School of Mathematics & Statistics University of New South Wales

Honours projects in Pure Mathematics, 2027

Below is a list of some possible honours supervisors, their research areas and suggestions for possible honours projects. The options are far from limited to what is included here so have a look and see who does mathematics that you find interesting and chat to them. For a complete list of supervisors, see

https://www.maths.unsw.edu.au/currentstudents/honours-pure-mathematics

Thomas Britz

- **How to count.**
  Counting might be the first and simplest mathematics that we learn - but it is also some of the most interesting and challenging mathematics. There are several powerful counting methods, such as the Inclusion-Exclusion Principle and Polya Counting, but many counting problems require particular ingenuity and creativity. The art of counting, Enumerative Combinatorics, is beautiful, fun, and challenging.
- **Algebraic Combinatorics.**
  Algebra and combinatorics frequently overlap and interact, and their intersection is Algebraic Combinatorics. This is a deep and wide field of mathematics that uses abstract algebra to address combinatorial problems and which uses combinatorial methods to yield algebraic results.

Tim Buttsworth

Geometric analysis and symmetries.
Many important problems in geometry, topology and physics can be phrased in terms of constructing and studying Riemannian manifolds with special curvature properties. In this project, students will use tools from analysis and algebra to study the geometry of such Riemannian manifolds, and possibly discover new examples.

Daniel Chan

My research interests are in algebra and algebraic geometry, though I have also supervised honours theses in number theory and topology. To get an idea of some of the mathematics I am interested in, you can check my YouTube channel which is aimed at beginning honours students

http://www.youtube.com/c/DanielChanMaths

If anything there is of interest to you, we can find an appropriate honours project in that area. Below is a sample of some honours projects. You may also wish to check my webpage

http://web.maths.unsw.edu.au/~danielch

for other projects and past honours theses.

- **Kleinian singularities and the McKay correspondence.**
  Let G be a finite subgroup of $SL_2(\mathbb{C})$ which acts naturally on $\mathbb{C}^2$. Then the space of G-orbits $\mathbb{C}^2/G$ is a complex surface in $\mathbb{C}^3$ which is not smooth. These Kleinian singularities are interesting objects in mathematics. In this project we look at the McKay correspondence which relates the representation theory of G to the geometry of $\mathbb{C}^2/G$. The theory involves an interesting mix of geometry, group theory and ring theory.

- ***D*-modules.**
  The set of differential operators $\sum p_i(x)\frac{d^i}{dx^i}$ forms a non-commutative ring called the first Weyl algebra A_1. It arises naturally in the theory of differential equations. In this project, we look at higher dimensional analogues A_n and their rather subtle module theory. One highlight is Bernstein's spectacular purely algebraic solution to a question of Gelfand's in complex analysis.

Jie Du

- **Quivers and their representations.**
  A quiver is a network which consists of vertices and oriented edges (or arrows) connecting vertices. A representation of a quiver is a collection of finite dimensional vector spaces indexed by the vertices together with a collection of linear transformations indexed by the arrows. This project investigates the representations of quivers, especially the classification of indecomposable ones. We will also look at some applications to Lie theory such as semisimple complex Lie algebras and quantum groups.

- **Symmetric groups and their representations/affine version.**
  A symmetric group is the permutation group on n letters. It has many fascinating properties in both combinatorics and representation theory. This project will start with an understanding of some basic structure of the symmetric group. This includes the length function, Young subgroups and their shortest coset representatives, and the Robinson-Schensted correspondence. Then we move on looking at the representation theory of the symmetric group or the structure of affine symmetric group, an infinite but tameable version of the symmetric group, and their associated Hecke algebras.

