

An Analytical Solution for Parameter Selection for Polishing Spherical Surfaces

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Abstract. This paper establishes an analytical solution for describing the trajectories of abrasives in polishing spherical surfaces when the motions of the tool and workpiece involve three independent angular velocities. As an example of its applications, the solution is used to show the effect of angular velocities on the trajectory coverage uniformity on a spherical surface – a key aspect for the control of a uniform material removal in a polishing operation. This solution provides a theoretical tool for the parameter selection of such machining processes.

Introduction

Abrasive polishing is a central manufacturing process for achieving precise and smooth finish of complex surfaces, such as planar, spherical, aspherical and free-form surfaces. However, the control of the material removal in polishing these complex surfaces is complicated. For example, the quality of a polished surface of a workpiece is largely affected by the interaction conditions between the workpiece and abrasive particles, including the properties of the materials involved, the load applied, the temperature variation, and in some cases, the chemical reactions at the work-tool interfaces.

In the polishing of a surface, a key aspect is to realise a uniform material removal over the whole area of interest, so that the geometrical accuracy of the surface generated by a precision shaping process before polishing, such as forming, cutting and grinding, can be precisely maintained. To this end, one should at least be able to understand and control the interaction frequency between an abrasive and a workpiece surface. The first step is therefore to describe the trajectories of motion of an abrasive on a surface to polish.

Sun and Zhang et al [1] used the finite element method to analyze the contact status of the polishing of precision optical lenses, including contact pressure distributions and polishing trajectories. They also carried out experimental investigations and dimensional analysis [2], and obtained a simple empirical formula to evaluate the material removal in the polishing of optical spherical surfaces of hydrophilic polymer materials. However, a numerical analysis can hardly provide an exact diagram for describing the continuous variations of abrasive trajectories with the change of kinetic parameters of a polishing process; whereas the applicability of an experimental model is often limited by the range of its testing conditions.

To overcome these difficulties, this paper will develop an analytical solution for describing the trajectories of abrasives in polishing a spherical surface and demonstrate the effect of angular velocities on the trajectory uniformity.

Solution of the Spatial Polishing Trace

Model Description. A schematic representation of the polishing process of a spherical surface, consisting of two bodies in contact, is shown in Fig. 1, where Body A, representing a workpiece (or a tool), has a spherical surface of radius R , revolving about a fixed axis at an angular velocity, ω_1 , and Body B, representing a tool (or a workpiece), has a hemispherical surface of the same radius of Body

A, revolving about its symmetric axis at an angular velocity, ω_2 , and simultaneously, making a pendulum motion about the centre of Body A at an angular velocity, ω_3 . Since the surfaces of Bodies A and B have the same radius (R) as defined above, these two surfaces are always in contact sliding during the motions of the two bodies.

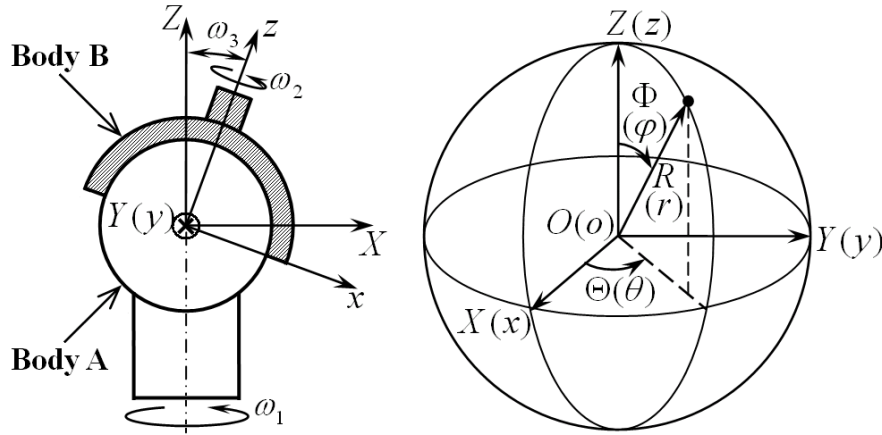


Fig. 1 An illustration of the polishing process and the corresponding coordinate systems.

