Multilateral Index Number Methods for Consumer Price Statistics

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EMG Workshop, 25 November 2022

Contributions of Jan de Haan to the EMG Workshop

Jan is unable to present this year due to illness. Previous presentations:

- 2005: The Re-design of the Dutch CPI
- 2006: A Matched-Model Geometric Mean Index for Supermarket Scanner Data
- 2007: Hedonic Price Indexes: A comparison of imputation, time dummy and other approaches
- 2008: Reducing Drift in Chained Superlative Price Indexes for Highly Disaggregated Data
- 2009: Does the Ivancic-Diewert-Fox Method Work for Seasonal Goods?
- 2010: An Appraisal-Based Generalized Regression Estimator of House Price Change
- 2011: Decomposing Changes in Aggregate Loan-to-Value Ratios

Contributions of Jan de Haan to the EMG Workshop

- 2012: Scanner Data and the Treatment of Quality Change in Rolling Year GEKS Price Indexes
- 2013: Online Data, Fixed Effects and the Construction of High-Frequency Price Indexes
- 2014: Accounting for Spatial Variation of Land Prices in Hedonic Imputation House Price Indexes
- 2017: Towards Resolving Some New and Old Problems in Price Measurement
- 2018: Scanner Data in the CPI: The imputation CCDI index revisited
- 2019: Pondering over the CES Price Index
- 2021: House Price Indexes: A Comparison of Repeat Sales and Other Multilateral Methods

Background

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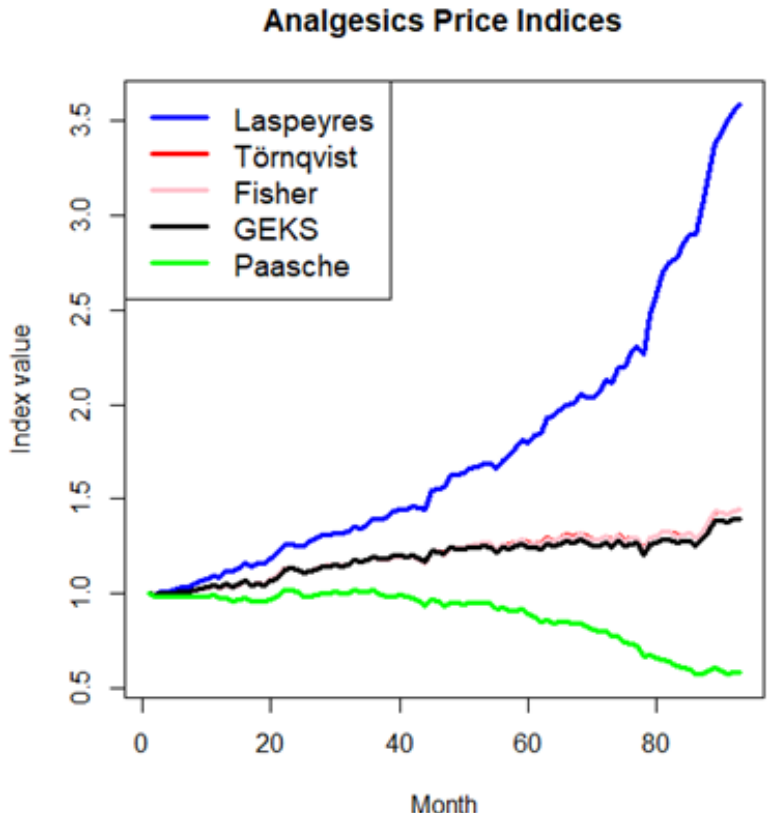
- The report also benefited from discussions and collaborations with Erwin Diewert, Jan de Haan and Shipei Zeng over the years

Why Multilateral Indexes?

- Chaining indexes is desirable because we want accurate measures of inflation between consecutive periods and we have product churn – don't want the basket to get out of date.
- But chaining with transaction level data can lead to massive chain drift. Example using Dominick's supermarket data:



Why Multilateral Indexes?

- Multilateral indexes were introduced to this context for the purpose of controlling chain drift.
- The GEKS index in the previous figure is a multilateral method (GEKS-Fisher) calculated over the full sample, which by definition does not suffer from chain drift bias; it satisfies the (weak) multiperiod identity test.
- Deviations from this index (the black line) then indicate chain drift through deviating from a chain-drift-free index. Note how the chain drift bias is especially dramatic for indexes commonly used by NSIs, the Laspeyres and Paasche indices.

Why Multilateral Indexes?

- With non-reviseable CPIs, need a method for extending the series – simply expanding the window for multilateral indexes can result in a re-writing of history.
- But extending the series can mean that the resulting series suffers from some chain drift, but not the kind of "explosive" chain drift that standard index numbers can exhibit with transactions level data.

Where Did the Idea Come From?

Ivancic, L. W.E. Diewert and K.J. Fox (2011), "Scanner Data, Time Aggregation and the Construction of Price Indexes," *Journal of Econometrics* 161, 24–35.
(Published version of a 2009 Discussion Paper)

Introduced:

- the idea of multilateral indexes to control chain drift,
- using GEKS and weighted Time Product Dummy multilateral methods,
- using splicing of Rolling Windows for extending.

Where Did the Idea Come From?

