Errata: Solution Sheet 4, May 25, 2012

Answers

1. $f(4) = 6$. Generally, $f(n) = (n-1)!$

2. $(22) * (35) = 770$ is the only combination where both numbers are over 18.

3. (a) $50 + 20 - 10 = 60$. This is the number of numbers *with* a factor in common with 100. So the answer should be $100 - 60 = 40$.

   (b) $25 * 102 + 10 * 105 - 5 * 110 = 3050$. This is the sum of numbers *with* a factor in common with 100. So the answer should be $1 + 2 + \cdots + 100 - 3050 = 2000$

4. $x = 0, 11$ are the 'fixed points' (solve $x_{n+1} = x_n$), but there are many ways to get to either 0 or 11. Rewrite the recurrence relation so that $x_{n+1} = (x_n - 5)^2 - 25$. Then you can write x_{n+1} in terms of x_0:

$$x_{n+1} = \Big(\big((x_0 - 5)^2 - 30\big)^2 - \ldots - 30\Big) - 25$$

   Where there are $n-1$ 30's. So if $x_{n+1} = 0$, re-arrangement gives

$$x_0 = 5 \pm \sqrt{30 + \sqrt{30 + \ldots \sqrt{30 \pm 5}}}$$

   Similarly for $x_{n+1} = 11$

$$x_0 = 5 \pm \sqrt{30 + \sqrt{30 + \ldots \sqrt{30 \pm 6}}}$$

5. $x = y = z$, so solve $x + x^2 = 2$.