Two right-handed Cartesian coordinate systems, a global coordinate system $OXYZ$ and a moving coordinate system $oxyz$, are introduced to describe the motions of the two bodies as illustrated in Fig.1. The two coordinate systems share the same origin located at the center of the spherical surface and the Y axis is perpendicular to the paper. For the moving coordinate system $oxyz$, y -axis is coincided with the pendulum motion axis of Body B and z -axis overlaps with the axis of rotation (ω_2) of Body B. The Z -axis of the global coordinate system $OXYZ$ is coincided with the revolving axis of Body A. Without losing the generality, $oxyz$ is assumed to be coincided with $OXYZ$ at the initial time $t = t_0$. The spherical coordinates (R, Θ, Φ) in $OXYZ$ and (r, θ, φ) in $oxyz$ are shown in the right part of Fig. 1.

For simplicity, the angular velocities $\omega_1, \omega_2, \omega_3$ are assumed to be independent of time, i.e., they are constants during a polishing. The pendulum motion of Body B in contact sliding with the surface of Body A has a maximal swing angle of 2α . Hence the cycle period of the swing motion is $T = 4\alpha / \omega_3$.

Analytical Solution. Consider two overlapping points at the initial time $t = t_0$, A and B , where A is fixed on the surface of Body A with coordinate $(R, \Theta_0^A, \Phi_0^A)$ in $OXYZ$ and B is fixed on the surface of Body B with coordinate $(r, \theta_0^B, \varphi_0^B)$ in $oxyz$. As described before, Bodies A and B have the same radius so that $r = R$. In the above notations, superscript A denotes Point A and superscript B denotes Point B . Since $OXYZ$ and $oxyz$ are coincident at $t = t_0$, $(R, \Theta_0^B, \Phi_0^B) = (R, \theta_0^B, \varphi_0^B)$.

At time t , Point A moves to its new position $(R, \Theta_t^A, \Phi_t^A)$ in $OXYZ$ where

$$\Theta_t^A = \Theta_0^A + \omega_1 t, \Phi_t^A = \Phi_0^A, \quad (1)$$

while Point B moves to its new position $(R, \theta_t^B, \varphi_t^B)$ in $oxyz$ where

$$\theta_t^B = \theta_0^B + \omega_2 t, \varphi_t^B = \varphi_0^B. \quad (2)$$

Here, for simplicity, let the initial time $t_0 = 0$ and subscript t denote a quantity at time instant t .

On the other hand, the Cartesian coordinate components of Point B at instant t in $oxyz$ are

$$\begin{aligned} x_t^B &= R \cos(\theta_0^B + \omega_2 t) \sin \varphi_0^B \\ y_t^B &= R \sin(\theta_0^B + \omega_2 t) \sin \varphi_0^B \\ z_t^B &= R \cos \varphi_0^B \end{aligned} \quad (3)$$

Therefore, the Cartesian coordinate components of Point B in $OXYZ$ are

$$\begin{aligned} X_t^B &= x_t^B \cos \Omega_t - z_t^B \sin \Omega_t \\ Y_t^B &= y_t^B \\ Z_t^B &= x_t^B \sin \Omega_t + z_t^B \cos \Omega_t \end{aligned} \quad (4)$$

where Ω_t is the angle from Z -axis to z -axis,

$$\Omega_t = \begin{cases} \omega_3(t - nT) & nT - T/4 \leq t < nT + T/4 \\ -\omega_3(t - nT - T/2) & nT + T/4 \leq t < nT + 3T/4 \end{cases} \quad (5)$$

