Constraint Equations in Cadastral Modelling

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Cadastral modelling uses the cadastral data including bearings and distances of boundary lines to calculate coordinates of boundary corners and associated marks, and to evaluate the data. This paper shows that at sites where several boundary corners are intended to be on a straight line a Least Squares adjustment of the boundary data may cause the points to move away from one straight line.

Methods of constraining boundary corners to stay on straight lines, when that is desired, in a Least Squares solution of cadastral boundary data are described in this paper. The LS process and equations are given for constraining points to stay on a straight line by using fixed 180° angle observations or by using parameter constraint equations. Also, considered are multiple points on a line, parallel line constraints and constraints to keep boundary points on a circular arc, where appropriate. An example is given to demonstrate the application and effect of the constraints.

Keywords*:* Constraint Equations, Cadastral, Least Squares Modelling

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INTRODUCTION

This paper investigates one of the tasks in the coordination of cadastral points using boundary dimension data from Deposited Plans (DP). The discussion is specifically related to the New South Wales cadastre, but may have wider applications. The research presented in this paper aims to show possible least squares (LS) calculations to provide options and tools, not to suggest which boundary definition practices to adopt or how to create or upgrade a Digital Cadastral Database (DCDB). Boundary survey definition questions and many DCDB matters are beyond the scope of this research, but a few issues are mentioned.

The paper was written in response to requests from surveyors to investigate cadastral modelling particularly with regard to matters such as straight line constraints. It is written in a style that is hopefully readable for professional surveyors, programmers, and students to read, as well as for academics and researchers.

Coordination of a cadastre and DCDB are matters that have been studied and debated, often passionately, for many years. Software for implementing the coordination of cadastres includes: CAD with surveying modules, LS network adjustment, and specialist cadastral modelling software within GIS or as standalone software. Anecdotally, many surveyors calculate their cadastral boundary definition surveys with coordinates and have done so for many years. However coordinates of all cadastral points and related marks are not currently publicly recorded in New South Wales (NSW) on DPs. Some surveyors do maintain their observations and coordinates in files for later use with future nearby surveys and thus over time they build a useful data set of coordinated cadastral points and marks that is not always available to other surveyors.

It is also beyond the scope of this paper to thoroughly investigate all the topics of coordination of cadastres or to expound significantly on them. However it is important to place the technical LS aspects of this paper in the broader context of coordinated cadastres. So briefly, some questions and points of concern are listed here, without an attempt to give good answers or to suggest best solutions.

What is the purpose of the coordination of the cadastral points and what accuracy is required? Is its purpose boundary definition, or GIS, or mapping? One application may be to generate coordinates that are suitable for finding cadastral survey marks by GNSS or traversing, as part of a surveyor's search and preparatory investigations prior to survey at the site. Another application might be to generate coordinates and plans suitable for planning new infrastructure. Do we want millimetre accuracy, or coordinates accurate to a few centimetres (e.g. lines that lay within the material of a fence), or an accuracy guided by the diameter of a hole that we might dig to find a buried survey mark, optic fibre cable or gas line, etc., or an accuracy depending on the scale of a mapping product? Is the purpose of the coordination to upgrade a DCDB, and if so what are the consequences on other layers such as utilities that might be tied to a DCDB? The answers to the questions on application and accuracy could influence the method chosen for data analysis.

Suppose a straight road boundary was created with several lots fronting it, thus several corners lie along a straight line. Further, some of the original corner marks may not be found by subsequent surveys of these lots. These later surveys may place additional cadastral reference marks (RM) along the road. The addition of information from new DPs in an area, which create new boundaries or perhaps change the dimensions of existing boundaries, may influence the coordinate calculations. Now when data from all the DPs of this road are combined, should we keep the line with a number of corner points on it straight as that was the original surveyor's intention, or allow small bends at each corner along the road frontage? Of course the answer partly depends on how far *off line* the intermediate points are located.

Another consideration: if for example, a straight road is shown on a DP as being 66 feet (one chain) wide, should we maintain the road width by constraining the solution so that both road frontage boundaries are parallel and 66 feet apart? Coordinate calculations could similarly allow for curved boundaries, so that radial and tangential boundary lines are constrained to their original intentions.

