Telescope Axes Surveys

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Abstract

The survey of the connection of VLBI telescopes to conventional survey control marks is discussed. Both the practical field measurement procedures and rigorous least squares analysis of the results are described. The telescopes are not assumed to have: axes which intersect; axes which are in perfect alignment; or axes which are metal rods whose centre lines can be determined by direct measurement. The 3D orientations of the centrelines of the axes are determined. The equations for the coordinates of the 'reference point' of the VLBI telescope are given. An equation for the shortest distance between the two axes is also given. The techniques discussed in this paper have other 3D monitoring applications, particularly monitoring structures with axes or centrelines.

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1. Introduction

Why survey the axes of radio telescopes? One reason for a survey connection from nearby control marks to radio telescopes is to monitor movements in the reference point of radio telescopes. Another reason is to connect VLBI observations to other types of survey measurements such as GPS.

If a telescope's axes are assumed to have been constructed perfectly, for example that vertical axes are truly vertical, then the survey is much simpler. However this paper will not make the assumption of perfect construction. The survey and analysis solves for the orientation of the axes. If the axes are then found to be truly constructed the subsequent work is simplified. However it is recommended that at some stage a survey should determine the true orientation of the axes.

This paper also investigates the propagation of errors so that not only the coordinates of the reference point (or vectors to nearby survey control marks) are determined but also some estimates of the quality of the coordinates (eg their error ellipses).

This paper is only one part of the total effort required to combine VLBI and other survey data. Harvey & Stolz (1985), Harvey (1985) and Harvey (1986) have investigated some other aspects. For high accuracy geodynamic studies all these aspects and more have to be considered. Surveyors in this type of work should be cautious before claiming they can use VLBI to monitor, for example, tide gauges at the centimetre level. On its own VLBI is a very good measuring tool for accurate geodynamic studies where the telescope sites are considered as large geodetic control ('trig.') stations. Note that it is the connection of VLBI to conventional surveys or GPS that yields problems.

2. VLBI Background

There are three types of VLBI antennas: those with a fixed vertical axis (AZEL); those with a fixed axis pointing to the celestial pole (HADEC); and those with a fixed horizontal axis (X-Y). Whilst antennas are of various types their collecting areas (dish) are all approximately symmetric around an axis. On this axis is a point (X in fig. 1) which may be called the electrical centre and it is the point to which the VLBI observations (delay and rate) refer. The electrical centre is not suitable as a reference point because the dish axis moves as an antenna points in different directions.

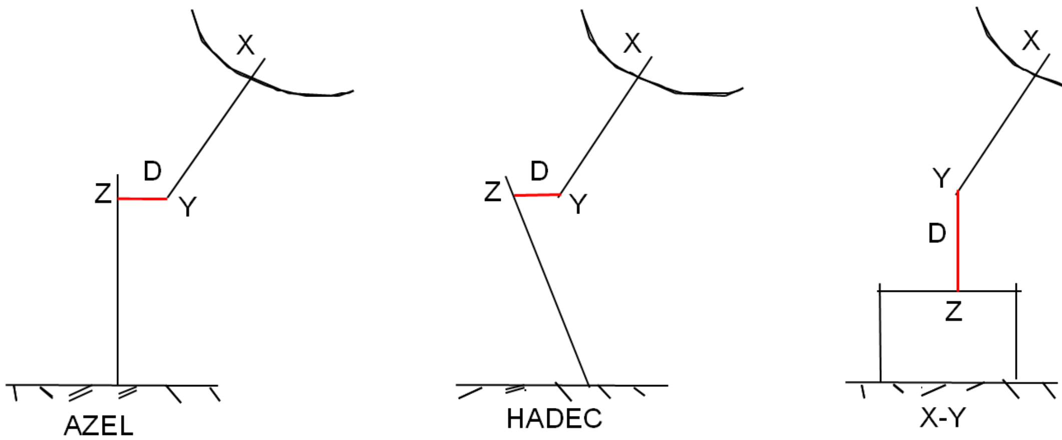

Figure 1. Axis Offset Diagrams.

Any point on the fixed axis would be suitable as the reference point. The difference between the pivot of the dish axis (Y in fig. 1) and the electrical centre will be constant provided instrumental delays are independent of antenna orientation. This difference is absorbed in the adjustment of the VLBI data.

