MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 15, August 29, 2016

1. Find all possible values of $n \geq 1$ for which there exist n consecutive positive integers whose sum is a prime number.

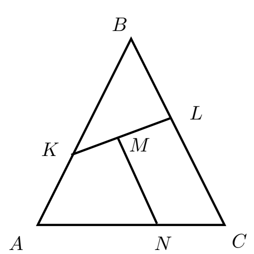
2. Each term of a sequence of natural numbers is obtained from the previous term by adding to it its largest digit. What is the maximal number of successive odd terms in such a sequence?

3. Johnny writes down quadratic equation
$$ax^2 + bx + c = 0$$
with positive integer coefficients a, b, c. Then Pete changes one, two, or none "+" signs to "−". Johnny wins, if both roots of the (changed) equation are integers. Otherwise (if there are no real roots or at least one of them is not an integer), Pete wins.

   Can Johnny choose the coefficients in such a way that he will always win?

4. On a 6×6 chess board there is a chess piece (a king) place in the lower left hand corner. In how many different ways can he reach the top right hand corner if he is never allowed to move "backwards"; that is, he can move to a neighhouring square either upwards or to the right or diagonally upwards to the right?

5. Points K and L are chosen on the sides AB and BC of the isosceles $\triangle ABC$ ($AB = BC$) so that $AK + LC = KL$. A line parallel to BC is drawn through midpoint M of the segment KL, intersecting side AC at point N. Find the value of $\angle KNL$.



6. $ABCD$ is a trapezium in which AB is parallel to DC, with $AB = BC = DA = 1$ and $CD = 1 + \sqrt{2}$. Let E be a point on AD such that we can fold the trapezium along a line passing through E so that A falls on CD. Find the maximum possible length of DE.

Senior Questions

1. On the graph of a polynomial with integral coefficients are two points with integral coordinates. Prove that if the distance between these two points is integral, then the segment connecting them is parallel to the x-axis.

2. What is the remainder when x^{2016} is divided by $(x-1)(x-2)$?

3. 17 people correspond by mail, each one with the 16 others. Only three different topics are discussed and each pair of correspondents dealswith only one of these. Show that there are at least three people who write to one another about the same topic.