To get a more realistic precision of measured refractivity, some relevant data have been added to the bottom part of Table 5. Three of the four data sets considered were measured at ambient conditions only (Essen & Froome (1951), Essen (1953) and Liebe et al. (1977b)) and, thus, did not allow the determination of K_2 and K_3 from the data. But, the precisions of the measured N_w or $(N_w T/p_w)$ at the specified temperature and water vapour pressure are valid (as are the quoted N_w) and give a good indication of the measurement precision. Boudouris' curve fit was revisited by Thayer (1974). The precision shown in the table for the Boudouris data is that of the $(N_w T/p_w)$ data about a linear regression fit. The precisions listed against Liebe et al. (1977b) also refers to a measured $(N_w T/p_w)$ value. The bottom part of Table 5 shows the excellent agreement between the precisions achieved by Boudouris, Essen, Froome and Liebe et al. Comparing these with the precisions shown for the other data sets (last column, upper part of Table 5) shows the incompatibility of the two sets of precision data. The

discrepancy between the precision values of Essen & Froome and those of the other data was already pointed out by Hill et al. (1982, Appendix).

As Hill et al. (1982, Appendix) explained, the separate averaging of the water vapour refractivity coefficients K_2 and K_3 is not satisfactory, since the values of K_2 and K_3 from different sources can differ significantly whereas the refractivity values computed from them can agree quite well. (See Table 3 for proof.) Hill et al. (1982) noted that 'it would seem preferable, but perhaps impossible, to fit (a curve) to a compilation of available experimental values of refractivity'. Since most sources do not provide the original measurements, independently averaging the K_2 and K_3 values is, however, the most direct approach. The criteria for the selection of the five data sets shown in the upper part of Table 5 have already been discussed. Next, a suitable weighting scheme has to be adopted. We know from Thayer's (1974) investigation of Boudouris' (1963) coefficients that the stated precisions of the coefficients (and those of the N_w at standard

| Author(s) | Date | Temp Range | Freq. | Weight p | K_2 | St. Dev. | K_3 $\times 10^3$ | St. Dev. $\times 10^3$ | Prec N_w at 20°C 13.33 hPa |
|-------------------------------|-------|---------------|--------|---------------|--|-------------|---------------------------|---------------------------|------------------------------------|
| | | [°C] | [GHz] | | [ppm K/hPa] | | [ppm K ² /hPa] | | [% of N_w] |
| Stranathan | 1935 | 21-189 | 0.5 | 1 | 72.86 | ± 7.05 | 373.6 | ± 2.5 | 0.65 |
| Groves & Sudgen | 1935 | 110-211 | 1.0 | 1/9 | 77.75 | ± 21.70 | 374.2 | ± 9.7 | 2.76 |
| Hurdis & Smyth | 1942 | 111-248 | 0.8 | 1/9 | 61.47 | ± 21.70 | 376.5 | ± 9.6 | 2.77 |
| Birnbaum & Chatterjee | 1951 | 25-103 | 9,24.8 | 1 | 69.43 | ± 13.02 | 377.4 | ± 4.3 | 1.13 |
| Boudouris | 1958 | 0-65 | 7-12 | 1 | 71.9684 | ± 10.50 | 375.406 | ± 3.0 | 1.30 |
| Weighted Mean | | | | | 71.2952 (± 1.3)* | | 375.463 (± 0.76)* | | |
| Essen & Froome | 1951 | amb | 24 | | | | | | 0.16 |
| Essen | 1953 | amb | 9 | | | | | | 0.25 |
| Boudouris | 1958 | 0-65 | 7-12 | | Prec. curve fit (Thayer 1974), $N_w T / P_w$ | | | | 0.16 |
| Liebe, Gimmestad, Hopponen | 1977b | amb | 61 | | Freq. independent component, $N_w T / P_w$ | | | | 0.14 |

Table 5: Experimental refractivity coefficients K_2 and K_3 for water vapour reported by different authors. (* = precision of weighted mean coefficient. Table 3 shows the precisions of N_w calculated with these values for two ambient conditions and with correlation coefficients $\rho = 0$ and $\rho = -0.5$.)

condition) do not give a good indication of the precision of any derived N_w values. Because of the correlation between K_2 and K_3 , the derived N_w values are better than the precisions in the upper part of Table 5 suggest.

Whereas the values by Stranathan (1935), Birnbaum & Chatterjee (1951) and Boudouris (1958) are based on very extensive experiments, well documented and cover the temperature range of interest in geodesy and surveying, the refractivity values of Groves & Sudgen (1935) and Hurdis and Smyth (1942) are based on a limited number of (8 to 11) measurements and were computed later by Birnbaum & Chatterjee (1952). Further considering the precisions listed in the upper part of Table 5, the assumption was made that the precision of the first, fourth and fifth data set was about the same and that the precision of the second and third set of coefficients was about 3 times worse. This relative weighting gives the weights listed in the table and was used to compute the weighted mean. The adjustment of the K_2 coefficients shows that the precision of a K_2 value of weight 1 is about ± 2.3 ppm K/hPa; similarly, the precision of a K_3 value of weight 1 is about $\pm 1.4 \times 10^3$ ppm K²/hPa. This means that the repeatability of the coefficients about the weighted mean is much better than the experimental precisions suggest. Realistic accuracy values for N_w values computed from K_2 and K_3 have been discussed in Section 3.1. The percentage values listed in the bottom half of Table 5 give another indication.

| Author(s) | Date | Frequency [GHz] | K_4 [ppm K/hPa] | St. Dev. [ppm K/hPa] |
|----------------------------|-------|--------------------|----------------------|-------------------------|
| Essen & Froome | 1951 | 24 | 133.172 | ± 0.27 |
| Heineken & Bruin | 1954 | 25 | 133.711 | ± 0.54 |
| Boudouris | 1958 | 9 | 133.037 | ± 0.27 |
| Newell & Baird | 1965 | 47.7 | 133.484 | ± 0.022 |
| Liebe, Gimmestad, Hopponen | 1977b | 61 | 133.511 | ± 0.150 |
| Weighted Mean | | | 133.4800 | ± 0.022 |

Table 6: Experimental refractivity coefficients K_4 for carbon dioxide reported by different authors. The values for the data sets Nos. 1, 2 and 3 were converted from Table 1 in Boudouris (1963), by multiplication of the measured refractivity by (273.13 K/1013.25 hPa).

Bevis et al. (1994) and Hasegawa & Stokesberry (1975) did not tabulate values of the refractivity coefficient of carbon dioxide. The main source of the values in Table 6 is

Boudouris (1963, Table 1(4)) who gave 13 refractivity values for CO₂ at 0°C and 760 mm Hg. Of these 13 values, 2 were omitted because they were measured at optical wavelengths, 7 because they were converted from measurements of the dielectric constant and one that is obviously in error. The remaining three values (for the data sets Nos. 1, 2 and 3 in Table 6) were then converted to K₄ coefficients by multiplication of the measured refractivity by (273.13 K/1013.25 hPa). The fourth and fifth values of K₄ in Table 6 were taken from the respective original publications. The weighted mean value of K₄ is 133.4800 ± 0.022 ppm K/hPa when using weights that are inversely proportional to the squares of the quoted standard deviations. The (a posteriori) variance factor of the adjustment is 1.031; this indicates that the agreement between the data in Table 6 is consistent with their quoted precisions.

Using the weighted mean coefficients from Tables 4 to 6, the following four-term formula can be constructed:

$$N_r = 77.6681 \frac{p_{d-c}}{T} + 71.2952 \frac{p_w}{T} + 375463 \frac{p_w}{T^2} + 133.4800 \frac{p_c}{T} \quad (17)$$

Using Eq. (15b), the older value of 300 ppm (0.03%) for the content of carbon dioxide and p_c as 0.0003 p_d, the refractivity coefficient for dry air with a CO₂ content of 300 ppm becomes

$$K_1 \frac{p_d}{T} = \{77.6681 + 0.0003 (133.4800 - 77.6681)\} \frac{p_d}{T} = 77.6848 \frac{p_d}{T} \quad (18)$$

The final three-term equation with weighted mean coefficients for air with 0.03% (300 ppm) content of CO₂ then is:

$$N_r = 77.6848 \frac{p_d}{T} + 71.2952 \frac{p_w}{T} + 375463 \frac{p_w}{T^2} \quad (19a)$$

where the dry air (including carbon dioxide) pressure p_d (= p_{tot} - p_w) and the partial water vapour pressure p_w are taken in hPa and the temperature T in K. The accuracy of the dry air refractivity component N_d is nominally 0.012% of N_d or, more conservatively (see Section 3.1), 0.02% of N_d. A realistic value for the accuracy of the water vapour component (N_w) is about 0.15% of N_w, as indicated by the values in the

bottom part of Table 5. A comparison of this with other formulae can be found in Tables 7 and 8.

Recomputing the K_1 term for the carbon dioxide content of 375 ppm (0.0375%) expected around the year 2004 gives the final form of the 'weighted mean' formula for the non-dispersive radio wave refractivity N_r (in units of K and hPa)

$$N_r = 77.6890 \frac{P_d}{T} + 71.2952 \frac{P_w}{T} + 375463 \frac{P_w}{T^2} \quad (19b)$$

5. Computer Routine (1 Hz to about 1 THz, ∞ m to 0.3 mm)

A practical model that simulates the complex refractive index for the propagation calculation of electromagnetic waves through the atmosphere has been developed by Liebe et al. over many years. The **Millimetre-Wave Propagation Model (MPM)** is as a program for frequencies below 1000 GHz in the atmosphere running on personal (IBM compatible) computers. The MPM 'consists of 44 oxygen and 30 local water resonance lines, of non-resonant spectra for dry air and of an empirical water vapour continuum that reconciles experimental discrepancies' (Liebe et al. 1992). The model is applicable for barometric pressures between 0 and 1200 hPa, ambient temperatures between -100 and +50°C, relative humidity between 0 and 100% and suspended water droplets and ice particle densities between 0 and >5 g/m³. (Other versions of MPM also model rainfall conditions.) The complete (complex) refractivity model is as follows (Liebe et al. 1992, Liebe et al. 1993, Liebe 1996):

$$N = N_D + N_V + N_{W,I} \quad (20)$$

Complex parameters are shown in bold type. Only the real-part is required for the computation of the refractivity. The complex refractivity $N_{W,I}$ of suspended water droplets and ice particles (for example in fog and in clouds) is of no direct interest in this context. Details can be found in Liebe et al. (1992). The complex dry-air refractivity N_D is computed (in ppm) from (Liebe et al. 1992, Liebe 1996)

$$N_D = N_d + \sum_k S_k F_k + S_o F_o + i S_n F_n'' \quad (21)$$

where N_d is the non-dispersive (dry-air) term, k is the index of the 44 oxygen resonances, S_k is the line strength and F_k the complex spectral shape function. S_o and F_o model the non-resonant refractivity below 10 GHz from the oxygen relaxation spectrum. The $S_n F_n''$ term models the pressure-induced nitrogen absorption above 100 GHz. The MPM computes the refractivity of atmospheric water vapour N_V (in ppm) as follows (Liebe et al. 1992, Liebe et al. 1993, Liebe 1996):

$$N_V = N_v + \sum_l S_l F_l + S_o F_o + N_C \quad (22)$$

where N_v is the non-dispersive water vapour refractivity, l is the index of the 30 local water resonances, S_l is the line strength and F_l the shape function. The continuum refractivity N_C models contributions over and above the 30 local lines and is partly based on the work by Hill (1988).

The MPM makes use of spectral data and is supported by many laboratory measurements to validate and enhance the overall performance of the model. The authors note that MPM dry-air absorption values agree with measured ones at the 1% level. 'Model predictions involving water vapour and water droplets are estimated to lie in the 10 percent range' (Liebe et al. 1992). MPM does not consider the weak spectra of trace gases such as O_3 , CO and N_2O nor does it provide an input for the CO_2 content. Presumably, a carbon dioxide content of 0.03% is assumed and included in the dry-air non-dispersive term. Table 1 shows the historical development of the coefficients K_1 , K_2 and K_3 used by Liebe and collaborators in the MPM and their likely source.

The MPM93 is generally available (in a number of variants), fully documented and includes an executable file for IBM PCs as well as the FORTRAN source code. It can be downloaded from the web site of the Institute of Telecommunication Sciences (<http://www.its.blrdoc.gov> -> Resources -> Anonymous ftp -> pub/ -> mpm93/ -> refrac/ or directly from <ftp://ftp.its.blrdoc.gov/pub/mpm93/refrac>). The REFRAC software is the most useful variant of MPM93 for use in geodesy and surveying.

It has been mentioned in Section 3.5, that Hill (2000) prepared FORTRAN77 software routines (IR_N) for the calculation of the phase and group refractive indices of air and its gaseous constituents in connection with a JPL/NASA research project. The source code can be obtained (on CD) from J. M. R ueger. It should be noted that there is

presently no manual for, nor a description of, IR_N and that some programming by the user is required before IR_N can be used.

6. Comparison of Formulae

Tables 7 and 8 show a comparison of the **Millimetre-wave Propagation Model 1993** (MPM, non-dispersive refractivity (N_o) only) with the simple formulae by Essen & Froome (1951, as adopted by IUGG in 1960 and 1963), Boudouris (1963), a formula based on coefficients determined by Liebe et al. (1977b), the formula recommended by the International Telecommunication Union in 1986 (after Bean & Dutton 1966 and Smith & Weintraub (1952)), a new formula based on the best available coefficients (Eq. (16a), after Boudouris (1963) and Newell & Baird (1965)) and a new formula based on the best average coefficients (Eq. 19a).

| T | PWVP | MPM93 | E&F51 IUGG'60 | Liebe 1977 | Boud 1963 | Best Available | Best Average | CCIR'86 |
|------|----------------|----------------|---------------------------|----------------------------|---------------------------|-----------------------------|-----------------------------|---------------------------|
| [°C] | P_w [hPa] | N_r [ppm] | Eq. (1) N_r [ppm] | Eq. (11) N_r [ppm] | Eq. (4) N_r [ppm] | Eq. (16a) N_r [ppm] | Eq. (19a) N_r [ppm] | Eq. (6) N_r [ppm] |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| 60 | 199.26 | 902.2 | 892.8 | 902.2 | 903.5 | 903.7 | 903.4 | 903.0 |
| 45 | 95.85 | 597.1 | 592.2 | 597.1 | 597.7 | 598.0 | 597.8 | 597.4 |
| 30 | 42.43 | 428.3 | 426.0 | 428.4 | 428.5 | 428.8 | 428.7 | 428.3 |
| 15 | 17.04 | 346.0 | 345.0 | 346.1 | 346.0 | 346.3 | 346.3 | 345.9 |
| 0 | 6.10 | 314.8 | 314.3 | 314.9 | 314.6 | 315.0 | 315.0 | 314.6 |
| -15 | 0.00 | 300.8 | 300.8 | 300.9 | 300.6 | 301.0 | 300.9 | 300.6 |
| -30 | 0.00 | 319.3 | 319.3 | 319.5 | 319.1 | 319.5 | 319.5 | 319.1 |

Table 7: Radio wave refractivity N_r (in parts per million) computed with the **Millimetre-Wave Propagation Model** and a number of simple formulae at selected temperatures, 1000 hPa total pressure, 300 ppm CO₂ content and 100% relative humidity.

