Calculation of Control Surveys on the Map Grid of Australia and new Geocentric Datum

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David Elford and Craig Turner were final year Geomatic Engineering students at UNSW while working on the foundations of this paper.

ABSTRACT

An example of least squares calculations of a small control survey with GPS and traditional survey measurements on Sydney's Middle Harbour is given in this paper. Students and programmers can see, and follow, the steps required. The new Australian geocentric map grid coordinates are used.

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INTRODUCTION

This paper contains an example of calculations of a control survey carried out on the Geocentric Datum of Australia (GDA) with coordinates on the Map Grid of Australia (MGA). There are two reasons for this, firstly the Inter-governmental Committee on Surveying and Mapping has recommended that Australia convert to the GDA by 2000. Secondly, very few numerical examples have been published that show practicing surveyors, students and programmers the steps inside software that calculates least squares adjustments of control surveys. This paper attempts to give an example using the new geocentric coordinates.

Showing all the steps in a worked example of least squares adjustment takes a lot of space. So this paper is limited to only part of a real survey and not all the analysis considerations are discussed.

Much could be said about the analysis of this data set, for example: choice of model equations and parameters, observation preprocessing, standard deviations and correlation's of observations, statistical analysis of output etc. They are important topics but are beyond the scope of this paper, see Harvey (1994) for more details.

NETWORK AND DATA DESCRIPTION

The network contains 6 points and 38 observations being a mix of GPS and traditional survey data. This type of survey is becoming more commonplace for control surveys because GPS can observe long and non-intervisible lines and traditional survey observations can observe at sites with limited sky visibility and (currently) often with better precision.

The survey was carried out on the shores of Sydney's Middle Harbour, pictures and a plan are given below. The observations and coordinates are given later. The coordinates of the fixed points are estimates and the results given in this paper are for educational purposes only.

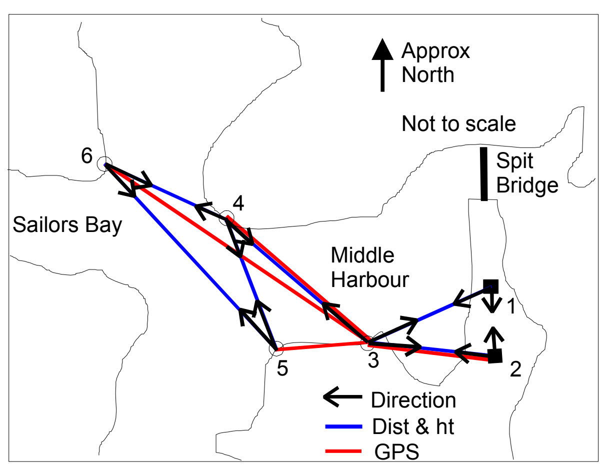

Figure 1. Sketch plan of network

Figure 2. SSM 57110, point 3

Figure 3. SSM 57112, point 4

Figure 4. SSM 68977, point 5

DESIGN QUESTIONS

There are several ways to adjust this data. Firstly, we chose least squares because we wanted to keep the changes to our observations (corrections, residuals) as small as possible. Secondly, we could choose to solve for coordinate parameters in XYZ or ENH or latitude, longitude and height (on an ellipsoidal surface or in 3D space). In this paper we show the latitude, longitude and height in 3D space adjustment. Various approaches have been used to check our answers and commercial programs have also been used for testing. Those people who want to adjust their control surveys on a plane surface or purely on grid coordinates may wish to use the data in this paper and compare their results with those below. Shortis & Seager (1994) provide equations for an alternative approach with calculations on the map grid.

Most of the geodetic equations used in this paper are well documented elsewhere. Instead of quoting an original source for each equation we suggest the reader see, for example, Vanicek & Krakiwsky (1986) or Leick (1995) and the publications in their reference lists.

To keep the paper as short and simple as possible we have not solved for geoid separation, transformation or refraction parameters and have not applied deflection of the vertical corrections. Also, only one iteration of the Least Squares is shown in detail. Final results, including coordinates, are also shown. Partial derivatives (coefficients), observed-computed terms (OMC), and statistical input have been shown step by step for only one line, a line with all types of observations involved.

The numbers shown below have been rounded off for display purposes, the actual calculations used values stored in the computer, so if intermediate calculations are attempted using numbers shown below then slightly different answers may be obtained due to round off errors.

