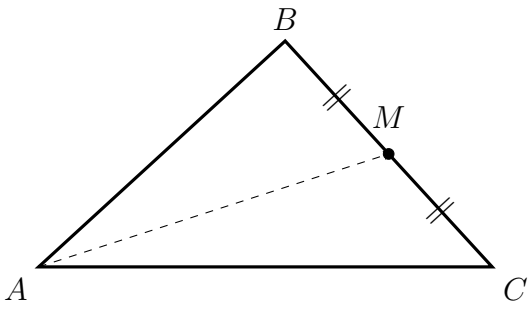
**MATHEMATICS ENRICHMENT CLUB.**
**Problem Sheet 11, August 6, 2018**

1. Let $ABC$ be a triangle, with $AM$ being one of its medians. Prove that the perpendicular distances from $B$ and $C$ to the line through $AM$ are equal[1].




2. (a) Show that 120 is a divisor of $n^5 - 5n3 + 4n$ for every integer $n$.
   (b) Show that 49 is not a divisor of $n^2 + n + 2$ for any integer $n$.

3. Three people, A, B and C, entered a competition. After the event, A reported "B was second, C was first." B said, "A was second, C was third." C said, "A was first, B was third." If each report contains one true statement and one falsehood, which of A or B performed better in the competition?

4. (a) Write 0.75 in base 2.
   (b) Write 0.96875 in base 2.
   (c) By writing the infinitely long sum

$$\frac{1}{2} + \frac{1}{4} + \cdots \frac{1}{2^k} + \cdots$$

   in base 2, deduce its value.

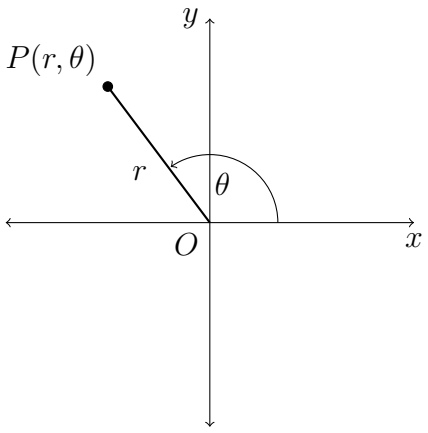
5. Find all pairs of integers $x$ and $y$ such that $x^3 - y^3 = 1729$. Show that there are no others.

[1]Adapted from AP Kiselev *Kiselev's Geometry: Planimetry*, Tr. A Givental, 2006.

**Senior Questions**

1. *An alternative coordinate system.* Usually, the coordinates of a point in the number plane are given using rectangular coordinates. Coordinates can also be given using a polar coordinate system.

   Suppose we have a point $P$ lying in the number plane. The polar coordinates of $P$ are given as $(r, \theta)$, where $r$ is the length of the ray $OP$, and $\theta$ is the angle (in radians) formed between $OP$ and positive $x$ axis measured in the counter-clockwise direction.



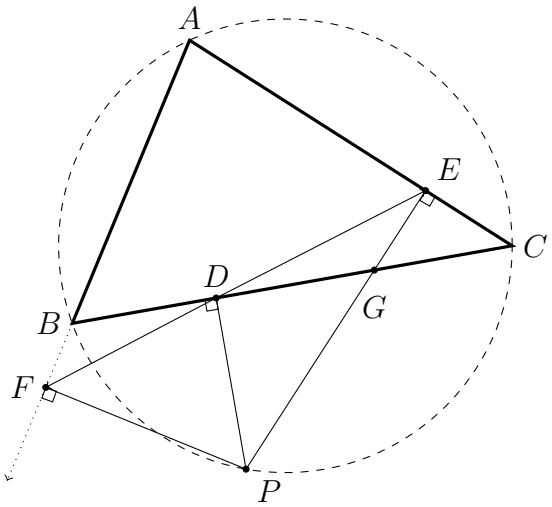

   (a) Convert the following points from polar to rectangular coordinates:

   i. $\left(\sqrt{2}, \frac{\pi}{4}\right)$ ii. $\left(1, \frac{3\pi}{2}\right)$ iii. $\left(\sqrt{3}, \frac{5\pi}{3}\right)$. iv. $\left(2, \frac{7\pi}{6}\right)$

   (b) The equation of a curve can also be given in terms of polar coordinates, usually in the form $r = f(\theta)$. For example, the equation of the unit circle in polar coordinates is $r = 1$. On separate axes, draw the graphs of the following curves given in polar form.

   i. $r = \theta$
   ii. $r = \cos(2\theta)$
   iii. $r = \sin(3\theta)$
   iv. $r = 1 + 2\cos\theta$

2. **The Simson Line** Let $ABC$ be any triangle. Let $P$ be a point on the circumscribed circle of $\triangle ABC$. Let $D$, $E$, and $F$ be the feet of the perpendicular from $P$ to the sides of the triangle (extended as necessary). Prove that $D$, $E$, and $F$ are collinear. [2]




[2]This question is adapted from R. Hartshorne, *Geometry: Euclid and Beyond*, p 61