A total pressure (p_{tot}) of 1000 hPa exactly was used for the comparison. To be consistent with the historical equations, the Eqs. (16a) and (19a) for a CO₂ content of 0.03% (300 ppm) were used (rather than Eqs. (16b) and (19b) for 375 ppm carbon dioxide). The relative humidity was set at 100% for temperatures between 0°C and 60°C. The saturation water vapour pressures used for the computations of Columns 4 to

9 are shown in Column 2 and were taken from Rüeiger (1990, 1996, Appendix B, after Goff & Gratch 1946). MPM uses relative humidity as input and converts it to partial water vapour pressure using the equations of Goff & Gratch (1946).

| T | PWVP P _w | Prec Eq. 16a ρ=0.0 | Prec Eq. 16a ρ=0.995 | (4-3) E&F51 minus MPM93 | (5-3) Lie77 minus MPM93 | (6-3) Bou63 minus MPM93 | (7-3) Eq. 16a minus MPM93 | (8-3) Eq. 19a minus MPM93 | (9-3) CCIR '86 minus MPM93 |
|------|------------------------|--------------------------|----------------------------|----------------------------------|----------------------------------|----------------------------------|------------------------------------|------------------------------------|-------------------------------------|
| [°C] | [hPa] | [ppm] | [ppm] | [ppm] | [ppm] | [ppm] | [ppm] | [ppm] | [ppm] |
| (1) | (2) | (10) | (11) | (12) | (13) | (14) | (15) | (16) | (17) |
| 60 | 199.26 | ±8.3 | ±1.2 | -9.4 | 0.0 | 1.4 | 1.6 | 1.3 | 0.9 |
| 45 | 95.85 | ±4.2 | ±0.5 | -4.8 | 0.0 | 0.6 | 0.9 | 0.7 | 0.3 |
| 30 | 42.43 | ±2.0 | ±0.2 | -2.3 | 0.1 | 0.2 | 0.5 | 0.4 | 0.0 |
| 15 | 17.04 | ±0.9 | ±0.1 | -1.0 | 0.1 | 0.0 | 0.3 | 0.3 | -0.1 |
| 0 | 6.10 | ±0.3 | ±0.0 | 0.0 | 0.1 | -0.1 | 0.2 | 0.2 | -0.2 |
| -15 | 0.00 | ±0.0 | ±0.0 | 0.0 | 0.1 | -0.2 | 0.2 | 0.2 | -0.2 |
| -30 | 0.00 | ±0.0 | ±0.0 | 0.0 | 0.1 | -0.2 | 0.2 | 0.2 | -0.2 |

Table 8: Comparison of the non-dispersive part of the radio wave refractivity N_r (in parts per million) from the Millimetre-Wave Propagation Model and a number of simple formulae at selected temperatures, 1000 hPa total pressure, 300 ppm CO₂ content and 100% relative humidity.

The precision of the new 'best available' formula (Eq. 16) was predicted using the propagation law of variances and the given precisions of the constants K_1 , K_2 and K_3 . Column 10 in Table 8 gives the precision without consideration of the correlation between the constants K_2 and K_3 (correlation coefficient $\rho = 0.0$). Column 11 in Table 8 uses a correlation coefficient of -0.995 between the two constants to compute the covariance between them. This correlation coefficient was obtained from a repetition of Boudouris' curve fit.

Column 12 of Table 8 shows clearly that the Essen & Froome equation (1951) differs significantly from the other three models at high temperature and humidity. The water vapour refractivity used by Essen & Froome is clearly not optimal. On the other hand, the Essen & Froome formula agrees marginally better than any other with Liebe's MPM below freezing point and at zero humidity. Not unexpectedly, the formula ('Liebe77') based on the K_1 , K_2 and K_3 terms by Liebe et al. (1977b) agrees very well with the MPM (see Column 13, Table 8); the differences do not exceed 0.15 ppm between -30°C and +60°C. Boudouris' formula agrees slightly better with the MPM93 than Eq. (16a)

derived above (from the 'best available' coefficients). The differences are, however, smaller than 1 ppm at temperatures below 50°C. Eq. (16a) gives values that are, on average, 0.3 ppm higher than those of Boudouris. This is expected since Newell & Baird's K_1 constant is slightly larger than that of Boudouris and as the Eq. (16a) (see Column 7 in Table 7) uses the same K_2 and K_3 constants as Boudouris' equation (Column 6, Table 7). The new Eq. (19a) (see Column 16 in Table 8) shows similar deviations from MPM93 that are similar to those of Boudouris' equation and Eq. (16a). At negative temperatures and zero humidity, the Boudouris' formula (see Eq. (4) in this paper) and the new Eqs. (16a) and (19a) have offsets from the MPM93 that are of the same magnitude but different sign. The differences in Columns 14 to 17 of Table 8 compare better with the precision values in Column (11), which take account of the (mathematical) correlation between K_2 and K_3 , than with those in Column (10), which do not. It follows that the correlation between K_2 and K_3 should not be ignored when predicting the precision of computed refractivity.

Table 8, in Columns 14 to 16, also shows that the MPM93 seems to underestimate the radio refractive index at high humidities and high temperatures when compared to the four alternative formulae investigated.

7. Conclusions

The radio refractive index formula (after Essen & Froome) recommended in Resolution No. 1 of the 13th General Assembly of IUGG (Berkley 1963) is now clearly out of date, as the Column 12 in Table 8 shows. With new absolute and relative measurements of the mid-infrared to radio wave refractive index of air being available, and with considerable advances having been made with the computation of anomalous refractivity in the mid-infrared to radio wave spectrum, there is a clear need to rescind the 1963 IUGG resolutions and to propose a more appropriate alternative.

Since a number of authors, particularly in the field of geodesy (e.g. Herring 1992, Jarlemark 1994, Mendes 1999), have started using the radio refractive index formula and coefficients of Thayer (1974), it is stressed again that the coefficients K_1 , K_2 and K_3 proposed by Thayer (1974) should **not** be used. It has been explained in Section 2 that Thayer's K_1 and K_2 have been derived, at least in part, from optical data. Regarding Thayer's K_1 coefficient, Hill (1996, p. 267) notes that 'the great accuracy of dispersion measurements in the visible and near-infrared is lost in such an extrapolation'. Hill

(1996) states, in relation to Thayer's K_2 and K_3 coefficients, that 'the contributions to refraction by water vapour cannot be extrapolated to the infrared and radio regions because of the strong contribution by the infrared resonances of water vapour'.

On the basis of Tables 7 and 8, it is not possible to choose a 'best' simple formula for hand calculations. If the MPM is taken as reference, then a formula of the type of Eq. (19), with the constants K_1 , K_2 and K_3 determined by Liebe et al. (1977b), produces the best agreement. On the other hand, if the newly derived 'best available' and 'best average' formulae (see Eqs. (16) and (19)) are taken as reference, then the MPM needs some fine tuning. The relative merits of the K_2 and K_3 values by Boudouris (1963) and Liebe et al. (1977b) as well as of the K_1 values by Newell & Baird (1965) and Liebe et al. (1977b) need to be established before a final decision on an appropriate course of action can be taken.

| Temp. | -30°C | -15°C | 0°C | +15°C | +30°C | +45°C | +60°C |
|---|-------|-------|-------|-------|--------|-------|--------|
| SWVP (hPa) | 0.38 | 1.65 | 6.12 | 17.04 | 42.434 | 95.86 | 199.26 |
| At Sea Level, p = 1000 hPa: | | | | | | | |
| RH = 0 % | 0.06 | 0.06 | 0.056 | 0.05 | 0.05 | 0.05 | 0.05 |
| RH = 15% | 0.06 | 0.06 | 0.07 | 0.08 | 0.11 | 0.16 | 0.26 |
| RH = 50% | 0.07 | 0.07 | 0.09 | 0.13 | 0.23 | 0.42 | 0.76 |
| RH = 75% | 0.07 | 0.07 | 0.11 | 0.18 | 0.32 | 0.61 | 1.11 |
| RH = 100% | 0.07 | 0.08 | 0.12 | 0.22 | 0.42 | 0.80 | 1.47 |
| At 4000 m above Sea Level, p = 600 hPa: | | | | | | | |
| RH = 0% | 0.04 | 0.04 | 0.03 | 0.03 | 0.03 | 0.03 | |
| RH = 15% | 0.04 | 0.04 | 0.04 | 0.06 | 0.09 | 0.14 | |
| RH = 50% | 0.04 | 0.05 | 0.06 | 0.11 | 0.21 | 0.40 | |
| RH = 75% | 0.04 | 0.05 | 0.08 | 0.15 | 0.30 | 0.59 | |
| RH = 100% | 0.04 | 0.05 | 0.10 | 0.19 | 0.40 | 0.78 | |

Table 9: Estimated precision, in absolute terms (in parts per million, ppm), of radio refractive index formulae at different ambient temperatures, humidities and pressures based on a relative precision of $\pm 0.02\%$ of dry air refractivity (N_d) and $\pm 0.2\%$ of water vapour refractivity (N_w).

It has been shown in Section 3.1 that the most meaningful precision information is quoted in percent of the refractivity N_i or in percent of the refractivity coefficient K_i . The estimated (absolute) precisions (in parts per million, ppm) of the radio refractive index formulae are given in Table 9 for two cases, namely 'sea level' and '4000 m above sea level'. Tables 3 and 4 indicate that the assumed relative precision of $\pm 0.02\%$ for the

dry air refractivity (N_d) and of $\pm 0.2\%$ for the water vapour refractivity (N_w) are realistic and, possibly, somewhat conservative.

The Millimetre-Wave Propagation Model (MPM) of the (US) National Telecommunications and Information Administration (NTIA) in Boulder presently best meets the IAG requirements for a computer procedure for the computation of the phase refractive index of radio and millimetre waves. Mr. M. Cotton of NTIA/ITS has started work for an IAG version of the MPM derivative program REFRAC, provisionally named MPM-IAG. Some changes to MPM suggested by Hill (1988, Eq. 9, Table II) have already been implemented. Work for the input of a variable CO_2 content has begun. It will be based on $K_4 = 133.5 \pm 0.15$ (K/hPa) and used a default content of 375 ppm. The coefficients K_1 to K_3 of the continuum formula used in MPM might require some fine tuning after consideration of Tables 7 and 8. The user interface of MPM-IAG has been changed from that of REFRAC to simplify the use of the program.

Further questions to be addressed are that of using the BIPM saturation water vapour pressure formula (and enhancement factor) as used in Ciddor (1996) and Ciddor & Hill (1999) and the question of the suitability or necessity of compressibility factors. It is possible, that an introduction of compressibility factors into MPM and REFRAC-IAG could compromise the integrity of the package. The poor agreement of the compressibility correction of water vapour with experimental data (see Fig. 1) is of concern and should be investigated further. The significance of the difference between the signal velocity and the phase velocity in geodetic measurements through the atmosphere needs also to be established.

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**Some Resolutions Adopted by the
International Association of Geodesy (IAG)
during its 22nd General Assembly
Birmingham, U.K., 19 - 30 July 1999**

(see <http://www.gfy.ku.dk/~iag/resolutions/> for complete listing)

Comptes Rendus

**RESOLUTIONS ADOPTED AT THE XXII GENERAL ASSEMBLY
BY THE IUGG ASSOCIATIONS**

International Association of Geodesy

RESOLUTION 3

The International Association of Geodesy,

- recognising
1. the accuracy of the instrumentation used for terrestrial electronic distance measurement and for measurements to satellites has improved greatly since IUGG adopted a resolution on the refractive index of air in 1963,
 2. new absolute and relative measurements of the refractive index of air have been made since 1963,
 3. more accurate refractive index formulae have been developed and older formulae have been found to be in error since 1963,
 4. the international temperature scale was revised in 1990, and
 5. a carbon dioxide content of air of 300 ppm is no longer appropriate noting the continuum dispersion formulae used by the recommendation below do not account for the effects of anomalous refractivity due to molecular resonances in the visible and near-infrared,
- recommends
1. sub paragraphs (a) and (b) of Resolution No. 1 of the 13th General Assembly of IUGG (Berkeley 1963) be cancelled
 2. the group refractive index in air for electronic distance measurement to better than one part per million (ppm) with visible and near infrared waves in the atmosphere be computed using the computer procedure published by Ciddor & Hill in *Applied Optics* (1999, Vol.38, No.9,1663-1667) and Ciddor in *Applied Optics* (1996, Vol. 35, No.9, 1566-1573),
 3. the following closed formulae be adopted for the computation of the group refractive index in air for electronic distance measurement (EDM) to within 1 ppm with visible and near infrared waves in the atmosphere:

$$N_L = (n_L - 1) 10^6 = \left(\frac{273.15}{1013.25} \frac{N_t P}{T} \right) - \frac{11.27e}{T}$$

where N_L is the group refractivity of visible and near infrared waves in ambient moist air, T is the temperature in Kelvin (ITS-90), $T = 273.15 + t$, t is the temperature in degrees Celsius ($^{\circ}\text{C}$), p the total pressure in hectopascal (hPa) and e the partial water vapour pressure in hectopascal (hPa) and n_L is the corresponding group refractive index.

The group refractivity N_g of standard air with 0.0375 % CO_2 content at $T = 273.15 \text{ K}$ (0°C), $p = 1013.25 \text{ hPa}$, $e = 0.0 \text{ hPa}$ is as follows:

$$N_g = (n_g - 1) 10^6 = 287.6155 + \frac{4.88660}{\lambda^2} + \frac{0.06800}{\lambda^4}$$

where λ is the carrier wavelength of the EDM signal (in micrometre,) and n_g the corresponding group refractive index. These closed formulae deviate less than 0.25 ppm from the accurate formulae (see (2) above) between -30°C and $+45^{\circ}\text{C}$, at 1000 hPa pressure, 100% relative humidity (without condensation) and for wavelengths of 650 nm and 850 nm, for example. The 1 ppm stated before makes some allowance for anomalous refractivity and the uncertainty in the determination of the atmospheric parameters. Where required, the phase refractivity N_{ph} of standard air with 0.0375 % CO_2 content at $T = 273.15 \text{ K}$ (0°C), $p = 1013.25 \text{ hPa}$, $e = 0.0 \text{ hPa}$ may be calculated as follows:

$$N_{ph} = (n_{ph} - 1) 10^6 = 287.6155 + \frac{1.62887}{\lambda^2} + \frac{0.01360}{\lambda^4}$$

where λ is the carrier wavelength of the signal (in micrometre,) and the corresponding phase refractive index.

Sponsored by the Ad-Hoc Working Group on Refractive Indices of Light, Infrared and Radio Waves in the Atmosphere (convener: J. M. R ueger) of the IAG Special Commission SC3 on Fundamental Constants (SCFC).

RESOLUTION 4

The International Association of Geodesy,

- recognising
1. the ultimate limit set by anomalous refractivity, due to absorption lines, to the accuracy of any continuous visible and near infrared refractive index formula, particularly the group refractive index,
 2. the scarcity, particularly in the near infrared, of the absolute refractivity measurements of dry air and moist air, on which present dispersion formulae are based, and
 3. the scarcity of direct measurements of the group refractive index, and
- noting
- the preliminary work done towards computing the magnitude of anomalous phase and group refractivity in the visible and near infrared,
- recommends
1. further work on the effect of absorption lines on the phase and group refractive indices of air so as to compute the magnitude of anomalous refractivity for specific instruments and, ideally, provide software to correct for such effects, and
 2. new absolute measurements of the refractivity of the constituent gases of the atmosphere (including water vapour) under non-laboratory conditions, with special emphasis on near infrared wavelengths.