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COORDINATES

Coordinates of known points

Fixed coordinates of the points on the Map Grid of Australia (MGA), which is a Transverse Mercator projection, are assumed to be:

Point	MGA East	MGA North	Name
1	337675.093	6257970.269	SSM87451
2	337733.104	6257870.969	SSM22768

Approximate coordinates of other points

Point	MGA East	MGA North	Name
3	337185	6257712	SSM57110
4	336690	6258157	SSM57112
5	336867	6257689	SSM68977
6	336232	6258477	SSM22575

Heights

No gravity or astronomic azimuth, latitude or longitude observations were available. So N (geoid-ellipsoid separation, not north coordinate) values were calculated by interpolation of AUSLIG Geodesy's (AUSLIG,1997) precise geoid for the Australian region, known as AUSGEOID93. "The absolute accuracy of these AUSGEOID93 values is estimated to be better than 0.5 metre, while the relative accuracy has been estimated as 2-5 parts per million (2-5 mm per km) … An N value interpolated from the AUSGEOID93 grid will generally only differ from a rigorously computed N value by a few cm. " (AUSLIG,1997).

AUSGEOID93 N values range <3cm across the site and refer to WGS84, not GRS80, ellipsoid. However, for the purposes of this paper, a constant value of 22.86m was chosen to represent all points. AUSLIG (1997) also give deflections of the vertical, but no deflection of the vertical corrections were applied in the work below (we expect values of several seconds in this area but they vary by less than 0.4" across the network).

The AHD heights (H) of points 1 and 2 are known, other points have approximate AHD heights. The ellipsoidal heights of the points are obtained as follows.

$h_1 = N_1 + H_1 = 22.86 + 2.732 = 25.592$ m

Similarly for other points:

Point	AHD H (m)	N (m)	Ellipsoidal h (m)
1	2.732	22.86	25.592
2	4.403	22.86	27.263
3	0.8	22.86	23.660
4	4.0	22.86	26.860
5	0.7	22.86	23.560
6	1.3	22.86	24.160

Convert coordinates

Redfearn's formulae, as implemented in AUSLIG's spreadsheet (AUSLIG, 1997), were used to convert between latitude & longitude and easting & northing, with the following constants. The process is similar to that used for AMG coordinate conversions but with different ellipsoid parameters.

Ellipsoid:	GRS80
Semi major axis (a)	6,378,137.000 m
Flattening (f)	1/ 298. 257 222 101
False easting	500,000 m
False northing	10,000,000 m
Central Scale factor (K_0)	0.9996
Zone width	6°
Eccentricity (e^2) = $2f - f^2$	= 0. 006 694 380

Point	Latitude	Longitude
1	-33° 48' 21.64352"	151° 14' 46.77449"
2	-33° 48' 24.89826"	151° 14' 48.96402"
3	-33° 48' 29.75397"	151° 14' 27.54982"
4	-33° 48' 15.03716"	151° 14' 08.60200"
5	-33° 48' 30.32389"	151° 14' 15.17149"
6	-33° 48' 04.39692"	151° 13' 51.01080"

Even though we chose coordinate parameters to be ellipsoidal latitude, longitude and height it is helpful for later calculations to also calculate the earth centered X Y Z coordinates of each point. For point 1, the first step is to calculate radii of curvature:

$$\nu_1 = \frac{a}{\sqrt{1-e^2\sin^2\phi_1}} = 6384756.074 \text{ m}$$

$$\rho_1 = \frac{a(1-e^2)}{(1-e^2\sin^2\phi_1)^{3/2}} = 6355184.095 \text{ m}$$

Often the symbols M and N are used, but we chose ν and ρ to avoid confusion with the geoid ellipsoid separation and North coordinates. Similar calculations for other points yield:

Pt	ν (m)	ρ (m)
1	6384756.074	6355184.095
2	6384756.387	6355185.028
3	6384756.853	6355186.420
4	6384755.440	6355182.201
5	6384756.908	6355186.583
6	6384754.419	6355179.151

Next we calculate the Cartesian coordinates:

$$X_1 = (\nu_1+h_1)\cos\phi_1\cos\lambda_1 = -4651118.768 \text{ m}$$
$$Y_1 = (\nu_1+h_1)\cos\phi_1\sin\lambda_1 = 2552079.134 \text{ m}$$
$$Z_1 = \{(1-e^2)\,\nu_1+h_1\}\sin\phi_1 = -3528601.849 \text{ m}$$

Pt	X (m)	Y (m)	Z (m)
1	-4651118.768	2552079.134	-3528601.849
2	-4651098.159	2552003.590	-3528686.105
3	-4650757.588	2552444.958	-3528808.412
4	-4650746.584	2552994.855	-3528433.419
5	-4650595.765	2552719.312	-3528822.946
6	-4650686.743	2553478.182	-3528159.499

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CALCULATIONS FOR LINE 3 TO 4

Least Squares calculations require the observations and estimates of their precision, approximate starting values for parameters (mainly point coordinates), partial derivatives (coefficients) and OMC terms (the differences between the observations and the equivalent values calculated from the starting coordinates). Harvey (1994) gives a fuller explanation of least squares and further examples of the application of least squares.

We show all the steps in all these calculations for just one line, from point 3 to point 4. This line contains all four types of observations dealt with in this paper. All the other observations and the results of their calculations are presented in summary form.