Research and workshops supported by Australian Research Council Grants based at UNSW Sydney, in collaboration with the Australian Bureau of Statistics and Statistics Netherlands:

- 2003-2005 Australian Research Council Linkage Grant, for "Can Electronic Point-of-Sale (POS) Data Improve the Australian Consumer Price Index," K.J. Fox and R.J. Hill. Industry Partner: Australian Bureau of Statistics

- 2006-2008 Australian Research Council Linkage Grant, for "Scanner Data in the Consumer Price Index: How to expand and improve their use," K.J. Fox, J. de Haan, P-H. van Mulligen and M. Silver. Industry Partners: Australian Bureau of Statistics and the Central Bureau of Statistics (Netherlands)

An example of a positive outcome from NSI engagement with academics.

Now Implemented in CPIs of Multiple Countries

- Australia
- New Zealand
- Netherlands
- Norway
- Belgium
- Luxembourg
- Other countries soon!

On-going research into performance of alternative:

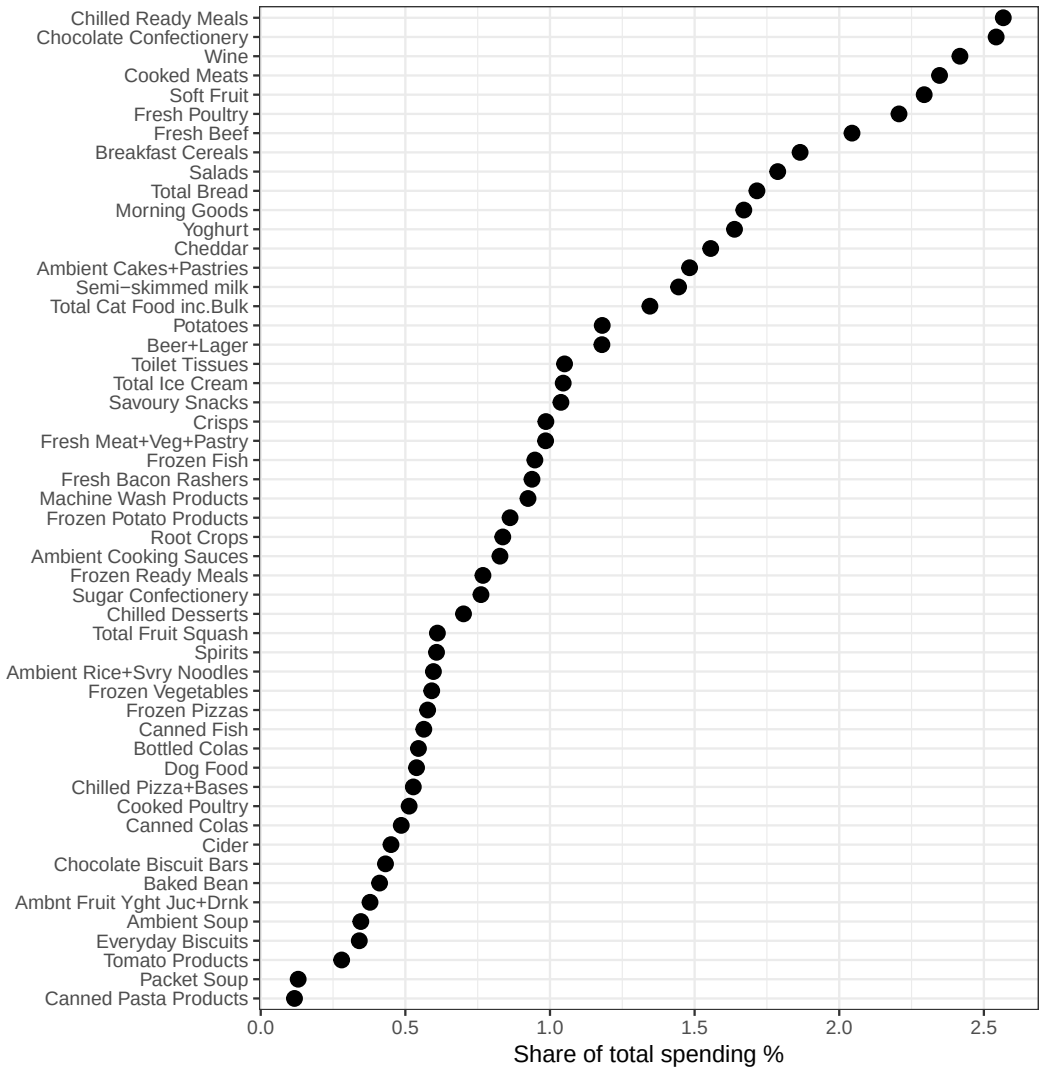
- multilateral indexes,
- splicing techniques,
- window lengths.