Sponsored by the Ad-Hoc Working Group on Refractive Indices of Light, Infrared and Radio Waves in the Atmosphere (convener: J. M. R ueger) of the IAG Special Commission SC3 on Fundamental Constants (SCFC).

International Association of Geodesy

**Report
of the
Ad-Hoc Working Party on**

**REFRACTIVE INDICES OF LIGHT, INFRARED
AND RADIO WAVES IN THE ATMOSPHERE**

of the

IAG Special Commission SC3 – Fundamental Constants (SCFC)

1993 – 1999

J. M. Rüeger

Convener
(Australia)

Presented

at the

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Abstract

This ad-hoc working group of the IAG Special Commission (SC3) has carefully reviewed the progress in the measurement and modelling of the refractive index of air. For light and near infrared waves, a closed simple formula is proposed for electronic distance measurement (EDM) to not better than one part per million (1 ppm) precision. For EDM of higher precision, a computer routine is proposed that uses the full Lorentz-Lorenz relationship, the new temperature scale of 1990, the BIPM density equations and the recently revised water vapour refractivity. The group notes that the perceived accuracy of any continuum formula is invalidated by anomalous refractivity near absorption lines. Further work on the effect of absorption lines on the phase and group refractive indices of air is strongly recommended, as are new absolute measurements of the refractivity of the constituent gases of the atmosphere (including water vapour) at non-laboratory conditions.

1. Introduction

In 1991, at the 20th General Assembly of the International Association of Geodesy (IAG) in Vienna, it was suggested that new IUGG resolutions on refractive indices be prepared for adoption at a future General Assembly of IUGG. An *Ad-hoc Working Party on Refractive Indices of Light, Infrared and Radio waves in the Atmosphere* was formed in 1993 under the umbrella of the International Association of Geodesy (IAG) Special Commission SC3 – Fundamental Constants (SCFC).

The last resolutions of the International Union of Geodesy and Geophysics (IUGG) on refractive indices date back to 1963. For light waves, the 1963 IUGG resolution recommended two interchangeable formulae for standard air by Barrell & Sears (1939) and Edlén (1953) as well as a simplified interpolation formula to ambient condition based on Barrell & Sears (1939). (To recommend two competing formulae was not such a good idea, considering that uniformity was to be achieved.) The formulae provided group refractive indices accurate to 0.1 ppm at standard conditions for carrier wavelengths between 185 and 644 nm. They were thought to give an accuracy of 2 ppm at ambient conditions. This was appropriate in 1963 when electro-optical distance meters were specified at $\pm(10 \text{ mm} + 2.5 \text{ ppm})$. Today, the best commercial electro-optical distance meters have a precision of $\pm(0.1 \text{ mm} + 0.1 \text{ ppm})$. Also, the state-of-the-art measurement of atmospheric parameters gives computed refractive indices with standard deviations of 1 to 5×10^{-8} (see Matsumoto et al. 1988, Birch & Downs 1993, for example). The formulae adopted in 1963 are not accurate enough for today's precision measurements, do not include post 1953 refractivity measurements and have not been designed for the near infrared spectrum where most distance meters operate today. This situation has led to the use of a variety of "non-approved" formulae in geodesy and surveying.

This report summarises the activities of the Ad-Hoc Working Party of the IAG Special Commission SC3 – Fundamental Constants (SCFC) for the period 1993 to 1999. In short, the working party was considering recent determinations of the refractive index of air, the carbon dioxide content of air, the extension of the formulae into the infrared and the implementation of the Lorentz-Lorenz relationship. So far, the working party concentrated on the refractive index in the visible and near infrared spectrum, because of an apparent lack of interest from geodesists working with GPS and VLBI. Work on proposals for refractive index formulae for the radio and millimetre waves has started, however, and some suggestions have been arrived at.

Three resolutions have been prepared by the working party for adoption at the 22nd General Assembly of IUGG (1999). They are reprinted in the Appendix. The first resolution recommends two solutions for the refractive index of light and near infrared waves, namely a simple closed formula (for the reduction of measurements of not better than one part per million) and a computer routine (for the reduction of measurements to better than one part per million). The second resolution makes recommendations on further work on the refractive index of light and infrared waves, with special emphasis on anomalous refractivity due to absorption lines. The third resolution recommends further work on the refractive index of infrared and radio waves.

The views expressed in this report are those of the author, but are based on consultations with the members of the group. The working party plans to publish a joint paper on its work soon. The extensive literature section includes references mentioned to in the text as well as other literature relevant to the work of the group.

2. Membership

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3. Refractive Index of Light and Near Infrared Waves in the Atmosphere

3.1 Phase, Group, Signal and Pulse Velocity and Corresponding Refractivity

The phase refractive index (and the phase velocity) is required for the reduction of measurements with the classic Michelson interferometer and the HP (Doppler) interferometer. Traditionally, it has been assumed that electronic distance meters (EDM instruments) using amplitude, frequency or phase modulation of the carrier of flight time measurements require the group refractive index (and the group velocity). The same group refractive index is also required for the pulse distance meters used in surveying and geodesy and instruments using polarisation modulation, such as the Kern Mekometer ME 5000. (The latter is equivalent to a phase modulation as the phase of the vertical polarisation is changing against the phase of the

horizontal polarisation.) White light interferometers, beat interferometers and interferometers modulated by a second wavelength also require the group index. As the next paragraph shows, the *group velocity concept* is an approximation that applies only if there are no absorption lines near or within the signal spectrum.

The term *signal velocity* was first defined by Brillouin (1960). Only recently, Oughstun et al. (1989) gave a corrected definition of signal velocity for absorptive and dispersive media. Based on Oughstun's work, Hill (in Ciddor & Hill 1999) derived the limiting conditions of the applicability of group velocity and, thus, group refractivity. Within these limits, where dispersive and absorptive distortions are negligible, the signal and the group velocities are the same. Beyond these limits, the terms *group velocity* and *group refractive index* lose their meaning because of the distortion of the wave packets. In these cases, Hill suggests to evaluate the signal velocity for specific EDM instruments using the algorithms developed by Xiao & Oughstun (1998). An alternative way accounting for the influence of resonances on (the phase and) group refractive index of air was proposed by Galkin & Tatevian (1997). A numerical comparison of the two methods has not (yet) been carried out.

The term *velocity of pulse propagation* is being used by Mandel & Wolf (1995, p. 822, Eq. 16.3-25). The members of the working party believe that this phenomenon likely applies to very short pulses in condensed media (with very non-linear dispersion) and to highly dispersive and/or absorbing media. The pulse velocity after Mandel & Wolf (1995) is, thus, assumed to be irrelevant to the work of the working party.

3.2 Continuum Refractivity (Dispersion)

After some lengthy discussions on the relative merits of the Edlén (1966) and the Peck & Reeder (1972) dispersion equations, preference was eventually given to Peck & Reeder. In consequence, Ciddor based his computer procedures on it (Ciddor 1996, Ciddor & Hill 1999). The advantages of the Peck & Reeder (1972) formulation are outlined in the Section on the "Two-Term Dispersion Formula" of their 1972 paper. Their arguments are convincing, particularly as the working party wants a formula that extends into the NIR. The Peck & Reeder 1972 equation differs from the Edlén 1966 equation in the following ways: (a) seven new (relative) measurements between 723 nm and 1530 nm, not used by Edlén, accuracy 1 part in 10^9 , (b) renormalised 8 earlier data (Edlén uses initial values), (c) two earlier values (used by Edlén) omitted, (d) root mean squares of fit of 51 out of 59 data is $\pm 1.7 \times 10^{-9}$ whereas Edlén is $\pm 3.1 \times 10^{-9}$ (between 0.23 μm and 1.69 μm), (e) P&R has an average offset of 0.2×10^{-9} in the infrared whereas the Edlén 1966 equation has a 4.3×10^{-9} offset. This is clearly not acceptable for a state-of-the-art equation for geodesy.

Galkin & Tatevian indicated that they are not entirely happy with the accuracy and documentation of the original data used by Edlén and Peck & Reeder Formula. They have repeated the curve fit to the Peck & Reeder data and did not find a better solution. Galkin & Tatevian (1997a) pointed out that each absorption line in the visible and near infrared causes a small upward shift of the continuum dispersion curve towards larger wavelengths. These small increases of the refractivity with increasing wavelength cannot be modelled by the Sellmeier equations used by Peck & Reeder (and Edlén). One could add a linear term to the standard 2-3 term Sellmeier equations or add additional Sellmeier terms to the standard 2-3 term formula for resonances at about 0.7 μm , 2.5 μm and 8 μm , for example. All existing dispersion formulae could also be improved by introducing the recent absolute and relative refractivity data (e.g. data by Matsumoto, Birch & Downs, Bönsch & Potulski) and by using a better mathematical model of the curve fit that allows data sets, that were converted from relative to absolute, to float up

and down as a group. Before any new curve fit one would also have to investigate the possible effect of absorption lines on old refractivity measurements.

3.3 Refractivity of Moist Air

Birch & Downs (1988a, 1988b, 1989) found an error in the original water vapour refractivity measurements of Barrell & Sears (1939). As all subsequent refractivity data were scaled to Barrell & Sears, this error is present in most existing refractivity formula. The findings by Birch & Downs were confirmed by Beers & Doiron (1992) and Bönsch & Potulski (1998). The latter confirm Birch and Downs data to within 1 part in 10^8 . The Ciddor formulae (Ciddor 1997, Ciddor & Hill 1999) are based on Birch & Downs' new moist air refractivity data.

All new data were established at laboratory conditions. In support of outdoor measurements, new absolute measurements of water vapour refractivity over a wide range of temperatures and pressures are strongly recommended.

3.4 Lorentz-Lorenz Relationship

The limitations of the Lorentz-Lorenz relationship were referred to by Owens (1967) and by Ciddor & Hill (1999). The latter note that there is no difference in terms of "practical precision" between the Lorentz-Lorenz equation (Böttcher & Bordewijk 1978, Eq. 12.2, p. 286) and the corrected Lorentz-Lorenz equation (Böttcher & Bordewijk 1978, Eq.12.16, p.292) at atmospheric densities. The magnitude of the difference was not quantified. Ciddor believes the difference to be at the 1 part in 10^{12} level. According to Hill, the corrected L-L equation is only required for dense media.

Ciddor (1996) did not fully implement the Lorentz-Lorenz (L-L) relation in the new computer procedure for the refractive index of light and near infrared waves. Ciddor (1996) stated that the difference between his solution and the full LL relationship does not exceed 2 parts in 10^9 at an altitude of 2000 m above sea level. Later, Hill computed a difference of 5 parts in 10^9 for an altitude of 4500 m. Ciddor's newest computer procedure (Ciddor & Hill 1999) now includes the full (uncorrected) L-L relationship.

3.5 Anomalous Refractivity

Anomalous refractivity may be defined as the deviation from the smooth continuum dispersion curve near absorption lines. It is the contribution from anomalous dispersion by resonances to the total refractivity of air. As discussed in Section 3.1, the signal velocity after Oughstun et al. (1989) and Xiao & Oughstun (1998) will give directly the total refractivity (continuum and anomalous component) near absorption lines. The approach followed by Galkin and Tatevian (1997a, 1997b) provides only the contribution (to the total refractivity) from anomalous dispersion by resonances. Their concept is similar to the one used by Hill for infrared and radio waves (Hill et al 1980a 1980b).

According to Galkin & Tatevian (1997), the original phase refractivity data may be affected by up to 1 part in 10^9 due to local resonance effects. Anomalous group refractivity may reach 0.7 ppm. However, there are errors that are very much larger (several ppm!) than the published value of 0.7 ppm. Unfortunately, the magnitude of anomalous phase and group refractivity is still unknown for most wavelengths covered by the Ciddor formula. A correcting term for anomalous refractivity in Ciddor's formula is required; it could be obtained from a software package for PCs based on Galkin & Tatevian's work and suitable spectroscopic data.

Galkin & Tatevian see a certain need for a general purpose software for the computation of anomalous refractivity for wide spectrum instruments. They presently consider the following absorption lines for inclusion: N₂, O₂, CO₂ plus water vapour. Other contenders are SO, CO, NO. They believe that CO₂ and O₂ can be ignored between 0.75 μm and 0.95 μm and that the water absorption lines listed in HITRAN are sufficient. It would be very useful to know the magnitude of the correction and, possibly, the average correction.

3.5.1 Single Line (Laser) Instruments

As far as the visible and NIR spectrum is concerned, the modelling of anomalous phase and, in particular, group refractivity is still an open question. According to Galkin & Tatevian, very strong absorption lines can affect the phase refractive index by about one part in 10⁻¹² to 10⁻¹³ in the visible spectrum. Near 0.77 μm, there is an effect of 1 part in 10⁻¹¹ due to an oxygen line. The group refractive index is affected more by five to six orders of magnitude.

Ideally, one would like to see something like Liebe's PC-based MPM software package (e.g. Liebe 1989). As most IR instruments are not single wavelength instruments but have an (approximately) Gaussian distribution of wavelengths, the software would also have to cater for this. Investigations of anomalous group refractivity are of great interest for the following gas laser wavelengths (narrow band): 441.6 (HeCd), 632.8 nm (HeNe), 1064 nm and 532 nm (Nd:YAG). The first two are used for terrestrial dual-colour distance measurements and the latter for satellite ranging. Even though anomalous refractivity can be computed by hand, software is required if many lines are to be computed. Galkin & Tatevian do not predict a good accuracy of such computations because the (assumed) Lorentzian line shape is a model and its far-wings a problem. There is also some doubt if the AFCL HITRAN atlas lists resonances with less than 5% absorption. In the whole, Galkin and Tatevian believe that a correction for stabilised laser instruments is feasible.

3.5.2 Broadband (Diode) Instruments

Considering that, presently, about 99% of all commercial distance meters operate in the near infrared (NIR), it would be of great importance to know the effect of water vapour resonances on the group refractive index of these instruments. It is likely that the magnitude of anomalous refractivity on infrared (IR) distance meters will be reduced by the fact that they feature a Gaussian spread of the emission wavelength, usually about ±20-30 nm about the centre wavelength (at 50% power points). But this would have to be demonstrated.

Considering the 220 instrument types listed in Rüeger (1996), three popular wavelengths emerge: 820 nm, 860 nm and 910 nm. Not shown in the list are the emerging AlGaInP visible laser diodes (VLD) operating between 650 to 690 nm. It would be of great interest to know the magnitude of anomalous group refractivity at these wavelengths (670 nm, 820 nm, 860 nm, 910 nm), assuming spectral widths (at 50% power) of 3.5 nm and 40 nm for laser diodes and high radiance emitting diodes, respectively. It is known (Rüeger 1996) that the wavelengths are temperature dependent (about 0.25 to 0.35 nm/°C) and that the actual wavelength of a diode may differ by as much as 15 nm from the nominal value. So, any computations of anomalous refractivity will only be able to indicate the magnitude of the problem for diode instruments.

Galkin & Tatevian correctly point out that all depends on the number and width of the absorption lines relative to the spectral width of the EDM instrument. The effect of a narrow resonance line would be significantly reduced by a broad spectral width instrument whereas even a broad spectral width instrument would suffer greatly from anomalous refractivity if

multiple absorption lines were to be distributed evenly and tightly over the spectral range of the instrument. Galkin & Tatevian note that there are more than 4000 water resonance lines between 0.75 μm and 0.95 μm .