Preliminary values calculated from starting coordinates

$\Delta X_{34} = X_4 - X_3 \quad = \quad 11.004$ m
$\Delta Y_{34} = Y_4 - Y_3 \quad = 549.897$ m
$\Delta Z_{34} = Z_4 - Z_3 \quad = 374.993$ m

Bearing (take care with quadrant) (Leick, 1995):

$$\alpha_{34} = \tan^{-1}\left(\frac{-\sin\lambda_3 \Delta X_{34} + \cos\lambda_3 \Delta Y_{34}}{-\sin\phi_3\cos\lambda_3\Delta X_{34} - \sin\phi_3\sin\lambda_3\Delta Y_{34} + \cos\phi_3\Delta Z_{34}}\right)$$

= 312.93° = 312°56'02.9"

Slope distance:

$$s_{34} = \sqrt{\Delta X_{34}^2 + \Delta Y_{34}^2 + \Delta Z_{34}^2} = 665.678\text{m}$$

Slope angle (Leick, 1995):

$$\theta_{34} = \sin^{-1}\left(\frac{\cos\phi_3\cos\lambda_3\Delta X_{34} + \cos\phi_3\sin\lambda_3\Delta Y_{34} + \sin\phi_3\Delta Z_{34}}{s_{34}}\right)$$

= 0.272°

To use equations with zenith angles instead of slope angles see Strang & Borre (1997). To work in units of " and mm for corrections to ϕ λ h we need the following unit conversion factor:

$U = (3600*180/\pi)/1000 = 648/\pi \cong 206.264$

Distance observations

We enter slope distance, corrected for instrument calibration and refraction (first velocity correction), and we enter instrument height (h_i) and target height (h_t). There is no need to reduce it to the distance between ground marks or to the ellipsoid, grid or sea level.

At	To	Distance	Hi	Ht
1	3	547.222 1.480	1.700	
3	2	566.518	1.698	1.660
3	4	666.493	1.698	1.630
4	5	500.700	1.629	1.633
4	6	558.406	1.629	1.632
5	6	1012.101	1.635	1.633

Calculate Cartesian coordinate components between the instrument and target axes:

$\Delta X_{34}' = (\nu_4+h_4+h_t)\cos\phi_4\cos\lambda_4 - (\nu_3+h_3+h_i)\cos\phi_3\cos\lambda_3$
$\quad = 11.054$m
$\Delta Y_{34}' = (\nu_4+h_4+h_t)\cos\phi_4\sin\lambda_4 - (\nu_3+h_3+h_i)\cos\phi_3\sin\lambda_3$
$\quad = 549.870$m
$\Delta Z_{34}' = \{(1-e^2)\nu_4+h_4+h_t\}\sin\phi_4 - \{(1-e^2)\nu_3+h_3+h_i\}\sin\phi_3$
$\quad = 375.031$m

$$s'_{34} = \sqrt{\Delta X'^2_{34} + \Delta Y'^2_{34} + \Delta Z'^2_{34}} = 665.678\text{m}$$

$OMCs_{34} = (\text{obs dis}_{34} - s'_{34}) *1000 = +815$ mm

Partial derivatives (Vanicek & Krakiwsky, 1986):
In the partial derivative equations for slope distances new values of α and θ could be calculated using $\Delta X'$ $\Delta Y'$ $\Delta Z'$ at the instrument and target axes.

$$\frac{\partial dis_{34}}{\partial\phi_3} = -(\rho_3 + h_3)\cos\alpha_{34}\cos\theta_{34} / U = -20987\text{mm/"}$$

$$\frac{\partial dis_{34}}{\partial\lambda_3} = -(\nu_3 + h_3)\cos\phi_3\sin\alpha_{34}\cos\theta_{34} / U = 18831 \text{ mm/"}$$

$$\frac{\partial dis_{34}}{\partial h_3} = -\sin\theta_{34} = -0.0047 \text{ unitless}$$

$$\frac{\partial dis_{34}}{\partial\phi_4} = -(\rho_4 + h_4)\cos\alpha_{43}\cos\theta_{43} / U = 20988 \text{ mm/"}$$

$$\frac{\partial dis_{34}}{\partial\lambda_4} = -(\nu_4 + h_4)\cos\phi_4\sin\alpha_{43}\cos\theta_{43} / U = -18830\text{mm/"}$$

$$\frac{\partial dis_{34}}{\partial h_4} = -\sin\theta_{43} = 0.0048 \text{ unitless}$$

Standard deviations of the distances are estimated to be $\pm$(2mm + 1ppm).

Standard deviation = 2 + 1*(665.678/1000) = $\pm$2.67mm
Variance = (Standard deviation)^2 = 7.1 mm^2.

The distance 3 to 4 is our third observation, the relevant term in the P matrix (the inverse of the variance covariance matrix of the observations) is:
$\quad P_{3,3} = 1/\text{variance} = 0.14$
The rest of the third column and third row of P contains 0.

