

SURVEYING PUZZLES

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ABSTRACT

A gymnasium for the mind with surveying puzzles and exercises is presented in this paper. It is intended for students to develop basic survey related problem solving skills and gain experience and confidence with surveying problems. It is also for older surveyors to keep their mind “fit” and to enjoy the use of their abilities accumulated over years of problem solving experience. Three types of puzzles are included: numerical puzzles related to our measurement and calculation work; visual puzzles related to our map and plan use; and word puzzles because surveyors write papers and reports, and to provide a variety of exercises.

INTRODUCTION

Crosswords, puzzles and problems often appear in recreational sections of newspapers and magazines. Sometimes they appear in technical publications. Educational games (e.g. the Least Squares Treasure Hunt described in [1]) and puzzles are valuable contributors to surveyors' education. So they are more than mere recreational activities. Here a series of problems, specifically related to surveying, are presented to challenge readers and to serve a serious purpose that is to maintain or improve the abilities of our minds. Various dictionaries define puzzles as problems designed to amuse or challenge by presenting difficulties to be solved by ingenuity or patient effort, or as something intricate, baffling or confusing that is to be solved. They are sometimes made purposely perplexing to test one's ingenuity.

A famous book on Survey Computations by Horner [2] contained many surveying problems and solutions for the education of surveyors. Many of his problems require the application of some mathematical, trigonometrical or geometric principle. One of Horner's problems is included here in a modified form. Martin Gardner, the author of many books and problems published in the Scientific American magazine and elsewhere calls them Mathemagical problems and solutions. Perhaps the problems in this paper could be called Geomagical problems?

Three types of problems or puzzles are included: numerical puzzles for our measurement and calculation work; visual puzzles for our map and plan use; and word based puzzles. Surveyors should have an advantage over the general public when solving the problems in this paper because they build on surveyors' prior knowledge and experience. The problems should therefore be more interesting for surveyors than those in books or newspapers or web sites. Sudoku™ and KENKEN™ puzzles are not included in this paper. They are survey oriented but are widely available elsewhere.

Why is it important for surveyors to attempt these problems? Are they worth the effort? “*What jogging is to the body, thinking is to the brain. The more we do it, the better we become.*” N. Yoshigahara as quoted in [3] which also reports on mentally challenging tasks such as solving puzzles as ways to slow down the decline that comes with age, and mental illnesses. Further he comments that doing the same types of tasks, or puzzles, will improve your ability at those tasks but that new mental challenges in a diverse range of tasks are also desirable to exercise the brain and enhance it. Readers might find ideas for more surveying related problems from other puzzles or from problem themed books. Creating new puzzles is also a useful exercise for readers.

It is possible to attempt the questions in this paper in any order, based on your preferences, but it is good exercise to do a variety of types of puzzles. Practicing only one type of problem will make you better at those problems, and they become easier, but ‘building only one set of muscles’ may not be so good for your mental health. Each question is given a star rating, where one star (*) indicates most surveyors can solve the puzzle without difficulty, and three stars (***) indicates quite challenging problems, but there are answers to them all!

Sometimes while working on problems you will get a sudden insight (the light bulb moment); that can be very satisfying. How do you get these creative insights? You might also wish to read some of the extensive writings by de Bono (e.g. [4]) and by others on this topic. They may also come unconsciously while relaxing or doing some menial task. So it might be worth having a rest if you are ‘stuck’ while trying to solve a problem. Many of the problems in this paper require deduction using perseverance and prior knowledge. There are no puzzles here that require much creativity in their solutions.

THE ANSWERS

Whether to include answers to the puzzles in this paper, or not to, has required some consideration. Surely some frustrated readers will want to know how to tackle perhaps one of the problems, and they might well learn something by reading a solution. Publishing a solution will also prove that a problem is solvable and that the author hasn’t made an error in the question. However, there are a few reasons why answers are not included here.

Publishing answers with the questions would make reading and studying this paper easier, but the intention is to exercise the mind. Also, it is likely that readers will find more pleasure in finding the answers themselves if they know they can not simply read them. Another way to view this concept is to say it is the journey that is important not the destination. That is, the mental exercise of seeking the solution is more valuable than knowing or finding the solution.

Surveyors need to be able to find correct answers in their surveys, and to know they are correct, unlike school students who check their mathematical problems’ solutions “with answers in the back of the book”. Surveyors learn to check their results themselves. Perhaps a subsequent publication will present worked solutions to these problems.

So, without the benefits of published answers, here are some comments and quotes about solution methods that might help.

Part of Horner's introduction [2] states: "...users of this compilation should tackle each problem by their own methods in the first instance as most of those supplied are capable of several methods of solution, ... solution by the addition of a few construction lines. ... if you cannot solve a problem ... plot it".

De Bono [4] discusses using creativity and lateral thinking to solve problems. He suggests that with hindsight good solutions to problems may well seem simple, clear and perhaps better than first attempts at problem solutions. However the problems in this paper can mostly be solved by persistent determination, logical processes, and prior knowledge. The author welcomes any reader's contributions of surveying related problems that can be described briefly and require creative or lateral solutions.

NUMBER AND EQUATION PUZZLES

Surveyors take pride in producing good quality results, so they specialise in finding errors and removing them or preventing them. In this section the puzzles initially focus on finding errors in data, and then puzzles move onto deriving equations. It is intended for readers to use their mind and mental arithmetic, with perhaps pen and paper for sketches and some small calculations for the early questions.

Cadastral boundary calculation problems can be solved with modern software such as CAD packages with survey computation components, or spreadsheets with equation solving routines, or by writing computer programs to trial every possible answer until a correct answer is found. This paper prefers to avoid computer based approaches and requests readers to exercise their minds. So, in some of the numerical problems calculators or computers can be used for the calculations but not as a solution tool. With the current technology we often solve problems with a series of calculation steps determining intermediate results as we progress. It may require more mental effort to solve a problem by deriving one or a few equations that yield the results in fewer calculation steps with actual data. So the latter is encouraged in this paper. The ability to do mental arithmetic calculations is useful. Build your confidence, speed and accuracy with practice.

These problems use plane geometry so you can safely ignore earth curvature, the atmosphere, and instrument effects. Unless otherwise stated all distances are in metres.

*Q1. * Network distances*

A new surveying technology that measures horizontal distances from a rover to stationary transmitters at known points was used to set up an indoor survey network. The network was in a prime industrial site around a concrete slab that is a perfect square with area exactly 169 m². The Surveyor set up a rover (at R) and measured its position from transmitters at the corners of the concrete that are labelled clockwise A, B, C and D. The recorded distances were:

$$\text{Distances: } RA = 6.403, \quad RB = 8.944, \quad RC = 2.042, \quad RD = 10.296$$

One of them is wrong, which one? Explain your reasons. This is not a multiple choice question. The important aspect of this problem is to give reasons why the distance is not

correct, it is not sufficient to merely identify the erroneous distance. Don't use a calculator or computer. Hint: don't focus on the millimetres; the error is much larger than that.

The rover then moved to another position (S) and measured the following distances. Again one is wrong, which one and (importantly) why?

Distances: SA = 18.606, SB = 11.402, SC = 14.866, SD = 10.198

Comment: We could solve these problems by Least Squares ($\min \Sigma v^2$) as in [5] and look at the resulting coordinates of the rover and the residuals to the distances. A Least Squares solution for point S assuming the coordinates of A, B, C and D are perfectly known and that all distances are of equal precision, takes many iterations to converge. Eventually it gives coordinates of S that are clearly grossly wrong! It also gives large residuals to all the distances even though only one of them is incorrect. The observation with the largest residual is not the erroneous distance. So the lesson here is that the Surveyor would be better to use his/her brain to investigate the problem before using or relying on a computed Least Squares solution. Alternatively, an L1 norm solution ($\min \Sigma |v|$) as in [6], quickly and correctly identifies the erroneous distance, does not give large residuals to the "good" distance observations and gives valid coordinates for S.

*Q2. * Spot the errors*

This question was originally presented in [5].

A) Find a single large error in the following list of distances (in metres) in a student's survey network. A plan of the network is not available and is not required.

From	To	Distance	From	To	Distance
1	4	299.725	6	5	131.640
1	9	108.437	6	7	61.446
1	2	69.351	6	11	62.152
2	4	230.375	7	6	61.446
2	10	130.113	7	8	74.243
2	1	69.351	7	10	41.223
3	4	53.581	7	11	75.795
3	11	126.855	8	7	74.243
4	1	299.725	8	9	62.078
4	2	230.374	8	11	143.506
4	3	53.580	9	1	108.436
4	5	166.876	9	8	92.079
5	6	131.639	9	11	152.444
5	4	166.874	10	2	130.114
5	11	143.698	10	7	41.224
11	3	126.853	11	7	75.795
11	5	143.697	11	8	143.506
11	6	62.151	11	9	152.447

B) Find three (3) large errors in the following list of height differences (in metres) measured by students in an EDM/theodolite survey network:

From	To	Height Diff	From	To	Height Diff
902	505	-5.954	902	578	2.496
902	903	1.654	903	902	-1.680
903	003	1.736	903	002	5.223
002	903	-5.237	002	904	0.970
002	905	1.879	905	002	-1.565
905	904	-0.588	906	908	1.294
906	049	-5.066	905	906	-2.369
906	905	3.371	049	906	-5.068
049	004	4.636	909	904	0.367
904	907	-0.798	904	909	-0.372
904	905	0.586	904	002	-0.973
901	004	0.486	901	907	1.886
901	003	-1.773	003	901	1.758
003	903	-1.741			

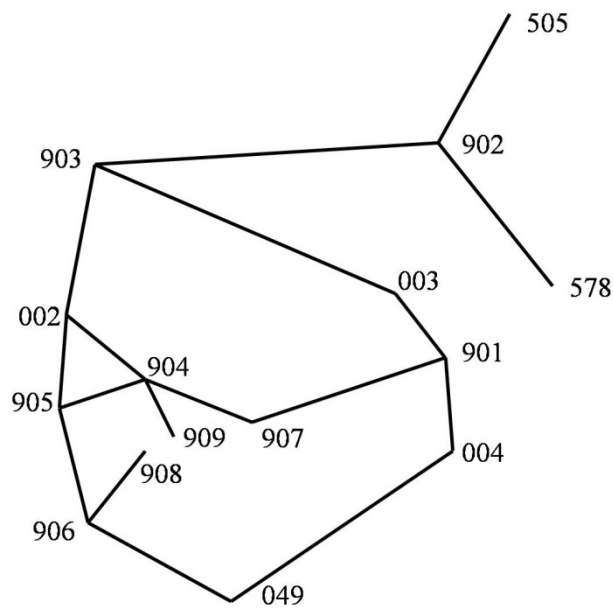


Fig. 1. Plan of network used in Q2 part B.

