

Solution Sheet 9, May 28, 2012

Answers

1.

$$\frac{20 \times \frac{66}{100} + 30 \times \frac{56}{100}}{50} = \frac{3}{5} = 60\%$$

2. Expand $(1 + x + x^2 + x^3)^5$ and take the coefficient of x^{12}

3. (a) Each slice of the clock is 30° . Hence $\angle XOY = 30n$ for $n = 0, 1, \dots, 11$. Triangle XYO is isosceles, so $\angle XYO = \angle YXO = \frac{|180-30n|}{2}$.

(b) Let r be the clock radius. Then area $A_{12}A_2A_6A_8} = \sqrt{3}r^2$, and area $A_{12}A_3A_6A_9} = 2r^2$. Hence the ratio is $\sqrt{3}/2$.

4. Let $n = \sqrt[3]{x + \sqrt{x^2 + 1}} + \sqrt[3]{x - \sqrt{x^2 + 1}}$. Then $x = \frac{n^3 - 3n}{2}$. Any integer value of n will result in a corresponding integer value for x .

5. (a) Start with $a + b > 2\sqrt{ab}$, and work your way backwards until you get $(a - b)^2 > 0$

(b) Use part a three times with different values for a, b chosen from x, y, z .