Our report

- Discuss and evaluate the properties of different multilateral index numbers
- Use Kantar data (over 2013-2019) and IndexNumR package in R to empirically quantify the difference between different price indices across ***many*** product categories
- Assess
  - index number methods
  - splicing techniques
  - window lengths
- Also illustrate some empirical determinants of chain drift with UK data

Data from 30,000 Households, 2013-2019

Figure: Share of total spending on product groups



Introduction

- Availability of ***scanner data*** potentially allows for the use of many millions of observations on prices and quantities to be used in price index construction.
- But how to calculate price indices with these data?
  - ***Fixed base*** indices quickly become unrepresentative due to product churn
  - Chaining of bilateral indices (Lowe, Laspeyres, Fisher, Törnqvist etc.) can lead to "explosive" indexes, or ***chain drift***
- Solution is ***multilateral*** indexes that satisfy the (weak) multiperiod identity axiom; if prices and quantities in period 4 are identical to those in period 1, then the following should hold when the indexes are chained:

$$P^{1,2}\,P^{2,3}\,P^{3,1} = 1$$

Multilateral indices

Various options - GEKS

$$\mathbb{P}^{\tau}_{GEKS-F} = \prod_{t} \left[P^{\tau,t}_{F}\right]^{1/T} \quad \text{where } P^{\tau,t}_{F} \text{ is a Fisher index}$$

$$\mathbb{P}^{\tau}_{CCDI} = \prod_{t} \left[P^{\tau,t}_{Tq}\right]^{1/T} \quad \text{where } P^{\tau,t}_{Tq} \text{ is a Törnqvist index}$$

$$\mathbb{P}^{\tau}_{GEKS-W} = \prod_{t} \left[P^{\tau,t}_{W}\right]^{1/T} \quad \text{where } P^{\tau,t}_{W} \text{ is a Walsh index}$$

Or Geary-Khamis

$$b_n = \sum_{t} \left(\frac{q^t_n}{q_n}\right)\left(\frac{p^t_n}{\mathbb{P}^t_{GK}}\right) \quad \text{for } n = 1, \ldots, N$$

$$\mathbb{P}^t_{GK} = \frac{\mathbf{p}^{t'}\mathbf{q}^t}{\mathbf{b}'\mathbf{q}^t} \quad \text{for } t = 1, \ldots, T.$$

The "linking problem"

- When new months are included in the index, past price changes will need to be revised
- One solution is *rolling window* to link indices in different windows $1, \ldots, T$ and $2, \ldots, T+1$ through a link period
  - 'roll forward' index $\mathbb{P}$ by one period to get $\widetilde{\mathbb{P}}$
  - use an overlapping period, s, to extend the index

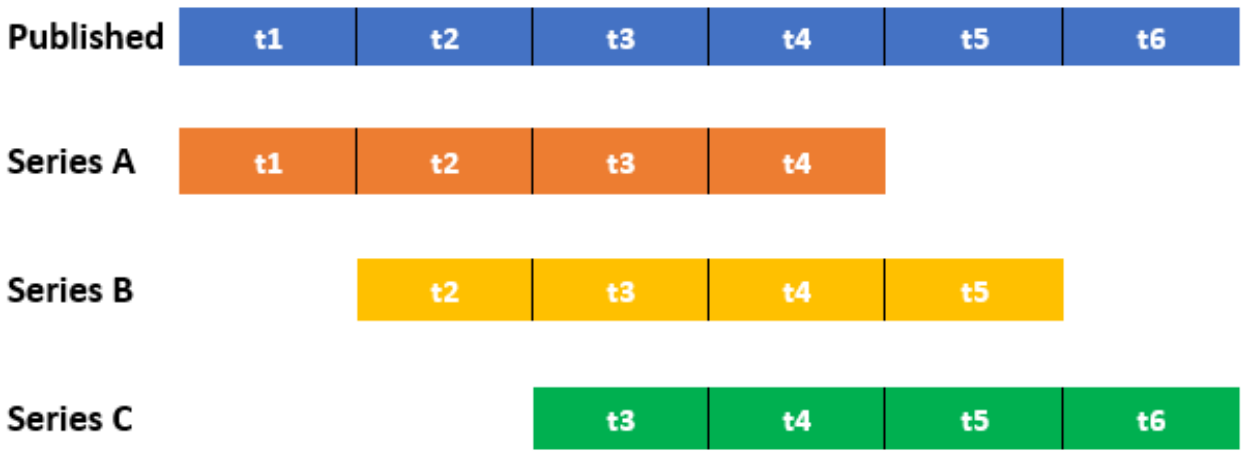
$$\rho^{T+1}(s) = \frac{\mathbb{P}_s}{\mathbb{P}_1} \frac{\widetilde{\mathbb{P}}^{T+1}}{\widetilde{\mathbb{P}}^s},$$

Splicing methods

- Many possible choices of s
  - **Movement splice**: $s = T$ (Ivancic, Diewert and Fox 2011)
  - **Window splice**: $s = 2$ (Krisnich 2016, JOS)
  - **Half splice**: $s = \frac{T}{2}$ (or, when T is an odd number $s = \frac{T+1}{2}$) (de Haan 2015)
  - **Mean splice**: geometric mean over each choice $s = 2, \ldots, T$ so $\rho^{T+1}(\text{mean}) = \prod_{s=2}^{T} \left(\rho^{T+1}(s)\right)^{1/(T-1)}$ (Diewert and Fox 2020, JBES)
- Fixed Base Expanding Window: chain through a base month (e.g. Dec). Window of months in each new period, which includes base month, is expanded as year progresses.