*Q3. ** Derivation of equations for Cadastral peg-out*

As an example, a cadastral lot is shown in figure 2 below. It has a road frontage boundary with a bearing of $2^{\circ}04'10''$ and the lot's southern side boundary has bearing $77^{\circ}35'50''$. A rectangular house (13m wide and 9.8m deep) is to be built parallel to the road boundary and as close as possible to both the road and the side boundary. The Local Council specifies houses must be no closer than 8m to the street and no closer than 1m to a side boundary.

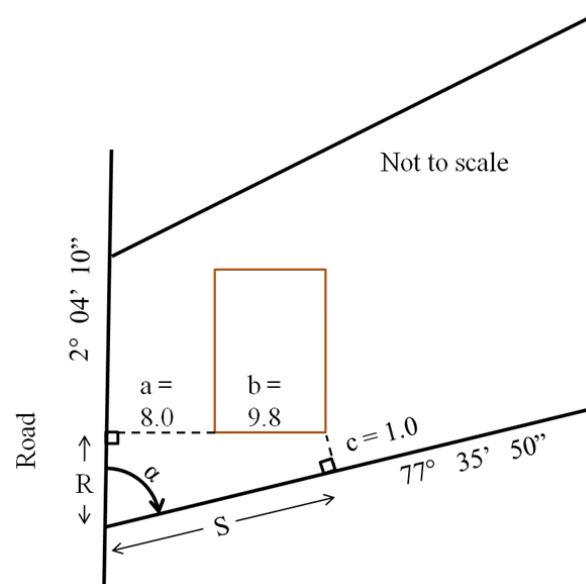


Fig. 2. Example cadastral lot

Our problem is to derive general equations (working for a variety of data values) for the distances R and S along the road and side boundaries that could be used to “peg-out” the nearest corner of the house from the corner of the lot. Derive simple equations of the form $R = f(a, b, c, \alpha)$ and $S = f(a, b, c, \alpha)$ that require a small number of elements in the equation, where α is the given angle, a is the street offset, b is the house depth, and c is the side boundary clearance. Check that your equations are valid and the solution correct for the special case when $\alpha = 90^{\circ}$. We need not concern ourselves with $\alpha = 0^{\circ}$ or 180° .

Comment: Currently surveyors could solve this problem with spreadsheet equation solvers or CAD software using a series of steps such as a loop with two missing distances, or intersection by bearings. However, here we want to exercise our brains to derive two equations that simply and directly give the required values using only the given data, with no intermediate calculation steps.

*Q4: *** Horner's Area puzzle*

This problem is based on a question in [2] but the data has been modified. In figure 3 the area of the polygon ABCDA is 2 ha and the length CD is 40 m. The bearings of each line, clockwise from North, are also shown in figure 3. From the data supplied, find a series of equations to give the lengths of the boundaries DA and CB.

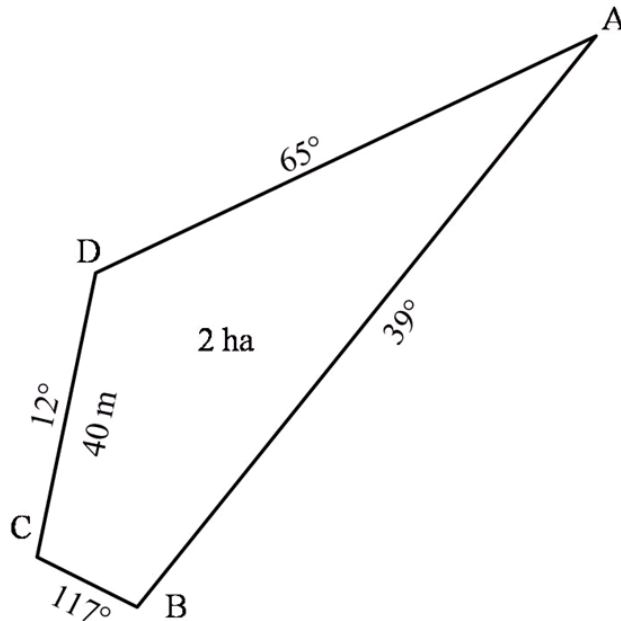


Fig. 3. Horner's polygon

Q5. * 'Crossnumber' puzzle

This puzzle is like a traditional crossword puzzle but the answers are numbers, not letters or words. It tests surveying based mental arithmetic and memory of conversion constants. New students may need to search text books or web sites to find the constants; older surveyors should be able to remember them. Try this question without using calculator or computer. If possible do the intermediate working in your head rather than on paper. Some answers merely require approximate values and some numbers should be rounded off to fit the spaces available.

The decimal point or negative sign, where relevant, occupies a space but commas are not used. Do not write units, or ° ' " symbols in the answer spaces. A sample answer might look like figure 4. Your puzzle is figure 5.

		-			3
		3			1
	1	.	2	3	0
		7			8
2		4			
3	7	7	1	2	3

Fig. 4. Example crossnumber

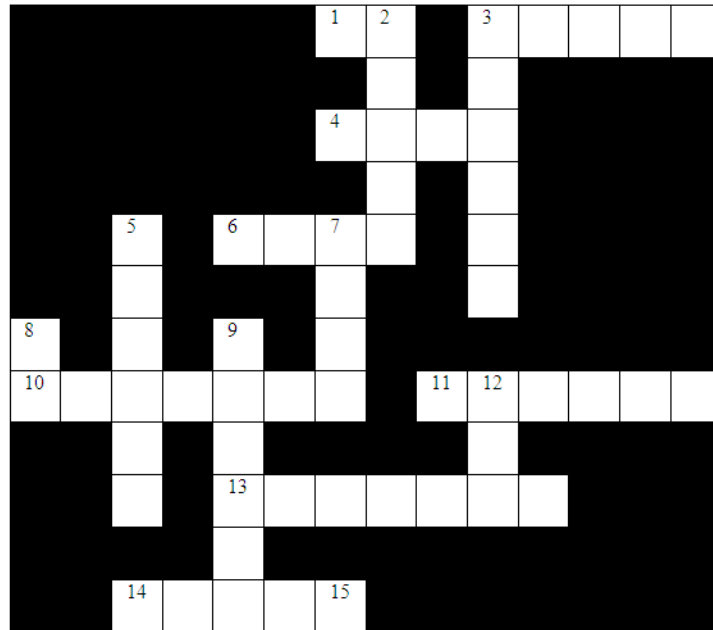


Fig. 5. Crossnumber puzzle

	ACROSS		DOWN
1	A change of 1" of latitude produces how many metres, approximately, at the surface of the earth? (rounded)	2	Convert 50 feet to metres (not US feet)
3	Distance, in metres, subtended by an angle of 1" after 200 metres (rounded)	3	Conversion constant, 1 foot = ? metres (exactly, not US feet)
4	Distance, in metres, subtended by an angle of 1' after 100 metres (rounded)	5	$\sin 315^\circ$
6	What does 20m on the ground plot as on a plan with a scale of 1:500 (in metres)?	7	$\sqrt{((0.03)^2 + (0.05)^2)}$ (rounded)
10	$360^\circ - 82^\circ 33' 44''$	8	$360^\circ - (94^\circ 23' 45'' + 265^\circ 35' 33'')$ in seconds
11	Number of seconds in 1 radian = $1/\sin(1'')$ (rounded)	9	$180^\circ + 82^\circ 33' 44''$ Answer in degrees, minutes and seconds, but don't include symbols
13	Pi, π (rounded)	12	$\sin 30^\circ$
14	$\sqrt{2}$ (rounded)	14	A change of 3 ppm represents how many millimetres change in an EDM distance of 4 km?
		15	Half of 1'26"

*Q6. ** Hansen's Problem*

This famous problem is named after Peter Hansen (1795–1874) who worked on the geodetic survey of Denmark in the 1800s. Four of the angles in a braced quadrilateral shown in figure 6 were measured and the distance DC is known. In this problem you may use a computer (but not a program designed for this problem) or calculator. You may use a series of steps to calculate all the other distances for the following data:

$$DC = 400 \text{ m} \quad \alpha = 35^\circ \quad \beta = 27^\circ \quad \gamma = 38^\circ \quad \delta = 42^\circ$$

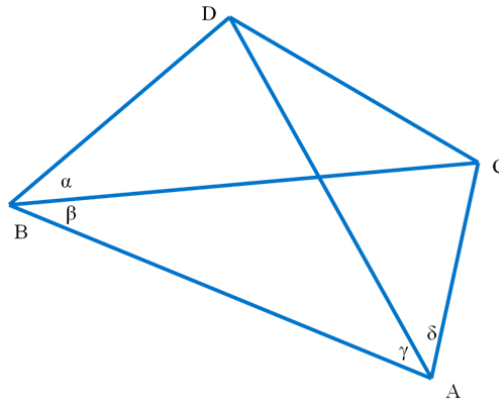


Fig. 6. Braced quadrilateral, not to scale

VISUAL PROBLEMS

*Q7. * Contour plan*

Figure 7 is a contour plan like those from a mine site. Contour values, contour interval and scale are not available. Is it more likely to be a stockpile or a quarry/open pit? Give reasons.