Direction observations

At	To	Mean Direction		
1	3	0°	00'	00.0"
1	2	266	14	24.3
2	3	119	23	26.3
2	1	193	56	36.0
3	2	0	00	00.7
3	4	236	45	34.0
3	1	348	18	42.0
4	6	0	00	00.0
4	3	186	59	15.7
4	5	214	18	44.3
5	6	0	00	00.0
5	4	18	07	09.0
6	4	100	15	28.0
6	5	116	26	59.0

Directions do not need to be reduced to 0° on the 'RO' or swung to approximate bearings. Least squares will determine the best fit swing (orientation) as a parameter.

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Calculate starting value of orientation for first direction in an arc:

$\Omega_3 = \alpha_{32}$ - obs dir_{32} = 74°48'17.9" - 0°00'00.7"
= 74°48'17.2"

then apply that orientation value to all other directions in the arc, so

$OMCdir_{34}$ = obs $dir_{34} - a_{34} + \Omega_3$
= 236°45'34.0" - 312°56'02.9" + 74°48'17.2" = -4932"

Corrections to directions, for deflections of the vertical, are constant at any one site. We are not including zenith angle observations, where corrections are a function of the azimuth of each line, and we are not solving for astronomic latitude, longitude or deflections of the vertical as parameters. So our orientation parameter will include the 'Laplace azimuth' correction for our directions.

Partial derivatives (Vanicek & Krakiwsky, 1986):

$$\frac{\partial dir_{34}}{\partial \phi_3} = \frac{(\rho_3 + h_3)\sin\alpha_{34}}{s_{34}\cos\theta_{34}} = -6990 \text{ unitless}$$

$$\frac{\partial dir_{34}}{\partial \lambda_3} = \frac{-(\nu_3 + h_3)\cos\phi_3\cos\alpha_{34}}{s_{34}\cos\theta_{34}} = -5429 \text{ unitless}$$

$$\frac{\partial dir_{34}}{\partial h_3} = 0$$

$$\frac{\partial dir_{34}}{\partial \phi_4} = \frac{(\rho_4 + h_4)\sin\alpha_{43}}{s_{34}\cos\theta_{43}} = 6989 \text{ unitless}$$

$$\frac{\partial dir_{34}}{\partial \lambda_4} = \frac{-(\nu_4 + h_4)\cos\phi_4\cos\alpha_{43}}{s_{34}\cos\theta_{43}} = 5429 \text{ unitless}$$

$$\frac{\partial dir_{34}}{\partial h_4} = 0 \qquad \frac{\partial dir_{34}}{\partial \Omega_3} = -1 \text{ unitless}$$

Standard deviations of the direction observations (sd) are estimated to be ±1.7" and centring error at instrument (i) and target (t) to be ± 1mm. Line 3-4 is 666m long so:

Variance = $(sd)^2 + (U^* \text{ i mm /L m})^2 + (U^*\text{t mm/L m})^2$
$Variance_{dir34} = (1.7)^2 + (U^*1/666)^2 + (U^*1/666)^2 = 3.08$
Standard deviation = $\sqrt{3.08}$ = ±1.76"
$P_{12,12} = 1/3.08 = 0.32$
Direction 34 is the 12th observation. The rest of the 34th column and 34th row of P contains 0.

GPS observations

At	To		(m)	s (mm)	correlations	
3	2	ΔX	-333.402	8.2	xy -0.63	xz 0.69
3	2	ΔY	-444.401	4.2		yz -0.50
3	2	ΔZ	110.849	5.7		
3	4	ΔX	11.369	9.6	xy -0.72	xz 0.42
3	4	ΔY	550.763	9.7		yz -0.36
3	4	ΔZ	375.141	7.5		
3	5	ΔX	162.067	11.7	xy -0.74	xz 0.79
3	5	ΔY	274.763	9.9		yz -0.72
3	5	ΔZ	-14.474	11.8		
3	6	ΔX	71.329	17.	xy -0.89	xz 0.83
3	6	ΔY	1033.793	12.		yz -0.83
3	6	ΔZ	648.836	12.		

The standard deviations of the GPS vectors obtained from preprocessing software were about ±1 to 2 mm. We have increased them to the values shown above, see Rizos (1997) for reasons and methods.

In this paper we ignore any scale and rotational differences between these WGS84 vectors and the GDA datum.