- **The classical and quantum Schur-Weyl duality.**
  For a finite dimensional space V over the complex number field $\mathbb{C}$, there are commutative actions on the r-fold tensor product $V^{\otimes r}$ of V by the general linear group $G = GL(V)$ and the symmetric group S_r on r letters. This induces two algebra homomorphisms

  $$\phi : \mathbb{C}G \to \mathrm{End}(V^{\otimes r}) \quad \text{and} \quad \psi : \mathbb{C}S_r \to \mathrm{End}(V^{\otimes r}).$$

  The classical Schur-Weyl duality shows that

  $$\mathrm{im}(\phi) = \mathrm{End}_{\mathbb{C}S_r}(V^{\otimes r}) \quad \text{and} \quad \mathrm{im}(\psi) = \mathrm{End}_{\mathbb{C}G}(V^{\otimes r}).$$

  If the field $\mathbb{C}$ is replaced by an arbitrary field k, then the duality continues to hold but the proof is much harder. This project will investigate this duality in the quantum case (which is already known) and in the affine case (which is partially known).

Gary Froyland

- **Topics in dynamical systems, ergodic theory, and Riemannian geometry.**
  Ergodic theory is the study of the dynamics of ensembles of points, in contrast to topological dynamics, which focusses on the dynamics of single points. Theoretical projects are available in dynamical systems, ergodic theory, and/or differential geometry, aiming at developing new mathematics to analyse the complex behaviour of nonlinear dynamical systems. Depending on your background, these projects may involve mathematics from ergodic theory, functional analysis, measure theory, Riemannian geometry, stochastic processes, and nonlinear and random dynamical systems.

- **Differential and spectral geometry with applications to fluid mixing.**
  Techniques from differential geometry and spectral geometry (via Laplace-type operators) have recently been shown to be particularly effective for analysing complex dynamics in a variety of theoretical and physical systems. This project will focus on developing and extending powerful techniques to extract important geometric and probabilistic dynamical structures from fluid-like models. If desired, application areas include the ocean (an incompressible fluid) and the atmosphere (a compressible fluid). This project will involve dynamical systems, differential geometry, and PDEs.

- **Operator-theoretic and differential geometric approaches to machine learning.** This project will develop new mathematical and computational approaches to analyse either low- or high-dimensional data. Operator-theoretic methods will be explored, including the use of transfer operators, dynamic Laplace operators, and Laplace-Beltrami operators, which extract dominant dynamic and geometric modes from the data. In a combined theoretical and numerical direction, this project will investigate the construction of these operators from high-dimensional data using dynamic and geometric kernel methods. A possible application is the analysis of global scalar fields obtained from satellite imagery such as sea-surface temperature to extract climate oscillations such as the El Niño Southern Oscillation and the Madden–Julian Oscillation. This project will use ideas from dynamical systems, functional analysis, and Riemannian geometry.

Catherine Greenhill

My research interests are in probabilistic, algorithmic and asymptotic combinatorics. Feel free to talk to me (or email me) about possible Honours topics, or to suggest an idea of your own.

- **Pulling graphs apart.**
  Suppose you take a complete graph on n vertices, with all possible edges present. Is it possible to decompose the edge set into a union of disjoint triangles? Two obvious necessary conditions are that 3 must divide the number of edges, and that the degree of every vertex must be even (so n must be odd). But it turns out that these necessary condition are also sufficient. This is a *graph decomposition* problem, with many generalisations that have been studied by many authors. The proofs typically involve clever combinatorial constructions.

  Now suppose you take a random graph on n vertices such that every vertex has degree d, where d is some constant. There have been a sequence of results showing how to pull such a random regular graph apart into random regular factors of various kinds. These results are known as *contiguity arithmetic*, and shed light on the structure of random regular graphs. The proof involves a method known as *small subgraph conditioning*.

  This Honours topic can be tailored to the student's interest: it could be a study of graph decomposition problems, or of contiguity arithmetic results, or both. In particular, a very recent result by Delcourt and Postle lies in the intersection of these two areas, and could be a focus of study.