3.6 Carbon Dioxide Content and Dispersion

A number of standard laboratories have started to use a default CO₂ content of laboratory air of 400 ppm or 450 ppm. Most standard laboratories now measure the actual content and correct for it. A CO₂ content of 450 ppm is clearly too high for measurements in the open (see Taylor 1994, for example). Rüeger (1998) has considered the matter and suggests 375 ppm (likely to be current in 2004) for all fixed CO₂ content formulae. The proposed resolutions for simple, closed formulae are based on this assumption. The accurate formulae have an input for the actual carbon dioxide content.

The computer procedures for the accurate computation of the refractive index of air for light and near infrared waves by Ciddor (1996) and Ciddor & Hill (1999) use the Birch and Downs (1994) implementation of Edlén's (1966) CO₂ correction. The deviation of the CO₂ term used from that of Old et al. (1971) and Simmons (1978) is less than 1 part in 10⁸. Even so, the Old et al.'s (1971) dispersion formula is more appropriate, as it includes the resonance effects from 4.25 μm and, thus, extrapolates much better into the infrared.

The inclusion of Old's CO₂ refractivity equation in the proposed computer procedures for the accurate refractivity is of value, in particular when two-colour measurements and group refractivity are concerned. This work has begun and will be documented in the "Guidelines for the Implementation" of the Ciddor (1996) and Ciddor & Hill (1999) procedures, as foreshadowed in the proposed resolution.

3.7 Water Vapour near Saturation

The proposed simple refractivity equation and the computer procedure for a more accurate refractivity include water vapour terms. When the relative humidity increases above 70% to 90%, small water droplets may begin to form and the air may become a mixture of gases and liquid water in form of an aerosol. When operating in high humidity, it must be noted that the proposed formulae do not take account of aerosols (liquid water droplets). The resolutions cover this aspect by specifying that the formulae apply to "non-condensing" conditions only.

It might be prudent to assume the claimed accuracy of the formulae to be valid only for relative humidities below 90%. The refractive index of water (rain) in the context of electronic distance measurement (EDM) was recently investigated by Rüeger (1996b, 1999). Drizzle (1 mm/h) changes the total refractivity by about 0.03 ppm whereas heavy rain (20 mm/h) changes it by 0.3 ppm. This indicates that small quantities of liquid water will not significantly affect the continuum refractivity.

4. Resolutions on the Refractive Index of Air for Visible and NIR Waves

4.1 First Resolution

The working party has prepared two resolutions on the refractive index of air for visible and NIR waves. The first paragraph of the first resolution recommends a computer procedure for measurements to better than one part per million. The computer procedures are fully

documented in Ciddor (1996) and Ciddor & Hill (1999). The Ciddor formulas produce refractivities to a few parts in 10^8 . They match recently reported measurements within the experimental error and are expected to be reliable over very wide ranges of atmospheric conditions and wavelength. These formulas use the BIPM density equations that are valid over ranges of at least -40 to $+100$ °C, 800 to 1200 hPa and 0 to 100% relative humidity, and so include all practical atmospheric conditions. The formulas apply over the wavelength range from below 350 nm to above 1300 nm and, thus, cover the wavelengths at which modern surveying instruments operate. The equations take into account all known factors (except for suspended aerosols, atmospheric contaminants such as oil vapours and the effects of absorption lines) and embody the latest values of physical parameters and units.

The basic form of the phase refractive index formula used by Ciddor is as follows:

$$N_{\text{amb}} = \frac{\rho_a}{\rho_{\text{asx}}} N_{\text{asx}} + \frac{\rho_{\text{wv}}}{\rho_{\text{wvs}}} N_{\text{wvs}} \quad (1)$$

where N_{amb} = refractivity of ambient moist air
 N_{asx} = refractivity of dry standard air at standard conditions (15°C, 1013.25 hPa, x ppm CO₂)
 N_{wvs} = refractivity of water vapour at standard conditions (20°C, 13.33 hPa)
 ρ_{asx} = density of standard air (with x ppm CO₂)
 ρ_{wvs} = density of standard water vapour
 ρ_a = density of dry component of ambient air
 ρ_{wv} = density of water vapour component of ambient air

| T | PWVP | CIDDOR | OWENS | EDLÉN | IUGG | Differences | | |
|------|----------------|---------------------------|---------------------------|---------------------------|---------------------------|-------------|-------|-------|
| [°C] | P_w [hPa] | N_L $\times 10^{-8}$ | N_L $\times 10^{-8}$ | N_L $\times 10^{-8}$ | N_L $\times 10^{-8}$ | (4-3) | (5-3) | (6-3) |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |

$\lambda = 650$ nm, CO₂ content = 300 ppm (0.03%), total pressure = 1000 hPa

| | | | | | | | | |
|-----|--------|----------|----------|----------|----------|--------|---------|--------|
| 60 | 199.26 | 23626.92 | 23537.14 | 23413.97 | 23564.92 | -89.78 | -212.95 | -62.00 |
| 45 | 95.85 | 25066.14 | 25018.14 | 24978.36 | 25042.27 | -48.00 | -87.78 | -23.87 |
| 30 | 42.43 | 26484.65 | 26462.35 | 26453.76 | 26479.96 | -22.30 | -30.89 | -4.69 |
| 15 | 17.04 | 27955.58 | 27946.75 | 27946.76 | 27957.72 | -8.83 | -8.82 | +2.14 |
| 0 | 6.10 | 29536.31 | 29533.76 | 29533.96 | 29538.14 | -2.55 | -2.35 | +1.83 |
| -15 | 0.00 | 31283.16 | 31284.67 | 31280.87 | 31281.10 | +1.51 | -2.29 | -2.06 |
| -30 | 0.00 | 33221.68 | 33223.60 | 33215.56 | 33210.86 | +1.92 | -6.12 | -10.82 |

$\lambda = 850$ nm, CO₂ content = 300 ppm (0.03%), total pressure = 1000 hPa

| | | | | | | | | |
|-----|--------|----------|----------|----------|----------|--------|---------|--------|
| 60 | 199.26 | 23199.45 | 23112.02 | 22986.56 | 23156.05 | -87.43 | -212.89 | -43.40 |
| 45 | 95.85 | 24629.37 | 24582.71 | 24542.14 | 24614.12 | -46.66 | -87.23 | -15.25 |
| 30 | 42.43 | 26032.10 | 26010.60 | 26001.80 | 26030.60 | -21.50 | -30.30 | -1.50 |
| 15 | 17.04 | 27482.34 | 27473.94 | 27473.96 | 27485.00 | -8.40 | -8.38 | +2.66 |
| 0 | 6.10 | 29038.32 | 29036.07 | 29036.32 | 29039.46 | -2.25 | -2.00 | +1.14 |
| -15 | 0.00 | 30756.90 | 30758.62 | 30754.88 | 30753.44 | +1.72 | -2.02 | -3.46 |
| -30 | 0.00 | 32662.81 | 32664.95 | 32657.05 | 32650.64 | +2.14 | -5.76 | -12.17 |

Table 1: Comparison of visible and NIR group refractivity N_L (in parts per 10^8) from the Ciddor (1996) and a number of earlier formulae. The Owens'97 data are based on Owens (1967, Eqs. (30), (31), (42)); the Edlén'66 data are based on Edlén (1966, Eqs. (1) and (22), as converted from phase to group, and Eq. (12)). The IUGG'63 data follow from a subset of the equations recommended by the IUGG Resolution of 1963 (see Rüeger 1996, Eqs. (5.15) and (5.13)).

The proposed resolution refers to "Guidelines for Implementation". These will summarise the computer procedures of Ciddor (1996) and Ciddor & Hill (1999), as amended to include the

Old et al. (1971) carbon dioxide dispersion formula, because of its appeal and theoretical basis in the infrared. This requires a switch from a differential correction to the computation of the total CO₂ contribution, thus leading to the summation of three terms (rather than the sum of two shown in Eq. (1)), namely one each for dry air (free of carbon dioxide and water vapour), carbon dioxide and water vapour.

The second paragraph of the first resolution recommends a closed formula for measurements of not better than 1 ppm. For the latter, a subset of the equations recommended by the 1963 IUGG Resolution is being used, as amended for a value of 375 ppm for the CO₂ content of the atmosphere. Table 1 shows that the chosen simple formula performs much better (at high humidity) than two formulae (OWENS'67 and EDLÉN'66) previously thought to be superior. The excellent agreement in Table 1 is due, in part, because of the use of the same values (exactly) of carbon dioxide content and partial water vapour in all formulae. Practising surveyors and geodesists will use differing equations for the saturation water vapour pressure, might omit the enhancement factor and will, typically, ignore any deviation of the actual CO₂ content from the default value. For these reasons and because anomalous refractivity can amount to more than 0.7 ppm and the measurement of the atmospheric parameters can easily introduce errors of similar magnitude, the closed formula is only suggested for measurements of not better than one part per million. Please note that Table 1 is based on 300 ppm CO₂ content, because all old formulae assume this content. The new formula is based on a carbon dioxide content of 375 ppm.

4.2 Second Resolution

The second resolution on the refractive index of air for visible and NIR waves suggests further work on some open questions. Firstly, the effect of absorption lines on the phase and group refractive indices of air needs to be evaluated to be able to quantify the magnitude of anomalous refractivity for specific instruments, or, in other words, the difference between the group refractivity and the signal refractivity. The numerical agreement between the approaches of Galkin & Tatevian (1997) and Oughstun (1991) should be investigated. Ideally, software should be written to evaluate the magnitude of these effects and, where possible, to correct for such effects. Such software may require better and more spectroscopic data of weak absorption lines.

As suggested earlier, it would be very useful if additional absolute and, possibly, relative measurements of the refractivity of the constituent gases of the atmosphere (including water vapour) be carried out at *non-laboratory conditions*, with special emphasis on near infrared wavelengths. In addition, measurements could be made in some doubtful points (for example 644 nm) of the visible spectrum.

Furthermore it should be considered to include the post-P&R data (including, in a suitable manner, relative measurements) in a revised P&R formula and to use the latter in an amended Ciddor formula. Initial steps for an amended P&R formula have been carried out by Galkin & Tatevian.

5. Refractive Index of Radio and Infrared Waves in the Atmosphere

This section is a summary of the review carried out by Rüeiger (1998b) and subsequent discussions with the working group members. Refer to Rüeiger (1998b) for more details.

5.1 Accuracy

Boudouris (1963) and Hartmann (1991) quoted an accuracy of the radio refractivity of 0.5% (for moist air, $p_w = 27$ hPa) whereas Thayer (1974) claims 0.02% for dry air and 0.05% for "extremely moist air". Walter (1990 p.101, quoting Liebe 1983) states an accuracy of 0.05 ppm. As the accuracy of the radio wave refractivity is heavily dependent on the water vapour content, it is best to quote the accuracy of the dry and the wet terms separately, as done by Thayer (1974). The widely differing views on the accuracy of the water vapour refractivity may be because some authors ignore the mathematical correlation between the two water vapour terms (K_2 and K_3) when applying the propagation law of variances whereas others do not. (Refer to the differences between Columns 8 and 9 in Table 2 below, for example. Boudouris' (1963) accuracy estimate seems to be on the conservative side and Thayer's (1974) estimate (for the accuracy of the water vapour terms) on the optimistic side.

5.2 Continuum Formulae and Compressibility Factors

Thayer (1974, Eq. (1)) suggested a closed formula (with compressibility factors Z) of the following form:

$$N_r = (n_r - 1) \times 10^{-6} = K_1 \frac{P_d}{T} Z_d^{-1} + K_2 \frac{P_w}{T} Z_w^{-1} + K_3 \frac{P_w}{T^2} Z_w^{-1} \quad (2)$$

The same author noted that the omission of compressibility factors leads to errors in the radio wave refractivity of 0.04 ppm in the dry term and 0.1 ppm in the wet term at high humidities. The former is of the same magnitude as the accuracy of the dry terms whereas the latter is half of the wet term accuracy claimed by Thayer and a twentieth of the accuracy quoted by others. In consequence, it is suggested to propose a closed formula for geodesy and surveying without compressibility factors.

The International Radio Consultative Committee (CCIR) of the International Telecommunication Union (ITU), at the 16th Plenary Assembly, Dubrovnik 1986, in Recommendation 453-1, considering the necessity of using a single formula, unanimously recommended the following formula for the refractive index of radio waves:

$$N = (77.6/T) (P + 4810 (e/T)) \quad (3)$$

with e , P in hPa, T in K. Report 563-3 gives an equivalent formula (after Bean and Dutton), with an error of less than 0.5% for frequencies of less than 100 GHz. The formula adopted by CCIR does not comprise compressibility factors and only one 'dry' term and one 'wet' term. The second (K_2) term of Eq. (2) is missing.

Some authors did use compressibility factors when reducing their measured refractivities to standard conditions whereas others did not. So, one could argue that coefficients K_i that were reduced with compressibility factors can be used directly in formulae with compressibility factors. On the other hand, coefficients K_i that were determined without compressibility factors can be used directly in formulae without compressibility factors. Considering Eq. (2), it becomes clear that the coefficients K_i in formulae without compressibility factors really cannot be numerically the same as those in formulae with compressibility factors. This aspect must yet be verified in all known formulae of interest in this context.

It might be desirable that precision formulae use compressibility factors Z_i and the matching K_i constants for dry (carbon dioxide free) air, water vapour and carbon dioxide terms even though the compressibilities change refractivity by less than the claimed accuracy of the formulae. To be consistent with the recommended formulae for the visible and NIR waves, the

compressibility factors should be computed with the BIPM formulae. See Ciddor (1996) for reference.

5.3 Carbon Dioxide

Following the recommendations in the proposed resolutions on the refractivity of light and near infrared waves, it is again suggested to use a 375 ppm fixed CO₂ content in all new closed formulae, like those for hand calculations. This has been implemented in Eq. (6) (Rüeger 1998b). The change from 300 to 375 ppm changes the radio wave refractivity by only 0.01 ppm at standard conditions. To be consistent, it is suggested that precise formulae and models should include the CO₂ content as input variable. A default of 375 ppm CO₂ could be implemented in MPM as well as an optional CO₂ content input.

5.4 Upper Frequency Limit for Closed Refractivity Formula

Different authors quote different upper limits for the non-dispersive region of radio wave refractivity. For example, Hartmann (1991) quoted 5 GHz, Thayer (1974) 20 GHz and Boudouris (1963) 30 GHz. These limits vary because of the magnitude of spurious effects tolerated and the accuracy of formulae assumed by these authors. Rüeger (1998b) suggested 1 GHz as cut-off as Liebe (1996) suggests additional terms for dispersive refractivity above 1 GHz. This limit is sufficiently removed from the nearest resonance frequencies of water vapour and oxygen. Liebe (1983) and Hartmann (1991) list, for example, H₂O resonances at 22.23 GHz, 67.81 GHz, 119.99 GHz, 321.22 GHz, and O₂ resonances between 53.59 GHz and 66.30 GHz and at 118.75 GHz. Further evaluation of the magnitude of the effect of absorption lines might permit to extend the validity range of simple closed formulae to 5 GHz or 10 GHz.

5.5 Dispersive Refractivity (to 1 THz)

As Hill et al. (1980) and Liebe (1989) did, dispersive refractivity can be added to experimental refractivity. Liebe's Millimetre-Wave Propagation Model (MPM) is the only operational model to account for 44 oxygen and 30 local water resonance lines plus an empirical water vapour continuum to offset experimental discrepancies. The CO₂ resonance at 15 µm should be sufficiently distant to cause anomalous refractivity between 1 Hz and 1THz.