Splicing on the published series (Chessa, 2021)

- Same as before but splice on published series not previously calculated window



- Window: $blue_{t6} = (green_{t6}/green_{t3})/(yellow_{t5}/yellow_{t3})blue_{t5}$
- "WISP": $blue_{t6} = blue_{t3} \times (green_{t6}/green_{t3})$
- Year-on-year index of published series same as year-on-year index of new window

Splicing on the published series: Some observations

- Interest is often in monthly price indices (e.g. for monetary policy), not year-on-year.
- Any index (e.g. fixed base Laspeyres) can be used to update the published series and keep the year-on-year growth rate in the published series the same as the new index. Multilateral indexes are not needed.
- Updating using splicing factors from adjacent windows seems more appropriate, as the product coverage is likely to be more similar between adjacent windows.

Problems with splicing

- Splicing allows the index to be updated without altering the series that has been published
- But reintroduces chain drift
- Extent of chain drift for different index numbers, extension methods and window lengths is an ***empirical question***
- We assess this in UK data using household scanner data (52 product categories)

Transitory products

- Many products are seasonal (Easter eggs, Christmas cakes), new products appear etc.
- Multilateral indexes down-weight seasonal products in month-to-month price changes
- One solution is to *impute* prices of goods with zero quantities
- GK and Walsh are *invariant* to imputation. Goods with zero quantity do not contribute to index.
    - Problem if anything other than inflation adjusted carry forward prices used (e.g. hedonic imputation)

ONS framework

- ONS has published a framework to rank indexes
- Weights properties of different indexes alongside other considerations (ease of explanation, computation etc.)
- Slightly favours ***Geary-Khamis with FBME extension method***

Our conclusions

- ONS Framework weights were informed by consultation with stakeholders, but still subjective; small changes lead to re-ranking of methods
- GK + FBME win points for 'ease of understanding' + international use
  - CCDI + mean splice also used internationally
  - CCDI relative ease of understanding is usually regarded as good
- Weight is put on fact GK index is an additive method, meaning that the corresponding quantity index can be easily decomposed.
  - But GK is not a simple weighted sum - 'weights' change with prices
  - Decompositions available for CCDI (Webster and Tarnow-Mordi 2019, JOS)

Our conclusions

- GK is found to be relatively sensitive to extension method chosen and link period (consistent with other findings in the literature, e.g. Lamboray 2017, Van Loon and Roels 2018)
- GK invariant to price imputation, which is problematic for NSIs wanting to e.g. use hedonic imputation methods.
- ***Empirical and theoretical considerations favour CCDI index with mean splice and 25-month rolling window***

Index values relative to CCDI index (all months 2013-2019)