Experienced Registered Surveyors do not always agree about a particular boundary's definition. For example, *When redefining lot boundaries, never connect across roads* (Azimuth, 2007a) and *Stick to your side of the road – use marks on the other side of the road when all the marks on your side of the road are gone ... and heaven forbid as a last resort use marks on the other side of the road around an angle!!!* (Azimuth, 2007b). These views are different from those who would argue to maintain the original plan's intentions which might include straight lines, original width roads etc. The views also imply some expected difficulty with road frontages. For boundary definition purposes we also need to consider the hierarchy of evidence that prefers monuments to measurements and other matters.

Further related questions include: Should we include data from all plans of the site or just the latest plans? How can we best manage the storage and transfer of data and results? How should we manage the possibility that coordinates of a cadastral point may change with time?

LEAST SQUARES MODELLING

Least Squares is a mathematical method suitable for adjusting observations containing random errors. Using boundary dimension data, which is derived from measurements that are processed and often rounded, and implied data from many plans in an area in a single coordination adjustment usually gives sufficient data redundancy to provide data quality indicators and might help resolve inconsistencies and errors. If there is a limited amount of redundant data in an area it is quite difficult to locate errors. Thus it is useful to include data from many plans of surveys in the project area. It is also important to include appropriate weighting (input standard deviations) considering the accuracy standards of the era when the survey was done and other evidence. However, it would be inappropriate to generalise here about the appropriate weighting of individual data.

The main LS topic for this paper is the analysis of data sets where cadastral boundary lines contain several points that were initially intended to be on a straight line (ie on-line). However, when all the cadastral data in that region are adjusted the points do not remain *on-line*, i.e. bends are introduced. A simple example is described

Figure 1. *Part of Cadastral Plan DP 255841 (obtained from Department of Lands, New South Wales). Reduced in size compared to original.*

later. There are at least four approaches. Firstly, do nothing and let the lines bend. Secondly, add additional *observations* of 0° or 180° angles with large weighting that force the lines to remain straight. Thirdly, introduce constraint equations to the LS model. The second and third of these methods are discussed in this paper. A fourth method is to determine a line of best fit for points intended to stay *online*. This latter method is described in Merritt (2006).

Once straight lines are dealt with appropriately we may then consider parallel straight line constraints e.g. road frontages.

EXAMPLE

The purpose of the following simple example is to demonstrate some aspects of the problem of LS estimation of cadastral point coordinates. The example does not aim to prove or support any method, nor does it represent the complexity of the often very large DCDB projects. This example uses data from NSW cadastral plan DP 255841, surveyed in 1977 (Figure 1).

The legibility of cadastral plans is important in cadastral modelling and some plans, especially very old plans, can be difficult to read. Figure 1 shows part of a plan and has been reduced in size to fit the page. For the purposes of this paper Figure 1 shows the context of the boundaries used as example data below; it is not necessary to be able to read the data in Figure 1. Figure 1 shows the subdivision of land north of Quebec Street. The lots in this plan were sold and houses were built in approximately the 1980s. However front fences were not built on most of the lots and 30 years later some of the original corner pegs placed by the survey for this DP still exist as do most of the reference marks. This is a somewhat rare occurrence. Incidentally, the marks existing in 2006 of the entire area in Figure 1 were surveyed and connected into a MGA (Map Grid of Australia) control network, but this paper does not require or use those results.

Some of the data in this plan are redrawn in Figure 2 and are used for example calculations in this paper.

The DP created lots 31 to 33, but the road boundary of Quebec St was created as a straight line in an earlier plan. Thus the intention was a straight line from point 1 to point 10 as labelled in Figure 2.