If the dish axis and the fixed axis intersect then the reference point is the point of intersection. In this case no axis offset corrections need to be applied to the VLBI observations. If the axes do not intersect, the reference point (Z in fig. 1) is defined by convention to be the intersection of the fixed axis with the plane perpendicular to the fixed axis which contains the moving (dish) axis. Some antennas have axis offsets as large as 15m, though offsets larger than about 7m are rare. It is possible to solve for the axis offset in a VLBI adjustment. However ground measurements of D are usually adopted. The VLBI adjustment yields 3D vector baselines which refer to the reference points of each telescope.

Since VLBI telescopes can be used as control stations for geodynamics they need to be stable or their movements monitored. One cause of movement is that a telescope may deform due to the force of the wind or deform as it points in different directions due its large weight. Also, the sun may heat one side of telescope and cause that side to expand more than the other side. Hung et al (1976) estimated that a 40°C temperature change would produce about 9mm movement of the reference point of one VLBI site that has a 64m diameter dish. The effect is predominantly to raise or lower the reference point. McGinness et al (1979) estimated the displacements of the reference point of another VLBI site with a 26m diameter dish, due to bearing 'runout', temperature change, and wind loading, to be less than a few millimetres. Therefore VLBI sites should be surrounded by a ground network of several control marks. Repeated observations should be made, both to locate the reference point and to monitor local geophysical and structural motions.

3. Survey Measurement Techniques

The VLBI reference point needs to be connected to the ground survey network for the reasons mentioned earlier. Clearly, the connection should be at least as reliable as any other line in the survey network. The connection can be surveyed with an accuracy of a few millimetres provided instrument calibration, observation procedure, and modelling corrections are to a high standard. However gross errors in the measurements, such as that reported by Thomas et al (1976), need to be eliminated. A misidentification of the VLBI reference point will not be detected in a ground adjustment. But it will cause errors in a combined adjustment using the VLBI vectors and ground surveys. Orientation, scale and reference coordinate systems for the connection are discussed by Harvey & Stolz (1985).

To determine the position of the reference point, observations have to be made to determine the centrelines of the antenna axes. These observations should be made with the antenna pointing in different directions to discover whether the reference point is independent of the direction of antenna pointing. No axis should be assumed to be in correct alignment, and the axes should not be assumed to intersect.

3.1 Difficulties

There are several practical difficulties in the survey. Among these are -

1) There is not necessarily any physical component at the reference point.

2) The VLBI reference point is located in the midst of the structural components of the telescope that obstruct many survey observations. The reference point of those telescopes surrounded by radomes will be difficult to observe because of the physical obstruction. Radomes are large metallic domes (shaped like golf balls) that surround and cover the whole radio telescope. To make theodolite or EDM observations through a radome requires holes to be bored for each line of site.

3) Usually the reference point, or any nearby point, cannot be conveniently occupied by surveying instruments.

4) It is not possible to observe astronomic latitude, longitude or azimuth at a mark below the reference point because the view of the stars is obstructed by the telescope.

5) Many VLBI sites use very large radio telescopes where the reference point can be tens of metres above the ground. If control marks are close to the telescope then steep observations, which are more severely affected by dislevelment of the theodolite, are required. If control marks are further from the telescope then the longer lines of sight imply less precise coordinates for the reference point.

3.2 Survey methods

Some methods of surveying the reference point will now be discussed.

1) One method is to establish a mark near ground level and directly beneath the reference point. The position of the reference point is usually determined by accepting that the constructed VLBI telescope perfectly agrees with the engineering design. Lines of sight through doorways or through holes bored in the structure, or both, connect the ground mark to the exterior site(s). A few exterior sites should be used to provide redundancy. The horizontal position of the ground mark and the height difference to the reference point are

then measured. The measurement of height difference is not necessarily simple or error free, especially for large antennas. The desired measurements can be obtained by levelling from nearby stable towers or by observing zenith angles from exterior sites to the reference point. It is usually impossible to measure the height difference reliably by hanging a tape (or by vertical EDM) from the "engineers' guess" of the position of the reference point to a ground mark because of obstruction by parts of the telescope. If zenith angles are observed care must be taken to reduce refraction and theodolite errors. Thus for many VLBI sites the use of a ground mark directly beneath the reference point may not be a practical or effective method.