5.6 Phase, Group and Signal Velocity

In geodesy and surveying, it has been generally assumed that the refractive index of radio waves is not dependent on the carrier frequency and, thus, only the phase refractive index must be considered. Microwave electronic distance measurements (EDM) used frequency modulation techniques to derive the distance measurements. GPS signals are code modulated but reconstructed carrier waves are usually used for the actual measurement in geodesy. It could be argued that, since some of the radio wave spectrum is dispersive, the concept of group or signal refractivity might have to be introduced where the propagation of modulated waves is used for measurements. If the visible spectrum can be taken as a guide then the anomalous group refractivity can be 100'000 times larger than the anomalous phase refractivity.

The question now arises if the concept of group velocity or signal velocity has to be introduced in the dispersive regions of the radio wave spectrum. According to Hufford, "signal velocity" is already used for the wave propagation in wave guides. Hufford (1987) did compute the millimetre-wave pulse distortion by a single absorption line simulating the terrestrial atmosphere

and, therein, gives a reference to Trizna & Weber (1982), which also discuss the signal velocity for pulse propagation in a medium with resonant anomalous dispersion.

Further investigation and quantification of the difference between phase and signal velocity in the frequency ranges 1 Hz to 1 GHz (non-dispersive) and 1 GHz to 1THz (dispersive) are clearly required.

5.7 Anomalous Refractivity Effect in Historic Data

Most radio wave refractivity data were measured above 1 GHz, were dispersive refractivity starts to have an effect. For example: Birnbaum & Chatterjee (1952; 9.28 GHz, 24.8 GHz), Boudouris (1963; 7 GHz to 12 GHz), Newell & Baird (1965; 47.7 GHz), Liebe (1969; 22.235 GHz), Liebe et al (1977; 53.5 to 63.5 GHz). The last two determinations accounted for anomalous refractivity whereas the first three did not.

To get consistent data for the derivation of a dispersion-free radio wave refractivity, it would be of great benefit if anomalous refractivity were removed from experimental data. In particular, it would be worthwhile to remove anomalous refractivity from the values in Newell & Baird's Table III (1965) for dry air and oxygen. The poor agreement of Froome's oxygen value could be entirely due to dispersive oxygen refractivity.

6. Refractivity for IR and Radio Waves: Work to Date and Pending Work

6.1 Formula for Hand Calculations (1 Hz to about 1 GHz, ∞ m to 0.3 m)

It is valuable to have a relative simple and closed solution for the refractive index of radio waves for easy calculation with pocket calculators and personal computers. The equations given here are empirical, based on experiment and ignore the non-ideal gas behaviour (compressibility) of air.

When ignoring compressibility factors, the refractivity N_r of radio waves (in ppm) can be expressed as:

$$N_r = (n_r - 1) \times 10^6 = K_1' \frac{p_{d-c}}{T} + K_2 \frac{p_w}{T} + K_3 \frac{p_w}{T^2} + K_4 \frac{p_c}{T} \quad (4)$$

where p_{d-c} ($= p_d - p_c = p_{tot} - p_w - p_c$) is the (partial) pressure of the dry and carbon-dioxide-free air, p_d is the (partial) pressure of the dry air ($= p_{tot} - p_w$), p_w is the partial water vapour pressure, p_c is the partial carbon dioxide pressure, the K_i are constants and T the temperature (in K). The coefficient K_1' is the constant K_1 without the CO_2 component. Because of its polar nature, water vapour has a density and a density-temperature term.

Based on the coefficients by Boudouris (1963: K_2 and K_3) and Newell & Baird (1965: K_1 and K_4) Rüeger (1998b) constructed a three-term equation for air with 0.03% (300 ppm) CO_2 content:

$$N_r = (n_r - 1) \times 10^6 = 77.691 \frac{p_d}{T} + 71.97 \frac{p_w}{T} + 375406 \frac{p_w}{T^2} \quad (5)$$

where the dry air (including carbon dioxide) pressure p_d ($= p_{tot} - p_w$) and the partial water vapour pressure p_w are taken in hPa and the temperature T in K. The accuracy of an equivalent equation is estimated by Boudouris (1963, p. 661) to be within 0.5% for temperatures between

-50°C and +40°C, (total) pressures between 187 and 1013.25 hPa, partial water vapour pressures between 0 and 27 hPa and frequencies between 1 Hz and 30 GHz.

Recomputing the K_1 term for the carbon dioxide content of 375 ppm (0.0375%), expected to be current around the year 2004, gives the final form of a possible new formula for the non-dispersive radio wave refractivity N_r (after Boudouris-Newell-Baird, in units of K and hPa)

$$N_r = (n_r - 1) \times 10^6 = 77.695 \frac{P_d}{T} + 71.97 \frac{P_w}{T} + 375406 \frac{P_w}{T^2} \quad (6)$$

A comparison of this equation with other formulae can be found in Table 2.

6.2 Computer Routine (1 Hz to about 1 THz, ∞ m to 0.3 mm)

A practical model that simulates the complex refractive index for the propagation calculation of electromagnetic waves through the atmosphere has been developed by Liebe et al. over many years. The Millimetre-Wave Propagation Model (MPM) is a program for frequencies below 1000 GHz in the atmosphere running on personal (IBM compatible) computers. The MPM "consists of 44 oxygen and 30 local water resonance lines, of non-resonant spectra for dry air and of an empirical water vapour continuum that reconciles experimental discrepancies" (Liebe et al. 1992). The model is applicable for barometric pressures between 0 and 1200 hPa, ambient temperatures between -100 and +50°C, relative humidity between 0 and 100% and suspended water droplets and ice particle densities between 0 and >5 g/m³. (Other versions of MPM also model rainfall conditions.) The complete (complex) refractivity model is as follows (Liebe et al. 1992, Liebe et al. 1993, Liebe 1996):

$$\mathbf{N} = \mathbf{N}_D + \mathbf{N}_V + \mathbf{N}_{W,I} \quad (7)$$

where the complex dry-air refractivity is denoted by \mathbf{N}_D , the refractivity of atmospheric water vapour by \mathbf{N}_V and the complex refractivity of suspended water droplets and ice particles by $\mathbf{N}_{W,I}$. Complex parameters are shown in bold type. Only the real-part is required for the computation of the refractivity. The refractivity of suspended water droplets and ice particles (for example in fog and in clouds) is of no direct interest in this context. The complex dry-air refractivity \mathbf{N}_D is computed (in ppm) from (Liebe et al. 1992, Liebe 1996)

$$\mathbf{N}_D = N_d + \sum_k S_k \mathbf{F}_k + S_o \mathbf{F}_o + i S_n \mathbf{F}_n \quad (8)$$

where N_d is the non-dispersive (dry-air) term, k is the index of the 44 oxygen resonances, S_k is the line strength and \mathbf{F}_k the complex spectral shape function. S_o and \mathbf{F}_o model the non-resonant refractivity below 10 GHz from the oxygen relaxation spectrum. The $S_n \mathbf{F}_n$ term models the pressure-induced nitrogen absorption above 100 GHz. The MPM computes the refractivity of atmospheric water vapour \mathbf{N}_V (in ppm) as follows (Liebe et al. 1992, Liebe et al. 1993, Liebe 1996):

$$\mathbf{N}_V = N_v + \sum_l S_l \mathbf{F}_l + S_o \mathbf{F}_o + N_C \quad (9)$$

where N_v is the non-dispersive water vapour refractivity, l is the index of the 30 local water resonances, S_l is the line strength and \mathbf{F}_l the shape function. The continuum refractivity N_C models contributions over and above the 30 local lines and is partly based on the work by Hill (1988).

The MPM makes use of spectral data and is supported by many laboratory measurements to validate and enhance the overall performance of the model. The authors note that MPM dry-air absorption values agree with measured ones at the 1% level. "Model predictions involving water vapour and water droplets are estimated to lie in the 10 per cent range" (Liebe et al. 1992). MPM does not consider the weak spectra of trace gases such as O₃, CO and N₂O nor

does it provide an input for the CO₂ content. Presumably, a carbon dioxide content of 0.03% is assumed and included in the dry-air non-dispersive term.

6.3 Comparison of Formulae

Table 2 shows a comparison of the Millimetre-Wave Propagation Model (MPM, non-dispersive refractivity (N_d) only) with the simple formulae by Essen & Froome (1951), Boudouris (1963), a formula based on Liebe's (1977) coefficients and the B-N-B formula developed above (Eqs. (5) and (6), after Boudouris (1963) and Newell & Baird (1965)). At a later stage, it might be of interest to compare Eq. (3) adopted by the International Telecommunications Union.

A total pressure (p_{tot}) of 1000 hPa exactly was used for the comparison. To be consistent with the historical equations, the B-N-B formula for a CO₂ content of 0.03% (300 ppm) was used (Eq. (5) rather than Eq. (6)). The relative humidity was set at 100% for temperatures between 0°C and 60°C. The saturation water vapour pressures used for the computations of Columns 4, 5, 6 and 7 are shown in Column 2 and were taken from Rüeiger (1990, 1996, Appendix B, after Goff & Gratch 1946). MPM uses relative humidity as input and converts it to partial water vapour pressure using Goff & Gratch (1946). The precision of the new B-N-B formula (Eq. (6)) was predicted using the propagation law of variances and the given precisions of the constants K_1 , K_2 and K_3 . Column 8 gives the precision without consideration of the correlation between the constants K_2 and K_3 . Column (9) uses a correlation coefficient of 0.995 between the two constants to compute the covariance between them. (The correlation coefficient was obtained from a repeat of Boudouris' curve fit.)

| T | PWVP | MPM93 | E&F51 | Liebe77 | Boud63 | B-N-B | Prec | Prec | Prec | Prec | Prec | Prec |
|------|--------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| [°C] | P_w | N_r | N_r | N_r | N_r | N_r | B-N-B | B-N-B | (4-3) | (5-3) | (6-3) | (7-3) |
| | [hPa] | $\times 10^{-6}$ | $\times 10^{-6}$ | $\times 10^{-6}$ | $\times 10^{-6}$ | $\times 10^{-6}$ | $\times 10^{-6}$ | $\times 10^{-6}$ | $\times 10^{-6}$ | $\times 10^{-6}$ | $\times 10^{-6}$ | $\times 10^{-6}$ |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) |
| | | | | | | | \pm | \pm | | | | |
| 60 | 199.26 | 902.2 | 892.9 | 902.2 | 903.5 | 903.7 | 8.3 | 1.2 | -9.2 | 0.0 | 1.4 | 1.6 |
| 45 | 95.85 | 597.1 | 592.3 | 597.1 | 597.7 | 598.0 | 4.2 | 0.5 | -4.8 | 0.0 | 0.6 | 0.9 |
| 30 | 42.43 | 428.3 | 426.0 | 428.4 | 428.5 | 428.8 | 2.0 | 0.2 | -2.3 | 0.1 | 0.2 | 0.5 |
| 15 | 17.04 | 346.0 | 345.0 | 346.1 | 346.0 | 346.3 | 0.9 | 0.1 | -1.0 | 0.1 | 0.0 | 0.3 |
| 0 | 6.10 | 314.8 | 314.3 | 314.9 | 314.6 | 315.0 | 0.3 | 0.0 | -0.5 | 0.1 | -0.1 | 0.2 |
| -15 | 0.00 | 300.8 | 300.7 | 300.9 | 300.6 | 301.0 | 0.0 | 0.0 | -0.1 | 0.1 | -0.2 | 0.2 |
| 30 | 0.00 | 319.3 | 319.2 | 319.5 | 319.1 | 319.5 | 0.0 | 0.0 | -0.1 | 0.0 | -0.2 | 0.2 |

Table 2: Comparison of the non-dispersive part of the radio wave refractivity N_r (in parts in per million) from the Millimetre-Wave Propagation Model (MPM'93) and a number of simple formulae.

Table 2 shows clearly that the Essen & Froome equation (1951), adopted by IUGG in 1963, differs significantly from the other three models at high temperature and humidity. The Essen & Froome formula agrees better than any other with Liebe's MPM below freezing point and at zero humidity. The water vapour refractivity used by Essen & Froome is not optimal. Not unexpectedly, the formula ("Liebe77") based on the K_1 , K_2 and K_3 terms by Liebe et al. (1977b) agrees very well with the MPM. The differences in Column 11 of Table 2 do not exceed 0.15 ppm between -30°C and +60°C. Boudouris' formula agrees slightly better with the MPM than the Boudouris-Newell-Baird formula derived above. The B-N-B formula gives values that are, on average, 0.3 ppm higher than those of Boudouris. This is expected as Newell & Baird's K_1 constant is slightly larger than that of Boudouris and as the B-N-B

formula (Column 7) uses the same K_2 and K_3 constants as Boudouris (Column 6). It is also evident that the Boudouris and the B-N-B formulae have offsets from the MPM of the same magnitude but of different sign at negative temperatures and zero humidity. The differences (6)-(3) and (7)-(3) compare better with the precision values in Column (9), which take account of the (mathematical) correlation between K_2 and K_3 , than with those in Column (8), which don't. It follows that the correlation between K_2 and K_3 should not be ignored when predicting the precision of computed refractivity.

It follows from Table 2 that it might be appropriate the Liebe 1977 formula if MPM'93 were to be adopted for precision measurements in geodesy. On the other hand, if the new B-N-B formula of Eq. (6) were adopted as a simple equation, then it might be necessary to change MPM and REFRAC-IAG (see below) accordingly.

6.4 Proposal for REFRAC-IAG

The Millimetre-Wave Propagation Model (MPM) of the (US) National Telecommunications and Information Administration (NTIA) in Boulder presently best meets the IAG requirements for a computer procedure for the computation of the phase refractive index of radio and millimetre waves. Mr. M. Cotton of NTIA/ITS has started work for an IAG version of the MPM derivative program REFRAC, provisionally named REFRAC-IAG. Some changes to MPM suggested by Hill (1988, Eq.9, Table II) have already been implemented. Work for the input of a variable CO_2 content has begun. It will be based on $K_4 = 133.5 \pm 0.15$ (K/hPa) and used a default content of 375 ppm. The user interface of REFRAC-IAG has been changed from that of REFRAC to simplify the use of the program.

Further questions to be addressed are that of using the BIPM saturation water vapour pressure formula (and enhancement factor) as used in Ciddor (1996) and Ciddor & Hill (1999), the sources and history of Liebe's K_1 , K_2 and K_3 terms and the question of the suitability or necessity of compressibility factors. It is possible, that an introduction of compressibility factors into MPM and REFRAC-IAG could compromise the integrity of the package.

7. Resolutions on the Refractive Index of Air for Radio and NIR Waves

The third resolution proposed for adoption at the 22nd General Assembly of IUGG notes that the radio refractive index formula (after Essen & Froome) recommended in Resolution No. 1 of the 13th General Assembly of IUGG (Berkley 1963) is now clearly out of date, as the Column 10 in Table 2 shows. With new absolute and relative measurements of the mid-infrared to radio wave refractive index of air and considerable advances having been made with the computation of anomalous refractivity in the mid-infrared to radio wave spectrum, there is a clear need to amend the 1963 IUGG resolutions.

Although some progress has been made towards new recommendations for a simple closed formula and a more accurate computer procedure, more work is clearly required. Sections 5 and 6 indicate the progress made and the open questions that remain. In particular, the need for compressibility factors needs to be investigated and the significance of the difference between the signal velocity and the phase velocity in geodetic measurements through the atmosphere established. The ad hoc working party is looking for interested scientists that are able to contribute to this work on a new recommendation for the mid-infrared to radio (phase and signal) refractive index in air. Depending on the choice of a best possible simple formula, the coefficients K_1 , K_2 and K_3 used by MPM and REFRAC-IAG might have to be adjusted.