LEAST SQUARES CALCULATION EXAMPLE

Consider lots 31, 32, and 33 in Figure 2. At this stage the adjoining lot boundaries, reference marks, MGA or subsequent

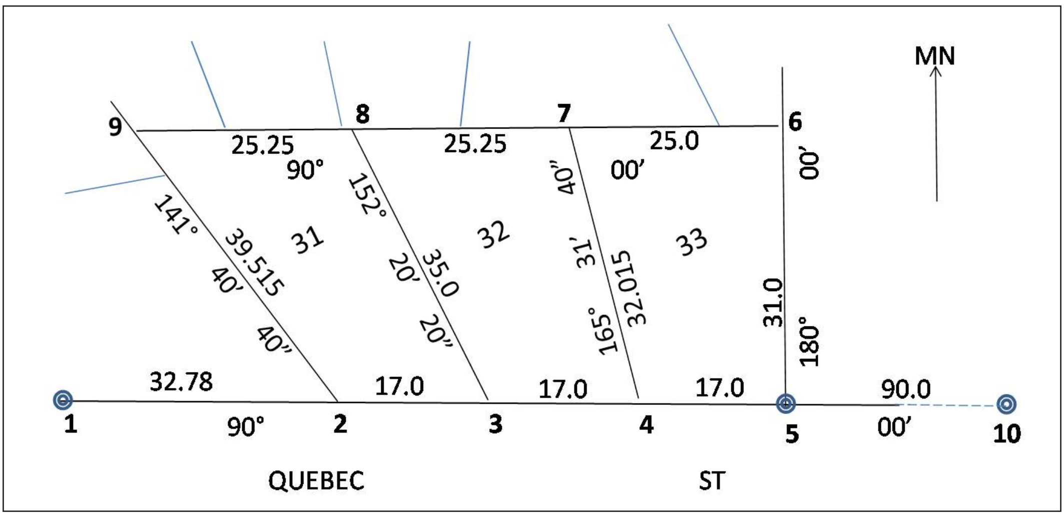

Figure 2. *Data from part of DP 255841*

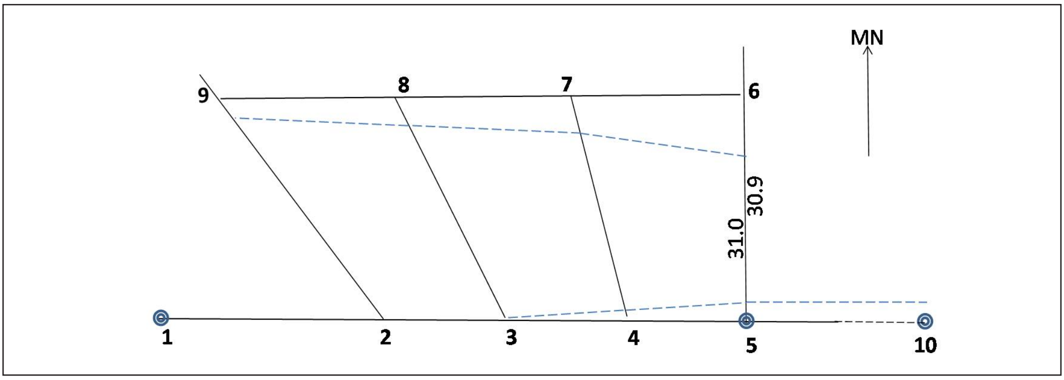

Figure 3. *Effect of two plans with different dimensions for line 5 to 6. Not to scale. Dashed lines represent an exaggerated view of the position of the adjusted boundaries.*

surveys of marks found are not considered. Standard deviations of the input data, for the purposes of these examples, have been set at about 0.3 times the least count of the rounded data on the plan. The data all come from one DP and are self consistent. The loop miscloses are smaller than 5 mm in each lot. The lengths of the lines have not been considered when assigning the standard deviations of the bearings. We considered the quality of measurements but also the reliability of marks and occupations measured to by different surveyors e.g. did they really measure to the same point? So, the standard deviations of the distances are set to ±2 mm and the bearings to ±7″. What effect does this choice of *weightings* or standard deviations of input data have on the results? Generally small changes in standard deviations do not have a major effect on the adjusted coordinates.

A standard LS adjustment of the data for the above three lots does not yield significant bends in either the front or rear lines of the lots. Point 1 only, was held fixed at arbitrary coordinates. If the bearings are treated as sets of direction observations from each point and solved for an orientation parameter at each corner the results are not significantly different to those above.