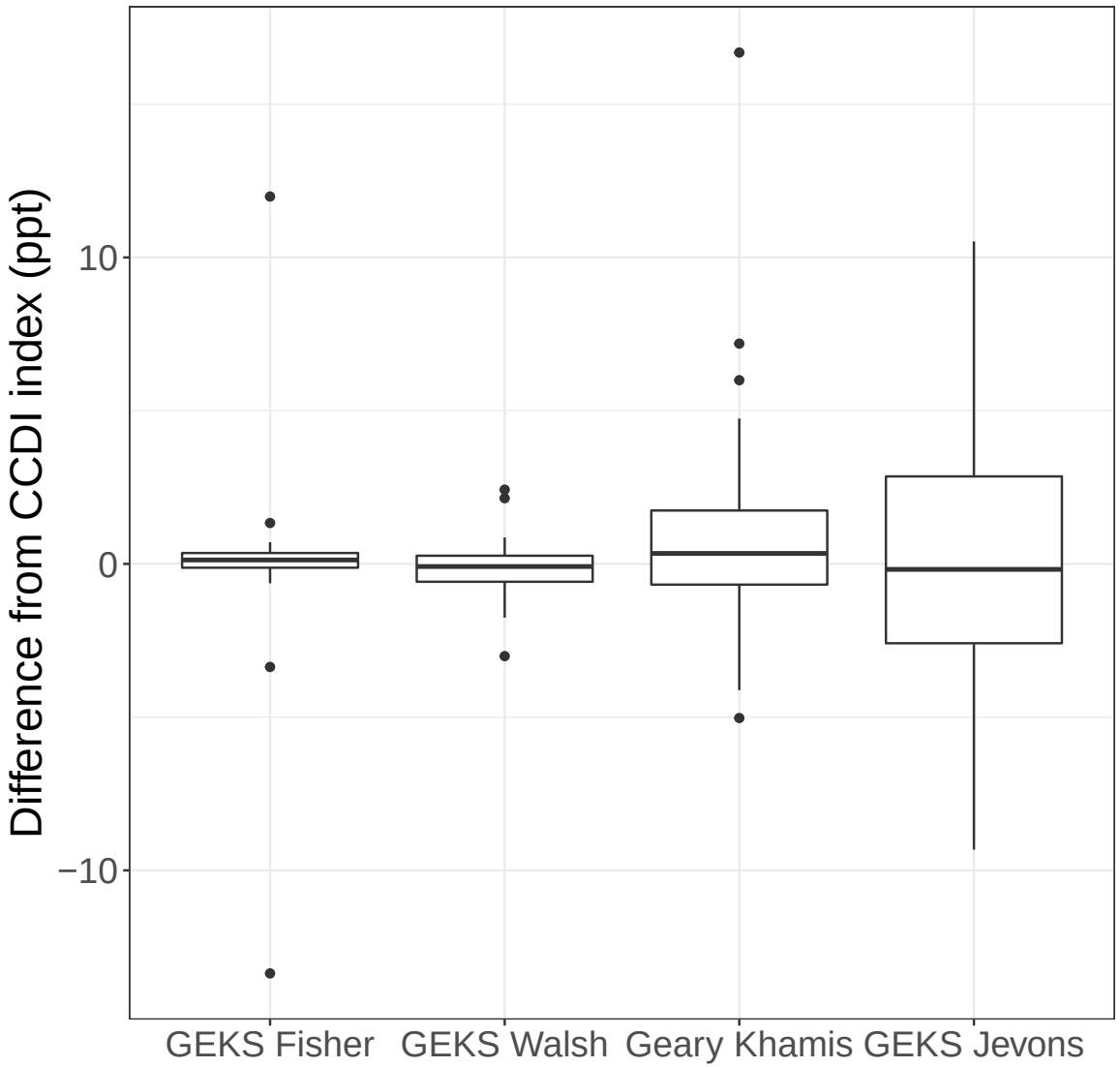
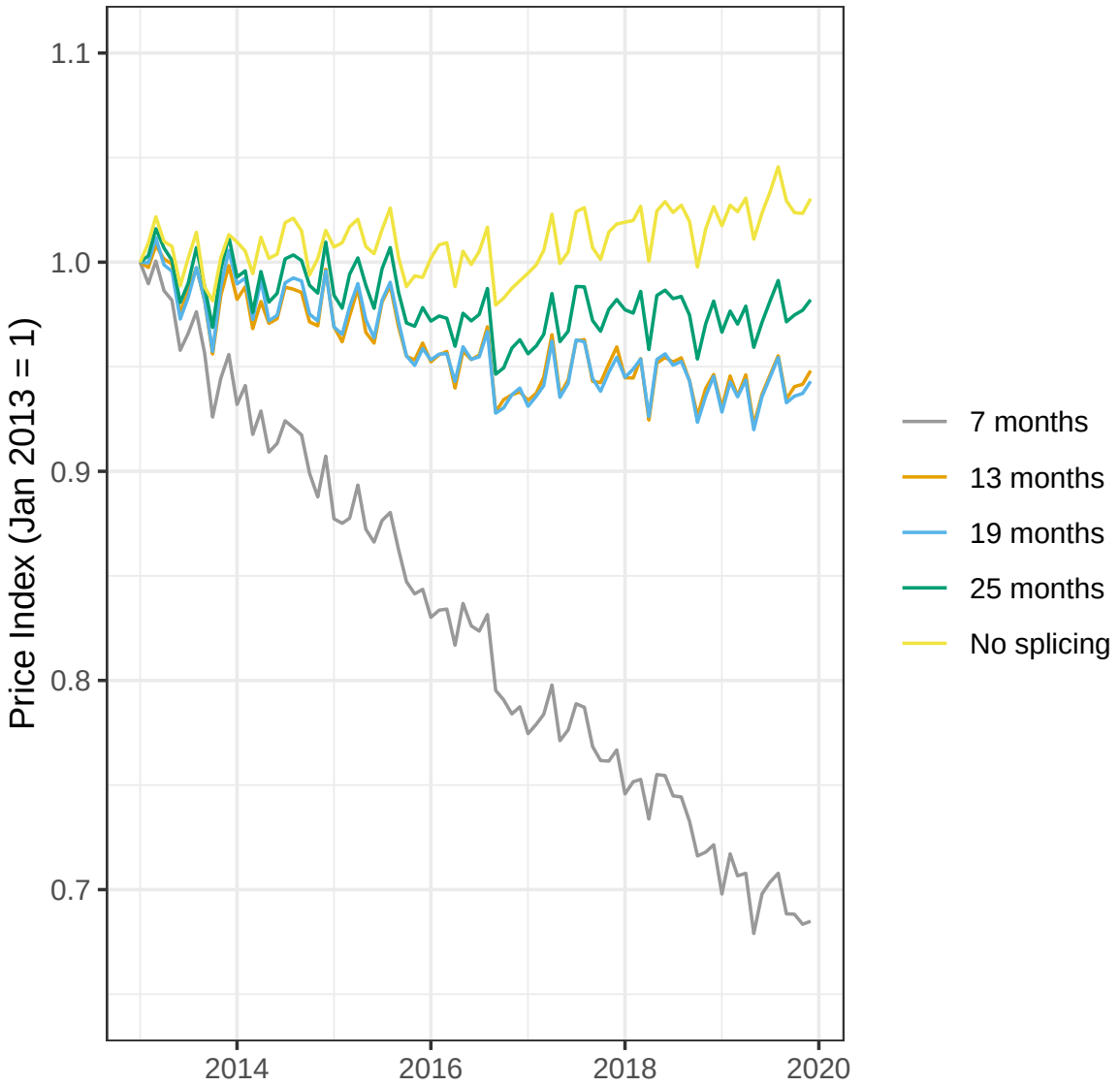