- **Combinatorial Markov chains.**
  A Markov chain is a simple stochastic process which is "memoryless", in that the next transition depends only on the current state, and not on the history of the chain. We consider discrete time Markov chains on finite state spaces which have exponentially large size (with respect to some parameter). If the Markov chain is ergodic then it converges to a unique stationary distribution. But how quickly does this convergence occur? There are a few methods for bounding this convergence rate, either to show that the chain "mixes rapidly" and hence can be used for efficient sampling, or that the chain "mixes torpidly" and converges exponentially slowly to the stationary distribution. The aim of the project is to survey the topic, learning about some of the results

which have been proved and the methods used to prove them. (You don't need to be familiar with probability theory, as only simple discrete probability theory is required, which is essentially just counting.)

Pinhas Grossman

I am interested in a few related but distinct areas of research. Some areas that I can supervise projects in are the following.

- **Operator algebras:** A possible project in this area would be to give an exposition of the free central limit theorem of Voiculescu's free probability theory, and discuss its connections to the theory of random matrices.
- **Modular tensor categories:** A possible project in this area is to explicitly decompose certain concrete examples of modular data into representations of the finite group $\mathrm{SL}(2, \mathbb{Z}/n\mathbb{Z})$, where n is the conductor of the category (following a theorem of Ng-Schauenberg that this is always possible).

  Planar algebras: A possible project in this area is to describe the Kazhdan-Wenzl classification of Type A tensor categories in terms of planar algebras.

David Harvey

My research is mainly on algorithmic and computational problems in number theory.

I am happy to supervise projects on any topic in number theory (with or without a computational component), and some areas of algebraic geometry. Here are some examples:

- Zeta functions of algebraic curves and varieties
- p-adic numbers and applications, e.g., Hasse–Minkowski theorem
- Rational points on elliptic curves
- Theory of algebraic curves
- Higher reciprocity laws
- Gröbner bases
- Riemann zeta function, Dirichlet L-functions

Also, I usually have available some more specific computational problems that would be appropriate for an Honours student with a strong programming background.

Potential Honours students are welcome to drop in for a chat (best to email me first).

Jonathan Kress

- **Symmetry algebras of superintegrable systems.**
  Superintegrable systems are Hamiltonian systems with an excess of symmetries. These symmetries form non-trivial algebraic structures that have been studied in connection with the classical special functions, orthogonal polynomials and the recently discovered 'exceptional orthogonal polynomials. The aim of this project is to understand these symmetry algebras and how they are related to special functions.

- **Conformally equivariant quantisation.**
  An observable of a classical Hamiltonian system is a functions of the its position and momenta that does not change as the system evolves. Quantising such a system involves replacing the classical observables with differential operators that commute with the Hamiltonian operator. However, in general, there is no unique way to do this. Imposing additional structure, such as conformal covariance, on both the classical and quantum systems can provide a way to resolve this problem. The aim of this project is to understand what this means and study some specific examples.

Anita Liebenau

I work in the following areas: extremal and probabilistic combinatorics, asymptotic enumeration.

- **Extremal graph theory.**
  Typical questions in this area are the following. What is the largest number of edges that a graph can have without containing a triangle? What is the smallest d such that every graph G with minimum degree at least d contains a Hamilton cycle? What is the largest number of edges that a graph G can have without containing a cycle of length 8 as a subgraph? The first two are resolved and known as Mantel's and Dirac's theorem. We lack satisfying answers for the third one. A project in this area could have a focus on probabilistic methods, on algebraic or geometric constructions, or on algorithmic aspects.

- **Graph Ramsey theory.**
  Ramsey theory can be described as finding order in big-enough chaos. The driving question of the field is the following: What is the smallest number n such that every red/blue-colouring of the edges of the complete graph on n vertices contains a monochromatic copy of the complete graph on k vertices? This number is called the Ramsey number of K_k. The known lower and upper bounds are unsatisfyingly far apart. Moreover, no deterministic construction reaches the lower bound given by the probabilistic method. That is, if we draw a graph uniformly at random from all n-vertex graphs then this graph is a good candidate for a lower bound with a very high probability. Yet, we have no idea how to construct such a graph deterministically.