Hypothetically, say another surveyor in a later plan for the lot to the east of lot 33, shows the distance from point 5 to point 6 (Figure 2) to be 30.9m i.e. 0.1m less than shown above, but with the same angles at the corners. The second surveyor's determination might be based on marks found or for other reasons. Differences like this are not uncommon, but a large discrepancy was selected here to highlight the effects. A least squares solution using data from both plans with a minimally constrained solution (point 1 is held fixed but not point 10), did cause a bend at point 5, as follows. Compared to the previous solution, point 4 moves 3 mm north, point 5 moves 7 mm north, point 10 moves 7 mm north, point 6 moves 22 mm south, points 7, 8 and 9 move south by smaller amounts. An exaggerated view of the shifts is shown in Figure 3. Of course, some outliers are flagged in the solution but these are not dealt with here.

When point 10 is held fixed as well as point 1, there is still a bend of the front and rear lot lines. The magnitudes of the shifts are similar to those above.

In this particular example some lines have cardinal bearings; the bearing of line 1 to 10 is exactly 90°. So a simple solution for this particular data would hold fixed the Northing coordinate of points 1, 2, 3, 4, 5 and 10. Solving for the Easting only would hold all points on line. But that is a special case, not applicable to all surveys. This *Easting only* solution for this data set left the street frontage, 1 to 10, as a straight line and moved points 9 to 6 south by 10 to 26 mm.

The discrepancies from straight lines shown in this example may or may not be significant depending on the application of the coordinates. Again, this paper does not suggest whether the lines should be constrained to be straight or not. This paper merely explains the technical aspects and leaves the decisions to the surveyor.

ANGLE CONSTRAINTS

Another way to hold lines straight is to add fictitious observations of 180° angles at certain points and to give these *observations* very small standard deviations (s) e.g. ±0.01″, thus causing these observations in practical terms to be fixed. Their residuals (v) and v/s will be close enough to zero. We do not set s = 0, i.e. perfectly held fixed, because that would cause divide by zero errors in the LS calculations.

With the data in Figures 2 and 3, a 180° constraint angle was added only at point 5 (i.e. ∠ 1-5-10) then points 1, 5 and 10 did stay on line but points 2, 3 and 4 were not on this line, they had offsets of a few millimetres. An alternative to applying a 180° angle e.g. ∠ 1-5-10 is to apply a 0° angle e.g. ∠ 5-1-10.

If there are more than three points on line e.g. points 1-2-3-4-5, one option is to add an angle at each intermediate point e.g. ∠ 1-2-3 = 180°, ∠ 2-3-4 = 180°, ∠ 3-4-5 = 180°, or ∠ 1-2-5 = 180°, ∠ 1-3-5 = 180°, and ∠ 1-4-5 = 180°. With the data in Figures 2 and 3 the points 2, 3, 4, and 5 were given fixed 180° angles and the line did stay straight from 1 through to 10. The back (northern) corners of the lots did, of course, shift.

Next, the rear lot boundaries are also constrained. The points 2, 3, 4, 5, 7 and 8 were given *observed* 180° ±0.01″ angles. The solution for this data can be compared with the adjusted coordinates from the first solution as shown in Figure 4. If the data in DP 255841 relevant to the lots north of *our* lots 31, 32, and 33 was included, then the line from 6 to 9 would include 8 points.

Effect of constraints on VF

What effect does adding these extra observations have on the estimated Variance Factor (VF), residuals, outliers, degrees of freedom and other aspects of LS? In the above context constraint angles are not derived from direction observations and are thus not correlated, and none of the other bearing or distance *observations* from the DPs are assumed to be correlated. So

$$VF = \frac{\sum \left(v/s \right)^2}{n-u} \tag{1}$$

If *fixed* angles are added their v/s is virtually zero, however the residuals and v/s of other observations will generally be different to the values if fixed angles had not been included. So adding fixed angles does contribute to the numerator in equation 1. If the fixed angles are counted as observations, then n-u increases (n-u = the degrees of freedom = number of observations − number of unknown