2) Another method is to build a temporary tower over the antenna so that a survey instrument could be set up directly above the reference point. Astronomic observations could be made to determine deflections of the vertical and the height measurements would be less obstructed. Clearly, the tower would need to be stable.

3) A better approach may be to determine the coordinates of the reference point by observing a 3D ground network surrounding the telescope. A target should be placed on each end of the moving axis and of the fixed axis. The targets should then be observed as the telescope is rotated about this axis. Note that the axis itself should not move. If a target does not move then it is truly on the centreline of the axis. However it is not usually possible to rotate the telescope through 360° about each axis, especially with equatorial mount telescopes. Moreover there may not be any structure on the axis! Some telescopes rotate on a circular arc track of several metres radius with no physical structure at the centre.

If targets can be placed on the centreline of an axis then horizontal directions, zenith angles, and perhaps distances can be observed to each target or, the targets could be observed by photogrammetry. Repeat the measurements for a variety of positions of the axes and from a few control marks in the immediate surrounds of the telescope. Each site should observe simultaneously, if possible, to minimize errors due to movement of the telescope. Unfortunately, due to obstructions, the targets may not be visible from many control marks. Measurements could be made under different sun and wind exposure conditions. Then compute the axis offset, the inclination of each axis, and the position of the reference point with respect to the local geodetic network, from an adjustment of these observations, which would include many redundancies. If the axis offset can be determined by direct measurement it would strengthen the solution for the position of the reference point. This method is suitable for locating the reference point of all types of telescope including a transportable antenna, provided targets can be placed on the centrelines of axes and that these targets can be seen from control marks.

The observation times should be scheduled to minimize the effects of time-dependent atmospheric anomalies. Variations in the refraction effect on horizontal direction, zenith angle and EDM distance measurement during the observation period can be monitored by placing a fixed target beside the telescope and at a similar height to the reference point. This fixed target should be observed regularly during the total observation period to monitor changes in direction, zenith angle and distance. The height difference could be monitored by precise levelling. Then use the variations in the observations to determine the magnitude of refraction effects and correct the observations to other targets.

The direction of the local vertical for all sites within a few hundred metres of the telescope is, generally, adequately determined by interpolation. However this assumption could be checked by making a detailed geoid map of the area by gravimetry.

4) This method uses similar measurements to those in (3) above. If targets cannot be placed on the centrelines of axes or these targets cannot be seen from control marks then the following procedure might be required. Mount targets on the telescope structure at some distance from an axis, for example on the track mentioned above (see figures 2 and 3). Then rotate the radio telescope about this one axis. The target scribes a circle if there is no wobble in the axis. Rotate the telescope more than 360° if possible then calculate position and orientation of the axis and investigate for wobble errors. If there is wobble then it can be studied from these same measurements.

If there is more than one target then the distances between the targets stay the same even though the coordinates of the targets change. Frequently the amount of rotation about an axis is restricted to less than 200°. In this case it is especially important to use more than one target.

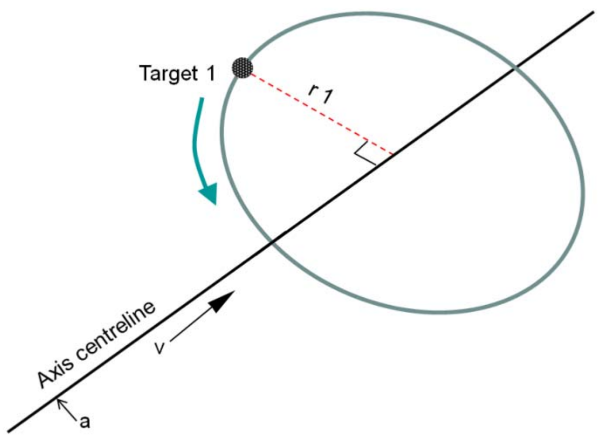

Figure 2. Rotation of a target about an axis.

If theodolite/EDM measurements are used instead of photogrammetry then generally the target prisms cannot be pointed to the instrument, so distances have to be corrected for the misalignment. Misalignment may also seriously affect the strength of the returned EDM signal, so locate the instrument carefully. Misalignment corrections are also needed if prisms are used as targets for sighting of directions and zenith angles.