8. Outlook

The ad-hoc working party considers its work on the continuum refractive index of visible and near infrared waves completed. Some possible improvements to the dispersion equation have been indicated all the same. From now onwards, the ad-hoc working party proposes to concentrate on the effects of absorption lines in the visible and near infrared regions of the spectrum on one hand and on the radio refractive index on the other. It is hoped that resolutions on the remaining aspects can be formulated well before the next General Assembly. Experts, particularly in the field of radio wave refractivity, that are able to contribute to the work are invited to join the group.

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**Proposed
RESOLUTIONS
OF THE
INTERNATIONAL UNION OF GEODESY AND GEOPHYSICS**

(related to Geodesy) on the occasion of its
22nd General Assembly (IUGG 99), 18 to 30 July 1999, Birmingham, UK

RESOLUTION No. A

The International Union of Geodesy and Geophysics,

recognising that:

- a) the accuracy of the instrumentation used for terrestrial electronic distance measurement and for measurements to satellites has improved greatly since IUGG adopted a resolution on the refractive index of air in 1963;
- b) new absolute and relative measurements of the refractive index of air have been made since 1963;
- c) more accurate refractive index formulae have been developed and older formulae have been found to be in error since 1963;
- d) the international temperature scale was revised in 1990; and
- e) a carbon dioxide content of air of 300 ppm is no longer appropriate;

Cancels:

sub paragraphs (a) and (b) of Resolution No. 1 of the 13th General Assembly of IUGG (Berkley 1963);

noting that:

the continuum dispersion formulas used by the recommendation below do not account for the effects of anomalous refractivity due to molecular resonances in the visible and near-infrared;

recommends that:

- a) the group refractive index in air for electronic distance measurement to better than one part per million (ppm) with visible and near infrared waves in the atmosphere be computed using the computer procedure published by Ciddor & Hill in *Applied Optics* (1999, Vol.38, No.9,1663-1667) and Ciddor in *Applied Optics* (1996, Vol. 35, No.9, 1566-1573). Guidelines for the implementation will be published separately. The listed papers also include a computer procedure for the calculation of the phase refractive index.
- b) the following closed formulas be adopted for the computation of the group refractive index in air for electronic distance measurement (EDM) to not better than 1 ppm with visible and near infrared waves in the atmosphere:

$$N_L = (n_L - 1) \times 10^6 = \left(\frac{273.15}{1013.25} \times \frac{N_g \times p}{T} \right) - \frac{11.27 e}{T}$$

where N_L is the group refractivity of visible and near infrared waves in ambient moist air, T is the temperature in kelvin (ITS-90), $T = 273.15 + t$, t is the temperature in degrees Celsius ($^{\circ}\text{C}$), p the total pressure in hectopascal (hPa) and e the partial water vapour pressure in hectopascal (hPa).

The group refractivity N_g of standard air with 0.0375% CO_2 content at $T = 273.15 \text{ K}$ (0°C), $p = 1013.25 \text{ hPa}$, $e = 0.0 \text{ hPa}$ is as follows

$$N_g = (n_g - 1) \times 10^6 = 287.6155 + \frac{4.88660}{\lambda^2} + \frac{0.06800}{\lambda^4}$$

where λ is the carrier wavelength of the EDM signal (in micrometre, μm) and n_g the corresponding group refractive index.

These closed formulas deviate less than 0.25 ppm from the accurate formulas (see (a) above) between -30°C and $+45^{\circ}\text{C}$, at 1000 hPa pressure, 100% relative humidity (without condensation) and for wavelengths of 650 nm and 850 nm, for example. The 1 ppm stated before makes some allowance for anomalous refractivity and the uncertainty in the determination of the atmospheric parameters.

Where required, the phase refractivity N_{ph} of standard air with 0.0375% CO_2 content at $T = 273.15 \text{ K}$ (0°C), $p = 1013.25 \text{ hPa}$, $e = 0.0 \text{ hPa}$ may be calculated as follows

$$N_{ph} = (n_{ph} - 1) \times 10^6 = 287.6155 + \frac{1.62887}{\lambda^2} + \frac{0.01360}{\lambda^4}$$

where λ is the carrier wavelength of the signal (in micrometre, μm) and n_{ph} the corresponding phase refractive index.

Sponsored by the Ad-Hoc Working Group on *Refractive Indices of Light, Infrared and Radio Waves in the Atmosphere* (convener: J. M. R ueger) of the IAG Special Commission SC3 on Fundamental Constants (SCFC).
15 June 1999

RESOLUTION No. B

The International Union of Geodesy and Geophysics,

recognising that:

- a) the accuracy of any continuum refractive index formula for the visible and near infrared spectrum is ultimately limited by anomalous refractivity due to absorption lines, particularly for the group refractive index;
- b) present dispersion formulae are based on very few absolute refractivity measurements of dry air and moist air, particularly in the near infrared; and
- c) very few direct measurements of the group refractive index are available;

noting that:

preliminary work on the computation of the magnitude of anomalous phase and group refractivity in the visible and near-infrared has been done;

recommends that:

- a) further work on the effect of absorption lines on the phase and group refractive indices of air be carried out in order to be able to quantify the magnitude of anomalous refractivity for specific instruments and, ideally, to provide software to correct for such effects; and
- b) new absolute measurements of the refractivity of the constituent gases of the atmosphere (incl. water vapour) be carried out at non-laboratory conditions, with special emphasis on near infrared wavelengths.

Sponsored by the Ad-Hoc Working Group on *Refractive Indices of Light, Infrared and Radio Waves in the Atmosphere* (convener: J. M. Rüeger) of the IAG Special Commission SC3 on Fundamental Constants (SCFC).
15 June 1999

RESOLUTION No. C

The International Union of Geodesy and Geophysics,

recognising that:

- a) the radio refractive index formula recommended in sub paragraph (c) of Resolution No. 1 of the 13th General Assembly of IUGG (Berkeley 1963) has not been generally adopted and is now obsolete;
- b) new absolute and relative measurements of the mid-infrared to radio wave refractive index of air have been made since 1963;
- c) considerable advances have been made with the computation of anomalous refractivity in the mid-infrared to radio wave spectrum;
- d) the international temperature scale was revised in 1990; and
- e) a carbon dioxide content of air of 300 ppm is no longer appropriate;

noting that:

preliminary work on a new recommendation on the radio refractive index has been done;

recommends that:

interested scientists contribute to the work on a new recommendation for the mid-infrared to radio refractive index in air.

Sponsored by the Ad-Hoc Working Group on *Refractive Indices of Light, Infrared and Radio Waves in the Atmosphere* (convener: J. M. Rüeger) of the IAG Special Commission SC3 on Fundamental Constants (SCFC).
15 June 1999

REFRACTIVE INDEX FORMULAE FOR ELECTRO-OPTICAL DISTANCE MEASUREMENT

FORMULES POUR L'INDICE DE RÉFRACTION POUR LA MESURE DE DISTANCE OPTO-ÉLECTRONIQUE

BRECHUNGSINDEX-FORMELN FÜR DIE ELEKTRO-OPTISCHE DISTANZMESSUNG

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ABSTRACT

The refractive index formulae adopted in 1963 by the International Union of Geodesy and Geophysics (IUGG) and the International Association of Geodesy (IAG) are being reviewed. The key developments over the last 25 years are discussed. State-of-the-art formulae for routine and precise electronic distance measurement (EDM) using visible and near infrared waves are presented. One of the formula recommended in 1963 remains the preferred formula for the reduction of routine measurements. A new formula for precision EDM is discussed and some of the remaining limitations are outlined.

RÉSUMÉ

Les formules pour l'indice de réfraction, adoptées par Union Internationale de Géodésie et Géophysique (IUGG) et par l'Association Internationale de Géodésie (AIG) en 1963, sont en train d'être révisées. Les développements-clés depuis sont discutés. Des formules actuelles pour la mesure de routine et de précision de distance électronique avec des ondes visibles et infra-rouges sont aussi présentées. Une des formules recommandée en 1963 continue à être la formule préférée pour la réduction des mesures de routine. L'article présente aussi une nouvelle formule pour la mesure de distance de précision et les limitations qui restent à résoudre.

ZUSAMMENFASSUNG

Die von der Internationalen Union für Geodäsie und Geophysik (IUGG) und von der Internationalen Assoziation für Geodäsie (IAG) im Jahre 1963 beschlossenen Brechzahl-Formeln werden überarbeitet. Die seitherige Entwicklung wird diskutiert und zeitgemässe Formeln für die elektro-optische Routine- und Präzisions-Entfernungsmessung vorgeschlagen. Eine der im Jahre 1963 empfohlenen Formeln wird nach wie vor für die Reduktion routinemässiger elektronischer Distanzmessungen bevorzugt. Eine neue Formel für Präzisionsdistanzmessungen wird diskutiert und einige verbleibende Probleme besprochen.

Proceedings, Commission 5 (Positioning and Measurement), 21st International Congress (FIG'98), International Federation of Surveyors, 19-25 July 1998, Brighton, UK, Paper TS28/1, pp. 191-208

INTRODUCTION

The last resolutions of the International Union of Geodesy and Geophysics (IUGG) on refractive indices date back to 1963. For light waves, the 1963 IUGG resolution recommends two interchangeable formulae for standard air by Barrell & Sears (1939) and Edlén (1953) as well as a simplified interpolation formula to ambient condition based on Barrell & Sears (1939). (To recommend two competing formulae was not such a good idea, considering that uniformity was to be achieved.) The formulae provided group refractive indices accurate to 0.1 ppm at standard conditions and at carrier wavelengths between 185 and 644 nm. They were thought to give an accuracy of 2 ppm at ambient conditions. This was appropriate in 1963 when electro-optical distance meters were specified at $\pm(10 \text{ mm} + 2\text{-}5 \text{ ppm})$. Today, the best commercial electro-optical distance meters have a precision of $\pm(0.1 \text{ mm} + 0.1 \text{ ppm})$. Also, the state-of-the-art measurement of atmospheric parameters gives computed refractive indices with standard deviations of 1 to 5×10^{-8} (see Matsumoto et al. 1988, Birch & Downs 1993, for example). The formulae adopted in 1963 are not accurate enough for today's precision measurements, do not include post 1953 refractivity measurements and have not been designed for the near infrared spectrum where most distance meters operate today. This situation has led to the use of a variety of "non-approved" formulae in geodesy and surveying.

In 1991, at the 20th General Assembly of the International Association of Geodesy (IAG) in Vienna, it was suggested that new IUGG resolutions on refractive indices be prepared for adoption at a future General Assembly of IUGG. An ad-hoc working party was formed in 1993 under the umbrella of the International Association of Geodesy (IAG) Special Commission SC3 – Fundamental Constants (SCFC). The aim of this working party is to prepare new resolutions for refractive index formulae for measurements (with an accuracy of better than one part per million) in geodesy for adoption at a forthcoming IUGG General Assembly. Amongst other things, the working party is considering recent determinations of the refractive index of air, the carbon dioxide content of air, the extension of the formulae into the infrared and the implementation of the Lorentz-Lorenz relationship. Present core members of the working party are: P. E. Ciddor (Australia), Y. S. Galkin (Russia), R. J. Hill (USA), H. Matsumoto (Japan), J. M. Rieger (Australia), R. A. Tatevian (Russia).

So far, the working party has concentrated on the refractive index in the visible and near infrared spectrum, because of an apparent lack of interest from geodesists working with GPS and VLBI. (Some proposals for the radio and millimetre waves have been made, however. Because of the constraints on the length of the paper, these aspects will be reported elsewhere.) This paper summarises the progress made in refractive index formulae for light and infrared waves since 1960 and lists the best formulae available today for routine and precision EDM. The views expressed in this interim report are those of the author. The published work of the working party is included, but does not cover all matters in discussion. The working party will report on its work at the next IUGG General Assembly in 1999 and, hopefully, will then also be able to propose some resolutions.

CARBON DIOXIDE CONTENT OF AIR

At Mauna Loa in Hawaii, the average carbon dioxide content of air was 338 ppm (0.0338%) in 1980 and 360 ppm (0.0360%) in 1994, with annual cycles of 6 ppm peak-to-peak (Heimann 1996). In the short term, this annual increase of 1.57 ppm per year extrapolates to a CO₂ content of 366 ppm in 1998, 369 ppm in 2000 and 375 ppm in 2004. In the long term, the CO₂ content will depend on the result of international agreements for the reduction of greenhouse gases. Depending on the adopted Intergovernmental Panel on Climate Change (IPCC) reduction scenarios, Taylor (1994) predicts CO₂ contents of 385–430 ppm in 2030, 385–485 ppm in 2050 and 365–610 ppm in 2100. It is suggested to use a carbon dioxide content of 0.0375% in all new fixed CO₂ content formulae for outdoor measurements. Precision formulae should have an input for the ambient CO₂ content.

These values naturally apply to outdoor measurements. It should be noted, however, that vegetation can locally increase the CO₂ concentration. McCoy et al. (1969) reported on a nocturnal 900 ppm CO₂ content near vegetation. Birch & Downs (1988b) reported on CO₂ contents of 400 to 800 ppm in laboratories, with a maximum of 1200 ppm (Birch & Downs 1989). They adopted a default value of 450 ppm in their 1993 (laboratory) equation.

HISTORY OF REFRACTIVE INDEX FORMULAE

The by no means exhaustive summary of the new developments is used to demonstrate the need for a new equation or equations. The new developments include absolute or relative refractivity measurements, new theoretical considerations and new refractive index formulae based on old and new refractivity data. The additions to the knowledge on refractive indices are discussed, in principle, in chronological order. Work by the same author(s) is grouped together, however, for easier reading.

Peck & Khanna (1962) measured the refractive index of air at eleven infrared wavelengths between 703.4 nm and 2058.7 nm to about 0.1×10^{-8} and normalised them to the refractivity of 27790.10×10^{-8} at 546.2258 nm. The values were up to 0.5×10^{-8} higher than those given by the Edlén (1953) formula. Peck & Reeder (1972) reported new measurements at eight infrared wavelengths from 724 nm to 1530 nm, which were again scaled to Edlén's (1966) value at 546.2 nm. For the new dispersion formula, the authors omitted the 1962 Peck & Khanna data measured at 2.08 μm and 0.70 μm (because they showed anomalies) and the value by Rank et al. (1958) at 407.8 nm (because interpolated and not measured). A new two-term (four-parameter) formula for wavelengths from 0.230 to 1.690 μm was derived. The authors demonstrate that their new formula has lesser average offsets from the data than Edlén's 1966 formula in all bands covered. The root mean square (rms) error of the fit is better in all selected bands other than that of the Erickson data where their rms is $\pm 0.047 \times 10^{-8}$ against Edlén's $\pm 0.030 \times 10^{-8}$. The most significant improvement over Edlén's 1966 formula is in the near infrared, from 0.72 μm to 1.69 μm .

Erickson (1962) carried out new measurements to test the independence of relative refractivity from water vapour and to test the dependence of relative refractivity on pressure. He found that the relative refractivity of dry air changes less than 3 ppm (or 1×10^{-9} in n) for a pressure change of 506 hPa for wavelengths between 389 and 644 nm. From measurements between 361 and 644 nm, Erickson derived a dispersion formula for relative refractivity of water vapour. A conservative figure of $\pm 4 \times 10^{-5}$ is quoted for the standard deviation of the means of the measurements of relative water vapour refractivity (or about 4×10^{-8} in n at 100 percent humidity and 30°C).