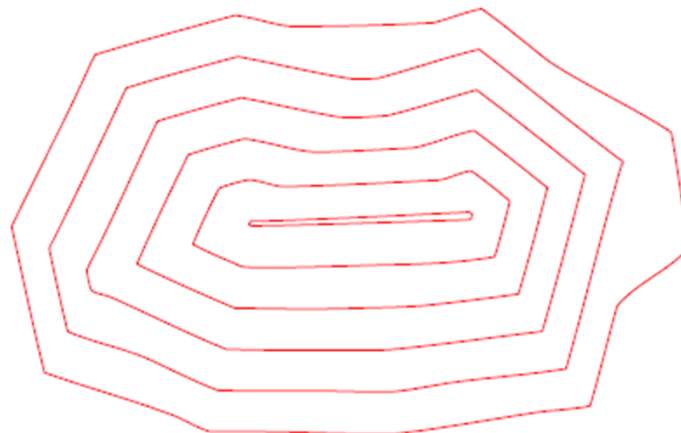


Fig. 7. Mine site contours

Q8. ** Contour puzzles

Surveyors create contours lines from points with 3D coordinates spread over a surface. Sometimes the points are evenly spaced and sometimes the points are positioned to reliably capture the nature of the surface such as where slope changes significantly. The contour lines can be determined from a triangulated mesh placed on the points. Surveyors know the method well and it is described in many basic surveying textbooks. This puzzle is the reverse of that familiar process. Here we are given the contours and are asked to find the heights of points. First an example of this new type of puzzle and its solution is presented and then two puzzles are provided for the reader to solve.

This type of puzzle was created as a visual surveying related problem which is simple to understand but can be challenging to solve. In these contour puzzles there are 23 points regularly spaced across a surface in a pattern of mostly equilateral triangles. The heights (or elevations) of the points are integers from 1 to 23 with no two points having the same elevation. Thus each of the numbers 1 to 23 is used once and only once. The pattern of the triangular mesh is shown below in figure 8. A sample puzzle is shown in figure 9 as an example, and its solution in figure 10.

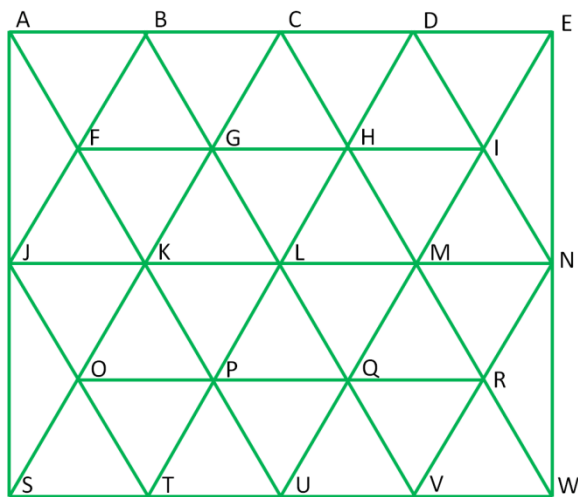


Fig. 8. Triangle mesh joining labelled points

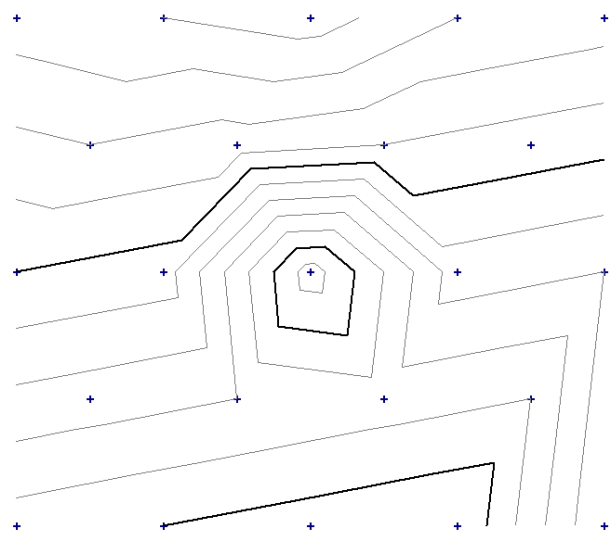


Fig. 9. Sample puzzle

In these puzzles the contour lines are shown, but are not labelled. The contour lines in raw form are a series of straight lines. They have not been smoothed into curved lines. The 10 and 20 contour lines are thicker to make the solution easier. The contour interval is 2 and only even contour lines are shown. Note that it is possible for there to be more than one 10 contour line on a puzzle, and similarly more than one of some of the other contours. In the colour versions of these puzzles, the mesh is shown light green, the 10 and 20 contour lines are thick brown, and the other contour lines are thinner brown. In the B&W versions of these puzzles, the mesh is not shown, the 10 and 20 contour lines are thick black, and the other contours are thinner grey.

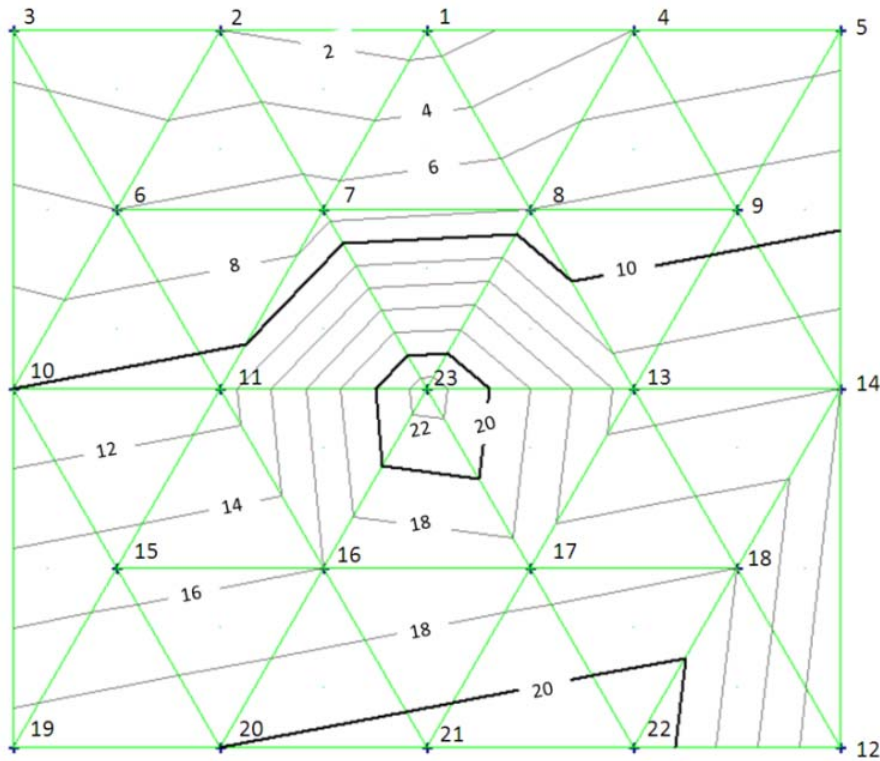


Fig. 10. Sample puzzle solution showing mesh, heights of points labelled and contours labelled.

Hints for solving these problems:

- Label the points A to W as shown in the mesh in figure 8. Contours were created using the lines shown in figure 8, e.g. AB, AF etc. Do not use other joins between points e.g. BK, LU etc that are not mesh lines to solve the puzzle.
- The ground surface has a constant slope along any mesh line, it does not undulate up and down between points, and so the contours must be either steadily increasing or decreasing between two points. This helps to label the contours. (The ground can't be horizontal between two points because no two points have the same height in our puzzles).
- Contours will pass through the point of the same height, for example the 10 contour passes through the point with a height of 10. But a point that is on a contour may not have a line coming from it if it is exactly at a local peak or exactly at the bottom of a hole, then the contour is a dot. In those cases use interval spacing between contours to decide on the height of a point.

The two puzzles to solve are given below in figures 11 and 12.