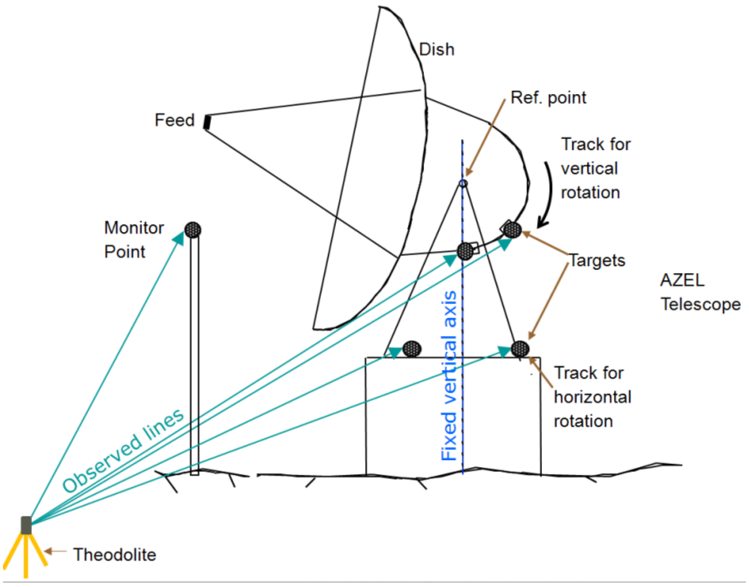

Figure 3. Possible survey method.

4. Solution for Target Coordinates.

The coordinates of the positions of the targets can be determined by standard least squares programs for determining 3D coordinates of survey networks from survey measurements or by common close range photogrammetry bundle adjustment software. The adjusted coordinates and, importantly, their covariance matrix can then be input to subsequent least squares programs to solve for the position and orientation of each axis. Then a third set of calculations can be applied, using propagation of variance laws and the equations below to determine the coordinates of the reference point and its error statistics. Since programs to determine the 3D coordinates are readily available this procedure appears to be more practical than writing a larger special computer program that takes all measurements, and does each of the above three steps within the one program to produce the coordinates of the reference point.

5. Equations

5.1 Revision of basic model equations

The equation of a line in 3D space can be expressed in several forms, some are:

$$\frac{x-a_1}{v_1}=\frac{y-a_{21}}{v_2}=\frac{z-a_3}{v_3}$$

or $\quad x = a_1 + \lambda v_1 \qquad y = a_2 + \lambda v_2 \qquad z = a_3 + \lambda v_3$

or in vector notation $\mathbf{x} = \mathbf{a} + \lambda\, \mathbf{v}$

where v_1 is the direction cosine with respect to the x axis, that is the cosine of the angle between the line and the x axis;
v_2 is the direction cosine with respect to the y axis;
v_3 is the direction cosine with respect to the z axis;
x, y, z is any point on the line;
a_1, a_2, a_3 is one particular point on the line, and
λ is the distance between x, y, z, and a_1, a_2, a_3.

The vectors **a** and **v** are shown in figure 2. In this paper the x axis is north, y axis is east, and z axis is height and plane coordinates are assumed sufficient for the local network. The coordinates of the targets are assumed to be in the same local x y z system.

Now $v_1^2 + v_2^2 + v_3^2 = 1$ so the direction cosines are not independent (given two the third can be calculated). Instead of v_1, v_2, v_3 the orientation of a line can be defined by two independent parameters. They are, in familiar surveying terms, bearing (B) and zenith angle (Z) of the line:

$$V_1 = \sin Z \sin B \qquad v_2 = \sin Z \cos B \qquad v_3 = \cos Z$$

The equation for points on a cylinder, i.e. rotating about an axis with radius r constant is:

$$\{v_1(x-a_1) + v_2(y-a_2) + v_3(z-a_3)\}^2 = r^2$$

5.2 Zenith Angle and Bearing Model

This section gives the equations and matrices required for the least squares adjustment of points rotating about an axis, thus forming a cylinder in 3D coordinates. The adjustment solves for the equation of the axis (a point on the axis plus the bearing and zenith angle of the axis) and the radius of the circle formed by each target.

