MATHEMATICS ENRICHMENT CLUB.
### Problem Sheet 10, July 31, 2017

1. How many ways can we change $1.00 into coins of $5, 10, 20$ and/or 50 cents?

<div align="center">

(a) 46   (b) 47   (c) 48   (d) 49   (e) 50.

</div>

2. Ten darts are thrown onto a square dart board which is 3m by 3m. Prove that at least two of the darts land within $\sqrt{2}$m of each other.

3. (a) Let ABC be a triangle and M the middle point in the segment BC. We divide the triangle in two ABM and AMC, as represented in the picture of the left. If the area of ABM is $4cm^2$, what is the area of AMC?

   (b) Consider a cuadrilateral $ABCD$ and dived it into four pieces by uniting the middle point of each side with a common point P (as represented in the left picture). If three of the pieces have size $7cm^2$, $9cm^2$ and $12cm^2$, what is the size of the last piece?
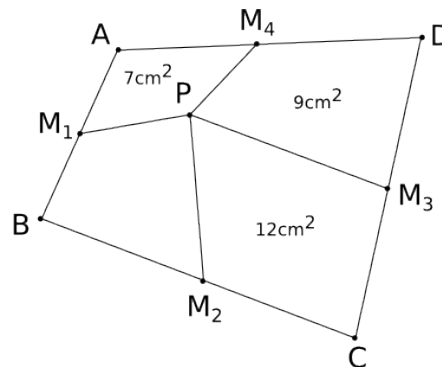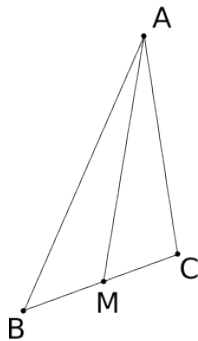
4. (a) Show that both 29 and 37 can be written as the sum of two squares, but that 30 and 31 cannot.

   (b) Show that $(a^2 + b^2)(c^2 + d^2) = (ac - bd)^2 + (ad + bc)^2$.

   (c) Use the previous formula to show how to write $1073 = 29 \times 37$ as the sum of two squares. In how many ways can 1073 be written as the sum of two squares?

5. Consider the numbers $1, 2, \ldots, 1000$. Show that among any 501 of them, two numbers exist such that one divides the other one.

Senior Questions

Imagine we have two regular dice (6-sided dice with numbers $1, 2, 3, 4, 5$ and 6 on its sides). If we roll them together and sum up the obtained values we will have a number between 2 and 12.

1. In how many ways can we obtain the number 4? What is the number that can be obtained in more different ways?

2. Is it possible to find two 6-sided dice B, C with the following property: for any integer k, the number of ways that rolling B and C together and summing them yields k is the same as the number of ways to obtain k when rolling two standard 6-sided dice together?

   Note: only positive integers on the sides of B and C are allowed.

3. Is the previous construction unique?