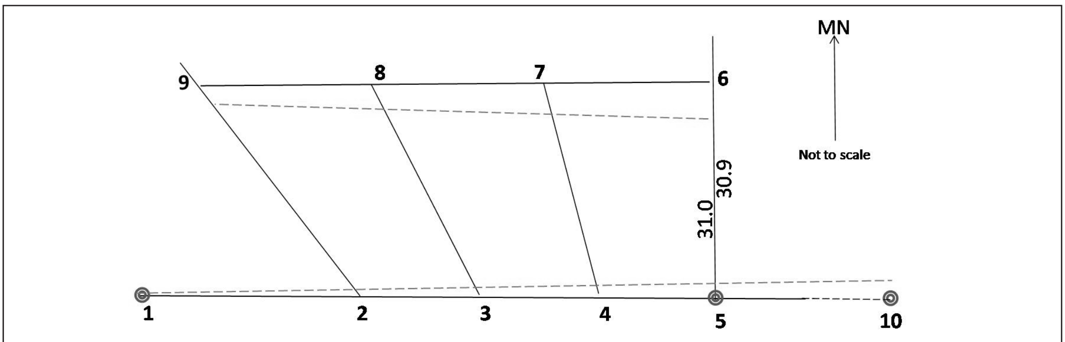

Figure 4. *Effect of two plans with different dimensions for line 5 to 6 with 180° constraint angles at points 2, 3, 4, 5, 7 and 8. Dashed lines represent an exaggerated view of the position of the adjusted boundaries.*

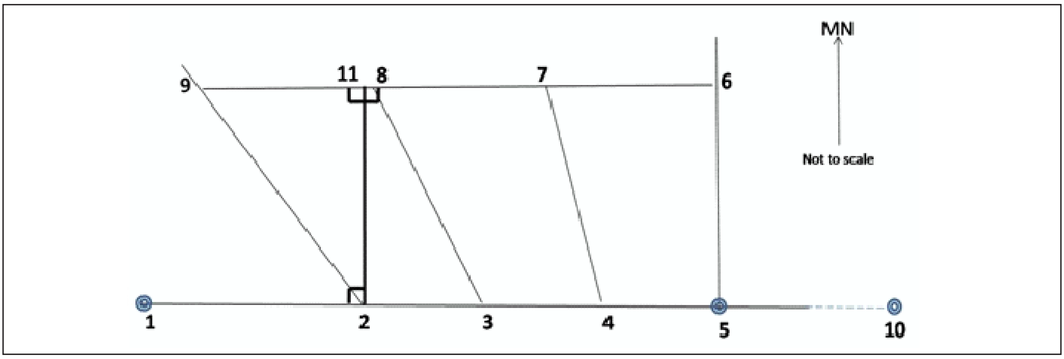

Figure 5. *Additional constraints to force parallel lines*

parameters). Here the number of pseudo *fixed* observations are added to the actual number of real observations. If n-u is large, as in a large data set, with a small percentage of fixed angles constraints, then a small error will not have significant consequences on VF in practice. However, care should be taken when interpreting VF when there are many constraint angles in a smaller adjustment.

Another consequence of including many constraints is that it will probably cause the magnitude of the residuals of other observations to increase because including constraint angles on many lines may create some tension in the network. In some cases this will cause more outliers to appear.

PARALLEL LINE CONSTRAINTS

How do we constrain lines to be parallel? For example, how do we constrain both sides of Quebec St to be parallel, or the front and rear boundaries of lots 31 to 33 to be parallel? In this particular example a simple method would be to hold the angles at points 5 and 6 *fixed* at 90° (the line 5 to 6 is at right angles to our lines of interest), but that is a special situation that does not always arise. So for the general case where there is no line joining the two parallel lines at 90°, this paper proposes that one solution is to add another line to the plan, add a new pedal point and add some 'fixed' 90° angles. This method could be applied to keep both sides of a road parallel even when there are no boundary lines shown on a plan across the road.

In the example, point 11 is added on the line 9-8-7 at a position perpendicular to one of the points on the line 1-10. Here point 2 is chosen, as shown in Figure 5, but any other point on this line could be selected. Approximate (starting) coordinates need to be entered for the two additional parameters for point 11, as for other points in the least squares adjustment.