The model equation (F) for point i to target J is:

$$0 = (x_i-a_1)^2+(y_i-a_2)^2+(z_i-a_3)^2 - \{\sin Z\sin B(x_i-a_1) + \sin Z\cos B(y_i-a_2) + \cos Z(z_i-a_3)\}^2 - r_J^2$$

Since the model equation includes parameters and more than one observation the combined (also called general) least squares method is used (Harvey, 1991).

If the order of the parameters is: $a_1\ a_2\ a_3\ Z\ B\ r_1\ r_2\ r_3\ .\ .\ .$
Then the A matrix (partial derivatives: $\partial F / \partial$parameters) for observed point i and target J:

$$A(i,1) = P\sin Z\sin B - 2(x_i - a_1)$$
$$A(i,2) = P\sin Z\cos B - 2(y_i - a_2)$$
$$A(i,3) = P\cos Z - 2(z_i - a_3)$$
$$A(i,4) = -P\{\cos Z\sin B(x_i-a_1)+\cos Z\cos B(y_i-a_2)-\sin Z(z_i-a_3)\}$$
$$A(i,5) = -P\{\sin Z\cos B(x_i - a_1) - \sin Z\sin B(y_i - a_2)\}$$

$A(i,5+J) \quad = -2r_J$

where $P = 2\{\sin Z \sin B(x_i - a_1) + \sin Z \cos B(y_i - a_2) + \cos Z(z_i - a_3)\}$

All other terms in the i^{th} row of A equal zero. The number of rows in the A matrix is the number of observed points and the number of columns is the number of parameters (= 5 + number of targets). I recommend that one of the components of a (i.e. a_1 or a_2 or a_3) be held fixed to overcome datum deficiency problems. In that case A(i,1) equals zero if a_1 is fixed, A(i,2) equals zero if a_2 is fixed, and A(i,3) equals zero if a_3 is fixed.

$B(i,3i-2) \quad = 2(x_i - a_1) - P\sin Z\sin B$
$B(i,3i-1) \quad = 2(y_i - a_2) - P\sin Z\cos B$
$B(i,3i) \quad = 2(z_i - a_3) - P\cos Z$

All other terms in the i^{th} row of B equal zero. The size of the B matrix is the number of points by three times the number of points.

In the first iteration calculate the correction term (sometimes called OMC term) by inserting the observed coordinates and the approximate values of the parameters in the model equation F above. That is

$$f_i = (x_i-a_1)^2+(y_i-a_2)^2+(z_i-a_3)^2 - \{P/2\}^2 - r_J^2$$

However in subsequent iterations there is an additional term because, in combined least squares, corrected observations are used in subsequent iterations. For an example of this practice see the book by CIS (1983 p. 141). The value for each f_i is the row of B times the residual vector v. (Do not confuse the residual vector v with the direction cosines v_1, v_2 and v_3). Now most of the row of B is zero; there are only three non zero values so:

$$f_i = (x_i-a_1)^2+(y_i-a_2)^2+(z_i-a_3)^2 - \{P/2\}^2 - r_J^2 + \{2(x_i - a_1) - P\sin Z\sin B\}\, v(3i-2) + \{2(y_i - a_2) - P\sin Z\cos B\}\, v(3i-1) + \{2(z_i - a_3) - P\cos Z\}\, v(3i)$$

Note that in the first iteration all terms in the residual vector equal zero so this equation is the same as f_i above, i.e. there is no correction.

5.3 Direction cosine (v_1 and v_2) model

This model is similar to the previous one but solves for v_1 and v_2 instead of Z and B. [Other pairs of the three direction cosines could be chosen.]

In this model a program should check that $-1 \le v_1 \le 1$, $-1 \le v_2 \le 1$, and that $(1 - v_1^2 - v_2^2) \ge 0$. The other direction cosine $v_3 = \sqrt{(1-v_1^2-v_2^2)}$.