  There are many directions to take here. One project could be to look at the deterministic constructions for lower bounds and see whether those graphs have other interesting properties. Another project would study the following question. What is the smallest number m of edges needed to guarantee that every graph G with m edges and every 2 colouring of the edges of G contains a monochromatic copy of a given tree T. How does the answer change if we replace "tree T" by "cycle C"?

- **Positional games on graphs.**
  In a Maker-Breaker game played on the edge set of a graph G two players claim edges of G alternately. Maker wins the game if at the end the graph consisting of Maker's edges has some predefined property (e.g. to contain a perfect matching). There are interesting connections to random graphs. A project in this area would start with a literature review of the area to learn existing methods and then apply them to a game where the answers are not known yet.

Kevin Limanta

My main research interest is in an intersection of algebraic combinatorics and discrete harmonic analysis. In general, I am interested in looking at integration theory and subsequently harmonic analysis

over some object X from a more algebraic point of view, by working with general fields of characteristic zero or finite fields. This has some algebraic combinatorics and coding theory component to it.

I am happy to supervise projects related to the following topics, although I am open to any other topic:

- Given a general field F and an algebraic curve S over F, can we have a notion of integration over S? If F is finite then this may give us a combinatorial type problem. If S is the unit circle in the affine plane A over F, this is closely connected to a family of integers called the super Catalan numbers. They appear in both the general field of characteristic zero and finite field of characteristic $p > 2$ case. Related to above, another possible direction would be to consider codes over the algebraic curve S. Loosely speaking, for any function f (usually polynomials), we are asking whether there exists a (proper) subset T of S such that the integral of f over T is equal to the integral of f over S. If so, can we always find the smallest subset?

- A combinatorial interpretation of the super Catalan numbers is still unknown to date, although there are some interpretations for specific cases. I am also interested in this.

Alina Ostafe

- **Arithmetic dynamics and unlikely intersections.**
  Dynamical systems generated by iteration of polynomials and rational functions is a classical area of mathematics with a rich history and a wide variety of results and applications. Recently, there has been substantial interest in arithmetical dynamical systems, meaning the iteration of rational functions over fields of number-theoretic interest.

  This project will focus on arithmetic properties of orbits, and in particular, in studying intersections of orbits with different structural sets such as the set of some special integers, subgroups, subfields, etc. This is a deep and very active area of research at the cross-roads of Diophantine and algebraic geometry and number theory.

- **Algebraic aspects of polynomial dynamical systems over finite fields.**
  This project focuses on the study of non-classical dynamical systems over finite fields and the study of atypical behaviour which is not present in standard constructions from complex dynamical systems. For example, some aspects of algebraic dynamical systems (ADS) to be studied within this project are: degree growth, irreducibility of iterates, trajectory length, etc. Such problems in ADS are of an intricate algebraic and number theoretic flavour, including the use of tools from number theory and algebra. Research into ADS has not only a high theoretical value, but also applied significance thanks to the great number of potential applications to many different areas of modern cryptography (for example in the construction of pseudorandom number generators, coding theory, Monte Carlo simulations, physics, etc. 3. Constructing good pseudorandom number generators: a cryptographic challenge

  For many cryptographic schemes it is very important to be able to generate random numbers as, for example, creating cryptographic keys which usually should be generated at random from a given keyspace. The algorithm for generating numbers that approximate the properties of random numbers is called a pseudo-random number generator (PRNG).

  PRNG's are deterministic algorithms with relatively few parameters that can easily be implemented on a computer. As such, PRNG's have been subjected to a rigorous theoretical analysis.

  Usually PRNGs use recursive procedures and yield sequences that are ultimately periodic. Some desirable properties of a sequence of pseudorandom numbers, depending on the application, are:

relatively large period length, it should have little intrinsic structure (linear complexity), it should have good statistical properties (uniform distribution), the generating algorithms should be efficient.

The aim of this project is to study such properties of classical and new recursive PRNGs. The area is an exciting mix of algebra, number theory and cryptography.

