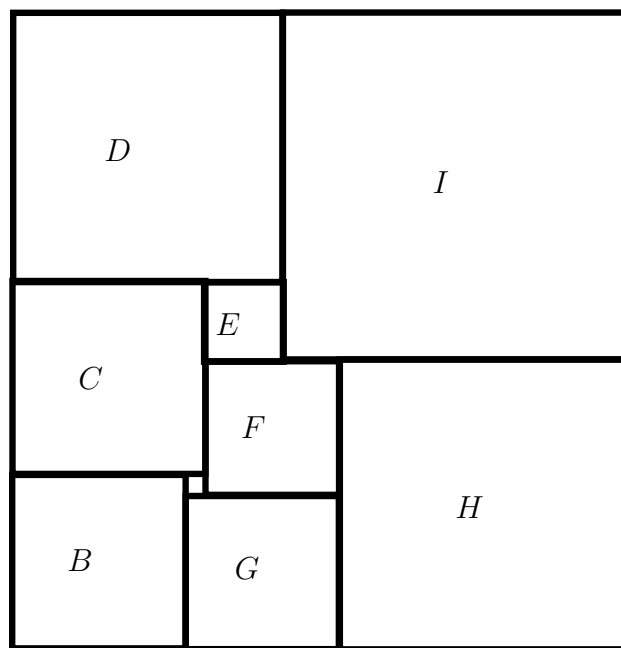




**MATHEMATICS ENRICHMENT CLUB.**

**Problem Sheet 11, August 5, 2014<sup>1</sup>**

1. In a plane lie 127 cogs. The teeth of cog 1 engage those of cog 2. The teeth of cog 2 engage those of cog 3 and so on. Finally the teeth of cog 127 engage those of cog 1. Can the cog wheels so arranged be turned?



2. Nine squares are arranged to form a rectangle as shown. The smallest square has area 1, find the total area of the rectangle.
3. I have two games of chance: 1) I flip a coin, if it comes up heads, you win. 2) I hold six pieces of string in a fist so that the tops stick out one side of my hand and the bottoms stick out the other side, but you can't tell which of the tops or which of the bottoms belong to the same piece of string. You tie the tops in pairs and the bottom in pairs and if you've made all pieces of string into one big ring, you lose. Which game is it more likely for you to win?

<sup>1</sup>Some problems from UNSW's publication *Parabola*, others from [www.brilliant.org](http://www.brilliant.org)

4. A right pyramid has square base  $ABCD$  with sides of length  $b$  and apex  $X$  at a height  $h$  above the base. What is the shortest distance an ant can travel when moving from  $A$  to  $C$ ?

5. Find the smallest  $n$  so that  $5^n + n^5$  is a multiple of 13.

6. On Wednesday, your teacher gives you a coin.

She says, “Keep flipping this coin until you see the pattern heads, tails, heads. Record the number of flips required to reach this pattern, and start flipping again (counting up from 1 again) until you see that pattern again. Record this second number, and start again and so on for the third and fourth until the end of the day. Finally, average all of the numbers you’ve recorded.”

Confused you still do as told like the obedient student you are.

On Thursday, the teacher gives you the coin again and says, “Do the EXACT same thing except you flip until you see the pattern heads, tails, tails.”

Is it expected that Thursday’s number will be equal to, higher than or lower than Wednesday’s?

### Senior Questions

1. The Mou He Fang Gui is a solid formed by the intersection of two perpendicular cylinders of radii  $r$ . Express the ratio of this volume to a sphere of radius  $r$ .

2. A napkin ring is formed by drilling a hole of length  $h$  through the centre of a sphere. Find the volume of the napkin ring.