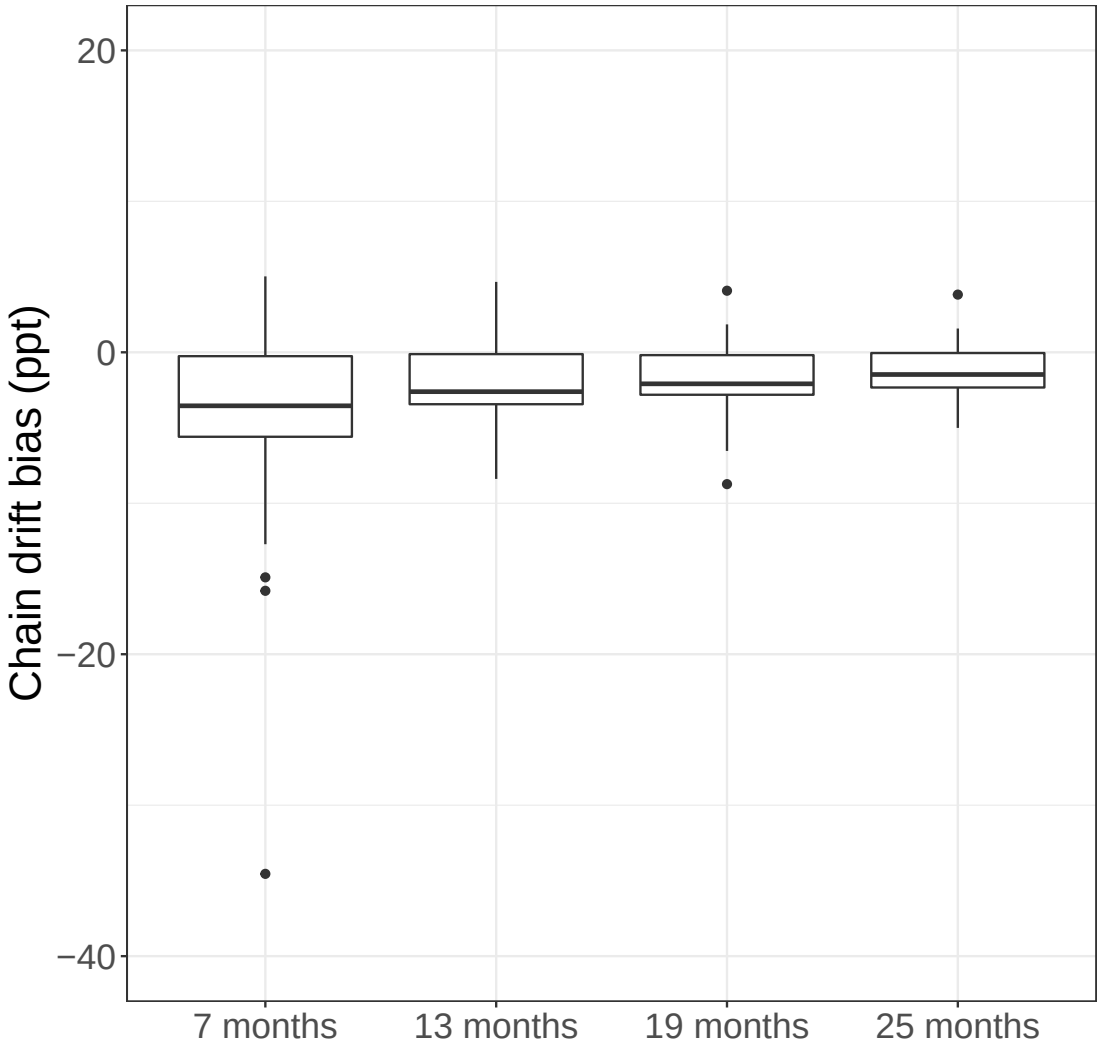