Alessandro Ottazzi

I propose the following honours projects:

- **Bergman Projection.** The Bergman projection is an integral operator that projects the space of square integrable functions onto square integrable and holomorphic functions. There are questions around characterizing the kernel of the operator as well as studying boundedness on different Lebesgue spaces. While traditionally studied on the unit disk, the Bergman projection has been analysed on domains with not necessarily smooth boundaries.
- **Mappings on Carnot groups.** Isometries and angle-preserving mappings on $\mathbb{R}^n$ are well known and they can be used to study Riemannian manifolds. Carnot groups are a class of metric Lie groups where the study of such mappings has been the topic of recent research. As $\mathbb{R}^n$ can be related to Riemannian manifolds, Carnot groups can be thought of as infinitesimal models of an ample class of spaces, namely sub-Riemannian manifolds.

Denis Potapov

- **Noncommutative Integration Theory and Multiple Operator Integrals.**
  Multiple Operator Integrals (MOI) is the modern tool in analysis which proved very efficient recently. The analysis research team at UNSW has recently succeeded resolving a few long standing and open problems with the help of MOI methods. I was pointed out by many of my colleagues and research collaborators that the MOI theory has reached the level of maturity which requires a good reference/graduate text book.

  In this project, I suggest to lay the foundation for a future book on Multiple Operator Integrals.
- **Dirac operators in modern analysis.**
  Differentiation operator has been the main example of the operator from Physics where the Multiple Operator Integration theory has been tested. Dirac operators on the other hand, are the higher dimensional version of the differentiation operator. In this project, I suggest to investigate a possibility of extension of some known applications of MOI methods to Dirac operator setting. Even though some results seems easily transferable to this setting; the others may bring some surprises.

Anna Romanov

My research is in geometric and categorical representation theory. Representation theory is a technique for studying groups, so the core of my research lies in algebra. (In other words, don't be fooled by the adjectives "geometric" and "categorical" – at heart, I am an algebraist.)

I am happy to supervise honours projects in topics related to my research. These topics can lean in three potential ways: geometric (D-modules), algebraic (representations of Lie groups or Lie algebras),

or categorical (categorification). Some specific ideas are below, but if there is another topic above that piques your interest, feel free to discuss it with me and we can create a project together.

- **Representation theory and quantum mechanics:** One of the most beautiful examples of the utility of representation theory comes from its predictive power in quantum mechanics. In particular, the symmetry of quantum system can be used to predict specific qualities of its quantum states. This can be seen quite concretely by examining the simplest example: the hydrogen atom and its symmetry group SO(3). A potential honours project is to describe this example in detail.

- **Whittaker modules for affine Lie algebras:** Affine Lie algebras are infinite-dimensional vector spaces with an additional structure allowing one to "multiply" elements (the Lie bracket). Because they are infinite-dimensional, their representation theory is complicated and not well-understood. Whittaker modules are a class of representations whose structure has been studied for finite Lie algebras, but are still poorly understood in the affine setting. Already in the simplest example of $\mathfrak{sl}(2)$, a surprising new phenomenon occurs. A potential honours project is to compute some examples of Whittaker modules for affine $\mathfrak{sl}(2)$ in order to shed light on this new affine phenomena.

- **Module categories associated to real Lie groups:**

  Categorical representation theory pursues the philosophy that certain algebraic patterns in representations can be understood as shadows of deeper phenomena happening on the level of categories. Applying this approach to the representation theory of real Lie groups leads to a class of "Lusztig-Vogan categories", which encapsulate crucial information about characters of unitary representations. These categories are not well-understood, even in the simplest examples. A potential honours project is to describe the structure of some Lusztig-Vogan categories corresponding to small Lie groups of type B and C, including a description of indecomposable objects and morphism spaces.

Alex Sherman

I am interested in the interface of representation theory, algebraic geometry, and tensor categories. I'm especially interested in phenomenon in positive characteristic, where familiar algebraic structures can behave in unexpected ways and many fundamental questions remain open. Here are some potential projects.

