

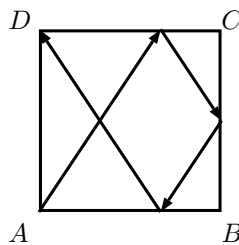
MATHEMATICS ENRICHMENT CLUB.

Problem Sheet 1, April 30, 2016

1. What is the last digit of $2^{4^{6^8}}$.
2. Shaun has to climb up 10 steps. He can either climb 1 step or 2 steps at each time. How many possible ways are there for Shaun to climb the stairs?
3. Consider the following logical statements:
 - (a) If this statement is true, then the other statement is false.
 - (b) If this statement is true, then the other statement is false.

How many of them is/are true?

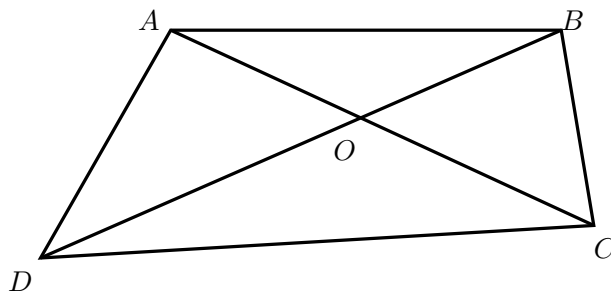
4. Let $S = |\sqrt{x^2 + 4x + 5} - \sqrt{x^2 + 2x + 5}|$, for some real number x . Find the maximum value of S .
5. Each of given 100 numbers was increased by 1. Then each number was increased by 1 once more. Given that the first time the sum of the squares of the numbers was not changed find how this sum was changed the second time.
6. A ball is projected from the bottom left corner of unit square $ABCD$ into its interior. We shall assume that the speed of the ball remains constant and it will continue bouncing off the edges until it arrives at a corner. For example, if the ball strikes $\frac{2}{3}$ of the way from D to C it will terminate at D .



Where must the ball strike on DC to finish at A ?

Senior Questions

1. Given that p is prime, when is $4^p + p^4$ prime?
2. Park's paths go along sides and diagonals of the convex quadrilateral $ABCD$. Alex starts at A and hikes along $AB - BC - CD$. Ben hikes along AC ; he leaves A simultaneously with Alex and arrives to C simultaneously with Alex. Chris hikes along BD ; he leaves B at the same time as Alex passes B and arrives to D simultaneously with Alex. Assuming the speeds of the hikers are constant, can it happen that Ben and Chris arrive at point O of intersection of AC and BD at the same time?



3. Find all real-valued functions f defined for x between 0 and 1 which satisfy

$$f(xy) = xf(x) + yf(y).$$