As part of the process in this example the points 2, 3, 4, 5, 7 and 8 would have 180° fixed angles to hold the line straight. Three further fixed angles and the coordinates of one extra point need to be added to the solution to keep the lines parallel. For example, angle ∠ 1-2-11 as 90°, angle ∠ 2-11-9 as 90°, and angle ∠ 6-11-2 as 90°. This will force point 11 to be on the line 9-6 and the line 9-6 to be parallel to the line 1-10. Other choices of the three additional angles are possible, e.g. ∠ 1-2-11 = 90°, ∠ 6-11-2 = 90° and ∠ 6-11-9=180°. In this example, if point 11 was chosen to be opposite to point 1 it would lie on an extension of the line west of point 9. Then a constraint angle, such as ∠ 9-11-6 = 0°, could be used as well as two other 90° fixed angles. Thus the pedal point does not have to lie on the second line, it can be on an extension of the line.

In this example, the result of constraining back and front boundaries to be straight and parallel yields two lines 30.981 m apart instead of 31.0 or 30.9 metres apart. It is not necessary to include a distance observation for the line 2-11. However if the two parallel lines are also required to be a constrained distance apart, as might be desired for both sides of a road, then a *fixed* distance for the pedal line could also be included. Distances can be held *fixed* by giving them a very small standard deviation.

As stated earlier, this constraining approach is optional and it might be used in boundary definition calculations for a new DP, or for setting up a DCDB. A surveyor doing boundary definition calculations for a new DP could investigate various aspects of the matter and consider the consequences of each constraint added. It might be wise to clearly distinguish in the input and output files which data are real observations and which are the constraint observations. On the other hand, if constraints are part of a large DCDB then this process may need to be automated; that is beyond the scope of this paper.

CONSTRAINT EQUATIONS

The angle observation constraint method described above requires one extra observation to keep three points on line, for each additional point on that line an extra observation is required. Alternatively, it is possible to extend the standard parametric LS method by adding constraint equations between parameters. Constraint equations in LS allow a physical law relating parameters to each other to be incorporated and to strengthen a solution by forcing some parameters to obey a rule or model (e.g. Mikhail, 1976; Vanicek and Krakiwsky, 1986).

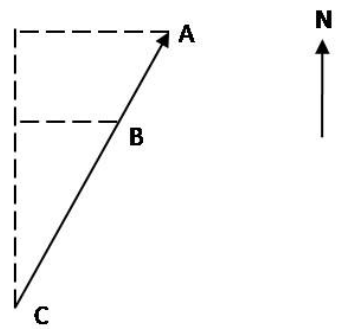

Figure 6.

For example, the heights of all points on the edge of a lake could be constrained so that they equal each other.

Constraint equations for points on a line

Consider points A, B, and C in Figure 6 to be on-line. Using the properties of similar triangles:

$$\frac{E_B - E_C}{N_B - N_C} - \frac{E_A - E_C}{N_A - N_C} = 0 \qquad (2)$$

This is similar to the equation for bearings, but does not need the tan^{-1} function. However if $\Delta N = 0$ or near zero (nearly EW lines) there is a divide by zero or near zero problem. An alternative form is $\frac{N_B - N_C}{E_B - E_C} - \frac{N_A - N_C}{E_A - E_C} = 0$ but that is weak for nearly NS lines. So we rearrange terms to remove divisions:

$$(E_B - E_C) \times (N_A - N_C) - (E_A - E_C) \times (N_B - N_C) = 0 \qquad (3)$$

This constraint that A, B and C are on-line has a similar effect to an angle ∠ A B C constrained to be 180°, however it does not add observations. The standard model equations for distance, bearing and angle observations, and their partial derivatives, are given in Harvey (2006) and elsewhere. The partial derivatives of this constraint equation (equation 3) for each of the parameters are simple, for example: $\frac{\partial f}{\partial E_A} = N_C - N_B$. A practical consideration for the LS solution is the units of the terms. If coordinates are in metres, then a *constant term* of equation 3 is in m^2 and the derivatives are in m, whilst a *constant term* of equation 2 would be unitless.