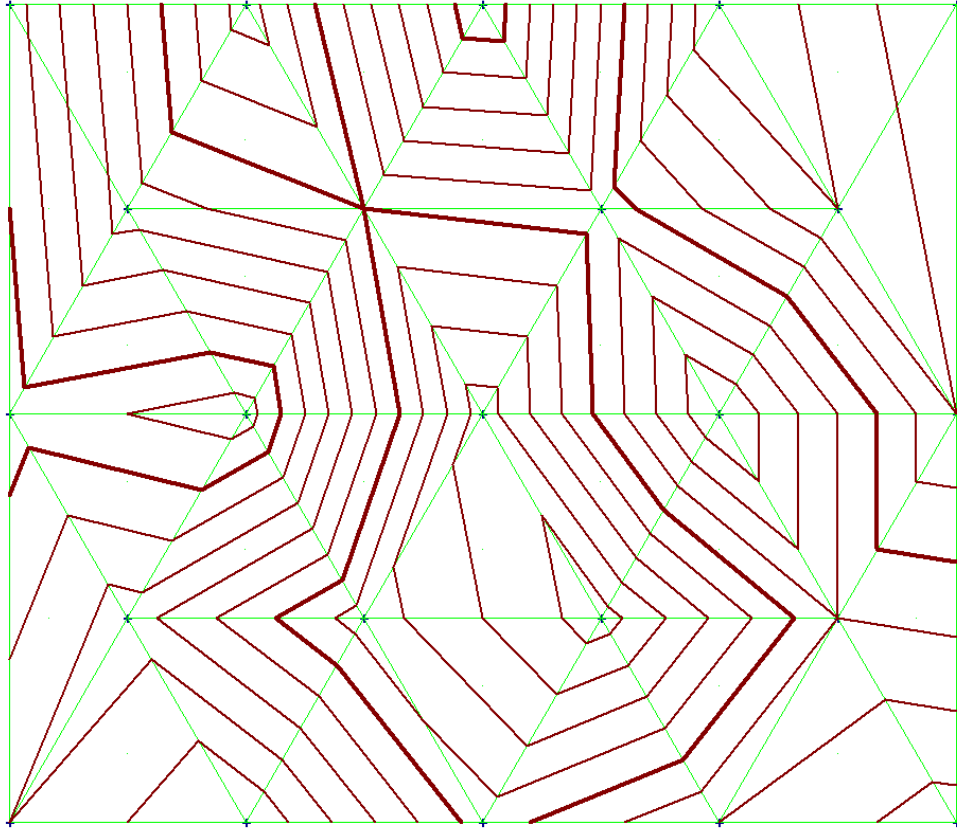


Fig. 11. Contour puzzle 1. ID 2006

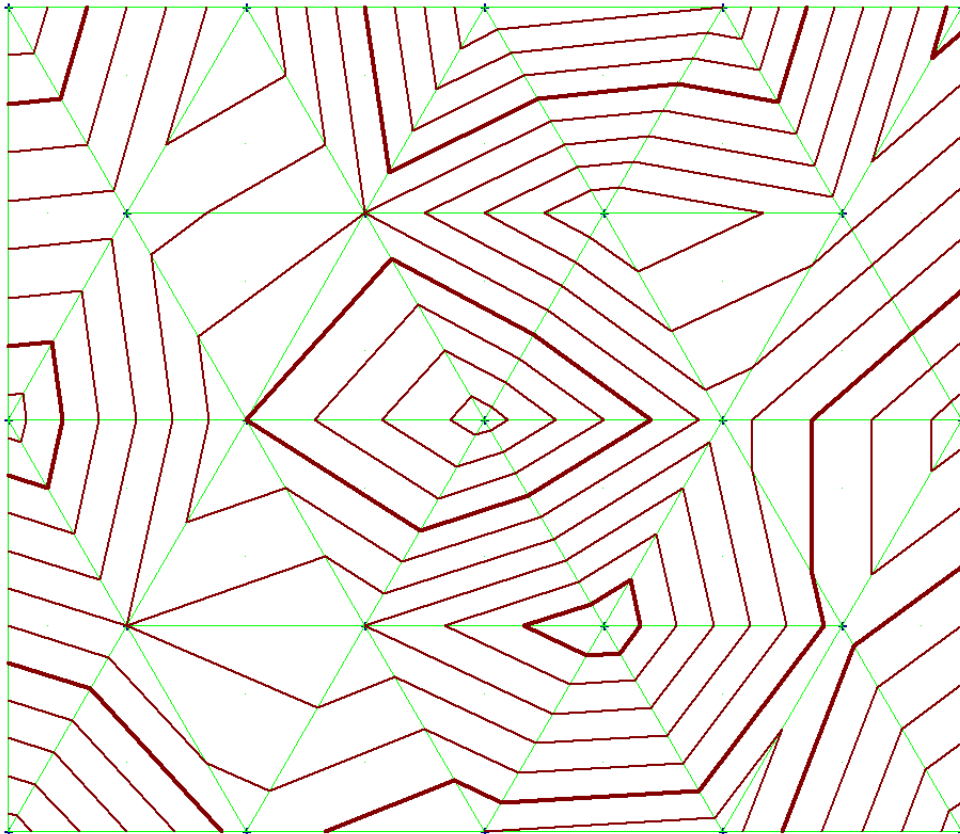


Fig. 12. Contour puzzle 2. ID 2007

Q9. ** Visual Memory Problem

A simple memory problem could show pictures of objects, list words or some numbers, and ask you to remember them and recite them later. Rudyard Kipling's book 'Kim' describes this type of problem. It is good to improve our memory abilities and our skills at map and plan reading, so for this purpose a portion of a map is shown in figure 13. But memorising field measurements as part of a survey is not encouraged because that is error prone and it is better to record data.

Study the map in figure 13 for a few minutes without reading the questions, and then answer the questions without looking back at figure 13.

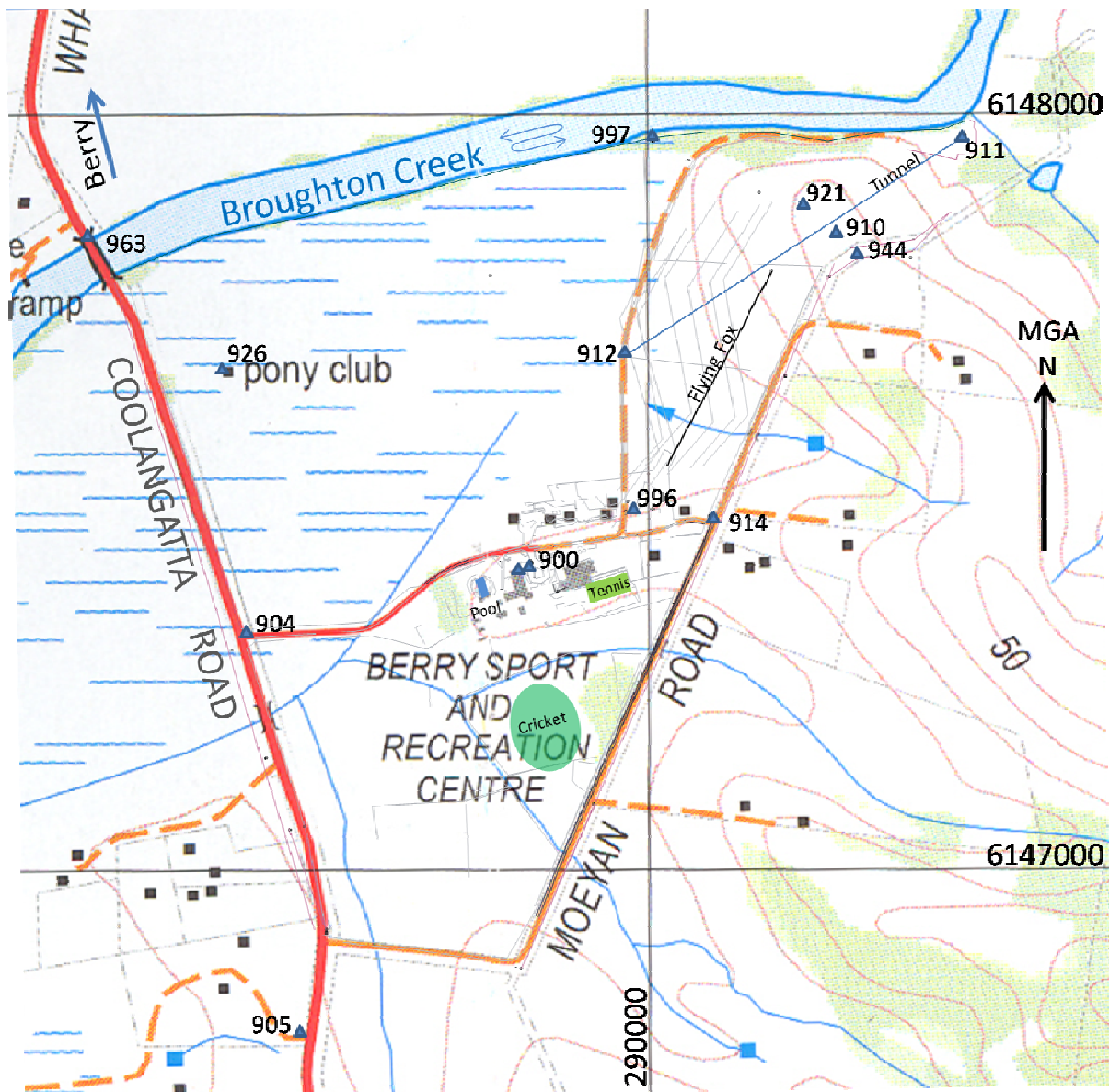


Fig.13. Memorise this map. (Parts of this image are derived from a map produced by the NSW Land and Property Management Authority – SIX website).

Map Questions:

- a) What is the name of the creek and which direction was it flowing?
- b) What were the names of the roads?
- c) What was the height above datum of some point in the area of the plan (any point or any contour)?
- d) What was the contour interval?
- e) What sporting facilities are available on the site?
- f) What is the spacing between grid lines?
- g) The location of several survey marks are shown by their three digit number beside a triangle symbol. What is the first digit of these survey mark numbers?
- h) What type of North is used?

*Q10. * Viewpoint and 3D drawings*

Pictorial views of an object, i.e. 2D representations of 3D objects, like isometric drawings are often used to communicate the appearance of new buildings.

A) Draw a 3D isometric view of the object shown below in figure 14, modified version of a problem from class notes of unknown source. A freehand drawing or sketch is sufficient.

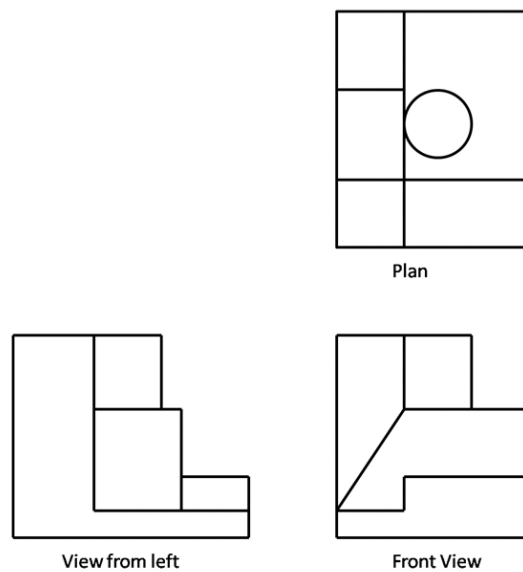


Fig.14. 2D to 3D drawing question

B) Figure 15 is a pictorial view of an object. What is the object and where do surveyors see this in practice? You may need to consider mentally rotating the object or changing your viewpoint to see it more conventionally.