$OMC\Delta X_{34}$ = (obs GPS $\Delta X_{34} - \Delta X_{34}$) * 1000
= (11.369 - 11.004)*1000 = +395 mm
$OMC\Delta Y_{34}$ = (obs GPS $\Delta Y_{34} - \Delta Y_{34}$) * 1000
= (550.763 - 549.897)*1000 = +866 mm
$OMC\Delta Z_{34}$ = (obs GPS $\Delta Z_{34} - \Delta Z_{34}$) * 1000
= (375.141 - 374.993)*1000 = +148 mm

$$\frac{\partial \Delta X_{34}}{\partial \phi_3} = (\rho_3 + h_3)\cos\lambda_3\sin\phi_3 / U = 15029 \text{ mm/"}$$

$$\frac{\partial \Delta X_{34}}{\partial \lambda_3} = (\nu_3 + h_3)\cos\phi_3\sin\lambda_3 / U = 12375 \text{ mm/"}$$

$$\frac{\partial \Delta X_{34}}{\partial h_3} = -\cos\phi_3\cos\lambda_3 = 0.728 \text{ unitless}$$

$$\frac{\partial \Delta X_{34}}{\partial \phi_4} = -(\rho_4 + h_4)\cos\lambda_4\sin\phi_4 / U = -15027 \text{ mm/"}$$

$$\frac{\partial \Delta X_{34}}{\partial \lambda_4} = -(\nu_4 + h_4)\cos\phi_4\sin\lambda_4 / U = -12377 \text{ mm/"}$$

$$\frac{\partial \Delta X_{34}}{\partial h_4} = \cos\phi_4\cos\lambda_4 = -0.728 \text{ unitless}$$

$$\frac{\partial \Delta Y_{34}}{\partial \phi_3} = (\rho_3 + h_3)\cos\lambda_3\sin\phi_3 / U = -8248 \text{ mm/"}$$

$$\frac{\partial \Delta Y_{34}}{\partial \lambda_3} = -(\nu_3 + h_3)\cos\phi_3\cos\lambda_3 / U = 22548 \text{ mm/"}$$

$$\frac{\partial \Delta Y_{34}}{\partial h_3} = -\cos\phi_3\sin\lambda_3 = -0.400 \text{ unitless}$$

$$\frac{\partial \Delta Y_{34}}{\partial \phi_4} = -(\rho_4 + h_4)\sin\lambda_4\sin\phi_4 / U = 8249 \text{ mm/"}$$

$$\frac{\partial \Delta Y_{34}}{\partial \lambda_4} = (\nu_4 + h_4)\cos\phi_4\cos\lambda_4 / U = -22547 \text{ mm/"}$$

$$\frac{\partial \Delta Y_{34}}{\partial h_4} = \cos\phi_4\sin\lambda_4 = 0.400 \text{ unitless}$$

$$\frac{\partial \Delta Z_{34}}{\partial \phi_3} = -(\rho_3 + h_3)\cos\phi_3 / U = -25601 \text{ mm/"}$$

$$\frac{\partial \Delta Z_{34}}{\partial \lambda_3} = 0$$

$$\frac{\partial \Delta Z_{34}}{\partial h_3} = -\sin\phi_3 = 0.556 \text{ unitless}$$

$$\frac{\partial \Delta Z_{34}}{\partial \phi_4} = (\rho_4 + h_4)\cos\phi_4 / U = 25602 \text{ mm/"}$$

$$\frac{\partial \Delta Z_{34}}{\partial \lambda_4} = 0$$

$$\frac{\partial \Delta Z_{34}}{\partial h_4} = \sin\phi_4 = -0.556 \text{ unitless}$$

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We take the correlations between ΔX, ΔY and ΔZ into our adjustment but ignore correlations between the components of one line and those of another. We construct a 3x3 symmetric variance covariance matrix as follows:

$s^2_{\Delta X34} = 9.6^2 = 92.2\ mm^2$

$s_{\Delta X34\ \Delta Y34} = \rho_{\Delta X34\ \Delta Y34}\ s_{\Delta X34}\ s_{\Delta Y34} = -0.72 * 9.6 * 9.7 = -67.0$

$$VCV = \begin{pmatrix} 9.6^2 & -.72*9.6*9.7 & .42*9.6*7.5 \\ & 9.7^2 & -.36*9.7*7.5 \\ sym & & 7.5^2 \end{pmatrix}$$

$$= \begin{pmatrix} 92.2 & -67.0 & 30.2 \\ & 94.1 & -26.2 \\ sym & & 56.3 \end{pmatrix}$$

$$P = VCV^{-1} = \begin{pmatrix} .024 & .016 & -.006 \\ & .022 & .002 \\ sym & & .022 \end{pmatrix}$$

Height difference observations

Most lines in this network were observed across the harbour. Instead of levelling we have used height differences obtained from reciprocal (non-simultaneous) zenith angle observations and EDM distances.

We are not including zenith angle observations because corrections or additional parameters would be needed to account for deflections of the vertical and refraction.