Edlén did revise his earlier formula in 1966. He saw no need to change the absolute values of Barrell & Sears (1939) since the Peck & Khanna (1962) data were 5.2×10^{-8} lower than Edlén (1953) and the Schlueter & Peck (1958) data 4.4×10^{-8} higher. The 1953 and 1966 Edlén dispersion curves cross at 605.8 nm. Erickson's (1962) data are based on this absolute value. The 1966 formula is based on new measurements by Rank (1958), Svensson (1960), Peck & Khanna (1962) and Erickson (1961, 1962). The wavelength range from 230 nm to 2059 nm now includes the near infrared. Refractivity as a function of temperature and pressure is based on the Lorentz-Lorenz relationship and the density and equation of state rather than the measurements of Barrell and Sears. The interpolation formula is valid for $+5^\circ\text{C} < t < +30^\circ\text{C}$ (laboratory conditions). The water vapour term is based on a fit of Erickson's relative data (361.1 to 644.0 nm) to Barrell & Sears absolute measurements at 1013.25 hPa, 30°C and 33.33 hPa of water vapour for wavelengths from 435.9 to 644.0 nm. The resulting equation is valid for conditions "not deviating too much from 20°C, 1013.25 hPa and 13.33 hPa" and, thus, does not suit field measurements very well.

Owens (1967) noted Edlén's (1966) statement that the precision of relative refractivity is 1×10^{-9} whereas the few absolute measurements indicate an accuracy of 5×10^{-8} . Owens

remarks that the range of Edlén's formula "does not cover all conditions that may be encountered in atmospheric work". After a critical analysis of available data and physical relationships, he derives a general and a simplified expression for the phase refractive index and a general and a simplified expression for the group refractive index. Deviations between the general and simplified phase refractive formulae do not exceed 0.002 ppm. The difference is less than 0.016 ppm in the case of general and simplified group refractive indices (at 45°C, 100% relative humidity and 1013.25 hPa). [Unfortunately, all formulae given by Owens use t (°C) = $K - 273.16$.] Owens equations are valid as follows: dry air: 0.2302–2.0586 μm , 240–330 K, 0–4000 hPa; water vapour: 0.3611–0.6440 μm , 250–320 K, 0–100 hPa; carbon dioxide: 0.2379–0.6910 μm , 240–330 K, 0–17 hPa.

Old et al. (1971) measured the absolute refractivity of CO₂ between 480 nm and 1817 nm and derived a formula for the compressibility. The dispersion of carbon dioxide is expressed as a 3-term (6-parameter) Sellmeier equation. The root-mean-square error of the fit is $\pm 3.0 \times 10^{-9}$.

Genike & Galkin (1975) pointed out the effect of molecular resonances on the group refractive index used in electronic distance measurement. The authors note that the dispersion in the visible spectrum is mostly caused by resonances in the far ultraviolet region of the spectrum and that the dispersion in regions far removed from the resonances has traditionally been modelled by smooth functions. They point out that numerous resonances occur in the visible and near infrared regions and illustrate the case with a diagram of the large variations in the transmittance of electromagnetic waves for the wavelengths between 890 nm and 930 nm. Relating absorption to refractive index shows phase refractive index fluctuations of 3×10^{-10} near 717.7 nm. Between 380 nm and 830 nm, the fluctuations (non-smoothness) of the phase refractive index varies from 1×10^{-9} to 1×10^{-12} . Even though the phase refractive index variations may, in the visible, be neglected, the fluctuations in the group refractive index, as used in electronic distance measurement, cannot be ignored. As the usual definition (Rayleigh) of the group refractive index is based on two waves of nearly equal frequency in a feebly dispersive medium, the authors derive a new formula for the group refractive index near weak molecular resonances and for low power radiation. Application of their new group refractive index formula shows that the maximum fluctuations of the group refractive index is about 100'000 larger than for the phase refractive index, that is 1×10^{-4} (100 ppm) to 1×10^{-7} (0.1 ppm) for wavelengths between 380 nm and 830 nm. The variations near 717.7 nm are shown in a diagram and amount to 1.2 ppm. Near resonance lines, even in the visible part of the spectrum, the group refractive index can change a few ppm for a change in wavelength of less than 0.1 nm. Clearly, these effects of weak resonance lines in the visible make it very difficult, if not impossible, to design a simple group refractive index formula for precision EDM.

Hill et al. (1980a 1980b 1981) computed the refraction and absorption fluctuations caused by temperature, humidity and pressure fluctuations for the frequency spectrum from radio waves to 5 μm (60 THz). The refractivity N is expressed as

$$N = N_c + N_a = N_d + N_w + N_a \quad (1)$$

where N_c is the continuum value and N_a the contribution from anomalous dispersion by infrared resonances. The continuum value N_c can be expressed as the sum of the dry air component N_d and the water vapour component N_w . The anomalous contribution by water vapour resonances (water vapour monomer only) is obtained from a line-by-line summation of the contributions of the individual resonance lines and covers the wavelength ranges 1.0 m to 5.682 μm and 5.102 μm to 4.854 μm . The phase refractive index can be less than unity. In consequence, the phase speed of light can exceed that in vacuum close to very strong resonances (e.g. near 30 μm). The strong dispersion keeps the group velocity less than that in vacuum (p. 1198). The effect of all infrared water vapour resonances on measurements at 633 nm are negligible (0.01×10^6). Hill & Clifford (1981) added that the relatively small effect of the near infrared resonances of oxygen and the refraction due to carbon dioxide is discussed by Zhevakin & Naumov (1967). Hill & Lawrence (1986) developed a simple formula for the dispersion and temperature dependence of water vapour refractivity between 7.8 μm and 19

μm based on the summation of UV and IR resonances. The fitted curve has a maximum error of 1.4 % and a rms of 0.7%. The authors point out that the graphs of anomalous refractivity published elsewhere must be consulted and that dispersion caused by resonances near the wavelength of interest must be added to the published equation. The final equation is scaled to coincide with Matsumoto's measurements at 10.6 μm .

Giacomo (1982) reported on the efforts of a working group of the Bureau International des Poids et Mesures (BIPM) for a unified approach in the determination of the density of moist air. Values for the molar gas constant, the molar mass of dry air, the molar mass of water are given and the density of moist air derived. Relative humidity (h), enhancement factor (f) and compressibility factor (Z) are defined. The saturation water vapour pressure is tabled between 0 and 27°C, the enhancement factor for 60-110 kPa and 0-30°C and the compressibility for 60-110 kPa and 15-27°C. The corresponding equations are given. A number of error budgets for the uncertainty of the density formula are given, the worst one being 13×10^{-5} . According to Davies (1992), the Comité International des Poids et Mesures (CIPM) adopted an amended equation for the determination of the density of moist air at its 80th Meeting (September 1991). This formula has been designated as the "1981/91 equation for the determination of the density of moist air". A three-parameter equation is used to compute the enhancement factor f , a nine-parameter equation for the compressibility factor Z and a four-parameter equation for water vapour pressure at saturation p_{sv} . The densities calculated with the 1991 formula are 3 parts in 10^5 smaller than those calculated with the 1981 equation, which is equivalent to 1 part in 10^8 in n . The uncertainty of the computed densities is between 0 and 13 parts in 10^5 , depending on interpretation.

Matsumoto (1982a 1982b) reported on the direct measurement of the group refractive index at 3.44 μm with an accuracy of $\pm 2 \times 10^{-7}$. The author notes that if the phase refractive index is accurate to 1.2×10^{-8} then the group refractive index would be accurate to only 6×10^{-7} . Edlén's 1966 formula for dry air is experimentally confirmed at 3 μm and at an uncertainty level of 1.2×10^{-8} . (Unfortunately, the measured value of relative refractivity between 0.633 and 3.39 μm is not stated.) The calculated refractivity (after Owens 1967) of moist air is found to be too high by 0.1 ppm. Later (Matsumoto 1984), the relative refractivity of dry air between 633 nm and 10 μm was measured and agreed with the calculated value within the measurement accuracy of 0.5%. The refractivity of water vapour was measured at six lines of the CO_2 laser near 10.0 μm with an accuracy of 1% with a 10.6/0.63 μm laser interferometer. The measured refractivity was 30% less than the value obtained by extrapolating the dispersion equation in the visible region". The refractivity of CO_2 gas was also measured (Matsumoto 1987) at 10 μm with an experimental error of 2% (8 parts in 10^6 of n). An existing CO_2 refractivity formula is confirmed at 10.0 μm within these experimental errors. Matsumoto and Zeng (1994) measured the refractive index at 532 nm and 18.3°C with two-colour interferometry to 1.4×10^{-7} . No significant differences from computed refractivities (Edlén) were found.

Deichl (1984) carried out a critical analysis of all simplifications used in the 1963 IUGG formulae. He found errors of 0.15 ppm at normal conditions and a systematic error of 0.3 – 0.4 ppm between light wave and microwave measurements and noted a difference of >0.4 ppm at 45°C and 100% relative humidity between Edlén's and Owens' humidity terms.

Birch & Downs (1988a 1988b 1989) measured the refractivity of dry air between 10 and 30°C and 20 to 115 kPa. The 99% uncertainty level of the measurements was estimated as $\pm 0.95 \times 10^{-8}$ and those of the computed values as 3.04×10^{-8} . New measurements of moist standard air at +20°C and $e = 4 - 20$ hPa indicate an error of 13×10^{-8} at 100% at 20°C in Edlén's water vapour term. The latter is attributed to physisorption effects in Barrel and Sears experiments. A modified Edlén equation is derived, which has an uncertainty of 3.3×10^{-8} after correction for prevailing CO_2 levels. The molar refraction of nitrogen, oxygen, argon, carbon dioxide, neon, helium, nitrous oxide, krypton and dry air was determined by Birch (1991) at 633 nm to 2 parts in 10^4 . Refractivity values are then derived. A correction for the refractivity of air for excess carbon dioxide is given as 1.45×10^{-10} per excess ppm of CO_2 .

In 1993, Birch & Downs describe a thorough, complete and far reaching revision of Edlén's 1966 formula. (Please note the corrections published in Birch & Downs 1994.) It is noted that, since 1966, improved data have become available for the density of air and the refractivity of water vapour. In addition, the temperature scale was redefined and the CO₂ content was found to be larger than 300 ppm. The revised equation is valid for "ambient conditions" over the wavelength range of 350 nm to 650 nm and 450 ppm CO₂ content. (The former may be defined as 10-30°C, 800 - 1200 hPa.) The authors note that there are indications that the uncertainty of the new formula may approach 1×10^{-8} . However, practical applications provide refractive indices with an uncertainty of about 1×10^{-7} due to the additional inaccuracies of the meteorological sensors. The modified Edlén humidity term proposed by Birch and Downs (1988a) is confirmed by Beers & Doiron (1992) based on line scale interferometer measurements at the National Institute of Standards and Technology, Gaithersburg, USA.

Using Peck & Reeder (1972) and Owens (1967), Rüeger (1990 Appendix A) developed a new group refractive index formula (Peck-Reeder-Owens). The dispersion of dry standard air was derived by converting Peck & Reeder's (1972) four-term phase refractive index formula to a group refractive index formula. The density of dry air, the density of water vapour and the dispersion of water vapour were taken from Owens' "simplified" formula.

Ciddor (1995) developed a new formula for precision distance measurements in geodesy and surveying. His phase and group refractivity formulae are presently the most accurate available. Like all other reported formulae for the visible and near infrared region of the spectrum, it does not model anomalous refractivity due to resonance lines. The refractive index is not expected to be better than $2-5 \times 10^{-8}$. The formula is discussed further below.

In 1996, Hill published an excellent review on the refractive index of atmospheric gases. Methane and nitrogen oxides have significant absorption lines that are, however, not significant at wavelengths larger than UV. So-called absorption windows permit the derivation of simple and accurate refractive index formulae for specific ranges of wavelengths such as the visible, the infrared and the radio wave windows. The dependence of refractivity on density of gases is described. It is noted that dispersion can be measured more accurately than absolute refractivity. The author lists the dispersion equations of Edlén (1966) and Peck & Reeder (1972). The effect on refractivity by water vapour cannot be neglected in the infrared. Line-by-line summation of the resonances of atmospheric molecules may be used between the visible and radio frequency windows. However, the line-by-line summation must be augmented by an extrapolation to the IR of the refractivity in the visible window to account for the resonances in the UV. The error in refraction calculations by line-by-line summation "is probably due to errors in the compilation of resonances parameters". In a table, the radio refractivities of nitrogen, carbon dioxide, water vapour, dry air, and oxygen are compared with extrapolated values from the visible. Dry air, nitrogen and oxygen have insignificant infrared dispersion resonances. Carbon dioxide and water vapour however have. The water vapour resonances in the infrared fully explain the difference between the extrapolated visible and the radio wave refractivity. The carbon dioxide difference is accounted for by the CO₂ rotational resonance near 15 μ m. The CO₂ dispersion formula used by Edlén (1966) is not appropriate for extrapolation to longer wavelengths whereas the formula by Old et al. (1971) is.

Galkin & Tatevian (1997a 1997b) continue the work of Genike & Galkin (1975). They note that there is a large number of molecular resonances in the visible and near infrared spectra due to most constituent gases of air, not just the trace gases. They show (Galkin & Tatevian 1997a) that the Rayleigh, Lorentz-Lorenz and Sellmeier formulae need to be extended to account for the anomalous refractivity due to resonances. The authors also point to problems near 100 percent relative humidity, where water microdrops are created that lead to an aerosol rather than a gas substance, and due to aerosol pollution, where even small concentrations can have an effect because of the 10^3-10^4 times larger refractivity. Galkin & Tatevian (1997b)

repeat that the group refractive index can be affected by more than several ppm due to the influence of resonance lines. They list eight weak oxygen resonances near 766 nm and eight weak oxygen resonance lines near the HeNe wavelength of 633 nm, for example. When using HeNe lasers (633 nm) for high precision EDM, the resonance effects on the group refractive index vary from +0.1 to -1.0 ppm between 632.789 nm and 632.853 nm (Galkin & Tatevian, 1997b, Fig. 3) and can change during measurements if the wavelength is not stable. The authors conclude that existing spectroscopic atlases must be extended to resonance lines of less than 0.01% absorption and must become more accurate if the smooth group refractive index is to be reliably corrected for the resonance effect of close and weak resonance lines.

FORMULA FOR HAND CALCULATIONS AND ROUTINE MEASUREMENTS

Considering that one of the formula adopted by IUGG (1963) compares extremely well with the new formula by Ciddor (1995) and is easy to use, a continuation of its use for routine work is appropriate. The following formula for the group refractivity for visible and near infrared waves at ambient conditions (Rüeger 1990 1996) is recommended

$$N_L = (n_L - 1) 10^6 = \left(\frac{273.15}{1013.25} \times \frac{N_g \times p}{(273.15 + t)} \right) - \frac{11.27 e}{(273.15 + t)} \quad (2)$$

where N_L is the group refractivity of visible and near infrared waves in ambient moist air, t the temperature in degree Celsius (ITS-90), p the total pressure in hectopascal (hPa) and e the partial water vapour pressure in hectopascal (hPa). The group refractivity N_g at standard conditions and 300 ppm CO₂ content is as follows (Barrell & Sears 1939):

$$N_g = (n_g - 1) 10^6 = 287.604 + \frac{4.8864}{\lambda^2} + \frac{0.0680}{\lambda^4} \quad (3)$$

Multiplication of all constants with the factor 1.000'040'07 (derived from the results of Ciddor's formulae for wavelengths of 650 nm and 850 nm at 0°C, 1013.25 hPa, 0.0 percent relative humidity and CO₂ contents of 300 ppm and 375 ppm), converts Eq. (3) to the new CO₂ content of 375 ppm (0.0375%):

$$N_g = (n_g - 1) 10^6 = 287.6155 + \frac{4.88660}{\lambda^2} + \frac{0.06800}{\lambda^4} \quad (4)$$

where λ is the carrier wavelength of the EDM signal (in micrometre, μm) and n_g the group refractive index in standard air at $t = 0^\circ\text{C}$, $p = 1013.25$ hPa, $e = 0.0$ hPa, 0.0375% CO₂ content. As Table 1 below shows, this formula (Eq. (3)) is used for the comparison at a CO₂ content of 300 ppm) deviates less than 0.24 ppm between -30°C and +45°C, 1000 hPa and 100% relative humidity from the Ciddor formula of 1995 for wavelengths of 650 nm and 850 nm.