These coordinate components can be transformed into the spherical components in $OXYZ$ as

$$\begin{aligned} R_t^B &= R \\ \Theta_t^B &= \sin^{-1} \left(Y_t^B / \sqrt{(X_t^B)^2 + (Y_t^B)^2} \right) \\ \Phi_t^B &= \cos^{-1} (Z_t^B / R) \end{aligned} \quad (6)$$

Therefore, Point B in the surface of Body B will be in contact with Point A in the surface of Body A at time t only if they have the same coordinate in $OXYZ$ at time t

$$\begin{aligned} \Theta_t^A &= \Theta_0^A + \omega_1 t = \Theta_t^B \\ \Phi_t^A &= \Phi_0^A = \Phi_t^B \end{aligned} \quad (7)$$

which means that

$$\begin{aligned} \Theta_0^A &= \Theta_t^B - \omega_1 t \\ \Phi_0^A &= \Phi_t^B \end{aligned} \quad (8)$$

Therefore, for a given Point B $(R, \Theta_0^B, \Phi_0^B)$, its spherical coordinate $(R, \Theta_t^B, \Phi_t^B)$ in $OXYZ$ at any time t is determined by Eq. (6), and its initial coordinate $(R, \Theta_0^A, \Phi_0^A)$ in the surface of Body A is determined by Eq. (8). Hence, the trajectory of Point B in the surface of Body A is defined.

Similarly, for a given Point A $(R, \Theta_0^A, \Phi_0^A)$ in the surface of Body A, its Cartesian coordinates in $OXYZ$ at time t are

$$\begin{aligned} X_t^A &= R \cos(\Theta_0^A + \omega_1 t) \sin \Phi_0^A \\ Y_t^A &= R \sin(\Theta_0^A + \omega_1 t) \sin \Phi_0^A \\ Z_t^A &= R \cos \Phi_0^A \end{aligned} \quad (9)$$

They can be transformed into the Cartesian coordinate in $oxyz$ as

$$\begin{aligned} x_t^A &= X_t^A \cos \Omega_t + Z_t^A \sin \Omega_t \\ y_t^A &= Y_t^A \\ z_t^A &= -X_t^A \sin \Omega_t + Z_t^A \cos \Omega_t \end{aligned} \quad (10)$$

The corresponding spherical coordinate components in $oxyz$ are

$$\begin{aligned} r_t^A &= R \\ \theta_t^A &= \sin^{-1} \left(y_t^A / \sqrt{(x_t^A)^2 + (y_t^A)^2} \right) \\ \varphi_t^A &= \cos^{-1} (z_t^A / R) \end{aligned} \quad (11)$$

Hence at time t , the initial coordinates of Point B , $(R, \Theta_0^B, \Phi_0^B)$, which is in contact with Point A can be written as

$$\begin{aligned} \theta_0^B &= \theta_t^A - \omega_2 t \\ \varphi_0^B &= \varphi_t^A \end{aligned} \quad (12)$$

which determines the trajectory of the given Point A in the surface of Body B at any time t .

As clarified before, the above derivation deals with constant angular velocities, $\omega_1, \omega_2, \omega_3$, during polishing.

Verification. To verify the above analytical solution, a commercially available finite element code, ANSYS [3], is employed to simulate the above mentioned polishing process. Bodies A and B are all modeled as rigid ones with the same boundary and initial conditions as defined in “Model Description” Section. The angular velocities are $\omega_1 = 1200 \text{rpm}$, $\omega_2 = 600 \text{rpm}$ and $\omega_3 = 50 \text{rpm}$, respectively. The maximum half-swing angle of Body B is $\alpha = 20^\circ$. The initial spherical coordinate of the point to simulate, i.e., Point A in the surface of Body A in $OXYZ$, is $(R, \Theta_0^A, \Phi_0^A) = (R, 90, 45)$. The polishing trajectories in the surface of Body B generated by Point A obtained by both the analytical solution and the FEM are plotted in Fig. 2, as projection in the polar coordinate plane. The total simulation time is one second. It is clear that the analytical solution is very close to FEM simulation result, showing that our analytical solution is correct.