The model equation (F) for point i to target J is:

$$0 = (x_i-a_1)^2+(y_i-a_2)^2+(z_i-a_3)^2 - \{v_1(x_i - a_1) + v_2(y_i - a_2) + [\sqrt{(1-v_1^2-v_2^2)}]\,(z_i - a_3)\}^2 - r_J^2$$

If the order of the parameters is: $a_1\ a_2\ a_3\ v_1\ v_2\ r_1\ r_2\ r_3\ \ldots$

Then the A matrix for point i and target J:

$$
\begin{array}{lll}
A(i,1) & = Pv_1 - 2(x_i - a_1) & \text{or} = 0 \text{ if } a_1 \text{ fixed} \\
A(i,2) & = Pv_2 - 2(y_i - a_2) & \text{or} = 0 \text{ if } a_2 \text{ fixed} \\
A(i,3) & = Pv_3 - 2(z_i - a_3) & \text{or} = 0 \text{ if } a_3 \text{ fixed} \\
A(i,4) & = -P\{(x_i - a_1) - v_1\,(z_i-a_3)/v_3\} & \\
A(i,5) & = -P\{(y_i - a_2) - v_2\,(z_i-a_3)/v_3\} & \\
A(i,5+j) & = -2r_J &
\end{array}
$$

where $P = 2\{v_1\,(x_i - a_1) + v_2\,(y_i - a_2) + v_3\,(z_i - a_3)\}$

All other terms in the i^{th} row of A equal zero.

$$
\begin{array}{ll}
B(i,3i-2) & = 2(x_i - a_1) - Pv_1 \\
B(i,3i-1) & = 2(y_i - a_2) - Pv_2 \\
B(i,3i) & = 2(z_i - a_3) - Pv_3
\end{array}
$$

All other terms in the i^{th} row of B equal zero.

Calculate the correction term f_i by inserting the observed coordinates and the approximate values of the parameters in the model equation F above. That is

$$
\begin{aligned}
f_i = (x_i-a_1)^2+(y_i-a_2)^2+(z_i-a_3)^2 - \{P/2\}^2 - r_J^2 + \{2(x_i - a_1) - Pv_1\}\ v(3i-2) + \\
\{2(y_i - a_2) - Pv_2\}\ v(3i-1) + \{2(z_i - a_3) - Pv_3\}\ v(3i)
\end{aligned}
$$

Note that in the first iteration all terms in the residual vector (v) equal zero so this equation is simpler in the first iteration.

Sometimes direction cosines (v_1, v_2 or v_3) are very close to 0 or 1. Care should be taken with numerical accuracies in any calculations where this occurs.

5.4 Misalignment models

If an axis is constructed close to a cardinal direction (eg horizontal, vertical, north-south) then it is possible to solve for the slight misalignments as small angles. For example, consider an axis that is close (within 60" say) to horizontal and is close to being north-south (i.e. 0° azimuth), then an approximate model is:

$$0 = (x_i-a_1)^2+(y_i-a_2)^2+(z_i-a_3)^2 - \{\alpha(x_i - a_1) + (y_i - a_2) + t(z_i - a_3)\}^2 - r_J^2$$

where α is the azimuth in radians and t is the tilt of the axis from horizontal.
This type of model uses the small angle approximations, that is $\sin\alpha \approx \alpha$ and $\cos\alpha \approx 1$.

5.5 Constraints

Each target lies at a different radius from an axis and lies in a different plane (normal to the axis). As well as solving for the parameters mentioned above (i.e. axis orientation and target radius from the axis), it is possible to add constraint equations to the least squares adjustment. It is possible to constrain the positions for a target to lie on a plane normal to the axis. Then the distance between any position of a target and the point on the axis (a_1, a_2, a_3) is constant. Constraint equations could be added to any of the previous models. The following equations show the one way of implementing the constraint in the zenith angle and bearing model.

There are two model equations (F) for point i to target J:

$$0 = (x_i-a_1)^2+(y_i-a_2)^2+(z_i-a_3)^2 - \{\sin Z\sin B(x_i-a_1) + \sin Z\cos B(y_i-a_2) + \cos Z(z_i-a_3)\}^2 - r_J^2$$

and

$$0 = \{\sin Z\sin B(x_i-a_1) + \sin Z\cos B(y_i-a_2) + \cos Z(z_i-a_3)\}^2 - m_J^2$$

where m is the distance from point a_1, a_2, a_3 to the pedal point of the target plane (the intersection of the plane containing the target positions and the axis).