- Lie algebras describe infinitesimal symmetries, and are fundamental to representation theory, geometry, and the language of modern physics. Their representation theory also has deep connections to number theory, combinatorics, and even differential equations. In this project we'd explore the beautiful world of semisimple Lie algebras, which have a beautiful classification in terms of the geometry of root systems and the combinatorics of Dynkin diagrams. There are many directions to go from here–one is the study of symmetric spaces, which have a rich geometric structure and are in correspondence with real forms. It would be interesting to understand the different known classifications of symmetric spaces, and relate them to one another in an explicit way.

- Categorical actions reveal connections between different representation categories by replacing linear operators with functors. A concrete starting point is provided by induction and restriction between representations of symmetric groups. This project will introduce categorical actions through these examples, before focusing on actions of the Heisenberg algebra and their relationship with affine Lie algebras. A possible research direction is to investigate how the exactness of a functor analogous to the Frobenius morphism reflects the structure of the categorical action.

- Symmetric tensor categories provide a common language for studying representations and their tensor products. In positive characteristic, they exhibit phenomena with no counterpart over the complex numbers. This project will introduce symmetric tensor categories through concrete examples, before studying the Verlinde categories Ver_{p^n}, which play a central role in major classification conjectures. An initial goal will be to understand different constructions of these categories and work out explicit examples. Depending on the student's interests, the project could develop towards comparing these constructions or exploring connections with representation theory and commutative algebra.

Igor Shparlinski

- **Dynamical systems of number theoretic origin.**
  Recently, there has been active interest in dynamical systems generated by iterations of various number theoretic functions reduced modulo a prime p. The following functions $x \mapsto 2^x \pmod p$ or $x \mapsto x^x \pmod p$ or $x \mapsto x! \pmod p$ are representative examples of such functions. The aim of the project is to study, theoretically and numerically, some natural properties of these maps, such as the number of fixed points, the distribution of orbit lengths and propogation from the origin.
- **Distribution of solutions of Diophantine equations and congruences in many variables.**
  The project is about studying the distribution of solutions to equations $f(x_1, \ldots, x_n) = 0$ and similar congruences with a polynomial $f \in Z[X_1, \ldots, X_n]$. This is a classical question of analytic number theory. The aim of the project is to study several versions of the above question in which the variables $(x_1, \ldots, x_n)$ are from domains of $\mathbb{R}^n$ with various geometric properties, such as boxes or domains with a smooth boundary or convex domains.

John Steele

- **Extending solutions of the Einstein equations.**
  Standard techniques for solving the field equations in General Relativity usually provide only a part of the space-time manifold. This project will study how we find the full manifold and what that might mean.
- **Generating the Kerr Black Hole**
  In 1964 Newman and Janis came up with a way of generating Kerr's spinning black hole solution from Schwarzschild's static black hole solution of the Einstein Equations. This idea can be used in other contexts, but its physical interpretation is still a matter of debate. This project will study the Newman-Janis trick, other methods of adding rotation to solutions of the Einstein equations in GR and how the symmetries of the solutions behave as the solutions are generated.
- **The Carroll Group**
  The Lorentz group of special relatively has a well known limit for the case where c, the speed of light, is sent to ∞, namely the group of Galilean transformations. Less well known is another limit due to Levy-Leblond and called by him the Carroll group (after Lewis Carroll), in which (in a certian sense) $c \to 0$. This project will consider the two groups, their structure, how they are related and some of their physical consequences.
- **Generating Solutions of the Einstein Equations.**
  Starting from one known solution of a differential equation it is sometimes possible to create others. Similar ideas apply to the Einstein Field Equations and this project will consider some of the approaches used.