At	To	Obs ΔH (m)	Calc ΔH (m)
1	3	-1.885	-1.932
4	6	-2.781	-2.700
3	4	3.188	3.200
3	2	3.561	3.603
5	6	0.565	0.600
5	4	3.332	3.300

$\Delta H_{34} = H_4 - H_3 = h_4 - N_4 - h_3 + N_3$

OMC $\Delta H_{34} = (\text{obs}\ \ \Delta H_{34} - \Delta H_{34}) * 1000$
$= \ (3.188 - 3.200) * 1000 = -12$ mm

$$\frac{\partial \Delta H34}{\partial \phi 3} = 0 \qquad \frac{\partial \Delta H34}{\partial \lambda 3} = 0 \qquad \frac{\partial \Delta H34}{\partial h3} = -1$$
$$\frac{\partial \Delta H34}{\partial \phi 4} = 0 \qquad \frac{\partial \Delta H34}{\partial \lambda 4} = 0 \qquad \frac{\partial \Delta H34}{\partial h4} = 1$$

Here the standard deviation of all the ΔH observations is assumed to be $\pm$7mm and not a function of line length. This gives a variance of 49 and $P_{35,35} = 1/7^2 = 0.02$ (ΔH_{34} is the 35^{th} observation).

LS VECTORS AND MATRICES FOR ALL OBSERVATIONS

For the other observations, calculations similar to those above yield the OMC vector, partials matrix **A**, and **P** matrix as shown below.

The b vector is the column vector of the OMC terms. The order of the terms corresponds with the order of the observations above. First, distance observations in mm, then direction observations in ", then GPS observations in mm, then ΔHt observations in mm. To save space the terms are listed in order rather than as a column.
(dist obs in mm) -6804, -4230, 815, 298, -359, 13,
(dir obs in ") 0, -4517, 0, -4780, 0, -4932, -267, 0, -94, -105, 0, -98, 0, -12,
(GPS obs in mm) 7170, -3040, -11457, 392, 845, 160, 239, 410, 59, 484, 569, -76,
(ΔHt in mm) 47, -81, -12, -42, -35, 32

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Matrix **A** is also called the design matrix, partial derivatives, or coefficients. **A** contains the partial derivatives with one line per observation (1 GPS vector = 3 observations). The observations are in the order listed in this paper. There is one column per parameter, in the order shown below (no columns for fixed points).

ϕ_3	λ_3	h_3	ϕ_4	λ_4	h_4	ϕ_5	λ_5	h_5	ϕ_6	λ_6	h_6	Ω_1 Ω_2 Ω_3 Ω_4 Ω_5 Ω_6	
-13896.	-22955.	-0.0034	0	0	0	0	0	0	0	0	0	0 0 0 0 0 0	
-8076.	-24820.	-0.0063	0	0	0	0	0	0	0	0	0	0 0 0 0 0 0	
-20987.	18830.	-0.0048	20988.	-18830.	0.0049	0	0	0	0	0	0	0 0 0 0 0 0	SDIS34
0	0	0	29001.	-8685.	0.0066	-29000.	8685.	-0.0066	0	0	0	0 0 0 0 0 0	
0	0	0	-18077.	20829.	0.0049	0	0	0	18078.	-20829.	-0.0048	0 0 0 0 0 0	
0	0	0	0	0	0	-24318.	15793.	-0.0005	24319.	-15793.	0.0007	0 0 0 0 0 0	
10238.	-4319.	0	0	0	0	0	0	0	0	0	0	-1 0 0 0 0 0	
0	0	0	0	0	0	0	0	0	0	0	0	-1 0 0 0 0 0	
10746.	-2436.	0	0	0	0	0	0	0	0	0	0	0 -1 0 0 0 0	
0	0	0	0	0	0	0	0	0	0	0	0	0 -1 0 0 0 0	
10746.	-2436.	0	0	0	0	0	0	0	0	0	0	0 0 -1 0 0 0	
-6990.	-5429.	0	6989.	5429.	0	0	0	0	0	0	0	0 0 -1 0 0 0	DIR34
10238.	-4319.	0	0	0	0	0	0	0	0	0	0	0 0 -1 0 0 0	
0	0	0	-9211.	-5571.	0	0	0	0	9210.	5571.	0	0 0 0 -1 0 0	
-6990.	-5429.	0	6989.	5429.	0	0	0	0	0	0	0	0 0 0 -1 0 0	
0	0	0	4289.	9980.	0	-4289.	-9979.	0	0	0	0	0 0 0 -1 0 0	
0	0	0	0	0	0	-3856.	-4137.	0	3855.	4138.	0	0 0 0 0 -1 0	
0	0	0	4289.	9980.	0	-4289.	-9979.	0	0	0	0	0 0 0 0 -1 0	
0	0	0	-9211.	-5571.	0	0	0	0	9210.	5571.	0	0 0 0 0 0 -1	
0	0	0	0	0	0	-3856.	-4137.	0	3855.	4138.	0	0 0 0 0 0 -1	
15029.	12375.	0.7284	0	0	0	0	0	0	0	0	0	0 0 0 0 0 0	
-8248.	22548.	-0.3998	0	0	0	0	0	0	0	0	0	0 0 0 0 0 0	
-25601.	0	0.5564	0	0	0	0	0	0	0	0	0	0 0 0 0 0 0	
15029.	12375.	0.7284	-15027.	-12377.	-0.7284	0	0	0	0	0	0	0 0 0 0 0 0	GPS34x
-8248.	22548.	-0.3998	8249.	-22547.	0.3999	0	0	0	0	0	0	0 0 0 0 0 0	GPS34y
-25601.	0	0.5564	25602.	0	-0.5564	0	0	0	0	0	0	0 0 0 0 0 0	GPS34z
15029.	12375.	0.7284	0	0	0	-15029.	-12376.	-0.7284	0	0	0	0 0 0 0 0 0	
-8248.	22548.	-0.3998	0	0	0	8249.	-22547.	0.3998	0	0	0	0 0 0 0 0 0	
-25601.	0	0.5564	0	0	0	25601.	0	-0.5564	0	0	0	0 0 0 0 0 0	
15029.	12375.	0.7284	0	0	0	0	0	0	-15025.	-12380.	-0.7284	0 0 0 0 0 0	
-8248.	22548.	-0.3998	0	0	0	0	0	0	8249.	-22547.	0.3999	0 0 0 0 0 0	
-25601.	0	0.5564	0	0	0	0	0	0	25603.	0	-0.5563	0 0 0 0 0 0	
0	0	1	0	0	0	0	0	0	0	0	0	0 0 0 0 0 0	
0	0	0	0	0	-1	0	0	0	0	0	1	0 0 0 0 0 0	
0	0	-1	0	0	1	0	0	0	0	0	0	0 0 0 0 0 0	HT34
0	0	-1	0	0	0	0	0	0	0	0	0	0 0 0 0 0 0	
0	0	0	0	0	0	0	0	-1	0	0	1	0 0 0 0 0 0	
0	0	0	0	0	1	0	0	-1	0	0	0	0 0 0 0 0 0	