To constrain three points to a straight line there is one constraint equation or one fixed angle. For each additional point on this line one additional fixed angle or constraint equation is required. If for example there are five points on a line, say points A, B, C, D and E (arranged in that order), then fixed angles could be applied at B, C and D. Alternatively, a constraint equation for each point B, C and D to be on the line AE could be:

$$(E_A-E_E) * (N_B-N_E) - (E_B-E_E) * (N_A-N_E) = 0$$

$$(E_A-E_E) * (N_C-N_E) - (E_C-E_E) * (N_A-N_E) = 0 \qquad (4)$$

$$(E_A-E_E) * (N_D-N_E) - (E_D-E_E) * (N_A-N_E) = 0$$

If it is possible to combine these equations to have one constraint equation for all five points then that would be advantageous for the software programmers, but the author has not yet found such a solution.

An example of a parallel line constraint equation, for the line 9 to 6 to be parallel to the line 1 to 5 in the example data, is:

$$(E_6 - E_9) \times (N_5 - N_1) - (E_5 - E_1) \times (N_6 - N_9) = 0 \qquad (5)$$

Additional constraint equations could be used to keep other points on these two lines.

Tong *et al* (2005) described a method based on observed digitised coordinates of corners of lots (not bearings and distances of boundary lines as used in this paper). Their constraints are the areas of lots and straight lines and right angle corners. Their constraint equations for lines use the bearings of lines (with arctan function) rather than the slope ratios as presented in this paper.

Sequential LS method with Constraint Equations

The equations for a parametric LS solution with parameter constraint equations applied sequentially are given below. See Vanicek and Krakiwsky (1986) and Mikhail (1976) and elsewhere, for derivations. Note that some authors call constraints *conditions* on the parameters.

Observation equations:

$F(X) - L = 0$

Partials of observation equations:

$A = \partial F/\partial x$

OMC terms

$b = -(F(x_a) - \ell)$

First step parameters:

$X' = x_a + \Delta x'$

where $\Delta x' = [A^T PA]^{-1} A^T P b = N^{-1} t$

Parameter constraint equations:

$Fc(X) = 0$

Partials of constraint equations:

$D = \partial Fc/\partial x$

Constraint *miscloses*:

$b_c = -Fc(x_a)$

Adjusted Parameters:

$X'' = X' - N^{-1}D^T(DN^{-1}D^T)^{-1}(b_c + D\,\Delta x')$

Quality (VCV Matrix) of parameters:

$Q_x = N^{-1} - N^{-1} D^T (DN^{-1} D^T)^{-1} DN^{-1}$

Residuals:

calculated observations from adj parameters − observations

VF:

$v^T Pv$ / (num. obs + num. constraint eqns − num. parameters)

These equations use approximate values of coordinates (x_a), and linearised partial derivatives in A. An iterative solution for non linear problems is performed as follows. The first iteration calculates $\Delta x'$ as in a standard adjustment, then calculates X'' by applying the constraints. The next iteration then uses X'' as the starting coordinates to determine a new $\Delta x'$, then constraints are applied again. It is recommended that good starting coordinates are used so that fewer iterations are required. It is expected that in our application the sequential constraint equations to maintain straight lines will not usually have a large effect on the solution and the iterative process.

The equations for X'' look computationally intensive, but the $(DN^{-1}D^T)^{-1}$ matrix is square and has only as many rows as there are constraint equations. For one constraint equation then $(DN^{-1}D^T)^{-1}$ is a single number. $N^{-1}D^T(DN^{-1}D^T)^{-1}(b_c + D\,\Delta x')$ is a vector as long as the number of parameters. The N^{-1} matrix is usually available because it is needed for the Qx of the parameters, but rarely the A matrix. Similarly Δx is available. The D matrix is usually relatively small compared to the A matrix. A practical consideration for the LS solution is the units of the terms in b_c and in D, they need to be consistent with the units of $\Delta x'$. If coordinates and $\Delta x'$ are in metres, then a b_c term for a constraint like equation 3 is in m^2, and the derivatives in D are in metres.

Note that the degrees of freedom of this solution is the number of observations plus the number of constraint equations minus the number of parameters. This is the same as the fixed angle approach that adds the number of fixed angles to the number of observations when determining the degrees of freedom previously.