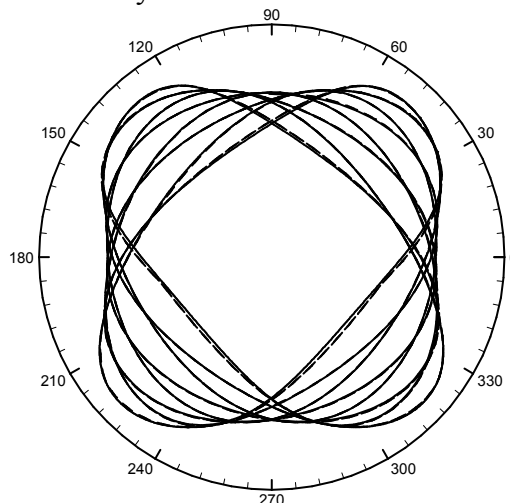


Fig. 2 Polishing trajectories on a spherical surface, where the solid line is the result from FEM and the dash line is from the analytical solution.

Results and Discussion

The analytical solution established above can be used to easily predict the effect of kinetic parameters, $\omega_1, \omega_2, \omega_3$, on the polishing quality of a spherical surface. As described previously, it is favorable that a combination of $\omega_1, \omega_2, \omega_3$ could produce an even coverage of trajectories of abrasive motions in a polishing process. For simplicity, assume that there are four abrasives fixed on four given points in the surface of Body B, i.e., $a(\varphi_0 = 70^\circ), b(\varphi_0 = 50^\circ), c(\varphi_0 = 30^\circ), d(\varphi_0 = 10^\circ)$. Their trajectories on the surface of Body A will show their polishing uniformity in an interval of polishing process. As a simple example to understand the effect of kinetic parameter combinations, let us focus on the variation of ω_1 in this discussion while keeping $\omega_2 = 1200 \text{ rpm}$ and $\omega_3 = 50 \text{ rpm}$ unchanged. In the calculations, the maximum half-swing angle is taken as $\alpha = 20^\circ$ and the total time of polishing is 5 seconds. To visualize the trajectories, they are projected onto plane XY as shown in Fig. 1.

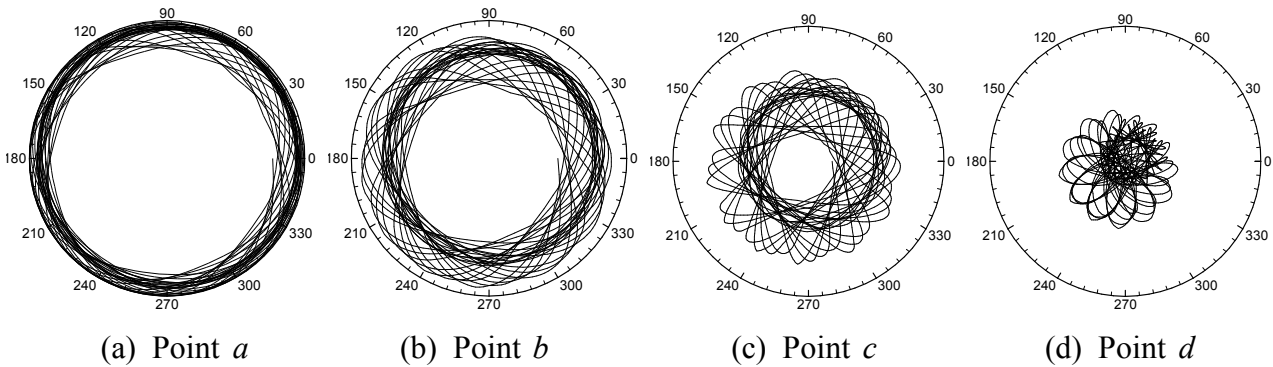


Fig. 3 Trajectories generated by individual abrasives at $\omega_1 = 340 \text{ rpm}$.