Illustration of chain drift: chocolate CCDI index with mean splice



Chain drift bias using different window lengths (CCDI using mean splice)



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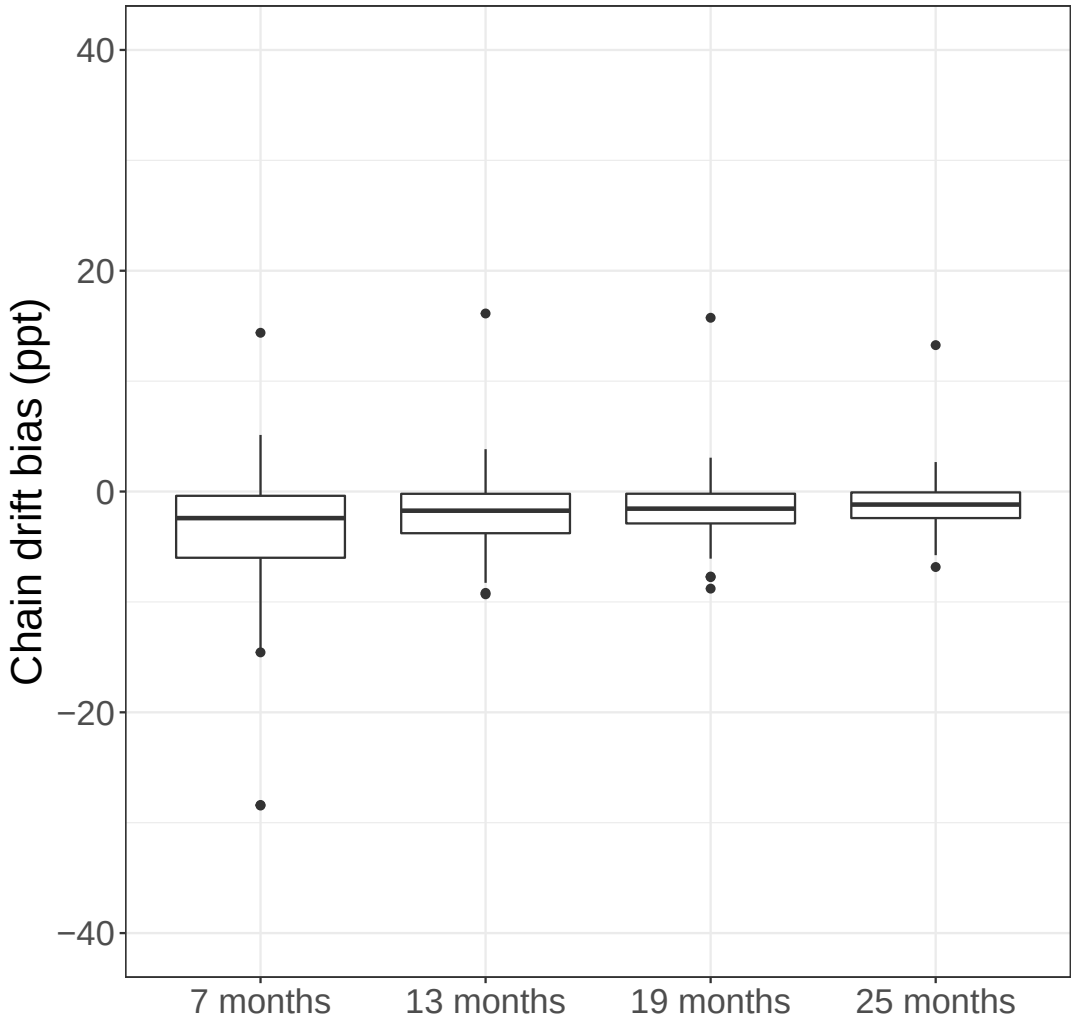
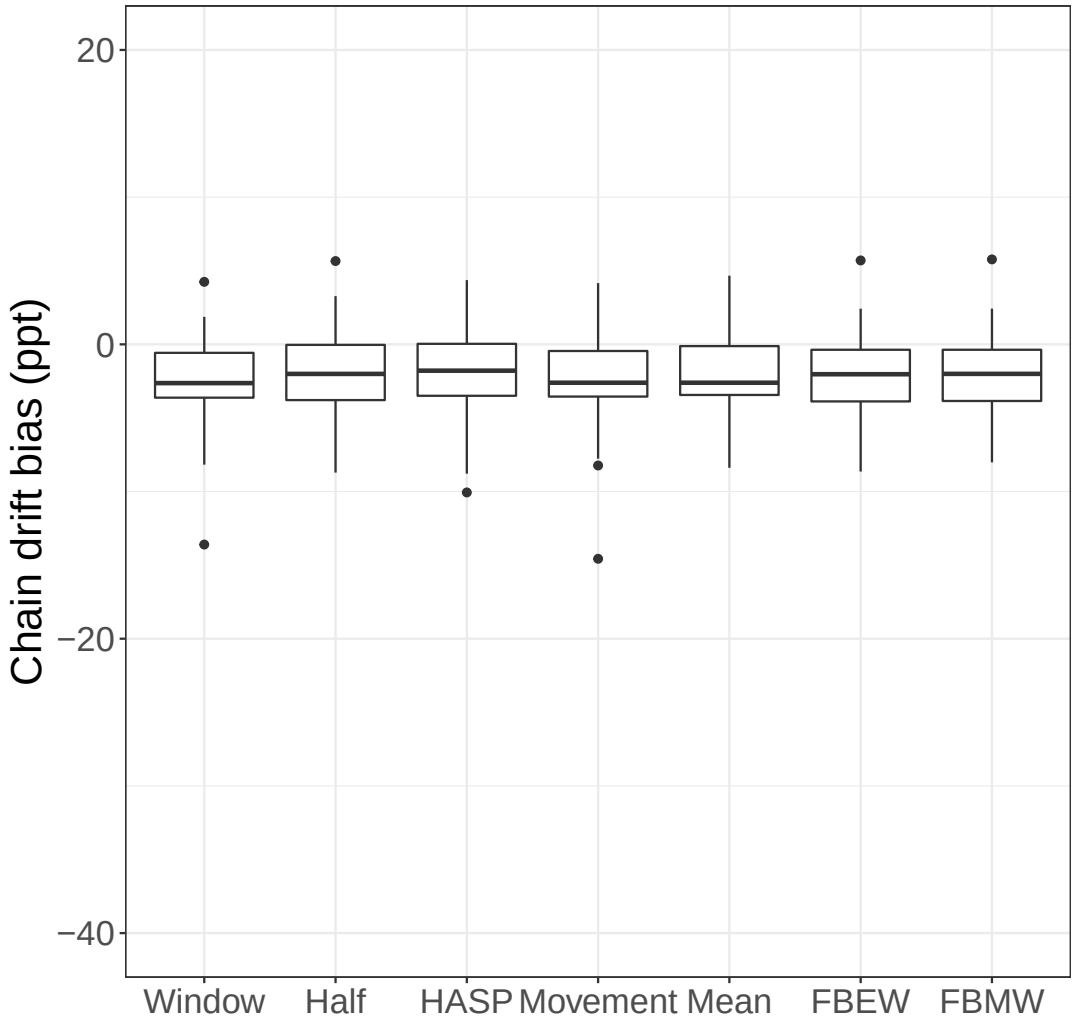


