

MATHEMATICS ENRICHMENT CLUB.¹

Problem Sheet 5, June 4, 2013

1. Express the number $0.504504504\dots$ as a fraction in lowest terms.
2. Yvonne and Znonimir play a game. They have a pile of 500 counters and each is allowed to remove 1,2,4,8,... counters from the pile, taking turns in so doing. The last person to take a counter loses. Assuming that both play using the best possible strategy at each go, who wins?
3. The last digit of 1997^{1997} is

(a) 1 (b) 3 (c) 5 (d) 7 (e) 9.

4. How many planes of symmetry has a rectangular box of dimensions $2 \times 2 \times 3$?
5. (a) Paul measured all 6 edges of a tetrahedron $ABCD$ and found them to be 1,3,4,5,6,8 cm. Can this be correct?
(b) Paul then measured the edges to be 2,3,4,5,6,8. If $AB = 2$ what is the length of CD ?
6. (a) Prove that the angle in a semicircle is right-angle.
(b) Show that if two chords of a circle mutually bisect each other, then they are both diameters.
(c) Complete the following statement: If a parallelogram is inscribed in a circle then

Senior Questions

1. Put $x = z - \frac{1}{z}$ and hence solve, in surd form, the cubic $x^3 + 3x = 1$.
2. Let ABC be a triangle with three medians intersecting at S . Let L, M be the midpoints of AC, AB respectively.
 - (a) Prove that the triangles LSC and MSB have equal areas.
 - (b) Given that LSC has area 100cm^2 , find the area of ABC .

¹Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres, Macquarie Uni.