MATHEMATICS ENRICHMENT CLUB.
Hint Sheet 15, September 2, 2014[1]

1. Wow, this was a lot harder than I thought it would be!

   Take the logarithm of both numbers, $\log(300!) = \log(300) + \log(299) + \cdots \log(2)$ which is the area under a bar graph where the bars are width 1 and height $\log(n)$. Plot this bar graph with a graph of $\log(x)$ and you'll see that the area of the bar graph is bigger than the area under $y = \log(x)$ from 1 to 300. Use this to find a lower bound for $\log(300!)$ which is bigger than $300 \log(100)$.

2. Each tile has to cover one white and one black square, pairing up the squares into black-white couples. Removing opposite corners removes two squares of the same colour, so the squares can't be paired into all black-white couples.

3. Use a process of elimination. All of 9!, 8! and 7! are larger than 3 digits, so the numbers cannot use these digits. Also, $6! = 720$ and our number cannot contain 7, 8 or 9, so 6 is out also. The biggest possible number is then $5! + 5! + 5! = 360$. Also, $4! + 4! + 4! < 100$ so we need to have at least one 5. And so on

4. The area of the larger semi-circle is the sum of the areas of the two smaller semi cirlces. The sum of the two crescents is the sum of the two smaller semi-circles plus the area of the trianlge minus the area of the larger semi-circle. So the two crescents have the same area as the area of the triangle.

5. For a game to be a draw, all 9 squares must be filled without anyone winning along the way. Now a game is not just the configuration at the end, but the order the o's and x's get put there. So count the number of ways of arranging the x's and o's so that there are no three-in-a-rows (there are 16), then for each of these arrangements, count the number of ways of playing the game to get there ($5! \times 4!$).

6. (a)

   (b)

[1]Some problems from UNSW's publication *Parabola*

Senior Questions

1. (a) Solve $\langle p_0, p_1 \rangle$ and show it equals zero.

   (b) Let $p_2(x) = a_2x^2 + a_1x + a_0$, and solve the equations $\langle p_2, p_0 \rangle = 0$ and $\langle p_2, p_1 \rangle = 0$ simultaneously.

   (c) Add up $\alpha_0 p_0 + \alpha_1 p_1 + \alpha_2 p_2$ and show the coefficients of the x^2, x and constant term can be any number.