Chain drift bias with different splicing methods (CCDI, 13 month window)



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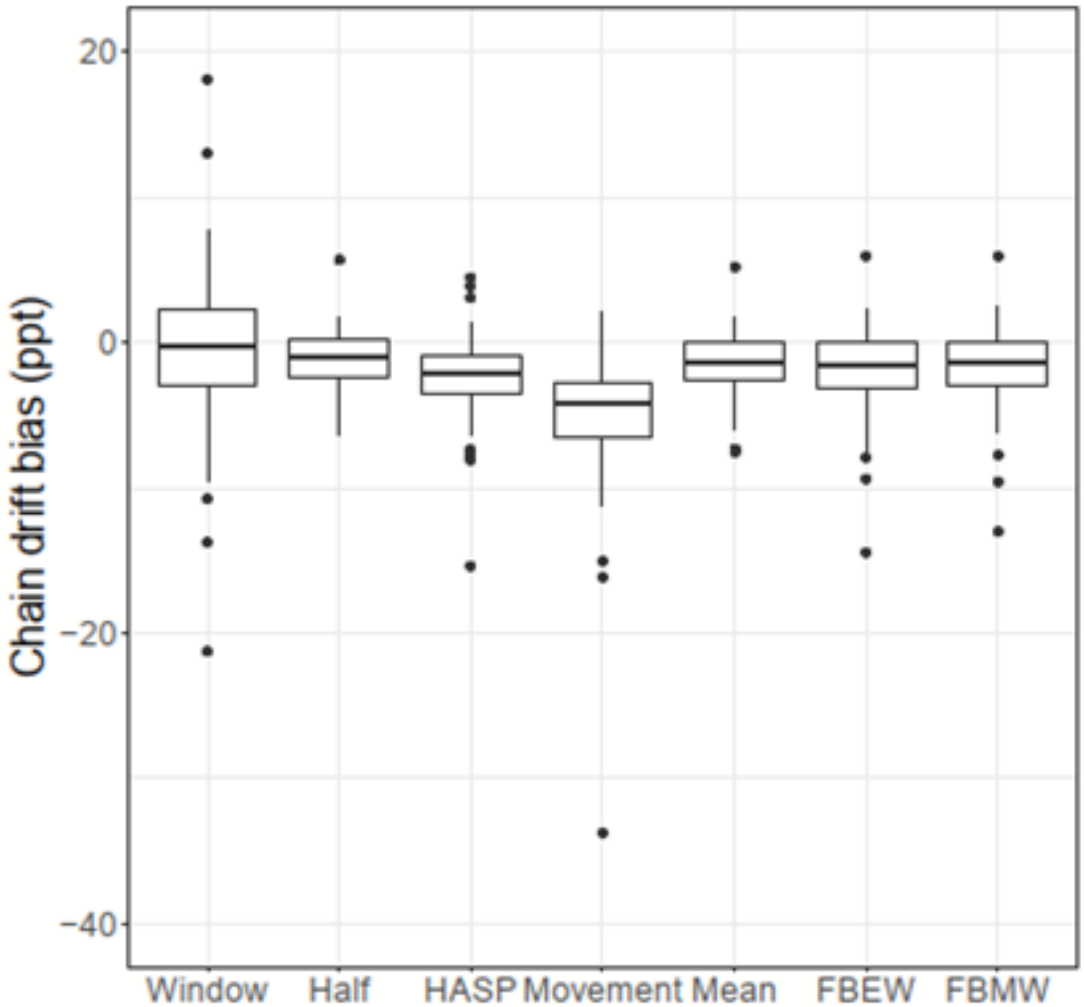

Also examine determinants of chain drift bias

- Products with high seasonality, or monthly and annual churn $\Longrightarrow$ more chain drift
- See also Melser and Webster (2020): identify life-cycle pricing as potential source of chain drift bias
- But these problems mostly solved by long window lengths (25 months or longer)

Summary

- Mean splice compromises over different link periods - will not be unlucky in choice of link period.
  - Empirically, mean splice reduces chain drift
  - Mean splice and the mean splice on the published series give essentially the same results in our applications. This near equivalence is not found for other splicing methods.
- CCDI can be decomposed
- Can use imputation for missing prices with CCDI
- Strong evidence is found in favour of using a 25-month window
- We also provide evidence on the drivers of chain drift and believe this is novel to the literature.

Research Directions

- When does product 'missingness' become a problem?
- Could the choice of splicing method depend on the timing of product entry and exit?
- More empirical examination of "similarity" linking methods using different data sets would aid in the understanding of their performance and their potential for implementation by NSIs.
- In the case of web-scraped data, if prices can be sampled according to their economic importance, is it better to apply a bilateral index to the resulting data or a multilateral index?