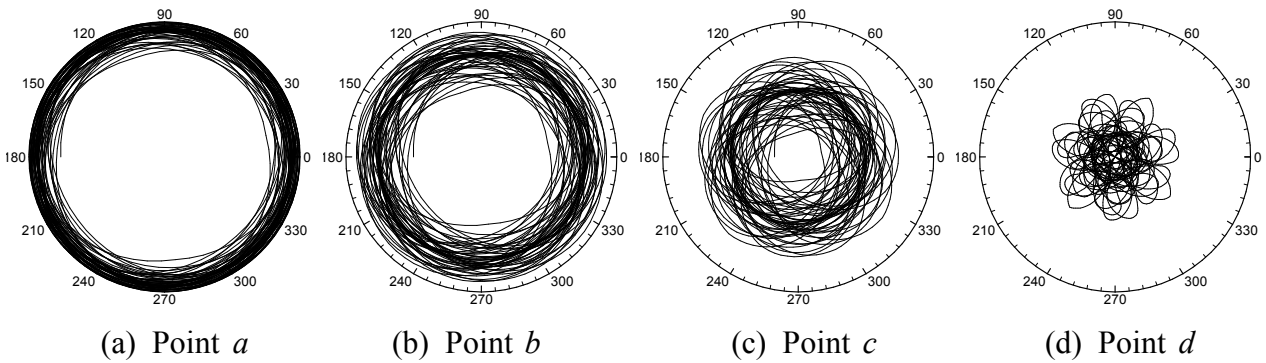


Fig. 4 Trajectories generated by individual abrasives at $\omega_1 = 170 \text{ rpm}$.

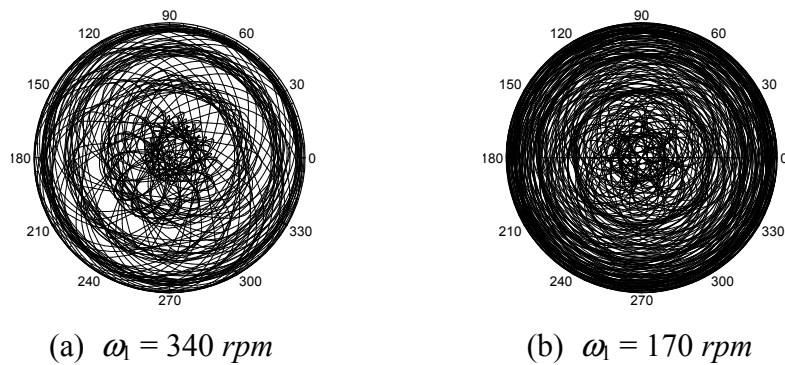


Fig. 5 Combined trajectories of the four abrasives.

Fig. 3 shows the trajectories of individual abrasives when $\omega_1 = 340 \text{ rpm}$ and Fig. 4 demonstrates those when $\omega_1 = 170 \text{ rpm}$. We can see clearly that only a variation of a single angular velocity, ω_1 ,

changes the abrasive trajectories significantly. The decrease of ω_1 from 340 *rpm* (Fig. 3) to 170 *rpm* (Fig. 4) makes the trajectory coverage area on the spherical surface much wider and more uniform. As a result, when the trajectories generated by four abrasives are combined together, the process with $\omega_1 = 340$ *rpm* (Fig. 5a) will leave many areas unpolished (the white areas), while that with $\omega_1 = 170$ *rpm* (Fig. 5b) will make a much more uniform polishing.

Conclusions

This paper has developed an analytical solution for describing the trajectories of individual abrasives at given locations on a tool surface when polishing spherical surfaces. The results show that carefully selected rotation speed can make the trajectories more evenly distributed over the whole polishing area. This analytical solution provides a theoretical tool for selecting polishing parameters.

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