P, a 38x38 matrix, contains all zeros except as follows. The first 20 diagonal terms = $1/s^2$ for distances and directions, in units of mm^{-2} and $''^{-2}$:

0.154 0.152 0.141 0.160 0.153 0.110 0.316 0.107 0.317 0.107 0.317 0.324 0.316 0.316 0.324 0.310 0.336 0.310 0.316 0.336

The structure of our **P** matrix looks like (except within P_{GPS}, all off diagonal terms are zero):

$$\begin{pmatrix} .154 & & & & & & \\ & .152 & & & & & \\ & & \ddots & & & & \\ & & & [P_{GPS}] & & & \\ & & & & \ddots & & \\ & & & & & .020 & \\ & & & & & & .020 \end{pmatrix}$$

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Then block diagonals of $\mathbf{P}_{GPS} = VCV^{-1}$ for GPS observations are (blank cells contain 0):

$$\begin{pmatrix}
.036 & .027 & -.026 & & & & & & & & & \\
.027 & .095 & .009 & & & & & & & & & \\
-.026 & .009 & .060 & & & & & & & & & \\
 & & & .240 & .016 & -.006 & & & & & & \\
 & & & .016 & .022 & .002 & & & & & & \\
 & & & -.006 & .002 & .022 & & & & & & \\
 & & & & & & .023 & .010 & -.012 & & & \\
 & & & & & & .010 & .025 & .008 & & & \\
 & & & & & & -.012 & .008 & .021 & & & \\
 & & & & & & & & & .019 & .017 & -.008 \\
 & & & & & & & & & .017 & .038 & .011 \\
 & & & & & & & & & -.008 & .011 & .026
\end{pmatrix}$$

Then the last 6 diagonal terms = $1/s^2$ for height differences in units of mm^{-2};

0.20 0.020 0.020 0.020 0.020 0.020

Once we have constructed the **A** and **P** matrices and b vector then we solve for corrections to our starting values for the parameters using the least squares solution equation:

$$\Delta x = [\, A^T PA]^{-1} A^T P\, b$$

The matrix algebra and solution method are not shown here, see Harvey (1994) for more details. Δx is a column vector with the following terms:

ϕ_3 λ_3 h_3	0.44848"	0.02619"	20.5mm
ϕ_4 λ_4 h_4	0.45423"	-0.01057"	14.4mm
ϕ_5 λ_5 h_5	0.44960"	0.00794"	-22.9mm
ϕ_6 λ_6 h_6	0.44334"	-0.00263"	-61.9mm

Orientations Ω_{1-6}

4488.3 4761.7 4757.9 -60.6 -67.6 -56.1 "

Adjusted values of the parameters are calculated using

$X = x_a + \Delta x$ For point 3

ϕ_3 = -33°48'29.75397" + 0.44848" = -33°48'29.30549"

λ_3 = 151°14'27.54982" + 0.02619" = 151°14'27.57601"

Ell. Height = 23.660m + 20.5 mm = 23.680 m

These adjusted coordinates are then used as starting values and the least squares is repeated. This continues until the Δx are insignificant.