As an example, the data in Figure 3 and one constraint equation that the points 1, 5 and 10 are on-line was solved using the above described process. The results of the solution with the parameter constraint equation (equation 2) were equivalent to those obtained by adding a *fixed* 180° angle ∠ 1-5-10 as discussed above.

An analyst can subjectively force a solution towards a desired outcome by using constraints, whether by angles or by parameter equations. Similar things can happen in Bayesian LS with prior weighted parameters or in Kalman Filtering. Be careful not to overdo this approach and end up with a solution that is not so dependent on the data. Perhaps the constraints should be loosened sometimes to see where the data *wants to go*. Sequentially applying constraints, as described above, allows an analyst to see the resultant coordinates, residuals, outliers etc in a solution without constraints and then another solution with the constraints. These solutions could be compared, though with very large data sets there maybe some reluctance to do so. In our example with two different values for the distance from point 5 to point 6, perhaps one of the distances was an error? In such a case a strongly constrained solution may make an error less obvious. The angle method of constraining straight lines can also be done *sequentially*, by doing one solution without the fixed angles, and another solution with the fixed angles.

FURTHER CONSIDERATIONS

There are many other considerations related to LS in a cadastral adjustment, this paper has investigated straight and parallel line constraints. Constraints for curved boundaries can be based around the knowledge that a circular curve is defined by its radius and centre. One way to constrain the radius is to treat it as a fixed, very small standard deviation, distance.

When multiple plans are used, should the bearings from each plan include an orientation parameter? Should there be one orientation parameter per plan, or per lot, or at each point on the plan? Should a scale factor parameter be determined for each plan? These matters and further examples with larger data sets have been considered by Ayers (2008).

Coordinated cadastres often involve many thousands of points and lines. The matrix calculations for a LS solution of large cadastral data sets with constraint equations can be huge. So the stability and efficiency of the solution needs to be considered.

Since almost all lines on a DP have a bearing and a distance we might consider the adjustment of ΔE and ΔN *observations* for each line. Gründig (1985) and Harvey (2006, section 8.5.3), combine the bearing and distance observations to create two new pseudo observations of ΔE and ΔN, where ΔE (=d sinβ) and ΔN (=d cosβ). The partial derivatives for these observation equations are simply -1, 0 or 1. This problem is linear and thus reduces the need for iteration in the LS solution. However the stochastic model needs consideration. For simplicity we might adopt the same standard deviation for both ΔE and ΔN for a line and assume that ΔE and ΔN are not correlated, but the appropriateness of that assumption depends on the equipment and techniques used in the survey. The choice of orientation parameter or not, or how many orientation parameters is beyond the scope of this paper. The addition of parameter constraint equations to this method is not difficult, but is not detailed in this paper. Exceptions to this method are offset distances shown on plans and road widths, which can usually be assumed to be at a bearing perpendicular to a line, and bearings without an accompanying observed distance to (usually distant) survey control marks which could be treated as a special case.

CONCLUSIONS

This paper has investigated the LS process and equations for constraining points to stay on a straight line by using fixed angle observations and by using parameter constraint equations. The investigation was extended to multiple points on-line, and to parallel line constraints. Another situation might be to constrain boundary points to lie on a circular arc, where appropriate, in some subdivisions.

Even if LS is a small part of a complete DCDB or boundary coordination software package, the practicality of programming the algorithm or method needs to be considered. It is easy to add a few more observations (e.g. 180° angles for straight lines), but adding constraint equations takes programming effort including perhaps redesign of the solution algorithms, and thus it is harder to implement initially. The fixed angle method is probably easier to implement in existing software and does not give different results to the parameter constraint equations method (of course, assuming both methods are correctly applied). The parameter constraint equations method does give intermediate results that allow analysts to interpret the effects of the constraints. A similar analysis can be done with the fixed angle method if two separate solutions, one with fixed angles and one without, are compared.

Cadastral modelling is a means of evaluating all the cadastral data in an area. It has consequences for the cadastre, for infrastructure and land development projects, and utilities layers in GIS. Surveyors can decide whether the application of the constraints, as presented in this paper, is useful for their survey area or not. Also, surveyors can decide whether the constraints in LS solutions of boundary data are applied in the field investigation stage or final stages of boundary definition.

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