In its most accurate form, the first velocity correction in electronic distance measurement can be expressed as (Rüeger 1990, 1996, Eqs. (6.2) and (6.5))

$$d = \left(\frac{n_{\text{REF}}}{n_L} \right) d' \quad (5)$$

where d' is the read-out distance (with instrument internal first velocity correction disabled), n_L is taken from Eqs. (2) and (4) and n_{REF} is the instrument specific reference refractive index

$$n_{\text{REF}} = \frac{c_0}{\lambda_{\text{MOD}} \times f_{\text{MOD}}} \quad (6)$$

where c_0 is the velocity of light in vacuum (299 792 458 m/s), λ_{MOD} is the fine modulation wavelength of the instrument ($2 \times$ unit length) and f_{MOD} the fine modulation frequency of the

instrument. Normally, the first velocity is expressed as an additive correction of the form (Rüeger 1990 1996)

$$K' = \left[C - \frac{D p}{(273.15 + t)} + \frac{11.27 e}{(273.15 + t)} \right] 10^{-6} \text{ d}' \quad (7)$$

where $C = (n_{\text{REF}} - 1) \times 10^6$, $D = (273.15/1013.25) \times N_g$ and N_g is taken from Eq. (4). The simplification used in the derivation of Eq. (7) introduces (additional) errors of less than 0.02 ppm (Rüeger 1990 1996).

COMPUTER ROUTINE FOR PRECISION MEASUREMENTS

The system of formulae proposed by Ciddor (1995) is presently the most appropriate for the computation of the group refractive index for precision electronic distance measurement in geodesy and surveying. It takes "into account all known factors (except for atmospheric contaminants such as oil vapours and the effects of absorption lines), embodies the latest values of all physical parameters and units and should be valid for all practical atmospheric conditions (-40°C to +100°C, 800 hPa to 1200 hPa and 0 to 100% relative humidity) and for wavelengths from below 350 nm to above 1300 nm."

The new formula by Ciddor (1995) is based on the dry-air dispersion formula of Peck & Reeder (1972), the water vapour dispersion formula of Owens (1967) adjusted to the new absolute moist air measurements of Birch and Downs (1993 1994) and the BIPM (Bureau International des Poids et Mesures) formulae for the density of moist air and saturation water vapour pressure (Davies 1992). It further includes the CO₂ content as a variable (CO₂ dispersion after Birch and Downs (1994), checked against Old et al. (1971)) and all temperatures in the new International Temperature Scale 1990 (ITS-90). The necessary conversions from old temperature scales are based on Preston-Thomas (1990).

The overall uncertainty (one standard deviation) of the refractive index is not expected to be better than 2.5×10^{-8} . The limiting uncertainty is that of density that is estimated by Giacomo to be 5.13×10^{-5} , which translates to an uncertainty of the refractive index of $1.4\text{--}3.5 \times 10^{-8}$. The uncertainty of the refractive index measurements is taken as a few parts in 10^{-8} . Assuming an uncertainty of 1×10^{-9} for the dispersion equations, then the uncertainty of the group refractive index would be 3×10^{-9} . The principle of Ciddor's (1995) computation of the refractivity at ambient conditions is as follows

$$N_{\text{amb}} = \frac{\rho_a}{\rho_{\text{asx}}} N_{\text{asx}} + \frac{\rho_{\text{wv}}}{\rho_{\text{wvs}}} N_{\text{wvs}} \quad (8)$$

where

- N_{amb} = refractivity of ambient moist air
- N_{asx} = refractivity of dry standard air at standard conditions (15°C, 1013.25 hPa, x ppm CO₂)
- N_{wvs} = refractivity of water vapour at standard conditions (20°C, 13.33 hPa)
- ρ_{asx} = density of standard air (with x ppm CO₂)
- ρ_{wvs} = density of standard water vapour
- ρ_a = density of dry component of ambient air
- ρ_{wv} = density of water vapour component of ambient air

Ciddor notes that the results of the above equation differ less than 2 parts in 10^{-9} in a worst case scenario from solutions with the direct Lorentz-Lorenz relationship. Numerical values for the phase refractive index at some specific conditions are given by Ciddor (1995). Please note that Ciddor's paper has two typographical errors (Ciddor 1996). (In Table 2, Line 2 ($t = 19.517$) the last entry should be 27394.3. In Table 3, Line 3 ($t = 40$, $p = 80$) the entry for Eq. (5) should be 19896.5.) Numerical values for the group refractive index at some (other) conditions are given below in Table 1. Ciddor gives the formulae necessary to compute all

components of Eq. (8) and instructions on the most appropriate sequence of the computations. A FORTRAN program for the Ciddor equation (as published) has been written at UNSW for (IBM compatible) personal computers and is available from the author. (Requests for the software should be sent to the author, together with a blank double-sided high-density 3.5 inch IBM-formatted floppy disk and a self-addressed address label.)

In an addendum to his paper, Ciddor (1966) noted that, since the publication of his paper, he has incorporated the full Lorentz-Lorenz relationship in his software. The revised equations may be published later. He further notes that the above Eq. (8) (Ciddor 1995 Eq. (5)), which assumes that the group refractive index is simply proportional to density, differs from the exact result by less than 1 part in 10^8 . He also states that the CO_2 dispersion formula used might be reviewed and that the effects of molecular resonances need further investigations.

| T | PWVP | CIDDOR | OWENS | EDLÉN | IUGG | Differences | | |
|--|----------------|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|---------------------------|---------------------------|---------------------------|
| [°C] | P_w [hPa] | 1995 N_L $\times 10^{-8}$ | 1967 N_L $\times 10^{-8}$ | 1966 N_L $\times 10^{-8}$ | 1963 N_L $\times 10^{-8}$ | (4-3) $\times 10^{-8}$ | (5-3) $\times 10^{-8}$ | (6-3) $\times 10^{-8}$ |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| $\lambda = 650 \text{ nm}, \text{ CO}_2 \text{ content} = 300 \text{ ppm (0.03\%)}, \text{ total pressure} = 1000 \text{ hPa}$ | | | | | | | | |
| 60 | 199.26 | 23626.92 | 23537.14 | 23413.97 | 23564.92 | -89.78 | -212.95 | -62.00 |
| 45 | 95.85 | 25066.14 | 25018.14 | 24978.36 | 25042.27 | -48.00 | -87.78 | -23.87 |
| 30 | 42.43 | 26484.65 | 26462.35 | 26453.76 | 26479.96 | -22.30 | -30.89 | -4.69 |
| 15 | 17.04 | 27955.58 | 27946.75 | 27946.76 | 27957.72 | -8.83 | -8.82 | +2.14 |
| 0 | 6.10 | 29536.31 | 29533.76 | 29533.96 | 29538.14 | -2.55 | -2.35 | +1.83 |
| -15 | 0.00 | 31283.16 | 31284.67 | 31280.87 | 31281.10 | +1.51 | -2.29 | -2.06 |
| -30 | 0.00 | 33221.68 | 33223.60 | 33215.56 | 33210.86 | +1.92 | -6.12 | -10.82 |
| $\lambda = 850 \text{ nm}, \text{ CO}_2 \text{ content} = 300 \text{ ppm (0.03\%)}, \text{ total pressure} = 1000 \text{ hPa}$ | | | | | | | | |
| 60 | 199.26 | 23199.45 | 23112.02 | 22986.56 | 23156.05 | -87.43 | -212.89 | -43.40 |
| 45 | 95.85 | 24629.37 | 24582.71 | 24542.14 | 24614.12 | -46.66 | -87.23 | -15.25 |
| 30 | 42.43 | 26032.10 | 26010.60 | 26001.80 | 26030.60 | -21.50 | -30.30 | -1.50 |
| 15 | 17.04 | 27482.34 | 27473.94 | 27473.96 | 27485.00 | -8.40 | -8.38 | +2.66 |
| 0 | 6.10 | 29038.32 | 29036.07 | 29036.32 | 29039.46 | -2.25 | -2.00 | +1.14 |
| -15 | 0.00 | 30756.90 | 30758.62 | 30754.88 | 30753.44 | +1.72 | -2.02 | -3.46 |
| -30 | 0.00 | 32662.81 | 32664.95 | 32657.05 | 32650.64 | +2.14 | -5.76 | -12.17 |

Table 1: Comparison of visible and NIR refractivity N_L (in parts per 10^8) from the Ciddor (1995) and a number of earlier formulae. The Owens'97 data are based on Owens (1967, Eqs. (30), (31), (42)); the Edlén'66 data are based on Edlén (1966, Eqs. (1), as and (22) converted from phase to group, and Eq. (12)). The IUGG data follow from Eqs. (2) and (3).

COMPARISON OF FORMULAE

Table 1 shows a comparison between the new Ciddor (1995) formula and the older Owens (1967), Edlén (1966) and IUGG (1963) formulae. The table lists the refractivity for two wavelengths, a total pressure of 1000 hPa and temperatures (in 15°C steps) between -30°C and $+60^\circ\text{C}$. As the old formulae all refer to a 300 ppm (0.03%) carbon dioxide content, the Ciddor formula was computed with the same value. As a worst case, the relative humidity was set to 100 percent at temperatures of 0°C and above. The saturation water vapour pressures (Rüeger 1990 1996, Appendix B, after Goff & Gratch), which were used as an input for all formulae, are listed for reference.

Evidently, the Edlén (1966) formula, that in the past has been recommended by some manufacturers and used by some users, performs worst at temperatures above $+15^\circ\text{C}$. This is

due to the poor water vapour modelling of the Edlén equation. (The latter was designed for laboratory conditions.) The error in the Edlén formulae amounts to 2.13 ppm at +60°C and 0.88 ppm at +45°C, which is clearly not acceptable for precise EDM, even though the extreme conditions are unlikely to occur in practice. The Owens (1967) formula performs about two times better at high temperatures and humidity, with errors of 0.90 ppm at +60°C and 0.48 ppm at +45°C. The results of the comparison of the Rüeiger (1990) equation is not shown in Table 1. This formula agrees with Owens' within 0.6×10^{-8} (and is usually closer to the Ciddor formula). The formulae by Edlén and Owens are equally affected by the errors in Barrell & Sears' absolute water vapour measurements. The latter were used as a reference by both authors and found to be in error (Birch & Downs 1988b 1989 1993 1994; Beers & Doiron 1992). The comparisons at zero humidity (see -15°C and -30°C) show a good agreement (0.02 ppm) for the Owens formula, which was, like the Ciddor formula, specially designed for outdoor measurements.

The IUGG formula of 1963, in the form given by Eqs. (2) and (3) has the smallest deviation from the Ciddor formulae at all temperatures above -15°C. It seems that the simplifications in Eq. (2) compensate for the systematic errors in the original equation. The errors of this formula do not exceed 0.6 ppm in normal conditions of the free atmosphere for pressures from 350 to 1000 hPa, temperatures from -30°C to +60°C and relative humidity from 0% to 100%.

The computations were repeated with a total pressure of 350 hPa (as on Mount Everest) for temperatures of 0°C, -15°C and -30°C. At this pressure, the agreement of the Owens and Edlén equations with the Ciddor equation is much better at -15°C and -30°C and zero humidity than at sea level pressure. With about 0.03 ppm, the agreement of the Owens and IUGG formulae is slightly worse at 0°C and a partial water vapour pressure of 6.10 hPa when compared with sea level pressure.

CONCLUSIONS

Because of the good agreement of the simple Eqs. (2) to (7) with the much more involved Ciddor equation (Eq. (8)), it is strongly recommended that Eqs. (5) or (7) together with Eqs. (2), (4) and (6) be used for the reduction of routine electronic distance measurements. It is also hoped that the manufacturers of EDM instrument will adopt the same Eqs. (5) or (7), with Eqs. (2), (4) and (6), for the real-time computation of the first velocity correction in instruments. These formulae now refer to a carbon dioxide content of 0.0375% (375 ppm, expected to become current in 2004) and are accurate to about 0.5 ppm.

For precision electro-optical distance measurements, the use of the Edlén (1966) formula should be discontinued because of its very poor performance at high temperatures and humidity. The best formula today is that of Ciddor (1995). It has the added advantage of an input for the actual CO₂ content of air. (Portable CO₂ meters with an accuracy of better than ±30-90 ppm are commercially available, for example from Väisälä and Edinburgh Sensors.) The Ciddor formula has already been and might be further amended to make it more rigorous. The overall and absolute accuracy of $\pm 2.5 \times 10^{-8}$ will not change, however.

One major issue has not been resolved yet, namely the errors in the above group refractive index formulae caused by resonance lines. According to Galkin & Tatevian, these errors can be as large as several ppm for EDM instruments having very narrow emission spectra (monochromatic) and centre wavelengths that are close to resonance lines (even very weak ones). To be able to compute this anomalous refractivity and to correct for it, a software package similar to the Millimetre-Wave Propagation Model (MPM) (Liebe 1989 1996) needs to be written, an extensive and accurate atlas of spectroscopic data is required and the emission wavelength of the distance meter must be known (under field conditions, at the time of the distance measurements and with very high accuracy). Galkin & Tatevian (1997b) noted that even the extensive HITRAN'96 molecular spectroscopic database (Rothman et al.

1996), which describes, amongst many others, 49444 water, 60802 carbon dioxide, 6292 oxygen and 120 nitrogen resonance lines is not accurate enough and does not list the weak resonance lines. When also considering that the exact emission wavelengths of EDM instruments are not usually known and tend to change with temperature and time, it becomes clear that the computation of anomalous group refractivity in the visible will not be easy nor likely possible in the very near future.

The author believes that, in practice, the problem of anomalous group refractivity in the visible and near infrared is greatly reduced by the fact that most distance meters, particularly those operating in the infrared, have quite broad emission spectra. This tends to average out anomalous refractivity. Even stabilised HeNe lasers will somewhat reduce anomalous refractivity because their servo loops tend to change their emission wavelengths in a cyclic pattern. As a start, it would be very worthwhile to get stability and emission data of a selection of current electronic distance meters and to predict the likely magnitude of the practical anomalous refractivity using available spectroscopic data. This should give at least some idea of the magnitude of the problem. Although there is some anecdotal evidence of different electro-optical distance meters giving different results, the magnitude of such problems tended to be below the several ppm predicted by Galkin & Tatevian.

The work of the IAG working party on the refractive index in air will be reported in more detail at a forthcoming General Assembly of IUGG and IAG. New resolutions may be proposed at that time. – A companion review of the refractive index of radio and millimetre waves will be published elsewhere.

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