

MATHEMATICS ENRICHMENT CLUB.¹

Problem Sheet 9, July 26, 2012

1. How many ways can we change \$1.00 into coins of 5,10,20 and/or 50 cents?

a. 46 *b.* 47 *c.* 48 *d.* 49 *e.* 50.

2. (a) What number is one less than 11010000 in base 3?

(b) What is the decimal representation of 220200_3 ?

3. If we expand $(2 + x)^{18}$ as a polynomial we obtain

$$(2 + x)^{18} = a_0 + a_1x + a_2x^2 + \dots + a_{18}x^{18},$$

where a_0, a_1, \dots, a_{18} are integers.

(Without using the binomial theorem), find a_0, a_1, a_{18} and $a_0 + a_1 + \dots + a_{18}$.

4. The hypotenuse of a right-angled triangle is 15 cm and the radius of the inscribed circle is 2cm. Find the perimeter of the triangle.

5. Show that if A is any subset containing $n + 1$ integers from the set $\{1, 2, 3, \dots, 2n\}$, then A contains (at least) 2 integers a and b such that a is a factor of b .

6. (a) How many diagonals are there in a regular pentagon $ABCDE$? Show that each diagonal is parallel to one of the sides.

(b) How many diagonals are there in a regular hexagon. Is there any result similar to that in (i) ?

7. $ABCD$ is a square of side length 1. Take P to be any (general) point inside the square and draw a line through p parallel to AD meeting AB and DC at Q, R respectively. Similarly draw a line through P parallel to AB meeting AD and BC at T and S .

Show that if the rectangle $AQPT$ has area larger than $\frac{1}{4}$ then the rectangle $PSCR$ has area less than $\frac{1}{4}$.

¹Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres, Macquarie Uni.