If the order of the parameters is: a_1 a_2 a_3 Z B r_1 m_1 r_2 m_2 r_3 m_3 . . .
Then the A matrix has two rows for point i (i=1,N) target j:

A(2i-1,1)	$= P\sin Z\sin B - 2(x_i - a_1)$	or = 0 if a_1 fixed
A(2i-1,2)	$= P\sin Z\cos B - 2(y_i - a_2)$	or = 0 if a_2 fixed
A(2i-1,3)	$= P\cos Z - 2(z_i - a_3)$	or = 0 if a_3 fixed
A(2i-1,4)	$= -P\{\cos Z\sin B(x_i - a_1) + \cos Z\cos B(y_i - a_2) - \sin Z(z_i - a_3)\}$	
A(2i-1,5)	$= -P\{\sin Z\cos B(x_i - a_1) - \sin Z\sin B(y_i - a_2)\}$	
A(2i-1,4+2j)	$= -2r_J$	
A(2i-1,5+2j)	$= -2m_J$	
A(2i,1)	$= -P\sin Z\sin B$	or = 0 if a_1 fixed
A(2i,2)	$= -P\sin Z\cos B$	or = 0 if a_2 fixed
A(2i,3)	$= -P\cos Z$	or = 0 if a_3 fixed
A(2i,4)	$= P\{\cos Z\sin B(x_i - a_1) + \cos Z\cos B(y_i - a_2) - \sin Z(z_i - a_3)\}$	
A(2i,5)	$= P\{\sin Z\cos B(x_i - a_1) - \sin Z\sin B(y_i - a_2)\}$	
A(2i,4+2j)	$= 0$	
A(2i,5+2j)	$= -2m_J$	

where $P = 2\{\sin Z\sin B(x_i - a_1) + \sin Z\cos B(y_i - a_2) + \cos Z(z_i - a_3)\}$

All other terms in the (2i) and (2i-1) row of A are zero. The size of the A matrix is number of points by number of parameters (= 5 + 2 * number of targets).

B(2i-1,3i-2)	$= 2(x_i - a_1) - P\sin Z\sin B$
B(2i-1,3i-1)	$= 2(y_i - a_2) - P\sin Z\cos B$
B(2i-1,3i)	$= 2(z_i - a_3) - P\cos Z$

$B(2i,3i-2) = P\sin Z \sin B$
$B(2i,3i-1) = P\sin Z \cos B$
$B(2i,3i) = P\cos Z$

All other terms in these rows of B equal zero. The size of the B matrix is number of points by three times the number of points.

In the first iteration calculate the correction term by inserting the observed coordinates and the approximate values of the parameters in the model equation F above. That is:

$$f_{2i-1} = (x_i-a_1)^2+(y_i-a_2)^2+(z_i-a_3)^2 - \{P/2\}^2 - r_J^2$$

$$f_{2i} = \{P/2\}^2 - m_J^2$$

In subsequent iterations there are additional terms similar to those discussed previously.

Further research is needed to answer the following questions. Should the constraint equation be implemented? Are there better ways to implement constraints? Should we implement the constraint that the distance between any two targets rotating about the same axis is constant?

6. Intersection of Axes

After the position and orientation of each axis has been determined it is possible to calculate the coordinates of the reference point. Many basic algebra text books give the equation for the shortest distance between two lines. This is useful to calculate the axis offset distance (D in Figure 1). However many texts do not give the equation for the coordinates of the points on the ends of the shortest line which connects the two axis centrelines. The reference point is at the end of the shortest connection that is on the fixed axis.

If one line (axis) passes through **a** parallel to **v** and the other axis passes through **b** parallel to **u** then the 3D coordinates of the reference point on the first line (given by **a** and **v**) is **c** where:

$$\mathbf{c} = \mathbf{a} + \quad ((\mathbf{b}-\mathbf{a}) \bullet ((\mathbf{v}\bullet\mathbf{u})\mathbf{u} - \mathbf{v})/(\mathbf{v}\bullet\mathbf{u})^2-1)\ \mathbf{v}$$

where the vectors $\mathbf{a} = (a_1\ a_2\ a_3)$, $\mathbf{b} = (b_1\ b_2\ b_3)$, $\mathbf{c} = (x_C\ y_C\ z_C)$, $\mathbf{v} = (v_1\ v_2\ v_3)$, and $\mathbf{u} = (u_1\ u_2\ u_3)$. The • indicates the dot product of two vectors.

