



MATHEMATICS ENRICHMENT CLUB.

Problem Sheet 17, September 8, 2015¹

1. If the number is made from $a \neq 1$, then $aaa\dots$ is divisible by a and thus not prime.

Suppose the number has a non-prime number of digits, then we can factor the number of digit this number has into $q \times p$ where q and p are integers. But then we can ‘split’ the number up into q blocks of p -length digits; i.e

$$\underbrace{(x + 2)^3}_{\text{text 1}}$$

2. Find the smallest positive integer such that it is a multiple of 9 and it has no odd digits.
3. Let m and n be positive integers. Find the number of ordered pairs (m, n) such that the expression $(m - 8)(m - 10) = 2^n$ is satisfied.
4. Three children Ann, Borus and Charlie sit at the round table and eat nuts. Children have more than 3 nuts. At the beginning Ann owns all nuts. If Ann has even number of nuts, she divides them into two equal parts and gives to Borus and Charlie and if the number of her nuts is odd, then she eats 1 nut and then does the same. Then the next child (one by one, around the table) does the same: divides all his (or her) nuts between two others eating one nut in the process, if it is necessary. And so on. Prove that:
 - (a) at least 1 nut will be eaten,
 - (b) the children won’t eat all nuts.
5. D is the midpoint of the side BC of triangle $\triangle ABC$. E and F are points on CA and AB respectively, such that BE is perpendicular to CA and CF is perpendicular to AB . If $\triangle DEF$ is an equilateral triangle, then show that $\triangle ABC$ is not equilateral.
6. Each cell of a 29×29 table contains one of the integers $1, 2, 3, \dots, 29$, and each of these integers appears 29 times. The sum of all the numbers above the main diagonal is equal to three times the sum of all the numbers below this diagonal. Determine the number in central cell of this table.

¹Some problems from UNSW’s publication *Parabola* and the *Tournament of Towns in Toronto*.

Senior Questions

1. The triangular numbers are given by $T_n = 1 + 2 + \cdots + n$ for n a positive integer ($T_1 = 1$).

Discovery and prove a formula for

$$T_n \left(\frac{1}{T_1} + \frac{1}{T_2} + \cdots + \frac{1}{T_n} \right)$$

2. A, B, C and D are points on the parabola $y = x^2$ such that AB and CD intersect on the y -axis. Determine the x -coordinate of D in terms of the x -coordinates of A, B and C , which are a, b and c respectively.
3. Let $f(x)$ be a polynomial of nonzero degree. Can it happen that for any real number a , an even number of real numbers satisfy the equation $f(x) = a$?