The Qx matrix, standard deviations of results, error ellipses, variance factor, etc. are also calculated. Our final results are shown below.

FINAL RESULTS

3D Network Adjustment by Program: Elfy Version 1.41

There are 12 coordinate parameters, 6 orientation parameters and 0 transformation parameters. There are 4 GPS baselines, 14 directions, 6 slope distances and 6 height differences. Reference Ellipsoid: GRS80.

v = residual = correction
s = input standard deviation of observation, incl. centring and ppm where applicable.

```
Type At To   Adjusted      v         s
SDIS  1  3   547.2207m  -1.35mm  2.55mm
SDIS  3  2   566.5173m  -0.67mm  2.57mm
SDIS  3  4   666.4921m  -0.90mm  2.67mm
SDIS  4  5   500.6988m  -1.16mm  2.50mm
SDIS  4  6   558.4042m  -1.82mm  2.56mm
SDIS  5  6  1012.1027m   1.72mm  3.01mm
DIR   1  3  244 26 32.02  -0.17"   1.8"
DIR   1  2  150 40 56.97   0.49"   3.1"
DIR   2  3  256 07 47.91   0.47"   1.8"
DIR   2  1  330 40 55.75  -1.39"   3.1"
DIR   3  2   76 07 59.81   0.86"   1.8"
DIR   3  4  312 53 29.04  -3.22"   1.8"
DIR   3  1   64 26 42.70   2.45"   1.8"
DIR   4  6  305 54 27.25   0.82"   1.8"
DIR   4  3  132 53 39.60  -2.53"   1.8"
DIR   4  5  160 13 12.53   1.81"   1.8"
DIR   5  6  322 05 56.78  -1.48"   1.7"
DIR   5  4  340 13  8.87   1.61"   1.8"
DIR   6  4  125 54 37.03  -1.13"   1.8"
DIR   6  5  142 06 10.23   1.07"   1.7"
GPS   3  2  -333.4165m -14.51mm  8.20mm
            -444.3963m   4.69mm  4.20mm
             110.8483m  -0.71mm  5.70mm
GPS   3  4    11.3678m  -1.21mm  9.60mm
             550.7635m   0.49mm  9.70mm
             375.1563m  15.33mm  7.50mm
GPS   3  5   162.0600m  -7.01mm 11.70mm
```

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			274.7557m	-7.32mm	9.90mm
			-14.4727m	1.31mm	11.80mm
GPS	3	6	71.3319m	2.92mm	17.00mm
			1033.7923m	-0.66mm	12.00mm
			648.8396m	3.57mm	12.00mm
HT_D	1	3	-1.8853m	-0.34mm	7.00mm
HT_D	4	6	-2.7766m	4.37mm	7.00mm
HT_D	3	4	3.1914m	3.38mm	7.00mm
HT_D	3	2	3.5563m	-4.66mm	7.00mm
HT_D	5	6	0.5615m	-3.51mm	7.00mm
HT_D	5	4	3.3381m	6.13mm	7.00mm

Variance Factor: 1.48

ADJUSTED MGA COORDINATES (ZONE 56)

Point	East(m)	North(m)	AHDHeight(m)
3	337185.551	6257725.832	0.847
4	336689.614	6258171.007	4.038
5	336867.085	6257702.868	0.700
6	336231.821	6258490.674	1.261

Standard Deviations & Error Ellipses (mm)

Point	E	N	H	S-Maj/Min		Brg
3	1.7	2.7	4.3	2.8	1.6	163
4	2.9	4.1	7.0	4.1	2.9	8
5	4.1	4.3	8.4	4.4	3.9	33
6	3.8	5.1	8.5	5.4	3.4	24

GEOGRAPHIC COORDINATES

Pt.	Sth Latitude	Longitude	Ell Ht(m)
1	33 48 21.6435	151 14 46.7745	25.592
2	33 48 24.8983	151 14 48.9640	27.263
3	33 48 29.3054	151 14 27.5804	23.707
4	33 48 14.5824	151 14 08.5963	26.898
5	33 48 29.8739	151 14 15.1840	23.560
6	33 48 03.9530	151 13 51.0130	24.121

CARTESIAN COORDINATES

Pt.	X(m)	Y(m)	Z(m)
1	-4651118.768	2552079.134	-3528601.849
2	-4651098.159	2552003.590	-3528686.105
3	-4650764.743	2552447.987	-3528796.953
4	-4650753.375	2552998.750	-3528421.797
5	-4650602.683	2552722.742	-3528811.426
6	-4650693.411	2553481.779	-3528148.113

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