Fedor Sukochev

- **Noncommutative integration theory.**
  This topic covers noncommutative integration theory both with respect to the trace and with respect to a state/weight. These projects involve studying geometry of various bimodules associated with von Neumann algebras (like noncommutative L_p-spaces).
- **Noncommutative Probability Theory.**
  This topic studies noncommutative probability theory, both with respect to the trace and "free version of probability" a la Voiculescu.
- **Classical probability theory.**
  This project involves studying norms of sums of independent and conditionally independent random variables in various spaces of measurable functions.
- **Noncommutative Geometry.**
  More precisely, this topic involves the part of noncommutative geometry which deals with singular traces and their applications. These projects may require a student also to delve into the theory of ideals of compact operators and their geometry.
- **Banach Space geometry.**
  This topic involves the study of Banach space geometry and related parts of noncommutative analysis, including differentiating operator-valued functions and applications to Mathematical Physics. Sometimes, these projects may also be relevant to some problems from perturbation theory and their applications in Mathematical Physics (e.g. Krein's spectral shift function, Koplienko's spectral shift function and their higher dimensional analogues).

Behrouz Taji

My research area is algebraic geometry. I am also interested in complex differential geometry. Some specific topics that I am happy to supervise for an honours project are the following:

- birational geometry, including the Minimal Model Program
- derived categories in algebraic geometry
- Hodge theory and Griffiths' variation of Hodge structures
- stability notations for sheaves and their analytic counterparts
- Kähler-Einstein metrics
- classification of algebraic varieties through numerical methods
- Fano varieties and their classification
- Hodge theoretic aspects of D-modules

Mircea Voineagu

There are possible projects in the areas of algebraic topology, category theory, algebraic geometry and homological algebra. For example:

- **Equivariant homotopy theory** In this project we investigate different models of doing homotopy theory on a topological space that has a finite group action. The main invariant of these theories is Bredon cohomology which is an object introduced in 80' and this invariant will be the subject of the project. Many of the possible properties of Bredon cohomology are still to be proved as extensions from the classical case, so the project can touch areas of the current research.

- **Applied Algebraic Topology** This is a project in the very new and exciting field of applications of algebraic topology in data analysis. In this project we analyze theoretically (and practically) different models of invariants on topological sets associated to concrete data sets. For example persistent homology, Mapper etc. represents way to understand data derived from biology in topological terms of connected components, 1- dimensional holes etc. For example, a 1-dimensional hole in a biological data (a circle) can be viewed as a possible clustering set around a hole and may have biological significance. The practical component of this project is analyzing neurodegenerative diseases such as autism etc from the point of view of topology. In collaboration with BABS we interpret the possible results from the biological point of view. For the practical component of this project a basic knowledge of programming language R is a plus. For the theoretical part one can analyze zig-zag persistence and its stability or be interested in an equivariant persistence invariant.

- **K-theory and algebraic geometry.**
  This is a project in algebraic geometry and algebraic K-theory. K-theory of an algebraic variety X is a set of algebraic invariants $K_i(X)$ based on the category of locally free sheaves on X. The Chow ring of an algebraic variety is constructed using all irreducible subvarieties of X with a product given by their intersection. A famous theorem of Grothendieck asserts that, with $\mathbb{Q}$-coefficients, $K_0(X)$ is ring-isomorphic with the Chow ring of X. This theorem was recently generalized to all higher K-groups by the introduction of motivic cohomology ring that vastly generalizes the Chow ring. In this project we will investigate various constructions, ideas and recent applications of motivic cohomology groups in algebraic geometry. It will be a deep interplay between algebraic geometry and other fields like algebraic topology and homological algebra.

Lee Zhao

My main research interest lies in the field of analytic number theory; that is, using analytic tools to answer number theoretic questions. The following are some potential honours project problems on which I will be happy to supervise students.

- **Analytic theory of L-functions** The theory of L-functions lies at the center of analytic number theory and emerged from the study of distribution of prime numbers. We will look to understand the analytic properties (locations of zeros, mean-values, etc.) of various kinds of L-functions.

- **Exponential and character sums** Exponential and character sums are extremely useful in analytic number theory as many problems can be reduced to the estimations of these sums. We will study the classical theory of these sums and how they relate to problems in number theory.

I will also be happy to discuss other potential projects in number theory.