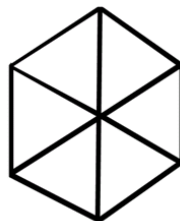


Fig.15. 3D to 2D, what is this?

WORD PROBLEMS

Q11. Nine Letter Word Puzzles

Several nine letter puzzles are shown below using words familiar to many surveyors. For example, the letters RSNVYIEUG can be rearranged to form the word SURVEYING. These puzzles follow some of the conventions of the nine letter word puzzles popular in newspapers, e.g. no plurals. Commonly, readers are asked to find as many words as possible from the letters in the puzzle, with at least four letters in each word and all words are to include the central shaded letter. However, here we ask that you find only the nine letter word, not all the shorter subset words.

A) ** To make the conventional nine letter problem simpler and more applicable for surveyors the puzzles here are made of words familiar to many surveyors and the letters are ordered in a pattern. So solving any one of these word puzzles will help you solve the others. Which of the puzzles below does not follow the pattern of the others? What are the words?

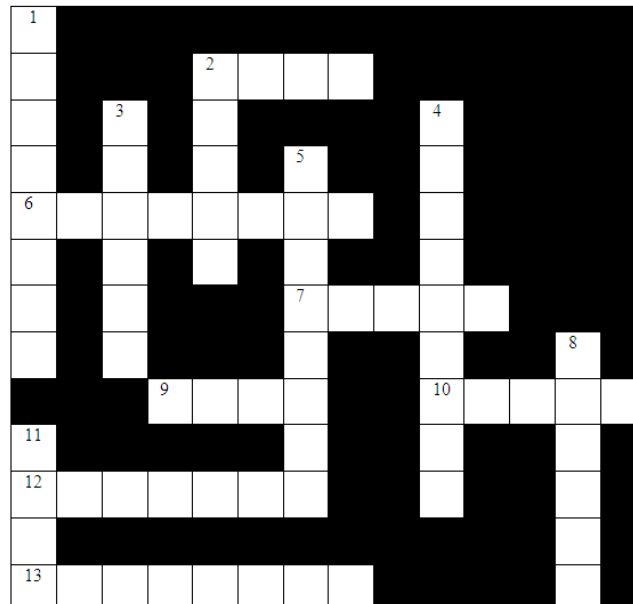
E	T	E	L	A	T	T	I	O	T	U	D	P	O	C
M	R	B	U	E	C	A	N	E	I	E	L	T	E	S
O	R	A	C	L	A	V	E	L	G	N	O	E	L	E

B) *** In the following list of jumbled surveying words, there is no pattern to the random rearrangement of letters. Resist the temptation to find a computer program to work out the answers for you. Spelling of English words sometimes varies around the world, but that shouldn't cause too many problems with the following listed words.

ACPIHELRSR	ALDLHOSEE	ARNMFRTSO	BDESDVUII	CATARALSD
CHRYERIHA	CIALLOETM	CNRTETESI	DATGEDARU	DELILDNAS
DOUONRECT	EERREECFN	EEPLOSECT	ENAORTTHS	EOALCTNRE
EPLREVOKE	ESMNDNIOI	ETNMINLAG	FLAIIDQUE	GTYIUIBMA
ICACTRNEO	IIATAODRN	ILLPOIDES	INMCKIAET	INTOQTAOU
ITSEEALLT	MRAHCEBNK	MEPRANTEN	NIOERAETT	ODREITICN
PASNDELCA	PISIENROC	QUMNPIEET	RAEILTACB	RAILACOBP
RAVCUURTE	RETOSNECI	RFSHOTGIE	RSGENIAUM	RWEHTONTS
SEOURERNO	SETETAIDM	SOTUSTWEH	SUEHATOTS	TCODRRC EE
TDDNRAUNE	TMRRAPAE E	TPMEERIRE	TRLEEMOIK	TSEENVENE
ULMTAPTHI	VAIONEIDT	XIETEOPNM	YNIMENIDT	YSUOSMPMI

Q12. * Crossword puzzle

A simple small conventional crossword puzzle is shown below. Crossword puzzles are not as difficult to create as our previous puzzles.



	ACROSS		DOWN
2	GPS plus, acronym	1	Used by photogrammetrists and avoided by surveyors
6	Trenches dug near a peg along the boundary lines of an old NSW land parcel	2	Famous scientist and surveyor from long ago
7	Gunter's distance measuring device	3	SPHERICAL ... The sum of three perfect angles of a triangle on a sphere - 180°
9	Important electronics inside EDM and GPS, acronym	4	Measure of repeatability of observations
10	Combination of theodolite FL and FR zenith angle observations detect this error of the zero mark	5	The part of a survey loop that we want to be small. Sometimes spelt with URE at the end, but not here.
12	A metric unit of area	8	Equals 60 minutes, and often four years
13	The M in UTM projections	11	Tidal boundary, acronym

CONCLUSIONS

Several problems have been presented to challenge and exercise the minds of surveyors. It is claimed that challenging the mind with difficult problems or with new types of problems is beneficial to the maintenance or growth of the person's mind. This is especially relevant for those people whose daily work is repetitive or familiar or involves mainly administrative tasks. A surveyor or student who has successfully completed all the questions is now in a position to comment on whether these puzzles should be considered as recreational activities or as a serious contribution to published surveying literature. A reader who found all these problems simple is welcome to create better problems or try to learn skills or knowledge in fields new to them. Readers who want more puzzles should see the addendum or the UNSW School of Surveying and Spatial Information System's web sites.

ACKNOWLEDGEMENTS

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ADDENDUM – MORE PUZZLES

I have not found a journal that wants to publish the above paper, so now there is no page limit. That allows me to include some more puzzles here.

*Q13. ** Phase Ambiguities Puzzle*

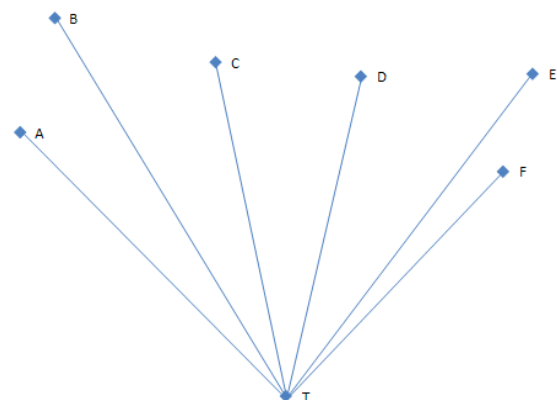
Many documents have been written over the years to explain GPS carrier phase ambiguities and solution methods. Often it helps students to understand a concept if they actually do something. The following puzzle provides an opportunity to do some calculations on a similar problem.

EDM has a somewhat similar approach to measuring distances. Before GPS and EDM surveyors used a 100m steel band to measure precise distances. Metric steel bands often have brass markers every 2m and they have a 3m tape attached to the zero end of the band. The tape is called the reader and it is graduated in the reverse direction, usually every millimetre. So to measure a distance surveyors placed a brass marker on one end of the line and read the 3m tape at the other end of the line. The distance is the sum of the two readings. This is similar to a GPS carrier signal, but here the cycle length is the spacing between the brass markers. If we do not record the number at the brass marker then we have an ambiguity problem.

In this puzzle we work in 2D coordinates and have a 2 metre cycle length. The coordinates of our known points A to F (our satellites) are ‘perfect’ and are given in the table below.

Point	E (x)	N (y)	Distance from T:	Marker dist.	Dist. on reader
A	335.80	647.29		a	0.88
B	336.61	652.18		b	1.04
C	340.36	650.29		c	0.37
D	343.75	649.68		d	1.79
E	347.76	649.79		e	0.93
F	347.07	645.61		f	0.86

From the table, the distance from T to A is $a + 0.88$ and a is an integer. Similarly, the table gives the ambiguous distances from T to the other known points. Find the integer values of $a, b, c, d, e,$ and f and the coordinates of T. We know that T is south of all the points A to F and that all the distances are less than 20m.



Not to scale, sketch

You may use a computer and any method you wish. But how do you know if you have the correct answers? In this data set all distances are reliable to less than 1cm, so when you have correct coordinates the residuals from least squares, or the differences between the calculated and observed distances are less than 1cm for all lines. You could try every combination of integers from 0 to 20, but is there a better way to do it?