Often the two axes are designed to be mutually perpendicular. When they are perpendicular $v\bullet u = 0$. So if they are perpendicular then:

$$\mathbf{c} = \mathbf{a} + ((\mathbf{b}-\mathbf{a})\bullet\mathbf{v})\ \mathbf{v}$$

Now if $a_1\ a_2\ a_3$ are the coordinates of the pedal point on the first (fixed) axis with cosines $v_1\ v_2\ v_3$, and if $b_1\ b_2\ b_3$ are the coordinates of the pedal point on the second (moving) axis with cosines u_1 u_2 u_3, then the coordinates of the reference point x_C (east), y_C (north), and z_C (height) are:

$$x_C = a_1 + Lv_1 \qquad y_C = a_2 + Lv_2 \qquad z_C = a_3 + Lv_3$$

$$L = \frac{(a_1 - b_1)(v_1 - VDUu_1) + (a_2 - b_2)(v_2 - VDUu_2) + (a_3 - b_3)(v_3 - VDUu_3)}{VDU^2 - 1} \quad \text{where} \quad VDU = v_1u_1 + v_2u_2 + v_3u_3$$

If the zenith angle and bearing of each axis have been determined then the direction cosines are calculated from:

$$\begin{array}{ll} v_1 = \sin Z_1 \sin B_1 & u_1 = \sin Z_2 \sin B_2 \\ v_2 = \sin Z_1 \cos B_1 & u_2 = \sin Z_2 \cos B_2 \\ v_3 = \cos Z_1 & u_3 = \cos Z_2 \end{array}$$

The axis offset, D, that is the shortest distance between the two axes, is:

$$\text{Axis Offset:} \quad D = \frac{(a_1 - b_1)(v_2u_3 - v_3u_2) + (a_2 - b_2)(v_3u_1 - v_1u_3) + (a_3 - b_3)(v_1u_2 - v_2u_1)}{\sqrt{(v_2u_3 - v_3u_2)^2 + (v_3u_1 - v_1u_3)^2 + (v_1u_2 - v_2u_1)^2}}$$

Once the parameters of the axes have been estimated, and their VCV matrix determined the coordinates of the reference point are calculated as above. However the coordinates alone are not enough. The precision of these coordinates should also be calculated. The equations for x_C, y_C and z_C above and the propagation law of variances can be used to find the standard deviations of the coordinates of the reference point and any correlations. Similarly, the precision of the calculated axis offset distance can be found from the propagation of variances.

These calculations require the equations for x_C, y_C and z_C to be differentiated. Some partial derivatives require a lot of calculation and there are many terms in the derivation. The derivatives of x, y, z, and axis offset with respect to a_1, a_2, a_3, b_1, b_2, b_3 are straightforward and can be calculated directly. The derivatives with respect to zenith angle and bearing are more complicated. Since standard deviations, correlations and error ellipse parameters are only required to one or two significant figures, approximate values of the partial derivatives are suitable. It is quite efficient, in terms of computer computation, to calculate $\frac{\Delta x}{\Delta ZA}$ as an approximation to $\frac{\partial x}{\partial ZA}$. Use $\frac{\Delta x}{\Delta ZA} = \frac{(x'-x)}{sZA}$ where s_{ZA} is the standard deviation of ZA and x' is the value of the x coordinate of the reference point calculated from ZA + s_{ZA} instead of ZA. In a computer program x' can be simply calculated by a call to a function or subroutine in the same way x is calculated.

7. Summary

There are several problems when measuring the connection between survey control marks and VLBI telescopes. These problems include non intersecting axes, axes not correctly aligned (eg not vertical), no physical mark at the reference point, sometimes no mechanical axis for physical determination of centreline, large structures and obstructions to lines of sight, steep lines of sight, and reciprocal observations to remove refraction are not possible. A survey method has been proposed to overcome most of these problems. The proposed method involves observing to targets which are rotated about the telescope axes (by pointing the VLBI telescope in different directions).

The equations and some matrices for the least squares adjustment to solve for the orientation of the axes and position of the reference point have been presented. The proposed method yields the precision of the results as well as the coordinates and does not assume the telescope has been perfectly constructed.

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