



MATHEMATICS ENRICHMENT CLUB.

Problem Sheet 7, June 13, 2016

1. The average age of 10 members of a committee is the same as it was 4 years ago, because an old member has been replaced by a young member. Find how much younger is the new member?
2. Evaluate $1^2 + 2^2 + 3^2 + \dots + (n - 1)^2 + n^2$, where n is some positive integer. Hint: consider the expression $(1 + i)^3 - i^3$.
3. Find the greatest power of 21 which divides $25!$.
4. Four points are located in a plane. For each point, the sum of the distances to the other three is calculated; and these four sums are found to be the same. Determine all possible configuration of the four points.
5. Let $n^2 - 3n - 126$ be a perfect square, where n is some integer. Find all possible values of n .
6. (a) How many rectangles of integer side lengths are there in a 6×7 rectangle?
(b) How many squares of integer side length are there in a 6×7 rectangle?

Senior Questions

1. A sequence is defined as follows: $a_1 = a_2 = a_3$, and for all positive integers n , $a_{n+3} = a_{n+2} + a_{n+1} + a_n$. Given that $a_{28} = 60903017$, $a_{29} = 11201821$ and $a_{30} = 20603361$. Find the last 3-digits of $\sum_{k=1}^{28} a_k$.
2. Given that x is the last 2 digits of the sum:

$$\sum_{n=0}^{100} n!.$$

Find the last 2 digits of

$$\sum_{m=0}^{100} m^x.$$

3. On an island there are a number of towns, and a number of roads linking the towns. Each town is the junction of exactly three roads. A traveler sets out along a road from one town, and at the next town takes the left hand road of the two available. At the following town the right hand road is taken, and so on, with left and right turns alternating. Prove that at some stage the traveler must return to the first town.