Q14. More Contour puzzles

Find the heights of all 23 points shown with a cross (+) in the contour map shown in Figure A1. The contour interval is 5 and the 10 and 20 contours are darker. The heights of all points are the integers (whole numbers) from 1 to 23. No two points have the same height. Assume that the ground surface has a uniform slope along any mesh line. The contours were created from a triangular mesh joining the points as shown:

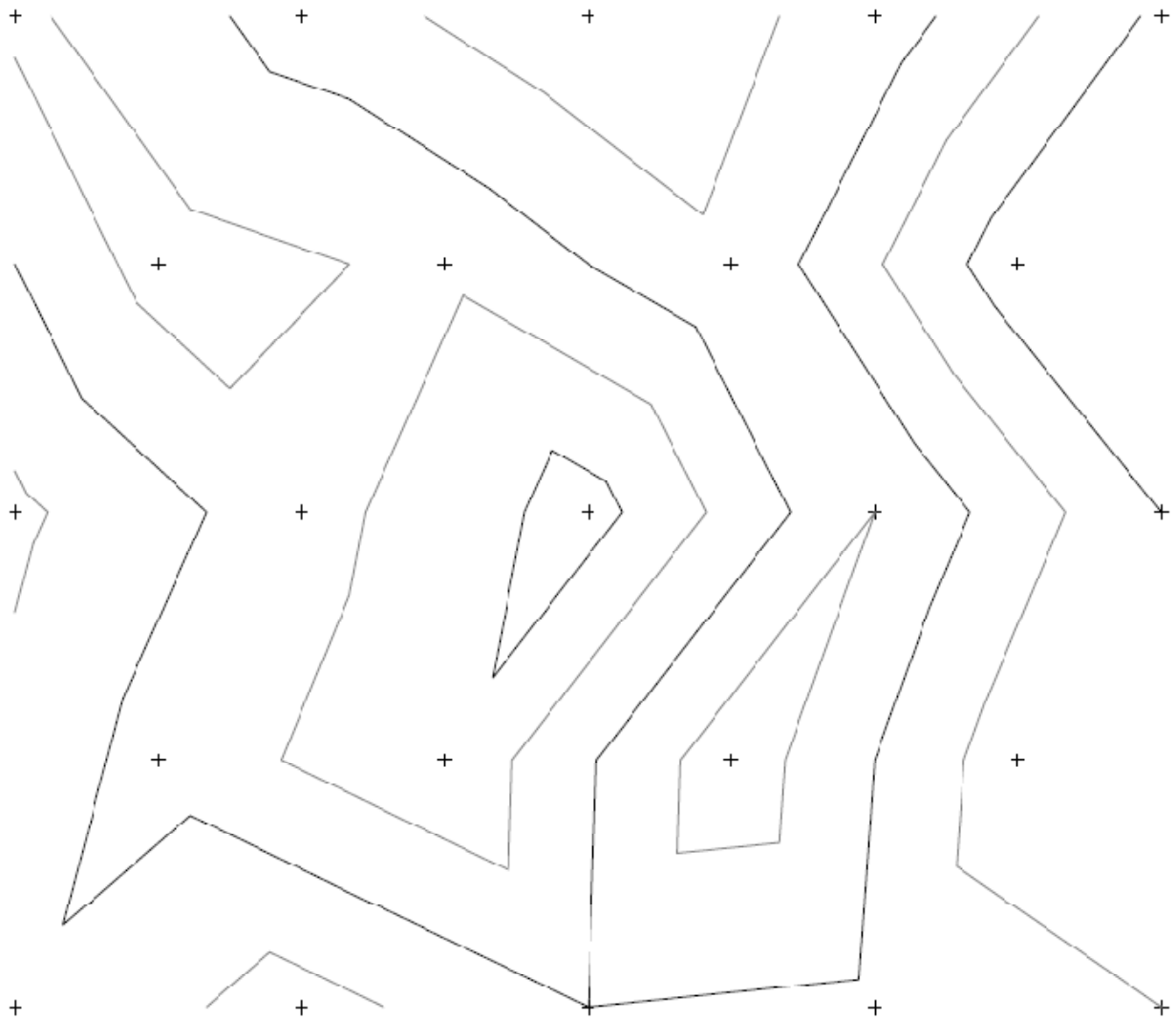
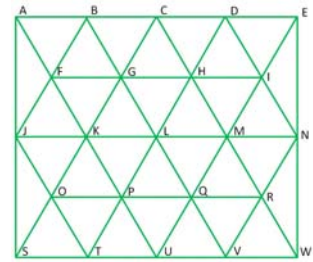


Figure A1. Contour puzzle with a contour interval 5. ID 5003

Find the heights of all 23 points shown with a cross (+) in the contour map with 'smoothed' contours shown in Figure A2. The contour interval is 2 and the 10 and 20 contours are darker. The heights of all points are the integers (whole numbers) from 1 to 23. No two points have the same height. Assume that the ground surface has a uniform slope along any mesh line. The contours were created from a triangular mesh joining the points as shown:

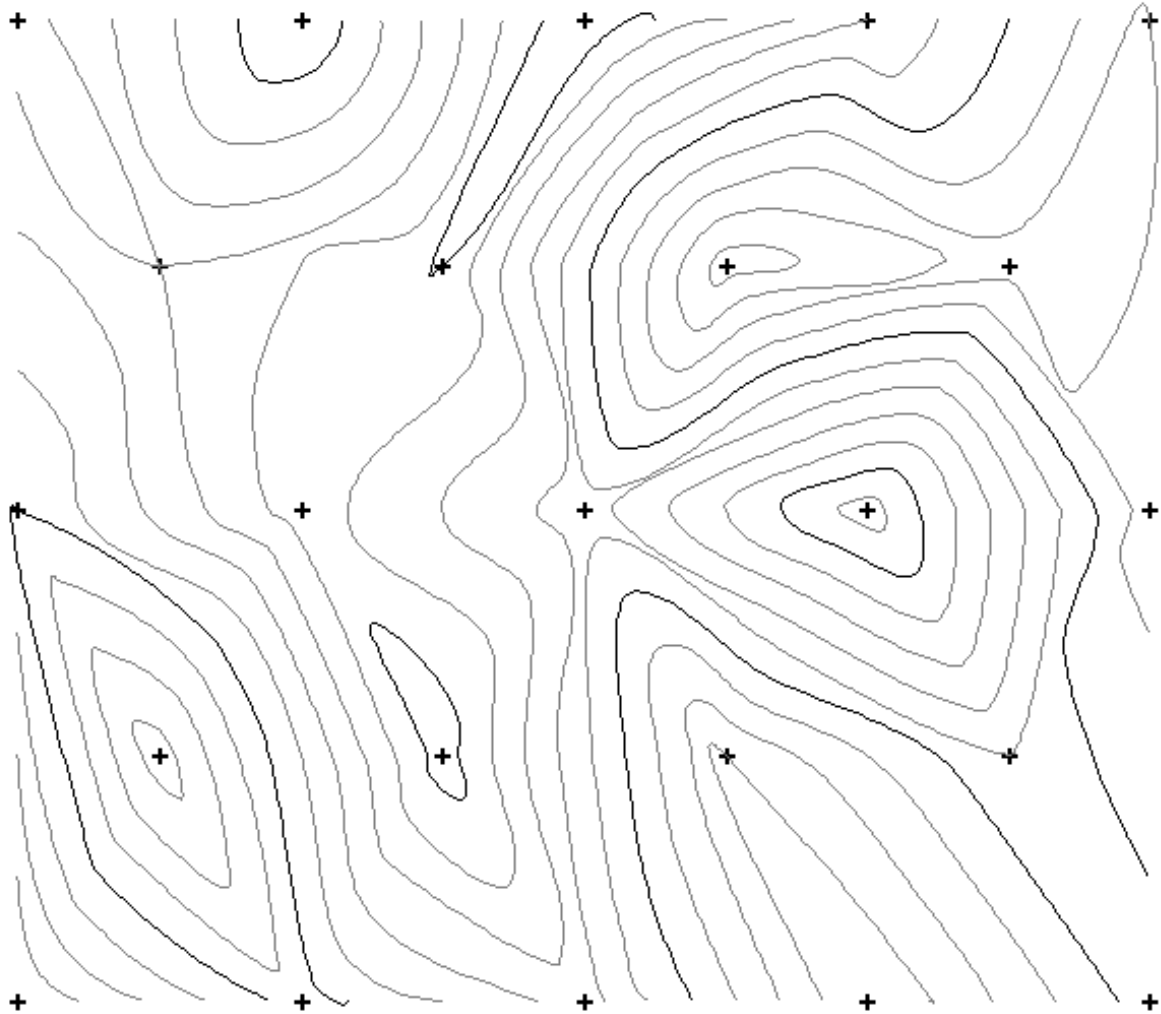
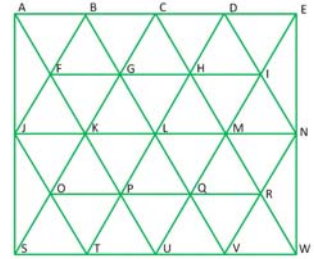


Figure A2. Contour puzzle with contours smoothed. Contour interval 2. ID 2002

Find the heights of all 23 points shown with a cross (+) in the contour map shown in Figures A3 to A8. The contour interval is 2 and the 10 and 20 contours are darker. The heights of all points are the integers (whole numbers) from 1 to 23. No two points have the same height. Assume that the ground surface has a uniform slope along any mesh line. The contours were created from a triangular mesh joining the points as shown:

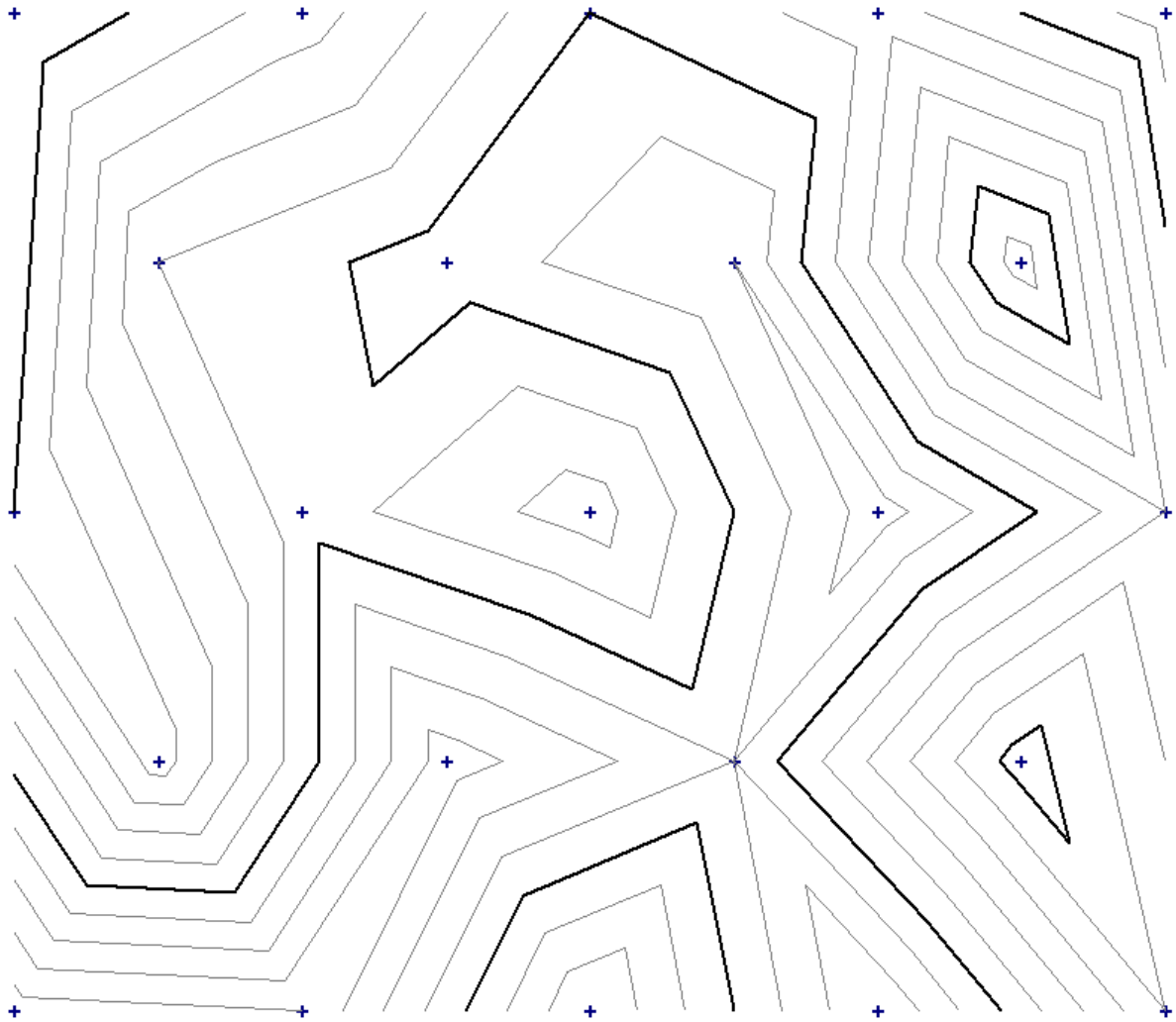
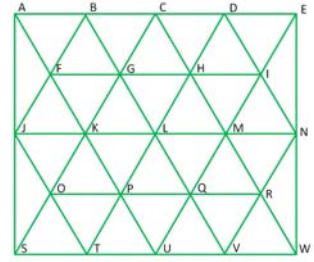


Figure A3. Contour puzzle, ID 2004

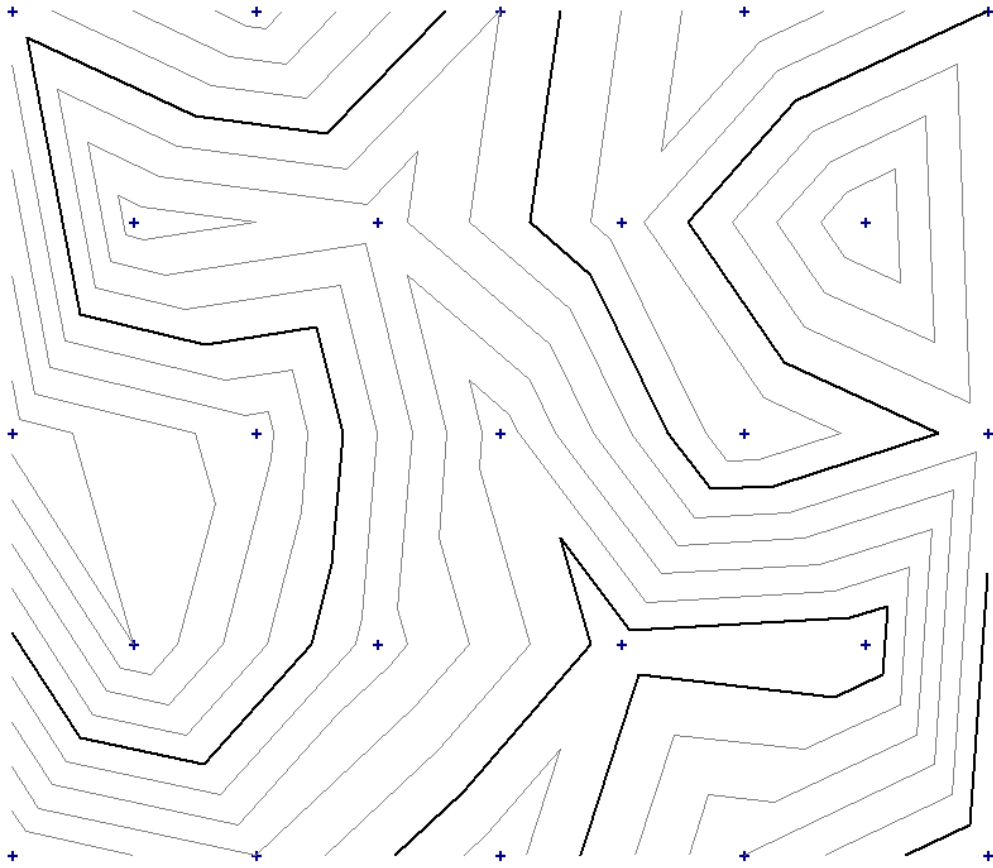


Figure A4. Contour puzzle, ID 2008

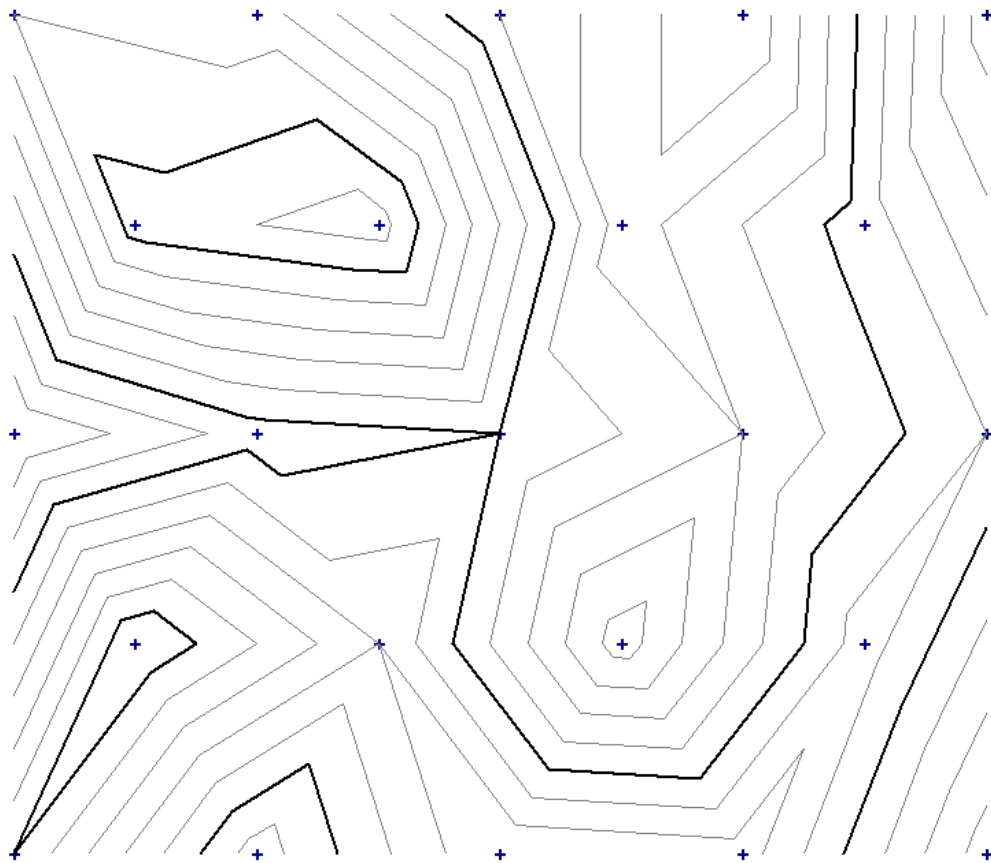


Figure A5. Contour puzzle, ID 2009

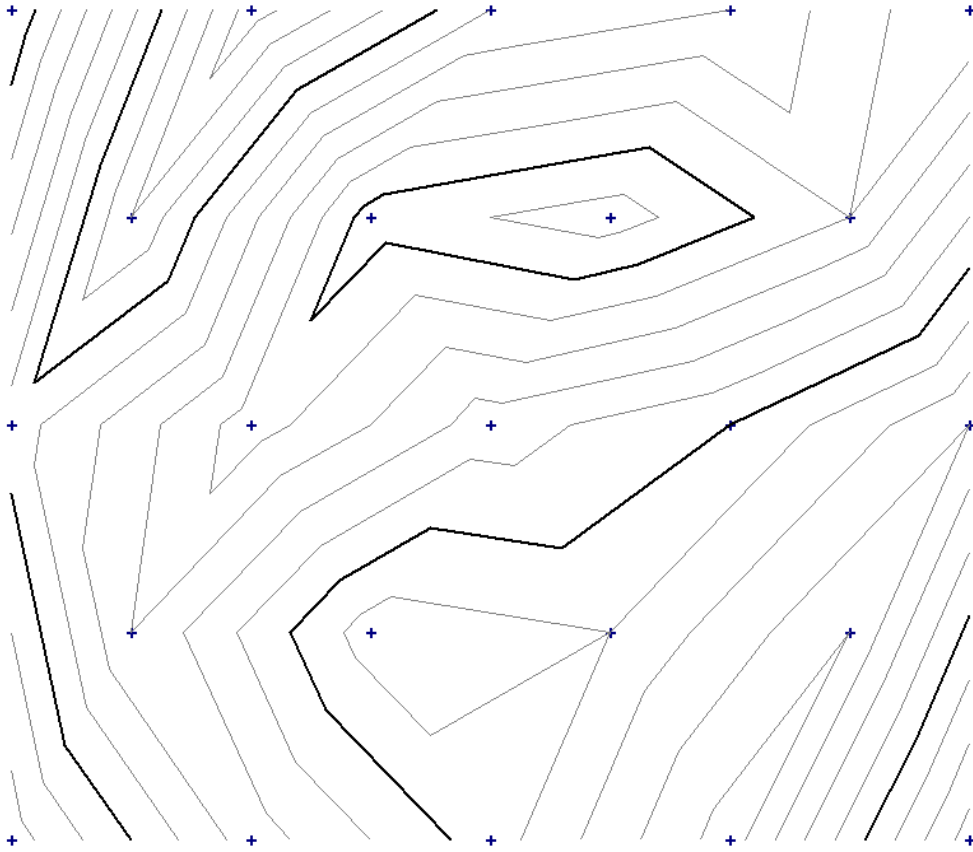


Figure A6. Contour puzzle, ID 2010

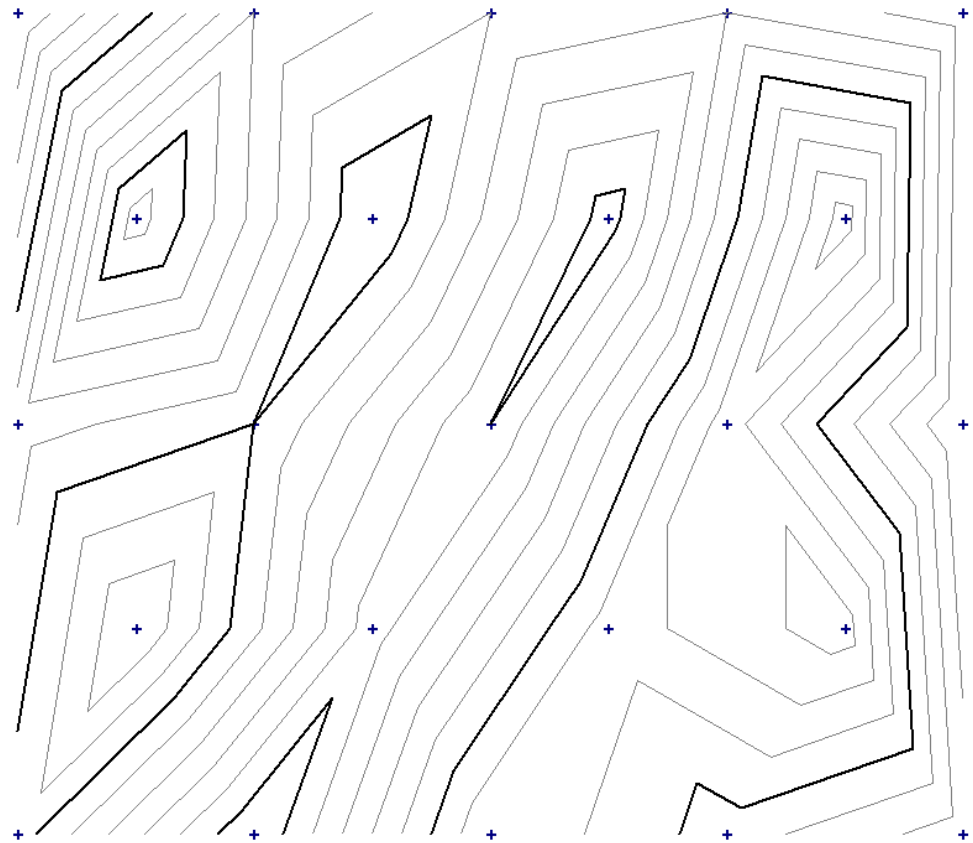


Figure A7. Contour puzzle, ID 2011

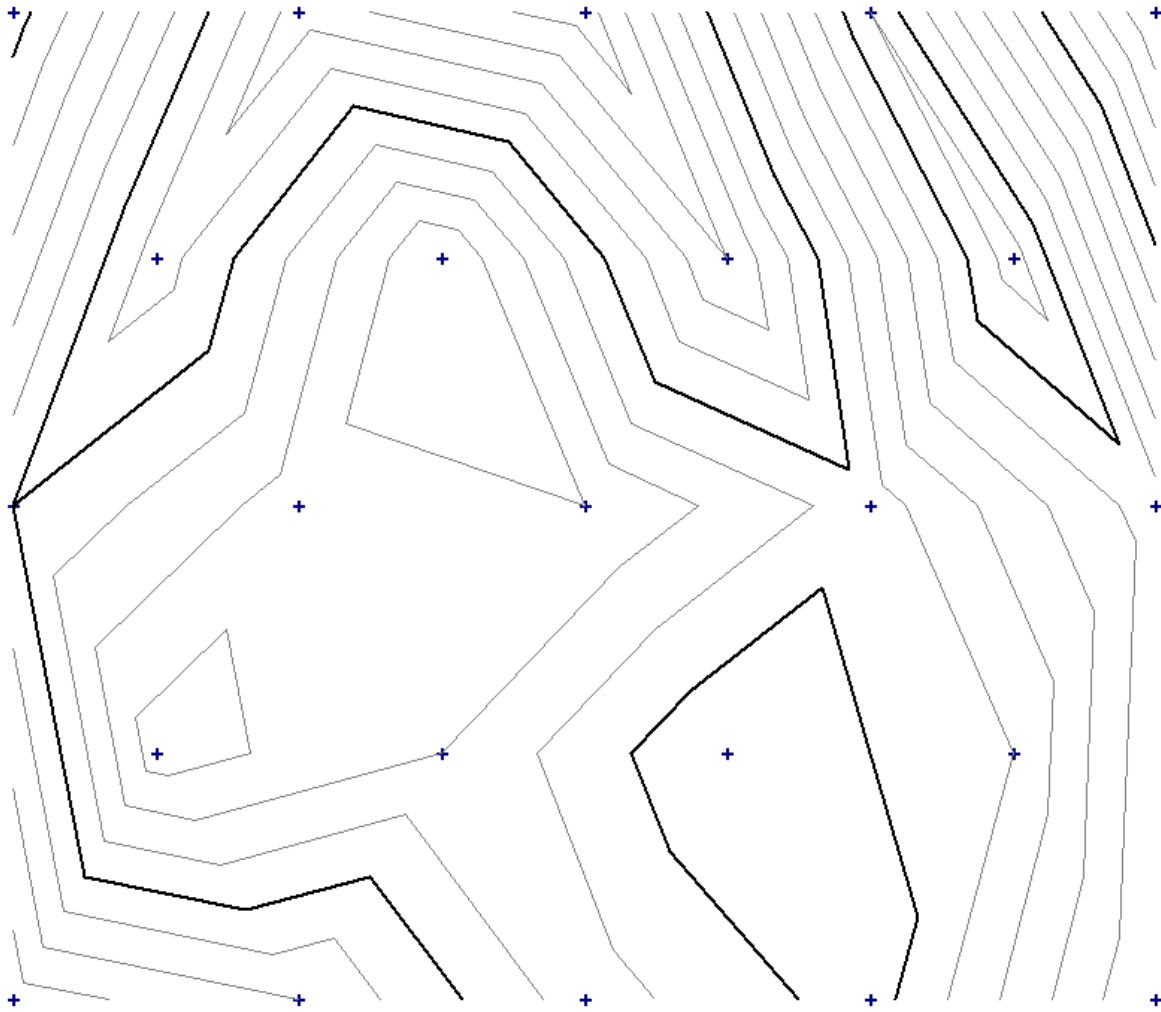


Figure A8. Contour puzzle, ID 2012

*Q15. ** Surveyor Logic Puzzle*

Recently I found a logic puzzle with a surveying theme at the website: [www.iowasurveyor.com/puzzles/Surveyor Logic Puzzle.pdf](http://www.iowasurveyor.com/puzzles/Surveyor_Logic_Puzzle.pdf) I found it interesting and challenging, and solved it. So I copied it and reproduce it here for your educational benefit.

Five surveyors worked in a growing town. One was employed with the city while the other four worked for private companies. You could always tell who had surveyed a property because each surveyor preferred to use a different colour of ribbon on his stakes and also used different monuments.

Determine where each surveyor was employed, which size & type of monument he used and his ribbon preference.

1. Charlie, who didn't work for the City, didn't use pipes. The Survey Solutions surveyor didn't use the ½-inch monument. Bob, who didn't work for Rocket Land Surveying, used rebar, but not the 1-inch size.
2. Ed, who didn't use rebar, used a larger monument than the surveyor who used blue ribbon.

3. The two $\frac{3}{4}$ -inch size monuments were a pipe and the monument set by Blue Moon Surveying. Archie from Survey Solutions didn't use rebar. Doug used a $\frac{3}{4}$ -inch monument, but it wasn't a pipe.
4. One of the surveyors who used rebar also used green ribbon. The City surveyor used rebar.
5. The two surveyors who used pipe were the one who also used orange ribbon and the Zodiac Engineering surveyor. One surveyor's rebar was the $\frac{1}{2}$ -inch size.
6. The surveyor who used pink ribbon also used rebar, but not the $\frac{1}{2}$ -inch size. The square bar wasn't set by the surveyor who used yellow ribbon.

Surveyor	Employer	Ribbon Colour	Monument Type	Monument Size
Archie				
Bob				
Charlie				
Doug				
Ed				